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ABSTRACT

The U.S. economy is characterized by large, longer term regime shifts in asset values relative to macroeconomic fundamentals. These movements coincide with shifts in the real federal funds rate in excess of a measure of the natural rate of interest, and in equity market return premia. We specify and estimate a novel DSGE model and find that the regime shifts coincide with equally important shifts in the parameters of a monetary policy rule that have long-lasting effects on the real rate of interest. Changes in the conduct of monetary policy also affect return premia, in the direction consistent with a reach for yield.

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1 Introduction

There is growing evidence that the real values of long-term financial assets fluctuate sharply in response to the actions and announcements of central banks. This includes the value of the stock market, a perpetual asset that endures indefinitely. But this creates a puzzle. Asset pricing theories can generally rationalize such large responses only if market participants believe that something related to the conduct of monetary policy will have a highly persistent influence on real variables.¹ Yet the notion that monetary policy could have long-lived effects on real variables is contravened by an agglomeration of foundational New Keynesian macro theories, which imply that monetary policy shocks should have only short-lived effects on real variables.² Empirical evidence appears consistent with this (Christiano, Eichenbaum, and Evans (2005)). But if this is so, how, why, and to what extent could monetary policy possibly influence long-lived assets?

This paper presents new evidence on these questions. We first show that the U.S. economy is characterized by quantitatively large, decades-long regime shifts in asset values relative to macroeconomic fundamentals. These movements coincide with equally important regime shifts in the level of the real federal funds rate in excess of a commonly used measure of the “natural” rate of interest, a variable referred to hereafter as the *monetary policy spread*, or *mps* for short. Regimes in which asset valuations are persistently high coincide with persistently low or negative values for the *mps*, while regimes in which valuations are persistently low coincide with persistently high values for the *mps*. The estimation identifies two subperiods characterized by low valuations and a high *mps*: 1978:Q4 to 2001:Q3, and 2006:Q2 to 2008:Q2. The first period spans the Volcker disinflation and its aftermath, while the second follows 17 consecutive Federal Reserve rate increases that left the nominal funds rate standing at 5.25% in June of 2006. All other subperiods of the sample are identified as high valuation/low *mps* regimes.

Our second result is that the high valuation/low *mps* regimes coincide with lower equity market return premia, consistent with a “reach for yield” in equity markets. Specifically, in a switch from a high to low *mps* regime, the estimated present discounted value of future return premia on the aggregate stock market, as well as that of several equity characteristic portfolios, simultaneously fall to lower levels. Moreover, the return premia of evidently riskier, higher Sharpe ratio portfolios, such as those that go long in value stocks or stocks that have recently appreciated the most, fall more than those of evidently less risky, lower Sharpe ratio portfolios, such as those that go long in growth stocks or stocks that have recently appreciated the least.

Taken together, this evidence suggests that low frequency movements in short-term real interest rates are directly linked to low frequency regime shifts in asset valuations and equity

¹We define a “real variable” here as any non-nominal variable, including risk premia and credit spreads.

²For a review of New Keynesian models, see Galí (2015).

return premia. But how much of these movements can be attributed to monetary policy? To address this question, the second part of the paper presents a dynamic, stochastic, general equilibrium (DSGE) model and estimation. Specifically, we develop a novel New Keynesian framework that, unlike canonical New Keynesian models described above, has the potential to generate highly persistent (though not permanent) departures from monetary neutrality. At the same time, our framework nests several special cases in which monetary non-neutrality could be small in magnitude, short-lived, or absent entirely, depending on parameter values. The full framework is solved and all parameters and latent states freely estimated using Bayesian methods, with the parameters of the policy rule estimated under flat priors. The estimation uses data on inflation expectations from the Michigan Survey of Consumers (SOC), the nominal federal funds rate, output growth, and inflation.

Along most dimensions, the model we propose is isomorphic to the prototypical New Keynesian model of Galí (2015), chapter 3. As in the prototypical model, this one has three Gaussian shocks: a demand or preference shock, a shock to the natural level of output, and a monetary policy shock. But we make two key modifications to this framework.

First, unlike the prototypical model, we allow for regime changes in the conduct of monetary policy. These take the form of shifts in the parameters of the nominal interest rate rule that include both the inflation target and the activism coefficients governing how strongly the monetary authority responds to deviations of inflation from target and to economic growth. Such changes in what we will call the *conduct* of monetary policy give rise to movements in the nominal interest rate that are conceptually distinct from those generated by the monetary policy *shock*, an innovation in the policy rate that is uncorrelated with inflation, economic growth, and shifts in the policy rule parameters. Second, we allow the evolution of beliefs about trend inflation to be potentially influenced by both an adaptive learning component as well as a signal about the central bank’s inflation target. For the adaptive learning component, we assume a representative agent forms expectations about future inflation using a constant gain learning algorithm, following the evidence using survey data established in Malmendier and Nagel (2015). To ensure that model expectations evolve in a manner that closely aligns with observed expectations, we map the learning algorithm into data by filtering observations on survey expectations of inflation over time. Overall perceived trend inflation is then a weighted average of the trend implied by the constant gain learning rule and the central bank’s inflation target. A weight of less than one on the target could arise either because the target is imperfectly observed, or because central bank announcements about the target are not viewed as fully credible. Because the weights on the two terms are freely estimated, our approach allows us to directly assess the importance of learning and imperfect information about the inflation target.

The results of estimating this model imply that the stance of monetary policy differs

markedly across the previously estimated valuation/*mps* regimes. Specifically, we find that the high valuation/low *mps* subperiods coincide with what we will refer to as a *dovish regime* characterized by a comparatively higher inflation target and less responsiveness to inflation relative to growth, while the low valuation/high *mps* subperiods coincide with a *hawkish regime* characterized by a lower inflation target and greater responsiveness to inflation relative to growth. As discussed below, the hawkish regime prevails in the Volcker years and during most of Greenspan’s tenure at the Federal Reserve through the end of 2001. But our estimates imply that, since the end of 2001, the conduct of monetary policy has been resolutely dovish.

How important are these changes in the way monetary policy is conducted for real interest rates and equity market risk premia? To address this question, we use our model estimates to identify movements in real variables that are attributable *solely* to the conduct of monetary policy, i.e., to regime changes in the policy rule. Several results from this exercise are noteworthy.

First, the estimates imply that changes in the conduct of monetary policy generate large and persistent fluctuations in the short-term real interest rate that last for decades. By contrast, monetary policy *shocks* have far more transitory effects, consistent with the evidence in Christiano, Eichenbaum, and Evans (2005). Second, almost all of the downward trend in the real interest rate observed since 1980 can be explained by regime changes in the conduct of monetary policy. This occurs because the policy rule parameters exhibit a decisive shift toward more hawkish values around the time of Volcker’s appointment, but then exhibit an equally decisive shift back to more dovish values in the aftermath of the near collapse of Long Term Capital Management, the tech bust in the stock market, and the 9/11 terrorist attacks. The conduct of monetary policy has remained dovish since, with the exception of a brief interlude from 2006:Q2-2008:Q2. Third, we distinguish the roles of regime changes in the inflation target from those in the activism coefficients and find that both are important. In particular, we show that, if the inflation target had changed over time but there had been no coincident change in the activism coefficients, fluctuations in real rates and output growth would have been substantially dampened throughout the post-1980 period. Fourth, our estimate of perceived trend inflation closely follows the adaptive learning rule, which plays a crucial role in the results. Indeed, if perceived trend inflation is counterfactually set equal to the inflation target, regime changes in the conduct of monetary policy are shown to have no affect on the real interest rate. Finally, we tie these results back to the estimates of equity return premia studied earlier and find that all premia are strongly positively correlated with the component of the real interest rate that is attributable solely to changes in the conduct of monetary policy, consistent with a reach for yield. By contrast, the residual component of the real interest rate exhibits much smaller correlations, reinforcing evidence of a distinct monetary policy role in equity premia over time.

The rest of the paper is organized as follows. The next section discusses related literature. Section 3 discusses the estimation of a joint Markov-switching system for asset valuations and the monetary policy spread and investigates whether the high valuation/low *mps* regimes are characterized by lower risk premia in equity market assets. Section 4 describes the DSGE model, explains how it is solved and estimated, and presents results of that estimation. Section 5 concludes. A large amount of additional material, test results, and a detailed data description have been placed in an Appendix for online publication.

2 Related Literature

The research in this paper touches on several different strands of literature that connect monetary policy to movements in asset values. Although not focused specifically on announcement effects, our work is related to a growing body of evidence that finds the values of long-term financial assets respond to the actions and announcements of central banks.³ Economists have proposed various explanations for these responses, including the revelation of private central bank information and the response of risk premia. Yet no matter what the channel, asset pricing models can typically only rationalize such large responses if something associated with the announcement is expected to have a highly persistent influence on real variables or risk premia.⁴ It is unclear why the actions of central banks might have such prolonged effects on return premia but not on interest rates. Our work contributes to this literature by finding evidence of regime changes in the conduct of monetary policy that have long-lasting effects on both short-term interest rates and equity market return premia. One interpretation of the evidence on central bank announcements is that the announcements are, in part, noisy signals about the possibility of a regime change in the conduct of monetary policy.

Our empirical findings also relate to a theoretical literature in which shifts downward in the risk-free interest rate coincide with shifts downward in risk premia, as in those models that can be broadly characterized as having a “reaching for yield” channel (e.g., Rajan (2006); Rajan (2013); Diamond and Rajan (2012); Farhi and Tirole (2012); Drechsler, Savov, and Schnabl (2014); Piazzesi and Schneider (2015); Acharya and Naqvi (2016); Coimbra and Rey (2017); Hanson, Lucca, and Wright (2018)). Alternatively, a decline in real rates driven by monetary policy could increase the fraction of wealth held by more risk tolerant investors, as in Kekre

³See Cochrane and Piazzesi (2002), Hanson and Stein (2015), Gertler and Karadi (2015), Gilchrist, López-Salido, and Zakrajšek (2015), Boyarchenko, Haddad, and Plosser (2016), Jarocinski and Karadi (2019), and Cieslak and Schrimpf (2019). These studies follow on earlier work finding a link between monetary policy surprises and short-term assets in high frequency data (Cook (1989); Bernanke and Kuttner (2005); Gürkaynak, Sack, and Swanson (2005)). A separate literature studies the timing of when premia in the aggregate stock market are earned in weeks related to FOMC-cycle time (Lucca and Moench (2015), Cieslak, Morse, and Vissing-Jorgensen (2015)).

⁴For reviews of frontier asset pricing models, see Cochrane (2005) and Campbell (2017).

and Lenel (2019), driving down return premia. In this paper we use the terminology “reach for yield” as short-hand for the phenomenon of declining risky asset return premia when real interest rates decline that could occur for any reason (e.g., a representative agent who becomes less risk averse or there is a shift in the composition of wealth owned by less risk averse agents). Our findings contribute to this literature by showing that persistently high asset valuations and persistently low return premia are associated with evidence of persistent regime changes in the conduct of monetary policy.

A separate body of theoretical work addresses the low and declining interest rates of recent decades with implications for risk premia that are opposite to what would be consistent with a reach for yield type phenomenon. In these theories, declining real rates are the result of shocks that increase the fraction of wealth held by more risk averse or more pessimistic investors, implying that risk premia rise rather than fall as interest rates decline (e.g., Barro and Mollerus (2014); Caballero and Farhi (2014); Hall (2016)). Thus, asset valuations in these theories can only be higher if the decline in the risk-free rate exceeds the rise in risk premia. We present evidence from equity markets that the low interest rate regimes we document coincide with lower equity market risk premia, consistent with theories that have a “reaching for yield” type mechanism. Our findings for stock market returns in this regard are reminiscent of recent evidence of reaching for yield in the Treasury market (e.g., Hanson and Stein (2015)), by U.S. prime money funds (e.g., Di Maggio and Kacperczyk (2015)), and by U.S. corporate bond mutual funds (Choi and Kronlund (2015)). The evidence in these papers pertains to heavily intermediated asset classes. By contrast, our evidence pertains to equity market portfolios, an asset class ostensibly held by retail investors and households, as well as intermediaries.

Finally, our work is related to previous research that has found evidence of infrequent regime changes in the parameters of an estimated monetary policy rule (e.g., Clarida, Gali, and Gertler (2000); Lubik and Schorfheide (2004); Bianchi (2013)). Unlike this work, we use a more recent sample and estimate whether there are *joint* regime changes in asset valuations and the *mps* that coincide with regime shifts in the policy rule and risk premia. We also present new evidence, vis-a-vis this literature, of changes in the policy rule parameters toward more dovish monetary policy that occurred at beginning of the 21st century.

This latter body of work also helps to motivate why we use regime switching over alternative procedures such as e.g., slowly drifting means, to document joint variation in valuations and policy rates. The papers cited in the previous paragraph use regimes to identify different phases of US monetary history. It is quite natural to model changes in the conduct of monetary policy as occurring with discrete regime changes. Different Chairs of the Federal Reserve bring their own views and priorities to the conduct of monetary policy, suggesting that data on policy interest rates is likely to be better described as being drawn from a mixture of distinct distributions with infrequent transitions between them, rather than a single distribution where

a transition occurs each period. The regime-switching approach therefore helps us evaluate a key hypothesis of the paper, namely whether longer run shifts in the conduct of monetary policy are associated with changes in discount rates, valuations, and return premia.

3 Regimes in Valuations, Interest Rates, and Risk Premia

This section describes how we model and estimate joint regimes in asset valuations and the *mps* using a Markov-switching model, and how we evaluate whether these regimes are associated with movements in risk premia. Before discussing the Markov-switching estimation, we begin by presenting some preliminary evidence that helps motivate the evidence for long-lived regimes in these variables.

3.1 Motivating Evidence

Figure 1 plots the behavior over time of a key instrument of monetary policy, namely the real federal funds rate, measured for the purposes of this plot as the nominal rate minus a four quarter moving average of inflation. The left panel plots this series along with an estimate of the natural real interest rate, from Laubach and Williams (2003). The natural rate of interest measures the component of the real rate whose fluctuations cannot be attributed to monetary policy.⁵ The figure shows that there are important lower-frequency fluctuations in the real federal funds rate over the full sample, but little long-term trend. By contrast, the natural rate of interest exhibits a clear downward trend over the entire sample. The right panel of the figure plots the spread between the real funds rate and the Laubach and Williams (2003) natural rate of interest, a variable we refer to as the monetary policy spread. Denote the time t value of this spread mps_t .⁶ The data are quarterly and span the sample 1961:Q1-2017:Q3.⁷ While there is no secular trend downward in real interest rates over the full sample, there is, however, a noticeable downward trend in both the real interest rate and the *mps* since about 1980, a point we come back to below.

Next, Table 1 reports the correlations between the real interest rate or the *mps* and different asset valuation metrics. These correlations are reported for the raw series, and for

⁵Such estimates involve theoretical restrictions on parameters of this component of real rates. In Laubach and Williams (2003) these restrictions amount to estimates of the level of the real rate that consistent with no change in inflation.

⁶ mps_t is computed as

$$FFR_t - (\text{Expected Inflation})_t - r_t^*,$$

where FFR is the nominal federal funds rate and where expected inflation is a four quarter moving average of inflation. r_t^* is the natural rate of interest from Laubach and Williams. The quarterly nominal funds rate is the average of monthly values of the effective federal funds rate.

⁷The 1961 start date is dictated by the availability of the natural rate of interest measure.

components of the raw series that retain fluctuations with “medium” term cycles, defined to be cycles that take between 8 and 50 years to complete, and “business” cycles, defined to take between 1.5 and 8 years. Panel A reports these correlations with $-cay_t$, the negative of the log consumption-wealth variable of Lettau and Ludvigson (2001) (LL hereafter), one of the broadest asset valuation metrics available. With cay_t , asset values are measured relative to two macroeconomic fundamentals: log consumption “ c_t ” and log labor income “ y_t .” The “ a_t ” is total household net worth, which is highly correlated with the return on the aggregate stock market. We use $-cay_t$ to put asset values in the numerator, and refer to it simply as a “wealth” ratio. Columns B-D consider alternative valuation ratios each of which has some measure of stock market wealth in the numerator. Panel B uses the Shiller price-earnings ratio⁸, Panel C uses the price-dividend ratio for the corporate sector, and Panel D uses the price-earnings ratio for the corporate sector.

Several results in Table 1 stand out. First, correlations between the valuation ratios and either the real funds rate or the *mps* are all negative at medium-term frequencies. Thus, over cycles of 8-50 years, persistently high valuations tend to coincide with indicators of monetary policy that are persistently more accommodative. By contrast, the correlations are all positive at business cycle frequencies and generally weaker in absolute terms.

Second, in all cases, the absolute correlation between the valuations and the *mps* is greater than that between valuations and the real interest rate itself. Thus, purging the funds rate of the component estimated to be unrelated to monetary policy leads to greater negative comovement, which is suggestive that monetary policy as opposed to real rates *per se* play a role in this correlation.

Third, the largest absolute correlation is with $-cay_t$, which has a -0.83 correlation with the real interest rate and a -0.84 correlation with the *mps* at medium-term frequencies. This is followed by correlations of -0.49 and -0.60, respectively, with the corporate sector price-dividend ratio, -0.19 and -0.30 with the Shiller price-earnings ratio, and -0.20 and -0.30 with the corporate sector price-earnings ratio. This finding, namely that lower frequency movements in cay_t are more highly correlated in absolute terms with short-term interest rates than are other valuation ratios, is consistent with prior evidence that cay_t picks up more variation in discount rates than do stock market valuation ratios (Lettau and Ludvigson (2005)). This occurs due to the offsetting effects of fluctuations in measures of expected stock market cash flow growth on stock market valuation ratios that have little influence on cay_t .⁹ That is, some variation in expected stock market returns appears to be positively correlated with expected growth in stock market cash flows, but not with expected growth in c_t or y_t . We observe this

⁸<http://www.multpl.com/shiller-pe/>

⁹A similar finding occurs using other valuation metrics when they are measured relative to macro fundamentals. See Greenwald, Lettau, and Ludvigson (2019).

mechanism at work in the current data in Panel E of Table 1. At medium-term frequencies, decreases in the real interest rate or *mps*, which tend to drive stock market valuation ratios up, are associated with increases in the earnings share of output, which tend to drive them down. Since cay_t is not as subject to this type of confounding cash flow effect, and since discount rate movements are at the core of what we investigate in this study, we use $-cay_t$ as a measure of valuations in our formal econometric analysis. We discuss this estimation next.

3.2 Markov-Switching Estimation

This section presents results for a joint Markov-switching model of breaks in the mean of cay and an instrument of monetary policy. As a cleaner indicator of monetary policy, we use the *mps* rather than the real federal funds rate, in order to purge the latter of the trending natural rate component that has nothing to do with monetary policy. We first describe an econometric model of regime switches in the mean of cay_t . We then introduce a similar relation for the *mps*. Finally, we explain how we jointly estimate regime changes in the means of the two variables.

The log valuation variable cay_t is derived from an approximate formula for the log consumption to aggregate (human and non-human) wealth ratio, and its relationship with future growth rates of a_t and/or future growth rates of c_t and y_t can be motivated from an aggregated household budget constraint.¹⁰ An approximate expression linking c_t , a_t , and y_t to expected future returns to asset wealth, consumption growth, and labor income growth may be derived to yield

$$cay_t \equiv c_t - \gamma_a a_t - \gamma_y y_t \approx \alpha + \mathbb{E}_t \sum_{i=1}^{\infty} \rho_w^i ((1 - \nu) r_{a,t+i} - \Delta c_{t+i} + \nu \Delta y_{t+1+i}),$$

where ν is the steady state ratio of human wealth to asset wealth and $r_{a,t}$ is the log return to asset (non-human) wealth. Theory typically implies that c_t , a_t , and y_t should be cointegrated, or that the linear combination of variables in cay_t should be covariance stationary.

In the standard estimation without regime shifts in any parameters, the above stationary linear combination of c_t , a_t , and y_t may be written

$$cay_t^{FC} \equiv c_t - \gamma_a a_t - \gamma_y y_t = \alpha + \epsilon_t^{FC}, \quad (1)$$

where the parameters to be estimated are α , γ_a , and γ_y . The residual ϵ_t^{FC} is the mean zero stationary linear combination of these data, referred to as the cointegrating residual. Note that ϵ_t^{FC} is not in general an i.i.d. shock. The superscript “*FC*” stands for “fixed coefficients” to underscore the fact that no parameters in this relation are time-varying.

¹⁰This formula is derived under several assumptions described in LL and elaborated on in Lettau and Ludvigson (2010). If labor income is modeled as the dividend paid to human capital, we get the formulation below.

In this paper, we estimate a Markov-switching version of this variable, analogously written as

$$cay_t^{MS} \equiv c_t - \beta_a a_t - \beta_y y_t = \alpha_{\xi_t} + \epsilon_t^c, \quad (2)$$

where $\epsilon_t^c \sim N(0, \sigma_{MS}^2)$. The intercept term, α_{ξ_t} , is a time-varying mean that depends on the existence of a latent state variable, ξ_t , presumed to follow a two-state Markov-switching process with transition matrix \mathbf{H} . Thus α_{ξ_t} assumes one of two discrete values, α_1 or α_2 . The choice of two regimes is not crucial, but provides a readily interpretable way to organize the data into a low and a high valuation regimes. The residual ϵ_t^c is a stationary, continuous-valued random variable by assumption. The slope coefficients β_a and β_y are analogous to γ_a and γ_y in the fixed coefficient regression (1). They are denoted differently to underscore the point that the coefficients in (1) and (2) are not the same, just as the parameters α and α_{ξ_t} , and the residuals ϵ_t^{FC} and ϵ_t^c are not the same. Because our procedure jointly recovers the slope coefficients β_a and β_y , the timing of regime changes, and, as an implication, the decomposition of cay_t^{MS} into α_{ξ_t} and ϵ_t^c , all three statistical objects can differ.

We combine the estimation of changes in the mean of cay_t^{MS} with an isomorphic model for mps_t to estimate a joint Markov-switching model with synchronized regimes. Specifically, we assume that regime changes in the mean of cay_t^{MS} coincide with regime changes in the mean of the mps :

$$mps_t = r_{\xi_t} + \epsilon_t^r, \quad (3)$$

where $\epsilon_t^r \sim N(0, \sigma_r^2)$. Unlike cay_t^{MS} , mps_t is an observed variable. Thus, in this case we only need to estimate the Markov-switching intercept coefficient r_{ξ_t} . It is worth emphasizing that the same latent state variable, ξ_t , is presumed to follow a two-state Markov-switching process with transition matrix \mathbf{H} , controls both changes in α_{ξ_t} and r_{ξ_t} . Thus the regimes are synchronized across the two means.

The econometric model may be succinctly stated as a Markov-switching regression system with synchronized regimes:

$$\begin{aligned} c_t &= \alpha_{\xi_t} + \beta_a a_t + \beta_y y_t + \epsilon_t^c \\ mps_t &= r_{\xi_t} + \epsilon_t^r \\ \epsilon_t^c &\sim N(0, \sigma_{MS}^2), \quad \epsilon_t^r \sim N(0, \sigma_r^2) \end{aligned}$$

where ξ_t is a latent variable that follows a Markov-switching process with transition matrix \mathbf{H} . Denote the set of parameters to be estimated collectively with the vector

$$\boldsymbol{\theta} = (\alpha_{\xi_t}, \beta_a, \beta_y, r_{\xi_t}, \sigma_{MS}^2, \sigma_r^2, \text{vec}(\mathbf{H}))'.$$

We use Bayesian methods with flat priors to estimate the model parameters in (2) and (3) over the period 1961:Q1-2017:Q3. The sequence $\xi_t = \{\xi_1, \dots, \xi_T\}$ of regimes in place at

each point is unobservable and needs to be inferred jointly with the other parameters of the model. Estimates of α_{ξ_t} and r_{ξ_t} are formed by weighting their two estimated values by their state probabilities at each point in time. Let T be the sample size used in the estimation and let the vector of observations as of time t be denoted \mathbf{Z}_t . Let $P(\xi_t = i | \mathbf{Z}_T; \boldsymbol{\theta}) \equiv \pi_{t|T}^i$ denote the probability that $\xi_t = i$, for $i = 1, 2$, based on information that can be extracted from the whole sample and knowledge of the parameters $\boldsymbol{\theta}$. We refer to these as the smoothed regime probabilities. We may decompose cay_t^{MS} into two components, a discrete-valued time-varying mean and a continuous-valued random variable:

$$cay_t^{MS} = c_t - (\beta_a a_t + \beta_y y_t) = \bar{\alpha}_t + \epsilon_t^c \quad (4)$$

$$\bar{\alpha}_t = \sum_{i=1}^2 \pi_{t|T}^i \alpha_i. \quad (5)$$

thus $\bar{\alpha}_t$ is the probability-weighted average of the Markov-switching means. An analogous bifurcation exists for mps_t , where r_{ξ_t} may be computed as $\bar{r}_t = \sum_{i=1}^2 \pi_{t|T}^i r_i$.

The posterior distribution of the empirical model (2) and (3) and the corresponding regime probabilities $\pi_{t|t}^i$ and $\pi_{t|T}^i$ are obtained by computing the likelihood using the Hamilton filter (Hamilton (1994)), and combining it with priors. Since we use flat priors, the posterior coincides with the likelihood. Our estimate of cay_t^{MS} and its decomposition into $\bar{\alpha}_t$ and ϵ_t^c , and of mps_t into \bar{r}_t and ϵ_t^r , use the posterior mode of the parameter vector $\boldsymbol{\theta}$ and the corresponding regime probabilities. Uncertainty about the parameters, or about any transformation of the model parameters, is characterized using a Gibbs sampling algorithm. The full statement of the procedure and sampling algorithm is given in the Appendix.

The variable cay_t^{MS} may be interpreted as log inverse asset valuation ratios, akin to a log dividend-price ratio as opposed to log price-dividend ratio. For brevity, we refer to cay_t^{MS} as an inverse “wealth” ratio, or equivalently define the log *wealth ratio* as $-cay_t^{MS} = -[\epsilon_t^c + \bar{\alpha}_t]$. Thus, a high α_i corresponds to a low wealth ratio, since $c_t - \beta_a a_t - \beta_y y_t$ is high whenever a_t is low relative to $c_t - \beta_y y_t$. In population ϵ_t^c and ϵ_t^{FC} are mean zero random variables, thus the intercept term $\bar{\alpha}_t$ gives the mean of the inverse wealth ratios.¹¹

Table 2 reports the parameter estimates, while Figure 2 reports the probability of regime 1 over time for the Markov-switching intercepts α_{ξ_t} and r_{ξ_t} based on the posterior mode parameter estimates.

The results show that the sample is divided into three subperiods characterized by the two regimes for α and r . Regime 1 is a high α /high r regime with posterior mode point estimates equal to $\hat{\alpha}_1 = -0.7239$ and $\hat{r}_1 = 0.0111$. The posterior mode estimates for the low α /low r regime 2 are $\hat{\alpha}_2 = -0.7500$ and $\hat{r}_2 = -0.0252$. Since a high α regime for cay corresponds to a

¹¹In a finite sample, ϵ_t^{MS} and ϵ_t^{FC} are not necessarily mean zero because of the leads and lags of the first differences included in the DLS regression used to correct for finite sample biases—see the Appendix. In population these variables are mean-zero by definition.

low valuation ratio, we refer to *high α regime 1* as the *low asset valuation/high mps regime*, and *low α regime 2* as the *high asset valuation/low mps regime*.

The overall sample is divided into regime subperiods using the most likely estimated regime sequence, a T -dimensional vector denoted ξ^T .¹² Table 3 shows the regime subperiods based on this definition. The low asset valuation/high *mps* regime prevails for a prolonged period of time from 1978:Q4 to 2001:Q3, during which the smoothed probability that $\alpha = \hat{\alpha}_1$ and $r = \hat{r}_1$ is very close to unity. By contrast, the pre-1978 and most of the post-2001 subsample are high asset valuation/low *mps* regimes, where the probability that $\alpha = \alpha_1$ and $r = \hat{r}_1$ is virtually 0. The low valuation/high *mps* regime briefly reappears from 2006:Q1 to 2008:Q2 following a string of 17 target federal funds rate hikes by the Federal Reserve that began on June 30, 2004 and ended with the nominal rate standing at 5.25% on the 29th of June 2006. The target funds rate target remained above 4% until January 2008, when it was lowered to 3%.

Table 2 reports summary statistics for the differences $\hat{\alpha}_1 - \hat{\alpha}_2$ and $\hat{r}_1 - \hat{r}_2$, along with percentiles of their posterior distributions. The 90% credible sets for $\hat{\alpha}_1 - \hat{\alpha}_2$ and $\hat{r}_1 - \hat{r}_2$ are non-zero and positive, indicating that the data favor changes in the mean of the inverse wealth ratio and the *mps*. The two regimes are stationary but persistent, as indicated by the estimated diagonal elements of the transition matrix \mathbf{H} , also reported in Table 2.

To give a visual impression of the properties of these regimes, Figure 3 plots $-cay_t^{MS}$ and the *mps* over time. Also plotted as horizontal lines are the values $-\bar{\alpha}_t$ and \bar{r}_t that arise in each regime over the sample. The figure shows that the wealth ratio $-cay_t^{MS}$ fluctuates around two distinct means in five separate periods of the sample, a high mean in the early part of the sample, a low mean in the middle, a low mean in the shorter subperiod from 2006:Q1 to 2008:Q2, and a high mean again at the end of the sample. The *mps* is a mirror image, fluctuating around a low mean in the early part of the sample, a high mean in the middle, and, with the exception of 2006:Q1 to 2008:Q2, a low mean the latter part of the sample.

Several narrative “events” in monetary history are labeled in the *mps* plot of Figure 3. The first occurrence of the high asset valuation/low *mps* regime from 1961:Q1 to 1978:Q3 coincides with the run-up of inflation in the 1960s and 1970s and low real interest rates. Researchers have concluded that monetary policy failed to react aggressively to inflation during those years (Clarida, Gali, and Gertler (2000); Lubik and Schorfheide (2004); Sims and Zha (2006); Bianchi (2013)). This is labeled the “Burns Accommodation,” after Arthur Burns who chaired the Federal Reserve Board over much of this subperiod. Real interest rates increased significantly during the “Volcker disinflation” and remained high for a prolonged period of time, coinciding with low valuations and high *mps*. The beginning of second occurrence of the high asset valuation/low *mps* regime is labeled the “Greenspan Put,” in Figure 3 after the perceived attempt of Chairman Greenspan to prop up securities markets in the wake of the IT bust, a

¹²The Appendix describes how the most likely regime sequence is computed from the filtered probabilities.

recession, and the aftermath of 9/11, by lowering interest rates and (allegedly) resulting in a perception of put protection on asset prices. The high valuation/low *mps* subperiod at the end of the sample overlaps with the explicit forward guidance “low-for-long” policies under Chairman Bernanke that promised in 2011 to keep interest rates at ultra low levels for an extended period of time, possibly longer than that warranted by a 2% inflation objective.

3.3 Reaching for Yield?

A persistently low mps_t is associated with persistently high asset valuations. A natural question is whether a low *mps* affects only the riskless real interest rate, or whether it also prompts investors to “reach for yield,” (RFY) thereby driving the return premium component of the discount simultaneously rate down, further stimulating risky asset valuations.¹³ We begin with a loglinear framework to illustrate how interest rate regimes could be revealed in return premia in a cross-section.

3.3.1 A Loglinear Framework

One way to assess the possibility of RFY is to exploit differences across assets. A change in discount rates driven by the riskless rate alone influences all assets in the same way, regardless of their riskiness. By contrast, RFY implies that, as investors attempt to shift portfolio allocations toward riskier/higher return assets in low interest rate environments, return premia adjust. Thus a change in discount rates accompanied by RFY will have pricing effects that differ across assets, depending on the riskiness of the asset. As interest rates move from high to low, RFY implies a greater increase in the market value, relative to fundamentals, of higher return/higher Sharpe ratio assets than it does for lower return/lower Sharpe ratio assets. Equivalently, risk premia should fall more for riskier assets. We investigate this possibility here, using data on the market portfolios and equity characteristic stock portfolios that exhibit large cross-sectional variation in return premia. An extension of the approach is also used to study the behavior of the risk premium on the aggregate stock market, as explained below.

To motivate the analysis, consider a log-linearization that follows Vuolteenaho (2000) and constructs earnings from book-market and return data using clean surplus accounting. Let B_t denote book value and M_t denote market value, and let the logarithm of the book-market ratio $\log(B_t/M_t)$ be denoted θ_t . Vuolteenaho (2000) shows that θ_t of an asset or portfolio can be decomposed as:

$$\theta_t = \sum_{j=0}^{\infty} \rho^j \mathbb{E}_t r_{t+1+j} + \sum_{j=0}^{\infty} \rho^j \mathbb{E}_t f_{t+1+j} - \sum_{j=0}^{\infty} \rho^j \mathbb{E}_t e_{t+1+j}^* \quad (6)$$

¹³In what follows we use the terms “risk” premia and “return” premia interchangeably to refer to the expected return on an asset in excess of the risk-free rate. We remain agnostic as to whether the observed premia are attributable to genuine covariance with systematic risk factors or mispricing, or both.

where $\rho < 1$ is a parameter, and r_{t+1+j} , f_{t+1+j} , and e_{t+1+j}^* stand for log excess return, log risk-free rate, and log earnings, respectively.¹⁴ In other words, the logarithm of the book-market ratio θ_t depends on the present discounted value (PDV) of expected excess returns (risk premia), expected risk-free rates, and expected earnings.

Given two portfolios x and y , the spread in their book-market ratios, $\theta_{x,t} - \theta_{y,t}$, is given by:¹⁵

$$\underbrace{\theta_{x,t} - \theta_{y,t}}_{\text{Spread in BM ratios}} = \underbrace{\sum_{j=0}^{\infty} \rho^j \mathbb{E}_t (r_{x,t+1+j} - r_{y,t+1+j})}_{\text{PDV of spread in risk premia}} - \underbrace{\sum_{j=0}^{\infty} \rho^j \mathbb{E}_t (e_{x,t+1+j}^* - e_{y,t+1+j}^*)}_{\text{PDV of spread in expected earnings}}$$

Note that the risk-free rate has no affect on this spread, since all portfolios are affected in the same way by the risk-free rate. Instead only the risk premium differential and expected earnings differential affect the book-market spread. Since RFY pertains only to the return premium differential, we adjust the book-market spread for the spread in expected earnings to isolate the return premium differential:

$$\underbrace{\theta_{x,t} - \theta_{y,t} + \sum_{j=0}^{\infty} \rho^j \mathbb{E}_t (e_{x,t+1+j}^* - e_{y,t+1+j}^*)}_{\text{Spread in BM ratios adjusted for earnings} \equiv \tilde{\theta}_{x,t} - \tilde{\theta}_{y,t}} = \underbrace{\sum_{j=0}^{\infty} \rho^j \mathbb{E}_t (r_{x,t+1+j} - r_{y,t+1+j})}_{\text{PDV of the spread in risk premia}} \quad (7)$$

The above expression shows that the spread in book-market ratios adjusted for expected future earnings is equal to the PDV of the spread in expected excess returns, or risk premia.

