NBER WORKING PAPER SERIES

A SIMPLER THEORY OF OPTIMAL CAPITAL TAXATION

Emmanuel Saez
Stefanie Stantcheva

Working Paper 22664
http://www.nber.org/papers/w22664

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
September 2016, Revised April 2017

We thank Alan Auerbach, Stephen Coate, Emmanuel Farhi, Mike Golosov, Roger Gordon, Louis Kaplow, Ben Lockwood, Andreas Peichl, Thomas Piketty, Chris Sleet, Joel Slemrod, Matthew Weinzierl, Nicolas Werquin, Daniel Waldenström, and numerous seminar participants for useful discussions and comments. We acknowledge financial support from the MacArthur Foundation, and the Center for Equitable Growth at UC Berkeley. We thank Nina Roussille for research assistance. The views expressed herein are those of the authors and do not necessarily reflect the views of the National Bureau of Economic Research.

A Simpler Theory of Optimal Capital Taxation
Emmanuel Saez and Stefanie Stantcheva
NBER Working Paper No. 22664
September 2016, Revised April 2017
JEL No. H20,H21

ABSTRACT

This paper develops a theory of optimal capital taxation that expresses optimal tax formulas in sufficient statistics. We first consider a simple model with utility functions linear in consumption and featuring heterogeneous utility for wealth. In this case, there are no transitional dynamics, the steady-state is reached immediately and has finite elasticities of capital with respect to the net-of-tax rate. This allows for a tractable optimal tax analysis with formulas expressed in terms of empirical elasticities and social preferences that can address several policy issues. These formulas have the advantage of being easily taken to the data to simulate optimal taxes, which we do using U.S. tax return data on labor and capital incomes. Second, we show how these results can be extended to a much broader class of utility functions and models. The same types of formulas carry over.

Emmanuel Saez
Department of Economics
University of California, Berkeley
530 Evans Hall #3880
Berkeley, CA 94720
and NBER
saez@econ.berkeley.edu

Stefanie Stantcheva
Department of Economics
Littauer Center 232
Harvard University
Cambridge, MA 02138
and NBER
sstantcheva@fas.harvard.edu

1 Introduction

The public debate has long featured an important controversy about the proper design of capital taxation. Arguments typically center around an equity-efficiency trade-off: who owns the capital and how strongly would capital react to higher taxes? The economics literature has developed dynamic, complex models, which have emphasized different results depending on the structure of individual preferences and shocks, the government's objective, and the policy tools available. The difficulty is that some of the highly salient questions in the policy debate on capital taxation have been very difficult to address in these complex models. A few examples are how to take into account income shifting between the capital and labor income bases, different types of capital assets, heterogeneity in agents' preferences or returns to capital, nonlinear taxation, and broader social fairness and equity considerations. Bridging the gap between economic theory and the policy debate seems especially important in the current context with growing income and wealth inequality, and where a large fraction of top incomes comes from capital income (Saez and Zucman, 2016; Piketty et al., 2016).

The goal of this paper is to connect the theory of capital taxation to the public debate by providing a framework in which many policy questions related to capital taxation can be addressed. This framework permits the derivation of robust optimal capital tax formulas expressed in terms of elasticities of capital supply with respect to the net-of-tax rate of return that can be estimated in the data, and distributional considerations which society has. The aim is to build a model which generates an empirically realistic response of capital to taxes (e.g.: non infinite elasticities to taxes), is sufficiently tractable to yield results for a variety of policy topics related to capital taxation, but general enough for these results to be robust to a broader set of models.

We start in Section 2 with a simple model in continuous time with the following two ingredients: First, individuals derive utility from wealth. We provide several microfoundations for this wealth in the utility specification: bequest motives, entrepreneurship, services from wealth, motivated beliefs, and social norms. It implies that the steady-state features finite supply elasticities of capital with respect to tax rates.[1] Second, utility of consumption is linear so that there are no consumption smoothing issues and individual responses to tax changes are immediate.[2] While very useful to analyze insurance issues (as in the New Dynamic Public Finance

[1]The magnitude of capital income elasticities is an empirical question. Our model nests the case of infinite steady state elasticities from earlier models as a special case. Other possible modeling devices to obtain finite elasticities would be introducing uncertainty as in the Aiyagari (1995) model or discount rates that depend on consumption (as in Judd (1985)). We argue that utility of wealth is much simpler and fits the data better in Section 2.2, but do consider these alternative models in Section 4.

[2]Anticipated tax reforms do not create any effect until they actually take place, which greatly simplifies

literature), reviewed below), consumption smoothing due to concave utility seems, at a first pass, less important for thinking about taxation of top incomes, where most of the capital is ultimately concentrated, and long-run taxation.[3]

While we generalize this model later on to allow for concave utilities (and the anticipatory and sluggish responses of capital to taxation they generate), the simpler version with linear consumption is extremely tractable and amenable to studying a wide range of issues about optimal capital taxation, such as nonlinear capital taxation, income shifting, cross-elasticities between capital and labor income, consumption taxation and others. It highlights the main forces shaping capital taxation which are obscured in more complex models. We can describe four sets of findings that we obtain by putting this newly gained simplicity to use.

First, we derive formulas for optimal linear and nonlinear capital income taxation that can be expressed in terms of the elasticity of the supply of capital income with respect to the net-of-tax rate of return, the shape of the capital income distribution, and the social welfare weights at each capital income level. We also derive formulas which take into account policy issues that have traditionally been hard to deal with in dynamic optimal capital tax models. These include, among others, joint-preferences and cross-elasticities between capital and labor, economic growth, heterogeneous returns to capital across individuals, and different types of capital assets and heterogeneous tastes for each of them.

Second, we derive a formula for the optimal tax on comprehensive income (labor plus capital income) that takes exactly the same form as the traditional optimal labor income formula. This formally justifies the use of the optimal labor income formulas to discuss optimal income taxation as has been done without rigorous justification in a number of studies (e.g., Diamond and Saez (2011)). The comprehensive income tax is the fully optimal tax if there is perfectly elastic income shifting between the labor and capital income bases when labor and capital are taxed differentially.

Third, we can analyze consumption taxation in this model as well by making the assumption

the analysis by eliminating the need to model anticipation effects and expectations about policy (unlike in the Chamley (1986) and Judd (1985) theory where unanticipated capital taxes are desirable while pre-announced long-distance capital taxes are not).

[3]To draw the analogy to the labor income tax literature, responses of labor to taxes are also part of a dynamic decision process if we acknowledge longer-term and slowly adjusting margins such as occupational choice and human capital acquisition. Two strands of the literature have thought of labor taxation in a dynamic way: the heterogeneous agents macro literature as in Jones et al. (1993) and the modern New Dynamic Public Finance literature reviewed below. While providing very useful insights, it has been more challenging to use this theory for policy guidance. The missing piece in optimal capital tax theory that we propose here is an approach that is dynamic, but can yield a static-equivalent model, which abstracts from transitional dynamics and as was adopted for labor income following the seminal contribution of Mirrlees (1971).

that real wealth (i.e., the purchasing power of wealth) enters individual utilities. In this case, a consumption tax makes people accumulate more nominal wealth so that their steady-state real wealth is unchanged. Hence, consumption taxation ends up being equivalent to labor taxation plus an initial wealth levy. It is thus not a sufficient tool to address capital inequality when there is two-dimensional heterogeneity as the data presented in Section 3 seems to suggest. The social welfare criterion required to justify a pure labor tax (or equivalently a pure consumption tax) is that all inequalities in capital are fair, which is a very strong requirement.

Fourth, our approach is very amenable to considering a broader range of justice and fairness principles related to capital taxation, through the use of generalized social welfare weights as in Saez and Stantcheva (2016). Given the prevalence of discussions about fairness and equity with regard to capital taxation, having a tractable way to incorporate broader and more diverse equity considerations is key. We consider several salient ethical standpoints from the policy debate. To give just one example, if differences in capital are considered fully fair (i.e., the generalized social welfare weights are uncorrelated with capital and capital income is not a tag) the optimal capital tax is zero.[4]

In Section 3, we put our formula in sufficient statistics to use by calibrating optimal taxes based on U.S. tax data on labor and capital income. Because capital income is much more concentrated than labor income, we find that, if the supply elasticities of labor and capital with respect to tax rates were the same, the top tax rate on capital income would be higher than the top tax rate on labor income. The model highlights which elasticities should fruitfully be estimated in the data, including the cross-elasticities between capital and labor (Section 2.4.5) and the elasticities and cross-elasticities for different types of capital assets (Section 2.4.8).

In Section 4, we show that the tax formulas obtained in the specific model of Section 2 carry over to a much broader class of models, including many of the models with concave utility for consumption used in the previous literature on capital income, as long as the elasticity of the capital income tax base is appropriately defined. We can thus systematically and consistently compare the elasticities and tax rates that arise in those key models. Qualitatively, the lessons and intuitions from the simpler model still apply. If responses of capital to taxes are very fast, then the quantitative implications of our simpler model are also still valid. If responses are slower, the elasticity of capital to taxes builds up slowly over time, which improves the equity-efficiency trade-off from the government's point of view in the short-run, and leads to higher

[4]We can also capture horizontal equity preferences, which take priority over vertical equity considerations and which penalize systems that treat people with the same ability to pay differently, the case in which wealth is a tag, or the case in which differences in wealth are considered unfair (following the theory laid out by Saez and Stantcheva (2016)).

optimal capital taxes.[5]

All proofs and various extensions are gathered in the Online Appendix.

Related work on capital taxation: Most importantly, our paper contributes to the core public economics literature on capital taxation by proposing a natural, tractable, and unified framework for some of the key policy questions explored by, among others, Gordon (1992), Gordon and Slemrod (2000), Gale, Hines, and Slemrod (2001), Kaplow (2001), Gordon, Kalambokidis, and Slemrod (2004), Slemrod (2007) and Auerbach, Hines, and Slemrod (2007). Our paper creates a closer link between the theoretical and empirical literatures on capital taxation by providing robust, sufficient statistics formulas that can make use of existing empirical estimates of the effects of taxation on capital income.

Our discussion of the equity issues involved in capital taxation is strongly connected to Kaplow and Shavell (2003), Kaplow and Shavell (2004), and Kaplow and Shavell (2007) and to the use of social welfare weights as Lockwood and Weinzierl (2015) and Lockwood and Weinzierl (2016) (for labor taxation).

Our paper is also related to a long-standing macro literature studying capital taxation. The stark result in Chamley (1986) and Judd (1985) – that in the long-run the optimal capital tax should be zero– arises because the anticipation elasticity to a long-run tax increase is infinite (see Piketty and Saez (2013b) and our Appendix). This result has generated a stream of subsequent work aimed at exploring its robustness to alternative settings and assumptions.[6] Aiyagari (1995) introduced uncertainty, which generates a finite anticipatory elasticity of capital and positive optimal capital taxes. We precisely compare our findings with wealth in the utility to these benchmark models in Section 4. Farhi (2010) considers the role of incomplete markets for capital taxation. In Albanesi and Sleet (2006), who use a mechanism design framework with private information, wealth is taxed because it emerges as a sufficient statistic for past history that the optimal tax should condition on.

Two forms of capital taxation are bequest or estate taxation and corporate taxation, studied in two complementary strands of the literature. Piketty and Saez (2013b) show that the Chamley-Judd result does not apply when elasticities are finite and there is two-dimensional heterogeneity in both capital (or bequest) and labor income.[7] Farhi and Werning (2013) consider estate taxation with heterogeneous altruism. Yang and De Nardi (2016) quantitatively study

[5] Whether exploiting the sluggishness of capital in the short-run to set higher taxes is a sound approach to optimal policy is questionable.

[6] In a recent paper, Straub and Werning (2015) call into question the validity of the Chamley-Judd result.

[7] See also Piketty and Saez (2012).

estate taxation (see also De Nardi (2004)). Optimal corporate taxation is studied theoretically in Chetty and Saez (2010) and Gordon and Dietz (2010) among others.

Closely related is the theory of optimal taxation of entrepreneurs – indeed, one of our proposed microfoundations for the "wealth in the utility" specification is entrepreneurship. The key papers here are Cullen and Gordon (2007), Cullen and Gordon (2006), and Gordon and Lee (2005).

We use our model to theoretically address the long-standing issue of consumption taxes, also taken up in Kaplow (2008) and Kaplow (1995).

Following Golosov, Tsyvinski, and Werquin (2014), we take a variational approach (see also Werquin (2016) and Golosov, Tsyvinski, and Werquin (2016), and Sachs, Tsyvinski, and Werquin (2016)). Their important contribution makes it possible to express the elasticities in our formulas in terms of the underlying structural elasticities. Findeisen and Sachs (2017) study redistribution and insurance with simpler tax instruments.

We currently abstract from a few issues, which could fruitfully be merged into our framework. The first is the role of idiosyncratic shocks and the resulting insurance problem that individuals and the government face plays a prominent role in Battaglini and Coate (2008b) or Golosov, Tsyvinski, and Werquin (2014). This is done in order to gain in tractability to deal with salient policy issues of interest other than insurance. Important considerations that shape capital taxation are also the political economy issues in Farhi, Werning, Sleet, and Yeltekin (2012) or Battaglini and Coate (2008a). Finally, behavioral issues may play a role for capital taxation, as in Kaplow (2015a), Kaplow (2015b), and Kaplow (2011).

In Section 2.2 we provide further references on the microfoundations for wealth in the utility.

2 A Simpler Model of Capital Taxation

In this section, we present a simpler model of capital taxation. The key simplification comes from having utility linear in consumption, which implies immediate convergence to the steady state. The key additional component is to introduce wealth in the utility, which allows for smooth responses of capital to taxation. This model usefully highlights the key efficiency-equity trade-off for capital taxation, often obscured in more complex models.

2.1 Model Setup

Time is continuous. Individual i has instantaneous utility with functional form $u_i(c,k,z) = c + a_i(k) - h_i(z)$, linear in consumption c, increasing in wealth k with $a_i(k)$ increasing and concave, and with a disutility cost $h_i(z)$ of earning income z increasing and convex in z. The individual index i can capture any arbitrary heterogeneity in the preferences for work and wealth, as well as in the discount rate δ_i. We justify the assumption of wealth in the utility in great detail below. The discounted utility of i from an allocation $\{c_i(t), k_i(t), z_i(t)\}_{t\geq 0}$ is:

$$V_i(\{c_i(t), k_i(t), z_i(t)\}_{t\geq 0}) = \delta_i \cdot \int_0^\infty [c_i(t) + a_i(k_i(t)) - h_i(z_i(t))]e^{-\delta_i t}dt. \tag{1}$$

We normalize utility by the discount rate δ_i so that an extra unit of consumption in perpetuity increases utility by one unit uniformly across all individuals. The net return on capital is r. At time 0, initial wealth of individual i is k_i^{init}. For any given time-invariant tax schedule $T(z, rk)$ based on labor and capital incomes, the budget constraint of individual i is:

$$\frac{dk_i(t)}{dt} = rk_i(t) + z_i(t) - T(z_i(t), rk_i(t)) - c_i(t). \tag{2}$$

$T'_L(z, rk) \equiv \partial T(z, rk)/\partial z$ denotes the marginal tax with respect to labor income and $T'_K(z, rk) \equiv \partial T(z, rk)/\partial(rk)$ denotes the marginal tax with respect to capital income.

The Hamiltonian of individual i at time t, with co-state $\lambda_i(t)$ on the budget constraint, is:

$$H_i(c_i(t), z_i(t), k_i(t), \lambda_i(t)) = c_i(t)+a_i(k_i(t))-h_i(z_i(t))+\lambda_i(t)\cdot[rk_i(t)+z_i(t)-T(z_i(t), rk_i(t))-c_i(t)].$$

Taking the first order conditions, the choice $(c_i(t), k_i(t), z_i(t))$ is such that:

$$\lambda_i(t) = 1, \quad h'_i(z_i(t)) = 1 - T'_L(z_i(t), rk_i(t)), \quad a'_i(k_i(t)) = \delta_i - r(1 - T'_K(z_i(t), rk_i(t))), \quad \text{and}$$

$$c_i(t) = rk_i(t) + z_i(t) - T(z_i(t), rk_i(t)).$$

In this model, $(c_i(t), k_i(t), z_i(t))$ jumps immediately to its steady-state value (c_i, k_i, z_i) characterized by $h'_i(z_i) = 1 - T'_L, a'_i(k_i) = \delta_i - r(1 - T'_K), c_i = rk_i + z_i - T(z_i, rk_i)$. This is achieved by a Dirac quantum jump in consumption at instant $t = 0$, so as to bring the wealth level from the initial k_i^{init} to the steady state value k_i. Because of this immediate adjustment and the lack of transition dynamics, we have that:

$$V_i(\{c_i(t), k_i(t), z_i(t)\}_{t\geq 0}) = [c_i + a_i(k_i) - h_i(z_i)] + \delta_i \cdot (k_i^{init} - k_i),$$

where the last term $(k_i^{init} - k_i)$ represents the utility cost of going from wealth k_i^{init} to wealth k_i at instant 0, achieved by the quantum Dirac jump in consumption.

Heterogeneous wealth preferences and a smooth steady state. Wealth accumulation in this model depends on the heterogeneous individual preferences, as embodied in the taste for wealth $a_i(\cdot)$ and in the impatience δ_i. It also depends on the net-of-tax return $\bar{r} = r(1 - T'_K(z, rk))$: capital taxes discourage wealth accumulation through a substitution effect (there are no income effects). Because of a possibly arbitrary heterogeneity in preferences for capital, steady state wealth holdings are heterogeneous across individuals and capital exhibits a smooth behavior in the steady state, with a finite elasticity of capital supply with respect to the net-of-tax return.

The wealth-in-the-utility feature puts a limit on individuals' impatience to consume. Intuitively, with linear consumption and no utility for wealth, the individual would like to consume all his wealth at once at time 0 (if $\delta_i > \bar{r}$). With utility of wealth, there is value of keeping some wealth. At the margin, the value lost in delaying consumption $\delta_i - \bar{r}$ is equal to the marginal value of holding wealth $a'_i(k)$ and the optimum for capital holding is interior. Note that we need to impose the condition that $\delta_i > \bar{r}$ for all individuals to avoid wealth going to infinity.[8]

Instant adjustments to the steady state and equivalence to the static model: With utility linear in consumption, there are no consumption smoothing considerations. As a result, all dynamic adjustments occur instantaneously and there are no transitional dynamics.

The dynamic model of equation (1) is mathematically equivalent to a static representation. I.e., the optimal choice (c_i, k_i, z_i) from the dynamic problem also maximizes the static utility equivalent:

$$U_i(c_i, k_i, z_i) = c_i + a_i(k_i) - h_i(z_i) + \delta_i \cdot (k_i^{init} - k_i), \tag{3}$$

subject to the static budget constraint $c_i = rk_i + z_i - T(z_i, rk_i)$.

Therefore a social welfare objective based on the original discounted utility V_i from equation (1) is equivalent to a social welfare objective based on the static equivalent U_i from equation

[8] In practice, wealth does not go to infinity because of shocks to the rate of return or to preferences (Piketty (2011, 2014)). The treatment of the case with uncertainty is relegated to Section 4.

(3). It also seems natural to impose a constraint $k \geq 0$ for those who do not like wealth (i.e., who have $a_i(k) \equiv 0$). Such individuals optimally choose $k = 0$ and behave entirely like in the static labor supply model.

Announced vs. unannounced tax reforms: With linear utility of consumption and the resulting lack of transitional dynamics, announced and unannounced tax reforms have exactly the same effect. If at time $t = 0$ a capital tax reform is announced to take place at time T, there is no behavioral response until the actual time of the reform. At time T, the capital stock jumps to its new steady level thanks to a Dirac quantum jump in consumption, exactly as in the unannounced tax reform case. The same optimal taxes apply in the short-run and long-run. As a result, as long as the tax on the return to capital is bounded (e.g. limited to 100%), issues of policy commitment and policy discretion are irrelevant in our model.[9]

2.2 Foundations of Wealth in the Utility

That there must be benefits from wealth other than consumption was already recognized by Weber, Keynes, and Smith among others. Max Weber called the phenomenon of individuals valuing wealth *per se* the "capitalist spirit" (Weber, 1958).[10] Keynes (1931) regretted people's "love of money as a possession." In Keynes (1919), he also lamented that "the duty of saving became nine-tenths of virtue and the growth of the cake the object of true religion," the cake being total wealth. Even more important was his observation that saving was seemingly only done for the sake of holding wealth. "Saving was for old age or for your children; but this was only in theory–the virtue of the cake was that it was never to be consumed, neither by you nor by your children after you."

Already Smith (1759) lamented that wealth could lend social status and moral prestige.[11] Wealth can be used by people as a very visible – even ostentatious– signal of one's innate abilities

[9] There is no temptation to increase the tax rate on capital returns unannounced, as individuals adjust instantaneously, so that the gain from such a tax hike goes to zero. If unanticipated wealth levies are allowed then the capital stock can always be expropriated. In our time continuous model, a wealth levy can be approximated by an infinite tax on capital income for an infinitesimal time. If the capital tax rate is bounded (say at 100%), wealth levies are ruled out. If wealth levies are anticipated, they can be fully avoided in our model with a suitable Dirac quantum consumption just before the wealth levy followed by a corresponding Dirac quantum saving just after the wealth levy.

[10] Weber (1958) viewed it as a result of Protestant values promoting saving, frugality, and capital accumulation.

[11] "This disposition to admire, and almost to worship, the rich and the powerful, and to despise, or, at least, to neglect persons of poor and mean condition, [...] is, at the same time, the great and most universal cause of the corruption of our moral sentiments. That wealth and greatness are often regarded with the respect and admiration which are due only to wisdom and virtue; and that the contempt, of which vice and folly are the only proper objects, is often most unjustly bestowed upon poverty and weakness, has been the complaint of moralists in all ages."

and strengths.[12] Conspicuous consumption is one example of an attempt to signal status and wealth to others, presumably because there are benefits from being perceived as wealthy.[13]

The assumption of wealth in the utility can be micro-founded and justified by the empirical evidence.[14] Wealth in the utility ultimately means relaxing the restrictive assumption that wealth only brings utility benefits through the sheer consumption flow that can be bought with it. While wealth will eventually be consumed by oneself or one's heirs, there are "warm glow" or "joy of ownership" benefits from having it even without spending it, which limit the impatience to consume it.

There is no compelling empirical evidence that a model with only utility for consumption captures microeconomic behavior better than the model with wealth in the utility. Quite the contrary, it has been shown that the standard Bewley models as in Aiyagari (1994) cannot match the empirical wealth distribution. First, precautionary savings in themselves cannot rationalize high wealth holdings at the top without "the capitalist spirit" (Carroll, 1997, 2000; Quadrini, 1999). Second, it is difficult to generate a saving behavior that makes the distribution of wealth much more concentrated than that of labor earnings (Benhabib and Bisin, 2016).[15] Third, as we show in Section 3, there is an important two-dimensional heterogeneity in capital and labor income: even conditional on labor income, capital income is unequally distributed, which means a second dimension of heterogeneity, in addition to differences in labor earnings ability, is required.

We next discuss formally four possible microfoundations for wealth in the utility.

[12] Social status concerns due to wealth may lead to externalities and to corrective taxation, which could be an interesting extension for future research.

[13] Christophera and Schlenker (2000) show in a randomized experiment, that people perceived to be wealthier are also perceived to be more able and talented (see also Dittmar (1992)).

[14] The technical reason for it is that the standard dynamic model with only utility for consumption leads to a degenerate steady state, where $\delta_i = \delta = \bar{r}$. This precludes heterogeneity in time preferences and implies an infinite elasticity of capital to taxes in the steady-state. Introducing utility for wealth is, however, not the only way and our derived tax formulas – expressed in terms of sufficient statistics – do not depend on it. Indeed, in section 4, we discuss two other assumptions used in the literature to obtain non-degenerate (and more realistic) responses of capital to taxes: introducing uncertainty, as in Aiyagari (1995), or consumption-dependent discount rates $\delta_i(c_i)$ as in Judd (1985). As argued in this section, a model with only heterogeneity or stochasticity in labor earnings does not fit the data well.

[15] That households want to keep wealth for purposes other than consumption is also suggested by behavior in retirement: very little wealth is annuitized, especially among the very wealthy, many assets are still available at death, and indeed, wealthy households do not appear to be rapidly de-accumulating wealth closer to their death.

2.2.1 Bequest Motive

The wealth in the utility specification can arise from bequest motives. With a warm-glow bequest motive, if an agent dies at date T, his utility is:

$$V_i(T) = \int_0^T u_i(c_i(t))e^{-\rho_i t}dt + e^{-\delta_i T}\phi_i(k_i(T)), \tag{4}$$

where ρ_i is the discount rate of agent i and $\phi_i(k_i(T))$ is the warm glow utility from the bequest $k_i(T)$ left at time T. If the death time T is stochastic and follows a Poisson process with rate p_i for agent i, then, as in the "perpetual youth" model of Yaari (1965) and Blanchard (1985), the utility can be rewritten in infinite horizon with:

$$V_i = \int_0^\infty e^{-(\rho_i+p_i)t} \cdot [u_i(c_i(t)) + p_i \cdot \phi_i(k_i(t))]\, dt. \tag{5}$$

This amounts to our wealth in the utility formulation with $\delta_i = \rho_i + p_i$ and $a_i(k_i(t)) = p_i \cdot \phi_i(k_i(t))$.

