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Optimal Tax Administration  
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**ABSTRACT**

This paper sets out a framework for analyzing optimal interventions by a tax administration, one that parallels and can be closely integrated with established frameworks for thinking about optimal tax policy. At its heart is a summary measure of the impact of administrative interventions—the “enforcement elasticity of tax revenue”—that is a sufficient statistic for the behavioral response to such interventions, much as the elasticity of taxable income serves as a sufficient statistic for the response to tax rates. Amongst the applications are characterizations of the optimal balance between policy and administrative measures, and of the optimal compliance gap.

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# 1 Introduction

This paper sets out some simple analytics of optimal tax administration, focusing on three questions.

The first concerns the usefulness, or otherwise, of the concept of the ‘compliance gap:’ the difference between the amount of tax legally due and that actually collected. This is an intuitively appealing indicator of the effectiveness of a revenue administration—with clear advantages, for instance, over the simple comparison of cost-revenue ratios (administration (and/or compliance) costs relative to revenue raised) that has traditionally been a focus in assessing the performance of tax administrations. Reflecting this appeal, the calculation and analysis of compliance gaps has become a major focus of concern in the last few years—and interest continues to grow: they are now regularly produced for a range of taxes in the U.S. by the Internal Revenue Service (IRS, 2012) and in the U.K. by HMRC, for example, and for the VAT in the member states of the European Union.<sup>1</sup> While many technical questions arise in measuring and analyzing compliance gaps, a more fundamental criticism is leveled by Gemmell and Hasseldine (2014). They stress that its mechanical construction abstracts from behavioral responses, which means, for instance, that measures which reduce the compliance gap may also reduce real activity and hence perhaps even revenue: an activity may be worth undertaking if the associated tax collected is below that legally due, but not if that tax is fully remitted. The welfare impact of administrative interventions thus cannot be inferred simply from associated changes in the compliance gap. Given too the costs of implementing such interventions, for both government and taxpayers, it is clear that—while much analysis often proceeds on the implicit presumption that, whatever the compliance gap is, it is too big—the optimal compliance gap is not zero. The question then arises: What is the ‘optimal’ compliance gap? More generally, how can we know whether an observed compliance gap is too big or too small?

The limitations of the compliance gap and other indicators in assessing the welfare impact of administrative actions point to a second and still more basic question: What do policy makers need to know about the costs and effectiveness of administrative interventions in order to set them optimally? The analogous question on the policy side has received considerable attention, with much of the recent literature focusing on the circumstances in which the elasticity of a reported tax base with respect to the net-of-tax statutory rate<sup>2</sup>—is a sufficient statistic for the choice of marginal tax rate. An extensive empirical literature focuses on estimating this elasticity, particularly for income taxes, in which setting it is known as the elasticity of taxable income.<sup>3</sup> But what of the administration side? The characterization of optimal interventions has

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<sup>1</sup>See HMRC (2015) and European Commission (2015) and, on tax gap analysis more generally, Keen (2013) and IMF (2015).

<sup>2</sup>That is, unity minus the tax rate.

<sup>3</sup>An extensive review of the literature on the elasticity of taxable income is provided by Saez, Slemrod and Giertz (2012).

certainly received some attention (starting with the classic treatment in Mayshar (1991)), but leaves open the question of whether there are concepts analogous to the taxable income elasticity that might prove equally useful in guiding empirical work on the under-studied issue of the proper extent and design of administrative interventions.

The third question is at the heart of much practical policy debate. Policy makers facing a need to raise more revenue have broadly two alternatives: to raise rates, or to devote more resources to improving tax compliance. Rhetoric on this abounds, but theory has provided little guidance on this most basic of policy choices: Is it better to raise an additional dollar of revenue by raising statutory tax rates or by strengthening tax administration so as to improve compliance? Closely related to this is the question of how constraints on one dimension of tax system design affect the optimal choice of the other: if, for instance, tax administration is weaker than would ideally be the case, does that call for higher tax rates than would otherwise be optimal, or for lower?

The key to the answers to these questions set out in this paper is the concept of the *enforcement elasticity of tax revenue*: the responsiveness of revenue collected to administrative interventions (one such elasticity, in principle, for each instrument of administration). This, it will be seen, is the administration-side analogue to the elasticity of taxable income, acting, in the same way, as a sufficient statistic for the behavioral impact of administrative interventions that encompasses effects on the levels of both true and concealed activities. At an optimum, the enforcement elasticity is equated to a straightforward variant of the usual cost-revenue ratio, a simple rule that also provides a clear role for a quantity that, with little theoretical rationale, has been a center of attention in the traditional literature on (and practice of) tax administration (dating back at least to Sandford (1973)). And the choice between policy and administrative measures turns on the balance between these two elasticities, along with an even simpler form of the cost-revenue ratio. The optimal compliance gap is characterized, in a benchmark case, by a simple inverse elasticity rule, the relevant elasticity in this case being that of evasion with respect to administrative intervention; more generally, the optimal gap also reflects the distinct reactions to administrative actions of both real earnings (as stressed by Gemmell and Hasseldine (2014)) and (legal) avoidance.

Section 2 establishes core results in a simple benchmark case, and section 3 then considers a variety of extensions. Section 4 assesses the state of empirical knowledge on enforcement elasticities, and section 5 concludes.

## 2 Analyzing Tax Administration: A Simple Framework

To start with a simple and standard case (along lines similar to both Chetty (2009) and Slemrod (2001))—taking this as a metaphor for wider circumstances in which both earning and concealing a tax base are costly—consider a representative individual with quasi-linear preferences (this being the first of several assumptions relaxed in the next section) of the form

$$W = x - \varphi(l) + v(g) \tag{1}$$

where  $x$  denotes private consumption,  $l$  hours worked and  $g$  public spending from which the consumer directly benefits;  $\varphi$  and  $v$  are both strictly increasing and, respectively, strictly convex and concave. Public spending is financed by a proportional tax on income at rate  $t$ ,<sup>4</sup> so that consumption is given by

$$x = wl - t.(wl - e) - c(e, \alpha) \tag{2}$$

where  $w$  is the (exogenous) wage rate,  $e$  the amount of income not revealed to the tax authorities,  $c$  the private costs associated with that failure to reveal (discussed further in a moment), while  $\alpha$ —central in what follows—is some enforcement parameter<sup>5</sup> that is at the control of the tax administration. This last is defined so that higher values of  $\alpha$  increase both the private costs of inaccuracy in revealing income and the marginal cost of inaccuracy, so that  $c_\alpha > 0$  and  $c_{e\alpha} > 0$ .<sup>6</sup> For the present, we take  $\alpha$  to be a single continuous variable, such as the probability of audit or the ease of remitting payment. In practice, of course, there are many types of intervention available to tax authorities, and, somewhat less straightforward to handle, many important administrative reforms (such as the introduction of a large taxpayer unit) that are best thought of as discrete changes. These two possibilities are among the extensions considered in Section 3.

