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ABSTRACT

What a country has done in the past, and what other countries are doing in the present, can feedback for good or for ill in debt markets. We develop a simple model of sovereign bond markets with global investors and endogenous information acquisition about fundamental default probabilities. This model displays hysteresis and contagion in sovereign bond spreads. Small fundamental shocks in one country can induce investors to acquire information, generating price volatility and increased risk premia. These changes may also induce investors to rebalance their portfolio, generating market segmentation and information acquisition in seemingly unrelated economies. Information regimes may persist over time, requiring large improvements in fundamentals to return to more stable bond spread conditions.

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1 Introduction

The behavior of sovereign debt markets is challenging to understand, particularly during crises with spiking bond yield spreads. This leads us to examine a novel source for these crises: changes in the information about sovereign default risk acquired by the agents that lend to these sovereigns. Along with this, we consider a novel mechanism for contagion: shifts in the information regime in one country may generate incentives for investors to reallocate funds and acquire information in other countries, thereby altering the pricing of default risk globally. In this framework, small shocks to fundamentals in one country can trigger waves of information acquisition in other countries, generating endogenous market segmentation and sharp increases in bond yield volatility in seemingly unrelated countries.

Sovereign risk premia frequently exhibit sudden fluctuations without obvious changes in underlying fundamentals. They may also react differently to a given change in fundamentals at different points in time. Nevertheless, there seems to be history dependence in the borrowing conditions faced by different countries: the same change in fundamentals can have different effects in different countries, and these differences are persistent over time. Indeed, a given country's *past* behavior seems to matter for how sovereign spreads react to changes in current fundamentals. (For discussion and documentation of these facts see Reinhart, Rogoff, and Savastano (2003), Tomz and Wright (2007), Broner, Martin, and Ventura (2010), Tomz and Wright (2013) and Aguiar and Amador (2014)).[1]

Aguiar et al. (2016) document that there are several important common components that drive sovereign spreads globally. However, they find that only a limited share of these components can be accounted for by global financial factors or changes in the risk-free rate. These large and largely unexplained common components suggest some form of contagion. Indeed, sovereign debt crises tend to be highly correlated across countries and sovereign spreads (the sovereign's cost of external funding) tend to co-move strongly. The most recent example is the 2010 debt crisis in Europe, but similar forces were at play in the wave of debt crises initiated by Poland in 1981, Mexico in 1994, Thailand in 1997, Russia in 1998, and Argentina in 2001.[2]

[1]There is a large quantitative literature that seeks to account for sovereign spreads and defaults; see for example Aguiar and Gopinath (2006), Arellano (2008), Chatterjee and Eyigungor (2012), Hatchondo and Martinez (2009). Aguiar et al. (2016) surveys this literature.

[2]For the recent European crisis, Beirne and Fratzscher (2013) use information on 31 advanced and

Previous work has attempted to explain contagion by appealing to different types of linkages between countries. One branch of the literature focuses on *real linkages*. For example, trade in goods or financial assets between countries may transmit negative shocks from one country to the next and lead to co-movements in sovereign spreads. A second branch of the literature focuses on *belief linkages* through learning and herding. In this view, contagion is driven by the correlation of beliefs about fundamentals in different countries, so that bad news about one country make investors pessimistic about other countries. Of course, a prerequisite for belief correlation to cause contagion is that observations about one country hold information about other countries. This again requires correlation in fundamentals across countries, or the existence of a common unobservable variable linking all countries. Finding evidence for structural linkages across countries is therefore crucial in providing support for the many theories of contagion that rely on them. Problematically, however, it is often difficult to empirically identify linkages that are plausibly powerful enough to induce the degree of contagion observed in many debt crisis episodes.

This in turn lead to a third set of explanations that rely on self-fulfilling debt crises either through feedback effects as Calvo (1988) and Lorenzoni and Werning (2013), and rollover problems as in Cole and Kehoe (1996), Aguiar et al. (2015), and Bocola and Dovis (2015). Aguiar et al. (2016) consider a one-country model and report that it is surprisingly difficult to explain the large amount of volatility we observe in sovereign spread data by including self-fulfilling debt crises. To explain contagion, this literature requires a correlated structure of sunspots to induce simultaneous roll-over crisis episodes in many countries at the same time.

These deficiencies in the literature motivate us to explore another novel form of linkages between countries that stem not from country fundamentals (the *demand* side) but rather from the investment and information acquisition decisions of investors (the *supply* side). To the extent that investors are exposed to several countries sovereign bond risk, changes in one country's fundamentals induce a portfolio reallocation that can naturally spill over to spreads in another country. A reenforcing factor is the change in information acquisition. An investor may have more incentives to obtain information about the country suffering the change in fundamentals, making its bond prices more volatile as a reflection of more information. This may lead investors with-

emerging economies and find that there was a sharp and simultaneous increase in sovereign spreads in both European and non-European countries. For a survey of previous crises see Reinhart and Rogoff (2009).

out information to reduce their exposure to this additional price risk, moving funds to a different country, which alters their incentives to acquire information about those other countries. This information spillover channel reinforces the simple portfolio reallocation channel.

We examine the potential of this new portfolio-information channel in a model of sovereign bond markets with multiple countries and two key elements. First, there is a global pool of risk-averse investors who freely allocate funds across sovereign bond markets. Second, these investors can choose to produce information about countries' fundamentals at a cost. This information is valuable because informed investors can use it to outbid uninformed investors in particularly attractive states of the world. In equilibrium, this benefit is exactly offset by the cost of becoming informed.

Our first result is that the flow of capital across countries can generate contagion in spreads even in the absence of any real linkages, correlation of fundamentals, or belief updating about one country due to equilibrium outcomes in another country. Specifically, when investor preferences exhibit *prudence* (that is, $u'''(c) > 0$, as is the case for CRRA utility functions), an increase in the probability of default in one country increases sovereign spreads for all sovereign bonds held by the investor. This is because an increase in the default risk of a given country increases the *background risk* inherent in the entire portfolio of sovereign bonds, and thereby reduces the investor's appetite to invest in sovereign debt more generally. Hence, sovereign bond prices fall across in all countries when one country becomes more likely to default. If this effect is sufficiently large and the increase in spreads is severe enough, it may no longer be feasible for countries to roll over their debt, causing a wave of debt crises.

This basic contagion result relies only on investor prudence and the fact that there is a common pool of investors for all countries. Hence it does *not* rely on changes in investors' wealth (as in Kyle and Xiong (2001) or Goldstein and Pauzner (2004)), borrowing constraints (as in Yuan (2005)) or short-selling constraints (as in Calvo and Mendoza (1999)). Indeed, contagion stems only from the portfolio rebalancing of prudent investors in response to an increase in the riskiness of a subset of investment assets. Broner, Gelos, and Reinhart (2004) provide empirical evidence about the importance of portfolio effects for contagion, while Lizarazo (2013) and Broner, Lorenzoni, and Schmukler (2013) discuss the importance of risk aversion for explaining the behavior of sovereign spreads.

Our second result is that the option to produce information about countries' fundamentals can generate multiple equilibria. In particular, an *uninformed equilibrium* in which no investor acquires information about the country's fundamentals may co-exist with an *informed equilibrium* in which some investors do acquire information about the country's fundamentals. These information regimes have real effects: taking as given the stochastic process for fundamentals, the average level and the volatility of spreads differ across regimes. In the uninformed equilibrium, spreads are stable and low on average, because investors are relatively insensitive to fundamental variations that they are uninformed about. In the informed equilibrium, in contrast, spreads are volatile and high on average, because investors are informed about fundamental variations and strongly react to their realization, demanding high risk premia in bad states of the world. For this reason, sovereigns strictly prefer an uninformed equilibrium to an informed equilibrium. The same is true for investors, because information acquisition is costly and information rents stem from rent-seeking at the expense of other investors. That is, the informed equilibrium is Pareto-inferior to the uninformed equilibrium whenever they co-exist. [3]

The second result magnifies the first result, as changes in fundamentals not only induce contagion in sovereign spreads but also in information regimes. Take two countries. When fundamentals in Country 1 change and induce information acquisition, any investors who remain uninformed about Country 1 reallocate funds towards Country 2 (segmentation) and away from sovereign bonds (due to higher background risk). As those investors become more exposed to Country 2, however, they have stronger incentives acquire information about Country 2. As a result, more information in one country induces segmentation and thereby spurs information acquisition in other countries. Instability, then, can also be contagious. This is our third key result. The finding is consistent with Van Nieuwerburgh and Veldkamp (2010), who show that information acquisition may lead to segmentation and further incentives to acquire information, rationalizing features deemed as anomalous in asset markets. In our paper the source of information incentives is quite different, as it relies on how countries raise funds and it applies to contagion of the mean and volatility of sovereign spreads. As in Calvo and Mendoza (2000), moreover, we highlight the role

[3]This result is driven in part by the fact information does not affect any real production or allocation decisions in our setting. Accordingly, there are no real benefits of information. More generally, of course, information may have benefits in terms of disciplining governments or allocating funds to productive investment opportunities. We assume away those benefits to focus on the forces behind information acquisition. Any real benefit of information will naturally make the uninformed equilibrium less desirable.

of information on sovereign crises. In contrast to them, however, our emphasis is on the contagion of information acquisition incentives. This also differentiates our work from Fostel and Geanakoplos (2008), who provide a theory of price contagion across asset classes that is based on leverage.

An important upshot from our analysis is that, because investors' optimal portfolio choice and the information regime *jointly* determine the mapping from country fundamentals to sovereign bond spreads and crises, there need not exist a unique mapping from economic fundamentals to spreads in sovereign bond markets even in the absence of roll-over crises driven by coordination failures. Indeed, since investors choose their portfolio by taking the fundamentals and information regimes in all countries into account, the mapping from fundamentals to prices in one country depends on equilibrium outcomes in all other countries. To the extent that a given pool of investors prices sovereign bonds in multiple countries, understanding contagion and default risk therefore requires a "global" view of bond markets.

Finally, to the extent that informational regimes are persistent (in that there is a change in regime if the only if the old regime can no longer be sustained), only large changes in fundamentals can force transitions across regimes. This refinement, which is conservative in that it restricts the frequency of transitions to those that are strictly necessary, implies that a country starting out in an uninformed equilibrium begins to attract informed investors only if its fiscal situation worsens substantially, while a country starting out in an informed equilibrium must improve dramatically to discourage informed investors. In the absence of large shocks, two countries may therefore face different informational regimes and spreads even when their current fundamentals are similar. As a result, a country's *past* sins or virtues may be important determinants of *current* borrowing conditions. This *hysteresis* in spreads implies that understanding contagion and default requires a "historical" view of bond markets.

We begin by studying a single-country model in Section 2. We show how information acquisition can generate equilibrium multiplicity and discuss its effects on bond yields. In Section 3 we extend the results to two countries, and discuss contagion through the portfolio channel and the information spillover channel. In Section 4 we interpret the dynamics of sovereign spreads in the recent European debt crisis through the lens our model. Section 5 concludes. All proofs are in Appendix A.

2 A Single Country Model

2.1 Setting

Environment: There are two periods, $t = 1, 2$. The economy is populated by a government and a unit mass of investors. Investors receive an endowment of wealth W in the first period and derive utility from consumption c in the second period. Their preferences over consumption are ordered by a strictly concave utility function $u(c)$ that satisfies the Inada conditions. In the first period, investors decide whether to invest in a safe asset that has gross return 1 (storage) or in risky debt issued by the government.

The government is risk-neutral and has legacy debt D coming due in period 1. This debt is owed to previous investors outside of the model. The government has no income in period 1, and receives stochastic income Y in period 2. The government must roll over the debt in period 1 in order to service its creditors. We assume that it does so by issuing pure-discount bonds using a discriminatory price auction.[4] In this auction, investors specify combinations (possibly menus) of prices P and quantities B of bonds that they wish to purchase. The government sells debt to the highest bidder until it either exhausts the bids or sells enough to roll over its debt. If the government cannot roll over its debt then it must default on initial investors, a situation we call a *debt crisis*.

In period 2, the debt issued in period 1 comes due, and the government decides whether to service the debt using income Y or to default. If the government defaults in either period, it can abscond with $(1-\theta)Y$ in income, where $\theta \in [0, 1]$ is the cost of default in terms of lost income. We assume that this cost of default is independent of whether the government defaults in period 1 or period 2. The government's objective is to maximize its second period consumption. Since the government is just seeking to roll over its debt during the first period, it will always do so if it can, reserving the decision to default for the second period.

While the realization of Y is drawn in period 2 from a distribution $F(Y)$, the realization of θ is drawn in period 1 from a discrete distribution with S elements $\Theta =$

[4] Brenner, Galai, and Sade (2009) show that a majority of countries in their sample of 84 countries use discriminatory auctions to sell bonds. Their sample includes 83% of OECD countries. Our price-discriminating auction works in a similar fashion to limit order books in markets intermediated by brokers/dealers; see Parlour and Seppi (2008) for a discussion of limit orders.

$\{\theta_1, .., \theta_S\}$, such that $\theta_1 > .. > \theta_s > .. > \theta_S$. The realization of θ is unknown to both the government and the investors in period 1. Investors can choose to learn the realization of θ in period 1 at a utility cost K. Or, they and the government can wait until it revealed for free in period 2. Since the emphasis of our paper is on the strategic behavior of investors in terms of acquiring information. We therefore abstract from the government acquiring information about θ and strategically signaling this information through its behavior.

Auctions: We now describe the auction market used by the government to raise funds in period 1. While investors are ex-ante identical, they may end up being either *informed* or *uniformed* depending on whether they choose to acquire information about θ or not. We denote by $n \in [0, 1]$ the fraction of informed investors and by P the marginal price of government debt in period 1.

When there are informed investors, this marginal price will depend upon the realized θ because informed investors condition their bids on θ. In this case, we denote the marginal price by $P(\theta)$. Without loss of generality, we take as given that informed investors always offer the marginal price $P(\theta)$ in state θ, and summarize their bidding behavior by the (conditional) quantity that they wish to purchase at that price, $B^I(\theta)$. The uninformed traders instead may find it advantageous to bid a menu of price-quantity pairs. That is, given the set of potential marginal prices $\{P(\theta_1), ..., P(\theta_S)\}$, they choose the quantities to bid at each one of these prices. Let $B^U(\theta)$ denote the amount that an uninformed trader bids if he chooses to bid at price $P(\theta)$. Whether the uninformed end up purchasing the bonds they offered to buy ultimately depends on which state of the world is realized. Specifically, the uninformed bids' are accepted whenever they offer a price that is above the realized marginal price.

Taking as given that the government can roll over its debt in period 1, the auction arrangement leads to the following government budget constraint for each θ.

$$nB^I(\theta)P(\theta) + (1-n) \sum_{\left\{\widehat{\theta}:P(\widehat{\theta}) \geq P(\theta)\right\}} B^U(\widehat{\theta})P(\widehat{\theta}) = D. \tag{1}$$

The first term on the left-hand side is the informed investors' bond purchases. The second term on the left-hand side is the uninformed investors' bond purchases, taking into account that their bids are accepted whenever they offer to buy bonds at a price above the realized marginal price $P(\theta)$.

For a given θ, the government cannot roll over the debt in period 1 if

$$nB^I(\theta)P(\theta) + (1-n) \sum_{\left\{\widehat{\theta}:P(\widehat{\theta})\geq P(\theta)\right\}} B^U(\widehat{\theta})P(\widehat{\theta}) < D.$$

As a result, it must default. We will refer to this case as a *debt crisis*.

If the government did not default in period 1, its debt coming due in period 2 is

$$R(\theta) = nB^I(\theta) + \sum_{\left\{\widehat{\theta}:P(\widehat{\theta})\geq P(\theta)\right\}} (1-n)B^U(\widehat{\theta})$$

If the government does not default in period 2, its payoff in period 2 is $Y - R(\theta)$. If it does default in period 2 its payoff is $(1-\theta)Y$. This leads to a simple cut-off rule: the government defaults in period 2 if and only if $Y < \bar{Y}(\theta)$, where

$$\bar{Y}(\theta) \equiv \frac{R(\theta)}{\theta}. \tag{2}$$

Given this default rule, the realized return to an investor on each bond is 1 if $Y \geq \bar{Y}(\theta)$ and 0 otherwise, and the bond's default probability is $1 - \Pr\left\{Y \geq \bar{Y}(\theta)\right\}$. So long as the total amount of debt coming due, $R(\theta)$, is weakly decreasing in θ (the higher the cost of default, the higher the price of debt and the less debt comes due in period 2) the default cut-off is strictly decreasing in θ and the default probability is weakly decreasing in θ. That is, defaults are less likely when the cost of default is high. A lower default probability in turn decreases the total bond issuance required to roll over the debt, which is again consistent with lower rollover costs and prices.

