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CAPITAL SHARE DYNAMICS WHEN FIRMS INSURE WORKERS

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ABSTRACT

Although the aggregate capital share for U.S. firms has increased, the firm-level capital share has decreased on average. The divergence is due to the largest firms. While these mega-firms now produce a larger output share, their labor compensation has not increased proportionately. We develop a model in which firms insure workers against firm-specific shocks. More productive firms allocate more rents to shareholders, while less productive firms endogenously exit. Increasing firm-level risk delays the exit of less productive firms and increases the measure of mega-firms, raising the aggregate capital share and lowering it on average. We present evidence supporting this mechanism.

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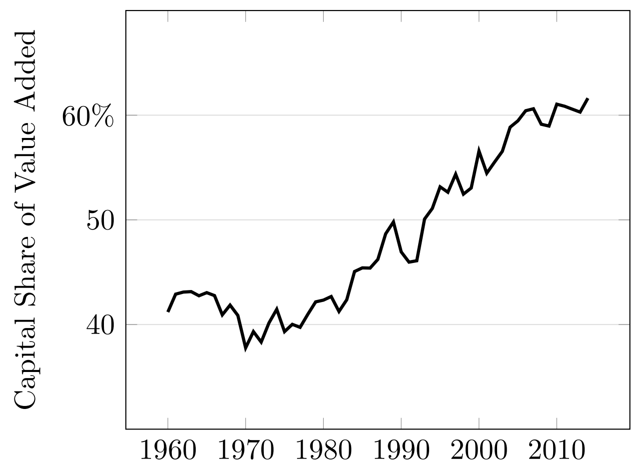
Over the last decades, publicly traded U.S. firms have experienced a large increase in firm-specific volatility of both firm-level cash flow as well as returns (see, e.g., Campbell, Lettau, Malkiel, and Xu, 2001; Comin and Philippon, 2005; Xiaolan, 2016; Bloom, 2014; Herskovic, Kelly, Lustig, and Van Nieuwerburgh, 2015). At the same time, the share of total value added that accrues to the owners of these firms (i.e., the aggregate capital share) has also increased (see Karabarbounis and Neiman, 2014; Piketty and Zucman, 2014). We find that the aggregate factor shares are largely determined by the firm-level factor shares of the largest U.S. firms in the right tail of the size distribution. These mega-firms have experienced substantial increases in their capital share, even though the capital share at the average U.S. firm has decreased.

The aggregate factor share dynamics in the U.S. economy are well understood, but the firm-level factor share dynamics are not. Between 1960 and 2010, the U.S. labor share of total output in the non-farm business sector of the U.S. economy has shrunk by 15%. This phenomenon does not appear to be limited to the U.S. (see, e.g., Piketty and Zucman, 2014). In the universe of U.S. publicly traded firms, we find that the capital share, measured as total operating income divided by total value added (plotted in Figure 1), has increased from 40% to 60% since 1980. Our key empirical contribution is to show that the increase in the capital share is concentrated among the largest, publicly traded firms in the U.S. Figure 2 shows the relationship between firm size and the ratio of capital income to sales (which is a measure of the capital share of profits). In 1970, there was essentially no relation between firm size and capital income to sales ratio. By 2010, this ratio was strongly increasing in size. This shift caused the average and aggregate capital share to diverge: The equal-weighted average capital share of publicly traded companies has declined since the 1980s.

We develop an equilibrium model that links these observations regarding volatility and factor share and provides novel implications for national income accounting. Our model demonstrates that when shareholders insure workers against idiosyncratic risk, capital shares vary substantially over the size distribution of firms, with the largest and most productive firms having the highest capital share. We show that these compositional changes in firm-level capital shares have first-order implications for the aggregate capital share in two ways. First, we show that in the context of our model, a increase in idiosyncratic volatility coupled with an increase in economic rents can quantitatively explain both the shift in the aggregate capital share and the average. Second, we examine the cross sectional implications of our model in the Compustat sample. Consistent with our model, industries that experienced the larger increase in idiosyncratic volatility experienced the larger increase in the capital share.

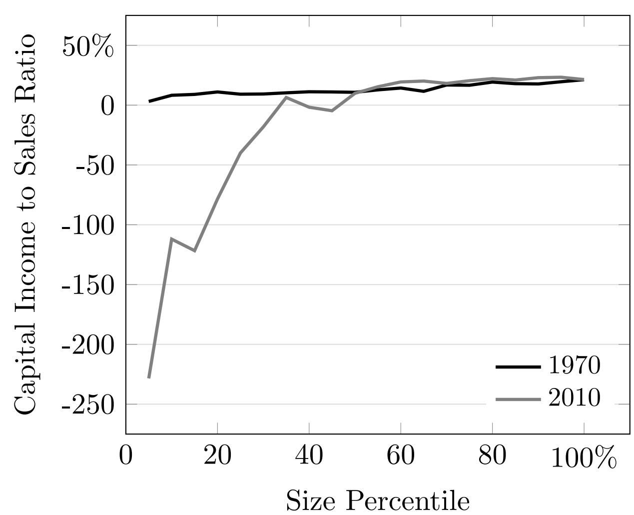
Shareholders of publicly traded firms can diversify away idiosyncratic firm-specific risk, while risk-averse workers cannot; therefore, it is efficient to provide workers with insurance

Figure 1: Aggregate Capital Share of Total Value Added for Public Firms.



The figure presents the aggregate capital share for all firms in the Compustat public firms database. Source: Compustat/CRSP Merged Fundamentals Annual (1960-2014). Aggregate capital share = $\sum_i$ Operating Income$_i$ divided by $\sum_i$ VA$_i$ for each year.

Figure 2: Firm-Level Capital Income to Sales Ratio by Size.



This figure presents the relation between the capital income to sales ratio and firm size for all firms in the Compustat public firms database. Firm size is measured as total assets. Each point represents the within-bin average of the ratio after grouping firms into 20 size bins. Source: Compustat/CRSP Merged Fundamentals Annual (1960-2014).

against firm-specific risk. We analyze a simple compensation contract in an equilibrium model of industry dynamics (see, e.g., Hopenhayn, 1992). This contract pays workers a fixed wage while allocating the remainder of the profits to shareholders. The level of compensation is set in equilibrium to capture the value of ex ante identical firms. Ex post, these firms are subject to permanent idiosyncratic shocks that lead some firms to increase in size and productivity while others decrease and potentially exit. We use this model as a laboratory to analyze the impact of changes in firm-level risk on the distribution of rents.

Standard national income accounting, applied in this model, yields a new perspective on capital share dynamics. The worker's compensation is set such that the net present value of starting a new firm, computed by integrating over all paths using the density for a new firm, is zero. In contrast, national income accounts integrate only over all firms that are currently active using the stationary size distribution, without discounting. As a result, the aggregate capital share calculation puts more probability mass on the right tail than the NPV calculation. As firm-level risk increases and the right tail of the firm size distribution grows, workers capture a smaller fraction of aggregate rents ex post, even though they capture all of the ex ante rents. This effect is partly offset by a larger mass of unprofitable firms in the left tail of the stationary size distribution. However, in our model, an increase in firm-level risk invariably increases the capital share. Only when the workers receive equity-only compensation is the capital share invariant to changes in firm-level volatility.

At the heart of this mechanism is the selection effect that arises by measuring the distribution of rents while excluding firms that have endogenously exited.[1] The capital share computed in national income accounts produces a biased estimate of the ex ante profitability of new firms. Moreover, an increase in selection increases the size of this bias. This effect explains the measured divergence between aggregate compensation and profits: Compensation is tied to ex ante profitability, not to ex post realized profits. This result also has a natural insurance interpretation. When idiosyncratic risk increases, workers effectively pay a larger idiosyncratic insurance premium ex post to shareholders. The increase in this ex post premium leads to an increase in the aggregate capital share, even though the shareholders are risk-neutral and receive zero rents ex ante.

This selection effect is quantitatively important for capital share dynamics. In a calibrated version of our model, we find that an increase in the size of economic rents (see, e.g., Furman and Orszag, 2015), from 20% to 40% together with an increase in volatility

[1] Jovanovic (1982) is the first study of selection in an equilibrium model of industry dynamics. Selection has also been found to be quantitatively important. Luttmer (2007) attributes about 50% of output growth to selection using a model with firm-specific productivity improvements, selection of successful firms, and imitation by entrants. This effect is closely related to Hopenhayn (2002)'s observation that selection biases average Tobin's Q estimates for industries above one.

from 20% to 40% replicates the increase in the aggregate capital share and also replicates the decrease in the average capital share. We note that our calibration exercises compare outcomes in stationary equilibria. Thus, although the increase in idiosyncratic volatility in the data occurs mostly in the earlier half our sample, we should not expect an increase in the capital share in the data to immediately reflect the calibrated increase in the stationary equilibrium.

To provide further evidence in support of our model, we conduct a variety of empirical tests. We show that the aggregate capital share increase is driven by the largest firms by plotting the capital income to sales ratio within firm size quantiles echoing Gabaix (2011)'s observation that we need to study the behavior of large firms to understand macroeconomic aggregates. In the smallest size quantile, the average capital income to sales ratio has decreased from around 10% to less than −100% from 1960 to 2010. In contrast, this same ratio in the largest quantile has remained almost unchanged at around 10%. Thus the capital share has become more dispersed across the size distribution. We also show that this effect is most pronounced in the health products and technology industry, which also experienced a large increase in volatility. To directly asses the link between industry level idiosyncratic volatility and the dispersion in the capital share, we regress the industry level capital income to sales ratio on idiosyncratic volatility. We show that a within-industry increase in idiosyncratic volatility is associated with an decrease in the average capital income to sales ratio.

Our paper intersects with three distinct strands of the literature. First, we use insights from recent work on firm size distribution. In a series of papers, Luttmer (2007, 2012) characterizes the stationary size distribution of firms when firm-specific productivity is subject to permanent shocks.[2] Firms incur a fixed cost of operating a firm. The selection effect of exit at the bottom of the distribution informs the shape of the stationary size distribution, which is a Pareto distribution with an endogenous tail index. Our work explores the impact of changes in the stationary size distribution on the distribution of rents in our laboratory economy.

Second, we embed an optimal risk-sharing contract in our analysis. There is a large literature on optimal risk-sharing contracts between workers and firms (see Thomas and Worrall, 1988; Holmstrom and Milgrom, 1991; Kocherlakota, 1996; Krueger and Uhlig, 2006; Lustig, Syverson, and Nieuwerburgh, 2011; Lagakos and Ordoñez, 2011; Berk and Walden, 2013; Eisfeldt and Papanikolaou, 2013; Xiaolan, 2016). This literature has analyzed the trade-off

[2]Other work on characterizing the firm size distribution includes Miao (2005); Gourio and Roys (2014); Moll (2016). Perla, Tonetti, Benhabib, et al. (2014) examine the endogenous productivity distribution in an environment where firms choose to innovate, adopt new technology, or keep producing with old technology.

between insurance and incentives. We analyze the case of two-sided limited commitment on the part of the firm and the skilled worker, similar to Ai and Li (2015); Ai, Kiku, and Li (2013). There is strong evidence that firms insure workers. Guiso, Pistaferri, and Schivardi (2005) were the first to study insurance within the firm using U.S. microdata, and they find that firms fully insure workers against transitory shocks, but not against permanent shocks (see also Rute Cardoso and Portela, 2009; Fuss and Wintr, 2009; Lagakos and Ordoñez, 2011; Friedrich, Laun, Meghir, and Pistaferri, 2014; Fagereng, Guiso, and Pistaferri, 2017, for foreign evidence). Xiaolan (2016) finds direct evidence of increased cash flow volatility for firms that provide better insurance to workers. Lagakos and Ordoñez (2011) find that the wages of low-skilled workers are more responsive to shocks than those of high-skilled workers. In our model, unskilled labor does not benefit from insurance. In a model with systematic shocks, Eisfeldt and Papanikolaou (2013) show that the outside options of skilled workers increase with positive systematic shocks, which in turn increases the skilled workers' share of firm profits and the riskiness of shareholder equity.

When we introduce moral hazard and other frictions that hamper risk sharing, our mechanism will be mitigated. However, we show that when we allow workers to have some exposure to firm performance, our primary results remain unchanged. The selection mechanism still applies as long as a firm's owners provide some insurance to its workers and as long as the firm can exit when productivity declines. Gabaix and Landier (2008); Edmans, Gabaix, and Landier (2009) find that equilibrium CEO compensation in a competitive market for CEO talent is composed of a cash component and an equity component. For our key results, we analyze the implications of this class of contracts.

Third, our paper contributes to the growing literature on the decline in the labor share of output. Karabarbounis and Neiman (2014) argue that this decline is due to a decrease in equipment prices that leads firms to substitute capital for labor. This mechanism does not predict a divergence between the average labor share and the aggregate labor share that we document in the data. One exception is the introduction of heterogeneous, size-dependent technology choices in the last two decades, but not before that. Elsby, Hobijn, and Şahin (2013) show that labor share decreases the most among industries exposed to import shocks, and this indicates that the decline may be due to the offshoring of labor.[3]

The rest of this paper is organized as follows: Section 1 describes new stylized facts.

[3]In more recent work that is preceded by our paper, others have documented similar evidence: Autor, Dorn, Katz, Patterson, and Reenen (2017) argue that the decrease in labor share is the result of the low labor share at "superstar" firms, while Kehrig and Vincent (2017) also find similar evidence using establishment-level data in the manufacturing industry. This evidence is consistent with ours, in that we also predict that relatively large and productive firms will have high capital shares. However, we provide a model-based explanation that relies on firm-level risk and selection.

Section 2 describes the benchmark model that we use as a laboratory. Section 3 considers a simple endowment version of this economy in which workers are completely insured. We derive the stationary firm size distribution in this benchmark model, and we describe its implications for the aggregate capital share. Section 4 considers a large class of compensation contracts that allow for performance sensitivity. Finally, Section 5 analyzes the capital share in the full version of our economy with unskilled labor and physical capital. Section 6 uses a calibrated version of our economy as a laboratory to explore the quantitative effect of changes in volatility on factor shares. Finally, Section 7 presents new empirical evidence on U.S. capital share dynamics, and we conclude by showing that compensation inequality has not kept pace with size inequality.

1 New Stylized Facts: U.S. Factor Share Dynamics, Volatility and Firm Size Distribution

1.1 Factor Share Dynamics

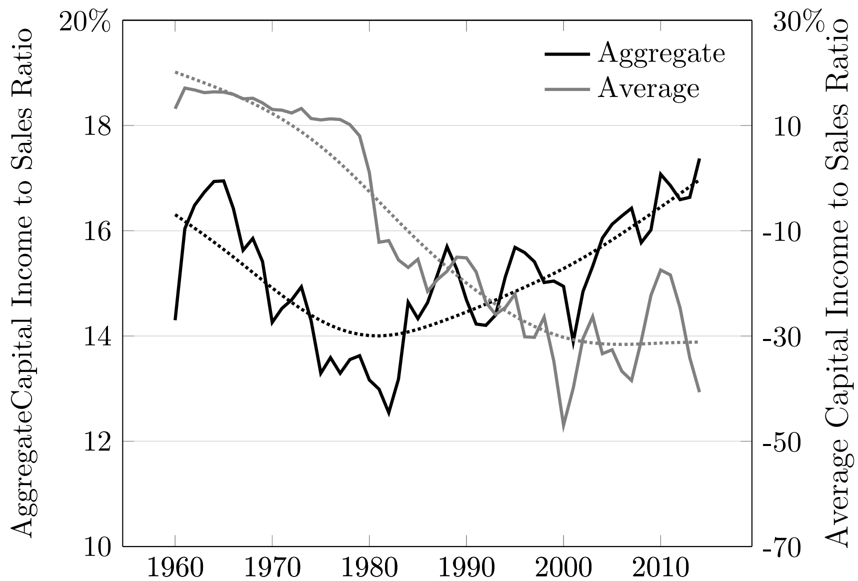
To measure capital share at the firm level, we use widely available accounting data from the Compustat/CRSP Merged Fundamentals Annual. The sample extends from 1960 to 2014. We exclude financial firms that have SIC codes in the interval 6000-6799, and we exclude firms whose sales, employee numbers and total asset values are negative. We measure the aggregate capital share of output as the ratio of aggregate capital income to aggregate value added. Capital income is measured as operating income before depreciation (OIBDP); OIBDP equals sales minus operating expenses including the cost of goods sold, labor costs, and other administrative expenses. Value added is computed as the sum of OIBDP and XLR, which records staff expenses. We provide further details on the data in Appendix A.

