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ABSTRACT

We present an equilibrium search model that parsimoniously rationalizes the use of auctions as a sales mechanism for new-in-box goods--a frequent occurrence in online retail markets--and analyze whether the existence of these auctions is welfare enhancing relative to a market consisting only of posted prices. Buyers have a deadline by which the good must be purchased, and sellers choose between auctions and posted-price mechanisms. As the deadline approaches, buyers increase their bids and are more likely to buy through posted-price listings. The model predicts equilibrium price dispersion even for new, homogeneous goods. Using data on one million auction and posted-price listings for new-in-box items on eBay.com, we find robust evidence consistent with our model. As predicted, bidders increase their bids from one auction to the next, equilibrium price dispersion exists, and auctions and posted-price listings coexist. Fitting the model to the data, we find that retail auctions increase total welfare by 1.8% of the average retail price if listing fees exactly cover platform costs, but reduce welfare by 2.3% if listing fees are pure profit

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1 Introduction

The choice of sales mechanism is a critical component of the design of many markets. Auctions are commonly thought of as a useful mechanism for achieving price discovery for non-standard, idiosyncratic goods, while posted prices are often thought of as an appropriate mechanism for standardized, homogeneous goods where price discovery is unnecessary. In this light, it is puzzling that online auction sites (such as eBay, Taobao, Tophatter, and others) frequently offer multiple auction listings of new, standardized products that are readily available at retail outlets. For example, eBay sold over 82 million new-in-box items via auction over one year, totaling to 1.6 billion dollars in sales.¹ Beyond simply being redundant to posted-price alternatives, these auctions also typically close at prices well below those of identical posted-price sales, calling into question the efficiency of offering such auctions. In this paper, we provide a new, parsimonious theoretical argument for the coexistence of these mechanisms. We then demonstrate that the model generates a range of patterns observed in online retail data and use the model to evaluate the welfare implications of the existence of these auctions.

The heart of the paper is an equilibrium search model. Buyers in the model have unit demand for a homogeneous item and encounter second-price sealed-bid auctions for the item at random intervals, or may purchase the item any time at a posted price. Each buyer is time sensitive, facing an individual-specific deadline by which he must obtain the good. These deadlines can be interpreted literally (*e.g.* the item is needed for a particular event, like a vacation, birthday, birth of a newborn), but more generally, they represent a limit on how long a consumer is willing to spend procuring a good. For instance, the cost of auction participation could increase with time if customers cannot sustain the same level of attention to the auctions, or become increasingly frustrated with repeatedly losing auctions. Alternatively, the consequence of not winning could deteriorate with time. For instance, customers could be shopping for a replacement part (such as an engine timing belt or bicycle tube) that hasn't yet failed but is increasingly likely to do so.

Deadlines introduce non-stationarity into the search problem and cause bids to increase with the duration of search. If the consumer reaches his deadline, he will buy at the posted price, but before his deadline, he might win an auction at a lower price.² Thus, his bid is shaded down from the true valuation, with more time remaining providing more opportunity and hence greater shading. This time sensitivity also generates price dispersion. In any given auction, buyers differ in their bids because some buyers need the object today more than others (*i.e.* are closer to their deadlines). Rather than assuming that this heterogeneity is exogenously given, as most auction models do, we derive this distribution of buyer deadlines endogenously based on the rate at which bidders win and exit the market.

¹Numbers are computed from internal eBay data between October 1st, 2013, and September 30th, 2014. eBay.com defines “new” to mean “a brand-new, unused, unopened, undamaged item in its original packaging (where packaging is applicable).”

²Throughout we will refer to a buyer as “he” and a seller as “she”.

Our model also allows sellers to choose whether to list their product in an auction or with a posted price. Auction participants never bid more than the posted price, ensuring that the auction will always earn less revenue, but auctions sell faster than posted-price listings. These competing effects are balanced in a mixed strategy equilibrium, allowing identical sellers to be indifferent between the two selling mechanisms.

