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THE DISCOUNTED EULER EQUATION:  
A NOTE

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Alisdair McKay, Emi Nakamura, and Jón Steinsson  
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### **ABSTRACT**

We present a simple model with unemployment risk and borrowing constraints which yields a “discounted Euler equation.” This feature of the model mutes the extent to which news about far future real interest rates (i.e., forward guidance) affects current outcomes. We show that this simple model approximates the outcomes of a rich model with uninsurable income risk and borrowing constraints in response to a forward guidance shock. The model is simple enough to be easily incorporated into standard DSGE models. We illustrate this with an application to the zero lower bound.

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# 1 Introduction

Recent work has highlighted that standard monetary business cycle models imply that far future forward guidance has implausibly large effects on current outcomes (Carlstrom, Fuerst, and Paustian, 2015; Del Negro, Giannoni, and Patterson, 2013). In earlier research, we showed that this implausible behavior of standard models results from the highly forward looking nature of the standard consumption Euler equation (McKay, Nakamura, and Steinsson, 2016). To see this, consider the response of consumption to a forward guidance announcement by the central bank in a standard New Keynesian model. A key equation in this type of model is the standard consumption Euler equation, which once it has been log-linearized, takes the form

$$\hat{C}_t = \mathbb{E}_t \hat{C}_{t+1} - \sigma(i_t - \mathbb{E}_t \pi_{t+1} - r_t^n), \quad (1)$$

where  $\hat{C}_t$  denotes consumption,  $i_t$  denotes the nominal interest rate,  $\pi_t$  denotes inflation,  $r_t^n$  is the natural real rate of interest, i.e., the real interest rate that would prevail with flexible prices (all measured as percent deviations from steady state),  $\mathbb{E}_t$  denotes the expectations operator, and  $\sigma$  is the intertemporal elasticity of substitution. Solving this equation forward yields

$$\hat{C}_t = -\sigma \sum_{j=0}^{\infty} E_t(i_{t+j} - \mathbb{E}_{t+j} \pi_{t+j+1} - r_{t+j}^n). \quad (2)$$

Notice that there is no discounting in the sum on the right hand side of this equation. This implies that news about future real interest rates at any horizon—however far in the future—has the same effect on current consumption.<sup>1</sup>

This prediction is sensitive to the presence of precautionary savings and liquidity constraints. In McKay, Nakamura, and Steinsson (2016), we presented a heterogeneous agents New Keynesian model in which the response of current consumption to far future forward guidance is greatly muted relative to the standard representative agent New Keynesian model.<sup>2</sup> A drawback of the model presented in that paper is that it is considerably more complicated to solve than a standard New Keynesian model.

In this note, we present a simple model with unemployment risk and borrowing constraints which yields a “discounted Euler equation.” The discounting in the Euler equation means that the

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<sup>1</sup>Here it is important to remember that consumption will return to its pre-shock steady state in the long run. In other words, temporary monetary policy actions have no long-run effects in this model.

<sup>2</sup>Kaplan, Moll, and Violante (2016) present similar results in a richer model. Werning (2015) presents conditions under which the response of consumption to interest rates in a heterogeneous agents model will be the same as in a representative agents model.

effects of future real interest rates on current consumption are muted relative to the standard New Keynesian model.<sup>3</sup> We show that this simple model approximates the outcomes of our richer model with uninsurable income risk and borrowing constraints in response to a forward guidance shock. The model is simple enough to be easily incorporated into standard DSGE models. We illustrate this with an application to the zero lower bound.

## 2 Discounted Euler Equation Model

Consider a simplified version of the model presented in McKay, Nakamura, and Steinsson (2016), in which idiosyncratic productivity is i.i.d. and takes one of two values each period, which we refer to as employed ( $z = 1$ ) and unemployed ( $z = 0$ ). The only income of the unemployed comes from home production in amount  $m$ . Recall that agents are not allowed to borrow in the model. In this version of the model, we furthermore assume for tractability that there are no assets available.<sup>4</sup>

We show in appendix A that under these assumptions we can derive the following log-linearized consumption Euler equation

$$\hat{C}_t = \alpha \mathbb{E}_t \hat{C}_{t+1} - \sigma(i_t - \mathbb{E}_t \pi_{t+1} - r_t^n). \quad (3)$$

This equation differs from the standard consumption Euler equation because of the parameter  $\alpha < 1$ . Solving this equation forward yields

$$\hat{C}_t = -\sigma \mathbb{E}_t \sum_{j=0}^{\infty} \alpha^j (i_{t+j} - \mathbb{E}_{t+j} \pi_{t+j+1} - r_{t+j}^n). \quad (4)$$

Notice that the effect of interest rates  $j$  periods in the future on current consumption is discounted by a factor  $\alpha^j$ . For this reason, we refer to equation (3) as the “discounted Euler equation.” The presence of  $\alpha < 1$  implies that far future interest rate changes have much smaller effects on current consumption than near term interest rate changes.