On the empirical side, De Nardi (2004) shows quantitatively that a model with a bequest motive can both explain large wealth holdings at the top and better match the lifecycle profiles of savings. Cagetti and De Nardi (2007) combine a bequest motive with a model of entrepreneurship, also discussed next.

2.2.2 Entrepreneurship

Wealth in the utility can also arise from a model of entrepreneurship. Entrepreneurship has been used as a key explanatory factor for the shape of the wealth distribution according to Quadrini (1999) and Quadrini (2000).[16]

In this model, there is a utility flow from running a business, which captures the non-pecuniary private benefits net of the effort or disutility costs of being an entrepreneur. Non-pecuniary benefits or costs from entrepreneurship have been shown to be substantial and important explanations for occupational choice (Hamilton, 2000; Hurst and Pugsley, 2010). Entrepreneur i receives a return on their capital r_i (Section 2.4.7 deals formally with heterogeneous returns).[17] For instance, if $a_i(k) = \eta_i k_i^\gamma/\gamma$ with $\gamma < 1$, and there is a linear tax on capital τ_K, entrepreneur i would chose a capital level such that: $r_i(1-\tau_K) = \delta_i - \eta_i k_i^{\gamma-1}$.

More generally, the wealth in the utility specification can apply to agents managing a wealth

[16] A useful extension for future research would be to have stochastic returns to capital.
[17] It is possible for $a_i(k)$ to be on net negative in which case we need $\delta_i < r_i$.

portfolio. This is an activity which entails not only a financial return, but also potential non-pecuniary benefits and/or time and effort costs.

2.2.3 Service Flows From Wealth

Capital is embodied in tangible or financial assets, which yield service flows. One salient example is housing, which yields a stream of utility in terms of housing services. But even financial assets provide utility in the form of security or potential liquidity beyond and above their financial return.

In that sense, our model resembles "money in the utility" models. Money is a special asset that yields zero nominal return and high liquidity. Other capital assets have different liquidities and returns. As explained in Poterba and Rotemberg (1987), wealth as held in the form of different assets is akin to other durables in that it provides services (e.g., security or liquidity) even when it is not consumed. Whether those services from wealth enter utility the exact same way as other durable goods is an empirical question that would merit careful estimation of utility functions. The authors argue that "many goods provide different 'types' of utility" and that to single out wealth services as being "unworthy of inclusion in a consumer's utility seems arbitrary at best."

The utility flows from assets are widely documented in the finance literature as being needed to better fit the financial data. Examples of papers which model housing capital as both an asset with returns and as a consumption good providing utility flows are Piazzesi et al. (2007), Stokey (2009), and Kiyotaki et al. (2011). The latter specifically assigns a different utility to renting a house and owning a house (e.g., the owner can modify the house to fit their own taste, which yields utility), which is exactly in the spirit of our specification.

In Section 2.4.8, we explicitly consider differentiated taxation of various types of capital assets.

2.2.4 Motivated Beliefs, Intrinsic Motivation, and Reputation Concerns

A wealth in the utility specification can also be justified through "motivated beliefs" as analyzed in the recent contribution by Bénabou and Tirole (2016). These "motivated beliefs," of which there are many different types, fulfill various psychological roles, such as self-confidence, moral self-esteem, or anxiety reduction, among others. Bénabou and Tirole (2016) provide abundant empirical evidence in favor of such beliefs.

Wealth in the utility can also arise from a social norm that assigns to an individual with a given amount of wealth a certain reputation or moral value, as already suggested by the quote from Smith (1759) above and as modeled explicitly by Bénabou and Tirole (2011). Motivated beliefs and social norms can directly be captured in our framework through the valuation function for wealth $a_i(k)$, the shape of which will depend on the exact nature of the psychological or behavioral phenomenon under consideration.

This links our framework to the rich literature on prosocial behavior, social norms, reputation concerns, and intrinsic motivation (as formalized, among others, in Bénabou and Tirole (2006)). Note that motivated beliefs or social norm impacts can be heterogeneous across individuals (since $a_i(\cdot)$ is indexed by i, which allows for arbitrary heterogeneity) which is one of the flexibilities of our framework.

We give some concrete examples among the many possible ones. If people expect others to perceive them as more able, more successful, or more patient if they exhibit more wealth, there is an "affective" motivated belief (Bénabou and Tirole, 2016) that arises out of the concern to make oneself look better. This will make people want to hold on to wealth for reasons other than just future consumption flow or consumption smoothing. A "functional" motivated belief would arise if, as seems to be the case in reality, others tend to be nicer and offer more favors to us if we are perceived as wealthier, presumably because we *could* potentially spend our wealth and thus yield benefits to others, even if we do not actually spend it in the given moment. The utility flow from wealth could also be due to an internal reputation concern or what Bénabou and Tirole (2016) call "self-signaling": each agent would like to believe that he is able, or a saver, or patient enough to sacrifice early consumption; accumulating wealth self-signals these qualities.

In Bénabou and Tirole (2006) and Bénabou and Tirole (2011), people care about their reputation among others and would like to be perceived as possessing the qualities valued in their society, such as high talent, patience, and an ability to succeed. If these attributes are not directly observable, people may be judged by others based on their observable labor and capital incomes. The valuation of wealth $a_i(k)$ by agent i could thus reflect the valuation of the reputation gain that capital confers.[18]

Bénabou and Tirole (2003) model the notion of intrinsic motivation: applied to our case it would mean that agents save and accumulate wealth in part because of an intrinsic motivation and not just for the financial return, much the same way that we believe that at least part

[18] Social perceptions may be based on psychological reasons or social stereotypes and not depend on the tax policy (i.e., not be based on an individuals' optimality conditions). Otherwise, there may be externalities and a corrective role for the tax system. This is a very interesting question to explore in future research, much as Bénabou and Tirole (2006) and Bénabou and Tirole (2011) do for the provision of incentives.

of one's work (labor) is done out of intrinsic motivation rather than for the external financial reward.

2.3 Optimal Tax Formulas

The government sets the time invariant tax $T(z, rk)$, subject to budget-balance, to maximize its social objective:

$$SWF = \int_i \omega_i \cdot U_i(c_i, k_i, z_i) di, \quad (6)$$

where $\omega_i \geq 0$ is the Pareto weight on individual i. We denote by $g_i = \omega_i \cdot U_{ic}$ the social marginal welfare weight on individual i. With utility linear in consumption, we have $g_i = \omega_i$. Without loss of generality, we further normalize the weights to sum to one over the population so that $\int_i \omega_i di = 1$. We first consider linear taxes and then turn to nonlinear taxes.

2.3.1 Optimal Linear Capital and Labor Taxation

We start by studying the optimal linear taxes at rates τ_K and τ_L on capital and labor income. Recall that $\bar{r} \equiv r \cdot (1 - \tau_K)$ denotes the net-of-tax return on capital. The individual maximizing choices are such that $a_i'(k_i) = \delta_i - \bar{r}$ and $h_i'(z_i) = 1 - \tau_L$ so that k_i depends positively on $\bar{r}$ and z_i depends positively on $1 - \tau_L$. For budget-balance, tax revenues are rebated lump-sum and the transfer to each individualindividual is $G = \tau_K \cdot rk^m(\bar{r}) + \tau_L \cdot z^m(1 - \tau_L)$ where $z^m(1 - \tau_L) = \int_i z_i di$ is aggregate labor income that depends on $1 - \tau_L$ and $k^m(\bar{r}) = \int_i k_i di$ is aggregate capital which depends on $\bar{r}$. The government chooses τ_K and τ_L to maximize social welfare SWF in (6), with $c_i = (1 - \tau_K) \cdot rk_i + (1 - \tau_L) \cdot z_i + \tau_K \cdot rk^m(\bar{r}) + \tau_L \cdot z^m(1 - \tau_L)$ and $U_i(c_i, k_i, z_i) = c_i + a_i(k_i) - h_i(z_i) + \delta_i \cdot (k_i^{init} - k_i)$.

Let the elasticity of aggregate capital k^m with respect to $\bar{r}$ be denoted by e_K and the elasticity of aggregate labor income z^m with respect to the net of tax rate $1 - \tau_L$ be e_L. Because there are no income effects, we have $e_L > 0$ and $e_K > 0$. Standard optimal tax derivations using the individuals' envelope theorems for the choice k_i yield:

$$\frac{dSWF}{d\tau_K} = rk^m \cdot \left[\int_i \omega_i \cdot \left(1 - \frac{k_i}{k^m}\right) di - \frac{\tau_K}{1 - \tau_K} \cdot e_K \right]$$

The social marginal welfare weight on individual i is $g_i = \omega_i$. At the optimal τ_K, we have $dSWF/d\tau_K = 0$, leading to the following proposition.

Proposition 1. ***Optimal linear capital tax.*** *The optimal linear capital tax is given by:*

$$\tau_K = \frac{1-\bar{g}_K}{1-\bar{g}_K+e_K} \quad \textit{with} \quad \bar{g}_K = \frac{\int_i g_i \cdot k_i}{\int_i k_i} \quad \textit{and} \quad e_K = \frac{\bar{r}}{k^m} \cdot \frac{dk^m}{d\bar{r}} > 0. \tag{7}$$

The optimal labor tax can be derived exactly symmetrically:

$$\tau_L = \frac{1-\bar{g}_L}{1-\bar{g}_L+e_L} \quad \text{with} \quad \bar{g}_L = \frac{\int_i g_i \cdot z_i}{\int_i z_i} \quad \text{and} \quad e_L = \frac{1-\tau_L}{z^m} \cdot \frac{dz^m}{d(1-\tau_L)} > 0. \tag{8}$$

Remarks:

The optimal capital tax is zero if $\bar{g}_K = 1$ or $e_K = \infty$. $\bar{g}_K = 1$ happens when there are no redistributive concerns along the capital income dimension (g_i is uncorrelated with k_i).

We discuss social preferences embodied in the social welfare weights g_i in Section 2.4.1. Briefly, as long as wealth is concentrated among individuals with lower social marginal welfare weights (such that g_i is decreasing in k_i and, hence $\bar{g}_K < 1$) the optimal capital tax is strictly positive.

We can also recover a few benchmark cases. The revenue maximizing tax rates (which arise when $\bar{g}_K = 0$ and $\bar{g}_L = 0$) are

$$\tau_K^R = \frac{1}{1+e_K} \quad \text{and} \quad \tau_L^R = \frac{1}{1+e_L}. \tag{9}$$

2.3.2 Optimal Nonlinear Separable Taxes

We now turn to the nonlinear tax system separable in labor and capital income, characterized by the tax schedules $T_L(z)$ and $T_K(rk)$. The individual's budget constraint is given by:

$$c_i = rk_i - T_K(rk_i) + z_i - T_L(z_i), \tag{10}$$

so that utility is:

$$U_i(c_i, k_i, z_i) = rk_i - T_K(rk_i) + z_i - T_L(z_i) + a_i(k_i) - h_i(z_i) + \delta_i \cdot (k_i^{init} - k_i). \tag{11}$$

The first-order conditions characterizing the individual's choice of capital and labor income are:

$$a_i'(k_i) = \delta_i - r(1 - T_K'(rk_i)) \quad \text{and} \quad h_i'(z_i) = 1 - T_L'(z_i).$$

We denote the average relative welfare weight on individuals with capital income higher than rk, by $\bar{G}_K(rk)$ and the average relative welfare weight on individuals with labor income higher than z, by $\bar{G}_L(z)$:

$$\bar{G}_K(rk) = \frac{\int_{\{i:rk_i \geq rk\}} g_i di}{P(rk_i \geq rk)} \quad \text{and} \quad \bar{G}_L(z) = \frac{\int_{\{i:z_i \geq z\}} g_i di}{P(z_i \geq z)}. \tag{12}$$

Let the density distributions of capital and labor income be, respectively, $h_K(rk)$ and $h_L(z)$ and the cumulatively distributions be $H_K(rk)$ and $H_L(z)$. Define the local Pareto parameters of the capital and labor income distributions as:

$$\alpha_K(rk) \equiv \frac{rk \cdot h_K(rk)}{1 - H_K(rk)} \quad \text{and} \quad \alpha_L(z) \equiv \frac{z \cdot h_Z(z)}{1 - H_Z(z)}.$$

Clearly, the income distributions and local Pareto parameters depend on the tax system.[19] The local elasticity of k with respect to the net of tax return $r(1 - T'_K(rk))$ at income level rk is denoted by $e_K(rk)$, while the local elasticity of z with respect to $1 - T'_L(z)$ is denoted by $e_L(z)$.

Because wealth and labor choices are separable, due to the lack of income effects and separable preferences, each tax satisfies the standard Mirrlees (1971) formula and can be expressed in terms of elasticities as in Saez (2001), as shown in the next proposition (the proof is in appendix).

Proposition 2. ***Optimal nonlinear capital and labor income taxes.***

The optimal nonlinear capital and labor income taxes are:

$$T'_K(rk) = \frac{1 - \bar{G}_K(rk)}{1 - \bar{G}_K(rk) + \alpha_K(rk) \cdot e_K(rk)} \quad \textit{and} \quad T'_L(z) = \frac{1 - \bar{G}_L(z)}{1 - \bar{G}_L(z) + \alpha_L(z) \cdot e_L(z)}. \tag{13}$$

Asymptotic Nonlinear Formula. In Section 3 we show that capital income is very concentrated, with top 1% capital income earners earning more than 60% of total capital income. The asymptotic formula when $rk \to \infty$ in (13) is likely relevant for most of the tax base.

$$T'_K(\infty) = \frac{1 - \bar{G}_K(\infty)}{1 - \bar{G}_K(\infty) + \alpha_K(\infty) \cdot e_K(\infty)}. \tag{14}$$

[19] Technically, in the definition of the local Pareto parameters, the densities $h_K(rk)$ and $h_L(z)$ should be replaced by the "virtual densities" $h^*_K(rk)$ and $h^*_L(z)$ defined as the densities at rk and z that would arise if the nonlinear tax system were replaced by the linearized tax system at points rk and z (see Saez (2001) for complete details).

The revenue maximizing rate obtains if $\bar{G}_K(\infty) = 0$.

Optimal linear tax rate in top bracket. It is also easy to derive a formula for the optimal linear tax rate in the top bracket above a given capital income threshold. The formula takes the standard form $\tau_K^{top} = (1-\bar{g}_K^{top})/(1-\bar{g}_K^{top}+a_K^{top}\cdot e_K^{top})$ with $\bar{g}_K^{top}$ the average social marginal welfare weight in the top bracket, e_K^{top} the elasticity in the top bracket, and a_K^{top} the Pareto parameter in the top bracket. The Pareto parameter is defined as $a_K^{top} = \frac{E[k_i|k_i \geq k^{top}]}{E[k_i|k_i \geq k^{top}]-k^{top}}$ where k^{top} is the threshold for the top bracket. This formula is the same as in labor income tax theory (Saez, 2001). As capital income is so concentrated, it has even wider applicability (see our numerical simulations below).

2.4 Policy Issues

We now put this newly gained simplicity to use, and consider how our framework can shed light on several salient issues in the public debate about capital taxation.

2.4.1 Ethical Considerations

We start by discussing four ethical standpoints which are often encountered in the public debate, and what level of capital tax they would imply. We can do this because our approach in terms of sufficient statistics is very amenable to the use of "generalized social welfare weights" g_i as in Saez and Stantcheva (2016), which can better capture the normative considerations which are relevant for capital taxation.[20]

Inequality in wealth deemed unfair: Inequality in wealth is viewed by some as unfair if they perceive wealth accumulation to be the result of preferences for wealth, higher patience, or higher returns on capital. Higher patience could for instance be considered a skill that allows some individuals to accumulate more and be better off in the long-run, much in the same way that a higher earning ability allows people to earn more and be better off in the traditional optimal labor income tax model. Higher returns on capital could be perceived as "luck." In that case, redistributing from wealth lovers to non-wealth lovers could be deemed socially desirable.[21] Social welfare weights g_i are then decreasing in k_i. For linear taxes, then, $\bar{g}_K < 1$ and $\tau_K > 0$.

[20]The generalized social welfare weights are given by $g_i = g(c_i, k_i, z_i; x_i^b, x_i^s)$ where x_i^b is a vector of characteristics which enter both utility and the weights, while x_i^s is a vector of characteristics that only enters the weights. This allows to introduce a gap between individual preferences and social considerations. Hence, it allows for a wider range of normative considerations to be taken into consideration than with standard welfare weights.

[21]The case for this argument may be even stronger if wealth comes from inheritances.

Inequality in wealth deemed fair: Conversely, some may consider inequality in wealth fair and irrelevant for redistribution. In this case, social welfare weights do not depend on k_i and are uncorrelated with k_i. People supporting this view may argue that higher wealth comes from a higher taste for savings (rather than consuming). It is through the sacrifice of earlier consumption that an individual has accumulated wealth. There is no compelling reason to redistribute "from the ant to the grasshopper" because the grasshopper had the same opportunity to save. In this case, if we further assume that wealth k_i is uncorrelated with other characteristics affecting social welfare weights (see discussion just below), then $\bar{g}_K = 1$ and $\tau_K = 0$.[22]

Wealth as a tag: Wealth can be a marker and tag for a characteristic that society cares about, but that taxes cannot directly condition on. In this case, g_i may not depend on k_i directly (as discussed in the previous paragraph), but is correlated with k_i, leading to $\bar{g}_K \neq 1$. For instance, society may care about equality of opportunity and may want to compensate people from poorer backgrounds for their difficult start in life. Even if society does not care about tastes for wealth and wealth per se, higher wealth could be a tag for a richer family background. For example and following Saez and Stantcheva (2016), if $g_i = 1$ for people from a low background and is zero for others, then $\bar{G}_K(rk)$, the average social welfare weight on those with capital income above rk will be the representation index of those from a low background among individuals with capital income above rk. If people with high capital income come disproportionately from wealthy backgrounds, then $\bar{G}_K(rk)$ is less than one, leading to a positive nonlinear capital income tax rate using formula (13).

Similarly, wealth can be a tag for earnings ability. Suppose there is inequality in both capital and labor income, but that the government only cares about the latter, so that g_i only depends on z_i and $T_L(z_i)$. If capital and labor income are uncorrelated, then $\bar{g}_K = 1$ and the optimal τ_K is zero. If they are positively correlated, then $\bar{k} < 1$ and hence $\tau_K > 0$: in this case, high wealth individuals also have higher labor income on average, and wealth acts as a form of tag.[23]

Horizontal equity concerns. Horizontal equity concerns mean that society does not want to treat differently people with the same "ability to pay." The key issue, which involves non-trivial value judgements, is to define "ability to pay" is. It could be total income, capital income, labor

[22] This case may also arise if, as in Coate (1995) the government represents high-income agents (who may be altruistic).

[23] We can here draw a parallel to the optimal transfer literature, which focuses heavily on the use of (imperfect) tags, such as in the key papers by Besley and Coate (1992) and Besley and Coate (1995).

income, or even the consumption of some particular goods. For instance, should ability to pay be measured by labor income only?

On the affirmative side are those who criticize the "double taxation" of income, first in the form of earned labor income and then in the form of an additional tax on capital income earned on savings out of labor income. In addition "equality of opportunity" type of arguments for savings (as opposed to equality of outcomes, in analogy to labor taxation) state that conditional on a given labor income, everybody has the same opportunities to save. This is the view that the grasshopper and the ant, with the same labor income, simply made different choices the consequences of which they have to bear.

On the negative side, an increase in returns on assets more generally would benefit savers and, conditional on a given labor income, individuals with a strong preference for wealth could end up with much higher incomes in the rate of return on capital is high. Indeed, in conceptual debates about the desirability of taxing capital income in the tax law and economics literature, proponents of the tax tend to use high rate of return scenarios (e.g., Warren (1980)) while opponents tend to use low rate of return scenarios (e.g., Weisbach and Bankman (2006)).

Overall, the most natural concept seems total income $y = z+rk$. A higher return on capital r is an advantage for wealth lovers, but this advantage is taken into account in the comprehensive income concept. With strong horizontal equity preferences, this justifies the comprehensive income tax (barring a Pareto improvement of providing a component specific tax break) (see Online Appendix A.5).[24]

2.4.2 Economic Growth

How would economic growth affect the optimal capital tax rate? Suppose that there is technological progress at an exogenous rate $g > 0$, leading to economic growth, so that all per capita variables grow at rate $g > 0$. We can perform the normalization that: $\tilde{z}(t) = z(t)e^{-gt}, \tilde{k}(t) = k(t)e^{-gt}, \tilde{c}(t) = c(t)e^{-gt}$. To sustain a balanced growth path with quasi-linear utility, the sub-utility functions need to take the form $h_{ti}(z(t)) = e^{gt} \cdot h_i(\tilde{z}(t))$ and $a_{ti}(k(t)) = e^{gt} \cdot a_i(\tilde{k}(t))$. We also assume that $T_t(z(t), rk(t)) = e^{gt} \cdot T(\tilde{z}(t), r\tilde{k}(t))$.

[24] An alternative case is if labor income inequality is viewed as fair while capital income inequality is viewed as unfair. In that case, a pure capital income tax should be used first up to revenue maximizing and, only then should a labor tax be added if more revenue is needed.

The discounted normalized utility should now be written as:

$$V_i(\{c_i(t), k_i(t), z_i(t)\}_{t\geq 0}) = \delta_i \cdot \int_0^\infty [c_i(t) + a_{ti}(k_i(t)) - h_{ti}(z_i(t))]e^{-\delta_i t}dt$$
$$= \delta_i \cdot \int_0^\infty [\tilde{c}_i(t) + a_i(\tilde{k}_i(t)) - h_i(\tilde{z}_i(t))]e^{-(\delta_i - g)t}dt.$$

The budget constraint of individual i is:

$$\dot{k}_i(t) = rk_i(t)+z_i(t)-T(z_i(t), rk_i(t))-c_i(t) \quad \text{i.e.} \quad \dot{\tilde{k}}_i(t) = (r-g)\tilde{k}_i(t)+\tilde{z}_i(t)-T(\tilde{z}_i(t), r\tilde{k}_i(t))-\tilde{c}_i(t).$$

Hence, this problem is mathematically equivalent to our earlier problem. Similar derivations show that the normalized solution $(\tilde{c}_i, \tilde{k}_i, \tilde{z}_i)$ for individual i at any time $t > 0$ is such that:

$$h'_i(\tilde{z}_i) = 1-T'_L(\tilde{z}_i, r\tilde{k}_i) \quad \text{and} \quad a'_i(\tilde{k}_i) = \delta_i - r(1-T'_K(\tilde{z}_i, r\tilde{k}_i)) \quad \text{and} \quad \tilde{c}_i = (r-g)\tilde{k}_i+\tilde{z}_i-T(\tilde{z}_i, r\tilde{k}_i).$$

The actual levels of (c_i, k_i, z_i) are then simply equal to: $(\tilde{c}_i \cdot e^{gt}, \tilde{k}_i \cdot e^{gt}, \tilde{z}_i \cdot e^{gt})$.

Again, $(\tilde{k}_i, \tilde{z}_i)$ immediately jumps to its steady-state value through an instantaneous Dirac quantum jump in consumption and wealth at date 0. We have:

$$V_i(\{\tilde{c}_i, \tilde{k}_i, \tilde{z}_i\}_{t\geq 0}) = \frac{\delta_i}{\delta_i - g} \cdot \left[\tilde{c}_i + a_i(\tilde{k}_i) - h_i(\tilde{z}_i) + (\delta_i - g) \cdot (k_i^{init} - \tilde{k}_i)\right]$$
$$= \frac{\delta_i}{\delta_i - g} \cdot \left[(r-g)\tilde{k}_i + \tilde{z}_i - T(\tilde{z}_i, r\tilde{k}_i) + a_i(\tilde{k}_i) - h_i(\tilde{z}_i)\right] + \delta_i \cdot (k_i^{init} - \tilde{k}_i)$$

Therefore, with growth, maintaining normalized wealth $\tilde{k}_i$ requires saving $g \cdot \tilde{k}_i$ in perpetuity, hereby lowering consumption by $g \cdot \tilde{k}_i$.

Intuitively, with economic growth, maintaining a given level of normalized wealth (put differently, a given wealth per capita) requires higher savings and hence reduced consumption. Suppose the economy moves from $g = 0$ to $g > 0$ at time t_0. At time t_0, there is no jump in wealth as normalized wealth is not affected by g. The equation for V_i above shows that wealth lovers (who choose a high $\tilde{k}_i$) gain relatively less than non wealth lovers (who choose for example $\tilde{k}_i = 0$). Economic growth benefits those with no capital more than wealth lovers owning capital.