We start by examining the time series dynamics of the capital share of U.S. publicly traded firms as measured in the Compustat/CRSP Merged Fundamentals dataset. As we document in Figure 1, the aggregate capital share for this firms has increased from 41% to 62% between 1970 and 2010.

However, this trend is not operative for a typical U.S. firm. To demonstrate this, we analyze firm-level data. Our measure of *firm-level* value added can be negative. Since capital income can also be negative, the ratio of capital income to value added at the firm level is not an informative measure of the firm's capital share that can be readily compared across firms.[4] Instead, we use the ratio of capital income to sales as a proxy for *firm-level* capital shares

[4] Over the past decade, 15% of public firms have negative value added. Dropping negative value-added firms arbitrarily truncates the left tail of firm size distribution.

Figure 3: Average and Aggregate Capital Income to Sales Ratio



The capital income to sales ratio equals the ratio of operating income to sales. Aggregate capital income to sales ratio = $\sum_i$ Operating Income$_i$ divided by $\sum_i$ Sales$_i$ for each year. Average capital income to sales ratio = mean (Operating Income divided by Sales) for each year. The dotted lines are the HP-filtered trends. Source: Compustat/CRSP Merged Fundamentals Annual (1960-2014).

in our firm-level analysis to deliver a well ordered estimate. Our key empirical contribution is to show that the increase in the capital share is concentrated among the largest, publicly traded firms in the U.S. Figure 2 shows the relationship between firm size and the ratio of capital income to sales (which is a measure of the capital share of profits). In 1970, there was essentially no relation between firm size and capital income to sales ratio. By 2010, this ratio was strongly increasing in size. This shift caused the average and aggregate capital share to diverge: The equal-weighted average capital share of publicly traded companies has declined since the 1980s.

This is confirmed in Figure 3, which plots the average and aggregate capital share as a fraction of sales in the sample of publicly traded firms. The average ratio of capital income to sales is the cross-sectional mean of the capital income to sales ratio for a given year. The aggregate ratio equals the sum of capital income (OIBDP) across all the firms divided by aggregate sales. The large declines in the average capital income to sales ratio are driven mostly by small firms that have negative operating margins. The average ratio drops from 13% in 1960 to -40% in 2014, while the aggregate ratio increases with a less dramatic scale, from 14% in 1960 to 17% in 2014. The initial decline in the aggregate capital income to sales ratio disappears when we correct the capital income measure for the expensing of R&D by adding it back to operating income. All of our empirical results remain robust to this adjustment.[5]

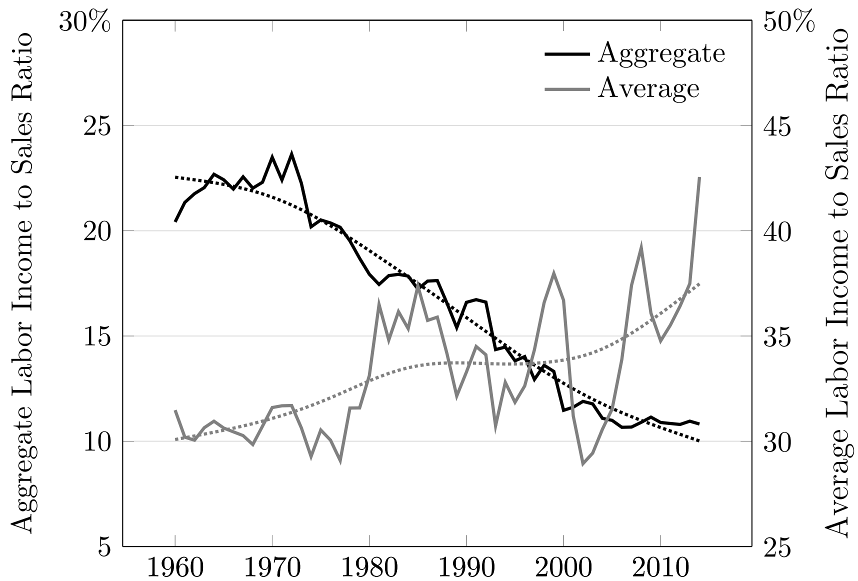
We also compute measures of the labor income share. One drawback of the Compustat data is the lack of comprehensive labor expense data: XLR in Compustat is sparse, having only roughly 13% firm-year observations in the sample. To address this weakness, we adopt Donangelo (2016)'s imputation procedure to construct the extended labor cost (extended XLR) for firms that failed to report staff expenses. To implement this measure, we group firms into one of 17 industries and then sort them into 20 size groups within an industry based on their total assets, thus obtaining a total of 340 industry-size cells. We first estimate the average labor cost per employee (XLR/EMP) within each industry/size cell for each year using the available XLR observations. We then use this estimate to impute labor costs to firms that have missing XLR data as the number of employees times the average labor cost per employee of the same industry/size cell during that year.[6]

The aggregate labor income to sales ratio in the non-farm business sector has declined by

[5]The empirical evidence using capital income to sales ratios after correcting income for R&D expenses is available upon request. We did not use this adjusted measure as our main measure, because R&D expenses include wages to R&D employees.

[6]We follow Donangelo (2016) and use the Fama-French 17 industry classifications. The result is robust to using 2-digit SIC codes. To check that our results are not an artifact of this imputation procedure, we also report the capital income as a fraction of sales. This measure of the capital share does not rely on the imputation.

Figure 4: Aggregate and Average Labor Income to Sales Ratio



Labor income to sales ratio is the ratio of estimated staff expenses (XLR) to sales. Aggregate labor income to sales ratio = $\sum_i \text{XLR}_i$ divided by $\sum_i \text{Sales}_i$ for each year. Average labor income to sales ratio = mean (XLR divided by Sales) for each year. The dashed lines are the HP-filtered trends. Source: Compustat/CRSP Merged Fundamentals Annual (1960-2014).

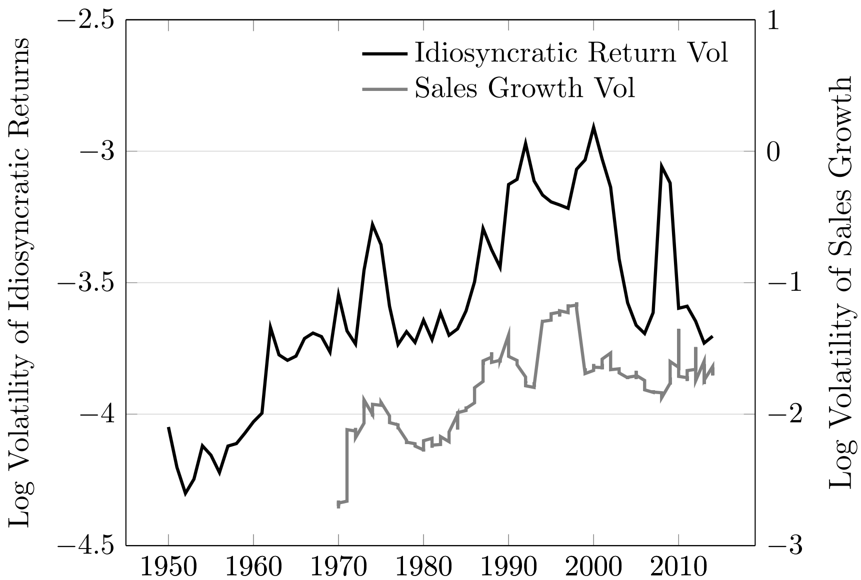
15%. However, the average labor share of output did not decline in our sample of publicly traded firms. Figure 4 shows the time series of both the average and the aggregate labor income to sales ratios in our sample using the estimated labor cost. The average labor share rises from 32% in 1960 to 40% in 2014, while the aggregate labor share (labor income to sales ratio) drops from 22% to 11% during the same period. Our alternative measure of aggregate labor share, the ratio of labor income to value added, also declines over the same sample period, from 59% to 40% (1 minus capital share from Figure 1). Our measure of the ratio of labor income to value added is lower than the BEA's, because our sample include only publicly traded firms. As we show in section B of the appendix, the aggregate labor share in the non-publicly-traded sector actually increased over the last few decades. As a consequence, the drop in the aggregate labor share that we record in the publicly traded sector exceeds that for the U.S. economy as a whole.

1.2 Firm-level Volatility and the Firm Size Distribution

Over the same period, U.S. firms experienced a dramatic increase in volatility. Figure 5 plots the log of firm-level volatility, computed as the equal-weighted average of the log volatility of the idiosyncratic component of stock returns or cash flows for all U.S. publicly traded firms. These volatility measures have more than doubled over the period 1950-2000 (see, e.g., Campbell et al., 2001; Comin and Philippon, 2005; Xiaolan, 2016; Bloom, 2014; Herskovic et al., 2015). After 2000, firm-level volatility levels off. Naturally, firm-level volatility informs the firm size distribution. As the volatility increases, firms in the right tail grow larger. Figure 6 indicates that over the time period 1960 to 2010, the power law coefficient, which measure the probability of firm size exceeding a threshold $\xi : Pr(\text{Size} > \bar{x}) \propto x^{-\xi}$, decrease from 1.37 (1.61) to 0.91 (1.14) when measuring size by total assets (sales).

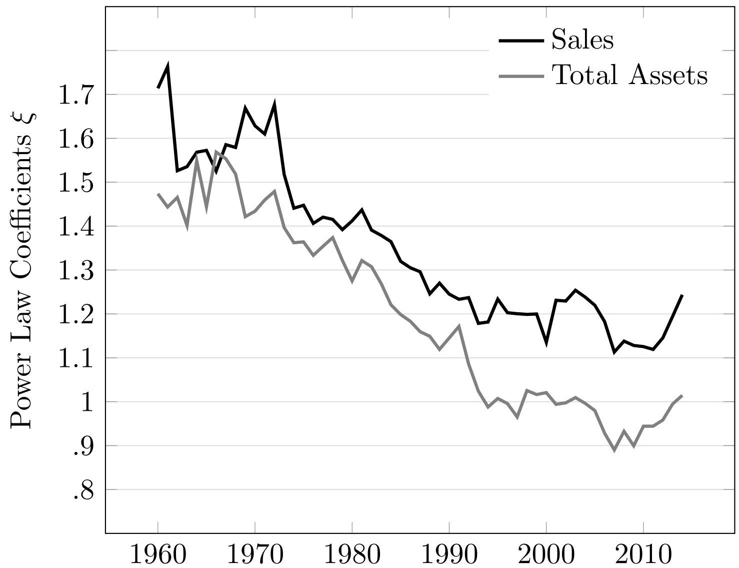
To summarize, we document a divergence in the moments of the firm-level capital and labor share distribution that is broadly consistent with the mechanism we now highlight in our model. Specifically, the trends we observe in the data are consistent with changes in firm-level volatility causing a shift in the distribution of firm size that favors the owners of capital. As firm-level volatility increases, the aggregate and average capital shares in the model will diverge.

Figure 5: Firm-Level Volatility of U.S. Public Firms



The black line indicates annualized idiosyncratic firm-level stock return volatility. Idiosyncratic returns are constructed within each calendar year by estimating a Fama French 3-factor model using all observations within the year. Idiosyncratic volatility is then calculated as the standard deviation of residuals of the factor model within the calendar year. We obtain the time series of idiosyncratic volatility by averaging across firms at each year. The gray line indicates the firm-level cash flow volatility estimated for all CRSP/Compustat firms using the 20 quarterly year-on-year sales growth observations for the calendar years. The idiosyncratic sales growth is the standard deviation of residuals of a factor specification. The factors for sales growth are the first three major principal components. Source: CRSP 1960-2014 and Compustat/CRSP Merged Fundamentals Annual 1950-2014.

Figure 6: Power Law Coefficient of Firm Size: 1960-2014



The variable ξ is the power law exponent: $Pr(size > x) = kx^{-\xi}$, where x is *Total Assets* or *Sales* of the firm. We estimate ξ year-to-year. Following literature, we estimate the slope of right tail using the top n firms in a given year, and n is identified by the top 95% cutoff in x. Source: Compustat/CRSP Merged Fundamentals Annual (1960-2014).

2 A Dynamic Model of Industry Equilibrium with Entry and Exit

In this section, we present a model to rationalize the facts we present in Figures 1 and 2 and Section 1. In our model, firms produce cash flows according to a simple production function. Importantly, the shareholders of a given firm hold an option to cease operations when productivity falls. This is the classic abandonment option. As is standard in the real options literature, increasing the volatility of the firms' cash flows increases the value of the option to wait to abandon, thus lowering the threshold of productivity at which the firm ceases operations.

We characterize the stationary distribution of firms, given the solution to the optimal abandonment problem. Increasing (idiosyncratic) cash flow volatility leads more firms to delay abandonment and to survive long enough to become highly productive. Thus, the average of the capital share of profits across firms increases in volatility.

2.1 Technology and Preferences

The economy is populated by a measure of ex ante identical firms, and each firm operates a standard production technology. A given firm i with productivity X_{it} has a single skilled worker, rents physical capital K_{it}, and employs unskilled labor L_{it}. The total output produced by this firm is given by

$$Y_{it} = X_{it}^{\nu} F(K_{it}, L_{it})^{1-\nu},$$

where F is homogeneous of degree one and $0 < \nu < 1$. The parameter ν governs the decreasing returns to scale at the firm level. Lucas refers to ν as the span of the control parameter of the firm's manager. Atkeson and Kehoe (2005) show that a decrease in competition in a model with imperfect competition is equivalent to an increase in ν in our model, and thus we interpret ν as a measure of the level of economic rents in the economy. The aggregate supply of physical capital and unskilled labor is denoted by k and l, respectively.

Firm productivity evolves according to

$$dX_{it} = \mu X_{it} dt + \sigma X_{it} dZ_t^i - X_{it} dN_{it}; \quad \text{for } X_{it} > X_{\min}, \tag{1}$$

where Z_{it} is a standard Brownian motion independent across firms, N_{it} is a Poisson process with intensity λ, and $X_{\min} > 0$ is some minimum level of productivity. If $dN_{it} = 1$, or if X_{it} reaches $X_{\min}$, X_i jumps to zero and the firm exits. The process N_{it} gives rise to what is often referred to as an exogenous death rate of firms, and it is necessary to guarantee the existence of a stationary distribution of firms for all parametrizations of the model. Since all firms are identical up to their current level of productivity, we omit the subscript i for the remainder of the discussion.

Each firm is owned by an investor, and each firm requires one skilled worker to operate. We assume that investors are risk-neutral and discount cash flows at the risk-free rate of $r > \mu$, while skilled workers value a stream of payment $\{c_t\}_{t\geq 0}$ according to the following utility function:

$$U(\{c_t\}_{t\geq 0}) = E\left[\int_0^{\infty} e^{-rt} u(c_t) dt\right],$$

where $u'(c) \geq 0$ and $u''(c) < 0$. We normalize the measure of skilled workers in the economy to one.

Firms can enter and exit the economy at the discretion of their owners. When a firm exits, its owner receives the liquidation value of the firm, which we normalize to zero, and its skilled worker immediately re-enters the skilled labor market. There is a competitive fringe of shareholders waiting to create new firms. When a shareholder creates a new firm, she matches with a skilled worker, then pays a cost P for the technology blueprint to begin production. After creating a new firm, the firm's initial productivity is drawn from a Pareto distribution with a density of

$$f(X) = \frac{\rho}{X^{1+\rho}}; \quad X \in [X_{\min}, \infty).$$

This distribution implies that the log-productivity of an entering firm is exponentially distributed with parameter $\rho > 1$, and it simplifies the characterization of equilibrium that follows. We denote the rate at which new firms are created by ψ_t. Note that this implies that the entry rate at a given point X is $\psi_t f(X)$.

