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ABSTRACT

We look into the impact of measurement error in capital on the estimation of production functions. We introduce an identification scheme and an estimation procedure that jointly deals with measurement error in capital and the standard simultaneity bias due to unobserved productivity shocks. We use lagged investment to instrument for potentially mis-measured capital stock, while conditioning on the part of productivity that is persistent. Our estimation routine nests standard approaches in the literature, such as Ackerberg, Caves, and Frazer (2015). It requires no additional data as investment is usually collected with other producer level data. We show through Monte-Carlo experiments that a 40 percent measurement error in capital yields capital coefficients that are biased downward by a factor of two. We illustrate our approach using data for three distinct economies: China, India and Chile; which experienced radically different processes of capital and productivity dynamics. We find capital coefficients that are typically two times larger than those using standard approaches that only control for simultaneity.

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1 Introduction

Production functions are a central component in a variety of economic analyses. However, these often first need to be estimated using data on individual production units, and there is a vast literature on how to deal with the challenges of measurement of output and inputs, and the identification in the presence of unobserved productivity shocks.[1] While it is well known by practitioners that the measurement of capital assets poses a problem for the estimation of production functions, there is scant evidence on the severity of this problem, and in addition how to address it.[2] There is reason to believe that, more than any other input in the production process, there are severe errors in the recording of a producer's capital stock. These errors can be potentially large and extremely difficult to reduce through improved collection efforts since the errors pile up over time through the capital accumulation process.

The role of measurement error in capital in forming conclusions about the drivers of productivity growth is perhaps well-known by economists, but they have not been the focus of much analysis. Two separate JEL articles have been (partly) dedicated to discuss productivity measurement, and the role of measurement error in inputs – see Bartelsman and Doms (2000) and Syverson (2011). But there is unfortunately not much guidance for empirical work in how to deal with it. This paper shows that commonly used estimation techniques in the productivity literature fail in the presence of plausible amounts of measurement error in capital. We propose a simple solution that requires no additional data, while nesting the most common approaches in the literature, and hereby allowing to correctly measure the role of capital accumulation as a major potential driver of productivity growth (at the aggregate and producer level).

The presence of measurement error in capital is, perhaps indirectly, reflected in the well-documented fact that when estimating production functions with firm fixed-effects, capital coefficients are extremely low, and sometimes even negative.[3] Another interpretation is that capital is a fixed factor of production, and, therefore, the variation left in the time series is essentially noise. However, this also implies that changes in capital, which capture depreciation, is

[1]See Olley and Pakes (1996); Levinsohn and Petrin (2003); Ackerberg, Caves, and Frazer (2015) for some famous examples.

[2]Griliches and Mairesse (1998); Becker, Haltiwanger, Jarmin, Klimek, and Wilson (2006) for instance.

[3]Griliches and Mairesse (1998) state, "*In empirical practice, the application of panel methods to micro-data produced rather unsatisfactory results: low and often insignificant capital coefficients and unreasonably low estimates of returns to scale.*".

potentially contaminated by measurement error. Indeed, in an in-depth study of measurement issues related to capital, Becker, Haltiwanger, Jarmin, Klimek, and Wilson (2006) find that different ways of measuring capital that ought to be equivalent, such as using perpetual inventory methods or inferring capital investment from the capital producing sectors, lead to different results for a variety of outcomes, such as parameter estimates of the production function, and the investment and capital patterns.

Recent advances in econometric modelling and estimation techniques have introduced control functions methods whereby unobserved productivity is replaced by a function of inputs, including capital, other inputs and productivity shifters (Ackerberg, Benkard, Berry, and Pakes, 2005). This function is unknown, and therefore has to be estimated alongside the production function. This greatly limits the joint-treatment of the endogeneity of inputs and measurement error of inputs, in one internally consistent framework (see Kim, Petrin, and Song (2016) for a treatment of measurement error in such a setting). Furthermore, the *control function* approach fundamentally relies on the productivity shock to be orthogonal to the capital stock. The intuition being that capital is slow to respond, and can not be adjusted within a period. Therefore, any (potential) measurement error in capital is captured in the empirical measure of the productivity shock, violating the orthogonality condition, leading to a bias in the capital coefficient.

The point of departure of this paper is to revisit the impact of measurement error in the capital stock, while preserving the main features of the control function approach. In doing so, we make two contributions. First we exploit the implicit (log) linearity of the production function and the underlying productivity process, present in the most common approaches, to deal with both the simultaneity bias and measurement error in inputs. Using control function techniques allows us to isolate the productivity shock, on which we can form moments for estimation. The moment conditions can be formed in a very flexible way to account for both measurement error in inputs, here capital, and various model specifications, including the speed of adjustment of inputs and, the market structure of output and input markets. Second, we propose to use (lagged) investment to identify the marginal product of capital, rather than the year-on-year variation in capital stock, where poorly measured depreciations attenuate the capital coefficient. In other words, rather than using the entire within-producer time-series variation in the capital stock to identify the capital coefficient, we rather rely on measured capital changes that line up with instances where producers invest.

Very few papers consider the role of measurement error in capital, or in inputs more generally. Van Biesebroeck (2007) evaluates the performance of various production function estimators, including the so-called control function approaches, in the presence of measurement error, although not with a specific focus on measurement error in capital. He compares various methods in the presence of log additive mean-zero independent and standard normally distributed errors to all inputs, measurement error in output and input prices. While his focus is on the bias in the estimated coefficients, we provide an estimator that is robust to the presence of such measurement error, in the context of endogenous input choices.

An often used alternative to production function estimation is the so-called factor-share approach. In general, and in order to accommodate imperfect competitive output markets, the output elasticities are directly computed in data through the ratio of an input's expenditure over total costs.[4] The latter includes the payments to capital which rely not only on an estimate of the user cost of capital, but also on a measure of the capital stock. Therefore, the factor-share approach is subject to a potential measurement error problem as well.[5]

2 Measurement error in capital

In this section, we discuss the relevance of obtaining the correct parameter value of the capital coefficient for *any* productivity analysis. We then discuss in greater detail what the potential sources of measurement error in capital are, and how we incorporate measurement error into the estimation of the production function. This discussion leads us to conclude that (lagged) investment is a natural instrument for the recorded capital stock.

2.1 Relevance for productivity analysis

Dealing with measurement error in capital is critical to obtain reliable estimates of the capital coefficient. We hereby revisit an older and important debate on the relative importance of

[4]See Foster, Haltiwanger, and Syverson (2008) and De Loecker, Eeckhout, and Unger (2020) for applications of the approach.

[5]Under constant returns to scale, no adjustment cost and perfect competitive output markets, revenue shares (the ratio of an input's expenditure in total revenue) can be used to estimate the production function (Asker, Collard-Wexler, and De Loecker, 2014), while under imperfect competition, the user cost of capital — which is rarely measured directly — needs to be observed.

capital accumulation and total factor productivity growth in explaining periods of high-growth throughout the world, at various points in time. The essence of the debate can be traced back to the measurement of the capital stock, and its growth, in the national accounts. If a country, a region, or an individual producer for that matter, experience a growth in output, it can come from factor accumulation or productivity growth. Economists care greatly as to which one it is, for the plain and simple reason that one is free, and the other is not. Factor accumulation, in the case of capital, requires investment, on top of offsetting depreciation, and it is privately-held, and purchased in the capital market at prevalent market prices; and thus imply a user cost of capital. In contrast, if output rises due to productivity growth, say due to technological progress, all factors become more productive.

This debate between productivity and factor accumulation was particular lively in the aftermath of the East Asian growth miracle. The industrial revolution in several East Asian countries, during (roughly) 1960 and 1990, lead to periods of rapid growth; Taiwan, Singapore, Hong Kong and Korea recorded average annual output-per-worker growth of 4.3, 4.2, 4.7 and 4.9 percent, respectively. The conflicting views on the source of his phenomenal output growth are reflected in several prominent papers. Hsieh (2002) put forward evidence, using duality theory, that indeed the source behind the output growth was TFP, while Young (1995) concluded that factor accumulation was the main driver behind this success.

Against this background, a separate literature developed taking the newly arrived micro-datasets to estimate production functions. These data cover the very same variables, output and input data, as in the analysis of Young (1995) and Hsieh (2002), but instead of aggregated to the level of a country (by means of inspecting the national accounts), they are recorded at the level of an individual producer. A major focus was on developing approaches that can deal with the transmission bias, leading to otherwise biased factor shares of the various inputs. This literature, starting with Olley and Pakes (1996), was also interested in productivity growth episodes, and detecting the drivers behind it. One important take-away from this literature is that the estimate of the factor share of capital is greatly affected by whether we recognize that a producers's input choices are systematically related to productivity, and that as industries mature a non-random set of producers is forced to exit. In the case of US telecom equipment producers, Olley and Pakes (1996) report a capital coefficient of 0.35, compared to the within-estimator (the benchmark at the time) of 0.06. While no attempts in this study, or in any of the subsequent work, were made to connect this to aggregate implications, it is clear that any conclusion, on what the drivers of output growth might be, will be highly sensitive to which point estimate is used. In a similar type of analysis Collard-Wexler and De Loecker (2015) demonstrate the importance of the point estimate on capital in detecting the productivity effects from the introduction of a new technology, in their case the minimill in steel production.

While we broadly relate to the literature on the relative importance of productivity and capital, as prominently featured in the literature mentioned above, we do not engage in a revision of a particular debate, but our findings on the magnitude of the capital coefficient do impact this discussion. Furthermore, we expect measurement error in capital to be problematic for any analysis of micro-level correlations of productivity. If we systematically underestimate the factor share of capital, we also underestimate its marginal product. This underestimate of the role of capital will be loaded into the productivity residual. Thus, any firm characteristic that is correlated with capital, such as firm size, export participation, research and development or management, among others, will falsely appear to be associated with higher productivity.

2.2 Sources of Measurement Error in Capital

Capital stock is typically measured in one of two ways, using either book value, or the perpetual inventory method (PIM, hereafter). The book value of capital is measured using direct information on the value of capital, as recorded in a firm's balance sheet. The PIM adds up depreciated past investments, and thus, requires data on investment, and recorded depreciations to construct capital stock. Of course, these approaches are related since the book value of capital is frequently the outcome of firms themselves applying the PIM in their internal accounting. PIM is the most common approach to construct capital stock series in census data, while accounting-based filing typically supply the book value of tangible fixed assets (e.g. Orbis and Computstat are two well-known examples); see Becker, Haltiwanger, Jarmin, Klimek, and Wilson (2006) for an excellent overview. PIM measures the capital stock of a particular asset K_a using:

$$K_{at} = \sum_{j=0}^{\infty} \theta_{ajt} I_{at-j}, \tag{1}$$

where θ_{ajt} is the weight at time t of asset a of vintage j and, thus, captures the depreciation profile, and I_{at-j} is the real gross investment of vintage j. Literally applying equation (1) is virtually impossible, even when we rely on the highest quality dataset, such as the U.S. Census of Manufacturers. Instead, applied work typically relies on a more familiar law of motion for capital:

$$K_{it}^{e} = (1 - \delta_{st})K_{it-1}^{e} + I_{it-1}, \tag{2}$$

where we now calculate current capital stock for a more aggregated asset e, such as equipment and buildings, and rely on an industry-wide depreciation rate for assets δ_{st}, where s indicates the industry, i indexes the producer, and t is year. Finally, real gross investment expenditure is ideally corrected for sales and the retirement of capital assets.