Short-sale prohibition: We assume that private investors cannot short the government's bond. We do so for two reasons. First, shorting the bond is not equivalent to pledging to deliver a unit of the bond at a later date. Rather, it requires committing to the same state-contingent payoff profile as the government. But in order to do this, the private investor would need access to exactly the sort of commitment technology as the government (taxation and non-anonymity, for example). Second, only the uninformed may have an incentive to short-sell the bond in any equilibrium with both informed and uninformed investors, as the informed can always buy bonds at the correct marginal price. The counterparty to a short-sale attempt by the uninformed must therefore be an informed investor. But an informed investor is only willing to buy the bond when it is weakly underpriced. The very fact that an offer to short-sell is accepted would therefore reveal information that the state of the world is such that the uninformed would not want to short-sell at the relevant marginal price in the first place. Hence there is no scope for short-selling in our setting.

We now turn to describing investors' decision problems in detail.

Investors' problem: An informed investor knows θ and takes the marginal price of debt $P(\theta)$ as given. The maximization problem is thus

$$U^I(\theta) = \max_{B^I(\theta)\geq 0} \quad u\left(W + \left[1 - P(\theta)\right] B^I(\theta)\right) \Pr\left\{Y \geq \bar{Y}(\theta)\right\} + u\left(W - P(\theta)B^I(\theta)\right)\left[1 - \Pr\left\{Y \geq \bar{Y}(\theta)\right\}\right] - K. \tag{3}$$

The first-order condition with respect to $B^I(\theta)$ is

$$u'\left(W + \left[1 - P(\theta)\right] B^I(\theta)\right)\left[1 - P(\theta)\right] \Pr\left\{Y \geq \bar{Y}(\theta)\right\} + u'\left(W - P(\theta)B^I(\theta)\right)\left[-P(\theta)\right]\left[1 - \Pr\left\{Y \geq \bar{Y}(\theta)\right\}\right] \leq 0, \tag{4}$$

and with strict equality if $B^I(\theta) > 0$.

An uninformed agent must choose how much to bid at each one of the possible marginal prices $P(\theta)$ without knowing θ. The maximization problem of an uninformed agent thus is

$$U^U = \max_{\{B^U(\widehat{\theta}_1),\ldots,B^U(\widehat{\theta}_S)\}} \sum_{\theta\in\Theta} \Pr(\theta) \left\{ \begin{array}{c} u\left(W + \sum_{\{\widehat{\theta}:\widehat{\theta}\geq\theta\}} \left[1 - P(\widehat{\theta})\right] B^U(\widehat{\theta})\right) \Pr\left\{Y \geq \bar{Y}(\theta)\right\} + \\ u\left(W - \sum_{\{\widehat{\theta}:\widehat{\theta}\geq\theta\}} P(\widehat{\theta})B^U(\widehat{\theta})\right)\left[1 - \Pr\left\{Y \geq \bar{Y}(\theta)\right\}\right] \end{array} \right\}. \tag{5}$$

The first-order condition for $B^U(\widehat{\theta})$ is

$$\sum_{\{\theta:\theta\leq\widehat{\theta}\}} \Pr\left\{\theta\right\} \left\{ \begin{array}{c} u'\left(W + \sum_{\{\theta':\theta\leq\theta'\leq\widehat{\theta}\}} \left[1 - P(\theta')\right] B^U(\theta')\right)\left[1 - P(\widehat{\theta})\right] \Pr\left\{Y \geq \bar{Y}(\theta)\right\} + \\ u'\left(W - \sum_{\{\theta':\theta\leq\theta'\leq\widehat{\theta}\}} P(\theta'^U(\theta')\right)\left[-P(\widehat{\theta})\right]\left[1 - \Pr\left\{Y \geq \bar{Y}(\theta)\right\}\right] \end{array} \right\} \leq 0. \tag{6}$$

This condition holds as an equality if $B^U(\widehat{\theta}) > 0$. Because the bids of the uninformed are not state-contingent, bids at a given price affects the first-order conditions at other prices. Optimal bids thus satisfy the system of equations (6) for all $\widehat{\theta}$.

Finally, the decision to become informed must be consistent with individual optimality. That is, if an interior fraction $n \in (0, 1)$, of investors chooses to become informed, investors must be indifferent between becoming informed and staying uninformed. If there are no informed investors, then it must be weakly preferred to remain uninformed. If there are no uninformed investors, then it must be weakly preferred to become informed. Formally,

$$\sum_{\theta} \Pr\{\theta\} U^I(\theta) - K \begin{array}{c} \leq \\ = \\ \geq \end{array} U^U \begin{array}{c} \text{if } n = 0, \\ \text{if } n \in (0,1), \\ \text{if } n = 1. \end{array} \tag{7}$$

We now formally define the equilibrium.

Definition 1 *An* ***equilibrium*** *consists of a set of cut-offs $\bar{Y}(\theta)$, prices $P(\theta)$, quantities for the informed and uninformed ($B^I(\theta)$ and $B^U(\theta)$ respectively) and a fraction of informed investors (n) such that the following conditions are satisfied.*

1. *The bond market clears in the first period for each state according to (1), unless $P(\theta) = 0$, $\bar{Y}(\theta) = \infty$ and there is a debt crisis in state θ.*

2. *The cut-offs $\bar{Y}(\theta)$ satisfy the threshold condition (2).*

3. *The choices of $B^I(\theta)$ and $B^U(\theta)$ are solutions to the informed and uniformed investors' problems (first-order conditions (4) and (6), respectively).*

4. *The fraction of informed investors n satisfies the indifference condition* (7). *The country is in an informed equilibrium when $n > 0$ and in an uninformed equilibrium when $n = 0$.*

There are a variety of equilibria. This is in part because the price of government debt affects the likelihood of repayment, which can in turn rationalize different prices of debt. For example, *no-lending* with $P(\theta) = 0$ and $\bar{Y}(\theta) = \infty$ for all θ is always an equilibrium. At a zero price the government will not be able to rollover its debt and therefore must default, this in turn rationalizes the zero price. This multiplicity is well-known since the work of Calvo (1988) and Cole and Kehoe (1996). Next we present a simplified *special case* to characterize the other (potentially multiple, because of information acquisition) equilibria in a tractable and intuitive way.

Simplifications: To illustrate the key forces determining the pricing of debt and the decision to become informed, we make the following simplifying assumptions. First, there are two possible costs of default ($S = 2$). That is, $\theta \in \{\theta_L, \theta_H\}$ with $0 < \theta_L < \theta_H < 1$. We say that θ_H is the "*good state*" and θ_L is the "*bad state*," with $\Pr(\theta_H) = a$ and $\Pr(\theta_L) = 1 - a$. Second, we assume that there are three possible income realizations in period 2: $Y \in \{Y_L, Y_M, Y_H\}$, with $Y_L < Y_M < Y_H$ and $(Y_L) = x$ and $\Pr(Y_M) = z$. Finally, we assume that the parameter values for θ, Y and prices are such that default cutoffs are always *independent of changes in prices*. This assumption allows us to focus on the direct effects of price changes on information acquisition, abstracting from the indirect effect through changes in default probabilities induced by the same price changes. Formally,

Assumption 1

$$Y_L < \bar{Y}(\theta_H) < Y_M < \bar{Y}(\theta_L) < Y_H$$

Given this assumption, we can focus on two possible states of the world. When the cost of default is high, the country only defaults when the income is low, which implies a default probability of $\kappa_H \equiv x$. We call this the "good state." When the cost of default is low, the country only repays when the income is high, which implies a default probability of $\kappa_L \equiv x + z$. We call this the "bad state."

Assumption 1 concerns both exogenous and endogenous variables since it depends on the bond prices $P(\theta_L)$ and $P(\theta_H)$. Its advantage is that fixes the period 2 default probabilities and allows them to be treated parametrically. It will become clear that a more general setting in which falling bond prices lead to higher default probabilities would further strengthen the channels we highlight. We therefore choose to treat default probabilities parametrically so as to cleanly discuss these channels. Moreover, we show that endogenous information acquisition is sufficient to generate multiple equilibria even when default probabilities are fixed and there is no feedback from prices to default.

2.2 Characterization of Equilibria

We now characterize equilibrium in our baseline setting. We begin by studying the *uninformed equilibium* in which no investor acquires information about the state of the world. Next, we study the *informed equilibrium* in which some investors decide to become informed about the state of the world. Finally, we describe that informed and uninformed equilibrium may co-exist, giving rise to equilibrium multiplicity, and show how changes in fundamentals can trigger switches across equilibrium types.

2.2.1 Uninformed Equilibrium

Define the expected probability of default as

$$\widehat{\kappa} \equiv ax + (1-a)(x+z).$$

Since there is no information about the country's state, there is a single marginal price $P = P(\theta)$ for all θ. Given this price, the first-order condition (4) pins down the demand for bonds B as

$$\frac{u'(W + [1-P]B)}{u'(W - PB)} = \frac{P\widehat{\kappa}}{(1-P)(1-\widehat{\kappa})}. \tag{8}$$

The next proposition characterizes how this demand reacts to changes in parameters.

Proposition 1 *The investors' demand of sovereign bonds is decreasing in the price of the bond P and its expected default probability $\widehat{\kappa}$.*

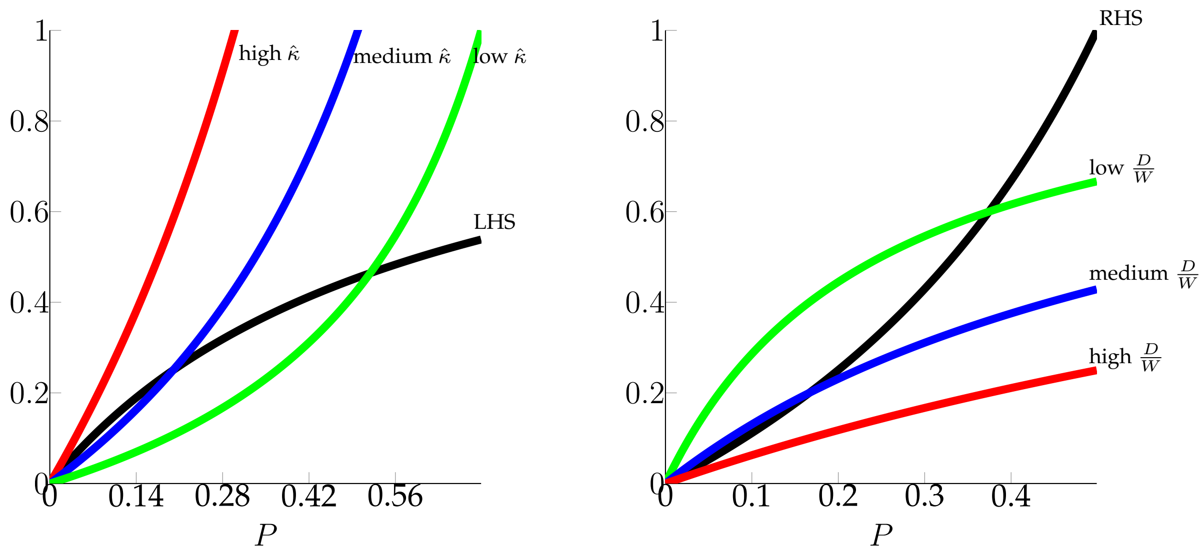
Substituting the government's budget constraint $PB = D$ into (8) implies that the equilibrium price P^* is defined by

$$\frac{u'(W - D + \frac{D}{P^*})}{u'(W - D)} = \frac{P^*\widehat{\kappa}}{(1-P^*)(1-\widehat{\kappa})}. \tag{9}$$

Proposition 2 *A debt crisis in the first period (this is, $P^* = 0$) is always an equilibrium. If there exist equilibria with $P^* > 0$, then the equilibrium price in the highest-price equilibrium decreases with the probability of default $\widehat{\kappa}$ and with the country's debt D.*

The comparative statics for the maximum sustainable equilibrium price are intuitive. The price falls if an increase in the probability of default reduces the demand for the sovereign bond, and when an increase in total debt increases its supply. The first panel of figure 1 shows these effects on demand and supply for the sovereign bond. The left-hand side of equation (9) is in black and the right-hand side in different colors for three different levels of $\widehat{\kappa}$. The equilibrium price is determined by the intersection of the two curves. The higher is the expected probability of default, the higher is the right-hand side and the smaller is the price P in equilibrium. When $\widehat{\kappa}$ is large enough, the only feasible equilibrium is $P^* = 0$ and there is a debt crisis.

Figure 1: Effect of z and D on Information in Equilibrium



The second panel of figure 1 shows the right hand side of equation (9) in black and the right hand side in different colors for three different levels of D/W. As before, the equilibrium price is determined by the intersection of the two curves. The higher is the relative indebtedness of the country, the higher is the left hand side and the smallest the price P in equilibrium. When D/W is large enough, the only feasible equilibrium is a $P^* = 0$ and there is a debt crisis.

In our setting, information acquisition is a choice. As a result, an uninformed equilibrium is sustainable only if no single investor has an incentive to pay the utility cost K to become informed. Because no individual agent's bidding behavior impacts equilibrium prices, the benefit of acquiring information in the uninformed equilibrium is the ability to make optimal state-contingent bids at the fixed marginal price P^*.

Naturally, an investor wants to buy more bonds when he learns that the state is good. This is immediate from the first order condition (8) evaluated at P^* and κ_H, as the optimal quantity is decreasing in the probability of default $\kappa_H < \widehat{\kappa}$. Accordingly, he would like to buy less bonds when he learns that the state is bad.

Formally, the expected benefits from acquiring information gross of the information cost K are thus

$$\chi^U \equiv a\left[U(B(\kappa_H, P^*)) - U(B(\widehat{\kappa}, P^*))\right] + (1-a)\left[U(B(\kappa_L, P^*)) - U(B(\widehat{\kappa}, P^*))\right],$$

As an informed investor can always replicate his uninformed bid, χ^U is weakly positive. An uninformed equilibrium is feasible if

$$K > \chi^U \geq 0.$$

The expected gross benefits of acquiring information are (i) increasing in z, (ii) non-monotonic in a and (iii) increasing in D/W. To see why, note that the difference between the optimal informed bid in each state and the optimal uninformed bid (that is, $B(\kappa_s, P^*) - B(\widehat{\kappa}, P^*)$) is increasing in the absolute difference $\kappa_s - \widehat{\kappa}$. Since $\widehat{\kappa} - \kappa_H = (1-a)z$ and $\kappa_L - \widehat{\kappa} = az$, this difference increases with z. Second, the absolute difference is maximized at intermediate levels of a. In the extremes, when $a = 0$ or $a = 1$, $U(B(\kappa_L, P)) = U(B(\widehat{\kappa}, P))$ and $\chi^U = 0$. Finally, the larger D/W the larger the investors' exposure to the risky asset. These higher stakes increases the benefit of knowing the probability of default in each state.

2.2.2 Informed Equilibrium

We now turn to characterizing the informed equilibrium in which a strictly positive fraction of investors acquires information.The critical difference to the uninformed equilibrium is that the informed equilibriums feature as many marginal prices as states of the world because informed investors make state-contingent bids. In this sense, the existence of informed investors thus changes the *structure* of the equilibrium.

Given that there are two states of the world, we have two prices: $P_L \equiv P(\theta_L)$ and $P_H \equiv P(\theta_H)$. We can then rewrite the first order condition (4) as

$$\frac{u'(W + [1 - P_s]B_s^I)}{u'(W - P_s B_s^I)} = \frac{P_s \kappa_s}{(1 - P_s)(1 - \kappa_s)} \tag{10}$$

where $\kappa_s \in \{\kappa_L, \kappa_H\}$ are the expected probabilities of default in each state s and $P_s \in \{P_L, P_H\}$ are the prices in each state s.

The next proposition describes the properties of the informed investors' demand for sovereign bonds in each state. The results are analogous to Proposition 1 in our discussion of the uninformed equilibrium.

Proposition 3 *The informed investors' demand of sovereign bonds in each state s is decreasing in the price of the bond P_s and its expected default probability κ_s.*

The crucial distinction between investor types is that the uninformed must post the same (menu of) bids for each state of the world. Because the auction is discriminatory, any bids submitted at the high-state marginal price P_H will thus *also* be accepted in the low state where the realized marginal price is $P_L < P_H$. If the uninformed do not submit any bids at P_H instead, they fail to purchase bonds in the high state. This exposes the uninformed to sources of bidding risk: (i) the risk of overpaying in the low state, and (ii) the risk of failing to participate in the high-state.