Upon matching with a skilled worker, an investor in a new firm offers a long term contract to the skilled worker before the realization of the firm's productivity and the firm's payment of the cost P. The skilled worker can reject the contract, at which point she is instantaneously matched with a new firm. Formally, this contract can be denoted by a process $\{c_t\}_{t\geq 0}$, which determines a payment to the skilled worker of c_t at time t. We assume that the investor cannot commit to continue operations or to pay the skilled worker after the firm has ceased operations. We also assume that the skilled worker can choose to exit the contract and match with a new firm at any time, and we assume that she does not have access to a savings technology. This contracting environment features a two-sided limited commitment problem similar to Ai et al. (2013) and Ai and Li (2015). Importantly, the outside option of the skilled worker will depend on the value of starting a new firm, which is endogenously determined in equilibrium. Eisfeldt and Papanikolaou (2013) consider a similar mechanism to explore the implications of the division of the surplus between shareholders and skilled workers for the cross-section of returns.

2.2 The Investors Problem

We denote the utility that the skilled worker receives upon entering this market by U_0, which is also the skilled worker's reservation utility. At the inception of the contract, the investor and the skilled worker take U_0 as exogenously given, although it will be determined in equilibrium by the market for skilled workers. The investor will continue operations as long as doing so yields a positive present value. This means that the investor's value for operating the firm is the solution to a standard *abandonment option*. Specifically, the investor operates the firm until a stopping time denoted by τ. The investor's problem is thus

$$\max_{K,L,\tau,c} E\left[\int_0^\tau e^{-rt}(Y_t - c_t - \kappa K_t - wL_t)dt\right], \tag{2}$$

such that

$$U_0 \leq E\left[\int_t^\tau e^{-r(s-t)}u(c_s)ds + e^{-r(\tau-t)}U_0\right] \text{ for all } t > 0. \tag{3}$$

Intuitively, the skilled worker's limited commitment constraint given in Equation (3) must be binding as promising her a greater continuation at any point in time can only reduce the investor's value for the firm. As a result, the skilled worker's value for the contract is constant over time, and it is without loss of generality that we restrict attention to contracts that offer the skilled worker a fixed wage of c until the firm exits, at which point the skilled worker re-enters the market and receives her outside option.

2.3 Equilibrium

We focus our analysis on equilibria in which the measure of firms at any given level of productivity is stationary. We denote the stationary distribution of log-productivity by $\phi(x)$, where $x = \log(X)$ throughout.

Definition 1. A stationary equilibrium consists of a rental rate κ for physical capital, a demand for physical capital as a function of productivity $K(X)$, a wage rate w for unskilled labor, a demand for unskilled labor $L(X)$ as a function of X, a compensation c^* for the skilled workers, an entry rate of new firms ψ^*, an exit policy for the shareholder $\bar{X}$, and a stationary distribution $\phi(x)$, such that

1. The exit policy $\bar{X}$ solves the investor's problem given by (2) and (3).
2. The stationary distribution $\phi(x)$ is consistent with the entry rate of new firms of ψ and with the exit policy $\bar{X}$.
3. The markets for physical capital, unskilled labor, and skilled workers clear

$$\int_{X_{\min}}^\infty K(x)\phi(x)dx = k, \quad \int_{X_{\min}}^\infty L(x)\phi(x)dx = l, \quad \text{and} \int_{X_{\min}}^\infty \phi(x)dx = 1.$$

4. Creating a new firm leaves the investor with zero expected NPV:

$$\int_{X_{\min}}^\infty V(X;c)f(X)dX = P.$$

Conditions 1-3 are standard equilibrium conditions. Condition 4 derives from the existence of the competitive fringe of investors waiting to create new firms. If an investor in a new firm offers a contract that leaves her with positive ex ante expected NPV, then the skilled worker will reject it because she can simply re-enter the market and instantaneously match with a new firm. Thus, Condition 4 is equivalent to allocating all the ex ante bargaining power to the skilled worker. This in turn determines the level of skilled worker compensation. An alternative definition for Condition 4 would be to allocate some bargaining power to the investor; however, doing so will not qualitatively change the results.

3 An Endowment Economy

To demonstrate the main forces behind our results, we start by analyzing an endowment version of this economy in which we abstract from physical capital and unskilled labor, or equivalently, setting the level of rents $\nu = 1$. In this version of the economy, firm-level output is determined by firm-level productivity $Y_t = X_t$. Thus, we can simplify the investor's problem to

$$V(X;c) = \max_{\tau} E\left[\int_0^{\tau} e^{-rt}(X_t - c)dt | X_0 = X\right], \tag{4}$$

where $V(X;c)$ is the value of operating a firm with current productivity X, given a skilled worker contract c. The payment c to the skilled worker then acts as a fixed cost or operating leverage. As such, the investor in a given firm will choose to exit if productivity X is low enough, following the classic problems of optimal abandonment considered in the real options literature as in Brennan and Schwartz (1985) or optimal default as in Leland (1994). Without a loss of generality, we can restrict attention to firm exit times that are given by threshold rules of the form

$$\tau = \inf\{t | X_t \leq \bar{X} \text{ or } dN_t = 1\}$$

for some $\bar{X} \geq 0$.

3.1 Equilibrium Analysis

In this section, we characterize the stationary equilibrium of the model and study its implications for national income accounting. We first solve for the firm value function and exit policy of the investor. An application of Ito's formula and the dynamic programming principle imply that $V(X;c)$ must satisfy the following ordinary differential equation:

$$(r+\lambda)V(X;c) = X - c + \mu X \frac{\partial}{\partial X} V(X;c) + \frac{1}{2}\sigma^2 X^2 \frac{\partial^2}{\partial X^2} V(X;c), \tag{5}$$

with the boundary conditions

$$V(\bar{X}(c);c) = 0, \tag{6}$$

$$\frac{\partial}{\partial X} V(\bar{X}(c);c) = 0, \tag{7}$$

$$\lim_{X\to\infty} \left| V(X;c) - \left(\frac{X}{r+\lambda-\mu} - \frac{c}{r+\lambda} \right) \right| = 0. \tag{8}$$

Equations (6) and (7) are value matching and smooth pasting conditions that delineate the optimal exercise boundary for the abandonment option. Equation (8) arises because as X_t tends to infinity, abandonment occurs with zero probability, and the value of the firm must tend toward the present value of a growing cash flow less a fixed cost.

The solution to Equations (5)-(8) is given by

$$\bar{X}(c) = \frac{\eta}{\eta+1} \frac{c(r+\lambda-\mu)}{r+\lambda} \tag{9}$$

$$V(X;c) = \frac{X}{r+\lambda-\mu} - \frac{c}{r+\lambda} - \left(\frac{\bar{X}(c)}{r+\lambda-\mu} - \frac{c}{r+\lambda} \right) \left(\frac{X}{\bar{X}(c)} \right)^{-\eta} \tag{10}$$

where

$$\eta = \frac{\mu - \frac{1}{2}\sigma^2 + \sqrt{(\mu - \frac{1}{2}\sigma^2)^2 + 2(r+\lambda)\sigma^2}}{\sigma^2}$$

is the positive root of the fundamental quadratic for Equation (5). Note that an increase in firm-level volatility σ invariably lowers the abandonment threshold, simply because an increase in volatility raises the option value of keeping the firm alive. This feature of the abandonment option plays a key role in our analysis. Its importance becomes apparent when we discuss the stationary distribution of firm size. Specifically, an increase in firm-level volatility leads to an increase in the mass of firms that delay exit, thus increasing the mass of firms that have low productivity as well the mass of firms that survive long enough to achieve high productivity.

Given the solution for firm value conditional on a skilled worker's wage c, as well as our assumption about the distribution of productivity of new firms, we can solve for the equilibrium compensation in a closed form. We have

$$c^* = \left(\frac{P(r+\lambda)(\rho-1)(\rho-\eta)}{\eta} \left(\frac{\eta(r+\lambda-\mu)}{(\eta+1)(r+\lambda)} \right)^{\rho} \right)^{-\frac{1}{\rho-1}}. \tag{11}$$

The derivation of c^* is given in Section C of the Appendix.

In order for the distribution to remain stationary, the expected change via inflow and outflow in the measure of firms at a given level of x must equal the measure of firms that exogenously die at the rate λ, less the measure of firms that endogenously enter at the rate $\psi g(x)$. This leads to the following Kolmogorov forward equation for the stationary distribution of log productivity $\phi(x)$:

$$\frac{1}{2}\sigma^2\phi''(x) - \left(\mu - \frac{1}{2}\sigma^2\right)\phi'(x) - \lambda\phi(x) + \psi g(x) = 0, \tag{12}$$

where $g(x) = \rho e^{-\rho x}$ is the density of initial log productivity x for entering firms. A similar argument gives a boundary condition for $\phi(x)$ at the exit barrier $\bar{x} = \log \bar{X}$

$$\phi(\bar{x}) = 0. \tag{13}$$

The final equation that determines the stationary distribution of firm size is given by the market clearing condition for skilled workers:

$$\int_{\bar{x}}^{\infty} \phi(x)dx = 1. \tag{14}$$

The solution to equations (12)-(14) is given by

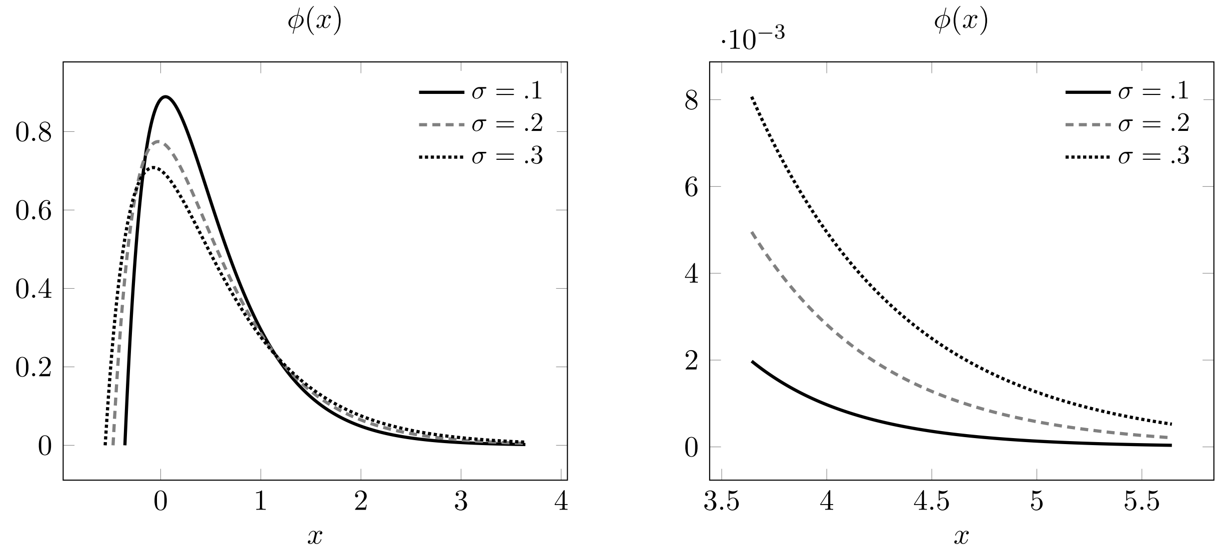
$$\phi(x) = \frac{\rho\gamma}{\rho-\gamma}\left(e^{-\gamma(x-\bar{x})} - e^{-\rho(x-\bar{x})}\right) \tag{15}$$

for $x \in [\bar{x}, \infty)$, where $\gamma = \frac{-(\mu-\frac{1}{2}\sigma^2)+\sqrt{(\mu-\frac{1}{2}\sigma^2)^2+2\sigma^2\lambda}}{\sigma^2}$. This solution also allows us to characterize the aggregate entry rate of new firms:

$$\psi = \frac{\gamma(\rho(\mu-\frac{1}{2}\sigma^2)+\frac{1}{2}\rho^2\sigma^2-\lambda)}{\rho-\gamma}e^{\rho\bar{x}}. \tag{16}$$

We note that our assumption about the density of productivity of entering firms allows for the simple closed form solutions shown above. The general solution to the ODE given in Equation (12) is exponential. By assuming that $g(x)$ is exponential as well, we are left with a solution to Equation (12) for which it is possible to solve the boundary condition given in

Figure 7: The stationary distribution of log-productivity



Parameter values: $\sigma = .1, .2, .3$, $r = 5\%, \mu = 2\%, \lambda = .05, \rho = 3$, and $P = 1$.

Equation (13).

Figure 7 plots the stationary distribution of firm productivity for different levels of σ. The other parameters are calibrated at $r = 5\%, \mu = 2\%, \lambda = .05, \rho = 3, P = 1$. As σ increases, the stationary distribution shifts to the left and becomes more diffuse, with a fatter right tail. This shift to the left is due to the fact that as firm-level volatility increases, the value of the option to wait to exit also increases, and the optimal point at which the investor chooses to exit necessarily decreases.

The effect of firm-level volatility on the shape of $\phi(x)$ visible in figure 7 is borne out by examining the higher-order moments of $\phi(x)$. Table 1 reports the standard deviation, the skewness, and the kurtosis of the log size distribution as we increase σ. As σ increases, the right skewness increases from 0.12 to 2.74, and the excess kurtosis of the log size distribution increases from 0.15 to 7.31. This overall widening of the distribution, with a fattening of the right tail originates from two effects: First, there is a direct effect of σ on the dispersion of the distribution of firm size. When firm-level productivity is more volatile, the stationary distribution of firms must be more dispersed. This is evident by examining the dependence of γ on σ. The second effect operates through the abandonment option. When the option to wait to exit becomes more valuable, more firms delay their exit. As a result, more firms survive long enough to become highly productive, and the right tail of the distribution widens. In the next section, we show that this effect has important implications for national income accounting.

Table 1: Higher-order moments of the log-size distribution implied by the model

σ	Mean	Std. Dev.	Skew.	Kurt.
.1	1.879	0.700	0.120	0.151
.2	1.493	0.696	2.186	5.631
.3	1.181	0.789	2.742	7.310

Moments of the stationary distribution of log-productivity for $\sigma = .1, .2$, and .3. Parameter values: $r = 5\%, \mu = 2\%, \lambda = .05, \rho = 3, p = 1$.

3.2 National Income Accounting

Armed with this stationary distribution, we can conduct national income accounting within our model for a range of σ. Specifically, we calculate the aggregate capital share and the average firm's capital share, respectively:

$$\begin{aligned}\text{Capital Share of Profits} = \Pi &= \frac{\int_{\bar{x}}^{\infty}(e^x - c)\phi(x)dx}{\int_{\bar{x}}^{\infty} e^x\phi(x)dx}, \\ &= 1 - \frac{c}{\int_{\bar{x}}^{\infty} e^x\phi(x)dx},\end{aligned}$$

$$\begin{aligned}\text{Average Capital Share of Profits} &= \int_{\bar{x}}^{\infty}\left(\frac{e^x - c}{e^x}\right)\phi(x)dx. \\ &= 1 - \int_{\bar{x}}^{\infty}\frac{c}{e^x}\phi(x)dx,\end{aligned}$$

We note that our expressions for both aggregate and average capital share are gross of the costs of starting new firms. If these costs are included, the expressions become less transparent but the results of the analysis below do not change.

To develop an intuition for the effect of a comparative static change in idiosyncratic volatility on the aggregate capital share, it is useful to decompose the expression into its constituent parts. The denominator of the second term Π is the total profits to all firms in the economy, and it is given by

$$\int_{\bar{x}}^{\infty} e^x\phi(x)dx = \left(\frac{\gamma}{\gamma-1}\right)\left(\frac{\rho}{\rho-1}\right)\bar{X}.$$

The numerator of the second term is the total compensation paid to skilled workers, and it is given by

$$c = \left(\frac{(r+\lambda)(\eta+1)}{\eta}\right)\left(\frac{1}{r+\lambda-\mu}\right)\bar{X}.$$

It suffices to normalize these terms by $\bar{X}$, since it is a common factor in both. As σ increases, the total profits in the economy, normalized by the minimum productivity of active firms $\bar{X}$, increases because the right tail of the stationary distribution of x becomes wider.