This immediately raises a few measurement issues. First, this approach requires an initial stock of capital, K_{e0}, at the date on which production started. Second, investment price deflators are rarely available at the producer-level — these are typically computed at the industry level. This is a problem, since asset mix can be differ considerably across producers within the same industry. Third, depreciation rates are assumed to not vary across producers and vintage of the capital stock, which again creates measurement error in capital. Aggregating over heterogeneous assets with a common depreciation factors is, thus, expected to introduce noise in the capital stock measure, as well. Moreover, depreciation does not simply follow a fixed factor, and this all is further compounded by reported depreciation being governed by tax treatment of depreciation rather than by economic depreciation.[6]

In contrast, investment is more precisely recorded through the purchases of various capital goods and services in a given year. This is in stark contrast to capital stock, which is accumulated over time, and this further exacerbates the problem. While, to some extent, every input, including labor and intermediate inputs, is subject to measurement error, capital is distinct in this dimension.

The use of book value as recorded in a producer's balance sheet is also subject to measurement error. In principle, one can rely on both measures — the book value and the constructed capital stock using PIM — and see how they line up. In the U.S. Census data on manufacturing, such as the Annual Survey of Manufacturing and the Census of Manufacturers, these perpetual inventory and direct assets measures differ by 15 to 20 percent (see Becker, Haltiwanger, Jarmin, Klimek, and Wilson (2006)). This suggests a reasonable amount of measurement error

[6]For example, when regulators set electricity rates (see, for instance, Progress Energy – Carolinas (2010)), they often have hundreds of pages of asset-specific depreciations depending on the lifespan of a boiler, car, truck, or building, and these depreciation rates typically have fairly intricate time series patterns.

in capital that is likely to be persistent over time. Given that we see measurement error even in the highest-quality data sources such as the U.S. Census of Manufacturers, capital measurement error may be more prevalent for datasets covering developing countries. In the latter, we are often precisely interested in identifying factors driving productivity growth, and the (mis) allocation of resource; therefore, accurately measuring the marginal products, and capital growth is of first-order importance. Finally, intangible capital is an increasing part of total capital stock (see Corrado, Hulten, and Sichel (2009) and Haskel and Westlake (2017)). For all the issues in measuring physical capital, such as buildings or machinery, measuring intangible capital such as research and development or brand equity, is substantially harder.

In the next section, we introduce the identification and associated estimation strategy. In Section 4, we turn to a series of Monte Carlo analysis, to demonstrate the performance of our estimator. We illustrate our approach, in Section 5, by contrasting the estimated capital coefficients using our procedure to traditional approaches. We do this for three distinct datasets that have a long tradition in the productivity literature, including China's NBES, the Indian census of manufacturing (ASI) and a sample of Chilean manufacturing producers.

3 Identification and Estimation Strategy

We present the identification strategy to accommodate both unobserved productivity shocks and measurement error in capital. This strategy suggests a straightforward estimation routine that does not require additional data, and does not not add any complexity to the standard estimation routines such as the class considered by ACF.

3.1 Identification under measurement error and productivity shocks

We consider a production function of the type:

$$q_{it} = f(m_{it}) + \beta_l l_{it} + \beta_k k_{it} + \omega_{it}, \tag{3}$$

where $q_{it}, l_{it}, k_{it}, m_{it}$ and ω_{it} denote (log) output, labor, capital, materials and productivity, respectively. This captures the class of Hicks-neutral production functions including the gross output and Leontief Cobb-Douglas specifications – i.e., $f(m_{it}) = \beta_m m_{it}$ and $f(m_{it}) = 0$, respectively.[7] In levels these two production functions are given by

$$Q_{it} = \begin{cases} M_{it}^{\beta_m} L_{it}^{\beta_l} K_{it}^{\beta_k} \Omega_{it} & \text{(Gross-Output)}, \\ \min\left\{\beta_m M_{it}, L_{it}^{\beta_l} K_{it}^{\beta_k} \Omega_{it}\right\} & \text{(Leontief)}. \end{cases} \tag{4}$$

In what follows we consider the standard DGP considered by ACF that allows to identify the production function that is perfect competitive output markets and competitive input markets with common factor prices. This allows us to focus on the role of measurement error in capital. In subsection 3.22 we briefly discuss departures from this perfect competition framework.

The laws of motion. Capital and productivity follow the (standard) law of motions:

$$K_{it} = (1-\delta)K_{it-1} + I_{it-1} \tag{5}$$

$$\omega_{it} = \rho\omega_{it-1} + \xi_{it} \tag{6}$$

This is a departure from the literature, which typically only assumes a first-order (exogenous) Markov process, that allows for a non-linear process of the form $\omega_{it} = g(\omega_{it-1}) + \xi_{it}$. In practice, typically an AR(1) process is used. Moreover, when the properties of new estimators, such as ACF, are evaluated via Monte-Carlo experiment, these also assume an AR(1) process.[8]

Measurement error in capital When we turn to the actual solution and implementation of our estimator, we rely on the commonly assumed errors-in-variable structure, where the observed log of the capital stock (k) is the sum of the log true capital stock (k^*) and the measurement error (ϵ^k):

$$k_{it} = k_{it}^* + \epsilon_{it}^k, \tag{7}$$

[7] To avoid the unlikely case that the data on materials is generated by unexpected large swings in the price of materials after labor and capital are installed, such that the Leontief FOC would fail to hold, we do not consider the more general Leontief structure $G(M_{it})$, but rather $\beta_m M_{it}$.

[8] While we consider an AR(1) process, the approach goes through for higher-order AR processes of the form $\sum_{p=1}^{P} \rho_p \omega_{t-p}$. It thus remains an empirical matter how to evaluate the trade-off between allowing for non-linearities and higher-order AR terms. Our approach does, however, accommodate endogenous processes of the form $\omega_{it} = \rho_1\omega_{it-1} + \rho_2 d_{it-1}$, where d_{it} is an action taken by the firm to impact future productivity, e.g. export status or innovation. See De Loecker (2013) for a discussion, and Braguinsky, Ohyama, Okazaki, and Syverson (2015) for a recent application.

where we use the * notation to denote variables measured without error – the one that is typically observed by the firm – and the unstarred notation to denote the observed value. This linear form of measurement error allows us to use IV to rid the model of bias from error-in-variables.

We refer to this representation, loosely, as the *reduced form* for the various measurement error sources we have described. We assume that ϵ^k_{it} is classical measurement error — i.e. it is uncorrelated with true capital stock k_{it}. We do, however, allow for ϵ^k_{it} to be serially correlated over time (within a producer). Since capital is constructed using historical information on the cost of assets, it is unlikely that there is no serial dependence in measurement error of the value of assets.

We further assume that this measurement error is orthogonal to last period's investment choice:

$$\mathbb{E}(i_{it-1}\epsilon^k_{it}) = 0. \tag{8}$$

Throughout we denote (log) investment as i_{it}, and we observe investment without error. [9] Thus, the main premise is that investment at time $t-1$ is informative about the capital stock at time t, conditional on lagged capital, but is not correlated with the measurement error in capital ϵ^k_{it}. This makes lagged investment a valid instrument, as lagged investment is, by the presence of the law of motion on capital, predictive of the current capital stock, and this assumption guarantees that it is orthogonal to the measurement error. This condition holds by construction if the measurement error is not serially correlated. However,we do allow for serially correlated measurement error in capital as the likely source behind measurement error is due errors in depreciation, which naturally give rise to dependency over time.

Of course, merely using lagged investment as an IV would not work: given that investment choices depend on unobserved productivity, the exclusion restriction is violated. Our approach is specifically designed to jointly treat the two sources of endogeneity: simultaneity and errors in variables.

Simultaneity with Errors-in-Variables Let us consider the empirical counterpart of the production function, where we follow the literature and include measurement error in output

[9] We can allow for measurement error in investment, as long as this measurement error is not correlated with the measurement error in capital ϵ^k_{it}, and if investment enters without error in the PIM calculation. This is precisely what our Monte Carlo simulations accommodate.

(ϵ_{it})[10]

$$q_{it} = \beta_l l_{it} + \beta_k k_{it} + \eta_{it}, \tag{9}$$

where η_{it} is a composite error term composed of productivity, and measurement error in capital and output:

$$\eta_{it} \equiv \omega_{it} - \beta_k \epsilon^k_{it} + \epsilon_{it}. \tag{10}$$

The search for an instrument (z_{it}) in this setting consists of satisfying both :

$$\mathbb{E}(k_{it} z_{it}) \neq 0 \text{ and } \mathbb{E}(\eta_{it} z_{it}) = 0. \tag{11}$$

This implies that not only that the instrument needs to be orthogonal to the capital and output measurement error $(\epsilon + \epsilon^k)$, but also orthogonal to the productivity term ω — to avoid the standard simultaneity bias. The literature has long recognized that the latter is difficult to find, since candidates that are correlated with inputs are, more often than not, also correlated with productivity, and therefore fail to satisfy the exclusion restriction. Input prices often fall in this category, since they are expected to be correlated with the inputs, but at the same likely correlated with productivity. Our setup adds a second source of endogeneity through the presence of capital measurement error, and we now explain how our approach can deal with both sources of endogeneity – i.e., $\mathbb{E}(k_{it}\omega_{it}) \neq 0$; the simultaneity problem, and $\mathbb{E}(k_{it}\epsilon^k_{it}) \neq 0$; the errors-in-variable problem.

It is instructive to consider how treating each source of endogeneity separately leads to a biased estimate of the capital coefficient (and potentially also the labor coefficient – more on this below). In fact deploying a *pure IV* approach using lagged investment would result in using the moment condition $\mathbb{E}(\eta_{it} i_{it-1}) = 0$. The investment instrument is strong, as $\mathbb{E}(i_{it-1} k_{it}) > 0$ due to the capital accumulation process. However, this IV approach would yield an upward bias since η captures productivity (ω), and we expect investment to be increasing in productivity $\mathbb{E}(i_{it-1}\omega_{it}) > 0$, through the persistent part of the productivity shock – i.e. the inequality reduces to $\mathbb{E}(i_{it-1}\omega_{it-1}) > 0$.[11]

[10]The error in output relates to true output as follows: $q_{it} = q^*_{it} + \epsilon_{it}$ where q^* is true output and q denotes measured output.

[11]Indeed, in our monte-carlo experiements in section 4 the IV estimator is higher than the OLS estimator. This pattern is mimicked by our applications, so we present this IV estimator, both to illustrate the bias it causes, but also, to show what is being leveraged in the data to yield consistently higher capital coefficients.

Following the standard ACF approach would lead to a moment condition on the productivity shock that still contains the measurement error in capital and would set the following moment condition equal to zero to identify β_k:

$$\mathbb{E}(\tilde{\xi}_{it} k_{it}) = 0, \tag{12}$$

where $\tilde{\xi}_{it} \equiv \xi_{it} - \beta_k \epsilon^k_{it} + \rho \beta_k \epsilon^k_{it-1}$. The presence of measurement error, at both t and $t-1$ makes it impossible to identify the capital coefficient.

