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THE RACE BETWEEN MACHINE AND MAN: IMPLICATIONS OF TECHNOLOGY FOR GROWTH, FACTOR SHARES AND EMPLOYMENT

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ABSTRACT

The advent of automation and the simultaneous decline in the labor share and employment among advanced economies raise concerns that labor will be marginalized and made redundant by new technologies. We examine this proposition using a task-based framework in which tasks previously performed by labor can be automated and more complex versions of existing tasks, in which labor has a comparative advantage, can be created. We characterize the equilibrium in this model and establish how the available technologies and the choices of firms between producing with capital or labor determine factor prices and the allocation of factors to tasks. In a static version of our model where capital is fixed and technology is exogenous, automation reduces employment and the share of labor in national income and may even reduce wages, while the creation of more complex tasks has the opposite effects. Our full model endogenizes capital accumulation and the direction of research towards automation and the creation of new complex tasks. Under reasonable conditions, there exists a stable balanced growth path in which the two types of innovations go hand-in-hand. An increase in automation reduces the cost of producing using labor, and thus discourages further automation and encourages the faster creation of new complex tasks. The endogenous response of technology restores the labor share and employment back to their initial level. Although the economy contains powerful self correcting forces, the equilibrium generates too much automation. Finally, we extend the model to include workers of different skills. We find that inequality increases during transitions, but the self-correcting forces in our model also limit the increase in inequality over the long-run.

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1 Introduction

The accelerated automation of tasks performed by labor raises concerns that new technologies will make labor redundant (e.g., Brynjolfsson and McAfee, 2012, Akst, 2014, Autor, 2015). The recent declines in the share of labor in national income and the employment to population ratio in the U.S. (e.g., Karabarbounis and Neiman, 2014, and Oberfield and Raval, 2014) are often interpreted to support the claims that, as digital technologies, robotics and artificial intelligence penetrate the economy, workers will find it increasingly difficult to compete against machines, and their compensation will experience a relative or even absolute decline. Yet, we lack a comprehensive framework incorporating such effects, as well as potential countervailing forces.

The need for such a framework stems not only from the importance of understanding how and when automation will transform the labor market, but also from the fact that similar claims have been made, but have not always come true, about previous waves of new technologies. Keynes famously foresaw the steady increase in per capita income during the 20th century from the introduction of new technologies, but incorrectly predicted that this would create widespread technological unemployment as machines replaced human labor (Keynes, 1930). In 1965, economic historian Robert Heilbroner confidently stated that "as machines continue to invade society, duplicating greater and greater numbers of social tasks, it is human labor itself—at least, as we now think of 'labor'—that is gradually rendered redundant"(quoted in Akst, 2014). Wassily Leontief was equally pessimistic about the implications of new machines. By drawing an analogy with the technologies of the early 20th century that made horses redundant, he speculated that "Labor will become less and less important... More and more workers will be replaced by machines. I do not see that new industries can employ everybody who wants a job"(Leontief, 1952).

This paper is a first step in developing a conceptual framework to study how machines replace human labor and why this might (or might not) lead to lower employment and stagnant wages. Our main conceptual innovation is to propose a unified framework in which tasks previously performed by labor are automated, while at the same time more complex versions of existing tasks in which labor has a comparative advantage are created.[1] The importance of these new complex tasks is well illustrated by the technological and organizational changes during the Second Industrial Revolution, which not only involved the replacement of the stagecoach by the railroad, sailboats by steamboats, and of manual dock workers by cranes, but also the creation of new labor-intensive tasks. These new tasks generated jobs for a new class of engineers, machinists, repairmen, and conductors as well as of modern managers and financiers involved with the introduction and operation of new technologies (e.g., Landes, 1969, Chandler, 1977, and Mokyr, 1990).

Today, while industrial robots, digital technologies and computer-controlled machines replace labor, we are once again simultaneously witnessing the emergence of new tasks ranging from engineering and programming functions to those performed by audio-visual specialists, executive assistants, data

[1] Herein lies our answer to Leontief's analogy: the difference between human labor and horses is that humans have a comparative advantage in new and more complex tasks. Horses did not.

administrators and analysts, meeting planners or computer support specialists. Indeed, during the last 30 years, new tasks and new job titles account for a large fraction of U.S. employment growth. To document this fact, we use data from Lin (2011) that measures the share of new job titles—in which workers perform newer tasks than those employed in more traditional jobs—within each occupation. In 2000, about 70% of the workers employed as computer software developers (an occupation employing one million people at the time) held new job titles. Similarly, in 1990 a radiology technician and in 1980 a management analyst were new job titles. Figure 1 shows that for each decade since 1980, employment growth has been greater in occupations with more new job titles. The regression line shows that occupations with 10 percentage points more new job titles at the beginning of each decade grow 5.05% faster over the next 10 years (standard error=1.3%). From 1980 to 2007, total employment in the U.S. grew by 17.5%. About half (8.84%) of this growth is explained by the *additional* employment growth in occupations with new job titles relative to a benchmark category with no new job titles.[2]

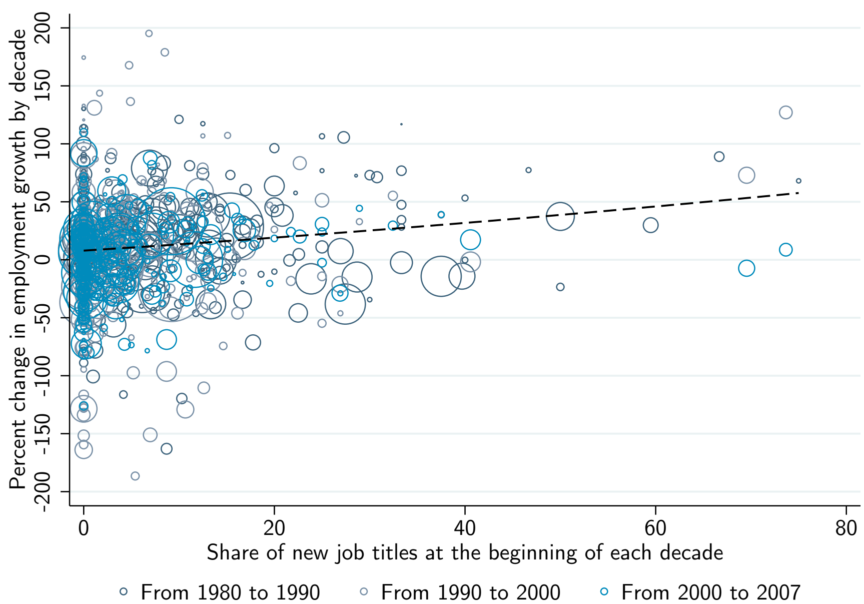

Figure 1: Employment growth by decade plotted against the share of new job titles at the beginning of each decade for 330 occupations. Data from 1980 to 1990 (in dark blue), 1990 to 2000 (in blue) and 2000 to 2007 (in light blue, re-scaled to a 10-year change). Data source: See Appendix B.

We start with a static model in which capital is fixed and technology is exogenous. There are two types of technological changes: the automation of existing tasks and new complex tasks in which labor has a comparative advantage. Our static model provides a rich but tractable framework to study how automation and the creation of new complex tasks impact factor prices, factor shares in national income and employment. Automation allows firms to produce tasks previously performed by labor

[2]The data for 1980, 1990 and 2000 are from the U.S. Census. The data for 2007 are from the American Community Survey. Additional information on the data and our sample is provided in Appendix B, where we also document in detail the robustness of the relationship depicted in Figure 1.

with capital, while the creation of new complex tasks allow firms to replace old tasks by new variants in which labor has a higher productivity. In contrast to the more commonly-used models featuring factor-augmenting technologies, here automation always reduces the share of labor in national income and employment, and may even reduce wages. Conversely, the creation of new complex tasks always increases wages, employment and the share of labor, and may even reduce the rate of return to capital. These comparative statics follow because factor prices are determined by the range of tasks performed by capital and labor, and exogenous shifts in technology alter the range of tasks performed by each factor (see also Acemoglu and Autor, 2011).

We then embed this framework in a dynamic economy in which capital accumulation is endogenous, and we characterize restrictions under which the model delivers balanced growth—which we take to be a good approximation to economic growth in the United States and the United Kingdom over the last two centuries. The key restrictions are that there is exponential productivity growth from the creation of new tasks and that the two types of technological changes—automation and the creation of new complex tasks—advance at equal paces. A critical difference from our static model is that capital accumulation responds to permanent shifts in technology in order to keep the interest rate constant. Thus, the dynamic effects of technology on factor prices depend on the response of capital accumulation as well. We show that the response of capital ensures that the productivity gains from both automation and the introduction of new complex tasks fully accrue to labor (the relatively inelastic factor) and increase overall wages in the long run—a feature we refer to as the *productivity effect*. Although real wages increase due to the productivity effect, automation always reduces the labor share.

Our full model endogenizes the rates of improvement of these two types of technologies by marrying our task-based framework with a directed technological change setup. This full version of the model remains tractable, and under natural assumptions, generates asymptotically stable balanced growth with equal advancement of the two types of technologies. If automation runs ahead of the creation of new complex tasks, market forces induce a slowdown of subsequent automation and countervailing advances in the creation of new complex tasks. As a result, in the long run, the share of labor in national income and employment return to their initial levels. The economics of these self-correcting forces are instructive and highlight a crucial new force: a wave of automation pushes down the effective cost of producing with labor. When technology is endogenous, this discourages further efforts to automate additional tasks and pushes the economy to redirect its research efforts towards the creation of new (labor-intensive) tasks.[3]

In our model, the stability of the balanced growth path implies that periods in which automation runs ahead of the creation of new complex tasks tend to self-correct. Contrary to the increasingly widespread concerns discussed above, our model raises the (theoretical) possibility that rapid automation need not signal the demise of labor, but might simply be a prelude to a phase of new technologies favoring labor. In addition, our analysis clarifies the long-run implications of different types of tech-

[3]Stability in addition requires that the productivity effect is not too strong; otherwise, automation would have a large impact on wages and could potentially discourage the creation of new tasks.

nological shocks. For example, if a wave of automation is triggered by a change in the innovation possibilities frontier (that is, in the technology for creating new technologies) that makes it easier to automate tasks, the economy will settle in a new balanced growth path with a greater share of tasks performed by capital, lower employment and lower labor share.

The final implication of our full model concerns the efficiency of the market equilibrium. In addition to the standard inefficiencies due to monopoly markups and appropriability problems in endogenous technological change models, our analysis identifies a new source of inefficiency that pushes towards too much automation and too few new tasks being created. This inefficiency arises because automation, which enables firms to economize on wage payments, responds to high wages; when some of the wage payments accruing to workers are rents (e.g., efficiency wages or quasi-rents created by labor market frictions), there will be more automation than what the social planner would desire, and technology becomes inefficiently biased towards replacing labor.

We also consider two extensions of our model. First, we introduce heterogeneity in skills, and assume that skilled labor has a comparative advantage in new complex tasks, which we view as a natural assumption.[4] Because of the comparative advantage of skilled workers relative to the unskilled in higher-index tasks, automation directly takes jobs from unskilled labor and thus increases inequality. Similarly, because skilled workers have a comparative advantage in new complex tasks, the creation of such tasks at first increase inequality as well. However, inequality increases tend to reverse themselves over longer periods as new tasks are standardized and can employ unskilled labor more productively. This extension formalizes the intuitive idea that both automation and the creation of new complex tasks increase inequality in the short run, but also points out that, over the long run, the self-correcting forces in our economy limit the increase in inequality. Our second extension modifies our baseline patent structure and reintroduces the creative destruction of the profits of previous innovators, which was absent in our main model but is often assumed in the endogenous growth literature. The results in this case are similar, but the conditions for uniqueness and stability of the balanced growth path are more demanding.

Our paper relates to several literatures. It can be viewed as a combination of task-based models of the labor market with directed technological change models.[5] Task-based models have been developed both in the economic growth and labor literatures, dating back at least to Roy's seminal work (1955). The first important recent contribution is Zeira (1998), which proposed a model of economic growth based on capital-labor substitution and constitutes a special case of our model. Acemoglu and Zilibotti (2000) developed a simple task-based model with endogenous technology and applied it to the study of productivity differences across countries, illustrating the potential mismatch between new technologies and the skills of developing economies (see also Zeira, 2006, Acemoglu, 2010). Autor, Levy and Murnane (2003) suggested that the increase in inequality in the U.S. labor market reflects the

[4] This assumption builds on Schultz (1965) (see also Greenwood and Yorukoglu, 1997, Caselli, 1999, Galor and Moav, 2000, Acemoglu, Gancia and Zilibotti, 2010, and Beaudry, Green and Sand, 2013).

[5] On directed technological change and related models, see Acemoglu (1998, 2002, 2003a, 2003b, 2007), Kiley (1999), Caselli and Coleman (2006), Gancia (2003), Thoenig and Verdier (2003) and Gancia and Zilibotti (2010).

automation and computerization of routine, labor-intensive tasks.[6] Our static model is most similar to Acemoglu and Autor (2011). Our full framework extends this model not only because of the dynamic equilibrium incorporating directed technological change, but also because tasks are combined with a general elasticity of substitution, and because the equilibrium allocation of tasks critically depends both on factor prices and the state of technology.[7]

Three papers from the economic growth literature that are particularly related to our work are Acemoglu (2003), Jones (2005), and Hemous and Olson (2015). The first two papers develop growth models in which the aggregate production function is endogenous and, in the long run, adapts to make balanced growth possible. In Jones (2005), this occurs because of endogenous choices about different combinations of activities/technologies being used. In Acemoglu (2003), this long-run behavior is a consequence of directed technological change in a model of factor-augmenting technologies. Our task-based framework here is a significant departure from this model, especially since it enables us to address questions related to automation, its impact on factor prices and its endogenous evolution. In addition, our framework provides a more robust economic force ensuring the stability of the balanced growth path: while in models with factor-augmenting technologies stability requires an elasticity of substitution between capital and labor that is less than 1 (so that the more abundant factor commands a lower share of national income), we do not need such a condition in this framework.[8] Finally, Hemous and Olson (2015) develop a model of automation and horizontal innovation with endogenous technology, and use it to study consequences of different types of technologies on inequality. High wages (in their model for low-skill workers) encourage automation. But unlike our model, the unbalanced dynamics that this generates are not countered by other types of innovations in the long run.[9]

The rest of the paper is organized as follows. Section 2 presents our task-based framework in the context of a static economy. Section 3 introduces capital accumulation and clarifies the conditions for balanced growth in this economy. Section 4 presents our full model with endogenous technology and establishes, under some plausible conditions, the existence, uniqueness and stability of a balanced growth path with two types of technologies advancing simultaneously. Section 5 examines the efficiency of equilibrium composition of new technologies. Section 6 considers the two extensions mentioned above. Section 7 concludes. Appendix A contains the proofs of some of the proofs omitted in the text,

[6] Acemoglu and Autor (2011), Autor and Dorn (2013), Jaimovich and Siu (2014), Foote and Ryan (2014), Burstein and Vogel (2012), and Burstein, Morales and Vogel (2014) provide various pieces of empirical evidence and quantitative evaluations on the importance of the endogenous allocation of tasks to factors in recent labor market dynamics.

[7] Acemoglu and Autor's model, like ours, is one in which a discrete number of labor types are allocated to a continuum of tasks. Costinot and Vogel (2010) develop a complementary model in which there is a continuum of skills and a continuum of tasks. See also the recent paper by Hawkins, Ryan and Oh (2015), which shows how a task-based model is more successful than standard models in matching the co-movement of investment and employment at the firm level.

[8] The role of technologies replacing tasks in this result can be seen by noting that with factor-augmenting technological changes, the impact on relative factor prices is ambiguous and the direction of innovation may be dominated by a strong market size effect (e.g., Acemoglu, 2002). Instead, in our model, the difference between factor prices regulates the future path of technological change and thus generates a powerful force that ensures stability.

[9] Kotlikoff and Sachs (2012) develop an overlapping generation model in which automation may have long-lasting effects because it reduces the wages of current workers, and via this channel, also depresses their savings and capital accumulation.

while Appendix B, which is not for publication, contains the remaining proofs, some additional results and the details of the empirical analysis presented above.

2 Static Model

We start with a static version of our model with exogenous technology, which will enable us to introduce our main setup in the simplest fashion and characterize the impact of different types of technological change on factor prices.

2.1 Environment

The economy contains a unique final good Y, produced by combining a continuum of tasks, $y(i)$, of unit measure with an elasticity of substitution $\sigma \in (0, \infty)$. Namely,

$$Y = \left(\int_{N-1}^{N} y(i)^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}}. \tag{1}$$

All tasks and the final good are produced competitively. This formulation will enable us to model the upgrading of the quality (productivity) of the unit measure of tasks as an increase in N. The fact that the limits of integration run between $N-1$ and N also imposes that the measure of tasks used in production always remains at 1, and a new more complex task replaces or upgrades the lowest-index task.[10]

Each task is produced combining labor or capital with a task-specific intermediate $q(i)$, which embodies the technology used both for production and for the possible automation of tasks. In preparation for our full model in Section 4, we assume that property rights to each intermediate are held by a technology monopolist which can produce it at the marginal cost $\mu\psi$ in terms of the final good, where $\mu \in (0,1)$ and $\psi > 0$. The technology for each intermediate can be copied by a fringe of competitive firms, which can replicate the intermediate at a higher marginal cost of ψ. We assume that μ is such that the unconstrained monopoly price of an intermediate is greater than ψ, ensuring that the unique equilibrium price for all types of intermediates in the presence of the competitive fringe will be a limit price at ψ.

All tasks can be produced by labor. On the other hand, we model the technological constraints on automation by assuming that there exists $I \in [N-1, N]$ such that tasks $i \leq I$ are *technologically automated* in the sense that they can feasibly be produced by capital as well. Though tasks $i \leq I$ are technologically automated, in equilibrium they do not need to be produced with capital. Whether they will or not depends on relative factor prices as we describe below. Conversely, tasks $i > I$ are not

[10] This formulation imposes that once a new task is created at N, it will automatically be utilized and as a consequence, will replace the lowest available task located at $N-1$. In Section 3, we provide conditions under which firms will indeed prefer to utilize such tasks immediately (see footnote 13). The reason why, once adopted, they replace older tasks is technological: as already noted, there is a unit measure of tasks, so newly created tasks are simply higher productivity versions of already existing tasks; moreover, we are also assuming that task i is not compatible and will not be used together with tasks $i' < i-1$ (see also footnote 18).

technologically automated. Independently of factor prices, they cannot be produced by capital and must be produced with labor.

Tasks $i > I$ can only be produced using labor, and their production function takes the form

$$y(i) = B\left[\eta q(i)^{\frac{\zeta-1}{\zeta}} + (1-\eta)\left(\gamma(i)l(i)\right)^{\frac{\zeta-1}{\zeta}}\right]^{\frac{\zeta}{\zeta-1}}, \tag{2}$$

where $\gamma(i)$ denotes the productivity of labor in task i, $\zeta \in (0, \infty)$ is the elasticity of substitution between intermediates and labor, $\eta \in (0, 1)$ is the distribution parameter of this constant elasticity of substitution production function, and finally, B is a normalizing constant set as $B \equiv (1-\eta)^{\zeta/(1-\zeta)}$ to simplify the algebra.

Tasks $i \leq I$ can be produced using labor or capital, and their production function takes the form

$$y(i) = B\left[\eta q(i)^{\frac{\zeta-1}{\zeta}} + (1-\eta)\left(k(i) + \gamma(i)l(i)\right)^{\frac{\zeta-1}{\zeta}}\right]^{\frac{\zeta}{\zeta-1}}. \tag{3}$$

All of the parameters are thus common between the production function of tasks above and below the threshold I, with the only difference being that those with $i \leq I$ can be produced by capital as well as labor. This feature is embedded in (3) via the assumption that capital and labor are perfect substitutes in the production of these tasks.[11]

Throughout, we assume that $\gamma(i)$ is strictly increasing, so that labor has strict *comparative advantage* in tasks with a high index. In the next section, we strengthen this assumption by imposing a parametric form for $\gamma(i)$, which will ensure that the productivity gains from the creation of new tasks generate balanced growth (see in particular, equation (12)). The key implication of the strict comparative advantage of labor in high-index tasks is that, in equilibrium, there will exist some threshold task $I^* \leq I$ such that all tasks $i \leq I^*$ are produced with capital, while all tasks $i > I^*$ are produced with labor (see below).[12]

Figure 1 depicts the resulting allocation of tasks to factors and also shows how, as already noted, the creation of new complex tasks replaces existing tasks from the bottom of the distribution.

In the static model, we take the capital stock to be fixed at K (we will endogenize it via household saving decisions in Section 3). In addition, since we wish to study the impact of new technologies not just on factor prices but also on employment, we assume that the employment level is given by a quasi-labor supply taken to be an increasing function of the wage rate W relative to capital payments RK, i.e., $L^s\left(\frac{W}{RK}\right)$. This quasi-labor supply curve thus implies that as the wage rate increases relative to payments to capital, the employment level increases as well. As we show in Section 3, the assumption that the level of employment depends on the ratio $\frac{W}{RK}$ and not simply on wages ensures that this

[11] A simplifying feature of the technology described in equation (3) is that capital has the same productivity in all tasks, while labor has a different productivity. This assumption could be relaxed at the cost of additional complexity in our notation.

Another, perhaps more important simplifying assumption is that high-index tasks can be produced with just labor. Having these tasks combine labor and capital would have no impact on our main results, and in Appendix B, we show how adding an additional layer of capital-labor substitution in (2) and (3) has no substantive impact on our results.

[12] Without loss of generality, we impose that when indifferent firms use capital. This explains our convention of writing that all tasks $i \leq I^*$ (rather than $i < I^*$) are produced using capital.

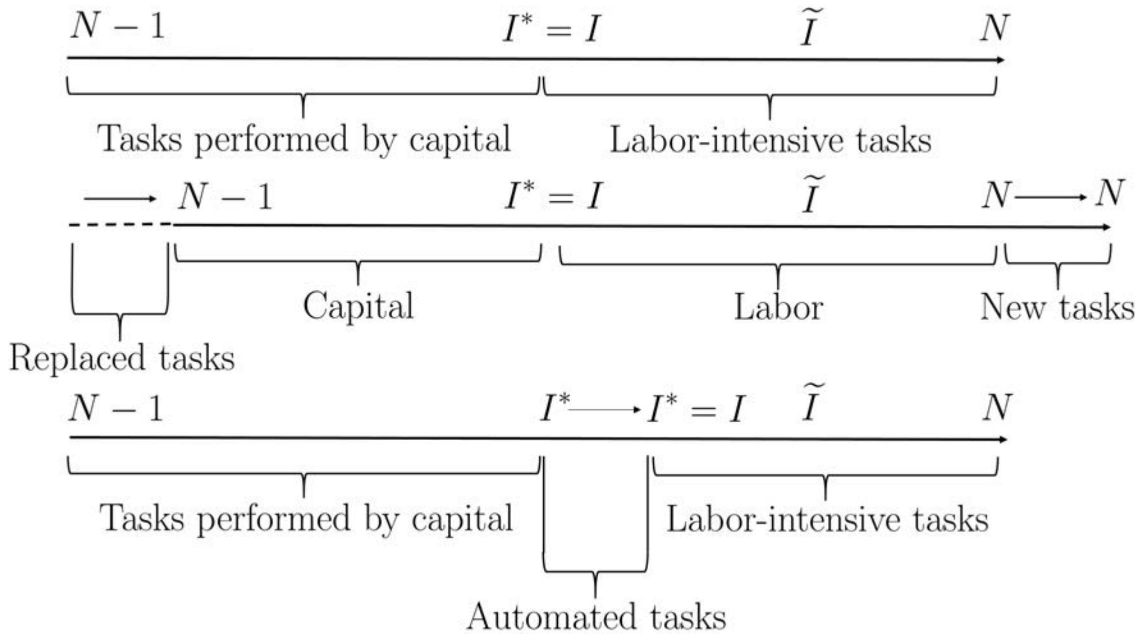

Figure 2: The task space and a representation of the effect of introducing new complex tasks (middle panel) and automating existing tasks (bottom panel).

quasi-labor supply will be consistent with balanced growth (see Acemoglu 2003). Though we impose this as a reduced-form in the text, it is straightforward to derive it from various micro foundations as we do in Appendix B.

With this specification of the supply of factors, capital and labor market clearing imply the conditions

$$\int_{N-1}^{N} k(i)di = K, \text{ and } \int_{N-1}^{N} l(i)di = L^s\left(\frac{W}{RK}\right).$$

2.2 Equilibrium in the Static Model

We now characterize the equilibrium in the static model, which can be summarized by the wage rate, W, the rental rate of capital (rental rate for short), R, and the equilibrium threshold I^*.

We proceed by characterizing the unit cost of producing each task as a function of factor prices and the automation possibilities represented by I. Because tasks are produced competitively, their price, $p(i)$, will be equal to the minimum unit cost of production:

$$p(i) = \begin{cases} c^u\left(\min\left\{R, \frac{W}{\gamma(i)}\right\}\right) \equiv \left[\left(\frac{\eta}{1-\eta}\right)^{\zeta}\psi^{1-\zeta} + \min\left\{R, \frac{W}{\gamma(i)}\right\}^{1-\zeta}\right]^{\frac{1}{1-\zeta}} & \text{if } i \le I, \\ c^u\left(\frac{W}{\gamma(i)}\right) \equiv \left[\left(\frac{\eta}{1-\eta}\right)^{\zeta}\psi^{1-\zeta} + \left(\frac{W}{\gamma(i)}\right)^{1-\zeta}\right]^{\frac{1}{1-\zeta}} & \text{if } i > I. \end{cases} \tag{4}$$

Here $c^u(\cdot)$ is the unit cost of production for task i, derived from the task production functions, (2) and (3). This unit cost also depends on the price of intermediates, ψ, but we suppress this dependence to simplify the notation. The unit cost for tasks $i \le I$ is written as a function of $\min\left\{R, \frac{W}{\gamma(i)}\right\}$ to reflect the fact that capital and labor are perfect substitutes in the production of automated tasks. In these tasks, firms will choose whichever factor has a lower effective cost—where the effective cost for labor is $W/\gamma(i)$ in view of the fact that the productivity of labor in task i is $\gamma(i)$.

The expression for $p(i)$ immediately implies that, given strict comparative advantage, there is a threshold $\widetilde{I}$ such that tasks below $I^* = \min\{I, \widetilde{I}\}$ will be produced using capital and the remaining more complex tasks will be produced using labor. Specifically, whenever $R > W/\gamma(i)$ and $i \leq I$, the relevant task is produced using capital, and otherwise it is produced using labor.[13] Since $\gamma(i)$ is strictly increasing, this implies that there exists a threshold $\widetilde{I}$ at which, if technologically feasible, firms would be indifferent between using capital and labor. Namely, at task $\widetilde{I}$, we have that $R = W/\gamma(\widetilde{I})$, or

$$\frac{W}{R} = \gamma(\widetilde{I}). \tag{5}$$

This threshold represents the task for which the costs of producing with capital or labor are equal. Without any other constraints, the cost-minimizing allocation of factors is to produce all tasks $i < \widetilde{I}$ with capital. However, if $\widetilde{I} > I$, firms will not be able to use capital all the way up to task $\widetilde{I}$ because of the constraint imposed by the available automation technology. For this reason, the equilibrium threshold below which tasks are produced using capital is given by

$$I^* = \min\{I, \widetilde{I}\}.$$

To fully characterize the static equilibrium, we next derive the demand for factors in terms of the (endogenous) threshold I^*. We choose the final good as the numeraire, which from (1) gives the demand for task i as

$$y(i) = Y p(i)^{-\sigma}. \tag{6}$$

From equations (4) and (6), equilibrium levels of task production can be written as

$$y(i) = \begin{cases} Yc^u \left(\min\left\{R, \dfrac{W}{\gamma(i)}\right\}\right)^{-\sigma} & \text{if } i \leq I, \\ Yc^u \left(\dfrac{W}{\gamma(i)}\right)^{-\sigma} & \text{if } i > I. \end{cases}$$

From the technologies in equations (2) and (3), it follows that the demand for capital and labor in each task is given by:

$$k(i) = \begin{cases} Yc^u (R)^{\zeta-\sigma} R^{-\zeta} & \text{if } i \leq I^*, \\ 0 & \text{if } i > I^*. \end{cases}$$

and

$$l(i) = \begin{cases} 0 & \text{if } i \leq I^*, \\ Y\gamma(i)^{\zeta-1} c^u \left(\dfrac{W}{\gamma(i)}\right)^{\zeta-\sigma} W^{-\zeta} & \text{if } i > I^*. \end{cases}$$

[13] This discussion reveals an asymmetry in our treatment of automation and new labor-intensive tasks. As already noted in footnote 10, we have assumed that the latter type of technology is always used when it is created (and hence we have not distinguished N, N^* and $\widetilde{N}$). This is because, as we show in Proposition 3, in the interesting part of the parameter space, where the interest rate is not too small (which in turn results from the discount rate in our full model, ρ, being above some threshold $\overline{\rho}$), all new labor-intensive technologies will be used immediately, whereas all new automation technologies may or may not be utilized immediately depending on the relative state of the two types of technologies.

Thus, capital and labor market clearing yield the following equilibrium conditions:

$$Y(I^* - N + 1)c^u(R)^{\zeta-\sigma}R^{-\zeta} = K, \tag{7}$$

and

$$Y\int_{I^*}^{N} \gamma(i)^{\zeta-1}c^u\left(\frac{W}{\gamma(i)}\right)^{\zeta-\sigma} W^{-\zeta}di = L^s\left(\frac{W}{RK}\right). \tag{8}$$

Though the set of equations characterizing an equilibrium are relatively straightforward, the substitution between factors (capital or labor) and intermediates (the $q(i)$'s) makes the relative demands for factors non-homothetic, opening the way to counterintuitive results. For instance, automation, which increases the productivity of capital, may end up raising the demand for labor more than the demand for capital. In what follows, we focus on the economically relevant case in which, at given factor proportions, automation reduces the relative demand for labor. The next assumption ensures this by limiting the extent of departures from homotheticity.

Assumption 1: One of the following three conditions holds:

1. $\left(\frac{\gamma(N-1)}{\gamma(N)}\right)^{2+\sigma+\eta} > |\sigma - \zeta|$, or
2. $\zeta \to 1$, or
3. $\eta \to 0$.

When $\sigma = \zeta$, the elasticities of substitution between tasks and between factors in the production of tasks are equal, ensuring homotheticity. Thus the first option in Assumption 1 requires that this departure from homotheticity is small relative to the inverse of the productivity gains from new tasks (where $\gamma(N)/\gamma(N-1)$ measures these productivity gains). The second option corresponds to the case where the task production function becomes Cobb-Douglas, which implies that intermediates account for a constant share of costs and also ensures homotheticity. Finally, the third option guarantees homotheticity as well because it makes the share of intermediates in the task production function small. The next proposition completes the characterization of the equilibrium under Assumption 1.

