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# THE UPSIDE-DOWN ECONOMICS OF REGULATED AND OTHERWISE RIGID PRICES

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## ABSTRACT

A version of the Becker-Lancaster characteristics model featuring quality-quantity tradeoffs reveals a number of surprising market behaviors that can result from price regulations that are imposed on competitive markets for products that have adjustable non-price attributes. Quality need not clear a competitive market in the same way that prices do, because quality can reduce the willingness to pay for quantity. Producers can benefit from price ceilings, at the expense of consumers. Price ceilings can result in quality-degradation "death spirals" that would not occur under quality regulation or excise taxation. The features of tastes and technology that lead to such outcomes are summarized with pairwise comparisons of (not necessarily constant) elasticities.

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Although not always highly visible outside of Communist countries, price regulations apply to a large fraction of economic transactions, even in the United States. There are, of course, controls on apartment rents and taxi fares in major cities, and minimum wages for low-skill workers. A number of states regulate interest rates on loans with usury laws and the federal government regulates interest and insurance rates with redlining prohibitions and antidiscrimination rules. Basic telephone and cable TV rates are regulated. Outside the state of Nevada, the price of sex is legislated to be zero. Price controls are the norm in the health sector, which by itself is already a sixth of the U.S. economy. Much modern research on business cycles features "sticky" prices, and the technology sector includes several markets with natural constraints on monetary prices (Lanier 2014): these are not exactly regulated prices but potentially share many of their economic characteristics.[1]

The textbook model of price ceilings says that binding ceilings reduce expenditure and the quantity traded in competitive markets, primarily by queuing or a random allocation mechanism. Price ceilings are supposed to benefit buyers, especially if the ceiling is not too far from the unregulated price.[2] These results are special, and misleading as to the economic mechanisms that might deliver them.

Following Cheung (1974), Murphy (1980), Leffler (1982), Raymon (1983), Barzel (1997), and Ippolito (2003), we assume that, although a price regulation prohibits competition on price, other forms of competition among buyers are not necessarily prohibited.[3] Practically all goods and services have non-price dimensions (hereafter, "quality") that can be and are distorted by a binding price ceiling. The quality dimensions include the time, place, or pleasantness of delivery. It could be the durability or reliability of the product, or the number of advertisements attached to it. Or the amount of customers' time that is required to acquire, finish, maintain, or consume the

[1] The degree of price stickiness can also be affected by regulation. For instance, item-pricing laws increase menu costs of changing prices, and result in less frequent price adjustments (Levy, et al. 1997). Unregulated industries with sticky prices may also have different cost structures than industries with regulated prices (Telser 2009).

[2] See, for example, Lee and Saez (2012) and Bulow and Klemperer (2012) for recent citations of this result, and possible qualifications of it.

[3] See also Telser (1960) who explains self-imposed pricing restrictions on the basis of non-price competition.

good. Or the size of the package. Quality responses to price ceilings help suppliers be compliant with the regulation.

There is considerable scope for adjustment of non-price attributes that would permit a regulated market to comply with a price ceiling without necessarily supplying less quantity because sellers spend considerable amounts as they attempt to make their product more attractive to buyers. Take apartments, for which it is sometimes said that the purchase price of land and structure equals the expected present value of the rental income to be received from tenants. In fact, about half of the revenues obtained from tenants is spent on short-run variable inputs rather than financing the structure's purchase or initial construction. Figure 1 shows the claims on national tenant-occupied housing output for 2006, as reported by Mayerhauser and Reinsdorf (2007). Almost half of housing output went to intermediate goods and services (e.g., realtor and advertising activities) and depreciation (a proxy for normal repairs and maintenance). Another five percent went to labor (largely management), and about three percent went to compensate landlords for holding vacant units. Landlords could adjust any of these items in order to reduce the ratio of costs to revenue.[4]

When non-price product attributes are adjustable, the impacts of a ceiling on quantity, quality, and the surplus of buyers and sellers have little to do with the supply and demand for the controlled good by comparison to not having/producing the good at all. On the demand side, it is not the same when price falls by regulation as when it changes due to a reduction in the marginal costs of producing the services delivered by the controlled good. On the supply side, it is not the same when price falls by regulation as when it falls due to a reduction in the buyers' marginal willingness to pay for the services delivered by the controlled good. Even when the curves are properly adjusted to reflect changes in non-price attributes, the usual supply and demand diagram is not

[4] Also note that costs can, in effect, be negative. This was typically the case in the market for broadcast radio programming, where listeners paid no money but tolerated advertisements, which allowed broadcasters to cover their costs. The zero price for broadcast radio programming was set by technology rather than regulator or statute, but the example illustrates how an industry can function and competition occur without sellers' covering their costs exclusively from customer revenues.

suitable for welfare analysis. These are our primary disagreements with textbook treatments of price controls, and begin to indicate why our results are so different.[5]

A price ceiling in a competitive market might increase the quantity sold because there is a quality-quantity tradeoff.[6] Holding expenditure constant, a ceiling prohibits low quantities. Take, for example, retail fruit and vegetable sales. Absent regulations, suppliers spend resources to preserve, cull, and promptly deliver their produce inventories so that the consumer receives fresh items. With a price ceiling set on, say, a per-ounce basis, suppliers cut down on their quality-enhancing expenditures and thereby reduce the fraction of the produce obtained by the consumer that is edible. Consumers with a price-inelastic demand for edible produce purchase more total produce because the survival rate of purchased produce is reduced by the price ceiling. A variety of goods from apartments to light bulbs to doctor appointments have this feature that the unregulated market serves customers with less, but more expensive, quantity because that quantity is efficiently managed to provide the maximum value for the customer's dollar. Our model does not assume that controlled goods necessarily have such ease of substitution between quality and quantity, but these examples begin to show why the textbook predictions may not be reliable.

To the extent that supply slopes up, producers tend to benefit, relative to the unregulated allocation, from the increase in quantity and lose from the reduction in quality. Indeed, we find a simple supply-elasticity condition that indicates whether a price ceiling net redistributes from consumers to producers, or vice versa. For some of the same reasons, the possibility for producer gains is still present even when the equilibrium quantity impact of a price ceiling is not positive.

Many studies before ours have noted that regulated or rigid prices can result in less quality as buyers compete by accepting less of the non-price attributes. Economist and experienced price regulator John Kenneth Galbraith (1980) explained why regulators

[5] On the geometry of, and conclusions regarding, market surplus, we also disagree with Spence (1975), Frech and Samprone (1980), Ippolito (2003), and others. See Section IV below.
[6] Murphy (1980) concludes that a price ceiling might increase quantity sold, but, without featuring the quantity-quality tradeoff, does not examine other consequences of it. Leffler's (1982) discussion of price ceilings does emphasize the tradeoff, and notes that quality can reduce the demand for quantity. Neither paper provides clear conditions for determining the sign of a price regulation's impact on quantity or on the position of the demand curve.

have difficulty preventing it.[7] Assar Lindbeck (1971, p. 39) noted the "deterioration of the housing stock" that results from rent control, adding that "next to bombing, rent control seems in many cases to be the most efficient technique so far known for destroying cities."[8] In discussing price controls during the Nixon administration, Barzel (1997, p. 20) noted that "[f]or many commodities the price controls caused inconveniences: fewer sales were made on credit, a smaller variety of goods was available, and free delivery was less frequent." Caps on physicians' fees are said to result in shorter appointments and longer wait times (Frech 2001). Numerous scholars, including Welch (1974), Hall (1982), Holzer, Katz and Krueger (1991), and Ippolito (2003) have noted that minimum wage laws may affect the non-pecuniary attributes of jobs. Frech and Samprone (1980) find that price regulation in the insurance industry affects the supply of non-price attributes. Boudreaux and Ekelund (1992) and Hazlett and Spitzer (1997) document that deregulating cable rates led to price increases driven by quality upgrades in the package (measured by the number of channels, program costs, etc.), whereas reregulation was accompanied by a dramatic drop in viewer ratings, which suggests a loss of quality. Gresham's Law says that currency-price regulations degrade the quality of money. It is also noted that queues can result from price ceilings, and take away from the customer experience (Taylor, Tsui and Zhu 2003, McCloskey 1985). But few of these, even those attempting to document the welfare costs of non-price rationing (e.g., Besley, Hall, and Preston (1999), Deacon and Sonstelie (1985), Hassin and Haviv (2003)), note that the supply of quantity shifts down, or that the willingness to pay for quantity may increase as buyers compete to accept less quality.[9] The supply effects have

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[7] He cites the famous example of candy-bar price controls during World War II, to which manufacturers responded by putting less candy in each bar. Regulators hoped that they could prevent this reaction by setting the price ceiling based on the weight shown on the package, but failed to anticipate that, prior to controls, each candy package actually contained more weight than indicated, so that weight per package could be reduced while complying with the regulation.

[8] See Block and Olsen (1981) and Moon and Stotsky (1993) for evidence on this point.

[9] Regarding retail gasoline price controls, Barzel's (1997, p. 21) did conclude that supply shifts down, noting that "[d]uring the period of price controls, market participants were able to alter the levels of gasoline transaction attributes not controlled by the government," such as lowering octane levels, excluding additives, shortening station operating hours, and requiring cash payment in order to reduce costs. However, Barzel assumes that adjustments of non-price attributes necessarily reduce the consumer's quantity demanded at any given price, which is contrary to our produce/lightbulbs/doctor appointments examples, and dramatically affects the results. See also Hall (1982) and our discussion of the Jevons (1866) paradox.

been noted in articles on "pure quality competition" (Abbott 1953, Gal-Or 1983) and in studies of specific industries in which competition occurs primarily in terms of quality (Steiner 1952, Koelln and Rush 1993), but our purpose is to provide a general model that can represent a variety of non-price attributes and connect the impact of price regulations to properties of tastes and technology.

Using a comparatively compact notation, previous results can be succinctly organized and clarified, and surprising new ones obtained. The effects of price regulations on quantity, expenditure, and the allocation of surplus between (identical) buyers and (identical) sellers are shown to depend on simple pairwise comparisons of (not necessarily constant) elasticities describing the economic environment. Price regulations create interdependencies among market participants, even though we assume that neither tastes nor technology are interdependent. The results differ remarkably from the previous literature, and presumably could differ even more in a model that had heterogeneous buyers, heterogeneous sellers, or imperfect competition, in addition to the endogenous product attributes featured here.

For conciseness, the scope of price regulations considered here is limited in three ways. First, the rest of this paper refers to ceilings, but not floors.[10] Our framework applies to price floors too, but ignoring them removes numerous provisos, inversions, etc., from the discussion. Also, the contrast between our results and previous ones are less subtle with ceilings than floors. Second, we do not consider price ceiling regulations that also specify the amount supplied. For example, supply could be conscripted, in which case yet additional factors are necessary to make predictions about the equilibrium quantity (Mulligan and Shleifer 2005, Mulligan 2015). Or the price regulation could also specify a rationing mechanism that itself restricts quantities, such as limiting how many items each household can buy (Taylor, Tsui and Zhu 2003). These are different than the competitive environment described here, but they are rarely described by the textbook analysis, either. Third, this paper features regulation-induced changes in non-price attributes that, holding price and expenditure constant, primarily affect the services consumers receive from the controlled good, rather than affecting the resources that the consumer has available for consuming other goods. The featured case encompasses the

[10] We also abstract from the case in which price ceilings become floors through regulatory capture.

examples cited above: the price regulation is misspecified in the sense that it normalizes expenditure with a quantity (say, ounces of produce received from a retailer) that is different from what consumers ultimately value from the controlled good (edible ounces of produce). In the latter model, not treated in this paper, the price regulation is misspecified in that some of the expenditure on the controlled good occurs downstream of the price regulation, so that compliance is achieved by moving production downstream.

Section I of the paper introduces our model of the taste, technology, and market structure in a single industry, which is the standard competitive model except that quantity and quality are combined in a production function to produce the services desired by the industry's customers. Section II considers a quality regulation both for its intrinsic interest and that it highlights some of the price-regulation results. Sections III and IV have conclusions about the positive and redistribution effects of price ceilings, respectively. Section V concludes.

## I. Quantity and quality as intermediate inputs

We follow the literature and specify a continuously differentiable production function $Y(n,q)$ as a function of quantity $n$ and quality $q$.[11] A contribution of this paper is to show how the properties of $Y(\cdot)$ relate to the consequences of price ceilings.

Define the (quality-) conditional cost function as

$$c(Y,q) = \min_n g(n,q) \quad s.t. \; Y(n,q) = Y \tag{1}$$

where the continuously differentiable function $g > 0$ reflects the resource costs of producing goods of the specified quality and quantity.[12] $q$ and $n$ are scalars.

[11] This quality-quantity specification is, of course, an application of Becker (1965) and Lancaster (1966). See also the discussion in Dreze and Hagen (1978) and Dixit (1979). Raymon (1983) applies the characteristics model to price ceilings, but does not report any comparative statics for quantities and assumes that (a) $Y = nq$ and (b) the industry has perfectly elastic factor supplies.

The price regulation puts a ceiling on per-unit-quantity expenditures (more on this below). Regarding the relationship between quality and regulatory compliance, this paper assumes (subscripts denote partial derivatives):

Assumption A $g_q$, $g_{nq}$ and $Y_q$ are positive in the relevant range.

$Y_q > 0$ is just a normalization so that "quality" refers to more services rather than less. Assumption A rules out zero first derivatives with respect to quality in order to examine situations in which compliance with the price ceiling can be achieved by adjusting non-price product attributes in a direction that makes each unit quantity fundamentally less valuable. As noted long ago by Becker and Lewis (1973), a distinctive feature of quality-quantity tradeoffs relative to other economic tradeoffs is that the price of quantity increases with quality, and vice versa. Assumption A captures this with its positive cross derivative $g_{nq}$.

The impacts of the price ceiling are closely related to the comparative statics with respect to $q$, beginning from the unregulated quality level, in the direction of less quality. We make assumptions about various consequences of adjusting quality and quantity:

Assumption B $g_n$ and $n$ are positive in the relevant range. $g_{qq}$ and $g_{nn}$ are nonnegative. The partial elasticity of $g$ with respect to $n$ is at least one. $g_{nq}$ is no less than $g_q/n$. $Y_n$ and $Y_{nq}$ are positive.