Denote the *adjusted* (for expected earnings) book-market ratio for portfolio x in regime i with a tilde as

$$\tilde{\theta}_{x,t}^i \equiv \theta_{x,t}^i + \sum_{j=0}^{\infty} \rho^j \mathbb{E}_t e_{x,t+1+j}^{*i}.$$

Let x denote a high return premia portfolio while y denotes a low return premia portfolio. Reaching for yield implies that, in a shift from a high ($i = 1$) to low ($i = 2$) interest rate regime, the adjusted book-market ratio of x should fall more than y , implying $(\tilde{\theta}_{x,t}^1 - \tilde{\theta}_{x,t}^2) - (\tilde{\theta}_{y,t}^1 - \tilde{\theta}_{y,t}^2) > 0$, or that the *difference-in-difference* of adjusted book-market ratios should be positive across regimes:

$$(\tilde{\theta}_{x,t}^1 - \tilde{\theta}_{y,t}^1) - (\tilde{\theta}_{x,t}^2 - \tilde{\theta}_{y,t}^2) > 0. \quad (8)$$

Thus RFY implies that the *spread* in the *adjusted* book-market ratios between the high return/high risk portfolio and the low return/low risk portfolio should be greater in regime 1 than in regime 2. Equivalently, in a switch from a high to low interest rate regime, the PDV of risk premia on the high return premium portfolio should fall more than that of the low return premium portfolio.

¹⁴Specifically, e^* is the log of 1 plus the earnings-book ratio, adjusted for approximation error. See Vuolteenaho (2000).

¹⁵This derivation follows Cohen, Polk, and Vuolteenaho (2003) and imposes the approximation that ρ is constant across portfolios. Cohen, Polk, and Vuolteenaho (2003) find that the approximation error generated by this assumption is small.

3.3.2 Risk Premia Spreads on Equity Portfolios

We now investigate empirically whether the spread in adjusted book-market ratios between assets with different risk/return profiles is statistically different across the two regimes. To do so, we need to estimate the risk-premia on these portfolios across the regimes. For this, we estimate a Markov-switching vector autoregression (MS-VAR) using portfolio data. The parameters of the MS-VAR are permitted to potentially undergo structural shifts, but we impose the previously estimated regime sequence ξ^T for the conditional means of cay^{MS} and the mps on the MS-VAR while allowing the parameters characterizing the different regimes as well as the transition matrix to be freely estimated. Note that the objective of this exercise is to establish whether risk premia differ across the two *previously estimated* asset valuation-policy rate regimes. We therefore deliberately “tie our hands” by forcing the regime sequence for the MS-VAR to correspond to breaks in α_{ξ_t} and r_{ξ_t} . There is no implication that risk premia must necessarily show evidence of structural change. All parameters other than the regime sequence are freely estimated and could in principle show no shift across the previously estimated regimes.

MS-VARs can be used to estimate the difference-in-difference (8) for any two stock portfolios with different average return premia, or used to compute the PDV of risk premia $\sum_{j=0}^{\infty} \rho^j \mathbb{E}_t r_{t+1+j}$ for any one asset such the aggregate stock market portfolio in excess of the short-term interest rate. We do both exercises here.

We present results on how the difference-in-difference has behaved over time and across our regimes using data on stock market portfolios that exhibit large cross-sectional variation in return premia. Specifically, we use the equity return data available from Kenneth French’s Dartmouth website on portfolios formed from double-sorting all stocks in the AMEX, NYSE, NASDAQ into categories on the basis of five size categories and five BM categories, and alternatively single-sorting into 10 momentum categories based on recent past return performance.¹⁶ We then use CRSP/Compustat to construct the BM ratios of the corresponding portfolios. It is well known that high BM portfolios earn much higher average returns than low BM portfolios, exhibiting a value-spread, especially in the small size quintiles. Along the momentum dimension, recent past winner stocks earn much higher returns than recent past losers. These statistical facts are evident from Table 4, which reports sample statistics for returns on two value spread portfolios—those long in the extreme value portfolio (highest BM ratio) and short in the extreme growth portfolio (lowest BM ratio) while being in the smallest and second smallest size quintile. The same is reported for a momentum spread portfolio—the portfolio that is long in the extreme winner portfolio (M10) and short in the extreme loser portfolio (M1). The annualized Sharpe ratio for the smallest value spread portfolio is 0.60 with an annualized mean

¹⁶http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

return of 10%. Similarly, the momentum spread portfolio has an annualized Sharpe ratio of 0.63 and annualized mean return of over 15%. Both of these strategies have much higher annualized Sharpe ratios and average return premia than the CRSP value-weighted stock market return in excess of the three-month Treasury bill return, where the corresponding numbers are 0.36 and 6%, respectively.

We estimate a single MS-VAR for the two value spread portfolios and the momentum spread portfolio along with other data that are predictor variables for the returns on these portfolios, chosen on the basis of an Akaike Information Criterion (AIC) selection procedure. The variables included in the MS-VAR are: (a) the momentum return spread, i.e., the difference between the excess return of the extreme winner (M10) portfolio and the excess return of the extreme loser (M1) portfolio; (b) the value return spread (S1), i.e., the difference between the excess return of the small (size 1) high BM portfolio and the excess return of the small (size 1) low BM portfolio; (c) the value return spread (S2), i.e., the difference between the excess return of the size 2 high BM portfolio and the excess return of the small size 2 low BM portfolio; (d) the momentum BM spread: the difference between the logarithm of the BM ratio of the M10 and M1 portfolios; (e) the value BM spread (S1): The difference between the logarithm of the BM ratio of the small (size quintile 1) high book-market portfolio and the logarithm of the BM ratio of the small (size 1) low book-market portfolio; (f) the value BM spread (S2): the difference between the logarithm of the BM ratio of the size quintile 2 high book-market portfolio and the logarithm of the BM ratio of the size 2 low book-market portfolio; (g) the real federal funds rate (nominal federal funds rate minus inflation); (h) the excess return on the small value portfolio.¹⁷ The sample for this estimation is 1964:Q1-2017:Q3.¹⁸ We follow a similar procedure to form an estimate of the PDV of risk premia on the overall stock market, $\sum_{j=0}^{\infty} \rho^j \mathbb{E}_t r_{t+1+j}$, but estimate a separate MS-VAR for this purpose because the relevant predictor variables are different from those using the equity-characteristic portfolios.¹⁹

¹⁷The BM spreads are included because they represent the natural valuation ratios for the portfolio return spreads that we are trying to predict (Cohen, Polk, and Vuolteenaho (2003)). The real FFR and the excess return on the small value portfolio are selected based on the Akaike Information Criterion (AIC) among a set of possible additional regressors. The five Fama/French factors (Fama and French (2015)) are considered as possible additional regressors, but not selected based on the AIC. The Online Appendix provides additional details on the variable selection procedure.

The data on BM ratios for individual portfolios are constructed from CRSP and Compustat exactly as the Fama-French portfolio returns are constructed. We mimic the selection and breakpoints of this construction and compute the book-market ratio of each portfolio.

¹⁸This is shorter than the sample used previously because reliable data for book-market ratios are not available prior to 1964:Q1.

¹⁹The MS-VAR specification for the market risk premium includes the following variables: (a) the market excess return, computed as the difference in the CRSP value-weighted stock market return (including dividend redistributions) and the three-month Treasury bill rate; (b) $-cay^{MS}$; (c) following Campbell and Vuolteenaho (2004), the small stock value spread (log-difference in the book to market ratio of the S1 value and S1 growth portfolio); (d) the SMB factor from Fama and French; (e) the HML factor from Fama and French. These variables are included because they improve the AIC criterion.

To exploit the heterogeneity across these portfolios, we begin by computing the *regime average* values of the adjusted BM *spreads* between the high and low return premia portfolios, $\bar{\theta}_{xy}^i \equiv \bar{\theta}_x^i - \bar{\theta}_y^i$, for each regime i . The regime average value of $\bar{\theta}_{xy}^i$ is defined as the expected value of $\tilde{\theta}_{xy,t}$, conditional on being in regime i today and on the variables of the VAR being equal to their conditional steady state mean values for regime i . The Appendix gives formal expressions for the regime average, and explains how they are computed from the MS-VAR parameters.

Table 5 reports the median and 68% credible sets for $\bar{\theta}_{xy}^i$, computed from each draw of the VAR parameters from the posterior distribution. The high (x) and low (y) return premia portfolios along the BM dimension are always the extreme value (highest BM) and the extreme growth portfolio (lowest BM), respectively, in each size category. Likewise, the high and low return premia portfolios along the momentum dimension are always the extreme winner (M10) and extreme loser portfolio (M1). For the market risk premium, x is the market return and y is the risk-free rate. The third row reports the analogous values for the regime average of the difference-in-difference of the PDV of risk premia between the high and low return premia portfolios across the two regimes, i.e., the difference $(\tilde{\theta}_{x,t}^1 - \tilde{\theta}_{y,t}^1) - (\tilde{\theta}_{x,t}^2 - \tilde{\theta}_{y,t}^2)$, as implied by the MS-VAR estimates. The last row reports the posterior probability that risk premia decline in the low *mps* regime, computed as the percentage of draws from the posterior distribution of regime averages for which risk premia are lower in regime 2 than in regime 1. To interpret the table, keep in mind that regime 1 is the low asset valuation/high interest rate regime, while regime 2 is the high asset valuation/low interest rate regime.

Table 5 shows that the median values of the adjusted BM spreads $\bar{\theta}_{xy}^i$ between the high and low return premia portfolios are positive in both regime 1 and regime 2. This is not surprising because portfolios that have higher return premia should have lower market-to-book values, holding fixed expected earnings. Ostensively riskier portfolios have a higher PDV return premium on average, regardless of the regime (see (7)). More significantly, the spreads are larger in the low valuation/high *mps* regime 1 than in the high valuation/low *mps* regime 2, implying that the difference-in-difference (“diff-in-diff”) across regimes is always positive. In a shift from high to low *mps* regime, the adjusted book-market ratios of high return/high risk portfolios fall more than do those of low return/low risk portfolios. Put differently, the return premia of evidently riskier/higher return assets decline more in environments with persistently high aggregate wealth ratios and low policy interest rates than do less risky/lower return assets.

The third row of Table 5 reports the 68% posterior credible intervals in parentheses for the diff-in-diff. Although the 68% posterior credible sets include negative values for the BM spreads and the market risk premium (though not the momentum spread), this does not imply that negative values are likely. The posterior distribution of the diff-in-diff displays substantial

negative skewness and, as a consequence, the posterior probability assigned to positive values, i.e., to a decline in premia during the high valuation/low *mps* subperiods, is quite high in all cases: 81%, 90%, 70%, and 64%, for the market premium, the momentum spread, the S1 BM spread, and the S2 BM spread, respectively. The odds that premia decline in the high valuation/low *mps* regime is over 4 to 1 for the market premium, over 9 to 1 for the momentum spread, over 2.3 to 1 and 1.8 to 1 for the S1 and S2 BM spreads, respectively. In short, the mass of probability overwhelmingly favors one particular interpretation, namely that return premia are lower rather than higher in the high valuation/low *mps* subperiods, consistent with a reach for yield.

We also estimate the PDV of risk premia as it evolves over the sample, estimated as the time t VAR forecasts, i.e. conditional expected values, of the PDV on the right-hand-side of (7).²⁰ Given the posterior distribution of the VAR parameters, these forecasts have their own posterior distribution. Figure 4 reports the median values of these forecasts as solid (blue) lines, while the regime averages are indicated by the dashed (red) lines. Although the risk premia are volatile, it is evident that they fluctuate around distinct means across the regimes. The portfolio estimated risk premia reach lows or near-lows in the post-millennial period, after shooting up briefly in the aftermath of the financial crisis of 2007-2008. Estimated risk premia return to low levels in the post-crisis ZLB period. These results are supportive of a channel that implies an increased appetite for risk-taking in low interest rate environments.

4 MS-DSGE Model

The foregoing results suggest that persistent low real interest rate environments characterized by low or negative monetary policy spreads are associated with high asset valuations and lower equity return premia. But, as discussed, macro models and macro estimates imply that monetary policy shocks have at most short-run effects on the real economy. This raises the question: could these findings be attributed to monetary policy, and if so why and how?

4.1 Model Description

To address these questions, we construct and estimate a New-Keynesian Markov-switching, dynamic, stochastic, general equilibrium (MS-DSGE) model. As we show below, this model has the capacity to exhibit long-lasting (though not permanent) effects of monetary policy on the real economy due to two distinctive features: sticky expectations and regime changes in the conduct of monetary policy. But the framework nests several special cases in which monetary non-neutrality could be small in magnitude, short-lived, or absent entirely. Thus the extent

²⁰The expectation $\mathbb{E}_t(\cdot) \equiv \mathbb{E}(\cdot|\mathbb{I}_t)$, computed as above where \mathbb{I}_t includes knowledge of the regime in place at time t , Z^t , and the VAR parameters for each regime.

to which the model exhibits long-lasting real effects depends on parameters that are freely estimated.

Outside of these two distinctive features, the model we propose is isomorphic to the prototypical New Keynesian model of Galí (2015), Chapter 3.

The behavior of households is summarized by a linearized Euler equation for a representative agent taking the form

$$y_t = \tilde{\mathbb{E}}_t(y_{t+1}) - \sigma \left[i_t - \tilde{\mathbb{E}}_t(\pi_{t+1}) - r_{ss} \right] + d_t \quad (9)$$

where y_t is detrended log output, i_t is the short-term nominal interest rate, $\tilde{\mathbb{E}}_t(\pi_{t+1})$ is expected inflation from the point of view of agents in the model, r_{ss} is the steady state real interest rate, and d_t is a demand shock that follows an AR(1) process $d_t = \rho_d d_{t-1} + \sigma_d \varepsilon_d$, $\varepsilon_d \sim N(0, 1)$. The parameter σ is the inverse of the intertemporal elasticity of substitution of the representative household.

The relation between inflation and the output gap is controlled by a New-Keynesian Phillips curve:

$$\pi_t - \bar{\pi}_t = \beta \tilde{\mathbb{E}}_t[\pi_{t+1} - \bar{\pi}_t] + \kappa [y_{t-1} - y_{t-1}^*] \quad (10)$$

where $\bar{\pi}_t \equiv \tilde{\mathbb{E}}[\pi_t | \mathbb{I}_t]$ denotes the perceived long term value of inflation that depends on \mathbb{I}_t , the information available to agents at time t . We discuss the way expectations are formed below. The parameter κ denotes the slope of the Phillips curve and depends on the extent of nominal rigidities in the economy. The variable y_t^* denotes the natural level of output defined as the level that would prevail absent nominal rigidities. Thus, $y_{t-1} - y_{t-1}^*$ is the output gap at time $t - 1$. We assume an AR(1) process for : $y_t^* = \rho_{y^*} y_{t-1}^* + \sigma_{y^*} \varepsilon_{y^*}$, $\varepsilon_{y^*} \sim N(0, 1)$.

The central bank obeys the following nominal interest rate rule:

$$\begin{aligned} i_t - \left(r_{ss} + \pi_{\xi_t}^T \right) &= (1 - \rho_{i, \xi_t}) \left[\psi_{\pi, \xi_t} \left(\pi_t - \pi_{\xi_t}^T \right) + \psi_{\Delta y, \xi_t} (y_t - y_{t-1}) \right] \\ &\quad + \rho_{i, \xi_t} \left[i_{t-1} - \left(r_{ss} + \pi_{\xi_t}^T \right) \right] + \sigma_i \varepsilon_i, \quad \varepsilon_i \sim N(0, 1). \end{aligned} \quad (11)$$

The rule implies that the central bank responds to deviations of inflation from the target and to detrended output growth. An important feature of this policy rule, and a departure from the prototypical model, is that it allows for regime shifts in the inflation target $\pi_{\xi_t}^T$ and in the activism coefficients ψ_{π, ξ_t} , and $\psi_{\Delta y, \xi_t}$ that govern how strongly the central bank responds to deviation from the target and to economic growth. The rule also allows for potential regime shifts in the autocorrelation coefficient ρ_{i, ξ_t} . These coefficients are modeled as following a Markov-switching process governed by the discrete random variable ξ_t and will be discussed below. Note the interest rate rule is written in deviations from the steady state conditional on being in a particular regime dictated by ξ_t . This means that, once inflation reaches the desired target, the economy stabilizes around it, absent shocks.

Expectations about inflation are formed using a learning algorithm as follows. First, we follow Malmendier and Nagel (2015) (MN) and assume that agents form conditional expectations about inflation using an autoregressive process, $\pi_t = \alpha + \phi\pi_{t-1} + \eta_t$. We further assume that agents must learn about the parameter α , while they have invariant beliefs about the autoregressive parameter ϕ .²¹ Learning implies that, each period, agents form a belief, $\tilde{\alpha}_t$, about α that is updated over time. Updating not only affects beliefs about next period inflation, it also affects beliefs about long-term trend inflation. Define *perceived trend inflation* to be the $\lim_{h \rightarrow \infty} \tilde{\mathbb{E}}_t[\pi_{t+h}]$ and denote it by $\bar{\pi}_t$. Given the presumed autoregressive process that agents use to form expectations about inflation, $\bar{\pi}_t$ is equal to unconditional mean of inflation under the agent's subjective measure, i.e., $\bar{\pi}_t = \tilde{\mathbb{E}}[\pi_t] = (1 - \phi)^{-1} \tilde{\alpha}_t$. Taken together, these assumptions imply that agents' expectations of one step ahead inflation, $\tilde{\mathbb{E}}_t[\pi_{t+1}]$, is a weighted average of perceived trend inflation and current inflation:

$$\tilde{\mathbb{E}}_t[\pi_{t+1}] = \tilde{\alpha}_t + \phi\pi_t = (1 - \phi) \bar{\pi}_t + \phi\pi_t. \quad (12)$$

The evolution of beliefs about $\tilde{\alpha}_t$ and $\bar{\pi}_t$ potentially reflects both an adaptive learning component as well as a signal about the central bank's inflation target. For the adaptive learning component, we follow evidence in MN that the University of Michigan Survey of Consumers (SOC) mean inflation forecast is well described by a constant gain learning algorithm. For the signal component, we assume that beliefs could be partly shaped by additional information the agent receives about the current inflation target. This signal could reflect the opinion of experts (as in MN) or a credible central bank announcement. Combining these two yields updating rules for $\tilde{\alpha}_t$ and $\bar{\pi}_t$ that are a weighted averages of two terms:

$$\tilde{\alpha}_t = (1 - \gamma^T) \left[\underbrace{\tilde{\alpha}_{t-1} + \gamma(\pi_t - \phi\pi_{t-1} - \tilde{\alpha}_{t-1})}_{\tilde{\alpha}_t^{CG}} \right] + \gamma^T \left[(1 - \phi) \pi_{\xi_t}^T \right] \quad (13)$$

$$\bar{\pi}_t = (1 - \gamma^T) \left[\underbrace{\bar{\pi}_{t-1} + \gamma(1 - \phi)^{-1}(\pi_t - \phi\pi_{t-1} - (1 - \phi)\bar{\pi}_{t-1})}_{\bar{\pi}_t^{CG}} \right] + \gamma^T \left[\pi_{\xi_t}^T \right] \quad (14)$$

The first terms in square brackets, $\tilde{\alpha}_t^{CG}$ and $\bar{\pi}_t^{CG}$, are the recursive updating rules implied by constant gain learning, where γ is the constant gain parameter that governs how much last period's beliefs $\tilde{\alpha}_{t-1}$ and $\bar{\pi}_{t-1}$ are updated given new information, π_t . (The Appendix provides additional details on constant gain learning.) The second terms in square brackets captures the

²¹In principle one could introduce learning about ϕ as well. We forgo doing this in order to keep the estimation tractable, since the most important learning aspects in the model involve those parameters such as α that bear most closely on trend inflation.

effect of the signal about the current inflation target $\pi_{\xi_t}^T$. If $\gamma^T = 1$, the signal is completely informative and the agent's belief about trend inflation is the same as the inflation target. If $\gamma^T = 0$, the signal is completely uninformative and the agent's belief about trend inflation depends only on the adaptive learning algorithm. Overall perceived trend inflation is therefore a weighted average of the trend implied by the constant gain learning rule and the central bank's inflation target. A weight of less than one on the target could arise either because the target is imperfectly observed, or because central bank announcements about the target are not viewed as fully credible. Note that the parameter γ^T is closely related to the speed with which the agent learns about a new inflation target. Since γ^T is freely estimated, we can empirically assess the magnitude of this speed and its role in macroeconomic fluctuations.

Agents form expectations about detrended output using a simple backward looking rule:

$$\tilde{\mathbb{E}}_t(y_{t+1}) = \delta y_{t-1}. \quad (15)$$

Unlike inflation, agents do not perceive a moving mean for detrended output. This assumption is consistent with the equilibrium of the model implying that the central bank cannot have a permanent effect on real activity. The Online Appendix proves that monetary neutrality holds in the very long run.

Using equations (12), (14), and (15), we substitute out $\tilde{\mathbb{E}}_t[\pi_{t+1}]$, $\bar{\pi}_t$, and $\tilde{\mathbb{E}}_t(y_{t+1})$ in the model equations (9), (10), and (11) to obtain the following system of equations:

1. Real activity

$$y_t = \delta y_{t-1} - \sigma [i_t - \phi \pi_t - (1 - \phi) \bar{\pi}_t - r_{ss}] + d_t.$$

2. Phillips curve:

$$\pi_t = \bar{\pi}_t + \frac{\kappa}{1 - \beta\phi} [y_{t-1} - y_{t-1}^*].$$

3. Monetary policy rule with changes in target:

$$\begin{aligned} i_t - (r_{ss} + \pi_{\xi_t}^T) &= (1 - \rho_{i,\xi_t}) \left[\psi_{\pi,\xi_t} \left(\pi_t - \pi_{\xi_t}^T \right) + \psi_{\Delta y,\xi_t} (y_t - y_{t-1}) \right] \\ &\quad + \rho_{i,\xi_t} \left[i_{t-1} - (r_{ss} + \pi_{\xi_t}^T) \right] + \sigma_i \varepsilon_i, \quad \varepsilon_i \sim N(0, 1). \end{aligned}$$

4. Law of motion for d_t :

$$d_t = \rho_d d_{t-1} + \sigma_d \varepsilon_d, \quad \varepsilon_d \sim N(0, 1).$$

5. Law of motion for y_t^* :

$$y_t^* = \rho_{y^*} y_{t-1}^* + \sigma_{y^*} \varepsilon_{y^*}, \quad \varepsilon_{y^*} \sim N(0, 1).$$

6. Perceived trend inflation:

$$\bar{\pi}_t = [1 - \gamma^T] [\bar{\pi}_{t-1} + \gamma(1 - \phi)^{-1} (\pi_t - \phi \pi_{t-1} - (1 - \phi) \bar{\pi}_{t-1})] + \gamma^T \pi_{\xi_t}^T.$$

4.2 MS-DSGE Estimation

We estimate the model using Bayesian methods. Parameter uncertainty is characterized using a random walk Metropolis–Hastings algorithm. The parameters of the policy rule, $\pi_{\xi_t}^T$, ψ_{π,ξ_t} , $\psi_{\Delta y,\xi_t}$ and ρ_{i,ξ_t} , are permitted to switch between two regimes according to a Markov-switching process. Since we are interested in whether the previously estimated regimes characterized by persistently high (low) valuations and low (high) *mps* could be attributable to monetary policy, we force the regime sequence for the policy rule parameters to correspond to that for α_{ξ_t} and r_{ξ_t} reported in Table 3. But the parameters characterizing the policy regimes as well as the transition matrix are freely estimated.²² Note that there is no implication from this procedure that the parameters of the policy rule must necessarily show evidence of structural change. Indeed, the estimation uses flat priors for all policy rule parameters, so not only are the parameters permitted to show no shift across the valuation/*mps* regimes, there is no prior imposed that predisposes the estimates to implying that they should shift.

As data, we use four observable series: real per-capita gross domestic product (GDP) growth, inflation, the nominal federal funds rate (FFR), and the mean inflation expectation from the SOC. By using inflation expectations, we ask the model to generate a realistic behavior for inflation expectations. Specifically, we map the perceived law of motion of inflation into the Michigan survey. Since we have only three shocks to match four observable variables, we allow for observation errors on all shocks.

The sample spans the period 1961:Q1 to 2017:Q3, in line with our estimates for the regimes in the means of *cay* and the *mps*. We use the full sample of data, including observations from the zero lower bound (ZLB) period. The Appendix shows that our findings on the long-lasting real effects of changes in the conduct of monetary policy are robust to replacing the FFR either with an estimated shadow rate, or with the one-year Treasury bill rate. The reason for this is that the policy rule regime changes we uncover are not mainly tied to the ZLB period.

The Appendix also provides a detailed description of the data, the model solution, and estimation.

4.3 MS-DSGE Estimation Results

Table 6 reports the prior and posterior distributions for the model parameters. For the policy rule parameter estimates for $\pi_{\xi_t}^T$, ψ_{π,ξ_t} , $\psi_{\Delta y,\xi_t}$ and ρ_{i,ξ_t} , where we use flat priors, a key finding is that the previously estimated regime subperiods (given in Table 3) are associated with quantitatively large changes in the estimated policy rule. Regime 1, which is the low valuation/high *mps* regime is a *hawkish* policy regime characterized by a low inflation target $\pi_{\xi_t}^T$ and

²²We use the regime sequence $\hat{\xi}^T = \{\hat{\xi}_1, \dots, \hat{\xi}_T\}$ that is most likely to have occurred, given our estimated posterior mode parameter values for θ . See the Appendix for details.

strong activism ψ_{π,ξ_t} against deviations of inflation from the target relative to activism $\psi_{\Delta y,\xi_t}$ on growth. Regime 2, which is the high valuation/low *mps* regime is a comparatively *dovish* regime characterized by an inflation target that is significantly higher and activism against inflation that is significantly lower. In fact, for regime 2, the 90% credible set for ψ_{π,ξ_t} includes 1, the threshold generally associated with the “Taylor principle” (Taylor (1993)), which prescribes that the central bank should raise nominal rates by more than one-for-one in response to deviations of inflation from target thereby raising the real rate and reducing inflationary pressure. The activism coefficient $\psi_{\Delta y,\xi_t}$ for output growth and the autoregressive parameter ρ_{i,ξ_t} are more similar across the two regimes.

These findings indicate that the policy rule parameters shifted to hawkish values in 1978:Q4 around the time of Volcker’s appointment, consistent with an older empirical literature (e.g., Clarida, Gali, and Gertler (2000)). But we find here that, starting 2001:Q4, there was a decisive shift back to the dovish policy rule that has, with the exception of a brief interlude from 2006:Q2-2008:Q2, remained in place to the end of our sample in 2017:Q3.

Shifts in the policy rule parameters across the two regimes are large in magnitude. Table 7 reports the posterior distribution for the differences in the parameters across regimes. The mode of the distribution of the difference in the quarterly inflation targets across is around 2%. This large value implies a difference in the annualized inflation target across regimes of almost 8%. The 90% credible set also indicates strong statistical evidence in favor of a quantitatively large difference in the inflation target across the two regimes. Similarly, the posterior distribution for the difference in the inflation activism coefficient ψ_{π,ξ_t} is centered on 1.2 with posterior credible sets that bounded well away from zero, confirming evidence of a change in the degree of activism aimed at stabilizing inflation around the desired target. Finally, the posterior distributions for the difference in activism $\psi_{\Delta y,\xi_t}$ on growth and in the autoregressive parameter ρ_{i,ξ_t} show only weak evidence of change in these parameters. Thus, overall, we can conclude that the two regimes present evidence of sizable shifts across the previously estimated regimes in the relative importance of inflation and economic growth in the policy rule and a large shift in the tolerable level of inflation.

For the non-policy-rule parameters, it is worth emphasizing that the estimates imply a very high level of inertia in inflation expectations. The constant gain parameter γ controlling the speed with which beliefs about long-term inflation are updated with new information on inflation is estimated to be quite low. Furthermore, the parameter γ^T controlling the information flow about the central bank target is estimated to be very low. Taken together, these findings imply that agents revise their beliefs about long term inflation only very slowly over time and mostly based on past realizations of inflation rather than on the inflation target itself.

Figure 5 shows how the model-implied series track their empirical counterparts. In general, observation errors play little to no role in the dynamics of the model implied series. Most

important for the application here, the dynamics of the model-implied series for one-period inflation expectations tracks the SOC series virtually without error. This is relevant since inflation expectations play a key role in the model’s predictions, as we show below. Figure 5 underscores the extent to which those predictions are predicated on empirically relevant expectations of inflation. The other model-implied series also track their empirical counterparts fairly closely. In particular, since the model fits the FFR and inflation expectations well, it also fits the real rate as measured by the difference in the two. For inflation, there are a handful of high-frequency spikes that the model is not well positioned to capture. A richer model could account for these spikes, but since the scope of our investigation is a study of lower frequency shifts in the policy rule, we do not view this as an important drawback of the framework. We now ask what these estimates imply for how monetary policy affects real interest rates and risk premia.

4.3.1 The Conduct of Monetary Policy and the Real Interest Rate

To investigate the importance of changes in the conduct of monetary policy in driving real interest rates over our sample, we consider a number of simulations presented in several figures below. All figures present the values of the variables at the estimated posterior mode parameter values.

Figure 6 shows the results of a simulation in which the observables and estimated state vector are taken as they were at the beginning of our sample with all Gaussian shocks are shut down, so that the only source of variation in the variables arises from changes in the conduct of monetary policy, i.e., from changes in the policy rule parameters. The simulation therefore isolates the contribution of changes in the inflation target and policy rule activism coefficients to fluctuations in aggregate variables. Note that these isolated movements are the only ones other than the policy shock that the model stipulates can be purely the result of the behavior of the monetary authority. Monetary policy also affects the propagation of the two non-policy shocks, as well as the policy shock itself, but these effects are, by contrast to changes in the policy rule, not solely the result of changes in monetary policy. The portion of movements in macro variables that can be directly associated with changes in the conduct of monetary policy are shown in blue (solid) lines in Figure 6. The figure also considers two counterfactual simulations to analyze the relative importance of changes in the target and changes in the activism coefficients. The orange (dashed) line assumes that monetary policy starts under the dovish regime and no subsequent regime change occurs. The black (dotted) line assumes that changes in the target occurred, but that the slope coefficients in the policy rule coefficients always remain as in the dovish, high inflation target regime.

A series of noteworthy results emerge from these exercises. First, if the central bank had always maintained a high target with low response to deviations of inflation from the target,

the economy would not have experienced the drop in inflation that occurred in the early 1980s. Instead, inflation would have kept increasing. What is more relevant and less obvious is the behavior of the real federal funds rate. The right panel of Figure 6 shows that changes in the conduct of monetary policy generate fluctuations in the real interest rate that lasts for decades. Comparing the estimated case with the orange dashed line that counterfactually assumes no policy rule changes occurred in our sample, it is evident that the real FFR would have been substantially more stable without the policy rule changes.

Figure 6 also shows that large, persistent swings in short-term interest rates driven by the conduct of monetary policy were the outcome not only of shifts in the inflation target, but also of regime changes in the activism coefficients of the policy rule. This may be observed by comparing the black dotted line, which shows a counterfactual in which the inflation target changes but there are no accompanying changes in the activism coefficients, with the baseline estimation shown in the blue solid line. Comparing the two, we see that the sharp increases in real rates associated with Volcker would have been far smaller had the activism coefficients remained constant. This happens because the hawkish regime exhibits both a lower inflation target and increased activism against deviations from the target. This combination—a lower target and aggressive action to quickly reach the lower target—generates a greater increase in the real interest rate than what a lower target alone would imply. So in the counterfactual simulation where the inflation target changes but the activism coefficients do not, the increase in the real FFR is significantly smaller and this comes at the cost for the central bank of having to take longer to rein in inflation and inflation expectations. A similar result holds in the short hawkish regime that precedes the Great Recession.

Figure 6 also shows that shifts to hawkish regimes under Volcker and in the pre-Great Recession subperiod contributed not only to sharp increases in real rates in those episodes but also to sharp contractions in output growth during the recessions that followed. In both cases the results show that these regimes with lower inflation targets, especially because they were coupled with more aggressive responses to deviations from the target, contributed to sharp contractions in real output.

The next figure investigates how important changes in the conduct of monetary policy were in generating the secular decline in real interest rates observed since 1980. For this purpose, we begin a simulation with the economy as it was in 1980:Q1, when inflation had reached its peak in our sample but just before the real interest rate reached its peak, which occurred in 1980:Q3. To isolate the effects of changes in the monetary policy rule, we set all Gaussian shocks after 1980:Q1 to zero. Thus, the simulation again focuses on the estimated portion of movements in the macroeconomic variables that can be directly associated with changes in the conduct of monetary policy, this time starting with the economy as it was right after Volcker was appointed. These movements are shown in blue (solid) lines in Figure 7. The actual values

for each series are shown in red (dashed) lines. The right panel of Figure 7 shows that the sharp run-up in real rates in the 1980s, and almost all of its downward trend since that time, is estimated to be attributable to changes in the conduct of monetary policy. Changes in the conduct of monetary policy do not, however, track the higher frequency fluctuations in the real rate. For example, there is a sharp transitory decline in real rates right after the Great Recession that is not associated with a shift in the policy rule parameters, but is instead attributed to a combination of the model’s Gaussian shocks.

Figure 8 explains the role of learning in generating the persistent movements in the real interest rate. For this purpose, we consider an alternative counterfactual simulation in which agents receive a perfect signal about the central bank’s inflation target, in contrast to what the parameter estimates actually imply. As explained above, this corresponds to the case $\gamma^T = 1$. This value is highly counterfactual, since the estimated parameter value of $\hat{\gamma}^T = 0.013$ implies that expectations of trend inflation as implied by the SOC data place virtually no weight on the actual target. Under the counterfactual scenario, the perceived trend value of inflation coincides in every period with the actual inflation target $\bar{\pi}_t = \pi_{\xi_t}^T$. In both the counterfactual and the baseline, the observables and estimated state vector are taken as they were at the beginning of our sample with all Gaussian shocks shut down, so that the only source of variation is changes in the conduct of monetary policy. The solid blue line is again the baseline estimated portion of movements in the macroeconomic variables that can be directly associated with changes in the conduct of monetary policy in full model, while the dashed line corresponds to the counterfactual case with perfect information about the central bank’s target.

Figure 8 shows that learning is crucial to the long-lasting effects of monetary policy changes on the real interest rate. In the counterfactual economy, inflation jumps immediately to the new target and we do not observe any effect on the real interest rate. Inflation jumps in the counterfactual case because the central bank does not have to “convince” agents about the new desired level of inflation. Thus it is the interaction between changes in the anti-inflationary stance of the central bank and sticky expectations that generates long-lasting fluctuations in the real interest rate.