Proposition 1 (Equilibrium in the static model) *Suppose that Assumption 1 holds. Then, given a range of tasks $[N-1, N]$, automation technology $I \in (N-1, N]$, and capital stock K, there exists a unique equilibrium characterized by factor prices, W and R, and threshold tasks, $\widetilde{I}$ and I^*, such that: (i) $\widetilde{I}$ is determined by equation (5) and $I^* = \min\{I, \widetilde{I}\}$; (ii) all tasks $i \leq I^*$ are produced using capital and all tasks $i > I^*$ are produced using labor; (iii) the capital and labor market clearing conditions, equations (7) and (8), are satisfied; and (iv) factor prices satisfy the ideal price index condition:*

$$(I^* - N + 1)c^u(R)^{1-\sigma} + \int_{I^*}^{N} c^u\left(\frac{W}{\gamma(i)}\right)^{1-\sigma} di = 1. \tag{9}$$

Proof. All of the expressions in this proposition follow from the preceding derivations. Existence and uniqueness are proved in Appendix A. ■

Figure 3 illustrates the unique equilibrium described in Proposition 1. The equilibrium is represented by the intersection of an upward and downward-sloping curve in the (ω, I) space, which determines $\omega \equiv \frac{W}{RK}$ and I^*. The downward-sloping curve, $\omega(I^*, N, K)$, corresponds to the relative demand for labor, which we obtain by combining the market clearing conditions for capital and labor, (7) and (8), together with the ideal price index condition, given in equation (9). Assumption 1 ensures that the relative demand curve $\omega(I^*, N, K)$ is decreasing in I^*—that is, automation reduces the relative demand for labor. The upward-sloping curve represents the cost-minimizing allocation of capital and labor to tasks represented by equation (5), with the constraint that the equilibrium level of automation can never exceed I.

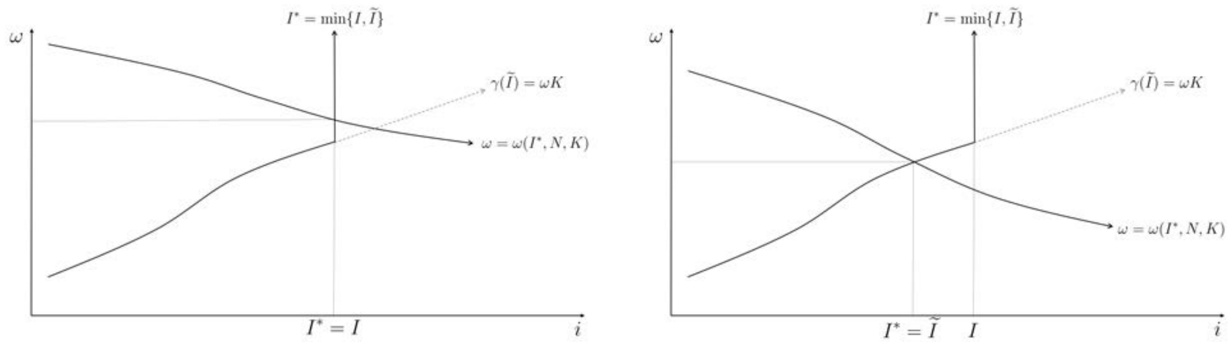

Figure 3: Static equilibrium. The left panel depicts the case in which $I^* = I < \widetilde{I}$ so that the allocation of factors is constrained by technology. The right panel depicts the case in which $I^* = \widetilde{I} < I$ so that the allocation of factors is not constrained by technology.

The figure distinguishes between the two cases highlighted above. In the left panel, we have $I^* = I < \widetilde{I}$ and the allocation of factors is constrained by technology, while the right panel plots the case in which $I^* = \widetilde{I} < I$ and firms choose the cost-minimizing allocation given factor prices.

The next proposition gives a complete characterization of comparative statics. In what follows, $\frac{\partial \omega}{\partial I^*}, \frac{\partial \omega}{\partial N}$ and $\frac{\partial \omega}{\partial K}$ denote the partial derivatives of $\omega(I^*, N, K)$.

Proposition 2 (Comparative statics) *Suppose that Assumption 1 holds.*[14]

- *If $I^* = I < \widetilde{I}$—so that the allocation of tasks to factors is constrained by technology—then:*
  - *the impact of technological change on relative factor prices is given by*

$$\frac{d \ln(W/R)}{dI} = \frac{d \ln \omega}{dI} = \frac{1}{\omega} \frac{\partial \omega}{\partial I^*} < 0, \frac{d \ln(W/R)}{dN} = \frac{d \ln \omega}{dN} = \frac{1}{\omega} \frac{\partial \omega}{\partial N} > 0$$

  - *the impact of capital on relative factor prices is given by*

$$\frac{d \ln(W/R)}{d \ln K} = \frac{d \ln \omega}{d \ln K} + 1 = \frac{1 + \varepsilon_L}{\sigma_{SR} + \varepsilon_L} > 0,$$

[14] In this proposition, we do not explicitly treat the case in which $I^* = I = \widetilde{I}$ in order to save on space and notation, since in this case left and right derivatives with respect to I are different.

where $\sigma_{SR} \in (0, \infty)$ is the short-run elasticity of substitution between labor and capital holding the allocation of factors to tasks fixed, and is given by a weighted average of σ and ζ.

  – *Moreover, if σ is sufficiently large, $\frac{d\ln W}{dI}, \frac{d\ln R}{dN} < 0$. Otherwise, $\frac{d\ln W}{dI}, \frac{d\ln R}{dN} \geq 0$.*

- *If $I^* = \widetilde{I} < I$—so that tasks are allocated to factors in the unconstrained cost minimizing fashion—then*

  – *the impact of technological change on relative factor prices is given by*

$$\frac{d\ln(W/R)}{dI} = \frac{d\ln\omega}{dI} = 0, \quad \frac{d\ln(W/R)}{dN} = \frac{d\ln\omega}{dN} = \frac{\frac{1}{\omega}\frac{\partial\omega}{\partial N}}{1 - \frac{1}{\omega}\frac{\partial\omega}{\partial I^*}\frac{1}{\varepsilon_\gamma}} > 0 \text{ and}$$

  – *the impact of capital on relative factor prices is given by*

$$\frac{d\ln(W/R)}{d\ln K} = \frac{d\ln\omega}{d\ln K} + 1 = \left(\frac{1+\varepsilon_L}{\sigma_{SR}+\varepsilon_L}\right)\frac{1}{1 - \frac{1}{\omega}\frac{\partial\omega}{\partial I^*}\frac{1}{\varepsilon_\gamma}} > 0,$$

  where $\varepsilon_\gamma = \frac{d\ln\gamma(I)}{dI} > 0$ is the semi-elasticity of the comparative advantage schedule. Here, the medium run elasticity of substitution $\sigma_{MR} \in (0, \infty)$ is

$$\sigma_{MR} = (\sigma_{SR} + \varepsilon_L)\left(1 - \frac{1}{\omega}\frac{\partial\omega}{\partial I^*}\frac{1}{\varepsilon_\gamma}\right) - \varepsilon_L > \sigma_{SR}.$$

  – *Moreover, if σ is sufficiently large, $\frac{d\ln R}{dN} < 0$. Otherwise, $\frac{d\ln R}{dN} \geq 0$.*

- *Finally, in both parts of the proposition, the labor share and employment move in the same direction as ω.*

Proof. See Appendix A. ■

The main implication of Proposition 2 is that the two types of technological changes—automation and the creation of new complex tasks—have polar implications. Automation, corresponding to an increase in I, reduces W/R, the labor share and employment (unless $I^* = \widetilde{I} < I$ and firms are not constrained by technology in their automation choice), while the creation of new tasks, corresponding to an increase in N, raises W/R, the labor share and employment.

These comparative static results are illustrated in Figure 3: automation moves us along the relative labor demand curve in the technology-constrained case shown in the left panel (and has no impact in the right panel), while the creation of new tasks shifts out the relative labor demand curve in both cases.

Another important implication of Proposition 2 is that when $I^* = I$, automation—an increase in I— can reduce real wages. This happens because automation expands the range of tasks performed by capital and contracts the set of tasks performed by labor. This last feature, combined with the diminishing returns to the quantity of a task in the aggregate production function, (1), puts downward pressure on the wage, but is counteracted by a positive effect coming from the fact that tasks are q-complements in (1). This positive effect is weaker when σ is greater, explaining why the overall impact of automation on the wage rate is negative when σ is large.[15] Likewise, when σ is large the creation of new complex tasks—that is, an increase in N—can reduce the rental rate.

These results are major consequences of the task-based framework developed here. With factor-augmenting technologies, technological improvements always increase the price of both capital and labor, but this is no longer the case when technological change alters the range of tasks performed by both factors (see also Acemoglu and Autor, 2011).[16] Furthermore, as it is well known, with factor-augmenting technologies, whether different types of technological improvements are biased towards one factor or the other depends on the elasticity of substitution. But this too is different in our task-based framework, where automation is *always* capital-biased (that is, it reduces W/R), while the creation of new complex tasks is *always* labor-biased (that is, it increases W/R).

A final implication of Proposition 2 is that the short-run elasticity of substitution between capital and labor differs from the "medium-run" elasticity. The short-run elasticity, σ_{SR}, applies when the range of tasks allocated to capital and labor is held fixed (as in the case where $I^* = I$). The medium-run elasticity, σ_{MR}, applies when the allocation of factors to different tasks responds to changes in factor prices (as in the case where $I^* = \widetilde{I}$).[17]

Though Proposition 2 provides a complete characterization of the responses of relative factor prices, factor shares and employment to automation and the creation of new complex tasks, the results are qualitative and the explicit expressions for the partial derivatives are complicated. This problem arises because of the aforementioned non-homotheticity in the demand for capital and labor. In two of the special cases mentioned above, when $\eta \to 0$ (the share of intermediates goes to zero), and when $\zeta \to 1$ (intermediates represent a constant share of the production of tasks), we can provide an explicit characterization of the equilibrium and the comparative statics. We further simplify the presentation of these results by taking $L(\omega) = L$ (and $\varepsilon_L = 0$), so that the quasi-labor supply coincides with the inelastic labor supply in the economy.

[15] We could have a negative impact of automation on wages even for moderate values of σ. For example, if $\sigma = \zeta = 1$ and $K/Y < 2.72$, automation reduces the marginal product of labor.

[16] For instance, with a constant returns to scale production function and two factors, capital and labor are $q-$complements. Thus, capital-augmenting technologies always increases the marginal product of labor . To see this, let $F(A_K K, A_L L)$ be such a production function. Then $W = F_L$, and $\frac{dW}{dA_K} = KF_{LK} = -LF_{LL} > 0$ (because of constant returns to scale).

[17] Another noteworthy corollary of this proposition is that a long-run negative association between capital accumulation and the labor share is not sufficient to conclude that σ—the elasticity of substitution between labor and capital—is above 1 (e.g., as argued by Karabarbounis and Neiman, 2014). This reasoning is valid only when technology is factor-augmenting, but not when the allocation of tasks to factors is endogenous. In particular, when automation responds to an increase in the capital stock, we could have a situation in which capital accumulation reduces the labor share, regardless of whether $\sigma \lesseqgtr 1$. For example, when $\sigma = \zeta = 1$ and $I^* = I$, as we show in Corollary 1, factor shares in our model are entirely independent of σ, and just depend on the extent of automation and the creation of new tasks.

In both of these special cases we obtain a revealing expression for aggregate output:

$$Y = \left[(I^* - N + 1)^{\frac{1}{\hat{\sigma}}} K^{\frac{\hat{\sigma}-1}{\hat{\sigma}}} + \left(\int_{I^*}^{N} \gamma(i)^{\hat{\sigma}-1} di \right)^{\frac{1}{\hat{\sigma}}} L^{\frac{\hat{\sigma}-1}{\hat{\sigma}}} \right]^{\frac{\hat{\sigma}}{\hat{\sigma}-1}}, \tag{10}$$

where $\hat{\sigma} \equiv \eta + (1-\eta)\sigma$ (which also implies that when $\eta \to 0$, we have $\hat{\sigma} = \sigma$).

This expression emphasizes that, in these special cases, output is a constant elasticity of substitution aggregate of capital and labor (with the short-run elasticity of substitution between capital and labor, σ_{SR}, simply being equal to $\hat{\sigma}$). Critically, the distribution parameters are endogenous and depend on the state of the two types of technologies in the economy. Indeed, automation increases the share of capital and reduces the share of labor in this aggregate production function, while the creation of new complex tasks does the opposite.

In these cases, the relative demand for labor can be obtained directly by differentiating (10):

$$\ln \omega = \left(\frac{1}{\hat{\sigma}} - 1 \right) \ln K + \frac{1}{\hat{\sigma}} \ln \left(\frac{\int_{I^*}^{N} \gamma(i)^{\hat{\sigma}-1} di}{I^* - N + 1} \right). \tag{11}$$

In these two special cases, equation (11) provides a more explicit characterization of the comparative statics derived in Proposition 2.

Corollary 1 *Suppose $\eta \to 0$ or $\zeta \to 1$. Then, there exists a unique equilibrium, and*

- *If $I^* = I < \widetilde{I}$:*

$$d \ln \omega = \left(\frac{1}{\hat{\sigma}} - 1 \right) d \ln K + \frac{1}{\hat{\sigma}} \left[\frac{\gamma(N)^{\hat{\sigma}-1}}{\int_{I^*}^{N} \gamma(i)^{\hat{\sigma}-1} di} + \frac{1}{I^* - N + 1} \right] dN - \frac{1}{\hat{\sigma}} \left[\frac{\gamma(I^*)^{\hat{\sigma}-1}}{\int_{I^*}^{N} \gamma(i)^{\hat{\sigma}-1} di} + \frac{1}{I^* - N + 1} \right] dI.$$

- *If $I^* = \widetilde{I} < I$:*

$$d \ln \omega = \left(\frac{1}{\hat{\sigma} + \Lambda/\varepsilon_\gamma} - 1 \right) d \ln K + \frac{1}{\hat{\sigma} + \Lambda/\varepsilon_\gamma} \left[\frac{\gamma(N)^{\hat{\sigma}-1}}{\int_{I^*}^{N} \gamma(i)^{\hat{\sigma}-1} di} + \frac{1}{I^* - N + 1} \right] dN,$$

where

$$\Lambda \equiv \frac{\gamma(I^*)^{\hat{\sigma}-1}}{\int_{I^*}^{N} \gamma(i)^{\hat{\sigma}-1} di} + \frac{1}{I^* - N + 1} > 0,$$

and $\hat{\sigma} \equiv \eta + (1-\eta)\sigma$.

In this corollary, the difference between the short-run and the medium-run elasticity of substitution can be seen quite clearly: $\sigma_{SR} = \hat{\sigma}$, and $\sigma_{MR} = \hat{\sigma} + \Lambda/\varepsilon_\gamma$. Moreover, the effect of shifts in technology are the same as in Proposition 2.

3 Dynamics, Balanced Growth and the Productivity Effect

In this section, we extend our model to a dynamic economy in which the evolution of the capital stock is determined by the saving decisions of a representative household. We then investigate the conditions under which the economy admits a balanced growth path (BGP), where output, the capital stock and wages grow at a constant rate. We conclude by discussing the long-run effects of automation on wages, the labor share and employment, and by highlighting an important "productivity effect," which stems from capital accumulation and creates a force from automation towards higher wages.

3.1 Balanced Growth

In order to ensure balanced growth, we need to put more structure on the comparative advantage schedule. Specifically, because balanced growth is driven by technology, and in this model sustained technological change comes from the creation of new complex tasks, constant growth requires the productivity gains from new tasks to be exponential:

$$\gamma(i) = e^{Ai} \text{ with } A > 0. \tag{12}$$

We impose this functional form in the remainder of the paper.[18] We also impose a simplified version of Assumption 1 under this functional form:

Assumption 1$'$: One of the following three conditions are satisfied:

1. $e^{-(2+2\sigma+\eta)A} > |\sigma - \zeta|$, or
2. $\zeta \to 1$, or
3. $\eta \to 0$.

We start by assuming exogenous paths for technological change, given by $\{I(t), N(t)\}$, and we define

$$n(t) \equiv N(t) - I(t)$$

as a summary measure of the state of technology. Automation reduces $n(t)$, and conversely, as more new complex tasks are created, $n(t)$ increases. We simplify the discussion and notation by assuming that $I^*(t) = I(t)$, so that newly automated tasks are immediately produced with capital. We discuss the conditions that ensure $I^*(t) = I(t)$ in Proposition 3 below.

Let $\{I(t), N(t), K(t)\}$ denote the path of the state variables: technology and capital. Also, let $\{R(t), W(t), Y(t), C(t)\}$ denote the path of factor prices, the equilibrium output, and the representative

[18] As usual we could have imposed this functional form only asymptotically, but we simplify the analysis and exposition by imposing it at all times.

Notice also that in this dynamic economy, as in our static model, the productivity of capital is the same in all automated tasks. This does not, however, imply that any of the previously automated tasks can be used regardless of N. As N increases, as emphasized by equation (1) and in footnote 10, the set of feasible tasks shifts to the right, and only tasks above $N - 1$ remain compatible with and can be combined with those currently in use.

household's consumption at each period. We assume that the representative household's preferences over consumption paths, $\{C(t)\}$, are given by

$$\int_0^\infty e^{-\rho t}\frac{C(t)^{1-\theta}-1}{1-\theta}dt. \tag{13}$$

The resource constraint of the economy takes the form

$$\dot{K}(t) = Y(t) - C(t) - \delta K(t) - \psi\mu\int_{N-1}^{N} q(i,t)di,$$

where $Y(t)$ continues to be given by (1), and δ is the depreciation rate of capital. Recall also that $\psi\mu$, with $\mu \in [0,1]$, parametrizes the marginal cost of producing intermediates (we maintain an exogenous markup of $1-\mu \geq 0$ for intermediate goods, which plays no important role until the next section).

We characterize the equilibrium in terms of the normalized variables $y(t) \equiv Y(t)/\gamma(I(t))$, $k(t) \equiv K(t)/\gamma(I(t))$, and $c(t) \equiv C(t)/\gamma(I(t))$. We also define two relevant normalizations for wages: $w_I(t) \equiv W(t)/\gamma(I(t))$, which is the effective wage at the least complex task that has not yet been automated, and $w_N(t) \equiv W(t)/\gamma(N(t))$, which is the effective wage at the newest, most complex task. Finally, as in our static model, $R(t)$ denotes the rental rate, and the interest rate on savings is $r(t) = R(t) - \delta$.

At each point in time, technology and capital, $n(t)$ and $k(t)$, fully determine output and factor prices as in the static equilibrium. Specifically, the market clearing conditions for capital and labor, (7) and (8), and the ideal price index condition, (9), give the following equilibrium conditions:

$$y(t)(1-n(t))c^u(R(t))^{\zeta-\sigma}R(t)^{-\zeta} = k(t),$$

$$y(t)\int_0^{n(t)}\gamma(i)^{\zeta-1}c^u\left(\frac{w_I(t)}{\gamma(i)}\right)^{\zeta-\sigma}w_I(t)^{-\zeta}di = L^s\left(\frac{w_I(t)}{R(t)k(t)}\right),$$

$$(1-n(t))c^u(R(t))^{1-\sigma} + \int_0^{n(t)}c^u\left(\frac{w_I(t)}{\gamma(i)}\right)^{1-\sigma}di = 1$$

Proposition 1 coupled with Assumption $1'$ guarantees that the equilibrium rental rate can be uniquely written as $R(t) = R(n(t),k(t))$. We also denote the normalized output net of intermediate costs by $f^E(n(t),k(t))$ and the growth rate of $\gamma(I(t))$ by $g(t)$.

Using this notation, we can describe the dynamic equilibrium of our model as paths for $c(t)$ and $k(t)$ that satisfy the Euler equation,

$$\frac{\dot{c}(t)}{c(t)} = \frac{1}{\theta}(R^E(n(t),k(t)) - \delta - \rho) - g(t), \tag{14}$$

the resource constraint,

$$\dot{k}(t) = f^E(n(t),k(t)) - c(t) - (\delta + g(t))k(t), \tag{15}$$

and the household's transversality condition,

$$\lim_{t\to\infty}(k(t)+\Pi(t))e^{-\int_0^t(\rho-(1-\theta)g(s))ds} = 0, \tag{16}$$

where $\Pi(t)$ denotes the (normalized) net present value of corporate profits, which results from the presence of the monopoly markup. The dynamic equilibrium is defined for any exogenous path for technology, which is fully summarized by $\{n, g\}$.

Consider an exogenous path for technology in which $\dot{I} = \dot{N} = \Delta$ so that both automation and the creation of new complex tasks advance in tandem. Along such a path $n(t) \to n$ and $g(t) \to A\Delta$. Figure 4 presents the phase diagram for the system of differential equations (14) and (15) in this case, which fully determines the structure of the dynamic equilibrium. The structure of the above system resembles the standard neoclassical growth model, except that we use a different normalization and the exogenous markups on intermediate goods introduce a wedge between R and $\partial f^E / \partial k$.

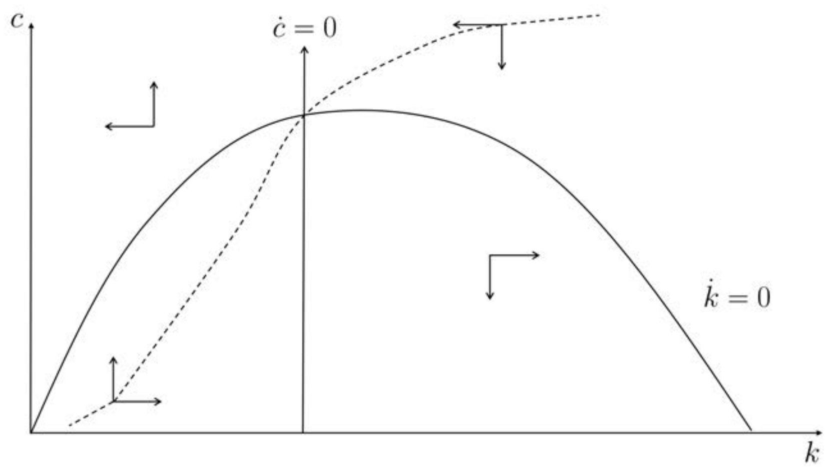

Figure 4: Balanced growth path and dynamic equilibrium when technology is exogenous and satisfies $n(t) \to n$ and $g(t) \to A\Delta$.

We define a BGP as an allocation in which $Y(t), C(t), K(t)$ and $W(t)$ grow at a constant rate and $R(t)$ is constant. The next proposition characterizes the conditions under which the asymptotic behavior of this economy converges to a BGP.

Proposition 3 (Dynamic equilibrium with exogenous technological change) *Suppose that Assumption 1′ holds. There exists a threshold $\overline{\rho}$ such that for $\rho > \overline{\rho}$, we have:*

1. *There exists $\overline{n}$ such that for $n(t) < \overline{n}$, we have $I^*(t) < I(t)$, while for $n(t) \geq \overline{n}$, $I^*(t) = I(t)$.*

2. *Suppose that there exists T such that for $t \geq T$, $n(t) \in [\overline{n}, 1]$. Then a BGP exists (and is unique) if and only if asymptotically $\dot{N} = \dot{I} = \Delta$, so that $\lim_{t\to\infty} n(t) = n$. In this BGP, $I^*(t) = I(t)$; $Y(t), C(t), K(t)$ and $W(t)$ grow at a constant rate $g = A\Delta$; and R is constant.*

3. *Suppose instead that there exists T such that for $t \geq T$, $n(t) < \overline{n}$. Then a BGP exists (and is unique) if and only if asymptotically $\dot{N} = \Delta$. In this BGP $I^*(t) < I(t)$ and $N(t) - I^*(t) \to \overline{n}$; $Y(t), C(t), K(t)$ and $W(t)$ grow at a constant rate $g = A\Delta$; and R is constant.*

4. *Moreover, given such a path of technological change (with $n(t) \in [\overline{n}, 1]$ or $n(t) \leq \overline{n}$ for all $t \geq T$), the dynamic equilibrium is unique. Starting from any level of capital the economy converges to the unique BGP.*

Proof. See Appendix A. ■

The key implication of Proposition 3 is that balanced growth results from the simultaneous process of automation and the creation of new complex tasks. But the proposition also highlights that this process needs to be "balanced" itself: the two types of technologies need to advance at the same rate so that $\lim_{t\to\infty} n(t) = n \geq \overline{n}$ (in the more interesting case where automated tasks are immediately produced with capital). Concerns about automation we cited in the Introduction notwithstanding, the proposition shows that a process of continuous automation is compatible with balanced growth.

The additional requirement in Proposition 3, that $\rho > \overline{\rho}$, ensures that the long-run equilibrium interest rate is not too close to 0. Provided that this is case, newly created complex tasks will be immediately utilized and produced with labor. If, instead, ρ were close to zero, capital would be so cheap that new complex tasks might remain unutilized (at least for a while) in a BGP. Thus, the condition $\rho > \overline{\rho}$ defines the relevant range of parameters for our investigation (and it is in this range that $\overline{n}$ is well-defined). Appendix A also characterizes the equilibrium when $\rho < \overline{\rho}$.[19]

Proposition 3 can be further illustrated and strengthened in the two special cases considered in the previous section; $\eta \to 0$ or $\zeta \to 1$. Suppose that, as required in part 2 of the proposition, we have $\dot{N} = \dot{I} = \Delta$. The aggregate production function in equation (10) can be simplified to $Y(t) = f(K(t), B(t)L)$, with

$$B(t) = \left(\int_{I(t)}^{N(t)} \gamma(i)^{\hat{\sigma}-1} di\right)^{\frac{1}{\hat{\sigma}-1}} = e^{AI(t)} \left(\frac{e^{A(\hat{\sigma}-1)n(t)} - 1}{A\hat{\sigma} - 1}\right)^{\frac{1}{\hat{\sigma}-1}},$$

so that $B(t)$ grows at a rate $A\Delta$. This example shows that the net effect of balanced automation and the creation of new technologies is to augment labor. Intuitively, technology becomes purely labor augmenting on net because labor and capital perform a fixed share of tasks, and the creation of new tasks directly increases the productivity of labor. This special case of our model also provides a direct connection between Proposition 3 and Uzawa's Theorem, which implies that balanced growth requires a representation of the production function with purely labor-augmenting technological change (e.g., Acemoglu, 2009, or Grossman, Helpman and Oberfield, 2016).

3.2 The Productivity Effect

We now study the dynamic implications of a permanent decline in $n(t)$, which in this dynamic setup corresponds to automation running ahead of the creation of new tasks. Because in the short run capital is fixed, the short-run implications of this change in technology are the same as in our static analysis in the previous section. However, over the long run, capital accumulation responds to the shift in technology in such a way as to keep the interest rate constant at its initial level, and in consequence, all the productivity gains from automation will accrue to the relatively inelastic factor, labor. Thus,

[19] The conditions $\rho > \overline{\rho}$ and $\lim_{t\to\infty} n(t) \geq \overline{n}$ are not restrictive. Focusing on a standard annual parametrization of our model with $\theta = 1$, $\delta = 0.06$, $g = 0.016$, $\sigma = 0.5$, $\zeta = 0.2$ (so that the elasticity of substitution between capital and labor lies between 0.5 and 0.2), $A = 2$, $\eta = 0.5$ and $\psi = 0.9$, we obtain $\overline{\rho} = 0.012$, so that the standard value of the discount rate, $\rho = 0.05$, is comfortably above this threshold. These parameters also imply $\overline{n} = 0.56$.

the dynamic economy underscores the role of another economic force, which we call the *productivity effect*: automation, by enabling the substitution of the cheaper capital for labor, increases productivity and thus the demand for labor and wages.[20] The productivity effect has been present in our analysis so far (in particular, it is because of this effect that automation need not reduce real wages in the short run), but as we show next, due to the induced capital accumulation, it becomes more powerful in the long run.

The next proposition characterizes the long-run impact of automation on factor prices and shares.

Proposition 4 (Long-run comparative statics) *Suppose that technology evolves exogenously and satisfies* $\lim_{t\to\infty} n(t) = n$*, Assumption 1′ holds, and* $\rho > \overline{\rho}$ *(where* $\overline{\rho}$ *was introduced in Proposition 3). Also, let* $\overline{n}$ *be the threshold introduced in Proposition 3. Then we have:*

1. *For* $n < \overline{n}$*, small changes in* n *do not affect the asymptotic behavior of the economy.*

2. *For* $n > \overline{n}$*, we have that:*

   - *The long-run rental rate* $R = \lim_{t\to\infty} R(t)$ *is equal to* $\rho + \delta + \theta g$*, and is thus independent of the extent of automation given by* n*. The capital stock adjusts in the long run so that* $R^E(n, k) = \rho + \delta + \theta g$*.*
   - *Long-run effective wages,* $w_I(n) = \lim_{t\to\infty} w_I$ *and* $w_N(n) = \lim_{t\to\infty} w_N$*, depend only on* n*. Moreover,* $w_I(n)$ *is increasing and* $w_N(n)$ *is decreasing in* n*.*
   - *The long-run labor share,* ω*, and the employment rate are increasing in* n*.*

Proof. See Appendix A. ■

This proposition illustrates the role of the productivity effect, and why this effect becomes more important in the long run. In the BGP, capital adjusts to keep the rental rate fixed at $R = \rho + \delta + \theta g$, and thus independent of the extent of automation summarized by n. In consequence, although in the short run the productivity gains from both technologies accrue to both factors, in the long run they only accrue to the inelastic factor—labor.

More specifically, the long-run behavior of wages is fully determined by the ideal price index condition, (9), which we can rewrite as

$$1 = (1-n)c^u(\rho+\delta+\theta g) + \int_0^n c^u \left(\frac{w_I}{\gamma(i)}\right)^{1-\sigma} di = (1-n)c^u(\rho+\delta+\theta g) + \int_0^n c^u \left(w_N\gamma(i)\right)^{1-\sigma} di. \quad (17)$$

Taking derivatives, we obtain

$$\begin{aligned} w_I'(n) =& A w_I(n) \frac{c^u(\rho+\delta+\theta g)^{1-\sigma} - c^u(w_N(n))^{1-\sigma}}{c^u(w_I(n))^{1-\sigma} - c^u(w_N(n))^{1-\sigma}} > 0, \text{ and} \\ w_N'(n) =& A w_N(n) \frac{c^u(\rho+\delta+\theta g)^{1-\sigma} - c^u(w_I(n))^{1-\sigma}}{c^u(w_I(n))^{1-\sigma} - c^u(w_N(n))^{1-\sigma}} < 0. \end{aligned} \quad (18)$$

[20] This is similar to the productivity or efficiency effect in models of offshoring such as Grossman and Rossi-Hansberg (2008), Rodriguez-Clare (2010) and Acemoglu, Gancia and Zilibotti (2015), which results from the substitution of cheaper foreign labor for domestic labor in certain tasks.

The signs of $w_I'(n)$ and $w_N'(n)$ can be derived from the following observations: since we are in the region in which $\rho > \overline{\rho}$ and $n > \overline{n}$, we have $w_I(n) > \rho + \delta + \theta g > w_N(n)$ (which is equivalent to both types of technologies being immediately utilized). These inequalities then imply that $w_I'(n) > 0$ and $w_N'(n) < 0$.