Now, let us consider the numerator. The value-matching condition, which pins down $\bar{X}$, implies that the present value of compensation c to a given skilled worker must equal the present value of all future gross cash flows to the firm at the moment that productivity reaches $\bar{X}$. Thus, the expression for c given above states that the total compensation to skilled workers is the present value of all the gross cash flows that are forgone by an exiting firm. This present value, normalized again by $\bar{X}$, also increases in σ for the same reason that total profits increase: There is a greater measure of future paths of the firm that result in high productivity. However, these high future draws of productivity are discounted at the rate r, so their effect on the total compensation paid to skilled workers is smaller than their effect on total profits. This intuition implies that the capital share of profits should be increasing in σ.

To show that this intuition is in fact correct, we can combine the terms above to derive the following simple, closed-form expression for Π:

$$\Pi = 1 - \left(\frac{r+\lambda}{r+\lambda-\mu}\right)\left(\frac{\rho-1}{\rho}\right)\left(\frac{\gamma-1}{\gamma}\right)\left(\frac{\eta+1}{\eta}\right). \tag{17}$$

We can calculate the derivative of Π with respect to the volatility parameter σ:

$$\frac{\partial \Pi}{\partial \sigma} = -\left(\frac{r+\lambda}{r+\lambda-\mu}\right)\left(\frac{\rho-1}{\rho}\right)\left[\left(\frac{\eta+1}{\eta}\right)\frac{1}{\gamma^2}\frac{\partial \gamma}{\partial \sigma} - \left(\frac{\gamma-1}{\gamma}\right)\frac{1}{\eta^2}\frac{\partial \eta}{\partial \sigma}\right]. \tag{18}$$

So, $\partial\Pi/\partial\sigma$ is positive if and only if

$$\eta(\eta+1)\frac{\partial \gamma}{\partial \sigma} \leq \gamma(\gamma-1)\frac{\partial \eta}{\partial \sigma}. \tag{19}$$

It is straightforward to show that $\eta(\eta+1)\frac{\partial \eta}{\partial \sigma} \leq 0$ and $\gamma(\gamma-1)\frac{\partial \gamma}{\partial \sigma} \leq 0$, so to verify (19) is equivalent to verifying

$$\frac{\eta(\eta+1)\frac{\partial \gamma}{\partial \sigma}}{\gamma(\gamma-1)\frac{\partial \eta}{\partial \sigma}} \geq 1. \tag{20}$$

One can show that

$$\frac{\eta(\eta+1)\frac{\partial\gamma}{\partial\sigma}}{\gamma(\gamma-1)\frac{\partial\eta}{\partial\sigma}} = \frac{\sqrt{(\mu-\frac{1}{2}\sigma^2)^2+2(r+\lambda)\sigma^2}}{\sqrt{(\mu-\frac{1}{2}\sigma^2)^2+2\lambda\sigma^2}} > 1, \tag{21}$$

which verifies that $\partial\Pi/\partial\sigma > 0$. Hence, in our model, the aggregate capital share always increases as volatility increases, as long as $r > 0$.

The expression given in Equation (21) validates our intuition about the effect of discounting on the relative sensitivity of both firm value and total stationary profits to changes in idiosyncratic volatility. The strictly positive sign of the comparative static requires that the discount rate r is positive. To understand this effect, it is helpful to consider the limiting case of no discounting. As r approaches zero, the ex ante average value of a firm (normalized by r) approaches the aggregate value of all payments to investors:

$$\begin{aligned}
\lim_{r\to 0}\int_{X_{\min}}^{\infty} rV(X)f(X)dX &= \lim_{r\to 0}\int_{X_{\min}}^{\infty} E\left[\int_0^{\tau} re^{rt}(X_t-c)dt|X_0=X\right]f(X)dX \\
&= \lim_{r\to 0}\int_0^{\infty}\int_{\bar{x}}^{\infty} re^{rt}(e^x-c)\phi_t(x)dxdt \\
&= \int_{\bar{x}}^{\infty}(e^x-c)\phi(x)dx.
\end{aligned}$$

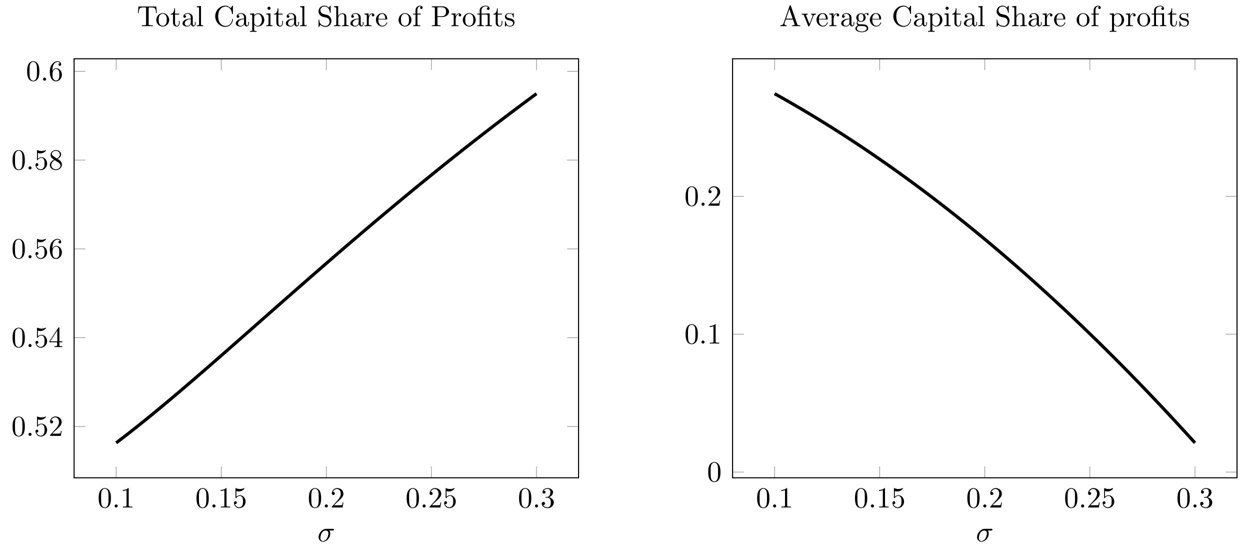
where

$$\phi_t(x) = \frac{\partial}{\partial x}\int_{\bar{x}}^{\infty} E[\mathbb{1}(x_t > x)\mathbb{1}(t \le \tau)|x_t = y]g(y)dy$$

is the distribution of log productivity x_t for a firm, given an initial value drawn from $g(\cdot)$ that is conditional on the firm not having yet exited. Intuitively, as r approaches zero, the present value of all future cash flows is given by the expectation of cash flow in the limit as t approaches infinity (i.e., in the stationary distribution). Returning to our intuition, the total compensation paid to skilled workers is then proportional to total profits, and σ has no effect on the capital share.

Figure 8 plots a numerical example. This figure plots the total and average capital share of profit as a functions of σ. We use the following parameter values: $r = 5\%, \mu = 2\%, \lambda = .05, \rho = 3, p = 1$. We can see that the total capital share of profits is increasing in σ, while the average capital share of profits is decreasing. The intuition is as follows: As σ increases, the value of the option to delay abandonment increases, hence the optimal threshold at which firms exit decreases. Holding the total measure of firms fixed, this means that the distribution of profits becomes more dispersed. The increase in the mass of firms in the right tail of the firm size distribution increases the total profit share because the profit share

Figure 8: The total and average capital share of profit as a function of σ.



Parameter values: $r = 5\%, \mu = 2\%, \lambda = .05, \rho = 3, p = 1$.

measures the ex post profitability of existing firms. This is effectively a selection bias. The profit share of entering firms is determined by setting the NPV of the investor's stake in the firm to zero. This NPV calculation integrates over all possible future paths for firm-level productivity, including those that lead the investor to choose to exit. In contrast, the stationary distribution of existing firms only considers firms that have survived. Surviving firms necessarily have a higher capital share of profits, otherwise the investor would have chosen to exit.

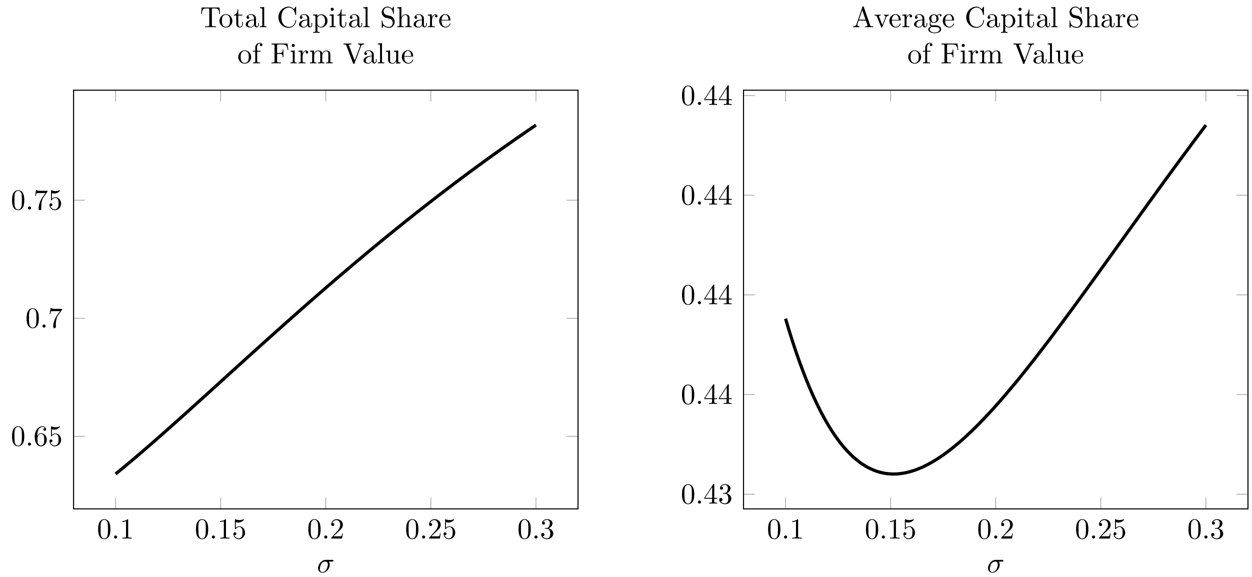
Our model also makes a novel prediction about the capital share at the average firm. The increase in the mass of firms that delay their exit means that more firms will have a low capital share. Thus, an increase in firm-level volatility can decrease the average profit share. This is in contrast to the effect one would expect to see if the increase in the total capital share of profits is due to a higher growth rate in the value of capital relative to wages that may follow the substitution of capital for labor. In that case, one would expect both the total and average capital share to increase. Figure 9 plots the total and average capital share of firm value derived from the model, with similar results as for profits.

Finally, we examine the comparative statics of the capital share with respect to the entry parameter. First, we have

$$\frac{\partial \Pi}{\partial \rho} = -\left(\frac{r+\lambda}{r+\lambda-\mu}\right)\left(\frac{\gamma-1}{\gamma}\right)\left(\frac{\eta+1}{\eta}\right)\frac{1}{\rho^2} < 0.$$

To understand this comparative static, note that an increase in ρ means that the right tail

Figure 9: The total and average capital share of firm value as a function of σ.



Parameter values: $r = 5\%, \mu = 2\%, \lambda = .05, \rho = 3, p = 1$.

of the entry distribution becomes thinner and entering firms are smaller on average. This in turn implies that the capital share decreases, because smaller firms have lower capital shares.

4 Pay for Performance

In this section, we allow for some exposure in the skilled worker's compensation to firm performance. This exposure could arise for a variety of reasons. For example, there could be a firm-level agency conflict between the skilled worker and investors, or the investor could be risk averse. In either case, the optimal contract will call for the skilled worker to bear some exposure to firm performance, either for incentive purposes or to improve risk sharing. The precise form of the optimal contract will depend on the nature of the agency problem or the exact preferences of the skilled workers and investors.[7] One possible concern thus far with our results may be that this exposure could mitigate the insurance nature of the relationship between firms' owners and their skilled workers, thus decreasing or reversing the effect of firm-level volatility on the capital share of profits. Rather than solve directly for an optimal contract for a particular problem, we assume that the skilled worker's contract

[7] Edmans et al. (2009) derive CEO compensation in a competitive equilibrium with a talent assignment and a moral hazard problem.

takes the following simple affine form

$$c_t = \beta X_t + w. \tag{22}$$

The sensitivity β of the skilled worker's payment c_t to the level of productivity is determined by either the severity of the agency problem or the nature of the risk-sharing problem, and it is exogenous from the standpoint of our model. The fixed wage w is set in equilibrium in the same manner as total wages are set above. This contract has the advantage of being particularly tractable to analysis in the context of our model of equilibrium.

For a given fixed wage w, the investor's problem is

$$\max_{\tau} \left[\int_0^{\tau} e^{-rt}((1-\beta)X_t - w)dt \right]. \tag{23}$$

Again, standard arguments imply that the investor's value function $V(X)$ must satisfy the following ODE:

$$(r+\lambda)V = (1-\beta)X - w + \mu X V' + \frac{1}{2}\sigma^2 X^2 V'', \tag{24}$$

with the boundary conditions

$$V(\bar{X}) = 0, \tag{25}$$

$$V'(\bar{X}) = 0, \tag{26}$$

$$\lim_{X\to\infty} \left| V(X) - \left(\frac{(1-\beta)X}{r+\lambda-\mu} - \frac{w}{r+\lambda} \right) \right| = 0. \tag{27}$$

This problem is essentially the same as the problem given in Equations (5)-(8), up to a scaling of the leading term by a factor of $(1-\beta)$. Thus, the solution to Equations (24)-(27) is

$$\bar{X} = \left(\frac{1}{1-\beta}\right)\left(\frac{\eta}{\eta+1}\right)\frac{w(r+\lambda-\mu)}{r+\lambda}$$

$$V(X) = \frac{(1-\beta)X}{r+\lambda-\mu} - \frac{w}{r+\lambda} - \left(\frac{(1-\beta)\bar{X}}{r+\lambda-\mu} - \frac{w}{r+\lambda}\right)\left(\frac{X}{\bar{X}}\right)^{-\eta},$$

where η is defined as above.

Given the solution for the investor's value, we can apply the investor's zero ex ante profit condition to determine the fixed component of the skilled worker's equilibrium contract.

This calculation yields

$$w^* = \left(\frac{P(r+\lambda)(\rho-1)(\rho-\eta)}{\eta} \left(\frac{\eta(r+\lambda-\mu)}{(1-\beta)(\eta+1)(r+\lambda)} \right)^{\rho} \right)^{-\frac{1}{\rho-1}} . \tag{28}$$

Comparing Equations (11) and (28) reveals that the fixed component of the equilibrium affine contract is just the equilibrium wage under full insurance scaled by a function of β. Thus, the investor's problem under the affine contract is identical to the problem under full insurance when the firm's productivity is scaled by a factor of $1-\beta$. The stationary distribution of firm productivity is unaffected by our assumption of affine contracts, up to a shifting of the optimal abandonment threshold (i.e., the left support of the stationary distribution). Thus, we can again calculate the total capital share of profits in the stationary distribution to obtain

$$\Pi = (1-\beta) \left(1 - \left(\frac{r+\lambda}{r+\lambda-\mu} \right) \left(\frac{\rho-1}{\rho} \right) \left(\frac{\gamma-1}{\gamma} \right) \left(\frac{\eta+1}{\eta} \right) \right) . \tag{29}$$

Comparing Equations (17) and (29) shows that the total capital share profits under the affine contract depends on γ and η, hence also on σ in the same manner as the total capital share of profits under full insurance. In other words, allowing the skilled worker to share in the success of successful firms does not change our main qualitative results.