4.3.2 The Dynamic Effects of Policy Rule Changes Versus Policy Shocks

Given the above results, it is of interest to compare the dynamic effects of a monetary policy shock with those of a monetary policy regime change. To do so, Figure 9 presents two sets of estimated impulse response functions. In the top row, we assume that the economy is initially in the dovish regime and consider the case of the monetary authority attempting to curb inflation, as exemplified by the Volcker disinflation. The blue solid line in the top row reports responses to a two standard deviation *contractionary* (i.e., positive) monetary policy shock and no policy rule regime change. The black dashed line in the top row reports impulse responses

to a regime change from the dovish to the hawkish regime, with all Gaussian shocks (including the monetary policy shock) set to zero. The figure shows that monetary policy shocks are comparatively short-lasting, consistent with evidence in Christiano, Eichenbaum, and Evans (2005). By contrast, the effects of a regime change in the policy rule are long-lived and last for many decades. This occurs because of the high degree of stickiness in inflation expectations implied by the model estimates. When the central bank curbs inflation, it lowers the inflation target and simultaneously moves aggressively toward that goal by intensifying its response to deviations from the target. But because inflation expectations are sticky, the sharp increase in the real interest rate that results remains elevated for decades.

Because the model is nonlinear, the duration of these effects can differ depending on whether we begin in a dovish or hawkish regime. So in the lower row of Figure 9 we now assume that the economy is initially in the hawkish regime and we consider the case of the monetary authority attempting to lift inflation, as exemplified by the post Great Recession era. In this case, the blue solid line shows the impulse responses to a two standard deviation *expansionary* (i.e., negative) monetary policy shock and no regime change in the conduct of monetary policy. The black dashed line shows responses to a regime shift from the hawkish to the dovish regime, with all Gaussian shocks set to zero. We again observe that the effects on the real interest rate of a policy rule regime change are extremely long lived and are even more persistent than the curbing inflation case. The reason is that, under the dovish regime, the estimated policy rule implies that the central bank responds less aggressively to deviations of inflation and output from their targets as indicated by the estimated activism coefficients ψ_{π, ξ_t} and $\psi_{\Delta y, \xi_t}$, which are smaller in the dovish regime. Thus, when the central bank seeks to lift inflation as opposed to curb it, it does so more gradually than in the hawkish regime. The result is that the real interest rate remains perturbed from its steady state for longer following a shift to the dovish monetary policy regime compared to a shift to the hawkish regime.

It should be emphasized that in either the lifting or curbing inflation case, the effects on the real rate of a pure regime change in the conduct of monetary policy are extremely long lived, lasting more than 90 years in both cases, in sharp contrast to a monetary policy *shock*. The reason monetary policy shocks have more short-lived effects is because, even in the dovish regime, the activism coefficient ψ_{π, ξ_t} is still large enough to allow the central bank to quickly stabilize the real rate in response to any shock that moves inflation away from target, including in response to monetary policy shocks. (This coefficient is larger still in the hawkish regime, implying even faster stabilization.) By contrast, it is a truism that there is no reason for the central bank to stabilize an intentional change in the stance of monetary policy. It follows that the extent to which regime changes in monetary policy persist in their real effects depends entirely on the speed with which agents adapt their expectations about long-term inflation. Our finding that agent's expectations adapt very slowly over time implies that changes in the

conduct of monetary policy have effects that last for many decades.

4.3.3 Dovish Monetary Policy and Reach for Yield in Equity?

Finally, we return to the question of whether a regime shift to the dovish monetary policy rule is associated with a shift in equity market return premia. Figure 10 plots the present discounted value of expected excess returns for the portfolios analyzed in Section 3.3 (dashed line, right axis) together with the identified component of the real interest rate that we estimate is attributable solely to regime changes in the monetary policy rule (solid line, left axis). The Figure exhibits a noticeable positive comovement between the two series, suggesting that when monetary policy is dovish and the real interest rate is consequently low, expected excess returns are also lower, consistent with a reach for yield.

To confirm this visual impression, the second column of Table 8 reports the correlation between the PDV of excess returns of the different equity characteristic portfolios and the identified component of the real interest rate driven solely by changes in the policy rule, denoted RIR_t^{MPR} in the table. This correlation is highly positive, equal to 0.88 for the momentum spread, 0.82 for the market excess return, 0.75 for the value spread in the small size quintile, and 0.70 for the value spread in the second smallest size quintile. Thus, the conditional risk premium on the market and on several equity characteristic portfolios all exhibit evidence consistent with a reach-for-yield in response to the long-lasting effects of changes in the conduct of monetary policy. The table shows that correlation of premia with RIR_t^{MPR} is systematically larger than that with the residual component of the real interest rate, $RIR_t - RIR_t^{MPR}$, and thus also larger than with the real interest rate itself (RIR_t). This shows that shifts in the monetary policy stance play an important role in generating the positive correlation between premia and the real interest rate, but that other movements in the real rate do not have this property. This may be because persistent low or high interest rate environments driven by shifts in the conduct of monetary policy have effects that last for decades, in contrast to movements in real rates driven by other factors. Indeed, there is no evidence consistent with a reach for yield when considering the interest rates movements driven by any of the Gaussian shocks, which have effects that are far more transitory than those associated with changes in the conduct of monetary policy.

5 Conclusion

The evidence in this paper shows that the U.S. economy is characterized by large, longer term regime shifts in asset values relative to macroeconomic fundamentals that arise concurrently with equally important shifts in the level of the short-term real interest rate in excess of a commonly used measure of the “natural” rate of interest, a variable we refer to as the monetary

policy spread, *mps*. The results identify two subperiods of the sample characterized by low valuations and a high *mps*: the period 1978:Q4 to 2001:Q3, and the period 2006:Q2 to 2008:Q2. The first subperiod of the sample spans the Volcker disinflation and its aftermath, while the second subperiod follows 17 consecutive Federal Reserve rate increases that left the nominal funds rate standing at 5.25% in June of 2006. All other subperiods, including the entire period of the sample after the Great Recession, are identified as high valuation/low *mps* regimes. We further document that the high valuation/low *mps* regimes are associated with lower equity market return premia, consistent with a reach for yield.

To investigate what part of these findings could be attributable to monetary policy, we solve and estimate a New Keynesian Markov-switching DSGE model with two distinctive features: adaptive learning in inflation expectations augmented by a possible signal about the inflation target, and regime shifts in the conduct of monetary policy, measured as time-variation in the parameters of an interest rate rule. The overall learning rule is mapped into inflation expectations data from the University of Michigan Survey of Consumers and fits the actual expectations series with negligible error.

Estimates of this model imply that the conduct of monetary policy differed markedly across the valuation/*mps* subperiods, with the high valuation/low *mps* subperiods coincident with a dovish policy rule characterized by a comparatively higher inflation target and less responsiveness in the estimated policy rule to inflation relative to growth.

We use estimates from the DSGE model to identify movements in the real variables that are attributable solely to the conduct of monetary policy. Several findings are noteworthy. First, regime changes in the conduct of monetary policy generate persistent fluctuations in the short-term real interest rate that last for decades. By contrast, monetary policy *shocks* have far more transitory effects. Second, the estimates imply that virtually all of the downward trend in the real interest rate observed since the early 1980s can be attributed to regime changes in the conduct of monetary policy. This occurs because the policy rule parameters exhibit a decisive shift toward more hawkish values around the time of Volcker's appointment to the Federal Reserve, but then exhibit an equally decisive shift back to dovish values in the aftermath of 9/11. They have remained there since, with the exception of a brief interlude from 2006:Q2-2008:Q2. Third, the estimates imply that perceived trend inflation closely follows an adaptive learning rule and thus adjusts very slowly to signals about changes in the inflation target. This plays a crucial role in the results. Indeed, if perceived trend inflation is counterfactually set to equal the estimated inflation target, regime changes in the conduct of monetary policy have no effect on real interest rates. Finally, conditional return premia on the market and on several equity characteristic portfolios are all strongly positively correlated with the component of the short-term real interest rate attributable to changes in the conduct of monetary policy, consistent with a reach for yield type phenomenon. By contrast, fluctuations in the more

transitory residual component of the real rate are not associated with important changes in equity return premia.

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Tables and Figures

	Overall	Medium	Business
Panel A: Correlations with $-cay_t$			
Real interest rate	-0.41	-0.83	0.25
Monetary policy spread	-0.52	-0.84	0.16
Panel B: Correlations with Shiller PE ratio			
Real interest rate	-0.30	-0.19	0.22
Monetary policy spread	-0.13	-0.30	0.18
Panel C: Correlations with Corp. PD ratio			
Real interest rate	-0.22	-0.49	0.22
Monetary policy spread	-0.25	-0.60	0.19
Panel D: Correlations with Corp. PE ratio			
Real interest rate	-0.28	-0.20	0.39
Monetary policy spread	-0.04	-0.30	0.29
Panel E: Correlations with Earnings-NVA ratio			
Real interest rate	-0.54	-0.38	-0.27
Monetary policy spread	-0.35	-0.46	-0.16

Table 1: Results under “Medium” use series filtered to retain fluctuations with cycles between 8 and 50 years; “Business” retains cycles x , $1.5 \leq x \leq 8$ years. r^* is from Laubach and Williams (2003). Monetary policy spread = $FFR_t - \text{Expected Inflation}_t - r_t^*$, where expected inflation is a four period moving average of inflation. Corp. PD ratio is the ratio of market equity (ME) to net dividends for the corporate sector from the Flow of Funds. Corp. PE ratio is the ratio of ME to after-tax profits of the corporate sector. NVA is net-value-added for the nonfinancial corporate sector. The sample spans 1961:Q1-2017:Q3.

Parameter	Mode	Mean	5%	95%
α_1	-0.7239	-0.7121	-0.7796	-0.6465
α_2	-0.7500	-0.7376	-0.8034	-0.6717
r_1	0.0111	0.0132	0.0097	0.0165
r_2	-0.0252	-0.0244	-0.0266	-0.0222
$\alpha_1 - \alpha_2$	0.0262	0.0255	0.0212	0.0296
$r_1 - r_2$	0.0363	0.0376	0.0342	0.0411
β_a	0.2762	0.2721	0.2414	0.3014
β_y	0.7619	0.7657	0.7286	0.8042
σ_c	0.0128	0.0143	0.0130	0.0157
σ_r	0.0141	0.0135	0.0123	0.0150
\mathbf{H}_{11}	0.9793	0.9696	0.9306	0.9943
\mathbf{H}_{22}	0.9830	0.9785	0.9539	0.9950

Parameter Estimates: cay^{FC}

α	γ_a	γ_y
-0.8132	0.2048	0.8551
(0.0448)	(0.0186)	(0.0233)

Table 2: Parameter estimates. The top panel reports posterior modes, means, and 90% error bands of the parameters of the Markov-switching cointegrating relation. Flat priors are used on all parameters of the model. The lower panel reports parameter estimates for the fixed coefficient cointegrating relation. Standard errors are in parantheses. The sample is quarterly and spans the period 1961:Q1 to 2017:Q3.

	1961Q1-1978Q3	1978Q4-2001Q3	2001Q4-2006Q1	2006Q2-2008Q2	2008Q3-2017Q3
Regime	HV/LMPS (2)	LV/HMPS (1)	HV/LMPS (2)	LV/HMPS (1)	HV/LMPS (2)

Table 3: Estimated regime sequence. The table reports the most likely regime sequence based on the posterior mode estimates. HV/LMPS: High-Valuation/Low Monetary Policy Spread regime (Regime 1). LV/HMPS: High-Valuation/Low Monetary Policy Spread regime (Regime 2).

Annualized Sharpe Ratios and Mean Returns					
Portfolio	SR	Mean	Portfolio	SR	Mean
Market	0.3665	0.0623	V-G (S1)	0.5966	0.0994
W-L	0.6261	0.1517	V-G (S2)	0.3279	0.0545

Table 4: The table reports annualized Sharpe ratios, "SR," and mean returns, "Mean," for the stock market and different portfolios. The Sharpe ratio is defined to be the unconditional mean return divided by the standard deviation of the portfolio return. The long-short portfolios "V-G" are the value-growth portfolios in a given size quintile, S1=smallest, S2= second smallest. long-short portfolios "W-L" are the winner-loser portfolio. For each size category, the return of the V-G portfolio portfolio return is the difference between the return on the extreme value (highest BM ratio) and the return of the extreme growth portfolio (lowest BM ratio). The return of the W-L portfolio return is the difference in returns between the extreme winner (M10) and the extreme loser (M1). All returns are computed at quarterly frequencies but the Sharpe ratios and mean returns are reported in annualized units. The sample spans the period 1964:Q1-2017:Q3.

Breaks in Market Premium and Book-Market Ratio Spreads				
	Market	W-L	Val-Gr (S1)	Val-Gr (S2)
Regime 1	1.5896 (0.8960,2.2558)	4.0432 (2.8912,5.2514)	2.6647 (1.7120,3.6701)	1.4875 (0.6118,2.4214)
Regime 2	1.2848 (0.5652,1.9219)	3.4000 (2.2660,4.5196)	2.4460 (1.6679,3.2698)	1.3577 (0.6063,2.1193)
Diff-in-Diff	0.2987 (-0.0367,0.7089)	0.6011 (0.1462,1.2409)	0.1958 (-0.1865,0.6593)	0.1251 (-0.2431,0.5429)
Prob. decline	0.81	0.90	0.70	0.64
Odds ratio	4.26	9.00	2.33	1.78

Table 5: The first two rows report the regime averages for the present discounted value of market expected excess returns and the spread in the present discounted value of portfolio expected excess returns. The columns labeled "Val-Gr" report the spreads for portfolios sorted along the book-market dimension, in a given size category (extreme value minus extreme growth). The columns labeled "W-L" report the spreads for portfolios sorted along the recent past return performance dimension (extreme winner minus extreme loser). The row labeled "Diff-in-Diff" reports the difference between these spreads across the two wealth ratio/interest rate regimes. The numbers in each cell are the median values of the statistic from the posterior distribution while in parentheses we report 68% posterior credible sets. The last row reports the probability that premia decline when moving from the low valuation regime to the high valuation regime. These probabilities are obtained by computing the fraction of draws from the posterior distribution for which the premia under the high valuation regime are lower than the premia under the low valuation regime.

	<i>Mode</i>	<i>Mean</i>	5%	95%	<i>Type</i>	<i>Para₁</i>	<i>Para₂</i>
π_1^T	0.8516	0.8411	0.7038	0.9642	<i>U</i>	0	10
$\psi_{\pi,1}$	2.3164	2.8146	1.9184	4.2444	<i>U</i>	0	10
$\rho_{i,1}$	0.8913	0.9080	0.8597	0.9494	<i>B</i>	0.5	0.2
$\psi_{\Delta y,1}$	2.6387	3.6663	1.9532	6.2615	<i>U</i>	0	10
π_2^T	2.8794	2.9040	2.7017	3.1626	<i>U</i>	0	10
$\psi_{\pi,2}$	1.1089	1.1146	0.8266	1.4120	<i>U</i>	0	10
$\rho_{i,2}$	0.8978	0.9264	0.8579	0.9804	<i>B</i>	0.5	0.2
$\psi_{\Delta y,2}$	1.2320	2.6661	0.8990	6.5637	<i>U</i>	0	10
γ	0.0017	0.0019	0.0008	0.0032	<i>B</i>	0.05	0.02
γ^T	0.0132	0.0131	0.0110	0.0152	<i>B</i>	0.2	0.1
σ	0.7970	1.1462	0.5406	2.0439	<i>G</i>	2	1
δ	0.9062	0.9008	0.8048	0.9696	<i>B</i>	0.9	0.05
β	0.7696	0.7156	0.5270	0.8909	<i>B</i>	0.8	0.1
κ	0.0343	0.0317	0.0143	0.0520	<i>G</i>	0.4	0.2
ρ_d	0.7589	0.8208	0.6731	0.9368	<i>B</i>	0.5	0.2
ρ_{y^*}	0.9457	0.9177	0.8474	0.9695	<i>B</i>	0.5	0.2
ϕ	0.8057	0.8041	0.7924	0.8149	<i>B</i>	0.5	0.2
r_{ss}	0.2540	0.3002	0.1210	0.5581	<i>G</i>	0.5	0.25
Δy_{ss}	0.3738	0.4126	0.3427	0.4912	<i>G</i>	0.5	0.2
σ_d	0.6033	0.6569	0.5431	0.7987	<i>U</i>	0	10
σ_i	0.1865	0.1950	0.1782	0.2153	<i>U</i>	0	10
σ_{y^*}	2.7349	3.7875	2.1213	7.4099	<i>U</i>	0	10
$\sigma_{oe,\Delta GDP}$	0.2897	0.2857	0.2339	0.3366	<i>U</i>	0	10
$\sigma_{oe,INFL}$	1.2294	1.2514	1.1557	1.3547	<i>U</i>	0	10
$\sigma_{oe,FFR}$	0.0000	0.0006	0.0000	0.0015	<i>U</i>	0	10
$\sigma_{oe,EXP}$	0.0686	0.0696	0.0517	0.0853	<i>U</i>	0	10

Table 6: This table reports the posterior mode, mean, and 90N stands Normal, G for Gaussian, and B for Beta, U for Uniform. For the Beta, Normanl, and Gaussian distributions, the first parameter and second parameter correspond to mean and standard deviation, respectively. For the uniform distribution they correspond to the lower and upper bound. The sample spans the period 1961:Q1-2017:Q3.

	<i>Mode</i>	<i>Mean</i>	5%	95%
$\pi_1^T - \pi_2^T$	-2.0278	-2.0629	-2.4647	-1.7533
$\psi_{\pi,1} - \psi_{\pi,2}$	1.2074	1.7001	0.6990	3.1387
$\rho_{i,1} - \rho_{i,2}$	-0.0065	-0.0184	-0.0911	0.0576
$\psi_{\Delta y,1} - \psi_{\Delta y,2}$	1.4067	1.0001	-3.3289	4.4279

Table 7: This table reports the posterior mode, mean, and 90monetary policy rule parameters across the two regimes. The sample spans the period 1961:Q1-2017:Q3.

	RIR_t	RIR_t^{MPR}	RIR_t^{RES}
Market Excess Return	0.43	0.82	0.09
Momentum Spread	0.52	0.88	0.18
Value Spread (S1)	0.51	0.75	0.23
Value Spread (S2)	0.46	0.70	0.20

Table 8: Corr. between PDV of market risk premium (rp) or portfolio rp spreads with RIR_t^{MPR} vs. movements in real rates driven by Gaussian shocks (RIR_t^{RES}). Market is the PDV of the market rp, Momentum W-L is the PDV of the difference in the Winner-Loser risk premia; Value Spread is the PDV of the difference in the High-Low book-market ratio portfolios in the smallest (S1) and next to smallest (S2) size quintiles. RIR_t is defined as FFR minus expected inflation (based on the model). The sample spans 1961:Q1 - 2017:Q3.

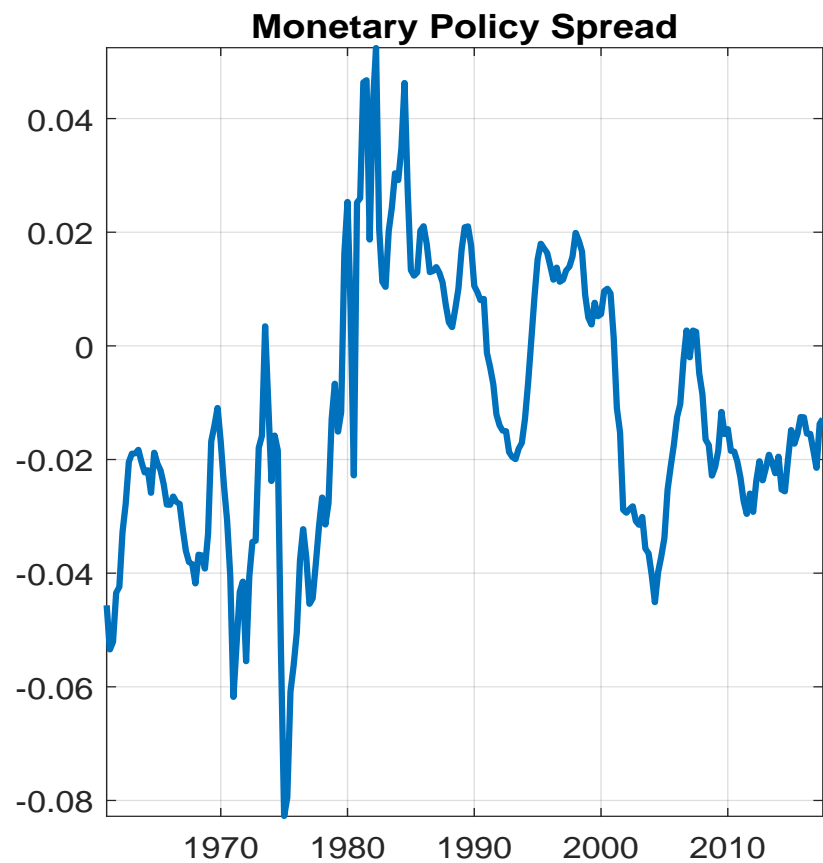
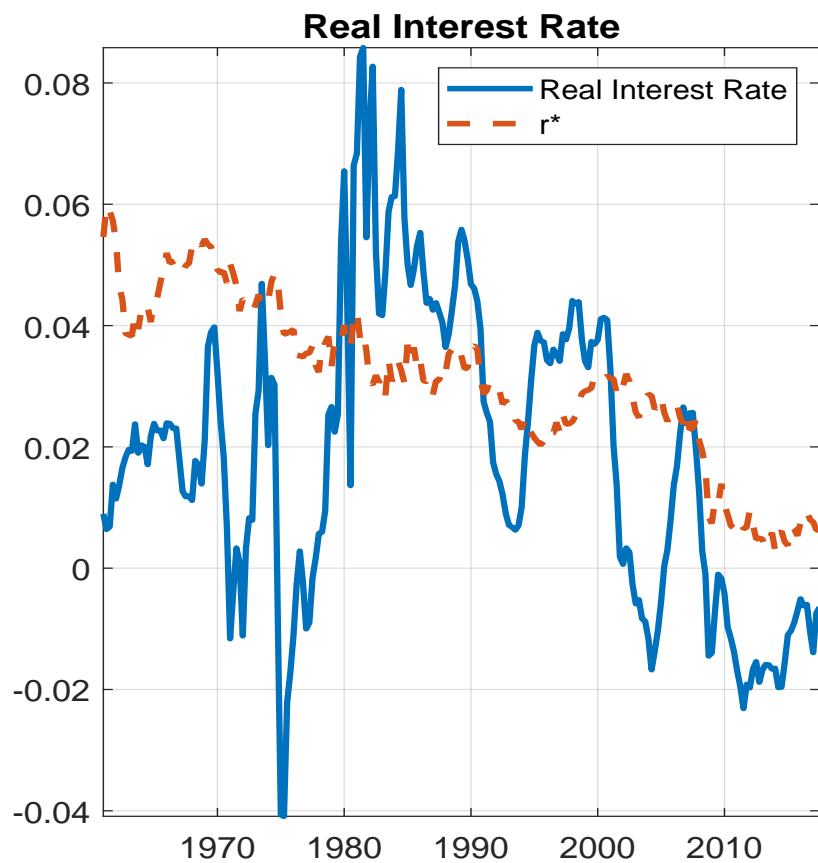


Figure 1: The real interest rate is the difference between the nominal federal funds rate (FFR) and expected inflation, where expected inflation is computed as a four quarter moving average of inflation. The monetary policy spread is defined as $MPS_t \equiv FFR_t - Expected\ Inflation_t - r_t^*$, where r_t^* is the natural rate of interest from Laubach and Williams (2003). The sample spans the period 1961:Q1-2017:Q3.

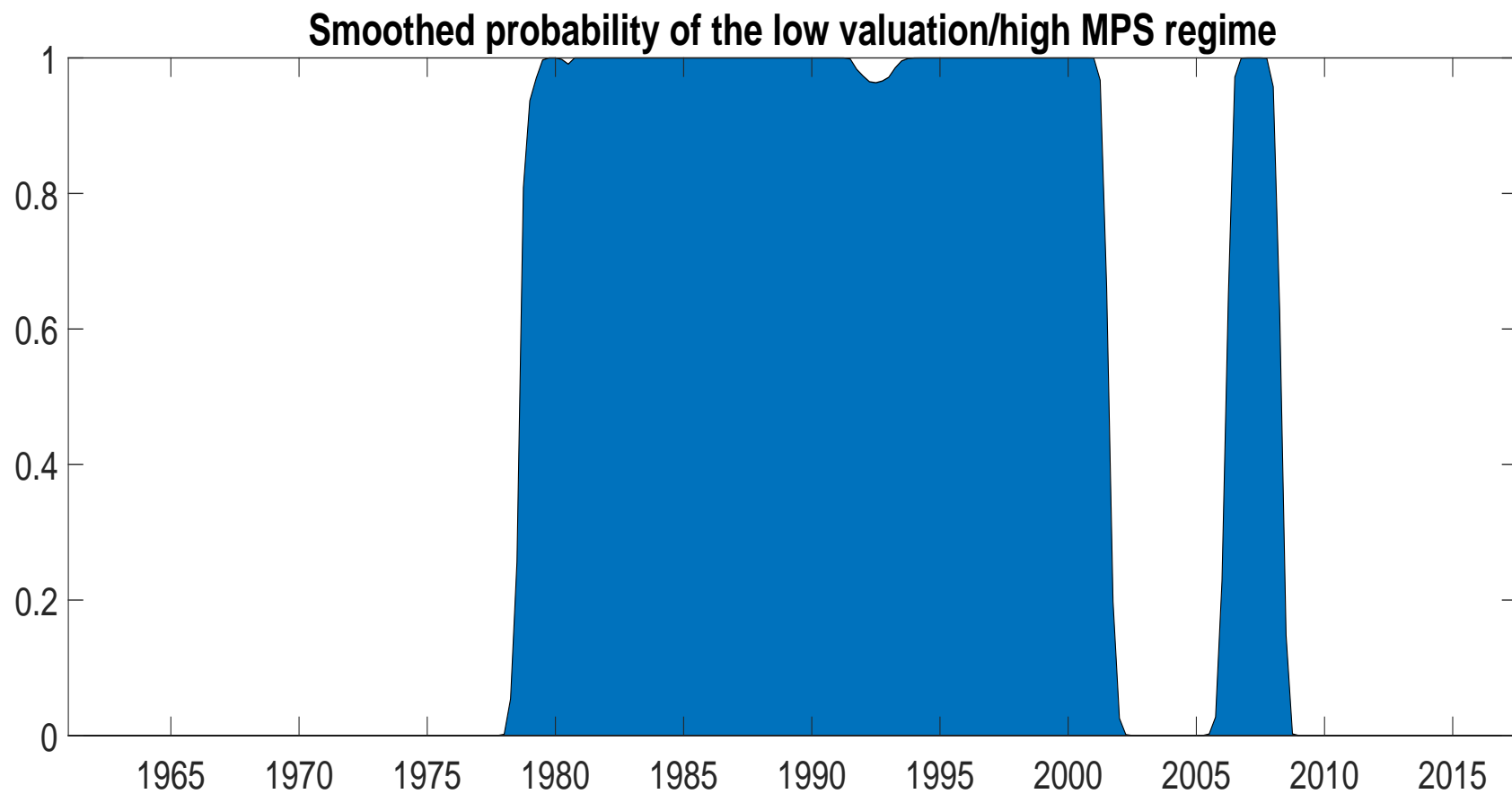


Figure 2: Smoothed probabilities of the low asset valuation/high monetary policy spread regime. The sample is quarterly and spans the period 1961:Q1 to 2017:Q3.

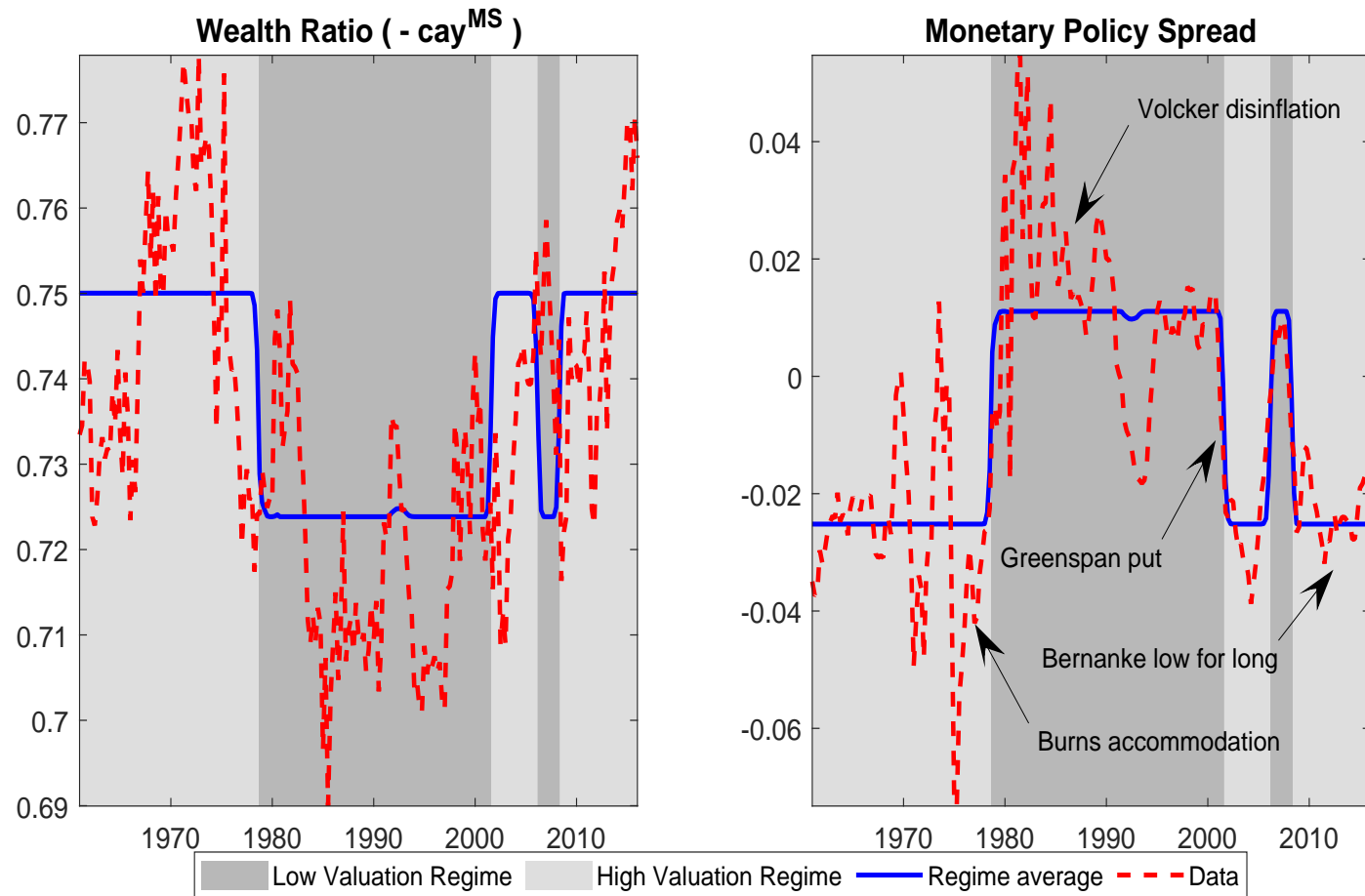


Figure 3: Figure plots the wealth ratio ($-cay^{MS}$) and the monetary policy spread $MPS_t \equiv FFR_t - Expected\ Inflation_t - r_t^*$. The series for r_t^* is from Laubach and Williams (2003). The solid line corresponds to the estimated mean at the posterior mode. The sample spans 1961:Q1-2017:Q4.

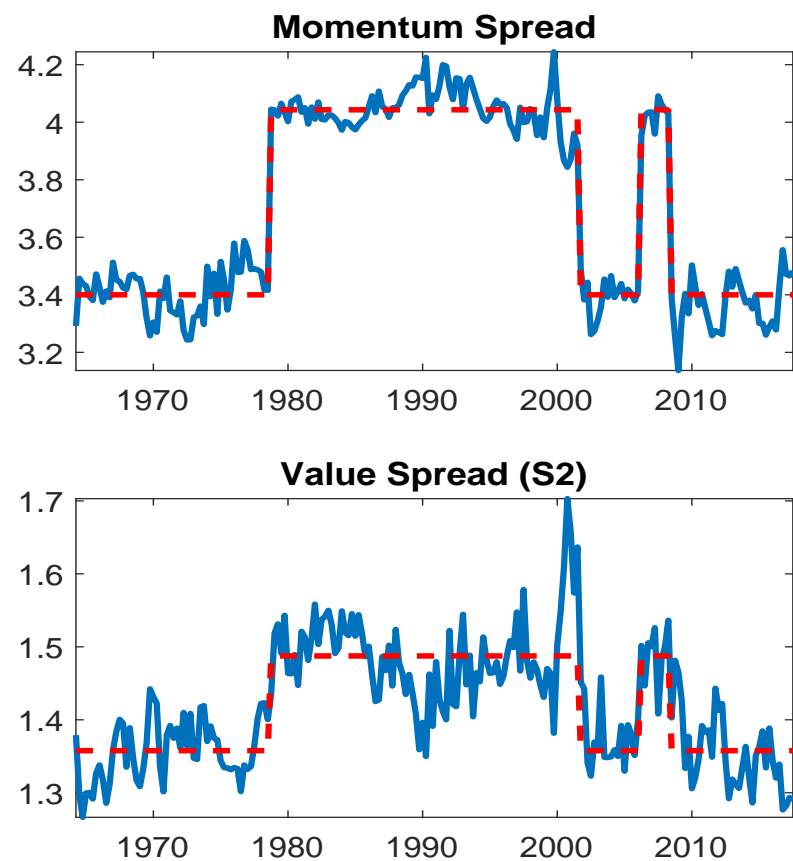
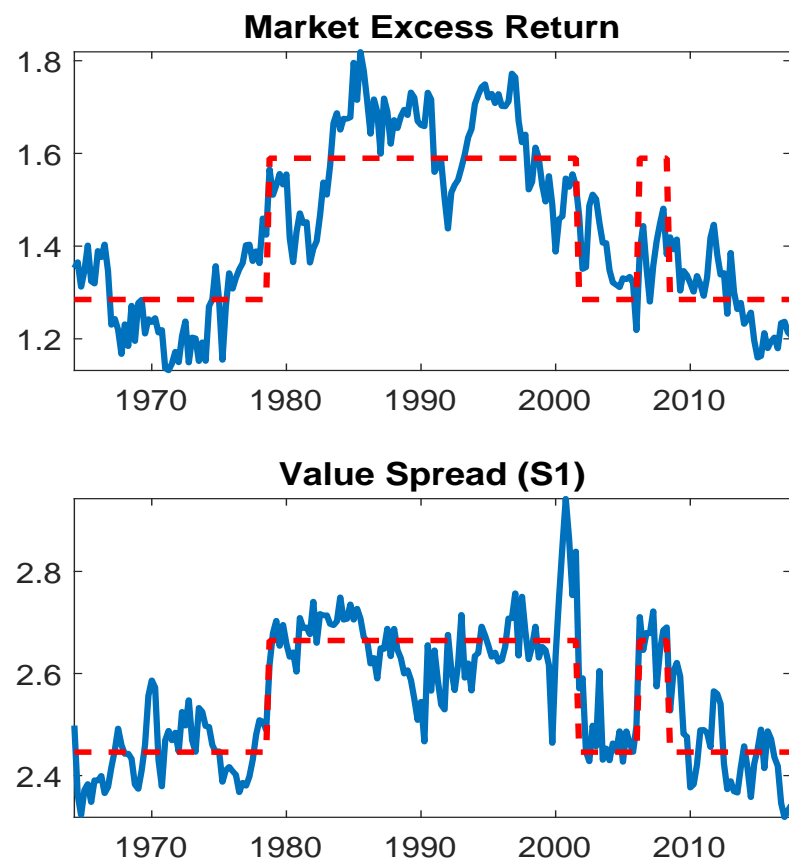


Figure 4: Evolution of Risk Premia. The figure reports the evolution of the PDV of risk premia for the stock market and three different spread portfolios. The blue solid line reports the evolution of the risk premia over time, while the red dashed line corresponds to the conditional steady state of the PDV based on the regime in place. Both are computed by taking into account the possibility of regime changes. The sample spans the period 1964:Q1-2017:Q3.