$g_n$ must be positive because quantity is not free. The elasticity restriction in Assumption B allows for upward-sloping supply. It is sometimes convenient to summarize the production function $Y$ and cost function $g$ with,

$$\sigma(n,q) \equiv \frac{Y_n(n,q)Y_q(n,q)}{Y_{nq}(n,q)Y(n,q)}\,, \qquad \frac{\theta(n,q)}{\theta(n,q)+1} \equiv \frac{g_q(n,q)/n}{g_{nq}(n,q)} \leq 1 \tag{2}$$

[12] For $q$ small enough relative to $Y$, there may not be any quantity that satisfies $Y = Y(n,q)$. However, Assumption C below guarantees that an unregulated equilibrium $(Y,q)$ pair would have a quantity satisfying the constraint.

$\sigma(n,q)$ is a combination of the elasticity of substitution between inputs at allocation $(n,q)$ and the returns to scale of $Y$ in the two inputs at that point. If $Y$ exhibits constant returns, or is a Cobb-Douglas function with any returns to scale, then $\sigma(n,q)$ is just the elasticity of substitution at allocation $(n,q)$. In the fruit/vegetable example from our introduction, one might take $n$ to be the number of ounces of produce that the customer obtains at retail, $q$ as the fraction of those ounces that are edible, and $Y = nq$ as the number of edible ounces. In this case, $\sigma$ is the same constant for all $(n,q)$ and equal to one. This paper shows how the intuition from the produce example can be applicable to production functions with a lot less substitution between quality and quantity.

We refer to $\theta(n,q)$ as the "price elasticity of the supply of quality" because the numerator of its definition is an average cost – the per unit cost of adding quality to all units sold – and the denominator is the marginal effect of expanding quantity on the marginal cost of quality.[13] We show how $\theta$ is an indicator of whether a price ceiling stifles competition among buyers, or among sellers, and thereby indicates the incidence of the regulation.

Let $n(Y,q)$ denote the quantity achieving the minimum (1) for a given quality amount $q$. The impact of quality on quantity is therefore the sum of a scale and a substitution effect:

$$\frac{dn}{dq} = n_Y \frac{dY}{dq} + n_q \tag{3}$$

The substitution effect $n_q$ is negative by Assumption A. In other words, the substitution effect by itself – moving along an isoquant for $Y$ in the $[n,q]$ plane – says that regulation might *increase quantity* by reducing quality, even if quality and quantity are not particularly good substitutes in the production function in the sense of having an elasticity of substitution between zero and one. The scale effect is a movement from one isoquant to another. As shown below, the scale effect on quantity has the opposite sign

[13] For example, if $\theta$ were a constant, then the cost function $g$ would have to have the form $g(n,q) = C_n(n) + [nf(q)]^{(1+\theta)/\theta}$. The $C_n$ term can be interpreted as the cost of supplying raw quantity (without "any" quality) and the square-bracket term the cost of adding quality to all of the $n$ units produced. See also the appendix.

of the cross derivative $c_{qY}$, which can be positive, negative, or zero without violating Assumption A or B.

To assess the direction and magnitude of the scale effect, it helps to concisely describe the efficient amount of services of the controlled good corresponding to any quality level $q$:

$$\max_{Y} u(Y, I - c(Y, q)) \tag{4}$$

where $I$ is the consumer's income that is used to finance $Y$ and other goods.[14] We restrict the preference function $u$ so that:

<u>Assumption C</u> The preferences $u$ for $Y$ and other goods are (a) sufficiently smooth that the demand for $Y$ is continuous, (b) such that the marginal willingness to pay for any $Y > 0$ is finite, (c) such that a nonnegative amount of $Y$ is efficient, and (d) such that $Y$ is not a Giffen good. $u$ is increasing in both arguments. $u$ is concave enough in both arguments that the quality-constant demand for quantity slopes down in the price-quantity space.

With Assumption C, the average and marginal value of consuming $Y$ are different, although we do not rule out the possibility that the two values are close, as they would be as the $u$ function becomes approximately linear in $Y$. In this sense, our preference setup is more general than some of previous studies of quality that assume that any one consumer obtains the same marginal and average value from a purchase of a given quality because he is limited to purchase only one unit.[15] As we show below, cases with

[14] This formulation includes the income effects of changes in total surplus, but does not include any income effect from the redistribution of surplus between consumers and producers of the controlled good. This assumption can be justified (a) for brevity, (b) as representing an economy where the owners of the factors of production are also consumers of the controlled good, or, especially, (c) the demand for the controlled good has negligible income effects (our approach in Assumption D below). See also Spence (1975), Dixit (1979), and many others writing on product quality without income effects.

[15] Bulow and Klemperer's (2012) paper examines the one-unit case, which they suggest to be applicable to "rental housing, health care, and minimum wages." Although we agree that it is uncommon for one family to have multiple rental houses or one worker to have more than one job, sometimes it is of interest to model the duration of time that a rented house is occupied or a job is held, and to do so without assuming that marginal and average values are the same.

significant differences between marginal and average value have some of the opposite results.[16]

The unconstrained efficient allocation is described by maximizing (4) with respect to both $q$ and $Y$. Although they are not featured in this paper, increases in the preference for $Y$, or multiplicative reductions in the cost function $g$, would increase the efficient quality or quantity or both, according to the shape of the expansion path shown in [$n$,$q$] plane.

## II. Competitive equilibrium with regulated quality

This paper is about price regulations rather than quality regulations, but the latter are both of intrinsic interest and highlight some of the economic effects of the former. We therefore begin with the case in which quality is limited to $\bar{q}$ by regulation rather than market forces. For brevity, our discussion of quality regulation refers only to the case in which the quality ceiling $\bar{q}$ is binding, so that market participants effectively take quality as given. Given $\bar{q}$ and consumers' outside income $I$, we therefore define a quality-regulated equilibrium as an output level $Y$, a quantity $n$, a price $p$, and profit amount $a$ such that (i) $Y$ and $n$ maximize $u(Y, I + a - pn)$ subject to $Y = Y(n, \bar{q})$ and taking $p$, $\bar{q}$, and $a$ as given, and (ii) $n$ maximizes $a = pn - g(n, \bar{q})$ taking $p$ and $\bar{q}$ as given.

Note that, for a given quality level $\bar{q}$, the price $p$ refers to the revenue per unit quantity, and not revenue per unit output. Our quality-regulated equilibrium is competitive in the sense that consumers and producers each take the price $p$ as given. At the equilibrium price, the same quantity $n$ is both utility maximizing and profit maximizing.

16 To be clear, we disagree with Spence's (1975, p. 417) assertion that it is "inessential" to assume that "each consumer buys only one unit of the good," even if values are heterogeneous across consumers. His assumption that average and marginal are the same at the consumer level is the source of the differences between his competitive results and ours.

### II.A. The supply and demand for the services provided by the controlled good

The quality-regulated equilibrium quantity and output is efficient given the quality level. The necessary and sufficient first-order conditions relating the equilibrium output $Y$ with the regulated quality $\bar{q}$ therefore follow from the maximization (4):

$$M\big(Y, I - c(Y, \bar{q})\big) = \lambda \tag{5}$$

$$c_Y(Y, \bar{q}) = \lambda \tag{6}$$

where $M$ denotes the marginal rate of substitution between $Y$ and other goods. $\lambda$ is the shadow price of the services produced by the controlled good with a quality-regulated equilibrium value of $p/Y_n$.

Without any reduction in $Y$, a quality reduction must increase quantity. Conversely, in order for a quality reduction to be associated with a quantity reduction, it must have a scale effect that is in the right direction and large enough to offset the quality-quantity substitution effect. The conditions (5) and (6) already suggest four separate reasons why a regulated quality reduction might not reduce $Y$:

Case MC The quality regulation does not raise the marginal cost of $Y$.

Case IY The demand for $Y$ is inelastic to its (shadow) price.

Case JM The conditional cost function is not convex (in quality) so that the regulation causes a jump in the mix of production inputs.

Case IE An income effect on $Y$-demand more than offsets the shadow-price effect.[17]

[17] The quality ceiling could reduce consumer income and $Y$ is a sufficiently inferior good. Or the quality ceiling increases consumer income and $Y$ is a sufficiently normal good.

Although the unregulated quality minimizes conditional cost $c(Y,q)$ with respect to quality, it does not necessarily minimize marginal cost. For the same reason, a quality limit that is binding for consumers cannot reduce conditional cost $c$, but it may reduce the shadow price $\lambda$. Without further assumptions about the functions $g$ and $Y$, we cannot assume that a regulated quality reduction reduces scale even if the demand for $Y$ is sensitive to its shadow price.[18]

Suppose, as just an example, that the conditional cost function were multiplicatively separable in $Y$ and $q$. This is equivalent to saying that there is a single efficient quality level that is independent of scale $Y$. The unregulated quality minimizes both $c$ and $c_Y$, and the first-order effect of a quality ceiling on $Y$ and $\lambda$ is zero (Case MC) even though the ceiling's quality-quantity substitution effect is not. If, instead, quality were an inferior input in the production of $Y$, then a regulated quality that is below, but near enough to, the unregulated equality would increase $Y$ – necessarily with more quantity – and thereby add to the quality-quantity substitution effect. Even if quality were a normal input, the quality regulation would not affect $Y$ if the demand for $Y$ were inelastic with respect to its shadow price (Case IY).

Case JM is frequently ruled out for analytical convenience, but the failure of the second-order conditions is more likely with quality-quantity tradeoffs than with many other economic tradeoffs because quantity and quality multiply each other in costs (Hirshleifer 1955, Theil 1952, Becker and Lewis 1973). Case JM says that the quantity jumps up, and quality jumps down, in response to a quality regulation, whereas Assumption C says that the demand for $Y$ does not jump. In the neighborhood of the jump, the substitution effect dominates the scale effect because the former is a discrete change whereas the latter is continuous.

Case IE features income effects on the demand for $Y$, which can go in either direction. Because of the ambiguous sign, likely second-order magnitude, and that the previous literature's positive analysis does not emphasize income effects, the rest of this paper abstracts from income effects too. Assumption D formalizes this and, to prevent our presentation from getting too long, also rules out Case JM.

[18] Recall that the conditional cost function, and therefore its $Y$ derivative, depends only on the "technology" $g()$ and $Y()$, and not on "preferences" $u$.

Assumption D $Y$-demand is income inelastic: the marginal rate of substitution function $M$ depends only on $Y$, and not on the consumption of other goods. The conditional cost function is convex in quality.

Note that Assumption $D$ does not rule out equilibrium effects of ceilings on $Y$, just those that occur through an income effect. With this assumption, the (not necessarily constant) magnitude of the price elasticity of demand for $Y$ is:

$$\eta(Y) \equiv -\frac{M(Y)}{M'(Y)Y} \tag{7}$$

In order to refer to elasticities, we normalize $Y$ so that it is positive in the relevant range. It follows from (2) and (7) that $\eta$ and $\sigma$ are both positive.

## II.B. The supply and demand for quantity

The quality-regulated equilibrium can equivalently be described in terms of the supply and demand for quantity:

$$g_n(n,\bar{q}) = p = M\big(Y(n,\bar{q})\big)Y_n(n,\bar{q}) \tag{8}$$

where $p$ is the price that consumers pay for each unit quantity that they consume. Although, for the moment, $p$ is not an object of regulation (quality is), the equivalent representation (8) helps to link the consequences of quality and price regulations.

Each of the functions from (8) can be drawn in the [$n$,$p$] plane, as in Figures 2 and 3. In this context, we refer to them as the marginal cost and willingness-to-pay curves, respectively. Assumption B requires that the marginal cost curve slopes up (or be horizontal). Assumption A requires that a lower curve marginal cost curve corresponds to a lesser quality level. Assumption C requires that the willingness-to-pay curve slopes down.

None of the assumptions requires that quality increase the willingness to pay at all points, or any points, on the curve. As an example consistent with light bulbs and grocery-store produce, consider $Y = nq$, with a $Y$-demand function $M$ that has a finite negative slope everywhere. At the demand choke point $n = Y = 0$, the willingness to pay is $M(0)\bar{q}$, which necessarily increases with the quality ceiling $\bar{q}$ because the consumer gets more output from a high-quality good than a low-quality one. But the high-quality good also moves the consumer further down his $Y$-demand curve, which reduces his marginal willingness to pay for $Y$. As a result, in the neighborhood of the choke point, the high-quality demand curve is above and steeper than the low-quality one. If any point on the $Y$-demand curve has $\eta < \sigma$ (the latter is one in this example), then the high-quality willingness-to-pay curve could cross the low-quality one from above. A consumer of higher-quality goods gets more output per unit quantity but, at the crossing point, $\eta = \sigma$ and his low valuation of output results in a willingness to pay for quantity that is the same as it would be if he had been consuming low-quality goods.

More generally, the direction of the effect of quality on the willingness to pay at any point on the quantity-demand curve is the sign of $(\eta - \sigma)$ at the same point. Wherever the difference is negative, consumers are more willing at the margin to substitute quantity, rather than other goods, for quality: a tighter quality ceiling increases their willingness to pay at that point. When the difference is positive, a quality ceiling reduces the willingness to pay.[19] If $Y$ demand also has a satiation point, which we assumed for the purposes of drawing Figure 2, then willingness-to-pay curves corresponding to different qualities must cross – that is have points with $\eta > \sigma$ as well as points with $\eta < \sigma$ – because it takes more quantity to reach satiation with low quality than with high. It is possible that the curves cross more than once. Conversely, the only way to have the high-quality curve always above (below) the low-quality curve is for Y-demand to have no satiation (choke) point, respectively.[20] A constant-elasticity demand

[19] The positive-difference case conforms with the Jevons (1866) paradox: increasing quality (say, the productivity of coal) increases the willingness to pay for each pound of coal because it sufficiently expands the use of coal-sourced energy.