While allowing skilled workers to share in some of the gains of successful firms does not change the aggregate dynamics of the capital share, it does have important implications for the distribution of the labor share across income levels. Income inequality has been rising, as observed by Piketty and Saez (2003) and Guvenen and Kuruscu (2007). Given this fact, the share of output that accrues to the top decile of the income distribution could have increased. That is, the income shares could have become more unequal. Our model is consistent with rising income share inequality when we allow skilled workers to share in the gains of successful firms via the affine contracts we consider in this section. In this case, the distribution of skilled worker pay essentially inherits the properties of the distribution of firm productivity (or size). As volatility increases, the most successful firms account for a larger share of total output. And, since the managers of these firms receive pay in proportion to productivity, their pay is also a larger fraction of total output.

5 The Full Production Economy

In this section, we analyze the full production economy. We maintain our assumptions about the preferences of investors and skilled workers and the structure of entry and exit in the economy from our basic model. Given these assumptions, it is still optimal to offer skilled workers a fixed wage. Also, investors face the same basic exit decision as in the endowment economy, up to an adjustment to net profits for the payments to physical capital and labor. Investors will thus choose to exit when productivity falls below some threshold $\bar{X}$. Note that since firms rent physical capital and unskilled labor in spot markets, a given firm's demand for these inputs will be a function of its current productivity.

5.1 Equilibrium Analysis

To characterize equilibrium, we begin by considering the allocation of physical capital and unskilled labor across active firms. Given spot rates for physical capital and unskilled labor and some current level of productivity, a given firm chooses capital and labor to maximize profits net of the rental payments to physical capital and wages to unskilled labor:

$$(K_t, L_t) = \arg\max_{K,L} \left\{ X_t^{\nu} F(K, L)^{1-\nu} - wL - \kappa K \right\}.$$

The homogeneity of the production function F implies that the solution (K_t, L_t) of the maximization above is linear in X_t. Market clearing then implies that physical capital and unskilled labor are allocated across firms according to the following linear allocation rule:

$$K_t = \frac{k}{\widehat{X}} X_t,$$
$$L_t = \frac{l}{\widehat{X}} X_t,$$

where

$$\widehat{X} = \int_{\bar{x}}^{\infty} e^x \phi(x) dx$$

is the average productivity in the economy given the stationary distribution of log productivity $\phi(x)$. This allocation rule implies that the output of any given firm is a linear function of aggregate output:

$$Y_t = \frac{y}{\widehat{X}} X_t,$$

where $y = \widehat{X}^{\nu} F(k, l)^{1-\nu}$ is aggregate output. As a result, a firm's gross earnings (operating profits) are proportional to X_t:

$$Y_t - wL_t - \kappa K_t = \frac{\nu y}{\widehat{X}} X_t.$$

For convenience, we let $\widehat{F} = \frac{\nu y}{\hat{X}}$. We refer to $\widehat{F}$ as the equilibrium rents normalized by (average) productivity $\hat{X}$.

Having determined the allocation of physical capital and unskilled labor, we can now analyze the investor's optimal abandonment decision. Thus, we can simplify the investor's problem to

$$V(X; c, \hat{F}) = \max_{\tau} E\left[\int_0^{\tau} e^{-rt}\left(\widehat{F}X_t - c\right) dt | X_0 = X\right], \tag{30}$$

where $V(X; c, \hat{F})$ is the value of operating a firm with current productivity X given a skilled worker contract c and rents $\widehat{F}$. The solution technique for the investor's problem is essentially the same as in the case with constant physical capital and labor up to a change in the coefficients in the ODE and determination of the optimal abandonment threshold. Given c and $\widehat{F}$, $V(X; c, \widehat{F})$ must satisfy the following ordinary differential equation

$$(r+\lambda)V(X; c, \widehat{F}) = \widehat{F}X - c + \mu X \frac{\partial}{\partial X} V(X; c, \widehat{F}) + \frac{1}{2}\sigma^2 X^2 \frac{\partial^2}{\partial X^2} V(X; c, \widehat{F}), \tag{31}$$

with the boundary conditions

$$V(\bar{X}(c, \widehat{F})); c, \widehat{F})) = 0, \tag{32}$$

$$\frac{\partial}{\partial X} V(\bar{X}(c, \widehat{F})); c, \widehat{F})) = 0, \tag{33}$$

$$\lim_{X\to\infty}\left| V(X; c, \widehat{F}) - \left(\frac{\widehat{F}X}{r+\lambda-\mu} - \frac{c}{r+\lambda}\right)\right| = 0, \tag{34}$$

where $\bar{X}(c, \widehat{F})$ is the abandonment threshold given c and $\widehat{F}$. Conditions (32) and (33) are the standard value-matching and smooth-pasting conditions, while Condition (34) arises because as X_t tends toward infinity, abandonment occurs with zero probability, as in the simple model we analyzed above. The smooth-pasting condition (33) need only hold if $\bar{X}(c, \widehat{F}) > X_{\min}$.

The solution to Equations (31)-(34) is given by

$$\bar{X}(c, \widehat{F}) = \left(\frac{\eta}{\eta+1}\right)\left(\frac{r+\lambda-\mu}{r+\lambda}\right)\left(\frac{c}{\widehat{F}}\right) \tag{35}$$

$$V(X; c, \widehat{F}) = \frac{\widehat{F}X}{r+\lambda-\mu} - \frac{c}{r+\lambda} - \left(\frac{\widehat{F}\bar{X}(c, \widehat{F})}{r+\lambda-\mu} - \frac{c}{r+\lambda}\right)\left(\frac{X}{\bar{X}(c, \widehat{F})}\right)^{-\eta}, \tag{36}$$

where η is as defined above.

Next, we consider equilibrium compensation. As in the endowment economy, c is set so as to give zero surplus to the investors for starting a new firm. Given the solution to the investor's value function and the Pareto entry distribution of new firms, it is straightforward to solve the investor's ex ante zero profit condition for the equilibrium c. We have

$$c^* = \left(\frac{P(r+\lambda)(\rho-1)(\rho-\eta)}{\eta} \left(\frac{\eta(r+\lambda-\mu)}{(\eta+1)(r+\lambda)\hat{F}} \right)^{\rho} \right)^{-\frac{1}{\rho-1}}. \tag{37}$$

Comparing the equilibrium managerial wage in the production economy to the endowment economy, we see that the two are identical up to an adjustment for equilibrium rents $\widehat{F}$.

5.2 Stationary Size Distribution

Finally, we consider the equilibrium distribution of productivity $\phi(x)$ as well as rents $\widehat{F}$, given an exit threshold of $\bar{X}$. Note that since X_t has the same dynamics as in the endowment economy model, the form of the stationary distribution for productivity is unchanged. The equilibrium average productivity is then

$$\widehat{X}(\bar{X}) = \frac{\bar{X}\gamma\rho}{(\gamma-1)(\rho-1)}. \tag{38}$$

This in turn implies that equilibrium rents are

$$\widehat{F} = \nu \left(\frac{\bar{X}\gamma\rho}{(\gamma-1)(\rho-1)} \right)^{\nu-1} F(k,l)^{1-\nu}. \tag{39}$$

An equilibrium is then characterized by a solution $(\bar{X}, \widehat{F})$ to Equations (35), (37), and (39). One can show that such a solution exists and is unique.

5.3 National Income Accounting

As in the endowment model, we can conduct national income accounting within our model. Specifically, we can calculate the aggregate capital share of output as

$$\begin{aligned} \text{Capital Share of Output} = \Pi &= \frac{y - wl - c}{y} \\ &= 1 - (1-\nu)(1-\alpha(k,l)) - \frac{c}{y}, \end{aligned}$$

where $1 - \alpha(k,l) = \frac{1}{F(k,l)^{-1}} \frac{\partial F(k,l)}{\partial l}$ is the elasticity of the production function F with respect to unskilled labor. In other words, the capital share of output is one minus the total labor share, where the labor share aggregates the share of output that accrues to unskilled and skilled labor. Using the definition of $\widehat{F}$ and Equations (35) and (38), we can express the total output as

$$y = \frac{\widehat{F}\widehat{X}}{\nu} = \left(\frac{\gamma\rho}{(\gamma-1)(\rho-1)}\right)\left(\frac{\eta}{\eta+1}\right)\left(\frac{r+\lambda-\mu}{r+\lambda}\right)\left(\frac{c}{\nu}\right), \tag{40}$$

so that the total output y is linear in c. Thus, the total capital share of output simplifies to

$$\Pi = 1-(1-\nu)(1-\alpha(k,l))-\nu\left(\frac{r+\lambda}{r+\lambda-\mu}\right)\left(\frac{\rho-1}{\rho}\right)\left(\frac{\gamma-1}{\gamma}\right)\left(\frac{\eta+1}{\eta}\right). \tag{41}$$

This expression is essentially the same as in the endowment economy less the unskilled labor share of output and an adjustment to the skilled worker's share of output for the elasticity of output with respect to productivity. Importantly, the comparative static of total capital share with respect to idiosyncratic volatility σ will have the same positive sign in both the endowment economy and the production economy. Intuitively, the share of output devoted to unskilled labor is determined by the shape of the production function and does not depend on aggregate rents or production, except through the aggregate quantity of physical capital and unskilled labor. At the same time, the *share* of output devoted to the skilled workers is determined by the equilibrium exit policy of firms, and it does not directly depend on aggregate production. Thus, the capital share of output does not directly depend on aggregate output except through the aggregate quantity of physical capital and unskilled labor.

6 Quantitative Experiments in the Calibrated Model

In this section, we explore the quantitative implications of our model. We calibrate the economy to match the empirical moments of the distribution of the capital share of output across firms in the U.S. Compustat sample. We then consider the effects of changes in the underlying parameters to quantify the effect of our mechanism on the aggregate and average capital share, as well as labor share.

6.1 Calibration

We first calibrate the model to match the aggregate moments from the sample of U.S. publicly traded firms over the period from 1960 to 1970. Panel A in Table 2 reports the moments we set out to match. One caveat is that we are not able to use value added to compute the firm-level capital share because the large portion of firms have negative value

Table 2: Benchmark Calibration

The table reports our benchmark calibration. Panel A reports target moments in the data and the implied moments from our production model. The data moments are computed from the sample Compustat/CRSP Merged Fundamentals Annual from 1960 to 1970. The sample excludes firms that have SIC codes from 6000 to 6799. Panel B reports the calibrated parameters. Panel C reports the preset parameters. Firm-level value added VA_i is OIBDP plus Extended XLR. To deal with negative values, we identify the minimum operating income (OIBDP) for each year, and we increase the value added of all firms by the absolute value of the minimum OIBDP$\times$(1+1%). The average capital share is computed using OIBDP divided by the adjusted value added. The standard deviation and skewness of the capital share is also estimated using the adjusted value added measure. The aggregate capital share is calculated using the unadjusted value added. The capital share at the exit threshold $\bar{X}$ is measured as the average capital shares three years prior to delisting.

Panel A: Capital Share Moments 1960-1970		
	Data	Model
Average Capital Share	0.208	0.246
Aggregate Capital Share	0.419	0.318
Standard Deviation of Capital Share	0.152	0.082
Capital Share at Exit	0.076	0.064
Entry Rate	-	0.013
Panel B: Calibrated Parameters		
σ	0.2	Idiosyncratic Vol
μ	0.02	Firm Growth
λ	0.055	Exogenous Exit Rate
ρ	3.5	Entrants Firm Size Distribution
α	0.27	Aggregate Physical Capital Share of Output
Panel C: Preset Parameters		
r	0.05	Discount Rate
k/l	1	Capital/Labor Ratio
p	1	Sunk Cost
ν	0.2	Share of Rents in GDP (Atkeson and Kehoe, 2005)

added. When estimating the average capital share for calibration, we use the adjusted value added instead of sales at the firm level as denominators. This allows us to obtain the empirical moments of the firm-level capital share that are more consistent with our theoretical counterpart without having to drop the negative value added observations.[8] Further details on the data are provided in Appendix A.

Our calibrated model successfully matches the average and aggregate capital share of output, and the capital share at the exit threshold. Panel B reports the parameters we chose to match these moments. Panel C reports preset parameters. We set the volatility $\sigma = 0.2$ to match the average idiosyncratic sales volatility. The drift parameter μ is chosen to match

[8]Although in the empirical exercise, we use income to sales ratios to proxy factor shares, our model does not have the theoretical counterpart of income to sales ratios.

the average TFP growth rate of 2%. We calibrate the death rate λ and the parameter ρ, which governs the entry distribution, to match the cross-sectional standard deviation of the firm-level capital share and the capital share at the exit threshold $\bar{X}$. Our calibration of λ and ρ also produces a reasonable exit rate or entry rate of the public firms at 1.34%; the average IPO rate is 3.4% in the 1980-2015 sample.[9] We chose $\nu = 0.2$, the share of GDP accounted for by rents, following Atkeson and Kehoe (2005).

Our baseline calibration considers the case of full insurance for skilled workers. As a robustness check, we calibrate a version of the full production economy in which the skilled workers sign performance-sensitive contracts; the results are reported in Appendix D. The main quantitative results survive.

6.2 Quantitative Experiments

Next, we use the benchmark calibration of the model to conduct the series of experiments reported in Table 3. In Panel A, we report the changes in firm-level capital share distribution when increasing volatility σ, the entry parameter ρ, and the rent share ν respectively. The parameter σ is 20% per annum in the benchmark calibration. Firm-level volatility has increased dramatically over the past five decades (see Comin and Philippon, 2005; Xiaolan, 2016; Herskovic et al., 2015).

Volatility When we double the volatility, the model predicts a decline in the average capital share of output of 6.6 percentage points and an increase in the aggregate capital share of output of 1.4 percentage points. These numbers mask large changes in the distribution of rents. In the benchmark calibration of our model, the owners only collect 13.9% of total rents at the average firm, but they collect 49.7% of aggregate rents. To translate the change in the capital share of output to a change in the capital share of rents, we must consider that these rents are a fraction of the output given by ν. Thus, doubling volatility increases the aggregate share of rents collected by owners by 7.2 pps. (roughly 1.4 pps divided by $\nu = .2$), while decreasing the average share of rents by 33 pps (roughly 6.6 pps divided by $\nu = .2$).

Discount Rate We use a discount rate of 5% in the baseline calibration. Our analytical results imply that a larger discount rate would amplify the effects of changes in volatility on the aggregate capital share. To demonstrate this, we also report moments when the discount rate is 10%. As expected, the increase in the aggregate share is larger when the discount

[9] IPO rates before 1980 are not available. Fama and French (2004) suggests that the IPO before 1979 is much lower.

rate is higher, because higher discount rates imply a larger gap between the ex ante and ex post calculations of firm profitability.

Entry We also consider the effects of increasing ρ, the parameter that governs the Pareto entry distribution, as well as the size of rents in the economy ν. Increasing ρ decreases the mass of the right tail of the entry distribution, thus decreasing both the average and the aggregate capital share, counter to the movements we find in the data.