Formally, we can rewrite the uninformed investors' first-order condition (6) for the low-price bid B_L^U as

$$P_L\kappa_L u'(W - P_H B_H^U - P_L B_L^U) = (1-P_L)(1-\kappa_L)u'(W + (1-P_H)B_H^U + (1-P_L)B_L^U), \quad (11)$$

and for the high-price bid B_H^U as

$$\begin{aligned} &a\left[P_H\kappa_H u'(W - P_H B_H^U)\right] + (1-a)\left[P_H\kappa_L u'(W - P_H B_H^U - P_L B_L^U)\right] = \\ &a\left[(1-P_H)(1-\kappa_H)u'(W(1-P_H)B_H^U)\right] \\ &\qquad +(1-a)\left[(1-P_H)(1-\kappa_L)u'(W + (1-P_H)B_H^U + (1-P_L)B_L^U)\right]. \qquad (12) \end{aligned}$$

These conditions reflected the aforementioned overpayment risk in that the uninformed investors end up purchasing B_H^U no matter the state. The next proposition describes general properties of the total expenditures on sovereign debt by uninformed investors vis-a-vis those of the informed.

Proposition 4 *Compared to informed investors, uninformed investors spend more in the bad state and less in the good state.*

The intuition for this result is as follows. On the one hand, uninformed investors overpay for a fraction $\frac{B_H^U}{B_L^U + B_H^U}$ of the debt that they purchase in the bad state. If uninformed investors were to spend the same amount as informed investors in the bad state, they would incur the same losses as the informed in case of default but receive smaller gains in case of repayment. The marginal benefits of spending more in the bad state are thus larger than the marginal costs, which induces the uninformed to spend more than informed in the bad state. On the other hand, the uninformed must spend whatever they spend in the good state in the bad state as well. As they are overexposed to sovereign debt in the bad state, they would rather reduce their exposure in the good state when compared to informed investors.

Since $B_H^U < B_H^I$, it may be the case that uninformed investors find it optimal to bid nothing at the high price (or indeed would want to short the high-price bond if doing so were feasible). We refer to the set of parameters under which $B_H^U = 0$ (that is, parameters under which the short-selling constraint binds and uninformed investors refrain from bidding for the high-price bond), as the *partial participation region*. Conversely, we refer to the set of parameters under which $B_H^U > 0$ as the *full participation region*.

In the partial-participation region, the uninformed can infer that the state of the world must be low whenever their (low-price) bids are accepted. As a result, they can bid on the low-state bond *as if* they were informed about the probability of default, and thus bid the same as informed investors. This follows directly from setting $B_H^U = 0$ in the first-order condition (11) and comparing it with (10). All information rents in the partial-participation region thus stem from informed investors' buying bonds in *both* states of the world. In the full-participation region, these *participation rents* are augmented by *price rents* because the uninformed buy at the high price no matter the state of the world.

We analyze the impact of information acquisition in two steps. First, we take the fraction of informed investors n to be exogenous, and show how information rents and prices vary with n. Next, we endogenize n by exploiting a free-entry condition that states investors must be indifferent between being informed and uninformed.

For a given n, prices in each state are determined by equalizing total demand with with the supply of sovereign bond D in both states. The government budget constraints for the high and low state, respectively, are thus

$$nP_H B_H^I + (1-n)P_H B_H^U = D \tag{13}$$

and

$$nP_L B_L^I + (1-n)[P_H B_H^U + P_L B_L^U] = D. \tag{14}$$

In Figure 2 we illustrate how the prices P_H and P_L depend on the fraction of informed investors n in the economy. As a reference point, we also plot the bond price in uninformed equilibrium P_U. The economy is in the full-participation region when $n < 0.7$, and transitions to the partial-participation region when $n \geq 0.7$.

Proposition 5 *The good state price, P_H, increases with the fraction of informed investors, n.*

This is straightforward to see from equation (14). Since informed investors demand more than uninformed investors in the good state, an increase in the mass of informed investors boosts total bond demand in this state. P_L typically decreases with n, but this is not as straightforward to show. There are two competing effects at work. On the one hand, informed investors demand less in the bad state than uninformed investors, which pushes down prices. On the other hand, an increase in P_H can in principle increase the *total* demand from the uninformed, as they now buy a fraction of their low-state bonds at this higher price.

In the partial participation region instead, P_L is independent of n. The reason is that the uninformed now operate under de-facto symmetric information, and so both investor types behave symmetrically. In contrast, P_H continues to increase in n because a larger mass of informed investors pushes up demand and prices in the good state. Indeed, this *cannibalization effect* among informed investors leads P_H to grow faster than in the full-participation region.

The partial-participation region begins when n is large because the price gap $P_H - P_L$ is larger when there are more informed investors. A larger price grap in turn exposes uninformed investors to more overpayment risk. As the gap grows, this force becomes sufficiently strong so as to deter the uninformed from bidding at the high price entirely.

Figure 3 depicts the utility of informed investors (U^I) and uninformed investors (U^U) in the informed equilibrium as a function of n. For reference, we also plot investor utility in the uninformed equilibrium in blue. Investor utility does not vary directly with n. Instead, the effect of increased information operates through prices.

Figure 2: Prices and the Fraction of Informed Investors

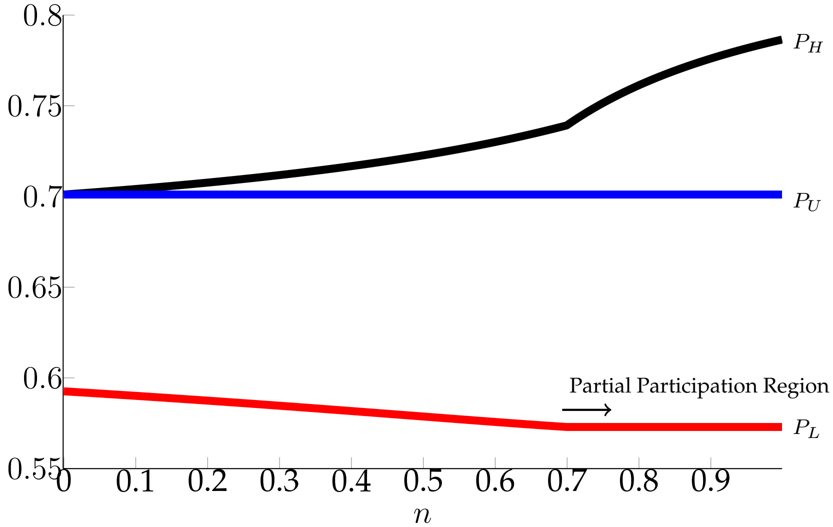
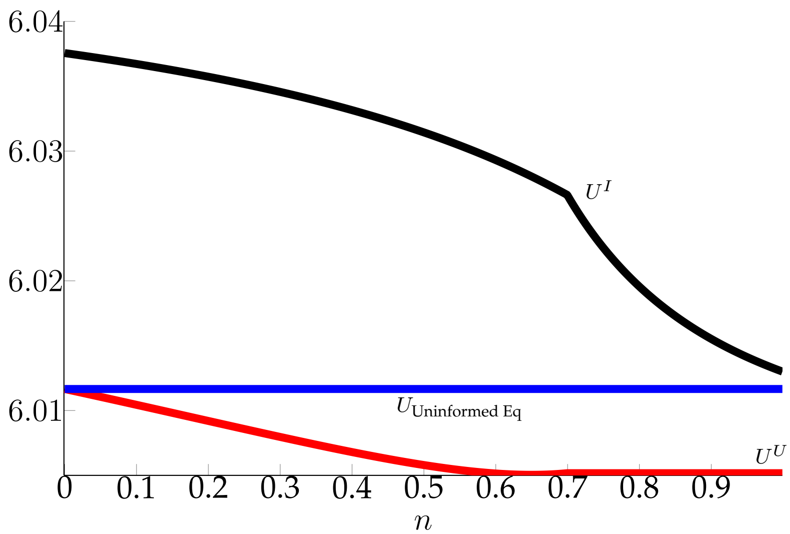


Figure 3: Utilities and the Fraction of Informed Investors



Several features are worth noting. First, the utility of informed (uninformed) investors in the informed equilibrium gross of the information cost is always above (below) the utility of investors in the uninformed equilibrium. This does not imply, however, that informed investors are better-off in the informed equilibrium, since they have to incur utility costs to become informed in the first place. Once we account for these costs, the relevant utility level for *all* investors is that of uninformed investors. Critically, investor utility is lower in the informed equilibrium than in the uninformed equilibrium because information is costly to acquire and only serves to extract rents from uninformed investors. Second, the utility of both informed and uninformed investors decline with n. The reason is that a higher fraction of informed investors pushes up prices in the aggregate and reduces the rents earned by all investors.

To complete the characterization of the informed equilibrium, we now impose the free-entry condition that investors are indifferent between becoming informed and remaining uninformed to pin down the equilibrium fraction of informed investors. In Figure 4 we plot the utility gap between investors $U^I(n) - U^U(n)$. The point of intersection with the information cost K defines n^*. The utility gap may be non-monotonic in n, giving rise to two informed equilibria: one with relatively low n^* and one with relatively high n^*. Whenever this is the case, we always focus on the latter because (i) it is the only *stable* informed equilibrium, and (ii) only stable informed equilibria exist when the utility gap is strictly decreasing in n. Taking this restriction as given, we now discuss the forces that determine the shape of the utility gap.

Sources of information rents. There are two kinds of information rents: *participation rents* and *price rents*. Participation rents accrue because the informed buy more bonds in the good state, and bonds are always priced such that investors earn inframarginal rents on their purchases. Price rents accrue because the informed buy bonds at the marginal price P_L in the bad state, while the uninformed buy $\widehat{B}_L^U = B_H^U + B_L^U$ bonds in the bad state at a higher average price $\widehat{P}_L = \left(\frac{B_H^U}{\widehat{B}_L^U}\right) P_H + \left(\frac{B_L^U}{\widehat{B}_L^U}\right) P_L$. This average price is large and close to P_H when uninformed investors bid heavily at the high-state price, i.e. when B_H^U is large relative to B_L^U.

Suppose first that we are in the full-participation region where the uninformed buy bonds in both states. Information rents depend on n as follows. When there are few informed investors and n is small, uninformed investors must participate heavily in the good state to clear the market. As a result, the portfolios of the two types are similar in the high state, and participation rents are small. On the other hand, uninformed investors buy a large fraction of their low-state bonds at P_H rather than P_L, and so price rents are large because $\widehat{P}_L$ is close to P_H.

How do these information rents evolve as n grows? We showed above that an increase in n leads to an increase in P_H. This hurts both types of investors, and leads to a reduction in bond purchases in the good state. Yet the speed at which portfolios adjust is asymmetric. The uninformed reduce their purchases of good-state bonds by more than the informed because (i) they end up purchasing the high-priced bonds in both states of the world, and (ii) higher P_H means that the price gap $P_H - P_L$ at which the uninformed overpay in the low state is increasing. This disproportionate retreat from the good state leads to *increased participation rents*.

Next, consider the effects of an increase in n on price rents. There are three forces. First, an increase in n leads to a reduction in P_L because the uninformed spend more in the low state than the informed. This price reduction benefits both groups of investors, but differentially so. Holding the uninformed's portfolio fixed, a widening price gap $P_H - P_L$ leads to increased price rents. Yet precisely because the uninformed adjust their portfolio optimally, growing P_H and falling P_L induce the uninformed to reallocate funds away from the high-price bond. Because the uninformed only get to buy at P_L whenever the state is indeed bad, they can buy low-state bonds *as if* they were informed. As a result, the uninformed's low-state portfolio grows closer to that of the informed. At the margin, this effect allows the uninformed to benefit disproportionately from falling P_L. The effect is reinforced by the fact that the gap $\widehat{P}_L - P_L$ falls *endogenously* as the informed shift their purchases to the bad-state bond and the weight on the high-state price P_H falls. As a result, there are *reduced price rents* for the informed.

The net change in utility gap after an increase in n is then determined by the relative change in participation and price rents. Net increases in the utility gap are more likely when the informed retreat from the good state more slowly, as the bad-state portfolio of the uninformed remains sufficiently different from that of the informed and price rents shrink slowly even as participation rents increase. Moreover, the utility gap is more likely to be increasing when n is small, as small n necessitates heavy participation of the uninformed in the high state, and thus sufficiently different low-state portfolios.

The utility gap in the full-participation region may therefore display an inverted-U shape whenever price rents decline slowly, but is strictly decreasing in n when price rents fall sharply. The non-monotonicity is more likely for large D and low z, because both contribute to a slower retreat of the uninformed from the high state: higher D increases the infra-marginal rents earned by both types of investors by depressing price levels, and makes withdrawing from the good state more costly; lower z means that the high-state bond is less overpriced because the default probabilities in the two states are relatively transparent.

The forces shaping the utility gap are particularly transparent in the partial-participation region. Because the uninformed buy bonds at the correct marginal price in the low state and do not buy any bonds in the high state, their low-state portfolio is the same as that of the informed. As a result, there are no price rents. Yet there are large participation rents because the uninformed do not participate in the high-state at all. At the same time, partial participation means that the uninformed are insulated against high-state price increases. Increases in n therefore do not lead to a further reallocation of funds away from the good state and accordingly do not boost participation rents. Instead, price increases in the high state now only hurt the informed, leading to a *cannibalization* of rents among the informed. In the partial-participation region, increases in n thus unequivocally lead to a reduction in the utility gap.

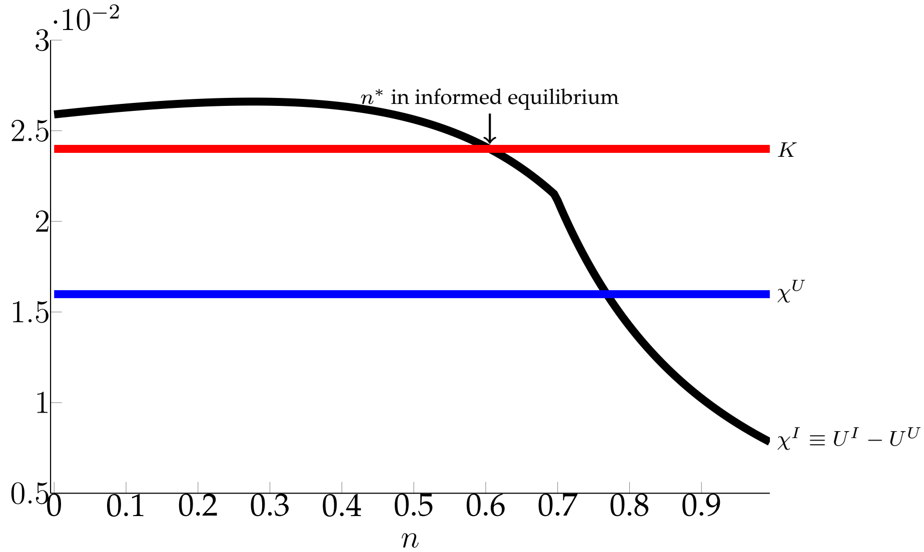
2.2.3 Multiplicity

We now show that uninformed and informed equilibrium can coexist. Figure 4 depicts an example where there is a unique informed equilibrium (i.e. the utility gap is strictly decreasing), and this informed equilibrium co-exists with an uninformed equilibrium. That is, the utility gap in the informed equilibrium is exactly equal to the cost of information ($\chi^I = K$) at some $n^* > 0$, but the gains from becoming informed in the uninformed equilibrium do not outweigh the costs ($\chi^U < K$).

The economic force underpinning this multiplicity is that the fundamental benefit of becoming informed is *the ability to buy the bond at the right marginal price in all states*. For this reason, the value of information depends on how different marginal prices are across states of the world. In an uninformed equilibrium, prices are independent of the state of the world. A newly informed investor can thus only adjusts quantities, but earns no rents from buying at the correct marginal price. An informed equilibrium instead features an endogenous price gap $P_H - P_L > 0$ that the newly informed investor can exploit: in addition to adjusting quantities, he also no longer overpays for the bond in the low state. The very presence of informed investors thus boosts the incentives for further information acquisition by generating a price rent that can be exploited at the expense of uninformed investors.