Our approach, however, relies on the following conditional orthogonality condition:

$$\mathbb{E}(\eta_{it} i_{it-1} | \omega_{it-1}) = 0, \tag{13}$$

which with help from the assumed laws of motion (equations (5) and (6), respectively) leads to the following moment condition that identifies the capital coefficient.

$$\mathbb{E}(\tilde{\xi}_{it} i_{it-1}) = 0. \tag{14}$$

This expression highlights that we require (lagged) investment to be orthogonal to contemporaneous measurement error in capital (stated in equation (8)).[12]

The labor coefficient is identified using standard moment conditions (depending on what one is willing to assuming about labor market frictions and adjustment costs). We do, however, expect the labor coefficient to be biased in the presence of measurement error in capital, even under the standard ACF approach, and this for the simple reason that labor and capital are correlated. Under the Cobb-Douglas production technology, assumed throughout the literature and in this paper, the two inputs are positively correlated and if there are no unobserved productivity shocks, one can show that this will lead to an upward bias of the labor coefficient. The intuition behind this result is that labor will pick up the true capital variation through the positive correlation of labor and (true) capital, and thus overestimate the impact of labor on output variation.[13] This has further implications for returns to scale. Correcting for the presence of measurement error in capital leads to ambiguous returns to scale implications; depending both on the severity of the error in capital recording and the strength of the correlation between labor and capital in the data.

[12] Unlike the approach introduced by Olley and Pakes (1996) where only firms that invest can be used to estimate the production function, our approach has the potential to treat the zero investment firms as information about the change in capital that is potentially contaminated with error.

[13] See chapter 11 of Maddala and Lahiri (1992) and Pischke (2007) for a formal treatment of the signal-to-noise bias in multivariate regression analysis.

3.2 Estimation

We present the main estimation algorithm that nests the standard approach in the literature, but considers serially correlated measurement error in capital.

3.2.1 Benchmark

The estimation procedure consists of two steps, and follows the discussion in ACF closely.[14] The main difference is that we exploit the (log) linear production structure in capital, labor and productivity, which dictates a (log) linear control for productivity. The log linearity of the production function guarantees that the control for productivity (through the cost minimizing first-order condition used to control for unobserved productivity) will also be log-linear, and allow to treat the error in capital measurement. Under our baseline specification of the Leontief production function, the cost minimizing material choice gives a simple log-linear control for productivity given by:

$$\omega_{it} = \ln \beta_m + m_{it} - \beta_l l_{it} - \beta_k k_{it} \text{ (Leontief)}. \tag{15}$$

Note that this *control* is independent of the presence input price variation, and imperfect competitive output or input markets. It merely states that the firm is optimally deploying the mix of inputs given the technology. Alternatively, under the gross output production function we follow ACF-LP and invert the material demand equation and (also) obtain a log linear expression for productivity given by:

$$\omega_{it} = -\ln \beta_m + (1 - \beta_m) m_{it} + p_t^m - p_t - \beta_l l_{it} - \beta_k k_{it} \text{ (Gross-Output)}, \tag{16}$$

where all lower cases denotes logs, and p^m and p are the material input and output price, respectively.[15] Note that in either the Leontief or Gross-Output, we obtain log-linear control functions.[16]

[14] The Not-for-Publication Appendix A provides precise details on these estimators, and our webpage has code for these in STATA.

[15] In this setting the input demand equation does depend on the specific features of output and input markets, such as input price variation and imperfect competition.

[16] If we replace the productivity shock in the production function with the control for productivity (regardless of which production function we consider), we are left with a regression of output net of material variation on a constant (and time dummies) and the measurement error in output: $q_{it} - m_{it} = c + \epsilon_{it}$. This is a standard finding

In both specifications, the first step consists of running a simple linear IV-regression where capital is instrumented with lagged investment:

$$\begin{aligned} q_{it} &= \theta_t + \theta_l l_{it} + \theta_k \hat{k}_{it} + \theta_m m_{it} + \tilde{\epsilon}_{it} \\ &= \phi_{it} + \tilde{\epsilon}_{it}, \end{aligned} \tag{17}$$

where θ_t captures the constant term and the time-varying prices (under the gross output specification), $\tilde{\epsilon}_{it} = \epsilon_{it} - \beta_k \epsilon_{it}^k$, and we use θ_h ($h = \{l, m, k\}$) to denote the reduced-form parameters, on each input, capturing both the production function coefficients and the parameters governing the control for unobserved productivity, and $\hat{k}_{it}$ denotes the predicted capital stock from the first-stage IV.[17]

The moment conditions introduced in the previous section can now be formed by observing that the joint productivity-measurement error of capital term (conditional on the parameter vector $\boldsymbol{\beta}$) is obtained using $\omega_{it}(\boldsymbol{\beta}) - \beta_k \epsilon_{it}^k = \phi_{it} - \beta_l l_{it} - \beta_k k_{it}$. Using the law of motion on productivity the moment conditions can be constructed to estimate the labor and capital coefficient, respectively:

$$\mathbb{E}\left[\tilde{\xi}_{it}(\boldsymbol{\beta}) \begin{pmatrix} l_{it-1} \\ i_{it-1} \end{pmatrix}\right] = 0. \tag{18}$$

In contrast with the standard approach (ACF) where $\mathbb{E}(\tilde{\xi}_{it} k_{it}) = 0$ is used to identify the capital coefficient, we rely on lagged investment to be orthogonal to the productivity shock. Our approach, however, preserves the flexibility of the ACF approach with respect to selecting alternative DGP, and in the absence of measurement error both estimators (ACF and ours) coincide.

The identification and estimation of the material coefficient, in case of the gross-output specification follows immediately from the assumption of perfect competitive output markets.

in the literature, see for instance, footnote 17 on page 2444 of ACF. However, the norm is to run a regression of output on labor, capital and materials. It is worth weighing the pros and cons. From a purely applied point of view, it is very strong to suggest that the variation in materials in sufficient to capture output, and it seems sensible to include labor, capital and estimate a coefficient on materials, all to eliminate the measurement error in output. From a pure theoretically point of view, it is sufficient to run a regression of output net of materials and recover the constant term. The advantage is that the measurement error in capital is not affecting the estimate of predicted output (used subsequently to isolate the productivity shock). Running this regression would then constitute the first stage for both technologies, and is robust to measurement error in capital. Note that this result is a consequence of modeling the productivity shock as a log-additive shock to a Cobb-Douglas input aggregator.

[17] Measured capital is predicted using lagged investment, labor, materials and year dummies.

Taking the first-order condition of profits with respect to materials immediately generates $\beta_m = \frac{P^m M}{PQ}$, as an estimate. Due to the potential measurement error in output (ϵ), taking the average of the material-revenue share would yield a biased estimate. Instead, we take the median material revenue share across the sample to obtain a consistent estimate of β_m. While the coefficient on materials is often not of direct interest in the case of the Leontief production function, it is given by the ratio of (physical) materials-to-output, $\frac{M}{Q}$, where again we consider the median across firms and time.[18]

4 Monte Carlo Analysis

We evaluate our estimator in a series of Monte Carlo analyses in which the main interest lies in comparing the capital coefficient across methods as we increase the level of measurement error in capital. We follow Ackerberg, Caves, and Frazer (2015) closely, starting with their data-generating process, and add measurement error to capital. We depart from ACF by adding time-varying investment costs in the investment policy function in order to generate additional time-series variation in capital stock.

We refer the reader to Appendix B for the details of the underlying model of investment, but the main features of our setup are as follows. We rely on a constant returns to scale production function with a quadratic adjustment cost for investment. This model yields closed-form solutions for both labor and capital, where labor is set using a static first-order condition given firm-specific wages.[19] Productivity and wages follow an AR(1) process, and we consider a quadratic adjustment cost for investment: $\phi_{it} I_{it}^2$, where ϕ_{it} — which should be considered as the price of capital — itself follows a first-order Markov process. We solve the model in closed form, extending the work in Syverson (2001) and discussed in Appendix B.2, and this generates our perfectly measured Monte Carlo dataset on output, inputs, investment and productivity.

We then overlay measurement error on this dataset composed of AR(1) processes with

[18] The presence of measurement error in output thus invalidates relying on average factor shares. See Appendix A.1 for more details and also see Asker, Collard-Wexler, and De Loecker (2014) and De Loecker, Eeckhout, and Unger (2020) for an application of this estimator.

[19] ACF also deal with two other data-generating processes, other than the approach we described (called DGP1 in their paper). In Appendix B.4, we also consider optimization error in labor (DGP2) and an interim productivity shock between labor and materials as in ACF (DGP3), along with optimization error in labor. We find similar results from any of the DGP's considered by ACF.

normally distributed shocks:

$$\begin{aligned}\epsilon_{it} &= \rho^q \epsilon_{it-1} + u^q_{it} \\ \epsilon^k_{it} &= \rho^k \epsilon^k_{it-1} + u^k_{it},\end{aligned} \tag{19}$$

where $u^q \sim \mathcal{N}(0, \sigma^2_q)$, and $u^k \sim \mathcal{N}(0, \sigma^2_k)$. In other words, we allow for serially correlated measurement error in output and capital.

We analyze on the impact of increasing ϵ^k, which is governed by the variance σ^2_k. We distinguish between the role of measurement error *within* a given Monte Carlo, and the overall distribution of estimated coefficients *across* 1,000 Monte Carlo runs.

Table B.1 in the Appendix shows the parameters used in our Monte Carlo. We pick the same parameters for the size of the dataset, production function, and processes for productivity and wages as in ACF. For the process for the price of capital, denoted ϕ_{it}, we pick parameters that match the the cross-sectional dispersion of capital ($std.[k_{it}] = 1.6$) and the time-series variation in capital ($Corr.[k_{it} - k_{it-1}] = 0.93$) in the Annual Survey of Industries in India (discussed in Section 5), choosing an autocorrelation term for the process of ϕ of 0.9 and a shock variance of 0.3.[20]

Finally, we pick parameters for the measurement error in inputs and output. We choose a measurement error for output with a standard deviation of 30 percent, and a low autocorrelation of 0.2. For the error in capital (ϵ^k), we choose a serial correlation coefficient of 0.7, so a fairly high persistence, and a standard deviation of 0.2. Note that this assumption on the time series process for capital measurement error yields a difference between k and k^*, which has a standard deviation of 30 percent.

Table 1 presents the estimated capital and labor coefficients from this Monte Carlo exercise, with (true) parameters of the production: $\beta_k = 0.4$ and $\beta_l = 0.6$. We compare the performance of estimators that use investment to instrument for mismeasured capital proposed in this paper, that we call Control IV, and of those that do not, that we call Control estimator.

The Control IV estimator yields unbiased — i.e., the mean parameters are the same as the true ones — and, precise estimates, with a standard deviation of 0.02. In contrast, the standard Control function estimator yields downwardly biased estimates of the capital coefficient, with

[20] Indeed, it is this last moment that the ACF Monte Carlo has difficulty replicating: it predicts a serial correlation coefficient of capital of 0.997, which is much more than in any producer-level dataset we are aware of.

Table 1: Monte-Carlo Results
($\beta_k = 0.4, \beta_l = 0.6$)

Coefficient	OLS	FE	IV	Control	Control IV
Capital	0.04	0.03	0.07	0.32	0.40
(s.e.)	(0.01)	(0.01)	(0.01)	(0.03)	(0.02)
Labor	0.95	0.90	0.94	0.52	0.60
(s.e.)	(0.01)	(0.01)	(0.01)	(0.02)	(0.02)

Notes: Mean and Standard deviation over 1,000 Monte-Carlo replications presented of DGP 1 in ACF with capital measurement error. Control refers to the control function approach used in ACF, while control IV refers to the control function that uses investment as an instrument. The estimates without control functions are OLS, IV with capital instrument by investments, FE with firm fixed effects.

an average of 0.32 versus a true value of 0.4. As in ACF, the OLS estimator for capital is downwardly biased because of the simultaneity problem, with $\beta_k^{OLS} = 0.04$. The IV estimator is similarly biased, but shows a substantially larger capital coefficient than the OLS ones. Figure 1 plots the distribution of estimates of both the labor and capital coefficient showing that the Control and Control IV estimates have similar variances, but there is very little overlap in the distributions of the estimated coefficients.