[20] To be clear, we say that there is a choke point if (a) the willingness to pay for $Y$ is finite at $Y = 0$ and (b) $Y(0,q) = 0$ for any quality in the relevant range. We say that there is a satiation point if (a) the willingness to pay for $Y$ is zero for a finite amount of $Y$ and (b) the satiation service level can be achieved with finite amounts of quantity and quality. Here we use satiation and choke points to help describe the global properties of the demand system, but note that a satiation point is of practical interest in those markets where the price ceiling is zero (i.e., buyers are prohibited from paying the sellers).

curve is an example of one without a choke point and is consistent with no positive impact of quality on the willingness to pay for quantity at any point on the curve.

The magnitude $\eta$ of the price elasticity of Y-demand is infinite at the choke point and zero at the satiation point. If the substitution elasticity in the production function were constant, as in the case of Figure 2, then quality would tend to increase the willingness to pay at low quantities, where $\eta$ tends to exceed $\sigma$, and increase the willingness to pay at high quantities, where $\eta$ tends to be less than $\sigma$. Because the existence and location of the crossing point depends only on the properties of the preference functions $Y$ and $M$, that point could be on either side of the unregulated equilibrium point. Our Figure 2 shows the case where the crossing point is to the left of the unregulated equilibrium and a quality ceiling locally increases the willingness to pay by reducing quality. In other words, the consumer's demand for the final output is locally relatively inelastic and he reacts to a quality ceiling by purchasing a greater quantity in order to maintain something close to the unregulated output level.

A potentially large segment of the willingness-to-pay schedule can have $\sigma = \eta$, as would be the case if the elasticity of substitution $\sigma$ were a constant for all $(n,q)$ and a segment of the Y-demand curve had a constant elasticity of the same magnitude. We show this case in Figure 3A in order to focus on supply effects. Because quality and quantity interact in costs, a quality limit shifts the marginal cost schedule down by reducing quality. The willingness-to-pay schedule does not shift in the relevant range, and the equilibrium result is more quantity and a lower price.

It follows that, as long as supply is at least a small bit price sensitive, $\sigma \geq \eta$ is sufficient but not necessary for a quality reduction to increase the quantity purchased. An equilibrium quantity reduction would require that $\eta$ be enough greater than $\sigma$ that the scale effect of the regulation on quantity be in the right direction and of sufficient magnitude to offset the substitution effect. In other words, in order for the regulation to reduce the equilibrium quantity, quality changes must, in the neighborhood of the unregulated allocation, reduce the willingness-to-pay schedule more than they reduce the marginal cost schedule. Conversely, for any functions $Y$ and $g$ satisfying our assumptions A-D, there exists preferences $u$ such that the equilibrium impact of a quality regulation is to increase the quantity sold and to increase the willingness to pay for quantity in the relevant range.[21] These surprising results are not solely a matter of the degree of substitution between quantity and quality.

It is helpful to consider the demand and supply for $n$ alongside the demand and supply for $Y$, as in Figures 3A and 3B. The prices shown in the two charts are different: $p$ in 3A and the shadow price $\lambda$ (= $p/Y_n$) in 3B. The willingness-to-pay-function in Figure 3A would shift with quality whereever $\sigma$ is different from $\eta$, but the Figure 3B's demand curve is independent of quality because it is just a graph of the consumer's marginal rate of substitution $M(Y)$ versus the services amount $Y$. As noted above, a quality reduction shifts Figure 3A's supply curve $g_n(n,q)$ down because $g_{nq} > 0$. Both figures, especially Figure 3B, are drawn for the $c_{Yq} = 0$ case in which quality is neither a normal nor an inferior input.[22] As a result, a quality change in either direction shifts up Figure 3B's supply curve and the shift is only second order.

If quality is either normal or inferior, then $c_{Yq}$ is negative or positive at the efficient allocation, respectively. However, because the impact of quality on the total cost of producing the efficient services amount is still second order, there still must be a point on Figure 3B's supply curve, with services less than the efficient amount, where $c_{Yq}$ is zero. A quality ceiling therefore rotates Figure 3B's supply curve around that point. The rotation is counterclockwise (clockwise) if quality is a normal (inferior) input, respectively.

In the inferior case, a quality ceiling therefore reduces the marginal cost of producing the efficient services amount even though it does not reduce the total cost. The equilibrium result of a quality ceiling is therefore more services $Y$ and more quantity $n$. This is a case in which the scale and substitution effects on quantity go in the same direction. The surprising effect of regulation on quantity is not necessarily a mere "relabeling" of how the services $Y$ are produced with $q$ and $n$, but may also reflect a regulation-induced reduction in the marginal (but not average) cost of producing those services.

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[21] Specifically, as $\eta$ approaches zero, the locus of equilibrium combinations of $q$ and $n$ is just an isoquant of $Y$, which must slope down in the $[n,q]$ plane.

[22] Figure 3B's supply curve is a graph of the marginal conditional cost $c_Y(Y,q)$, holding $q$ fixed. To be clear, because the marginal cost of quality depends on quantity, we do not define "normal input" with respect to an expansion path with $Y$'s MRS constant, but rather with respect to an expansion path that equates the MRS in $Y$ to the MRT in $g$.

## II.C. The regulated-market multiplier defined

A quality regulation, at least, does not move the market along a supply curve, but rather shifts it and may result in more quantity. This is the source of many of our results, so it helps to examine, in addition to $\sigma$ and $\eta$, the properties of $g$, $Y$, and $u$ that determine the magnitude of the quantity impact. We define $\beta$ to be the ratio of the equilibrium quantity impact to the shift in the supply curve measured in the quantity dimension. Algebraically, that ratio depends on the shapes of the model's three primitive functions $u$, $Y$, and $g$:

$$\beta(n,q) \equiv \frac{D_q(n,q)}{D_n(n,q)} \frac{g_{nn}(n,q)}{g_{nq}(n,q)} \tag{9}$$

$$D(n,q) \equiv M\big(Y(n,q)\big)Y_n(n,q) - g_n(n,q) \tag{10}$$

where subscripts denote partial derivatives. $D(n,q)$ denotes the gap between the willingness to pay and the marginal cost of quantity, which has an equilibrium value of zero.[23] For each unit reduction in quality, equilibrium quantity therefore changes by $D_q/D_n$ while the marginal cost curve shifts $-g_{nq}$ in the price dimension and $g_{nq}/g_{nn}$ in the quantity dimension. Also note that measuring the magnitude of the various derivatives with respect to any monotone transformation of quality, rather than quality itself, would not affect the magnitudes of $\beta$, $\sigma$, and $\eta$.[24]

$\beta = 0$ when the supply of quantity is perfectly elastic ($g_{nn} = 0$). Otherwise, $\beta$'s sign depends on whether the price ceiling moves equilibrium quality and quantity in opposite directions ($\beta > 0$) or in the same direction ($\beta < 0$). The intermediate case shown in Figure 3A has $\sigma = \eta$ – quality does not shift the willingness to pay in either direction –

[23] $D_n < 0$ is therefore the difference between the willingness-to-pay function's slope and the marginal cost curve's slope. As explained below, the sign of $D_q$ is ambiguous. $D$ is related to Cheung's (1974) concept of non-exclusive income.

[24] Note that the sign and magnitude of $\sigma$ would be different if a monotone transformations of $Y$ were measured rather than $Y$ itself. In many examples, $Y$ is measureable and therefore its cardinal properties have empirical content. Moreover, monotone transformations of $Y$ and $u$ that leave invariant the reduced form valuation $u(Y(n,q))$ have no effect on the comparisons between $\sigma$ and $\eta$ that are emphasized in this paper.

so that $\beta$ is just a function of the relative slopes of the marginal cost and willingness-to-pay curves:

$$\beta(n,q) \to \left[1 - \frac{\frac{d}{dn} M\big(Y(n,q)\big) Y_n(n,q)}{g_{nn}(n,q)}\right]^{-1} \in [0,1] \tag{11}$$

where the fraction's numerator is the slope of the willingness-to-pay curve and the denominator is the marginal cost curve's slope. At one extreme, the supply of quantity is fixed, and the market multiplier is one. At the other extreme, the marginal cost curve is horizontal and the market multiplier is zero. Both of these results for Figure 3A, and results for marginal cost curves that are neither horizontal nor vertical, are akin to results from tax incidence because quality changes are shifting marginal cost without shifting demand.

Although not shown in Figure 3A, $\sigma$ can exceed $\eta$ by enough that a regulated quality reduction *increases* the price per unit because it sufficiently shifts the willingness-to-pay function. $\beta$ exceeds one in such cases, and quality ceilings have different effects than price ceilings do, because the former raises price and the latter reduces it. Our analysis of price ceilings therefore begins with further examination of $\beta$, distinguishes comparative statics at allocations with $\beta < 1$ from those with $\beta \geq 1$, and explains why β can be interpreted as a "market multiplier."

## III. Competitive equilibrium with regulated prices

We ultimately want to examine the consequences of regulations that constrain prices but do not effectively constrain all of the non-price attributes of the controlled good. Following Murphy (1980), Leffler (1982), Raymon (1983), Barzel (1997), and Ippolito (2003), our model has an equilibrium quality, rather than an equilibrium price, that coordinates the consumers and producers. We define a price-regulated equilibrium that is competitive in the sense that consumers and producers each take the quality $q$ as given. Each consumer would prefer to be able to make greater-than-equilibrium-quality purchases at the regulated price, but no producer has an incentive to supply that extra quality. Each producer would prefer to be able to sell less-than-equilibrium quality at the regulated price, but no consumer has an incentive to accept less quality.

### III.A. Equilibrium defined

Formally, given a price ceiling $\bar{p}$ and consumers' outside income $I$, a price-regulated equilibrium is an output level $Y$, a quantity $n$, a quality level $q$, and profit amount $a$ such that (i) $Y$ and $n$ maximize $u(Y, I + a - \bar{p}n)$ subject to $Y = Y(n, q)$ and taking $\bar{p}$, $q$, and $a$ as given, and (ii) $n$ maximizes $a = \bar{p}n - g(n, q)$ taking $\bar{p}$ and $q$ as given. We assume that the price ceiling is binding and build that assumption into the definition above.[25]

Both our quality-regulated equilibrium and price-regulated equilibrium have consumers and producers each "choosing" a quantity, taking price and quality as given. $D(n,q) = 0$, as defined in equation (10), is zero in either case. The difference – nontrivial as we show below – is whether price or quality is set by regulation, with the other coordinating the two sides of the market.

A level curve of $D(n,q) = 0$ can be displayed in the $[n,q]$ plane together with level curves for $Y$ and $g$, and the former's slope shows a lot about the comparative static $dn/d\bar{p}$. Moreover, (the inverse of) that slope is readily decomposed into scale and substitution effects:

$$\frac{D_q}{-D_n} = -\frac{Y_q(n,q)}{Y_n(n,q)} + \frac{c_{Yq}(Y(n,q),q)}{D_n/Y_n(n,q)} \tag{12}$$

The first term on the RHS of (12) is the slope of the isoquant, and thereby represents the substitution effect shown in equation (3). The second term represents the remaining quantity and quantity changes that involve changing isoquants. The second

[25] The appendix offers a more detailed description of revenues and costs and what each buyer and seller understands to be his consequence of accepting a quality level that is different from the equilibrium value. Because we primarily consider price ceilings below the price that prevails absent regulation, we refer to comparative statics with $d\bar{p} < 0$ as "tightening the ceiling" and comparative statics with $d\bar{p} > 0$ as "relaxing" it.

term has the opposite sign of the cross derivative $c_{Yq}$, and therefore can be positive, negative or, as in Case MS, zero. Case IY also features a special case of the second term, namely that the term goes to zero as the term's denominator becomes large.

## III.B. Comparative statics with the market multiplier

A quality reduction among a subset of suppliers would cause their customers to change the quantity that they buy. If the supply of quantity is not perfectly elastic, this change affects the market's marginal cost of quantity according to the marginal rate of substitution in the marginal cost function $g_n(n,q)$, which is $g_{nn}/g_{nq}$. The direction and magnitude of this price impact is therefore measured by the market multiplier function $\beta$ that we defined above (equation (9)). Moreover, using (12), we can decompose the market multiplier into substitution and scale effects:

$$\beta(n,q) = \left(\frac{Y_q}{Y_n} + \frac{c_{Yq}}{-D_n/Y_n}\right)\frac{g_{nn}}{g_{nq}} \tag{13}$$

Note that the market multiplier depends on all three primitive functions $u$, $Y$, and $g$, but the preference function $u$ enters only through the $D_n$ term and with an ambiguous sign because $c_{Yq}$ can have either sign.

Assuming for the moment that the equilibrium quantity and quality are differentiable with respect to the price ceiling, the comparative statics for the system $D(n,q) = 0$ and $\bar{p} = g_n(n,q)$ with respect to $\bar{p}$ are:

$$\frac{dn}{d\bar{p}} = \frac{D_q/(-D_n)}{g_{nq}}\frac{1}{1-\beta} \tag{14}$$

$$\frac{dq}{d\bar{p}} = \frac{1}{g_{nq}}\frac{1}{1-\beta} \tag{15}$$

In an unregulated market, a quantity-for-quality substitution among a subset of the sellers would, through the price mechanism, cause the rest of the market to substitute quality for quantity. It can have the opposite effect in the regulated market because the higher marginal cost of quantity makes it more difficult for market participants to comply with the price ceiling. In other words, when $\beta > 0$, a price ceiling in a *competitive* market creates an element of strategic complementarity in quality choices. Quality-quantity substitution by a subset of consumers induces the rest of the market to adjust in the same direction, even though we assume no interdependency in preferences. The competitive analysis of price ceilings therefore resembles Becker's (1991) and Becker and Murphy's (2003) competitive analysis of "social interactions" in which each buyer's willingness to pay for the social good is increasing with the number of other buyers who are purchasing that good. The complementarity among market participants is especially strong when $\beta > 1$, when the multiplier changes the signs of the derivatives (14) and (15). Hereafter we refer to $\beta$ as the "regulated-market multiplier", or "market multiplier" for short.[26]

Figure 4A graphs the locus of price-regulated equilibrium quality-price combinations, holding constant the taste and technology functions $u$, $Y$, $g$.[27] The locus slopes up if and only if $\beta < 1$. We draw one downward-sloping portion on the quality interval $q \in [q_2,q_1]$, where $\beta > 1$, although for some taste and technology functions there not be any downward-sloping portion (there also could be multiple parts with $\beta > 1$). The companion Figure 4B shows the locus of equilibrium quantity, assuming that supply is neither perfectly elastic nor perfectly inelastic. It is, qualitatively, the horizontally mirrored image of Figure 4A wherever $\beta > 0$ and thereby in those cases closely resembles the demand curve drawn by Becker (1991, Figure 2). The point $(n_1, \bar{p}_1)$ in Figure 4B represents the same equilibrium as the point $(q_1, \bar{p}_1)$ in Figure 4A. The same relation holds for $(n_2, \bar{p}_2)$ and $(q_2, \bar{p}_2)$. Because the market multiplier does not have to be

[26] Becker and Murphy's (2003) study of demand interactions for social goods refers to $\beta$ as a "social multiplier." The goods in our model are, by assumption, not "social," but inter-consumer complementarities are created by the combination of price regulation and competition. Also, we do not consider imperfect competition in this paper, but the reader may guess that the presence of a market multiplier is one reason why a price ceiling can be more harmful in a competitive market than an imperfectly competitive one.