We refer to  $c$  interchangeably as costs of compliance or of concealment. The former is the more familiar term in the literature (by which we will calibrate some of our results), and calls to mind the costs that may be incurred by the effort to reveal taxable income with full accuracy (a complex form to complete, for instance), so that  $c_e < 0$ . The latter term, on the other hand, calls to mind the costs incurred in hiding income from the tax authorities (which might reflect not only the risk of penalty, distortion of commercial

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<sup>4</sup>There is, thus, no demogrant; the implications of nonlinear taxation are considered after Proposition 1 below.

<sup>5</sup>Describing administrative interventions in terms of enforcement might seem to neglect the encouragement of voluntary compliance that many tax administrations see as a core part of their work. But this usage is for brevity only, and the framework here can be interpreted to encompass, for example, not only what Alm (2014) calls the 'enforcement paradigm' of tax administration but also his 'service' and 'trust' paradigms. Measures which encourage voluntary compliance, for example, such as prepopulating returns, can be seen as increases in  $\alpha$  that make it easier to be honest (equivalently, harder to be dishonest), so that  $c_{e\alpha} > 0$ , just as is assumed below.

<sup>6</sup>Derivatives are indicated by subscripts for functions of several variables, and by primes for functions of just one.

activities and the like, but an intrinsic preference for honesty), in which case  $c_e > 0$ . Both concerns can be accommodated within the general formulation here. It might well be, for instance, that  $c$  is U-shaped in  $e$ , with some least-cost level of under-declaration relative to which it is costly to be either less or more dishonest. No taxpayer, however, will under-declare at a point at which  $c_e < 0$ , since a little more concealment would then reduce both tax paid and non-tax costs  $c$ . For simplicity, we therefore assume throughout that  $c_e > 0$  and  $c_{ee} > 0$ .

Substituting (2) into (1), with  $g$  exogenous, the individual's choices of hours worked and concealment are characterized, respectively, by the necessary conditions  $(1 - t)w - \varphi'(l) = 0$  and  $t - c_e(e, \alpha) = 0$  (so that, as just noted,  $c_e > 0$  in equilibrium), which define solutions  $l(t, w)$  and  $e(t, \alpha)$ , with  $e_\alpha = -c_{e\alpha}/c_{ee} < 0$ . Note that labor supply is independent of enforcement, reflecting the independence of concealment costs from income; this means that the Gemmell-Hasseldine (2014) critique cannot be captured in this setting, a point returned to later.

Administration costs are taken to be simply  $a(\alpha)$ , with  $a' > 0$ , so that the government's budget constraint is

$$g + a(\alpha) = t.(wl - e), \quad (3)$$

which, combined with (1) and (2), implies social welfare of

$$W(t, \alpha) = wl - t.(wl - e) - c(e, \alpha) - \varphi(l) + v(t.(wl - e) - a(\alpha)), \quad (4)$$

the assumption here being that private concealment costs are real social costs, not transfers (such as fines); this too will be relaxed later.

For the government's choice of tax rate, differentiating in (4) and using the envelope property gives the necessary condition<sup>7</sup>

$$W_t = -z + v'.(z + tz_t) = 0, \quad (5)$$

where

$$z(t, \alpha) \equiv wl(t, w) - e(t, \alpha) \quad (6)$$

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<sup>7</sup>It is sufficient for the second-order condition to be satisfied that  $z_{tt}$  (the sign of which depends on third derivatives of  $l(t)$  and  $c(e, \alpha)$ ) be non-positive.

denotes taxable income. Rearranging this gives the well-known condition

$$\frac{t}{1-t} = \left( \frac{v' - 1}{v'} \right) \frac{1}{E(z, 1-t)}, \quad (7)$$

where  $E(m, n)$  denotes the elasticity of  $m$  with respect to  $n$ . This is the familiar result that the elasticity of (reported) taxable income,  $E(z, 1-t)$ , is a sufficient statistic for the marginal welfare impact of a change in the tax rate, with a higher elasticity implying, all else equal, an optimally lower tax rate. This evidently remains true in the presence of administration costs of the form  $a(\alpha)$ . By way of illustration, suppose (here and in later examples) that the marginal social value of spending  $v'$  is 1.2, and the elasticity of taxable income is 0.25 (around the mid-point for the long-run elasticity from the review in Saez et al. (2012)). Then (7) implies an optimal (tax-inclusive) tax rate of 40 percent.

### Optimal administrative interventions and the enforcement elasticity

Now, however, the government also determines the extent of its enforcement activity,  $\alpha$ . For this less studied margin of choice, the necessary condition<sup>8</sup> from (4), again using envelope properties, is

$$W_\alpha = -c_\alpha + v' \cdot (tz_\alpha - a_\alpha) = 0, \quad (8)$$

so that the additional revenue gained from stricter enforcement is equated to the associated additional compliance and administration costs, with the latter weighted more heavily than the former because they need to be paid for from distorting taxation. This is a standard condition for the optimal size of a tax administration,<sup>9</sup> one immediate and familiar implication being that the real resource cost to the private sector of complying with its tax obligations means that it is generally not optimal to take enforcement up to the point at which tax revenue, net of administration costs, is maximized (unless the marginal social value of additional revenue is infinitely large). What does not seem to have been noted, however, is that this condition can be expressed as a simple elasticity rule: an administration side analogue to (7). For this, define

$$\phi \equiv \frac{\alpha(c_\alpha/v') + \alpha a_\alpha}{tz} \quad (9)$$

to be the *adjusted marginal cost-revenue ratio*, in the sense that the numerator is a linear approximation<sup>10</sup> to the sum of compliance and administration costs, with a greater weight on the latter for the reason just

<sup>8</sup>The second-order condition is satisfied if  $z_{\alpha\alpha} \leq 0$ , which again depends on third derivatives.

<sup>9</sup>This condition is implicit in Mayshar (1991).