Let us consider linear taxes on capital for simplicity, with again $\bar{r} = r(1 - \tau_K)$. If $\bar{r} < g$, then wealth lovers would hold more wealth, but have lower consumption than those with less wealth. Conversely, if $\bar{r} > g$, then wealth lovers would hold more wealth and also have higher consumption. In a world in which society disregards wealth per se and cares mostly about consumption (i.e., social welfare weights are based on consumption c only), $\bar{\tau}_K = 1 - g/r$ may be a natural upper bound on the capital tax. This discussion connects with the famous r vs. g discussion at the heart of Piketty (2014).[25]

2.4.3 Jointness in preferences for labor and capital

There could be jointness in the preferences for work and wealth, which introduces cross-elasticities between the capital and labor taxes. It is indeed reasonable to think that work incentives could be affected by wealth.[26]

The discounted utility is:

$$V_i(\{c_i(t), k_i(t), z_i(t)\}_{t\geq 0}) = \delta_i \int_0^\infty [c_i(t) + v_i(k_i(t), z_i(t))]e^{-\delta_i t}dt, \tag{15}$$

with $v_i(k, z)$ increasing concavely in k and decreasing concavely in z. With linear taxes τ_K and τ_L, the budget constraint of individual i is:

$$\frac{dk_i(t)}{dt} = \bar{r}k_i(t) + (1 - \tau_L) \cdot z_i(t) + r\tau_K k^m(t) + \tau_L z^m(t) - c_i(t).$$

The choice $(c_i(t), k_i(t), z_i(t))$ for individual i at any time $t > 0$ is such that:

$$-v_{iz}(k_i(t), z_i(t)) = 1 - \tau_L, \qquad v_{ik}(k_i(t), z_i(t)) = \delta_i - \bar{r},$$

$$\text{and} \quad c_i(t) = \bar{r}k_i(t) + (1 - \tau_L) \cdot z_i(t) + r\tau_K k^m(t) + \tau_L z^m(t).$$

The dynamic model is again equivalent to the static specification:

$$U_i(c_i, k_i, z_i) = c_i + v_i(k_i, z_i) + \delta_i(k_i^{init} - k_i).$$

Denote by $e_{L,(1-\tau_K)} \equiv \frac{(1-\tau_K)}{z^m} \cdot \frac{dz^m}{d(1-\tau_K)}$ the cross-elasticity of average labor income to the net-of-tax return and by $e_{K,(1-\tau_L)} \equiv \frac{(1-\tau_L)}{rk^m} \cdot \frac{d(rk^m)}{d(1-\tau_L)}$ the cross-elasticity of average capital income to the net-of-tax labor tax rate.

Proposition 3. ***Optimal labor and capital taxes with joint preferences.*** *With joint preferences, the optimal linear capital tax (respectively, labor tax) taking the labor tax (respec-*

[25] Another paper that explores the link between capital taxation and the $r - g$ concept is Fuest et al. (2015).
[26] This is the reason for an asset test in the case of disability insurance in Golosov and Tsyvinski (2006).

tively, the capital tax) as given is:

$$\tau_K = \frac{1 - \bar{g}_K - \tau_L \frac{z^m}{rk^m} e_{L,(1-\tau_K)}}{1 - \bar{g}_K + e_K} \quad \textit{and} \quad \tau_L = \frac{1 - \bar{g}_L - \tau_K \frac{rk^m}{z^m} e_{K,(1-\tau_L)}}{1 - \bar{g}_L + e_L}.$$

The formula for each tax applies even if the other tax is not optimally set. The effects of jointness in preferences on the optimal labor and capital taxes depend on the complementarity or substitutability of preferences for capital and labor. If having more capital decreases the cost of work, then $e_{L,(1-\tau_K)} > 0$ and, at any given τ_L, the capital tax should optimally be set lower.

2.4.4 Comprehensive Income Tax System $T(z + rk)$

An important policy question is how the tax rate should be set if all income – whether stemming from capital or labor– were to be treated the same way for tax purposes. In many countries, most "ordinary" capital income, such as interest from a standard savings account, is taxed like labor income. Within our framework, we can easily solve for the optimal nonlinear tax on comprehensive income $y \equiv rk + z$, of the form $T_Y(y)$, i.e., for the optimal system within the class of tax systems that treat capital and labor income perfectly symmetrically. We then discuss when such a tax system is optimal. In this case, the optimal tax formula turns out to take the same form as in Mirrlees (1971) and Saez (2001).

Define the average welfare weight on individuals with total income higher than y as:

$$\bar{G}_Y(y) = \frac{\int_{\{i: y_i \geq y\}} g_i di}{P(y_i \geq y)}. \tag{16}$$

Let $h_Y(y)$ and $H_Y(y)$ be the density and cumulative distribution functions of the total income distribution. $\alpha_Y(y) \equiv \frac{y h_Y(y)}{1 - H_Y(y)}$ is the local Pareto parameter for the distribution of total income y and $e_Y(y)$ is the elasticity of total income to the net of tax rate $1 - T'_Y(y)$ at point y.

Using the envelope theorem, we obtain a standard optimal tax formula on full income.

Proposition 4. ***Optimal tax on comprehensive income.***

(i) The optimal nonlinear tax on comprehensive income (labor and capital income) $y = rk + z$ is given by:

$$T'_Y(y) = \frac{1 - \bar{G}_Y(y)}{1 - \bar{G}_Y(y) + \alpha_Y(y) \cdot e_Y(y)}.$$

(ii) The optimal linear tax on comprehensive income is:

$$\tau_Y = \frac{1-\bar{g}_Y}{1-\bar{g}_Y+e_Y}. \tag{17}$$

$$\text{with} \quad \bar{g}_Y \equiv \frac{\int_i g_i y_i}{y^m} = \frac{z^m \bar{g}_L + r k^m \bar{g}_K}{z^m + r k^m} \quad \text{and} \quad e_Y \equiv \frac{dy^m}{d(1-\tau_Y)} \frac{(1-\tau_Y)}{y^m} = \frac{(z^m e_L + r k^m e_K)}{z^m + r k^m}. \tag{18}$$

A tax system based on comprehensive income may be optimal for equity reasons (discussed in Section 2.4.1) or for efficiency reasons, due to the existence income shifting opportunities between the capital and labor income bases (in Section 2.4.5).

2.4.5 Income Shifting

A salient issue in the policy debate is the possibility of shifting income between the labor and capital bases.

To model this, suppose that individuals can shift an amount of labor income x from the labor to the capital tax base at a utility cost $d(x)$, increasing and convex in x. Hence, if reported labor income at time t is $z_i^R(t)$, we have $x_i(t) = z_i(t) - z_i^R(t)$. The aggregate shifted amount at time t is $x^m(t) \equiv \int_i x_i(t)di$. We consider linear taxes in this section.

We can easily show that in this case again, the dynamic and static problems are equivalent. The discounted normalized utility of individual i,

$$V_i(\{c_i(t), k_i(t), z_i(t), x_i(t)\}_{t\geq 0}) = \delta_i \cdot \int_0^\infty [c_i(t) + a_i(k_i(t)) - h_i(z_i(t)) - d_i(x_i(t))] e^{-\delta_i t} dt,$$

under the budget constraint:

$$\dot{k}_i(t) = \bar{r}k_i(t) + (1-\tau_L)z_i(t) - c_i(t) + (\tau_L - \tau_K)x_i(t) + \tau_L(z^m(t) - x^m(t)) + \tau_K(rk^m(t) + x^m(t)),$$

is equivalent to the static model:

$$U_i(c,k,z,x) = c + a_i(k) - h_i(z) - d_i(x) + \delta_i \cdot (k^i_{init} - k),$$

subject to the static budget constraint $c = \bar{r}k + (1-\tau_L)z + (\tau_L - \tau_K)x + \tau_L(z^m - x^m) + \tau_K(rk^m + x^m)$. This static model of tax shifting was analyzed in Piketty and Saez (2013a). The individual's choice is characterized by the following conditions:

$$h_i'(z_i) = 1 - \tau_L \quad \text{and} \quad a_i'(k_i) = \delta_i - \bar{r},$$

$$d_i'(x_i) = \tau_L - \tau_K \quad \text{and} \quad c_i = \bar{r}k_i + (1-\tau_L)z_i + (\tau_L - \tau_K)x_i + \tau_L(z^m - x^m) + \tau_K(rk^m + x^m).$$

Hence, labor income is a function $z_i(1-\tau_L)$ of the net-of-tax rate, capital income is a function of the net-of-tax return $\bar{r}$, and shifted income is a function $x(\Delta\tau)$ of the tax differential $\Delta\tau \equiv \tau_L - \tau_K$.

In the same way that we previously defined the distributional factors for capital and labor income in (7) and (8), we can define the distributional factor for shifted income as: $\bar{g}_X = \int_i \omega_i x_i / z^m$. As long as the distributional factor $\bar{g}_X$ is small enough (in a way made precise in the proof in the Appendix) so that allowing income shifting is not an attractive way of redistributing income, we have the following results.

Proposition 5. ***Optimal Labor and Capital Taxes with Income Shifting.***

i. If $e_K > e_L \frac{(1-\bar{g}_K)}{(1-\bar{g}_L)}$, then $\frac{1-\bar{g}_L}{1-\bar{g}_L+e_L} \geq \tau_L > \tau_K \geq \frac{1-\bar{g}_K}{1-\bar{g}_K+e_K}$ and conversely, if $e_K < e_L \frac{(1-\bar{g}_K)}{(1-\bar{g}_L)}$, then $\frac{1-\bar{g}_L}{1-\bar{g}_L+e_L} \leq \tau_L < \tau_K \leq \frac{1-\bar{g}_K}{1-\bar{g}_K+e_K}$.

ii. If there is no shifting, the linear tax rates are set according to their usual formulas in (7) and (8).

iii. If shifting is infinitely elastic, then the tax differential $\Delta\tau$ goes to 0 and $\tau_K = \tau_L = \tau_Y = \frac{1-\bar{g}_Y}{1-\bar{g}_Y+e_Y}$ where $\bar{g}_Y = \frac{z^m \bar{g}_L + rk^m \bar{g}_K}{z^m + rk^m}$ is the distributional factor of total income, and $e_Y = \frac{(z^m e_L + rk^m e_K)}{z^m + rk^m}$ is the elasticity of total income.

Thus, as long as there is shifting with a finite elasticity, the labor and capital taxes are compressed toward each other, away from their optimal values with no shifting. With an infinite shifting elasticity, the optimum is to set a comprehensive tax on full income $y = rk + z$, as solved for in (17). Strong shifting opportunities, with elasticities tending to infinity, can thus provide a justification for a tax based on total comprehensive income which is orthogonal to the social ethical considerations discussed in Section 2.4.1.

2.4.6 Consumption taxation

Can a consumption tax achieve more redistribution than a wealth tax and be more progressive than a tax on labor income? Our simple model allows us to cleanly assess the role of and the scope for a consumption tax.

Let us define real wealth as wealth expressed in terms of purchasing power, or, equivalently, wealth as normalized by the price of consumption. It seems natural that individuals should care about real wealth, rather than nominal wealth, for the real economic power or status that it confers. As long as individuals care about real wealth, a consumption tax is equivalent to a tax on labor income augmented with a tax on initial wealth as in the standard model with no utility for wealth (see e.g., Kaplow (1994); Auerbach (2009)). Hence, the consumption tax cannot achieve a more equal steady state than the labor tax. In the simplest case with a linear consumption tax, it is immediate to see this equivalence.[27]

If the tax exclusive rate is t_C, so that the implied price of consumption is $1 + t_C$, the equivalent tax inclusive rate is τ_C, which is such that $1 - \tau_C = 1/(1 + t_C)$. Real wealth is here $k^r = k \cdot (1 - \tau_C)$ and flow utility is $u_i = c + a_i(k^r) - h_i(z)$. The budget constraint of the individual becomes $\dot{k} = [\bar{r}k + z - T_L(z)] - c/(1 - \tau_C) + G$, where $G = \tau_L z^m + \tau_K r k^m + t_C c^m$ is the lump-sum transfer rebate of tax revenue. The budget constraint can be rewritten in terms of real wealth as: $\dot{k}_r = \bar{r}k^r + (z - T_L(z)) \cdot (1 - \tau_C) + G \cdot (1 - \tau_C) - c$.

In real terms, the consumption tax τ_C then just adds a layer of taxes on labor income, leaving $\bar{r}$ unchanged. For the individual, the steady state (i.e., the static model) $(\bar{r}, T_L, \tau_C)$ is equivalent to $(\bar{r}, \hat{T}_L, \tau_C = 0)$ with $\hat{T}_L$ such that $z - \hat{T}_L(z) = (z - T_L(z)) \cdot (1 - \tau_C)$.

The difference between these two tax systems is that consumption taxation also taxes initial wealth by reducing its real value from k_i^{init} to $k_i^{r,init} = (1 - \tau_C) \cdot k_i^{init}$. This means that a consumption tax does successfully tax initial wealth, but has no long term effect on the distribution of real wealth. If the government undoes this initial wealth redistribution by giving a lump-sum transfer $\tau_C \cdot k_i^{init}/(1 - \tau_C)$ to an individual i with initial wealth holdings k_i^{init}, the equivalence between a consumption tax system $(\bar{r}, T_L, \tau_C)$ and a modified labor tax system with no consumption tax $(\bar{r}, \hat{T}_L, \tau_C = 0)$ becomes complete both in the dynamic consumer problem, the steady-state of the consumer, and the intertemporal government budget. Hence we have:

Proposition 6. ***Equivalence of consumption taxes and labor taxes.*** *A linear consumption at inclusive rate τ_C is equivalent to a tax on labor income combined with a tax on initial wealth.*

To refute a common fallacy on the redistributive power of consumption taxes, suppose that there is no initial wealth (and, hence, no need for a compensating transfer if a consumption tax were to be introduced) and that labor income is inelastic and uniform across individuals.

[27] With a progressive consumption tax, the equivalence is less immediate, but nevertheless present and we consider this case in Online Appendix A.6.

Differences in wealth then only arise from differences in tastes for wealth. It is clear that a pure labor income tax achieves no redistribution in this setting: it just taxes the inelastic and equal labor income and rebates it back as an equal lump-sum transfer to all individuals. If there were a consumption tax in this setting, those with higher preferences for wealth would end up having higher income, higher consumption, and pay higher taxes than those with lower preferences for wealth. But recall that the consumption tax is fully equivalent to the labor income tax in this setting and that the labor income tax achieves no redistribution. Thus, while wealth lovers look like they pay higher taxes in the steady state on their higher consumption, this is because they paid less taxes while building up their wealth at instant 0. This initial wealth accumulation is what gives them higher steady state consumption in the first place. Wealth lovers build up more nominal wealth with consumption taxation so that their real wealth is the same as under the equivalent labor income tax (and no consumption tax). With a consumption tax only, wealthy individuals pay more taxes in steady state, but they also accumulate more nominal wealth so that inequality in *real* wealth is unaffected in the steady state.

It is hence important to draw a distinction between the observed cross-section and the lifetime distribution of resources. In our simple model, in the cross-sectional steady-state, the consumption tax looks redistributive, when, in reality, it is not.

2.4.7 Heterogeneous Returns to Capital

In practice, individuals may have very different returns on their wealth. Financially savvy people may be able to hold optimized portfolios with higher returns for instance. Higher wealth individuals empirically seem to reap a higher return, potentially because of smarter investments or economies of scale in financial management (Piketty, 2014). Entrepreneurs investing their capital in a business may have different abilities for running their business and generating returns.

With heterogeneous returns to capital, the full dynamic model with utility as in (1) subject to the budget constraint in (2), where r is replaced by a heterogeneous return r_i is again equivalent to the same static model as above, with the following budget constraint: $c_i = r_i(1-\tau_K)k_i + (1-\tau_L)z_i + \tau_K \int_i r_i k_i(\bar{r}_i) + \tau_L z^m(1-\tau_L)$.

At the optimal τ_K, we have $dSWF/d\tau_K = 0$, so that:

$$\tau_K = \frac{1-\bar{g}_{rK}}{1-\bar{g}_{rK}+e_{rK}} \quad \text{with} \quad \bar{g}_{rK} = \frac{\int_i g_i \cdot r_i k_i}{\int_i r_i k_i} \quad \text{and} \quad e_{rK} = \frac{(1-\tau_K)}{\int_i r_i k_i} \cdot \frac{d\int_i r_i k_i}{d(1-\tau_K)} > 0.$$

Heterogeneous returns do not affect the formula in terms of sufficient statistics, $\bar{g}_{rK}$ and e_{rK}. However, they may affect our ethical judgments on taxes, especially if there is a systematic correlation (as discussed in Piketty (2014)) between wealth and the return on wealth.

Different returns on capital could be perceived as unfair: for a given amount of sacrificed consumption, some individuals reap higher returns, much like for a given amount of sacrificed leisure, some individuals reap a higher labor income in the standard labor tax model. Redistribution across individuals with different returns may then be perceived as desirable, even conditional on total capital income.[28]

2.4.8 Different Types of Capital Assets

Another issue which would be very difficult to handle in standard dynamic capital tax models is that, in practice, there is not just one single type of capital, but rather different assets, with different liquidity and payoff patterns. Moreover, individuals may have heterogeneous tastes for different assets. Our model is flexible enough to incorporate different types of capital assets and heterogeneous preferences for them. Thanks to the direct utility component for wealth here, we can rationalize why people would hold assets with different returns above and beyond the standard risk-return trade-off considerations. For instance, a home can yield direct utility benefits. Government bonds or shares in one's own company may also have an individual-specific value, if people care about the national or company-specific contribution that their capital makes.

Consider J assets with different returns denoted generically by r^j, taxes τ_K^j, and net-of-tax return $\bar{r}^j$. Iindividualndividual i holds a level k_i^j of asset j, with initial level $k_i^{init,j}$. For simplicity, assume exogenous and uniform labor income z. The static utility equivalent for individual i can feature joint preferences in the assets:

$$U_i = c_i + a_i(k_i^1, .., k_i^J) + \delta_i \cdot \sum_{j=1}^{J} (k_i^{init,j} - k_i^j),$$

[28]Put differently, someone with a high r_i (a "luck" shock) should be deemed less deserving than someone with a high k_j (a higher consumption sacrifice) conditional on $r_i k_i = r_j k_j$. On the other hand, if returns are deemed fair, then social welfare weights should be the same conditional on $r_i k_i = r_j k_j$ (regardless of whether the high capital income comes from a higher capital stock or a higher return on capital).

with the budget constraint:

$$c_i = \sum_{j=1}^{J} \bar{r}^j k_i^j + z + \sum_{j=1}^{J} \tau_K^j r^j k^{m,j}.$$

It is straightforward to derive the tax rates on each asset, analogous to the formula for capital and labor taxes with joint preferences in Section 2.4.3:

Proposition 7. ***Different types of capital with heterogeneous, joint preferences.*** *The optimal tax on capital asset j, given all other tax rates τ_K^s for $s \neq j$ (not necessarily optimally set) is given by:*

$$\tau_K^j = \frac{1 - \bar{g}_K^j - \sum_{s \neq j} \tau_K^s \frac{k^{m,s}}{k^{m,j}} e_{K^s,(1-\tau_K^j)}}{1 - \bar{g}_K^j + e_K^j} \tag{19}$$

$$\textit{with} \quad \bar{g}_K^j = \frac{\int_i g_i \cdot k_i^j}{\int_i k_i^j}, \quad e_K^j = \frac{\bar{r}^j}{k^{m,j}} \cdot \frac{dk^{m,j}}{d\bar{r}^j} > 0, \quad \textit{and} \quad e_{K^s,(1-\tau_K^j)} = \frac{\bar{r}^j}{k^{m,s}} \cdot \frac{dk^{m,s}}{d\bar{r}^j}. \tag{20}$$

The tax on each type of capital asset is first determined by the two standard considerations of equity and efficiency. Indeed, with no cross-elasticities,[29] the formulas are simply:

$$\tau_K^j = \frac{1 - \bar{g}_K^j}{1 - \bar{g}_K^j + e_K^j}.$$

Assets with higher elastiticities (e_K^j) should be taxed less. Those with a higher redistributive impact, i.e., for which holdings are concentrated among high welfare weight individuals ($\bar{g}_K^j$ high) should be taxed less, all else equal.[30] Society may have very different value judgements regarding different assets, embodied in very different weights $\bar{g}_K^j$, leading to different optimal tax rates.

Second, the efficiency cost of taxing asset j depends on its cross-elasticities with other assets and its fiscal spillovers to the other assets' tax bases. If the asset is complementary to many other assets the efficiency cost of taxing it may be much larger than the own-price elasticity.

In addition, if the government cannot freely optimize the tax rate on some asset s, then, when asset j and asset s are complements ($e_{K^s,(1-\tau_K^j)} > 0$), the higher existing tax on asset s

[29]This case arises with separable utilities across different assets: $a_i(k_i^1, ...k_i^J) = \sum_{j=1}^{J} a_i^j(k_i^j)$.

[30]Conversely, assets equally distributed ($\bar{g}_K^j \approx 1$) should not be taxed much for redistributive purposes.

would push towards a lower optimal tax on asset j.

2.4.9 The aggregate capital stock and an endogenous return to capital

An often discussed policy question is that, in practice, the return to capital may not be exogenously given by r and may endogenously depend on an aggregate production function $F(K, L)$ where $K = \int_i k_i di$ is aggregate capital and $L = \int_i l_i di$ is aggregate labor, with l_i the effective labor supplied by individual i. Earnings are equal to $z_i = w \cdot l_i$ with $w = F_L$ the wage per unit of effective labor. $r = F_K$ is the marginal return to capital.

A direct application of the Diamond and Mirrlees (1971) theory implies that the optimal tax formulas for capital and labor would be unchanged with an aggregate production function. In other words, optimal tax rates depend solely on the supply side elasticities and general equilibrium price effects are irrelevant. The intuition is simple: consider for instance increasing τ_L. This creates an indirect transfer from capital owners to labor (human capital) owners because a lower labor supply depresses the endogenous returns to capital and increases the returns to labor. However, this transfer can be offset at no fiscal cost through a higher capital tax such that the post return to capital is unchanged relative to the situation in which the labor tax was not increased.

Thanks to the Diamond-Mirrlees theory, the question of how to tax capital holdings of different individuals can be treated separately from the question about the optimal aggregate capital stock.

3 Numerical Application to U.S. Taxation

In this section, we give empirical content to the optimal tax rates derived in Section 2. One of the advantages of our method is that the sufficient statistics that appear in the optimal tax formula provide a clear link to the data. We use IRS tax data for 2007 on labor and capital income distributions.[31] We follow the conventions of Piketty and Saez (2003) to define income and percentile groups. The individual unit is the tax unit defined as a single person with dependents if any or a married couple with dependents if any. Capital income is defined as all capital income components reported on individual tax returns, and includes dividends, realized capital gains, taxable interest income, estate and trust income, rents and royalties, net profits from businesses (including S-corporations, partnerships, farms, and sole proprietorships).

[31] We choose 2007 as this is the most recent year of publicly available micro-level US tax data available before the Great Recession. By September 2016, the most recent year available was 2010.

Labor income is defined as market income reported on tax returns minus capital income defined above. It includes wages and salaries, private pension distributions, and other income.[32] We recognize that the tax based income components we use to classify capital and labor incomes do not perfectly correspond to economic capital and labor incomes.[33] Yet, any tax system that taxes capital and labor separately has to use the existing tax based income components. For simplicity, any negative income is set at zero. In aggregate, capital income represents 26% of total income and labor income represents 74% of total income (see Figure 2). As our theory boils down to a static model, it is directly suited for thinking through optimal taxation of annual labor and capital income, as actual income tax systems operate.

3.1 Empirical Distributions of Capital and Labor Income

Three key facts about the distributions of labor and capital income stand out.

i. Capital income is more unequally distributed than labor income.

The distributions of both labor and capital income (and, thus, of total income) exhibit great inequalities, but capital income is much more concentrated than labor income, as shown in the Lorenz curves in Figure 1. The top 1% people as ranked by capital income earn 63% of all capital income, while the bottom 80% earn essentially zero capital income.

ii. At the top, total income is mostly capital income.

At the top of the income distribution total income comes mostly from capital income. Figure 2 shows capital and labor income as a fraction of total income for the full population (P0-P100) and for several subgroups as ranked by total income. At the top of the income distribution, capital comes close to 80% of total income.

iii. Two-dimensional heterogeneity in both labor and capital income.

There is an important two-dimensional heterogeneity in labor and capital income. Conditional on labor income, capital income continues to exhibit a lot of inequality. Figure 3 plots the Lorenz curves for capital income (the cumulative share of capital income owned by those below each percentile of the capital income distribution), but conditional on being in four groups according to labor income: all individuals, the bottom 50% by labor income, the top 10% by labor income and the top 1% by labor income. Even conditional on labor income, there is still a very large concentration of capital income.

[32] Our definition of capital income is broad (and correspondingly, our definition of labor income is narrow), as business profits are actually a mix of labor and capital income.

[33] See Piketty et al. (2016) for an attempt to reconstruct the economic capital and labor incomes starting from tax data.

3.2 Optimal Separable Tax Schedules

3.2.1 Methodology

We first start by considering the optimal separable tax schedules for capital and labor income of the form $T_L(z_i)$ and $T_K(rk_i)$, making use our sufficient statistics non-linear formulas derived in Section 2.3.2.

We assume constant elasticities for labor and capital income, denoted by, respectively, e_L and e_K.[34] Starting from the micro-level IRS tax data, we invert individuals' choices of labor and capital income, given the current U.S. tax system to obtain the implicit latent types which are consistent with these observed choices and these constant elasticities. The distribution of types is hence such that, given the constant behavioral elasticities and the actual U.S. tax schedule, the capital and labor income distributions match the empirical ones (Saez (2001) developed this methodology in the case of optimal labor income taxation). We then fit non-parametrically the distribution of latent types. We repeat the same procedure for total income.

At the top, the distributions of labor, capital, and total income exhibit constant hazard rates and approximate a Pareto distribution with tail parameters denoted by, respectively, a_L, a_K, and a_Y. The empirical Pareto parameters are plotted in Figure 4 for labor, capital, and total income. For labor income the Pareto parameter is around $a_L = 1.6$, for capital income it is $a_K = 1.38$, and for total income it is $a_Y = 1.4$ (given that the tail of total income is mostly capital income).