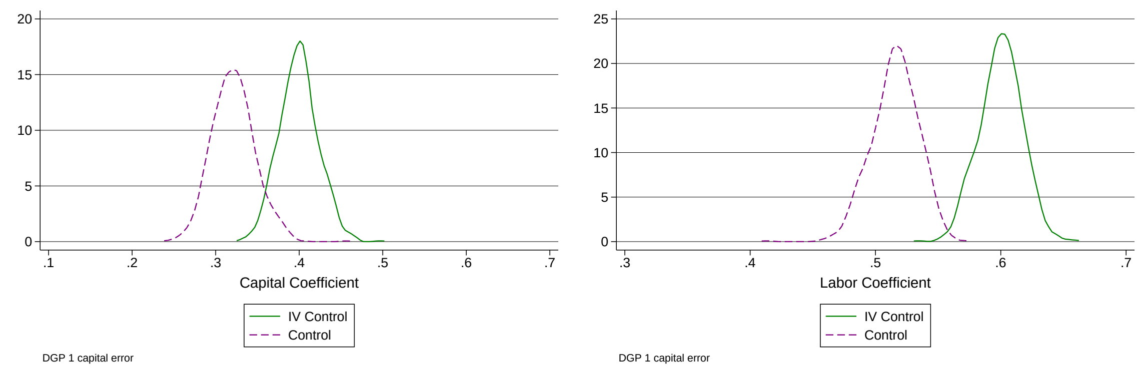
4.1 The impact of measurement error

In Figure 2, we plot the average estimate of β_k over 100 replications against the variance of capital measurement error σ_k for both the Control IV and Control estimators. This Monte Carlo simulation shows that standard estimator become progressively more biased away from the true value of β_k as the measurement error in capital increases. It is of course difficult to guess the relevant range of this variance, but the main takeaway is that our IV-based estimator is insulated from this problem. The simulations do suggest that standard methods deliver an estimate of half the magnitude for a standard deviation in the capital measurement error σ_k of about 0.2, which corresponds to a standard deviation between k and k^* of 0.28 in the stationary distribution.

It is important to note that our estimator is robust to capital measurement error, while still undoing the simultaneity bias that typically plagues the production function estimation.

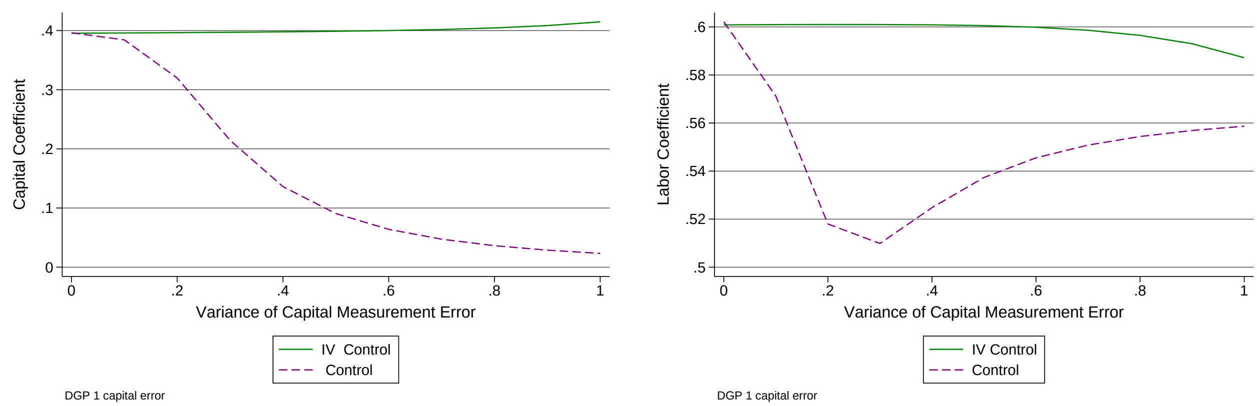
Figure 1: The distribution of the estimated capital and labor coefficient in a Monte Carlo ($\beta_k = 0.4$, $\beta_l = 0.4$)

(a) Capital (b) Labor



Note: We plot the distribution of the estimated capital coefficient across 1000 Monte Carlo replications, with $\beta_k = 0.4$, and $\sigma_k^2 = 0.2$.

Figure 2: Impact of measurement error on capital coefficient in a Monte Carlo ($\beta_k = 0.4$):



Note: We plot the estimated capital coefficient as a function of the variance in the capital measurement error (σ_k^2). Average of 100 Monte Carlo replications per value of σ_k.

Therefore, applying our estimator when the capital stock is accurately measured also provides consistent estimates of the production function coefficients.

4.2 Alternative sources of measurement error

As discussed before, we have considered what we refer to as a reduced form for the measurement error in capital. That is, we consider the standard representation of an errors-in-variable, whereby the measurement error is (log) additive — here $k + \epsilon^k$. In Appendix C, we discuss an alternative source of measurement error in capital, derived structurally from the measurement error in depreciation rates: $K_{it} = (1-\delta_{it})K_{it-1}+I_{it}$ and $\delta_{it} = (\delta+\epsilon^d_{it})$; i.e., there is measurement error in depreciation rates.

This form of measurement error does not map into a log-additive structure, and, we evaluate our estimator in the presence of this alternative setup. The main takeaway from Appendix Figure C.1 is that our estimator outperforms the other approaches but, given the formal violation of the moment conditions, leads to a small bias of the capital coefficient for large values of the variance of the capital measurement error.

The evidence from the Monte Carlo unequivocally favors our estimator in the presence of measurement error in capital and, moreover, suggests that the bias can be quite severe for moderate measurement error in capital. To verify how large this problem is in real data, we now apply our estimator, exactly as performed in our Monte Carlo analysis, to three different datasets of manufacturing plants, in India, China and Chile.

5 Applications to plant-level data

We illustrate our approach by applying it to three plant-level datasets. The first dataset we use is from the Annual Survey of Industries from India. This a plant-level survey for over 600,000 plants over a twenty-year period. Allcott, Collard-Wexler, and O'Connell (2016) have previously used and described this dataset. The second dataset is the Chilean census, as used in Pavcnik (2002) and Levinsohn and Petrin (2003), and covers plants in the Chilean manufacturing sector for the period 1979-1986. Finally, we evaluate the impact of our correction to the Chinese manufacturing database provided to us by Brandt, Van Biesebroeck, and Zhang (2012). All variables are deflated using industry-specific price deflators and Appendix D describes each dataset briefly, and presents basic summary statistics.

These three datasets have been used extensively to study productivity dynamics, but at the same time have distinct features related to the measurement of capital. The data on Chinese establishments report the book value of plants and investment, while the Indian and the Chilean census data report both the book value and the (constructed) capital stock using the perpetual inventory method. In addition, the economic environments are different in important ways. There is substantial investment during the process of economic transition in China, while in India and Chile the presence of small unproductive firms and persistent frictions paints a very different productivity dynamic over time. We expect these differences to materialize in the estimated coefficient, and in the role and importance of measurement error in the capital stock.

Throughout, we compare estimated production function coefficients (on labor and capital) from our approach to those obtained by simple OLS, IV (without the simultaneity control) and the control function approaches including the investment IV one presented in this paper. We estimate a separate capital and labor coefficient for each industry in each country. We do this for both the structural Leontief model ($q = \beta_l l + \beta_k k$) and for the more usual gross-output production function ($q = \beta_l l + \beta_m m + \beta_k k$) where we use a first-order condition to estimate the material coefficient. We focus our discussion primarily on the gross-output specification, since it uses a production function that is far more commonly studied in the empirical literature, but the pattern of results is similar for the Leontief specification.

We start by reporting the median for the various estimators in Table 2, but also include the 25th and 75th percentile across industries to give an idea of the range of estimates. We confirm a well-known result in the literature that using fixed effects lowers the capital coefficient substantially. In the case of the Gross-Output production function, the capital coefficient falls from a median of 0.12 for OLS to 0.06 for firm FE in India, and from 0.24 to 0.12 for Chile, with no difference for China. By itself, this does not conclusively show that there is measurement error in capital. However, if capital is fixed over a long period of time, we cannot identify its marginal product using the time series variation within producers. China is a clear case where capital is changing at an unprecedented rate, quite atypical from the capital evolution of most countries. Our next specification, IV (investment), considers a two-stage least squares regression of output on capital and labor, where we instrument for capital with lagged investment. We find substantially higher capital coefficients compared to OLS, of 0.06 versus 0.18 in China, 0.12 versus 0.17 in India, and 0.10 versus 0.22 in Chile, respectively. This reinforces our prior that instrumenting for capital with lagged investment may lead to a higher capital coefficient. However, investment and unobserved productivity are very likely to be positively correlated, so

Table 2: Production Function Coefficients

Gross Output with FOC Materials

	China		India		Chile	
	Capital	Labor	Capital	Labor	Capital	Labor
OLS	0.06	0.12	0.12	0.25	0.10	0.43
	[0.04 0.09]	[0.09 0.17]	[0.09 0.15]	[0.18 0.32]	[0.04 0.06]	[0.33 0.51]
FE	0.06	0.11	0.06	0.19	0.05	0.30
	[0.03 0.09]	[0.07 0.15]	[0.04 0.09]	[0.14 0.24]	[0.08 0.13]	[0.22 0.38]
IV	0.18	0.03	0.17	0.16	0.22	0.19
	[0.11 0.23]	[0.00 0.06]	[0.14 0.25]	[0.12 0.21]	[0.19 0.37]	[0.03 0.40]
Control	0.05	0.12	0.09	0.27	0.09	0.38
	[0.02 0.10]	[0.07 0.21]	[0.07 0.11]	[0.23 0.341]	[-0.01 0.12]	[0.33 0.57]
IV Control	0.12	0.10	0.19	0.19	0.16	0.21
	[0.08 0.17]	[0.04 0.19	[0.12 0.20]	[0.17 0.30	[0.13 0.35]	[0.03 0.59]

Leontief

	China		India		Chile	
	Capital	Labor	Capital	Labor	Capital	Labor
OLS	0.28	0.45	0.38	0.70	0.24	0.89
	[0.22 0.33]	[0.38 0.54]	[0.36 0.80]	[0.64 0.80]	[0.19 0.31]	[0.85 0.97]
FE	0.26	0.40	0.17	0.60	0.12	0.71
	[0.22 0.30]	[0.36 0.45]	[0.16 0.30]	[0.36 0.45]	[0.07 0.14]	[0.66 0.81]
IV	0.64	0.13	0.60	0.40	0.55	0.45
	[0.54 0.74]	[0.46 0.99]	[0.54 0.74]	[0.46 0.99]	[0.45 0.74]	[0.15 0.65]
Control	0.25	0.60	0.33	0.76	0.22	1.01
	[0.10 0.40]	[0.33 0.89]	[0.27 0.41]	[0.65 0.88]	[0.14 0.30]	[0.84 1.06]
IV Control	0.31	0.73	0.44	0.68	0.29	0.97
	[0.20 0.42]	[0.46 0.99]	[0.20 0.42]	[0.46 0.99]	[0.12 0.38]	[0.87 1.32]

Notes: We report the median coefficient across all industries for each dataset. In parenthesis we show the 25th and 75th percentile for each coefficient. Leontief specification is $Q_{it} = \min\{\beta_m M_{it}, L_{it}^{\beta_l} K_{it}^{\beta_k} \Omega_{it}\}$, and Gross Output with FOC Materials is given by $M_{it}^{\beta_m} L_{it}^{\beta_l} K_{it}^{\beta_k} \Omega_{it}$, where the material coefficient is computed using the median of the material revenue share ($\beta_m = Median[\frac{P_t^M M_{it}}{P_t Q_{it}}]$).

the increase in the capital coefficient in the IV regression might also reflect the endogeneity of investment.