<sup>10</sup>Given the further reasonable assumptions that  $c(e, 0) = a(0) = 0$ .

noted. Then it is straightforward to write (8) as

$$\phi = E(z, \alpha), \tag{10}$$

where, for reasons we comment on further below,  $E(z, \alpha)$  will be referred to as the *enforcement elasticity of tax revenue*. Thus:

PROPOSITION 1: *At an optimum, the enforcement elasticity of tax revenue is equal to the adjusted marginal cost-revenue ratio.*

The enforcement elasticity is thus a sufficient statistic for the behavioral response to administrative measures in much the same way that the more familiar elasticity of taxable income is sufficient in respect of rate changes. As in that case, the distinct effects on declared and concealed incomes are immaterial, with only the impact on reported taxable income  $z$  mattering. And the reason is the same: this is all that is relevant for the government’s revenue concern, and marginal shifts between the two have no impact on consumer welfare. Summarized as an elasticity, this effect is then balanced against the associated marginal implementation costs, recognizing the difference that administration costs are socially more costly than compliance costs. (Proposition 1 can of course also be expressed as an administration-side inverse elasticity rule, a twin to the Ramsey rule or (7) above, equating the adjusted marginal revenue-cost ratio to the inverse of the enforcement elasticity).

A word on terminology. Given the assumption here of a proportional income tax, the elasticity of reported taxable income  $E(z, \alpha)$  is identical to the elasticity of tax collected (gross of administration costs)  $T$ , say, where in this case  $T = tz$ . In more general circumstances, it is the latter,  $E(T, \alpha)$ , that is the relevant quantity; as is easily seen by reworking the analysis above for the case of a piecewise linear tax schedule.<sup>11</sup> Hence it is the elasticity of tax revenue that is relevant for administrative actions, rather than that of taxable income which is the focus in considering the choice of tax rate.

Notable too is that Proposition 1 establishes a clear analytical significance for the cost-revenue ratios that much of the literature on tax administration has traditionally been focused on. What is needed, ideally, are estimates of the *marginal* compliance and administration costs associated with changes in enforcement interventions. These, however, are not readily available. Some progress can be made by supposing—temporarily—that implementation costs are linearly homogenous in enforcement efforts, so that  $\alpha c_\alpha = c(\alpha)$  and  $\alpha a_\alpha = a(\alpha)$ . Then the right hand side of (9) is exactly the sum of compliance and administration costs relative to revenue

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<sup>11</sup>In this case, tax paid can be written as  $T\{z(\alpha, t)\} = t(z - h) + T_0$ , where  $h$  is the lower bound of the reported income range over which the marginal rate  $t$  applies and  $T_0$  is the tax owed on income of  $h$ . Quasi-linearity of preferences means that the first-order conditions both of the individual and for welfare maximization remain as above. Since  $tz_\alpha = T_\alpha$ , rearranging (8) and dividing by  $T$  gives  $\phi^* = E(T, \alpha)$ , where  $\phi^* \equiv (\alpha(c_\alpha/v') + a_\alpha)/T$ .

collected, with the former divided by the marginal value of public funds—which are quantities that have received considerable attention in the literature. We do not attempt a comprehensive review of the extensive literatures on estimating compliance<sup>12</sup> and administration costs,<sup>13</sup> but to put flesh on Proposition 1 take as illustrative values that may reasonably appropriate in thinking about the personal income tax in the United States: the cost of the IRS being about 0.6 percent of revenue collected, we take  $a/z = 0.006$  and, following Slemrod (2004), who puts the cost of complying with the personal income tax in the United States at around 11 percent of revenues, take  $c/z = 0.011$ .<sup>14</sup> With  $v' = 1.2$  as above, Proposition 1 then implies an enforcement elasticity at the optimum of around 0.1; an elasticity higher than this would warrant additional spending on implementation.

More plausibly, however, and as assumed in the analytics, one might suppose both compliance and administration costs to be convex in  $\alpha$ , in which case  $\phi$  exceeds this adjusted sum of implementation costs. Proposition 1 then implies that, at an optimum,

$$\frac{(c(e, \alpha)/v') + a(\alpha)}{tz} \leq E(z, \alpha). \quad (11)$$

Observed adjusted average implementation costs thus set a lower bound on what the enforcement elasticity must be if the observed position is optimal. But all this, of course, begs the question of the likely value of the enforcement elasticity, which is taken up in section 4.

### The optimal compliance gap

As noted in the introduction, tax administrations are now showing great interest in measuring compliance gaps: the difference between tax legally due and that collected. In the current context, that gap is  $g \equiv$

<sup>12</sup>Estimates of compliance costs have been made in many countries, but comparability between them is problematic because of differences in methodology as well as the conceptualization of compliance costs itself (for example, whether these should be conceived of as pertaining to social or to private costs, the latter implying that one should subtract private benefits of tax remittance such as interest from delayed tax remittance) and which kinds of taxes are referred to. Evans (2003) provides an overview of this literature as of more than a decade ago, revealing that the range of estimates is huge, from 0.45 percent of revenue for U.K. excise taxes to between 49 percent and 56 percent for the personal income tax in India, with the estimates being on average much higher in developing countries, followed by Australia and New Zealand, then the United States and Canada, with the lowest estimates from Europe. None of these studies seriously attempts to estimate the *marginal* compliance costs with respect to tax-system instruments. Slemrod (1989) uses micro data from a taxpayer survey of compliance costs to understand its determinants. The analysis suggests that the taxpayer hours part of compliance costs go up with a taxpayer's marginal tax, but not to a significant degree; the dollars spent for professional assistance does have a positive significant association with the marginal tax rate, but in one of the two specifications this appears only for itemizers.

<sup>13</sup>OECD (2015, Table 5.4) presents values of  $a/z$  for 53 countries for 2013. These are not directly comparable because the definitions of both  $a$  and  $z$  vary across countries. For the U.S. and U.K. the numbers are 0.47 percent and 0.73 percent respectively. Of the 31 OECD countries with comparable data, the median is 0.85 percent. For the 19 non-OECD countries for which data are provided, the median is 0.87 percent. The report does not investigate the marginal administrative cost with respect to tax enforcement instruments.

<sup>14</sup>Note that the administrative cost ratio here refers to all taxes that the IRS handles, while the compliance cost ratio refers to the income tax alone. Using them in conjunction is not problematic so long as the cost ratios for income and other taxes are broadly similar.

$te/twl = e/wl$ . Noting from (6) that the enforcement elasticity can be written as

$$E(z, \alpha) = - \left( \frac{e}{z} \right) E(e, \alpha), \quad (12)$$

where  $E(e, \alpha) < 0$  is the elasticity of evasion with respect to enforcement, and that  $e/z = g/(1-g)$ , it follows from (10) that the result in Proposition 1 implies that:

PROPOSITION 2: *At an optimum, the compliance gap is given by*

$$\frac{g}{1-g} = \frac{-\phi}{E(e, \alpha)}. \quad (13)$$

The optimal compliance gap is thus characterized by a simple inverse elasticity rule, with the relevant elasticity being the enforcement elasticity of evasion  $E(e, \alpha)$ , and the factor of proportionality being the adjusted marginal cost-revenue ratio.