[27] It is a graph of $p = M(Y(n,q))Y_n(n,q)$, but only for combinations $(n,q)$ that are a regulated equilibrium for some $p$.

positive, especially for low ceilings, we show two upward-sloping parts in Figure 4B. One of them slopes up because $\beta > 1$ and the other because $\beta < 0$.[28]

At $\beta < 1$ allocations, the comparative statics are qualitatively the same as they are for a quality-regulated equilibrium because tightening the price ceiling involves a reduction in the quality limit experienced by consumers. It follows that $\sigma$ may be more or less than $\eta$, but in the former case it follows from section II's results that $dn/d\bar{p}$ is negative or, if supply is completely inelastic, zero.

The $\beta > 1$ allocations are the most different from the quality-regulated results shown in Section II. They occur only where $\sigma > \eta$.[29] A substitution of quantity for quality, which suppliers implement as they attempt to comply with the ceiling, increases the equilibrium marginal cost of quantity by affecting factor prices (see also the Appendix) and thereby frustrates suppliers' adjustments. Any regulated equilibrium on this portion is unstable in the sense that a small reduction in the price ceiling that induces suppliers to cut their product quality must, in order to result in a market price that is compliant with the new price ceiling, involve a quality reduction great enough to be on an upward-sloping part of the curve. Assuming that an actual controlled market is better represented by a stable equilibrium than an unstable one, then the differentiable comparative statics (14) and (15) do not apply and our price-regulation analysis has some resemblance with (special cases of) insurance premium "death spiral" models in which a relatively efficient allocation can be supported as a competitive equilibrium, but that

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[28] When $g_{nn} > 0$, $dn/dp$ can also be written as $-\frac{\beta}{1-\beta}\frac{1}{g_{nn}}$, which is negative only in the interval $\beta \in (0,1)$. In drawing Figures 4A and 4B, we assume that the consumer's first-order condition $D(n,q) = 0$ is sufficient for describing utility maximization and that the marginal rate of substitution between quantity and quality in production $Y$ diminishes more rapidly than does the corresponding marginal rate of substitution in cost $g$. As in Becker's (1991) model, the nonmonotonic relationship between price and quantity shown in Figure 4B is therefore not the result of failures of the second-order conditions of competitive market participants. Those failures are possible too, and discussed below.

[29] Also note from equation (9) that either sign of $(\beta-1)$ is consistent with scale effects in either direction (i.e., $c_{Yq}$ of either sign). For example, cases MC and IY are both cases with zero scale effect but are consistent with either $\beta \in [0,1)$ or $\beta \geq 1$, according to the elasticity of the supply of quantity. Although our Assumption D rules out Case JM, we note here that JM is consistent with either a positive, negative, or zero social multiplier (JM's jump can be represented as a gap in Figures 4A and 4B's schedules for those quantities and qualities that are skipped by the jump).

equilibrium is unstable because equilibrium pricing is inefficient (Feldman and Dowd 1991).[30]

Nothing is shown or assumed in Figures 4A and 4B about the price, quality, or quantity that would prevail without regulation. In theory, the multiplier formula (9) could be evaluated at the unregulated quantity and quality. The result says little about unregulated comparative statics, but it would be informative about some of the consequences of imposing a price regulation on that market. Figure 4C illustrates with a zoomed-in version of Figure 4A for the case in which the market multiplier exceeds one at the unregulated allocation shown as *U*. A price ceiling introduced below, but close to, the unregulated price, the regulation would (a) induce a discrete quality reduction (from $q_u$ to $q_r << q_u$), (b) harm consumers, (c) benefit producers, (d) cause a discrete loss in social surplus,[31] and, if the supply of quantity were at all responsive to prices, (e) increases expenditure and the quantity sold. To prove the second and third points, note that, absent regulation, a consumer chooses quality $q_u$ and pays $p_u$ per unit quantity, even though he could obtain $q_r$ more cheaply (namely, at a discount of $(q_u - q_r)g_q/n$ per unit). In effect, a price ceiling close to $p_u$ forces each consumer to accept quality $q_r$ without receiving the discount that is available absent regulation.[32] Meanwhile, producers benefit from the price ceiling because they deliver less quality and get essentially the same price per unit, thereby getting more surplus from the first $n_u$ units they produce and getting a nonnegative surplus on the remaining $(n_r - n_u)$ units.[33]

To prove the remaining points, note that small quality reductions are not enough to comply with a price ceiling, regardless of how close it is to the unregulated price $p_u$, because quality reductions by each supplier frustrate the compliance attempts by the

[30] Although it is not the case for the situation shown in Figures 4A-4C, it is theoretically possible that no stable price-regulated equilibrium exists (any unregulated equilibrium is stable, and unique). However, in any application with a *Y*-demand curve that has a choke point with a finite negative slope, $\eta$ approaches infinity as one moves along that demand curve toward the choke point, which means that $\beta < 1$ in that neighborhood. In other words, willingness-to-pay schedules consistent with Figures 4A-4C may be look like those drawn in Figure 2.

[31] Note that the regulation induces a discrete movement along the conditional cost function in the quality dimension, away from the conditional-cost-minimizing quality.

[32] The algebraic proof uses the consumer's value function $\mathrm{v(q)} \equiv \max_{\mathrm{n}} \mathrm{u}\big(\mathrm{Y(n,q)}\big) - \bar{\mathrm{p}}\mathrm{n}$, which, given $\bar{p}$, is strictly increasing in the quality level $q$.

[33] Because $\beta < 1$ at the allocation *R*, further reductions in the ceiling may reduce producer surplus below what it is at *R*, and perhaps even below what it is at *U*.

others. Quality must fall at least to $q_r$. $R$ is a regulated equilibrium for a price ceiling that is near the unregulated price, and therefore has essentially the same marginal cost of quantity as the unregulated equilibrium does. Because (i) the marginal cost schedule $g_n(n,q)$ is increasing in both arguments and (ii) $q_r < q_u$, expenditure and quantity at allocation $R$ must exceed what they are at allocation $U$ unless the supply of quantity is completely inelastic to price, in which case $n_r = n_u$. These results for quantity, expenditure, and the allocation of surplus are our first of several that are essentially opposite of the textbook analysis, where a price ceiling benefits consumers (and, if supply is competitive, reduces quantity) as long as the ceiling is close enough to the unregulated price.

Consider Figure 4C again. A price ceiling of $\bar{p} \in (p_u, \bar{p}_2)$ introduced to the unregulated and efficient market $U$ might have no effect, because the unregulated equilibrium price and marginal cost $g_n$ are less than such a ceiling. However, for a regulated market with a ceiling at (or nearby and below) $p_u$, relaxing its ceiling to a level in the interval $(p_u, \bar{p}_2)$ may not result in the efficient allocation. An individual seller does not, given the factor prices prevailing at $R$, have an incentive to supply as much quality as $q_u$ because he would need to charge more than $\bar{p}_2$, which would be in violation of the regulation. The problem is that quantity-quality substitution that resulted in the quality level $q_r$ makes the marginal unit of quantity more expensive to produce than it is in the unregulated economy. In order to willingly supply the efficient quality, an individual seller must not only see the price regulation relaxed above $p_u$, but also anticipate that the other sellers will supply the efficient quality, rather than the quality level between $q_2$ and $q_u$ that corresponds to the relaxed ceiling and is part of a stable regulated equilibrium. We leave a rigorous dynamic analysis for future research, and here just note that Figure 4C might have some of the foundations for a conclusion that the effects of price regulation depend not only on tastes and technologies, but also the market's prior regulatory history.

At first glance, it might seem that quality is isomorphic with price in that either by itself could coordinate the demand and supply of quantity, albeit less efficiently than price and quality would together. This is true if $\beta$ were everywhere less than one, because then the "supply" of quantity ($g_n(n,q) = p$) would cross the "demand" ($M(Y(n,q))Y_n(n,q) = p$) only once in the $[n,q]$ plane. Moreover, a price regulation would amount to a quality regulation, just in different units. But, if there are regions where $\beta >$ 1, then there exist price ceilings $p$ such that the supply and demand cross multiple times, even though the second-order conditions for utility and profit maximization are satisfied. This is a fundamental difference between prices and quality as allocators of quantity and a difference between quality regulations and price regulations.[34]

## III.C. Welfare costs that are worse than first order

The social welfare losses from a quality regulation are second-order because consumer willingness to pay is smooth and the unregulated equilibrium has a quality level that minimizes total conditional costs $c(Y,q)$. This resembles the textbook model where price regulations create second-order losses. However, if the unregulated allocation has $\beta > 1$, then it is unstable as a price-regulated equilibrium. A price ceiling below the unregulated price level, no matter how close, produces a discrete reduction in quality and therefore in social welfare. As shown above, consumers are discretely worse off and producers may be better off.

These welfare results are not only directionally different from the textbook analysis, they are of an entirely different character. Indeed, they are different from most tax analyses, where imposing a small tax on an otherwise efficient market creates only second-order welfare losses.[35] The reaso is that, say, an excise tax creates a gross-of-tax price that is automatically indexed to marginal cost. In contrast, a price regulation is typically not indexed to marginal cost and thereby cannot prevent discrepancies between price and marginal cost that are arbitrarily large.[36]

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[34] For other differences, see Telser (1987) and Weitzman (1974).

[35] Although rarely analyzed, tax rates that are indexed to market conditions could result in multiple equilibria and "multiplier" comparative statics. One such tax is the "Rising-Tide Tax System" (Burman, et al. 2006) that proposes to index the rate of taxation of high earnings to market outcomes for the high earners. The paper containing the proposal and analysis thereof fails to note that high tax rates might make skills more scarce, and thereby result in a feedback loop in which rising tax rates and falling skills quantities mutually reinforce each other (we owe this point to Kevin M. Murphy).

[36] This result resembles Hayek's (1945) exposition of the socially important role of market prices in coordinating human activity. See also the appendix.

Figure 5 illustrates the distinction, under the assumption that $g_{nn} > 0$, which means that the supply of quantity is less than perfectly elastic. The horizontal axis measures the amount by which the price ceiling $\bar{p}$ is set below the unregulated price $p_u$. Allocations to the left indicate ceilings that are close to the unregulated price while those to the right indicate more severe price ceilings. The vertical axis measures the impact of the ceiling on various outcomes. The green and red curves describe the impact when the market multiplier $\beta$ is at least as large as one at the unregulated allocation. Regulated quantity $n$ and expenditure $\bar{p}n$ (green curve) are each discretely higher than its unregulated counterpart, although they tend to decline as the ceiling gets more severe.[37] Total surplus $u$, consumer surplus $(u + g - \bar{p}n)$, and quality $q$ are each discretely less than its unregulated counterpart (see the red curve). They continue to decline with further increases in the ceiling. Compare the green and the red curves, which relate to multipliers of at least one, with the black and blue curves, respectively, which relate to multipliers less than one. In the latter case, each of the outcomes is close to its unregulated counterpart (i.e., the origin) as long as the price ceiling is close enough. Moreover, with $\beta < 1$, the marginal effect of the ceiling on total surplus is zero in the neighborhood of the unregulated allocation (see the gray curve).

A full analysis of efficient and robust redistribution is beyond the scope of this paper, but Figure 5 already suggests that such an analysis must account for the different character of the redistribution that occurs for $\beta < 1$ and $\beta > 1$. If the sign of $(\beta - 1)$ were unknown, consumers' expected loss from a price ceiling could well be negative even though a gain were far more likely than a loss, because the amount lost conditional on losing is of a different order of magnitude than the amount gained conditional on gaining.

Note that Barzel (1997), Glaeser and Luttmer (2003) and others have argued that price ceilings create first-order social losses due to the rationing mechanism used to resolve the "shortage." These allocative losses have been ruled out in our approach, which treats all consumers as identical and has no shortage (unless the shortage is interpreted as a non-price product attribute – see below). In other words, a large market

[37] Although not shown in Figure 5, there may be a range where quantity increases at the margin with ceiling severity because the ceiling has not yet sufficiently increased the marginal cost of $Y$.

multiplier is an additional reason why the losses from price regulation need not be second order.

### III.D. Example: Quantity is in fixed supply

The sign of ($\beta - 1$) depends on the direction in which level curves of the marginal cost function $g_n(n,q)$ cross the level curves of the willingness to pay (for quantity) function $M(Y(n,q))Y_n(n,q)$ in the $[n,q]$ plane. The former slope down or, in the limit of price-inelastic supply of quantity, vertical. $\beta$ therefore exceeds one if and only if the latter level curves are both sloping down and flatter than the level curves of former.