To capture social preferences for redistribution, we assign exogenous weights g_i which decline in observed disposable income at the current tax system, i.e., such that the weight for individual i in the data is equal to $g_i = 1/((z_i + rk_i)(1-\tau^{US}) + R^{US})$ where $\tau^{US} = 25\%$ and R^{US} mimic the U.S. average tax rate on total income and demogrant. Such weights decline to zero as income goes to infinity, implying that optimal top rates are given by the asymptotic revenue maximizing tax rates derived earlier.

[34] For labor income, as is well known, this requires a disutility of work of the form $h_i(z) = z_i^0 \cdot (z/z_i^0)^{1+1/e_L}/(1+1/e_L)$ where z_i^0 is exogenous potential earnings equal to actual earnings when the marginal labor income tax rate is zero. Similarly, for capital income, this requires a utility of wealth of the form $a_i(k) = \delta_i \cdot k - r \cdot k_i^0 \cdot (k/k_i^0)^{1+1/e_K}/(1+1/e_K)$ where k_i^0 is exogenous potential wealth equal to actual steady state wealth when the marginal capital income tax rate is zero. This disutility of wealth function has to depend on the discount rate δ_i and the rate of return r. It is first increasing and then decreasing in wealth k. However, in equilibrium, the individual always chooses k_i in the increasing portion of the $a_i(k)$ function.

3.2.2 Results

Panels (a) and (b) in Figure 5 show, respectively, the optimal marginal labor income tax as a function of labor income and the optimal marginal capital income tax as a function of capital income, each for three different values of the elasticity parameters, namely 0.25, 0.5, and 1. We use a range of possible elasticities given the uncertainty coming out of the empirical literature (see Saez et al. (2012) for a recent survey).

The optimal labor and capital income taxes both follow closely the shape of the empirical Pareto parameter from Figure 4. The labor income tax hence takes the familiar shape as in Saez (2001) and naturally is lower when the elasticity of labor income to the net of tax rate is higher.

The capital income tax schedule is new. Because capital is so concentrated, the asymptotic nonlinear tax rate, which approximates the linear top tax rate, as explained in Section 2.3.2, kicks in very rapidly, covering the vast majority of the capital income tax base. Above the top 1%, the optimal marginal tax rate on capital income is essentially constant, so that the nonlinear tax schedule at the top is very well approximated by a linear tax rate. Naturally, the level of that optimal linear top tax rate depends inversely on the elasticity of capital income to the net of tax return.

Because capital income is more concentrated than labor income, the Pareto parameter for capital income is lower than for labor income, leading to a higher top tax rate for capital income than for labor income when the elasticities e_L and e_K are the same. In another words, e_K would need to be significantly higher than e_L to justify imposing the same top tax rate on capital and labor incomes.

3.3 Optimal Comprehensive Tax Schedule

We then turn to exploiting the optimal tax on comprehensive income, $T_Y(y)$, with $y = z + rk$, making use of the nonlinear formulas derived in Section 2.4.4 in terms of sufficient statistics. We repeat the same procedure outlined above for labor and capital income, assuming that the elasticity of total income e_Y is constant. We again consider three possible values. Panel (c) in Figure 5 plots the optimal marginal tax rate $T'_Y(y)$ as a function of total income y.

The optimal marginal tax rate on total income has a shape similar to that on labor income. Often, in numerical applications of the Mirrlees (1971) labor income tax model, total income is used for the calculations. We can here rigorously compare the resulting two schedules. The asymptotic top tax rate on total income is closest to the asymptotic top tax rate on capital income from panel (b) as capital income dominates labor income among top incomes.

4 Generalized Model

In this section, we generalize the results from the previous simple model to the case with an arbitrary concave utility. We start by deriving optimal taxes and show that the formulas from Section 2 still apply in this generalized model with transitional dynamics, as long as the elasticity of the tax base – which now features slow adjustments– is appropriately taken into account. The qualitative lessons we drew in the simpler model are hence valid and it is only the quantitative implications of the elasticities that differ. The faster the responses of capital to tax changes, and the more quantitatively robust all previous results from Section 2 are. If responses are slow, then the government can tax more in the short run, when taking advantage of the sluggish adjustments of capital. Exploiting the slow responses may, however, be normatively unappealing. We also compare our results to those of earlier models, making use of the unifying tax formulas we obtain that are widely applicable.

4.1 Generalized wealth in the utility model

In the generalized model with concave utility for consumption and wealth in the utility, the discount rate of individual i is δ_i and his instantaneous utility is $u_i(c_i(t), k_i(t), z_i(t))$. With time-invariant taxes $T(rk, z)$, individual i choices $(c_i(t), k_i(t), z_i(t))$ converge to a steady state characterized by:

$$u_{ik}/u_{ic} = \delta_i - r(1 - T'_K), \qquad u_{ic} \cdot (1 - T'_L) = -u_{iz}, \quad \text{and} \quad c_i = rk_i + z_i - T(z_i, rk_i).$$

Aggregating across individuals, in the steady state, capital has a finite elasticity with respect to the net-of-tax return. Conditional on labor income, wealth is heterogenous across individuals due to differences in the taste for capital (embodied in the utility u_i) and in impatience (embodied in the discount rate δ_i). Relative to the simpler model in Section 2, consumption smoothing considerations now kick in, the convergence to the steady state is no longer instantaneous and there are transitional dynamics.

The government maximizes a standard dynamic social welfare function equal to:[35]

$$SWF = \int_i \omega_i V_i(\{c_i(t), k_i(t), z_i(t)\}_{t\geq 0}) di, \tag{21}$$

$$\text{where} \quad V_i(\{c_i(t), k_i(t), z_i(t)\}_{t\geq 0}) = \delta_i \cdot \int_{t=0}^{\infty} u_i(c_i(t), k_i(t), z_i(t)) e^{-\delta_i t} dt.$$

4.1.1 Long-run budget neutrality

It is useful to think of the case in which the *quantitative* results from the simpler model of Section 2 would carry over, in addition to the qualitative results. The formulas are in sufficient statistics, so that, if social fairness views are held constant, only the elasticities will be affected by the underlying behavioral responses.

Suppose that the government's tax system has to be budget neutral in the long-run steady state. Suppose, however, that it does not necessarily have to be budget neutral in the short-run and that the government can absorb the transitional surplus and deficits within some limit through a buffer fund or through debt. Suppose further that the management of this buffer fund and debt is done separately, so that the distributional impacts from it are not counted in the tax problem (it may, of course, affect the cost of public funds, and, hence, indirectly affect the tax problem). Then, all quantitative results from the simpler model in Section 2 carry over unaffected. The exact same formulas apply with using the same steady-state elasticities. The transitional surplus/deficit in funds will be smaller the faster the adjustment of capital.

4.1.2 Optimal linear tax formulas in the generalized model

We now revert to the standard case in which budget neutrality needs to hold in all periods. The government budget constraint is:

$$\int_i T(rk_i(t), z_i(t)) di \geq E$$

where E is some exogenous non-transfer spending or revenue requirement.

[35] Maximizing the individuals' steady state welfare $SWF = \int_i \omega_i \cdot u_i(c_i, k_i, z_i) di$ is paternalistic and does not respect the envelope theorem. An infinitesimal change in wealth dk_i has a positive effect on individual i steady state instantaneous utility equal to $(u_{ic} r(1 - T'_K) + u_{ik}) dk_i = u_{ic} \delta_i dk_i$ where the equality comes from the steady state condition $u_{ik}/u_{ic} = \delta_i - r(1 - T'_K)$. This artificially creates a positive welfare effect that will tend to lower the optimal capital income tax. Intuitively, increasing wealth looks good because the steady state "forgets" that accumulating wealth requires to sacrifice past consumption.

For given linear taxes on capital and labor income, τ_K and τ_L – the revenues from which are rebated to individuals in a lump-sum fashion every period– the economy converges to a steady state. To simplify the presentation, let us assume that at time 0 the economy starts from a steady state with tax rates (τ_K, τ_L). We consider a small reform $d\tau_K$ that takes place at time 0 (and is, hence, unanticipated). We are going to derive conditions such that the small reform has zero first order effect on welfare, which effectively implies that the initial tax rate τ_K is optimal.[36]

Let $e_K(t)$ be the elasticity of aggregate capital in period t, $k^m(t)$, with respect to the net of tax rate $\bar{r}$, i.e.: $e_K(t) = (\bar{r}/k^m(t)) \cdot (dk^m(t)/d\bar{r})$. This elasticity converges to the steady state elasticity e_K. In contrast to Section 2, the convergence is not immediate because individuals smooth consumption and hence adjust their wealth slowly. Hence, under regularity assumptions, $e_K(t)$ starts at zero at $t = 0$ and then builds up with t until it converges to $e_K > 0$. We define as well the elasticity of aggregate labor income $z^m(t)$ to the net of tax return on capital as $e_{L,(1-\tau_K)}(t) = (\bar{r}/z^m(t)) \cdot (dz^m(t)/d\bar{r})$.

We define the social marginal welfare weight on person i as $g_i = \omega_i u_{ic}|_{t=0}$ and we assume without loss of generality (by normalizing the Pareto weights ω_i) that they sum to one: $\int_i g_i di = 1$. Using the envelope theorem (i.e., that behavioral responses dk_{ti} can be ignored when computing the change in individual welfare dV_i), we can consider the welfare impact of the small tax change and derive the optimal linear capital tax rate.

Proposition 8. ***Optimal linear capital tax in the Steady State.***

The optimal linear capital income tax takes the form:

$$\tau_K = \frac{1 - \bar{g}_K - \tau_L \frac{z^m}{k^m} \bar{e}_{L,1-\tau_K}}{1 - \bar{g}_K + \bar{e}_K} \quad \textit{with} \quad \bar{e}_K = \int_i g_i \delta_i \int_{t=0}^{\infty} e_K(t) \cdot e^{-\delta_i t} dt, \tag{22}$$

$$\bar{e}_{L,(1-\tau_K)} = \int_i g_i \delta_i \int_{t=0}^{\infty} e_{L,(1-\tau_K)}(t) \cdot e^{-\delta_i t} dt, \quad \textit{and} \quad \bar{g}_K = \int_i g_i \cdot k_i / k^m.$$

The formula is qualitatively exactly the same as in the simpler model in Section 2.4.3. The quantitative difference lies in the elasticity of the capital tax base $\bar{e}_K$ which replaces e_K from Section 2 and the elasticity of the labor tax base $\bar{e}_{L,(1-\tau_K)}$ which replaces $e_{L,(1-\tau_K)}$. $\bar{e}_K$ is the average of build up elasticities that converge to the long-run elasticity e_K. Hence, typically $\bar{e}_K < e_K$. In addition, the same cross-elasticity effects already discussed in Section 2.4.3 still

[36]It is also possible to start from an arbitrary tax system (τ_{K0}, τ_{L0}) and away from the steady state, and then derive the optimal unanticipated new tax system (τ_K, τ_L) implemented at time 0 that maximizes social welfare. The formulas would be similar but would require keeping track across time of all variables that converge slowly to the new steady-state, requiring more cumbersome notations.

apply here: all else equal, and at any positive labor income tax rate τ_L, if capital and labor income are complements (so that $e_{L,(1-\tau_K)}(t) > 0$), the optimal capital tax is pushed down relative to the case with no cross-elasticities.

4.1.3 Generalizing the results from the simpler model

The optimal fully nonlinear tax system with transitional dynamics is more complex and derived in Online Appendix A.4. Here, we consider the much simpler tax system with a linear labor income tax at rate τ_L and a capital income tax with constant tax rate τ_K for capital income above rk^{top}.[37]

Let $e_K^{top}(t)$ to be the average elasticity of total capital income of those individuals with capital income above threshold rk^{top}. It is measured at time t following a small reform of the top bracket tax rate $d\tau_K$ taking place at time 0. The elasticity is weighted by capital income. Let $e_{L,1-\tau_K}(t)$ be the elasticity of labor income of those individuals with capital income above threshold rk^{top}.

Proposition 9. ***Optimal top capital tax rate in the steady state.***

The optimal top capital tax rate above capital income level rk^{top} takes the form:

$$\tau_K^{top} = \frac{1 - \bar{g}_K^{top} - \tau_L \cdot \frac{z^m}{r(k^{m,top} - k^{top})} \cdot \bar{e}_{L,(1-\tau_K)}}{1 - \bar{g}_K^{top} + a_K^{top} \cdot \bar{e}_K^{top}},$$

with $\bar{e}_K^{top} \equiv \int_i g_i \delta_i \int_{t=0}^{\infty} e_K^{top}(t) \cdot e^{-\delta_i t} dt$. $\bar{g}_K^{top} = \frac{\int_{i:k_i \geq k^{top}} g_i \cdot (k_i - k^{top})}{\int_{i:k_i \geq k^{top}} (k_i - k^{top})}$ is the average capital income weighted welfare weight in the top capital tax bracket, and $a_K^{top} = \frac{k^{m,top}}{k^{m,top} - k^{top}}$ is the Pareto parameter of the capital income distribution. $\bar{e}_{L,(1-\tau_K)} \equiv \int_i g_i \delta_i \int_{t=0}^{\infty} e_{L,(1-\tau_K)}(t) \cdot e^{-\delta_i t} dt$.

We can also generalize the other results from Section 2.4. The optimal tax on total income $y_i = rk_i + z_i$ takes the same form as in Proposition 4 with the long-run elasticity e_Y replaced by the total elasticity of the income tax base, taking into account the transitional adjustments, $\bar{e}_Y = \int_i g_i \delta_i \int_0^{\infty} e_Y(t) \cdot e^{-\delta_i t} dt$. Similarly it is straightforward to generalize the results in subsections 2.4.7 and 2.4.8. Regarding the latter, with transitional dynamics, the government will be more tempted to tax more heavily assets which are slower to adjust (holding fixed the long-run elasticity e_K^j and the distributional factor $\bar{g}_K^j$).

[37] The capital income tax schedule below rk^{top} can be nonlinear. As we saw in Section 3, capital income is very strongly concentrated, so that even for the fully nonlinear optimal tax schedule, the asymptotic tax rate applies for most of the capital income tax base. Therefore, this constant top tax rate is without much loss of generality relative to the fully nonlinear capital income tax system.

4.1.4 Discussion

If the responses of capital to tax changes are very fast, then $\bar{e}_K$ is very close to the steady state elasticity e_K, as in Section 2.3.1. In this case, the quantitative implications of our simple and generalized models will be similar as well. With fast adjustments of capital, our previous results are robust.

Empirically, policy makers in general worry about capital adjustments happening very quickly following tax changes by, for instance, capital flights abroad (Johannesen, 2014).[38] Companies can modify their dividend payouts quickly to changes in dividend taxation for the sake of their shareholders (Chetty and Saez, 2005; Alstadsaeter and Fjaerli, 2009).[39]

If responses are slow on the other hand, then $\bar{e}_K < e_K$. In the short-run, the equity-efficiency trade-off for capital taxation looks more favorable if individuals are not able to adjust their capital as quickly as with linear utility. As a result, and considering formula (22), the government can tax more by taking advantage of these slow responses in the short-run.

However, exploiting such slow responses does not seem very appealing from a normative perspective. A well-designed policy cannot or should not endlessly exploit short-run adjustment costs. If nothing else, this will create a commitment problem for the government as it will always look favorable to unexpectedly increase taxes on existing capital. Using the long-term elasticity seems to be the soundest approach from a public policy perspective.

A brief comparison to labor taxation can be enlightening here as well. The Mirrlees (1971) model can be narrowly interpreted as a labor supply model with the elasticity of hours of work to taxes. It can also be interpreted more broadly as a model of earnings supply incorporating long-run responses of human capital accumulation or occupational choice. For labor too there is a short-run elasticity in which hours are adjusted, and a long-run, potentially larger, elasticity based on skill choice or occupational choice. The same formulas – which we routinely use– carry over simply substituting the short-run labor supply elasticity by the long-run elasticity of earnings e_L with $\tau_L = (1-\bar{g}_L)/(1-\bar{g}_L+e_L)$. The exact same reasoning applies to capital. There

[38] Johannesen (2014) shows that the introduction of a withholding tax for EU individuals with Swiss bank accounts led immediately, within two quarters of the reform, to a drop of 30-40% in deposits. Empirical evidence on the short-run versus long-run responses of capital to taxes is very difficult to come by. Saez et al. (2012), surveying the literature on taxable income elasticities, argue that the long-term responses, although particularly important in the case of a dynamic decision such as capital are understudied. Slemrod and Shobe (1989) is an exception, trying to estimate the short-term (transitory) and long-term (permanent) effect of tax changes on capital gains realizations. They find that the first year response has an elasticity of 2.38, while the long-run elasticity is slightly lower, at 1.75.

[39] These authors find that the introduction of the Norwegian shareholder income tax led to immediate effects on payouts, emphasizing that capital income can react very quickly and flexibly to tax changes.

is a short-run capital income elasticity (where past savings decisions are fixed) and a long-run capital income elasticity where savings have fully adjusted. The issue of government wanting to tax existing capital is similar to the issue of government wanting to tax existing human capital. In this view, a meaningful way to think about policy is to not exploit the transitional dynamics, but to assume the long-run elasticity applies.

4.2 Anticipated Reforms

In this section, we extend the analysis to anticipated reforms, that occur at time $T > 0$. With anticipated reforms, if individuals have heterogeneous discount rates, the timing of the reform (T) has non-trivial welfare consequences. We thus suppose for this section only that $\delta_i = \delta$ for all i. This will also allow an easier comparison to earlier models from the literature in section 4.3. Appendix A.2.2 provides the formal derivations. We again assume that we start form a steady state at time 0 with time invariant linear taxes (τ_K, τ_L).

We consider a change $d\tau_K$ in the tax rate τ_K that takes place at time $T \geq 0$ and is announced at time 0. Individuals start changing their consumption and wealth accumulation decisions at time 0 in anticipation of the reform. We denote again by $e_K(t) = (\bar{r}/k^m(t)) \cdot (dk^m(t)/d\bar{r})$ the elasticity of aggregate capital in period t. $e_K(t)$ converges again toward the steady state elasticity e_K as $t \to \infty$. Following Piketty and Saez (2013b), we denote by $\bar{e}_K = \delta \int_{t=0}^{\infty} e_K(t) e^{-\delta(t-T)} dt$ the total elasticity of the present discounted value of the capital tax base (as of time T as the tax change starts at time T). We can split this total elasticity into pre-reform responses with elasticity e_K^{ante} and post-reform responses with elasticity e_K^{post}:

$$\bar{e}_K = \underbrace{\delta \int_{t<T} e_K(t) e^{-\delta(t-T)} dt}_{e_K^{ante}} + \underbrace{\delta \int_{t \geq T} e_K(t) e^{-\delta(t-T)} dt}_{e_K^{post}}. \tag{23}$$

The sluggish adjustment post-reform typically implies that $e_K^{post} < e_K$. In the previous section with unanticipated reforms, we had $\bar{e}_K = e_K^{post} < e_K$. In Section 2, $\bar{e}_K = e_K^{post} = e_K$ since responses were instantaneous (whether the reform was anticipated or not).

For anticipated reforms, the optimal linear capital income tax, starting from a steady state, takes the form:

$$\tau_K = \frac{1 - \bar{g}_K - \tau_L \frac{z^m}{rk^m} \bar{e}_{L,1-\tau_K}}{1 - \bar{g}_K + \bar{e}_K}, \quad \text{with} \tag{24}$$

$$\bar{e}_K = \delta \int_{t=0}^{\infty} e_K(t) \cdot e^{-\delta(t-T)} dt, \quad \bar{e}_{L,1-\tau_K} = \delta \int_{t=0}^{\infty} e_{L,1-\tau_K}(t) \cdot e^{-\delta(t-T)} dt, \quad \text{and} \quad \bar{g}_K = \int_i g_i \cdot k_i / k^m.$$

Anticipation effects add the elasticity component e_K^{ante} to the total elasticity, so that the appropriate elasticity to use in the formula is $\bar{e}_K = e_K^{ante} + e_K^{post}$.[40]

In Online Appendix A.3, we show that in our model with wealth in the utility, the anticipation elasticity will be infinite for $T \to \infty$ with full certainty. While this would lead to a zero optimal capital tax rate, it does not occur except in a particularly unrealistic policy setting, namely if the reform is announced infinitely in advance with perfect certainty. It also breaks down with uncertainty: the anticipation elasticity is then finite.[41]

4.3 Comparison with Earlier Models

We next compare the optimal capital tax rates in our model to those in three benchmark models of capital taxation: the Aiyagari model (Aiyagari, 1995), the Chamley-Judd model (Chamley, 1986; Judd, 1985), and the Judd endogenous discount rate model (Judd, 1985). While these papers mostly focus on anticipated reforms, we can consider both anticipated and unanticipated reforms for each model, which lead to quantitatively different optimal tax rates. The goal of this section is to show the robustness of our formula. In the end, what matters for optimal tax policy are the elasticities properly defined. Conditional on the elasticities, the primitives of the model are largely irrelevant. Table 1 summarizes the elasticities and optimal tax rates for these different models and for different reforms.

4.3.1 Comparison to the Aiyagari Model with Uncertainty

We first consider the Aiyagari (1995) model with uncertainty, in discrete time. Individual per-period utility is $u_{ti} = u_{ti}(c_{ti})$. Earnings z_{ti} are stochastic and exogenous for simplicity, and we assume no labor income tax $\tau_L = 0$. Again, the discount rate δ is homogeneous across individuals.

Assume a standard structure for the stochastic process of earnings z_{ti} and preferences u_{ti} so that, under a time invariant tax rate τ_K, the economy converges to an ergodic steady state with

[40] Labor income also exhibits pre-reform anticipation cross-elasticities.

[41] It is important to distinguish which results arise from the primitives of each model versus from the reforms considered. Both in our wealth-in-the-utility model and the Chamley-Judd model, anticipated reforms at $T \to \infty$ generate infinite anticipation elasticities. As argued, such reforms rarely occur in practice. However, the Chamley-Judd model also generates infinite steady state elasticities, whereas our model features a non-degenerate steady state with smooth responses of capital to taxes and wealth heterogeneity.

a time invariant distribution for $(u_{ti}, k_{ti}, c_{ti})_{i \in I}$ independent of the distribution of initial wealth. All derivations are in Appendix A.2.3.

The Aiyagari paper considers an anticipated tax reform at time T. If T is sufficiently large, so that anticipation responses only start once the economy is in its ergodic steady state, then Piketty and Saez (2013b) show that the optimal linear capital tax rate takes the form:[42]

$$\tau_K = \frac{1 - \bar{g}_K}{1 - \bar{g}_K + \bar{e}_K} \quad \text{with} \quad \bar{e}_K = e_K^{ante} + e_K^{post}, \tag{25}$$

where, e_K^{ante} and e_K^{post} are the equivalents of the elasticities in the previous section in discrete time: $e_K^{ante} = \frac{\delta}{1+\delta} \sum_{t<T} \left(\frac{1}{1+\delta}\right)^{t-T} e_{Kt}$, and $e^{post} = \frac{\delta}{1+\delta} \sum_{t \geq T} \left(\frac{1}{1+\delta}\right)^{t-T} e_{Kt}$. Hence our previous formula in (24) exactly applies (setting the labor cross-elasticity $e_{L,1-\tau_K}$ to zero with inelastic labor). The quantitative implications may, however, be different.

First, we show that the steady state elasticity e_K is finite, exactly like in our model with wealth in the utility since the uncertainty effectively smoothes the response of capital to taxes. Second, the anticipation elasticity is also finite for any T, so that the Aiyagari model has a non-zero optimal capital tax rate even in the long-run steady state (i.e., for anticipated reforms with $T \to \infty$. This would also be true in our wealth-in-the-utility model if we added uncertainty. Third, whether the tax rate given by (24) is higher or lower in the wealth-in-the-utility model relative to the Aiyagari (1995) model is ambiguous and depends on whether uncertainty generates larger and/or faster responses of capital to tax rates than does wealth in the utility.

4.3.2 The Chamley-Judd model

In the Chamley-Judd model (Chamley, 1986; Judd, 1985), individuals have a standard utility $u(c_{it}, z_{it})$ and there is no uncertainty. Formula (24) also applies in the Chamley-Judd model, but the elasticities implied by that model are quantitatively different.

First, the steady state is degenerate unless $\delta = \bar{r}$, which means that in the steady state, any change in the capital income tax rate leads to an infinite response. Hence, $e_K = \infty$ and the optimal capital tax in the steady state is zero. By contrast, in our model the steady state elasticity is always finite and the steady state non-degenerate. Second, as shown in Piketty and Saez (2013b), the anticipation elasticity e_K^{ante} is also infinite when $T \to \infty$, leading to a zero optimal tax rate.

[42] We relax this assumption in Appendix A.2.3.