In this version of the model, the unemployed households are liquidity constrained, while the employed households are not. The presence of  $\alpha$  in equation (3) results from the fact that with some probability the currently employed households will be unemployed next period and next period’s expected marginal utility is therefore partly determined by the exogenous marginal utility in the

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<sup>3</sup>Interestingly, such a formulation of the consumption Euler equation has actually been used in policy calculations by the Central Bank of Norway, to combat the forward guidance puzzle. We thank Oistein Roisland for pointing this out to us.

<sup>4</sup>Krusell, Mukoyama, and Smith (2011) and Ravn and Sterk (2013) have also used this approach to formulate tractable incomplete markets models.

unemployed state. For this reason, the currently employed households put some probability on the outcome that they will not benefit from an increase in next period’s consumption and therefore do not increase current consumption one-for-one with next period’s expected consumption.<sup>5</sup>

In the richer incomplete markets model presented in McKay, Nakamura, and Steinsson (2016), the response of output to real interest rate changes differs from the response with complete markets both because the response declines with the horizon of forward guidance and also because the response to contemporaneous changes in the real rate is smaller. The discounted Euler equation can match the first of these differences with an appropriate value for  $\alpha$ . To match the second of these differences we can change the value of the intertemporal elasticity of substitution  $\sigma$  in the discounted Euler equation.

Figure 1 shows that the discounted Euler equation—equation (3)—with  $\alpha = 0.97$  and  $\sigma = 3/8$  provides a good approximation to the response of output to a real interest rate shock 20 quarters in the future in the baseline incomplete markets model analyzed in McKay, Nakamura, and Steinsson (2016).<sup>6</sup> The approximation is nearly perfect up until the time that the interest rate changes. What the discounted Euler equation misses is the fall in consumption after the interest rate shock. This fall is due to redistribution of wealth in the incomplete markets model (from households with high MPCs to households with low MPCs), which the discounted Euler equation does not capture.

### 3 Application to the ZLB

We now illustrate the importance of discounting in the Euler equation using an application to the Zero Lower Bound (ZLB). We follow the classic paper by Eggertsson and Woodford (2003) in how we model the ZLB, but show that our “discounted Euler Equation” model yields substantially different conclusions.

First, using the aggregate resource constraint we rewrite the Euler equation in terms of the

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<sup>5</sup>Pierragalli (2006) and Nistico (2012) provide an alternative micro-foundation for discounting in the Euler equation based on mortality shocks as in Blanchard’s (1985) perpetual-youth model. In their formulation, discounting only arises from aggregation over different generations, and to generate a quantitatively important deviation from the standard Euler equation, these authors assume counter-factually high death rates. Our approach rationalizes why long-lived consumers can have short planning horizons. Moreover, in Pierragalli and Nistico’s formulation, the discounting in the Euler equation is larger the larger is the amount of financial wealth in the economy and disappears when financial wealth is in zero-net supply. In contrast, in our full model, agents discount the future more when they have little liquid financial wealth to buffer shocks to income. Discounting of this kind does not arise in the spender-saver setup (see, e.g., Bilbiie, 2008).

<sup>6</sup>Recall that  $\sigma = 1/2$  in McKay, Nakamura, and Steinsson (2016). The factor by which we need to reduce  $\sigma$  in the simple model to match the model in McKay, Nakamura, and Steinsson (2016) is therefore 0.75.