We now discuss the comparative statics of equilibrium outcomes with respect to fundamentals, and show how shocks to fundamentals can trigger switches from in-

Figure 4: Equilibrium Multiplicity

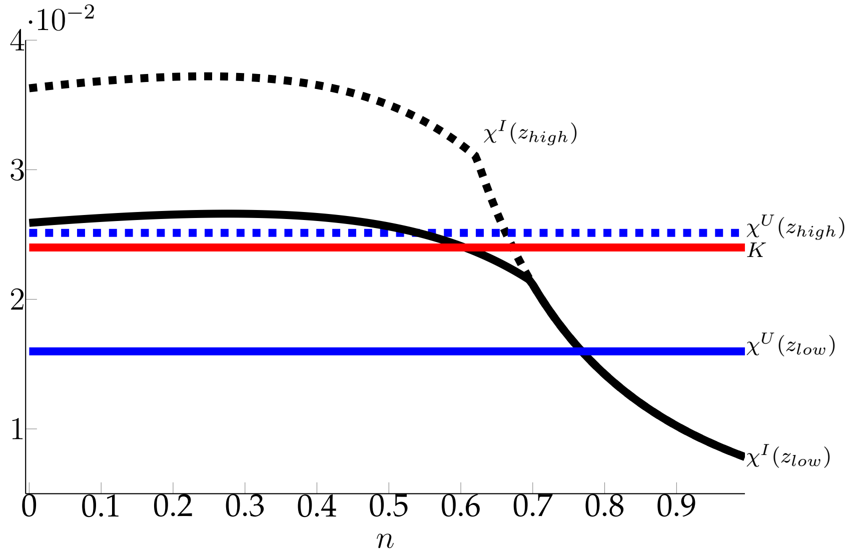


formed to uninformed equilibrium and vice versa. Specifically, we analyze changes in the countries' debt burden and default probability. Default probabilities can increase in three ways: an increase in the probability of a mediocre output z, an increase in the probability of a bad output x, and an increase in the probability of a bad state, a. Naturally, the ultimate source of the shock will have different implications for price levels, volatility and information acquisition.

We focus on analyzing an increase in z (a reduction in the probability of a high income realization and an increase in the probability of a mediocre realization). This change induces an increase in the gap between default probabilities in the low state ($\kappa_H = x$) and the high state($\kappa_L = x+z$) by boosting the default probability in the low state only. It thus represents an increase in the country's *downside risk*. Because the focus of our paper is on crises events, we therefore view changes in z as the key comparative static of interest. Figure 5 shows how the set of possible equilibria changes in response to an increase in z from $z_{low} = 0.15$ to $z_{high} = 0.195$.

The solid lines represent $z_{low} = 0.15$. The dotted lines $z_{high} = 0.195$. We find that an increase in the probability of default driven by an increase in z leads to more information acquisition. The intuition is that information is more valuable when there is more variation in default probabilities across states. In the uninformed equilibrium, an increase in z boosts the individual incentives to acquire information so as to buy

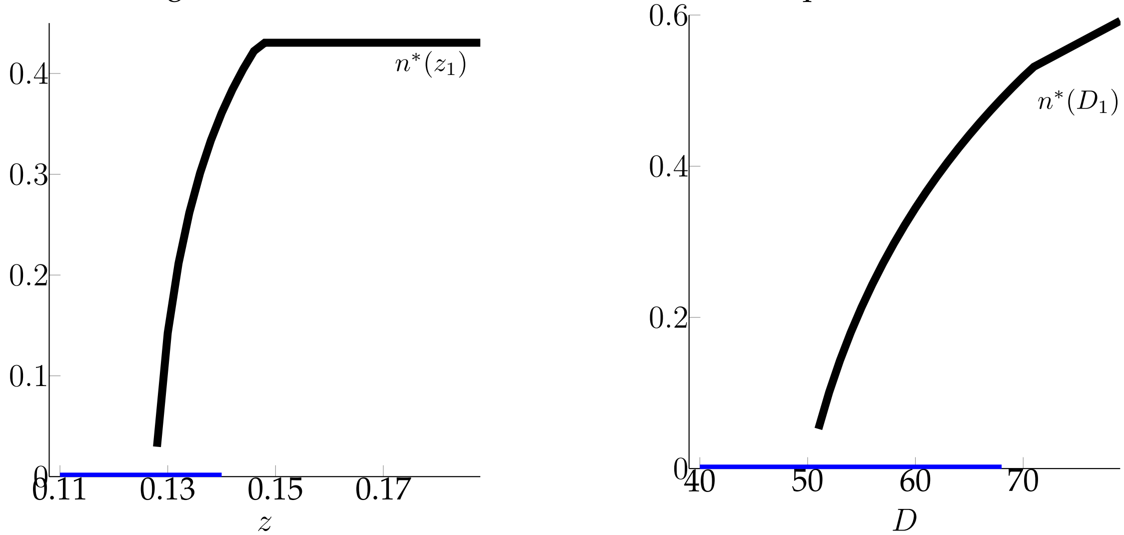
Figure 5: Effect of z on Equilibrium Multiplicity



less of the bond when the default probability is $x + z$ (low state) rather than x (high state). That is, χ^U increases. In this numerical example, this effect is large enough for the uninformed equilibrium to become unsustainable. In the informed equilibrium, increases in z grow the utility gap through its effect on price rents because the uninformed overpay by more when z is large. As a result, n^* must increase to equalize returns. We obtain a similar result when we compare two levels of indebtedness: an increase in per-capita debt D/W also increases the incentives to become informed in both equilibria. The intuition is that higher debt levels force investors to hold a larger fraction of their assets in risky debt, increasing the value of not overpaying for the bond. Figure 6 depicts these results graphically by plotting the equilibrium fraction of informed investors n^* as we vary z and D/W. Note that these two mechanisms naturally reinforce each other in a more general model in which default probabilities endogenously increase when increased indebtedness erodes bond prices.

Figure 7 plots the uninformed-equilibrium price P_U and the informed-equilibrium prices P_H and P_L as a function of z (evaluated at the equilibrium fraction of informed n^* at each fundamental z). The graph features three distinct regions. When z is low, there are low incentives to acquire information and only the uninformed equilibrium is sustainable. When z is high, there are large incentives to acquire information and only the informed equilibrium is sustainable. Both equilibria coex-

Figure 6: Effect of z and D on Information in Equilibrium



ist for intermediate values of z. Computing the weighted average of prices $E(P) = aP_H + (1-a)[\omega P_H + (1-\omega)P_L]$, where $\omega = \frac{(1-n)B_H^U}{(1-n)(B_H^U + B_L^U) + nB_L^I}$ is the fraction of low-state bonds bought at the high price, shows that the informed equilibrium is not only characterized by higher price volatility, but also by a lower average price $E(P)$. While more informed investors boost the price in the good state, the bad-state price falls disproportionately as the high marginal utility of wealth in a default state depresses investor's risk appetite. Accordingly, Figure 8 shows that the government's average debt burden is also higher in the informed equilibrium. That is, not only is increased information acquisition harmful to investors, it also hurts the government's fiscal position.

Remark on robustness. We emphasize that the co-existence of informed and uninformed equilibrium does not rely on the fact there are precisely zero informed investors when no investor spends resources to become informed. In particular, the same forces persist even in the presence of a (small) fraction of exogenously informed investors $\underline{n}$. While the presence of these investors would generate an exploitable price gap even in the "uninformed" equilibrium, this gap would be smaller than the one in an informed equilibrium in which an additional mass of investors endogenously acquires information. As a result, there continues to be a "burst" in the the value of information so long as utility gap is initially increasing in the total mass of informed investors. We have shown this to be the case as long as the uninformed retreat from

Figure 7: Equilibrium Prices

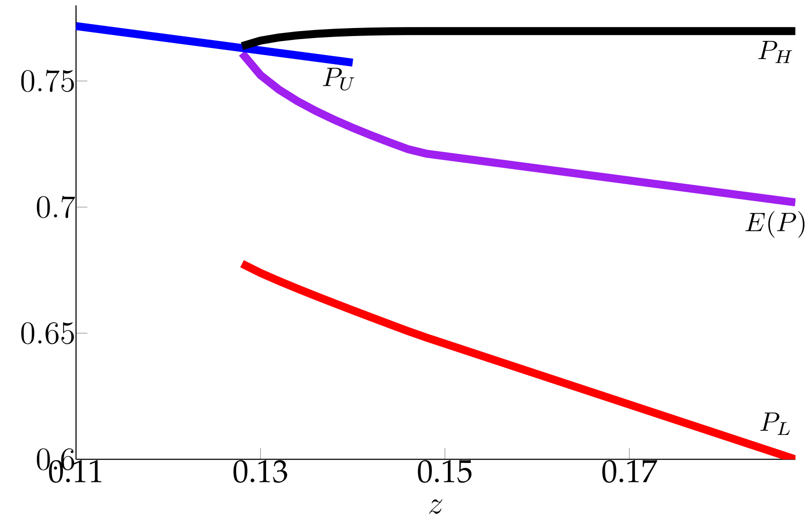
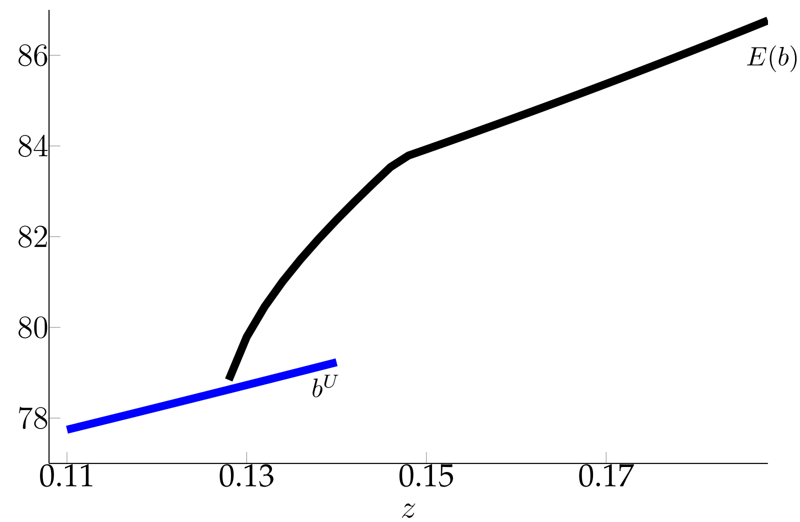


Figure 8: Equilibrium Debt Burden



high state sufficiently slowly. Indeed, in line with our discussion on the multiplicity of informed equilibria above, the "uninformed" equilibrium with exactly $\underline{n}$ exogenously informed investors would be an unstable informed equilibrium, while the informed equilibrium with additional endogenous information is stable.

2.2.4 Persistent Information Regimes and Bond Price Hysteresis

The equilibrium multiplicity characterized above has implications for interpreting how shocks to fundamentals affect a country's debt burden, as well as the volatility that countries experience in their sovereign spreads. Consider a simple and plausible equilibrium selection rule: a country remains in a given equilibrium (informed or uninformed) as long doing so is sustainable, but switches when it must. This "conservative" equilibrium selection greatly restricts the role of multiplicity, but necessarily introduces history dependence, or *hysteresis*, in that two countries with the same *current* fundamentals may face different bond prices, debt burdens, and volatility only because their past was different.[5] Such a rule also implies that small changes to fundamentals may sometimes lead to large price changes, while at other times they do not, and that some countries may have to undergo dramatic fiscal consolidations in order to benefit from a move to an uninformed regime, while those who have been less profligate in the past are given more leeway.

These results are also relevant in interpreting the mapping from fundamentals to sovereign debt prices. Periods of relative calm in bond prices do not necessarily imply that fundamentals are calm, as it may be that the country raises funds in an uninformed equilibrium in which prices are relatively insensitive to fundamentals. In contrast, turbulent periods do not necessarily imply that fundamentals have become much more turbulent than normal, as it may be that the country transitioned to an informed equilibrium in which prices are more sensitive to movements in fundamentals.

2.3 Information Rents in the Presence of Secondary Markets

Our analysis above assumes that government bonds are only traded once upon their issuance. In practice, of course, government bonds typically trade in deep and liquid secondary markets that dynamically incorporate investors' private information. This raises the question of whether informed investors can still earn information rents in the presence of such markets. To address this issue, we introduce a secondary market in which investors competitively trade government bonds bought during the primary market. This secondary market takes place after the primary market has

[5] A history-dependent selection criterion is formally proposed and solved by Cooper (1994).

closed, but before the government makes its default decision. To account for the fact that any private information held by investors during the initial auction may have been incorporated into prices or otherwise revealed, we assume that the state of the world θ is publicly observed prior to the opening of the secondary market, but after the closing of the primary market. This information structure implies that secondary market prices and quantities are state-contingent. We denote the secondary market prices in state $\theta \in \{L, H\}$ by ρ_θ. We relegate the full analysis to Appendix B and focus here on discussing informally the main implications of secondary markets for the robustness of our previous results.

We show that, even with secondary markets, there exist informed equilibria in which informed investors earn rents that exactly offset the cost of acquiring information. Moreover, the presence of secondary markets may exacerbate the price volatility in primary auctions, and *with* secondary markets there may exist informed equilibria that are not sustainable *without* secondary markets. The benefits of information in a setting with secondary markets arise from differences between primary and secondary market prices. This differential allows informed investors to capture arbitrage profits by buying low in the primary market and selling high in the secondary market, with the price gap endogenously determined to exactly deliver the required rents. The next proposition formalizes this basic insight

Proposition 6 *In any informed equilibrium ($n^* > 0$) the high-state primary market price is lower than the high-state secondary market price ($P_H < \rho_H$), while the low-state primary market price is equal to the low-state secondary market price ($P_L = \rho_L$).*

Informed investors thus earn arbitrage profits because there are price differences across primary and secondary markets *conditional on the state*. Compared to the model without secondary markets, information rents thus consist purely of price rents rather than participation rents, as the uninformed always end up holding bonds even when they do not purchase any in the primary market. Uninformed investors thus "pay" for the ability to make state-contingent bids by buying in the high-price secondary market, while informed investors pay for this ability directly by buying information.

The degree of arbitrage is modulated by two endogenous variables: the fraction of informed investors n and the primary market price P_H. Informed investors always fully exploit the arbitrage opportunity and buy as many primary market bonds as possible: $B_H^I = \frac{W}{P_H}$. The total supply of bonds in the secondary market is thus limited only by the wealth and mass of the informed investor pool. That is, arbitrage rents are decreasing in n because an increase in bond supply depresses prices. An increase in the primary market price in turn decreases arbitrage rents in two ways: by lowering the price gap $\rho_H - P_H$ and by reducing the number of primary market bonds each informed investor can afford to buy.

The extension thus shows that, even in the presence of secondary markets whose prices convey information about θ, there are incentives to acquire information as long as investors do not have access to unlimited arbitrage capital. Furthermore, these incentives may indeed exacerbate in occasions multiplicity of equilibria, contagion and volatility of prices.

3 Two-Country Model

So far we have studied the role of information in determining the funding conditions for a single country. We now extend the model and study two countries that raise funds by issuing debt to the same unit mass of investors. For simplicity, we abstract from secondary markets and assume that both countries are characterized by the same stochastic structure for income Y and default costs θ, albeit with possibly different parameters.

Our analysis is in two steps. First, we study the two-country problem in the absence of information acquisition. We show that negative shocks to one country's fundamentals increase spreads in both countries through a portfolio reallocation effect as long as investors exhibit prudence $(u'''(c) > 0)$. Second, we introduce endogenous information acquisition and show that there are complementarities across countries in the incentives to acquire information. This leads to a second form of contagion based on information regimes: shocks that induce local information acquisition may trigger global information acquisition, generating increased sensitivity to fundamentals and higher volatility in both countries.

We use the following structure to maintain tractability in our two-country setting. First, we assume that every investor is a household consisting of two members, each of whom is assigned to participate in the bond auction of one particular country. Second, auctions in both countries take place simultaneously, and the investor must split its wealth across its two members prior to the auctions. Third, each household member can acquire information about the country it has been assigned to, but members cannot communicate this information to each other prior to the auction. As a result, each member's bids are not explicitly contingent on the bids of the other member. Household members can, of course, coordinate on an information acquisition *strategy* ex ante.

These assumptions allow us to simplify the analysis by restricting the number of equilibrium prices. If all bids were contingent on the realization of the state and bids in the other country, each investor would have to optimally choose bids at J^2 potential marginal prices in each country, where J is the number of possible states of the world. Moreover, these optimal bids would be globally linked through the first-order conditions of uninformed investors, greatly complicating the demand system. Our structure reduces the number of marginal prices and bids to J in each country, but preserves the essential features of a two-country setting: investors must make a portfolio choice about which country to invest in, and the optimal portfolio and information acquisition choices depends on the information environment in each country.