Given the increasingly common practice of quantifying compliance gaps, the condition in (13) can be turned, potentially, to give some sense of the appropriateness of enforcement efforts: the aggregate compliance gap of 14.5 percent reported by the IRS,<sup>15</sup> for instance, can be optimal in this setting only if a 10 percent increase in spending on the IRS would reduce evasion by about 5 percent—and the gap ought to be lower if (and only if) the impact on evasion is greater. Judging whether or not this is likely to be the case, and applying Proposition 2 more generally, requires empirical knowledge of the responsiveness of evasion to administrative interventions, of which there is very little.<sup>16</sup>

### **Balancing policy measures and administrative interventions**

The conditions (5) and (8) for the optimal choice of tax rate and enforcement each hold, of course, even if the other policy is not optimally set. The nature of the implied relationship between the two, however, is not immediately obvious. Suppose, for instance, that administrative intervention  $\alpha$  is for some reason constrained below its level at the full optimum. Is the best response to set the statutory tax rate higher than at the full optimum, in an attempt to offset the weak administration? Or is it to set a lower tax rate, so as to limit the erosion of the base by under-controlled evasion? Applying the implicit function to the first-order condition for the choice of tax rate, equation (5), and assuming for simplicity that  $v'$  is constant, the sign of the slope of the best response  $t(\alpha)$ <sup>17</sup> is given by

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<sup>15</sup>Internal Revenue Service (2012).

<sup>16</sup>Gorodnichenko, Martinez-Vasquez and Peter (2012), which examines this in the context of the flat tax reform in Russia, is a rare example.

<sup>17</sup>The sign of the slope of the best response  $\alpha(t)$ , defined by equation (8), is of course the same. Stability of adjustment to the full equilibrium in line with best responses is implied by concavity of  $W(t, \alpha)$ .

$$W_{t\alpha} = -(v' - 1)e_\alpha - v'te_{t\alpha} \quad (14)$$

and hence, recalling that  $e_\alpha < 0$  :

PROPOSITION 3: *It is sufficient (but not necessary) for  $dt/d\alpha > 0$ , so that the statutory tax rate and enforcement are strategic complements, that  $e_{t\alpha} < 0$ .*

The direction of the relationship between the two policy variables is thus indeed ambiguous, with a key role played by  $e_{t\alpha}$ . If, as might plausibly be supposed, the marginal impact of a higher tax rate in encouraging evasion,  $e_t > 0$ , is dampened by more effective administration, the two are strategic complements: the best response to weaker administration is to set a lower tax rate than would otherwise be the case; conversely, the best response to a higher tax rate is greater administrative effort.

Combining the two necessary conditions, this analytical framework also gives some simple insights into one of most basic problems faced by policy makers: whether to finance additional public spending revenue by increasing statutory tax rates or by administrative strengthening to reduce noncompliance. Calculating the welfare impact of increasing  $\alpha$  while reducing  $t$  so as to leave unchanged public spending  $g$  (spending, that is, other than on administration itself) gives:<sup>18</sup>

PROPOSITION 4: *At the margin, it is better to raise additional revenue by strengthening enforcement than by raising the statutory tax rate if and only if*

$$E(z, \alpha) > \frac{\alpha(c_\alpha + a_\alpha)}{tz} - E(z, 1 - t) \left( \frac{t}{1 - t} \right) \left( \frac{\alpha c_\alpha}{tz} \right). \quad (15)$$

At a full optimum, of course, this condition holds as an equality.<sup>19</sup>The proposition speaks, rather, to the policy makers' problem of finding that optimum. It implies—as one might now expect—that, starting from some arbitrary initial position, the case for raising additional revenue by spending additional resources on enforcement rather than by raising tax rates is stronger the higher is the enforcement elasticity (because administrative measures are then more productive of revenue) and the higher is the usual elasticity of taxable income (because that means a higher welfare cost of a tax rate increase). Lower (unadjusted) marginal

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<sup>18</sup>That is, evaluating

$$dW = W_t \frac{dt}{d\alpha} |g + W_\alpha$$

using the implication of (3) that

$$\frac{dt}{d\alpha} |g = \frac{a_\alpha - tz_\alpha}{z + tz_t}$$

and assuming that  $tz_\alpha - a_\alpha > 0$  in the initial position.

<sup>19</sup>This can be seen by substituting for  $v'$  in (10) from (7).

implementation costs, relative to initial revenue, also naturally favor enforcement actions, but their composition matters: higher compliance costs matched by reduced administrative costs point towards the use of enforcement actions because, as noted earlier, their financing does not come from distortionary taxation. All else equal, a higher tax rate also favors enforcement.

This condition again invites illustrative calculations. With the same assumptions as above, for instance, at a (tax-inclusive) tax rate of 40 percent, enforcement is the preferred source of additional revenue if and only if the enforcement elasticity is 0.1 or more.

### 3 Extensions

The framework above is of course highly stylized. This section considers a series of generalizations, in each case doing so relative to that simple case, beginning with two that are very straightforward before turning to others that bring in more subtle considerations: transfer costs, multiple households, more general forms of concealment cost, and discrete reforms.

#### Multiple instruments, fixed administrative budget

Tax administrations of course have many types of administrative intervention at their disposal (numbers and intensity of desk and field audits, quality and availability of customer services, the intensity of debt recovery actions...). This is readily incorporated in the framework above. Thinking now of  $\alpha$  in (4) as a vector of instruments,  $\alpha \equiv (\alpha_1, \dots, \alpha_n)$ , the necessary condition on  $\alpha_k$  is that

$$E(z, \alpha_k) = \phi_k \tag{16}$$

where  $\phi_k = (\alpha_k(c_{\alpha_k}/v') + \alpha_k a_{\alpha_k})/tz$ . This gives a simple analogue of Proposition 1: at an optimum, the elasticity of revenue with respect to the  $k$ th instrument is equated to its associated adjusted marginal cost-revenue ratio. Proposition 2 also generalizes very readily: for each instrument, the ratio of the adjusted marginal cost-revenue ratio to the enforcement elasticity of evasion is equated to the common value of the (gap-exclusive) compliance gap.