The special case with inelastically-supplied quantity is potentially applicable to rent control and other price regulations where supply is fixed in the short run, but it also highlights some of reasons why $\beta$ could exceed one. Given $n$ and $p$, a price-regulated fixed-quantity equilibrium is a quality limit $x$ that satisifies $M(Y(n,q))Y_n(n,q) = p$. At the unregulated allocation, the market multiplier is:[38]

$$\beta(n,q) \rightarrow\ 1 + \left(\frac{\sigma(n,q)}{\eta(n,q)} - 1\right)\frac{n}{g_q(n,q) + xg_{qq}(n,q)}M\big(Y(n,q)\big)Y_{nq}(n,q) \qquad (16)$$

It follows that, with inelastic supply, the multiplier at $(n,q)$ exceeds one if and only if the elasticity $\sigma$ of substitution in production exceeds the magnitude $\eta$ of the price elasticity of $Y$-demand at that point.[39] Because, as shown in Section II, any continuous demand curve with a satiation point has points on it with $\eta < \sigma$, there must also be points with $\beta >$ 1.

The reason that the character of the multiplier hinges on a comparison of $\eta$ and $\sigma$ is that, holding quantity fixed, quality increases the willingness to pay if and only if $\eta >$ $\sigma$; so that the scale effect on willingness to pay exceeds the quality-quantity substitution

[38] We derive a multiplier for the inelastic supply case by (a) taking the definition (9), (b) assuming $g(n,q) = n^{1+\gamma}/(1+\gamma)+G(nq)$, and (c) taking the limit as $\gamma$ goes to infinity, holding constant the marginal cost at the unregulated allocation.

[39] In the more general case that the supply of quantity is at least somewhat sensitive to the price, $\eta < \sigma$ is necessary but not sufficient for $\beta > 1$. Or to put it another way, $\beta > 1$ means that $\sigma$ exceeds $\eta$ by enough to offset the degree to which the willingness to pay for $n$ decreases with $n$.

effect. When $\eta < \sigma$, quality reductions – implemented by suppliers as they attempt to comply with the price ceiling – increase consumers' willingness to compete on the basis of accepting low quality, which further reduces quality. There is not a stable equilibrium until a part of the parameter space is reached in which $\eta > \sigma$, such as the allocation $U$ shown in Figure 4C and the $\sigma < \eta$ allocations shown in Figure 2.

When the supply of quantity is fixed at $n$, Figures 4A and 4C are graphs of $M(Y(n,q))Y_n(n,q)$ versus $q$. If $\eta < \sigma$ at the unregulated equilibrium, then the unregulated price is in the interval $(\bar{p}_1, \bar{p}_2)$, and the unregulated quality in the interval $(q_2,q_1)$. A price ceiling close to the unregulated price discretely reduces quality to a level less than $q_2$ (specifically, a point on the curve that coincides with the price ceiling measured on the vertical axis) and has no effect on quantity. As noted above, consumers are unambiguously worse off because they are paying essentially the same but getting less quality. Producers are unambiguously better off because their revenue is essentially the same, but they have reduced their average costs by providing less quality. This is yet another result the opposite of the textbook analysis, where it is reported that producer surplus is lost, and consumer surplus is gained, in industries with price ceilings and inelastic supply, at least if the regulated price is close enough to the unregulated. This result does not even require that quality be a particular good substitute for quantity, as long as other goods are an even worse substitute.

## IV. Who benefits from price and quality ceilings?

Beginning from an allocation with $\beta > 1$, introducing a price ceiling close to the unregulated price, or marginally tightening one, results in discretely less consumer and social surplus and discretely more producer surplus. The purpose of this section is to also address the cases in which $\beta < 1$ and the price or quality ceiling is not necessarily near the unregulated equilibrium value. The two are related because a ceiling that is discretely below the unregulated value can be achieved by introducing a ceiling close to the unregulated value, followed by a sequence of marginal reductions in that ceiling.

The marginal cost curve $g_n(n,q)$ drawn in the $[n,p]$ plane (see Figure 3A) is shifted down by a quality ceiling, or by a price ceiling that results in less equilibrium quality. As shown in Figure 6, the equilibrium price change is a combination of the vertical distance $g_{nq}(n,q)dq$ of the marginal cost shift and the movement along that curve, which can be in either direction. Supposing for the moment that regulation has no quantity impact (i.e., $\beta$ = 0), as at the allocation $R_0$ shown in Figure 6, then producers are losing revenue $nd\bar{p} = ng_{nq}(n,q)dq$ but saving the total costs $g_q(n,q)dq$ that are shaded in the figure.[40] By our Assumption B, the net cannot be positive. Because movements down the marginal cost curve (i.e., $\beta < 0$) further reduce revenue more than total costs, it follows that producers cannot benefit from ceiling regulations without $\beta > 0$ at enough of the allocations between the regulated and unregulated allocations that the net impact on quantity is positive.

Now consider a quality ceiling that increases quantity enough that there is no price impact, as at the allocation $R_0$ in the figure. Here there is no change in revenue, but a reduction in costs. It follows that *producers cannot lose*, and consumers cannot gain, from quality ceilings unless $\beta < 1$ at enough of the allocations between the regulated and unregulated allocations that the net impact on price is negative. As shown in Section III, the same reasoning applies to the marginal tightening of a price ceiling: producers benefit and consumers lose unless $\beta < 1$.

For discrete changes in a price ceiling or, when $\beta \in (0,1)$, marginal changes in either type of ceiling regulation, the producer's benefit cannot be signed without more information about regulation's relative impacts on revenue and costs. The cost savings on the unregulated quantity is $g_{nq}(n,q)dq$, while the corresponding revenue loss is $n_u dp$.[41] If we define $\beta$ for discrete regulation changes the same way that we do for marginal changes – as the ratio of equilibrium price change to the amount of the shift of the marginal cost curve measured in the price dimension – the revenue loss on the unregulated quantity is $(1-\beta)n_u g_{nq}(n,q)dq$. The cost savings exceed the revenue loss if and only if:

[40] Although Figure 3B, which graphs supply and demand in the $[Y,\lambda]$ plane, is effective for measuring social surplus, it is less effective for measuring the allocation of surplus because the equilibrium shadow price is not necessarily what consumers pay sellers per unit $Y$. The latter does occur, however, when production function takes the form $Y(n,q) = ny(q)$, so that the shadow price $\lambda = p/Y_n$.

[41] Costs and revenue on the increment $(n_r-n_u)$ to quantity are essentially zero because price equals marginal cost.

$$\beta > \frac{1}{1+\theta} \tag{17}$$

The inequality (17) is a necessary and sufficient condition for producers to benefit from ceiling regulations, and a sufficient condition for consumers to lose.[42] Whenever producers gain from a tighter ceiling, consumers lose because the ceiling reduces total surplus.

Notice that the inequality (17) includes $\theta$, which we have called the "elasticity of supply of quality." The appendix to this paper has a special case of the model that illustrates the connection in more detail, but the tradeoff featured in (17) comes from the fact that our model has two potential sources of surplus for producers: quality and quantity. The unregulated equilibrium maximizes social surplus, but not producer surplus (producers compete!), which opens the possibility that regulation could change the mix of quantity and quality in a way that benefits producers. When quality is elastically supplied ($\theta$ large), producers are not harmed much by the quality reduction, and can make up for it if their production of quantity sufficiently expands ($\beta$ large).

The two directional possibilities are shown with black and blue curves in Figure 5. Both curves exhibit a first-order impact on producer surplus, by which we mean that, at the unregulated equilibrium, the marginal effect of reducing the ceiling is not zero. But producers gain from a tighter ceiling when the inequality (17) holds, which is the case represented by the black curve.

Because $\theta > 0$, Section III.D and the inequality (17) show that a positive $\beta$ – in other words, either supply is completely inelastic or the quality reduction that results from a ceiling is associated with a quantity increase – is necessary but not sufficient for producers to benefit. This is essentially the opposite of the textbook analysis (see also Bulow and Klemperer (2012)), where quantity reductions are taken as evidence that supply is price elastic and therefore that consumers may be losing from a price ceiling.

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[42] Murphy (1980) has a characteristics model of the supply side in which producers can benefit from price ceilings. In the rent control context, Autor, Palmer and Pathak's (2014) empirical results suggests that price ceilings in Cambridge, Massachusetts harmed producers. But note that Cambridge rent control enforcement included conscription – such as the taking of properties by the power eminent domain (Mulligan 2015) – which is not part of our model of price ceilings.

## V. Conclusions

This paper abstracts from the question of which buyers receive the goods in a regulated market, and previous studies have noted the first-order efficiency losses from the misallocations caused by ceilings. Nevertheless, we find that the welfare costs of price regulation can be worse than first order, and of a different character than the costs of excise taxes or quality regulations, because flexible prices may be needed to prevent beggar-thy-neighbor reactions among the sellers. In effect, decisions about non-price product attributes become complementary in the absence of a price mechanism, even though tastes and technology are not fundamentally complementary. The price-regulated equilibrium behavior need not be close to efficient even when the regulated price is.

If Bulow and Klemperer (2012) are correct that price ceilings on average harm both consumers and producers, then the existence of ceilings would suggest that support for the regulation among market participants, if any, would either be misguided or come from subsets of consumers or producers that are different enough from the average. Our model suggests an alternative: that producers may benefit *on average* from a price or quality ceiling because it softens the non-price (a.k.a., quality) competition that would otherwise dissipate some of their surplus. Moreover, markets with relatively inelastic supply are especially likely to feature this type of benefit for producers.

Real-world products have many non-price attributes, and our model is not detailed enough to predict the types and composition of quality adjustments that would occur. But those adjustments could include something like customer "waiting" if it reduces sellers' costs. Take, for example, the inventories that sellers have on hand. Low average inventories mean lower costs but more stock outs and thereby less average value for consumers. A customer encountering a stock out is waiting in the sense that he must defer his purchase until the seller replenishes the inventory (Yurukoglu, Liebman and Ridley 2016). Our model can capture this by treating seller average inventory levels as a non-price attribute $q$ that goes in the customer's production function $Y$. This approach contrasts with previous models in which, akin to an excise tax, waiting is an additional cost borne in part by buyers but (purportedly) yielding no benefit to sellers (McCloskey

1985, pp. 325-6, Frech and Lee 1987, Friedman 1990, Figure 17-4, Bulow and Klemperer 2012).

In contrast to the standard analysis of price controls, our model has no random or purely wasteful mechanism for resolving the "shortages" associated with price ceilings. Market participants in our model have no incentive to engage in such schemes, because sellers prefer to adjust non-price attributes in a way that reduces costs and buyers prefer to get a low-quality product than no product at all. A practical implication of our approach is that regulated-equilibrium quantity and quality reflect both supply and demand conditions, whereas the standard approach says that changes in demand only affect the amount of the shortage without affecting what sellers do.

We show that quality degradation can either increase or decrease buyers' willingness to pay at a given quantity, and provide an elasticity condition that describes which case applies in any particular situation. Even though strong assumptions are needed to guarantee the latter case, the former is largely absent from the literature, and is the source of some of the "upside-down" results. With lower supply costs and little or no reduction in the willingness to pay, *a price ceiling could increase the quantity traded*, especially when there is an inelastic demand for the services provided by the controlled good. It is even possible that the ceiling increases the services themselves.

More empirical work on these predictions is needed, especially with a framework that is consistent with the discontinuities suggested by the theory, but for now we point to a couple of examples that seem to confirm. One is the case of doctor appointments, where it has been suggested that ceilings on the price per visit results in patients' visiting the doctor more frequently for the same health condition. As Frech (2001, p. 338) puts it, Japanese patients "are often told to come back for return visits. And, even injections of drugs were often split in half to make two visits necessary." Indeed, we wonder whether price ceilings in health care increase spending in that market, rather than decrease it. Another is the case of rent control of pre-war premises in Hong Kong, which appears to have increased the number of leases and perhaps even the number of square feet under lease as tenants engaged in partial subletting and landlords rented to "rooftop squatters." Cheung (2016) interprets these practices as choices to reduce rent dissipation, but they are also consistent with the predictions of our model.

With sufficiently inelastic supply, pure non-price competition may have multiple equilibria, and that the transition from one to another might be heuristically described as a quality-degradation spiral with some resemblance to insurance premium "death spiral" models. In these cases, the incidence of price regulations is especially far from the standard analysis. A few studies such as Block and Olsen (1981), and experiences with communism itself, have shown that price ceilings can result in extraordinary quality degradation. Recent advocates of rent control, pointing to the case of modern Germany, also assert that ceilings do not always harm quality (Bourne 2014). Nevertheless, there do not appear to be many statistical analyses of actual price ceilings that formally attempt to confirm the existence of multiple competitive equilibria. Perhaps this absence is due to a paucity of real examples, or merely because this implication of competitive behavior had not yet been developed. But even if it were the former, perhaps understanding this potential of competitive behavior would help regulators to avoid creating any new ones.[43]

The direction of the quantity impact of price controls is sometimes used as a litmus test for whether the controlled market is competitive or not. A ceiling that increases quantity is supposed to reveal noncompetitive behavior and social gains from the ceiling. More work is needed to understand non-price adjustments in imperfect competition settings, but we can already say that, without additional information about tastes and technology, either direction of quantity impact is simultaneously consistent with perfect competition, with social harm, and with consumer harm from price regulations. A price ceiling that increases the quantity traded may only reveal that the market is substituting quantity for quality, and not that sellers were ever holding back supply.

[43] With respect to Gresham's Law, Rolnick and Weber (1986) confirm both possibilities: currency-market regulators often recognize that fixing prices can create multiple equilibria, but that sometimes the low-quality equilibrium is observed for small-denomination currencies.