Turning to the estimators that control for the simultaneity of inputs, and the impact of measurement error in capital. We find the capital coefficient increases from a median of 0.05 for an ACF control function to 0.12 for our control IV approach for China. Likewise, the median coefficient increases from 0.09 to 0.19 for India, and from 0.09 to 0.16 for Chile. The fact that the capital coefficient roughly doubles is striking, but completely in line with the predictions from our Monte-Carlo, say in figure 2, if the variance of the measurement error σ_k is around 0.2. In addition, the Leontief production function shows a similar pattern of increasing capital coefficients when moving from the control to the control IV approach. The capital coefficient increases from 0.25 to 0.31 for China, 0.33 to 0.44 for India, and 0.22 to 0.29 for Chile.

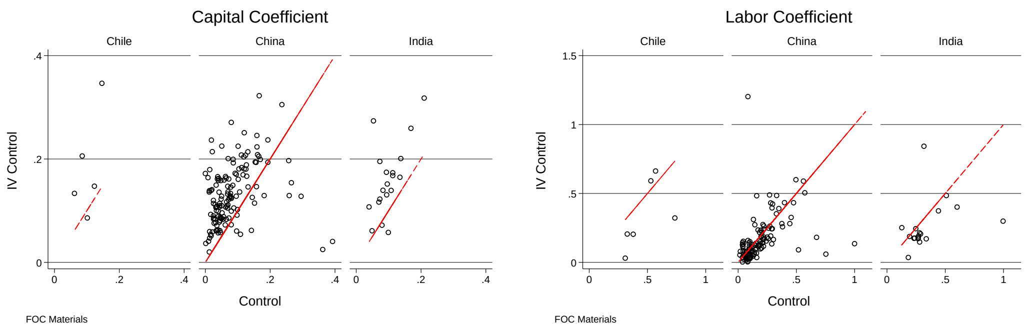
As for the labor coefficient, in general we find that the cases where the capital coefficient increases, the labor coefficient offsets this decrease. For instance, for the gross-output production function, the control function labor coefficient decreases from 0.12 to 0.10 when we use our IV approach in China, 0.27 to 0.19 for India, and 0.38 to 0.21 for Chile. Likewise, the labor coefficient in the IV estimates are lower than the OLS ones, falling from 0.12 to 0.03 for China, 0.25 to 0.16 for India, and 0.43 to 0.19 for Chile respectively for the case of the gross-output production function. Note as well that the Monte-Carlos were less dispositive as to the effect of capital measurement error on the labor coefficient.

Finally, in Figure 3 we plot the industry-specific capital and labor coefficients by country, with the top panel showing the gross-output production function, and the bottom panel showing the Leontief. The vertical axis shows the capital coefficient from our IV estimator, while the horizontal axis shows the capital coefficient that does not instrument with investment, and the red solid line is the 45 degree line.

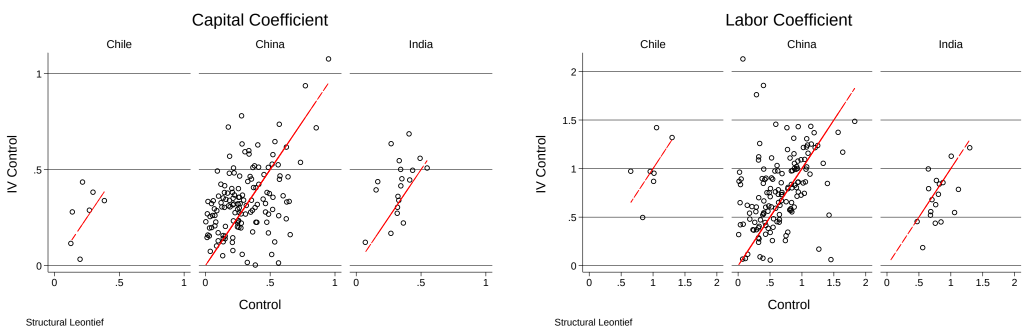
The bulk of the observations for the capital coefficient are above the 45-degree line (in red), indicating that our IV estimates are higher than the non-IV estimates for capital. This is particularly true for the Gross-Output production function. Note as well, that there is nothing mechanical about our higher capital coefficients: there are several industries where we find a lower capital coefficient using the IV control approach versus the control function. The labor coefficients are also a bit lower, but this pattern is less striking. These differences are also frequently statistically significant. We have computed the standard errors for the Control

Figure 3: Industry-specific Coefficients

(a) Gross-Output



(b) Leontief



Notes: Each observation is a two-digit industry (as classified by the respective national industry classification), and we plot the capital coefficient obtained from our procedure (i.e., IV) against the alternative control function. The red line is the 45-degree line.

and Control IV estimators using a bootstrap procedure, and report these in Table E in the Appendix. We test for the capital coefficient being larger in the IV Control estimator than the Control estimator at the 90 percent level, which happens in 15 percent of industries for the Leontief production function, and in 17 percent of industries for the Gross-Output production function. In contrast, the capital coefficient for the Control estimator is only larger than the Control IV estimate at the 90 percent level for only 4 percent of industries for the Leontief production function, and none of the industries for the Gross-Output.

Taking these results at face value suggests that measurement error in capital leads to obtaining capital coefficients that are significantly lower — i.e., half the magnitude. This has important consequences for any subsequent productivity analysis.

6 Concluding remarks

This paper revisits the estimation of production functions in the presence of measurement error in capital. Our starting point is that appropriately measuring capital is one of the most difficult tasks that go into estimating a production function. There is, however, rather surprisingly little work that deals directly with measurement error in capital, or any input for that matter.

Introducing an estimator that relies on a hybrid IV-control function approach, we build on what have now become standard techniques to address the simultaneity bias, and add an IV strategy to correct for the measurement error of capital. We propose a simple strategy that relies on investment to inform us about the marginal product of capital; specifically, we use lagged investment as an instrument for the capital stock while still controlling for the standard simultaneity bias. The latter is crucial to capture the well-known endogeneity concern that arises due the presence of unobserved productivity shocks. Our approach therefore captures leading estimators in the literature that have come to be widely used throughout various subfields of economics.

From an applied point, our estimator is easy to implement and adds no extra cost to the user, and allows for a great deal of flexibility in incorporating recent advances in the estimation of production function in the context of market imperfections, both in output and input markets, or the presence of adjustment frictions for factors.

Monte Carlo simulations show that our estimator performs well, even in cases of rather large measurement error. We also apply our estimator to Indian, Chilean and Chinese producer-level data. We estimate capital coefficients that are double those obtained with standard techniques. This indicates that correcting for measurement error in capital can be a first-order concern, and it has immediate implications for the literature that studies productivity dynamics, firm growth, investment, productivity dispersion, and the covariates of productivity growth. Indeed, any analysis where the conclusions rest on estimates of the marginal product of capital, by itself, or as an input into measurement of productivity, can be misleading in the presence of errors in the recording and measurement of the capital stock.

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A Factor shares and Alternative Environments

We present a more detailed discussion on the use of the factor share approach to identify and estimate the material coefficient in the baseline setup of our framework. Departures from perfect competitive output and input markets are briefly discussed and we offer an illustration under a specific model of product differentiation under common input prices, and highlight that the main features of our approach are robust to considering such an environment.

A.1 Factor shares

Under the standard DGP op perfect competition it is well-known that the output elasticity of a variable input in production can immediately be measured in the data using the *revenue share* of an input X:

$$\beta_X = \frac{(P^X X)_{it}}{(P_t \tilde{Q})_{it}}. \tag{A.1}$$

where $P^X X$ is the expenditure on the input X, and PQ is observed revenue, including the measurement error in output ($\tilde{Q}_t = Q_{it} \exp(\epsilon_{it})$). In the case of the Cobb-Douglas production function (either gross output if we wish to measure the coefficient on materials, or Cobb-Douglas in labor and capital, if we wish to measure the coefficient on labor), we have to confront the substantial dispersion in the revenue share in (any) dataset. Under the assumed setup this dispersion can only come from measurement error in output, and therefore we need to rely on an estimator that is robust to this.

The presence of measurement error in output then calls for a simple estimator:

$$\hat{\beta_X} = Median\left[\frac{(P^X X)_{it}}{(P\tilde{Q})_{it}}\right], \tag{A.2}$$

where the median is used instead of the mean, due to the error in the denominator of this expression. This estimator is a consistent estimator of β_X if we assume that the output measurement error satisfies $Med[\epsilon_{it} = 0]$. This is obtained by letting the median operator go through the observed revenue share:

$$\frac{(P^X X)_{it}}{P_t Q_{it}} \frac{1}{\exp(Median[\epsilon_{it}])}, \tag{A.3}$$

where we use the property of the exponential being a monotone function. Thus, our estimator proposed in equation (A.2) is a consistent estimator of β_X. Likewise, we can estimate the material coefficient β_m under the gross output production function, and under Leontief the estimator considers the median of the output share ($\frac{M_{it}}{\tilde{Q}_{it}}$).

Under the gross output specification, a special case presents itself whereby only the capital coefficient is required to be estimated using GMM. Using the estimates $\hat{\beta}_l$ and $\hat{\beta}_m$, we can net out the contribution of these inputs to output: $y_{it} = q_{it} - \beta_l l_{it} - \beta_m m_{it}$. We can then proceed with our estimation routine relying on y_{it} as the measure of output (instead of q_{it} in the first-stage). This discussion outlines how control function methods can be combined with factor-share approaches in our method. However, the factor share approaches crucially relies on the assumptions of flexible input choice and competitive output markets.

A.2 Alternative environments

In most settings, we observe firms charging different prices for their output, and paying different prices for inputs, which leads to an additional complication since researchers typically have access to only (deflated) revenues and expenditures on inputs. We believe this to be a very important concern. The starting point of our analysis, however, is to assume we have correctly converted the revenue and expenditure data to the comparable units in a physical sense, and this is precisely the setup of Ackerberg, Caves, and Frazer (2015).

A related but distinct concern is the presence of imperfect competitive output and input markets. As discussed in the review article by De Loecker and Goldberg (2014), both observations lead to two challenges: unobserved output and input price variation plagues identification, and second, the control for unobserved productivity needs to be adjusted to accommodate the departures from perfect competition and identical prices across firms. The approach suggested in this paper is thus valid under commonly assumed models of imperfect competition, either when adopting the Leontief technology, or in the case of the gross-output production function by extending the input demand equation as in De Loecker and Warzynski (2012) and De Loecker, Goldberg, Khandelwal, and Pavcnik (2016). We discuss briefly, and illustrate it with a few leading cases, how our approach can be adjusted to incorporate departures from the standard setup in the main text of the paper.