The behavior of the effective wages also implies that, following a permanent increase in $I(t)$ (to $I(t) + \nu$ with $\nu > 0$ for all $t \geq T$), $W(t)$ will eventually rise above its initial trajectory. Likewise, the creation of new complex tasks increases wages in the sense that a permanent increase in $N(t)$ (to $N(t) + \nu$ with $\nu > 0$ for all $t \geq T$) will increase $W(t)$ above its initial path.[21]

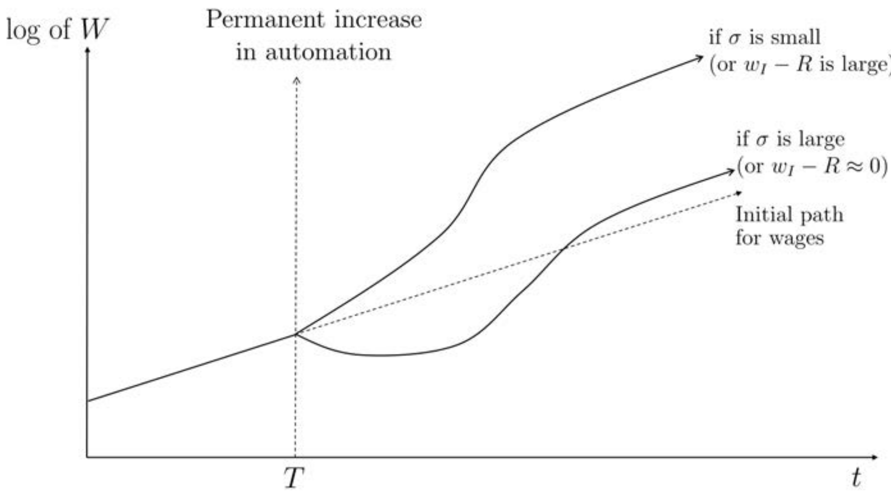

Figure 5: Dynamic behavior of wages following a permanent increase in automation.

In summary, although the results from the static model continue to apply in the short run when capital does not adjust, because of the productivity effect, the potential negative impact of automation on the equilibrium wage level disappears in the long run. This is illustrated in Figure 5, which plots the dynamic behavior of the equilibrium wage, W, following a permanent decline in n. Even though the equilibrium wage may fall in the short run following a surge in automation (if σ is sufficiently large, as established in Proposition 2), it necessarily increases in the long run because of the induced capital accumulation and the productivity effect.[22] The duration of the period with stagnant or depressed wages depends on θ, which determines the speed of capital adjustment.[23]

[21] The first claim follows because $W(t) \to w_N(n)\gamma(N(t))$, and due to the productivity effect, automation raises $w_N(n)$. The second claim follows because $W(t) \to w_I(n)\gamma(I(t))$, and again due to the productivity effect, the creation of new complex tasks raises $w_I(n)$.

[22] The expression for $w_I(n)$ also shows that when $w_I(n) \approx R$—which corresponds the case where productivity gains from automation are modest—an increase in automation always reduces wages in the short run and leaves them approximately unchanged in the long run. This is because, in this case, productivity gains from automation are limited, and thus the long-run benefit of the inelastic factor, labor, is also limited.

[23] This comparative static serves as an alternative explanation for what historian Robert C. Allen termed the "Engel's pause;" the period covering the first half of the 19th century in Britain (see Allen, 2009). During this period, real wages stagnated while output per worker and capital profits increased. Wages started increasing in the second half of the 19th century. These patterns are consistent with the dynamic predictions of our model following a wave of automation.

Importantly, Proposition 4 also establishes that, while the wage rate increases in the long run with automation (a decrease in n), the long-run labor share and employment decrease with automation. In fact, Appendix B shows that, following a wave of automation, if $\sigma_{SR} < 1$, capital accumulation mitigates the short-run response of the labor share but does not fully offset it; while, if $\sigma_{SR} > 1$, capital accumulation further depresses the labor share. In light of these results, the recent decline in the labor share and the employment to population ratio can be interpreted as a consequence of automation outpacing the creation of new labor-intensive tasks over the last two decades. This phenomenon could be accompanied by stagnant or lower wages in the short-run while capital adjusts to its new BGP level.

4 Full Model: Tasks and Endogenous Technologies

The previous section established the existence of a BGP under the assumption that $\dot{N} = \dot{I} = \Delta$. But why should these two types of technologies advance at the same rate? This is the question at the center of our paper, and to answer it, we now develop our full model, which endogenizes the pace at which automation and the creation of new complex tasks proceeds.

4.1 Endogenous and Directed Technological Change

We endogenize technological change by allowing new intermediates, which embody the technologies that automate existing tasks or create new complex tasks, to be introduced by technology monopolists. We assume that successful innovations always achieve automation or the creation of new tasks in the order of the intermediate indices, $i \in [0, \infty)$. As a consequence, automation and the creation of new complex tasks correspond, respectively, to an increase in I and to an increase in N. We continue to assume that all intermediates, including those that have just been invented, can be produced at the fixed marginal cost of $\mu\psi$, and that the fringe of competitive firms forces the technology monopolists to price intermediates at ψ, which implies an exogenous per-unit profit of $(1 - \mu)\psi$.

In this section, we adopt a structure of intellectual property rights whereby automating or replacing an existing task is viewed as an infringement of the patent of the technology previously used to produce that task. Consequently, a firm automating a task must license or buy the relevant patent from the technology monopolist that supplied the intermediate good used in combination with labor to produce this task. Similarly, a firm introducing a new complex task, which is creating a new more complex version of an existing task, will have to obtain the relevant patent from the previous technology monopolist of that task. We finally assume that both purchases take place with the new inventors making a take-it-or-leave-it offer to the holder of the existing patent.

This game form ensures that each technology monopolist will receive the same flow of revenues regardless of whether its own intermediate is replaced or not—either as profits when he is operating or as payments for its patent when he is replaced. Moreover, because new entrants must compensate the incumbent technology monopolists that they replace, our patent structure removes the creative destruction of profits, which is present in other models of quality improvements under the alternative assumption that new firms do not have to respect the intellectual property rights of the technology on which they are building (e.g., Aghion and Howitt, 1992; Grossman and Helpman, 1991). In Section 6, we explore how our main results change when the intellectual property rights regime allows for the creative destruction of profits.

We assume that innovation of both types requires scientists.[24] There is a fixed supply of S scientists, which will be allocated to automation ($S_I(t) \geq 0$) or the creation of new tasks ($S_N(t) \geq 0$),

$$S_I(t) + S_N(t) \leq S.$$

When a scientist is employed in automation, she automates κ_I tasks per unit of time, and receives a wage W_I^S. When she is employed in the creation of new tasks, she creates κ_N new tasks per unit of time, and receives a wage W_N^S. However, there is also a comparative advantage structure to the allocation of scientists in that the cost of effort for scientists depends on their exact skills and the type of technology on which they are working. More specifically, we assume that when working in automation, scientist i incurs a cost of $\chi_I^i Y(t)/\lambda$, and when working in the creation of new tasks, she incurs a cost of $\chi_N^i Y(t)/\lambda$. These costs are multiplied with aggregate output in the economy to ensure balanced growth,[25] while $\lambda > 0$ parameterizes the importance of wage income for scientists relative to the effort cost. We assume that the distribution of $\chi_N^i - \chi_I^i$ among scientists is given by a continuous and strictly increasing distribution function G over a support containing 0 as an interior point. For notational convenience, we also adopt the normalization $G(0) = \frac{\kappa_N}{\kappa_I + \kappa_N}$. This formulation ensures that the allocation of scientists responds to the relative profitability of innovation in the two sectors in a "smooth" fashion, avoiding discontinuous shifts which otherwise complicate the analysis of dynamics. As $\lambda \to \infty$, we recover these discontinuous shifts, while in the case where $\lambda = 0$, a constant fraction$\frac{\kappa_N}{\kappa_I + \kappa_N}$ of scientists will work on automation regardless of the profitabilities of the two types of innovation (thus making technological change "undirected").

The productivity of scientists into two types of innovations, together with their comparative advantages, determine their wages in these two sectors, W_I^S and W_N^S. Since comparative advantage only affects the costs of effort and all scientists have the same productivity in the two sectors, the innovation possibility frontier of the economy can be summarized as

$$\dot{I}(t) = \kappa_I S_I(t), \text{ and } \dot{N}(t) = \kappa_N S_N(t). \tag{19}$$

4.2 Equilibrium with Endogenous Technological Change

To characterize the equilibrium with endogenous technological change, we need to compute the value functions that determine the net present discounted value accruing to monopolists from automation

[24]Focusing on an innovation possibilities frontier using just scientists, rather than variable factors such as in the lab-equipment specifications (see Acemoglu 2009), is convenient because it enables us to focus on the direction of technological change—and not on the overall amount of technological change—especially when we turn to the welfare analysis in the next section.

[25]Alternatively, these costs could be in terms of the scientists' wage with very similar implications.

Throughout we also assume that the costs of effort are sufficiently low that all scientists will work either in automation or the creation of new tasks.

and the creation of new complex tasks. We denote these value functions by $V_I(t)$ and $V_N(t)$, respectively. More specifically, $V_I(t)$ is the value of automating the task at $i = I(t)^+$ (i.e., the highest-indexed task that has not yet been automated, or more formally $i = I(t)+\varepsilon$ for ε arbitrarily small and positive). Likewise, $V_N(t)$ is the value of a new technology creating a more complex task at $i = N(t)^+$.

An equilibrium with endogenous technology is given by paths $\{K(t), N(t), I(t)\}$ for capital and technology (starting from initial values $K(0), N(0), I(0)$) paths $\{R(t)\ , W(t), W_I^S(t), W_N^S(t)\}$ for factor prices, paths $\{V_N(t), V_I(t)\}$ for the value functions of technology monopolists, and paths $\{S_N(t), S_I(t)\}$ for the allocation of scientists such that all markets clear, all firms, including prospective technology monopolists, maximize profits, the representative household maximizes its utility, and $N(t)$ and $I(t)$ evolve endogenously according to equation (19).

We start by characterizing the value functions for technology monopolists. Suppose that in this equilibrium $n(t) > \bar{n}$, so that $I^*(t) = I(t)$ and new automated tasks start being produced with capital immediately. The flow profits accruing to a technology monopolists selling an intermediate good $q(i)$ for an automated task i are

$$\pi_I(t,i) = (1-\mu)\left(\frac{\eta}{1-\eta}\right)^{\zeta} \psi^{1-\zeta} Y(t) c^u (R(t))^{\zeta-\sigma}. \tag{20}$$

Intuitively, these profits come from the ability of firms to produce task i using capital, which is embodied in the intermediate good provided by the technology monopolist. Similarly, the flow profits accruing to a technology monopolist that sells an intermediate good $q(i)$ for a non-automated technology (for labor with productivity productivity $\gamma(i)$ in the corresponding task) are

$$\pi_N(t,i) = (1-\mu)\left(\frac{\eta}{1-\eta}\right)^{\zeta} \psi^{1-\zeta} Y(t) c^u \left(\frac{W(t)}{\gamma(i)}\right)^{\zeta-\sigma}. \tag{21}$$

It is then straightforward to compute the offer that a monopolist with a new technology (embodied in its intermediate good) automating task I^+ at time t needs to make to the firm currently holding the patent for the (labor-intensive) technology of that intermediate. This offer will be given by the net present discounted value of the profit streams, discounted using the path of future interest rates, that the existing patent-holder would obtain

$$(1-\mu)\left(\frac{\eta}{1-\eta}\right)^{\zeta} \psi^{1-\zeta} \int_t^{\infty} e^{-\int_0^{\tau}(R(s)-\delta)ds} Y(\tau) c^u \left(\frac{W(\tau)}{\gamma(I)}\right)^{\zeta-\sigma} d\tau.$$

Similarly, the offer that a monopolist with a new technology (embodied in its intermediate good) allowing the creation of a new complex task N^+ at time t needs to make to the firm currently holding the patent for the old technology for task $N-1$ (which is necessarily being produced with capital) is

$$(1-\mu)\left(\frac{\eta}{1-\eta}\right)^{\zeta} \psi^{1-\zeta} \int_t^{\infty} e^{-\int_0^{\tau}(R(s)-\delta)ds} Y(\tau) c^u \left(R(\tau)\right)^{\zeta-\sigma} d\tau.$$

Since these are take-it-or-leave-it offers, the best response of the patent-holders is to accept them.[26]

[26] This expression is written by assuming that the patent-holder will also turn down subsequent less generous offers in the future. Writing it using dynamic programming and the one-step ahead deviation principle leads to the same conclusion.

Thus, we can compute the values of innovating and becoming a technology monopolist as:

$$V_I(t) = MY(t)\int_t^\infty e^{-\int_t^\tau (R(s)-\delta-g(s))ds}\left(c^u\left(R(\tau)\right)^{\zeta-\sigma} - c^u\left(w_I(\tau)e^{\int_t^\tau g(s)ds}\right)^{\zeta-\sigma}\right)d\tau, \tag{22}$$

and

$$V_N(t) = MY(t)\int_t^\infty e^{-\int_t^\tau (R(s)-\delta-g(s))ds}\left(c^u\left(w_N(\tau)\frac{\gamma(n(\tau))}{\gamma(n(t))}e^{\int_t^\tau g(s)ds}\right)^{\zeta-\sigma} - c^u\left(R(\tau)\right)^{\zeta-\sigma}\right)d\tau, \tag{23}$$

with $M = (1-\mu)\left(\frac{\eta}{1-\eta}\right)^{\zeta}\psi^{1-\zeta}$. In what follows, define the normalized value functions $v_I \equiv V_I(t)/Y(t)$ and $v_N \equiv V_N(t)/Y(t)$, which will be convenient to work with, especially since as $t \to \infty$ they will only depend on n.[27]

The expressions for the value functions, $V_I(t)$ and $V_N(t)$, share a common form: they subtract the lower cost of producing a task with the factor for which the new technology is designed from the higher cost of producing the same task with the older technology. Because our structure of intellectual property rights removes the creative destruction effects, the profits from introducing new intermediates always depend on the alternative cost of producing a task with the older technology.

Competition among prospective technology monopolists to hire scientists implies that, when employed in automating existing tasks, a scientist earns a wage of $W_I^S(t) = \kappa_I V_I(t)$. Likewise, when employed in creating new complex tasks, a scientist earns a wage of $W_N^S(t) = \kappa_N V_N(t)$. These combined with the costs of effort specified above imply that the numbers of scientists working in automation and the creation of new tasks are

$$S_I(t) = SG\left(\frac{\kappa_I v_I - \kappa_N v_N}{\lambda}\right) \in [0, S], \qquad S_N(t) = S\left[1 - G\left(\frac{\kappa_I v_I - \kappa_N v_N}{\lambda}\right)\right] \in [0, S]. \tag{24}$$

Intuitively, whenever one of the two types of innovation is more profitable, more scientists will be allocated to that activity. Our formulation further implies that the allocation of scientists to the two different types of innovation is independent of the level of aggregate output, and guarantees that $\dot{n}(t) > 0$ if and only if $\kappa_N v_N > \kappa_I v_I$, and $\dot{n}(t) < 0$ if and only if $\kappa_N v_N < \kappa_I v_I$.

Using the same normalizations as in the previous section, we can represent the equilibrium with endogenous technology by a time path of the tuple $\{c, k, n, S_I, v_I, v_N\}$ such that:

- Consumption satisfies the Euler equation (14) coupled with the transversality condition in equation (16) (where in addition we can note that the net present value of corporate profits in equation (16) is simply $\Pi(t) = Iv_I + Nv_N$).

- Capital satisfies the resource constraint in equation (15).

- The gap between automation and the creation of new tasks, $n(t) = N(t) - I(t)$, satisfies:

$$\dot{n}(t) = \kappa_N S - (\kappa_I + \kappa_N)G\left(\frac{\kappa_I v_I - \kappa_N v_N}{\lambda}\right)S.$$

  This implies that $\dot{n} = 0$ if and only if $\kappa_I v_I = \kappa_N v_N$.

[27] To avoid confusion, and with a slight abuse of notation, we always write v_I and v_N as functions of n (and never explicitly as functions of time).

- The allocation of scientists satisfies the allocation rule in equation (24).
- The growth rate of $\gamma(I(t))$ and of aggregate variables is $g(t) = A\kappa_I S_I(t)$.
- The value functions that determine the allocation of scientists, $v_I(t), v_N(t)$, are given by (22) and (23).

A BGP is defined as in Proposition 3, as an allocation in which the normalized variables $c(t), k(t), n(t)$ and the rental rate $R(t)$ are constant—except that now n will be determined endogenously. The next proposition gives another one of the main results of the paper. It establishes conditions for the existence, uniqueness and asymptotic stability of a BGP in which there are both types of technological changes.

Proposition 5 (Equilibrium with endogenous technological change) *Suppose that $\sigma > \zeta$ and Assumption 1′ holds. Then there exist $\overline{\rho}$ and $\overline{A}$ such that for $\rho > \overline{\rho}$ and $A < \overline{A}$ (where $\overline{\rho}$ is as defined in Proposition 3), the following are true:*

1. *There exists $\overline{\kappa}$ such that for $\frac{\kappa_I}{\kappa_N} > \overline{\kappa}$, there is a BGP, where $\dot{N} = \dot{I} = \frac{\kappa_I \kappa_N}{\kappa_I + \kappa_N} S$, and Y, C, K and W grow at the constant rate $g = A\frac{\kappa_I \kappa_N}{\kappa_I + \kappa_N} S$, and the rental rate, R, the labor share and employment are constant. Along this path, we have $N(t) - I(t) = n^D$, with n^D determined endogenously from the condition $\kappa_N v_N = \kappa_I v_I$, and satisfying $n^D \in (\bar{n}, 1)$, where $\bar{n}$ is as defined in Proposition 3. In addition, there exists $\overline{\overline{\rho}} \geq \overline{\rho}$ such that if $\rho > \overline{\overline{\rho}}$, the BGP is unique.*

2. *Suppose that $\frac{\kappa_I}{\kappa_N} > \overline{\kappa}$ and $\rho > \overline{\overline{\rho}}$ so that the BGP is unique. When $\theta = 0$, the dynamic equilibrium is globally (saddle-path) stable. Moreover, there exists $\overline{\overline{A}} \leq \overline{A}$ such that when $A < \overline{\overline{A}}$, for any value of θ, the dynamic equilibrium is unique in the neighborhood of the BGP and is asymptotically (saddle-path) stable.*

3. *If, on the other hand, $\frac{\kappa_I}{\kappa_N} < \overline{\kappa}$, we have $\kappa_N v_N > \kappa_I v_I$ and there exists a globally stable BGP where all scientists are allocated to create new labor-intensive tasks. Therefore, asymptotically $n \to 1$, and the economy converges to a BGP in which all tasks are produced by labor and long-run growth is driven by the creation of new tasks.*

4. *For $\rho < \overline{\rho}$, there exists an asymptotically stable BGP where $\kappa_N v_N < \kappa_I v_I$, all tasks are produced with capital, and long-run growth is propelled by capital accumulation.*

Proof. See Appendix A. ■

The first important result contained in this proposition (part 1) is the existence of a BGP and its uniqueness (when $\rho > \overline{\overline{\rho}}$). The second critical result, established in part 2 of the proposition, is that this BGP is asymptotically stable and also globally stable when $\theta = 0$ (so that preferences have an infinite elasticity of intertemporal substitution). This result implies that there are powerful market forces pushing the economy towards the BGP.

These results are established under several conditions. First, we require $\sigma > \zeta$. This condition ensures that innovations are directed towards technologies that allow firms to produce tasks by using the cheaper (or more productive) factors. The profitability of introducing an intermediate that embodies a new technology will depend on the demand for the intermediate good. As the factor that needs to be combined with the intermediate to produce a given task (labor or capital) becomes cheaper, two effects come into play. First, the decline in costs allows firms to scale up their production, which increases the demand for the intermediate good. The extent of this positive scale effect is regulated by the elasticity of substitution σ: the greater is σ, the more powerful is this effect directing innovation towards technologies that work with cheaper factors. However, there is a countervailing force as well: as a factor becomes cheaper, it is substituted for the intermediate it is combined with, reducing the demand for the intermediate good that embodies the new technology. This countervailing substitution effect is regulated by the elasticity of substitution ζ: the greater is ζ, the more powerful is this effect discouraging innovations towards technologies that work with cheaper factors. The condition $\sigma > \zeta$ guarantees that the former, positive effect dominates, so that prospective technology monopolists have an incentive to introduce technologies that allow firms to produce tasks more cheaply. When the opposite holds and $\zeta > \sigma$, we could have a situation in which technologies that work with more costly factors are more profitable. Pathologically, in this case the net present discounted values from innovation would be negative.[28] The condition $\sigma > \zeta$ is not only theoretically necessary in our model but is also empirically plausible. Because intermediates embody the technology that allows firms to produce with certain factors, we expect the elasticity of substitution between factors and intermediates, ζ, to be very low, so that they are highly complementary. The condition $\sigma > \zeta$ thus ensures that, quite intuitively, intermediates are gross complements to factors they work with.

Second, we require that $A < \overline{A}$ to guarantee that the growth rate of the economy is not too high. If the growth rate is above the threshold implied by $\overline{A}$, the creation of new tasks is discouraged (even if current wages are low) because firms anticipate that wages will grow very rapidly, which will reduce the future profitability of these labor-intensive tasks. This requirement is strengthened to $A < \overline{\overline{A}}$ in the second part of the proposition when we consider local stability, which allows us to use Taylor approximations of the value functions.[29]

Third, as in Proposition 3, parts 1-3 of the proposition require $\rho > \overline{\rho}$. As discussed in that context, this assumption ensures that the interest rate is not too close to 0, which in turn guarantees that newly created complex tasks will immediately start being used with labor. The proposition further shows that when $\rho > \overline{\rho}$, and the other conditions in the proposition are satisfied, the long-run equilibrium endogenously involves $n > \bar{n}$. In this region there are productivity gains from automating tasks, and therefore a demand for automation. Instead, when $n < \bar{n}$, prospective monopolists have no incentives

[28] Take, for example, (22). Becaus a task performed by labor is being automated, $c^u\left(R(\tau)\right) < c^u\left(\frac{w(\tau)}{\gamma(I(t))}\right)$, and hence if we had $\zeta > \sigma$, the profit stream would be negative. Thus, in order to ensure positive incentives for innovation, the condition $\sigma > \zeta$ is imposed even for the existence result in part 1 of the proposition.

[29] The restriction that $A < \overline{A}$ also ensures that the net present discounted value of the representative household is finite and thus the transversality condition is satisfied. Moreover, this restriction also guarantees that the condition from Assumption 1′, $e^{-(2+2\sigma+\zeta)A} > |\sigma - \zeta|$, holds.

to automate tasks.

The economic forces ensuring the stability of the BGP in parts 1 and 2 of the proposition are intuitive. In a BGP, the normalized value of different types of innovations converges to $v_I(n) = \lim_{t\to\infty} \frac{V_I(t)}{Y(t)}$ and $v_N(n) = \lim_{t\to\infty} \frac{V_N(t)}{Y(t)}$. These limiting functions, $v_I(n)$ and $v_N(n)$, are fully determined by the technology gap n, as can be can be seen from equations (22) and (23). Moreover, in a BGP we must have $\kappa_I v_I(n^D) = \kappa_N v_N(n^D)$ so that $\dot{n} = 0$, which implies that both technologies should advance in parallel. When $\kappa_I v_I(n) > \kappa_N v_N(n)$, we instead have $\dot{n} < 0$, and when $\kappa_I v_I(n) < \kappa_N v_N(n)$, we have $\dot{n} > 0$. Neither of these possibilities is consistent with balanced growth.

Figure 6 draws the net present discounted value (normalized by output) of allocating scientists to creating new complex tasks or to automation when $\sigma > \zeta$, $\rho > \overline{\rho}$ and $A < \overline{A}$. The figure shows that, in the region where $n > \bar{n}$, as more tasks are automated (as n decreases), the value of additional automation, $v_I(n)$, decreases. This is the key economic force that generates stability in our model: greater automation reduces the effective wage in the next task to be automated, $w_I(n)$, relative to the interest rate (which is fixed and independent of technology). Consequently, the (normalized) long-run value of automating additional tasks declines with automation.

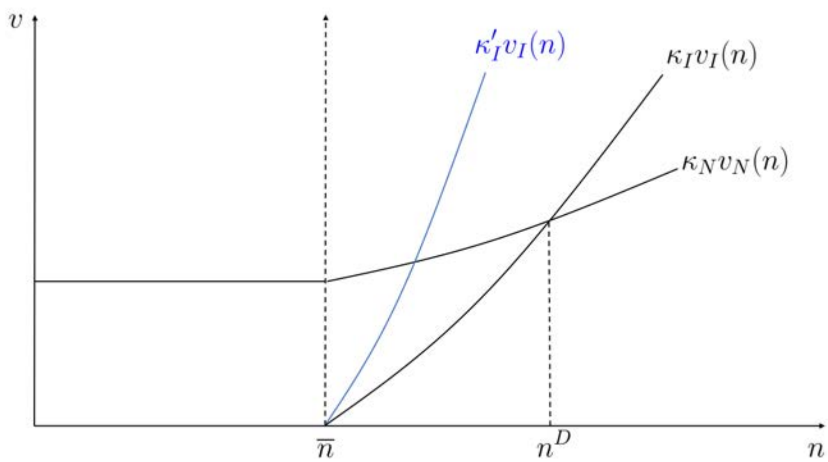

Figure 6: Determination of n^D in steady state.

Nevertheless, this property is not sufficient for the stability or uniqueness of a BGP. Figure 6 also shows that, asymptotically, an increase in automation (a decrease in n) reduces the value of creating new complex tasks, $v_N(n)$: because of the productivity effect, the effective wage paid in the most complex tasks, $w_N(n)$, also increases with automation. As a result, the (normalized) value of creating new complex tasks declines with automation, which generates a force towards instability and multiplicity.[30] The condition $\rho > \overline{\overline{\rho}}$ guarantees that the productivity effect of automation on the wage $w_N(n)$ is small, or equivalently, that the induced capital accumulation does not reverse the direction in

[30]The intuition for the productivity effect can be alternatively viewed as follows: automation induces the accumulation of capital, which by raising wages in the long run makes the creation of new labor-intensive tasks less profitable. Thus, the induced capital accumulation crowds out some of the self-correcting forces that stem from the response of technology to changes in factor prices. The condition $\rho > \overline{\overline{\rho}}$ ensures that this indirect capital accumulation effect does not dominate

which subsequent technological improvements respond to a wave of automation. In contrast, when this condition fails and $\rho \leq \overline{\overline{\rho}}$, we could have situations in which, asymptotically, an increase in automation raises the wage $w_N(n)$ so much that it discourages the introduction of new more complex tasks, paving the way to multiple steady states in our baseline model.

Observe also that the conditions we have discussed so far are not sufficient to guarantee that the curves for $\kappa_I v_I(n)$ and $\kappa_N v(n)$ intersect. The former always starts below the latter as shown in Figure 6, but may always remain below it. The last condition in Proposition 5, that κ_I/κ_N is sufficiently large, ensures that such an intersection takes place and thus there exists a unique "interior" BGP. This discussion then immediately leads to part 3 of the proposition, which establishes that when this condition does not hold, the long-run equilibrium will be one in which only new tasks are developed, and there is no automation. BGP is now achieved by continuous creation of more productive tasks (replacing older tasks), and hence, our economy in this case looks very similar to a standard "Schumpeterian" economy with growth propelled by quality improvements (e.g., Aghion and Howitt, 1992; Grossman and Helpman, 1991).

Finally, part 4 of the proposition shows that when $\rho < \overline{\rho}$, we have a balanced growth path in which all (except possibly a measure zero subset modem) of tasks are produced with capital, and the economy grows by accumulating capital.[31] This equilibrium can therefore be likened to Leontief's "horse equilibrium," because it makes labor redundant. Notably, however, such an equilibrium is possible only when the discount rate, and thus the long-run interest rate, is very low (recall from footnote 19 that under standard values of the other parameters, $\rho < \overline{\rho}$ would require the annual discount rate to be less than 0.012).

In summary, the critical economic force highlighted by Proposition 5 is that, differently from models with factor-augmenting technologies, it is factor prices—not primarily the market sizes—that guide the direction of technological change.[32] Consequently, there are stronger incentives to undertake the type of innovation that will work with the factor that has a lower effective cost.

Proposition 5 shows that shifts in technology, for example in the form of a series of new automation technologies (corresponding to an unanticipated decline in n), will set in motion self-correcting forces. Following such a change, there will be an adjustment process bringing the level of employment and the labor share back to their initial BGP values. This does not, however, imply that *all* shocks will leave the long-run prospects of labor unchanged. The next corollary shows that if there is a change in the innovation possibilities frontier that makes automation easier than before, the economy will move to a new BGP with lower employment and a lower share of labor in national income.

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the direct, stabilizing effect described in the previous paragraph.

[31] The qualifier "except possibly a measure zero subset" is introduced, since when $L(0) > 0$, there will still be positive labor supply even with very low wages, but asymptotically all of this labor will be employed in the highest-indexed task; along the BGP, output of this task will grow by allocating additional capital there as well.

Note also that this BGP is not necessarily globally stable, because the two curves in the right panel of Figure A2 in Appendix A may cross, in which case the interior BGP will be unstable, and there will be another locally stable BGP in which the economy behaves as in a Schumpeterian growth model as described in part 3.

[32] We should also note that this does not overturn the "weak bias" results in Acemoglu (2007), since these were derived in a setting that is general enough to nest the current environment.

Corollary 2 *Suppose that all the conditions in Proposition 5 are satisfied and there is a one-time permanent increase in κ_I/κ_N. Then the economy converges to a new BGP with lower n^D, lower employment and a lower share of labor in national income.*

This corollary follows by noting that an increase in κ_I/κ_N shifts the intersection of the curves $\kappa_I v_I(n)$ and $\kappa_N v(n)$ in Figure 6 to the left, leading to a lower value of n^D in the BGP. This will trigger an adjustment process in which the labor share and employment decline over time, but ultimately settle to their new steady-state values.

Together this corollary and Proposition 5 delineate the types of changes in technology that will trigger self-correcting dynamics: those driven by faster than usual arrival of automation technologies will do so, while those which alter the ability of the society to create new automation technologies will not (and thus will result in worse prospects for labor in the future).

5 Welfare

In this section we turn to an analysis of the efficiency of the equilibrium described in Proposition 5. Our main finding is that the presence of rents for workers, as captured by our quasi-labor supply, biases the composition of equilibrium technology towards too much automation and the creation of too few new complex tasks (and this is in addition to other distortions that exist in models of endogenous technology). Two complementary results illustrate this inefficiency. First, in Appendix B we characterize the constrained efficient allocation chosen by a social planner who is subject to the same innovation possibilities frontier, the same quasi-labor supply schedule and the constraint that wages equal the marginal product of labor. We then show how this constrained efficient allocation can be decentralized by taxes and subsidies. In addition to the usual wedges (taxes/subsidies) between the social planner's allocation and the decentralized equilibrium, workers' rents create an additional reason to tax automation relative to the creation of new complex tasks. Second, we show how welfare in the decentralized equilibrium can be improved by altering the composition of R&D in the direction of creating more new complex tasks and automating fewer of the existing ones.

Let $F^P(I, N, K, L)$ denote the aggregate output net of the costs of producing intermediates when the level of employment is L, the capital stock is K, the state of technologies is represented by N and I, and intermediates are priced at their marginal cost (which is the relevant net aggregate output expression for the social planner, since she would always price all intermediates at marginal cost). Because the level of employment is given by the quasi-labor supply $L = L^s(\omega)$, we have that in the constrained efficient allocation the resulting marginal products of labor and capital are fully determined by technology and capital, and we can write the resulting value of ω as $\omega^P(I, N, K)$.