To model this complex, dynamic market, we introduce two techniques common in the search literature but largely unexploited in the auction literature: analysis in continuous time, and steady state distributions. These techniques keep our model tractable and enable comparative static analysis, which is uncommon in dynamic auction models.³ Specified in a continuous time framework, the model’s equilibrium conditions can be translated into a solvable system of differential equations, which was the methodological innovation of Akin and Platt (2012).⁴ The restriction to steady state behavior implies that the distribution of buyers with respect to their deadline remains stable. Population dynamics are consistent with steady offsetting flows of incoming buyers and outgoing winners. This can best be interpreted as focusing on long-run behavior in a market fairly thick with buyers, which seems reasonable for the auctioning of standardized retail items. A variety of insights arise from the comparative statics analysis, such as the fact that market design in this environment should account for the endogeneity of bidders’ willingness-to-pay. We illustrate this with an increase in listing fees, which shifts sellers from auctions to posted prices and compresses the distribution of bids.

We examine the model’s predictions empirically using a new dataset of one million auction and posted-price listings of brand-new goods from 3,663 distinct products offered on eBay from October 2013 through September 2014. As predicted, auctions and posted-price mechanisms coexist for these products, with auctions representing 47% of sales. We find significant price dispersion both within auction sales (an interquartile range equal to 32% of the average price) as well as between auctions and posted-price listings (auction prices are 15% lower than posted-price sales). We also find that, as the model predicts, past losers tend to bid more in subsequent auctions (2.0% more on average), with the rate of increase rising with the length of search. To our knowledge, the empirical fact that repeat bidders raise their bids has not been previously identified in the literature. Furthermore, we find that repeat bidders are increasingly likely to bid on listings for which fast shipping options are available and are increasingly likely to win the auction relative to bidders who have not searched as long.

Having established these empirical facts, we use the data to estimate the model’s parameters, each of which are conveniently pinned down by easily computable moments of the data.

³Notable exceptions are Zeithammer (2006) and Said (2011), which feature comparative statics on reservation price with respect to the discount rate, the time between auctions, and (in the latter) the expected number of competitors. These move in the same direction as in our model.

⁴In the sequential auctions literature, only Ingster (2009) uses a continuous time model, but as bidder heterogeneity is exogenous there, our approach is unnecessary for that solution. Rather, the solution to the discrete time model in Said (2011) has more parallels to our method; there, the relevant state variable affecting bid shading is the (discrete) number of active bidders. The equilibrium conditions are then depicted as a system of first order difference equations.

Using these parameters, we simulate bidding functions and distributions of willingness to pay. We find that the model performs strikingly well in fitting key data patterns that are not used as identifying moments. We also show that ignoring the effects of time-sensitivity on bidders’ willingness to pay can lead to miscalculations of objects of interest such as bidder surplus: ignoring time-sensitivity and treating bidders as having a constant willingness to pay in subsequent auctions substantially overestimates bidder surplus, and ignoring bidding dynamics altogether underestimates surplus.

We use the estimated parameters to evaluate whether auctions for standardized goods offer any welfare advantages when posted-price listings are already available. These effects hinge on whether listing fees merely cover the cost of operating the platform or represent pure profit. In the former case, welfare is higher when both mechanisms operate because auctions shorten the time sellers spend on the market. In the latter case, eBay captures most of the value of sellers’ time on the market, and welfare would be higher if auctions were shut down.

While we see deadlines as a natural explanation for the key features of our data, they are not the only possible explanation for any one feature in isolation. Increasing bids over time, for example, can be also rationalized by a subtle story of bidders’ learning about the competition they face, which affects the option value of waiting for future auctions and motivates bidders to revise each subsequent bid. We present empirical evidence showing that bidder learning is unlikely to fully explain the pattern of increasing bids over time. Moreover, while separate explanations might exist for these patterns individually, buyer deadlines provide a single, unified explanation of all of the facts together.

1.1 Literature

Our paper contributes to the competing mechanisms literature, which considers the welfare implications of alternative trading mechanisms. Some of this literature focuses on the trade-offs between auctions and posted-price mechanisms. Julien et al. (2001), Einav et al. (2016), and Wang (1993) provide models in which one mechanism is strictly preferred over the other except in “knife-edge” or limiting cases. Etzion et al. (2006), Caldentey and Vulcano (2007), Hammond (2013), and Bauner (2015) present models in which both mechanisms coexist, but rely on ex-ante buyer or seller heterogeneity. In contrast, we show that both mechanisms may be used in equilibrium under generic parameters, even though buyers and sellers are homogeneous ex-ante.⁵

Our paper also contributes to explaining equilibrium price dispersion, where search frictions allow sellers to charge different prices for a homogeneous good. This behavior is rational