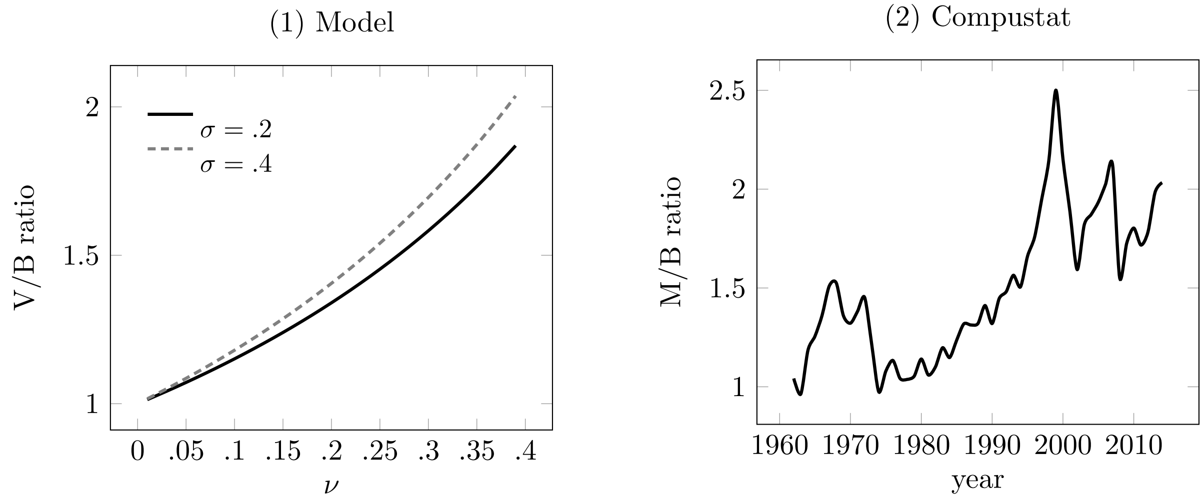
Rents We are also interested in the effect of increasing ν in our quantitative analysis. We start by doubling ν from its baseline value. As pointed out by Atkeson and Kehoe (2005), an increase in ν maps directly onto a decrease in competition in the class of models we consider (see Furman and Orszag, 2015; Barkai, 2016). Recently, many authors have documented evidence of an increase in intangible capital formed by U.S. corporations (e.g., Hall (2001), Corrado, Hulten, and Sichel (2009), Corrado and Hulten (2010), Eisfeldt and Papanikolaou (2014)). These empirical measures of intangible capital quantify the value of rents that accrue to the owners of the capital stock.[10]

In our model, doubling ν generates an increase in valuation ratios that is comparable to that in the data. Economic rents are back-loaded in the model, and hence the market value of firms rises relative to its replacement cost as ν rises (See Appendix E). We found that the market-to-book ratio has gone up dramatically over the last 5 decades in the U.S. As shown in Figure 10, the change in the share of economic rent ν has a monotonic impact on the aggregate market-to-book ratio in our model. The observed secular trend of the empirical valuation ratio, market value of total assets divided by the book value of total assets (Figure 10 (2)), is consistent with the a sizable increase in ν in the U.S. economy. As is clear from Table 3, doubling ν in our model leads to the increase market-to-book ratio by 0.567, which is also quantitatively comparable to the change observed in the data.

The quantitative effects of doubling ν is reported in Panel A of Table 3. Note that an increase in ν alone cannot account for the decrease in the average capital share. Interestingly, doubling ν, the share of GDP due to rents, only increases the aggregate capital share by 4.5 pps. simply because the owners only receive 13.9% of rents. The ν parameter, or the level of aggregate rents, has no direct effect on the actual distribution of rents between capital owners and skilled labor. However, the volatility mechanism operates through the division of the rents. Thus, when rents are larger, the effect of a change in volatility on the aggregate

[10] Falato, Kadyrzhanova, and Sim (2013) estimate that the intangible capital stock relative to total assets has increased from 20% in the end of 1970 to 80% in 2010, while Barkai (2016)'s calculations imply that economic rents have increased more than 200 pps.

Figure 10: Valuation Ratio



Parameter values in figure (1) is taken from our benchmark calibration Table 2. Figure (2) plots the ratio of the aggregate market value to the aggregate total assets. Market value of an individual firm i is measured as Total Assets + Stock Price × Common Shares Outstanding − Common/Ordinary Equities. Data Sources: Compustat/CRSP Merged Fundamentals Annual (1960-2014).

capital share of profits will be larger. Next, we consider a joint increase in both volatility and the size of rents.

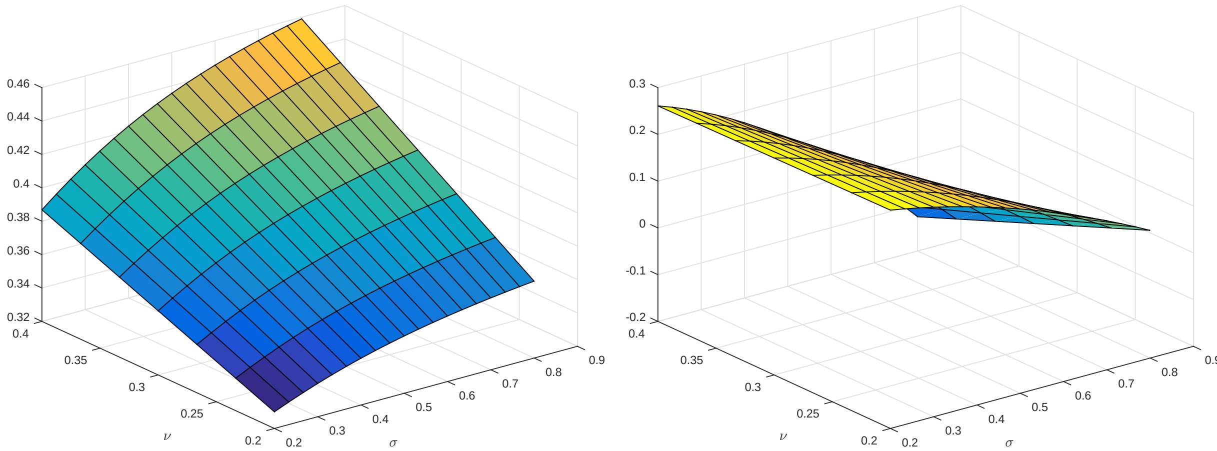
Rents and Volatility We report the effect of a joint increase in σ and ν in Panel B of Table 3. In this case, the model predicts an increase of 7.4 pps. in the aggregate capital share and a decrease of 15.9 pps. in the average capital share. When we increase the discount rate to 10%, the increase in the aggregate capital share is 9.3 pps. while the decrease in the average capital share is 9.2 pps. To visualize the joint effect of changes in volatility and the rents, Figure 11 plots the aggregate capital share (left panel) and the average capital share (right panel) against σ and ν. Increases in ν have a minor effect on the aggregate capital share, except when these are augmented by increases in σ. Finally, only increases in σ lower the average capital share.

To conclude, Table 4 considers the joint distribution of firm-level capital shares and size in the data and in the model. Our model matches the size pattern in average capital shares remarkably well: negative capital shares for the smallest firms and large capital shares for the largest firms. We conclude that this model can quantitatively match the main changes in the moments of factor shares, provided we consider an increase in volatility as well as a decrease in competition, which leads to larger rents.

Table 3: Changes in Capital Share Moments from 1960-1970 to 1990-2014.
The table reports changes in moments of firm-level capital share distribution over different sample periods and the changes in moments of capital share distribution from the transitory experiment of the model. For both panels, the "Data" column reports the differences between moments of capital share distribution in the periods of 1990-2014 and 1960-1970, and the "Model" column reports the differences in the moments of two stationary size distributions in response to changes in parameters starting from the benchmark calibration. In Panel A, we sequentially double σ, ρ, and ν; and we compute the moments of the new stationary size distribution. In Panel B, we double σ and ν jointly, and we compute the moments of the new stationary size distribution.

	Data	**Model**		
Panel A: Univariate Experiment				
		$\sigma \to 2\sigma$	$\rho \to 3\rho$	$\nu \to 2\nu$
		Discount Rate $r = 0.05$		
Average Capital Share	-0.118	-0.066	-0.029	-0.027
Aggregate Capital Share	0.138	0.014	-0.027	0.045
Capital Share at $\bar{X}$	-0.574	-0.190	0.000	-0.210
MB Ratio	0.533	0.066	-0.047	0.567
		Discount Rate $r = 0.10$		
Average Capital Share	-0.118	-0.043	-0.026	-0.006
Aggregate Capital Share	0.138	0.018	-0.024	0.057
Capital Share at $\bar{X}$	-0.574	-0.133	0.000	-0.167
MB Ratio	0.533	0.082	-0.059	0.727
Panel B: Multivariate Experiment				
		$\sigma \to 2\sigma$	$\sigma \to 2\sigma$	
		$\nu \to 2\nu$	$\nu \to 2\nu$	
		DR $r = 0.05$	DR $r = 0.10$	
Average Capital Share	-0.118	-0.159	-0.092	
Aggregate Capital Share	0.138	0.074	0.093	
Capital Share at $\bar{X}$	-0.574	-0.589	-0.432	
MB Ratio	0.533	0.741	0.945	

Figure 11: Aggregate and Average Capital Share



The figure plots the aggregate (left panel) and average (right panel) capital share against ν and σ. Parameter values: $r = 0.10, \mu = 0.02, \lambda = 0.055, \rho = 3.5, \alpha = 0.27$.

Table 4: Firm-level Capital Share and Size Distribution in Multivariate Experiment

The table reports the joint distribution of firm-level capital share and size (total assets). In Panel A, the "Data" row reports the average capital share for size quintiles in the sample period 1960-1970. The "Model" row reports the average capital share for size quintiles, computed from the benchmark calibration. In Panel B, the "Data" row reports the average capital share for size quintiles in the sample period 1990-2014. The "Model" row reports the average capital share for size quintiles, computed after doubling ν and σ. The discount rate r is 0.05.

Panel A: 1960-1970 Sample					
	Size Quintiles				
	-20%	20%-40%	40%-60%	60%-80%	80%-
Data	0.042	0.094	0.157	0.237	0.357
Model	0.131	0.196	0.247	0.297	0.361
Panel B: 1990-2014 Sample					
	Size Quintiles				
	-20%	20%-40%	40%-60%	60%-80%	80%-
Data	-0.119	-0.137	0.059	0.264	0.497
Model: $\sigma \to 2\sigma$	-0.009	0.101	0.186	0.267	0.357
Model: $\sigma \to 2\sigma$ $\nu \to 2\nu$	-0.291	-0.071	0.099	0.260	0.441

7 Empirical Evidence Linking U.S. Capital Share Dynamics and Idiosyncratic Volatility

In this section, we present empirical evidence on the joint dynamics of compensation, firm size, and the implied capital share dynamics. We show that the findings are largely consistent with the implications of our model.

7.1 Cross-Sectional Variation in Capital Share Dynamics

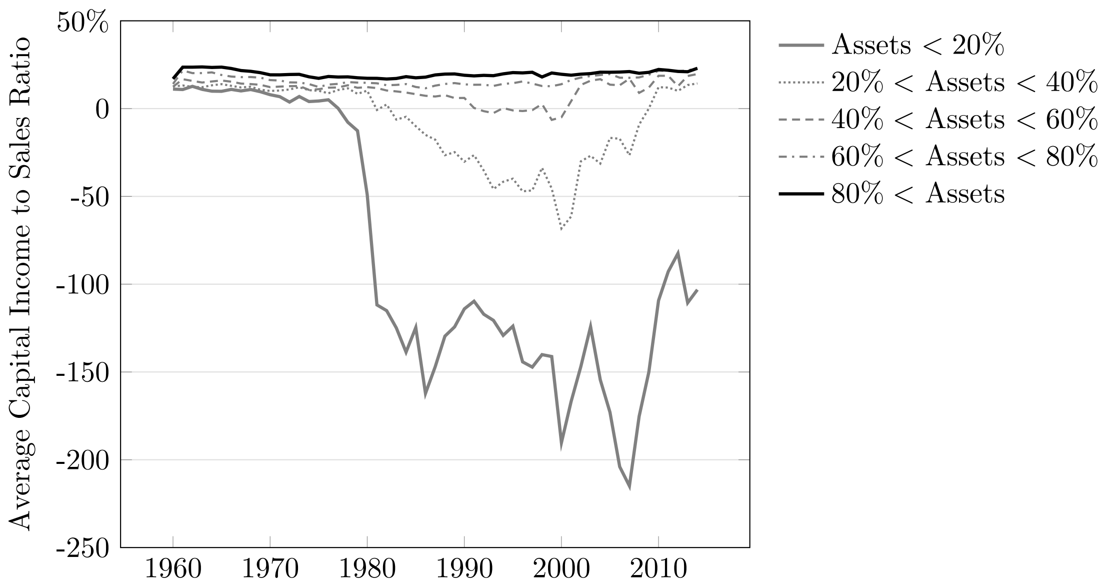
A key prediction of our model is that the distribution of capital share across firm size becomes more dispersed as idiosyncratic volatility increases. In particular, the capital share at the smallest firms declines. This occurs because small firms with low profitability delay exit when volatility increases. The cross-sectional variation in the capital shares bears this out. Over the period 1960-2014, firm-level volatility doubled, and the capital share at the smallest firms significantly decreased. Figure 12 presents the time series of the average capital income to sales ratio for different size quintiles.[11] All firms are sorted into five quintiles based on their total assets. Within each group, we compute the average capital income to sales ratio in each quintile. We emphasize two aspects of this figure: First, the capital income to sales ratio is increasing in firm size at each date. This fact is consistent with the core mechanism of our model; larger and more productive firms have higher capital shares ex post because their shareholders bear more risk than their skilled workers. Second, the average capital income to sales ratio tends to decline more in the smaller size quintiles, while it increases for the large firms (the last quintile). Taken together, these facts indicate that while the aggregate capital share has increased, this increase is driven exclusively by the largest firms. These facts are also consistent with our model. As volatility increases, the dispersion of the size distribution of firms increases. This in turn increases the dispersion in the distribution of capital shares since larger (and more productive) firms have larger capital shares.

The increase in the dispersion of capital shares across the firm size distribution is also present within industries. To demonstrate this, we repeat the exercise carried out in Figure 12 within four industry groups: consumer goods, manufacturing, health products and information, and computers and technology (i.e., high tech industries).[12] Specifically, we fix the definitions of industries over time, and we sort firms into five size quintiles within each

[11] Recall that since value added can be negative at the firm level, we use the capital income to sales ratio as a measure of the captal share at the firm level.

[12] The definitions of the consumer goods, manufacturing, and health products industry groups are adapted from the Fama-French five-industry classification, while the high tech industry definition is from the BEA Industry Economic Accounts. The high tech industry consists of computer and electronic products, publishing and software, information and data processing, and computer system design and related services.

Figure 12: Capital Income to Sales Ratio by Firm Size



This figure presents the average capital income to sales ratio by size over time. Size is measured by total assets, and the capital income to sales ratio is measured as operating income (OIBDP) divided by sales. For each year, firms are categorized into five groups based on their total assets, and we estimate the average capital income to sales ratio within each group for a given year. The sample is winsorized at 1%. The sample includes all Compustat firms (both active and inactive) for 1960-2014.

industry. Figure 13 plots the results. We find similar cross-sectional patterns within each industry: The dispersion of the capital income to sales ratio across size groups increases over the last five decades, while the more significant decline occurs in the smaller size quintiles. Interestingly, we observe a greater increase in dispersion of the capital income to sales ratio in the high tech and health products industries, which have relatively higher firm-level volatility. For example, we observe highly negative operating margins for small firms in the health products industry, a sector characterized by high volatility.