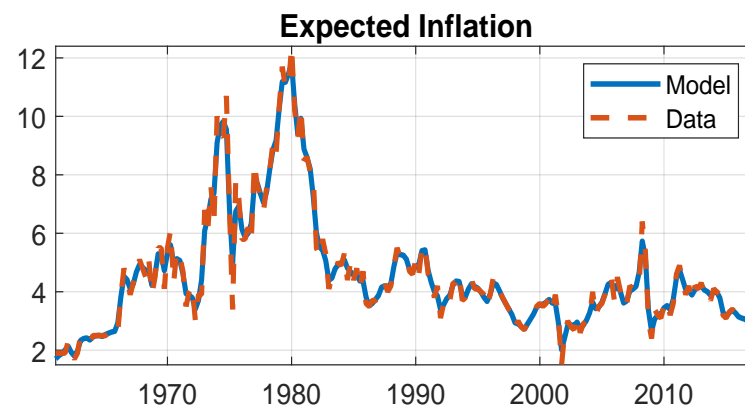
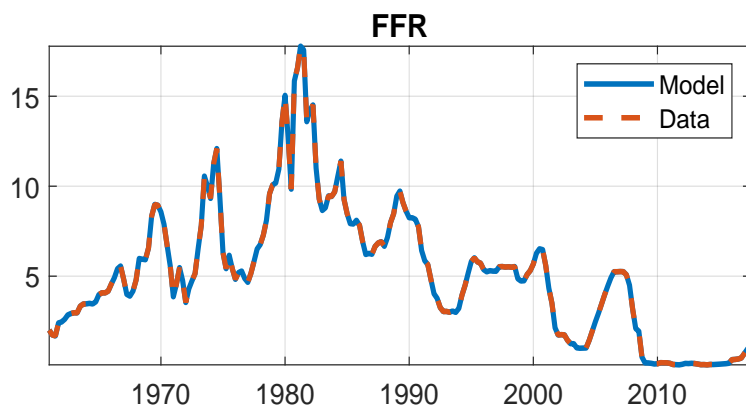
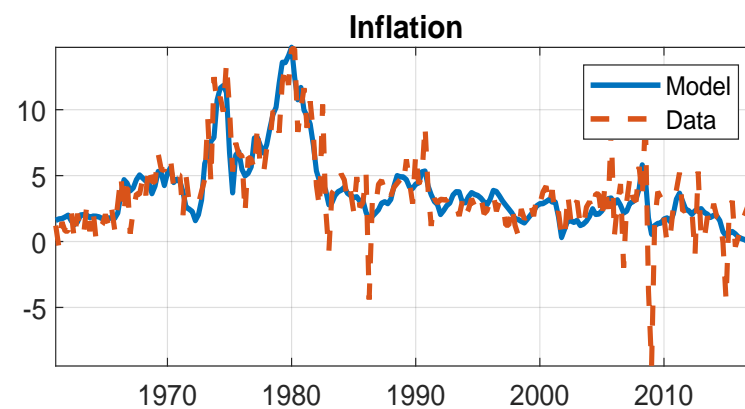
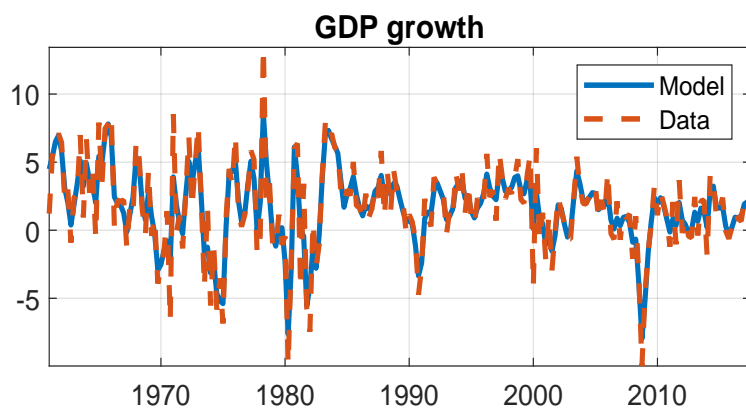


Figure 5: The figure reports the model implied series and the corresponding observed series. Expected inflation comes from the Michigan Survey of Consumers. The difference is due to observation errors. The sample spans 1961:Q1 - 2017:Q3.

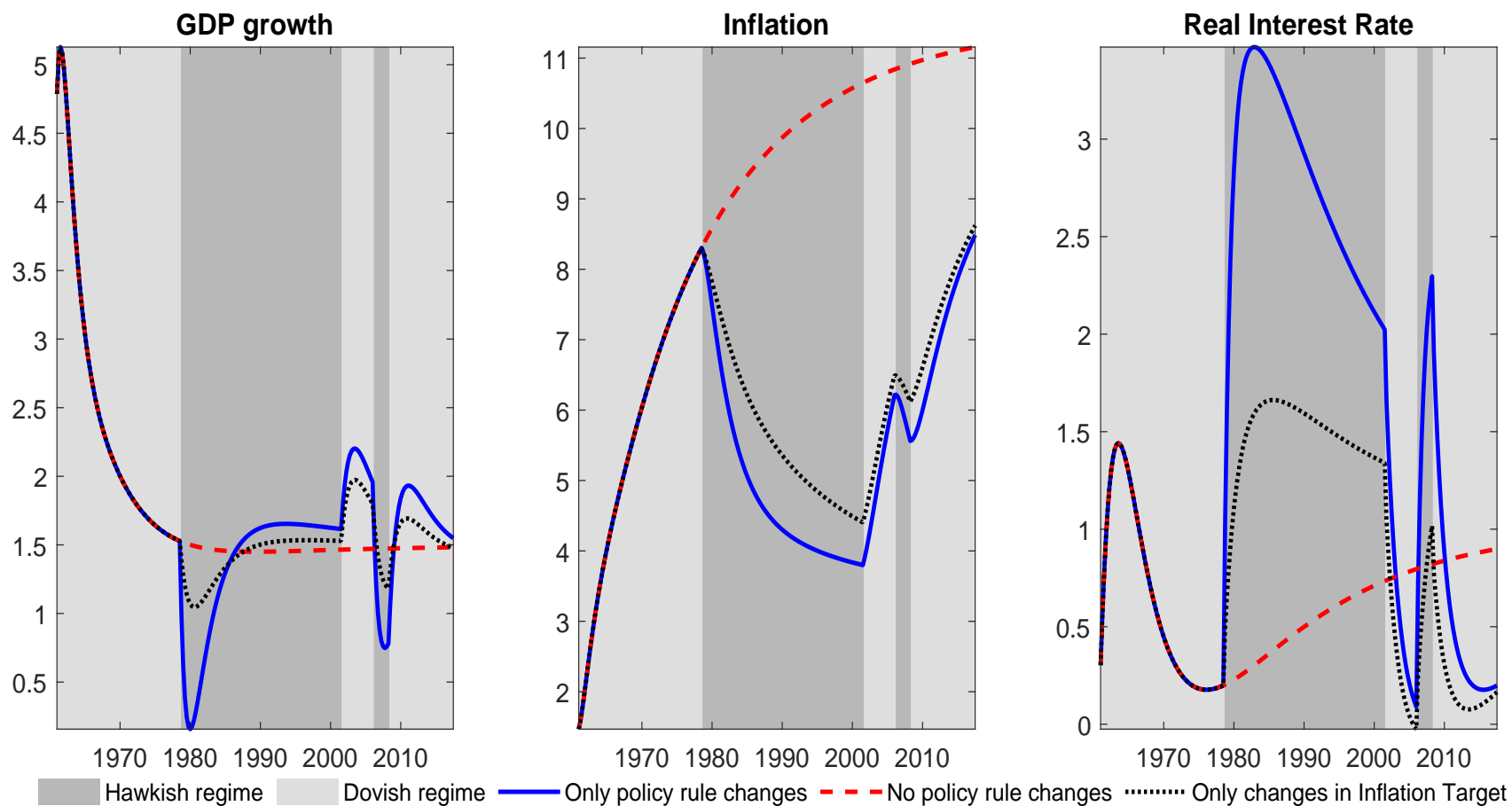


Figure 6: The blue line corresponds to the fluctuations generated by changes in both the target and the slope coefficients. The orange line assumes that monetary policy starts under the Dovish regime and no regime change occurs. Finally, the black dotted line assumes that changes in the target occurred, but that the slope coefficients in the Taylor rule coefficients always remain as in the Dovish-high target regime.

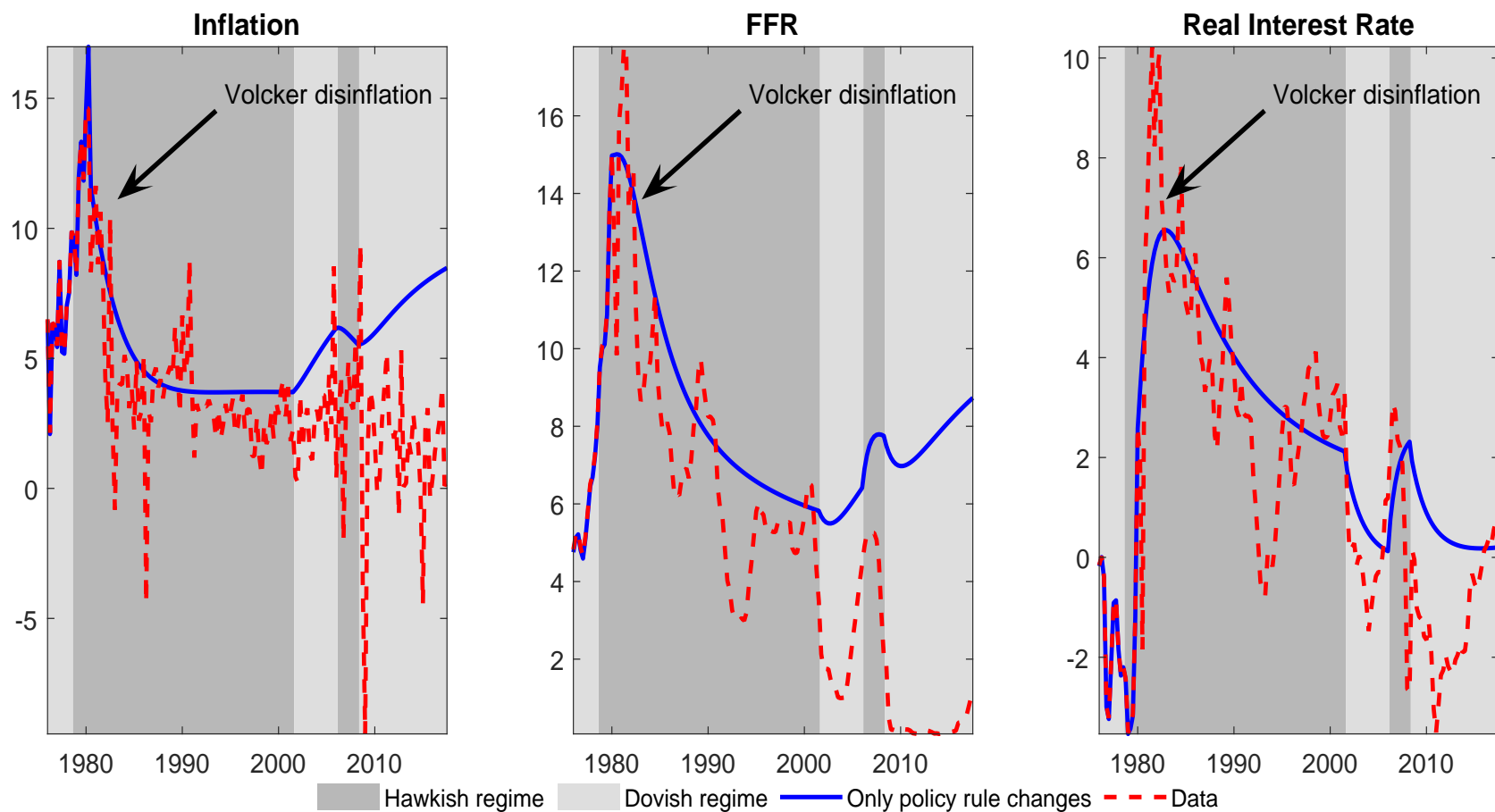


Figure 7: The Volcker disinflation. We start the economy as it was in 1980:Q1 and remove all Gaussian shocks that occurred after that period, but keep the estimated regime sequence. The dashed line corresponds to the data. The real interest rate is computed as the difference between the FFR and expected inflation. Expected inflation is obtained based on the model solution.

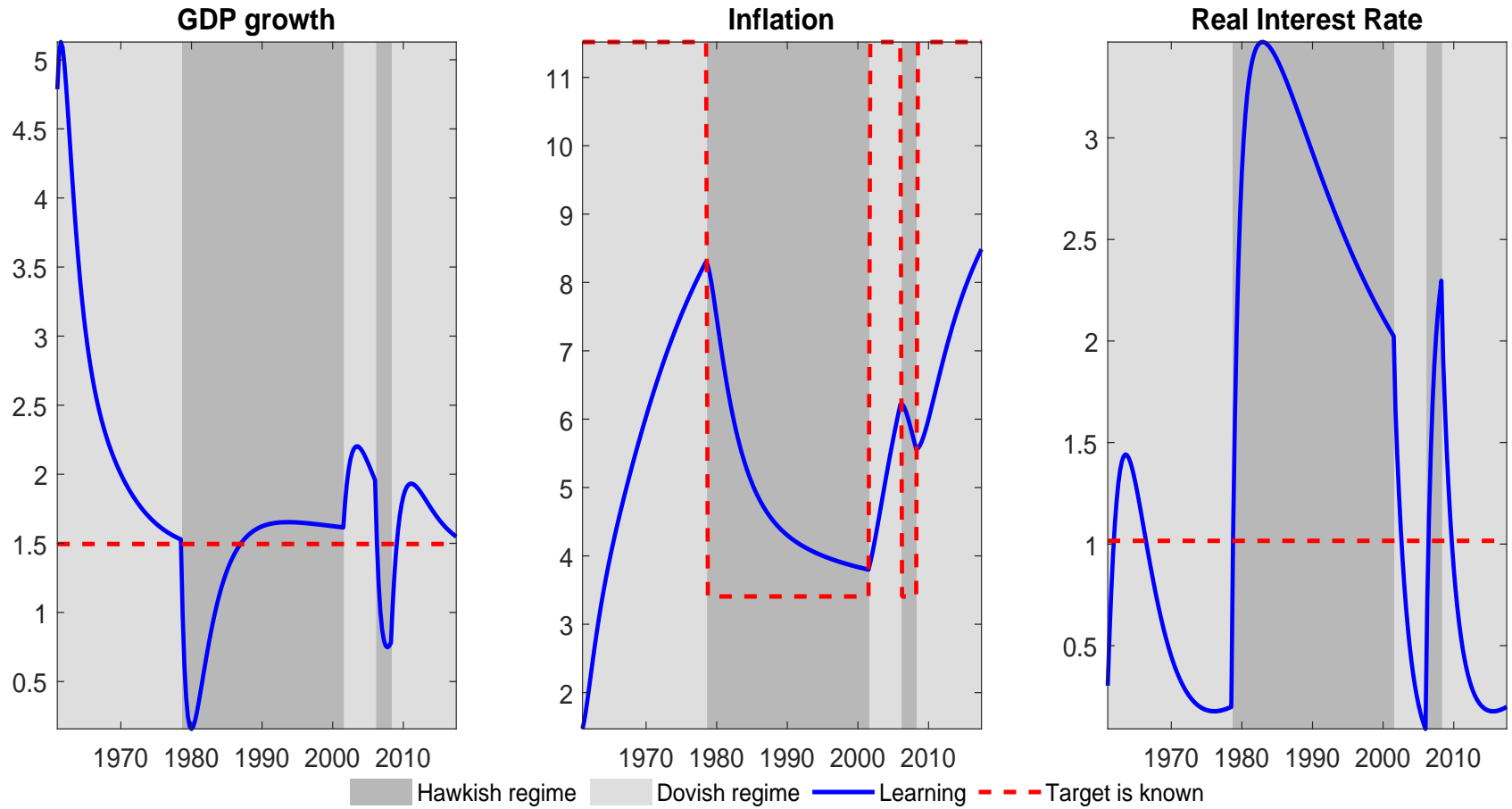


Figure 8: Perfect information about the target. The blue solid line shows estimated fluctuations generated only by changes in the policy rule (inflation target and slope coefficients) when agents learn about trend inflation. The orange dashed line shows a counterfactual in which the policy rule shifts but agents observe the inflation target. Dovish regime has a high target π^T and low activism against deviations from the target π^T . Hawkish regime has a low π^T and high activism against deviations from π^T . The sample spans 1961:Q1 - 2017:Q3.

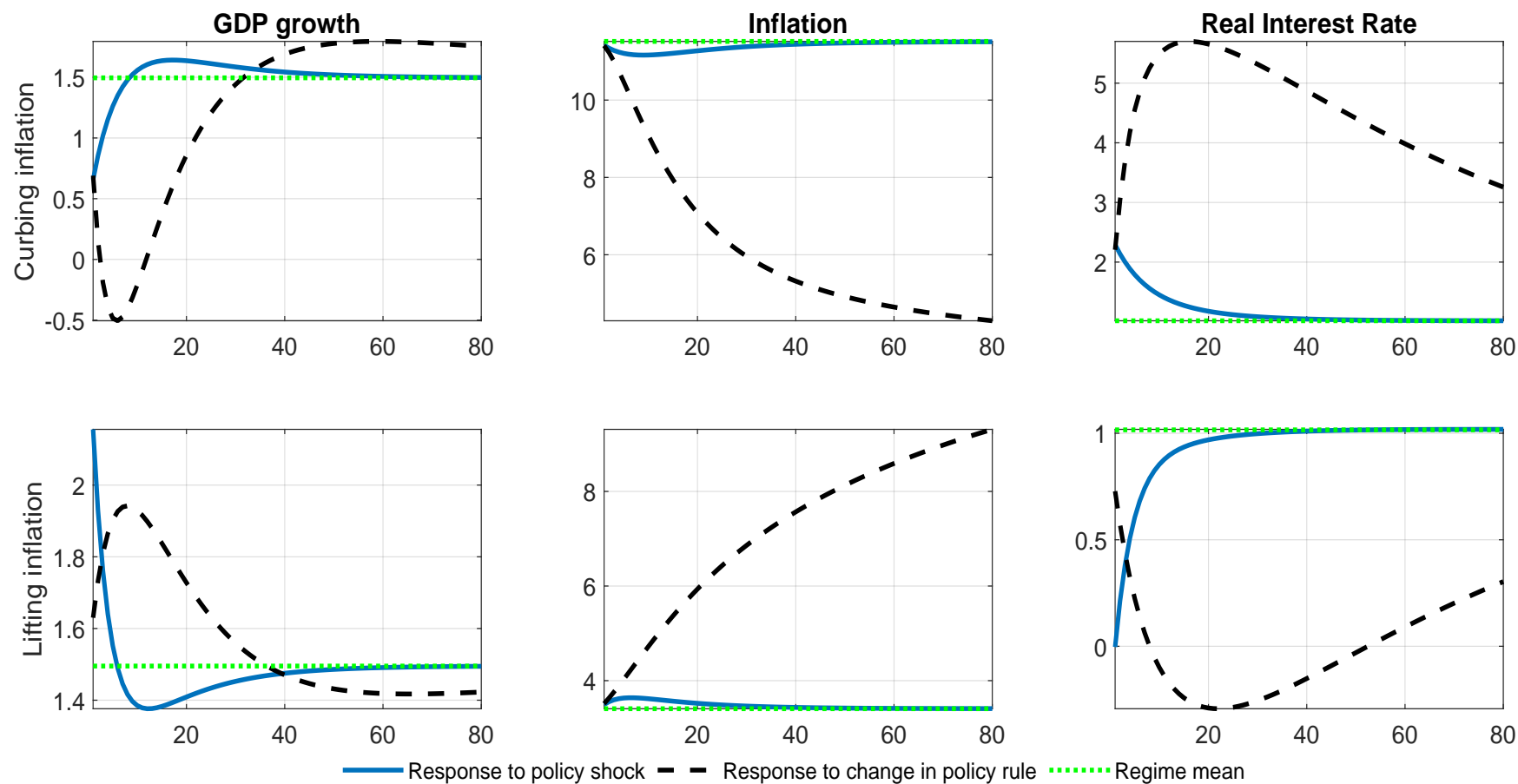


Figure 9: Top row: Curbing inflation. The economy is initially in the Dovish regime. The blue solid line presents the evolution of the macro variables in response to a two standard deviation contractionary monetary policy shock and no regime change. The black dashed line presents the evolution of the macro variables in response to a regime change from the Dovish regime to the Hawkish regime. **Lower row: Lifting inflation.** The economy is initially in the Hawkish regime. The blue solid line presents the evolution of the macro variables in response to a two standard deviation expansionary monetary policy shock and no regime change. The black dashed line presents the evolution of the macro variables in response to a regime change from the Hawkish regime to the Dovish regime.

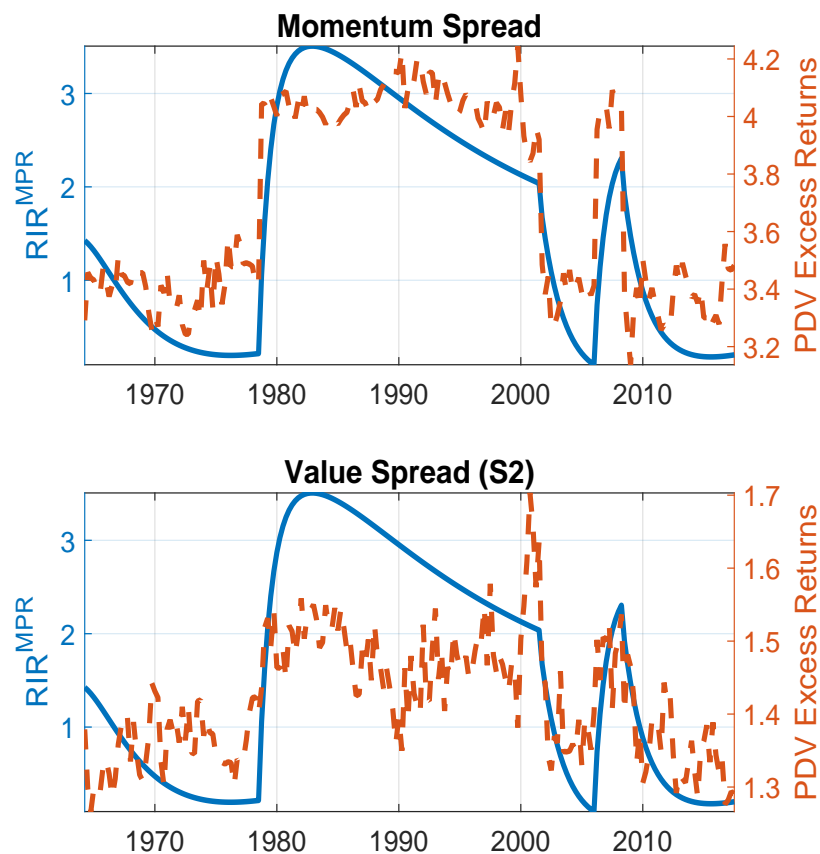
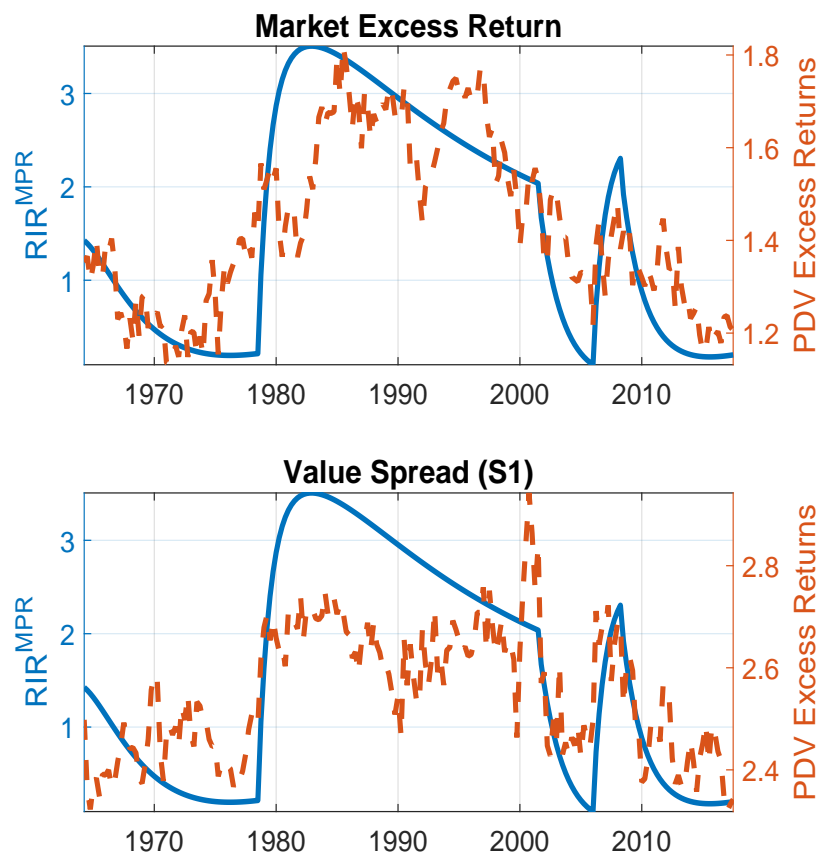


Figure 10: Excess returns and policy rule changes. The figure reports the time series of the present discounted value of expected excess returns for different portfolios (dashed line, right axis) together with fluctuations of the real interest rate due to changes in the monetary policy rule (solid line, left axis).

Appendix for Online Publication

Data Appendix

This appendix describes the data used in this study.

CONSUMPTION

Consumption is measured as either total personal consumption expenditure or expenditure on nondurables and services, excluding shoes and clothing. The quarterly data are seasonally adjusted at annual rates, in billions of chain-weighted 2005 dollars. The components are chain-weighted together, and this series is scaled up so that the sample mean matches the sample mean of total personal consumption expenditures. Our source is the U.S. Department of Commerce, Bureau of Economic Analysis.

LABOR INCOME

Labor income is defined as wages and salaries + transfer payments + employer contributions for employee pensions and insurance - employee contributions for social insurance - taxes. Taxes are defined as $[\text{wages and salaries}/(\text{wages and salaries} + \text{proprietors' income with IVA and CCADJ} + \text{rental income} + \text{personal dividends} + \text{personal interest income})]$ times personal current taxes, where IVA is inventory valuation and CCADJ is capital consumption adjustments. The quarterly data are in current dollars. Our source is the Bureau of Economic Analysis.

POPULATION

A measure of population is created by dividing real total disposable income by real per capita disposable income. Our source is the Bureau of Economic Analysis.

WEALTH

Total wealth is household net worth in billions of current dollars, measured at the end of the period. A break-down of net worth into its major components is given in the table below. Stock market wealth includes direct household holdings, mutual fund holdings, holdings of private and public pension plans, personal trusts, and insurance companies. Nonstock wealth includes tangible/real estate wealth, nonstock financial assets (all deposits, open market paper, U.S. Treasuries and Agency securities, municipal securities, corporate and foreign bonds and mortgages), and also includes ownership of privately traded companies in noncorporate equity, and other. Subtracted off are liabilities, including mortgage loans and loans made under home equity lines of credit and secured by junior liens, installment consumer debt and other. Wealth is measured at the end of the period. A timing convention for wealth is needed because the level of consumption is a flow during the quarter rather than a point-in-time estimate as is wealth (consumption data are time-averaged). If we think of a given quarter's consumption data as measuring spending at the beginning of the quarter, then wealth for the quarter should

be measured at the beginning of the period. If we think of the consumption data as measuring spending at the end of the quarter, then wealth for the quarter should be measured at the end of the period. None of our main findings discussed below (estimates of the cointegrating parameters, error-correction specification, or permanent-transitory decomposition) are sensitive to this timing convention. Given our finding that most of the variation in wealth is not associated with consumption, this timing convention is conservative in that the use of end-of-period wealth produces a higher contemporaneous correlation between consumption growth and wealth growth. Our source is the Board of Governors of the Federal Reserve System. A complete description of these data may be found at <http://www.federalreserve.gov/releases/Z1/Current/>.

CRSP PRICE-DIVIDEND RATIO

The stock price is measured using the Center for Research on Securities Pricing (CRSP) value-weighted stock market index covering stocks on the NASDAQ, AMEX, and NYSE. The data are monthly. The stock market price is the price of a portfolio that does not reinvest dividends. The CRSP dataset consists of $vwretx(t) = (P_t/P_{t-1}) - 1$, the return on a portfolio that doesn't pay dividends, and $vwretd_t = (P_t + D_t)/P_{t-1} - 1$, the return on a portfolio that does pay dividends. The stock price index we use is the price P_t^x of a portfolio that does not reinvest dividends, which can be computed iteratively as

$$P_{t+1}^x = P_t^x (1 + vwretx_{t+1}),$$

where $P_0^x = 1$. Dividends on this portfolio that does not reinvest are computed as

$$D_t = P_{t-1}^x (vwretd_t - vwretx_t).$$

The above give monthly returns, dividends and prices. The annual log return is the sum of the 12 monthly log returns over the year. We create annual log dividend growth rates by summing the log differences over the 12 months in the year: $d_{t+12} - d_t = d_{t+12} - d_{t+11} + d_{t+11} - d_{t+10} + \dots + d_{t+1} - d_t$. The annual log price-dividend ratio is created by summing dividends in levels over the year to obtain an annual dividend in levels, D_t^A , where t denotes a year hear. The annual observation on P_t^x is taken to be the last monthly price observation of the year, P_t^{Ax} . The annual log price-dividend ratio is $\ln(P_t^{Ax}/D_t^A)$. Note that this value for dividend growth is only used to compute the CRSP price-dividend ratio on this hypothetical portfolio. When we investigate the behavior of stock market dividend growth in the MS-VAR, we use actual dividend data from all firms on NYSE, NASDAQ, and AMEX. See the data description for MS-VARs below.

FLOW OF FUNDS EQUITY PAYOUT, DIVIDENDS, PRICE

Flow of Funds payout is measured as “Net dividends plus net repurchases” and is computed using the Flow of Funds Table F.103 (nonfinancial corporate business sector) by subtracting Net Equity Issuance (FA103164103) from Net Dividends (FA106121075). We define net repurchases

to be repurchases net of share issuance, so net repurchases is the negative of net equity issuance. Net dividends consists of payments in cash or other assets, excluding the corporation's own stock, made by corporations located in the United States and abroad to stockholders who are U.S. residents. The payments are netted against dividends received by U.S. corporations, thereby providing a measure of the dividends paid by U.S. corporations to other sectors. The price used for FOF price-dividend and price-payout ratios is "Equity," the flow of funds measure of equities (LM103164103).

PRICE DEFLATOR FOR CONSUMPTION AND ASSET WEALTH

The nominal after-tax labor income and wealth data are deflated by the personal consumption expenditure chain-type deflator (2005=100), seasonally adjusted. In principle, one would like a measure of the price deflator for total flow consumption here. Since this variable is unobservable, we use the total expenditure deflator as a proxy. Our source is the Bureau of Economic Analysis.

DATA FOR MS-VAR TO ESTIMATE RISK PREMIA

The variables included in the MS-VAR for the equity characteristics portfolio data are: (a) the momentum return spread, i.e., the difference between the excess return of the extreme winner (M10) portfolio and the excess return of the extreme loser (M1) portfolio; (b) the value return spread (S1), i.e., the difference between the excess return of the small (size 1) high BM portfolio and the excess return of the small (size 1) low BM portfolio; (c) the value return spread (S2), i.e., the difference between the excess return of the size 2 high BM portfolio and the excess return of the small size 2 low BM portfolio; (d) the momentum BM spread: the difference between the logarithm of the BM ratio of the M10 and M1 portfolios; (e) the value BM spread (S1): The difference between the logarithm of the BM ratio of the small (size quintile 1) high book-market portfolio and the logarithm of the BM ratio of the small (size 1) low book-market portfolio; (f) the value BM spread (S2): the difference between the logarithm of the BM ratio of the size quintile 2 high book-market portfolio and the logarithm of the BM ratio of the size 2 low book-market portfolio; (g) the real FFR (FFR minus inflation); (h) the excess return on the small value portfolio. We then use CRSP/Compustat to construct the BM ratios of the corresponding portfolios. PROVIDE DETAILS HERE.

The MS-VAR specification for the market risk premium includes the following variables: (a) the market excess return, computed as the difference in the CRSP value-weighted stock market return (including dividend redistributions) and the three-month Treasury bill rate; (b) $-cay^{MS}$; (c) the small stock value spread (log-difference in the book to market ratio of the S1 value and S1 growth portfolio); (d) the SMB factor from Fama and French; (e) the HML factor from Fama and French. These variables are obtained from Kenneth French's Dartmouth webpage.

DATA FOR MS-DSGE ESTIMATION

Inflation expectations are taken from the mean inflation forecasts of one year ahead inflation, provided by the University of Michigan Survey of Consumers. Our data sources for output growth are the NIPA tables constructed by the Bureau of Economic Analysis and the St. Louis Fed. Real GDP per capita is obtained by dividing nominal GDP (NIPA 1.1.5, line 1) by the GDP deflator (NIPA 1.1.4, line 1) and population. Population is measured as Civilian Non-institutional Population (CNP16OV) and downloaded from FRED, a website maintained by the Federal Reserve Bank of St. Louis. Inflation is measured as the quarter-to-quarter log-change of CPI. Both the CPI and the FFR series are downloaded from FRED, a website maintained by the Federal Reserve Bank of St. Louis. Expected inflation is the mean of the one-year-ahead expected inflation based on the Michigan survey. All variables are annualized.