The constrained efficient allocation maximizes the representative household's utility given by (13) subject to the endogenous evolution of the state variables:

$$\dot{K}(t) = F^P(N(t), I(t), K(t), L(t)) - C(t) - \delta K(t), \dot{N}(t) = \kappa_N S_N(t), \qquad \dot{I}(t) = \kappa_I S_I(t),$$

and because the planner faces the same quasi-labor supply schedule and labor demand relations, we also have:

$$L(t) \leq L^s(\omega^P(I, N, K)).$$

From the fact that the social planner maximizes (13) we can see that there is no utility loss or opportunity cost of supplying labor; the higher wages needed to employ additional workers are "quasi rents" (see Appendix B). Notice also that we have focused on the case in which the planner chooses $I^*(t) = I(t)$ (in the terminology of our previous section), so that all automated tasks are produced immediately with capital. As in the previous section, this will be the case when $\rho > \overline{\rho}$, since in this case the planner will always choose to operate in the region where $n > \overline{n}$.

We show in Appendix B that the solution to this maximization problem gives the constrained efficient allocation and can also be used to characterize the taxes and subsidies that need to be imposed on the decentralized equilibrium to achieve this constrained efficient allocation. In addition to the usual taxes and subsidies to internalize monopoly markups and technological externalities, the social planner will need to impose a tax on automation and a subsidy on the creation of new tasks in order to combat the tendency of the decentralized equilibrium to automate excessively. Intuitively, while the planner recognizes that automating tasks reduces employment (or creating new complex tasks increases it) and this has a first-order effect on workers (recall that there is no utility loss or opportunity cost of supplying labor), innovators do not internalize this externality.

Here, we demonstrate the presence of excessive automation in the decentralized equilibrium more directly. We show that starting from a decentralized allocation, the social planner can improve the allocation of resources by discouraging automation. The next proposition establishes this result when $\zeta \geq 1$ (which includes the tractable special case of our model in which $\zeta \to 1$). We also assume that intermediates are subsidized at the rate $1 - \mu$, which removes the main effect of monopoly markups, or equivalently that $\mu \to 1$.

Proposition 6 (Excessive automation) *Suppose that $\rho > \overline{\rho}$ as in Proposition 5, Assumption 1′ holds, and $\zeta \geq 1$. Moreover, suppose that intermediate goods are subsidized and can be purchased at their marginal cost (or equivalently $\mu \to 1$). Consider the decentralized equilibrium path described in Proposition 5, and which converges to a BGP with $N(t) - I(t) \to n^D$). Then there exists a feasible allocation with $n^P(t) \geq n(t)$ and $\lim_{t\to\infty} n^P(t) > n^D$ that achieves strictly greater welfare than the decentralized equilibrium.*

Proof. See Appendix B. ■

This proposition establishes that departing from the equilibrium in the direction of discouraging automation and encouraging the creation of new complex tasks improves welfare. Intuitively, because of the gap between the equilibrium wage and the opportunity cost of labor, redirecting research towards the creation of new complex tasks instead of automation has a positive first-order effect on workers, while it only has a second-order impact on the profits of prospective technology monopolists.

6 Extensions

In this section, we discuss two extensions. First we introduce heterogeneous skills, which allow us to analyze the impact of technological changes on inequality. Second, we study a different structure of intellectual property rights that introduces the creative destruction of profits.

6.1 Automation, New Tasks and Inequality

In this subsection, we introduce heterogeneous skills and study how automation and the creation of new tasks impact inequality. This extension is motivated by the observation that, because new tasks are more complex, their creation may favor high-skill workers. The natural assumption that high-skill workers have a comparative advantage in new complex tasks receives support from the data. For instance, the left panel of Figure 7 shows that in each decade since 1980, occupations with more new job titles had higher skill requirements in terms of the average years of schooling among employees at the start of each decade (relative to the rest of the economy). However, the right panel of the same figure also shows a pattern of "mean reversion" whereby average years of schooling in these occupations decline in each subsequent decade, most likely, reflecting the fact that new job titles became more open to lower-skilled workers over time.

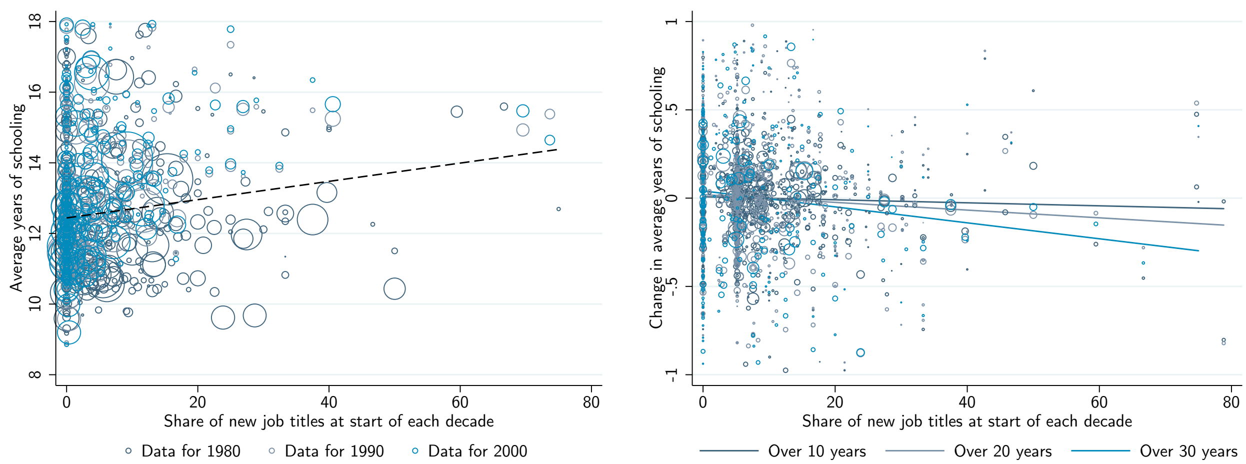

Figure 7: *Left panel:* Average years of schooling among employees against the share of new job titles at the beginning of each decade for 330 occupations. *Right panel:* Change in average years of schooling over the next 10 years (dark blue), next 20 years (blue) and next 30 years (in light blue) against the share of new job titles at the beginning of each decade. See Appendix B for data sources and detailed definitions.

We incorporate these features into our model by assuming that there are two types of workers: low-skill and high-skill. We introduce a pattern of comparative advantage that reflects our interpretation of the patterns in the data: the productivity of high-skill labor is the same as before,

$$\gamma_H(i) = e^{A_H i},$$

while the productivity of low-skill labor improves over time as new tasks become more "standardized".[33] In particular, for tasks $i \leq N(t)$, the productivity of low-skill workers is

$$\gamma_L(i,t) = \gamma_H(i) \cdot \Gamma(N(t) - i),$$

where $\Gamma(\cdot)$ is increasing, bounded above by 1, and satisfies $\Gamma(0) < 1$.

This structure implies that the productivity of low-skill labor in a new task (which means in task i at time t such that $N(t) = i$) starts at $\gamma_L(i,t) = \gamma_H(i)\Gamma(0) < \gamma_H(0)$. This productivity then increases as time passes. Since $\gamma_L(i,t)/\gamma_H(i) \equiv \Gamma(N(t) - i)$ is decreasing in i, high-skill labor has a comparative advantage at high-index tasks. This structure of comparative advantage ensures that there exists a threshold task M such that high-skill labor performs tasks in $[M, N]$, low-skill labor performs tasks in (I, M), and capital performs tasks in $[N - 1, M]$.

We denote the wages of high and low-skill labor by W_H and W_L, respectively. As before, we assume that there is a quasi-labor supply of high-skill labor given by $H^s(\frac{W_H}{RK})$, and a quasi-labor supply of low-skill labor given by $L^s(\frac{W_L}{RK})$, both of which have a constant and equal elasticity $\nu_L \geq 0$.

Proposition 7 (Automation, new tasks and inequality) *Suppose technology evolves exogenously and either one of $\sigma - \zeta$, $\zeta - 1$, or η is sufficiently close to 0.*

1. *Suppose that $\dot{N} = \dot{I} = \Delta$ and $I^*(t) = I(t)$ (and $A_H(1-\theta)\Delta < \rho$ so that net present discounted value of household income is finite). Then, the economy admits a unique BGP. In this BGP W_H and W_L grow at the same rate as the economy and the wage gap, w_H/w_L, remains constant. Moreover, both low-skill and high-skill workers perform a constant share of the tasks.*

2. *Given such a path of technological change, the dynamic equilibrium is unique starting from any initial condition and converges to the BGP.*

3. *The immediate effect of increases in both I and N is to raise the wage gap W_H/W_L. But the medium-run impact of an increase in N is to reduce inequality.*

Proof. See Appendix B. ■

A number of features are worth noting. First, this extended model generates not only an endogenous distribution of income between capital and labor, but also inequality between high-skill and low-skill workers. Here, inequality reflects the assumed structure of comparative advantage for workers of different skill levels in different tasks. The short-run comparative statics in the proposition imply that automation, by squeezing out tasks previously performed by low-skill labor, increases inequality between the two types of skills. Interestingly, because it is high-skill labor that has a comparative advantage in high-index tasks, the creation of new complex tasks also tends to increase inequality at

[33] This formulation captures the feature that new technologies and tasks are standardized over time (e.g., Acemoglu, Gancia and Zilibotti, 2010) or that low-skill workers require more time to adapt to new technologies (e.g., Schultz, 1965, Nelson and Phelps, 1966, Greenwood and Yorukoglu, 1997, Caselli, 1999, Galor and Moav, 2000, Beaudry, Green and Sand, 2013, and Goldin and Katz, 2008).

first. However, because tasks become standardized over time, which raises the productivity of low-skill workers, the medium-term implications of automation and the creation of new tasks are very different. The former increases inequality both in the short and the medium run. In contrast, the creation of new tasks increases inequality in the short run, but not in the medium run. In fact, low-skill workers gain relative to capital in the medium run from the creation of new tasks. Interestingly, inequality may be particularly high following a period of adjustment in which the labor share first declines—due to increases in automation—and then recovers—due to the introduction of new complex tasks. Inequality may remain high for a while, and only start declining after recently-introduced new tasks become sufficiently standardized.

6.2 Creative Destruction of Profits

In this subsection, we modify our baseline assumption on intellectual property rights, reverting to the classical setup in the literature in which new technologies do not infringe the patents of the products that they replace (Aghion and Howitt, 1992, and Grossman and Helpman, 1991). This assumption introduces creative destruction effects—the destruction of profits of previous inventors by new innovators. We will see that this alternative structure has little effect on the nature of a BGP in our model, but we require more demanding conditions to guarantee its uniqueness and stability.

Let us first define $V_N(t,i)$ and $V_I(t,i)$ as the values at time t of having introduced different technologies for the production of task i (respectively, new complex tasks and automation). The value functions satisfy the following Bellman equations:

$$r(t)V_N(t,i) - \dot{V}_N(t,i) = \pi_N(t,i) \qquad\qquad r(t)V_I(t,i) - \dot{V}_I(t,i) = \pi_I(t,i).$$

Here $\pi_I(t,i)$ and $\pi_N(t,i)$ denote the flow profits from automating and creating new complex tasks, respectively, which are given by the formulas in equations (20) and (21).

For a firm creating a new complex task i, let $T^N(i)$ denote the time at which it will be replaced by a technology allowing the automation of this task. Likewise, for a firm automating task i at time t, let $T^I(i)$ denote the time at which it will be replaced by a more complex technology using labor. Since firms anticipate these deterministic replacement dates, their value functions also satisfy the boundary conditions $V_N(T^N(i),i) = 0$ and $V_I(T^I(i),i) = 0$.

Using the Bellman equations together with the boundary conditions derived above, we obtain:

$$V_N(t) = V_N(N(t),t) = (1-\mu)\left(\frac{\eta}{1-\eta}\right)^{\zeta}\psi^{1-\zeta}\int_t^{T^N(N(t))} e^{-\int_t^{\tau}(R(s)-\delta)ds}Y(\tau)c^u\left(\frac{W(\tau)}{\gamma(N(t))}\right)^{\zeta-\sigma}d\tau,$$

$$V_I(t) = V_I(I(t),t) = (1-\mu)\left(\frac{\eta}{1-\eta}\right)^{\zeta}\psi^{1-\zeta}\int_t^{T^I(I(t))} e^{-\int_t^{\tau}(R(s)-\delta)ds}Y(\tau)c^u\left(\min\left\{R(\tau),\frac{w(\tau)}{\gamma(I(t))}\right\}\right)^{\zeta-\sigma}d\tau.$$

In addition, for reasons that will become readily clear, we modify the evolution of the technology frontier and assume that advances in automation take the form

$$\dot{I}(t) = \kappa_I\phi(n(t))S_I(t), \text{ and } \dot{N}(t) = \kappa_N S_N(t) \tag{25}$$

Here, the function $\phi(n(t))$ is included and assumed to be weakly increasing to capture the possibility that automating tasks closer to the frontier (defined as the highest available task) may be more difficult.

The characterization of the BGP is similar to that in Proposition 5 with (25) replacing (19). But there may exist multiple BGPs and additional assumptions on the function $\phi(n)$ need to be imposed to guarantee stability.

Proposition 8 (Equilibrium with creative destruction) *Suppose that $\sigma > \zeta$, Assumption 1′ holds, $\rho > \overline{\rho}$, $A < \overline{A}$ (where $\overline{\rho}$ and $\overline{A}$ are defined as in Proposition 5), and there is creative destruction of profits. Then:*

1. *There exist $\overline{\phi}$ and $\underline{\phi} < \overline{\phi}$ such that if $\phi(0) < \underline{\phi}$ and $\phi(1) > \overline{\phi}$, then there exists at least one (interior) stable BGP in which there is research in both automation and the creation of new complex tasks. In this BGP, we have $N(t) - I(t) = n^D$, $\kappa_N v_N(n^D) = \kappa_I \phi(n^D) v_I(n^D)$ and $\dot{N} = \dot{I} = \frac{\kappa_I \kappa_N \phi(n^D)}{\kappa_I \phi(n^D) + \kappa_N} S$. Also, Y, C, K and w grow at the constant rate based on $g = A \frac{\kappa_I \kappa_N \phi(n^D)}{\kappa_I \phi(n^D) + \kappa_N} S$, R is constant, and the labor share and employment are constant.*

2. *If $\phi(n)$ is constant, there is no asymptotically stable BGP with $n^D \in (0, 1)$. Any asymptotically stable equilibrium involves $n(t) \to 0$ or $n(t) \to 1$.*

Proof. See Appendix B. ■

The first part of the proposition follows from an analogous argument to that in the proof of Proposition 5, with the only difference being that, because of the presence of the function $\phi(n)$ in equation (25), the key condition determining a BGP becomes $\kappa_I \phi(n) v_I(n) = \kappa_N v_N(n)$. In addition, in a BGP, newly created tasks are automated after a period of length $T^N(N(t)) - t = \frac{n^D}{\Delta}$, and newly automated tasks are replaced by new complex ones after a period of length $T^I(I(t)) - t = \frac{1 - n^D}{\Delta}$. Here, $\Delta = \frac{\kappa_I \kappa_N \phi(n^D)}{\kappa_I \phi(n^D) + \kappa_N} S$ is the endogenous rate at which N and I increase. Thus, both types of innovations are replaced after a fixed length of time, which ensures that the creative destruction of profits does not change the balance of the incentives for innovation.

The major difference with our previous analysis is that, in the presence of the creative destruction of profits, $v_I(n)$ decreases with n—that is, $v_I(n)$ now increases when more tasks are automated and n declines—generating a new force towards instability. The reason is that, when n declines, automated tasks are replaced by new complex ones less frequently, because newly automated tasks are replaced after $\frac{1-n}{\Delta}$ units of time. This then increases the net present discounted value of profits for automating tasks. This effect is not compensated by changes in the flow profits from automation: because of the response in capital accumulation, the long-run interest rate, and hence the flow profits from automation, remains unchanged. Likewise, $v_N(n)$ increases with n—that is, $v_N(n)$ decreases when more tasks are automated—also contributing to instability. Intuitively, when n decreases, newly created tasks are automated more often, because newly created tasks are automated after $\frac{n}{\Delta}$ units of time, and this reduces the net present discounted value of profits from new tasks. The productivity effect contributes another force towards instability: when n decreases (there is greater automation) the effective wage in the newest tasks, $w_N(n)$, increases, further reducing the value from new tasks.

These observations imply that, if $\phi(n)$ were constant, the intersection between the curves $\kappa_N v_N(n)$ and $\kappa_I \phi(n) v_I(n)$ would give an unstable BGP. This result is in stark contrast to the asymptotic stability of the BGP characterized in Proposition 5. Economically, this difference is a consequence of the fact that, in contrast to our baseline model (and the socially planned economy), the creative destruction of profits implies that the incentives for prospective monopolists to innovate depend on the total revenue that a technology generates and not on the *incremental value* created by their innovation (which is the difference between these revenues and the revenues that the replaced technology generated). In our baseline model and in the constrained efficient allocation, the fact that automation reduced the incremental value of automating additional tasks was the key force generating stability, but this force is absent when innovators destroy the profits of previous inventors.

As shown in Figure 8, with the creative destruction of profits, stability imposes some restrictions on the $\phi(n)$ function, and specifically, stability requires the conditions that $\phi(0) < \underline{\phi}$ and $\phi(1) > \overline{\phi}$. This ensures that the first intersection of these two curves takes place in the region in which $\kappa_I \phi(n) v_I(n)$ is steeper than $\kappa_N v_N(n)$, and their interception yields an asymptotically stable BGP. As the figure also shows, even in this case there might be additional BGPs, each having different stability properties. Notice also that, as shown in Figure 8, the two cars can intersect more than once. If so, the second intersection is unstable, and there may also exist a corner equilibrium in which, as in Proposition 5 part 3, there is a BGP in which all tasks are produced with labor.

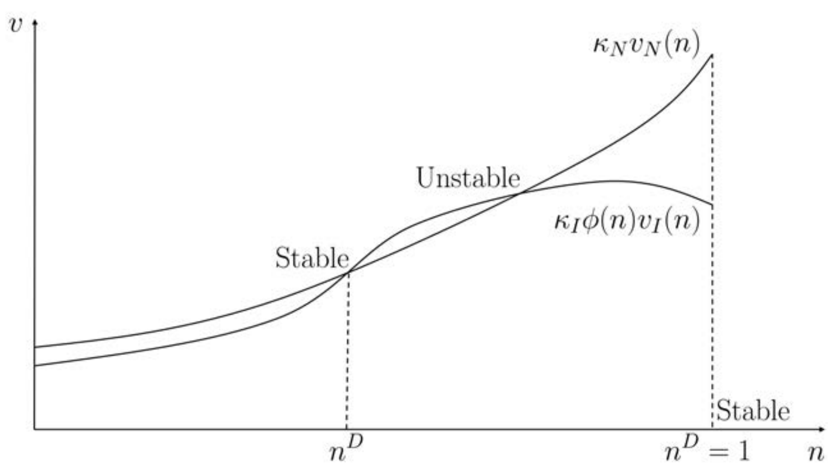

Figure 8: Determination of n^D when the structure of intellectual property rights features the creative destruction of rents. The model has an odd number of equilibria, which in the case depicted include a stable one at $n^D = 1$.

7 Conclusion

As more tasks performed by labor are being automated, concerns that these new technologies will make labor redundant have intensified. This paper developed a comprehensive framework in which these forces can be analyzed and contrasted with countervailing effects. At the center of our model is a task-based framework. Automation is modeled as the (endogenous) expansion of the set of tasks that can be performed by capital, thus replacing labor in tasks that it previously produced. The main new feature of our framework is that, in addition to automation, there is another type of technological change enabling the creation of new, more complex versions of existing tasks, and it is labor that tends to have a comparative advantage in these new tasks. We characterize the structure of equilibrium in such a model, showing how, given factor prices, the allocation of tasks between capital and labor is determined both by available technology and the endogenous choices of firms between producing with capital or labor.

One attractive feature of task-based models is that they highlight the link between factor prices and the range of tasks allocated to factors: when the equilibrium range of tasks allocated to capital increases (for example, as a result of automation), the wage relative to the rental rate and the share of labor in national income decline, and the equilibrium wage rate may also fall. Conversely, as the equilibrium range of tasks allocated to labor increases, the opposite result obtains. In our model, because the supply of labor is elastic, automation tends to reduce employment, while the creation of new tasks increases employment. These results highlight that, while both types of technological changes undergird economic growth, they have very different implications for the factor distribution of income and also for employment.

Our full model endogenizes the direction of research towards automation and the creation of new complex tasks, showing how this framework generates a BGP in which both types of innovations go hand-in-hand. Moreover, under reasonable assumptions, the dynamic equilibrium is unique and locally converges to the BGP. Underpinning this stability result is the impact of relative factor prices on the direction of technological change. The task-based framework— differently from the standard models of directed technological change which are based on factor-augmenting technologies—implies that as a factor becomes cheaper, this not only influences the range of tasks allocated to it, but also generates incentives for prospective technology monopolists to introduce technologies that allow firms to utilize this factor more intensively. These economic incentives then imply that by reducing the effective cost of labor in the least complex tasks, automation discourages further automation and generates a powerful self-correcting force towards stability.

Though market forces ensure the stability of the BGP, they do not necessarily generate the efficient composition of technology. If the elastic labor supply relationship results from rents (so that there is a wedge between the wage and the opportunity cost of labor), then there is an important and new distortion in the direction of technological change. Because firms value reducing the rents earned by workers, there is a natural bias towards excessive automation. On the other hand, because the planner recognizes that these rents are just transfers, she has weaker incentives to automate and replace labor with capital in additional tasks.

In addition to claims about automation leading to the demise of labor, several commentators are concerned about the inequality implications of automation and related new technologies. In one of our extensions, we have studied this question by introducing a distinction between low-skill and high-skill labor, with the latter having a comparative advantage in producing the new complex tasks. In this extension, both automation (which squeezes out tasks previously performed by low-skill labor) and the creation of new tasks (which directly benefits high-skill labor) will increase inequality. Nevertheless, the medium-term implications of the creation of new tasks could be very different, because new tasks are later standardized and used by low-skill labor. As a result of this effect, there exists a unique BGP in which not only the factor distribution of income (between capital and labor) but also inequality between the two skill types is endogenous but constant.

We consider our paper to be a first step towards a systematic investigation of different types of technological changes that impact capital and labor differentially. Several areas of research appear fruitful based on this first step. First, we introduced labor market distortions in this model in the form of a reduced-form quasi-labor supply curve. Going beyond this, an important set of issues center around how the process of automation and replacement of workers by capital interplays with the costly and potentially slow reallocation of workers across tasks and firms. We take some steps in this direction in our companion paper, Acemoglu and Restrepo (2016). Second, our model implies that it is always to tasks at the bottom that are automated; in reality, it may be those in the middle (e.g., Acemoglu and Autor, 2001). Ensuring a pattern of productivity growth consistent with balanced growth is more challenging in this case, though incorporating the possibility of such "middling tasks" being automated is an important generalization. Third, there may be major differences in the ability of technology to automate and also to create new tasks across industries (e.g., Polanyi, 1966, Autor, Levy and Murnane, 2003). An interesting step is to construct realistic models in which the sectoral composition of tasks performed by capital and labor as well as technology evolves endogenously and is subject to industry-level technological constraints (e.g., on the feasibility or speed of automation). Finally, and perhaps most importantly, our model highlights the need for additional empirical evidence on how automation takes place and how the incentives for automation and the creation of new tasks respond to policies and changes in the environment. One interesting direction would be to construct measures of automation and the creation of new tasks, potentially at the industry level, and then explore the impact on technology choices and innovation of industry-level variation in wages and institutional restrictions on capital-labor substitution.

Appendix A: Proofs

Proofs from Section 2

Proof of Proposition 1: We proceed in three steps. First, we show that I^*, N and K, determine unique equilibrium values for R, W and Y, thus allowing us to define the function $\omega(I^*, N, K)$ representing the relative demand for labor, which was introduced in the text. Second, we provide a lemma which ensures that $\omega(I^*, N, K)$ is decreasing in I^* (and increasing in N). Third, we show that $\min\{I, \widetilde{I}\}$ is nondecreasing in ω and conclude that there is a unique pair $\{\omega^*, I^*\}$ such that $I^* = \min\{I, \widetilde{I}\}$ and $\omega^* = \omega(I^*, N, K)$. This pair uniquely determines the equilibrium relative factor prices and range of tasks that get effectively automated.

Step 1: Consider I^*, N and K such that $I^* \in (N-1, N)$. Then, R, W and Y satisfy the system of equations given by capital and labor market clearing, equations (7) and (8), and the ideal price index, equation (9).

Taking the ratio of (7) and (8), we obtain

$$\frac{\int_{I^*}^{N} \gamma(i)^{\zeta-1} c^u\left(\frac{W}{\gamma(i)}\right)^{\zeta-\sigma} W^{-\zeta} di}{L^s\left(\frac{W}{RK}\right)(I^*-N+1)c^u(R)^{\zeta-\sigma}R^{-\zeta}} = \frac{1}{K} \tag{A1}$$

In view of the fact that L^s is nondecreasing and the function $c^u(x)^{\zeta-\sigma}x^{-\zeta}$ is decreasing everywhere in x (as it can be verified directly by differentiation), it follows that the left-hand side is decreasing in W and increasing in R . Therefore, (A1) defines an upward-sloping relationship between W and R, which we refer to as the relative demand curve.

On the other hand, inspection of equation (9) readily shows that this equation gives a downward-sloping locus between R and W as shown in Figure A1, which we refer to as the ideal price curve.

For a given I^*, N and K, the intersection point between the relative demand and the ideal price curves determines the equilibrium factor prices (if it exists and is unique).

Because the relative demand curve is upward sloping and the ideal price index curve is downward sloping, there can be at most one intersection. To prove that there always exists an intersection, observe that $\lim_{x\to 0} c^u(x)^{\zeta-\sigma}x^{-\zeta} = \infty$, and that $\lim_{x\to\infty} c^u(x)^{\zeta-\sigma}x^{-\zeta} = 0$. These observations imply that as $W \to 0$, the numerator of (A1) limits to infinity, and hence, so must the denominator, i.e., $R \to 0$. This proves that the relative demand curve starts from the origin. Similarly, as $W \to \infty$, the numerator of (A1) limits to zero, and so must the denominator (i.e., $R \to \infty$). This then implies that the relative demand curve goes to infinity as $R \to \infty$. Thus, the upward-sloping relative demand curve necessarily starts below and ends above the ideal price curve, which ensures that there always exists an intersection between these curves. The unique intersection defines the equilibrium values of W and R, and therefore the function $\omega(I^*, N, K) = \frac{W}{RK}$.

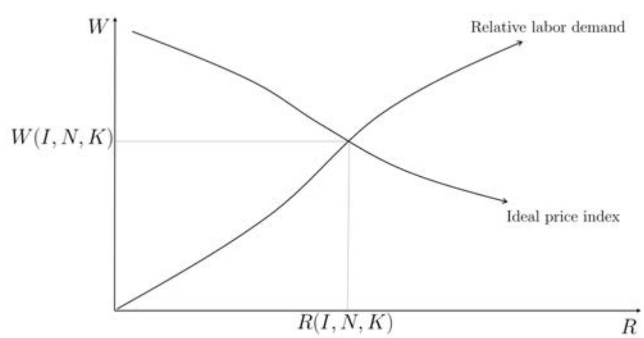

Figure A1: Construction of function $\omega(I^*, N, K)$.

Step 2: Step 2 follows directly from the following lemma, which we prove in Appendix B.

Lemma A1 *Suppose Assumption 1 holds. Then $\omega(I^*, N, K)$ is decreasing in I^* and is increasing in N.*

Step 3: We now we establish that $I^* = \min\{I, \widetilde{I}\}$ is uniquely defined. Since $\gamma(\widetilde{I}) = \omega K$, $\widetilde{I}$ is increasing in ω, and thus $I^* = \min\{I, \widetilde{I}\}$ is nondecreasing in ω. Consider the pair of equations $\omega = \omega(I^*, N, K)$ and $I^* = \min\{I, \widetilde{I}\}$ plotted in Figure 3. Because $\omega = \omega(I^*, N, K)$ is decreasing in I^* and $I^* = \min\{I, \widetilde{I}\}$ is increasing in ω, there exists at most a single pair (ω, I^*) satisfying these two equations (or a single intersection in the figure).

To prove existence, we again verify the appropriate boundary conditions. Suppose that $I^* \to N-1$. Then from (7), $R \to 0$, while $W > 0$, and thus $\omega \to \infty$. This ensures that the curve $\omega(I^*, N, K)$ starts above $I^* = \min\{I, \widetilde{I}\}$ in Figure 3. Since $I^* = \min\{I, \widetilde{I}\}$, it is bounded above by I, and cannot be below $\omega = \omega(I^*, N, K)$ at $I^* = I$, ensuring that there must exist a unique intersection between the two curves over the interval $I^* \in (N-1, I]$, which completes the proof of Proposition 1.■

Proof of Proposition 2: We first establish the comparative statics of ω with respect to I, N and K when both $I^* = I < \widetilde{I}$ and $I^* = \widetilde{I} < I$, and then turn to their effects on the level of factor prices.

Comparative statics with respect to I: The relative demand locus $\omega = \omega(I^*, N, K)$ does not directly depend on I. Thus, the comparative statics are entirely determined by the effect of changes in I on the $I^* = \min\{I, \widetilde{I}\}$ schedule in Figure 3. When $I^* = \widetilde{I} < I$, small changes in I have no effect as claimed in the proposition. Suppose next that $I^* = I < \widetilde{I}$. In this case, an increase in I shifts the curve $I^* = \min\{I, \widetilde{I}\}$ to the right in Figure 3. From Lemma A1, we have that $\omega(I^*, N, K)$ is decreasing in I^*. This shift the shift in I increases I^* and reduces ω—as stated in the proposition. Moreover, because $I^* = I$, we have

$$\frac{d\ln(W/R)}{dI} = \frac{d\ln\omega}{dI^*} = \frac{1}{\omega}\frac{\partial\omega}{\partial I^*} < 0,$$

where $\frac{\partial\omega}{\partial I^*}$ denotes the partial derivative of $\omega(I^*, N, K)$ with respect to I^*.

Comparative statics for N: From Lemma A1, changes in N only shift the relative demand curve up in Figure 3. Hence, when $I^* = I < \widetilde{I}$, we have

$$\frac{d\ln(W/R)}{dN} = \frac{d\ln\omega}{dN} = \frac{1}{\omega}\frac{\partial\omega}{\partial N} > 0,$$

where $\frac{\partial\omega}{\partial N}$ denotes the partial derivative of $\omega(I^*, N, K)$ with respect to N.

Turning next to the case where $I^* = \widetilde{I} < I$, note that the threshold task is given by $\gamma(I^*) = \omega K$. Therefore, $dI^* = \frac{1}{\varepsilon_\gamma} d\ln\omega$ (where recall that ε_γ is the semi-elasticity of the γ function as defined in the proposition). Therefore, $\frac{d\ln(W/R)}{dN} = \frac{d\ln\omega}{dN}$, and we can compute this total derivative as claimed in proposition:

$$\frac{d\ln\omega}{dN} = \frac{1}{\omega}\frac{\partial\omega}{\partial N} + \frac{1}{\omega}\frac{\partial\omega}{\partial I^*}\frac{1}{\varepsilon_\gamma}\frac{d\ln\omega}{dN} = \frac{\frac{1}{\omega}\frac{\partial\omega}{\partial N}}{1 - \frac{1}{\omega}\frac{\partial\omega}{\partial I^*}\frac{1}{\varepsilon_\gamma}}.$$

Comparative statics for K: The curve $I^* = \min\{I, \widetilde{I}\}$ does not depend on K, all comparative statics are entirely determined by the effect of capital on $\omega(I^*, N, K)$. An increase in K shifts up the relative demand locus in Figure A1 (this does not affect the ideal price index condition, which simplifies the analysis in this case), and thus increases W and reduces R . The impact on $\omega = \frac{W}{RK}$ depends on whether the initial effect on W/R has elasticity greater than one (since K is in the denominator).