4.3.3 The Judd endogenous discount factor model

In Judd (1985), the discount rate $\delta_i = \delta_i(c_i)$ depends smoothly on consumption. Utility is:

$$V_i(\{c_i(t), z_i(t)\}_{t\geq 0}) = \int_0^{\infty} u_i(c_i(t), z_i(t)) e^{-\int_0^t \delta_i(c_i(s))ds} dt.$$

In Appendix A.2.4, we derive the optimal linear tax formula starting from a steady state and considering an unanticipated reform, which is the same as in (22), except that $\bar{g}_K$ is redefined to take into account that the welfare impact of taxes now also goes through the discount factor $\delta_i(c_i)$ which depends on consumption:

$$g_i = \frac{\omega_i \frac{1}{\delta_i(c_i)}\left(u_{ic} + \frac{\delta_i'(c_i)}{\delta_i(c_i)} u_i\right)}{\int_i \omega_i \frac{1}{\delta_i(c_i)}\left(u_{ic} + \frac{\delta_i'(c_i)}{\delta_i(c_i)} u_i\right)} \quad \text{and} \quad \bar{e}_K = \int_i g_i \delta_i(c_i) \int_{t=0}^{\infty} e^{-\delta_i(c_i)t} e_K(t) dt.$$

Again, the faster capital adjusts, the closer $\bar{e}_K$ is to the long-run elasticity e_K. As with wealth-in-the-utility, the steady state of this model is non-degenerate, with $\delta_i(c_i(t)) = \bar{r}$ and generates a finite long-term elasticity e_K. In addition, as shown in Piketty and Saez (2013b), the anticipation elasticity e_K^{ante} is infinite, and hence the long-run optimal capital tax is zero.

5 Conclusion

In this paper we propose a tractable new model for capital taxation, which creates a link to the policy debate and empirical analysis. We first presented a simple model with linear utility for consumption and a concave wealth-in-the-utility component which generates immediate adjustments of capital in response to taxes, a non-degenerate, smooth response of capital to taxes, and allows for arbitrary heterogeneity in preferences for capital, work, and discount rates.

We derive formulas for optimal linear and nonlinear capital income taxation which are expressed in terms of the elasticity of capital with respect to the net-of-tax rate of return, the shape of the capital income distribution, and the social welfare weights at each capital income level. We put the simplicity of this model to use by considering a range of policy issues such as the cases with joint-preferences and cross-elasticities between capital and labor, economic growth, heterogeneous returns to capital across individuals, different types of capital assets and heterogeneous tastes for each of them, or optimal taxes on comprehensive income.

We show how our results are robust in a model with a general, concave utility function as long as the elasticities of the capital tax base are appropriately adjusted to take into account transitional dynamics and potentially slow adjustments. The qualitative lessons from the simpler model hence carry over. The faster the adjustment of capital to taxes, and the closer the quantitative results are as well.

We make use of our sufficient statistics formulas to numerically simulate optimal taxes based on U.S. tax data. Given how concentrated the distribution of capital is, the asymptotic tax rate for capital applies for the majority of capital income in the economy and should be higher than the top tax rate on labor income if the supply elasticities of labor and capital with respect to tax rates are the same. The theoretical framework we provide points to the key elasticities that need to be estimated in future work. These include the cross-elasticities between capital and labor and the elasticities and cross-elasticities of different types of capital assets, which it may be optimal to tax differently.

Our approach is very amenable to incorporating broader justice and fairness principles for capital taxation in an operational way. We discuss a range of ethical considerations regarding capital taxation that are salient in the policy debate. As long as, conditional on labor income, social marginal welfare weights depend directly on wealth (which is the case if wealth is perceived as unfairly distributed for many possible reasons) or are correlated with wealth (as in the case of the use of wealth as a tag), there is scope for capital taxation from an equity perspective. In future work, it would be very valuable to better understand society's equity considerations when it comes to capital taxation.

References

Aiyagari, R. (1994). Uninsured idiosyncratic risk and aggregate savings. *Quarterly Journal of Economics 109*, 659–684.

Aiyagari, R. (1995). Optimal capital income taxation with incomplete markets, borrowing constraints, and constant discounting. *Journal of Political Economy 103*(6), 1158–1175.

Albanesi, S. and C. Sleet (2006). Dynamic Optimal Taxation with Private Information. *Review of Economic Studies 73*(1), 1–30.

Alstadsaeter, A. and E. Fjaerli (2009). Neutral taxation of shareholder income? corporate responses to an announced dividend tax. *International Tax and Public Finance 16*(4), 571–604.

Auerbach, A. (2009). The choice between income and consumption taxes: A primer. In A. Auerbach and D. Shaviro (Eds.), *Institutional Foundations of Public Finance: Economic and Legal Perspectives*. Cambridge: Harvard University Press.

Auerbach, A., J. R. Hines, and J. Slemrod (2007). *Taxing Corporate Income in the 21st Century*. Cambridge University Press.

Battaglini, M. and S. Coate (2008a). A dynamic theory of public spending, taxation, and debt. *The American Economic Review 98*(1), 201–236.

Battaglini, M. and S. Coate (2008b). Pareto efficient income taxation with stochastic abilities. *Journal of Public Economics 92*(3), 844–868.

Bénabou, R. and J. Tirole (2003). Intrinsic and extrinsic motivation. *Review of Economic Studies 70*(3), 489–520.

Bénabou, R. and J. Tirole (2006). Incentives and prosocial behavior. *American Economic Review 96*(5), 1652–1678.

Bénabou, R. and J. Tirole (2011). Laws and Norms. *NBER Working Paper* (17579).

Bénabou, R. and J. Tirole (2016). Mindful economics: The production, consumption, and value of beliefs. *Journal of Economic Perspectives 30*(3), 141–164.

Benhabib, J. and A. Bisin (2016). Skewed wealth distributions: Theory and empirics. *NYU Working Paper*.

Besley, T. and S. Coate (1992). Workfare versus welfare: Incentive arguments for work requirements in poverty-alleviation programs. *The American Economic Review 82*(1), 249–261.

Besley, T. and S. Coate (1995). The design of income maintenance programmes. *The Review of Economic Studies 62*(2), 187–221.

Blanchard, O. (1985). Debt, Deficits, and Infinite Horizons. *Journal of Political Economy 93*, 223–247.

Cagetti, M. and M. De Nardi (2007). Entrepreneurship, Frictions, and Wealth. *Journal of Political Economy 114*, 835–870.

Carroll, C. D. (1997). Buffer-stock saving and the life cycle/permanent income hypothesis. *Quarterly Journal of Economics 112*, 1–56.

Carroll, C. D. (2000). Why do the rich save so much? In J. B. Slemrod (Ed.), *Does Atlas Shrug? The Economic Consequences of Taxing the Rich.* Harvard University Press.

Chamley, C. (1986). Optimal taxation of capital income in general equilibrium with infinite lives. *Econometrica 80*(3), 607–622.

Chetty, R. and E. Saez (2005). Dividend taxes and corporate behavior: Evidence from the 2003 dividend tax cut. *Quarterly Journal of Economics 120*(3), 791–833.

Chetty, R. and E. Saez (2010). Dividend and Corporate Taxation in an Agency Model of the Firm. *American Economic Journal: Economic Policy 2*(3), 1–31.

Christophera, A. N. and B. R. Schlenker (2000). The impact of perceived material wealth and perceiver personality on first impressions. *Journal of Economic Psychology 21*(1), 1–19.

Coate, S. (1995). Altruism, the samaritan's dilemma, and government transfer policy. *The American Economic Review 85*(1), 46–57.

Cullen, J. and R. Gordon (2006). Tax Reform and Entrepreneurial Activity. *Tax Policy and the Economy 20*, 41–72.

Cullen, J. and R. Gordon (2007). Taxes and Entrepreneurial Risk Taking: Theory and Evidence for the U.S. *Journal of Public Economics 91*(7-8), 1479–1505.

De Nardi, M. (2004). Wealth Inequality and Intergenerational Links. *Review of Economic Studies 71*, 743–768.

Diamond, P. and E. Saez (2011). The case for a progressive tax: from basic research to policy recommendations. *The Journal of Economic Perspectives 25*(4), 165–190.

Diamond, P. A. and J. A. Mirrlees (1971). Optimal taxation and public production i: Production efficiency. *The American Economic Review 61*(1), 8–27.

Dittmar, H. (1992). Perceived material wealth and first impressions. *British Journal of Social Psychology 31*(4), 379–391.

Farhi, E. (2010). Capital Taxation and Ownership when Markets are Incomplete. *Journal of Political Economy 118*(5), 908–948.

Farhi, E., C. Sleet, I. Werning, and S. Yeltekin (2012). Nonlinear capital taxation without commitment. *Review of Economic Studies 79*(4), 1469–1493.

Farhi, E. and I. Werning (2013). Estate taxation with altruism heterogeneity. *American Economic Review, Papers and Proceedings 103*(3), 489–495.

Findeisen, S. and D. Sachs (2017). Redistribution and Insurance with Simple Tax Instruments. *Journal of Public Economics 146*, 58–78.

Fuest, C., A. Peichl, and D. Waldenström (2015). Piketty's r-g model: wealth inequality and tax policy. *CESIfo Forum, No. 1, 1-10*.

Gale, W. G., J. R. Hines, and J. Slemrod (2001). *Rethinking Estate and Gift Taxation*. Brookings Institution Press.

Golosov, M. and A. Tsyvinski (2006). Designing optimal disability insurance: A case for asset testing. *Journal of Political Economy 114*(2), 257–279.

Golosov, M., A. Tsyvinski, and N. Werquin (2014). A variational approach to the analysis of tax systems. Working Paper 20780, NBER.

Golosov, M., A. Tsyvinski, and N. Werquin (2016). Recursive contracts and endogenously incomplete markets. In M. Woodford (Ed.), *Handbook of Macroeconomics, Vol.2.*, pp. 725–841. North-Holland.

Gordon, R. and M. Dietz (2010). Dividends and taxes. Working Paper 12292, National Bureau of Economic Research.

Gordon, R., L. Kalambokidis, and J. Slemrod (2004). Do we now collect any revenue from taxing capital income? *Journal of Public Economics 88*(5), 981 – 1009.

Gordon, R. and Y. Lee (2005). Tax Structure and Economic Growth. *Journal of Public Economics 89*(5-6), 1027–1043.

Gordon, R. H. (1992). Can capital income taxes survive in open economies? *The Journal of Finance 47*(3), 1159–1180.

Gordon, R. H. and J. Slemrod (2000). Are 'real' responses to taxes simply income shifting between corporate and personal tax bases? In J. Slemrod (Ed.), *Does Atlas Shrug? The Economic Consequences of Taxing the Rich*, pp. 240–280. Russell Sage Foundation and Harvard University Press.

Hamilton, B. H. (2000). Does entrepreneurship pay? an empirical analysis of the returns to self?employment. *Journal of Political Economy 108*(3), 604–631.

Hurst, E. and B. W. Pugsley (2010). Non pecuniary benefits of small business ownership. *Chicago Booth Working Paper*.

Johannesen, N. (2014). Tax evasion and swiss bank deposits. *Journal of Public Economics 111*, 46–62.

Jones, L. E., R. E. Manuelli, and P. E. Rossi (1993). Optimal taxation in models of endogenous growth. *Journal of Political economy 101*(3), 485–517.

Judd, K. L. (1985). Redistributive taxation in a simple perfect foresight model. *Journal of public Economics 28*(1), 59–83.

Kaplow, L. (1994). Taxation and risk taking: A general equilibrium perspective. *National Tax Association 47*(4), 789–798.

Kaplow, L. (1995). The Income Tax versus the Consumption Tax and the Tax Treatment of Human Capital. *Tax Law Review 51*, 35–46.

Kaplow, L. (2001). A framework for assessing estate and gift taxation. In Gale, Hines, and Slemrod (Eds.), *Rethinking Estate and Gift Taxation*, pp. 164–215. Brookings Institution.

Kaplow, L. (2008). Capital Levies and Transition to a Consumption Tax. In Auerbach and Shapiro (Eds.), *Institutional Foundations of Public Finance: Economic and Legal Perspectives*. Harvard University Press.

Kaplow, L. (2011). Targeted Savings and Labor Supply. *International Tax and Public Finance 18*, 507–518.

Kaplow, L. (2015a). Government policy and labor supply with myopic or targeted savings decisions. In J. Brown (Ed.), *Tax Policy and the Economy*. NBER and University of Chicago Press.

Kaplow, L. (2015b). Myopia and the Effects of Social Security and Capital Taxation on Labor Supply. *National Tax Journal 68*, 7–32.

Kaplow, L. and S. Shavell (2003). Fairness versus Welfare: Notes on the Pareto Principle, Preferences, and Distributive Justice. *Journal of Legal Studies 32*, 331–362.

Kaplow, L. and S. Shavell (2004). Any Non-welfarist Method of Policy Assessment Violates the Pareto Principle: Reply. *Journal of Political Economy 112*, 249–251.

Kaplow, L. and S. Shavell (2007). Moral Rules, the Moral Sentiments, and Behavior: Toward a Theory of an Optimal Moral System. *Journal of Political Economy 115*, 494–514.

Keynes, J. M. (1919). *The Economic Consequences of the Peace*. London: Macmillan.

Keynes, J. M. (1931). *Essays in Persuasion*. London: Macmillan.

Kiyotaki, N., A. Michaelides, and K. Nikolov (2011). Winners and losers in housing markets. *Journal of Money, Credit and Banking 43*(2-3), 255–296.

Lockwood, B. and M. Weinzierl (2015). De gustibus non est taxandum: Theory and evidence on preference heterogeneity and redistribution. *Journal of Public Economics 124*, 74–80.

Lockwood, B. and M. Weinzierl (2016). Positive and normative judgments implicit in us tax policy, and the costs of unequal growth and recessions. *Journal of Monetary Economics 77*, 30–47.

Mirrlees, J. (1971). An exploration in the theory of optimum income taxation. *The review of economic studies 38*, 175–208.

Piazzesi, M., M. Schneider, and S. Tuzel (2007). Housing, consumption and asset pricing. *?Journal of Financial Economics 83*, 531–569.

Piketty, T. (2011). On the long-run evolution of inheritance: France 1820-2050. *Quarterly journal of economics 61*(3), 1071–1131.

Piketty, T. (2014). *Capital in the Twenty-first Century*. Harvard University Press.

Piketty, T. and E. Saez (2003). Income inequality in the united states, 1913–1998. *The Quarterly Journal of Economics 118*(1), 1–41.

Piketty, T. and E. Saez (2012). A theory of optimal capital taxation. Working Paper 17989, NBER.

Piketty, T. and E. Saez (2013a). Optimal labor income taxation. In M. F. Alan J. Auerbach, Raj Chetty and E. Saez (Eds.), *Handbook of Public Economics, Vol. 5*, pp. 391–474. Amsterdam: North-Holland.

Piketty, T. and E. Saez (2013b). A theory of optimal inheritance taxation. *Econometrica 82*(4), 1241–1272.

Piketty, T., E. Saez, and G. Zucman (2016). Distributional national accounts: Methods and estimates for the united states. NBER Working Paper 22945, NBER.

Poterba, J. and J. Rotemberg (1987). Money in the utility function: An empirical implementation. In W. Barnett and K. Singleton (Eds.), *New Approaches to Monetary Economics.*, pp. 219–240. Cambridge University Press.

Quadrini, V. (1999). The Importance of Entrepreneurship for Wealth Concentration and Mobility. *Review of Income and Wealth 45*(1), 1–19.

Quadrini, V. (2000). Entrepreneurship, Savings and Social Mobility. *Review of Economic Dynamics 3*, 1–40.

Sachs, D., A. Tsyvinski, and N. Werquin (2016). Nonlinear Tax Incidence and Optimal Taxation in General Equilibrium. *Working Paper, Toulouse School of Economics*.

Saez, E. (2001). Using elasticities to derive optimal income tax rates. *Review of Economic Studies 68*(1), 205–229.

Saez, E., J. Slemrod, and S. Giertz (2012). The elasticity of taxable income with respect to marginal tax rates: A critical review. *Journal of Economic Literature 50*(1), 3–50.

Saez, E. and S. Stantcheva (2016). Generalized social marginal welfare weights for optimal tax theory. *American Economic Review 106*(1), 24–45.

Saez, E. and G. Zucman (2016). Wealth inequality in the united states since 1913: Evidence from capitalized income tax data. *Quarterly Journal of Economics 131*(2), 519–578.

Slemrod, J. (2007). Does the united states tax capital income? In H. J. Aaron, L. E. Burman, and C. E. Steuerle (Eds.), *Taxing Capital Income*, pp. 3–38. The Urban Institute Press.

Slemrod, J. and W. Shobe (1989). The tax elasticity of capital gains realizations: Evidence from a panel of taxpayers. *National Tax Journal 42*(4), 503–508.

Smith, A. (1759). *The Theory of Moral Sentiments.*

Stokey, N. (2009). Moving costs, nondurable consumption and portfolio choice. *?Journal of Economic Theory 144*, 2419?2439.

Straub, L. and I. Werning (2015). Positive Long Run Capital Taxation: Chamley-Judd Revisited. Working paper, mit.

Warren, A. C. (1980). Would a consumption tax be fairer than an income tax? *The Yale Law Journal Volume 89*(Number 6).

Weber, M. (1958). *The Protestant Ethic and the Spirit of Capitalism.* Charles Scribner and Sons, New York.

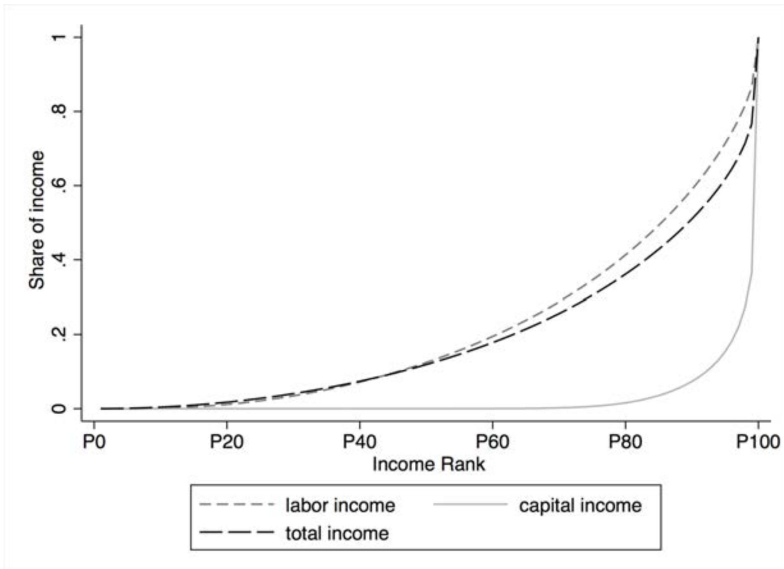
Weisbach, D. A. and J. Bankman (2006). The Superiority of an Ideal Consumption Tax over an Ideal Income Tax. *58 Stanford Law Review 1413*.

Werquin, N. (2016). Income taxation with frictional labor supply. *Toulouse School of Economics, Working Paper*.

Yaari, M. (1965). Uncertain lifetime, life insurance, and the theory of the consumer. *Review of Economic Studies 32*, 137–150.

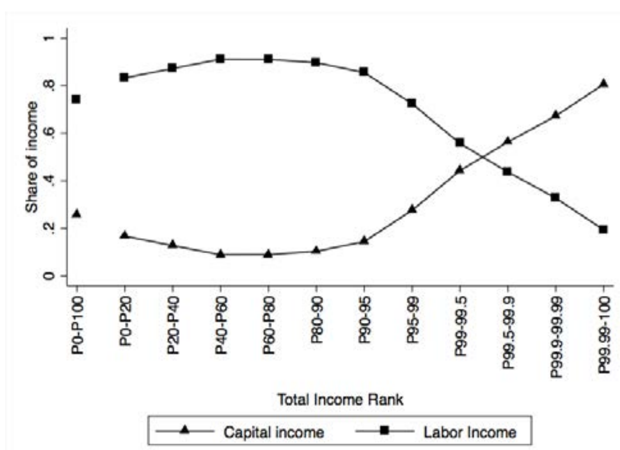
Yang, F. and M. De Nardi (2016). Wealth inequality, family background, and estate taxation. *Journal of Monetary Economics 77*, 130–150.

Figure 1: Lorenz Curves for capital, labor, and total income



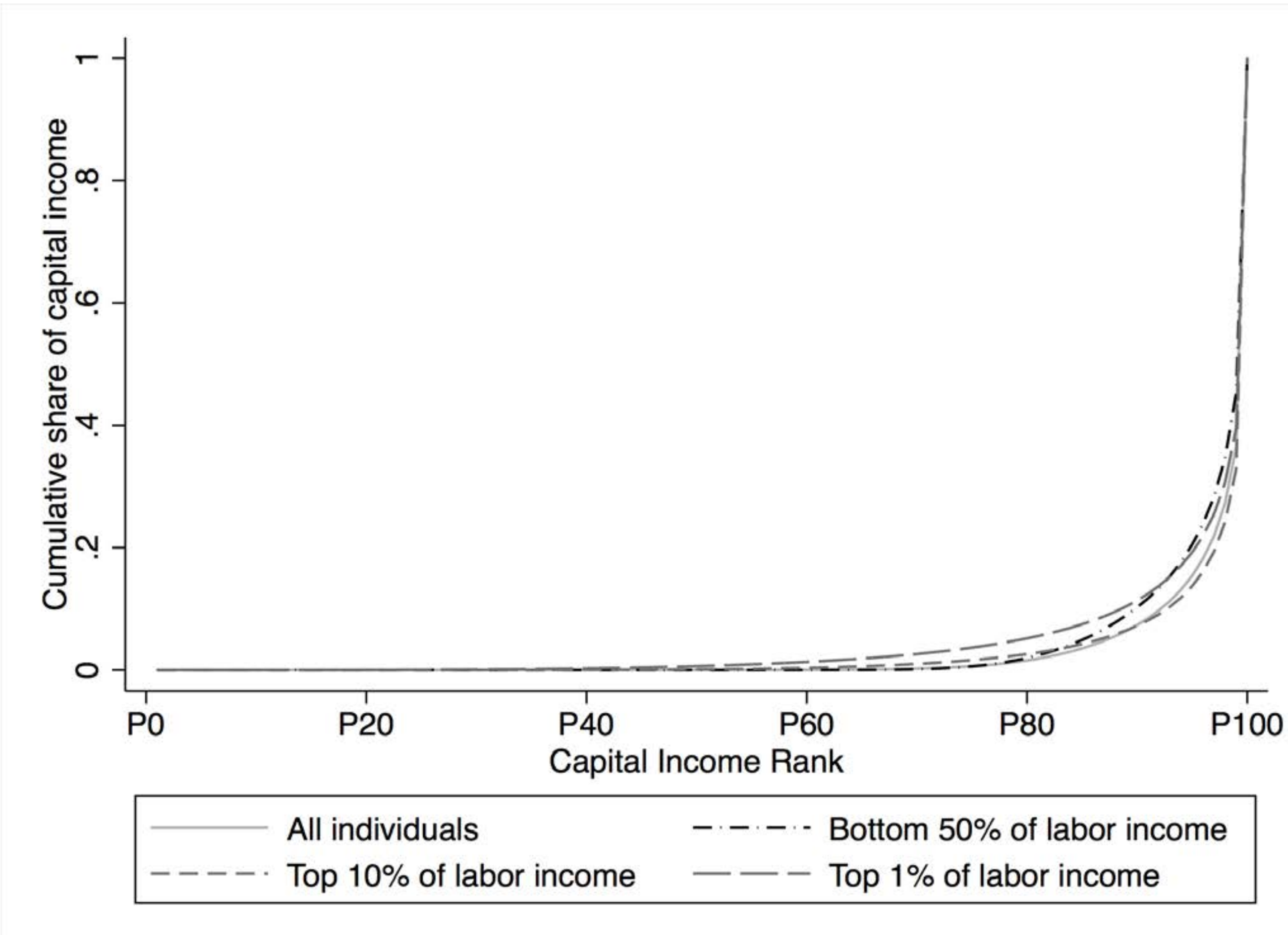
Notes: Computations based on IRS tax return data for year 2007. The figure represents the Lorenz curves for labor income, capital income, and total income (capital + labor income). The Lorenz curve is the cumulative share of income owned by those below each income percentile (x-axis). The distributions of both labor and capital income (and, thus, of total income) exhibit great inequalities, but capital income is much more concentrated than labor income.

Figure 2: Capital and labor incomes as a share of total income



Notes: Computations based on IRS tax return data for year 2007. The figure shows the composition of total income within several groups, ranked by total income, and marked on the horizontal axis. The first observation represents the overall population P0-P100. P0-P20 denotes the bottom 20% tax units, etc. At the top of the income distribution, most of total income comes from capital income.