3.1 Contagion Without Information Acquisition

We start by analyzing the two-country model without information acquisition in which both countries are in the uninformed equilibrium. Note that no-information-acquisition is always an equilibrium of the model with endogenous information for sufficiently high information costs. This case turns out to be a fairly straightforward extension of the one-country uniformed equilibrium. Since there are now three possible assets (storage, country 1's bonds and country 2's bonds) the household's maximization problem is

$$\max_{B_1,B_2} U = \widehat{\kappa}_1 \left[\widehat{\kappa}_2 u(W - P_1B_1 - P_2B_2) + (1-\widehat{\kappa}_2)u(W - P_1B_1 + (1-P_2)B_2)\right]$$
$$+(1-\widehat{\kappa}_1)\left[\widehat{\kappa}_2 u(W + (1-P_1)B_1 - P_2B_2) + (1-\widehat{\kappa}_2)u(W + (1-P_1)B_1 + (1-P_2)B_2)\right]$$

The first-order condition for bids in country i is

$$\frac{E_i(u'(+))}{E_i(u'(-))} = \frac{P_i \widehat{\kappa}_i}{(1-P_i)(1-\widehat{\kappa}_i)}$$

where

$$E_i(u'(-)) = \widehat{\kappa}_{-i} u'(W - P_i B_i - P_{-i} B_{-i}) + (1-\widehat{\kappa}_{-i}) u'(W - P_i B_i + (1-P_{-i}) B_{-i})$$

and

$$E_i(u'(+)) = \widehat{\kappa}_{-i} u'(W + (1-P_i) B_i - P_{-i} B_{-i}) + (1-\widehat{\kappa}_{-i}) u'(W + (1-P_i) B_i + (1-P_{-i}) B_{-i})$$

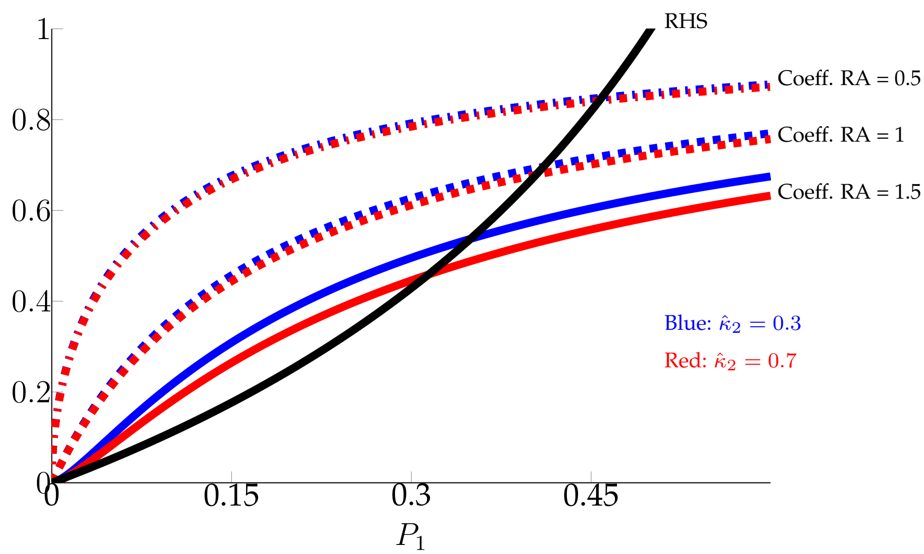
The next proposition shows that, when u satisfies constant relative risk aversion (CRRA), an increase in the expected default probability in one country reduces the sovereign price in the other country. This result holds despite the fact that bond returns in each country are i.i.d and there is no feedback other than through portfolio reallocation. The reason is *prudence* ($u''' > 0$): the "background risk" introduced by increased default risk in one country induces investors to hold less risky assets overall, depressing bond prices globally.

Proposition 7 *There is contagion (i.e.* $\frac{\partial P_i}{\partial \widehat{\kappa}_{-i}} < 0$*) when preferences are CRRA.*

Because contagion relies on prudence, the magnitude of contagion is increasing in the degree of prudence. For the CRRA utility function $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$, the coefficient of relative prudence is $(1+\gamma)$. To understand how prudence affects spillovers, we now specialize the first-order condition (17) to country 1, and ask how changes in country two's default probability $\hat{\kappa}_2$ affect country 1's bond price for various levels of γ. Figure 9 graphically depicts the first-order condition. The red and blue lines depict its left-hand side, consisting of the ratio of marginal utilities across default and no-default in country 1. Because the investor holds both bonds, this ratio generically depends the default probability in country 2. Blue represents $\hat{\kappa}_2 = 0.3$, while red represents $\hat{\kappa}_2 = 0.7$. The black line depicts its right-hand side, which is independent of γ and $\hat{\kappa}_2$.

We find three results. First, the larger prudence (and, thus, risk aversion), the lower the bond price for any $\hat{\kappa}_2$. Second, the larger $\hat{\kappa}_2$, the lower the bond price, for any given γ. Third, the larger prudence, the sharper the fall in country 1's bond price upon an increase $\hat{\kappa}_2$. In other words, the sensitivity of country 1 to country 2's fundamentals is increasing in the degree of investor prudence.

Figure 9: Contagion and Prudence



3.2 Contagion on the Informational Regime

We now turn to endogenous information acquisition with two countries. Given that household members do not share acquired information prior to bidding, the set of marginal prices in country i is independent of θ_j, the realized default cost in country j. We denote the marginal price in state of the world j in country i by P_{ij}, and investors' bids at this price in state θ_i by $B_{ij}(\theta_i)$. The investor's terminal wealth, or consumption, in state $s = (\theta_1, \theta_2, Y_1, Y_2)$ is

$$C(s) = W + \sum_{i=1}^{2} \sum_{j':\theta_{j'} \geq \theta_j} \left[\mathbb{I}(Y_i > \overline{Y}(\theta_{j'})) - P_{ij'} \right] B_{ij'}(\theta_{j'}),$$

where Y_i denotes country i's realized income. Accordingly, investors' utility gross of K, the cost of acquiring information is

$$\sum_{s} \Pr\{s\} U\left(C(s)\right), \tag{15}$$

where $\Pr\{s\}$ denotes the probability of state s. The *net* utility of the investor is (15) if he does not acquire information in any country, (15) minus K if he acquires information in one country, and (15) minus $2K$ if he acquires information in both. The investors' decision problem is to choose $\left\{\{B_{ij}(\theta_i)\}_{j=1,2}\right\}_{i=1,2}$ to maximize (15). Given that information is endogenous, investor bids must satisfy the measurability constraint

$$B_{ij}(\theta_j) = B_{ij}(\theta_{j'}) \text{ for all } j \text{ and } j'. \tag{16}$$

for any country i in which the investor did not acquire information. This condition ensures that investors bid the same amount in states of the world that they cannot distinguish.

We focus our analysis on demonstrating how the presence of informed investors in one country shapes the incentives to acquire information in the other country. To do so, we construct the *value of becoming informed* in four different types of equilibrium, and ask how this value varies with the global information environment. Whenever possible, we focus on symmetric equilibria in which the number of informed and the set of marginal prices is the same in both countries. As a result, the value of information is also symmetric. Such symmetric equilibria naturally exist when the country's fundamentals are identical, which we assume throughout. Figure 10 depicts our results graphically by plotting the value of information as a function of n.

The first case we consider is the uninformed equilibrium in which no investor acquires information in either country. As a result, there is a single marginal price in each country that is identical across countries. The value of becoming informed is the increase in utility obtained by a single new informed investor who takes as given that no other investors are informed about any country. We denote this value by χ_{UU}^{IU-UU} and depict it in blue.

The second case is an equilibrium in which there are informed investors in the "home" country, but we exogenously impose that no investor acquires information about the "foreign" country. In this asymmetric equilibrium, there is a single price in the foreign country, and two state-contingent prices in the home country. We compute the incentives to acquire information by comparing the payoffs of investors who are informed about the home country to those who are not, taking as given that no investor is informed about the foreign country. We denote the information incentives in this equilibrium by χ_{IU}^{IU-UU} and depict it in black.

The third case we consider in Figure 10 is a symmetric equilibrium in which $n \leq 0.5$ investors are informed about the home country, a further n investors are informed about the foreign country, but no investor is informed about both countries. Given that both countries are symmetric and fundamentals are i.i.d, the marginal benefit of acquiring information in one country is higher than that of acquiring information in the second country. As a result, no investor will acquire information in both countries until all investors are informed about at least one country. There are now two marginal prices in each country, and these state-contingent price are identical across countries. We compute the incentives to acquire information by comparing the utility obtained by those informed in one country and those informed in neither. We denote this value by χ_{II}^{IU-UU} and depict it in solid green.

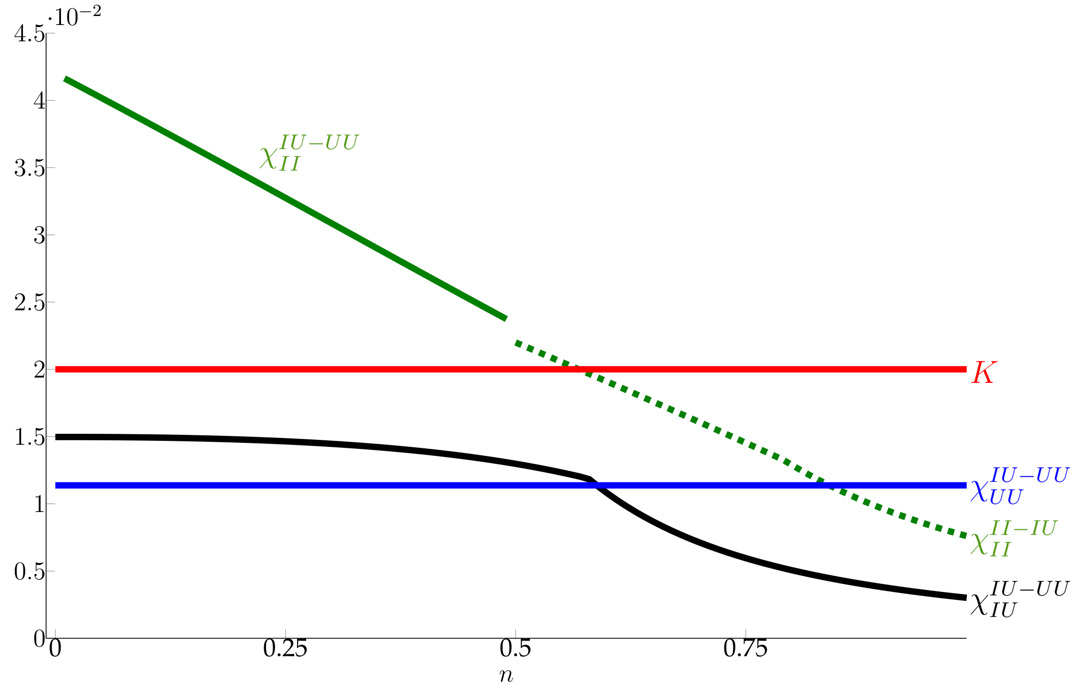
Once $n = 0.5$, all investors are informed about one of the two countries, with exactly 0.25 investors informed about each. For $n > 0.5$, we therefore construct the symmetric equilibrium in which $n - 0.5$ investors are informed about both countries, and $\frac{n-0.5}{2}$ of investors are each informed about one of the two countries. That is, all investors are informed about each country, and some investors are informed about both. As before, there is an (identical) set of two marginal prices in each country. χ_{II}^{II-IU}, plotted in dashed green, shows the incentives to acquire information in the second country by comparing the utility obtained by investors informed in both countries to that of investors informed in only one country.

Finally, the solid red line depicts K, the per-country cost of acquiring information.

We find that the incentives to acquire information in one country are strictly higher when there are informed investors in the other country than when there are not. The intuition is as follows: as long as there is a country without informed investors, uninformed investors can invest in the uninformed country without running the risk of buying the bonds at the wrong marginal price. The uninformed country thus acts as a "safe haven" by providing the opportunity to buy bonds without price risk. This dulls the uninformed's incentives to acquire information. When there are informed investors in both countries instead, the remaining uninformed investors cannot avoid paying excessive prices in some states of the world if they want to buy bonds, increasing their incentives to acquire information. The existence of informed investors thus begets further information acquisition.

We present an example in table 1 to show how the information environment shapes the individual investor's portfolio allocation. To do this, we fix the number of in-

Figure 10: Complementarity on Information Incentives



vestors who are informed about country 1 at 1/4 of the total. We consider two cases with respect country 2: no one is informed about this country and 1/4 of the total investors are informed about it. This example corresponds to $n = 0.25$ in figure 10, hence note that no investor is informed about both countries.

In the first part of the table we compare the total expenditures of an informed investor in the country in which he is informed to the expenditures in the country in which he is uninformed, as a fraction of his total wealth W. In the second part we make a similar comparison for an investor who is uninformed in both countries. The table illustrates several stark patterns. First, while the informed investors always tilt their investments toward the country they are informed about, this tilting is much stronger when the other country is also in the informed equilibrium (from 30.5% to 36.7%). Moreover, the reduction in their investment in country 2 (where they are uniformed) when there are informed investors also in country 2 (from 19.3% to 8.3%) is so strong that their overall average investment in risky bonds falls (from 49.8% to 45.0%).

When we look at the investment expenditures of investors who are uninformed in both countries, we see that they too tilt their investments away from a country which

Table 1: Average Expenditures by Information Regime: Informed vs. Uninformed

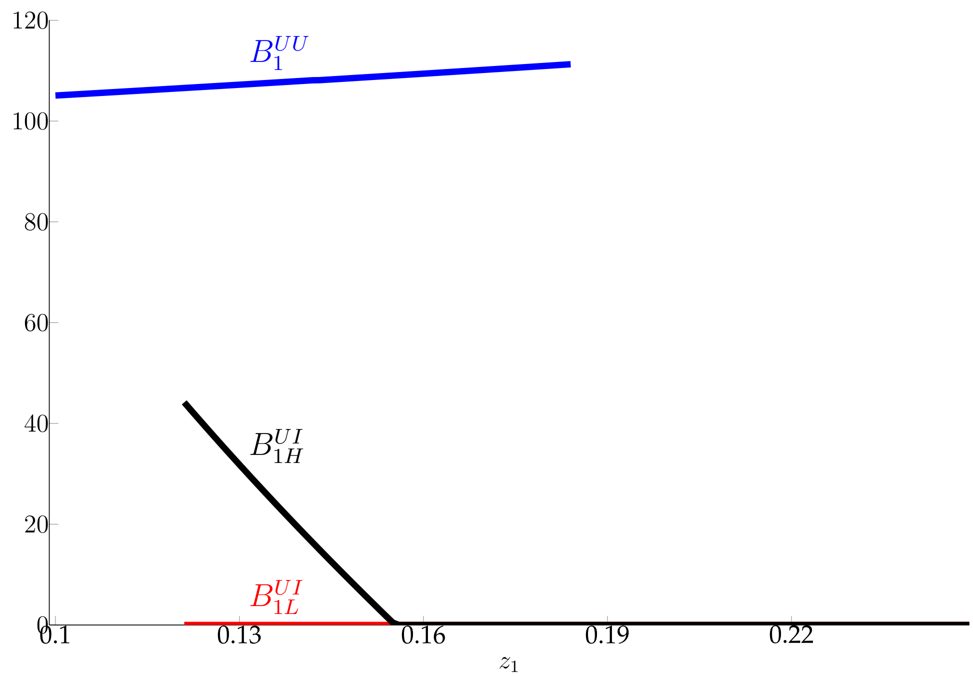
Informed in 1 only	EX_I^I	EX_U^I	$EX_I^I + EX_U^I$
$n_1 = \frac{1}{4}, n_2 = 0$	30.5%	19.3%	49.8%
$n_1 = n_2 = \frac{1}{4}$	36.7%	8.3%	45.0%
Uninformed in both	EX_I^U	EX_U^U	$EX_I^U + EX_U^U$
$n_1 = \frac{1}{4}, n_2 = 0$	11.0%	20.2%	31.2%
$n_1 = n_2 = \frac{1}{4}$	9.0%	9.0%	18.0%

is in the informed equilibrium. This leads them to invest more in a country without informed investors (11.0% in country 1 as opposed to 20.2% in country 2 where no investor is informed) and to invest symmetrically invest in both countries (9% in each) when there are the same fraction of informed investors participating. However, in the second case, the uninformed have very sharply reduced their average expenditures on risky bonds (from 31.2% to 18%).