Notice that the function $\omega(I^*, N, K)$ *already* incorporates the equilibrium labor supply response. To distinguish this supply response from the elasticity of substitution determined by factor demands, we define $\omega^L(I^*, N, K, L)$ as the static equilibrium for a fixed level of the labor supply L.

Once again using the notation $\frac{\partial \omega^L}{\partial K}\frac{K}{\omega^L} = \frac{1}{\sigma_{SR}} - 1$, and $-\frac{\partial \omega^L}{\partial L}\frac{L}{\omega^L} = \frac{1}{\sigma_{SR}}$, with σ_{SR} denoting the short-run elasticity of substitution between labor and capital, in the case where $I^* = I < \widetilde{I}$, we have

$$d\ln\omega = \left(\frac{1}{\sigma_{SR}} - 1\right) d\ln K - \frac{1}{\sigma_{SR}}\varepsilon_L d\ln\omega = \frac{1-\sigma_{SR}}{\sigma_{SR}+\varepsilon_L} d\ln K,$$

where we have used the fact that $\omega(I^*, N, K) = \omega^L(I^*, N, K, L^s(\omega))$. This establishes the claims about the comparative statics with respect to K when $I^* = I < \widetilde{I}$.

For the case where $I^* = \widetilde{I} < I$, we have

$$d\ln(W/R) = \frac{1-\sigma_{SR}}{\sigma_{SR}+\varepsilon_L} d\ln K + \frac{1}{\omega}\frac{\partial \omega}{\partial I^*}\frac{1}{\varepsilon_\gamma} d\ln(W/R) = \frac{1-\sigma_{SR}}{\sigma_{SR}+\varepsilon_L}\frac{1}{1-\frac{1}{\omega}\frac{\partial\omega}{\partial I}\frac{1}{\varepsilon_\gamma}} d\ln K.$$

This expression implies the formula in the proposition.

Effects on factor price levels: Consider an increase in I in the case in which $I^* = I < \widetilde{I}$. In Figure A1, this increase in automation rotates the relative demand curve clockwise around the origin, but also shifts up the downward-sloping ideal price curve — which reflects the gains in productivity accruing to both factors.

Although in general the effects on the wage level are ambiguous, when σ is large, the ideal price curve shifts by little and the effect through the relative demand curve dominates. In the limit in which $\sigma \to \infty$, the ideal price curve does not shift, which implies that when automation increases the wage level declines. The proofs of the results for the effects of N on the rental rate are analogous. ■

Proofs from Section 3

As in the main text, define $w_N(n) \equiv \lim_{t\to\infty} W(t)/\gamma(N(t))$ and $w_I(n) \equiv \lim_{t\to\infty} W(t)/\gamma(I^*(t))$. Notice that now we explicitly take into account the possibility that $I^*(t)$ might be different from $I(t)$.

Because in a BGP $R = \rho + \delta + \theta g$, these effective wages will just be functions of $n = \lim_{t\to\infty} N(t) - I(t)$. The next lemma characterizes their behavior in the BGP.

Lemma A2 (Behavior of effective wages $w_N(n)$ and $w_I(n)$) *There exists $\overline{\rho} > 0$ such that:*

1. *For $\rho > \overline{\rho}$, there exists $\overline{n} \in (0, 1)$ such that:*
    - *for $n \geq \overline{n}$, we have $I^* = I$, and for $n < \overline{n}$, we have $I^* < I$;*
    - *for $n \geq \overline{n}$, $w_N(n)$ is increasing and $w_I(n)$ is decreasing in n. Both wages are constant for $n < \overline{n}$;*
2. *For $\rho \leq \overline{\rho}$, there exists a different threshold $\widetilde{n} \in [0, 1)$ such that:*
    - *for $n \geq \widetilde{n}$, both technologies are used, while for $n < \widetilde{n}$, firms do not create or use new tasks (because labor is not productive or cheap enough compared to capital);*

- *for $n \geq \widetilde{n}$, $w_N(n)$ is increasing and $w_I(n)$ decreasing in n. Both wages are decreasing in n for $n < \widetilde{n}$.*

Proof: See Appendix B.

Before proceeding, we will now state a result claimed in the text as a corollary of Lemma A2:

Corollary A1 *Suppose $\rho > \overline{\rho}$. Then in the BGP, all new tasks will be produced with labor immediately.*

This corollary follows immediately by noting that, in this case, $\rho + \delta + \theta g > w_N(n)$ for all n. Note however that this conclusion does not hold for $\rho < \overline{\rho}$. In this case, for $n \leq \widetilde{n}$, we have $w_N(n) > \rho+\delta+\theta g$, which implies that new tasks will not be immediately produced with labor, which is more costly. This corollary justifies our focus in the main text on the case where $\rho > \overline{\rho}$, and hence the fact that we did not introduce a separate notation to account for the possibility that, when created, new complex tasks might not be immediately produced using labor.

We now return to the rest of the proof of Proposition 3.

Proof of Proposition 3: We prove each part of the proposition separately.

Part 1: Since $\rho > \overline{\rho}$, this part follows directly from Lemma A2.

Part 2: We start by proving the "if" part. Suppose that $\dot{N} = \dot{I} = \Delta$ and that $\lim_{t\to\infty} n(t) = n \geq \overline{n}$. Then the normalized variables converge to values that solely depend on n. Given the functional form of γ in (12), it follows that Y, C, K, W grow at the same rate $g = A\Delta$, while the rental rate, R, remains constant. This shows that balanced growth emerges when both technologies advance at the same speed.

For the "only if" part, note that in any BGP, Y, C, K and W must grow at some constant rate g, and thus y, c, k and w_I must also grow at some constant rate $\widetilde{g}$ while R remains constant at $\rho+\delta+\theta g$. If $\widetilde{g} = 0$, we have that y, c, k and w_I must converge. Because the behavior of these normalized variables only depends on $n(t)$, we must also have that $\lim_{t\to\infty} n(t) = n \in [\overline{n}, 1]$, which then implies that $\dot{N} = \dot{I}$. Moreover, $\dot{N} = \dot{I} = \Delta$ because Y, C, K and W grow at a constant rate. These observations show that to establish the "only if" part it is enough to show that $\widetilde{g} = 0$.

Suppose to obtain a contradiction that $\widetilde{g} < 0$. Then, for t large enough, we will have $w_I(t) < R(t)$. This implies that, at this point, newly automated tasks are not immediately produced with capital, which contradicts Lemma A2 (recall that $\rho > \overline{\rho}$ and $n(t) \in [\overline{n}, 1]$).

Next, suppose once again, to obtain a contradiction, that $\widetilde{g} > 0$. This implies that for t large enough, we will have $w_I(t)/\gamma(1) > R(t)$, which implies that $w_N(t) > R(t)$. At this point in time, newly created tasks are not immediately produced with labor, which contradicts Lemma B1 (since we have $\rho > \overline{\rho}$). This establishes the "only if"direction of the proof.

Part 3: We start by proving the "if" part. Suppose that $n(t) < \overline{n}$ for all $t > T$, and that $\dot{N} = \Delta$. Lemma A2 implies that $I^*(t) = \widetilde{I}(t) < I(t)$, and therefore, $w_I(t) = R(t)$.

Let us also define $n^*(t) \equiv N(t) - I^*(t)$. Then, from the same steps that we used in the main text, it follows that y, c, k, and w_I converge to values that depend only on $n^*(t)$. Because asymptotically,

$R(t)$ is constant and $w_I(t) = R(t)$, we must have that $n^*(t)$ converges to $\overline{n}$. Moreover, because the normalized variables converge to values that only depend on n^*, the economy achieves a BGP in which Y, C, K and W grow at the rate $A\Delta$—the same rate as $\gamma(N(t))$ and $\gamma(I^*(t))$—while the rental rate, R, remains constant.

For the "only if" part, note that in any BGP with $n(t) < \overline{n}$, we have that $n^*(t) = \overline{n}$. This implies that w_I, w_N, y, c, k converge to constant values. Because Y, C, K and W grow at some constant rate g, we must have that $\gamma(I^*(t))$ and $\gamma(N(t))$ also grow at this constant rate, which implies that $\dot{N} = \Delta$ and $g = A\Delta$, as claimed.

Part 4: Starting with any initial value of $k(0)$ and $n(0)$, the equilibrium behavior is given by equations (14), (16) and (15). This is identical to the equations characterizing dynamics in the canonical neoclassical growth model (see, for example, Proposition 8.5 and 8.6 in Acemoglu (2009)). Moreover, the condition that $\rho > \overline{\rho}$ guarantees that $\rho > A(1-\theta)\Delta$, which ensures that the transversality condition holds, establishing part 4. ■

Proof of Proposition 4: That there are no effects when $n \leq \overline{n}$ follows from Lemma A2. Next consider the case $n > \overline{n}$, in which technology changes the asymptotic behavior of the economy. The expressions and comparative statics for the wages follow from equation (18), which we derived in the proof of Lemma A2.

The asymptotic behavior of the labor share can be established by considering two separate cases. First, suppose that $\sigma_{SR} \leq 1$. Let $k_I(n)$ denote the steady-state value for $K/\gamma(I)$. We have $\omega(0, n, k_I(n)) \equiv \frac{w_I(n)}{(\rho+\delta+\theta g)k_I(n)}$. Differentiating this expression, we obtain

$$k_I'(n) = \frac{w_I'(n)\frac{1}{Rk} - \frac{\partial \omega}{\partial N}}{\frac{\omega}{k}\frac{1+\varepsilon_L}{\sigma_{SR}+\varepsilon_L}}.$$

Using this expression, we compute the total effect of technology on ω as

$$\frac{d\omega}{dn} = \frac{\partial \omega}{\partial N}\left(\frac{\sigma_{SR}+\varepsilon_L}{1+\varepsilon_L}\right) + \frac{w_I'(n)}{Rk}\left(\frac{1-\sigma_{SR}}{1+\varepsilon_L}\right).$$

Because $\frac{\partial \omega}{\partial N} > 0$ and $w_I'(n) > 0$, we have that, whenever $\sigma_{SR} \leq 1$, ω increases with n.

Next suppose that $\sigma_{SR} > 1$. Let $k_N(n)$ denote the steady-state value for $K/\gamma(N)$. We have that $\omega(-n, 0, k_N(n)) \equiv \frac{w_N(n)}{(\rho+\delta+\theta g)k_N(n)}$. Differentiating this expression, we have

$$k_N'(n) = \frac{w_N'(n)\frac{1}{Rk} + \frac{\partial \omega}{\partial I^*}}{\frac{\omega}{k}\frac{1+\varepsilon_L}{\sigma_{SR}+\varepsilon_L}} < 0.$$

We can then compute the total effect of technology on ω as

$$\frac{d\omega}{dn} = -\frac{\partial \omega}{\partial I^*}\left(\frac{\sigma_{SR}+\varepsilon_L}{1+\varepsilon_L}\right) + \frac{w_N'(n)}{Rk}\left(\frac{1-\sigma_{SR}}{1+\varepsilon_L}\right).$$

Because $\frac{\partial \omega}{\partial I^*} < 0$ and $w_N'(n) < 0$, we have that, whenever $\sigma_{SR} \geq 1$, ω increases with n. Since $\omega \equiv \omega(-n, 0, k_N(n)) \equiv \omega(0, n, k_I(n))$, We again conclude that, in the long run, ω always increases with n. ■

Proofs from Section 4

Let $v_N(n) \equiv \lim_{t\to\infty} V_N(t)/Y(t)$ and $v_I(n) \equiv \lim_{t\to\infty} V_I(t)/Y(t)$ be the normalized value functions, which in the BGP only depend on n.

Lemma A3 (Asymptotic behavior of the normalized value functions) *Suppose that $\sigma > \zeta$, and that the conditions required in Lemma A2 hold. Let $\overline{\rho}$ be as defined in Proposition 3. Then there exist thresholds $\overline{A}$ and $\overline{\overline{\rho}}$ such that:*

1. *For $\rho > \overline{\rho}$ and $A < \overline{A}$:*
   - *if $n < \overline{n}$, we have $\kappa_N v_N(n) > \kappa_I v_I(n) = \mathcal{O}(g)$;*
   - *if $n \geq \overline{n}$, $v_N(n)$ and $v_I(n)$ are strictly increasing in n;*
   - *if in addition $\rho > \overline{\overline{\rho}}$, then for $n \geq \overline{n}$, $v_I(n)/v_N(n)$ is increasing in n.*
2. *For $\rho \leq \overline{\rho}$:*
   - *for $n < \widetilde{n}$, we have $v_I(n) > 0$ and $v_N(n) \leq 0$;*
   - *for $n \geq \widetilde{n}$, both $v_N(n)$ and $v_I(n)$ are positive and strictly increasing.*

Proof. Let $g \equiv A\frac{\kappa_I\kappa_N}{\kappa_I+\kappa_N}S$ be the growth rate of the economy in the BGP. Suppose $\rho > \overline{\rho}$. Then for $n \geq \overline{n}$, we can write the value functions in the BGP as:

$$v_N(n) = M\int_0^\infty e^{-(\rho-(1-\theta)g)\tau}\left[c^u\left(w_N(n)e^{g\tau}\right)^{\zeta-\sigma} - c^u\left(\rho+\delta+\theta g\right)^{\zeta-\sigma}\right]d\tau,$$
$$v_I(n) = M\int_0^\infty e^{-(\rho-(1-\theta)g)\tau}\left[c^u\left(\rho+\delta+\theta g\right)^{\zeta-\sigma} - c^u\left(w_I(n)e^{g\tau}\right)^{\zeta-\sigma}\right]d\tau,$$

where we have defined $M \equiv (1-\mu)\left(\frac{\eta}{1-\eta}\right)^\zeta \psi^{1-\zeta}$. Thus, the value functions only depend on the unit cost of labor $w_N(n)$ and $w_I(n)$, and on the rental rate, which is equal to $\rho+\delta+\theta g$ in the BGP.

Now consider Taylor expansions of both of these expressions (which are continuously differentiable) around $g = 0$:

$$v_N(n) = \frac{M}{\rho}\left[c^u\left(w_N(n)\right)^{\zeta-\sigma} - c^u\left(\rho+\delta\right)^{\zeta-\sigma}\right] + \mathcal{O}(g),$$
$$v_I(n) = \frac{M}{\rho}\left[c^u\left(\rho+\delta\right)^{\zeta-\sigma} - c^u\left(w_I(n)\right)^{\zeta-\sigma}\right] + \mathcal{O}(g), \tag{A2}$$

where $\mathcal{O}(g)$ denotes terms that vanish when the growth rate, g, is small, which enables us to approximate the integrals (solving them out explicitly when $g = 0$). Moreover, when $n < \overline{n}$, automated tasks are not immediately produced with capital. Instead, capital is used when wages have grown enough. Because future wage growth provides the only incentive for automation, we have $v_I(n) = \mathcal{O}(g)$. On the other hand, the expression for $v_N(n)$ still applies and remains bounded away from zero. Thus, there exists $\overline{A} > 0$ such that for $A < \overline{A}$ (which guarantees that g is small), $\kappa_N v_N(n) > \kappa_I v_I(n) > 0$ as claimed.

Differentiating the value functions in (A2) when $n \geq \overline{n}$ immediately establishes that they are both strictly increasing (since $w_I(n) > \rho + \delta + \theta g > w_N(n)$ from Lemma A2). The implied behavior of the normalized value functions is depicted in Figure A2.

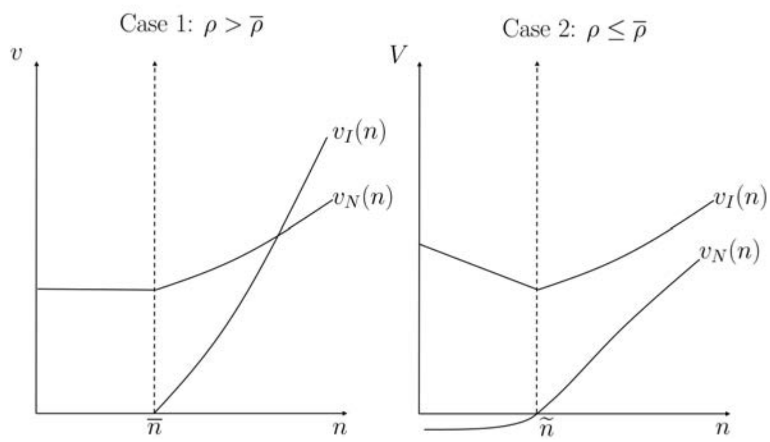

Figure A2: Behavior of value functions in steady state with respect to changes in $n = N - I$.

We now prove the existence of a threshold $\overline{\overline{\rho}}$, which guarantees that the curves $\kappa_N v_N(n)$ and $\kappa_I v_I(n)$ cross at most once. To prove this result, note that $\frac{v_I'(n)}{v_I(n)} > \frac{v_N'(n)}{v_N(n)}$ if and only if

$$c^u(w_I(n))^{\zeta-\sigma} s(I) \frac{c^u(\rho+\delta)^{1-\sigma} - c^u(w_N(n))^{1-\sigma}}{1-\sigma} \left(c^u(w_N(n))^{\zeta-\sigma} - c^u(\rho+\delta)^{\zeta-\sigma} \right) >$$
$$c^u(w_N(n))^{\zeta-\sigma} s(N) \frac{c^u(w_I(n))^{1-\sigma} - c^u(\rho+\delta)^{1-\sigma}}{1-\sigma} \left(c^u(\rho+\delta)^{\zeta-\sigma} - c^u(w_I(n))^{\zeta-\sigma} \right), \tag{A3}$$

where recall that $s(i)$ is the labor share in the production of task i.

For ρ sufficiently large, we have that $\rho + \delta \to w_I(n)^+$, and thus the inequality (A3) necessarily holds (notice that, while the right-hand side is strictly positive, the left-hand side always converges to zero). Thus, there exists a threshold $\overline{\overline{\rho}}$ such that for $\rho > \overline{\overline{\rho}}$ and $A < \overline{A}$, The single crossing condition (A3), which ensures that v_I/v_N is increasing in n, holds for all $n \geq \overline{n}$. (If the value of $\overline{\overline{\rho}}$ that ensures this property is strictly less than $\overline{\rho}$, then we simply set $\overline{\overline{\rho}} = \overline{\rho}$.)

The second part of the lemma has an analogous proof, with the behavior of normalized values given as in the right panel of Figure A2, and the details are omitted.. ■

Proof of Proposition 5:

Part 1: A BGP emerges if and only if $\dot{I} = \dot{N}$, and $n(t) = n^D$. To ensure that asymptotically $\dot{n} = 0$, in a BGP we must have

$$\kappa_I v_I(n) = \kappa_N v_N(n).$$

Thus, a BGP exists if and only if there exists a solution n^D to this equation.

Given $\rho > \overline{\rho}$ and $\overline{A} > A$, Lemma A3 implies that $\kappa_N v_N(\overline{n}) > \kappa_I v_I(\overline{n})$. This shows that at $\overline{n}$, the curve $\kappa_N v_N(\overline{n})$ is above $\kappa_I v_I(\overline{n})$. However, as the ratio $\frac{\kappa_I}{\kappa_N}$ increases—starting from zero—the curves $\kappa_I v_I(n)$ and $\kappa_N v_N(n)$ eventually cross at some point. This proves that for $\kappa > \overline{\kappa}$, there exists a BGP represented by the first intersection at $n^D \in (\overline{n}, 1)$ between these two curves. Figure 6 in the main text illustrates the determination of n^D in this case. Finally, if $\rho > \overline{\overline{\rho}}$, from Lemma A3 v_I/v_N is increasing in n, so n^D defined by $\kappa_I v_I(n^D) = \kappa_N v_N(n^D)$ is unique.

Part 2: We will first prove the global stability claim for $\theta = 0$, and then turn to local stability when $\theta > 0$.

Proof of stability of the unique BGP when $\theta = 0$: Suppose that $\theta = 0$. In this case, capital adjusts immediately and its equilibrium stock only depends on n, which becomes the unique state variable of the model. The rental rate is fixed at $R = \rho + \delta$ (or the interest rate is $r = \rho$), and the effective wages are given by $w_I(n)$ and $w_N(n)$.

Define next $v \equiv \kappa_I v_I - \kappa_N v_N$. Now starting from any $n(0)$, an equilibrium with endogenous technology is given by the path of (n, v) such that the evolution of the state variable is given by

$$\dot{n} = \kappa_N S - (\kappa_N + \kappa_I) G\left(\frac{v}{\lambda}\right) S,$$

and the difference of the normalized value functions v satisfies the forward looking differential equation:

$$\rho v - \dot{v} = M\kappa_I \left(c^u\left(\rho+\delta\right)^{\zeta-\sigma} - c^u\left(w_I\right)^{\zeta-\sigma}\right) - M\kappa_N\left(c^u\left(w_N\right)^{\zeta-\sigma} - c^u\left(\rho+\delta\right)^{\zeta-\sigma}\right) + \mathcal{O}(g),$$

and in addition the transversality condition (16) holds.

Let n^D denote the BGP value for $n(t)$ in the unique BGP. Since $\kappa_I v_I(n) - \kappa_N v_N(n)$ crosses zero only once at n^D in this case, equilibrium dynamics can be analyzed using the behavior of the value difference, v, as in Figure 6. We now prove that this BGP is globally stable. Figure A3 presents the phase diagram of the system in (v, n). Importantly, the locus for $\dot{v} = 0$ crosses $v = 0$ at n^D from below only once. This follows from the fact that $\kappa_I v_I'(n^D) > \kappa_N v_N'(n^D))$ (that is, $\kappa_I v_I(n)$ cuts $\kappa_N v_N(n)$ from below at n^D as shown in Figure 6).

The system of differential equations determining the behavior of n, v near this BGP can be linearized as:

$$\dot{n} = -\frac{(\kappa_N + \kappa_I)}{\lambda} G'(0) S v, \text{ and } \dot{v} = \rho v - Q,$$

where $Q > 0$ denotes the derivative of $-M\kappa_I c^u\left(w_I\right)^{\zeta-\sigma} + M\kappa_N c^u\left(w_N\right)^{\zeta-\sigma}$ with respect to n (this derivative is positive because $\kappa_I v_I(n)$ cuts $\kappa_N v_N(n)$ from below at n^D). Thus, the eigenvalues of the characteristic polynomial of this system add up to $\rho > 0$, and their product is given by $-Q\frac{(\kappa_N+\kappa_I)}{\lambda} G'(0) S < 0$. This implies that there is one positive and one negative eigenvalue, ensuring asymptotic (saddle-path) stability. It also follows from the same argument that for each $n(0)$, there is a unique $v(0)$ in the stable arm of the system, and thus guarantees uniqueness in the neighborhood of the BGP.

In order to show that globally all equilibria must be along the stable arm, we need to rule out other potential equilibrium paths. From the figure it is clear that if the equilibrium does not settle at n^D, it must reach the region with $\dot{v} > 0$ and $\dot{n} < 0$, or it must reach the region with $\dot{v} < 0$ and $\dot{n} > 0$. In the first case, v is strictly increasing and n is strictly decreasing, and hence there are no interior limit points. This implies $v \to \infty$ along any such path, and thus $v_I \to \infty$, which violates the transversality condition (16). In the second case, $v \to -\infty$ and $n \to 1$, which again analogously violates the transversality condition.

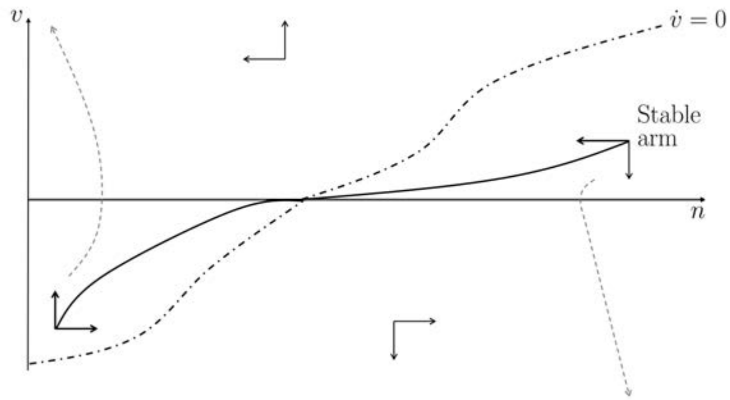

Figure A3: Phase diagram and global saddle path stability when $\theta = 0$.

Proof of local stability of the unique BGP when $\theta > 0$: Let us next turn to the case in which $\theta > 0$. An equilibrium is given by a solution to the following system of differential equations: Starting from any $n(0), k(0)$ the equilibrium path with endogenous technology is given by a tuple $\{n, k, c, v\}$ such that:

- The evolution of the state variables is given by

$$\dot{n}(t) = \kappa_N S - (\kappa_N + \kappa_I) G\left(\frac{v}{\lambda}\right) S,$$

- The paths for $c(t)$ and $k(t)$ satisfy the Euler equation,

$$\frac{\dot{c}(t)}{c(t)} = \frac{1}{\theta}(R^E(n(t), k(t)) - \delta - \rho) - \mathcal{O}(g)$$

  coupled with the transversality condition in equation (16), and the resource constraint,

$$\dot{k}(t) = f^E(n(t), k(t)) - c(t) - \delta k(t).$$

- The value function v satisfies the forward looking differential equation:

$$\rho v - \dot{v} = \kappa_I \pi_I(n, k) - \kappa_N \pi_N(n, k) + \mathcal{O}(g),$$

  with

$$\pi_N(n,k) = M\left(c^u\left(w_N^E(n,k)\right)^{\zeta-\sigma} - c^u\left(R^E(n,k)\right)^{\zeta-\sigma}\right)\pi_I(n,k) = M\left(c^u\left(R^E(n,k)\right)^{\zeta-\sigma} - c^u\left(w_I^E(n,k)\right)^{\zeta-\sigma}\right)$$

  denoting the flow profits for innovators. Here, $w_N^E = \frac{W}{\gamma(N)}$ and $w_I^E = \frac{W}{\gamma(I)}$, which also depend on the stock of capital outside of the BGP.

By continuity, there exists a threshold $\overline{\overline{A}} \leq \overline{A}$ such that, for $A < \overline{\overline{A}}$, the local behavior of the above system matches that of the system in which we take the limit $g \to 0$ in the above system of equations.

To simplify the notation, define the partial derivatives

$$Q_k = \kappa_I \frac{\partial \pi_I}{\partial k} - \kappa_N \frac{\partial \pi_N}{\partial k} > 0, Q_n = \kappa_I \frac{\partial \pi_I}{\partial n} - \kappa_N \frac{\partial \pi_N}{\partial n} > 0, Q_v = \frac{\kappa_I + \kappa_N}{\lambda} G'(0) S > 0.$$

evaluated at their BGP values. Then local equilibrium dynamics can be linearly approximated around the BGP as follows (where $n^D, v^D(=0), k^D$ and c^D designate the BGP values):

$$\begin{aligned}
\dot{n} =& - Q_v v \\
\dot{v} =& \rho v - Q_k[k(t) - k^D] - Q_n[n(t) - n^D], \\
\dot{c} =& \frac{c^D}{\theta} R_n^E[n(t) - n^D] + \frac{c^D}{\theta} R_k^E[k(t) - k^D] \\
\dot{k} =& f_n^E[n(t) - n^D] + (f_k^E - \delta)[k(t) - k^D] - [c - c^D].
\end{aligned}$$

Here, the f_n^E, R_n^E and f_k^E, R_k^E denote the partial derivatives of f and R with respect to n and k. The characteristic polynomial of the linearized system of differential equations (with all derivatives still evaluated at their BGP values) can be written as

$$P(\lambda) = \left| \begin{pmatrix} -\lambda & -Q_v & 0 & 0 \\ -Q_n & \rho - \lambda & 0 & -Q_k \\ \frac{c^D}{\theta} R_n^E & 0 & -\lambda & \frac{c^D}{\theta} R_k^E \\ f_n^E & 0 & -1 & f_k^E - \delta - \lambda \end{pmatrix} \right|,$$

or expanding it:

$$\begin{aligned}
P(z) = z^4 - z^3(f_k^E - \delta + \rho) + z^2 \left(-Q_v Q_n + \frac{c^D}{\theta} R_k^E + \rho(f_k^E - \delta) \right) \\
- z \left(Q_v(f_n^E Q_k - (f_k^E - \delta) Q_n) + \rho \frac{c^D}{\theta} R_k^E \right) + Q_v(R_n^E Q_k - R_k^E Q_n) \frac{c^D}{\theta}.
\end{aligned}$$

We now show that this polynomial has exactly two eigenvalues with positive real parts and two eigenvalues with negative real parts. First, note that $R_n^E Q_k - R_k^E\ Q_n > 0$, which is also the condition for the curve $\kappa_I v_I(n)$ cutting $\kappa_N v_N(n)$ from below. Indeed, the term $R_n^E\ Q_k - R_k^E\ Q_n > 0$ corresponds to the change in flow profits, $\kappa_I \pi_I - \kappa_N \pi_N$ that results from an increase in n when capital adjusts to keep the interest rate constant. Next, let z_1, z_2, z_3 and z_4 be the eigenvalues of the above system. Then $z_1 z_2 z_3 z_4 = Q_v(R_n^E\ Q_k - R_k^E\ Q_n)\frac{c^D}{\theta} > 0$. Moreover, $z_1 + z_2 + z_3 + z_4 = f_k^E - \delta + \rho > 0$ (which is the trace of the matrix $P(z)$). This implies that either there are exactly two eigenvalues with positive real parts and two eigenvalues with negative real parts, or all eigenvalues have positive real parts. We rule out the latter possibility by showing that the system has at least one negative real root.

To do so, we prove instead that the polynomial $P(-z)$ has at least one positive real root. Descartes' rule of signs, applied to the polynomial $P(-z)$, implies that this will be the case provided that at least one of the coefficients $Q_v(f_n^E Q_k - (f_k^E - \delta)Q_n + \rho \frac{c^D}{\theta} R_k^E$ and $-Q_v Q_n + \frac{c^D}{\theta} R_k^E + \rho(f_k^E - \delta)$ is negative. This is indeed the case as we can see by separately considering two cases. First, suppose that $f_K^E - \delta < 0$. Then

$$-Q_v Q_n + \frac{c^D}{\theta} R_k^E + \rho(f_k^E - \delta) < 0.$$

This follows from Proposition 2, which establishes that $Q_n > 0$ (i.e., as n increases—holding capital constant—the incentives to do automation increase). Suppose, alternatively, that $f_k^E - \delta > 0$. Then

$f_n^E Q_k - (f_k^E - \delta)Q_n < 0$, this also follows from Proposition 2, which ensures that an increase in n, holding output constant, raises the wage relative to the rental rate, or equivalently

$$Q_n - Q_k \frac{f_n^E}{f_k^E - \delta} > 0,$$

which implies that, when $f_k^E - \delta > 0$, we must also have $f_n^E Q_k - (f_k^E - \delta)Q_n < 0$, and thus

$$Q_v(f_n^E Q_k - (f_k^E - \delta)Q_n) + \rho \frac{c^D}{\theta} R_k^E < 0.$$

Because the model has two state variables and there are exactly two roots with negative real parts, equilibrium dynamics are asymptotically (saddle-path) stable.

Part 3: When $\kappa < \overline{\kappa}$, we have $\kappa_I v_I < \kappa_N v_N$ throughout, so $\dot{n} > 0$. Thus, asymptotically we have $n(t) = 1$ and the BGP is identical to that of an endogenous growth model with purely labor-augmenting technological change.