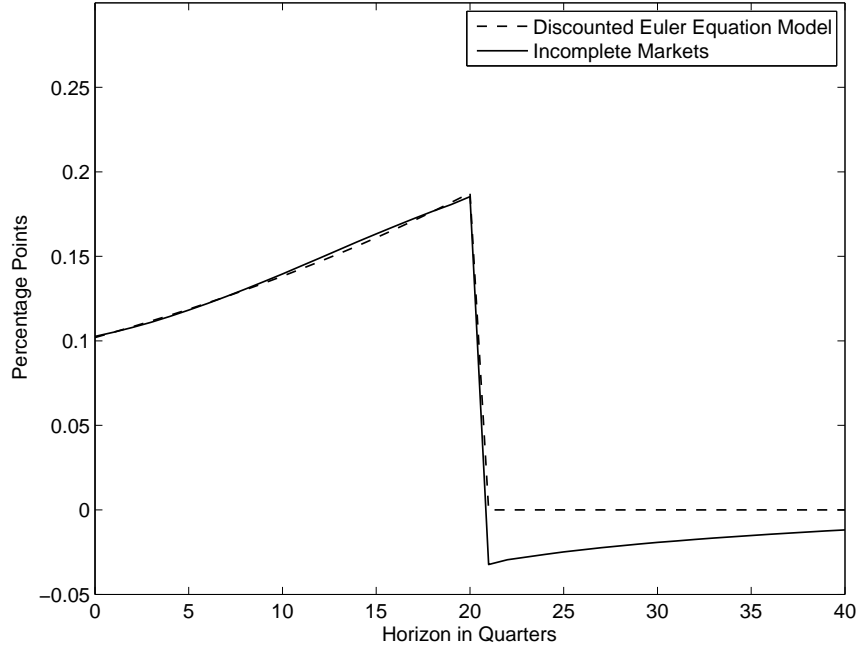


Figure 1: Response of output to a 50 basis point change in the expected real interest rate in quarter 20 in the incomplete markets model and in the discounted Euler equation model.

output gap  $\hat{Y}_t$ <sup>7</sup>

$$\hat{Y}_t = \alpha E_t \hat{Y}_{t+1} - \zeta \sigma (i_t - E_t \pi_{t+1} - r_t^n), \quad (5)$$

where  $\zeta$  is the factor by which we need to change  $\sigma$  to match reponse of output to a contemporaneous interest rate shock in baseline incomplete markets model in McKay, Nakamura, and Steinsson (2016) (i.e. the amount by which the effects of even contemporaneous interest rate shocks are muted in the incomplete markets model).

The Calvo Phillips curve is

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \hat{Y}_t. \quad (6)$$

Finally, we assume that the central bank follows a “naive” monetary policy,

$$i_t = \max(0, r_t^n + \phi \pi_t). \quad (7)$$

where  $\phi > 1$ . For comparability with Eggertsson and Woodford (2003), we assume that  $\beta = 0.99$ ,  $\sigma = 0.5$ , and  $\kappa = 0.02$ .

Following Eggertsson and Woodford (2003), we assume that the ZLB binds due to a shock that lowers the natural rate below zero and persists at the same negative value with probability  $\lambda$  each

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<sup>7</sup>In our model the level of output under flexible prices is constant so  $\hat{Y}_t$  is both the log-deviation of output from steady state and the output gap.

quarter. With probability  $1 - \lambda$ , it reverts back to normal. For simplicity, we assume that once the natural rate reverts back to normal, the zero lower bound on nominal interest rates never binds again in the future.

We start by solving for the level of the output gap and inflation after the shock has dissipated. Since we have assumed that the natural rate will never go negative again, it is feasible for the monetary authority to set  $i_t = r_t^n$  at all times after the shock dissipates. This implies that both the output gap and inflation will be zero at all times after the shock dissipates. Given this, it is easy to solve for the output gap and inflation while the shock persists. First, notice that all periods while the shock persists are identical since the probability of the shock reverting to normal does not change over time. This implies that output and inflation will be constant while the shock persists. We refer to the period during which the shock persists as the short run. Next, notice that in the short run  $E_t \hat{Y}_{t+1} = \lambda \hat{Y}_t$  and  $E_t \pi_{t+1} = \lambda \pi_t$  since with probability  $1 - \lambda$  the economy will revert to normal (in which case  $\hat{Y}_t = \pi_t = 0$ ). Using these facts and equations (6) and (5), a few steps of algebra (presented in Appendix B) yield

$$\pi_S = \frac{\kappa}{1 - \beta\lambda} \hat{Y}_S, \quad (8)$$

$$\hat{Y}_S = \frac{\zeta\sigma}{1 - \alpha\lambda - \frac{\zeta\sigma\lambda\kappa}{1 - \lambda\beta}} r_S^n, \quad (9)$$

where  $\pi_S$  and  $\hat{Y}_S$  denote inflation and the output gap in the short run, and  $r_S^n$  denotes the natural real rate of interest in the short run.