⁵The above papers also differ from ours in that each assumes a static setting. Kultti (1999) examines a dynamic setting, obtaining a continuum of payoff-equivalent equilibria in which wait times are equivalent in auctions vs. posted prices. In contrast, our model generates a unique equilibrium predicting that buyer wait times are shorter in posted-price listings than in auctions (and vice-versa for sellers). Bauner (2015) is only dynamic on the side of the seller, whose unsold product can be sold in the next period until a common deadline; but each buyer only participates in the market for one period.

for sellers when buyers differ in their search costs (Salop and Stiglitz, 1977; Stahl, 1989; Backus et al., 2014; Schneider, 2014) or are inattentive (Malmendier and Lee, 2011); or when sellers obfuscate (Ellison and Wolitzky, 2012) or do not honor previous quotes (Akin and Platt, 2014). Price dispersion in auctions has been documented by Chiou and Pate (2010) for gift cards and Einav et al. (2015) for items offered repeatedly by the same seller. We document price dispersion on the same order of magnitude across a wide variety of new-in-box items. We join Backus et al. (2014) in providing theoretical foundations for that price dispersion.⁶ Their search model features two simultaneous auctions, one of which can only be seen after costly search. Buyers differ in their search costs, leading to less competition in the obscure auction and frequent differences in closing prices across the auctions. This relates to simultaneous search models such as Burdett and Judd (1983), whereas ours resembles sequential search as in Diamond (1987). Our model delivers *pure price dispersion*, in the sense that even if all sellers are identical and buyers are ex-ante identical in their valuation and time to search, their ex-post-differing deadlines create a continuum of dispersed prices. This also creates *within-individual price dispersion*, since each buyer’s reservation price systematically changes with search duration (as it did for unemployed workers in Akin and Platt, 2012).

Our work also connects to the nascent literature on infinite sequential auctions (Zeithammer, 2006; Ingster, 2009; Said, 2011, 2012; Backus and Lewis, 2012; Bodoh-Creed, 2012; Hendricks et al., 2012; Hendricks and Sorensen, 2015), in which bidders shade their bids below their valuations, taking into account the continuation value of future participation.⁷ These papers, as well as ours, focus on dynamics between auctions rather than within an auction, which occur instantaneously via second-price sealed bidding.⁸ We contribute to this literature by providing an alternative source of heterogeneity among bidders. Rather than assuming buyer heterogeneity in willingness-to-pay is exogenous, our model generates this heterogeneity endogenously as some unlucky bidders search longer than others, and therefore are nearer their deadline and willing to pay more. A distinguishing prediction of our model, and one for which we find strong empirical evidence, is that a bidder’s bid will increase the longer the bidder has participated in auctions for a given item.

⁶A more basic source of dispersion arises due to occasional lack of competition. Kultti (1999) and Julien et al. (2001) demonstrate this in the extreme case where a bidder wins for free if he is alone, but if anyone else participates, they compete until the price equals their common valuation.

⁷This literature also focuses on online auctions. Earlier work considered a finite sequence of auctions (Milgrom and Weber, 2000; Engelbrecht-Wiggans, 1994; Jeitschko, 1999), which induces a common deadline for all potential buyers, beyond which the good cannot be obtained. In contrast, deadlines in our model are idiosyncratic, and the good is always available.

⁸The exceptions are Hendricks et al. (2012), where new bidders move last to avoid disclosing their arrival, and Said (2012), where each period features a multi-unit ascending auction. Within-auction dynamics have mostly been studied in the context of a single auction (Ambrus et al., 2014; Hopenhayn and Saeedi, 2015) or concurrent auctions (Peters and Severinov, 2006; Ely and Hossain, 2009). In those environments, bidders can benefit from incremental bidding or waiting till the last minute (*sniping*) rather than submitting their true valuation as their only bid (Roth and Ockenfels, 2002).