Part 4: As the right panel of Figure A2 shows, when $\rho \leq \overline{\rho}$, Lemma A3 implies that for $n \leq \widetilde{n}$ $\kappa_I v_I(n) > \kappa_N v_N(n)$. This observation implies that there is an asymptotically stable BGP with $n(t) = 0$. ■

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Appendix B (Not-For-Publication): Omitted Proofs and Additional Results

Details of the Empirical Analysis

Here we provide information about the data used in constructing Figures 1 and 7. We also provide a regression analysis documenting the robustness of the patterns illustrated in these figures.

Data: We use data on the demographic characteristics of workers and employment counts in each of the 330 consistently defined occupations proposed by David Dorn (see `http://www.ddorn.net/data.htm`). Our sources of data are the U.S. Censuses for 1980, 1990 and 2000, and the American Community Survey for 2007. We focus on the set of workers between 16 and 64 years of age.

Our measure of new job titles is taken from Lin (2011), who computes the share of new job titles within each occupation for 1980, 1990 and 2000. Lin defines new job titles by comparing changes across waves of the Dictionary of Occupational Titles, and also by comparing the 1990 Census Index of Occupations with its 2000 counterpart. The data is available for 329 occupations in 1980, and 330 occupations for 1990 and 2000. The data are available from his website https://sites.google.com/site/jeffrlin/newwork.

Detailed Analysis for Figure 1: To document the role of new job titles in employment growth, we estimate the regression

$$\ln E_{it+10} - \ln E_{it} = \beta N_{it} + \delta_t + \Gamma_t X_{it} + \varepsilon_{it}. \tag{B1}$$

Here, the dependent variable is the percent change in employment from year t to $t+10$ in each occupation i. We stack the data for $t = 1980, 1990, 2000$. For $t = 2000$, we use the change from 2000 to 2007 as the dependent variable and re-scale it to a 10-year change. In all regressions we include a full set of decadal effects δ_t, and in some models we also control for differential decadal trends that vary depending on observable characteristics of each occupation, $\Gamma_t X_{it}$. These characteristics include the share of workers in different 5-year age brackets and from different races (Black, Hispanic), and the share of foreign and female workers. These covariates flexibly control for demographic changes that may affect the labor supply that is relevant for each occupation. Finally, ε_{it} is an error term. Throughout, all standard errors are robust against arbitrary heteroskedasticity and serial correlation of the error term within occupations.

The coefficient of interest is β, which represents the additional employment growth in occupations with a large share of new job titles, N_{it}.

Panel A in Table B1 presents estimates of equation (B1). Column 1 contains no additional covariates (the number of observations in this column is 989. We miss one observation because Lin's measure only covers 329 occupations in 1980). Our estimates indicate that occupations with 10 percentage points more new job titles at the beginning of each decade grew 5.05% faster over the decade (standard error= 1.29%). If occupations with more new job titles did not grow any faster than the benchmark category with no novel jobs, employment growth from 1980 to 2007 would have been, on average, 8.66% instead of the actual 17.5%, implying that approximately 8.84% of the 17.5% growth is accounted for by new job titles as reported at the bottom rows of the panel.

In column 2 we control for the log of employment at the beginning of the decade (year t). The coefficient of interest increases slightly to 0.560 and continues to be precisely estimated. The log of employment at year t appears with a negative coefficient, which indicates that smaller occupations tend to grow more over time. The quantitative contribution of new tasks and job titles remains very similar to column 1, increasing slightly to 9.8%.

In column 3 we control for the trends that depend on the demographic covariates described above, which have little effect on the quantitative results. In column 4, we also control for the average years of schooling among workers in each occupation at the beginning of the decade. Although this covariate reduces the magnitude of the coefficient of the share of new job titles, our estimate for β remains highly significant. The contribution of new job titles is now estimated at 6.62% out of the 17.5% growth between 1980 and 2007.

Column 5 repeats the specification of column 4, but this time we reweight the data by the 1980 share of employment in each occupation. This weakens the relationship of interest, and the share of novel tasks and jobs is no longer statistically significant. However, this lack of significance is driven by a few large occupations that are outliers in the estimated relationship. (In contrast, there are no major outliers in the unweighted regressions reported in columns 1-4). These outliers include office supervisors, office clerks, and production supervisors; three occupations that had combined employment of about 4 million workers in 1980 and have been contracting since then. Though these occupations introduced a significant number of new job titles in 1980, they shed a large amount of workers in the subsequent years. In column 6, we exclude these three occupations from our analysis, and obtain a similar pattern to that of column 4.

Finally, in Panel B, we present a set of regressions, which are analogous to those in the top panel, but that focus on a long difference specification between 1980 and 2007. The overall patterns are very similar, and now the contribution of novel tasks and new job titles to the 17.5% growth in employment between 1980 and 2007 is between 7.27 and 8.5%.

Detailed Analysis for Figure 7: To document the presence of some amount of "standardization" of occupations with greater new job titles,

$$\Delta Y_{it} = \beta N_{it} + \delta_t + \Gamma_t X_{it} + \varepsilon_{it}. \tag{B2}$$

Here, the dependent variable is the change in the average years of schooling among workers employed in occupation i and measured over different time horizons (10 years, 20 years or 30 years). We stack the data for $t = 1980, 1990, 2000$, but our sample becomes smaller as we measure the change in average years of schooling over longer periods of time. We again use the ACS dayear 2007. The covariates and independent variable are the same ones that we used in equation (B1).

Panel A in Table B2 presents estimates of equation (B2). Columns 1 and 2 present models in which the dependent variable is the change in average years of education over a 10-year period (hence, we get 989 observations for 330 occupations). Columns 3 and 4 focus on the change in average years

Table B1: Differential employment growth in occupations with more new job titles

	Dep. var: Percent change in employment growth by decade.					
					Weighted by size	
	(1)	(2)	(3)	(4)	(5)	(6)
	Panel A: Stacked differences over decades.					
Share of new job titles at the start of decade	0.505***	0.560***	0.479***	0.378***	0.140	0.351***
	(0.129)	(0.127)	(0.136)	(0.139)	(0.163)	(0.132)
log of employment at start of decade		-0.031**	-0.045***	-0.042***	0.002	0.005
		(0.012)	(0.014)	(0.013)	(0.011)	(0.011)
Average years of schooling at start of decade				9.602***	8.231***	8.179***
				(1.864)	(1.809)	(1.793)
R-squared	0.03	0.04	0.11	0.14	0.13	0.15
Observations	989	989	989	989	989	980
Occupations	330	330	330	330	330	327
Employment growth from 1980-2007 in p.p.	17.5	17.5	17.5	17.5	17.5	17.5
Contribution of novel tasks and jobs	8.84	9.8	8.38	6.62	2.45	6.14
	Panel B: Long differences from 1980-2007.					
Share of new job titles in 1980	1.247***	1.398***	1.458***	1.192***	0.028	0.450
	(0.392)	(0.348)	(0.353)	(0.333)	(0.532)	(0.483)
log of employment in 1980		-0.150***	-0.183***	-0.163***	-0.044	-0.037
		(0.031)	(0.035)	(0.034)	(0.032)	(0.032)
Average years of schooling in 1980				21.779***	15.978***	15.855***
				(4.162)	(4.204)	(4.226)
R-squared	0.02	0.08	0.17	0.24	0.18	0.18
Observations	329	329	329	329	329	326
Occupations	329	329	329	329	329	326
Employment growth from 1980-2007 in p.p.	17.5	17.5	17.5	17.5	17.5	17.5
Contribution of novel tasks and jobs	7.27	8.155	8.50	8.29	0.16	2.17
Covariates:						
Decade fixed effects	✓	✓	✓	✓	✓	✓
Demographics × decade effects			✓	✓	✓	✓

Notes: The table presents 10-years stacked-differences estimates (Panel A) and long-differences estimates (Panel B) of the share of new job titles in an occupation on subsequent employment growth. The bottom row in each panel reports the observed growth and the share explained by growth in occupations with more new job titles. The bottom rows indicate additional covariates included in each model. In column 5 we reweight the data using the baseline share of employment in each occupation in 1980, and in column 6 we exclude three large employment categories that are outliers in the model of column 5. These include office supervisors, office clerks, and production supervisors. Standard errors robust against heteroskedasticity and serial correlation within occupations are presented in parentheses.

of education over a 20-year period. Columns 5 and 6 focus on the change in average years of education over a 30-year period. In the models presented in the even columns we include a full set of trends that are allowed to vary depending on the composition of employment in each occupational category.

Table B2: Reversal in skill content for occupations with more new job titles and in occupations that used to hire more educated workers.

	Dep. var: Change in average years of schooling.					
	Over 10 years		Over 20 years		Over 30 years	
	(1)	(2)	(3)	(4)	(5)	(6)
	Panel A: Change in occupations with new job titles.					
Share of new job titles at the start of decade	-0.085	-0.103	-0.219	-0.203	-0.452**	-0.411**
	(0.123)	(0.118)	(0.179)	(0.173)	(0.215)	(0.176)
R-squared	0.32	0.53	0.12	0.33	0.02	0.29
Observations	989	989	659	659	329	329
Occupations	330	330	330	330	329	329
	Panel B: Change in occupations with more educated workers.					
Average years of education at the start of decade	-0.030***	-0.077***	-0.028**	-0.102***	-0.083***	-0.149***
	(0.006)	(0.011)	(0.013)	(0.017)	(0.016)	(0.021)
R-squared	0.36	0.59	0.14	0.41	0.17	0.43
Observations	990	990	660	660	330	330
Occupations	330	330	330	330	330	330
Covariates:						
Decade fixed effects	✓	✓	✓	✓	✓	✓
Demographics × decade effects		✓		✓		✓

Notes: The table presents OLS estimates that explain the change in average years of schooling among workers employed in a given occupation. These changes are computed over 10 years (Columns 1 and 2), 20 years (Columns 3 and 4) and 30 years (Columns 5 and 6). In Panel A we explain the subsequent change in years of schooling as a function of share of new job titles in each occupation at the start of the decade. In Panel B we explain the subsequent change in years of schooling as a function of the years of schooling in each occupation at the start of the decade. The bottom rows indicate additional covariates included in each model. Standard errors robust against heteroskedasticity and serial correlation within occupations are presented in parentheses.

Our estimates indicate that, although occupations with more new job titles tend to hire more skilled workers initially, this pattern slowly reverts over time. Figure 7 shows that, at the time of their introduction, occupations with 10 percentage points more new job titles hire workers with 0.35 more years of schooling). But our estimates in Column 6 of Table B2 show that this initial difference in the skill requirements of workers slowly vanishes over time. 30 years after their introduction, occupations with 10 percentage points more new job titles hire workers with 0.0411 fewer years of education than the workers hired initially (standard error= 0.0176).

Relatedly, Panel B of the same table shows a similar pattern when we look at occupations that start with greater average years of schooling at the beginning of a decade. In particular, for each decade since 1980, employment growth has been faster in occupations with greater skill requirements—as measured by the average years of education among employees at the start of each decade (see Table B1). But estimating a version of (B2) with average years of schooling at the beginning of the decade on the right-hand side, we find significant mean reversion. For example, Column 6 shows that occupations that used to hire workers with one additional year of schooling workers reduce their average years of schooling by 0.149 years relative to baseline after 30 years (standard error=0.021).

Remaining Proofs from Section 2

We start by providing the proof of Lemma A1.

Proof of Lemma A1. As we have just seen, the equilibrium conditions that uniquely determine $W(I^*, N, K)$ and $R(I^*, N, K)$, are:

$$L^s\left(\frac{W}{RK}\right)(I^*-N+1)c^u(R)^{\zeta-\sigma}R^{-\zeta}-K\int_{I^*}^{N}\gamma(i)^{\zeta-1}c^u\left(\frac{W}{\gamma(i)}\right)^{\zeta-\sigma}W^{-\zeta}di=0$$

$$(I^*-N+1)c^u(R)^{1-\sigma}+\int_{I^*}^{N}c^u\left(\frac{W}{\gamma(i)}\right)^{1-\sigma}di=1.$$

Taking total derivatives with respect to I^*, N, the first equation yields

$$\begin{aligned}
&dN\left(L^s c^u(r)^{\zeta-\sigma}r^{-\zeta}+K\gamma(N)^{\zeta-1}c^u\left(\frac{W}{\gamma(N)}\right)^{\zeta-\sigma}W^{-\zeta}\right)\\
-&dI^*\left(L^s c^u(r)^{\zeta-\sigma}r^{-\zeta}+K\gamma(I^*)^{\zeta-1}c^u\left(\frac{W}{\gamma(I^*)}\right)^{\zeta-\sigma}W^{-\zeta}\right)\\
=&(d\ln W-d\ln r)\left(K\int_{I^*}^{N}\gamma(i)^{\zeta-1}c^u\left(\frac{W}{\gamma(i)}\right)^{\zeta-\sigma}W^{-\zeta}\left[s(i)(\sigma-\zeta)+\zeta\right]di+\varepsilon_L L^s(I^*-N+1)c^u(r)^{\zeta-\sigma}r^{-\zeta}\right)\\
+&d\ln r\left(K\int_{I^*}^{N}\gamma(i)^{\zeta-1}c^u\left(\frac{W}{\gamma(i)}\right)^{\zeta-\sigma}W^{-\zeta}\left[(s(i)-s_k)(\sigma-\zeta)\right]di\right).
\end{aligned}$$

Here, $s(i)$ is the share of labor in the production of task i and s_k is the share of capital in tasks produced with capital.

The last term in the above equation captures the non-homotheticity introduced by the presence of intermediate goods. In all the special cases in which $(s(i)-s_k)(\sigma-\zeta)=0$, the demand system is homothetic and the results outlined in Lemma A1 follow easily. These cases include the limits when $\zeta\to 1$ (and $s(i)=s_k$ are constant) or $\eta\to 0$ (and $s(i)=s_k=1$ are constant). Intuitively, in these cases the relative demand curve consists of a ray passing through the origin. However, when $(s(i)-s_k)(\sigma-\zeta)\neq 0$, the non-homotheticity becomes important and we need to take into account the movements in the ideal price curve to determine the behavior of relative factor prices.

Differentiation of the ideal price index condition gives us the equation:

$$\begin{aligned}
&dN\frac{1}{1-\sigma}\left(c^u\left(r\right)^{1-\sigma}-c^u\left(\frac{W}{\gamma(N)}\right)^{1-\sigma}\right)+dI\frac{1}{1-\sigma}\left(c^u\left(\frac{W}{\gamma(I^*)}\right)^{1-\sigma}-c^u\left(r\right)^{1-\sigma}\right)\\
=&(d\ln W-d\ln r)\left(\int_{I^*}^{N}s(i)c^u\left(\frac{W}{\gamma(i)}\right)^{1-\sigma}di\right)+d\ln r\left((I^*-N+1)s_k c^u(r)^{1-\sigma}+\int_{I^*}^{N}s(i)c^u\left(\frac{W}{\gamma(i)}\right)^{1-\sigma}di\right).
\end{aligned}$$

Combining both expressions, we can solve for $d\ln W - d\ln R$ as

$$(d\ln W - d\ln R)A = dNP_N - d_I P_I,$$

where:

$$\begin{aligned}
A =& \left(K\int_{I^*}^{N}\gamma(i)^{\zeta-1}c^u\left(\frac{W}{\gamma(i)}\right)^{\zeta-\sigma}W^{-\zeta}\left[s(i)(\sigma-\zeta)+\zeta\right]di + \varepsilon_L L^s(I^*-N+1)c^u(r)^{\zeta-\sigma}r^{-\zeta}\right)\\
&\times\left((I^*-N+1)s_k c^u(r)^{1-\sigma}+\int_{I^*}^{N}s(i)c^u\left(\frac{W}{\gamma(i)}\right)^{1-\sigma}di\right)\\
&-\left(\int_{I^*}^{N}s(i)c^u\left(\frac{W}{\gamma(i)}\right)^{1-\sigma}di\right)\times\left(K\int_{I^*}^{N}\gamma(i)^{\zeta-1}c^u\left(\frac{W}{\gamma(i)}\right)^{\zeta-\sigma}W^{-\zeta}\left[(s(i)-s_k)(\sigma-\zeta)\right]di\right),
\end{aligned}$$

$$\begin{aligned}
P_N =& \left(L^s c^u(r)^{\zeta-\sigma}r^{-\zeta}+K\gamma(N)^{\zeta-1}c^u\left(\frac{W}{\gamma(N)}\right)^{\zeta-\sigma}W^{-\zeta}\right)\\
&\times\left((I^*-N+1)s_k c^u(r)^{1-\sigma}+\int_{I^*}^{N}s(i)c^u\left(\frac{W}{\gamma(i)}\right)^{1-\sigma}di\right)\\
&-\frac{1}{1-\sigma}\left(c^u\,(r)^{1-\sigma}-c^u\left(\frac{W}{\gamma(N)}\right)^{1-\sigma}\right)\times\left(K\int_{I^*}^{N}\gamma(i)^{\zeta-1}c^u\left(\frac{W}{\gamma(i)}\right)^{\zeta-\sigma}W^{-\zeta}\left[(s(i)-s_k)(\sigma-\zeta)\right]di\right),
\end{aligned}$$

$$\begin{aligned}
P_I =& \left(L^s c^u(r)^{\zeta-\sigma}r^{-\zeta}+K\gamma(I^*)^{\zeta-1}c^u\left(\frac{W}{\gamma(I^*)}\right)^{\zeta-\sigma}W^{-\zeta}\right)\\
&\times\left((I^*-N+1)s_k c^u(r)^{1-\sigma}+\int_{I^*}^{N}s(i)c^u\left(\frac{W}{\gamma(i)}\right)^{1-\sigma}di\right)\\
&+\frac{1}{1-\sigma}\left(c^u\left(\frac{W}{\gamma(I^*)}\right)^{1-\sigma}-c^u\,(r)^{1-\sigma}\right)\times\left(K\int_{I^*}^{N}\gamma(i)^{\zeta-1}c^u\left(\frac{W}{\gamma(i)}\right)^{\zeta-\sigma}W^{-\zeta}\left[(s(i)-s_k)(\sigma-\zeta)\right]di\right).
\end{aligned}$$

We now show that, under the conditions of Lemma A1, we have that $A, P_N, P_I > 0$.

A sufficient condition for $A > 0$ is that:

$$\begin{aligned}
&\left(K\int_{I^*}^{N}\gamma(i)^{\zeta-1}c^u\left(\frac{W}{\gamma(i)}\right)^{\zeta-\sigma}W^{-\zeta}\left[s(i)(\sigma-\zeta)+\zeta\right]di\right)\times\left(\int_{I^*}^{N}s(i)c^u\left(\frac{W}{\gamma(i)}\right)^{1-\sigma}di\right)\\
\geq&\left(\int_{I^*}^{N}s(i)c^u\left(\frac{W}{\gamma(i)}\right)^{1-\sigma}di\right)\times\left(K\int_{I^*}^{N}\gamma(i)^{\zeta-1}c^u\left(\frac{W}{\gamma(i)}\right)^{\zeta-\sigma}W^{-\zeta}\left[(s(i)-s_k)(\sigma-\zeta)\right]di\right).
\end{aligned}$$

After canceling common terms on both sides of this inequality, it boils down to:

$$\int_{I^*}^{N}\gamma(i)^{\zeta-1}c^u\left(\frac{W}{\gamma(i)}\right)^{\zeta-\sigma}\left[s(i)(\sigma-\zeta)+\zeta\right]di \geq \int_{I^*}^{N}\gamma(i)^{\zeta-1}c^u\left(\frac{W}{\gamma(i)}\right)^{\zeta-\sigma}\left[(s(i)-s_k)(\sigma-\zeta)\right]di.$$

This can be rewritten as:

$$\int_{I^*}^{N}\gamma(i)^{\zeta-1}c^u\left(\frac{W}{\gamma(i)}\right)^{\zeta-\sigma}\left[s(i)(\sigma-\zeta)+\zeta-(s(i)-s_k)(\sigma-\zeta)\right]di \geq 0.$$

The last inequality always holds because $s(i)(\sigma-\zeta)+\zeta-(s(i)-s_k)(\sigma-\zeta) = \sigma s_k + \zeta(1-s_k) > 0$.

To determine the signs of P_N and P_I, we regroup terms as follows. First, we group the terms that are multiplied by s_k in the expression for P_I. To guarantee that these terms add up to a positive number, a sufficient condition is given by:

$$\left(L^s c^u(r)^{\zeta-\sigma} r^{-\zeta}\right) \times (I^* - N + 1) s_k c^u(r)^{1-\sigma}$$
$$\geq \left|\frac{\sigma-\zeta}{1-\sigma}\left(c^u\left(\frac{W}{\gamma(I^*)}\right)^{1-\sigma} - c^u(r)^{1-\sigma}\right)\right| \times \left(K\int_{I^*}^{N} s_k \gamma(i)^{\zeta-1} c^u\left(\frac{W}{\gamma(i)}\right)^{\zeta-\sigma} W^{-\zeta} di\right).$$

The relative demand for factors implies that

$$\left(L^s c^u(R)^{\zeta-\sigma} R^{-\zeta}\right) \times (I^* - N + 1) s_k c^u(R)^{1-\sigma} = s_k c^u(R)^{1-\sigma} K \int_{I^*}^{N} \gamma(i)^{\zeta-1} c^u\left(\frac{W}{\gamma(i)}\right)^{\zeta-\sigma} W^{-\zeta} di.$$

Therefore, we can rewrite the sufficient condition as:

$$s_k c^u(r)^{1-\sigma} K \int_{I^*}^{N} \gamma(i)^{\zeta-1} c^u\left(\frac{W}{\gamma(i)}\right)^{\zeta-\sigma} W^{-\zeta} di$$
$$\geq s_k \left|\frac{\sigma-\zeta}{1-\sigma}\left(c^u\left(\frac{W}{\gamma(I^*)}\right)^{1-\sigma} - c^u(r)^{1-\sigma}\right)\right| \times K \int_{I^*}^{N} \gamma(i)^{\zeta-1} c^u\left(\frac{W}{\gamma(i)}\right)^{\zeta-\sigma} W^{-\zeta} di.$$

After removing common terms on both sides, this condition becomes:

$$c^u(R)^{1-\sigma} \geq \left|\frac{\sigma-\zeta}{1-\sigma}\left(c^u\left(\frac{W}{\gamma(I^*)}\right)^{1-\sigma} - c^u(R)^{1-\sigma}\right)\right|. \tag{B3}$$

Second, we group the terms that are multiplied by $s(i)$ in the expression for P_I. To guarantee that these terms add up to a positive number, a sufficient condition is given by:

$$\left(K\gamma(I^*)^{\zeta-1} c^u\left(\frac{W}{\gamma(I^*)}\right)^{\zeta-\sigma} W^{-\zeta}\right) \times \left(\int_{I^*}^{N} s(i) c^u\left(\frac{W}{\gamma(i)}\right)^{1-\sigma} di\right)$$
$$\geq \left|\frac{\sigma-\zeta}{1-\sigma}\left(c^u\left(\frac{W}{\gamma(I^*)}\right)^{1-\sigma} - c^u(r)^{1-\sigma}\right)\right| \times \left(K\int_{I^*}^{N} s(i)\gamma(i)^{\zeta-1} c^u\left(\frac{W}{\gamma(i)}\right)^{\zeta-\sigma} W^{-\zeta} di\right).$$

After removing common terms and re-grouping, this condition becomes

$$\int_{I^*}^{N} s(i) W^{-\zeta}\left(\gamma(I^*)^{\zeta-1} c^u\left(\frac{W}{\gamma(I^*)}\right)^{\zeta-\sigma} c^u\left(\frac{W}{\gamma(i)}\right)^{1-\sigma} - \gamma(i)^{\zeta-1} c^u\left(\frac{W}{\gamma(i)}\right)^{\zeta-\sigma}\left|\frac{\sigma-\zeta}{1-\sigma}\left(c^u\left(\frac{W}{\gamma(I^*)}\right)^{1-\sigma} - c^u(R)^{1-\sigma}\right)\right|\right) \geq 0. \tag{B4}$$

We now show that, for this condition to hold, it suffices that

$$\left(\frac{\gamma(I^*)}{\gamma(i)}\right)^{\max\{\sigma,\zeta\}-1} c^u\left(\frac{W}{\gamma(i)}\right)^{1-\sigma} \geq \left|\frac{\sigma-\zeta}{1-\sigma}\left(c^u\left(\frac{W}{\gamma(I^*)}\right)^{1-\sigma} - c^u(R)^{1-\sigma}\right)\right|. \tag{B5}$$

We first show this in the case in which $\sigma > \zeta$. In this case, we have that

$$c^u\left(\frac{W}{\gamma(I^*)}\right)^{\zeta-\sigma} \geq c^u\left(\frac{W}{\gamma(i)}\right)^{\zeta-\sigma}\left(\frac{\gamma(i)}{\gamma(I^*)}\right)^{\zeta-\sigma}.$$

This follows from the fact that

$$\frac{c^u\left(\frac{W}{\gamma(i)}\right)}{c^u\left(\frac{W}{\gamma(I^*)}\right)} \geq \frac{\gamma(I^*)}{\gamma(i)},$$

an inequality which can be proven by straightforward differentiation of the function $c^u(x)/x$, which is weakly decreasing.

Plugging this in the inequality in equation (B4), we obtain the sufficient condition

$$\int_{I^*}^{N} s(i)W^{-\zeta}\gamma(i)^{\zeta-\sigma}\left(\gamma(I^*)^{\sigma-1}c^u\left(\frac{W}{\gamma(i)}\right)^{\zeta-\sigma}c^u\left(\frac{W}{\gamma(i)}\right)^{1-\sigma} - \gamma(i)^{\sigma-1}c^u\left(\frac{W}{\gamma(i)}\right)^{\zeta-\sigma}\left|\frac{\sigma-\zeta}{1-\sigma}\left(c^u\left(\frac{W}{\gamma(I^*)}\right)^{1-\sigma} - c^u\left(R\right)^{1-\sigma}\right)\right|\right),$$

which holds provided that

$$\left(\frac{\gamma(I^*)}{\gamma(i)}\right)^{\sigma-1}c^u\left(\frac{W}{\gamma(i)}\right)^{1-\sigma} \geq \left|\frac{\sigma-\zeta}{1-\sigma}\left(c^u\left(\frac{W}{\gamma(I^*)}\right)^{1-\sigma} - c^u\left(R\right)^{1-\sigma}\right)\right|.$$

This inequality coincides with the condition in equation (B5), completing this step of the proof.

Turning next to the case where $\zeta \geq \sigma$, because the cost function is increasing we have that:

$$c^u\left(\frac{W}{\gamma(I^*)}\right)^{\zeta-\sigma} \geq c^u\left(\frac{W}{\gamma(i)}\right)^{\zeta-\sigma}$$

Plugging this in the sufficient condition in equation (B4) yields:

$$\int_{I^*}^{N} s(i)W^{-\zeta}\left(\gamma(I^*)^{\zeta-1}c^u\left(\frac{W}{\gamma(i)}\right)^{\zeta-\sigma}c^u\left(\frac{W}{\gamma(i)}\right)^{1-\sigma} - \gamma(i)^{\zeta-1}c^u\left(\frac{W}{\gamma(i)}\right)^{\zeta-\sigma}\left|\frac{\sigma-\zeta}{1-\sigma}\left(c^u\left(\frac{W}{\gamma(I^*)}\right)^{1-\sigma} - c^u\left(R\right)^{1-\sigma}\right)\right|\right),$$

which holds provided that

$$\left(\frac{\gamma(I^*)}{\gamma(i)}\right)^{\zeta-1}c^u\left(\frac{W}{\gamma(i)}\right)^{1-\sigma} \geq \left|\frac{\sigma-\zeta}{1-\sigma}\left(c^u\left(\frac{W}{\gamma(I^*)}\right)^{1-\sigma} - c^u\left(R\right)^{1-\sigma}\right)\right|,$$

once again establishing (B5).

Third, we group the terms that are multiplied by $s(i)$ in the expression for P_N. To guarantee that these terms add up to a positive number, a sufficient condition is given by:

$$\left(K\gamma(N)^{\zeta-1}c^u\left(\frac{W}{\gamma(N)}\right)^{\zeta-\sigma}W^{-\zeta}\right)\times\left(\int_{I^*}^{N} s(i)c^u\left(\frac{W}{\gamma(i)}\right)^{1-\sigma}di\right)$$
$$\geq\left|\frac{\sigma-\zeta}{1-\sigma}\left(c^u\left(r\right)^{1-\sigma} - c^u\left(\frac{W}{\gamma(N)}\right)^{1-\sigma}\right)\right|\times\left(K\int_{I^*}^{N} s(i)\gamma(i)^{\zeta-1}c^u\left(\frac{W}{\gamma(i)}\right)^{\zeta-\sigma}W^{-\zeta}di\right).$$

This condition can be rewritten as:

$$\int_{I^*}^{N} s(i)W^{-\zeta}\left[c^u\left(\frac{W}{\gamma(i)}\right)^{1-\sigma}\gamma(N)^{\zeta-1}c^u\left(\frac{W}{\gamma(N)}\right)^{\zeta-\sigma} - \left|\frac{\sigma-\zeta}{1-\sigma}\left(c^u\left(R\right)^{1-\sigma} - c^u\left(\frac{W}{\gamma(N)}\right)^{1-\sigma}\right)\right|\gamma(i)^{\zeta-1}c^u\left(\frac{W}{\gamma(i)}\right)^{\zeta-\sigma}\right]di \geq 0.$$

In Step 1 of the proof of this proposition, we showed that $x^{-\zeta}c^u(x)^{\sigma-\zeta}$ is decreasing in x. This implies that $\gamma(N)^{\zeta}c^u\left(\frac{W}{\gamma(N)}\right)^{\zeta-\sigma} > \gamma(i)^{\zeta}c^u\left(\frac{W}{\gamma(i)}\right)^{\zeta-\sigma}$.