Over the shorter term at least, tax administrations may also face the constraint of an overall budget constrained below its fully optimal level. Adding to the government's problem the constraint (which we assume to bind) that  $a(\alpha) \leq \bar{a}$ , it is readily shown<sup>20</sup> that  $E(z, \alpha_i) - \phi_i > 0$  for all  $i$  and that for any two instruments  $\alpha_k$  and  $\alpha_j$ ,

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<sup>20</sup>Assuming that  $\alpha_j a_{\alpha_j} > 0$ .

$$\frac{E(z, \alpha_k) - \phi_k}{E(z, \alpha_j) - \phi_j} = \frac{\alpha_k a_{\alpha_k}}{\alpha_j a_{\alpha_j}}. \quad (17)$$

An administrative budget fixed below its optimal level thus requires that the revenue elasticity for each instrument optimally remains above its adjusted cost-revenue ratio, to an extent that is greater, all else equal, for instruments that are administratively more costly.

### General preferences

The key result on the significance of the enforcement elasticity, Proposition 1, remains unchanged if preferences are instead of the general form  $W(x, l, r)$ , except that the marginal valuation of the public good becomes  $W_r/W_x$  rather than  $v'$ ; and the same is true of all results that rely on the manipulation of the first-order conditions for welfare maximization (including Proposition 4 on the balance between policy and administrative measures). What does change are results that rely on the independence of real earnings from enforcement, which typically does not hold in this case; the implications of this emerge more clearly in the discussion of general implementation costs below.

### Transfer costs

Suppose that, as in Chetty (2009), some proportion  $\mu$  of the compliance costs  $c(e)$  incurred by the taxpayer involves not real resource use but rather transfers to others—such as, for instance, fines or any part of payments to tax advisers that represent rents; and that the social value of these exactly equals the cost to the taxpayer. The costs in (4) then need to be replaced by  $(1 - \mu)c(e)$ : only the non-transfer component is a genuine social cost. In this case, the elasticity of taxable income ceases to be a sufficient statistic for evaluating behavioral response to a tax rate change: as Chetty (2009) shows, it is replaced by weighted average of that and the elasticity of labor supply with respect to the tax rate.

From the administrative perspective, however, it is easily seen that the enforcement elasticity remains sufficient, the only change to the analysis above being that the clarification that the relevant marginal cost-revenue ratio  $\phi$  should reflect only the proportion  $1 - \mu$  of compliance costs that relate to use of real resources. With this natural amendment, Propositions 1 to 4 continue to apply as stated above.

### Many households

Now suppose that there are many households, differing (only) in their wage rate,  $w_h$ , so that, denoting indirect utility by  $v(w, t, \alpha)$ , the objective function becomes, in straightforward notation,  $\Omega[v(w_1, t, \alpha), \dots, v(w_n, t, \alpha)]$  for some social welfare function  $\Omega$ .

Maximizing with respect to  $t$ , the condition analogous to (7) now involves individual-specific elasticities of taxable income,  $z_h$ , weighted by corresponding social marginal valuations.<sup>21</sup> Maximizing with respect to  $\alpha$ , however, the independence of evasion from the wage rate and additive separability of  $u$  in spending  $g$  are readily seen to imply that equation (8) continues to apply, and hence so too do Propositions 1 through 3. This robustness of those results—with distributional considerations material to the relevance of the aggregate taxable income elasticity but not to that of the aggregate enforcement elasticity—is striking, though it is clearly sensitive to the commonality of the extent of evasion across all households, which is implied by the strong assumptions on the structure of implementation costs that we turn to next.

### More general implementation costs

The formulation of implementation costs  $c(\cdot)$  above is restrictive in at least three respects. First, it assumes that the costs of concealment are independent of earnings, which carries the implication that real effort  $l$  is unaffected by enforcement activities  $\alpha$ : one might instead expect it to be easier to hide any given dollar amount income the greater is true income. Second, it also fails to distinguish between evasion and avoidance (meaning by the latter the reduction of legal liability other than by reducing real earnings). This matters, in particular, in calculating compliance gaps, which usually—though not always<sup>22</sup>—consider only illegal non-payment. These two considerations point to concealment costs of the more general form  $c(wl, e, s, \alpha)$ , where  $s$  denotes the amount of income on which tax is avoided, so that reported taxable income is now

$$z = wl - e - s. \tag{18}$$

The third limitation is in assuming that administration costs are independent of the scale of the tasks facing the administration, one natural way of allowing for monitoring and processing costs being to now suppose that these take the more general form  $a(\alpha, z)$ , with  $a_\alpha > 0$ .

With these generalizations, the objective in (4) is replaced by

$$W(t, \alpha) = wl - t.(wl - e - s) - c(wl, e, s, \alpha) - \varphi(l) + v(t.(wl - e - s) - a(\alpha, wl - e - s)) \tag{19}$$

where now the individual's choices are described by functions  $l(t, \alpha)$ ,  $e(t, \alpha)$  and  $s(t, \alpha)$ .<sup>23</sup> With real labor decisions now affected by enforcement, and three choice variables, the comparative statics of behavioral response are evidently now much more complex. The appendix considers the special case in which there is no

<sup>21</sup>As discussed, for example, in Slemrod and Gillitzer (2014).

<sup>22</sup>In the U.K., for instance, official estimates of the VAT gap include avoidance.

<sup>23</sup>Dependence on the wage rate, inessential from here, is suppressed.

avoidance, showing that few clear-cut results emerge. Quite subtle restrictions on the structure of concealment costs  $c(wl, e, \alpha)$  are needed, for instance, even to be sure that  $E(z, 1 - t)$  and  $E(z, \alpha)$  are positive. There is thus a hint, for instance, not wholly implausible, of a Laffer-type effect by which enforcement can be taken to a point at which it reduces reported income, though of course—and as will be verified momentarily—this cannot be the case at an optimum.<sup>24</sup>

In this more general setting, the necessary condition on the choice of tax rate implies that

$$\frac{t - a_z}{1 - t} = \left( \frac{v' - 1}{v'} \right) \frac{1}{E(z, 1 - t)}. \quad (20)$$

The elasticity of taxable income thus remains sufficient; the only change relative to the usual result in (7) is that, all else equal, the optimal tax rate is higher: because dealing with taxable income is costly, the attractions of expanding it by reducing the tax rate are lower.

In terms of optimal enforcement, defining

$$\Phi \equiv \frac{\alpha(c_\alpha/v') + \alpha a_\alpha}{tz - za_z} \quad (21)$$

and rearranging the necessary condition on the choice of  $\alpha$  gives:

PROPOSITION 5: *In this more general setting, optimal enforcement is characterized by the condition*

$$\Phi = E(z, \alpha). \quad (22)$$

This differs only very modestly from the condition for the basic case in Proposition 1, the only change (comparing  $\Phi$  and  $\phi$ ) being that (approximated) costs are now expressed relative not to gross revenue  $tz$  but to revenue net of approximated marginal administration costs,  $t - za_z$ .