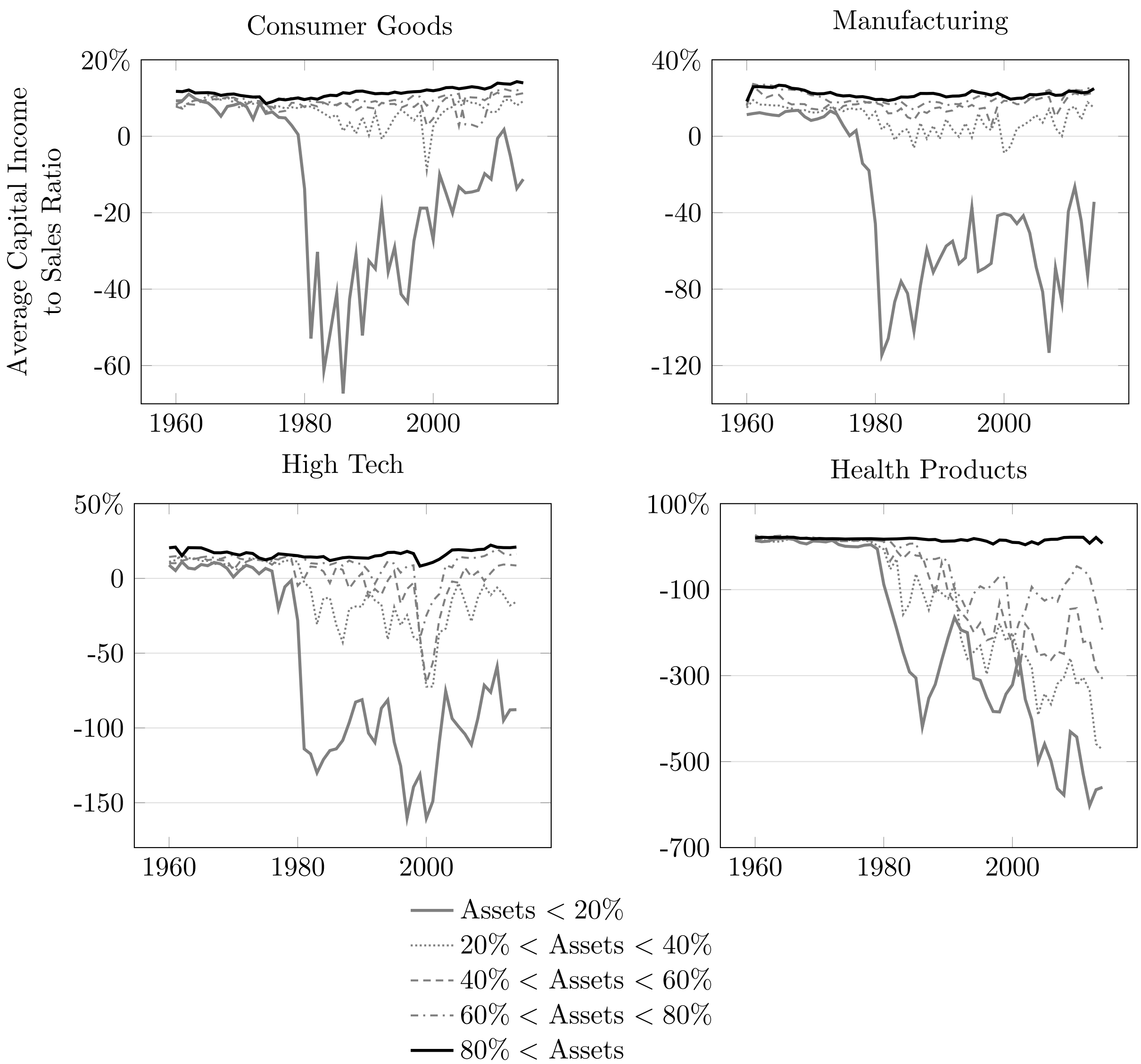
To provide more direct evidence on the link between volatility and the dispersion of the capital share within industries, we regress the industry average capital income to sales ratio on the average firm-level volatility within industry at the 2 digit SIC code level. The regression results are reported in Table 5. As shown in Column (1), a one standard deviation (0.013) increase in the firm-level stock return volatility corresponds to a 12% decline in the average capital income to sales ratio. In Column (2), the effect of firm-level sales growth volatility on capital share is less significant. The difference is likely because sales growth volatility is estimated using 5-year sales growth data instead of contemporaneous sales growth. However, the effect in Column (2) is economically sizable: A one standard deviation (0.204) increase in the sales growth volatility leads to a drop of more than 8% in the average capital income to sales ratio.

To provide further evidence that the patterns we see in the dynamics of capital share across the firm size distribution are consistent with firms delaying exit, we directly examine the capital share of firms close the exit boundary. Specifically, we investigate firms that exit the public domain due to poor company performance (e.g., liquidation, insolvency, bankruptcy) by obtaining the security delisting information from the CRSP U.S. Stock Event database. Figure 14 plots the capital to sales ratio three years before delisting for these firms. Consistent with our model, the average capital share of firms three years before delisting declines by almost 90 pps. from 1970 to 2014. This result remains largely unchanged if we consider the capital share five years before delisting.

8 Conclusion

We propose a mechanism by which an increase in firm-level volatility can have important effects on national income accounting. A firm's owner insures skilled workers against firm-level productivity shocks. As a result, the owner may choose to exit if productivity becomes too low. The level of the skilled workers' compensation is set based on the expected firm value, which necessarily integrates over paths that end in exit. In contrast, when accounting for income, one typically integrates over surviving firms that necessarily feature lower capital

Figure 13: Average Capital Share of Output: Industries



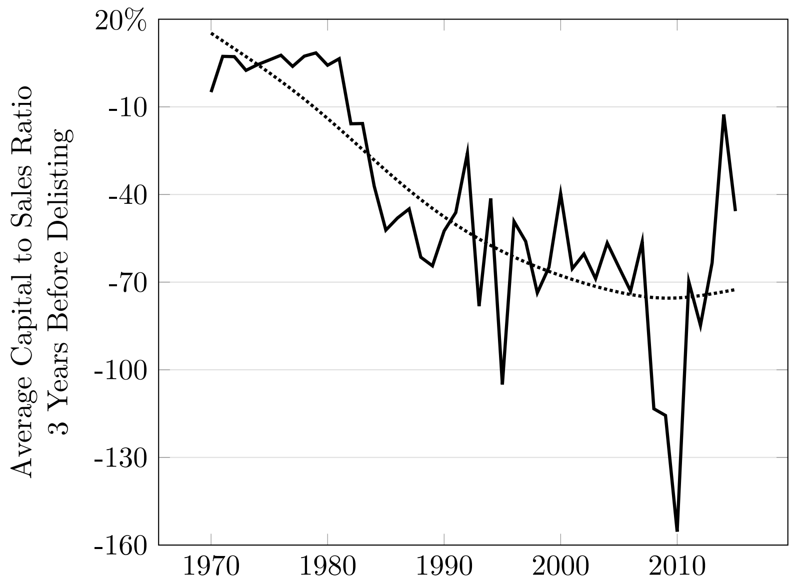
We use the revised Fama-French five-industry classification. Within each industry, we sort firms into five groups based on their total assets. The plot shows the average capital share within each size group for four different industries. Source: Compustat/CRSP Merged Fundamentals Annual (1960-2014).

Table 5: Average Capital Share and Idiosyncratic Volatility: Industry Level 1960 – 2014

The table reports the regression results of industry capital income/sales ratios on the average idiosyncratic volatility. The industry capital income/sales ratio is calculated as the average of capital income/sales ratios across firms within industry. *Idio.Vol(ret)* is the average log idiosyncratic stock return volatility within industry, and *Idio.Vol(sales)* is the average log idiosyncratic sales growth volatility within industry. *Tangibility* is the average of gross property, plant and equipment (PPEGT) to total assets (AT) ratio within industry. *M/B* ratio is the industry average market-to-book ratio within industry. Column (1) and column (2) define industry using 2-digit SIC code, and column (3) and column (4) define industry using 3-digit SIC code. The sample includes all firms in Compustat/CRSP merged database (both active and inactive), 1960-2014. The sample is winsorized at 1%. t statistics in parentheses, and * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

	(1)	(2)	(3)	(4)
	Capital Share 2-digit SIC		Capital Share 3-digit SIC	
Idio.Vol(ret)	-9.476***		-6.468***	
	(-2.96)		(-6.06)	
Idio.Vol(sales)		-0.393		-0.145*
		(-1.51)		(-1.80)
Tangibility	0.112	0.136	0.120**	0.090*
	(0.91)	(1.06)	(2.38)	(1.76)
M/B	-0.084**	-0.112**	-0.072***	-0.095***
	(-2.46)	(-2.31)	(-3.86)	(-4.56)
Constant	0.287***	0.264***	0.185***	0.210***
	(2.73)	(2.85)	(3.92)	(2.82)
Year FE	Y	Y	Y	Y
Industry FE	Y	Y	Y	Y
N	3,091	2,774	11,838	10,394
Ind. Group	65	63	252	249
R^2	0.156	0.169	0.083	0.059

Figure 14: Exit Threshold: Average Capital Income to Sales Ratio Five Years before Delisting



The plots present the average capital share of output five years before delisting. We define a firm's exit from the public firm domain by the use of delisting codes 400-490 and 550-591. The dotted line is the HP filtered trend. Source: Compustat/CRSP Merged Fundamentals Annual (1960-2014) and CRSP delisting code.

shares of profit. This leads to a difference between the aggregate capital share of income, which is calculated ex post, and the capital share of value at the origination of the firm, which is calculated ex ante. When firm-level volatility increases, this difference can increase, thus increasing the aggregate capital share while decreasing the average capital share. We present a calibration of our model that shows we can replicate the data with reasonable parameters. Finally, we present time series and cross-sectional evidence for Compustat firms that is consistent with our proposed mechanism.

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A Data Appendix

A.1 Data Construction

The Sample. Compustat/CRSP Merged Fundamental Annual contains widely available accounting data and stock return data for all publicly traded firms. The sample is from 1960-2014, and it includes all Compustat/CRSP firms (both active and inactive).[13] We exclude financial firms with SIC codes 6000-6799 for our main analysis, and we exclude firms that have negative sales, negative employee numbers, or negative total asset values. Last, we also exclude firms that indicate the currency code for Canadian dollars in order to focus on the sample of U.S. firms only. The sample is winsorized at 1%.

Construction of Main Variables. We measure a firm's capital income using operating income before depreciation (OIBDP). The capital share of output is defined as OIBDP/Sales.

Labor income is measured using the labor cost reported by the public firms. Since Staff Expenses (XLR) is not required to file for public firms, we obtain only sparse observations of labor cost directly from the database. Following Donangelo (2016), we construct the *extended labor cost* (extended XLR). First, we estimate the average labor cost per employee (XLR/EMP) within the industry-size group for each year. Industries are classified using the Fama-French 17-industry definition, and firms are sorted into 20 size groups based on their total assets, which yields a total of 340 industry-size groups. Then, the labor cost of a firm with missing XLR equals the number of employees multiplied by the average labor cost per employee of the same industry-size group during that year. We winsorize the extended XLR at 5% to exclude outliers from the approximation. We measure the labor share of output as extended labor cost (XLR)/Sales.

Value added (VA) is defined as OIBDP + extended XLR. We winsorize the VA at 5% to exclude outliers from the approximation of extended XLR. We also calculate capital share as OIBDP/VA and calculate labor share as extended XLR/VA. We also estimate the *adjusted* value added to deal with negative values. We identify the minimum operating income (OIBDP) for each year, and we increase the value added of all firms by the absolute value of the minimum OIBDP$\times$(1+1%). The adjusted VA is then OIBDP$\times$(1+1%) + extended XLR.

We measure firm-level volatility using both idiosyncratic cash flow volatility and idiosyncratic stock return volatility. Idiosyncratic stock returns are constructed within each year

[13] Using the Compustat/CRSP merged dataset gives us consistent sample for estimating stock return volatility and the delisting threshold. All our empirical results of capital share, capital income to sales ratio, and labor share remain the same using Compustat sample.

by obtaining the residual of a Fama-French 3-factor model using all observations within that year. Idiosyncratic stock return volatility is calculated as the standard deviation of residuals within that year. To obtain idiosyncratic cash flow volatility, we first estimate the first three major principal components of quarterly sales growth. The idiosyncratic cash flow volatility is the standard deviation of residuals of a sales growth factor specification using 20 quarterly year-on-year observations.

Industry Classification. We classify firms into five industries: *consumer goods, manufacturing, health products, high tech,* and *others.* The classification of consumer goods, manufacturing, and health products industries are taken from Fama-French 5-industry classification. The high tech industry category is defined following the definition of the information, computer, and technology industry classification from the BEA Industry Economic Accounts, which consists of computer and electronic products, publishing industries (including software), information and data processing services, as well as computer systems design and related services. We classified all remaining firms (including the finance industry) into other industries. The traditional industries in our paper consists of a combination of the consumer goods and manufacturing industries. To categorize the new economy industries or the highly intangible-intensive industries, we use our definition of high tech (ICT) industries.

Non-Publicly Traded Firms. We obtain the measure of capital income for non-publicly traded firms by subtracting the aggregate capital income of the Compustat firms from the aggregate capital income of the U.S. economy. The aggregate capital income is measured using NIPA table *Net Operating Surplus*, which measures the aggregate business income from production after subtracting labor costs, taxes on production and imports (less subsidies), and consumption of fixed capital (economic depreciation) from value added, but before subtracting financing costs (such as net interest) and business transfer payments. Net operating surplus is a profit-like measure that is conceptually closest to earnings before interest and tax (EBIT) in the NIPA tables (See Mead, Moulton, and Petrick (2004)).

A.2 Figures and Tables

In this section, we provide more details regarding the figures and tables in the paper.

Average and Aggregate Factor Share. Using the Compustat sample, we construct the average and aggregate factor shares as follows: For each year, aggregate factor share = $\frac{\Sigma_i \text{Factor Income}}{\Sigma_i \text{Output}}$, and the average factor share = $\Sigma_i \left(\frac{\text{Factor Income}}{\text{Output}}\right)/N$.

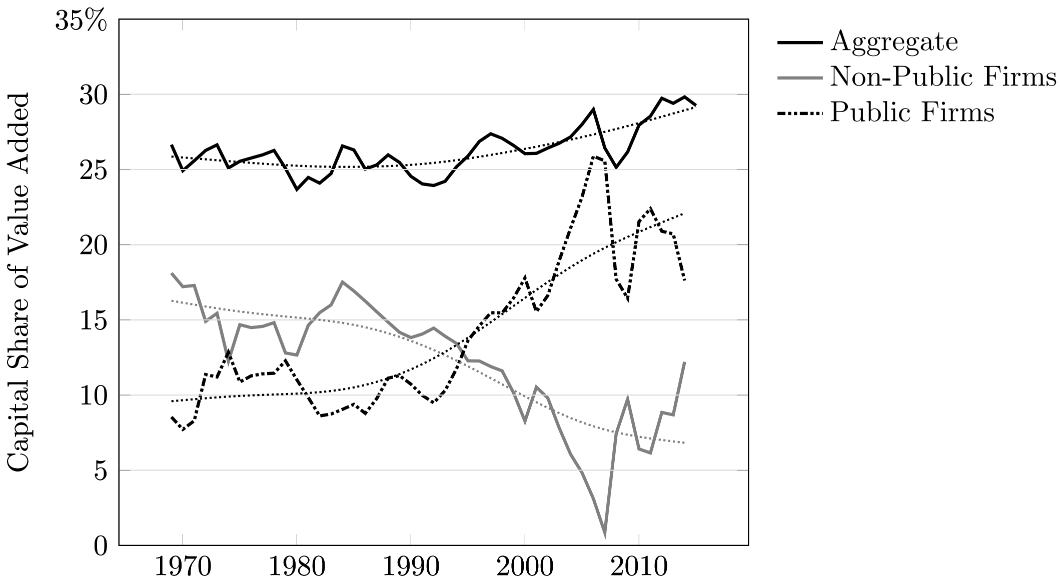
Size Groups. For each year, all firms are sorted into five groups based on their total assets. Within each group, we compute the average and aggregate labor share and capital share. Figure 12 and Figure 13 show the time series of the average and aggregate capital share of each size group.

Delisting Threshold. We obtain the security delisting information from the CRSP U.S. Stock Events database. The *delisting* Code is a 3-digit integer code that (a) indicates that a security is still trading or (b) provides a specific reason for delisting. For our interest, we consider delisting due to liquidation (delisting codes 400 to 490) and delisting by the current exchange for various reasons due to poor company fundamentals (e.g., insolvency, bankruptcy, insufficient capital (delisting codes 550 to 591)). Then, we calculate the average capital shares either three years or five years before delisting. Figure 14 shows the time series of the pre-delisting performance from 1970 to 2014.