Therefore, a sufficient condition for the terms that are multiplied by $s(i)$ in the expression for P_N to add up to a positive number is

$$\int_{I^*}^{N} s(i)\gamma(i)^{\zeta-1}c^u\left(\frac{W}{\gamma(i)}\right)^{\zeta-\sigma}W^{-\zeta}\times\left[\frac{\gamma(I)}{\gamma(N)}c^u\left(\frac{W}{\gamma(i)}\right)^{1-\sigma}-\left|\frac{\sigma-\zeta}{1-\sigma}\left(c^u\left(R\right)^{1-\sigma}-c^u\left(\frac{W}{\gamma(N)}\right)^{1-\sigma}\right)\right|\right]di\geq 0.$$

This expression implies that, to guarantee that these terms are positive, the following relationship would be sufficient

$$\frac{\gamma(I)}{\gamma(N)}c^u\left(\frac{W}{\gamma(i)}\right)^{1-\sigma}\geq\left|\frac{\sigma-\zeta}{1-\sigma}\left(c^u\left(R\right)^{1-\sigma}-c^u\left(\frac{W}{\gamma(N)}\right)^{1-\sigma}\right)\right|. \tag{B6}$$

Fourth, and lastly, we group the terms that are multiplied by s_K in the expression for P_N. To guarantee that these terms add up to a positive number, a sufficient condition is given by:

$$L^s c^u(r)^{\zeta-\sigma}r^{-\zeta}\times(I^*-N+1)s_k c^u(r)^{1-\sigma}$$
$$\geq\left|\frac{\sigma-\zeta}{1-\sigma}\left(c^u\left(r\right)^{1-\sigma}-c^u\left(\frac{W}{\gamma(N)}\right)^{1-\sigma}\right)\right|\times\left(K\int_{I^*}^{N}s_k\gamma(i)^{\zeta-1}c^u\left(\frac{W}{\gamma(i)}\right)^{\zeta-\sigma}W^{-\zeta}di\right).$$

The relative demand for factors implies that

$$L^s c^u(R)^{\zeta-\sigma}R^{-\zeta}(I^*-N+1)s_k c^u(R)^{1-\sigma}=s_k c^u(R)^{1-\sigma}K\int_{I^*}^{N}\gamma(i)^{\zeta-1}c^u\left(\frac{W}{\gamma(i)}\right)^{\zeta-\sigma}W^{-\zeta}di.$$

Therefore, we can rewrite the sufficient condition as:

$$s_k c^u(r)^{1-\sigma}K\int_{I^*}^{N}\gamma(i)^{\zeta-1}c^u\left(\frac{W}{\gamma(i)}\right)^{\zeta-\sigma}W^{-\zeta}di$$
$$\geq s_k\left|\frac{\sigma-\zeta}{1-\sigma}\left(c^u\left(\frac{W}{\gamma(N)}\right)^{1-\sigma}-c^u\left(r\right)^{1-\sigma}\right)\right|\times K\int_{I^*}^{N}\gamma(i)^{\zeta-1}c^u\left(\frac{W}{\gamma(i)}\right)^{\zeta-\sigma}W^{-\zeta}di.$$

After removing common terms on both sides, this condition becomes

$$c^u(R)^{1-\sigma}\geq\left|\frac{\sigma-\zeta}{1-\sigma}\left(c^u\left(\frac{W}{\gamma(N)}\right)^{1-\sigma}-c^u\left(R\right)^{1-\sigma}\right)\right|. \tag{B7}$$

To finalize the proof of the lemma, we use the fact that for any effective factor prices p_1, p_2 (the price per effective unit of labor or capital at any given task), we have

$$\left|\frac{\sigma-\zeta}{1-\sigma}\left(c^u\left(p_1\right)^{1-\sigma}-c^u\left(p_2\right)^{1-\sigma}\right)\right|\leq|\sigma-\zeta|c^u\left(\frac{W}{\gamma(N-1)}\right)c^u\left(\frac{W}{\gamma(N)}\right)^{-\sigma}. \tag{B8}$$

This inequality follows because $f(x)=\frac{1}{1-\sigma}x^{1-\sigma}$ is a concave function and the effective factor prices satisfy $p_1, p_2\in\left[\frac{W}{\gamma(N)},\frac{W}{\gamma(N-1)}\right]$.

Inequality (B8) then implies that to guarantee (B3), (B5), (B6) and (B7), the following would suffice

$$c^u(R)^{1-\sigma}\geq|\sigma-\zeta|c^u\left(\frac{W}{\gamma(N-1)}\right)c^u\left(\frac{W}{\gamma(N)}\right)^{-\sigma}.$$
$$\left(\frac{\gamma(I^*)}{\gamma(i)}\right)^{\max\{\sigma,\zeta\}-1}c^u\left(\frac{W}{\gamma(i)}\right)^{1-\sigma}\geq|\sigma-\zeta|c^u\left(\frac{W}{\gamma(N-1)}\right)c^u\left(\frac{W}{\gamma(N)}\right)^{-\sigma}.$$
$$\frac{\gamma(I)}{\gamma(N)}c^u\left(\frac{W}{\gamma(i)}\right)^{1-\sigma}\geq|\sigma-\zeta|c^u\left(\frac{W}{\gamma(N-1)}\right)c^u\left(\frac{W}{\gamma(N)}\right)^{-\sigma}.$$

These inequalities can be, in turn, rewritten as

$$\left(\frac{c^u(R)}{c^u\left(\frac{W}{\gamma(N-1)}\right)}\right)\left(\frac{c^u\left(\frac{W}{\gamma(N)}\right)}{c^u(R)}\right)^{\sigma} \geq |\sigma-\zeta|.$$

$$\left(\frac{\gamma(I^*)}{\gamma(i)}\right)^{\max\{\sigma,\zeta\}-1}\left(\frac{c^u\left(\frac{W}{\gamma(i)}\right)}{c^u\left(\frac{W}{\gamma(N-1)}\right)}\right)\left(\frac{c^u\left(\frac{W}{\gamma(N)}\right)}{c^u\left(\frac{W}{\gamma(i)}\right)}\right)^{\sigma} \geq |\sigma-\zeta|.$$

$$\frac{\gamma(I)}{\gamma(N)}\left(\frac{c^u\left(\frac{W}{\gamma(i)}\right)}{c^u\left(\frac{W}{\gamma(N-1)}\right)}\right)\left(\frac{c^u\left(\frac{W}{\gamma(N)}\right)}{c^u\left(\frac{W}{\gamma(i)}\right)}\right)^{\sigma} \geq |\sigma-\zeta|.$$

From the properties of unit cost functions, it follows that for all $p \in \left[\frac{W}{\gamma(N)}, \frac{W}{\gamma(N-1)}\right]$ we have that $c^u(p) \geq c^u\left(\frac{W}{\gamma(N-1)}\right)\frac{\gamma(N-1)}{\gamma(N)}$ and $c^u\left(\frac{W}{\gamma(N)}\right) \geq c^u(p)\frac{\gamma(N-1)}{\gamma(N)}$. Using these properties, it follows that the sufficient conditions above hold whenever

$$\left(\frac{\gamma(N-1)}{\gamma(N)}\right)^{1+\sigma} \geq |\sigma-\zeta| \qquad \left(\frac{\gamma(N-1)}{\gamma(N)}\right)^{\max\{\sigma,\zeta\}+\sigma} \geq |\sigma-\zeta| \qquad \left(\frac{\gamma(N-1)}{\gamma(N)}\right)^{2+\sigma} \geq |\sigma-\zeta|.$$

Thus a sufficient condition for all three inequalities to hold is

$$\left(\frac{\gamma(N-1)}{\gamma(N)}\right)^{2+2\sigma+\zeta} \geq |\sigma-\zeta|,$$

establishing the desired result. ■

Remaining Proofs from Section 3

Proof of Lemma A2. Let us first suppose that there exists $x \in [0,1]$ for which

$$w_I(x) > \rho + \delta + \theta g > w_N(x). \tag{B9}$$

This condition states that in the neighborhood of $n(t) = x$, both automation and the creation of new complex tasks are profitable, and thus newly created tasks will be immediately produced using labor, and newly automated tasks will be immediately produced using capital. We discuss the cases in which this inequality does not hold for any x below.

We proceed in several steps.

Step 1: We show that $w_I(n)$ is increasing and $w_N(n)$ decreasing in n for all $n \geq x$.

Rewrite the ideal price index condition (9), as in Section 3.2, substituting for the BGP value of the rental rate, $R = \rho + \delta + \theta g$, which yields (17). Differentiating the expressions for effective wages using this BGP value of the interest rate yields equation (18) in the main text.

Suppose first that $\sigma < 1$. Then, for $n \geq x$, the numerator of the first expression is positive, and the numerator of the second expression is negative, and their denominators are positive, and thus $w_I'(n) > 0 > w_N'(n)$. Suppose next that $\sigma > 1$. In this case, the signs of the numerators are flipped, but the denominators are negative, so we reach the same conclusion. From the Fundamental Theorem of Calculus, we can conclude that

$$w_I(n) > \rho + \delta + \theta g > w_N(n) \tag{B10}$$

for all $n \geq x$.

Step 2: We now show that the inequality (B10) cannot hold for all $n \in [0, x)$.

Using the same argument as in Step 1, we have that for $n < x$ and provided that (B10) holds, $w_I(n)$ continues to be increasing and $w_N(n)$ decreasing in n. To obtain a contradiction, suppose that equation (B10) holds for all $n \in [0, x)$. Then,

$$w_I(0) > \rho + \delta + \theta g > w_N(0),$$

but this is impossible, since at $n = 0$, $w_I(0) = w_N(0)$, thus yielding a contradiction.

Step 3: We next show that either there exists $\overline{n}$ or $\widetilde{n}$ as in the lemma, but both thresholds cannot exist simultaneously. Moreover, which threshold exists and is relevant depends on whether $\rho \lesseqgtr \overline{\rho}$. Since (B10) holds at $n = x$, but not at $n = 0$, and both $w_I(n)$ and $w_N(n)$ are continuous, there exists either $\overline{n}$ such that $w_I(\overline{n}) = \rho + \delta + \theta g$, or $\widetilde{n}$ such that $w_N(\widetilde{n}) = \rho + \delta + \theta g$, or both. We now show that only one of these cases may occur, and that ρ determines which case it is.

First, suppose that as we move from $n = x$ to the left, we reach $\overline{n}$ first. Because $R > w_I(n)$ for $n \leq \overline{n}$, in this region there are no incentives to use capital in automated tasks, which implies $I^* < I$. Further increases in I—or reductions in n—do not change the equilibrium allocation. Thus, for $n \leq \overline{n}$, $w_I(\overline{n})$ and $w_N(n)$ are constant, as shown in the left panel of Figure B1. This establishes that, in this case, $\rho + \delta + \theta g > w_N(n)$ for all $n \in [0, 1]$, and there is no threshold $\widetilde{n}$. In this case, for $n > \overline{n}$ we have that inequality (B10) holds, which implies that automated tasks are immediately produced with capital and newly created tasks are immediately produced with labor.

Suppose next that as we move from x to the left, we reach $\widetilde{n}$ first. For $n < \widetilde{n}$, we have that $w_N(n) > \rho + \delta + \theta g$, and thus newly created tasks that use labor are less productive than their old automated. Moreover, equations (18) and (B10) imply that both $w_I(n)$ and $w_N(n)$ are decreasing to the left of $\widetilde{n}$ as shown in the right panel of Figure B1. This establishes that, in this case, $w_I(n) > \rho + \delta + \theta g$ for all $n \in [0, 1]$, and there is no threshold $\overline{n}$.

Consequently, one and only one of $\overline{n}$ and $\widetilde{n}$ will be reached. We now show that which one of these two thresholds is reached first is determined by the discount rate, ρ. For ρ sufficiently small, we necessarily reach the threshold $\widetilde{n}$ first. This is the case depicted in the right panel of Figure B1. Moreover, because

$$\frac{\partial w_N(n)}{\partial \rho} = -\frac{1}{\gamma(n)} \frac{(1-n)c'(\rho + \delta + \theta g)c^u(\rho + \delta + \theta g)^{-\sigma}}{\int_0^n \frac{1}{\gamma(i)} c^u(w_I/\gamma(i))^{-\sigma} c^{u\prime}(w_I/\gamma(i))di} < 0,$$

as ρ increases the curve for $w_N(n)$ shifts down, while the curve for $\rho + \delta + \theta g$ shifts upwards in Figure B1. This implies that, as ρ increases, the interception between these curves, $\widetilde{n}$, shifts to the left and

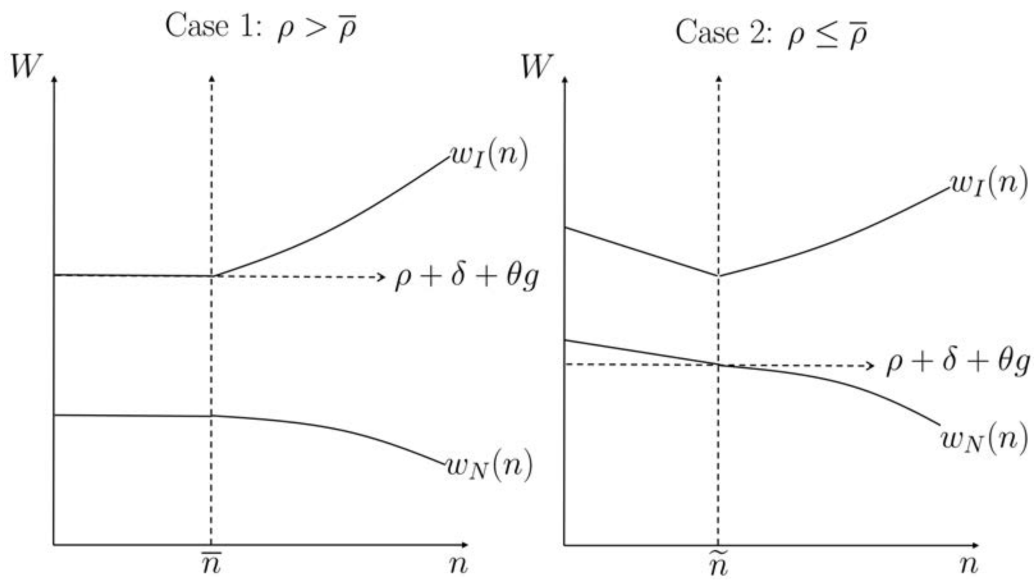

Figure B1: Behavior of unit costs of labor with respect to changes in $n = N - I$ in steady state.

for some ρ we will have that the curves $w_N(n)$ and $\rho + \delta + \theta g$ will intercept exactly at $\widetilde{n} = 0$. These observations imply that there exists a value $\overline{\rho}$ such that

$$w_N(0) = w_I(0) = \overline{\rho} + \delta + \theta g.$$

Let $\overline{\rho}$ denote the smallest ρ for which this is the case. The definition of $\overline{\rho}$ implies that, for $\rho < \overline{\rho}$ the threshold $\widetilde{n}$ is reached and we have the case depicted in the right panel of Figure B1. On the other hand, for $\rho > \overline{\rho}$ we necessarily have that the threshold $\overline{n}$ is reached (the curve $w_N(n)$ is below $\rho + \delta + \theta g$ for these values of ρ) and we have the case depicted in the left panel of Figure B1.

Step 4: We finalize the proof by dealing with the cases in which the inequality (B9) does not hold. Suppose first that the right inequality does not hold. Then we can simply define $\widetilde{n} = 1$, and the lemma applies as is. Suppose next that the left inequality does not hold. Then we define $\overline{n} = 1$, and the lemma applies as is. This concludes the proof of the lemma. ■

Constrained Efficient Allocation and Proofs from Section 5

In this part of the Appendix, we complete the characterization of the constrained efficient allocation. The constrained efficient allocation solves the maximization problem introduced in the main text. To simplify the notation, we denote the marginal product of labor and capital in the planner's allocation by $W^P(I, N, K) = \frac{\partial F^P}{\partial L}$ and $R^P(I, N, K) = \frac{\partial F^P}{\partial K}$, respectively.

The current value Hamiltonean for the planner's problem is given by:

$$H \equiv \frac{C^{1-\theta} - 1}{1 - \theta} + \mu_k \left(F^P(I, N, K, L) - \delta K - C\right) + \mu_L(L^s(\omega^P(I, N, K)) - L) + \mu_I \kappa_I S_I + \mu_N \kappa_N S_N.$$

Here, μ_N and μ_I denote the shadow values of the two types of technology, respectively, and μ_L and μ_K the shadow values of labor and capital. The maximum principle implies that these objects satisfy the necessary conditions:

$$\rho\mu_N - \dot{\mu}_N = \mu_K \frac{\partial F^P}{\partial N} + \mu_L \frac{\partial L^s}{\partial \omega}\frac{\partial \omega^P}{\partial N}, \qquad \rho\mu_I - \dot{\mu}_I = \mu_K \frac{\partial F^P}{\partial I} + \mu_L \frac{\partial L^s}{\partial \omega}\frac{\partial \omega^P}{\partial I},$$

$$\rho\mu_K - \dot{\mu}_K = \mu_K(R^P - \delta) + \mu_L \frac{\partial L^s}{\partial \omega}\frac{\partial \omega^P}{\partial K}, \qquad \mu_L = \mu_K W^P.$$

All the functions in the above differential equations are evaluated at their corresponding arguments at time t.

Moreover, the current value Hamiltonean associated with the planner's problem is concave, so these conditions (plus the Euler equation for consumption and the transversality condition) are sufficient for characterizing the constrained efficient allocation.

We start by providing formulas for $\frac{\partial F^P}{\partial N}$ and $\frac{\partial F^P}{\partial I}$. To derive these formulas, we rewrite the function $F^P(I, N, K, L)$ as:

$$\begin{aligned} F^p(I, N, K, L) \equiv \max_{k(i), l(i), q_K(i), q_L(i)} & \left[\int_{N-1}^{I} y^p(q_K(i), k(i))^{\frac{\sigma-1}{\sigma}} + \int_{I}^{N} y^p(q_L(i), \gamma(i)l(i))^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \\ & - \mu\psi \int_{N-1}^{I} q_K(i)di - \mu\psi \int_{I}^{N} q_L(i)di \\ & \text{subject to: } \int_{N-1}^{I} k(i)di = K, \text{ and } \int_{I}^{N} l(i)di = L. \end{aligned}$$

Here, $y^p(q_K(i), k(i))$ and $y^p(q_L(i), \gamma(i)l(i))$ denote the production of a task when using $q_K(i)$ and $q_L(i)$ units of the intermediate mixed with $k(i)$ units of capital or $\gamma(i)l(i)$ units of effective labor, respectively (see equations (2) and (3)). Notice that we have written this problem assuming that the planner chooses $I^* = I$ at all times, as we remarked in the proposition. B1. Also, notice that the multipliers for the restriction on total capital is R^p—the shadow rental rate—, and the multiplier for the restriction on total labor is W^p—the shadow wage.

Denote by $c^p(\cdot)$—rather than $c^u(\cdot)$—the unit cost of producing a task when intermediates are priced at their marginal cost. An application of the envelope theorem to the above problem yields

$$\begin{aligned} \frac{\partial F^p}{\partial I} =& \frac{\sigma}{\sigma-1} Y^{\frac{1}{\sigma}} \left[y^p(q_K(I), k(I))^{\frac{\sigma-1}{\sigma}} - y^p(q_L(I), \gamma(I)l(I))^{\frac{\sigma-1}{\sigma}} \right] \\ & - \mu\psi(q_K(I) - q_L(I)) - (R^p k(I) - W^p l(I)) \\ =& \frac{\sigma}{\sigma-1} Y \left[c^p\left(R^p\right)^{1-\sigma} - c^p\left(\frac{W^p}{\gamma(I)}\right)^{1-\sigma} \right] - Y \left[c^p\left(R^p\right)^{1-\sigma} - c^p\left(\frac{W^p}{\gamma(I)}\right)^{1-\sigma} \right] \\ =& \frac{1}{1-\sigma} Y \left[c^p\left(\frac{W^p}{\gamma(I)}\right)^{1-\sigma} - c^p\left(R^p\right)^{1-\sigma} \right]. \end{aligned}$$

Here, we have used the fact that the planner sets the level of production of a task to $c^p(\cdot)\, Y^{-\sigma}$, with $c^p(\cdot)$ its unitary cost.

Likewise,

$$\begin{aligned}\frac{\partial F^p}{\partial N} =&\frac{\sigma}{\sigma-1}Y^{\frac{1}{\sigma}}\left[y^p(q_L(N),\gamma(N)l(N))^{\frac{\sigma-1}{\sigma}}-y^p(q_K(N-1),k(N-1))^{\frac{\sigma-1}{\sigma}}\right]\\&-\mu\psi(q_L(N)-q_K(N-1))-(W^pl(N)-R^pk(N-1))\\=&\frac{\sigma}{\sigma-1}Y\left[c^p\left(\frac{W^p}{\gamma(N)}\right)^{1-\sigma}-c^p\left(R^p\right)^{1-\sigma}\right]-Y\left[c^p\left(\frac{W^p}{\gamma(N)}\right)^{1-\sigma}-c^p\left(R^p\right)^{1-\sigma}\right]\\=&\frac{1}{1-\sigma}Y\left[c^p\left(R^p\right)^{1-\sigma}-c^p\left(\frac{W^p}{\gamma(N)}\right)^{1-\sigma}\right].\end{aligned}$$

Let $\Psi_N(t)\equiv\frac{\mu_N(t)}{\mu_K(t)Y(t)}$ and $\Psi_I\equiv\frac{\mu_I(t)}{\mu_K(t)Y(t)}$ be the shadow discounted net present values of new technologies (in terms of additional net output they create). These values are then given by:

$$\begin{aligned}\Psi_I(t)=&\int_t^\infty e^{-\int_0^\tau\left(R^P\left(1+\omega^PL^s\varepsilon_L\frac{\partial\ln\omega^P}{\partial\ln K}\right)-\delta-g(s)\right)ds}\left(\frac{c^p(w_I^P(\tau))^{1-\sigma}-c^p(R^P(\tau))^{1-\sigma}}{1-\sigma}+s_L\varepsilon_L\frac{\partial\ln\omega^P}{\partial I}\right)d\tau\\\Psi_N(t)=&\int_t^\infty e^{-\int_0^\tau\left(R^P\left(1+\omega^PL^s\varepsilon_L\frac{\partial\ln\omega^P}{\partial\ln K}\right)-\delta-g(s)\right)ds}\left(\frac{c^p(R^P(\tau))^{1-\sigma}-c^p(w_N^P(\tau))^{1-\sigma}}{1-\sigma}+s_L\varepsilon_L\frac{\partial\ln\omega^P}{\partial N}\right)d\tau.\end{aligned}\tag{B12}$$

Here, s_L is the economy-wide labor share, w_I^P is the normalized wage $W^p/\gamma(I)$, and w_N^P is the normalized wage $W^p/\gamma(N)$.

These equations are analogous to the expressions for v_I and v_N in the decentralized equilibrium given by equations (22) and (23). In particular, notice that like prospective technology monopolists, the planner also values the automation of existing tasks or the introduction of new more complex tasks depending on the difference in production costs. Thus, our structure of intellectual property rights produces private incentives for innovation that share this common feature with the social value of such innovations. However, because technology monopolists only capture a non-constant share of the surplus that new tasks generate, the exact expressions for Ψ_I and Ψ_N differ from those of v_I and v_N. Besides these differences the expressions for the social value of introducing different technologies show that the planner also take into account their effect on employment, captured by the terms $s_L\varepsilon_L\frac{\partial\ln\omega^P}{\partial I}$ and $s_L\varepsilon_L\frac{\partial\ln\omega^P}{\partial N}$ in equation (B12). The social planner cares about this margin because changes in employment have a first-order positive effect on workers owing to the gap between wage and the opportunity cost of work.

Given the current values for $\Psi_I(t)$ and $\Psi_N(t)$, the optimal allocation of scientists to the two different types of research then satisfies

$$S_I(t)=SG\left(\frac{\kappa_I\Psi_I-\kappa_N\Psi_N}{\lambda}\right)\in[0,S]\qquad S_N(t)=S\left[1-G\left(\frac{\kappa_I\Psi_I-\kappa_N\Psi_N}{\lambda}\right)\right]\in[0,S].\tag{B13}$$

Following our characterization in Section 4, we denote the normalized output in the socially planned economy by $f^P(k,n)=F^P(I,N,K,L(\omega^P(I,N,K)))/\gamma(I)$, and the rental rate, $R^P(n,k)$ as functions of technology and capital. The constrained efficient allocation can be represented as a time path for the variables $\{c(t),k(t),n(t),S_I(t),\Psi_I(t),\Psi_N(t)\}_{t=0}^\infty$ such that:

- Consumption satisfies the Euler equation

$$\frac{\dot{c}(t)}{c(t)} = \frac{1}{\theta}\left(R^P(n(t), k(t))\left(1 + \omega^P L^s \varepsilon_L \frac{\partial \ln \omega^P}{\partial \ln K}\right) - \delta - \rho\right) - A\kappa_I S_I(t),$$

together with the transversality conditions

$$\lim_{t\to\infty} \mu_k K e^{-\rho t} = 0, \qquad \lim_{t\to\infty} I\mu_I e^{-\rho t} = 0, \qquad \lim_{t\to\infty} N\mu_N e^{-\rho t} = 0.$$

- Capital satisfies the resource constraint in equation

$$\dot{k}(t) = f^P(n(t), k(t)) - c(t) - (\delta + A\kappa_I S_I(t))k(t)$$

- The gap between automation and the creation of new tasks, $n(t) = N(t) - I(t)$, satisfies:

$$\dot{n}(t) = \kappa_N S - (\kappa_I + \kappa_N) G\left(\frac{\kappa_I \Psi_I - \kappa_N \Psi_N}{\lambda}\right) S$$

- The allocation of scientists satisfies the allocation rule in equation (B13).

- The social values of allocating scientists to develop different technologies, $\Psi_I(t)$ and $\Psi_N(t)$, satisfy equation (B12).

The following proposition summarizes the properties of the constrained efficient allocation and provides a set of taxes and subsidies that can be used to decentralize it.

Proposition B1 (Constrained efficient allocation and decentralization) *Under the same conditions as in Proposition 5, the constrained efficient allocation admits a unique BGP. Moreover, the constrained efficient allocation locally converges to this BGP, and if $\theta \to 0$, it globally converges to this BGP.*

Finally, the constrained efficient allocation can be decentralized by using the following sets of taxes and subsidies:

1. *A proportional subsidy at the rate $1-\mu$ on intermediate prices to remove the monopoly markups.*

2. *A proportional subsidy/tax of $\omega^P L^s \varepsilon_L \frac{\partial \ln \omega^P}{\partial \ln K}$ on gross interests on savings. This subsidy/tax corrects for the impact of capital on employment (this expression yields a positive subsidy when $\sigma_{SR} < 1$ and capital raises the labor share, and a tax in the opposite case).*

3. *Proportional subsidies/taxes on the profits of successful innovators who entered the market at time $t' \leq t$. These subsides/taxes correct for the technological externality generated by the two different types of innovation and appropriability problems.*

4. *A proportional subsidy on the profits of successful innovators who create new complex tasks. This subsidy is proportional to $s_L \varepsilon_L \frac{\partial \ln \omega^P}{\partial N} \geq 0$, and corrects for the fact that these technology monopolists do not take into account the positive effect of new complex tasks on the level of equilibrium employment.*

5. *A proportional tax on the profits of successful innovators who automate existing tasks. This tax is proportional to* $-s_L \varepsilon_L \frac{\partial \ln \omega^P}{\partial I} \geq 0$*, and corrects for the fact that these technology monopolists do not take into account the negative effect of automation on the level of equilibrium employment.*

Proof. Using the formula in equation (B12), we now establish the decentralization result by construction. First, assume the planner subsidizes a fraction $1-\mu$ to the price of intermediate goods, and sets a subsidy to interests on capital savings of $s_k = \omega^P L^s \varepsilon_L \frac{\partial \ln \omega^P}{\partial \ln K}$. This guarantees households discount future income at the socially optimal rate.

Now, we can decentralize the planner's allocation by subsidizing/taxing the period τ profits of a firm that automates task $I(t)$ at time t at the rate $s_I(t,\tau)$, and the period τ profits of a firm that introduces a new complex task $N(t)$ at time t at the rate $s_N(t,\tau)$.

With these subsidies, the value of automating jobs or creating new tasks is given by a small modification of equations (22) and (23), which takes into account that firms sell intermediates at a price ψ, but buyers perceive a price $\mu\psi$ because of the subsidy. These values also discount future profits as the same rate the planner does because of the subsidies/taxes to capital accumulation and the subsidies to the two different types of innovations, $s_I(t,\tau)$ and $s_N(t,\tau)$. Thus:

$$V_I(t) = Y(t)\int_t^\infty e^{-\int_0^\tau \left(R^P\left(1+\omega^P L^s \varepsilon_L \frac{\partial \ln \omega^P}{\partial \ln K}\right)-\delta-g(s)\right)ds} M \cdot \left(c^P\left(R^P(\tau)\right)^{\zeta-\sigma} - c^P\left(w_I^P(\tau)\frac{\gamma(I(\tau))}{\gamma(I(t))}\right)^{\zeta-\sigma}\right)\cdot(1+s_I(\tau))d\tau,$$

$$V_N(t) = Y(t)\int_t^\infty e^{-\int_0^\tau \left(R^P\left(1+\omega^P L^s \varepsilon_L \frac{\partial \ln \omega^P}{\partial \ln K}\right)-\delta-g(s)\right)ds} M \cdot \left(c^P\left(w_N^P(\tau)\frac{\gamma(N(\tau))}{\gamma(N(t))}\right)^{\zeta-\sigma} - c^P\left(R^P(\tau)\right)^{\zeta-\sigma}\right)\cdot(1+s_N(\tau))d\tau,$$

where recall that $M \equiv (1-\mu)\psi\left(\frac{\eta}{1-\eta}\right)^\zeta(\mu\psi)^{-\zeta}$. Here we used the cost function $c^p(\cdot)$ which takes into account that intermediates are already priced at their marginal cost.

The following subsidy-tax policy ensures that the incentives of successful innovators are aligned with the social value of their innovations and provide a way to decentralize the planner's allocation:

$$\begin{aligned}1+s_I(t,\tau) = &\left(\frac{1}{(1-\mu)\psi\left(\frac{\eta}{1-\eta}\right)^\zeta(\mu\psi)^{-\zeta}}\right)\times\left(\frac{c^p\left(R^P(\tau)\right)^{\zeta-\sigma}-c^p\left(w_I^P(\tau)\right)^{\zeta-\sigma}}{c^p\left(R^P(\tau)\right)^{\zeta-\sigma}-c^p\left(w_I^P(\tau)\frac{\gamma(I(\tau))}{\gamma(I(t))}\right)^{\zeta-\sigma}}\right)\\ &\times\left(\frac{\frac{c^p(w_I^P(\tau))^{1-\sigma}-c^p(R^P(\tau))^{1-\sigma}}{1-\sigma}}{c^p\left(R^P(\tau)\right)^{\zeta-\sigma}-c^p\left(w_I^P(\tau)\right)^{\zeta-\sigma}}\right)\times\left(\frac{\frac{c^p(w_I^P(\tau))^{1-\sigma}-c^p(R^P(\tau))^{1-\sigma}}{1-\sigma}+s_L\varepsilon_L\frac{\partial\ln\omega^P}{\partial I}}{\frac{c^p(w_I^P(\tau))^{1-\sigma}-c^p(R^P(\tau))^{1-\sigma}}{1-\sigma}}\right),\end{aligned}$$

and

$$\begin{aligned}1+s_N(t,\tau) = &\left(\frac{1}{(1-\mu)\psi\left(\frac{\eta}{1-\eta}\right)^\zeta(\mu\psi)^{-\zeta}}\right)\times\left(\frac{c^p\left(w_N^P(\tau)\right)^{\zeta-\sigma}-c^p\left(R^P(\tau)\right)^{\zeta-\sigma}}{c^p\left(w_N^P(\tau)\frac{\gamma(N(\tau))}{\gamma(N(t))}\right)^{\zeta-\sigma}-c^p\left(R^P(\tau)\right)^{\zeta-\sigma}}\right)\\ &\times\left(\frac{\frac{c^p(R^P(\tau))^{1-\sigma}-c^p(w_N^P(\tau))^{1-\sigma}}{1-\sigma}}{c^p\left(w_N^P(\tau)\right)^{\zeta-\sigma}-c^p\left(R^P(\tau)\right)^{\zeta-\sigma}}\right)\times\left(\frac{\frac{c^p(R^P(\tau))^{1-\sigma}-c^p(w_N^P(\tau))^{1-\sigma}}{1-\sigma}+s_L\varepsilon_L\frac{\partial\ln\omega^P}{\partial N}}{\frac{c^p(R^P(\tau))^{1-\sigma}}{1-\sigma}-c^p(w_N^P(\tau))^{1-\sigma}}\right).\end{aligned}$$

These subsidies/taxes can be separated into several components as illustrated by the way in which we have written them. First, profits from both types of innovations get a gross subsidy of

$$\left(\frac{1}{(1-\mu)\psi\left(\frac{\eta}{1-\eta}\right)^\zeta(\mu\psi)^{-\zeta}}\right) > 1.$$

This term captures the known fact that innovators only manage to extract a fraction of the surplus they generate.