### 3.1 How Deep is a ZLB Recession?

Our first result is that the discounted Euler equation model implies that the same shocks that generate a huge recession at the ZLB in the standard model, imply a much smaller recession in the discounted Euler Equation model. Eggertsson and Woodford (2003) present results for a shock that lowers the natural rate to  $r_S^n = -0.02$  (annualized) and reverts to normal with probability  $\lambda = 0.1$  (per quarter). They show that in the standard model, a shock of this size and persistence generates a very large recession—an output gap of -14.3%—accompanied by a large amount of deflation (-10.5%). In Table 1, we show that in the discounted Euler equation model, this same shock leads to a much more modest recession. The output gap is a mere -2.9%, and inflation falls by only 2.1%. Clearly, incorporating discounting of future interest rates radically alters the conclusions one comes to about the severity of the problem that the monetary authority faces with this type of shock.<sup>8</sup>

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<sup>8</sup>We set  $\alpha = 0.97$  and  $\zeta = 0.75$  as in Figure 1. We compare this to the standard case of  $\alpha = 1$  and  $\zeta = 1$ .

Table 1: How Deep is a ZLB Recession?

	Output	Inflation
Standard Model ( $\alpha = 1, \zeta = 1$ )	-14.3%	-10.5%
Discounted Euler Equation Model ( $\alpha = 0.97, \zeta = 0.75$ )	-2.9%	-2.1%
Response of output and inflation when the natural rate falls to -2% (annualized) with a 10% per quarter probability of returning to normal.		

### 3.2 When does the Deflationary Death Spiral Occur

Our second result is that the discounted Euler equation model implies that the economy is less prone to a “deflationary death spiral” at the ZLB. The strength of the deflationary forces in the standard model are due to a feedback loop that gets stronger the more persistent is the shock to the natural rate. The basic feedback loop results from the following chain of logic: The negative natural rate leads to a positive interest rate gap—a real rate that is higher than the natural rate—because the nominal rate can’t fall below zero. This leads output to fall, and if the shock is persistent it leads expectations of future output to fall, which in turn leads expected inflation to fall, which causes the current real rate to rise further, and current output to fall further, etc. The more persistent is the shock, the more it affects expected inflation and the stronger this feedback loop becomes.

It is well known that the strength of the deflationary forces associated with negative shocks to the natural real rate become infinitely strong—i.e. imply that the (log) output gap and inflation go to negative infinity—for even relatively modest levels of persistence. This can be seen by looking at the denominator of the expression for the short run output gap in equation (9). As this denominator goes to zero, the short run output gap goes to negative infinity. In the standard model (with  $\alpha = 1$  and  $\zeta = 1$ ), this occurs for a shock with an expected duration of 11 quarters.

In the discounted Euler equation model, the strength of the deflationary forces are muted and consequently the persistence of the ZLB shock needs to be greater for this “deflationary death spiral” to occur. This is depicted in Figure 2, which plots the drop in output for different levels of persistence of the ZLB shock. The solid line is the standard model, while the two broken lines are two calibrations of the discounted Euler equation model that match the baseline and high-risk calibrations of the incomplete markets model in McKay, Nakamura, and Steinsson (2016),