Leontief As discussed in the main text, under the Leontief technology the presence of imperfect competition does not affect the control. The reason is simple: regardless of input price differences, or imperfect competition in the product market, as long as firms cost minimize and produce at the optimal point where $F(L, K)\Omega = \beta_M M$, the control is valid.

Gross Output Under this technology, the control function approach is adopted and alternative environments (such as input price differences and imperfective product and factor markets) require additional terms to be included in the control to reflect the dependance of optimal input demand on these output and input market conditions. In general, let the vector z_{it} captures these factors, then $\omega_{it} = h(m_{it}, k_{it}, l_{it}, z_{it})$. A few leading cases that are of interest are: input price variation, demand-side heterogeneity (e.g. export demand, product-level demand differences), quality differences. See De Loecker and Goldberg (2014) and De Loecker, Goldberg, Khandelwal, and Pavcnik (2016) for a detailed discussion.

The FOC approach to estimating the material coefficient is, however, no longer valid under imperfect competitive output markets, and therefore to estimate the material coefficient β_m under the gross output specification, we can no longer rely on the factor share.This is precisely the approach of De Loecker and Warzynski (2012) to estimate markups: comparing the revenue share to the output elasticity of a variable input generates a measure of the markup. We therefore introduce an additional moment condition to identify the material coefficient – i.e., $\mathbb{E}(\tilde{\xi}_{it} m_{it-1}) = 0$. This requires additional structure to the problem by relying on observed, and serially correlated material prices, that are to be included in the control for productivity. Under Leontief the material coefficient remains as before.

Finally, our approach, like ACF, can also handle different timing assumptions on the choice of inputs. For instance, in the case in which labor faces adjustment costs, the labor input now constitutes a state variable, and labor choices will not entirely react to productivity innovation shocks ξ_{it}. In the case of one-period hiring, labor at time t and labor at $t-1$ are exogenous variables. The moments condition is then given by: $\mathbb{E}\left[\tilde{\xi}_{it} \left(\ l_{it} \ \right)\right] = 0$ Thus, the source of identification for the labor coefficient is then precisely that adjustment costs vary across firms, to the extent that these vary with the labor stock (see Bond and Söderbom (2005) for a discussion).

B Monte Carlo: details

In this section, we describe details of the Monte Carlo that we will use to evaluate the performance of our estimator.

Table B.1: Monte Carlo Parameters

<u>Data Size</u>		
Number of Firms	N= 1,000	Taken from ACF.
Time Periods	T= 10	
<u>Production Function Parameters</u>		
Capital Coefficient	$\beta_k = 0.4$	
Labor Coefficient	$\beta_l = 0.6$	
Depreciation Rate	$\delta = 0.2$	
Productivity Process	ρ_ω=0.7, $\sigma_\omega = 0.3$	
Wage Process	ρ_w=0.3, $\sigma_w = 0.1$	
Cost Capital ϕ	ρ_ϕ=0.9, $\sigma_\phi = 0.3$	Dispersion of Log Capital of 1.6 Autocorrelation of Capital of 0.93
<u>Measurement Error Parameters</u>		
Error in Capital	$\rho^k = 0.7$, $\sigma_k = 0.2$	High Persistence, and 30 percent measurement error in stationary distribution
Error in Output	$\rho^y = 0.2$, $\sigma_y = 0.3$	Low Persistence, and 30 percent measurement error in stationary distribution

We specify laws of motion for each of the variables in the data-generating process.

B.1 Timing

First, we specify the timing assumptions in our model. Investment is chosen with one period time to build. Materials are chosen statically — i.e., after the firm knows its productivity Ω_{it}. Labor is chosen statistically in for DGP 2 and DGP 3, and in an interim period for DGP 1 — i.e., part of the productivity shock is revealed before the firm makes its labor choice.

Second, there are four exogenous state variables, productivity A_{it}, wages W_{it}, output prices P_{it}, and the price of capital ϕ_{it}, which all have log AR(1) processes. The only endogenous state variable is capital.

Logged productivity A has a first-order Markov evolution:

$$a_{it} = \rho^a a_{it-1} + u^a_{it}, \tag{B.1}$$

where $u^a \sim \mathcal{N}(0, \sigma^2_a)$.

In addition, log wages have a first-order Markov process:

$$w_{it} = \rho^w w_{it-1} + u^w_{it}, \tag{B.2}$$

and likewise for the logged price for output (P):

$$p_{it} = \rho^p p_{it-1} + u^p_{it}, \tag{B.3}$$

where $u^w \sim \mathcal{N}(0, \sigma^2_w)$ and $u^p \sim \mathcal{N}(0, \sigma^2_p)$. For the purposes of the Monte Carlo, we will normalize $p_{it} \equiv 1$, the case of perfect competition.

B.2 Derivation of Investment Policy as in Syverson (2001)

In this section, we derive a closed form for the investment function in Syverson (2001), to show that we can allow a time-varying cost of capital ϕ_{it}. This derivation is very close to Syverson (2001), so our goal is merely to show that this model admits a time-varying cost of capital ϕ_{it}.

Firms have flow profits given by:

$$\Pi_{it} = P_{it} A_{it} L^{\alpha}_{it} K^{1-\alpha}_{it} - W_{it} L_{it} - \frac{\phi_{it}}{2} I^2_{it}, \tag{B.4}$$

where P is the price of output, A is physical productivity, W refers to firm specific wages, and I is investment.

The firm's value function V is given by:

$$\begin{aligned} V(P_{it}, A_{it}, K_{it}, W_{it}, \phi_{it}) = &\underset{L_{it}, K_{it}}{max}\, P_{it} A_{it} L^{\alpha}_{it} K^{1-\alpha}_{it} - W_{it} L_{it} \\ &+ \beta \mathbb{E}_{it} V(P_{it+1}, A_{it+1}, K_{it+1}, W_{it+1}, \phi_{it+1}) \\ \text{such that } &K_{it+1} = (1-\delta) K_{it} + I_{it} \end{aligned} \tag{B.5}$$

where δ is the depreciation rate of capital.

Labor is chosen using the usual first-order condition $\frac{\partial \Pi_{it}}{\partial L_{it}} = 0$:

$$\begin{aligned} P_{it} A_{it} \alpha L^{\alpha-1}_{it} K^{1-\alpha}_{it} &= W_{it} \\ \rightarrow L_{it} &= \left[\frac{\alpha P_{it} A_{it}}{W_{it}} \right]^{\frac{1}{1-\alpha}} K_{it} \end{aligned} \tag{B.6}$$

And, likewise, investment solves the Euler Equation, $\frac{\partial V}{\partial I} = 0$, giving,

$$\phi_{it} I_{it} = \beta \mathbb{E}_{it} V_K(P_{it+1}, A_{it+1}, K_{it+1}, W_{it+1}, \phi_{it+1}). \tag{B.7}$$

The envelope condition yields:

$$\begin{aligned} V_K(P_{it}, A_{it}, K_{it}, W_{it}, \phi_{it}) =& (1-\alpha) P_{it} A_{it} L_{it}^{\alpha} K_{it}^{-\alpha} \\ &+ (1-\delta)\mathbb{E}_{it} V_K(P_{it+1}, A_{it+1}, K_{it+1}, W_{it+1}, \phi_{it+1}). \end{aligned} \tag{B.8}$$

Substituting into the first-order conditions, the envelope condition becomes

$$\phi_{it} I_{it} = \beta \mathbb{E}_{it} \left[(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}} W_{it+1}^{-\frac{\alpha}{1-\alpha}} P_{it+1}^{\frac{1}{1-\alpha}} A_{it+1}^{\frac{1}{1-\alpha}} \right] + \beta(1-\delta)\mathbb{E}_{it} \phi_{it+1} I_{it+1}. \tag{B.9}$$

And then iterating this equation forward — i.e., replacing $\mathbb{E}_{it}\phi_{it} I_{it}$ with the right-hand side in equation (B.9) — yields:

$$\begin{aligned} \phi_{it} I_{it} =& \beta \mathbb{E}_{it} \left[(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}} W_{it+1}^{-\frac{\alpha}{1-\alpha}} P_{it+1}^{\frac{1}{1-\alpha}} A_{it+1}^{\frac{1}{1-\alpha}} \right] \\ &+ \beta(1-\delta)\mathbb{E}_{it} \beta \left[(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}} W_{it+2}^{-\frac{\alpha}{1-\alpha}} P_{it+2}^{\frac{1}{1-\alpha}} A_{it+2}^{\frac{1}{1-\alpha}} \right] \\ &+ [\beta(1-\delta)]^2 \mathbb{E}_{t+1} \phi_{it+2} I_{it+2} \\ \phi_{it} I_{it} =& \beta \mathbb{E}_{it} \left[(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}} W_{it+1}^{-\frac{\alpha}{1-\alpha}} P_{it+1}^{\frac{1}{1-\alpha}} A_{it+1}^{\frac{1}{1-\alpha}} \right] \\ &+ \beta(1-\delta)\mathbb{E}_{it} \beta \left[(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}} W_{it+2}^{-\frac{\alpha}{1-\alpha}} P_{it+2}^{\frac{1}{1-\alpha}} A_{it+2}^{\frac{1}{1-\alpha}} \right] \\ &+ [\beta(1-\delta)]^2 \beta \left[(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}} W_{it+3}^{-\frac{\alpha}{1-\alpha}} P_{it+3}^{\frac{1}{1-\alpha}} A_{it+3}^{\frac{1}{1-\alpha}} \right] \\ &+ [\beta(1-\delta)]^3 \mathbb{E}_{t+2} \phi_{it+3} I_{it+3}. \end{aligned} \tag{B.10}$$

Writing in the form of geometric series

$$I_{it} = \frac{\beta(1-\alpha)}{\phi_{it}} \alpha^{\frac{\alpha}{1-\alpha}} \mathbb{E}_{it} \sum_{j=0}^{\infty} \left\{ [\beta(1-\delta)]^j W_{it+1+j}^{-\frac{\alpha}{1-\alpha}} P_{it+1+j}^{\frac{1}{1-\alpha}} A_{it+1+j}^{\frac{1}{1-\alpha}} \right\} \tag{B.11}$$

Given that we assume that P_{it}, A_{it} and W_{it} follow the log-linear AR(1) process with normal error terms, the investment function becomes:

$$
\begin{aligned}
I_{it} = & \frac{\beta(1-\alpha)}{\phi_{it}} \alpha^{\frac{\alpha}{1-\alpha}} \mathbb{E}_{it} \sum_{j=0}^{\infty} \{[\beta(1-\delta)]^j W_{it}^{-\frac{\alpha\phi_w^{i+1}}{1-\alpha}} \prod_{s=0}^{j} (u_{t+1+i-s}^{w})^{-\frac{\alpha\phi_w^s}{1-\alpha}} P_{it}^{\frac{\phi_p^{i+1}}{1-\alpha}} \\
& \cdot \prod_{s=0}^{j} (u_{t+1+i-s}^{p})^{\frac{\phi_p^s}{1-\alpha}} A_{it}^{\frac{\phi_a^{i+1}}{1-\alpha}} \prod_{s=0}^{j} (u_{t+1+i-s}^{a})^{\frac{\phi_a^s}{1-\alpha}} \} \\
= & \frac{\beta(1-\alpha)}{\phi_{it}} \alpha^{\frac{\alpha}{1-\alpha}} \prod_{s=0}^{j} \{[\beta(1-\delta)]^j W_{it}^{-\frac{\alpha\phi_w^{i+1}}{1-\alpha}} \prod_{s=0}^{j} \mathbb{E}_{it} (u_{t+1+i-s}^{w})^{-\frac{\alpha\phi_w^s}{1-\alpha}} P_{it}^{\frac{\phi_p^{i+1}}{1-\alpha}} \\
& \cdot \prod_{s=0}^{j} \mathbb{E}_{it}[(u_{t+1+i-s}^{p})^{\frac{\phi_p^s}{1-\alpha}}] A_{it}^{\frac{\phi_a^{i+1}}{1-\alpha}} \prod_{s=0}^{j} \mathbb{E}_{it}[(u_{t+1+i-s}^{a})^{\frac{\phi_a^s}{1-\alpha}}] \}
\end{aligned}
$$