Computing cay^{MS}

Let \mathbf{z}_t be a 3×1 vector of data on c_t , a_t , and y_t , and k leads and k lags of Δa_t and Δy_t and let $\mathbf{Z}_t = (\mathbf{z}_t, \mathbf{z}_{t-1}, \dots, \mathbf{z}_1)$ be a vector containing all observations obtained through date t . To estimate the parameters of this stationary linear combination we modify the standard fixed coefficient dynamic least squares regression (DLS-Stock and Watson (1993)) regression to allow for shifts in the intercept α_{ξ_t} :

$$c_t = \alpha_{\xi_t} + \beta_a a_t + \beta_y y_t + \sum_{i=-k}^k b_{a,i} \Delta a_{t+i} + \sum_{i=-k}^k b_{y,i} \Delta y_{t+i} + \sigma^c \varepsilon_t^c \quad (\text{A1})$$

where $\varepsilon_t \sim N(0, 1)$.²³ The parameters of the econometric model include the cointegrating parameters and additional slope coefficients $\beta = (\beta_a, \beta_y, b)'$, where $b = (b_{a,-k}, \dots, b_{a,k}, b_{y,-k}, \dots, b_{y,k})'$, the two intercept values α_1 and α_2 , the standard deviation of the residual σ , and the transition probabilities contained in the matrix \mathbf{H} .

We combine the estimation of changes in the mean of cay_t^{MS} with an isomorphic model for the monetary policy spread. Specifically, we assume that regime changes in the mean of cay_t^{MS} coincide with regime changes in the mean of the mps :

$$mps_t = r_{\xi_t} + \epsilon_t^r, \quad (\text{A2})$$

where $\epsilon_t^r \sim N(0, \sigma_r^2)$. Importantly, unlike cay_t^{MS} , mps_t is an observed variable. Thus, in this case we only need to conduct inference about the MS intercept coefficient r_{ξ_t} . It is worth emphasizing that the same latent state variable, ξ_t , presumed to follow a two-state Markov-switching process with transition matrix \mathbf{H} , controls both changes in α_{ξ_t} and \bar{r}_{ξ_t} .

²³The DLS regression controls for leads and lags of the right-hand-side variables to adjust for the inefficiencies attributable to regressor endogeneity that arise in finite samples.

The model can then be summarized as follows:

$$\begin{aligned}
c_t &= \alpha_{\xi_t} + \beta_a a_t + \beta_y y_t + \sum_{i=-k}^k b_{a,i} \Delta a_{t+i} + \sum_{i=-k}^k b_{y,i} \Delta y_{t+i} + \sigma^c \epsilon_t^c \\
mps_t &= r_{\xi_t} + \sigma^r \epsilon_t^r \\
\epsilon_t^c &\sim N(0, 1), \quad \epsilon_t^r \sim N(0, 1)
\end{aligned}$$

where ξ_t is a hidden variable that follows a Markov-switching process with transition matrix \mathbf{H} . Collect all model parameters into a vector $\boldsymbol{\theta} = (r_{\xi_t}, \sigma^r, \beta, \alpha_{\xi_t}, \sigma^c, \mathbf{H})'$. The model can be thought as a multivariate regression with regime changes in which some of the parameters are restricted to zero.

Our estimate of cay_t^{MS} is based on the posterior mode of the parameter vector $\boldsymbol{\theta}$ and the corresponding regime probabilities. Collect the conditional probabilities $\pi_{t|t}^i = p(\xi_t = i | Y^t; \boldsymbol{\theta})$ for $i = 1, \dots, m$ into an $m \times 1$ vector $\pi_{t|t} = p(\xi_t | Y^t; \boldsymbol{\theta})$. The filtered probabilities reflect the probability of a regime conditional on the data up to time t , $\pi_{t|t} = p(\xi_t | Y^t; \boldsymbol{\theta})$, for $t = 1, \dots, T$, and are part of the output obtained computing the likelihood function associated with the parameter vector $\boldsymbol{\theta} = \{r_{\xi_t}, \sigma^r, \beta, \alpha_{\xi_t}, \sigma, \mathbf{H}\}$. They can be obtained using the following recursive algorithm given by the Hamilton filter:

$$\begin{aligned}
\pi_{t|t} &= \frac{\pi_{t|t-1} \odot \eta_t}{\mathbf{1}' (\pi_{t|t-1} \odot \eta_t)} \\
\pi_{t+1|t} &= \mathbf{H} \pi_{t|t}
\end{aligned} \tag{A3}$$

where η_t is a vector whose j -th element contains the conditional density $p(c_t, mps_t | \xi_t = j, x_{M,t}, x_{F,t}; \boldsymbol{\theta})$, i.e.,

$$p(c_t, mps_t | \xi_t = j, x_{M,t}, x_{F,t}; \boldsymbol{\theta}) = \frac{1}{\sqrt{2\pi\sigma^{c,2}}} \frac{1}{\sqrt{2\pi\sigma^{r,2}}} \exp \left[-\frac{[c_t - (\alpha_j x_{M,t} + \beta x_{F,t})]^2}{2\sigma^{c,2}} - \frac{[mps_t - r_j x_{M,t}]^2}{2\sigma^{r,2}} \right], \tag{A4}$$

the symbol \odot denotes element by element multiplication, and $\mathbf{1}$ is a vector with all elements equal to 1. To initialize the recursive calculation we need an assumption on the distribution of ξ_0 . We assume that the two regimes have equal probabilities: $p(\xi_0 = 1) = .5 = p(\xi_0 = 2)$.

The smoothed probabilities reflect all the information that can be extracted from the whole data sample, $\pi_{t|T} = p(\xi_t | Y^T; \boldsymbol{\theta})$. The final term, $\pi_{T|T}$ is returned with the final step of the filtering algorithm. Then, a recursive algorithm can be implemented to derive the other probabilities:

$$\pi_{t|T} = \pi_{t|t} \odot [\mathbf{H}' (\pi_{t+1|T} (\div) \pi_{t+1|t})]$$

where (\div) denotes element by element division.

In using the DLS regression (A1) to estimate cointegrating parameters, we lose 6 leads and 6 lags. For estimates of cay_t^{FC} , we take the estimated coefficients and we apply them to the

whole sample. To extend our estimates of cay_t^{MS} over the full sample, we can likewise apply the parameter estimates to the whole sample but we need an estimate of the regime probabilities in the first 6 and last 6 observations of the full sample. For this we run the Hamilton filter from period from -5 to $T + 6$ as follows. When starting at -5 , we assume no lagged values are available and the DLS regression omits all lags, but all leads are included. When at $t = -4$ we assume only one lag is available and the DLS regression includes only one lag, and so on, until we reach $t = 0$ when all lags are included. Proceeding forward when $t = T + 1$ is reached we assume all lags are available and all leads except one are available, when $t = T + 2$, we assume all lags and all leads but two are available, etc. Smoothed probabilities are then computed with standard methods as they only involve the filtered probabilities and the transition matrix \mathbf{H} .

Gibbs Sampling Algorithm

This appendix describes the Bayesian methods used to characterize uncertainty in the regression parameters. To simplify notation, we denote the vector containing all variables whose coefficients are allowed to vary over time $x_{M,t}$, while $x_{F,t}$ is used to denote the vector containing all the variables whose coefficients are kept constant. We then obtain:

$$c_t = \alpha_{\xi_t} x_{M,t} + \beta x_{F,t} + \sigma^c \varepsilon_t^c \quad (\text{A5})$$

$$mps_t = r_{\xi_t} x_{M,t} + \sigma^r \varepsilon_t^r \quad (\text{A6})$$

where, in our case, $\beta = [\beta_a, \beta_y, b_{a,-k}, \dots, b_{a,+k}, b_{y,-k}, \dots, b_{y,+k}]$ and the vector $x_{M,t}$ is unidimensional and always equal to 1.

Suppose the Gibbs sampling algorithm has reached the n -th iteration. We then have draws for $r_{\xi_t,n}, \sigma_n^r, \beta_n, \alpha_{\xi_t,n}, \sigma_n^c, \mathbf{H}_n$, and ξ_n^T , where $\xi_n^T = \{\xi_{1,n}, \xi_{2,n}, \dots, \xi_{T,n}\}$ denotes a draw for the whole regime sequence. The parameters for equations (A5) and (A6) can be drawn separately, while the regime sequence ξ_n^T requires a joint evaluation of the Hamilton filter. Finally, the transition matrix \mathbf{H}_n is drawn conditionally on the regime sequence.

Specifically, the sampling algorithm is described as follows.

1. **Sampling β_{n+1} :** Given $\alpha_{\xi_t,n}, \sigma_n^c$, and ξ_n^T we transform the data:

$$\tilde{c}_t = \frac{c_t - \alpha_{\xi_t,n} x_{M,t}}{\sigma_n^c} = \beta \frac{x_{F,t}}{\sigma_n^c} + \varepsilon_t = \beta \tilde{x}_t + \varepsilon_t.$$

The above is a regression with fixed coefficients β and standardized residual shocks. Standard Bayesian methods may be used to draw the coefficients of the regression. We assume a Normal conjugate prior $\beta \sim N(B_{\beta,0}, V_{\beta,0})$, so that the conditional (on $\alpha_{\xi_t,n}, \sigma_n^c$, and ξ_n^T) posterior distribution is given by

$$\beta_{n+1} \sim N(B_{\beta,T}, V_{\beta,T})$$

with $V_{\beta,T} = \left(V_{\beta,0}^{-1} + \tilde{X}'_F \tilde{X}_F \right)^{-1}$ and $B_{\beta,T} = V_{\beta,T} \left[V_{\beta,0}^{-1} B_{\beta,0} + \tilde{X}'_F \tilde{C} \right]$, where $\tilde{C} = (\tilde{c}_1, \dots, \tilde{c}_T)'$ and $\tilde{X}_F = (x_{F,1}, \dots, x_{F,T})'$ and $B_{\beta,0}$ and $V_{\beta,0}^{-1}$ control the priors for the fixed coefficients of the regression. Keeping in mind that the residuals have been normalized to have unit variance, with flat priors, $B_{\beta,0} = 0$ and $V_{\beta,0}^{-1} = 0$ and $B_{\beta,T}$ and $V_{\beta,T}$ coincide with the maximum likelihood estimates, conditional on the other parameters.

2. **Sampling** $\alpha_{i,n+1}$ for $i = 1, 2$: Given β_{n+1} , σ_n^c , and ξ_n^T we transform the data:

$$\tilde{c}_t = \frac{c_t - \beta_{n+1} x_{F,t}}{\sigma_n^c} = \alpha_{\xi_t} \frac{x_{M,t}}{\sigma_n^c} + \varepsilon_t = \alpha_{\xi_t} \tilde{x}_{M,t} + \varepsilon_t.$$

The above regression has standardized shocks and Markov-switching coefficients in the transformed data. Using ξ_n^T we can group all the observations that pertain to the same regime i . Given the prior $\alpha_i \sim N(B_{\alpha_i,0}, V_{\alpha_i,0})$ for $i = 1, 2$ we use standard Bayesian methods to draw α_i from the conditional (on β_{n+1} , σ_n^c , and ξ_n^T) posterior distribution:

$$\alpha_{i,n+1} \sim N(B_{\alpha_i,T}, V_{\alpha_i,T}) \text{ for } i = 1, 2$$

where $V_{\alpha_i,T} = \left(V_{\alpha_i,0}^{-1} + \tilde{X}'_{M,i} \tilde{X}_{M,i} \right)^{-1}$ and $B_{\alpha_i,T} = V_{\alpha_i,T} \left[V_{\alpha_i,0}^{-1} B_{\alpha_i,0} + \tilde{X}'_{M,i} \tilde{C}_i \right]$ where \tilde{C}_i and $\tilde{X}_{M,i}$ collect all the observations for the transformed data for which regime i is in place. The parameters $B_{\alpha_i,0}$ and $V_{\alpha_i,0}^{-1}$ control the priors for the MS coefficients of the regression: $\alpha_i \sim N(B_{\alpha_i,0}, V_{\alpha_i,0})$ for $i = 1, 2$. With flat priors, we have $B_{\alpha_i,0} = 0$ and $V_{\alpha_i,0}^{-1} = 0$ and $B_{\alpha_i,T}$ and $V_{\alpha_i,T}$ coincide with the maximum likelihood estimates, conditional on the other parameters.

3. **Sampling** $r_{i,n+1}$ for $i = 1, 2$: Given σ_n^r and ξ_n^T we transform the data:

$$\widetilde{mps}_t = \frac{mps_t}{\sigma_n^r} = r_{\xi_t} \frac{x_{M,t}}{\sigma_n^r} + \varepsilon_t = \alpha_{\xi_t} \tilde{x}_{M,t} + \varepsilon_t.$$

The above regression has standardized shocks and Markov-switching coefficients in the transformed data. Using ξ_n^T we can group all the observations that pertain to the same regime i . Given the prior $r_i \sim N(B_{r_i,0}, V_{r_i,0})$ for $i = 1, 2$ we use standard Bayesian methods to draw r_i from the conditional (σ_n^r and ξ_n^T) posterior distribution:

$$r_{i,n+1} \sim N(B_{r_i,T}, V_{r_i,T}) \text{ for } i = 1, 2$$

where $V_{r_i,T} = \left(V_{r_i,0}^{-1} + \tilde{X}'_{M,i} \tilde{X}_{M,i} \right)^{-1}$ and $B_{r_i,T} = V_{r_i,T} \left[V_{r_i,0}^{-1} B_{r_i,0} + \tilde{X}'_{r,i} \tilde{R}_i \right]$ where \tilde{R}_i and $\tilde{X}_{r,i}$ collect all the observations for the transformed data for which regime i is in place. The parameters $B_{r_i,0}$ and $V_{r_i,0}^{-1}$ control the priors for the MS coefficients of the regression: $r_i \sim N(B_{r_i,0}, V_{r_i,0})$ for $i = 1, 2$. With flat priors, we have $B_{r_i,0} = 0$ and $V_{r_i,0}^{-1} = 0$ and $B_{r_i,T}$ and $V_{r_i,T}$ coincide with the maximum likelihood estimates, conditional on the other parameters.

4. **Sampling** σ_{n+1}^c : Given β_{n+1} , $\alpha_{\xi_t, n+1}$, and ξ_n^T we can compute the residuals of the regression:

$$\tilde{c}_t = c_t - \beta_{n+1}x_{F,t} - \alpha_{\xi_t}x_{M,t} = \sigma^c \varepsilon_t.$$

With the prior that σ^c has an inverse gamma distribution, $\sigma^c \sim IG(Q_0, v_0)$, we use Bayesian methods to draw σ_{n+1}^c from the conditional (on β_{n+1} , $\alpha_{\xi_t, n+1}$, and ξ_n^T) posterior inverse gamma distribution:

$$\sigma_{n+1} \sim IG(Q_T^c, v_T), \quad v_T = T + v_0, \quad Q_T = Q_0 + E^c E^c$$

where E^c is a vector containing the residuals, T is the sample size, and Q_0 and v_0 control the priors for the standard deviation of the innovations: $\sigma^c \sim IG(Q_0, v_0)$. The mean of a random variable with distribution $\sigma^c \sim IG(Q_T^c, v_T)$ is Q_T/v_T . With flat priors we have $Q_0 = 0$ and $v_0 = 0$, and the mean of σ^c is therefore $(E^c E^c)/T$, which coincides with the standard maximum likelihood (MLE) estimate of σ^c , conditional on the other parameters.

5. **Sampling** σ_{n+1}^r : Given $r_{\xi_t, n+1}$ and ξ_n^T we can compute the residuals of the regression:

$$\widetilde{mps}_t = mps_t - r_{\xi_t}x_{M,t} = \sigma^r \varepsilon_t^r.$$

With the prior that σ^r has an inverse gamma distribution, $\sigma^r \sim IG(Q_0, v_0)$, we use Bayesian methods to draw σ_{n+1}^r from the conditional (on $r_{\xi_t, n+1}$ and ξ_n^T) posterior inverse gamma distribution:

$$\sigma_{n+1} \sim IG(Q_T^r, v_T), \quad v_T = T + v_0, \quad Q_T^r = Q_0 + E^r E^r$$

where E is a vector containing the residuals, T is the sample size, and Q_0 and v_0 control the priors for the standard deviation of the innovations: $\sigma^r \sim IG(Q_0, v_0)$. The mean of a random variable with distribution $\sigma^r \sim IG(Q_T^r, v_T^r)$ is Q_T^r/v_T^r . With flat priors we have $Q_0 = 0$ and $v_0 = 0$, and the mean of σ^r is therefore $(E^r E^r)/T$, which coincides with the maximum likelihood (MLE) estimate of σ^r , conditional on the other parameters.

6. **Sampling** ξ_{n+1}^T : Given $r_{\xi_t, n}$, σ_n^r , β_n , $\alpha_{\xi_t, n}$, σ_n^c , and \mathbf{H}_n , we can treat equations (A5) and (A6) as a multivariate regression in which some parameters are restricted to zero. This allows to obtain filtered probabilities for the regimes using the filter described in Hamilton (1994). Following Kim and Nelson (1999) we then use a Multi-Move Gibbs sampling to draw a regime sequence ξ_{n+1}^T .
7. **Sampling** \mathbf{H}_{n+1} : Given the draws for the MS state variables ξ_{n+1}^T , the posterior for the transition probabilities does not depend on other parameters of the model and follows a

Dirichlet distribution if we assume a prior Dirichlet distribution.²⁴ For each column of \mathbf{H}_{n+1} the posterior distribution is given by:

$$\mathbf{H}_{n+1}(:, i) \sim D(a_{ii} + \eta_{ii,n+1}, a_{ij} + \eta_{ij,n+1})$$

where $\eta_{ij,n+1}$ denotes the number of transitions from state i to state j based on ξ_{n+1}^T , while a_{ii} and a_{ij} the corresponding priors. With flat priors, we have $a_{ii} = 0$ and $a_{ij} = 0$.

8. If $r + 1 < R$, where R is the desired number of draws, go to step 1, otherwise stop.

These steps are repeated until convergence to the posterior distribution is reached. We check convergence by using the Raftery-Lewis Diagnostics for each parameter in the chain. See section below. We use the draws obtained with the Gibbs sampling algorithm to characterize parameter uncertainty in Table 2. The Gibbs sampling algorithm is used to generate a distribution for the difference between the two means in the same manner it is used to generate a distribution for any parameter. For each draw from the joint distribution of the model parameters, we compute the difference and store it. We may then compute means and/or medians, and error bands, as for any other parameter of interest.

Convergence Checks

The 90% credible sets are obtained making 2,000,000 draws from the posterior using the Gibbs sampling algorithm. One in every one thousand draws is retained. We check convergence using the methods suggested by Raftery and Lewis (1992) and Geweke (1992). For Raftery and Lewis (1992) checks, we target 90% credible sets, with a 1% accuracy to be achieved with a 95% minimum probability. We initialize the Gibbs sampling algorithm making a draw around the posterior mode. Sims and Zha (2006) point out that in Markov-switching models it is important to first find the posterior mode and then use it as a starting point for the MCMC algorithm due to the fact that the likelihood can have multiple peaks. The tables below pertain to convergence of the Gibbs sampling algorithm.

Book-to-Market Ratio

We use the methods and assumptions of the previous subsection to obtain the present value decomposition of the book to market ratio. Consider an MS-VAR:

$$Z_t = c_{\xi_t} + A_{\xi_t} Z_{t-1} + V_{\xi_t} \varepsilon_t$$

²⁴The Dirichlet distribution is a generalization of the beta distribution that allows one to potentially consider more than 2 regimes. See e.g., Sims and Zha (2006).

where Z_t is a column vector containing n variables observable at time t and $\xi_t = 1, \dots, m$, with m the number of regimes, evolves following the transition matrix \mathbf{H} . If the MS-VAR has more than one lag, the companion form can be used to recast the model as illustrated above.

Define the column vectors q_t and π_t :

$$q_t = [q_t^1, \dots, q_t^m]' , q_t^i = \mathbb{E}_0 (Z_t 1_{\xi_t=i}) , \pi_t = [\pi_t^1, \dots, \pi_t^m]' ,$$

where $\pi_t^i = P_0 (\xi_t = i)$ and $1_{\xi_t=i}$ is an indicator variable that is equal to 1 when regime i is in place and zero otherwise. The law of motion for $\tilde{q}_t = [q_t', \pi_t']'$ is then given by

$$\underbrace{\begin{bmatrix} q_t \\ \pi_t \end{bmatrix}}_{\tilde{q}_t} = \underbrace{\begin{bmatrix} \Omega & C\mathbf{H} \\ & \mathbf{H} \end{bmatrix}}_{\tilde{\Omega}} \begin{bmatrix} q_{t-1} \\ \pi_{t-1} \end{bmatrix}$$

where $\pi_t = [\pi_{1,t}, \dots, \pi_{m,t}]'$, $\Omega = bdiag (A_1, \dots, A_m) \mathbf{H}$, and $C = bdiag (c_1, \dots, c_m)$. Recall that:

$$\mathbb{E}_0 (Z_t) = \sum_{i=1}^m q_t^i = w q_t, \quad w = \begin{bmatrix} I_n, \dots, I_n \\ \underbrace{\hspace{1cm}}_m \end{bmatrix}$$

To compute the present value decomposition of the book-to-market ratio, define:

$$\begin{aligned} q_{t+s|t}^i &= \mathbb{E}_t (Z_{t+s} 1_{\xi_t=i}) = \mathbb{E} (Z_{t+s} 1_{\xi_t=i} | \mathbb{I}_t) \\ 1_x' &= [0, \dots, 1, \dots, 0, 0]', \quad mn = m * n \end{aligned}$$

where \mathbb{I}_t contains all the information that agents have at time t , including the probability of being in one of the m regimes. Note that $q_{t|t}^i = Z_t \pi_t^i$.

Now consider the formula from Vuolteenaho (1999):

$$\theta_t = \sum_{j=0}^{\infty} \rho^j \mathbb{E}_t r_{t+1+j} + \sum_{j=0}^{\infty} \rho^j \mathbb{E}_t f_{t+1+j} - \sum_{j=0}^{\infty} \rho^j \mathbb{E}_t e_{t+1+j}^*$$

Given that our goal is to assess if assets with different risk profiles are affected differently by the breaks in the long-term interest rates, we are going to focus on the *difference* between the book-to-market ratios. Specifically, given two portfolios x and y , we are interested in how the difference in their book-to-market ratios, $\theta_{x,t} - \theta_{y,t}$, varies across the two regimes:

$$\underbrace{\theta_{x,t} - \theta_{y,t}}_{\text{Spread in BM ratios}} = \underbrace{\sum_{j=0}^{\infty} \rho^j \mathbb{E}_t (r_{x,t+1+j} - r_{y,t+1+j})}_{\text{PDV of the difference in expected excess returns}} - \underbrace{\sum_{j=0}^{\infty} \rho^j \mathbb{E}_t (e_{x,t+1+j}^* - e_{y,t+1+j}^*)}_{\text{PDV of the difference in expected earnings}}$$

If then we want to correct the spread in BM ratios by taking into account expected earnings, we have:

$$\underbrace{\theta_{x,t} - \theta_{y,t} + \sum_{j=0}^{\infty} \rho^j \mathbb{E}_t (e_{x,t+1+j}^* - e_{y,t+1+j}^*)}_{\text{Spread in BM ratios corrected for earnings}} = \underbrace{\sum_{j=0}^{\infty} \rho^j \mathbb{E}_t (r_{x,t+1+j} - r_{y,t+1+j})}_{\text{PDV of the expected spread in excess returns}} \quad (\text{A7})$$

The spread in excess returns, $r_{xy,t} \equiv r_{x,t} - r_{y,t}$. Then the right hand side of (A7) can be computed as:

$$\begin{aligned} \sum_{j=0}^{\infty} \rho^j \mathbb{E}_t (r_{xy,t+1+j}) &= \sum_{j=0}^{\infty} \rho^j 1'_{r_{xy}} w q_{t+1+j|t} \\ &= 1'_{r_{xy}} w (I - \rho \Omega)^{-1} [\Omega q_{t|t} + C (I - \rho \mathbf{H})^{-1} \mathbf{H} \pi_{t|t}]. \end{aligned}$$

Therefore, we have:

$$\tilde{\theta}_{xy,t} \equiv \underbrace{\tilde{\theta}_{x,t} - \tilde{\theta}_{y,t} + \sum_{j=0}^{\infty} \rho^j \mathbb{E}_t (e_{x,t+1+j}^* - e_{y,t+1+j}^*)}_{\text{Spread in BM ratios corrected for earnings}} = 1'_{r_{xy}} w (I - \rho \Omega)^{-1} [\Omega q_{t|t} + C (I - \rho \mathbf{H})^{-1} \mathbf{H} \pi_{t|t}] \quad (\text{A8})$$

where we have used $\tilde{\theta}_{xy,t}$ to define the spread in BM ratios corrected for earnings.

Similar formulas are used to compute risk premia for the individual portfolios. The premium for a portfolio z coincides with the present discounted value of its excess returns:

$$\underbrace{\text{premium}_{z,t}}_{\text{Premium}} \equiv \underbrace{\sum_{j=0}^{\infty} \rho^j \mathbb{E}_t (r_{z,t+1+j})}_{\text{PDV of excess returns}} = 1'_{r_z} w (I - \rho \Omega)^{-1} [\Omega q_{t|t} + C (I - \rho \mathbf{H})^{-1} \mathbf{H} \pi_{t|t}], \quad (\text{A9})$$

where $1'_{r_z}$ is a vector used to extract the PDV of excess returns from a vector containing the PDV of all variables included in the VAR.

Regime Average We also compute the *regime average* value of $\tilde{\theta}_{xy,t}$. The regime average is defined as:

$$\bar{\tilde{\theta}}_{xy}^i \equiv 1'_{r_{xy}} w (I - \rho \Omega)^{-1} [\Omega \bar{q}_i + C (I - \rho \mathbf{H})^{-1} \mathbf{H} \bar{\pi}_i]$$

where $\bar{\pi}_i = 1_i$ and $\bar{q}_i \equiv [0, \dots, \bar{\mu}_i, \dots, 0]$ is a column vector that contains the conditional steady state of for the mean value of Z_t conditional on being in regime i , i.e., $\mathbb{E}_i(Z_t) = \bar{\mu}_i = (I_n - A_i)^{-1} c_i$, and zero otherwise. Recall that the conditional steady state, $\bar{\mu}_i$, is a vector that contains the expected value of Z_t conditional on being in regime i . Therefore, the vector captures the values to which the variables of the VAR converge if regime i is in place forever. Although none of our regimes are estimated to be absorbing states, this is still a good approximation for regimes that can be expected to persist for prolonged periods of time. Note that $\bar{\tilde{\theta}}_{xy}^i$ is computed by conditioning on the economy being initially at $Z_t = \bar{\mu}_i$ and in regime i , but taking into account that there might be regime changes in the future. Therefore, we can also think about $\bar{\tilde{\theta}}_{xy}^i$ as the expected value of $\tilde{\theta}_{xy,t}$, conditional on being in regime i today and on the variables of the VAR being equal to the conditional steady state mean values for regime i . Formally:

$$\bar{\tilde{\theta}}_{xy}^i = \mathbb{E} \left(\tilde{\theta}_{xy,t} | \xi_t = i, Z_t = \bar{\mu}_i \right). \quad (\text{A10})$$

Similarly, we can compute the *regime average* value of risk premia for an individual portfolio z , $\text{premium}_{z,t}$:

$$\overline{\text{premium}_z}^i \equiv 1'_{r_z} w (I - \rho \Omega)^{-1} [\Omega \bar{q}_i + C (I - \rho \mathbf{H})^{-1} \mathbf{H} \bar{\pi}_i]. \quad (\text{A11})$$

Formulas (A8), (A9), (A10), and (A11) are used in the paper to produce Figure 4 and Table 5. For each draw of the VAR parameters from the posterior distribution, we can compute the evolution of $\tilde{\theta}_{xy,t}$ and individual portfolio $premia_{z,t}$, by using (A8) and (A9). Thus, we obtain a posterior distribution for $\tilde{\theta}_{xy,t}$ and $premia_{z,t}$. The medians of these posterior distributions are reported as the blue solid lines in Figure 4. Similarly, for each draw of the VAR coefficients, we compute $\tilde{\theta}_{xy}^i$ and the difference $\tilde{\theta}_{xy}^1 - \tilde{\theta}_{xy}^2$. Thus, we obtain a posterior distribution for $\tilde{\theta}_{xy}^i$ and for the difference $\tilde{\theta}_{xy}^1 - \tilde{\theta}_{xy}^2$. The medians of the distribution of $\tilde{\theta}_{xy}^i$ and \overline{premia}_z^i for $i = 1, 2$, are reported in Figure 4 (red dashed line). Table 5 reports the median and the 68% posterior credible sets both for the distribution of $\tilde{\theta}_{xy}^i$, for $i = 1, 2$, and for the difference in these across regimes, $\tilde{\theta}_{xy}^1 - \tilde{\theta}_{xy}^2$. Finally, the last row of Table 5 reports the percentage of draws for which $\tilde{\theta}_{xy}^1 - \tilde{\theta}_{xy}^2 > 0$ and $\overline{premia}_z^1 - \overline{premia}_z^2 > 0$ as the probability that risk premia are lower in the high asset valuation/low interest rate regime than they are in the low asset valuation/high interest rate regime.

Variable Selection for VARs to Compute PDV of Risk Premia

We start with a series of fixed regressors that are relevant for predicting market excess returns or the return of the spread portfolios. To limit the size of the MS-VAR, we then use the Akaike information criterion (AIC) to decide whether to include some additional regressors. Specifically, we compute the AIC for the equation(s) that correspond(s) to the return(s) that we are trying to predict. We then choose the specification that minimizes the AIC.

Here are the details:

1. MS-VAR for the Market excess return:

Fixed regressors (all lagged): Market excess return, inverse valuation ratio based on cay^{MS} . The inverse valuation ratio is included because it represents a measure of asset valuation that can predict future stock market returns. Note that given that we are conditioning to the regime sequence obtained when estimating cay^{MS} , the intercept for the corresponding equation will adjust in a way to reflect the low frequency breaks identified above.

Possible additional variables to be chosen for the estimation based on the AIC: Value (small) spread (log-difference in the book to market ratio of the small value portfolios and the book to market ratio of the small growth portfolios), Real FFR, term yield spread, four of the five Fama and French factors (SMB, HML, RMW, CMA), cay (based on PCE, available on Martin Lettau's website.) Note that we do not include the market excess return from Fama and French (MKTMINRF) as a possible additional regressor because our dependant variable is the excess market return itself. Therefore, this variable is automatically included in the MS-VAR.

Additional regressors selected based on the AIC: Value Spread, and SMB and HML factors from Fama and French.

2. MS-VAR for (a) Momentum return spread: The difference between the excess return of the extreme winner (M10) portfolio and the excess return of the extreme loser (M1) portfolio; (b) Value return spread (S1): The difference between the excess return of the small (size 1) high BM portfolio and the excess return of the small (size 1) low BM portfolio; (c) Value return spread (S2): The difference between the excess return of the size 2 high BM portfolio and the excess return of the small size 2 low BM portfolio.

Fixed regressors (all lagged): (a) Momentum return spread; (b) Value return spread (S1); (c) Value return spread (S2); (d) Momentum BM spread: The difference between the logarithm of the BM ratio of the extreme winner (M10) portfolio and the logarithm of the BM ratio of the extreme loser (M1) portfolio; (e) Value BM spread (S1): The difference between the logarithm of the BM ratio of the small (size quintile 1) high book-market portfolio and the logarithm of the BM ratio of the small (size 1) low book-market portfolio; (f) Value BM spread (S2): The difference between the logarithm of the BM ratio of the size quintile 2 high book-market portfolio and the logarithm of the BM ratio of the size 2 low book-market portfolio.

Possible additional variables to be chosen for the estimation based on the AIC: Real FFR computed as the difference between FFR and Inflation, excess return of small growth portfolio, excess return of small value portfolio, five Fama-French factors (SMB, HML, RMW, CMA, MKTMINRF.)

Additional regressors selected based on the AIC: Real FFR and excess return of the small value portfolio.

Most Likely Regime Sequence

In this appendix we provide details on how to compute the most likely regime sequence. This most likely regime sequence is based on our estimates for the breaks in cay^{MS} and mps , and is taken as given in the portfolio MS-VAR and the MS-DSGE estimation. Specifically, we choose the particular regime sequence $\xi_n^T = \{\hat{\xi}_{1,n}, \dots, \hat{\xi}_{T,n}\}$ that is most likely to have occurred, given our estimated posterior mode parameter values for θ . This sequence is computed as follows. First, we run Hamilton's filter to get the vector of filtered probabilities $\pi_{t|t}$, $t = 1, 2, \dots, T$. The Hamilton filter can be expressed iteratively as

$$\begin{aligned}\pi_{t|t} &= \frac{\pi_{t|t-1} \odot \eta_t}{\mathbf{1}' (\pi_{t|t-1} \odot \eta_t)} \\ \pi_{t+1|t} &= \mathbf{H} \pi_{t|t}\end{aligned}$$

where η_t is a vector whose j -th element contains the conditional density $p(c_t | \xi_t = j, x_{M,t}, x_{F,t}; \theta)$, the symbol \odot denotes element by element multiplication, and $\mathbf{1}$ is a vector with all elements equal to 1. The final term, $\pi_{T|T}$ is returned with the final step of the filtering algorithm. Then, a recursive algorithm can be implemented to derive the other smoothed probabilities:

$$\pi_{t|T} = \pi_{t|t} \odot [\mathbf{H}' (\pi_{t+1|T} (\div) \pi_{t+1|t})]$$

where (\div) denotes element by element division. To choose the regime sequence most likely to have occurred given our parameter estimates, consider the recursion in the next to last period $t = T - 1$:

$$\pi_{T-1|T} = \pi_{T-1|T-1} \odot [\mathbf{H}' (\pi_{T|T} (\div) \pi_{T|T-1})].$$

We first take $\pi_{T|T}$ from the Hamilton filter and choose the regime that is associated with the largest probability, i.e., if $\pi_{T|T} = (.9, .1)$, where the first element corresponds to the probability of regime 1, we select $\hat{\xi}_T = 1$, indicating that we are in regime 1 in period T . We now update $\pi_{T|T} = (1, 0)$ and plug into the right-hand-side above along with the estimated filtered probabilities for $\pi_{T-1|T-1}$, $\pi_{T|T-1}$ and estimated transition matrix \mathbf{H} to get $\pi_{T-1|T}$ on the left-hand-side. Now we repeat the same procedure by choosing the regime for $T - 1$ that has the largest probability at $T - 1$, e.g., if $\pi_{T-1|T} = (.2, .8)$ we select $\hat{\xi}_{T-1} = 2$, indicating that we are in regime 2 in period $T - 1$, we then update to $\pi_{T-1|T} = (0, 1)$, which is used again on the right-hand-side now

$$\pi_{T-2|T} = \pi_{T-2|T-2} \odot [\mathbf{H}' (\pi_{T-1|T} (\div) \pi_{T-1|T-2})].$$

We proceed in this manner until we have a most likely regime sequence ξ^T for the entire sample $t = 1, 2, \dots, T$. Two aspects of this procedure are worth noting. First, it fails if the updated

probabilities are exactly (.5, .5). Mathematically this is virtually zero. Second, note that this procedure allows us to choose the most likely regime sequence by using the recursive formula above to update the filtered probabilities sequentially from T to time $t = 1$. This allows us to take into account the time dependence in the regime sequence as dictated by the transition probabilities.