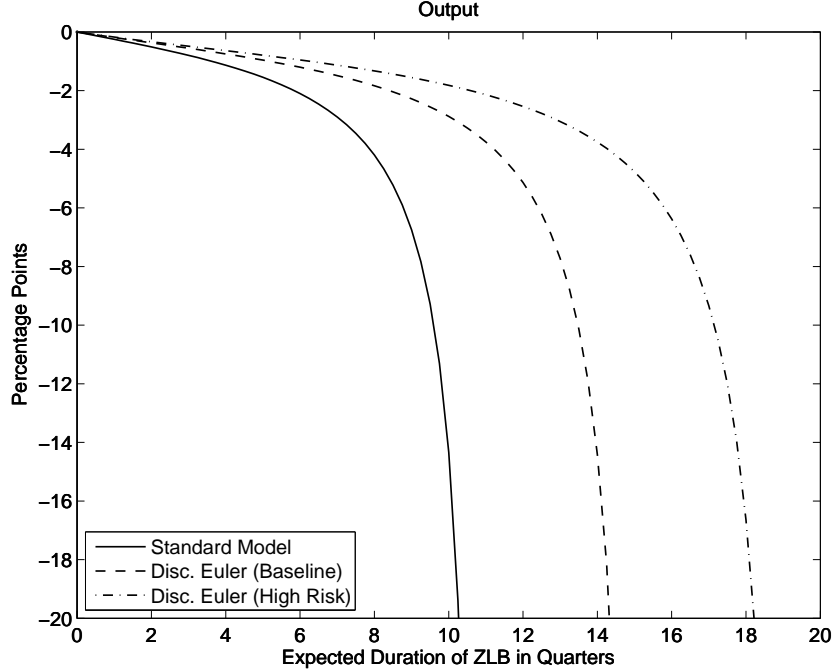


Figure 2: Response of output to shock that makes the natural real rate of interest -2% (annualized) for different levels of persistence of the shock (different values of  $\lambda$ ).

respectively.<sup>9</sup> The deflationary death spiral occurs only for shocks that are considerably more persistent in the discounted Euler equation model than in the standard model.

## 4 Conclusion

In this note, we develop simple microfoundations for a “discounted Euler equation” that is easy to incorporate into standard DSGE models. This discounted Euler equation implies that the effects of far future changes in real interest rates as well as far future changes in the natural real rate have much smaller effects on contemporaneous outcomes than in the standard New Keynesian model. We show that this has important implications for policy experiments when nominal interest rates are constrained by their zero lower bound.

<sup>9</sup>As we discuss in the text,  $\alpha = 0.97$  and  $\zeta = 0.75$  matches the baseline calibration of the incomplete markets model in McKay, Nakamura, and Steinsson (2016). We use the same approach to match the high-risk calibration with the discounted Euler equation model. This yields  $\alpha = 0.94$  and  $\zeta = 0.7$ .

## A A Simple Incomplete Markets Model

The discounted Euler equation (3) can be micro-founded with a simplified version of the model analyzed in McKay, Nakamura, and Steinsson (2016). The simplified model deviates from the full model in the following ways:

1. The idiosyncratic productivity shock takes just two values, which we will call employed ( $z = 1$ ) and unemployed ( $z = 0$ ).
2. Idiosyncratic productivity is i.i.d. across time:  $\Pr(z'|z) = \Pr(z')$ .
3. The supply of government debt is zero:  $B = 0$ .
4. Unemployed agents have access to a home production technology that produces final goods in amount  $m$ .
5. Firm dividends  $D_t$  are distributed only to the employed households.

This version of our model is analytically tractable because there is no wealth in the economy and there is a strict borrowing constraint,  $b' \geq 0$ . As the gross supply of assets is zero, there is no possibility of saving in equilibrium and so the distribution of wealth remains degenerate at zero.

### A.1 The Environment

Here we briefly summarize the model in McKay, Nakamura, and Steinsson (2016) including the simplifications above. The economy is populated by a unit continuum of ex ante identical households with preferences given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t e^{q_t} \left[ \frac{c_{h,t}^{1-\gamma}}{1-\gamma} - \frac{\ell_{h,t}^{1+\psi}}{1+\psi} \right],$$

where  $c_{h,t}$  is consumption of household  $h$  at time  $t$  and  $\ell_{h,t}$  is labor supply of household  $h$  at time  $t$ .  $q_t$  is an aggregate patience shock that determines the natural rate of interest and is given by  $q_t = q_{t-1} + r_{t-1}^n$  with  $q_0 = 0$ . Households are endowed with stochastic idiosyncratic employment status  $z_{h,t} \in \{0, 1\}$ , which is drawn i.i.d. with probabilities  $\Pr(z_{h,t} = 0) = u$  and  $\Pr(z_{h,t} = 1) = 1 - u$ . Employed households earn labor income  $W_t \ell_{h,t}$ , where  $W_t$  is the aggregate real wage. Unemployed households produce  $m$  units of the consumption good at home.

In this economy, a final good is produced from intermediate inputs according to the production function

$$Y_t = \left( \int_0^1 y_{j,t}^{1/\mu} dj \right)^\mu,$$

where  $Y_t$  denotes the quantity of the final good produced at time  $t$  and  $y_{j,t}$  denotes the quantity of the intermediate good produced by firm  $j$  in period  $t$ . The intermediate goods are produced using

labor as an input according to the production function

$$y_{j,t} = n_{j,t},$$

where  $n_{j,t}$  denotes the amount of labor hired by firm  $j$  in period  $t$ .