Since for $\epsilon \sim \mathcal{N}(0, \sigma^2)$, we have $\mathbb{E}(u^{\frac{\phi^s}{1-\alpha}}) = \exp(\frac{\sigma^2\phi^{2s}}{2(1-\alpha)^2})$; then, the investment function can be further simplified as:

$$
\begin{aligned}
I_{it} = & \frac{\beta(1-\alpha)}{\phi_{it}} \alpha^{\frac{\alpha}{1-\alpha}} \mathbb{E}_{it} \sum_{i=0}^{\infty} \{[\beta(1-\delta)]^j W_{it}^{-\frac{\alpha\phi_w^{i+1}}{1-\alpha}} \prod_{s=0}^{j} \exp\left(\frac{\alpha^2\sigma_w^2\phi_w^{2s}}{2(1-\alpha)^2}\right) P_{it}^{\frac{\phi_p^{i+1}}{1-\alpha}} \\
& \cdot \prod_{s=0}^{j} \exp\left(\frac{\sigma_p^2\phi_p^{2s}}{2(1-\alpha)^2}\right) A_{it}^{\frac{\phi_a^{i+1}}{1-\alpha}} \prod_{s=0}^{j} \exp\left(\frac{\sigma_a^2\phi_a^{2s}}{2(1-\alpha)^2}\right) \}.
\end{aligned}
\tag{B.12}
$$

B.3 Process for the Price of Capital

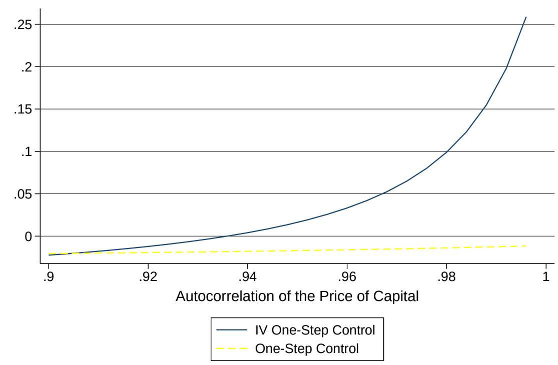
In the original Monte Carlo proposed by Ackerberg, Caves, and Frazer (2015), the authors extend the model proposed by Syverson (2001) by allowing for the price of capital ϕ to differ between firms — i.e., to allow the price for capital to be a firm-specific ϕ_i. This is important, in the context of their Monte Carlo, since it allows a higher cross-sectional dispersion of capital between firms than that generated by reasonable processes of productivity, given the patterns in standard producer-level data.

In this paper, we also need capital to move around more than it does in Ackerberg, Caves, and Frazer (2015), with a process that generates a serial correlation coefficient for capital of 0.99. Instead, we have the following AR(1) process:

$$
\phi_{it} = \rho^{\phi}\phi_{it-1} + \sigma^{\phi}u_{it}^{\phi}, \tag{B.13}
$$

where $u_{it}^{\phi} \sim \mathcal{N}(0, 1)$.

Figure B.1 shows the relationship between the autocorrelation of the price of capital and estimates of the capital coefficient using both the one and two-step estimators proposed in the paper. To make these estimates more comparable, when we change ρ^{ϕ}, we adjust σ^{ϕ} so that the stationary distribution



Notes: Autocorrelation of price of capital refers to the parameter ρ^{ϕ}, where the process for ϕ is $\phi_{it} = \rho^{\phi}\phi_{it-1} + u_{it}$. The variance term σ_k for measurement error is $\sigma_k = 0$. Average estimated coefficient over 100 Monte Carlo replications.

Figure B.1: Persistence of the price of capital ϕ; i.e., ρ^{ϕ}, and estimate of the capital coefficient

of ϕ, given by the usual formula for an AR(1) process with normal errors $\frac{\sigma}{\sqrt{1-\rho^2}}$, is unchanged. In panel a, showing one-step estimators, for a wide range range of ρ^{ϕ} parameters below one, our estimator performs very well. However, at very high levels of persistence of ϕ, our IV one-step estimate drops to 0.2. In contrast, the estimates for panel b, showing two-step estimators, do not change much as we vary the persistence of the price of capital ϕ.

B.4 Alternative Data-Generating Processes

We evaluate the performance of our estimator in two alternative data-generating processes (DGPs), as considered in ACF in their Monte Carlos.

- DGP 1:

  DGP 1 is the case considered in the main Monte Carlos in the paper. Note that labor is chosen a half-period before materials are picked. More precisely, labor is chosen at time $t - 0.5$, and materials are chosen at time t, where the productivity process is adjusted so that the stochastic process for $\omega_{it-0.5}$ is given by:

  $$\omega_{it} = \rho^{0.5}\omega_{it-0.5} + \xi^{b}_{it},$$

  where ξ^{b}_{it} is an appropriately adjusted normally distributed shock.

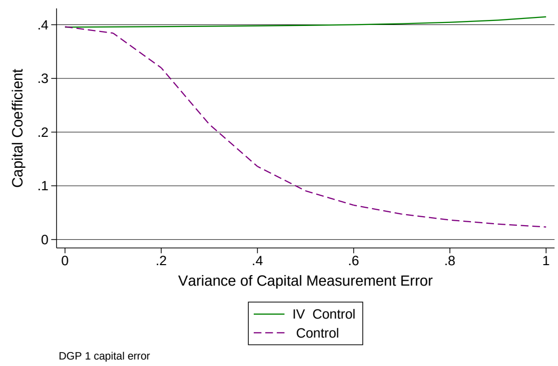
- DGP 2:

DGP2 refers to the case of optimization error in labor. The variance of the wage distribution is shut down, $\sigma_w = 0$, but instead, firms face an optimization error in labor. Thus, $l_{it} = l^*_{it} + \epsilon^l_{it}$ where $\epsilon^l_{it} \sim \mathcal{N}(0, 0.37)$.

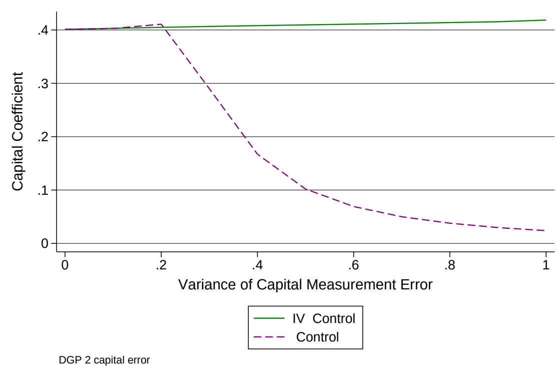
- DGP 3:

  DGP 3, has the same process as DGP 1, but adds in optimization error in labor,$l_{it} = l^*_{it} + \epsilon^l_{it}$, where $\epsilon^l_{it} \sim \mathcal{N}(0, 0.37)$, as in DGP 2.

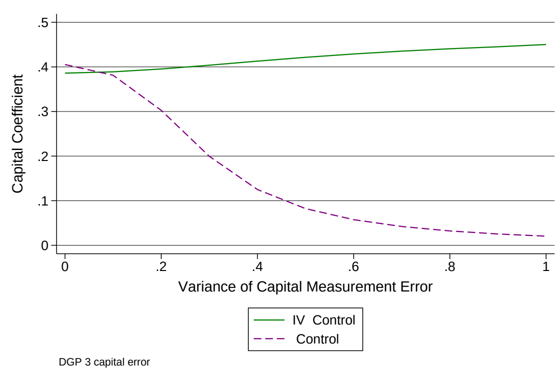
Figure B.2, below replicates Figure 2 for DGP 1, 2 and 3, and shows the sensitivity of our IV and non-IV two-step estimators to the measurement error in capital. Notice that the pattern that we document in the Monte Carlos for DGP 1 in the main paper is the same as what we find for these alternative DGPs: our estimator performs well with varying degrees of capital measurement error, while the standard control function approaches are biased for reasonable amounts of measurement error.



(a) Two-Step Control DGP 1



(b) Two-Step Control DGP 2



(c) Two-Step Control DGP 3

Note: We plot the estimated capital coefficient as a function of the variance in the capital measurement error (σ_k^2). Average of 100 Monte Carlo replications per value of σ_k. The true value of $\beta_k = 0.4$.

Figure B.2: Relationship between β_k and Measurement error σ_k in Capital for different DGPs

C Different Processes for Measurement Error in Capital

In our main specifications in this paper, we rely on the process for measurement error having the form:

$$k_{it} = k^*_{it} + \epsilon^k_{it}, \tag{C.1}$$

where ϵ^k_{it} is a mean zero measurement error, which may be serially correlated. While this is the standard formulation for an *errors-in-variables* structure, we contrast this approach to, what we refer to as, a structural derived measurement error for capital. After we introduce this setup, we perform an analogous Monte Carlo analysis to evaluate our estimator, and we make sure that both approaches are directly comparable through their implied variance in the measurement error of capital.

As discussed in Section 2, we consider the main source of the capital measurement error to stem from errors in depreciation $D_{it} = \delta_{it} K_{i-1t}$, where the correct measure is given by $d_{it} = \delta^*_{it} K^*_{it-1}$. Applying the same law of motion for capital as before, it is easy to show that the measured capital stock under the error in depreciation rates is given by:

$$K_{it} = \sum_{\tau=0}^{t} I^*_{i\tau} - \sum_{\tau=0}^{t} D_{i\tau-1}, \tag{C.2}$$

where we keep the assumption that we observe investment without error and, for simplicity, also the initial capital stock.[21] The source of measurement error is, thus, from the cumulative depreciation errors, and we capture as follows: $\delta_{it} = \delta^* + \epsilon^d_{it}$. Both the reduced-form and the structural approach generate a wedge between the measured and true capital stock, in levels K and K^*, respectively. After some algebra, we have a direct mapping between the structural measurement error, ϵ^d_{it}, and the reduced form measurement ϵ^k_{it}:

$$\epsilon^d_{it} = \frac{K^*_{it}}{K^*_0} \exp(\epsilon^k_{it}). \tag{C.3}$$

This relationship is important when comparing the performance of our estimator across both Monte Carlos: a much smaller variance in the depreciation error, ϵ^d_{it}, is needed to generate a certain variance of the classical measurement error, ϵ^k_{it}.[22]

C.1 Monte Carlo Analysis

All parameters of the Monte Carlo are the same as before, except that we parameterize $\epsilon^d_{it} \sim \mathcal{N}(0, \sigma_d)$. True depreciation is given by $D^*_{it} = \delta K^*_{i-1t}$. However, measured depreciation is given by $D_{it} = (\delta + \epsilon^d_{it}) K_{it}$.