Estimation of the MS-VAR

In this appendix we provide details on the estimation of the MS-VAR. Given that we take the regime sequence as given, we need only estimate the transition matrix and the parameters of the MS-VAR across the two regimes. The model is estimated by using Bayesian methods with flat priors on all parameters. As a first step, we group all the observations that belong to the same regime. Conditional on a regime, we have a fixed coefficients VAR. We can then follow standard procedures to make draws for the VAR parameters as follows.

Rewrite the VAR as

$$\begin{aligned} Y_{T \times n} &= X_{(T \times k)(k \times n)} A_{\xi_t} + \varepsilon_{T \times n}, \quad \xi_t = 1, 2 \\ \varepsilon_t &\sim N(0, \Sigma_{\xi_t}) \end{aligned}$$

where $Y = [Z_1, \dots, Z_T]'$, the t -th row of X is $X_t = [1, Z'_{t-1}, Z'_{t-2}]$, $A_{\xi_t} = [c_{\xi_t}, A_{1,\xi_t}, A_{2,\xi_t}]'$, the t -th row of ε is ε_t , and where $\Sigma_{\xi_t} = V_{\xi_t} V_{\xi_t}'$. If we specify a Normal-Wishart prior for A_{ξ_t} and V_{ξ_t} :

$$\begin{aligned} \Sigma_{\xi_t}^{-1} &\sim W(S_0^{-1}/v_0, v_0) \\ \text{vec}(A_{\xi_t} | \Sigma_{\xi_t}) &\sim N(\text{vec}(B_0), \Sigma_{\xi_t} \otimes N_0^{-1}) \end{aligned}$$

where $E(\Sigma_{\xi_t}^{-1}) = S_0^{-1}$, the posterior distribution is still in the Normal-Wishart family and is given by

$$\begin{aligned} \Sigma_{\xi_t}^{-1} &\sim W(S_T^{-1}/v_T, v_T) \\ \text{vec}(A_{\xi_t} | \Sigma_{\xi_t}) &\sim N(\text{vec}(B_T), \Sigma_{\xi_t} \otimes N_T^{-1}) \end{aligned}$$

Using the estimated regime sequence ξ_n^T we can group all the observations that pertain to the same regime i . Therefore the parameters of the posterior are computed as

$$\begin{aligned} v_T &= T_i + v_0, \quad N_T = X_i' X_i + N_0 \\ B_T &= N_T^{-1} (N_0 B_0 + X_i' X_i \hat{B}_{MLE}) \\ S_T &= \frac{v_0}{v_T} S_0 + \frac{T_i}{v_T} \hat{\Sigma}_{MLE} + \frac{1}{v_T} (\hat{B}_{MLE} - \hat{B}_0)' N_0 N_T^{-1} X_i' X_i (\hat{B}_{MLE} - \hat{B}_0) \\ \hat{B}_{MLE} &= (X_i' X_i)^{-1} (X_i' Y_i), \quad \hat{\Sigma}_{MLE} = \frac{1}{T_i} (Y_i - X_i \hat{B}_{MLE})' (Y_i - X_i \hat{B}_{MLE}), \end{aligned}$$

where T_i, Y_i, X_i denote the number and sample of observations in regime i . We choose flat priors ($v_0 = 0, N_0 = 0$) so the expressions above coincide with the MLE estimates using observations in regime i :

$$v_T = T_i, \quad N_T = X_i' X_i, \quad B_T = \hat{B}_{MLE}, \quad S_T = \hat{\Sigma}_{MLE}.$$

Armed with these parameters in each regime, we can make draws from the posterior distributions for $\Sigma_{\xi_t}^{-1}$ and A_{ξ_t} in regime i to characterize parameter uncertainty about these parameters.

Given that we condition the MS-VAR estimates on the most likely regime sequence, ξ_n^T , for cay^{MS} , it is still of interest to estimate the elements of the transition probability matrix for the MS-VAR parameters, \mathbf{H}^A , conditional on this regime sequence. Because we impose this regime sequence, the posterior of \mathbf{H}^A only depends on ξ_n^T and does not depend on other parameters of the model. The posterior has a Dirichlet distribution if we assume a prior Dirichlet distribution.²⁵ For each column of \mathbf{H}^A the posterior distribution is given by:

$$\mathbf{H}^A(:, i) \sim D(a_{ii} + \eta_{ii,r+1}, a_{ij} + \eta_{ij,r+1})$$

where $\eta_{ij,r+1}$ denotes the number of transitions from regime i to regime j based on ξ_n^T , while a_{ii} and a_{ij} the corresponding priors. With flat priors, we have $a_{ii} = 0$ and $a_{ij} = 0$. Armed with this posterior distribution, we can characterize uncertainty about \mathbf{H}^A . Note that the posterior \mathbf{H}^A will be in general different from the posterior distribution of \mathbf{H} because the former is based on a particular regime sequence ξ_n^T , while the latter reflects the entire posterior distribution for ξ_n^T . The estimated transition matrix \mathbf{H}^A can in turn be used to compute expectations taking into account the possibility of regime change (see the next subsection).

MS-DSGE Model

This section reports technical details about the MS-DSGE model.

Evolution of beliefs

This subsection describes the evolution of beliefs. We first describe the standard constant gain learning algorithm. We then introduce the signal about the current inflation target.

Constant Gain Adaptive Learning Suppose that agents believe that inflation evolves according to an AR(1) process:

$$\pi_t = \alpha + \phi\pi_{t-1} + \eta_t \tag{A12}$$

²⁵The Dirichlet distribution is a generalization of the beta distribution that allows one to potentially consider more than 2 regimes. See e.g., Sims and Zha (2006).

Perceived trend inflation $\bar{\pi}_t$ is defined as the $\lim_{h \rightarrow \infty} \tilde{\mathbb{E}}_t(\pi_{t+h})$, which equals the unconditional mean of inflation $\tilde{\mathbb{E}}(\pi_t)$. Taking expectations on both sides of equation A12 we find,

$$\tilde{\mathbb{E}}(\pi_t) = \alpha + \phi \tilde{\mathbb{E}}(\pi_t) \Rightarrow \tilde{\mathbb{E}}(\pi_t) = \bar{\pi}_t = \tilde{\alpha}_t / (1 - \phi),$$

where we plug in the value of $\tilde{\alpha}_t$ that agents perceive at t as the last step. Thus, the AR(1) process implies a one-to-one mapping between the perceived constant $\tilde{\alpha}_t$ and perceived trend inflation $\bar{\pi}_t$.

Individuals estimate $b \equiv (\alpha, \phi)'$ from past data following

$$\begin{aligned} R_t &= R_{t-1} + \gamma_t (x_{t-1} x'_{t-1} - R_{t-1}) \\ b_t &= b_{t-1} + \gamma_t R_t^{-1} x_{t-1} (\pi_t - b'_{t-1} x_{t-1}) \end{aligned} \quad (\text{A13})$$

where $x_t = (1, \pi_t)'$. The recursion is started at some point in the distant past. The sequence of gains $0 < \gamma_t < 1$ determines the degree of updating cohort s applies when faced with an inflation surprise at time t . For $\gamma_t = 1/t$ the algorithm represents a recursive formulation of an ordinary least squares estimation that uses all available data until time t with equal weights (see Evans and Honkapohja (2001)). For constant γ_t , it represents a constant-gain learning algorithm with exponentially decaying weights.

The previous specification simplifies if we assume that agents are only uncertain about the long term value of inflation, but not its persistence. If agents only learn about α and the recursion has started in the distant pass we have:

$$R_t = 1 \text{ if } R_{t-1} = 1 \quad (\text{A14})$$

$$\tilde{\alpha}_t = \tilde{\alpha}_{t-1} + \gamma_t (\pi_t - \phi \pi_{t-1} - \tilde{\alpha}_{t-1}) \quad (\text{A15})$$

To see the above, note that if ϕ were known, the agent would estimate α by running a regression of $\pi_t - \phi \pi_{t-1}$ on a constant, or a vector of ones. So $x_t = 1$ in every period and $R_t = R_{t-1} + \gamma_t (x_{t-1} x'_{t-1} - R_{t-1}) = R_t = R_{t-1} + \gamma_t (1 - R_{t-1})$. Starting value for $R = R_0 \Rightarrow R_1 = R_0 + \gamma(1 - R_0)$. Continuing to iterate, this converges to 1 no matter what R_0 as long as $0 < \gamma_t < 1$. Set $x_t = R_1 = 1$ in (A13) to get (A15).

With constant gain learning, the variable γ_t is a constant parameter that we denote γ . This implies:

$$\tilde{\alpha}_t = \tilde{\alpha}_{t-1} + \gamma (\pi_t - \phi \pi_{t-1} - \tilde{\alpha}_{t-1}). \quad (\text{A16})$$

Hereafter we assume that learning is constant gain. Using the relation between $\bar{\pi}_t$ and $\tilde{\alpha}_t$, we get:

$$\begin{aligned} \tilde{\alpha}_t &= (1 - \phi) \bar{\pi}_t \Rightarrow \\ (1 - \phi) \bar{\pi}_t &= (1 - \phi) \bar{\pi}_{t-1} + \gamma (\pi_t - \phi \pi_{t-1} - (1 - \phi) \bar{\pi}_{t-1}) \Rightarrow \\ \bar{\pi}_t &= \bar{\pi}_{t-1} + \gamma (1 - \phi)^{-1} (\pi_t - \phi \pi_{t-1} - (1 - \phi) \bar{\pi}_{t-1}), \end{aligned} \quad (\text{A17})$$

where the second equation above follows from (A16).

Signal About the Inflation Target In our model, we combine the constant gain learning algorithm described above with a signal about the central bank's inflation target, thereby allowing beliefs to be partly shaped by additional information the agent receives about the target. This signal could reflect the opinion of experts (as in MN), or a credible central bank announcement. If we use $\tilde{\alpha}_t^{CG}$ and $\tilde{\pi}_t^{CG}$ to denote the beliefs implied by the constant gain learning described above, we obtain modified updating rules for $\tilde{\alpha}_t$ and $\tilde{\pi}_t$ that are a weighted averages of two terms:

$$\begin{aligned}\tilde{\alpha}_t &= (1 - \gamma^T) \left[\underbrace{\tilde{\alpha}_{t-1} + \gamma (\pi_t - \phi \pi_{t-1} - \tilde{\alpha}_{t-1})}_{\tilde{\alpha}_t^{CG}} \right] + \gamma^T \left[(1 - \phi) \pi_{\xi_t}^T \right]. \\ \tilde{\pi}_t &= (1 - \gamma^T) \left[\underbrace{\tilde{\pi}_{t-1} + \gamma (1 - \phi)^{-1} (\pi_t - \phi \pi_{t-1} - (1 - \phi) \tilde{\pi}_{t-1})}_{\tilde{\pi}_t^{CG}} \right] + \gamma^T \left[\pi_{\xi_t}^T \right].\end{aligned}$$

The first terms in square brackets, $\tilde{\alpha}_t^{CG}$ and $\tilde{\pi}_t^{CG}$, are the recursive updating rules implied by constant gain learning as in (A16) and (A17). These terms are combined with two terms that involve the central bank's current inflation target $\pi_{\xi_t}^T$. Note that, since $\tilde{\alpha}_t = (1 - \phi) \tilde{\pi}_t$ under the autoregressive model, the term $(1 - \phi) \pi_{\xi_t}^T$ is simply the value of $\tilde{\alpha}_t$ that would arise if $\tilde{\pi}_t = \pi_{\xi_t}^T$. If $\gamma^T = 1$, the signal is completely informative and the agent's belief about trend inflation is the same as the inflation target. If $\gamma^T = 0$, the signal is completely uninformative and the agent's belief about trend inflation depends only on the learning algorithm. Thus, the resulting laws of motion for the beliefs are a weighted average of what would arise under constant gain learning and a term reflecting information about the current inflation target.

5.0.4 Expected inflation

Expected inflation from the point of view of the agents in the model is formed based on equation A12 and their beliefs about the constant α , i.e., $\tilde{\alpha}_t$. Specifically, we have

$$\begin{aligned}\tilde{E}_t [\pi_{t+1}] &= \tilde{\alpha}_t + \phi \pi_t \\ \tilde{E}_t [\pi_{t+2}] &= \tilde{\alpha}_t + \phi \tilde{\alpha}_t + \phi^2 \pi_t \\ \tilde{E}_t [\pi_{t+3}] &= \tilde{\alpha}_t + \phi \tilde{\alpha}_t + \phi^2 \tilde{\alpha}_t + \phi^3 \pi_t \\ \tilde{E}_t [\pi_{t+4}] &= \tilde{\alpha}_t + \phi \tilde{\alpha}_t + \phi^2 \tilde{\alpha}_t + \phi^3 \tilde{\alpha}_t + \phi^4 \pi_t\end{aligned}$$

where, in line with the learning literature, we have assumed that agents do not take into account that their beliefs might change in the future (i.e., they do not have anticipated utility).

Cumulative inflation over the next year is:

$$\begin{aligned}\tilde{E}_t[\pi_{t,t+4}] &= [4 + 3\phi + 2\phi^2 + \phi^3] \tilde{\alpha}_t + [\phi + \phi^2 + \phi^3 + \phi^4] \pi_t \\ &= [4 + 3\phi + 2\phi^2 + \phi^3] (1 - \phi) \bar{\pi}_t + [\phi + \phi^2 + \phi^3 + \phi^4] \pi_t\end{aligned}$$

where in the second row we have used the fact that $\bar{\pi}_t = \tilde{\alpha}_t / (1 - \phi)$. The general formulas are:

$$\begin{aligned}\tilde{E}_t[\pi_{t+h}] &= \tilde{\alpha}_t + \phi \tilde{\alpha}_t + \dots + \phi^{h-1} \tilde{\alpha}_t + \phi^h \pi_t \\ \tilde{E}_t[\pi_{t,t+h}] &= (1/h) \sum_{i=1}^h \tilde{E}_t[\pi_{t+i}]\end{aligned}$$

Using matrix algebra, we can express the *perceived* law of motion for inflation as:

$$\begin{bmatrix} \tilde{\alpha}_t \\ \pi_{t+1} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & \phi \end{bmatrix} \begin{bmatrix} \tilde{\alpha}_t \\ \pi_t \end{bmatrix} + \begin{bmatrix} 0 \\ \eta_{t+1} \end{bmatrix}$$

This is equivalent to:

$$\begin{bmatrix} \bar{\pi}_t \\ \pi_{t+1} \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 \\ 1 - \phi & \phi \end{bmatrix}}_{\Omega} \underbrace{\begin{bmatrix} \bar{\pi}_t \\ \pi_t \end{bmatrix}}_{e_{\pi\bar{\pi}} S_t} + \begin{bmatrix} 0 \\ \eta_{t+1} \end{bmatrix}$$

where once again we have used $\bar{\pi}_t = \tilde{\alpha}_t / (1 - \phi)$ and the matrix $e_{\pi\bar{\pi}}$ is used to extract both inflation π_t and the perceived long term inflation $\bar{\pi}_t$ from the state vector S_t . The latter formulation is used for the solution of the model since it is $\bar{\pi}_t$ rather than $\tilde{\alpha}_t$ that appears in the state space representation of the model. It follows that

$$\tilde{E}_t[\pi_{t,t+h}] = e_{\pi} \Omega (I - \Omega)^{-1} (I - \Omega^4) (e_{\pi\bar{\pi}} S_t)$$

where the vector e_{π} is used to extract inflation.

Long-run Monetary Neutrality

In a model with rational expectations, the relation between inflation and the output gap is controlled by a New-Keynesian Phillips curve:

$$\pi_t - \bar{\pi}_t = \beta \mathbb{E}_t[\pi_{t+1} - \bar{\pi}_t] + \kappa [y_{t-1} - y_{t-1}^*]$$

where $\bar{\pi}_t$ denotes the long term value of inflation that depends on the central bank's inflation target. The parameter κ denotes the slope of the Phillips curve and depends on the extent of nominal rigidities in the economy. Taking the unconditional expectation on both sides, we have:

$$\begin{aligned}\mathbb{E}[\pi_t - \bar{\pi}_t] &= \beta \mathbb{E}[\pi_{t+1} - \bar{\pi}_t] + \kappa \mathbb{E}[y_{t-1} - y_{t-1}^*] \\ \mathbb{E}[\pi_t] - \mathbb{E}[\bar{\pi}_t] &= \beta \mathbb{E}[\pi_t] - \beta \mathbb{E}[\bar{\pi}_t] + \kappa \mathbb{E}[y_{t-1} - y_{t-1}^*] \\ 0 &= \kappa \mathbb{E}[y_{t-1} - y_{t-1}^*]\end{aligned}$$

where we have used the fact that $\bar{\pi}_t = \mathbb{E}[\pi_t]$. Therefore, we have: $\mathbb{E}[y_{t-1}^*] = \mathbb{E}[y_{t-1}] = 0$.

With learning, long-term neutrality still holds:

$$\begin{aligned}\pi_t - \bar{\pi}_t &= \beta\phi[\pi_t - \bar{\pi}_t] + \kappa[y_{t-1} - y_{t-1}^*] \\ \mathbb{E}[\pi_t] &= \mathbb{E}[\bar{\pi}_t] + \frac{\kappa}{1 - \beta\phi}\mathbb{E}[y_{t-1} - y_{t-1}^*] \\ 0 &= \kappa\mathbb{E}[y_{t-1} - y_{t-1}^*]\end{aligned}$$

where we have used the fact that $\bar{\pi}_t = \mathbb{E}[\pi_t]$. Therefore, we have: $\mathbb{E}[y_{t-1}^*] = \mathbb{E}[y_{t-1}] = 0$.

Solution and Estimation of the Model

We can rewrite the system of equations as:

$$y_t = \delta y_{t-1} - \sigma[i_t - \phi\pi_t - (1 - \phi)\bar{\pi}_t - r] + d_t \quad (\text{A18})$$

$$\pi_t = \bar{\pi}_t + \frac{\kappa}{1 - \beta\phi}[y_{t-1} - y_{t-1}^*] \quad (\text{A19})$$

$$i_t - (r + \pi_{\xi_t}^T) = (1 - \rho_{i,\xi_t})\left[\psi_{\pi,\xi_t}(\pi_t - \pi_{\xi_t}^T) + \psi_{\Delta y,\xi_t}(y_t - y_{t-1})\right] \quad (\text{A20})$$

$$+ \rho_{i,\xi_t}\left[i_{t-1} - (r + \pi_{\xi_t}^T)\right] + \sigma_i \varepsilon_{i,t} \quad (\text{A21})$$

$$r_t^* = -(1/\sigma)(y_t^* - \delta y_{t-1}^* - d_t) + r \quad (\text{A22})$$

$$y_t^* = \rho_{y^*} y_{t-1}^* + \sigma_{y^*} \varepsilon_{y^*,t} \quad (\text{A23})$$

$$\bar{\pi}_t = [1 - \gamma^T][\bar{\pi}_{t-1} + \gamma(1 - \phi)^{-1}(\pi_t - \phi\pi_{t-1} - (1 - \phi)\bar{\pi}_{t-1})] + \gamma^T \pi_{\xi_t}^T \quad (\text{A24})$$

$$d_t = \rho_d d_{t-1} + \sigma_d \varepsilon_{d,t} \quad (\text{A25})$$

State space and parameter vectors

Define the parameter vectors θ_{ξ_t} and $\theta_{\xi_t}^c$ as

$$\begin{aligned}\theta_{\xi_t} &= [\delta, \sigma, \beta, \kappa, \psi_{\pi,\xi_t}, \psi_{\Delta y,\xi_t}, \rho_{i,\xi_t}, \rho_{y^*}, \gamma^T, \gamma, \phi, \rho_d]' \\ \theta_{\xi_t}^c &= [\pi_{\xi_t}^T, r]'\end{aligned}$$

and the state vector S_t and the vector of Gaussian shocks ε_t as

$$\begin{aligned}S_t &= [y_t, y_t^*, \pi_t, i_t, r_t^*, \bar{\pi}_t, d_t]' \\ \varepsilon_t &= [\varepsilon_{i,t}, \varepsilon_{y_t^*}, \varepsilon_{d,t}]', \quad \varepsilon_t \sim N(0, I)\end{aligned}$$

Let the matrix $Q = \text{diag}(\sigma_i, \sigma_{y^*}, \sigma_d)$ be a square matrix with the shock standard deviations on the main diagonal. Conditional on each regime, the system of equations can be rewritten using matrix notation:

$$\Gamma_0(\theta_{\xi_t}) S_t = \Gamma_c(\theta_{\xi_t}^c) + \Gamma_1(\theta_{\xi_t}) S_{t-1} + Q\varepsilon_t$$

Note that the vector $\Gamma_c \left(\theta_{\xi_t}^c \right)$ includes the inflation target for the corresponding regime.

Inverting the matrix $\Gamma_0 \left(\theta_{\xi_t} \right)$, we obtain the solution of the model as MSVAR:

$$S_t = C \left(\theta_{\xi_t}^c, \theta_{\xi_t} \right) + T(\theta_{\xi_t}) S_{t-1} + R(\theta_{\xi_t}) Q \varepsilon_t$$

where $C \left(\theta_{\xi_t}^c, \theta_{\xi_t} \right) = \Gamma_0^{-1} \left(\theta_{\xi_t} \right) \Gamma_c \left(\theta_{\xi_t}^c \right)$, $T(\theta_{\xi_t}) = \Gamma_0^{-1} \left(\theta_{\xi_t} \right) \Gamma_1 \left(\theta_{\xi_t} \right)$, and $R(\theta_{\xi_t}) = \Gamma_0^{-1} \left(\theta_{\xi_t} \right)$.

The solution of the model can be combined with an observation equation to estimate the model. Given that we know the regime sequence, we can estimate the model with a standard Kalman filter algorithm. The only caveat is that the associated transition equation varies over time. We thus have the following state space representation:

$$X_t = D + Z [S_t', y_{t-1}]' + U v_t \quad (\text{A26})$$

$$S_t = C \left(\theta_{\xi_t}^c, \theta_{\xi_t} \right) + T(\theta_{\xi_t}) S_{t-1} + R(\theta_{\xi_t}) Q \varepsilon_t \quad (\text{A27})$$

$$v_t \sim N(0, I) \quad (\text{A28})$$

where v_t is a vector of observation errors and U is a diagonal matrix with the standard deviations of the observation errors on the main diagonal. As said before, we condition on a regime sequence ξ_t , so the transition equation (A27) in at each point in time is known.

In our estimation, we use four observables: Real GDP per capita growth, Inflation, Federal Funds rate, and the mean of the Michigan survey one-year-ahead inflation forecasts. All variables are annualized. We have observation errors on all variables because we have 3 shocks for four observables.

Thus, the vector X_t is defined as:

$$\begin{bmatrix} \Delta GDP \\ Inflation \\ FFR \\ E(Inflation) \end{bmatrix} = \begin{bmatrix} \overline{\Delta GDP} \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 4y_t - 4y_{t-1} \\ 4\pi_t \\ 4i_t \\ [4 + 3\phi + 2\phi^2 + \phi^3] (1 - \phi) \bar{\pi}_t + [\phi + \phi^2 + \phi^3 + \phi^4] \pi_t \end{bmatrix} + \begin{bmatrix} v_t^y \\ v_t^\pi \\ v_t^f \\ v_t^e \end{bmatrix}$$

where in the last row we have used the fact that expectations for an agent in the model is:

$$\begin{aligned} \tilde{E}_t [\pi_{t,t+4}] &= [4 + 3\phi + 2\phi^2 + \phi^3] \tilde{\alpha}_t + [\phi + \phi^2 + \phi^3 + \phi^4] \pi_t \\ &= [4 + 3\phi + 2\phi^2 + \phi^3] (1 - \phi) \bar{\pi}_t + [\phi + \phi^2 + \phi^3 + \phi^4] \pi_t \end{aligned}$$

The mapping from the variables of the model to the observables can be written using matrix algebra. The vector D is then:

$$D = \begin{bmatrix} \overline{\Delta GDP} \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

The matrix Z is then:

$$Z = \begin{bmatrix} 4 & 0 & 0 & 0 & 0 & 0 & 0 & -4 \\ 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 & 0 & 0 & 0 \\ 0 & 0 & [\phi + \phi^2 + \phi^3 + \phi^4] & 0 & 0 & 0 & 0 & [4 + 3\phi + 2\phi^2 + \phi^3](1 - \phi) \\ & & & & & & & 0 \end{bmatrix}$$

Note that the matrix Z loads detrended output (y_t) and lagged detrended output (y_{t-1}).

The likelihood is computed with the Kalman filter and then combined with a prior distribution for the parameters to obtain the posterior. As a first step, a block algorithm is used to find the posterior mode, while a Metropolis-Hastings algorithm is used to draw from the posterior distribution.

Draws from the posterior are obtained using a standard Metropolis-Hastings algorithm initialized around the posterior mode. When working with models whose posterior distribution is very complicated in shape it is very important to find the posterior mode. Here are the key steps of the Metropolis-Hastings algorithm:

- Step 1: Draw a new set of parameters from the proposal distribution: $\vartheta \sim N(\theta_{n-1}, c\bar{\Sigma})$
- Step 2: Compute $\alpha(\theta^m; \vartheta) = \min\{p(\vartheta)/p(\theta^{m-1}), 1\}$ where $p(\theta)$ is the posterior evaluated at θ .
- Step 3: Accept the new parameter and set $\theta^m = \vartheta$ if $u < \alpha(\theta^m; \vartheta)$ where $u \sim U([0, 1])$, otherwise set $\theta^m = \theta^{m-1}$
- Step 4: If $m \leq n^{sim}$, stop. Otherwise, go back to step 1

The matrix $\bar{\Sigma}$ corresponds to the inverse of the Hessian computed at the posterior mode $\bar{\theta}$. The parameter c is set to obtain an acceptance rate of around 30%. We use four chains of 540,000 draws each (1 every 200 draws is saved). Convergence is checked by using the Brooks-Gelman-Rubin potential reduction scale factor using within and between variances based on the four multiple chains used in the paper.

The only aspect of the estimation that it is not traditional is that the transition equation (A27) varies over time. However, given that we estimate the model fixing the regime sequence, we can easily modify the standard Kalman filter to handle this change. Specifically, the modified Kalman filter is described as follows.

Given a sequence of regimes $\xi^T = \xi_1 \dots \xi_T$, the Kalman filter involves the following steps for each $t = 1 \dots T$:

1. Prediction:

$$S_{t|t-1} = C \left(\theta_{\xi_t}^c, \theta_{\xi_t} \right) + T(\theta_{\xi_t}) S_{t-1|t-1} \quad (\text{A29})$$

$$P_{t|t-1} = T(\theta_{\xi_t}) P_{t-1|t-1} T(\theta_{\xi_t})' + R(\theta_{\xi_t}) Q^2 R(\theta_{\xi_t})' \quad (\text{A30})$$

$$\eta_{t|t-1} = X_t - X_{t|t-1} = X_t - D - Z * S_{t|t-1} \quad (\text{A31})$$

$$f_{t|t-1} = Z P_{t|t-1} Z' + U^2 \quad (\text{A32})$$

2. Updating

$$S_{t|t} = S_{t|t-1} + K_t \eta_{t|t-1} \quad (\text{A33})$$

$$P_{t|t} = P_{t|t-1} - K_t Z P_{t|t-1} \quad (\text{A34})$$

where $K_t = P_{t|t-1} Z' f_{t|t-1}^{-1}$ is the Kalman gain.

The log-likelihood $\ln L$ is then obtained as:

$$\ln L = -.5 \sum_{t=1}^T \ln (2\pi f_{t|t-1}) - .5 \sum_{t=1}^T \eta_{t|t-1}' f_{t|t-1}^{-1} \eta_{t|t-1}.$$

Details about the solution. The matrices used to write the model in state space form are described below.

Equations:

$$\begin{aligned} y_t &= \delta y_{t-1} - \sigma [i_t - \phi \pi_t - (1 - \phi) \bar{\pi}_t - r] + d_t \\ \pi_t &= \bar{\pi}_t + \frac{\kappa}{1 - \beta \phi} [y_{t-1} - y_{t-1}^*] \end{aligned} \quad (\text{A35})$$

$$i_t - \left(r + \pi_{\xi_t}^T \right) = (1 - \rho_{i,\xi_t}) \left[\psi_{\bar{\pi},\xi_t} \left(\bar{\pi}_t - \pi_{\xi_t}^T \right) + \psi_{\pi,\xi_t} \left(\pi_t - \pi_{\xi_t}^T \right) + \psi_{\Delta y,\xi_t} (y_t - y_{t-1}) \right] \quad (\text{A36})$$

$$+ \rho_{i,\xi_t} \left[i_{t-1} - \left(r + \pi_{\xi_t}^T \right) \right] + \sigma_i \varepsilon_{i,t} \quad (\text{A37})$$

$$y_t^* = \rho_{y^*} y_{t-1}^* + \sigma_{y^*} \varepsilon_{y^*,t} \quad (\text{A38})$$

$$\bar{\pi}_t = [1 - \gamma^T] [\bar{\pi}_{t-1} + \gamma (1 - \phi)^{-1} (\pi_t - \phi \pi_{t-1} - (1 - \phi) \bar{\pi}_{t-1})] + \gamma^T \pi_{\xi_t}^T + \sigma_{\bar{\pi}} \varepsilon_{\bar{\pi},t} \quad (\text{A39})$$

$$d_t = \rho_d d_{t-1} + \sigma_d \varepsilon_{d,t} \quad (\text{A40})$$

We get:

$$\begin{aligned}
y_t &= \delta y_{t-1} - \sigma i_t + \sigma \phi \pi_t + \sigma(1 - \phi)\bar{\pi}_t + \sigma r + d_t \\
\pi_t &= \bar{\pi}_t + \frac{\kappa}{1 - \beta\phi} y_{t-1} - \frac{\kappa}{1 - \beta\phi} y_{t-1}^* \\
i_t - (r + \pi_{\xi_t}^T) &= (1 - \rho_{i,\xi_t})\psi_{\bar{\pi},\xi_t}\bar{\pi}_t - (1 - \rho_{i,\xi_t})\psi_{\bar{\pi},\xi_t}\pi_{\xi_t}^T \\
&\quad + (1 - \rho_{i,\xi_t})\psi_{\pi,\xi_t}\pi_t - (1 - \rho_{i,\xi_t})\psi_{\pi,\xi_t}\pi_{\xi_t}^T \\
&\quad + (1 - \rho_{i,\xi_t})\psi_{\Delta y,\xi_t}y_t - (1 - \rho_{i,\xi_t})\psi_{\Delta y,\xi_t}y_{t-1} \\
&\quad + \rho_{i,\xi_t}i_{t-1} - \rho_{i,\xi_t}(r + \pi_{\xi_t}^T) \\
&\quad + \sigma_i \epsilon_{i,t} \\
y_t^* &= \rho_y^* y_{t-1}^* + \sigma_{y^*} \epsilon_{y^*,t} \\
\bar{\pi}_t &= (1 - \gamma^T)\bar{\pi}_{t-1} + (1 - \gamma^T)\gamma(1 - \phi)^{-1}\pi_t \\
&\quad - (1 - \gamma^T)\gamma(1 - \phi)^{-1}\phi\pi_{t-1} \\
&\quad - (1 - \gamma^T)\gamma(1 - \phi)^{-1}(1 - \phi)\bar{\pi}_{t-1} \\
&\quad + \gamma^T \pi_{\xi_t}^T + \sigma_{\bar{\pi}} \epsilon_{\bar{\pi},t} \\
d_t &= \rho_d d_{t-1} + \sigma_d \epsilon_{d,t}
\end{aligned}$$

Equations with state variables at t on the LHS, everything else on the RHS, and re-ordered to match the state variable vector:

$$\begin{aligned}
y_t + \sigma i_t - \sigma \phi \pi_t - \sigma(1 - \phi)\bar{\pi}_t - d_t &= \delta y_{t-1} + \sigma r \\
y_t^* &= \rho_y^* y_{t-1}^* + \sigma_{y^*} \epsilon_{y^*,t} \\
\pi_t - \bar{\pi}_t &= + \frac{\kappa}{1 - \beta\phi} y_{t-1} - \frac{\kappa}{1 - \beta\phi} y_{t-1}^* \\
i_t - (1 - \rho_{i,\xi_t})\psi_{\bar{\pi},\xi_t}\bar{\pi}_t - (1 - \rho_{i,\xi_t})\psi_{\pi,\xi_t}\pi_t - (1 - \rho_{i,\xi_t})\psi_{\Delta y,\xi_t}y_t &= -(1 - \rho_{i,\xi_t})\psi_{\bar{\pi},\xi_t}\pi_{\xi_t}^T \\
&\quad - (1 - \rho_{i,\xi_t})\psi_{\pi,\xi_t}\pi_{\xi_t}^T \\
&\quad - (1 - \rho_{i,\xi_t})\psi_{\Delta y,\xi_t}y_{t-1} \\
&\quad + \rho_{i,\xi_t}i_{t-1} - \rho_{i,\xi_t}(r + \pi_{\xi_t}^T) \\
&\quad + \sigma_i \epsilon_{i,t} + (r + \pi_{\xi_t}^T) \\
\bar{\pi}_t - (1 - \gamma^T)\gamma(1 - \phi)^{-1}\pi_t &= (1 - \gamma^T)\bar{\pi}_{t-1} \\
&\quad - (1 - \gamma^T)\gamma(1 - \phi)^{-1}\phi\pi_{t-1} \\
&\quad - (1 - \gamma^T)\gamma(1 - \phi)^{-1}(1 - \phi)\bar{\pi}_{t-1} \\
&\quad + \gamma^T \pi_{\xi_t}^T + \sigma_{\bar{\pi}} \epsilon_{\bar{\pi},t} \\
d_t &= \rho_d d_{t-1} + \sigma_d \epsilon_{d,t}
\end{aligned}$$

Goal: matrix form with $\Gamma_0 S_t = \Gamma_C + \Gamma_1 S_{t-1} + \Psi Q \epsilon_t$. Γ_0 and Γ_1 are 6×6 matrices. Γ_C is 6×1 . Ψ is 6×4 .

State variables: $S_t = [y_t, y_t^*, \pi_t, i_t, \bar{\pi}_t, d_t]'$.

Stochastic variables: $Q = \text{diag}(\sigma_i, \sigma_{y^*}, \sigma_{\bar{\pi}}, \sigma_d)$.