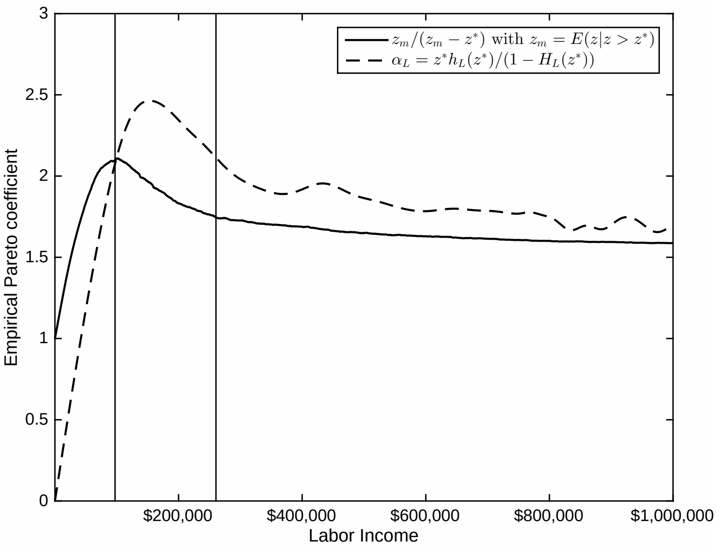
Figure 3: Two-dimensional heterogeneity:
Lorenz curves for capital, conditional in labor income



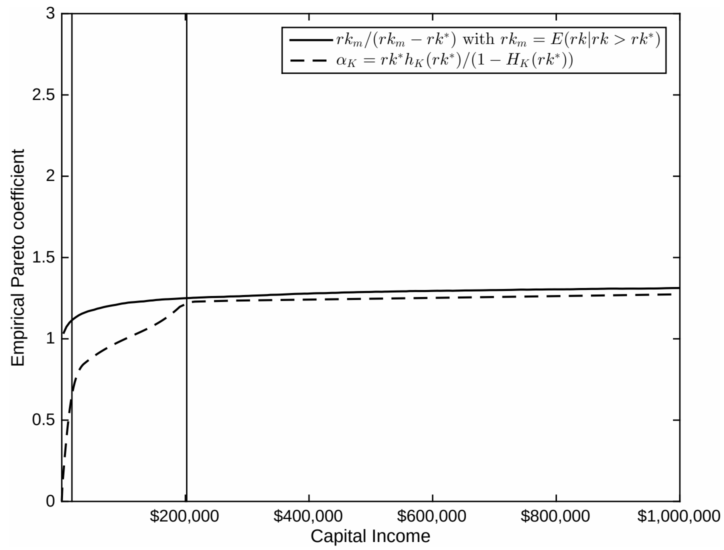
Notes: Computations based on IRS tax return data for year 2007. The figure depicts the Lorenz curves for capital income (the Lorenz curve is the cumulative share of capital income owned by those below each percentile of the capital income distribution), for four groups defined by labor income: all individuals, the bottom 50% by labor income, the top 10% by labor income and the top 1% by labor income. Even conditional on labor income, there are large inequalities in capital income. Put differently, there is a lot of two-dimensional heterogeneity in both labor and capital income.

Figure 4: Empirical Pareto parameters

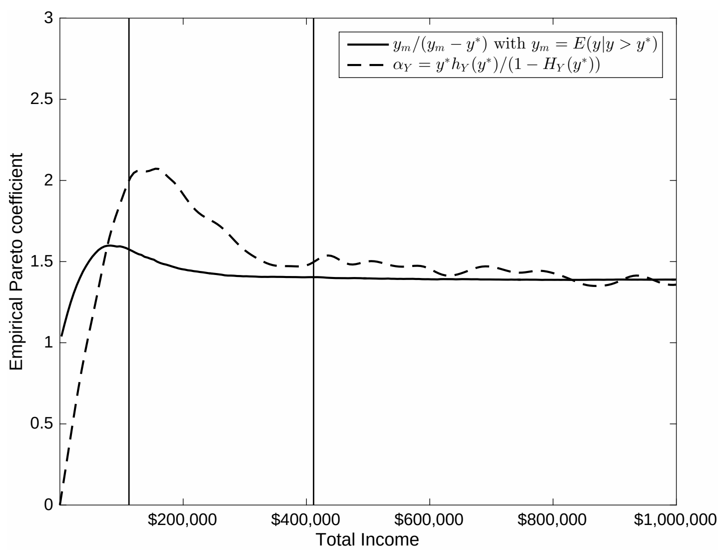
(a) Labor Income



(b) Capital Income



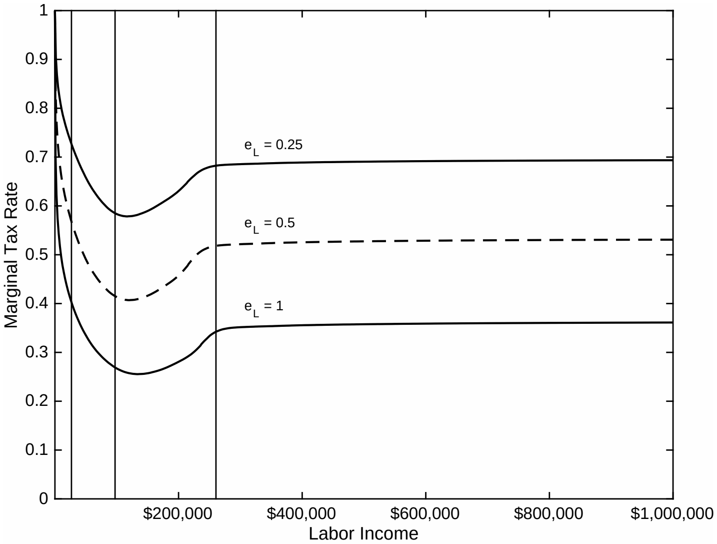
(c) Total Income



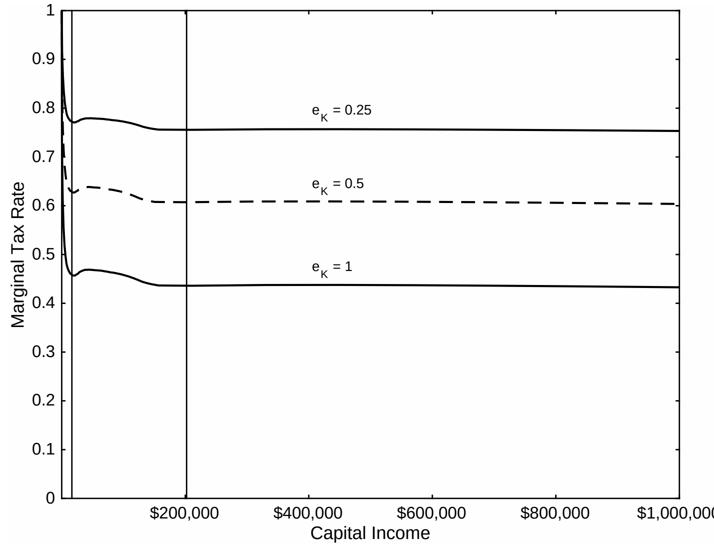
Notes: Computations based on IRS tax return data for year 2007. The figure depicts the empirical Pareto parameters for the labor income distribution (panel (a)), the capital income distribution (panel (b)) and the total income distribution (panel (c)). For labor income, we compute the top bracket Pareto parameter $z_m/(z_m - z^*)$ relevant for the optimal linear tax rate above z^* and the local Pareto parameter $\alpha(z^*) = z^* h_L(z^*)/(1 - H_L(z^*))$ where $h_L(z)$ is the density and $H_L(z)$ the cumulated distribution, which is relevant for the optimal nonlinear $T_L'(z^*)$. The x-axis depicts z^*. The vertical lines depict the 90th and 99th percentiles of each distribution. We repeat the same for capital income rk and total income $y = rk + z$. At the top, all three distributions are very well approximated by Pareto distributions with constant tail parameters of around $a_L = 1.6$ for labor, $a_K = 1.38$ for capital, and $a_Y = 1.4$ for total income.

Figure 5: Optimal Marginal Tax Rates

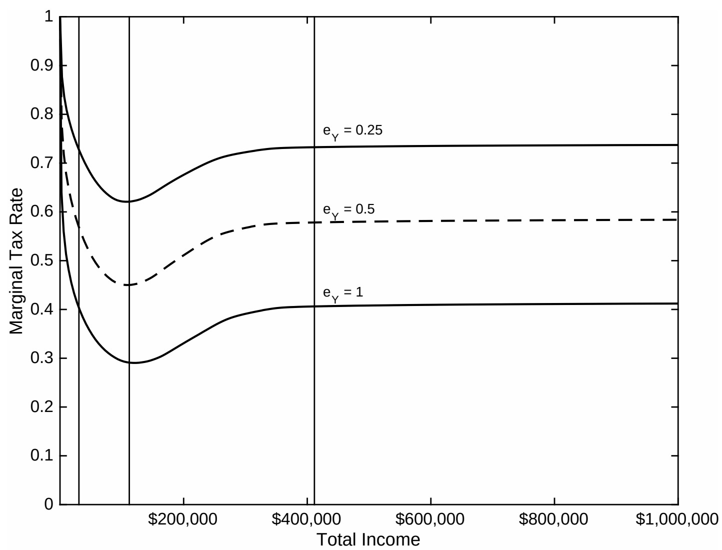
(a) Optimal labor income tax rate $T'_L(z)$



(b) Optimal capital income tax rate $T'_K(rk)$



(c) Optimal comprehensive income tax rate $T'_Y(rk + z)$



Notes: Computations based on IRS tax return data for year 2007. Optimal marginal tax rates based on the formulas in Section 2.3.2. Panel (a) plots the optimal marginal tax rate on labor income. Panel (b) plots the optimal marginal tax rate on capital income. Panel (c) plots the optimal marginal tax rate on total income. In each panel, optimal marginal tax rates are plotted for three different elasticity values: 0.25, .5, and 1. In each panel, the three vertical lines represent, respectively, the median, the top 10% and the top 1% thresholds of the 2007 the labor, capital, and total income distributions in the U.S. (the median capital income is zero).

Table 1: Comparison of Elasticities and Taxes in Capital Taxation Models

Utility (1)	Transitional Dynamics? (2)	Uncertainty or Certainty? (3)	Reform anticipated or unanticipated? (4)	Model (5)	e_K^{ante} (6)	e_K^{post} (7)	e_K (8)	$\bar{e}_K$ (9)	Optimal τ_K (10)
Wealth in the Utility	No	Certainty	Either	Section 2	0	$= e_K$	$< \infty$	$= e_K$	> 0
	Yes	Certainty	Anticipated	Section 4	∞	$< e_K$	$< \infty$	∞	0
			Unanticipated	Section 4	0	$< e_K$	$< \infty$	$< \infty$	> 0
Standard	Yes	Uncertainty	Anticipated	Aiyagari (1995)	$< \infty$	$< e_K$	$< \infty$	$\lesseqgtr e_K, < \infty$	> 0
			Unanticipated	Section 4	0	$< e_K$	$< \infty$	$< e_K$	> 0
	Yes	Certainty	Anticipated	Chamley-Judd	∞	$< e_K$	∞	∞	0
			Unanticipated	Section 4	0	$< e_K$	∞	$< \infty$	> 0
Endogenous $\delta(c_i)$	Yes	Certainty	Anticipated	Judd (1985)	∞	$< e_K$	$< \infty$	∞	0
			Unanticipated	Section 4	0	$< e_K$	$< \infty$	$< \infty$	> 0

Notes: This table presents a comparison of supply elasticities of capital with respect to the net-of-tax rate of return and optimal capital income tax rates across various models. Column (1) indicates the type of utility function. Column (2) indicates whether there are transitional dynamics (which is equivalent to whether the utility is linear vs. concave in consumption). Column (3) indicates whether there is uncertainty in future labor incomes and preferences. Column (4) indicates whether the tax reform determining the optimal tax rate is anticipated (in the long-distance future) or unanticipated (at time zero). Column (5) indicates the Section in the paper covering the model or whether an existing paper in the literature covers it. Columns (6)-(9) describes the magnitude of the four elasticities: e_K^{ante} the anticipation response elasticity, e_K^{post} the post-reform elasticity, e_K the long-run steady state elasticity. Recall that $\bar{e}_K = e_K^{ante} + e_K^{post}$ and typically $e_K^{post} < e_K$. Column (10) describes the sign and magnitude of the optimal linear tax rate τ_K on capital income. It is assumed that $\bar{g}_K < 1$ so that taxing capital income is desirable (absent any behavioral response). Adding wealth in the utility to the Aiyagari model does not change the predictions.

Online Appendix – Not For Publication

A.1 Proofs for Section 2

Proof of Proposition 2.

We derive the optimal capital tax. The optimal labor tax is derived exactly in the same way.

Consider a small reform $\delta T_K(rk)$ in which the marginal tax rate is increased by $\delta\tau_K$ in a small band from capital income rk to $rk+d(rk)$, but left unchanged anywhere else. This reform has a mechanical revenue effect, a behavioral effect, and a welfare effect.

The mechanical revenue effect above capital income rk is

$$d(rk)\delta\tau_K[1-H_K(rk)].$$

The behavioral effect comes only from taxpayers with capital income in the range $[rk, rk+d(rk)]$. Thanks to the linear utility (i.e., no income effects), taxpayers above rk do not respond to the tax rates since they do not face a change in their marginal tax rate. Taxpayers in the small band have a behavioral response to the higher marginal tax rate. They each reduce their capital income by $\delta(rk) = -e_K\delta\tau_K/(1-T_K'(rk))$ where e_K is the elasticity of capital income rk with respect to the net-of-tax return $r(1-T_K'(rk))$. As there are $h_K(rk)d(rk)$ taxpayers affected by the change in marginal tax rates, the resulting loss in tax revenue is equal to:

$$-d(rk)\delta\tau_K \cdot h_K(rk)e_K(rk)rk\frac{T_K'(rk)}{(1-T_K'(rk))},$$

with $e_K(rk)$, as defined in the text, the average elasticity of capital income in the small band.

The change in tax revenue is rebated lump-sum to all taxpayers. The value of this lump-sum transfer to society is $\int_i g_i = 1$ due to the absence of income effects (the lumpsum rebate also does not change any behavior with linear utility).

By definition of the average social marginal welfare weight above rk, $\bar{G}_K(rk)$, in (12), the welfare effect on the tax payers above rk who pay more tax $\delta\tau_K \cdot d(rk)$ is:

$$-\delta\tau_K \cdot d(rk)\int_{i:rk_i\geq rk} g_i = -\delta\tau_K \cdot d(rk)(1-H_K(rk))\bar{G}_K(rk).$$

At the optimum, the sum of the mechanical revenue effect, the behavioral effect, and the welfare effect needs to be zero, which requires that:

$$d(rk)\delta\tau_K \cdot \left[1 - H_K(rk) - h_K(rk) \cdot e_K(rk) \cdot rk \cdot \frac{T'_K(rk)}{1 - T'_K(rk)}\right] \\ - d(rk)\delta\tau_K \cdot (1 - H_K(rk)) \cdot \bar{G}_K(rk) = 0.$$

We can divide everything by $d(rk)\delta\tau_K$ and re-arrange to obtain:

$$\frac{T'_K(rk)}{1 - T'_K(rk)} = \frac{1}{e_K(rk)} \cdot \frac{1 - H_K(rk)}{rk \cdot h_K(rk)} \cdot (1 - \bar{G}_K(rk)).$$

Using the definition of the local Pareto parameter $\alpha_K(rk) = rkh_K(rk)/(1 - H_K(rk))$, we obtain the capital tax formula in the proposition. The optimal marginal labor tax formula is derived in the same way, replacing capital income rk with labor income z.

Proof of Proposition 3.

Let G be government revenue. The change in revenue from a change in the capital income tax $d\tau_K$ is:

$$dG = rk^m \left[1 - \frac{\tau_K}{1 - \tau_K} \cdot e_K - \frac{\tau_L}{1 - \tau_K} e_{L,(1-\tau_K)} \frac{z^m}{rk^m}\right] \cdot d\tau_K.$$

Hence the change in social welfare is:

$$\frac{dSWF}{d\tau_K} = \int_i g_i \left(-rk_i + \frac{dG}{d\tau_K}\right) = \left(\int_i g_i\right) \cdot \left(-\frac{\int_i g_i rk_i}{\int_i g_i} + \frac{dG}{d\tau_K}\right).$$

Setting this to zero and using the definition of $\bar{g}_K = \frac{\int_i g_i k_i}{\int_i g_i k^m}$, yields:

$$\tau_K = \frac{1 - \bar{g}_K - \tau_L e_{L,(1-\tau_K)} \frac{z^m}{rk^m}}{1 - \bar{g}_K + e_K},$$

which is the optimal capital tax formula with joint preferences and cross-elasticities. The optimal labor tax formula with cross elasticities can be derived exactly symmetrically.

Proof of Proposition 4.

The derivation of the optimal tax on comprehensive income follows exactly the proof of Proposition 2 above, replacing capital income rk with total income y.

Proof of Proposition 5.

The government maximizes:

$$
\begin{aligned}
SWF &= \int_i \omega_i U_i(c_i, k_i, z_i, x_i) \\
\text{with} \quad U_i(c_i, k_i, z_i, x_i) &= \bar{r}k_i + (1-\tau_L)z_i + (\tau_L - \tau_K)x_i + \tau_L(z^m - x^m) \\
&+\tau_K(rk^m + x^m) + a_i(k_i) - h_i(z_i) - d_i(x_i) + \delta_i \cdot (k_i^{init} - k_i).
\end{aligned}
$$

The first order conditions with respect to τ_L and τ_K are:

$$\int_i \omega_i(z^m - x^m - (z_i - x_i)) - \tau_L \frac{dz^m}{d(1-\tau_L)} - (\tau_L - \tau_K)\frac{dx^m}{d\tau_L} = 0,$$

$$\int_i \omega_i(rk^m + x^m - (rk_i + x_i)) - \tau_K r \frac{dk^m}{d(1-\tau_K)} - (\tau_L - \tau_K)\frac{dx^m}{d\tau_K} = 0.$$

Since x_i depends only on $\tau_L - \tau_K$, we have that: $\frac{dx^m}{d\tau_L} = -\frac{dx^m}{d\tau_K} = \frac{dx^m}{d(\tau_L - \tau_K)}$. Let $\Delta\tau \equiv \tau_L - \tau_K$. The FOCs can be rewritten as:

$$\frac{z^m - x^m - \int_i \omega_i(z_i - x_i)}{\frac{dz^m}{d(1-\tau_L)}} - \Delta\tau \frac{\frac{dx^m}{d(\tau_L - \tau_K)}}{\frac{dz^m}{d(1-\tau_L)}} = \tau_L,$$

$$\frac{rk^m + x^m - \int_i \omega_i(rk_i + x_i)}{r\frac{dk^m}{d(1-\tau_K)}} + \Delta\tau \frac{\frac{dx^m}{d(\tau_L - \tau_K)}}{r\frac{dk^m}{d(1-\tau_K)}} = \tau_K.$$

Let us simplify notation a bit and denote:

$$z' \equiv \frac{dz^m}{d(1-\tau_L)} \quad k' \equiv \frac{dk^m}{d(1-\tau_K)} \quad x' \equiv \frac{dx^m}{d(\tau_L - \tau_K)}.$$

Taking the difference of those two equations, we can express $\Delta\tau$ as

$$\Delta\tau\left(1 + x'\left(\frac{1}{z'} + \frac{1}{rk'}\right)\right) = \frac{z^m - x^m - \int_i \omega_i(z_i - x_i)}{z'} - \frac{rk^m + x^m - \int_i \omega_i(rk_i + x_i)}{rk'}, \tag{A1}$$

Since $\left(1 + x'\left(\frac{1}{z'} + \frac{1}{rk'}\right)\right) > 0$, the sign of $\Delta\tau$ is that of the right-hand side of the above expression.

$$\Delta\tau > 0 \Leftrightarrow \frac{z^m - x^m - \int_i \omega_i(z_i - x_i)}{z'} > \frac{rk^m + x^m - \int_i \omega_i(rk_i + x_i)}{rk'}.$$

Define the distributional factor of shifted income, by analogy to the distributional factors $\bar{g}_K$

and $\bar{g}_L$ for capital and labor income.

$$\bar{g}_X = \frac{\int_i \omega_i x_i}{z^m}.$$

The right-hand side of (A1) can be rewritten as:

$$RHS = \frac{1 - \frac{x^m}{z^m} - \bar{g}_L + \bar{g}_X}{\frac{e_L}{1-\tau_L}} - \frac{1 + \frac{x^m}{rk^m} - \bar{g}_K - \bar{g}_X \frac{z^m}{rk^m}}{\frac{e_K}{1-\tau_K}}.$$

Hence:

$$\Delta\tau > 0 \Leftrightarrow \frac{1 - \frac{x^m}{z^m} - \bar{g}_L + \bar{g}_X}{\frac{e_L}{1-\tau_L}} > \frac{1 + \frac{x^m}{rk^m} - \bar{g}_K - \bar{g}_X \frac{z^m}{rk^m}}{\frac{e_K}{1-\tau_K}}.$$

Suppose that $\bar{g}_X$ is small enough – otherwise, encouraging "shifting" may be good for distributional reasons. Formally, for $x^m > 0$,

$$\frac{x^m}{rk^m} - \bar{g}_X \frac{z^m}{rk^m} > 0 \quad \text{and} \quad \frac{x^m}{z^m} - \bar{g}_X > 0.$$

Conversely, for $x^m < 0$, we have $\bar{g}_X < 0$, and we assume that $\bar{g}_X$ is small relative to x^m in absolute value.

$$\frac{x^m}{rk^m} - \bar{g}_X \frac{z^m}{rk^m} < 0 \quad \text{and} \quad \frac{x^m}{z^m} - \bar{g}_X < 0.$$

We can then write:

$$\Delta\tau > 0 \Leftrightarrow e_K > e_L \cdot \frac{1-\tau_K}{1-\tau_L} \cdot \frac{\left(1 + \frac{x^m}{rk^m} - \bar{g}_K - \bar{g}_X \frac{z^m}{rk^m}\right)}{\left(1 - \frac{x^m}{z^m} - \bar{g}_L + \bar{g}_X\right)}.$$

If $\Delta\tau = 0$, there is no shifting and hence $x_i = 0$ for all i and $x^m = 0$, and hence $\bar{g}_X = 0$. Therefore,

$$\text{If} \quad \Delta\tau = 0: \quad e_K = e_L \frac{(1-\bar{g}_K)}{(1-\bar{g}_L)}.$$

If $\Delta\tau > 0$, then $x^m > 0$ and $e_K > e_L \frac{(1-\bar{g}_K)}{(1-\bar{g}_L)}$.

Conversely, if $\Delta\tau < 0$, then $x^m < 0$ and $e_K < e_L \frac{(1-\bar{g}_K)}{(1-\bar{g}_L)}$.

Thus:

$$\Delta\tau \lesseqqgtr 0 \Leftrightarrow e_K \lesseqqgtr e_L \frac{(1-\bar{g}_K)}{(1-\bar{g}_L)}.$$

We can now rewrite the FOCs as:

$$z^m(1 - \frac{x^m}{z^m} - \bar{g}_L + \bar{g}_X) - \Delta\tau x' = z^m e_L \frac{\tau_L}{1-\tau_L},$$

$$rk^m(1 + \frac{x^m}{rk^m} - \bar{g}_K - \bar{g}_X \frac{z^m}{rk^m}) + \Delta\tau x' = rk^m e_K \frac{\tau_K}{1-\tau_K}.$$

We distinguish three cases:

- If $e_K > e_L \frac{(1-\bar{g}_K)}{(1-\bar{g}_L)}$, then $\Delta\tau > 0$ and

$$e_L \frac{\tau_L}{1-\tau_L} < 1 - \frac{x^m}{z^m} - \bar{g}_L + \bar{g}_X < 1 - \bar{g}_L.$$

  and in this case:

$$e_K \frac{\tau_K}{1-\tau_K} > (1 + \frac{x^m}{rk^m} - \bar{g}_K - \bar{g}_X \frac{z^m}{rk^m}) > 1 - \bar{g}_K.$$

  So that the optimal tax rates with shifting are bracketed by their revenue maximizing rates.

- If there is no shifting, $x \equiv 0$ then revenue maximizing rates apply.

- If x' is very large (very sensitive shifting to any tax differential), then from equation (A1), we have that $\Delta\tau \approx 0$ and hence $\tau_L \approx \tau_K$. Summing the FOCs and using this equality yields $\tau_L = \tau_K = \tau_Y$ where τ_Y is the optimal linear tax rate on comprehensive income derived in Proposition 4.

Proof of Proposition 6.

Let us compare the following two regimes considered in the text:

Regime 1 – Consumption tax regime: $(\bar{r}, T_L, \tau_C)$, with an initial lump-sum transfer $\tau_C \cdot k_i^{init}/(1-\tau_C)$ to wealth holders with initial wealth k_i^{init}.

Regime 2 – No consumption tax regime: $(\bar{r}, \hat{T}_L, \tau_C = 0)$ with $(z - \hat{T}_L(z)) = (z - T_L(z)) \cdot (1 - \tau_C)$. Let $\tilde{k}_i$ denote the steady state wealth choice under this regime.

We will show that these regimes are equivalent in the steady state, in the consumer's dynamic optimization problem, and in the government's revenue raised, as claimed in the text.

Steady-state equivalence:

The budget constraint in regime 1 is: $\dot{k} = [\bar{r}k + z - T_L(z)] - c/(1-\tau_C) + G$, where $G = \tau_L z^m + \tau_K r k^m + t_C c^m$ is the lump-sum transfer rebate of tax revenue. The budget constraint can be rewritten in terms of real wealth as: $\dot{k}^r = \bar{r}k^r + (z - T_L(z)) \cdot (1-\tau_C) + G \cdot (1-\tau_C) - c$.

Utility is:

$$u_i = c_i + a_i(k_i^r) - h_i(z_i).$$

The first-order conditions of the individual are:

$$(1 - T_L'(z_i)) \cdot (1 - \tau_C) = h_i'(z_i), \quad a_i'(k_i^r) = \delta_i - \bar{r}.$$

Given that $(1 - \hat{T}_L'(z_i)) = (1 - T_L'(z_i)) \cdot (1 - \tau_C)$ for all z_i, the steady-state choices of labor income and real capital of the individual are unaffected. Using the steady state budget constraint, real consumption c_i is also not affected as long as the real lump-sum transfer $G \cdot (1 - \tau_C)$ is not affected, which we prove right below. The link between the two capital levels is: $\tilde{k}_i = (1 - \tau_C) \cdot k_i$ (since real steady state wealth is unaffected).

Equivalence of the dynamic consumer optimization problem.