## VI. Appendix: A Detailed Description of the Economic Environment

### VI.A. Equilibrium defined, with specification for off-equilibrium qualities

This appendix offers a more detailed interpretation of an economic environment described by (1) and (4), as well as showing what would be the consequences of an individual consumer or producer of dealing in a product quality that is different from the equilibrium. Two factors of production are required to produce the products in the controlled market, $Z_n$ and $Z_q$, with factor prices $w_n$ and $w_q$, respectively. $Z_n$ is used to produce the raw items, before any quality enhancements. Each raw item requires one unit of $Z_n$. $Z_q$ is used to make the quality enhancements. If $q$ is the quality level and $n$ is the number of items, then $G(q)n$ units of $Z_q$ are needed. $G$ is not necessarily monotonic in $q$, but for $q$ large enough it is increasing and unbounded. The surplus $a$ of the factor suppliers to the controlled market are:

$$a = \left[w_n + w_q G(q)\right]n - g(n,q) \tag{18}$$

The inverse factor supplies are $w_n = S_n(Z_n)$ and $w_q = S_q(Z_q)$. It follows that the cost function $g$ is the total factor resource cost of quantity and quality:

$$g(n,q) \equiv \int_0^n S_n(Z_n)dZ_n + \int_0^{nG(q)} S_q\left(Z_q\right)dZ_q \tag{19}$$

There is free entry in the business of hiring the factors of production to make the controlled good, so intermediate-supplier profits are zero and their revenues are equal to the revenue term in the surplus equation (18). Their revenue per unit quantity must be compliant with the price ceiling. It follows from the price ceiling constraint that there is a quality level $x$ such that only qualities $q \le x$ are available in the market. The intermediate suppliers are willing to sell goods of any quality strictly less than $x$ as long as they receive at least $[w_n+w_qG(q)]$ per unit.

Given a price ceiling $\bar{p}$ and consumers' outside income $I$, the more detailed equilibrium described in this appendix is an available-quality ceiling $x$, a pair of factor input prices $w_n$ and $w_q$, profits $a$, factor quantities $Z_n$ and $Z_q$, a quality level $q$, and a quantity $n$, such that:

(i) The available-quality ceiling $x$ is consistent with the price ceiling $\bar{p}$.

$$w_n + w_q G(x) = \bar{p} \tag{20}$$

(ii) Given the factor prices $w_n$ and $w_q$, the available quality ceiling $x$, and factor income $a$, the quantity $n$ and quality $q$ maximize utility:

$$\{n, q\} = \underset{n \geq 0, q \leq x}{\operatorname{argmax}}\, u\big(Y(n, q), I + a - [w_n + w_q G(q)]n\big) \tag{21}$$

(iii) Profits $a$ satisfy (18) and (19)

(iv) and, given the factor prices, factor supplies equal factor demands,

$$w_n = S_n(n) = S_n(Z_n), \qquad w_q = S_q\big(nG(q)\big) = S_q\big(Z_q\big) \tag{22}$$

Algebraically, this more detailed equilibrium description is eight unknown scalars described by eight equations. In the main text, our equilibrium has just four of these equations, which is why the main text has no explicit predictions for factor prices or factor quantities.

The unregulated equilibrium is the equilibrium corresponded to a ceiling of $\bar{p} = \infty$. Any ceiling less than infinity binds in the sense that it restricts quality choices ($x < \infty$), although those quality choices may be irrelevant because they are beyond the unregulated quality. An equilibrium does not exist if the price ceiling is so low than it does not cover the factor costs of even the lowest quality levels.

In this appendix's example, the marginal cost of quantity is:

$$g_n(n, q) = S_n(n) + S_q\big(nG(q)\big)G(q) \tag{23}$$

The conditional cost function is:

$$c(Y, q) = g(n(Y, q), q) \tag{24}$$

where $n(Y,q)$ is the quantity needed of quality $q$ needed to deliver services $Y$. A price or quality ceiling that increases quantity therefore changes the composition of producer surplus by moving the market up one of the factor-supply curves and down the other, as shown in Figure 7. Total producer surplus can be greater to the extent that quantity is increasing enough or that the surplus from selling the unregulated quality is small enough. These tradeoffs are summarized by the inequality (17) in the main text.

In order to look at the market multiplier in more detail, consider a price-regulated equilibrium, and normalize its price and quality to one. Now consider lowering the ceiling to $\bar{p} < 1$. This affects the equilibrium quality and quantity, and therefore the factor prices $w_n$ and $w_q$. If, hypothetically, there were a seller still supplying the good with quality one, he would find that the new ceiling both reduces the price received *and* changes the marginal cost of supplying the same good, which is $w_n + w_qG(1)$. The former comes directly from the regulation, but the latter comes from the compliance responses of the other suppliers. In order for the market multiplier to exceed one, the lower ceiling must (a) increase quantity (thereby increasing $w_n$) and (b) have a small enough effect on $w_q$ (e.g., $S_q{}' = 0$) that the net effect is to increase $w_n + w_qG(1)$. In this case, the quality reductions implemented by any group of sellers raise the marginal costs of all sellers and thereby further encourage quality reductions.

More generally, a price ceiling drives a wedge between the private and social benefits of supplying quality because factor prices respond to that behavior. Consider a "price-regulated planner" that was choosing quality $x$ and quantity $n$ subject to the constraint that the marginal cost of quantity cannot exceed $\bar{p}$. That planner's Lagrangian is:

$$\mathcal{L} \equiv u\big(Y(n,x)\big) - g(n,x) + [\bar{p} - g_n(n,x)]\lambda \tag{25}$$

The optimal quantity for the price-regulated planner satisfies:

$$M\big(Y(n,x)\big)Y_n(n,x) = \bar{p} + \lambda g_{nn}(n,x) \tag{26}$$

By comparison, our model's price-regulated equilibrium satisfies:

$$M\big(Y(n,x)\big)Y_n(n,x) = \bar{p} \tag{27}$$

The price-regulated planner and the market coincide only if (a) both factors are perfectly elastically supplied ($g_{nn}$ = 0) or (b) the price regulation is not binding ($\lambda$ = 0). The price-regulated planner's decision considers the fact that quantity choices affect the cost of compliance, whereas the decision of an individual seller (subject to regulation) does not.[44]

The price-regulated planner's condition (26) shows that, with special exceptions (a) and (b) noted above, the marginal cost of quantity exceeds $\bar{p} = g_n(n,q)$. In other words, regulated prices fail to reflect all of the relevant marginal costs, and this failure is the source of some of the most damaging market reactions to the regulation. Quality regulation does not fail in this way. If the planner were subject to quality regulation instead, she would be maximizing $u(Y(n,x))-g(n,x)$ with respect to quantity (only), just as the quality-regulated market does.

## VI.B. A special case that demonstrates the upside-down results

The cost function, production function, and utility function do not have to be exotic in order to obtain our model's unusual results. Here we assume that the factor supply functions each have a constant elasticity, $S_n(Z_n) = Z_n^{1/\gamma}/A$ and $S_q\big(Z_q\big) = Z_q^{1/\theta}B/A$, with $A$, $B$, $\gamma$, $\theta$ all positive, and the function $G(q)$ to be increasing and convex with an elasticity greater than one. In this case, the cost function is

$$g(n,q) = \frac{\gamma}{1+\gamma}\frac{n^{(1+\gamma)/\gamma}}{A} + B\frac{\theta}{1+\theta}\frac{[nG(q)]^{(1+\theta)/\theta}}{A} \tag{28}$$

We also take the marginal rate of substitution function $M(Y) = 1 - Y$ to be linear on the range $Y \in [0,1]$ and the production function to be $Y(n,q) = nq$. In this case, the

[44] Conversely, by making the equilibrium condition (8) or (27) rather (26), we assume that the industry's marginal costs are either constant or rising because of factors that are not perfectly elasticity supplied to the industry.

willingness to pay for quantity is $(1-nq)q$. The magnitude $\eta$ of the price elasticity of $Y$-demand is less than one, and therefore less than the elasticity of substitution $\sigma = 1$, at any allocation with $Y > \frac{1}{2}$. It follows that, beginning at any such allocation, reducing quality will increase the willingness to pay for quantity.

The expansion path in the $[n,q]$ plane is described by:

$$[qG'(q) - G(q)][G(q)]^{1/\theta}B = n^{1/\gamma - 1/\theta} \tag{29}$$

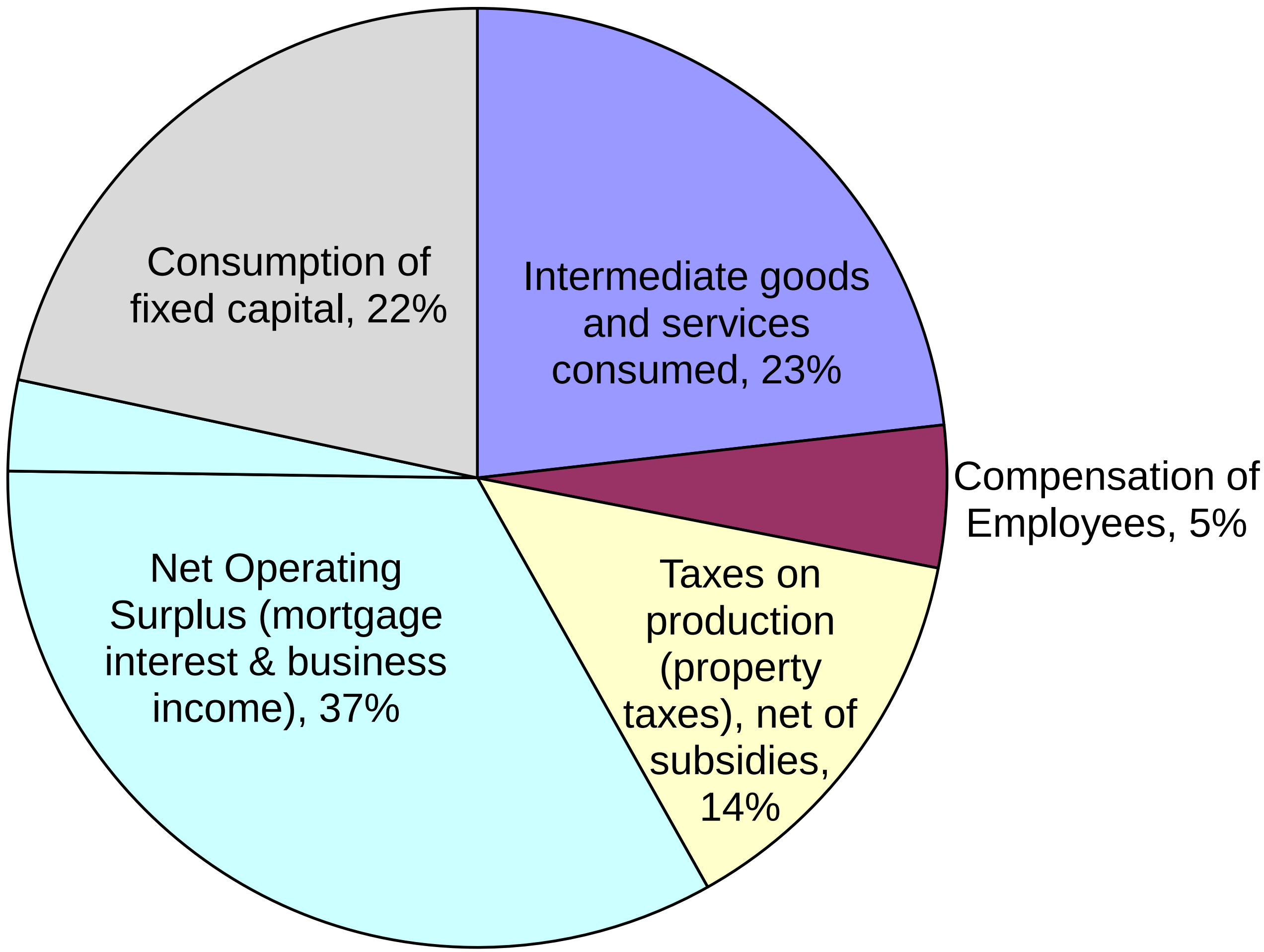
The expansion path therefore can have a slope of either sign, which is the same as the sign of $(\theta - \gamma)$. If $\theta < \gamma$, then a quality ceiling sufficiently close to the unregulated quality results in more equilibrium output $Y$.

The market multiplier $\beta$ can exceed one at the unregulated allocation. Take the special case of the cost function (28) that has a perfectly elastic supply of the quality factor $Z_q$ and an elasticity of supply of the quantity factor $Z_n$ that is 1/2:

$$g(n,q) = \frac{1}{5}\left(\frac{16}{27}n^3 + \frac{nq^5}{4}\right) \tag{30}$$

The efficient allocation is $n = \frac{3}{4}$ and $q = 1$. At this allocation, the marginal cost $g_n$ is $\frac{1}{4}$, the elasticity of $Y$-demand is $-1/3$, and the market multiplier is 24/23. There are three price-regulated equilibria with $g_n = \frac{1}{4}$: the unstable efficient allocation and the stable allocations with $(n,q)$ approximately equal to (0.84,0.36) and (0.74,1.02).

**Figure 1. Claims on gross tenant-occupied housing output, 2006**




Note: the smaller NOS piece is the part allocated to vacant units.