First, Γ_0 , which is for the time t state variables on the LHS. Empty cells are zero.

$$\Gamma_0 = \begin{bmatrix} & y_t & y_t^* & \pi_t & i_t & \bar{\pi}_t & d_t \\ y_t & 1 & & -\sigma\phi & \sigma & -\sigma(1-\phi) & -1 \\ y_t^* & & 1 & & & & \\ \pi_t & & & 1 & & -1 & \\ i_t & -(1-\rho_{i,\xi_t})\psi_{\Delta y,\xi_t} & & -(1-\rho_{i,\xi_t})\psi_{\pi,\xi_t} & 1 & -(1-\rho_{i,\xi_t})\psi_{\bar{\pi},\xi_t} & \\ \bar{\pi}_t & & & -(1-\gamma^T)\gamma(1-\phi)^{-1} & & 1 & \\ d_t & & & & & & 1 \end{bmatrix}$$

Next, Γ_1 , which is for the time $t-1$ state variables on the RHS. Empty cells are zero.

$$\Gamma_1 = \begin{bmatrix} & y_{t-1} & y_{t-1}^* & \pi_{t-1} & i_{t-1} & \bar{\pi}_{t-1} & d_{t-1} \\ y_t & \delta & & & & & \\ y_t^* & & \rho_y^* & & & & \\ \pi_t & \frac{\kappa}{1-\beta\phi} & -\frac{\kappa}{1-\beta\phi} & & & & \\ i_t & -(1-\rho_{i,\xi_t})\psi_{\Delta y,\xi_t} & & & \rho_{i,\xi_t} & & \\ \bar{\pi}_t & & & -(1-\gamma^T)\gamma(1-\phi)^{-1}\phi & & (1-\gamma^T)(1-\gamma) & \\ d_t & & & & & & \rho_d \end{bmatrix}$$

Ψ inserts the stochastic processes into each of the equations. Empty cells are zero.

$$\Psi = \begin{bmatrix} & \epsilon_{i,t} & \epsilon_{y^*,t} & \epsilon_{\bar{\pi},t} & \epsilon_{d,t} \\ y_t & & & & \\ y_t^* & & \sigma_{y^*} & & \\ \pi_t & & & & \\ i_t & \sigma_i & & & \\ \bar{\pi}_t & & & \sigma_{\bar{\pi}} & \\ d_t & & & & \sigma_d \end{bmatrix}$$

Finally, Γ_C collects all of the leftover constant terms on the RHS.

$$\Gamma_C = \left[\begin{array}{c|c} y_t & \sigma r \\ \hline y_t^* & 0 \\ \hline \pi_t & 0 \\ \hline i_t & (1 - \rho_{i,\xi_t})[r + \pi_{\xi_t}^T (1 - \psi_{\pi,\xi_t} - \psi_{\bar{\pi},\xi_t})] \\ \hline \bar{\pi}_t & \gamma^T \pi_{\xi_t}^T \\ \hline d_t & 0 \end{array} \right]$$

5.0.5 ZLB Robustness checks for MS-DSGE

In this appendix, we conduct two robustness checks to verify that our results are not distorted by the time spent at the zero lower bound (ZLB) in the aftermath of the financial crisis. First, we re-estimate our MS-DSGE model using the Wu-Xia (Wu and Xia (2016)) shadow rate. Second, we use the one-year Treasury yield instead of the federal funds rate in our estimation. The shadow rate is downloaded from Professor Wu's website, while the one-year Treasury yield is downloaded from FRED. The figures presented in this appendix show that the main result of the paper are not affected by using these alternative measures for the interest rate.

In the Wu and Xia model, the short-term interest rate is the maximum of the shadow federal funds rate and the lower bound on interest rates. Wu and Xia set this lower bound to 25 basis points because that was the rate paid on both required and excess reserve balances during the December 16, 2008, to December 15, 2015, period when the Federal Open Market Committee (FOMC) set the target range for the federal funds rate at 0 to 25 basis points. On December 16, 2015, the FOMC increased the rate paid on reserve balances to 50 basis points and the target range for the federal funds rate to 25 to 50 basis points. Once the lower bound is not binding anymore, the shadow rate coincides with the actual FFR.

The results of Wu and Xia are based on a multivariate version of the shadow rate term structure model (SRTSM) introduced by Black (1995). In the SRTSM, the observed short term rate is the maximum between a lower bound and the shadow rate. The shadow rate, in turn, is an affine function of a vector of state variables that follow a VAR process. Absent the lower bound, the model would be fully linear. Thus, the lower bound introduces a non-linearity in the mapping from the factors to the observed short term interest rate. The key idea behind the model and the work of Wu and Xia is that by observing the behavior of forward rates at different maturities, the researcher can back out a measure of the shadow short term interest rate. In other words, forward rates reflect the overall monetary policy stance and can be used to recover the implicit behavior of the shadow interest rate.

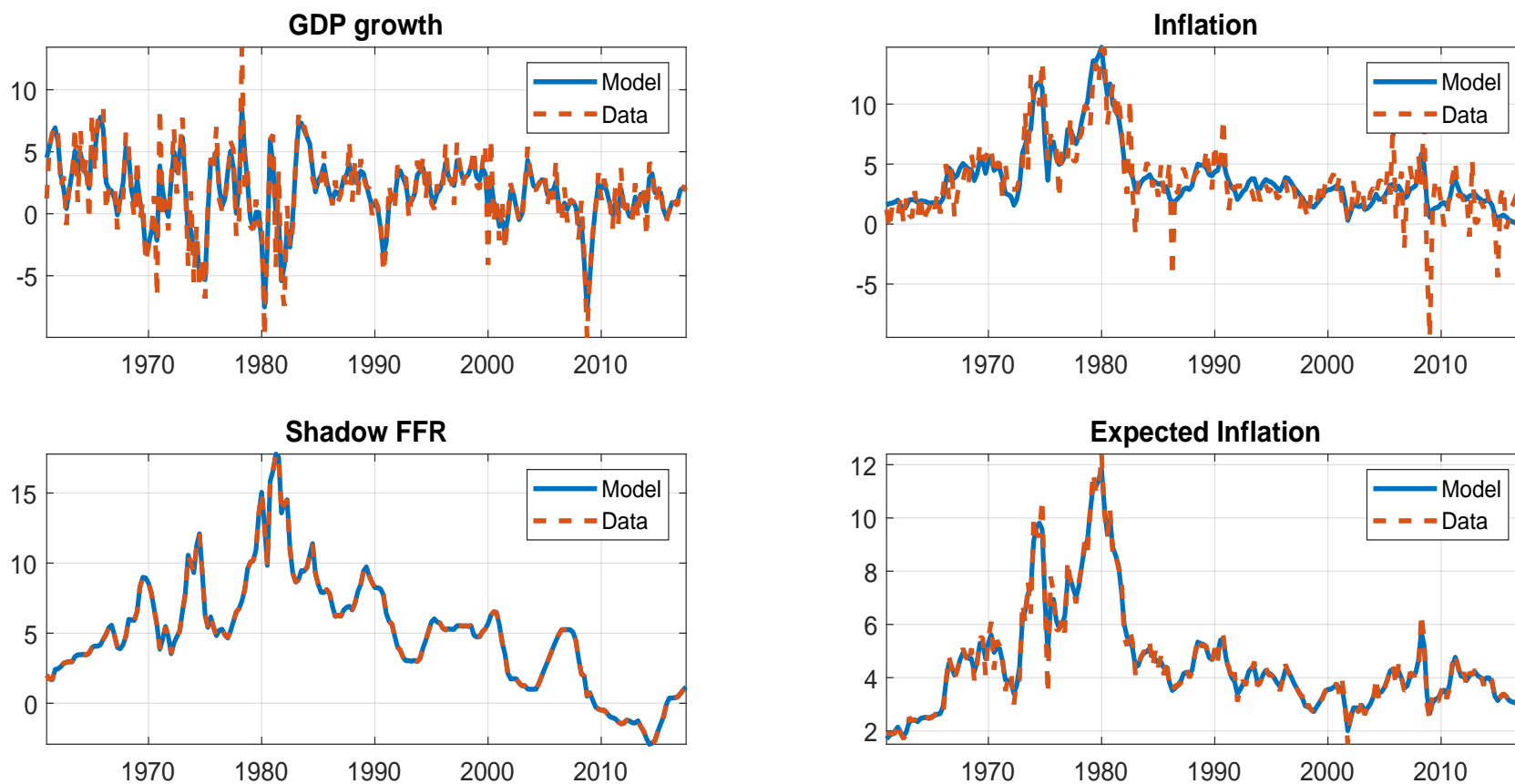


Figure A.1: Shadow rate estimates of the MS-DSGE model. The figure reports the model implied series and the corresponding observed series. Expected inflation comes from the Michigan Survey of Consumers. The difference is due to observation errors. The sample spans 1961:Q1 - 2017:Q3. Results are based on estimates obtained replacing the FFR with the Wu-Xia shadow rate when the ZLB is binding.

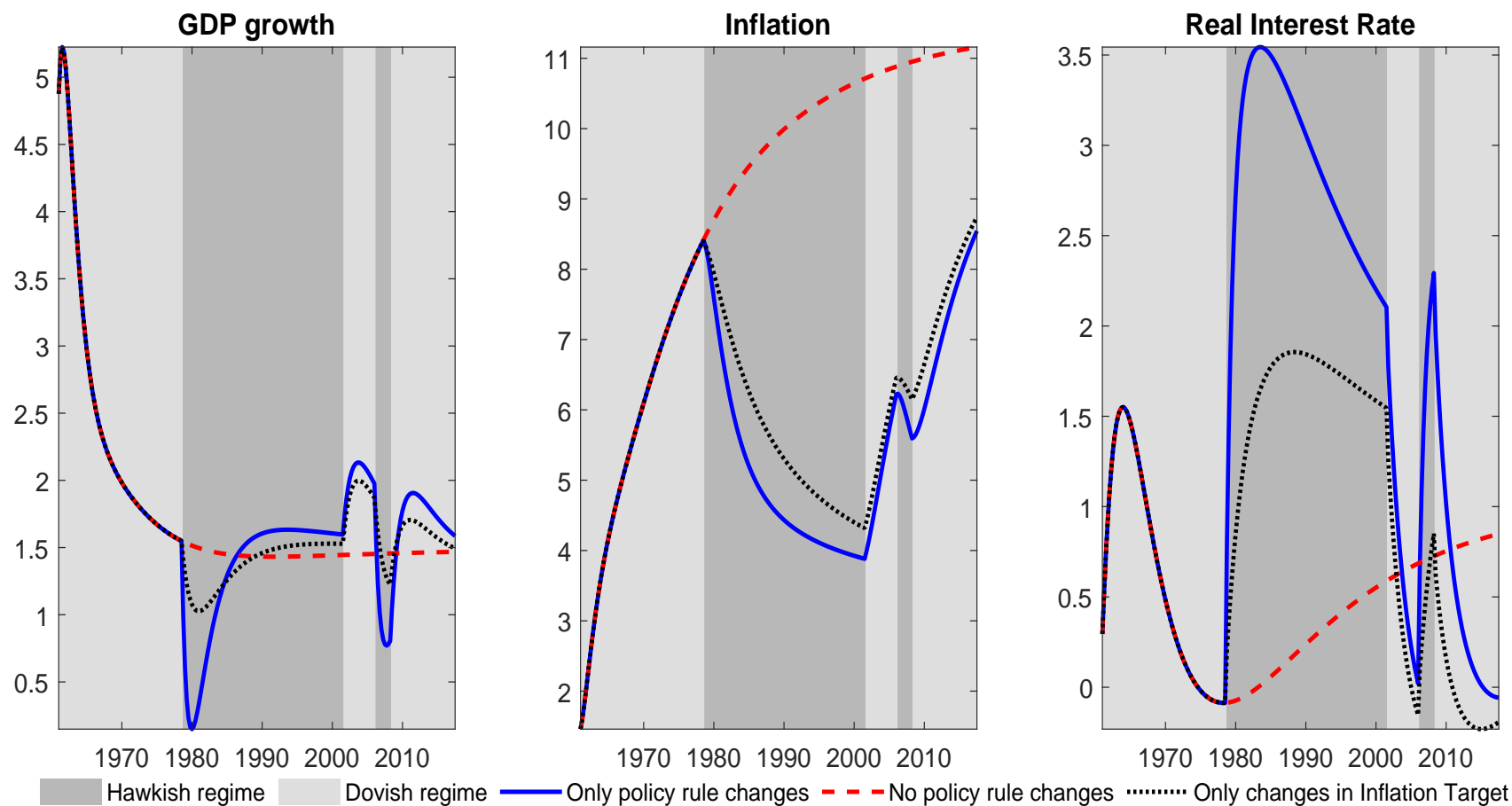


Figure A.2: Shadow rate estimates of the MS-DSGE model. The blue line corresponds to the fluctuations generated by changes in both the target and the slope coefficients. The orange line assumes that monetary policy starts under the Dovish regime and no regime change occurs. Finally, the black dotted line assumes that changes in the target occurred, but that the slope coefficients in the Taylor rule coefficients always remain as in the Dovish-high target regime. Results are based on estimates obtained replacing the FFR with the Wu-Xia shadow rate when the ZLB is binding.

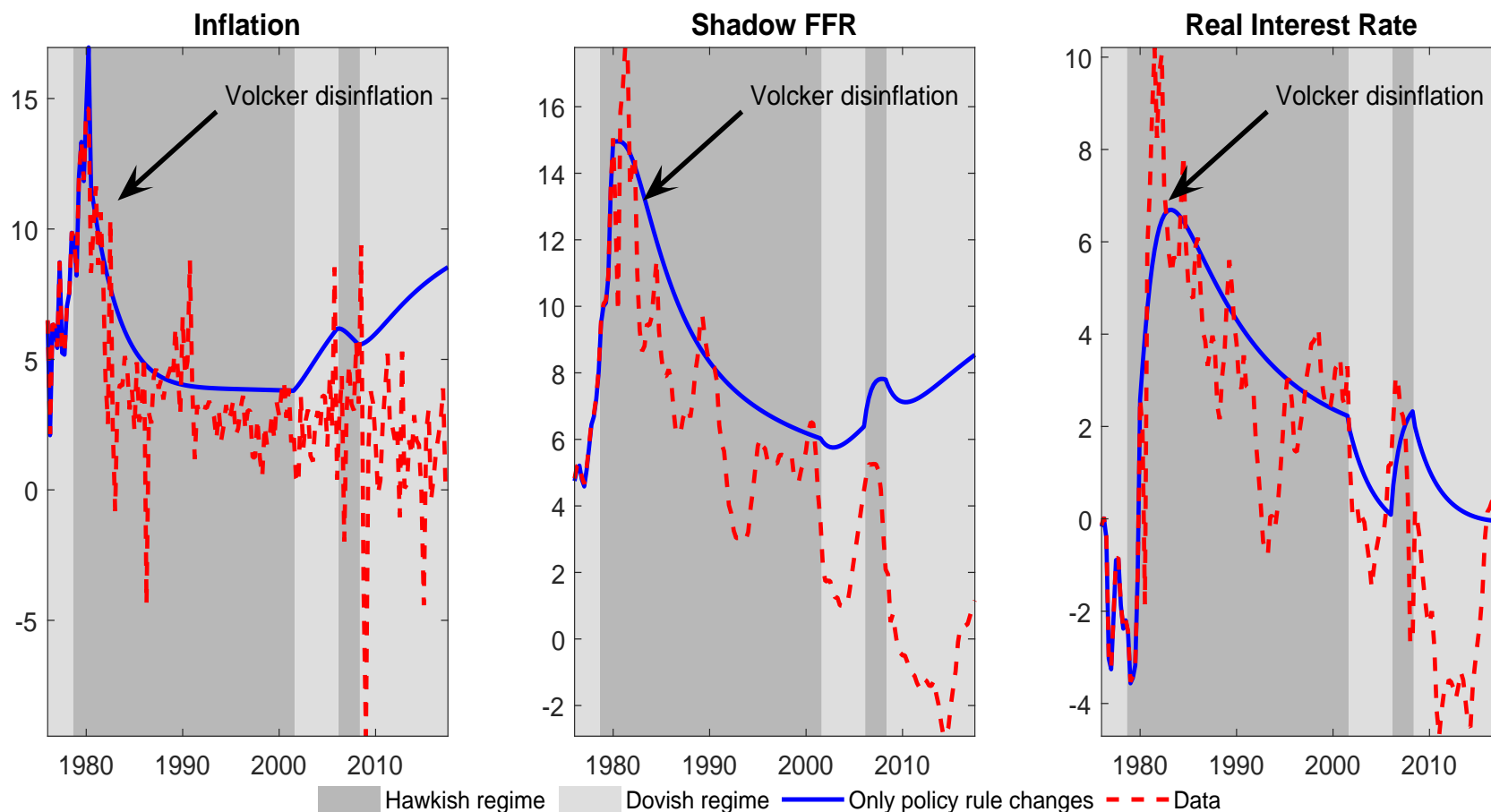


Figure A.3: Shadow rate estimates of the MS-DSGE model. The Volcker disinflation. We start the economy as it was in 1980:Q1 and remove all Gaussian shocks that occurred after that period, but keep the estimated regime sequence. The dashed line corresponds to the data. The real interest rate is computed as the difference between the FFR and expected inflation. Expected inflation is obtained based on the model solution. Results are based on estimates obtained replacing the FFR with the Wu-Xia shadow rate when the ZLB is binding.

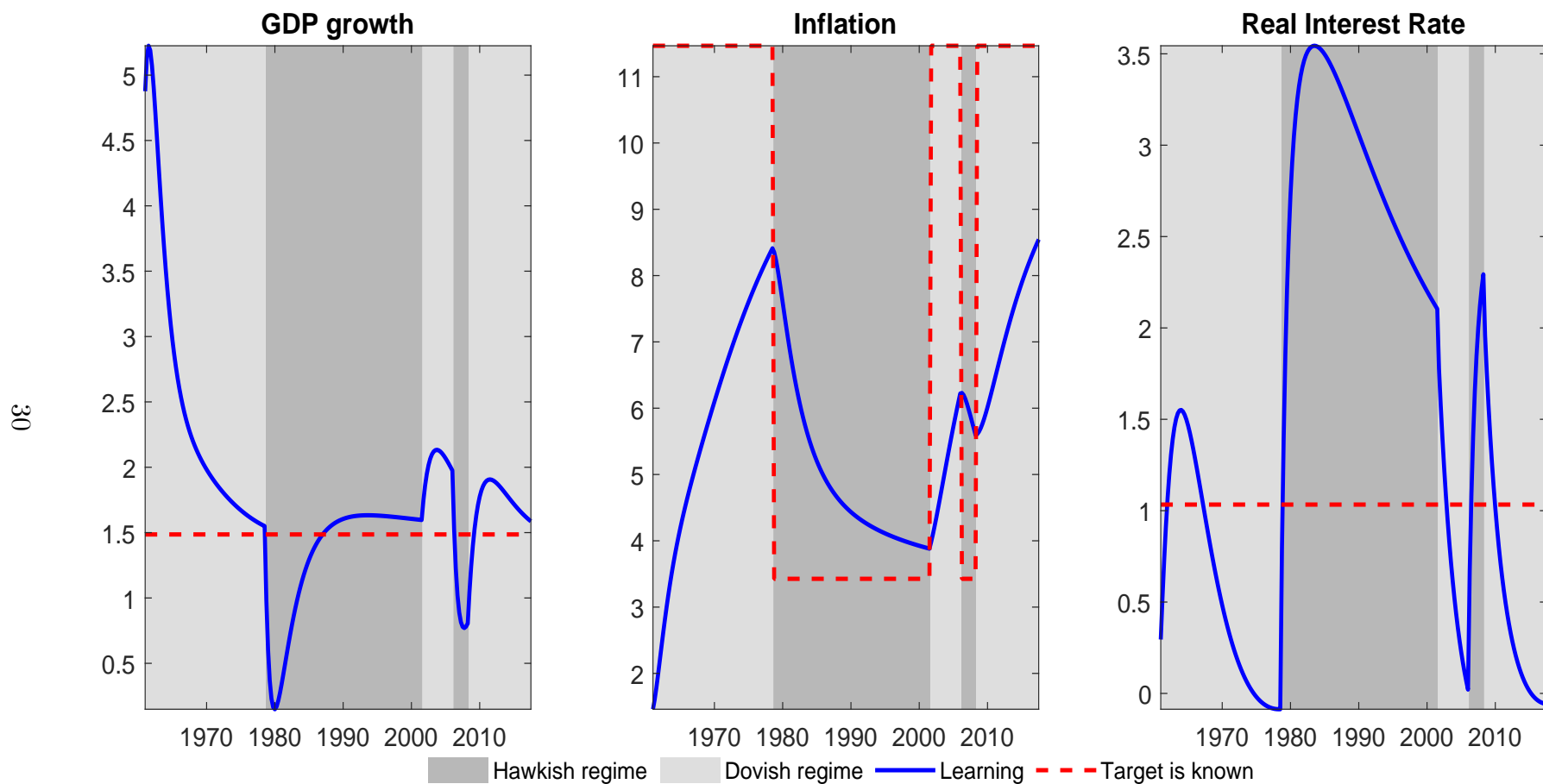


Figure A.4: Shadow rate estimates of the MS-DSGE model. Perfect information about the target. The blue solid line shows estimated fluctuations generated only by changes in the policy rule (inflation target and slope coefficients) when agents learn about trend inflation. The orange dashed line shows a counterfactual in which the policy rule shifts but agents observe the inflation target. Dovish regime has a high target π^T and low activism against deviations from the target π^T . Hawkish regime has a low π^T and high activism against deviations from π^T . The sample spans 1961:Q1 - 2017:Q3. Results are based on estimates obtained replacing the FFR with the Wu-Xia shadow rate when the ZLB is binding.

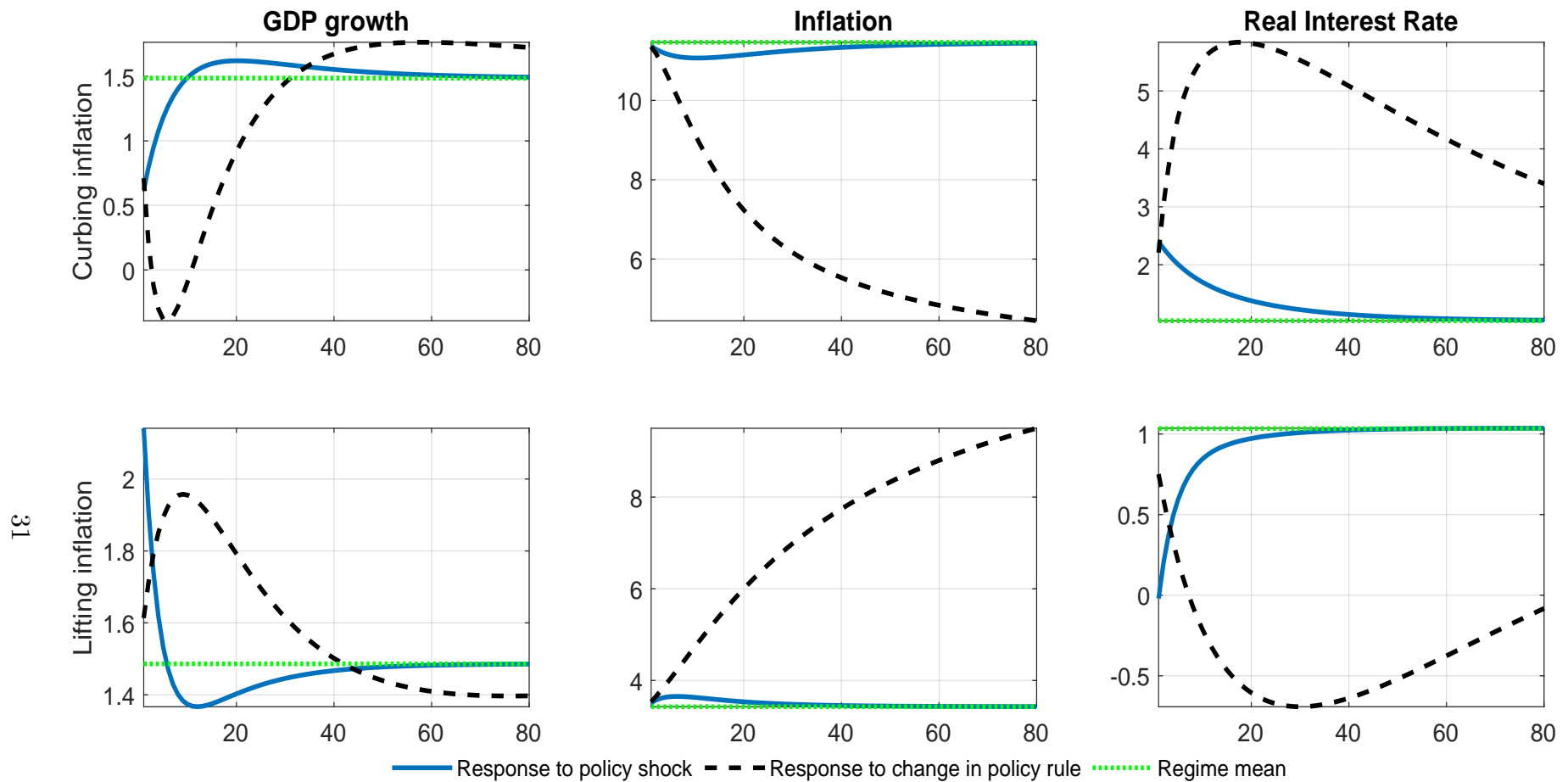


Figure A.5: Shadow rate estimates of the MS-DSGE model. Top row: Curbing inflation. The economy is initially in the Dovish regime. The blue solid line presents the evolution of the macro variables in response to a two standard deviation contractionary monetary policy shock and no regime change. The black dashed line presents the evolution of the macro variables in response to a regime change from the Dovish regime to the Hawkish regime. **Lower row: Lifting inflation.** The economy is initially in the Hawkish regime. The blue solid line presents the evolution of the macro variables in response to a two standard deviation expansionary monetary policy shock and no regime change. The black dashed line presents the evolution of the macro variables in response to a regime change from the Hawkish regime to the Dovish regime. Results are based on estimates obtained replacing the FFR with the Wu-Xia shadow rate when the ZLB is binding.

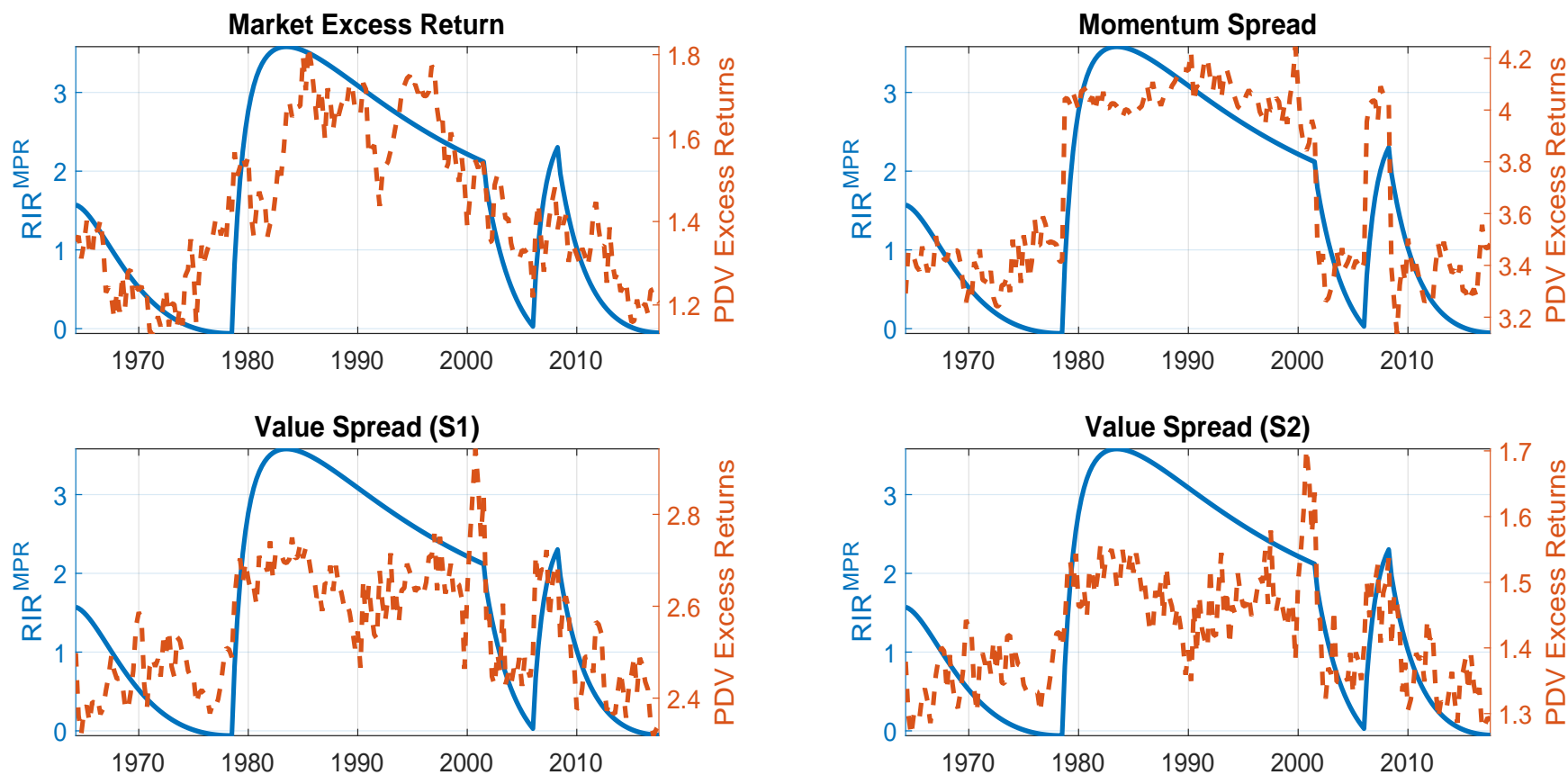


Figure A.6: Shadow rate estimates of the MS-DSGE model. Excess returns and policy rule changes. The figure reports the time series of the present discounted value of expected excess returns for different portfolios (dashed line, right axis) together with fluctuations of the real interest rate due to changes in the monetary policy rule (solid line, left axis). Results are based on estimates obtained replacing the FFR with the Wu-Xia shadow rate when the ZLB is binding.

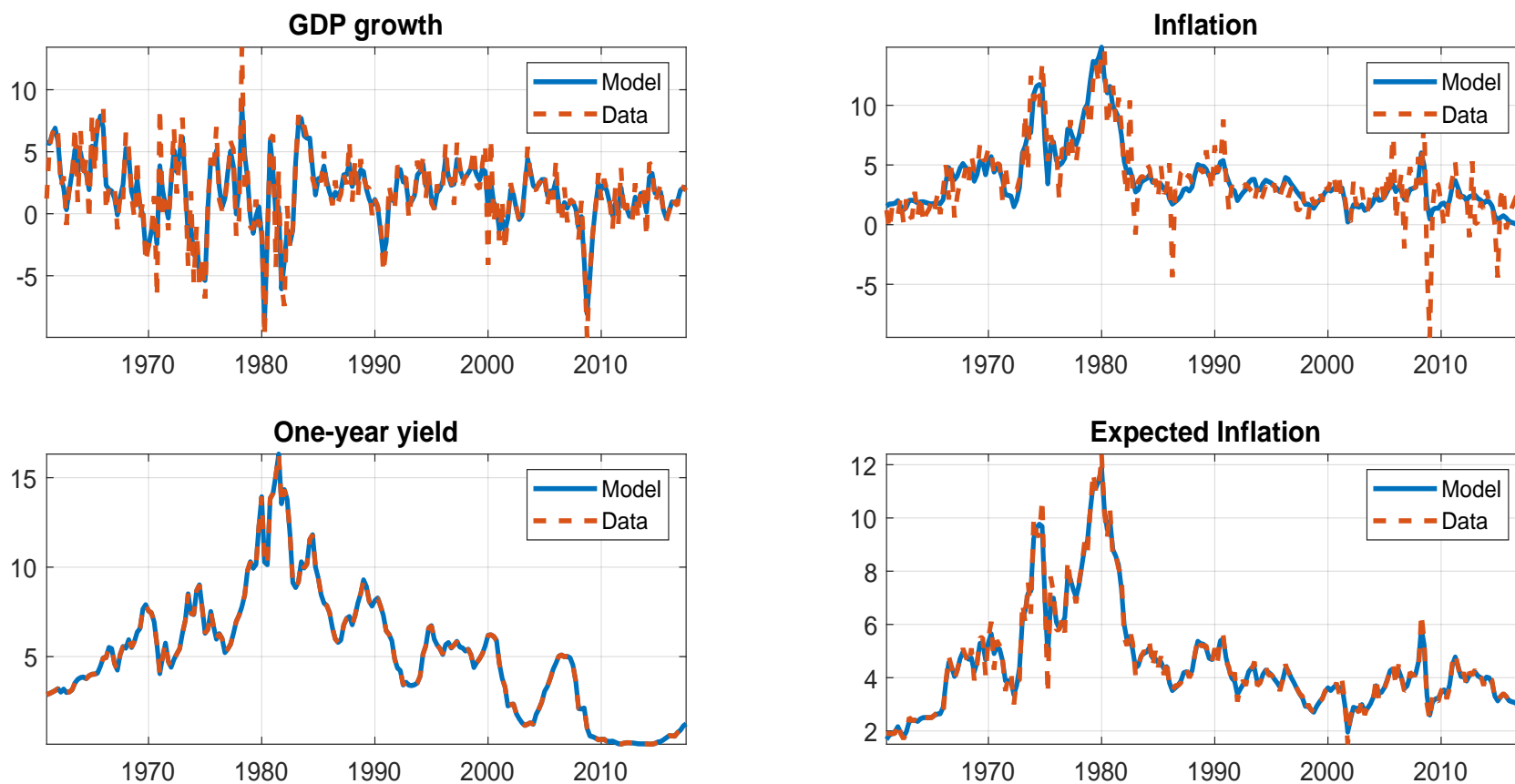


Figure A.7: One-year yield estimates of the MS-DSGE model. The figure reports the model implied series and the corresponding observed series. Expected inflation comes from the Michigan Survey of Consumers. The difference is due to observation errors. The sample spans 1961:Q1 - 2017:Q3. Results are based on estimates obtained using the one-year treasury yield instead of the FFR.