While the final good is produced by a representative competitive firm, the intermediate goods are produced by monopolistically competitive firms. The intermediate goods firms face frictions in adjusting their prices that imply that they can only update their prices with probability  $\theta$  per period as in Calvo (1983). These firms are controlled by a risk-neutral manager who discounts future profits at rate  $\beta$ . Whatever profits are produced are paid out immediately to the employed households with each of them receiving an equal share.

Households trade a risk-free real bond with real interest  $r_t$  between periods  $t$  and  $t+1$ . Borrowing constraints prevent these households from taking negative bond positions. There is no external supply of bonds. Finally, a monetary authority determines the nominal interest rate.

## A.2 Discounted Euler equation

As individual assets are constant at zero, it follows that all households of a given employment status must choose the same consumption. Let  $c_{e,t}$  be the consumption of the employed and  $c_{u,t}$  be the consumption of the unemployed agents. Moreover, the absence of opportunities to borrow and save implies that consumption must be equal to income for all individuals. The unemployed will therefore consume  $m$ . The Euler equation for employed agents is

$$c_{e,t}^{-\gamma} \geq \beta e^{r_t^n} (1 + r_t) \mathbb{E}_t \left[ (1 - u) c_{e,t+1}^{-\gamma} + u m^{-\gamma} \right]. \quad (10)$$

The Euler equation for an unemployed agent is

$$m^{-\gamma} \geq \beta e^{r_t^n} (1 + r_t) \mathbb{E}_t \left[ (1 - u) c_{e,t+1}^{-\gamma} + u m^{-\gamma} \right].$$

Notice that the right hand side of the Euler equation is independent of employment status as employment is i.i.d. across time. Therefore, if  $m < c_{e,t}$  the unemployed households must be constrained and their Euler equation will not hold with equality. We assume that  $m$  is low enough that  $m < c_{e,t}$  for all  $t$ .<sup>10</sup>

Following Krusell, Mukoyama, and Smith (2011) and Ravn and Sterk (2013), we will focus on the equilibrium of this economy in which the Euler equation of the employed holds with equality in

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<sup>10</sup>It is easy to relax the assumption that idiosyncratic productivity is i.i.d., however, with i.i.d. productivity it is especially easy to see that the unemployed will be constrained if  $m$  is low enough.

all periods. In this equilibrium, the employed households choose zero savings and are therefore up against their constraint. But the constraint does not bind in the sense that they would not strictly prefer to borrow more if allowed. There are other equilibria of this economy in which the Euler equation for the employed household holds with inequality (implying that the employed households would strictly prefer to borrow if allowed). We focus on the equilibrium in which the employed households' Euler equation holds with equality for the following reason. In the more realistic case in which the gross supply of assets (either internal or external) is positive there is a unique equilibrium in which the employed households' Euler equation holds with equality. Consider a sequence of such economies with smaller and smaller gross supplies of assets. The equilibrium we focus on is the unique equilibrium of the limiting economy for which the supply goes to zero.<sup>11</sup>

We assume that home production is not measured as part of GDP. Therefore measured aggregate consumption is

$$C_t = (1 - u)c_{e,t}. \quad (11)$$

Substituting into the employed Euler equation we have

$$(C_t/(1 - u))^{-\gamma} = \beta e^{r_t^n} R_t \mathbb{E}_t [(1 - u)(C_{t+1}/(1 - u))^{-\gamma} + um^{-\gamma}]. \quad (12)$$

Log-linearizing this equation and using the Fisher equation yields

$$\hat{C}_t = -\frac{1}{\gamma} [i_t - \mathbb{E}\pi_{t+1} + r_t^n] + \alpha \mathbb{E} [\hat{C}_{t+1}] \quad (13)$$

where

$$\alpha \equiv \frac{1}{1 + \frac{u}{1-u} \left( \frac{\bar{c}_e}{m} \right)^\gamma}, \quad (14)$$

and bars denote steady state values and hats denote log deviations from steady state. Notice that  $\alpha$  is decreasing in the probability of becoming unemployed,  $u$ , and the consumption impact of unemployment,  $\bar{c}_e/m$ . The parameter  $\sigma$  used in the main text is equal to  $1/\gamma$ .