Second, profits from automation get taxed at a gross rate

$$\frac{c^p\left(R^P(\tau)\right)^{\zeta-\sigma}-c^p\left(w_I^P(\tau)\right)^{\zeta-\sigma}}{c^p\left(R^P(\tau)\right)^{\zeta-\sigma}-c^p\left(w_I^P(\tau)\frac{\gamma(I(\tau))}{\gamma(I(t))}\right)^{\zeta-\sigma}}<1;$$

while profits from creating new complex tasks get subsidized at the rate

$$\frac{c^p\left(w_N^P(\tau)\right)^{\zeta-\sigma}-c^p\left(R^P(\tau)\right)^{\zeta-\sigma}}{c^p\left(w_N^P(\tau)\frac{\gamma(N(\tau))}{\gamma(N(t))}\right)^{\zeta-\sigma}-c^p\left(R^P(\tau)\right)^{\zeta-\sigma}}>1.$$

These taxes/subsidies correct for a *technological externality*: by inventing new tasks and increasing N, monopolists improve the quality of intermediates that future entrants are able to develop, which creates a positive externality on subsequent innovators. The opposite occurs for automation: by automating task I, new entrants will be forced to automate more complex tasks, in which they will obtain fewer profits. These taxes/subsidies depend on the time at which a task was introduced t—since they are a compensation (or charge) for all technologies built on top of them. This is why the subsidies that are needed to decentralize the planner allocation, $s_I(t,\tau)$ and $s_N(t,\tau)$, not only depend on the current time period, τ, but also on the time that the innovation took place, t.

Third, profits from automation get taxed/subsidized at a gross rate:

$$\frac{\frac{c^p(w_I^P(\tau))^{1-\sigma}-c^p(R^P(\tau))^{1-\sigma}}{1-\sigma}}{c^p\left(R^P(\tau)\right)^{\zeta-\sigma}-c^p\left(w_I^P(\tau)\right)^{\zeta-\sigma}};$$

while profits from creating new complex task get subsidized at a gross rate:

$$\frac{\frac{c^p(R^P(\tau))^{1-\sigma}-c^p(w_N^P(\tau))^{1-\sigma}}{1-\sigma}}{c^p\left(w_N^P(\tau)\right)^{\zeta-\sigma}-c^p\left(R^P(\tau)\right)^{\zeta-\sigma}}.$$

These subsidies account for the fact that, as the technology for producing tasks is a constant elasticity of substitution function, the monopolists who supply intermediate goods will charge a constant markup. When $\zeta=1$ (and $\sigma>1$), so that monopolists earn a constant fraction of the value of the task, these taxes/subsidies collapse to $\frac{1}{\sigma-1}\lesseqgtr 1$.

Finally, profits from automation get taxed at a gross rate:

$$\frac{\frac{c^p(w_I^P(\tau))^{1-\sigma}-c^p(R^P(\tau))^{1-\sigma}}{1-\sigma}+s_L\varepsilon_L\frac{\partial\ln\omega^P}{\partial I}}{\frac{c^p(w_I^P(\tau))^{1-\sigma}-c^p(R^P(\tau))^{1-\sigma}}{1-\sigma}}<1;$$

while profits from creating new complex task get subsidized at a gross rate:

$$\frac{\frac{c^p(R^P(\tau))^{1-\sigma}-c^p(w_N^P(\tau))^{1-\sigma}}{1-\sigma}+s_L\varepsilon_L\frac{\partial\ln\omega^P}{\partial N}}{\frac{c^p(R^P(\tau))^{1-\sigma}}{1-\sigma}-c^p(w_N^P(\tau))^{1-\sigma}}>1.$$

these taxes/subsidies correct for the fact that technology monopolists do not take into account the effect of technologies on the quasi-supply of labor. While, all else equal, automation reduces the supply of labor, the creation of new complex tasks increases it. Because at the margin increasing the level of employment improves welfare (given that it has no opportunity cost), the planner taxes profits from automation and subsidizes profits from the creation of new complex tasks.

Notice that the scientist allocation can be decentralized in many ways. In particular, since there is a fixed supply of scientists, we only need to get the *relative* expected profits from each type of innovation right. The particular decentralization outlined here guarantees the level of innovators' profits also matches the social value of innovation. Even if both types of technology end up being subsidized in equilibrium, this does not matter because the money can be recovered by taxing scientists ■

Remarks: Note first that in contrast to neoclassical models of capital taxation (e.g., Chamley, 1986 and Judd, 1985, but also see Straub and Werning, 2014), the decentralization of the constrained efficient allocation requires taxing or subsidizing capital accumulation. This is because the capital stock affects wages and thus the level of employment through the quasi-labor supply schedule. For instance, if $\sigma_{SR} < 1$, capital increases the labor share and employment in the short run (see Proposition 2) which is, as noted above, beneficial (recall that workers strictly prefer to work than not). Thus in this case, the social planner would subsidize capital accumulation. When $\sigma_{SR} > 1$, and capital reduces the labor share and employment, the opposite applies.

Second, the quality ladder structure in the creation of new complex tasks introduces a technological externality. By undertaking this type of innovation and thus increasing N, a technology monopolist also allows future innovators to create more productive new tasks (because $\gamma(N)$ is increasing). Automation creates an opposite and somewhat more subtle externality. Because capital has the same productivity in all automated tasks, there is no direct technological externality. But automation today forces future innovators to automate higher-indexed tasks, which are the ones in which labor has a comparative advantage (because $\gamma(I)$ is increasing), and this reduces the incremental profits of future innovators.

Finally and most importantly, the quasi-labor supply schedule creates an additional and novel distortion in the equilibrium relative to the constrained efficient allocation. Because firms do not internalize that the quasi-rents received by workers are transfers, they automate tasks taking into account the wage rate. In contrast, the social planner understand that these quasi-rents are transfers, and thus at the margin bases her automation decisions on the opportunity cost of labor rather than the market wage. Equivalently, because the planner recognizes that wages are above the opportunity cost of labor, she prefers to create more employment. In the market allocation, the resulting greater incentives of firms to automate tasks than what is socially optimal translate into too much R&D directed towards automation and too little R&D directed towards the creation of new complex tasks. For this reason, the social planner would like to encourage (subsidize) the creation of new complex tasks and discourage (tax) automation, as outlined in parts 4 and 5 of the proposition.

Proof of Proposition 6: Let $S_I(t)$ and $S_N(t)$ denote the allocation of scientists, and consider the allocation obtained by a small deviation $S_N^P(t)=\min\{S_N(t)+\nu,1\}$ and $S_I^P(t)=\max\{S_I(t)-\nu,0\}$ if $S_I(t)<1$, and $S_N^P(t)=S_N(t)$, $S_I^P(t)=S_I(t)$ otherwise.

Clearly, the new allocation satisfies $n^P(t)\geq n(t)$. Furthermore, we have that asymptotically $n^P(t)>n(t)$. We prove that for a small $\nu>0$, such deviation increases welfare and reduces the extent of automation.

For ν small enough, we have that the above allocation changes welfare by $\nu(\kappa_N\mu_N(t)-\kappa_I\mu_I(t))$, whenever $S_I(t),S_N(t)\in(0,1)$. Moreover, whenever the allocation of scientists is an interior one, we have $\kappa_N V_N(t)=\kappa_I V_I(t)$.

Thus, to prove that the reallocation of scientists away from automation increases welfare, it is enough to verify that, whenever $\kappa_N V_N(t)=\kappa_I V_I(t)$, we have $\kappa_N\mu_N-\kappa_I\mu_I>0$. This is equivalent to proving the inequality:

$$\frac{\Psi_N(t)}{\Psi_I(t)}>\frac{V_N(t)}{V_I(t)},$$

with all value functions evaluated at the decentralized equilibrium path.

Because we assumed that in the decentralized allocation the intermediate goods are subsidized at the rate $1-\mu$ (or equivalently, that $\mu\to 1$), unit costs are given by $c^P(\cdot)$, and factor prices are given by w_N^P,w_I^P and R^P.

Thus, we can compute Ψ_N and Ψ_I as:

$$\begin{aligned}
\Psi_I(t)=&Y(t)\int_t^\infty e^{-\int_0^\tau\left(R^P-\delta-g(s)\right)ds}\left(\frac{c^p(w_I^P(\tau))^{1-\sigma}-c^p(R^P(\tau))^{1-\sigma}}{1-\sigma}+s_L\varepsilon_L\frac{\partial\ln\omega^P}{\partial I}\right)d\tau,\\
\Psi_N(t)=&Y(t)\int_t^\infty e^{-\int_0^\tau\left(R^P-\delta-g(s)\right)ds}\left(\frac{c^p(R^P(\tau))^{1-\sigma}-c^p(w_N^P(\tau))^{1-\sigma}}{1-\sigma}+s_L\varepsilon_L\frac{\partial\ln\omega^P}{\partial N}\right)d\tau.
\end{aligned}$$

These are variants of the formula provided in equation (B12) in the main text, in which we now discount future welfare gains from technology at the household discount rate.

However, this implies the inequalities:

$$\begin{aligned}
\frac{\Psi_N(t)}{\Psi_I(t)}&=\frac{\int_t^\infty e^{-\int_0^\tau\left(R^P-\delta-g(s)\right)ds}\left(\frac{c^p(R^P(\tau))^{1-\sigma}-c^p(w_N^P(\tau))^{1-\sigma}}{1-\sigma}+s_L\varepsilon_L\frac{\partial\ln\omega^P}{\partial N}\right)d\tau}{\int_t^\infty e^{-\int_0^\tau\left(R^P-\delta-g(s)\right)ds}\left(\frac{c^p(w_I^P(\tau))^{1-\sigma}-c^p(R^P(\tau))^{1-\sigma}}{1-\sigma}+s_L\varepsilon_L\frac{\partial\ln\omega^P}{\partial I}\right)d\tau}\\
&>\frac{\int_t^\infty e^{-\int_0^\tau\left(R^P-\delta-g(s)\right)ds}\frac{c^p(R^P(\tau))^{1-\sigma}-c^p(w_N^P(\tau))^{1-\sigma}}{1-\sigma}d\tau}{\int_t^\infty e^{-\int_0^\tau\left(R^P-\delta-g(s)\right)ds}\frac{c^p(w_I^P(\tau))^{1-\sigma}-c^p(R^P(\tau))^{1-\sigma}}{1-\sigma}d\tau}\\
&>\frac{\int_t^\infty e^{-\int_0^\tau\left(R^P-\delta-g(s)\right)ds}\frac{c^p(R^P(\tau))^{1-\sigma}-c^p\left(w_N^P(\tau)\frac{N(\tau)}{N(t)}\right)^{1-\sigma}}{1-\sigma}d\tau}{\int_t^\infty e^{-\int_0^\tau\left(R^P-\delta-g(s)\right)ds}\frac{c^p\left(w_I^P(\tau)\frac{I(\tau)}{I(t)}\right)^{1-\sigma}-c^p(R^P(\tau))^{1-\sigma}}{1-\sigma}d\tau}\\
&\geq\frac{\int_t^\infty e^{-\int_0^\tau\left(R^P-\delta-g(s)\right)ds}\left(c^p\left(w_N^P(\tau)\frac{N(\tau)}{N(t)}\right)^{\zeta-\sigma}-c^p(R^P(\tau))^{\zeta-\sigma}\right)d\tau}{\int_t^\infty e^{-\int_0^\tau\left(R^P-\delta-g(s)\right)ds}\left(c^p(R^P(\tau))^{\zeta-\sigma}-c^p\left(w_I^P(\tau)\frac{I(\tau)}{I(t)}\right)^{\zeta-\sigma}\right)d\tau}\\
&=\frac{V_N(t)}{V_I(t)},
\end{aligned}$$

as we set out to prove.

The first inequality follows from the novel inefficiency introduced in this paper: the fact that labor gets rents in equilibrium pushes towards the underprovision of new tasks and excessive automation. The second inequality follows from the technological externality; which as explained above pushes towards the underprovision of new tasks. The last inequality follows by noting that:

$$\frac{\frac{c^p(R^P(\tau))^{1-\sigma}-c^p\left(w_N^P(\tau)\frac{N(\tau)}{N(t)}\right)^{1-\sigma}}{1-\sigma}}{c^p\left(w_N^P(\tau)\frac{N(\tau)}{N(t)}\right)^{\zeta-\sigma}-c^p(R^P(\tau))^{\zeta-\sigma}} \geq \frac{\frac{c^p\left(w_I^P(\tau)\frac{I(\tau)}{I(t)}\right)^{1-\sigma}-c^p(R^P(\tau))^{1-\sigma}}{1-\sigma}}{c^p(R^P(\tau))^{\zeta-\sigma}-c^p\left(w_I^P(\tau)\frac{I(\tau)}{I(t)}\right)^{\zeta-\sigma}}.$$

To prove this inequality, denote by $A_N = c^p\left(w_N^P(\tau)\frac{N(\tau)}{N(t)}\right)^{\zeta-\sigma}$, $A_I = c^p\left(w_N^P(\tau)\frac{I(\tau)}{I(t)}\right)^{\zeta-\sigma}$, $A_K = c^p(R$ $^p(\tau))^{\zeta-\sigma}$.

The last inequality is then equivalent to

$$\frac{h(A_K)-h(A_I)}{A_K-A_I} \geq \frac{h(A_N)-h(A_K)}{A_N-A_K},$$

where $h(x)=\frac{1}{1-\sigma}x^{\frac{1-\sigma}{\zeta-\sigma}}$. When $\zeta \geq 1$ this function is (weakly)convex, and the above inequality follows from the convexity of $h(\cdot)$ and the observation that $A_N > A_K > A_I$.

When $\zeta < 1$ the last inequality is reversed. Thus, in this case, the inequality $\frac{\Psi_N(t)}{\Psi_I(t)} > \frac{V_N(t)}{V_I(t)}$, holds only if ε_L is sufficiently large■

Proofs from Section 6

Proof of Proposition 7: Consider an exogenous path for technology in which $\dot{N} = \dot{I} = \Delta$ and $I^*(t) = I(t)$ so that $N(t) - I^*(t) = n$. We show that the economy admits a BGP in which $\dot{M} = \Delta$, and therefore $N(t) - M(t) = n - m$.

Denote the normalized wages of low-skill and high-skill labor by $w_L = W_L/\gamma(I^*)$ and $w_H = W_H/\gamma(I^*)$, respectively. Balanced growth requires that these two normalized wages converge to constant values, so that both grow at the same rate as the aggregate economy. A BGP must satisfy two additional conditions. First, because at time t firms are indifferent between producing task $M(t)$ with low-skill or high-skill workers, we must Have

$$\Gamma(N(t)-M(t)) = \Gamma(n-m) = \frac{w_L}{w_H}. \tag{B14}$$

Moreover, because $\Gamma(N(t)-i)$ is decreasing in i, low-skill workers have comparative advantage in low-index tasks, and thus will produce the tasks in $(I^*(t), M(t))$, while high-skill workers will produce the tasks in $[M(t), N(t)]$.

The second condition is that the wage gap must also be consistent with market clearing. The market clearing conditions are given by:

$$K = Y(I^* - N + 1)c^u(R)^{\zeta-\sigma}R^{-\zeta},$$
$$L^s\left(\frac{W_L}{RK}\right) = Y\int_{I^*}^{M}\gamma_L(i,t)^{\zeta-1}c^u\left(\frac{W_L}{\gamma_L(i,t)}\right)^{\zeta-\sigma}W_L^{-\zeta}di,$$
$$H^s\left(\frac{W_H}{RK}\right) = Y\int_{M}^{N}\gamma_H(i)^{\zeta-1}c^u\left(\frac{W_H}{\gamma_H(i)}\right)^{\zeta-\sigma}W_H^{-\zeta}di.$$

Using the normalized variables $k \equiv K/\gamma(I^*)$ and $y \equiv Y/\gamma(I^*)$, these conditions can be rewritten as

$$k = y(1-n)c^u(R)^{\zeta-\sigma}R^{-\zeta},$$
$$L^s\left(\frac{w_L\gamma}{Rk}\right)\gamma = y\int_0^m (\gamma_H(i)\Gamma(n-i))^{\zeta-1}c^u\left(\frac{w_L}{\gamma_H(i)\Gamma(n-i)}\right)^{\zeta-\sigma} w_L^{-\zeta}di,$$
$$H^s\left(\frac{w_H}{Rk}\right) = y\int_m^n \gamma_H(i)^{\zeta-1}c^u\left(\frac{w_H}{\gamma_H(i)}\right)^{\zeta-\sigma} w_H^{-\zeta}di.$$

We can also observe that, under our maintained assumptions, the demand for low-skill and high-skill labor is isoelastic with a negative elasticity of ε_D. In particular, this elasticity is equal to σ if $\sigma \to \zeta$ or $\eta \to 0$, or to $\sigma s + (1-s)$ if $\zeta \to 1$, with s the constant share of labor or capital in the production of a task.

Moreover, because we assumed that the supply of labor is isoelastic, we can write the relative demand for low-skill and high-skill labor as:

$$\frac{L^s(1)}{H^s(1)}\left(\frac{w_L}{w_H}\right)^{\nu_L} = \frac{\int_0^m(\gamma_H(i)\Gamma(n-i))^{\zeta-1}c^u\left(\frac{1}{\gamma_H(i)\Gamma(n-i)}\right)^{\zeta-\sigma}di}{\int_m^n \gamma_H(i)^{\zeta-1}c^u\left(\frac{1}{\gamma_H(i)}\right)^{\zeta-\sigma}di}\left(\frac{w_L}{w_H}\right)^{-\varepsilon_D} \tag{B15}$$

Combining equations (B14) and (B15), we conclude that there is a BGP if and only if there exists a threshold m such that:

$$\frac{L^s(1)}{H^s(1)}\Gamma(n-m)^{\nu_L+\varepsilon_D} = \frac{\int_0^m(\gamma_H(i)\Gamma(n-i))^{\zeta-1}c^u\left(\frac{1}{\gamma_H(i)\Gamma(n-i)}\right)^{\zeta-\sigma}di}{\int_m^n \gamma_H(i)^{\zeta-1}c^u\left(\frac{1}{\gamma_H(i)}\right)^{\zeta-\sigma}di}.$$

To show that this threshold always exists, notice that the left-hand side of the above equation decreases with m, while the right-hand side increases with m. Furthermore, when $m \to 0$ the right-hand side converges to zero, and when $m \to n$ the right-hand side converges to infinity. Thus, there is a unique value of $m \in (0,n)$ for which the above equality holds. This establishes our claim that there is a unique BGP.

We omit the proof of part 3 of the proposition, which establishes the effect of changes in technology on inequality. The required steps are similar to those in the proof of Lemma A1. Specifically, similar steps established that automation reduces the relative demand for low-skill labor, but at the same time the productivity gains it generates increase the demand for high-skilled labor, thus increasing inequality. On the other hand, an increase in N raises the demand for high-skill labor labor relative to the demand for low-skill workers, also increasing inequality. ■

Proof of Proposition 8: We provide formulas for the asymptotic behavior of the value functions in this case. From the Bellman equations provided in the main text, it follows that along a BGP we have

$$v_N(n) = M\int_0^{\frac{n}{\Delta}} e^{-(\rho-(1-\theta)g)\tau}c^u\left(w_N(n)e^{g\tau}\right)^{\zeta-\sigma}d\tau,$$
$$v_I(n) = M\int_0^{\frac{1-n}{\Delta}} e^{-(\rho-(1-\theta)g)\tau}c^u\left(\rho+\delta+\theta g\right)^{\zeta-\sigma}d\tau.$$

Here $\Delta = \frac{\kappa_I \kappa_N \phi(n^D)}{\kappa_I \phi(n^D)+\kappa_N} S$ is the endogenous rate at which both technologies grow in a BGP.

As before, a BGP requires that n^D satisfies

$$\kappa_I \phi(n^D) v_I(n^D) = \kappa_N v_N(n^D).$$

Using these formulas, the proof of the proposition follows the same steps as in the proof of Proposition 5. Using the same steps, we also obtain that the equilibrium in this case is locally stable whenever $\kappa_I \phi(n) v_I(n)$ cuts $\kappa_N v_N(n)$ from below. ■

When New Tasks Also Use Capital

In our baseline model, new tasks use only labor. This simplifying assumption facilitated our analysis, but is not crucial or even important for our results. Here we outline a version of the model where new tasks also use capital and show that all of our results continue to hold in this case. Suppose, in particular, that the production function for non-automated tasks is

$$y(i) = B\left[\eta q(i)^{\frac{\zeta-1}{\zeta}} + (1-\eta)\left(B_\nu (\gamma(i) l(i))^\nu k(i)^{1-\nu}\right)^{\frac{\zeta-1}{\zeta}}\right]^{\frac{\zeta}{\zeta-1}}, \tag{B16}$$

where $k(i)$ is the capital used in the production of the task, $\nu \in (0,1)$, and $B_\nu \equiv \nu^{-\nu}(1-\nu)^{-(1-\nu)}$ is a constant that is re-scaled to simplify the algebra.

Automated tasks $i \le I$ can be produced using labor or capital, and their production function takes the form

$$y(i) = B\left[\eta q(i)^{\frac{\zeta-1}{\zeta}} + (1-\eta)\left(k(i) + B_\nu (\gamma(i) l(i))^\nu k(i)^{1-\nu}\right)^{\frac{\zeta-1}{\zeta}}\right]^{\frac{\zeta}{\zeta-1}}. \tag{B17}$$

Comparing these production functions to those in our baseline model ((2) and (3)), we readily see that the only difference is the requirement that labor has to be combined with capital in all tasks (while automated tasks continue not to use any labor). Note also that when $\nu \to 1$, we recover the model in the main text as a special case. It can be shown using a very similar analysis to that in our main model that most of the results continue to hold with minimal modifications. For example, there will exist a threshold $\widetilde{I}$ such that tasks below $I^* \equiv \min\{I, \widetilde{I}\}$ will be produced using capital and the remaining more complex tasks will be produced using labor. Specifically, whenever $R \in \arg\min\left\{R, R^{1-\nu}\left(\frac{W}{\gamma(i)}\right)^\nu\right\}$ and $i \le I$, the relevant task is produced using capital, and otherwise it is produced using labor. Since $\gamma(i)$ is strictly increasing, this implies that there exists a threshold $\widetilde{I}$ at which, if technologically feasible, firms would be indifferent between using capital and labor. Namely, at task $\widetilde{I}$, we have that $R = W/\gamma(\widetilde{I})$, or

$$\frac{W}{R} = \gamma(\widetilde{I}).$$

This threshold represents the index up to which using capital to produce a task yields the cost-minimizing allocation of factors. However, if $\widetilde{I} > I$, firms will not be able to use capital all the way up to task $\widetilde{I}$ because of the constraint imposed by the available automation technology. For this reason, the equilibrium threshold below which tasks are produced using capital is given by

$$I^* = \min\{I, \widetilde{I}\},$$

meaning that $I^* = \widetilde{I} < I$ when it is technologically feasible to produce task $\widetilde{I}$ with capital, and $I^* = I < \widetilde{I}$ otherwise.

The demand curves for capital and labor are similar, with the only modification that the demand for capital also comes from non-automated tasks. Following the same steps as in the text, we can then establish analogous results. This requires the more demanding Assumption $1''$, which guarantees that the demand for factors is homothetic:

Assumption $1''$: One of the following three conditions holds:

1. $\sigma - \zeta \to 0$, or
2. $\zeta \to 1$, or
3. $\eta \to 0$.

The following proposition is very similar to Proposition 1, with the only difference being in the ideal price condition.

Proposition B2 (Equilibrium in the static model when $\epsilon \in (0,1)$) *Suppose that Assumption $1''$ holds. Then, for any range of tasks $[N-1, N]$, automation technology $I \in (N-1, N]$, and capital stock K, there exists a unique equilibrium characterized by factor prices, W and R, and threshold tasks, $\widetilde{I}$ and I^*, such that: (i) $\widetilde{I}$ is determined by equation (5) and $I^* = \min\{I, \widetilde{I}\}$; (ii) all tasks $i \leq I^*$ are produced using capital and all tasks $i > I^*$ are produced using labor; (iii) the capital and labor market clearing conditions, equations (7) and (8), are satisfied; and (iv) factor prices satisfy the ideal price index condition:*

$$(I^* - N + 1)c^u(R)^{1-\sigma} + \int_{I^*}^{N} c^u\left(R^{1-\nu}\left(\frac{W}{\gamma(i)}\right)^{\nu}\right)^{1-\sigma} di = 1. \tag{B18}$$

Proof. The proof follows the same steps as Proposition 1. ■

Comparative statics in this case are also identical to those in the baseline model (as summarized in Proposition 2) and we omit them to avoid repetition. The dynamic extension of this more general model is also very similar, and in fact, Proposition 3 applies identically, and is also omitted. To highlight the parallels, we just present the equivalent of Proposition 5.

Proposition B3 (Equilibrium with endogenous technology when $\nu \in (0,1)$) *Suppose that $\sigma > \zeta$ and Assumption $1''$ holds. Then there exist $\overline{\rho}$ and $\overline{A}$ such that for $\rho > \overline{\rho}$ and $A < \overline{A}$, the following are true:*

1. *There exists $\overline{\kappa}$ such that for $\frac{\kappa_I}{\kappa_N} > \overline{\kappa}$, there is a BGP, where $\dot{N} = \dot{I} = \frac{\kappa_I \kappa_N}{\kappa_I + \kappa_N} S$, and Y, C, K and W grow at the constant rate $g = A\frac{\kappa_I \kappa_N}{\kappa_I + \kappa_N} S$, and the rental rate, R, the labor share and employment are constant. Along this path, we have $N(t) - I(t) = n^D$, with n^D determined endogenously from the condition $\kappa_N v_N = \kappa_I v_I$, and satisfying $n^D \in (\bar{n}, 1)$. In addition, there exists $\overline{\overline{\rho}} \geq \overline{\rho}$ such that if $\rho > \overline{\overline{\rho}}$, the balance growth path is unique.*

2. *Suppose that $\rho > \overline{\overline{\rho}}$ so that the balance growth path is unique. Then, when $\theta = 0$, the dynamic equilibrium is globally (saddle-path) stable. Moreover, there exists $\overline{\overline{A}} \leq \overline{A}$ such that provided that $A < \overline{\overline{A}}$, and for any value of θ, the dynamic equilibrium is unique in the neighborhood of the BGP and is locally saddle-path stable.*

Proof. The proof of this result follows closely that of Proposition 5, especially exploiting the fact that the behavior of profits of automation and the creation of new tasks behave identically to those in the baseline model, and thus the value functions behave identically also. ■

Microfoundations for the Quasi-Labor Supply Function

We provide various micro-foundations for the quasi-labor supply expression used in the main text, $L^s\left(\frac{W}{RK}\right)$.

Efficiency wages: Our first micro-foundation relies on an efficiency wage story. Suppose that, when a firm hires a worker to perform a task, the worker could shirk and, instead of working, use her time and effort to divert resources away from the firm.

Each firm monitors its employees, but it is only able to detect those who shirk with probability q at each instant of time. If the worker is caught shirking, the firm does not pay wages and retains its resources. Otherwise, the worker earns her wage and a fraction of the resources that she diverted away from the firm.

In particular, assume that each firm holds a sum RK of liquid assets that the worker could divert, and that if uncaught, a worker who shirks earns a fraction $b(i)$ of this income. We assume that the sum of money that the worker may be able to divert is RK to simplify the algebra. In general, we obtain a similar quasi-supply curve for labor so long as these funds are proportional to total income $Y = R\,K + WL$.

In this formulation, $b(i)$ measures how untrustworthy worker i is, and we assume that this information is observed by firms. $b(i)$ is distributed with support $[0, \infty)$ and has a cumulative density function G. Moreover, we assume there is a mass L of workers. A worker of type $b(i)$ does not shirk if and only if:

$$W \geq (1-q)[W + b(i)RK] \rightarrow \frac{W}{RK}\frac{q}{1-q} \geq b(i).$$

Thus, when the market wage is W, firms can only afford to hire workers who are sufficiently trustworthy. The employment level is therefore given by:

$$L^s = G\left(\frac{W}{RK}\frac{q}{1-q}\right)L.$$

When $q = 1$—so that there is no monitoring problem—, we have that $G\left(\frac{W}{RK}\frac{q}{1-q}\right) = 1$, and the supply of labor is fixed at L for all wages $W \geq 0$. However, when $q < 1$—so that there is a monitoring problem—, we have that $L^s < L$. Even though all workers would rather work than stay unemployed, the monitoring problem implies that not all of them can be hired at the market wage. Notice that, though it is privately too costly to hire workers with $b(i) > \frac{W}{RK}\frac{q}{1-q}$, these workers strictly prefer employment unemployment.

Alternatively, one could also have a case in which firms do not observe $b(i)$, which is private information. This also requires that firms do not learn about workers. To achieve that, we assume that workers draw a new value of $b(i)$ at each point in time.

When the marginal value of labor is W, firms are willing to hire workers so long as the market wage $\widetilde{W}$ satisfies:

$$(W-\widetilde{W})G\left(\frac{\widetilde{W}}{RK}\frac{q}{1-q}\right)-(1-q)\left(\widetilde{W}+RK\int_{\frac{\widetilde{W}}{RK}\frac{q}{1-q}}^{\infty}bdG(b)\right)\geq 0.$$

This condition guarantees that the firm makes positive profits from hiring an additional worker, whose type is not known.

Competition among firms implies that the equilibrium wage at each point in time satisfies:

$$(W-\widetilde{W})G\left(\frac{\widetilde{W}}{RK}\frac{q}{1-q}\right)-(1-q)\left(\widetilde{W}+RK\int_{\frac{\widetilde{W}}{RK}\frac{q}{1-q}}^{\infty}bdG(b)\right)= 0.$$

This curve yields an increasing mapping from $\frac{W}{RK}$ to $\frac{\widetilde{W}}{RK}$, which we denote by

$$\frac{\widetilde{W}}{RK}=h\left(\frac{W}{RK}\right).$$

Therefore, the effective labor supply in this economy, or the quasi-supply of labor, is given by

$$L^s=G\left(\frac{\widetilde{W}}{RK}\frac{q}{1-q}\right)=G\left(h\left(\frac{W}{RK}\right)\frac{q}{1-q}\right)L.$$

As in the previous model, even though the opportunity cost of labor is zero, the economy only manages to use a fraction of its total labor.

Minimum wages: Following Acemoglu (2003), another way in which we could obtain a quasi-labor supply curve is if there is a binding minimum wage. Suppose that the government imposes a (binding) minimum wage $\widetilde{W}$ and indexes it to the income level and current employment, L. In particular, assume that

$$\widetilde{W}=M^{-1}(L)(RK+WL).$$

Here, $RK+WL$ represents the total income in this economy (net of intermediate goods' costs), and $M^{-1}(\cdot)$ is an increasing function with inverse $M(\cdot)$, which determines the indexation. Rearranging this expression we obtain:

$$L^s=M\left(\frac{\widetilde{W}}{RK+\widetilde{W}L^s}\right).$$

This functional equation defines the quasi-labor supply in this economy as the solution to the above functional equation. This quasi-labor supply is an increasing function of $\frac{\widetilde{W}}{RK}=\frac{W}{RK}$ (because the minimum wage is binding).

Additional References for Appendix B

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