The concentration of investment by the informed increases the benefits they gain from becoming informed. At the same time, as uninformed investors will shrink their participation in risky sovereign bonds as there is more information lowers their welfare and increases the relative gains from becoming an informed investor. These forces contribute the extent of cross-country information complementarities.

Figure 10 illustrates the strength of this equilibrium cross-country information complementarity. We choose the cost of information K to be such that, conditional on no investors being informed in the foreign country, only the uninformed equilibrium is sustainable. That is, no investor acquires information about the home country at cost K when no investor is informed about the foreign country. If instead we allow for endogenous information acquisition in both countries, a symmetric informed equilibrium exists in addition to the uninformed equilibrium. In this particular example, the informed equilibrium is such that 20% of investors are informed about both countries and the remaining 80% are informed about exactly one country. The scope for information acquisition in both countries thus boosts the degree of *equilibrium* information acquisition. We have already shown that information is reflected in higher spreads, debt burdens, and volatility. The cross-country information complementarity result we establish thus immediately leads to important implications for the contagion of sovereign debt crises: a switch to an informed equilibrium in one country may be enough to trigger information acquisition globally, raising the specter of rising yields and default risk.

Figure 11: Segmentation

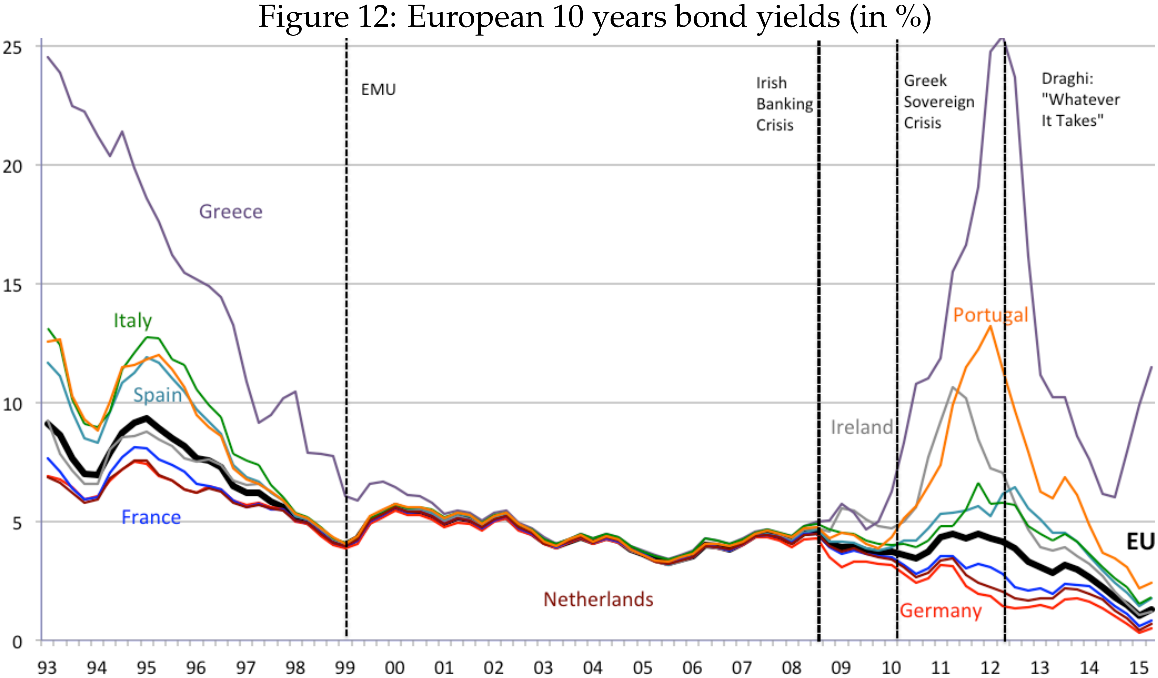


The impact of this mechanism on investment segmentation is corroborated in Figure 11. We plot the equilibrium bids of an investor who has acquired information in country 1 but not 2, in the symmetrically informed equilibrium in which n^* investors are informed about one of the two countries and $1 - 2n^*$ investors are uninformed about both. The black line depicts the investor's bids at the high marginal price in country 1 (the country in which he is uninformed), while the red line depicts his bids at the low marginal price in that country. For comparison, the blue line depicts the bids of an uninformed investor in the uninformed equilibrium. All bids are plotted as a function of $z_1 = z_2$, the increment in the bad state's default probability. The investor optimally invests less in the country in which he is uninformed when other investors *are* informed about this country. An increase in z_1, which leads to to an increase in information acquisition incentives and, thus, n^*, only strengthens this effect. Investors thus tend to invest more in the country in which they are less informationally disadvantaged, and the fact that they concentrate their holdings further boosts information acquisition incentives. These forces can be sufficiently strong that these investors invest exclusively in the country in which they are informed.

4 An illustration Based on the European Debt Crisis

We now illustrate our model's mechanism using the recent European Debt Crisis. Figure 12 shows that European sovereign bond yields exhibited substantial heterogeneity prior to the 1999 introduction of the Euro, but were stable and remarkably similar for almost a decade thereafter. A divergence of bond yields sets in with the 2008 collapse of Lehman Brothers and the ensuing Irish banking crisis. This divergence intensifies during 2010 and 2011 (during the so-called "Greek sovereign crisis"), with yields rising sharply for some countries (notably Greece, Ireland and Portugal), and declining for others (notably Germany, France and Netherlands). This "fanning out" of European bond yields stopped right after Mario Draghi's proclamation that the ECB "...is ready to do whatever it takes to preserve the Euro. And believe me, it will be enough." Since then, the yields of most countries started a process of convergence.

Figure 12: European 10 years bond yields (in %)



Source: Eurostat, EMU Convergence Criterion Database. Notes: As in Wright (2014), data are derived from secondary market information on prices of government bonds issued in local currency with a residual maturity of around 10 years.

Table 2: Regression Results

	Coeff.	s.d.
β_1	-0.204***	(0.023)
β_2	-0.174***	(0.050)
β_3	-0.023***	(0.008)
β_4	0.027***	(0.005)
R^2	0.64	
N	1493	
FE	Yes	

One explanation of this pattern is that bond yields closely reflect sharply diverging fundamentals after the 2008 global crisis, substantially deteriorating in countries like Greece and Ireland but improving in Germany, France and the Netherlands. Another explanation is that fundamentals did not diverge substantially, but that the *sensitivity* of yields to fundamentals increased, for example due to more private information acquisition.

To capture these explanations we run the following simple OLS regression

$$Yields_{it} = (\beta_1 + \beta_2 \mathbb{I}_c)\Delta GDP_{it} + (\beta_3 + \beta_4 \mathbb{I}_c)\left(\frac{Debt}{GDP}\right)_{it} + \eta_i + \eta_t + \epsilon_{it}$$

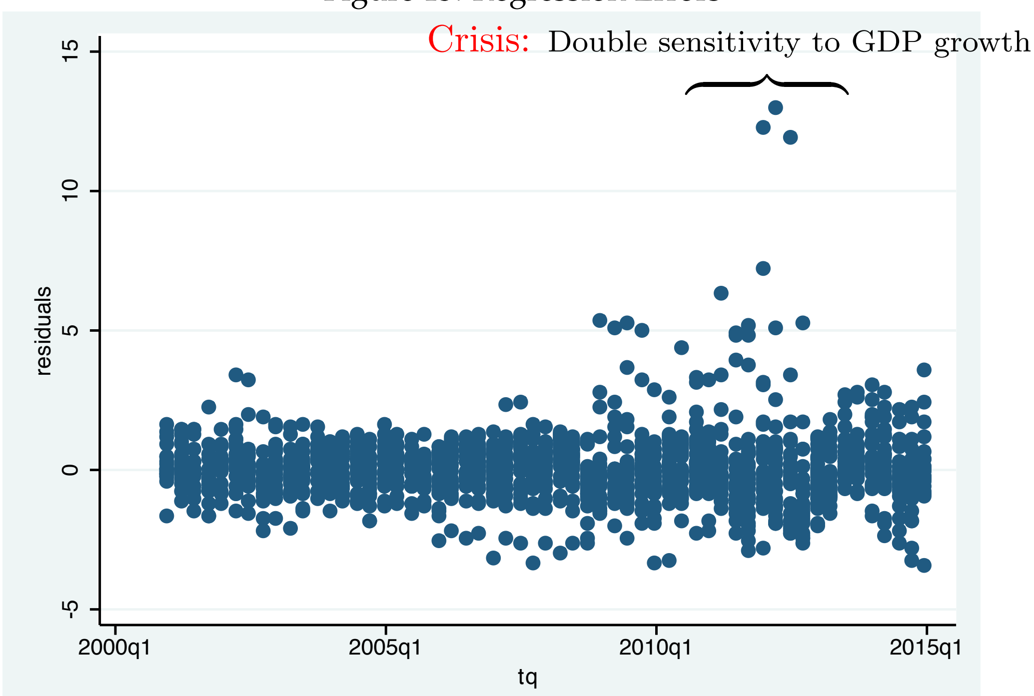
with yearly data from Eurostat for 28 european countries since 2000.[6] $Yields_{it}$ correspond to 10 year government bond yields for country i in year t. The observed fundamentals we include are the yearly change of real GDP per capita, ΔGDP_{it} and the outstanding level of public debt over GDP, $\left(\frac{Debt}{GDP}\right)_{it}$. We allow for country and year fixed effects, and for the possibility that the sensitivity of yields to fundamentals changes during crises. We capture this concern by the indicator $\mathbb{I}_c$, which is equal to 1 for the crisis years 2009-2013.

The regression results show that the the first explanation is partially correct, as GDP growth and debt over GDP are significant variables explaining the evolution of yields. Yet the second explanation plays a role as well, as the sensitivity of yields to GDP growth and debt over GDP increase significantly during the crisis. Yet even together, the explanatory power of fundamentals declines during the crisis, as evidenced by

[6]Countries are Austria, Belgium, Bulgaria, Croatia, Cyprus, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Ireland, Italy, Latvia, Lithuania, Luxembourg, Malta, Netherlands, Poland, Portugal, Romania, Slovakia, Slovenia, Spain, Sweden and United Kingdom.

evolution of the regression errors ϵ_{it} for 2009-2013. Figure 13 shows that these regression errors increased significantly between 2009 and 2013, precisely as yields fanned out.[7] More specifically, the standard deviation of the regression errors increased by a factor of three during the crisis when compared to normal times. Consistent with these results, Bocola and Dovis (2015) find that standard empirical models that tend to capture the evolution of yields in normal times are not able to accommodate their dynamics during the recent European sovereign crisis. This implies the divergence cannot be explained by the observed behavior of the usual fundamentals alone.

Figure 13: Regression Errors



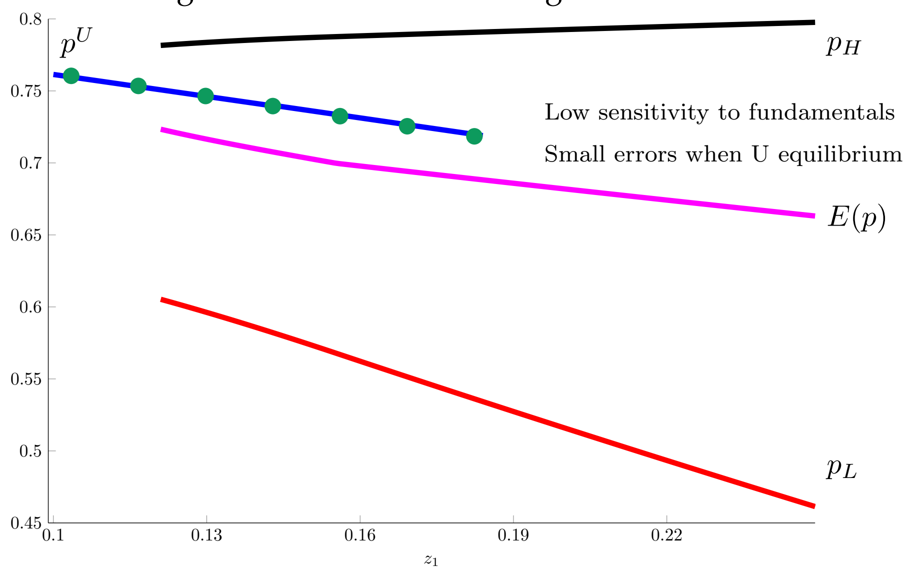
Our paper provides a novel interpretation of this residual: The divergence of the errors and the higher sensitivity to publicly observable fundamentals during crises may be due to increased private information acquisition by investors. Indeed, both the larger sensitivity to observed fundamentals and the larger errors from a regression based on those fundamentals can be rationalized by our model when one country suffers a shock that triggers a switch to a global informed equilibrium.

To see this, imagine a situation with seven countries with different z levels, which can be interpreted as the inverse of the GDP growth (the larger the GDP growth, the

[7]For more involved empirical analyses, but similar results, see Borgy et al. (2012), von Hagen, Schuknecht, and Wolswijk (2011) and Baldacci and Kumar (2010).

lower the expected probability of default for the country). Imagine also that during normal times these countries are all in an uninformed equilibrium. In Figure 14 this is captured by the seven green dots having a sovereign price according to p^U. As can be seen, prices are not very sensitive to fundamentals and can be perfectly explained by the observed fundamental z (in this extreme there would be no errors if running a regression as the one above).

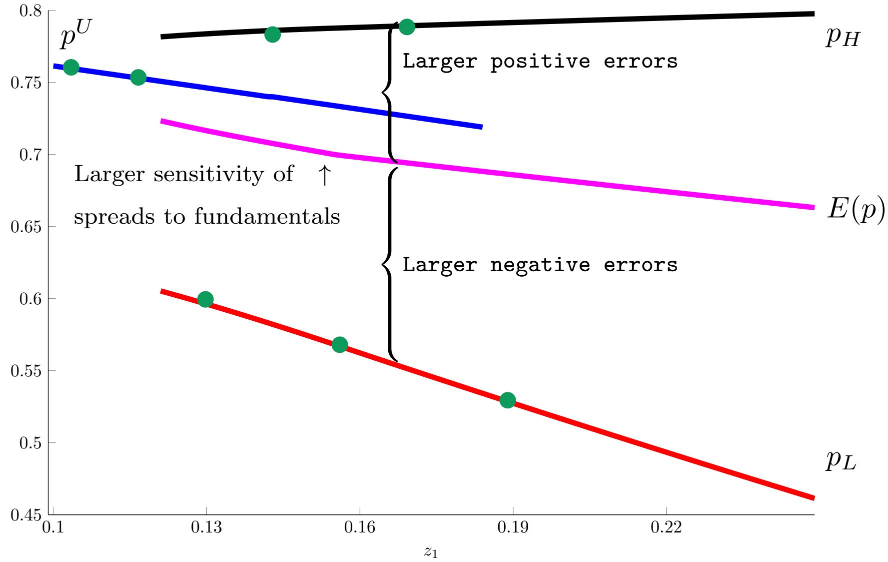
Figure 14: Simulation During Normal Times



Imagine now that the country with the largest z (or the smaller GDP growth) has a negative, relatively small, shock that reduces its GDP growth even more. The uninformed equilibrium becomes unsustainable for such a country, and investors begin acquiring information about its economy. Cross-country complementarities in information acquisiton then imply that investors may find it optimal to *also* learn about countries that were not directly hit by the initial shock. As a result, Figure 14 may change to Figure 15.

In Figure 15, the five countries with the lowest GDP growth (highest z) have moved to an informed equilibrium. Sovereign bond prices now reflect information not only about z but also about θ, which is not publicly observable and is not included in the regression. Indeed, information acquisition in our model may well involve learning about non-public shocks to variables that may not always be relevant for bond yields, such as the political cost of default, the health of the domestic financial institutions, or the exposure of domestic banks to certain assets. This has two implications. First,

Figure 15: Simulation During Crises



for some countries information about θ is "positive" (θ_H or high cost of default) and their price will be $p_H > p^U$. For other countries, information about θ is "negative" (θ_L or low cost of default) and their price will be $p_L < p^U$. This immediately implies that any regression model that uses standard publicly observable data to explain yields will have more errors during a crisis than in normal times, as there are variables not observed by the econometrician that enter into the pricing of debt.