The implied characterization of the optimal compliance gap changes more fundamentally. Recognizing that the income on which tax is legally payable is reduced by the amount of avoidance, this gap is now naturally expressed as

$$G \equiv \frac{e}{wl - s}, \quad (23)$$

while, recalling that  $z = wl - s - e$ , the enforcement elasticity can be expressed as a weighted average of the elasticities with respect to enforcement of evasion and of legally liable income:

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<sup>24</sup>Indeed equation (A.8) in the appendix leaves open the possibility that tighter enforcement might even increase the amount of evasion,  $e$ : by increasing the marginal value of true income in concealing evasion, it might lead to an increase in true income which then reduces the marginal cost of concealment and so tends to increase evasion—an effect that (absent other restrictions) it seems might dominate others tending to reduce evasion.

$$E(z, \alpha) = -\frac{e}{z}E(e, \alpha) + \left(\frac{wl - s}{z}\right)E(wl - s, \alpha). \quad (24)$$

Using (23) and (24) into (22) gives:

PROPOSITION 6: *In this more general setting, the optimal compliance gap satisfies*

$$\frac{G}{1 - G} = \frac{-(\Phi - E(wl - s, \alpha))}{E(e, \alpha) - E(wl - s, \alpha)}. \quad (25)$$

Relative to Proposition 2, there are two differences in the characterization of the optimal compliance gap.<sup>25</sup> The first is straightforward: to the extent that processing taxable income is costly, this (through the term in  $\Phi$ ) tends to imply a higher optimal gap, as one would expect. The second operates through the elasticity of taxable income,  $wl - s$ . If tougher enforcement leads to a reduction in legally taxable income, so that  $E(wl - s, \alpha) < 0$ , this implies an optimal compliance gap that is higher than the simple inverse elasticity rule of Proposition 2 implies. The reason is clear: additional enforcement is undesirable to the extent that it worsens the tax-induced reduction in true income  $wl$  (along the lines of the argument in Gemmell and Hasseldine (2014)) or simply shifts the taxpayer from evading to avoiding. Plausible though it may be to expect such effects, however, they are not theoretically assured, given the ambiguities noted above; the sign of  $E(s, \alpha)$ , for instance, seems likely to depend on whether evasion and avoidance are complements or substitutes.

### Discrete administrative reforms

Many administrative changes—both major, such as movement to a semi-autonomous revenue agency, and more routine, such as a change in reporting requirements—are best thought of discrete changes in  $\alpha$ , from  $\alpha_0$  to  $\alpha_1$ , say. To see how these can be accommodated in the present setting, it is clearest, in returning to the basic case of section 2, to suppose that  $v'$  is constant. With labor supply then unaffected by this reform, the change in welfare implied by (4) is, in obvious notation,

$$\Delta W \equiv W_1 - W_0 = (v' - 1)t\Delta z - v'\Delta a - \Delta c \quad (26)$$

where  $\Delta c \equiv c(e_1, \alpha_1) - c(e_0, \alpha_0)$ , rearranging which implies:

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<sup>25</sup>We are unable to sign  $E(e, \alpha) - E(wl - s, \alpha)$ , but assume it be strictly negative; re-interpretation of the proposition is straightforward if this is not the case, but with the counter-intuitive implication that, for instance, higher marginal administration costs  $a_\alpha$  imply an optimally smaller compliance gap.

PROPOSITION 7: *Assuming  $v'$  to be constant, a discrete administrative reform from  $\alpha_0$  to  $\alpha_1$  is welfare-improving if and only if*

$$\left(\frac{v' - 1}{v'}\right) \left(\frac{\alpha_1 \Delta z}{z_1 \Delta \alpha}\right) \geq \frac{\alpha_1 (\Delta a / \Delta \alpha) + \alpha_1 (\Delta c / v' \Delta \alpha)}{tz_1}. \quad (27)$$

Comparing with the condition in Proposition 1, the most obvious difference is that the (now arc) enforcement elasticity is scaled by the marginal value of public funds: the higher is that valuation, the more likely it is (even apart from the effect through the compliance cost term) that a project which raises revenue will increase welfare. This is perhaps as one would have expected, the puzzle rather being why this effect does not appear in the continuous case of Proposition 1. The explanation, and reconciliation between the two results, lies in the  $\Delta c$  term in (27). This captures not only the direct effect from the administrative reform, corresponding to  $c_\alpha$  in Proposition 1, but also that through the induced change in evasion, corresponding to  $c_e e_\alpha$ . The effect on concealment costs of the latter partly offsets that from the change in revenue; and, as an envelope property, does so exactly for a small reform, in which case (27), taken as an equality, reduces to the condition in Proposition 1.<sup>26</sup>

## 4 What do we know about enforcement elasticities?

The analysis above points to the centrality in designing administrative interventions of understanding and measuring enforcement elasticities—or, more generally, the marginal revenue and costs associated with administrative actions. Tax administrations themselves are of course well aware of this, and commonly cite the additional revenues that they could collect if given additional resources when making their budget submissions. But these claims are rarely made public, and in any event are doubtless to some degree self-serving. Beyond that, however, the issue seems to have received almost no direct empirical attention, in large part because of the difficulty of constructing well-identified and comprehensive estimates.

We are aware of only a few exceptions. Reviewing the evidence, Alm (2012) reports both econometric work and lab experiments as pointing to elasticities of reported income with respect to the audit rate (taking account only of the direct impact) of 0.1 to 0.2, with signs of a decreasing marginal effect. Direct attempts at estimation have been made by the Internal Revenue Service (IRS) in the U.S. In 1986, the then Research Director, Frank Malanga, published a paper that provided marginal revenue-to-cost ratios<sup>27</sup> for four separate

<sup>26</sup>To see this more precisely, write the right-hand side of (27) as

$$(t(e_0 - e_1) + c(e_1, \alpha_1) - c(e_0, \alpha_1)) - v'(z_1 - z_0) - v'(a_1 - a_0) - c(e_0, \alpha_0) + c(e_0, \alpha_1).$$

Dividing by  $\Delta \alpha$  and letting  $\Delta \alpha \rightarrow 0$ , the first term converges to  $-te_\alpha + c_e e_\alpha$ , which, from the necessary condition for the individual's choice of  $e$ , is zero; and the rest converges to the terms in Proposition 1.

<sup>27</sup>Broadly corresponding in the current setting to  $tz_\alpha / \alpha a_\alpha$ .