# Figure 2. The demand for raw quantity with a quality ceiling.

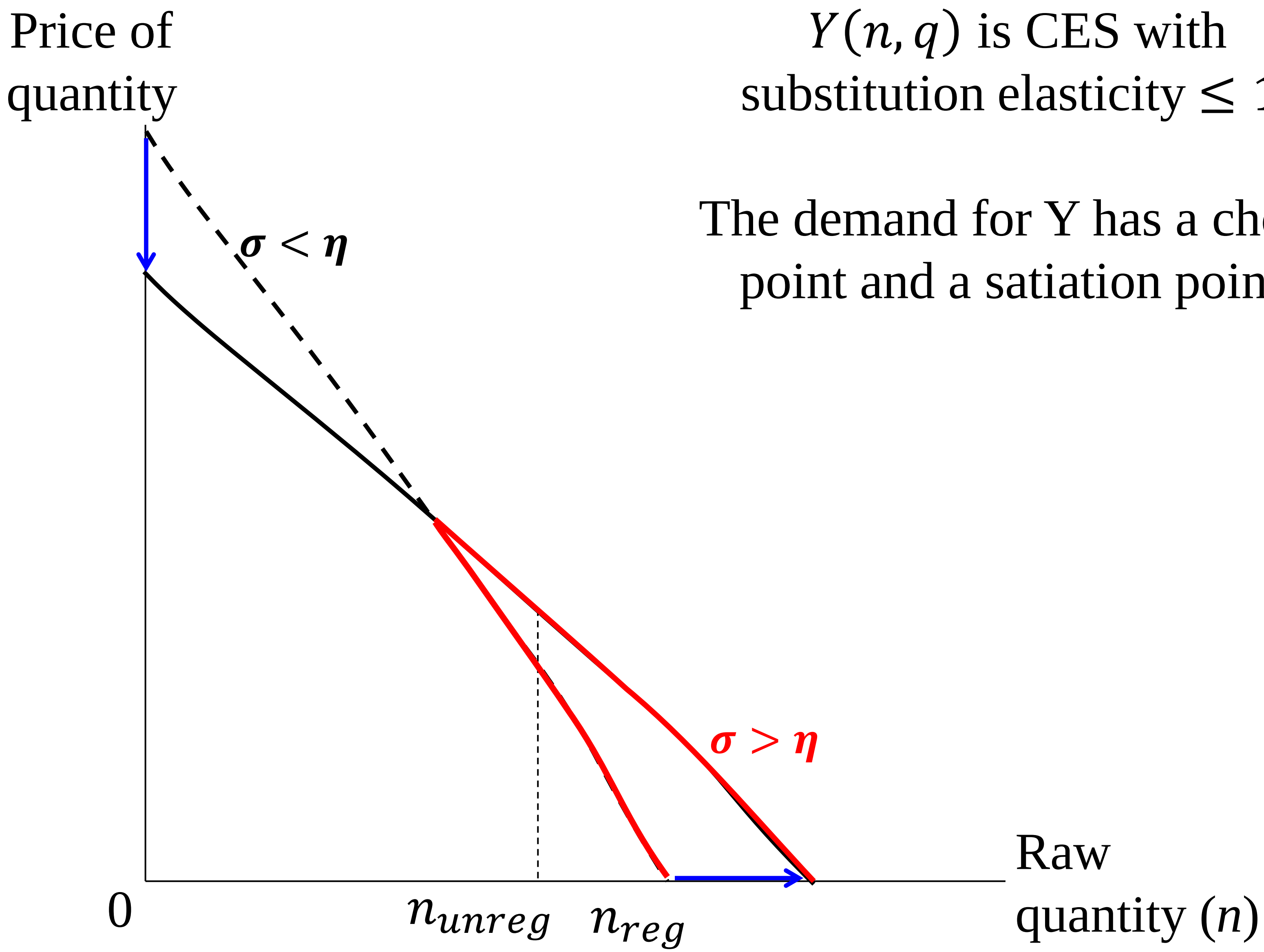


**Figure 3A. The raw quantity of the controlled good,**
with quality regulation and $\sigma = \eta$ on the relevant parts of demand.

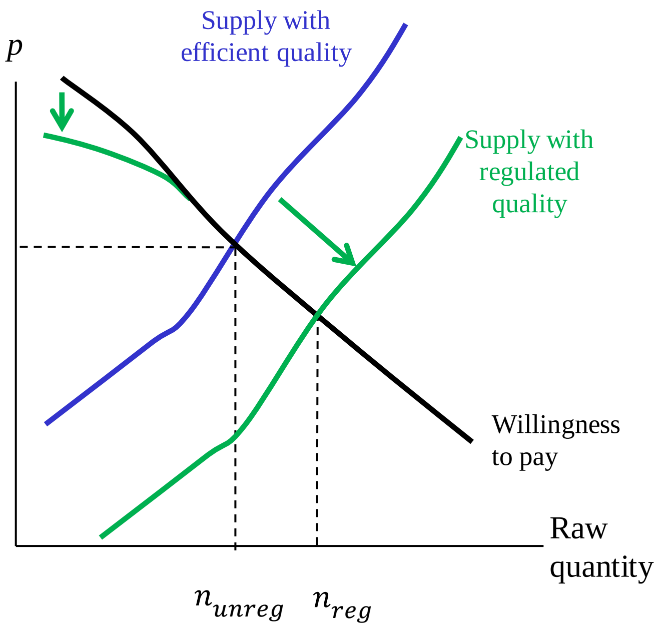
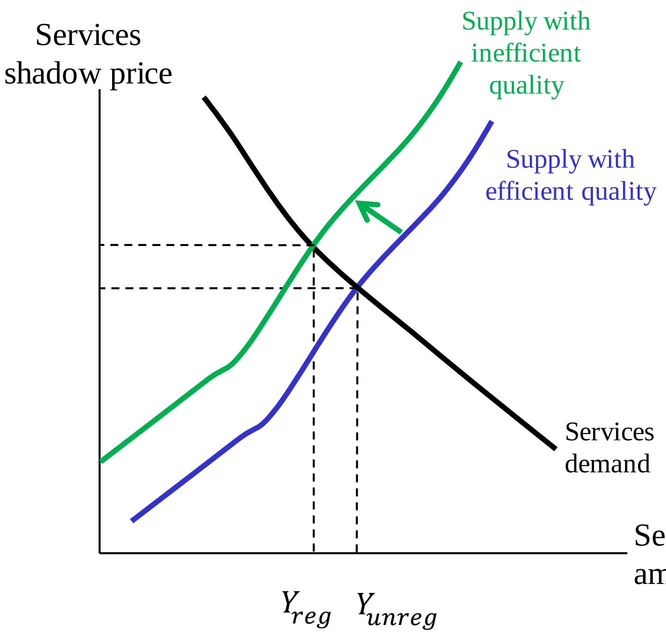



**Figure 3B. The services provided by the controlled good,**
with separable conditional cost: $Y$-supply shift is second order.



# Figure 4A. Equilibrium quality vs. the price ceiling

The role of the market multiplier

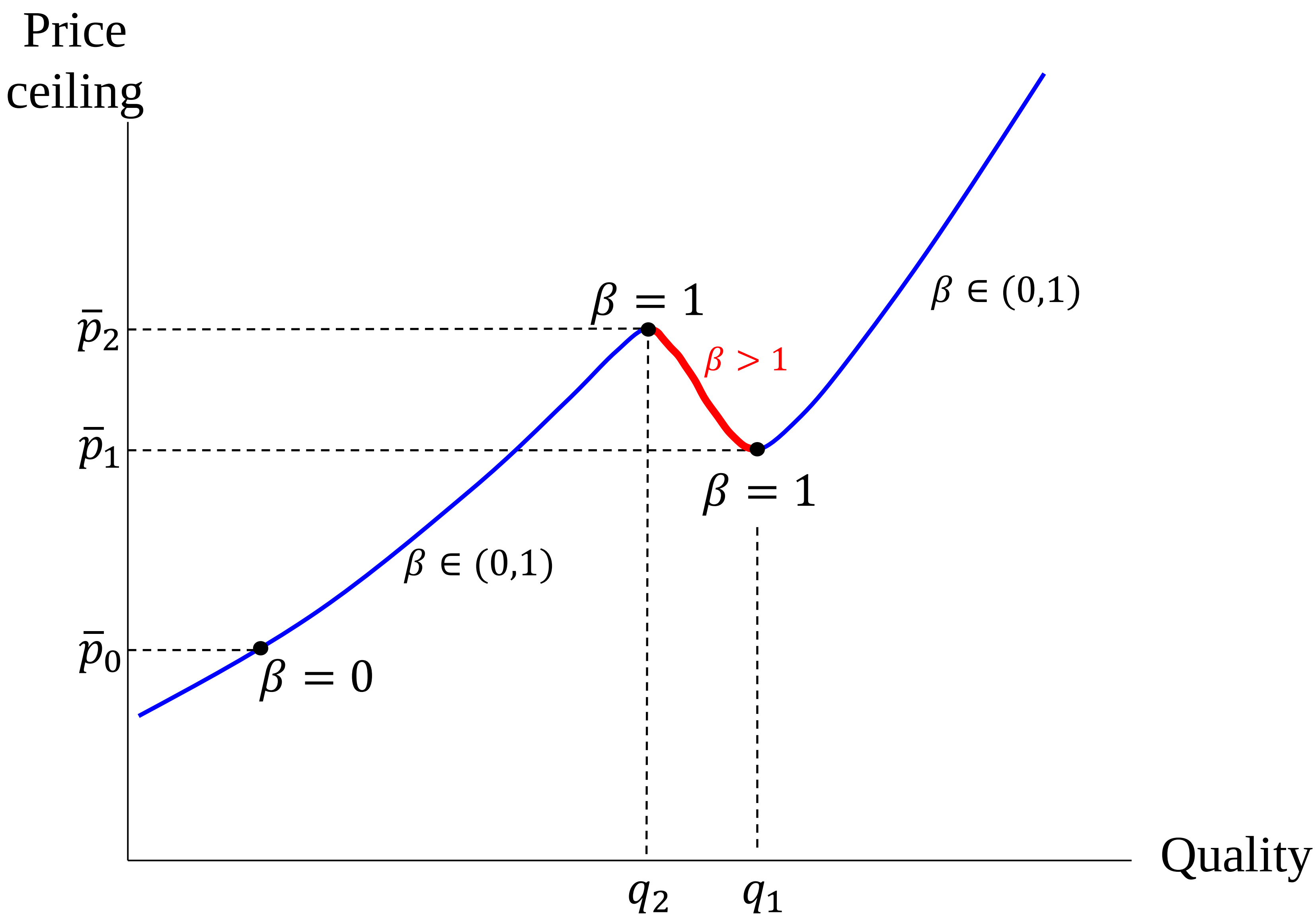
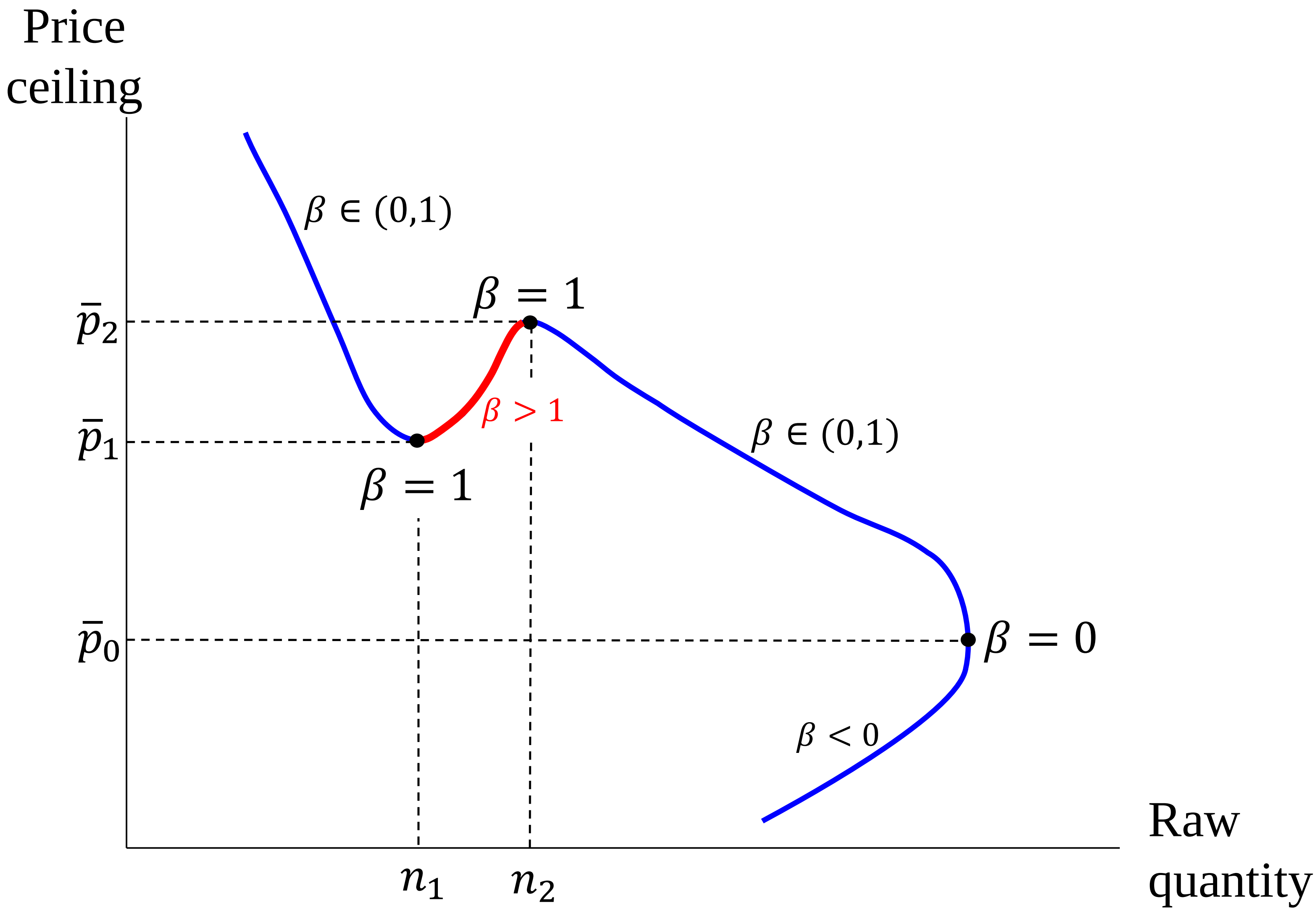