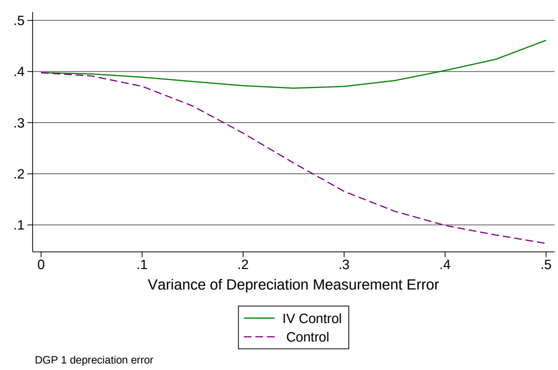
[21] This is without loss of generality for the purpose of this Appendix.

[22] We have traced this out in our Monte Carlo analysis, and we obtain about a 20-1 ratio — i.e., we obtain similar implied variances for the measurement error using either $\sigma^k = 0.2$ or $\sigma^d = 0.01$.

Figure C.1 presents the results of this Monte Carlo for both the IV Control method we propose and the Control method that does not use investment as an instrument. Plot the relationship between the mean estimate of β_k over 1,000 replications, as we vary method and a two-step , across the three data-generating process considered by ACF.

Figure C.1 shows that under a small amount of measurement error in depreciation — say, on the order of a 0.01 variance shock to a depreciation rate of 0.1 — the IV control function method we have proposed does fairly well, with mean estimates around the true value of 0.4. In contrast, the mean estimates that do not instrument with investment, show a drop of β_k to 0.3, with a 0.01 variance shock to the depreciation rate, in line with the previous results that we showed illustrating that measurement error in capital stock leads to downward bias on the capital coefficient.

Notice that Figure C.1 does not show that our IV Control function estimator is consistent for any value of measurement error in depreciation. Indeed, the process for measurement error in depreciation does not lead to the log additive error structure on capital that we need for estimation. Instead, our goal is merely to point out that our estimator might perform well for alternative measurement error structures, at least for some small deviations from our structure.



Note: We plot the estimated capital coefficient as a function of the variance in the capital measurement error (σ_k^2). Average of 100 Monte Carlo replications per value of σ_d. The true value of $\beta_k = 0.4$.

Figure C.1: Relationship between β_k and Measurement error σ_d in Depreciation

D Data Appendix

We apply our estimator to three datasets, covering manufacturing plants in China, India and Chile. There have been numerous productivity studies using these data, and, therefore, are completely standard in which variables are reported, and how they are constructed.

D.1 Chilean manufacturing

Annual plant-level data on all manufacturing plants with at least ten workers were provided by Chile?s Instituto Nacional de Estadistica (INE). These data, which cover the period 1979-1986, include production, employment, investment, intermediate input, and balance-sheet variables. Industries are classified according to the four digit ISIC industry code. Output and input price indices are constructed at the three digit industry and obtained directly from average price indices produced by the Central Bank of Chile. We directly observe total number of employees, total real value of production, total real intermediate input, total real book-value of fixed assets, gross additions to capital (i.e. investment). In total there are 37,600 plant-year observations reporting employment, with a minimum of 4,205 plants in 1983 and 5,814 plants in 1979. Following Levinsohn and Petrin (2003) we rely on the law of motion for capital where investment is productive within the year, reflecting the accounting standards of reported additions to capital stock (i.e. $K_t = (1-\delta)K_{t-1} + I_t$). See Pavcnik (2002), Levinsohn and Petrin (2003) and Asker, Collard-Wexler, and De Loecker (2014) for productivity studies using these data.

D.2 Chinese manufacturing

We Chinese manufacturing is described in detail in Brandt, Van Biesebroeck, and Zhang (2012). This is firm-level data for the period 1998–2007 that are the product of annual surveys conducted by the National Bureau of Statistics (NBS). The survey includes all industrial firms that are either state-owned, or are non-state firms with sales above 5 million RMB. There are plant-level identifiers that

There are 1,557,915 plant-by-year observations at 346,434 unique plants. For plants observed multiple times, only about 1 percent have gaps between yearly observations.

D.3 Indian manufacturing

We use India's Annual Survey of Industries (ASI) for establishment-level microdata; this dataset is described in more detail in Allcott, Collard-Wexler, and O'Connell (2016). Registered factories with over 100 workers (the "census scheme") are surveyed every year, while smaller establishments (the "sample scheme") are typically surveyed every three to five years. The publicly available ASI includes establishment identifiers that are consistent across years beginning in 1998, but we have plant identifiers going back to 1992. We have a plant-level panel for the entire 1992-2010 sample.

The ASI is comparable to manufacturing surveys in the United States and other countries. Variables include revenues, value of fixed capital stock, total workers employed, total costs of labor, and materials. Industries are grouped using India's NIC (National Industrial Classification) codes, which are closely related to SIC (Standard Industrial Classification) codes.

There are 615,721 plant-by-year observations at 224,684 unique plants. 107,032 plants will be immediately dropped from our estimators because they are observed only once. For plants observed multiple times, 60 percent of intervals between observations are one year, while 91 percent are five years or less.

The mean (median) plant employs 79 (34) people and has gross revenues of 139 million (20 million) Rupees, or in U.S. dollars approximately $3 million ($400,000).

E Standard Errors Across Estimators

Table E.1: Standard errors

Leontief

	China		India		Chile	
	Capital	Labor	Capital	Labor	Capital	Labor
OLS	0.02	0.03	0.01	0.01	0.03	0.05
FE	0.03	0.04	0.01	0.02	0.04	0.06
IV	0.05	0.05	0.02	0.03	0.09	0.13
Control	0.13	0.23	0.05	0.11	0.05	0.13
IV Control	0.12	0.24	0.09	0.19	0.20	0.45

Gross Output with FOC Materials

	China		India		Chile	
	Capital	Labor	Capital	Labor	Capital	Labor
OLS	0.01	0.01	0.00	0.01	0.01	0.03
FE	0.01	0.02	0.01	0.01	0.04	0.04
IV	0.02	0.02	0.01	0.01	0.05	0.07
Control	0.13	0.23	0.05	0.11	0.05	0.13
IV Control	0.12	0.24	0.09	0.19	0.20	0.45

Notes: We report the median standard error across all industries for each dataset. Leontief specification is $Q_{it} = \min\{\beta_m M_{it}, L_{it}^{\beta_l} K_{it}^{\beta_k} \Omega_{it}\}$, and Gross Output with FOC Materials is given by $M_{it}^{\beta_m} L_{it}^{\beta_l} K_{it}^{\beta_k} \Omega_{it}$, where the material coefficient is computed using the median of the material revenue share ($\beta_m = Median[\frac{P_t^M M_{it}}{P_t Q_{it}}]$).

A Estimators: code and implementation

In this section we describe the estimators proposed in this paper in great detail, enough so that they can easily be coded up by other researchers, and code for these estimators is also available in STATA.

A.1 Leontief

1. Estimate ϕ function.

$$q_{it} = \theta_l l_{it} + \theta_k k_{it} + \theta_m m_{it} + \epsilon_{it}$$

   by 2SLS using exogenous regressors $W = [l_{it}, i_{it-1}, m_{it}]$, and obtain $\hat{\phi}_{it} = \theta_l l_{it} + \theta_k k_{it} + \theta_m m_{it}$. Note that the control function (not IV) has $W_{it} = [l_{it}, k_{it}, m_{it}]$ instead — the OLS estimator.

2. For a parameter $\boldsymbol{\beta}$, minimize the criterion $\mathcal{Q}(\boldsymbol{\beta})$ using:

   (a) Compute $\omega_{it} = \hat{\phi}_{it} - \beta_k k_{it} - \beta_l l_{it}$

   (b) Estimate the AR(1) process for productivity, $\omega_{it} = \rho\omega_{it-1}$, by OLS, obtain $\hat{\rho}$. Recover productivity shock $\tilde{\xi}_{it} = \omega_{it} - \hat{\rho}\omega_{it-1}$ – where tilde refers to the presence of the measurement error in capital.

   (c) Compute $\mathcal{Q}(\boldsymbol{\beta})$ as the empirical analogue of the moment condition $\mathbb{E}\left[\tilde{\xi}_{it}\begin{pmatrix} l_{it-1} \\ i_{it-1}\end{pmatrix}\right] = 0$.

$$\mathcal{Q}(\boldsymbol{\beta}) = (\boldsymbol{\xi}\boldsymbol{z})'(\boldsymbol{z}'\boldsymbol{z})^{-1}(\boldsymbol{\xi}\boldsymbol{z}),$$

   where $\boldsymbol{\xi}$ denotes the stacked vector of $\tilde{\xi}_{it}$, and $\boldsymbol{z}$ denotes the stacked vector of $[i_{it-1}, l_{it-1}]$. Note that the control function (not IV) has $Z_{it} = [k_{it}, l_{it-1}]$ instead.

   (d) Find $\hat{\boldsymbol{\beta}}$ as the minimizer of $\mathcal{Q}(\boldsymbol{\beta})$.

A.2 Gross Output (FOC Materials)

1. Estimate β_m

$$\hat{\beta}_m = Median\left(\frac{P^M M_{it}}{PQ_{it}}\right).$$

2. Produce output y_{it} netted out from material contribution.

$$y_{it} = q_{it} - \hat{\beta}_m m_{it}$$

3. Estimate ϕ function.

$$y_{it} = \theta_l l_{it} + \theta_k k_{it} + \theta_m m_{it} + \epsilon_{it}$$

by 2SLS using exogenous regressors $W = [l_{it}, i_{it-1}, m_{it}]$, and obtain $\hat{\phi}_{it} = \theta_l l_{it} + \theta_k k_{it} + \theta_m m_{it}$.

4. For a parameter $\boldsymbol{\beta}$, minimize the criterion $\mathcal{Q}(\boldsymbol{\beta})$ using:

   (a) Compute $\omega_{it} = \hat{\phi}_{it} - \beta_k k_{it} - \beta_l l_{it}$

   (b) Estimate the AR(1) process for productivity, $\omega_{it} = \rho\omega_{it-1}$, by OLS, obtain $\hat{\rho}$. Recover productivity shock $\xi_{it} = \omega_{it} - \hat{\rho}\omega_{it-1}$.

   (c) Compute $\mathcal{Q}(\boldsymbol{\beta})$ as the empirical analogue of the moment condition $\mathbb{E}\left[\tilde{\xi}_{it}\begin{pmatrix} l_{it-1} \\ i_{it-1} \end{pmatrix}\right] = 0$.

   $$\mathcal{Q}(\boldsymbol{\beta}) = (\boldsymbol{\xi}\boldsymbol{z})'(\boldsymbol{z}'\boldsymbol{z})^{-1}(\boldsymbol{\xi}\boldsymbol{z}),$$

   where $\boldsymbol{\xi}$ denotes the stacked vector of $\tilde{\xi}_{it}$, and $\boldsymbol{z}$ denotes the stacked vector of $[i_{it-1}, l_{it-1}]$.

   (d) Find $\hat{\boldsymbol{\beta}}$ as the minimizer of $\mathcal{Q}(\boldsymbol{\beta})$.