IRS functions; these ranged between 7.0 (for examination) to 26.9 (for delinquent returns). The methodology is not spelled out, however; it does, though, explicitly exclude the deterrent effect. In more recent work of the same kind, Hodge et al. (2015) estimate, using IRS administrative data, the marginal revenue-to-cost ratio for seven different categories of correspondence (as opposed to face-to-face) audits, where each category relates to one or a small number of specific issues on the tax return (such as specific deduction items) or special issues that are often present on returns filed late. For each category, they rank returns by the calculated indicator or risk score, from most to least likely to reflect noncompliance. For each return, they then calculate (1) the revenue ultimately collected from the audit, appeals, litigation and collection processes and (2) the total (human) resources costs expended in all of those stage. From this information they can calculate the cumulative revenue and cost when moving from the ex-ante most productive audits to marginally less ex-ante productive ones, and from the cumulative totals approximate the marginal cost-revenue ratio at the current level of audits in each category. They calculate that the marginal revenue-cost ratio varies among the seven categories from 4.29 to 9.98; this—using also the data they provide on total revenues and costs—translates into enforcement elasticities of between 0.60 and 0.99.<sup>28</sup> Given also information on associated compliance costs, and a shadow value of public spending, prescriptions would then follow for both the optimal mix between these enforcement activities at any given total budget and the appropriate overall budget for each.<sup>29</sup>

The IRS still refers to marginal revenue yields. For example, the Administration’s fiscal 2016 budget request asserts that the \$421 million sought for traditional revenue-producing initiatives are estimated to generate \$2,799 million in annual enforcement revenue, “achieving an ROI [return on investment] of \$6.4 to \$1.0.” This number does not include the effect enhanced enforcement has in deterring non-compliance. Eleven specific initiatives for which funding was sought are estimated to have ROI’s that range from 1.8 to 20.6. Although this is an insightful methodology, neither the revenue nor the cost measures in this exercise capture the social benefits or cost of more enforcement. The revenue is only the “direct” revenue collected from the audited individual, and therefore excludes the “indirect” revenue collected because more audit coverage translates into a higher perception of detection, which in turn reduces noncompliance through the classic deterrent effect. Nor does it include a network deterrent effect that operates through the communication of information from those audited to others to whom they are connected. If the ratio of direct to indirect effects is uniform, the recommendations for resource re-allocation will be unchanged, but that is a strong and unconfirmed assumption. Moreover, determining the optimal aggregate amount of resources to allocate to auditing activities of all kinds does require taking in to account indirect as well as direct effects, so the

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<sup>28</sup>Calculated from Columns 5 and 6 of Table 5 in Hodge et al. (2015).

<sup>29</sup>Hodge et al. (2015) draw out the implications for the optimal mix at unchanged total administrative spending (thus ignoring compliance costs).

magnitude of this ratio is crucial. The central quantity for our model could be several times higher than the numbers reported above.<sup>30</sup>

IMF (2015) takes a top-down approach to estimating the evasion elasticity, by regressing—for a balanced panel of 27 countries (EU members and Japan) from 2000 to 2011—the estimated VAT compliance gap on total expenditure on tax administration and its lagged change (plus the estimated output gap).<sup>31</sup> The identification is within-country only, given the inclusion of country fixed effects. To address the endogeneity of administrative spending, in one specification the lagged change in spending is instrumented with the lagged output gap. The estimated impact of administrative spending on the compliance gap is significantly negative, as might be expected, while that on the change in spending (when instrumented) is insignificant, which is also plausible given that additional administrative interventions are likely to take some time to have effect. The estimated long-run enforcement elasticity is about 0.17.<sup>32</sup> The interpretation of the estimated impact of total administrative spending is not entirely clear, however, since this is the sum of spending on many different instruments, partly incurred in relation to taxes other than the VAT, and undoubtedly different countries are making different marginal changes in these instruments. It is thus very much a reduced form coefficient, and certainly there is no reason to believe the enforcement elasticity to be the same across all countries in the sample. One merit of the focus on aggregate compliance, however, is that in principle it includes deterrent effects. The analysis, in any event, suggests potential value in the use of multi-country time series of administrative data published regularly by the OECD.<sup>33</sup>

A new wave of empirical literature is using modern techniques to learn about the impact of enforcement techniques, although they have not yet produced estimates of the elasticities our model shows to be critical. For example, many tax authorities have recently proven willing to partner with researchers to design and implement randomized controlled trials to learn about the impact of certain aspects of tax enforcement. The most prominent example is threat-of-audit letters, pioneered by Slemrod et al. (2001), who analyzed the results of a randomized controlled experiment conducted by the State of Minnesota Department of Revenue. Kleven et al. (2011) conduct a similar audit experiment in Denmark; and Pomeranz (2015) uses a similar approach to explore the nature of firm’s compliance along VAT chains in Chile. Ortega and Scartascini (2015) investigate the impact of altering the delivery mechanism of such correspondence from the tax authority. Other recent research within a rapidly growing field of enquiry<sup>34</sup> investigates the impact of

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<sup>30</sup>Reviewing the evidence, Alm (undated) suggests that indirect effects can be 3-5 times as large as direct.

<sup>31</sup>The standard rate of VAT proves insignificant.

<sup>32</sup>Keen (2016) elaborates on this analysis.

<sup>33</sup>And now being compiled, for a wider set of countries, as a joint endeavor (the ‘International Survey of Revenue Administrations’) of the IMF, Inter American Center of Tax Administrations (CIAT), Intra-European Organisation of Tax Administrations (IOTA) and the OECD.

<sup>34</sup>On which, more generally, see for instance IMF (2015), Slemrod (2016) and Luttmer and Singhal (2014).

expanding information reporting. Carrillo et al. (2014) examine the effect of a change of the tax authority’s use of third-party information on reported firm revenues for the corporate tax in Ecuador. Slemrod et al. (2015) study the 2011 IRS initiative requiring credit-card companies and other third-party payment organizations to report electronic payments received by businesses.

These studies are a step, but just a step, toward providing the information needed to flesh out the normative models developed here. While they give a sense of what kinds of enforcement initiatives are likely to affect the level of evasion, they do not provide quantitative estimates of the effect. Nor do they help quantify the other element needed to characterize optimal tax administration: the resource costs—of administration and compliance—of implementing administrative change.