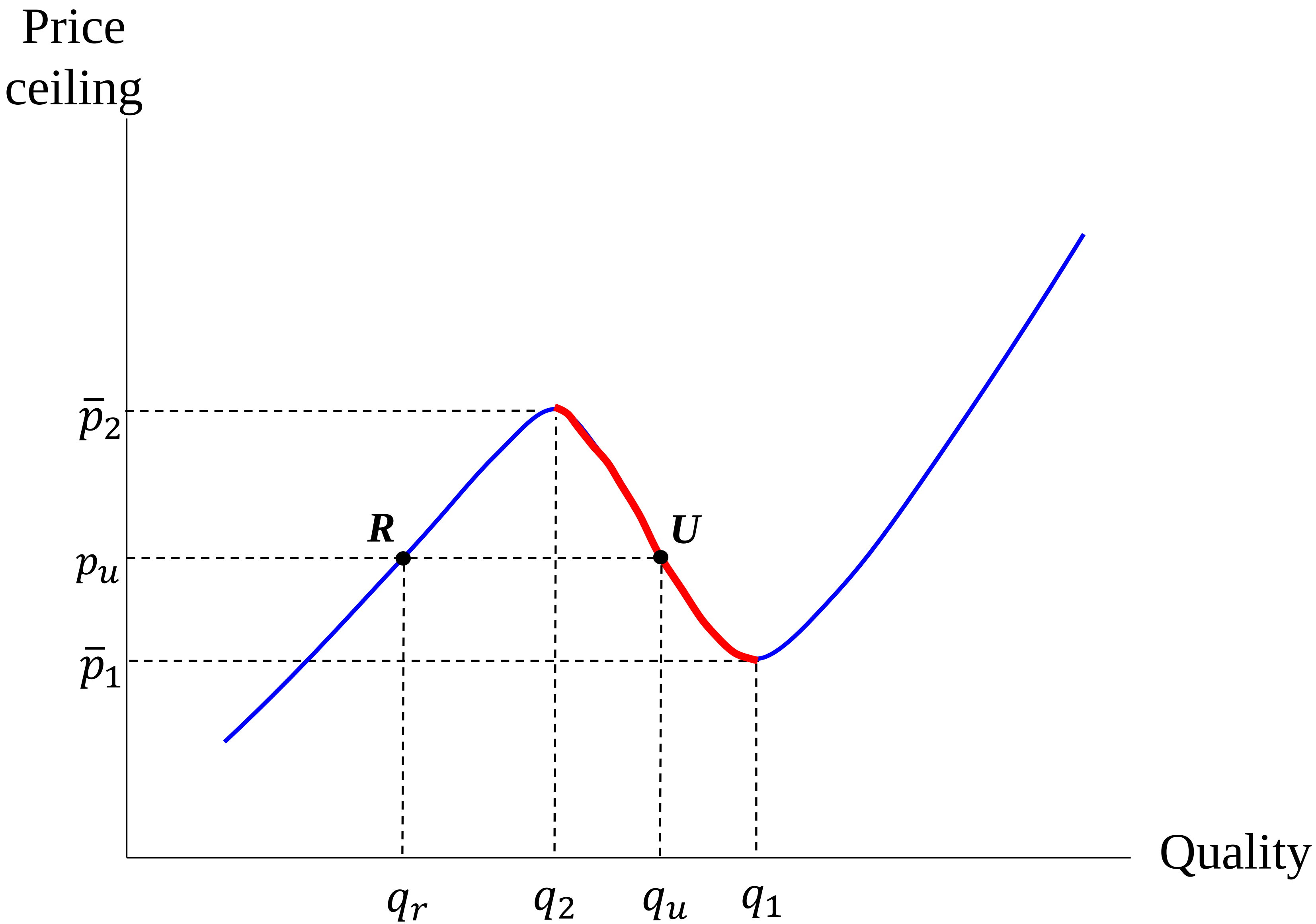
**Figure 4B. Equilibrium quantity vs. the price ceiling**

The role of the market multiplier, assuming $g_{nn} > 0$



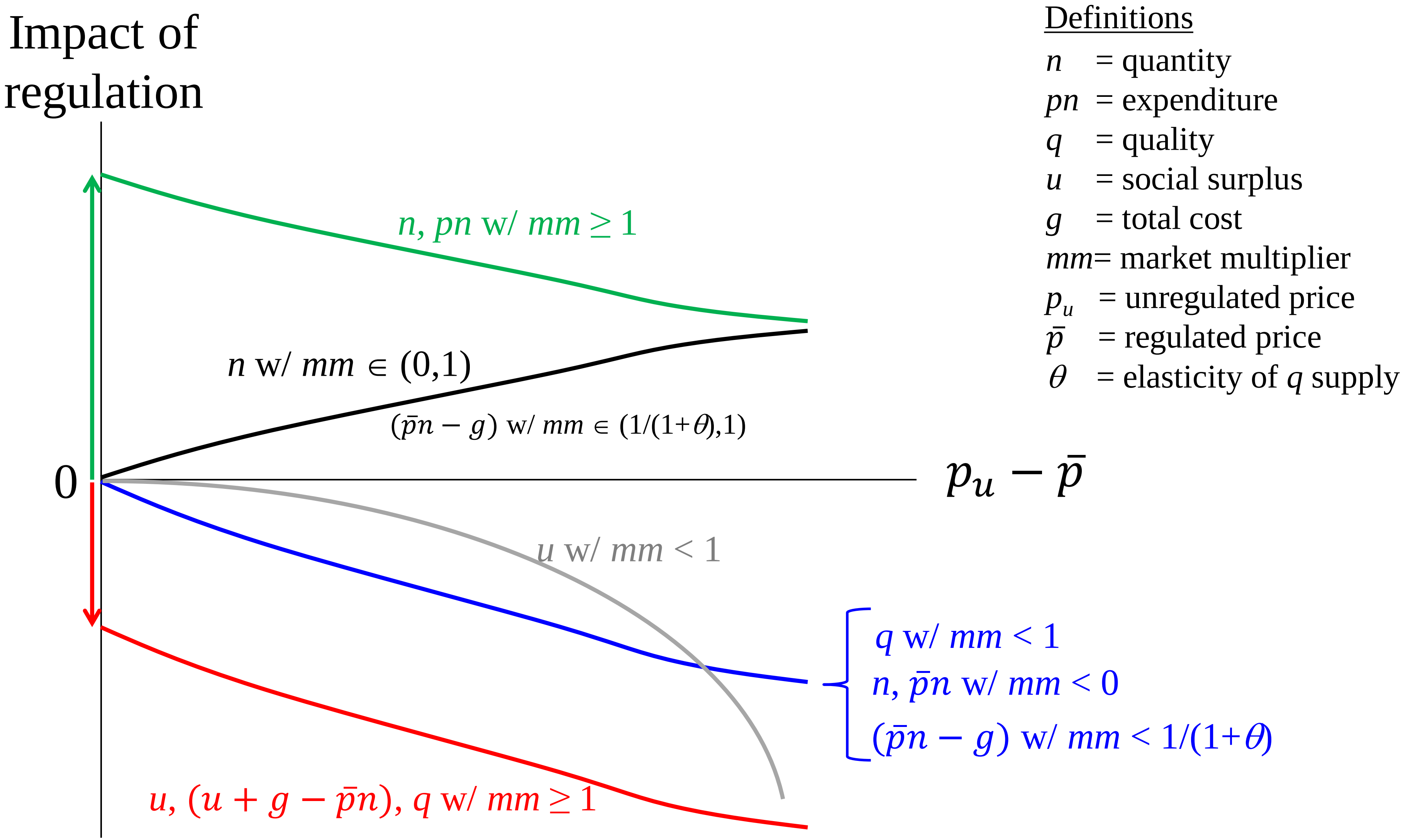
# Figure 4C. Equilibrium quality vs. the price ceiling

Example: the multiplier exceeds one at the unregulated allocation



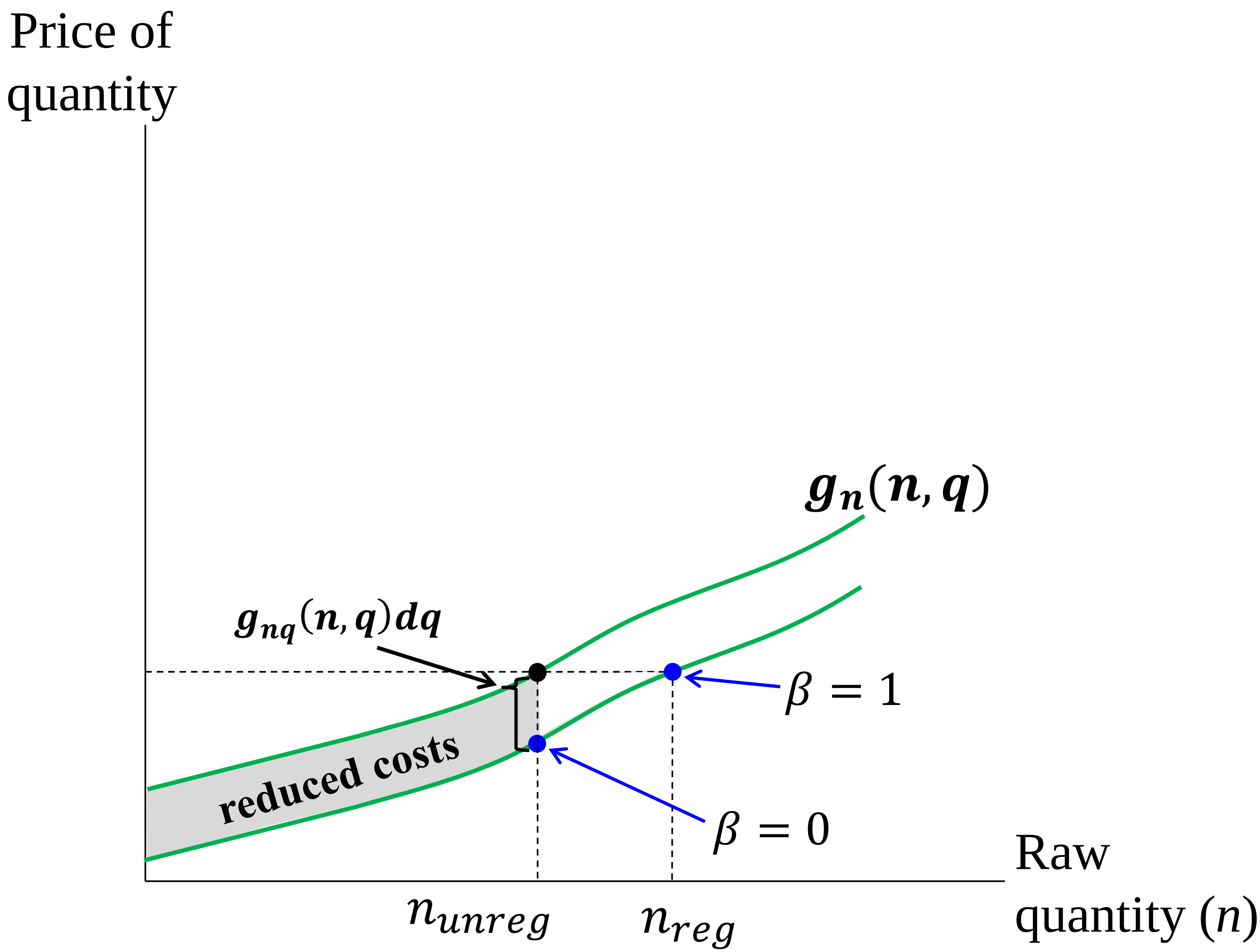
# Figure 5. Qualitative effects of price regulation

by the market multiplier value at the unregulated allocation



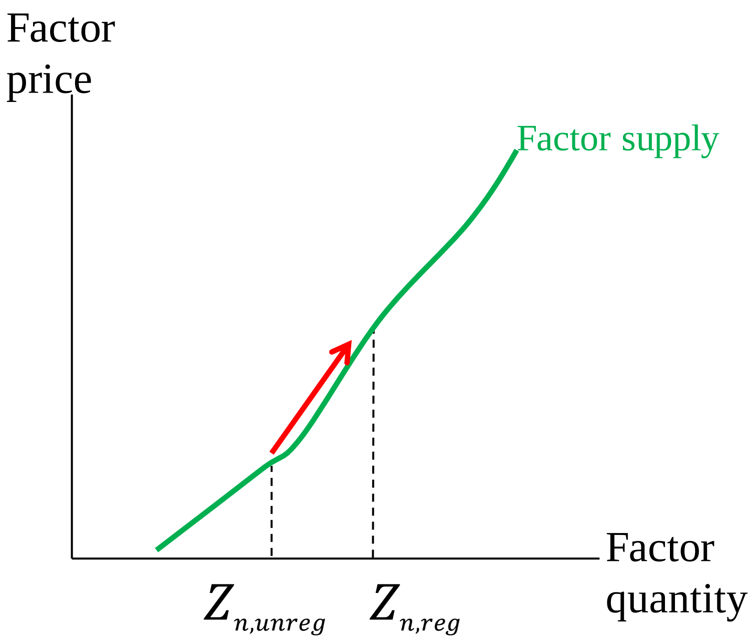

Note: Assumes that supply is not perfectly elastic

**Figure 6. Producer surplus**
with a quality or price ceiling.

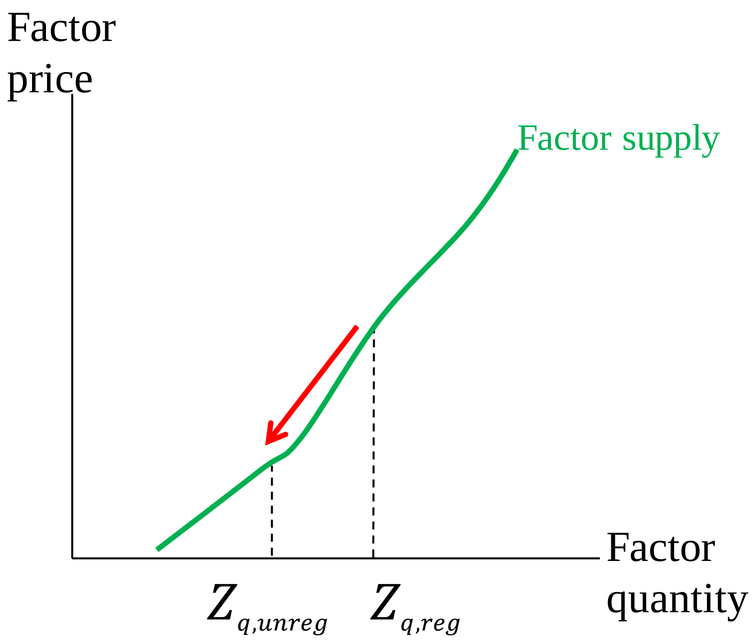


# Figure 7. Quality regulation changes the composition of producer surplus.

**Factors for producing raw quantity**




**Factors for adding quality**




$$g(n,q) = \int_0^n S_n(Z_n)dZ_n + \int_0^{nG(q)} S_q(Z_q)dZ_q$$

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