Second, since average price is lower in the informed equilibrium than in the uninformed equilibrium, having more countries in the informed equilibrium makes the sensitivity of prices to fundamentals z larger. Specifically, the relevant coefficient is represents the average effect on the blue p^U faced by uninformed countries, and the purple $E(p)$ faced by informed countries.

Moreover, our analysis of the two-country model shows that increases in information acquisition and default risk lead to the segmentation of investor portfolios, and to higher debt burdens overall. This is consistent with the evidence in Battistini, Pagano, and Simonelli (2013) and Cipollini, Coakley, and Lee (2015), who show that there was increased segmentation in the European sovereign market during the crisis period.

5 Conclusions

This paper constructs a simple model of portfolio choice with information acquisition in which a global pool of risk-averse investors holds sovereign debt issued by a number of different countries and information about some determinants of default are not easily observable and costly to acquire.

For a single country we have shown that the participation of informed investors (informed equilibrium) is more likely when the country is highly indebted and when there is more certainty about its fundamentals. An equilibrium in which a country raises funds from informed investors is Pareto-inferior, as investors obtain less utility and the country faces higher yields, bond price volatility, and debt levels.

Given that an informed and a uninformed equilibrium may coexist, small changes in fundamentals can generate large changes in realized spreads and debt burdens. If the selection of equilibrium is history-dependent (the country remains in a given equilibrium as long as this equilibrium is sustainable) then the sovereign price of two countries with the same fundamentals but different past may be substantially different.

Once we allow for many countries, there are two important sources of contagion. On the one hand, contagion of sovereign debt prices does not require fundamental linkages or common factors, just a common pool of investors that react to changes in fundamentals of each country and rebalance the portfolio. On the other hand, the information regime is also contagious, as one country moving to an informed equilibrium increases the incentives to acquire information about other countries, even in the absence of economies of scale to acquire information.

Our results show why it is not straightforward to interpret changes in sovereign debt prices as informative about the country's fundamentals, as they depend not only on publicly observable fundamentals but sometimes also on fundamentals that are not easily observable, as they depends not only on the country's own fundamentals but also on other countries' fundamentals, as they depend not only on the country's informational regime (and thus, potentially on past fundamentals) but also on other countries' informational regime.

We have highlighted the main forces behind information acquisition and the mapping from observable and non-observable fundamentals to sovereign debt spreads, but did not focus on their quantitative power. However, there are many reasons why we may expect these forces to be also quantitatively relevant. Just to mention a few magnifying forces: First, an endogenous probability of default that responds to bond prices itself would mean that small changes in information would have dramatic effects on prices and default risk. Moreover, there would also be a feedback effect across countries: an exogenous increase in default probability in one country induces a reduction of prices in several other countries, increasing the probabilities of default in all those countries, further reduction of prices, and so on. Second, fundamental linkages across countries naturally magnify contagion. Third, time varying prudence, for example because of time varying risk-aversion or time varying wealth, would magnify contagion and informationa acquisition during crises. Fourth, market segmentation can concentrate contagion in certain regions, buffering others. Finally, how a shock in a country changes the informational equilibrium in other countries depend on the structure of the costs to acquire information: if a country attracts informed investors and then makes it cheaper for them to acquire information about other similar countries, then it is more likely that those other countries also attract informed investors.

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A Appendix: Proofs

Proof of Proposition 1: Rewrite the first-order condition (8) as

$$F(B|P,\widehat{\kappa}) \equiv \frac{u'(W+[1-P]B)}{u'(W-PB)} - \frac{P\widehat{\kappa}}{(1-P)(1-\kappa)} = 0.$$

Define $u'(+) \equiv u'(W+[1-P]B)$ and $u'(-) \equiv u'(W-PB)$. Differentiating with respect to $\widehat{\kappa}$, $\frac{dB}{d\widehat{\kappa}}$ is negative because

$$\frac{\partial F}{\partial B} = \frac{(1-P)u''(+)u'(-) + Pu''(-)u'(+)}{u'^2(-)} < 0$$

and

$$\frac{\partial F}{\partial \widehat{\kappa}} = -\frac{P}{(1-P)(1-\widehat{\kappa})^2} < 0.$$

Differentiating with respect to P, $\frac{dB}{dP}$ is negative if

$$\frac{\partial F}{\partial P} = \frac{B}{u'^2(-)}[u''(-)u'(+) - u''(+)u'(-)] - \frac{\widehat{\kappa}}{(1-P)^2(1-\widehat{\kappa})} < 0$$

A sufficient condition for this to be the case is that $\frac{u''(-)}{u'(-)} \leq \frac{u''(+)}{u'(+)}$, which is always the case for CRRA and CARA preferences. **Q.E.D.**

Proof of Proposition 2: Define

$$F(P|\widehat{\kappa}) = \frac{u'(W-D+\frac{D}{P})}{u'(W-D)} - \frac{P\widehat{\kappa}}{(1-P)(1-\widehat{\kappa})}$$

An equilibrium price P^* satisfies $F(P^*|\widehat{\kappa}) = 0$. At the one extreme, $P^* = 0$ is always a solution. To see this, note that under the Inada condition $\lim_{c\to\infty} u'(c) = 0$ the first term is zero. Hence, $F(P = 0|\widehat{\kappa}) = 0$. At the other extreme, for $P = 1-\widehat{\kappa}$, $F(P = 1-\widehat{\kappa}|\widehat{\kappa}) < 0$. To see this, note that the first term is less than one and the second term is equal to one. The risk-neutral price $P = 1-\widehat{\kappa}$ is thus never part of an equilibrium under risk-aversion. If parameters are such that $F(P|\widehat{\kappa}) < 0$ for all $P \in (0, 1-\widehat{\kappa}]$, then the only equilibrium is given by $P^* = 0$. If $F(P|\widehat{\kappa}) > 0$ for some $P \in (0, 1-\widehat{\kappa}]$, then there are also other equilibria. Among these, the maximum sustainable P^* is such that $\frac{\partial F}{\partial P} < 0$ (recall that $F(P^*|\widehat{\kappa}) = 0$ and $F(P = 1-\widehat{\kappa}|\widehat{\kappa}) < 0$).

As a result, the maximum sustainable equilibrium price is decreasing in $\widehat{\kappa}$ and D/W because (i) $\frac{dP}{d\widehat{\kappa}} = -\frac{\frac{\partial F}{\partial \widehat{\kappa}}}{\frac{\partial F}{\partial P}}$ and $\frac{\partial F}{\partial \widehat{\kappa}} = -\frac{P}{(1-P)(1-\widehat{\kappa})^2} < 0$,

and (ii) $\frac{dP}{dD} = -\frac{\frac{\partial F}{\partial D}}{\frac{\partial F}{\partial P}}$ and $\frac{\partial F}{\partial D} = -\frac{\frac{1-P}{P}u''(+) + \frac{u'(+)}{u'(-)}u''(-)}{u'(-)} < 0$ **Q.E.D.**

Proof of Proposition 3: Identical to proof of Proposition 1, with default probabilities that are conditional to the realization of the aggregate state.**Q.E.D.**

Proof of Proposition 4: First, we prove that informed investors spend less than uninformed investors in the bad state. That is, we show $P_L B_L^I < P_H B_H^U + P_L B_L^U$. Suppose otherwise, so that $P_L B_L^I \geq P_H B_H^U + P_L B_L^U$. Then

$$P_L \kappa_L u'(W - P_L B_L^I) \geq P_L \kappa_L u'(W - P_H B_H^U - P_L B_L^U)$$

From the first-order conditions for informed investors in the bad state (10) and the first-order condition for uninformed investors at the marginal price for the bad state (11), this implies

$$u'(W + (1 - P_L)B_L^I) \geq u'(W + (1 - P_H)B_H^U + (1 - P_L)B_L^U)$$

or

$$B_L^I - (B_H^U + B_L^U) \leq P_L B_L^I - (P_H B_H^U + P_L B_L^U) < B_L^I - (\frac{P_H}{P_L} B_H^U + B_L^U)$$

where the second strict inequality is the result of $P_L < 1$. This is a contradiction for all $P_H > P_L$.

Second, we prove that informed investors spend more than uninformed investors in the good state. That is, $P_H B_H^I > P_H B_H^U$. Notice the first-order condition for uninformed investors when bidding at the marginal price for the good state (12) can be rewritten as

$$(1 - a)\left[P_H \kappa_L u'(W - P_H B_H^U - P_L B_L^U) - (1 - P_H)(1 - \kappa_L)u'(W + (1 - P_H)B_H^U + (1 - P_L)B_L^U)\right] =$$
$$a\left[(1 - P_H)(1 - \kappa_H)u'(W(1 - P_H)B_H^U) - P_H \kappa_H u'(W - P_H B_H^U)\right]$$

From equation (11) and $P_H > P_L$ the left hand side is positive. This implies

$$\frac{u'(W + [1 - P_H]B_H^U)}{u'(W - P_H B_H^U)} > \frac{P_H \kappa_H}{(1 - P_H)(1 - \kappa_H)}$$

Comparing with the first order conditions for informed investors in the good state (10), then $B_H^U < B_H^I$. **Q.E.D.**

Proof of Proposition 5: If the economy is in a *partial participation region*, market clearing for the good state is just

$$n P_H B_H^I = D$$

Increasing n is isomorphic to decreasing D, and as we showed in Proposition 2 this implies $\frac{dP_H}{dn} > 0$.

In contrast, if the economy is in a *full participation region*, market clearing for the good state is

$$nP_HB_H^I + (1-n)P_HB_H^U = D,$$

which we can rewrite it in terms of excess demand as

$$ED(P_H) = B_H^U + n(B_H^I - B_H^U) - \frac{D}{P_H} = 0.$$

Then

$$\frac{dP_H}{dn} = -\frac{B_H^I - B_H^U}{n\frac{\partial B_H^I}{\partial P_H} + (1-n)\frac{\partial B_H^U}{\partial P_H} - \left(-\frac{D}{P_H^2}\right)} > 0.$$

To see that this fraction is positive for the highest equilibrium price, note first that the numerator is positive, as we have shown that $B_H^I > B_H^U$. With respect to the denominator, however, as the slope of the demand (given by $n\frac{\partial B_H^I}{\partial P_H} + (1-n)\frac{\partial B_H^U}{\partial P_H}$) and of the supply (given by $-\frac{D}{P_H^2}$) are both negative, in principle the denominator could be positive or negative. For the highest price in equilibrium, however, the denominator is negative: when evaluated at $P_H = 1-\kappa$ there is an excess of supply, as $B_H^I = 0$ and $B_H^U = 0$ (then there is no demand), while the supply is given by $\frac{D}{1-\kappa}$. The highest price in equilibrium is computed at the highest price at which demand and supply equalize, which implies that $n\frac{\partial B_H^I}{\partial P_H} + (1-n)\frac{\partial B_H^U}{\partial P_H} < \left(-\frac{D}{P_H^2}\right) < 0$. **Q.E.D.**

Proof of Proposition 6: Begin with the high-state prices. Suppose for a contradiction that $P_H \geq \rho_H$. $P_H > \rho_H$ cannot be part of an equilibrium since all investors would prefer to wait and buy bonds in the secondary market. If $P_H = \rho_H$, uninformed investors strictly prefer to wait for the secondary market so as to make state-contingent bids once θ is revealed. We have already seen that uninformed investors can always make state-contingent bids on the low-state bond, since they only get to buy at P_L when the state of the world is indeed low. Conditional on $P_H = \rho_H$, individual optimality then dictates that the uninformed buy the same portfolio as the informed, and obtain the same gross utility. This is a contradiction with $n^* > 0$. Now turn to the low-state prices. We have already argued that both uninformed and informed can make state-contingent bids at P_L. Thus if $P_L \neq \rho_L$, *all* investors would either prefer to buy in the secondary markets only, or to buy in the primary market and sell in the secondary market. In both cases, market clearing fails. Hence $P_L = \rho_L$. **Q.E.D.**

Proof of Proposition 7: Impose resource constraints $P_1B_1 = D_1$ and $P_2B_2 = D_2$ for each country in the first order conditions. Denoting $R = P_1B_1 + P_2B_2 = D_1 + D_2$, write first-order conditions as

$$\frac{\widehat{\kappa}_{-i}u'(W - R + \frac{D_i}{P_i}) + (1-\widehat{\kappa}_{-i})u'(W - R + \frac{D_i}{P_i} + \frac{D_{-i}}{P_{-i}})}{\widehat{\kappa}_{-i}u'(W-R) + (1-\widehat{\kappa}_{-i})u'(W - R + \frac{D_{-i}}{P_{-i}})} - \frac{P_i\widehat{\kappa}_i}{(1-P_i)(1-\widehat{\kappa}_i)} = 0$$

For simplicity

$$\frac{\widehat{\kappa}_{-i}u'(+-)+(1-\widehat{\kappa}_{-i})u'(++)}{\widehat{\kappa}_{-i}u'(--)+(1-\widehat{\kappa}_{-i})u'(-+)}-\frac{p_i\widehat{\kappa}_i}{(1-p_i)(1-\widehat{\kappa}_i)}=0 \tag{17}$$

where the first argument of u' corresponds to the repayment or not of country i and the second argument to the repayment or not of country $-i$.

$$\frac{dP_i}{d\widehat{\kappa}_{-i}}=-\frac{\frac{\left[u'(+-)-u'(++)-(1-\widehat{\kappa}_{-i})\frac{D_i}{P_{-i}^2}\frac{\partial P_{-i}^2}{\partial\widehat{\kappa}_{-i}}u''(++)\right]}{E_i(u'(-))}-\frac{E_i(u'(+))}{E_i(u'(-))}\frac{\left[u'(--)-u'(-+)-(1-\widehat{\kappa}_{-i})\frac{D_{-i}}{P_{-i}^2}\frac{\partial P_{-i}^2}{\partial\widehat{\kappa}_{-i}}u''(-+)\right]}{E_i(u'(-))}}{-\frac{D_iE_i(u''(+))}{P_i^2E_i(u'(-))}-\frac{\widehat{\kappa}_i}{(1-P_i)^2(1-\widehat{\kappa}_i)}}$$

A sufficient condition for contagion ($\frac{dP_i}{d\widehat{\kappa}_{-i}}<0$) is that the denominator is negative – which is the case for the highest P_i^* in equilibrium, as we discussed in the one-country case – and that numerator is also negative. The numerator is negative when

$$\frac{\frac{u'(+-)-u'(++)}{1-\widehat{\kappa}_{-i}}-\frac{D_{-i}}{P_{-i}^2}\frac{\partial P_{-i}}{\partial\widehat{\kappa}_{-i}}u''(++)}{E_i(u'(+))}<\frac{\frac{u'(--)-u'(-+)}{1-\widehat{\kappa}_{-i}}-\frac{D_{-i}}{P_{-i}^2}\frac{\partial P_{-i}}{\partial\widehat{\kappa}_{-i}}u''(-+)}{E_i(u'(-))}$$

This condition holds because the relative change in the gains from bidding in country i is necessarily smaller than the relative change in the losses. As a result, bids are lower in country i, leading to a decline in the demand and, thus, prices. **Q.E.D.**

B Appendix: Extension to Secondary Markets

In this appendix we augment the baseline one-country model to include a secondary market stage. Secondary market trading occurs after the initial bond auction (primary bond market) but before the bonds pay out (default decision).

We take as given that all investors learn θ prior to the secondary market, and denote the associated state of the world by subscript $s\in\{L,H\}$. Let ρ_s denote the secondary market price in that state, and let B_s^i and b_s^i denote the primary and secondary market state-s bond purchases of agent type i, respectively. Let X_s^i denote agent i's total primary market *expenditure* on bonds in state s, and let $\hat{B}_s^i$ denote agent i's *total* bond holdings at the beginning of the secondary market in that state. As the state of the world is revealed prior to the secondary market, we can analyze the secondary market state-by-state, conditional on the primary market bond purchases by each agent. To this end, define the following variables:

$$\begin{aligned} W_s^i &= W - X_s^i \\ \tilde{W}_s &= nW_s^I + (1-n)W_s^U \\ \tilde{B}_s &= n\hat{B}_s^I + (1-n)\hat{B}_s^U, \end{aligned}$$

where W_s^i is the amount of safe assets that the agent enters the state-s secondary market with and the latter two variables represent aggregate safe assets and bond holdings in that state.