The law of motion in real-wealth equivalent, $\dot{k}^r = \bar{r}k^r + (z - T_L(z)) \cdot (1 - \tau_C) + G \cdot (1 - \tau_C) - c$, is the same in regime 1 and regime 2 as long as the real lump-sum transfer $(1 - \tau_C) \cdot G$ is the same, which we show below. The initial wealth after the lump-sum transfer $\tau_C \cdot k_{init}^i/(1 - \tau_C)$ from the government becomes $k_i^{init} + \tau_C \cdot k_i^{init}/(1 - \tau_C) = k_i^{init}/(1 - \tau_C)$, so that real wealth after the transfer is k_i^{init}, the same it was in the tax regime without a consumption tax.

Equivalence of government revenue.

In regime 1, there is first the initial cost of providing the lump-sum $\tau_C \cdot \int_i k_i^{init}/(1 - \tau_C)$ to all initial wealth holders. At the same time, the initial consumption change is taxed, which yields: $\tau_C \cdot \int_i (k_i^{init} - k_i)/(1 - \tau_C)$.

In real terms, this is worth:

$$A = -\tau_C \cdot \int_i k_i.$$

The nominal tax flow per period under this regime is (which is also equal to the lump-sum transfer per-period in nominal terms is G:

$$G = \frac{\tau_C}{1 - \tau_C} \int_i c_i + \int_i T_L(z_i) + \int_i \tau_K r k_i.$$

We can express consumption under this regime as:

$$c_i = (z_i - T_L(z_i))(1 - \tau_C) + \bar{r}(1 - \tau_C)k_i + G(1 - \tau_C).$$

and aggregate consumption as:

$$\int_i c_i = (1-\tau_C)\int_i (z_i - T_L(z_i)) + \bar{r}(1-\tau_C)\int_i k_i + G(1-\tau_C).$$

Solving for G using the definition of G and the expression for aggregate consumption yields:

$$G = \int_i T_L(z_i) + \frac{\tau_C}{1-\tau_C}\left(\int_i z_i + \bar{r}\int_i k_i\right) + \frac{1}{1-\tau_C}\int_i \tau_K r k_i.$$

In real terms, revenue is:

$$(1-\tau_C)\cdot G = (1-\tau_C)\int_i T_L(z_i) + \tau_C \int_i z_i + \tau_C \bar{r}\int_i k_i + \int_i \tau_K r k_i.$$

In Regime 2, the (real) revenue is:

$$\int_i \hat{T}_L(z_i) + \int_i \tau_K r \tilde{k}_i.$$

Using the map between the labor income taxes: $(z - \hat{T}_L(z)) = (z - T_L(z))\cdot(1-\tau_C)$, we obtain that the real revenue in Regime 2 is:

$$\int_i (\tau_C z + T_L(z)\cdot(1-\tau_C)) + \int_i \tau_K r \tilde{k}_i.$$

The difference between the per-period real revenue in regime 1 and that in regime 2 is hence: $\tau_C \int_i r k_i$. Recall that the initial change in revenue in regime 1 was $A = -\tau_C \cdot \int_i k_i$, which, converted into a per-period equivalent is exactly $A \cdot r = -\tau_C \cdot \int_i r k_i$ and cancels out perfectly the change in per-period revenue between the two regimes.

A.2 Proofs for Section 4

A.2.1 Generalized Model

Proof of Proposition 8

The steady state is characterized by: $u_{ik}/u_{ic} = \delta_i - r(1-T'_K)$, $u_{ic}\cdot(1-T'_L) = -u_{iz}$ and $c_i = rk_i + z_i - T(z_i, rk_i)$

With linear taxes, this simplifies to: $u_{ik}/u_{ic} = \delta_i - \bar{r}$, $u_{ic}\cdot(1-\tau_L) = -u_{iz}$ and $c_i = \bar{r}k_i + z_i(1-\tau_L)$.

First, consider the case with exogenous labor income. Let us assume that the economy has converged to the steady state. Consider a small reform $d\tau_K$ that takes place at time 0 and is unanticipated. Let us denote by $e_K(t)$ the elasticity of aggregate $k^m(t)$ with respect to $1-\tau_K$. $e_K(t)$ converges to e_K from the original analysis (the long-run steady state elasticity). Using the envelope theorem (i.e., behavioral responses dk_{ti} can be ignored when computing dV_i), the effect on the welfare of individual i is:

$$dV_i = d\tau_K \cdot \delta_i \Bigg[\int_0^\infty u_{ic}(c_i(t), k_i(t)) r k^m(t) \cdot e^{-\delta_i t} - \int_0^\infty u_{ic}(c_i(t), k_i(t)) r k_i(t) \cdot e^{-\delta_i t} \\ - \frac{\tau_K}{1-\tau_K} \int_0^\infty u_{ic}(c_i(t), k_i(t)) r k^m(t) e_K(t) \cdot e^{-\delta_i t} dt \Bigg].$$

In the steady state, $k^m(t)$ and $c_i(t)$, $k_i(t)$ are time-constant so that:

$$dV_i = d\tau_K \cdot r k^m \left[u_{ic}(c_i, k_i) - u_{ic}(c_i, k_i) \frac{k_i}{k^m} - \frac{\tau_K}{1-\tau_K} \delta_i u_{ic}(c_i, k_i) \int_0^\infty e_K(t) \cdot e^{-\delta_i t} dt \right].$$

The change in social welfare is $dSWF = \int_i \omega_i dV_i$ so that:

$$dSWF = \int_i d\tau_K \cdot r k^m \omega_i \left[u_{ic}(c_i, k_i) - u_{ic}(c_i, k_i) \frac{k_i}{k^m} - \frac{\tau_K}{1-\tau_K} \delta_i u_{ic}(c_i, k_i) \int_0^\infty e_K(t) \cdot e^{-\delta_i t} dt \right].$$

Recall the normalization of social welfare weights: $\int_i \omega_i u_{ic} = 1$ and $g_i = \omega_i u_{ic}$. Hence,

$$dSWF \propto 1 - \int_i \frac{g_i k_i}{k^m} - \frac{\tau_K}{1-\tau_K} \int_i \delta_i g_i \int_0^\infty e_K(t) \cdot e^{-\delta_i t} dt.$$

With endogenous labor supply, the change in individual i's welfare, dV_i:

$$dV_i = d\tau_K \cdot \delta_i \Bigg[\int_0^\infty u_{ic}(c_i(t), k_i(t), z_i(t)) r k^m(t) \cdot e^{-\delta_i t} - \int_0^\infty u_{ic}(c_i(t), k_i(t), z_i(t)) r k_i(t) \cdot e^{-\delta_i t} \\ - \frac{\tau_K}{1-\tau_K} \int_0^\infty u_{ic}(c_i(t), k_i(t), z_i(t)) r k^m(t) e_K(t) \cdot e^{-\delta_i t} dt \\ - \frac{\tau_L}{1-\tau_K} \int_0^\infty u_{ic}(c_i(t), k_i(t), z_i(t)) e_{L,1-\tau_K}(t) z^m(t) e^{-\delta_i t} dt \Bigg].$$

In the steady state, $k^m(t)$, $z^m(t)$, $c_i(t)$, $z_i(t)$, and $k_i(t)$ are time-constant, so that the change in individual i's utility is:

$$dV_i = d\tau_K \cdot r k^m \Bigg[u_{ic}(c_i, k_i . z_i) - u_{ic}(c_i, k_i, z_i) \frac{k_i}{k^m} \\ - \frac{\tau_K}{1-\tau_K} \delta_i u_{ic}(c_i, k_i, z_i) \int_0^\infty e_K(t) \cdot e^{-\delta_i t} dt - \frac{\tau_L}{1-\tau_K} \delta_i u_{ic}(c_i, k_i, z_i) \frac{z^m}{r k^m} \int_0^\infty e_{L,1-\tau_K}(t) \cdot e^{-\delta_i t} dt \Bigg].$$

and the change in social welfare is:

$$dSWF = \int_i \omega_i dV_i = \int_i d\tau_K \cdot rk^m \omega_i \Big[u_{ic}(c_i, k_i, z_i) - u_{ic}(c_i, k_i, z_i) \frac{k_i}{k^m}$$
$$- \frac{\tau_K}{1-\tau_K} \delta_i u_{ic}(c_i, k_i, z_i) \int_0^\infty e_K(t) \cdot e^{-\delta_i t} dt - \frac{\tau_L}{1-\tau_K} \delta_i u_{ic}(c_i, k_i, z_i) \frac{z^m}{rk^m} \int_0^\infty e_{L,1-\tau_K}(t) \cdot e^{-\delta_i t} dt \Big].$$

Using the normalization of social welfare weights: $\int_i \omega_i u_{ic} = 1$ and $g_i = \omega_i u_{ic}$.

$$dSWF \propto 1 - \int_i \frac{g_i k_i}{k^m} - \frac{\tau_K}{1-\tau_K} \int_i \delta_i g_i \int_0^\infty e_K(t) e^{-\delta_i t} dt - \frac{\tau_L}{1-\tau_K} \frac{z^m}{rk^m} \int_i \delta_i g_i \int_0^\infty e_{L,1-\tau_K}(t) e^{-\delta_i t} dt,$$

which yields the formula in the text.

Proof of Proposition 9

We consider the top tax rate τ_K on capital above threshold k^{top}. As r is uniform, this is equivalent to a top tax rate applying above capital income threshold rk^{top}. Let N denote the fraction of individuals above k^{top}. We again use the notation $k^{m,top}$ to denote the average wealth above the top threshold, i.e.:

$$k^{m,top} = \frac{\int_{i:k_i(t) \geq k^{top}} rk_i}{N},$$

Suppose we change the top tax rate on capital by $d\tau_K$. As defined in the text, let $e_K^{top}(t)$ be the elasticity of capital holding of top capital earners (the wealth elasticity of total wealth to the tax rate of those with capital income above rk^{top}). For all individuals above the cutoff, the change in utility is:

$$dV_i = d\tau_K \delta_i \Big[\int_0^\infty u_{ic}(c_i(t), k_i(t)) Nr(k^{m,top}(t) - k^{top}) e^{-\delta_i t} - \int_0^\infty u_{ic}(c_i(t), k_i(t)) r(k_i(t) - k^{top}) e^{-\delta_i t}$$
$$- \frac{\tau_K}{1-\tau_K} \int_0^\infty u_{ic}(c_i(t), k_i(t)) Nrk^{m,top}(t) e_K^{top}(t) \cdot e^{-\delta_i t} dt \Big].$$

Starting from the steady state, capital levels are constant so that:

$$dV_i = u_{ic} r(k^{m,top} - k^{top}) N d\tau_K \left[1 - \frac{(k_i - k^{top})}{(k^{m,top} - k^{top})N} - \frac{\tau_K}{1-\tau_K} a_K^{top} \int_0^\infty \delta_i e_K^{top}(t) \cdot e^{-\delta_i t} dt \right],$$

where $a_K^{top} = \frac{k^{m,top}}{(k^{m,top} - k^{top})}$.

For individuals below the cutoff, the change in utility is:

$$dV_i = u_{ic} r(k^{m,top} - k^{top}) N d\tau_K \left[1 - \frac{\tau_K}{1-\tau_K} a_K^{top} \int_0^\infty \delta_i e_K^{top}(t) \cdot e^{-\delta_i t} dt\right].$$

The change in social welfare is such that:

$$dSWF \propto 1 - \int_{i:k_i \geq k^{top}} g_i \frac{(k_i - k^{top})}{(k^{m,top} - k^{top})N} - \frac{\tau_K}{1-\tau_K} a_K^{top} \int_i g_i \delta_i \int_0^\infty e_K^{top}(t) \cdot e^{-\delta_i t} dt.$$

Let

$$\bar{g}_K^{top} \equiv \int_{i:k_i \geq k^{top}} g_i \frac{(k_i - k^{top})}{(k^{m,top} - k^{top})N} \quad \text{and} \quad \bar{e}_K^{top} \equiv \int_i g_i \delta_i \int_0^\infty e_K^{top}(t) \cdot e^{-\delta_i t} dt.$$

Then, we obtain the optimal tax rate τ_K such that $dSWF = 0$:

$$\tau_K = \frac{1 - \bar{g}_K^{top}}{1 - \bar{g}_K^{top} + a_K^{top} \bar{e}_K^{top}}.$$

With endogenous labor, let

$$e_{L,(1-\tau_K)}(t) = \frac{dz^m(t)}{d(1-\tau_K)} \frac{(1-\tau_K)}{z^m(t)} = \frac{dz^m(t)}{d\bar{r}} \frac{\bar{r}}{Nz^m(t)}.$$

be the elasticity of aggregate (average) labor income z^m with respect to the top capital tax rate, normalized by N, in the two bracket tax system.

For all individuals with capital income above the cutoff:

$$\begin{aligned} dV_i = d\tau_K \cdot \delta_i \Bigg[& \int_0^\infty u_{ic}(c_i(t), k_i(t), z_i(t)) N r(k^{m,top}(t) - k^{top}) \cdot e^{-\delta_i t} \\ & - \frac{\tau_L}{1-\tau_K} \int_0^\infty u_{ic}(c_i(t), k_i(t), z_i(t)) z^m(t) N e_{L,(1-\tau_K)}(t) \cdot e^{-\delta_i t} \\ & - \int_0^\infty u_{ic}(c_i(t), k_i(t), z_i(t)) r(k_i(t) - k^{top}) \cdot e^{-\delta_i t} \\ & - \frac{\tau_K}{1-\tau_K} \int_0^\infty u_{ic}(c_i(t), k_i(t), z_i(t)) N r k^{m,top}(t) e_K^{top}(t) \cdot e^{-\delta_i t} dt \Bigg]. \end{aligned}$$

Starting from the steady state, capital and labor income are constant over time:

$$dV_i = u_{ic} N r (k^{m,top} - k^{top}) d\tau_K \cdot \Bigg[1 - \frac{(k_i - k^{top})}{(k^{m,top} - k^{top})N}$$
$$- \frac{\tau_L}{1-\tau_K} \frac{z^m}{r(k^{m,top} - k^{top})} \int_0^\infty \delta_i e_{L,(1-\tau_K)}(t) \cdot e^{-\delta_i t} dt - \frac{\tau_K}{1-\tau_K} a_K^{top} \int_0^\infty \delta_i e_K^{top}(t) \cdot e^{-\delta_i t} dt\Bigg].$$

The change in social welfare is:

$$dSWF = \int_i \omega_i dV_i \propto 1 - \int_{i: rk_i \geq rk^{top}} g_i \frac{(k_i - k^{top})}{(k^{m,top} - k^{top})N}$$
$$- \frac{\tau_L}{1-\tau_K} \frac{z^m}{r(k^{m,top} - k^{top})} \int_i g_i \int_0^\infty \delta_i e_{L,(1-\tau_K)}(t) \cdot e^{-\delta_i t} dt - \frac{\tau_K}{1-\tau_K} a_K^{top} \int_i g_i \int_0^\infty \delta_i e_K^{top}(t) \cdot e^{-\delta_i t} dt.$$

Define $\bar{e}_K^{top}$, $\bar{e}_{L,(1-\tau_K)}$, and $\bar{g}_K^{top}$ as in the text. The optimal formula in the text is then obtained by rearranging the previous condition.

A.2.2 Anticipated Reforms in the Generalized Model

Consider anticipated reform to the capital income tax $d\tau_K$ at time $T > 0$. Capital and labor already start adjusting in anticipation of the reform before time T. The change in the utility of individual i is:

$$dV_i = d\tau_K \cdot \delta_i \Bigg[\int_T^\infty u_{ic}(c_i(t), k_i(t)) r k^m(t) \cdot e^{-\delta_i t} dt - \int_T^\infty u_{ic}(c_i(t), k_i(t)) r k_i(t) \cdot e^{-\delta_i t} dt$$
$$- \frac{\tau_K}{1-\tau_K} \int_0^\infty u_{ic}(c_i(t), k_i(t)) r k^m(t) e_K(t) \cdot e^{-\delta_i t} dt\Bigg].$$

In the steady state, $k^m(t)$ and $c_i(t)$, $k_i(t)$ are time-constant, hence we have:

$$dV_i = d\tau_K r k^m e^{-\delta_i T} \cdot \Bigg[u_{ic}(c_i, k_i) - u_{ic}(c_i, k_i) \frac{k_i}{k^m} - \frac{\tau_K}{1-\tau_K} \delta_i u_{ic}(c_i, k_i) \int_{t<T} e_K(t) \cdot e^{-\delta_i (t-T)} dt$$
$$- \frac{\tau_K}{1-\tau_K} \delta_i u_{ic}(c_i, k_i) \int_{t \geq T} e_K(t) \cdot e^{-\delta_i (t-T)} dt\Bigg].$$

As explained in the text, we assume homogeneous discount rates across individuals. Using that $\int_i g_i = \int_i u_{ci} \omega_i = 1$, we can write $dSWF = \int_i \omega_i dV_i$:

$$dSWF \propto= 1 - \int_i g_i \frac{k_i}{k^m} - \frac{\tau_K}{1-\tau_K} \delta \int_{t<T} e_K(t) \cdot e^{-\delta(t-T)} dt - \frac{\tau_K}{1-\tau_K} \delta \int_{t \geq T} e_K(t) \cdot e^{-\delta(t-T)} dt.$$

Defining the distributional factor $\bar{g}_K = \int_i g_i \frac{k_i}{k^m}$ and the anticipation elasticity e_K^{ante}, the post elasticity e_K^{post} and the total elasticity $\bar{e}_K = e_K^{ante} + e_K^{post}$, we obtain the optimal tax rate τ_K such that $dSWF = 0$:

$$\tau_K = \frac{1 - \bar{g}_K}{1 - \bar{g}_K + \bar{e}_K}.$$

With endogenous labor, the anticipation effects through the cross-elasticities can also start before the reform. The effect on labor is then also augmented by the anticipation cross-elasticities, yielding the elasticity $\bar{e}_{L.1-\tau_K}$ as defined in the proposition.

A.2.3 Aiyagari (1995) Model with and without anticipation effects

Note that all proofs below would be exactly the same as the proofs for wealth-in-the-utility if we reformulated it in discrete time, replacing the standard utility without wealth in the utility, $u_{ti}(c_{ti})$, by $u_{ti}(c_{ti}, k_{ti})$. This is done by letting u'_{ti} denote $\frac{\partial u_{ti}(c_{ti}, k_{ti})}{\partial c_{ti}}$ instead of $\frac{\partial u_{ti}(c_{ti})}{\partial c_{ti}}$.

We apply the envelope theorem, which states that the changes in the capital tax rate $d\tau_K$ only has a direct impact on utility through the direct reduction in consumption that it causes. Using this, and taking the derivative of the social welfare SWF with respect to $d\tau_K$ yields:

$$\begin{aligned}
dSWF &= \sum_{t<T} \left(\frac{1}{1+\delta}\right)^t \int_i \omega_i u'_{ti} \cdot (\tau_K r dk_t^m) + \sum_{t\geq T} \left(\frac{1}{1+\delta}\right)^t \int_i \omega_i u'_{ti} \cdot (r d\tau_K (k_t^m - k_{ti}) + \tau_K r dk_t^m) \\
&= -d\tau_K \Bigg(\frac{\tau_K}{1-\tau_K} \left[\sum_{t<T} \left(\frac{1}{1+\delta}\right)^t r k_t^m e_{Kt} \int_i \omega_i u'_{ti} + \sum_{t\geq T} \left(\frac{1}{1+\delta}\right)^t r k_t^m e_{Kt} \int_i \omega_i u'_{ti} \right] \\
&+ \sum_{t\geq T} \left(\frac{1}{1+\delta}\right)^t \int_i \omega_i u'_{ti} \cdot r(k_t^m - k_{ti}) \Bigg) \\
&= -d\tau_K \left(\frac{\tau_K}{1-\tau_K} \left[\sum_{t\geq 0} \left(\frac{1}{1+\delta}\right)^t r k_t^m e_{Kt} \int_i \omega_i u'_{ti} \right] - \sum_{t\geq T} \left(\frac{1}{1+\delta}\right)^t \int_i \omega_i u'_{ti} \cdot r(k_t^m - k_{ti}) \right).
\end{aligned}$$

If variables have already converged to their ergodic paths when the anticipation responses start: then all terms in e_{Kt} are zero before the steady state has been reached and hence, we can divide through by $\int_i \omega_i u'_{ti} k_t^m = \int_i g_i k_t^m$ which is constant across t. Thus:

$$dSWF \propto \frac{\tau_K}{(1-\tau_K)} \left(\frac{\delta}{1+\delta} \sum_{t<T} \left(\frac{1}{1+\delta}\right)^{t-T} e_{Kt} + \frac{\delta}{1+\delta} \sum_{t\geq T} \left(\frac{1}{1+\delta}\right)^{t-T} e_{Kt} \right) - 1 + \frac{\int_i g_i k_{ti}}{\int_i g_i k_t^m}.$$

Define the anticipation responses e_K^{ante}, the post-reform response e_K^{post}, and the total response $\bar{e}_K$ to be:

$$e_K^{ante} = \frac{\delta}{1+\delta}\sum_{t<T}\left(\frac{1}{1+\delta}\right)^{t-T} e_{Kt}, \quad e_K^{post} = \frac{\delta}{1+\delta}\sum_{t\geq T}\left(\frac{1}{1+\delta}\right)^{t-T} e_{Kt}, \quad \text{and} \quad \bar{e}_K = e_K^{ante} + e_K^{post}.$$

and the distributional factor $\bar{g}_K = \frac{\int_i g_i k_{ti}}{\int_i g_i k_t^m}$. Then we have as in the text that the optimal capital tax in the Aiyagari (1995) model is given by:

$$\tau_K = \frac{1-\bar{g}_K}{1-\bar{g}_K+\bar{e}_K}.$$

For the unanticipated reform at time $T=0$ that is studied in the text, assume that the economy is already in the steady state as of time 0, and set $e_K^{ante}=0$ so that:

$$\bar{e}_K = \frac{\delta}{1+\delta}\sum_{t\geq 0}\left(\frac{1}{1+\delta}\right)^{t} e_{Kt}.$$

If variables have not converged to their ergodic paths when the anticipation responses start: we have to take into account the transition of the marginal utilities and the capital stock across time.

$$dSWF = -d\tau_K\left(\frac{\tau_K}{(1-\tau_K)}\left[\sum_{t<T}\left(\frac{1}{1+\delta}\right)^t rk_t^m e_{Kt}\int_i \omega_i u'_{ti}\right] - \sum_{t\geq T}\left(\frac{1}{1+\delta}\right)^t \int_i \omega_i u'_{ti}\cdot r(k_t^m - k_{ti})\right).$$

Dividing by $\sum_{t\geq T}\left(\frac{1}{1+\delta}\right)^t \int_i \omega_i u'_{ti}\cdot k_t^m$ yields:

$$dSWF \propto \frac{\tau_K}{(1-\tau_K)}\left[\sum_{t<T}\left(\frac{1}{1+\delta}\right)^t k_t^m e_{Kt}\frac{\int_i \omega_i u'_{ti}}{\sum_{t\geq T}\left(\frac{1}{1+\delta}\right)^t \int_i \omega_i u'_{ti}\cdot k_t^m}\right]$$
$$-1+\sum_{t\geq T}\left(\frac{1}{1+\delta}\right)^t \frac{\int_i \omega_i u'_{ti}\cdot k_{ti}}{\sum_{t\geq T}\left(\frac{1}{1+\delta}\right)^t \int_i \omega_i u'_{ti}\cdot k_t^m}.$$

Now we have to redefine the average welfare weight as:

$$\bar{g}_K \equiv \sum_{t\geq T}\left(\frac{1}{1+\delta}\right)^t \frac{\int_i u'_{ti}\cdot k_{ti}}{\sum_{t\geq T}\left(\frac{1}{1+\delta}\right)^t \int_i u'_{ti}\cdot k_t^m},$$

and the total elasticity as:

$$\bar{e}_K = \sum_{t\geq 0}\left(\frac{1}{1+\delta}\right)^t k_t^m e_{Kt}\frac{\int_i u'_{ti}}{\sum_{t\geq T}\left(\frac{1}{1+\delta}\right)^t \int_i u'_{ti}\cdot k_t^m}.$$

With these redefined variables, the same formula holds.

A.2.4 Judd (1985) Model

In the Judd (1985) model, individual utility is:

$$V_i(\{c_i(t), z_i(t), k_i(t)\}_{t\geq 0}) = \int_0^\infty u_i(c_i(t), k_i(t), z_i(t))e^{-\int_0^t \delta_i(c_i(s))ds}dt.$$

The effect on V_i from a small change in the capital tax $d\tau_K$ is now:

$$\begin{aligned} dV_i = d\tau_K\Bigg[\int_{t=0}^\infty \Bigg(u_{ic}(c_i(t), k_i(t), z_i(t))e^{-\int_0^t \delta_i(c_i(s))ds} + \delta_i'(c_i(t))\int_t^\infty u_i(s)e^{-\int_0^s \delta_i(c_i(m))dm}ds\Bigg) \\ \times\left(rk^m(t) - rk_i(t) - \frac{\tau_K}{1-\tau_K}rk^m(t)e_K(t)\right)dt\Bigg].\end{aligned}$$

In the steady state, we can hence write dV_i as:

$$\begin{aligned} &d\tau_K r\left[\int_0^\infty \left(u_{ic}e^{-\delta_i(c_i)t} + \delta_i'(c_i)u_i e^{-\delta_i(c_i)t}\int_t^\infty e^{-\delta_i(c_i)(s-t)}ds\right)\left(k^m(t) - k_i(t) - \frac{\tau_K}{1-\tau_K}k^m(t)e_K(t)\right)dt\right] \\ &= d\tau_K r\Bigg[\left(u_{ic}\int_0^\infty e^{-\delta_i(c_i)t}dt + \delta_i'(c_i)u_i\int_0^\infty e^{-\delta_i(c_i)t}\frac{1}{\delta_i(c_i)}\right)\times[k^m(t) - k_i(t)] \\ &\quad - \int_0^\infty \left(u_{ic}e^{-\delta_i(c_i)t} + \delta_i'(c_i)u_i e^{-\delta_i(c_i)t}\int_0^\infty e^{-\delta_i(c_i)t}ds\right)\frac{\tau_K}{1-\tau_K}k^m(t)e_K(t)\Bigg] \\ &= d\tau_K r k^m\frac{1}{\delta_i(c_i)}\left(u_{ic} + \frac{\delta_i'(c_i)}{\delta_i(c_i)}u_i\right)\left[1 - \frac{k_i}{k^m} - \frac{\tau_K}{1-\tau_K}\delta_i(c_i)\int_0^\infty e^{-\delta_i(c_i)t}e_K(t)\right].\end{aligned}$$

We can hence see that the formulas from our model apply but with g_i and $\bar{e}_K$ as redefined in the text.