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<sup>11</sup>Suppose there is an outstanding supply of government debt in amount  $\varepsilon > 0$  financed by a lump sum tax. In this case, bond market clearing requires some household to hold this debt at the prevailing interest rate (which is set by the monetary authority). This household must be indifferent at the margin regarding how much they hold of the debt (i.e., their Euler equation must hold with equality). If instead the Euler equation of all households held with strict inequality, no household would want to hold the outstanding bonds. There would be excess supply of bonds and excess demand for consumption goods. Since firms must meet demand at posted prices, they would produce more, which would increase income and consumption and lead marginal utility of consumption to fall. This process would continue until the agents with the lowest marginal utility of consumption—the employed agents—were willing to hold the bonds. Thus, for any positive  $\varepsilon$  the Euler equation of the employed households will hold with equality in equilibrium and the equilibrium we focus on is the limit of these equilibria as  $\varepsilon$  goes to zero.

### A.3 Phillips curve

Market incompleteness does not change the Phillips curve. To see this, note that the labor supply condition for the employed is

$$c_{e,t}^{-\gamma} W_t = \ell_t^\psi.$$

Using the fact that aggregate labor supply is  $L_t = (1-u)\ell_t$  and aggregate consumption is given by equation (11), this last expression can be rewritten as

$$(1-u)^{\gamma+\psi} C_t^{-\gamma} W_t = L_t^\psi.$$

Log-linearizing this equation and using the aggregate resource constraint  $C_t = Y_t$  and a first-order approximation to the aggregate production function,<sup>12</sup>  $\hat{L}_t = \hat{Y}_t$ , yields

$$\hat{W}_t = (\psi + \gamma)\hat{Y}_t.$$

Market incompleteness does not change the relationship between inflation and marginal cost. To a first-order approximation, this relationship is

$$\pi_t = \beta E_t \pi_{t+1} + \frac{(1 - \beta(1 - \theta))\theta}{1 - \theta} \hat{M}C_t.$$

As the real marginal cost is the wage, we have

$$\pi_t = \beta E_t \pi_{t+1} + \frac{(1 - \beta(1 - \theta))\theta}{1 - \theta} (\psi + \gamma)\hat{Y}_t. \quad (15)$$

As the flexible price equilibrium is constant in our model,  $\hat{Y}_t$  is the output gap and (15) is the standard New Keynesian Phillips curve.

## B Algebra Behind Equations (8) and (9)

Consider first the Phillips curve:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \hat{Y}_t.$$

Since the output gap and inflation are constant at  $\hat{Y}_S$  and  $\pi_S$ , respectively, and  $E_t \pi_{t+1} = \lambda \pi_S$  in the short run, we have that

$$\pi_S = \beta \lambda \pi_S + \kappa \hat{Y}_S,$$

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<sup>12</sup>Here we abstract from the efficiency loss due to price dispersion, which is zero in a first-order approximation around a zero-inflation steady state.

which implies

$$\pi_S = \frac{\kappa}{1 - \beta\lambda} \hat{Y}_S \quad (16)$$

as long as  $\hat{Y}_S$  and  $\pi_S$  are finite.

Consider next the discounted Euler equation

$$\hat{Y}_t = \alpha E_t \hat{Y}_{t+1} - \zeta \sigma (i_t - E_t \pi_{t+1} - r_t^n).$$

Again, since the output gap and inflation are constant at  $\hat{Y}_S$  and  $\pi_S$ , respectively, and  $E_t \pi_{t+1} = \lambda \pi_S$  and  $E_t \hat{Y}_{t+1} = \lambda \hat{Y}_S$  in the short run, and, in addition, since the natural real rate is  $r_S^n$  in the short run, we have that

$$\hat{Y}_S = \alpha \lambda \hat{Y}_S + \zeta \sigma (\lambda \pi_S + r_S^n).$$

If we now use equation (16) to eliminate  $\pi_S$  from this equation we obtain

$$\hat{Y}_S = \alpha \lambda \hat{Y}_S + \zeta \sigma \left( \lambda \frac{\kappa}{1 - \beta\lambda} \hat{Y}_S + r_S^n \right),$$

which implies

$$\hat{Y}_S = \frac{\zeta \sigma}{1 - \alpha \lambda - \frac{\zeta \sigma \lambda \kappa}{1 - \beta\lambda}} r_S^n$$

as long as  $\hat{Y}_S$  is finite.

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