B.1 Optimal Secondary Market Bids

In each state s, agent i's secondary-market objective is to maximize:

$$u_s^i = \kappa_s \log\left(W_s^i - \rho b_s^i\right) + (1-\kappa_s)\log\left(W_s^i + \hat{B}_s^i + (1-\rho)b_s^i\right),$$

where κ_s denotes the default probability in state s. Taking first-order conditions with respect to b_s^i yields the secondary market demand function

$$b_s^i = \frac{W_s^i(1-\rho_s-\kappa_s) - \kappa_s\rho\hat{B}_s^i}{\rho_s(1-\rho_s)} \tag{18}$$

The secondary market clearing condition is

$$nb_s^I + (1-n)b_s^U = 0$$

Accordingly, the market-clearing secondary market price is

$$\rho_s^* = \frac{(1-\kappa_s)\tilde{W}_s}{\tilde{W}_s + \kappa\tilde{B}_s},$$

and is pinned down by agents' aggregate beginning-of-period bond holdings. Secondary market quantities are

$$b_s^{i\,*} = \frac{(\tilde{W}_s + \kappa_s\tilde{B}_s)(\tilde{B}W_s^i - \tilde{W}_s\hat{B}_s^i)}{\tilde{W}_s(\tilde{W}_s + \tilde{B}_s)} \tag{19}$$

Plugging this into the utility function gives

$$u_s^i = \kappa_s \log\left(W_s^i - \frac{(1-\kappa_s)(\tilde{B}W_s^i - \tilde{W}_s\hat{B}_s^i)}{\tilde{W}_s + \tilde{B}_s}\right) + (1-\kappa_s)\log\left(W_s^i + \hat{B}_s^i + \frac{\kappa_s(\tilde{B}W_s^i - \tilde{W}_s\hat{B}_s^i)}{\tilde{W}_s}\right)$$

The secondary-market indirect utility function thus is

$$u_s^i = \log\left(\frac{\tilde{W}_s(W_s^i + \hat{B}_s^i) + \kappa_s(\tilde{B}_s W_s^i - \tilde{W}_s \hat{B}_s^i)}{(\tilde{W}_s + \tilde{B}_s)^{\kappa_s}\tilde{W}_s^{1-\kappa_s}}\right), \tag{20}$$

and is, by definition, a function of primary-market quantities only.

B.2 Informed Equilibrium with Secondary Markets

From Proposition 6, prices must satisfy $\rho_H > P_H$ and $\rho_L = P_L$. High-state price differences across primary and secondary market allow the informed to capture arbitrage rents that rationalize their information cost. We now construct equilibria that satisfy these conditions. As before, let $n > 0$ denote the fraction of informed investors.

Begin by considering the optimal portfolio of an informed investor. Since $\rho_H > P_H$ and the informed can buy bonds state-by-state, every informed investor will buy as many bonds as possible in the high state. That is, $B_H^I = W/P_H$ and so $W_H^I = 0$. As a result, the secondary market demand of an informed investor is $b_H^I = -\frac{\kappa_H W}{(1-\rho_H)P_H}$. The informed's high-state secondary market *supply* is this strictly increasing in ρ_H and strictly decreasing in P_H. Given that there are no low-state price differences between primary and secondary market, the informed are indifferent between buying in either market. Indeed, the first-order condition (18) pins down the *total* demand in the low state: $b_L^I + B_L^I = \frac{W(1-P_L-\kappa_L)}{P_L(1-P_L)}$. We will later allocate this demand across primary and secondary markets so as to satisfy market clearing in both.

We now turn to the decision problem of an uninformed investor, taking as given that $P_L^* = \rho_L^*$. We begin by specializing the indirect utility functions given above to the case of the uninformed investor. Start with the low state. Because the uninformed purchase the high-price bonds they bid for in every state of the world, our definitions above imply that

$$\begin{aligned} W_L^U &= W - P_H B_H^U - P_L B_L^U \\ \hat{B}_L^U &= B_H^U + B_L^U \end{aligned}$$

Simplify notation by defining $\chi_L \equiv \log\left((\tilde{W}_L + \tilde{B}_L)^{\kappa_L}\tilde{W}_L^{1-\kappa_L}\right)$. Then the uninformed's secondary market indirect utility function in the low state is:

$$\begin{aligned} u_L^U = &\log\left(\tilde{W}_L\left[W + (1-P_H)B_H^U + (1-P_L)B_L^U\right] + \kappa_L\tilde{B}_L(W - P_H B_H^U - P_L B_L^U) - \kappa_L\tilde{W}_L(B_H^U + B_L^U)\right) \\ &- \chi_L \end{aligned}$$

Collecting terms and using the fact that $P_L = \rho_L = \frac{(1-\kappa_L)\tilde{W}_L}{\tilde{W}_L + \kappa_L\tilde{B}_L}$ implies that B_L^U drops out of this expression:

$$u_L^U = \log\left(W\left[\tilde{W}_L + \kappa_L \tilde{B}_L\right] + B_H^U\left[(1-\kappa_L)\tilde{W}_L - P_H\left(\tilde{W}_L + \kappa_L \tilde{B}_L\right)\right]\right) - \chi_L$$

That is, the uninformed's primary market bids in the low state are irrelevant for their utility because they can always choose to buy more (or less) in the secondary market *at the same price*. Accordingly, as was the case for the informed, the low-state secondary market demand funtion 18 pins down the *total* amount of bonds purchased at $P_L = \rho_L$ but is silent on whether this is done in the primary or secondary market. Given that secondary market quantities are chosen optimally, the uninformed's low-state utility thus depends only on the amount of primary-market bonds purchased at the high price, B_H^U.

Next turn to the uninformed's high-state secondary market indirect utility function. Our definitions are:

$$\begin{aligned} W_H^U &= W - P_H B_H^U \\ \hat{B}_H^U &= B_H^U \\ \chi_H &= \log\left((\tilde{W}_H + \tilde{b}_H)^{\kappa_H} \tilde{W}_H^{1-\kappa_H}\right) \end{aligned}$$

The uninformed's high-state utility conditional on bidding optimally in the secondary market is:

$$u_H^U = \log\left(\tilde{W}_H\left[W + (1-P_H)B_H^U\right] + \kappa_H \tilde{B}_H (W - P_H B_H^U) - \kappa_H \tilde{W}_H B_H^U\right) - \chi_H$$

Collecting terms gives:

$$u_H^U = \log\left(W\left[\tilde{W}_H + \kappa_H \tilde{B}_H\right] + B_H^U K_H\left[(1-\kappa_H)\tilde{W}_H - P_H\left(\tilde{W}_H + \kappa_H \tilde{B}_H\right)\right]\right) - \chi_H$$

The decision problem of the uninformed therefore is:

$$\max_{B_H^U} \qquad U^U = a u_H^U(B_H^U) + (1-a) u_L^U(B_H^U) \tag{21}$$

The first derivative of (21) with respect to B_H^U is:

$$\begin{aligned} \frac{\partial U^U}{\partial B_H^U} = a &\left(\frac{(1-\kappa_H)\tilde{W}_H - P_H\left(\tilde{W}_H + \kappa_H \tilde{B}_H\right)}{W\left[\tilde{W}_H + \kappa_H \tilde{B}_H\right] + B_H^U\left[(1-\kappa_H)\tilde{W}_H - P_H\left(\tilde{W}_H + \kappa_H \tilde{B}_H\right)\right]}\right) \\ +(1-a) &\left(\frac{(1-\kappa_L)\tilde{W}_L - P_H\left(\tilde{W}_L + \kappa_L \tilde{B}_L\right)}{W\left[\tilde{W}_L + \kappa_L \tilde{B}_L\right] + B_H^U\left[(1-\kappa_L)\tilde{W}_L - P_H\left(\tilde{W}_L + \kappa_L \tilde{B}_L\right)\right]}\right) \end{aligned}$$

Dividing through by $\tilde{W}_H + \kappa_H \tilde{B}_H$ and $\tilde{W}_L + \kappa_L \tilde{B}_L$, respectively, gives

$$\frac{\partial U^U}{\partial B_H^U} = \frac{a\left(\rho_H - P_H\right)}{W + B_H^U\left(\rho_H - P_H\right)} + \frac{(1-a)\left(\rho_L - P_H\right)}{W + B_H^U\left(\rho_L - P_H\right)} \tag{22}$$

The first term on the RHS is strictly positive because $\rho_H > P_H$. It follows that the second term must be negative. If not, then $B_H^U = W$, which cannot be part of an equilibrium, because then *all* agents would be trying to sell in the secondary market. As a result, we must have $P_H > \rho_L = P_L$.

There are two cases: (i) an interior equilibrium in which $B_H^U > 0$, and (ii) a corner equilibrium in which the uninformed do not buy any primary-market bonds in the high state, $B_H^U = 0$. We now discuss these in turn.

B.2.1 Corner equilibrium with $B_H^U = 0$

Begin by assuming that the uninformed do not buy any bonds in the high-state primary market, $B_H^{U*} = 0$. For this to be an optimal choice, we need $\frac{\partial U^U}{\partial B_H^U}|_{B_H^U=0} \leq 0$. By (22), this condition is

$$a\left(\rho_H - P_H\right) \leq (1-a)\left(P_H - \rho_L\right). \tag{23}$$

The uninformed thus do not participate in the high-state primary market whenever the price gap between primary and secondary market in the high-state is smaller than the price gap between high state and low state, adjusted for the probability of the high state.

Suppose that condition (23) indeed holds and the uninformed do not participate in the high-state primary market. We can then construct the competitive equilibrium in closed form. First note that the high-state primary market-clearing condition is:

$$nP_H B_H^I = nW = D$$

which immediately implies that

$$n^* = \frac{D}{W}$$

Note that because n^* is pinned down by market-clearing but P_H is not, P_H is the free variable that adjusts to fix the informed's rents at exactly K.

In the high-state secondary market, we then have $W_H^I = 0$, $W_H^U = W$, $\hat{B}^I = \frac{W}{P_H}$, and $\hat{B}^U = 0$. This implies that

$$\begin{aligned}
\tilde{W}_H &= nW_H^I + (1-n)W_H^U = (1-n)W = W - D \\
\tilde{B}_H &= nB_H^I + (1-n)B_H^U = n\frac{W}{P_H} = \frac{D}{P_H}.
\end{aligned}$$

Hence the secondary-market price is given by:

$$\rho_H^* = \frac{(1-\kappa_H)P_H(W-D)}{P_H(W-D)+\kappa_H D} \tag{24}$$

and is strictly *increasing* in P_H.

Next, we want to pin down the primary-market high-state price. We can derive this price from the utility indifference condition across investor types because, in the corner equilibrium, the uninformed and the informed behave symmetrically in the low state. All rents thus stem from the high state. The utility of the informed in the high state is:

$$u_H^I = \log\left((1-\kappa_H)\tilde{W}_H \frac{W}{P_H}\right) - \chi_H$$

The utility of the uninformed in the high state is:

$$u_H^U = \log\left(W\left(\tilde{W}_H + \kappa_H \tilde{B}_H\right)\right) - \chi_H$$

Using the definitions of $\tilde{W}_H$ and $\tilde{B}_H$ this becomes:

$$u_H^I = \log\left((1-\kappa_H)\frac{W(W-D)}{P_H}\right) - \chi_H$$

$$u_H^U = \log\left[W\left((W-D)+\kappa_H \frac{D}{P_H}\right)\right] - \chi_H$$

Given a utility cost of information of the form $K = \log(K_0)$, where K_0 is the information cost in consumption-equivalent terms, the utility indifference condition is:

$$a \cdot \left(u_H^I - U_H^U\right) - \log(K_0) = 0 \Leftrightarrow u_H^I - u_H^U - \log\left({K_0}^{\frac{1}{a}}\right) = 0$$

Solving for P_H implies that utility indifference holds if

$$P_H^* = \frac{1}{{K_0}^{\frac{1}{a}}}\left[1 - \kappa_H \frac{W + ({K_0}^{\frac{1}{a}} - 1)D}{W-D}\right]$$

Note that P_H^* is decreasing in K. Moreover, $\rho_H^* > P_H^*$ for any $K > 0$ and the price gap $\rho_H^* - P_H^*$ is increasing in K.

To complete the equilibrium characterization, we now need to derive the equilibrium price in the low state. As mentioned above, the uninformed's low-state decision problem is exactly the same as that of the informed. As a result, we must have that

$$b_L^U + B_L^U = \frac{W(1 - P_L - \kappa_L)}{P_L(1 - P_L)}.$$

Secondary market quantities are thus

$$b_L^U = \frac{W(1 - P_L - \kappa_L)}{P_L(1 - P_L)} - B_L^U \qquad \text{and} \qquad b_L^I = \frac{W(1 - P_L - \kappa_L)}{P_L(1 - P_L)} - B_L^I$$

Guess and verify that the solution is of the form $B_L^I = B_L^U = B_L^*$ and $b_L^I = b_L^U = 0$. That is, guess that both types of investors choose the same low-state primary market portfolio and do not trade in the low-state secondary market. Then

$$b_L^* = \frac{W(1 - \kappa_L - P_L)}{P_L(1 - P_L)},$$

and the primary market equilibrium price is

$$P_L^* = 1 - \frac{\kappa_L W}{W - D}$$

It follows that $W_L^I = W_L^U = W - P_L^* B_L^*$ and $\tilde{B}_L = B_L^*$. We can use (18) to verify that $b_L^I = b_L^U = 0$. Moreover, it is then straightforward to verify that $\rho^* = P_L^*$ as well. This completes the argument. Note that the low-state price is just the full information price – which is also true in the partial-participation region of the model without secondary markets.

B.2.2 Interior equilibrium with $B_H^U > 0$

We now turn to the case where the uninformed optimally choose to participate in the high-state primary market, that is $B_H^U > 0$. Taking as given that $P_L = \rho_L$, we must then solve for $\{P_L, \rho_H, P_H, b_L^I, B_L^I, b_H^I, B_H^I, b_L^U, B_L^U, b_H^U, B_H^U, n\}$. We now show that we can reduce this problem to a system of five equations in the five unknowns

$$\mathcal{U} = \{P_L, \rho_H, P_H, B_L^U, n\}.$$

Given $\mathcal{U}$, we can derive B_H^U from the first-order condition (22):

$$B_H^U = \frac{W\left[a(\rho_H - P_H) - (1 - a)(P_H - \rho_L)\right]}{(\rho_H - P_H)(P_H - \rho_L)} \tag{25}$$

We can then pin down b_H^U from (18). Equation (18) also pins down b_L^U conditional on B_L^U. We have therefore pinned down all uninformed quantities given $\mathcal{U}$.

Turning to the informed, we know that $B_H^I = \frac{W}{P_H}$. The low-state primary market clearing condition is $nP_L B_L^I + (1-n)\left(P_H B_H^U + P_L B_L^U\right) = D$. Rearranging gives

$$B_L^I = \frac{1}{nP_L}\left[D - (1-n)\left(P_H B^U + P_L B_L^U\right)\right].$$

Using the secondary market demand functions (18) then shows that all informed quantities are also pinned down by $\mathcal{U}$. We can then solve for the remaining endogenous variables $\mathcal{U}$ using the following five equations:

1. High-state primary market clearing: $nW + (1-n)P_H B_H^U = D$.
2. High-state secondary market clearing: $nb_H^I + (1-n)b_H^U = 0$.
3. Low-state secondary market clearing: $nb_L^I + (1-n)b_L^U = 0$.
4. Utility indifference: $u^I - K = u^U$.
5. No price differences in the low-state: $P_L = \rho_L = \frac{(1-\kappa_L)\tilde{W}_L}{\tilde{W}_L + \kappa_L \tilde{B}_L}$.

B.3 Illustration

We now provide an illustration of the equilibrium with secondary markets. To do so, Figure 16 contrasts primary market prices with and without secondary markets as a function of z. We use the same parameter values as in Figure 7 in the main text. Black denotes the high state and red denotes the low state. Solid lines depict the equilibrium without secondary markets, while dashed lines depict the equilibrium with secondary markets. The uninformed equilibrium price is in blue.

The illustration shows that the model with secondary markets can give rise to two results. First, primary market prices can be more volatile in the presence of secondary markets. In our example, the high-state price is higher throughout, while the low-state price is lower for relatively low values of z. As discussed above, the low-state price is the same when z is sufficiently large to trigger the partial-participation region without secondary markets. Second, an informed equilibrium with secondary markets exists even when an informed equilibrium without secondary markets does not exist. The illustration thus suggests that the advent of secondary market in and of itself may trigger information acquisition and price volatility. Countries with low z may therefore be hurt by secondary markets, while countries with high z, who are in an informed equilibrium regardless, may reap benefits from higher high-state prices without suffering any downsides.

Figure 16: Equilibrium Prices with and without Secondary Markets