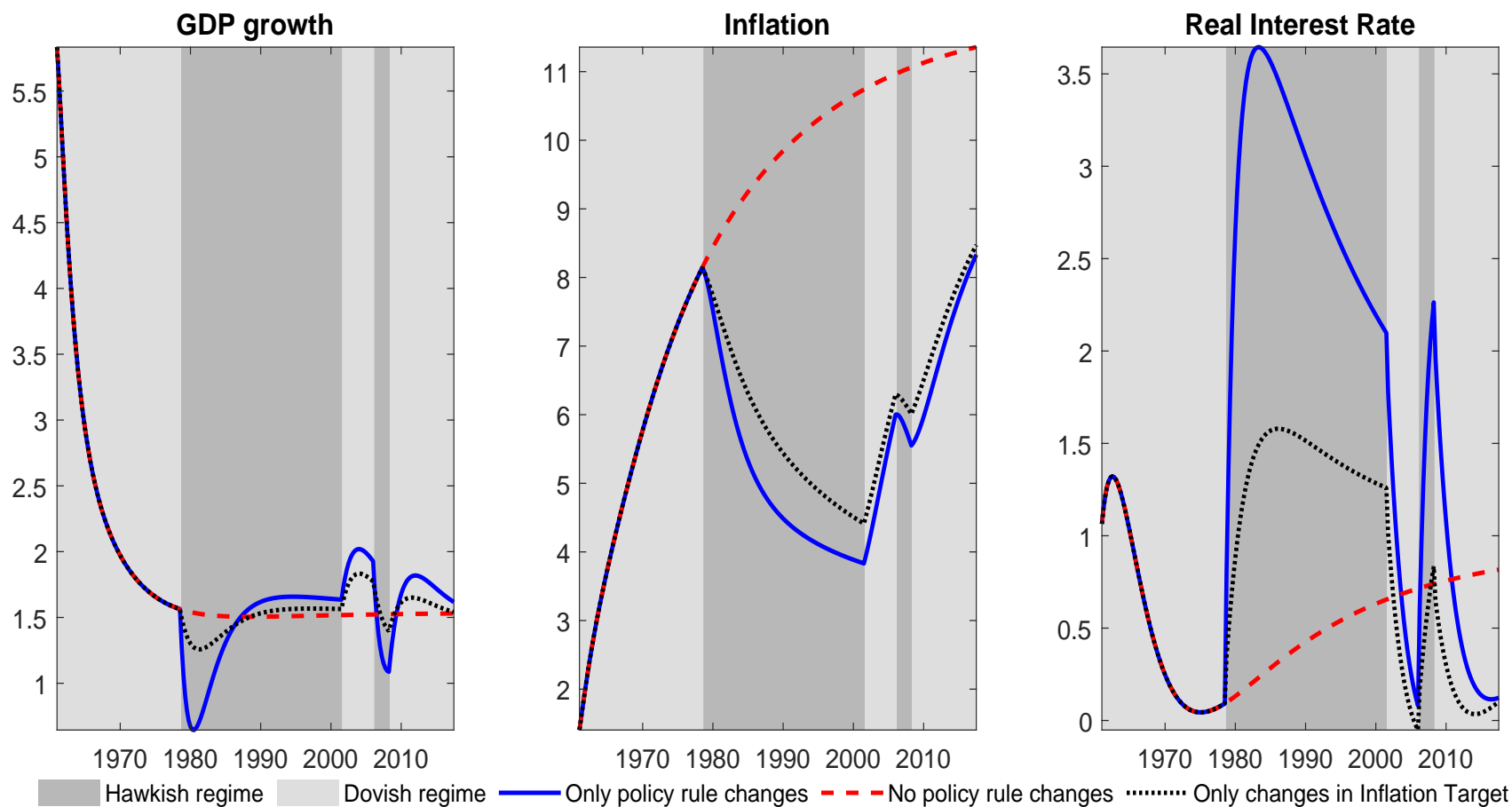


Figure A.8: One-year yield estimates of the MS-DSGE model. The blue line corresponds to the fluctuations generated by changes in both the target and the slope coefficients. The orange line assumes that monetary policy starts under the Dovish regime and no regime change occurs. Finally, the black dotted line assumes that changes in the target occurred, but that the slope coefficients in the Taylor rule coefficients always remain as in the Dovish-high target regime. Results are based on estimates obtained using the one-year treasury yield instead of the FFR.

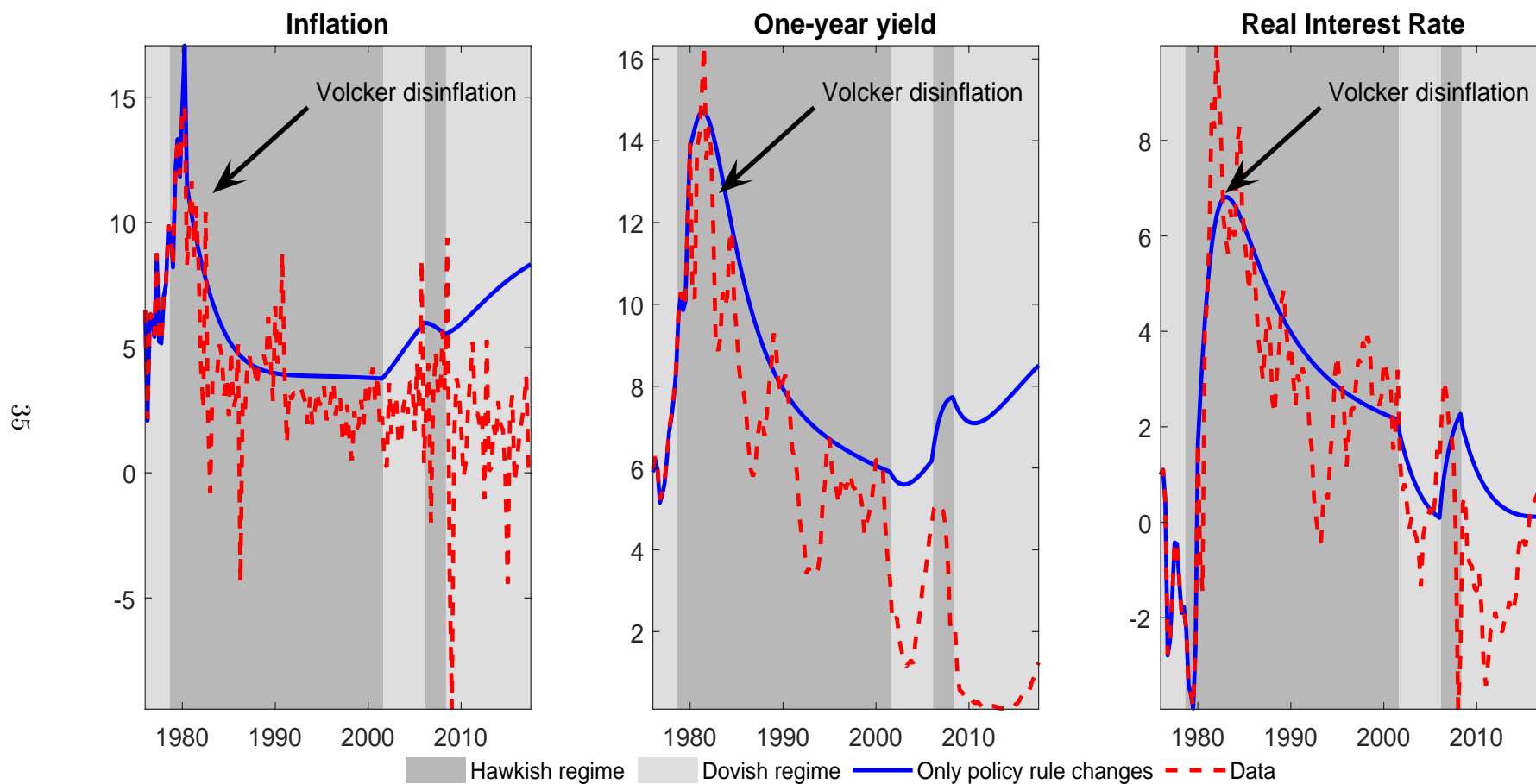


Figure A.9: One-year yield estimates of the MS-DSGE model. The Volcker disinflation. We start the economy as it was in 1980:Q1 and remove all Gaussian shocks that occurred after that period, but keep the estimated regime sequence. The dashed line corresponds to the data. The real interest rate is computed as the difference between the FFR and expected inflation. Expected inflation is obtained based on the model solution. Results are based on estimates obtained using the one-year treasury yield instead of the FFR.

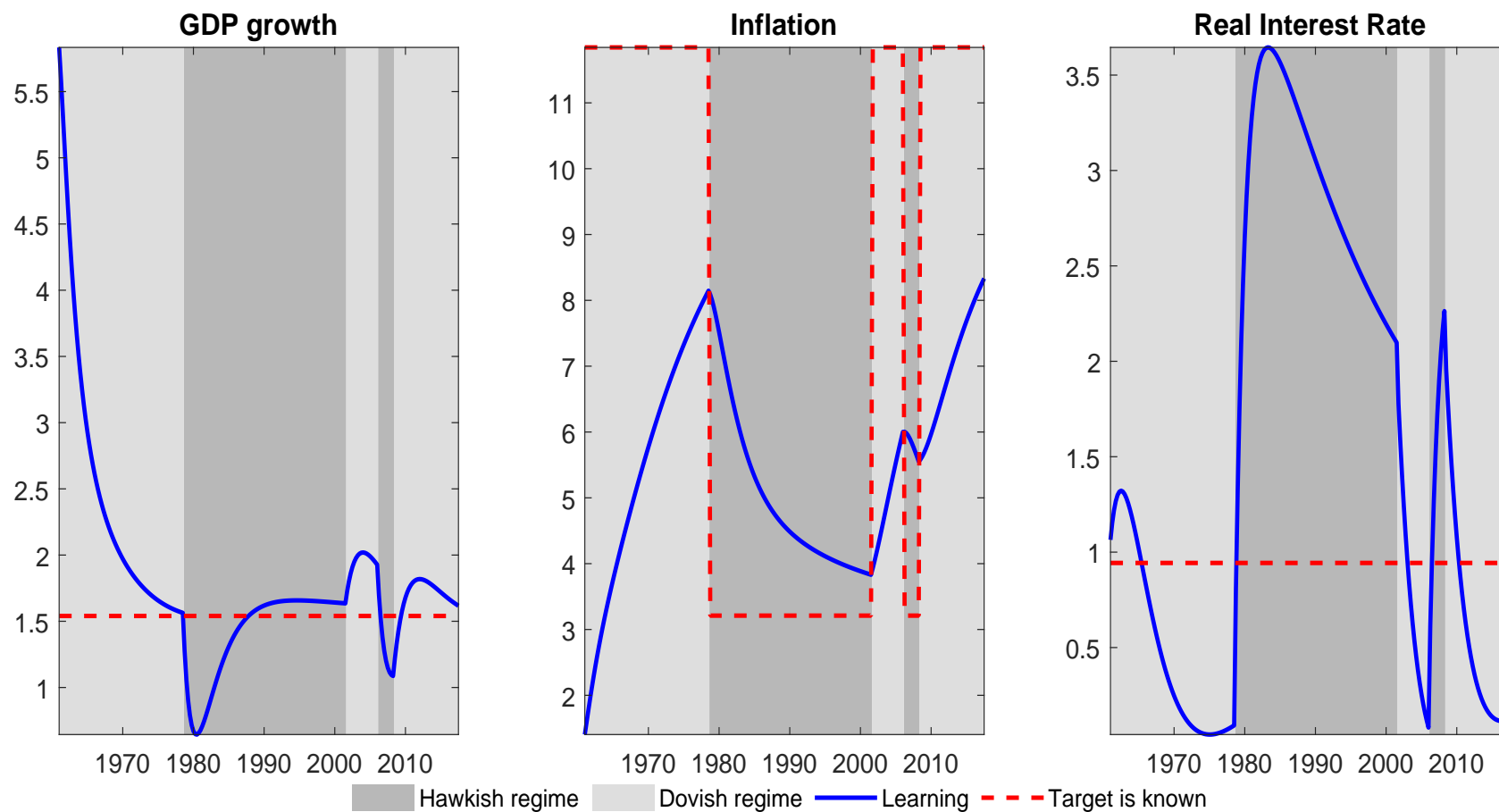


Figure A.10: One-year yield estimates of the MS-DSGE model. Perfect information about the target. The blue solid line shows estimated fluctuations generated only by changes in the policy rule (inflation target and slope coefficients) when agents learn about trend inflation. The orange dashed line shows a counterfactual in which the policy rule shifts but agents observe the inflation target. Dovish regime has a high target π^T and low activism against deviations from the target π^T . Hawkish regime has a low π^T and high activism against deviations from π^T . The sample spans 1961:Q1 - 2017:Q3. Results are based on estimates obtained using the one-year treasury yield instead of the FFR.

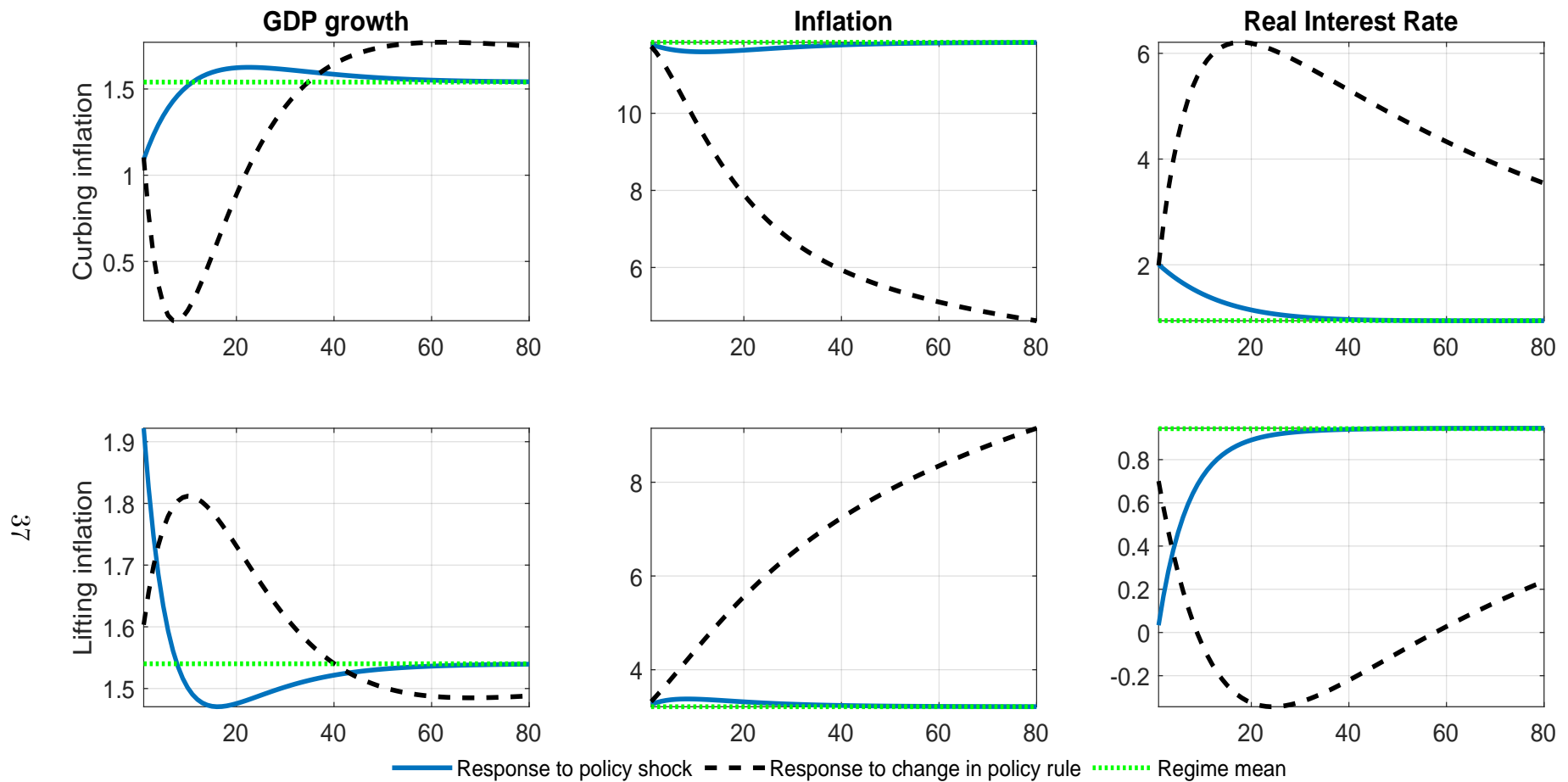


Figure A.11: One-year yield estimates of the MS-DSGE model. Top row: Curbing inflation. The economy is initially in the Dovish regime. The blue solid line presents the evolution of the macro variables in response to a two standard deviation contractionary monetary policy shock and no regime change. The black dashed line presents the evolution of the macro variables in response to a regime change from the Dovish regime to the Hawkish regime. **Lower row: Lifting inflation.** The economy is initially in the Hawkish regime. The blue solid line presents the evolution of the macro variables in response to a two standard deviation expansionary monetary policy shock and no regime change. The black dashed line presents the evolution of the macro variables in response to a regime change from the Hawkish regime to the Dovish regime. Results are based on estimates obtained using the one-year treasury yield instead of the FFR.

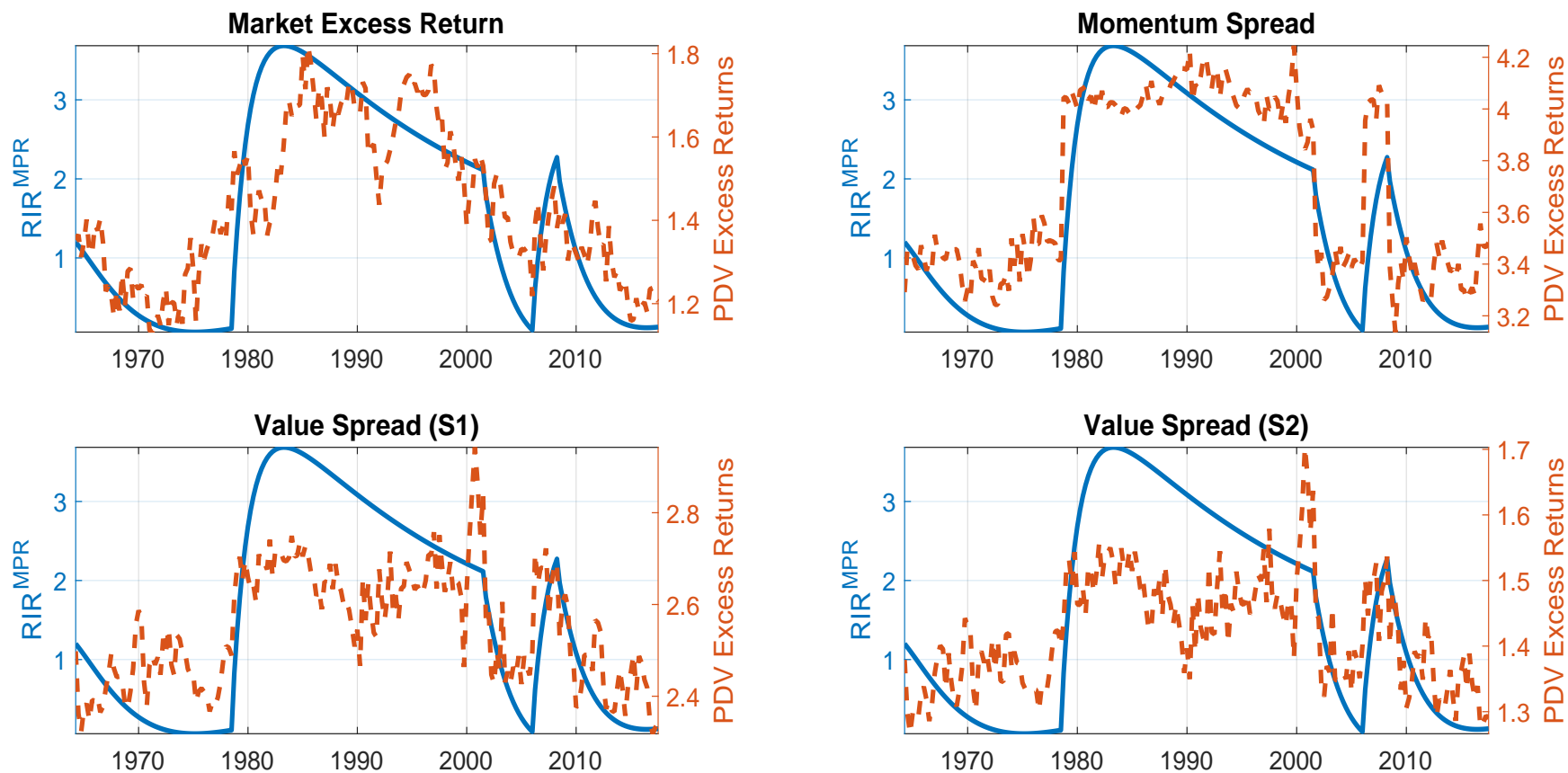


Figure A.12: One-year yield estimates of the MS-DSGE model. Excess returns and policy rule changes. The figure reports the time series of the present discounted value of expected excess returns for different portfolios (dashed line, right axis) together with fluctuations of the real interest rate due to changes in the monetary policy rule (solid line, left axis). Results are based on estimates obtained using the one-year treasury yield instead of the FFR.

Conditional Expectations and Economic Uncertainty

In this appendix we explain how expectations and economic uncertainty are computed for variables in the MS-VAR. More details can be found in Bianchi (2016). Consider the following first-order MS-VAR:

$$Z_t = c_{\xi_t} + A_{\xi_t} Z_{t-1} + V_{\xi_t} \varepsilon_t, \varepsilon_t \sim N(0, I) \quad (\text{A41})$$

and suppose that we are interested in $\mathbb{E}_0(Z_t) = \mathbb{E}(Z_t | \mathbb{I}_0)$ with \mathbb{I}_0 being the information set available at time 0. Note that the first-order VAR is not restrictive because any VAR with $l > 1$ lags can be rewritten as above by using the first-order companion form, and the methods below applied to the companion form.

Let n be the number of variables in the VAR of the previous Appendix section. Let m be the number of Markov-switching states. Define the $mn \times 1$ column vector q_t as:

$$q_t = [q_t^{1'}, \dots, q_t^{m'}]'$$

$mn \times 1$

where the individual $n \times 1$ vectors $q_t^i = \mathbb{E}_0(Z_t 1_{\xi_t=i}) \equiv \mathbb{E}(Z_t 1_{\xi_t=i} | \mathbb{I}_0)$ and $1_{\xi_t=i}$ is an indicator variable that is one when regime i is in place and zero otherwise. Note that:

$$q_t^i = \mathbb{E}_0(Z_t 1_{\xi_t=i}) = \mathbb{E}_0(Z_t | \xi_t = i) \pi_t^i$$

where $\pi_t^i = P(\xi_t = i) = P(\xi_t = i | \mathbb{I}_0)$. Therefore we can express $\mu_t = \mathbb{E}_0(Z_t)$ as:

$$\mu_t = \mathbb{E}_0(Z_t) = \sum_{i=1}^m q_t^i = w q_t$$

where the matrix $w = [I_n, \dots, I_n]$ is obtained placing side by side m n -dimensional identity matrices. Then the following proposition holds:

Proposition 1 *Consider a Markov-switching model whose law of motion can be described by (A41) and define $q_t^i = \mathbb{E}_0(Z_t 1_{\xi_t=i})$ for $i = 1 \dots m$. Then $q_t^j = c_j \pi_t^j + \sum_{i=1}^m A_j q_{t-1}^i h_{ji}$.*

It is then straightforward to compute expectations conditional on the information available at a particular point in time. Suppose we are interested in $\mu_{t+s|t} \equiv \mathbb{E}_t(Z_{t+s})$, i.e. the expected value for the vector Z_{t+s} conditional on the information set available at time t . If we define:

$$q_{t+s|t} = [q_{t+s|t}^{1'}, \dots, q_{t+s|t}^{m'}]'$$

where $q_{t+s|t}^i = \mathbb{E}_t(Z_{t+s} 1_{\xi_t=i}) = \mathbb{E}_t(Z_{t+s} | \xi_t = i) \pi_{t+s|t}^i$, where $\pi_{t+s|t}^i \equiv P(\xi_t = i | \mathbb{I}_t)$, we have

$$\mu_{t+s|t} = \mathbb{E}_t(Z_{t+s}) = w q_{t+s|t}, \quad (\text{A42})$$

where for $s \geq 1$, $q_{t+s|t}$ evolves as:

$$q_{t+s|t} = C \pi_{t+s|t} + \Omega q_{t+s-1|t} \quad (\text{A43})$$

$$\pi_{t+s|t} = \mathbf{H} \pi_{t+s-1|t} \quad (\text{A44})$$

with $\pi_{t+s|t} = [\pi_{t+s|t}^1, \dots, \pi_{t+s|t}^m]'$, $\Omega = \text{bdiag}(A_1, \dots, A_m)(\mathbf{H} \otimes I_n)$, and $C = \text{bdiag}(c_1, \dots, c_m)$, where e.g., c_1 is the $n \times 1$ vector of constants in regime 1, \otimes represents the Kronecker product and bdiag is a matrix operator that takes a sequence of matrices and use them to construct a block diagonal matrix.

Similar formulas hold for the second moments. Before proceeding, let us define the vectorization operator $\varphi(X)$ that takes the matrix X as an input and returns a column vector stacking the columns of the matrix X on top of one another. We will also make use of the following result: $\varphi(X_1 X_2 X_3) = (X_3' \otimes X_1) \varphi(X_2)$.

Define the vector $n^2 m \times 1$ column vector Q_t as:

$$Q_t = [Q_t^{1'}, \dots, Q_t^{m'}]'$$

where the $n^2 \times 1$ vector Q_t^i is given by $Q_t^i = \varphi[\mathbb{E}_0(Z_t Z_t' 1_{\xi_t=i})]$. This implies that we can compute the vectorized matrix of second moments $M_t = \varphi[\mathbb{E}_0(Z_t Z_t')]$ as:

$$M_t = \varphi[\mathbb{E}_0(Z_t Z_t')] = \sum_{i=1}^m Q_t^i = W Q_t$$

where the matrix $W = [I_{n^2}, \dots, I_{n^2}]$ is obtained placing side by side m n^2 -dimensional identity matrices. We can then state the following proposition:

Proposition 2 *Consider a Markov-switching model whose law of motion can be described by (A41) and define $Q_t^i = \varphi[\mathbb{E}_0(Z_t Z_t' 1_{\xi_t=i})]$, $q_t^i = \mathbb{E}_0[Z_t 1_{\xi_t=i}]$, and $\pi_t^i = P_0(\xi_t = i)$, for $i = 1 \dots m$. Then $Q_t^j = [\widehat{c}c_j + \widehat{V}V_j \varphi[I_k]] \pi_t^j + \sum_{i=1}^m [\widehat{A}A_j Q_{t-1}^i + \widehat{D}A C_j q_{t-1}^i] h_{ji}$, where $\widehat{c}c_j = (c_j \otimes c_j)$, $\widehat{V}V_j = (V_j \otimes V_j)$, $\widehat{A}A_j = (A_j \otimes A_j)$, and $\widehat{D}A C_j = (A_j \otimes c_j) + (c_j \otimes A_j)$.*

It is then straightforward to compute the evolution of second moments conditional on the information available at a particular point in time. Suppose we are interested in $\mathbb{E}_t(Z_{t+s} Z_{t+s}')$, i.e. the second moment of the vector Z_{t+s} conditional on the information available at time t . If we define:

$$Q_{t+s|t} = [Q_{t+s|t}^{1'}, \dots, Q_{t+s|t}^{m'}]'$$

where $Q_{t+s|t}^i = \varphi(\mathbb{E}_t(Z_{t+s} Z_{t+s}' 1_{\xi_t=i})) = \varphi(\mathbb{E}_t(Z_{t+s} Z_{t+s}' | \xi_t = i)) \pi_{t+s|t}^i$, we obtain $\varphi(\mathbb{E}_t(Z_{t+s} Z_{t+s}')) = W Q_{t+s|t}$. Using matrix algebra we obtain:

$$Q_{t+s|t} = \Xi Q_{t+s-1|t} + \widehat{D}A C q_{t+s-1|t} + \widehat{V}c \pi_{t+s|t} \quad (\text{A45})$$

$$q_{t+s|t} = C \pi_{t+s|t} + \Omega q_{t+s-1|t}, \quad \pi_{t+s|t} = \mathbf{H} \pi_{t+s-1|t}. \quad (\text{A46})$$

where

$$\begin{aligned} \Xi &= \text{bdiag}(\widehat{A}A_1, \dots, \widehat{A}A_m)(\mathbf{H} \otimes I_{n^2}), \quad \widehat{V}c = [\widehat{V}V + \widehat{c}c], \quad \widehat{c}c = \text{bdiag}(\widehat{c}c_1, \dots, \widehat{c}c_m), \\ \widehat{V}V &= \text{bdiag}(\widehat{V}V_1 \varphi[I_k], \dots, \widehat{V}V_m \varphi[I_k]), \quad \widehat{D}A C = \text{bdiag}(\widehat{D}A C_1, \dots, \widehat{D}A C_m)(\mathbf{H} \otimes I_n). \end{aligned}$$

With the first and second moments at hand, it is then possible to compute the variance s periods ahead conditional on the information available at time t :

$$\varphi [\mathbb{V}_t (Z_{t+s})] = M_{t+s|t} - \varphi [\mu_{t+s|t} \mu'_{t+s|t}], \quad (\text{A47})$$

where $M_{t+s|t} = \varphi (\mathbb{E}_t (Z_{t+s} Z'_{t+s})) = \sum_{i=1}^m Q_{t+s|t}^i = W Q_{t+s|t}$.

To report estimates of (A42) and (A47) we proceed as follows. Note that $\mu_{t+s|t} = \mathbb{E}_t (Z_{t+s}) = w q_{t+s|t}$ and $M_{t+s|t}$ depend only on $q_{t+s|t}$ and $Q_{t+s|t}$. Furthermore we can express (A43)-(A44) and (A45)-(A46) in a compact form as

$$\tilde{Q}_{t+s|t} = \tilde{\Xi}^s \tilde{Q}_{t|t} \text{ where } \tilde{\Xi} = \left[\begin{array}{c|cc} \Xi & \widehat{DAC} & \widehat{VcH} \\ \hline & \Omega & \frac{CH}{H} \end{array} \right], \quad (\text{A48})$$

where $\tilde{Q}_{t+s|t} = [Q'_{t+s|t}, q'_{t+s|t}, \pi'_{t+s|t}]'$. Armed with starting values $\tilde{Q}_{t|t} = [Q'_{t|t}, q'_{t|t}, \pi'_{t|t}]'$ we can then compute (A42) and (A47) using (A48). To obtain $\pi'_{t|t}$ recall that we assume that \mathbb{I}_t includes knowledge of the regime in place at time t , the data up to time t , Z^t , and the VAR parameters for each regime. Given that we assume knowledge of the current regime, $\pi_{t|t}^i \equiv P(\xi_t = i | \mathbb{I}_t)$ can only assume two values, 0 or 1. As a result $\pi'_{t|t}$ will be $(1, 0)$ or $(0, 1)$. As a result, and given $Z_t \in \mathbb{I}_t$, $q'_{t|t} = [q_{t|t}^{1'}, q_{t|t}^{2'}]'$ with $q_{t|t}^i \equiv \mathbb{E}_t (Z_t | \xi_t = i) \pi_{t|t}^i$, will be $[Z_t' \cdot 1, Z_t' \cdot 0]'$ or $[Z_t' \cdot 0, Z_t' \cdot 1]'$. Analogously, $Q'_{t|t} = [Q_{t|t}^{1'}, Q_{t|t}^{2'}]'$ with $Q_{t|t}^i \equiv \varphi (\mathbb{E}_t (Z_t Z_t' | \xi_t = i)) \pi_{t|t}^i$ will be $[\varphi (Z_t Z_t' \cdot 1)', \varphi (Z_t Z_t' \cdot 0)']'$ or $[\varphi (Z_t Z_t' \cdot 0)', \varphi (Z_t Z_t' \cdot 1)']'$.

Mean Square Stability

We consider the following MS-VAR model with n variables and $m = 2$ regimes:

$$Z_t = c_{\xi_t} + A_{1,\xi_t} Z_{t-1} + A_{2,\xi_t} Z_{t-2} + V_{\xi_t} \varepsilon_t, \quad \varepsilon_t \sim N(0, I) \quad (\text{A49})$$

where Z_t is an $n \times 1$ vector of variables, c_{ξ_t} is an $n \times 1$ vector of constants, A_{l,ξ_t} for $l = 1, 2$ is an $n \times n$ matrix of coefficients, $V_{\xi_t} V_{\xi_t}'$ is an $n \times n$ covariance matrix for the $n \times 1$ vector of shocks ε_t . The process ξ_t controls the regime that is in place at time t and evolves based on the transition matrix \mathbf{H} .

When estimating the MS-VAR we require the model to be mean square stable. Mean square stability is defined as follows:

Definition 1 *An n -dimensional process Z_t is mean square stable if and only if there exists an n -vector $\bar{\mu}$ and an n^2 -vector \overline{M} such that:*

- 1) $\lim_{t \rightarrow \infty} \mathbb{E}_0 [Z_t] = \bar{\mu}$
- 2) $\lim_{t \rightarrow \infty} \mathbb{E}_0 [Z_t Z_t'] = \overline{M}$

for any initial Z_0 and ξ_0 .

Mean-square-stability requires that first and second moments converge as the time horizon goes to ∞ . Under the assumptions that the Markov-switching process ξ_t is ergodic and that the innovation process ε_t is asymptotically covariance stationary, Costa, Fragoso, and Marques (2004) show that a multivariate Markov-switching model as the one described by (A49) is mean-square stable if and only if it is asymptotically covariance stationary. Both conditions hold for the models studied in this paper and are usually verified in economic models.

Costa, Fragoso, and Marques (2004) show that in order to establish MSS of a process such as the one described by (A49), it is enough to check MSS stability of the correspondent homogeneous process: $Z_t = A_{\xi_t} Z_{t-1}$. In this case, formulas for the evolution of first and second moments simplify substantially: $q_t = \Omega q_{t-1}$ and $Q_t = \Xi Q_{t-1}$. Let $r_\sigma(X)$ be the operator that given a square matrix X computes its largest eigenvalue. We then have:

Proposition 3 *A Markov-switching process whose law of motion can be described by (A49) is mean square stable if and only if $r_\sigma(\Xi) < 1$.*

Mean square stability allows us to compute finite measures of uncertainty as the time horizon goes to infinity. Mean square stability also implies that shocks do not have permanent effects on the variables included in the MSVAR.

Conditional Steady State

Consider a MS-VAR:

$$Z_t = c_{\xi_t} + A_{\xi_t} Z_{t-1} + V_{\xi_t} \varepsilon_t$$

where Z_t is a column vector containing n variables observable at time t and $\xi_t = 1, \dots, m$, with m the number of regimes, evolves following the transition matrix \mathbf{H} . If the MS-VAR has more than one lag, the companion form can be used to recast the model as illustrated above.

The conditional steady state for the mean corresponds to the expected value for the vector Z_t conditional on being in a particular regime. This is computed by imposing that a certain regime is in place forever:

$$\mathbb{E}_i(Z_t) = \bar{\mu}_i = (I_n - A_i)^{-1} c_i \quad (\text{A50})$$

where I_n is an identity matrix with the appropriate size. Note that unless the VAR coefficients imply very slow moving dynamics, after a switch from regime j to regime i , the variables of the VAR will converge (in expectation) to $\mathbb{E}_i(Z_t)$ over a finite horizon. If there are no further switches, we can then expect the variables to fluctuate around $\mathbb{E}_i(Z_t)$. Therefore, the conditional steady states for the mean can also be thought as the values to which the variables converge if regime i is in place for a long enough period of time.

The conditional steady state for the standard deviation corresponds to the standard deviation for the vector Z_t conditional on being in a particular regime. The conditional standard deviations for the elements in Z_t are computed by taking the square root of the main diagonal elements of the covariance matrix $\mathbb{V}_i(Z_t)$ obtained imposing that a certain regime is in place forever:

$$\varphi(\mathbb{V}_i(Z_t)) = (I_{n^2} - A_i \otimes A_i)^{-1} \varphi(V_{\xi_t} V_{\xi_t}') \quad (\text{A51})$$

where I_{n^2} is an identity matrix with the appropriate size, \otimes denotes the Kronecker product, and the vectorization operator $\varphi(X)$ takes a matrix X as an input and returns a column vector stacking the columns of the matrix X on top of one another.