B Public vs. Private

Our empirical analysis focusses on publicly traded firms. However, Davis, Haltiwanger, Jarmin, and Miranda (2007) find that non-farm, firm-level volatility in the private sector has declined in recent decades. According to the logic of our model, the aggregate capital share for private firms in the U.S. should not have increased over the same period of time. Using the NIPA net operating surplus (NOS), which is a profit-like measure that aggregates the overall operating income of the U.S. economy, we infer the operating income of private firms by subtracting the aggregate operating income of publicly traded firms (Compustat/CRSP firms) from NIPA net operating surplus. Figure 15 decomposes the aggregate capital share of total value added into a private and a public component. The component due to private firms, the ratio of the aggregate operating income of private firms to total value added, declined from 1969 to 2014, even though private sector has grown recently: the number of publicly listed firms has decreased by 14% since 1996 (Doidge, Karolyi, and Stulz (2015)).

Figure 15: Aggregate Capital Share of U.S.: Public versus Private Decomposition



This figure decomposes the aggregate capital share (black solid line) for all U.S. industries into the share due to private firms (gray solid line), and the share due to public firms (dashed black line). The universe of private firms covers all firms excluding publicly traded firms in Compustat. The aggregate capital share of U.S. value added is the ratio of NIPA's net operating surplus retrieved from FRED and total value added from the BEA's GDP-by-industry accounts. Private capital income is obtained by subtracting the aggregate capital income (earnings before interest and tax) reported by Compustat public firms from the NIPA net operating surplus. The private (public) share is the ratio of private (public) aggregate capital income and total value added. The dotted lines are the HP-filtered trends of the three time series of capital shares. Data sources: U.S. Bureau of Economic Analysis, Gross domestic income: Net operating surplus: Private enterprises [W260RC1Q027SBEA] (1969-2015). Bureau of Economic Analysis, GDP by Industry Accounts (1960-2015). Compustat/CRSP Merged Fundamentals Annual (1960-2014).

C Proofs

C.1 Derivation of Equilibrium Compensation

The equilibrium wage is given by the solution to the following equation:

$$\int_{\bar{X}(c)}^{\infty} V(X;c)f(X)dX = P,$$

where $\bar{X}(c)$ is given by Equation (9) and $V(X;c)$ is given by Equation (10). Note that this equation is equivalent to Condition 3 of Definition 1. First, after evaluating the integral on the left side, we have

$$\begin{aligned}
&\int_{\bar{X}(c)}^{\infty} \left(\frac{X}{r+\lambda-\mu} - \frac{c}{r+\lambda} - \left(\frac{\bar{X}(c)}{r+\lambda-\mu} - \frac{c}{r+\lambda} \right) \left(\frac{X}{\bar{X}(c)} \right)^{-\eta} \right) \frac{\rho}{X^{1+\rho}} dX \\
=&(\rho \bar{X}(c))^{\rho} \left(\frac{\bar{X}(c)}{(r+\lambda-\mu)(\rho-1)} + \frac{c}{(r+\lambda)\rho} + \frac{\bar{X}(c)(r+\lambda) - c(r+\lambda-\mu)}{(r+\lambda-\mu)(r+\lambda)(\eta+\rho)} \right) \\
=&\left(\frac{\eta(r+\lambda-\mu)}{(1+\eta)(r+\lambda)} \right)^{-\rho} \left(\frac{\eta}{(r+\lambda)(\rho-1)(\eta+\rho)} \right) c^{-(\rho-1)}.
\end{aligned}$$

Note that our assumption on the Pareto form $f(X)$ facilitates the computation of the integral shown above, because both $V(X;c)$ and $f(X)$ are power functions. This integral represents the expected value of the firm to the shareholder after paying the fixed cost but before realizing the initial productivity of the firm. Since $\rho > 1$, it is monotonically increasing in c, and we can solve to obtain the expression for equilibrium compensation given in (11).

C.2 Derivation of Stationary Distribution

The ODE for $\phi(x)$ has the following general solution:

$$\phi(x) = A_1 e^{\gamma_1 x} + A_2 e^{-\gamma_2 x} + A_3 e^{-\rho x}, \tag{42}$$

where γ_1 and γ_2 are given by

$$\gamma_1 = \frac{\mu - \frac{1}{2}\sigma^2 + \sqrt{(\mu - \frac{1}{2}\sigma^2)^2 + 2\sigma^2\lambda}}{\sigma^2} \tag{43}$$

$$\gamma_2 = \frac{-(\mu - \frac{1}{2}\sigma^2) + \sqrt{(\mu - \frac{1}{2}\sigma^2)^2 + 2\sigma^2\lambda}}{\sigma^2}. \tag{44}$$

First note that $\gamma_1 > 0$ implies that $A_1 = 0$. To ease notation, we drop the subscript on γ_2. Next, note that an application of the ODE gives

$$A_3 = -\frac{\rho\psi}{\frac{1}{2}\rho^2\sigma^2 + \rho(\mu - \frac{1}{2}\sigma^2) - \lambda}. \tag{45}$$

Finally, the boundary condition implies that

$$A_2 e^{-\gamma\bar{x}} + A_3 e^{-\rho\bar{x}} = 0,$$

so

$$A_2 = -A_3 e^{(\gamma-\rho)\bar{x}}. \tag{46}$$

The result in Equation (15) directly follows from the solution above and also from an application of the market clearing condition for skilled workers.

C.3 Derivation of Total and Average Capital Share of Profits.

We have

$$\begin{aligned}\Pi = \frac{\int_{\bar{x}}^{\infty}(e^x - c)\phi(x)dx}{\int_{\bar{x}}^{\infty} e^x\phi(x)dx} &= 1 - \frac{c\int_{\bar{x}}^{\infty}\phi(x)dx}{\int_{\bar{x}}^{\infty} e^x\phi(x)dx} \\ &= 1 - \frac{c}{\int_{\bar{x}}^{\infty} e^x\phi(x)dx},\end{aligned}$$

where the second step follows from the market clearing condition given in Equation (14). To continue the calculation, we have

$$\begin{aligned}\int_{\bar{x}}^{\infty} e^x\phi(x)dx &= \int_{\bar{x}}^{\infty} \frac{\rho\gamma}{\rho-\gamma}\left(e^{-(\gamma-1)x+\gamma\bar{x}} - e^{-(\rho-1)x+\rho\bar{x}}\right)dx \\ &= \bar{X}\left(\frac{\rho}{\rho-1}\right)\left(\frac{\gamma}{\gamma-1}\right) \\ &= \left(\frac{\rho}{\rho-1}\right)\left(\frac{\gamma}{\gamma-1}\right)\left(\frac{\eta}{\eta+1}\right)\left(\frac{r+\lambda-\mu}{r+\lambda}\right)c\end{aligned}$$

Substituting this expression into the expression for Π given above yields the desired result. The derivation of the average capital share is similar.

D Calibration of the Full Production Economy with Pay for Performance Contracts

In this section of Appendix, we report our benchmark calibration and the quantitative experiments in the full production economy with the skilled worker's compensation contract taking the affine form:

$$c_t = \beta Y_t + w.$$

Table 6 reports the parameters and moments of our benchmark calibration. We set the pay-performance sensitivity parameter β to 1% in the 1960-1970 period. Although there are intense debate and studies on the evolution of corporate top executives compensation, little has documented the pay-performance sensitivities of the average skilled workers in the firm. In the survey paper, Frydman and Jenter (2010) find only moderate[14] wealth-performance sensitivities for the top three executives in large public firms. The pay-performance sensitivities (less than 0.07%) tend to be lower than wealth-performance sensitivities. The average skilled workers' pay-performance sensitivities should also be much lower than that of top executives. Hence, without more direct evidence, we calibrate the sensitivity parameter β at 1% in our benchmark calibration, and show the robustness by setting β to 5% in our quantitative experiments. With some exposure in the skilled workers' compensation to firm performance, the model implies a higher α (0.3) in order to match the aggregate capital share, and a lower λ to generate a reasonable exit rate given that the real option channel is now weaken. Table 7 reports the results of our quantitative experiments, and Table 8 reports the joint size and capital share distribution.

[14] Average 13% of CEO compensation is from equity and options.

Table 6: Benchmark Calibration: Pay-for-Performance

The table reports our benchmark calibration with pay-for-performance compensation. Panel A reports target moments in the data and the implied moments from our production model. The data moments are computed from the sample Compustat/CRSP Merged Fundamentals Annual from 1960 to 1970. The sample excludes firms that have SIC codes from 6000 to 6799. Panel B reports the calibrated parameters. Panel C reports the preset parameters. Firm-level value added VA_i is OIBDP plus Extended XLR. To deal with negative values, we identify the minimum operating income (OIBDP) for each year, and we increase the value added of all firms by the absolute value of the minimum OIBDP$\times$(1+1%). The average capital share is computed using OIBDP divided by the adjusted value added. The standard deviation and skewness of the capital share is also estimated using the adjusted value added measure. The aggregate capital share is calculated using the unadjusted value added.

Panel A: Capital Share Moments 1960-1970		
	Data	Model
Average Capital Share	0.210	0.267
Aggregate Capital Share	0.419	0.380
Standard Deviation of Capital Share	0.152	0.092
Capital Share at Exit	0.076	0.050
Entry Rate	-	0.023
Panel B: Calibrated Parameters		
σ	0.2	Idiosyncratic Vol
μ	0.02	Firm Growth
λ	0.03	Exogenous Exit Rate
ρ	3.5	Entrants Firm Size Distribution
α	0.3	Aggregate Physical Capital Share of Output
Panel C: Preset Parameters		
r	0.05	Discount Rate
k/l	1	Capital/Labor Ratio
p	1	Sunk Cost
ν	0.2	Share of Rents in GDP (AK 2005)
β	1%	Pay for Output Sensitivity

Table 7: Changes in Capital Share Moments from 1960-1970 to 1990-2014.

The table reports changes in moments of firm-level capital share distribution over different sample periods and the changes in moments of capital share distribution from the transitory experiment of the model with pay-for-performance compensation. For both panels, the "Data" column reports the differences between moments of capital share distribution in the periods of 1990-2014 and 1960-1970, and the "Model" column reports the differences in the moments of two stationary size distributions in response to changes in parameters starting from the benchmark calibration. In Panel A, we sequentially double σ, ρ, and ν; and we compute the moments of the new stationary size distribution. In Panel B, we double σ and ν jointly, and we compute the moments of the new stationary size distribution.

	Data	**Model**		
Panel A: Univariate Experiment				
		$\sigma \to 2\sigma$	$\rho \to 3\rho$	$\nu \to 2\nu$
		Discount Rate $r = 0.05$; $\beta = 0.01$		
Average Capital Share	-0.118	-0.089	-0.028	-0.031
Aggregate Capital Share	0.138	0.012	-0.013	0.088
Capital Share at $\bar{X}$	-0.574	-0.239	0.000	-0.260
		Discount Rate $r = 0.05$; $\beta = 0.05$		
Average Capital Share	-0.118	-0.070	-0.022	-0.031
Aggregate Capital Share	0.138	0.010	-0.011	0.088
Capital Share at $\bar{X}$	-0.574	-0.189	0.000	-0.260
		Discount Rate $r = 0.1$; $\beta = 0.01$		
Average Capital Share	-0.118	-0.053	-0.023	-0.001
Aggregate Capital Share	0.138	0.013	-0.011	0.097
Capital Share at $\bar{X}$	-0.574	-0.147	0.000	-0.189
		Discount Rate $r = 0.1$; $\beta = 0.05$		
Average Capital Share	-0.118	-0.042	-0.018	-0.001
Aggregate Capital Share	0.138	0.010	-0.009	0.097
Capital Share at $\bar{X}$	-0.574	-0.116	0.000	-0.189
Panel B: Multivariate Experiment				
		$\sigma \to 2\sigma$	$\sigma \to 2\sigma$	$\sigma \to 2\sigma$
		$\nu \to 2\nu$	$\nu \to 2\nu$	$\nu \to 2\nu$
		DR $r = 0.05$	DR $r = 0.1$	DR $r = 0.1$
		$\beta = 0.01$	$\beta = 0.01$	$\beta = 0.05$
Average Capital Share	-0.118	-0.213	-0.109	-0.099
Aggregate Capital Share	0.138	0.113	0.124	0.121
Capital Share at $\bar{X}$	-0.574	-0.750	-0.492	-0.461

Table 8: Firm-level Capital Share and Size Distribution in Multivariate Experiment

The table reports the joint distribution of firm-level capital share and size (total assets) in the model with pay-for-performance compensation. In Panel A, the "Data" row reports the average capital share for size quintiles in the sample period 1960-1970. The "Model" row reports the average capital share for size quintiles, computed from the benchmark calibration. In Panel B, the "Data" row reports the average capital share for size quintiles in the sample period 1990-2014. The "Model" row reports the average capital share for size quintiles, computed after doubling ν and σ. The discount rate r is 0.05. We show results with $\beta = 0.01$ and $\beta = 0.05$.

Panel A: 1960-1970 Sample					
	Size Quintiles				
	-20%	20%-40%	40%-60%	60%-80%	80%-
Data	0.042	0.094	0.157	0.237	0.357
Model ($\beta = 0.01$)	0.134	0.212	0.272	0.329	0.390
Model ($\beta = 0.05$)	0.156	0.218	0.265	0.310	0.359
Panel B: 1990-2014 Sample					
	Size Quintiles				
	-20%	20%-40%	40%-60%	60%-80%	80%-
Data	-0.119	-0.137	0.059	0.264	0.497
Model: $\sigma \to 2\sigma, \beta = .01$	-0.045	0.089	0.190	0.283	0.377
Model: $\sigma \to 2\sigma, \beta = .05$	0.015	0.121	0.200	0.274	0.348
Model: $\sigma \to 2\sigma$ $\nu \to 2\nu, \beta = .01$	-0.406	-0.130	0.077	0.268	0.462
Model: $\sigma \to 2\sigma$ $\nu \to 2\nu, \beta = .05$	-0.346	-0.099	0.087	0.258	0.433

E Valuation Ratio

In this appendix, we solve for the aggregate valuation ratio of our production economy.

Given the homogeneity of F, the price of capital is given by solving the first order condition for the firm's problem and is given by

$$\kappa = (1-\nu)\left(\frac{F_1(k,l)}{F(k,l)}\right) y$$

where F_1 denotes the partial derivative of F with respect to it's first argument. Since F, k, and l do not depend on any of the parameters with which we want to take a comparative static, we can just let

$$\alpha = \left(\frac{F_1(k,l)}{F(k,l)}\right) k$$

to get that the aggregate rental payment to physical capital is

$$\kappa k = (1-\nu)\alpha y$$

which makes some intuitive sense. It's basically just the capital expenditure share of output that you get from a model with a homogeneous production function. To get the book value of physical capital, we can simply capitalize this number to get

$$\text{Book value of physical capital} = \frac{\kappa k}{r} = \frac{(1-\nu)\alpha y}{r}$$

Given equilibrium $\hat{F}$, c^*, $\bar{X}$ and the firm value $V(X,;c,\hat{F})$ from equation (31), the ratio of the aggregate of market value of rents is

$$\begin{aligned}
\hat{V} &= \int_{\bar{x}}^{\infty} V(X;c^*,\hat{F})\phi(x)dx \\
&= \frac{\hat{F}\hat{X}}{r+\lambda-\mu} - \frac{c^*}{r+\lambda} - \left(\frac{\widehat{F}\bar{X}^{1+\eta}}{r+\lambda-\mu} - \frac{c^*\bar{X}^{\eta}}{r+\lambda}\right)\int_{\bar{x}} e^{-\eta x}\phi(x)dx \\
&= \frac{\nu y}{r+\lambda-\mu} - \frac{c^*}{r+\lambda} - \left(\frac{\widehat{F}\bar{X}^{1+\eta}}{r+\lambda-\mu} - \frac{c^*\bar{X}^{\eta}}{r+\lambda}\right)\int_{\bar{x}} e^{-\eta x}\phi(x)dx
\end{aligned}$$

The ratio of the aggregate value of firms (cumulative of the aggregate value of physical capital) to book value of physical capital is then

$$\frac{\text{Market}}{\text{Book}} = 1 + \frac{r\hat{V}}{(1-\nu)\alpha y}$$