A.3 Steady State and Anticipation Elasticities

We now prove two further results.

Steady state elasticities are finite with wealth in the utility.

With a general utility and wealth in the utility, the first-order condition for agent i in the steady state is:

$$u_{ki} = (\delta_i - \bar{r})u_{ci}$$

In the steady state, the budget constraint is:

$$c_i = \bar{r}k_i + z_i$$

hence the steady state can be rewritten as: $(\delta_i - \bar{r})u_{ci}(\bar{r}k_i + z_i, k_i) = u_{ki}(\bar{r}k_i + z_i, k_i)$ which is a smooth function of k_i, as long as the function $u_i(c_i, k_i)$ is smooth and concave in consumption and capital. Hence, the responses of consumption and capital to the net-of-tax return $\bar{r}$ are smooth and non-degenerate. The same argument holds with endogenous labor supply, which is chosen smoothly.

Anticipation elasticities are infinite with wealth in the utility and certainty, but finite with uncertainty (with or without wealth in the utility).

We can also show that the anticipation elasticities to a reform $d\tau_K$ for $t \geq T$ is infinite when there is full certainty, even with wealth in the utility. The proof is as in Piketty and Saez (2013b) for the Chamley-Judd model (without wealth in the utility).

With full certainty, the first-order condition of the agent with respect to capital always holds:

$$u_{ci,t} = (1 + \bar{r})/(1 + \delta_i)u_{ci,t+1} + 1/(1 + \delta_i)u_{ki,t+1}$$

Suppose we start from a situation in a well-defined steady state: $(\delta_i - \bar{r})u_{ci} = u_{ki}$ where we have perfect consumption smoothing.

The intertemporal budget constraint is:

$$\sum_{t \geq 0} \left(\frac{1}{1+r}\right)^t c_{ti} + \lim_{t \to \infty} k_{ti} = \sum_{t \geq 0} \left(\frac{1}{1+r}\right)^t z_{ti} + k_{0i}$$

Consumption smoothing implies:

$$u_{ci}(\bar{r}k_i + z_i, k_i) = \lambda$$

for the multiplier λ on the budget constraint. Hence, $k_i^{\infty} = \lim_{t \to \infty} k_{ti} > 0$. Given that there is perfect consumption smoothing, using the budget constraint to solve for consumption yields:

$$c = \left(1 - \frac{1}{1+r}\right)\left(\sum_{t\geq 0}\left(\frac{1}{1+r}\right)^t z_{ti} + k_{0i} - k_i^\infty\right) \tag{A2}$$

Consider what happens if the capital tax rate increases by $d\tau_K > 0$ for $t \geq T$. The present discounted value of all resources, denoted by Y_i for agent i is:

$$Y_i = k_{i0} + \sum_{t=1}^{T}\left(\frac{1}{1+r}\right)^t z_{ti} + \sum_{t\geq T}\left(\frac{1}{1+\bar{r}}\right)^t z_{ti}$$

The change in resources evaluated at $\tau_K = 0$ is:

$$dY_i = \left(\frac{1}{(1+r)}\right)^T \sum_{t\geq T} t\left(\frac{1}{(1+r)}\right)^{t-T+1} z_{ti} d\tau_K \propto \left(\frac{1}{(1+r)}\right)^T d\tau_K$$

Hence, consumption pre-reform will shift down by a factor proportional to $\left(\frac{1}{(1+r)}\right)^T d\tau_K$. From the aggregated budget constraint we have that:

$$k_t^m = (1+r)^t k_0^m - c_0^m(1 + (1+r) + (1+r)^2 + ... + (1+r)^{t-1}) + (z_{t-1}^m + .. + (1+r)^{t-1} z_0^m)$$

Therefore, the change in the aggregate capital stock is:

$$dk_t^m = -dc_0^m \left(\frac{(1+r)^{t-1} - 1}{r}\right)$$

Recall that the change in consumption (from (A2)) is proportional to $\left(\frac{1}{(1+r)}\right)^T d\tau_K$. Hence:

$$dk_t^m \propto -\left(\frac{1}{(1+r)}\right)^T \left(\frac{(1+r)^{t-1} - 1}{r}\right) d\tau_K = -(1+r)^{-T}\left(\frac{(1+r)^{t-1} - 1}{r}\right) d\tau_K$$

Hence:

$$e_{Kt} \propto k_t^m (1+r)^{-T}\left(\frac{(1+r)^{t-1} - 1}{r}\right) d\tau_K$$

Recall that the anticipation elasticity e_K^{ante} is defined as:

$$e_K^{ante} = \frac{\delta}{1+\delta}\sum_{t<T}\left(\frac{1}{1+\delta}\right)^{t-T} e_{Kt} \propto \frac{\delta}{1+\delta}\sum_{t<T}\left(\frac{1}{1+\delta}\right)^{t-T} k_t^m (1+r)^{-T}\left(\frac{(1+r)^{t-1} - 1}{r}\right) d\tau_K$$

Since we have $\delta > r$, $\lim_{T\to\infty}\left(\frac{1+\delta}{1+r}\right)^T = \infty$, which makes the sum above (to which the anticipation elasticity is proportional) converge to infinity when T goes to infinity.

A.4 Optimal Nonlinear Taxes in the Generalized Model

Consider a small reform $\delta T_K(rk)$ in which the marginal tax rate is increased by $\delta\tau_K$ in a small band $[rk^*, rk^* + d(rk^*)]$, as in the proof of Proposition 2. Let us first derive the change in revenue from the reform in any period t. We start from the steady state. Since the reform has to be budget-neutral every period, the change in transfer to the agent will depend on the change in tax revenues at each period.

First, for any capital income rk above capital income rk^*, additional revenue equal to $d(rk^*)\delta\tau_K$ is raised. The total additional tax collected is $(1 - H_K(rk^*))d(rk^*)\delta\tau_K$. Second, for taxpayers in the small band $[rk^*, rk^* + d(rk^*)]$, the change in marginal tax rates generates changes in capital income through two channels. The first channel is a pure substitution effect due to the change $\delta\tau_K$ in marginal tax rates. The second channel is through the shift in capital income along the nonlinear tax schedule, which leads to an additional change in marginal tax rates equal to $dT_i' = T_K''(rk_i)\delta(rk_i, t)$. Let $e_K^c(rk, t)$ be the elasticity in period t at capital income level rk to a small change in the marginal tax rate that i) is unanticipated and occurs at time 0 and ii) lasts for all periods $t \geq 0$. $e_K^c(rk, t)$ is thus a policy elasticity. Formally, $e_K^c(rk, t) = \frac{dk}{d(1-T_K'(rk))}\frac{(1-T_K'(rk))}{k}$ where dk is the total change in capital for the reform described.

Hence the total change in capital income from tax payers in the small band is:

$$\delta(rk, t) = -e_K^c(rk^*, t)rk^*\frac{\delta\tau_K + T_K''(rk^*)\delta(rk, t)}{1 - T_K'(rk^*)}$$

Rearranging this yields:

$$\delta(rk, t) = -e_K^c(t)rk^*\frac{\delta\tau_K}{1 - T_K'(rk^*) + e_K^c(rk^*, t)rk^*T_K''(rk^*)}$$

The total behavioral effect in the small band is hence:

$$-e_K^c(rk^*, t)rk^*\frac{T_K'(rk^*)}{1 - T_K'(rk^*) + e_K^c(rk^*, t)rk^*T_K''(rk^*)}h_K(rk^*, t)\delta\tau_K d(rk^*)$$

Because we start from the steady state, the density is constant across time and $h_K(rk^*, t) = h_K(rk^*), \quad \forall t$.

Third, for taxpayers above rk^*, there is a change in the average tax liability. Let $\eta(rk, t)$ be the elasticity of capital income in period t at capital income level rk for a small change in virtual income that i) is unanticipated and occurs at time 0 and ii) lasts for all periods $t \geq 0$. $\eta(rk, t)$ is thus also policy elasticity.

There are again two channels: the direct impact of the tax change, equal to $\eta(rk, t)\delta\tau_K d(rk^*)$ and the indirect effect due to the move along the nonlinear tax schedule, which increases marginal tax rates by $dT_i' = T_K''(rk)\delta(rk, t)$. The total effect is hence:

$$\delta(rk, t) = \eta(rk, t)\frac{\delta\tau_K d(rk^*)}{1 - T_K'(rk) + rke_K^c(rk, t)T_K''(rk)}$$

Integrating over all taxpayers with incomes above the small band, the total tax revenue raised through this third component is:

$$\delta\tau_K d(rk^*)\int_{rk^*}^{\infty} -\eta(s, t)\frac{T_K'(s)}{1 - T_K'(s) + se_K^c(s, t)T_K''(s)}h_K(s)d(s)$$

The total change in revenue $dG(t)$ is:

$$\begin{aligned} d(rk^*)\delta\tau_K \cdot [(1 - H_K(rk^*)) - e_K^c(rk^*, t)rk^*\frac{T_K'(rk^*)}{1 - T_K'(rk^*) + rk^*e_K^c(rk^*, t)T_K''(rk^*)}h_K(rk^*) \\ + \int_{rk^*}^{\infty} -\eta(s, t)\frac{T_K'(s)}{1 - T_K'(s) + se_K^c(s, t)T_K''(s)}h_K(s)d(s)] \end{aligned} \tag{A3}$$

For agents below the small band, there is no change in the tax paid, but they benefit from the lump-sum rebate in revenue dG. For them $dT_i(t) = dG(t)$. Hence, the welfare impact for agents with $rk_i \leq rk^*$ is $\int_{i:rk_i<rk^*} g_i dG(t)di$. On the other hand, above the small band, agents receive the lump-sum increase in revenue $dG(t)$ but also pay an extra tax $\delta\tau_K d(rk^*)$, so that for them $dT_i(t) = (-\delta\tau_K d(rk^*) + dG(t))$. Hence, the welfare effect on agents above the small band is: $\int_{i:rk_i\geq rk^*} g_i(-\delta\tau_K d(rk^*) + dG(t))$. Thus the total change in welfare is:

$$\int_i \delta_i g_i \int_t dG(t)e^{-\delta_i t} - \int_{i:rk_i\geq rk^*} \delta_i g_i \int_t \delta\tau_K d(rk^*)e^{-\delta_i t}$$

Welfare weights g_i do not depend on time is because we start from a steady state (even if they are standard social welfare weights with $g_i = \omega_i u_{ci}$). We normalize $\int_i g_i = 1$.

Substituting for the change in revenue from (A3), the change in welfare is:

$$\begin{aligned} d(rk^*)\delta\tau_K \cdot [(1 - H_K(rk^*)) - \int_t e_K^c(rk^*, t)rk^*\frac{T_K'(rk^*)}{1 - T_K'(rk^*) + rk^*e_K^c(rk^*, t)T_K''(rk^*)}h_K(rk^*)\int_i \delta_i g_i e^{-\delta_i t} didt \\ - \int_t\int_{rk^*}^{\infty} \eta(s, t)\frac{T_K'(s)}{1 - T_K'(s) + se_K^c(s, t)T_K''(s)}h_K(s)ds \int_i \delta_i g_i e^{-\delta_i t}didt - \int_{i:rk_i\geq rk^*} g_i di] \end{aligned}$$

Thus at the optimum, the optimal marginal tax schedule is characterized by the differential equation:

$$(1-H_K(rk^*)) - \int_t e_K^c(rk^*,t)rk^* \frac{T_K'(rk^*)}{1-T_K'(rk^*)+rk^*e_K^c(rk^*,t)T_K''(rk^*)} h_K(rk^*) \int_i \delta_i g_i e^{-\delta_i t} di dt$$
$$- \int_t \int_{rk^*}^{\infty} \eta(s,t) \frac{T_K'(s)}{1-T_K'(s)+se_K^c(s,t)T_K''(s)} h_K(s) ds \int_i \delta_i g_i e^{-\delta_i t} di dt - \int_{i:rk_i \geq rk^*} g_i di = 0 \quad \text{(A4)}$$

A.5 Optimal Taxation with Horizontal Equity Concerns.

In this section, we formally consider optimal capital and labor taxation under horizontal equity concerns.

As derived in Section 2.4.4, the optimal revenue-maximizing rates are: $\tau_L^R = \frac{1}{1+e_L}$ and $\tau_K^R = \frac{1}{1+e_K}$. Without loss of generality, we suppose that capital is more elastic so that $\tau_K^R < \tau_L^R$. The optimal linear comprehensive tax on income is, as derived in (17):

$$\tau_Y = \frac{1-\bar{g}_Y}{1-\bar{g}_Y+e_Y} \quad \text{with} \quad \bar{g}_Y = \frac{\int_i g_i \cdot y_i}{\int_i y_i}$$

Suppose that the distribution of capital and labor income is dense enough, so that at every total income level $y = rk + z$, there are agents with $y = rk$ (capital income only) and $y = z$ (labor income only).

Generalized social welfare weights that capture horizontal equity concerns are such that:

(i) If $\tau_L = \tau_K$, then g_i are standard, for instance $g_i = u_{ci}$ for all agents. Any reform that changes taxes should put zero weight on those who after the reform are such that $\tau_L z_i + \tau_K rk_i < \max_j\{\tau_L z_j + \tau_K rk_j | z_j + rk_j = z_i + rk_i\}$, i.e., on those who pay less taxes at a given total income $y = rk_i + z_i$, or, equivalently, have the highest disposable income and consumption at any income. This means that if labor taxes are increased, $g_i = 0$ for those with any positive capital income at each total income level. Conversely, increasing capital taxes will yield $g_i = 0$ for those individuals with some labor income at each total income level.

(ii) If $\tau_L > \tau_K$, then all the social welfare weights are concentrated on those with $\tau_L z_i + \tau_K rk_i > \max_j\{\tau_L z_j + \tau_K rk_j | z_j + rk_j = z_i + rk_i\}$, i.e., on those agents with only labor income. Conversely, if $\tau_L < \tau_K$, all the social welfare weights are on agents with only capital income.

Suppose that, starting from a situation with $\tau_L = \tau_K$ we introduce a small tax break on capital income, $d\tau_K < 0$. Capital income earners now get an unfair advantage and all the weight is concentrated on those with no capital income (equivalently, everyone with $k_i > 0$ receives a weight $g_i = 0$). As a result, a small tax break on capital can only be optimal if it raises tax revenue and, hence, allows to lower the tax rate on labor income as well. This can only occur if $\tau_Y > \tau_K^R$, i.e., the optimal comprehensive tax rate is above the revenue-maximizing rate on capital income.

Proposition 1. ***Optimal labor and capital taxation with horizontal equity concerns.***

(i) If $\tau_Y \leq \tau_K^R$, taxing labor and capital income at the same comprehensive rate $\tau_L = \tau_K = \tau_Y$ is the unique optimum.

(ii) If $\tau_Y > \tau_K^R$, a differential tax system with the capital tax rate set to the revenue maximizing rate $\tau_K = \tau_K^R < \tau_L$ (with both τ_K and τ_L smaller than τ_Y) is the unique optimum.

Proof. Let us consider the two cases in turn.

(i) If $\tau_Y \leq \tau_K^R$.

To see why $\tau_L = \tau_K = \tau^*$ is an equilibrium, suppose that we tried to lower the tax rate on capital income. Then, all the weight will concentrate on people with only labor income, which will then in turn make it optimal to increase the tax on capital again.

This equilibrium is unique. There is no other equilibrium with equal taxes on capital and labor that can raise more revenue with a lower tax rate, by definition of τ_Y as the optimal rate on comprehensive income. There is also no equilibrium with non-equal tax rates on capital and labor. Suppose that we tried to set (without loss of generality) $\tau_K < \tau_L$. Then to raise enough revenue we would require that $\tau_K < \tau_Y < \tau_L$. Since capital owners are now advantaged, all the social welfare weight concentrates on people with only labor income. Since then a fortiori $\tau_K < \tau_K^R$, increasing τ_K would mean that more revenue would be raised, which would allow us to lower τ_L, which is good since all weight is on people with only labor income.

(ii) If $\tau_Y > \tau_K^R$.

In this case, the equilibrium has $\tau_K = \tau_K^R < \tau_Y$ and $\tau_Y > \tau_L > \tau_K^R$. Clearly this is an equilibrium since we cannot decrease τ_L without losing revenue and we cannot raise more revenue through τ_K (since it is already set at the revenue-maximizing rate for the capital tax base). In addition, we cannot decrease τ_K further without increasing τ_L, which is not desirable since it would benefit people capital income earners, who already receive a weight of zero.

This equilibrium is also unique. If we set $\tau_L = \tau_K$ equal, we should set them equal to τ_Y which is the optimal tax rate on comprehensive income. But then, since τ_K is now above its revenue maximizing rate, we could lower both τ_K and τ_L without losing revenues, so this would not be an equilibrium. On the other hand, as long as we set $\tau_K < \tau_L$, capital income earners get zero weight and the only possibility is to go all the way to $\tau_K = \tau_K^R$ since only people with only labor income have a non-zero weight.

□

As a result, horizontal equity concerns will be a force pushing towards the comprehensive income tax system derived in Section 2.4.4. In the text, we provided an efficiency argument in favor of a tax on comprehensive income (based on income shifting opportunities) while the argument here is based on equity considerations. With horizontal equity preferences, deviations from a comprehensive income tax system can only be justified if they raise more revenue and generate a Pareto-improvement, which drastically reduces the scope for them. In Saez and Stantcheva (2016) we argue that this is akin to a generalized Rawlsian principle whereby discrimination against some groups (e.g., capital owners versus labor providers) is only permissible if it makes the group discriminated against better off, i.e., if it generates a Pareto improvement.

A.5.1 Horizontal Equity with Nonlinear Taxation

The same reasoning as for linear taxation with horizontal equity also applies to nonlinear taxes. Starting from a comprehensive tax system $T_Y(z+rk)$ as derived in Section 2.4.4, lowering the tax rate on capital income, conditional on a given total income level, will generate a horizontal inequity and concentrate all social weight on those with no capital income conditional on that total income level. Such a preferential tax break for capital income earners will only be acceptable if it generates more revenue and allows to lower the tax rate on labor income as well. We show this below.

Formally, suppose that we start from the optimal tax on comprehensive income, $T_Y(rk+z)$, as derived in Section 2.4.4 which does not discriminate between capital and labor income conditional on total income. We say that a tax system unambigously favors capital (respectively, labor) at income level $y = rk + z$, if for any (rk, z) such that $y = rk + z$, and any $\varepsilon \in]0, z]$, $T_Y(rk, z) > T(rk + \varepsilon, z - \varepsilon)$ (having more capital income, conditional on a given total income leads to lower taxes). (Note that it may be the case that a tax system favors capital only at some y levels or only at some rk, z ranges..)

Denote a change in the tax by $\delta T(rk, z)$.

A deviation $\delta T(rk, z)$ is said to introduce horizontal inequity, if, starting from a comprehensive tax system $T_Y(rk+z)$, the resulting tax system $T_Y(z+rk)+\delta T(rk.z)$ cannot be expressed as $\tilde{T}_Y(rk + z)$ for some function $\tilde{T}_Y$.

With nonlinear taxes, we can again define the generalized social welfare weights as follows.

i) If there is a comprehensive tax $T_Y(z + rk)$, then everybody has standard weights, such as, for instance, $g_i = u_{ci}$. For any deviation $\delta T(rk, z)$ that introduces horizontal inequity, the weights concentrate on the agents who pay the highest tax at a given total income level, i.e., on those with $T_Y(z_i + rk_i) + \delta T(rk_i, z_i) = \max_j \{T_Y(z_j + rk_j) + \delta T(rk_j, z_j)|z_j + rk_j = rk_i + z_i\}$ (which is equivalent to putting all the weight on the agent(s) with lowest disposable income at any total income level).

Hence, the weights also need to depend on $\delta T(z, rk)$, the direction of the tax reform.

ii) If the tax is such that $T(rk, z)$ cannot be expressed as $\tilde{T}_Y(rk + z)$ for some function $\tilde{T}_Y$, then the weights concentrate on those with
$T(z_i, rk_i) = \max_j \{T(z_j, rk_j) | z_j + rk_j = rk_i + z_i\}$, i.e., on the agents which pay the highest tax (equivalently, have the lowest disposable income) conditional on total income.

Equilibria:

Suppose that, at the comprehensive tax rate, no small reform $\delta T(rk.z)$ that introduces horizontal equity and favors capital (according to our definitions above) can increase total tax revenues, i.e., for all $\delta T(rk, z)$ that favor capital and introduce horizontal inequity, the alternative tax system $\tilde{T}(rk, z) = T(rk + z) + \delta T(rk, z)$ is such that:

$$\int_i T_Y(rk_i(T) + z_i(T))di > \int_i \tilde{T}_Y(rk_i(\tilde{T}) + z_i(\tilde{T}))di$$

where naturally, the choices $z_i(T)$ and $k_i(T)$ depend on the tax system T. Then the unique equilibrium has the comprehensive tax system in place, as derived in 2.4.4. No horizontal inequity can be an equilibrium unless it introduces a Pareto improvement.

Suppose on the other hand that if the revenue maximizing tax rate on capital, $T_K^R(rk)$ were implemented, and a labor income tax $T_L(z)$ was used to complement it, more revenue could be raised than with the tax on comprehensive income $T_Y(rk, z)$ and the tax burden on all agents would be lower than under the comprehensive income tax. Then, the optimum is to set differential taxes on capital and labor income, with the capital tax at its optimal revenue-maximizing schedule. Horizontal inequity is an equilibrium because it generates a Pareto improvement.

A.6 Progressive Consumption Taxes

The progressive consumption tax is defined on an exclusive basis as $t_C(.)$ such that

$$\dot{k} = \bar{r}k + z - [c + t_c(c)]$$

Equivalently, we can again define the inclusive consumption tax $T_C(y)$ on pre-tax resources y devoted to consumption such that $c + t_c(c) = y$ is equivalent to $c = y - T_C(y)$, i.e., $y \to y - T_C(y)$ is the inverse function of $c \to c + t_c(c)$ and hence $1 + t'_C = 1/(1 - T'_C)$.

The case of a progressive consumption tax is most easily explained with inelastic labor income (possibly heterogenous across individuals). Real wealth k^r in the presence of the progressive consumption tax is:

$$k^r(k) = k - \frac{T_C(\bar{r}k + z) - T_C(z)}{\bar{r}}$$

Recall that real wealth is defined as nominal wealth adjusted for the price of consumption. There are to see why the above is the right expression. First, wealth k provides an income stream $\bar{r}k$ which translates into extra permanent consumption equal to the income minus the tax paid on the extra consumption $\bar{r}k - [T_C(\bar{r}k+z) - T_C(z)]$ which can be capitalized into wealth k^r by dividing by $\bar{r}$. If labor income is heterogeneous across agents, then $k^r(k, z)$ should also be indexed by z. Another way to see this is to ask what the capital k^r would be that would yield the same disposable income as the nominal capital under the consumption tax. Disposable income in terms of real capital k^r is $\bar{r}k^r - T_C(z)$. Disposable income expressed in terms of nominal capital is: $\bar{r}k - T_C(\bar{r}k + z)$. These two must be equal, which yields the expression for k^r above. k^r has three natural properties: with no consumption tax, real and nominal wealth are equal, $dk^r/dk = 1 - T'_C$, i.e., and extra dollar of nominal wealth is worth $1 - T'_C$ in real terms, and $k^r(0) = 0$.

In that case, we have in steady-state

$$c = \bar{r}k + z - T_C(\bar{r}k + z) = \bar{r}k^r + z - T_C(z)$$

and the first order condition for utility maximization is $a'_i(k^r) = \delta - \bar{r}$. Hence, real capital is chosen to satisfy the same condition as nominal capital when there is no consumption tax. Put differently, any consumption tax will be undone by agents in terms of their savings and will have no effect on the real value of their wealth held (and, hence, by definition of the real wealth, on their purchasing power). Hence, the consumption tax is equivalent to a tax on labor income only.

The equivalence is not exact with elastic labor supply, as in that case, the marginal consumption tax depends on the labor choice and the first-order condition for labor income is $h'_i(z) = 1 - T'_C(\bar{r}k + z) + a'_i(k^r)[T'_C(\bar{r}k + z) - T'_C(z)]/\bar{r}$.