## 5 Conclusions

This paper has set out a rigorous but straightforward framework for thinking about the optimal extent and nature of interventions by a tax administration, one that parallels, and can be closely integrated with, the powerful frameworks for thinking about optimal tax policy that have been developed over recent years. At its heart is a summary measure of the impact of administrative interventions—the elasticity of tax revenue with respect to some intervention—that we label the *enforcement elasticity of tax revenue*. This acts as a sufficient statistic for the behavioral response to administration in much the same way as the familiar elasticity of taxable income serves as a sufficient statistic for the response to tax rate changes. Indeed the sufficiency of the enforcement elasticity is, if anything, rather more robust than that of the taxable income elasticity: it remains a sufficient statistic for behavioral response, for instance, when concealment costs are partly transfers, while the elasticity of taxable income does not. Combined with information on marginal compliance and administration costs, and on the marginal social value of public spending, the enforcement elasticity is the quantity upon which many key issues in designing efficient tax systems have been shown to turn.

Some of the questions that can be addressed within this framework are classic ones—the optimal size of a tax administration, for instance—but others are new and surprisingly under-studied, such as the characterization of the optimal compliance gap and the substitutability or complementarity between enforcement measures and statutory rate increases as revenue-raising devices. A core result here, for instance, is a simple elasticity rule for efficient tax administration: at an optimum, the enforcement elasticity is equal to marginal compliance and administration costs relative to revenue raised (adjusting the latter to reflect their not being financed from distorting taxes). This approach thus also provides a clear rationale for all the effort that has been put into understanding compliance and administration costs, the relevance of which for good policy

making has often been taken as obvious rather than carefully articulated. This setting also enables a simple answer to the very basic question of whether it is better to raise an additional dollar by increasing statutory tax rates or by tighter administration: this turns out to depend only on the two key elasticities, those of enforcement and taxable income, and on aspects of compliance and administration costs. Not least, given the current focus of many tax administration on measuring tax (compliance) gaps, it not only raises the often-neglected question of how one might know if an observed compliance gap is 'too big' or 'too small', but also provides a clear answer: in the simplest case, this too turns on a comparison involving both the enforcement elasticity and the responsiveness of real activity to administrative intervention.

There are, of course, many limitations to this framework. Some would be fairly easy to accommodate, such as the possibility, stressed by many tax administrators, that the private benefits of compliance (such as improved access to credit markets) are significantly under-estimated by taxpayers. Others are more fundamental: in a many-household context, for example, the framework here attaches no importance to horizontal equity, either as a welfare concern in itself or in potentially shaping compliance behavior.

The greatest obstacle to practical implementation, however, is the current state of empirical knowledge. An extensive and long-established literature has produced much information on compliance and, especially, administration costs, but has focused much more on their levels than on how they vary with alternative interventions, which is what matters for the efficient design of those interventions and their comparison with policy measures. Most fundamental, however, is the lack of empirical information on enforcement elasticities. While tax administrations often recognize the importance of these elasticities in their budget submissions, they have received almost no direct attention in the academic literature, and the information that can be inferred indirectly is very limited. Looking forward, however, there are two reasons to be optimistic. For one thing, the analysis developed here tells us exactly what empirical quantities to look for, a necessary but clearly insufficient condition for reaching the goal of an implementable guide to policy. Second, a new wave of creative empirical researchers, often working in concert with enlightened tax administrations and armed with modern empirical research designs that promise compelling identification of causal impacts, are making clear progress. The framework set out here can, we believe, be helpful in guiding these efforts.

## A Appendix: Some comparative statics

With avoidance  $s$  suppressed, the necessary conditions for the taxpayer's problem of maximizing  $wl - t \cdot (wl - e) - c(wl, e, \alpha) - \varphi(l)$  are

$$w(1 - t) - c_\omega w - \varphi' = 0 \quad (\text{A.1})$$

$$t - c_e = 0 \quad (\text{A.2})$$

where  $\omega \equiv wl$ . Perturbing this system gives

$$\begin{bmatrix} -(c_{\omega\omega}w^2 + \varphi''), & -c_{\omega e}w \\ -c_{e\omega}, & -c_{ee} \end{bmatrix} \begin{bmatrix} dl \\ de \end{bmatrix} = \begin{bmatrix} w, & c_{\omega e}w \\ -1, & c_{e\alpha} \end{bmatrix} \begin{bmatrix} dt \\ d\alpha \end{bmatrix}. \quad (\text{A.3})$$

Inverting, and denoting by  $\Delta$  the determinant of the matrix on the left-hand side, which is strictly positive as a second-order condition, the effects of a tax rate change are given by

$$\Delta \frac{dl}{dt} = -(c_{ee} + c_{\omega e})w \quad (\text{A.4})$$

$$\Delta \frac{de}{dt} = (c_{\omega\omega} + c_{e\omega})w^2 + \varphi'' \quad (\text{A.5})$$

so that, for reported income  $z = wl - e$ ,

$$\Delta \frac{dz}{dt} = -(c_{ee} + c_{\omega\omega} + 2c_{\omega e})w^2 - \varphi''. \quad (\text{A.6})$$

The effects of tighter enforcement are given by

$$\Delta \frac{dl}{d\alpha} = (c_{\omega e}c_{e\alpha} - c_{ee}c_{\omega\alpha})w \quad (\text{A.7})$$

$$\Delta \frac{de}{d\alpha} = c_{e\omega}c_{\omega\alpha} \cdot w^2 - c_{e\alpha}(c_{\omega\omega}w^2 + \varphi''), \quad (\text{A.8})$$

from which

$$\Delta \frac{dz}{d\alpha} = [c_{e\alpha}(c_{\omega e} + c_{\omega\omega}) - c_{\omega\alpha}(c_{ee} + c_{e\omega})]w^2 + \varphi''. \quad (\text{A.9})$$

Natural assumptions in trying to sign these terms (beyond the convexity assumptions that  $c_{ee}$ ,  $c_{\omega\omega}$  and  $\varphi''$  are all strictly positive) are that  $c_{e\alpha} > 0$  (tighter enforcement raises the cost of concealing an additional dollar),  $c_{e\omega} < 0$  (higher true income makes it easier to conceal an additional dollar) and  $c_{\alpha\omega} < 0$  (higher true income blunts the impact of tougher enforcement on concealment costs). Even with this, however, little can be said.  $E(z, 1 - t)$  may be negative; and, if  $c_{\omega e}$  is sufficiently negative, so may  $E(z, \alpha)$ . Further assumptions are needed to sign these terms. The elasticity of taxable income will be strictly positive, for instance, if (and only if)  $c_{ee} + c_{e\omega} > -(c_{\omega\omega} + c_{\omega e})$ , so that increasing both true income and evasion by \$1 increases the marginal cost of concealment ( $c_e$ ) by more than it increase the marginal benefit of earnings in facilitating concealment ( $-c_\omega$ ).

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