2 Buyer Behavior

We first model buyers' choices. Consider a market for a homogeneous good in a continuous-time environment. Buyers enter the market at a constant rate of δ , seeking one unit of the good which is needed for consumption in T units of time. Buyers will enjoy utility x (measured in dollars) from the good at the time of consumption; if purchased early, that future utility is discounted at rate ρ . Thus, if the good is purchased with s units of time remaining until the buyer's deadline, his realized utility at the time of purchase is $xe^{-\rho s}$ minus the purchase price.⁹

The good is offered in second-price sealed-bid auctions that, from an individual buyer's perspective, occur at Poisson rate α . When such an auction occurs, the buyer joins a pool of participants for that auction with exogenous probability τ , reflecting the possibility that a buyer can be distracted from auction participation by other commitments. Each participant submits a bid and immediately learns the auction outcome, with the highest bidder winning and paying the second highest bid. Alternatively, at any time, a buyer can obtain the good directly at a posted price of z . We assume throughout that $x \geq z$, so that buyers weakly benefit from purchasing via the posted-price option. It is important to note that our model does not require that x , the final consumption utility, be identical across buyers; x may vary across buyers so long as it lies weakly above z .

Every buyer shares the same deadline T on entering the market, but because they enter the market at random times, they differ in their remaining time s . In any given auction, bidders do not know the number of other competing bidders and each bidder's state s is private information. However, the distribution of bidder types in the market (represented by cumulative distribution $F(s)$) is commonly known, as is the total stock of buyers in the market, a finite mass denoted H . Both $F(s)$ and H are endogenously determined.

The strategic questions for buyers are what bid to submit and when to purchase from the posted-price listings. This dynamic problem can be expressed recursively. Let $V(s)$ denote the discounted expected utility of a buyer with s units of time remaining until his deadline. Each participant submits a bid $b(s)$ that depends on his time remaining.¹⁰

⁹Section A of the Supplemental Appendix presents a more general version of the model in which buyers receive a fraction of their utility x immediately at the time of purchase. Qualitatively, this has minimal impact on equilibrium behavior.

¹⁰One abstraction in our model is that bidders do not infer any information about their rivals from prior rounds. Such information is unimportant if valuations are redrawn between each auction, but if valuations are persistent, this *leakage* of private information leads to further shading of bids (Backus and Lewis, 2012; Said, 2012). Leakage cannot occur in our model because encountering the same opponent in a future auction is a probability zero event (as in Zeithammer, 2006). This approximates a large market, where the probability of repeat interactions are too low to justify tracking hundreds of opponents. In our data, if a bidder encounters an opponent and later participates in another auction for the same item, he only has an 8.4% chance of encountering that opponent again.

The optimal behavior is to bid one's reservation value, setting:

$$b(s) = xe^{-\rho s} - V(s), \quad (1)$$

that is, the present value of the item minus the opportunity cost of skipping all future auctions. As in the standard second-price sealed-bid auction, this strategy is weakly dominant.¹¹ We assume that $b(s)$ is decreasing in s (willingness to pay increases as the deadline approaches), and later verify that this holds in equilibrium. In this section, the auctioneer is assumed to open the bidding at $b(T)$, which is relevant when only one buyer participates.¹²

We assume that the number of bidders in any given auction is determined by a draw from a Poisson distribution with mean $\lambda = \tau H$, so the probability that n bidders participate is $e^{-\lambda} \lambda^n / n!$. While this distribution literally governs the *total* number of participants per auction, it also describes (from the perspective of a bidder who has just entered) the probability that n *other* bidders will participate. This convenient parallel between the aggregate distribution (which enters expected revenue and the steady state conditions) and the distribution faced by the individual (which enters his expected utility) is crucial to the tractability of the model but is not merely abuse of notation. Myerson (1998) demonstrates that in Poisson games, the individual player would assess the distribution of other players the same as the external game theorist would assess the distribution for the whole game.

In light of this, a buyer's expected utility in state s can be expressed in the following Hamilton-Jacobi-Bellman (HJB) equation:

$$\begin{aligned} \rho V(s) = -V'(s) + \tau \alpha \left(\sum_{n=0}^{\infty} \frac{e^{-\lambda} \lambda^n}{n!} (1 - F(s))^n (xe^{-\rho s} - V(s)) \right. \\ \left. - e^{-\lambda} b(T) - \sum_{n=1}^{\infty} \frac{e^{-\lambda} \lambda^n}{n!} \int_s^T b(t) n (1 - F(t))^{n-1} F'(t) dt \right). \end{aligned} \quad (2)$$

We now explain the continuous-time formulation in (2) piece by piece.¹³ The left-hand side represents the flow of expected utility that a buyer with s units of time remaining receives each instant. The right hand side depicts potential changes in (net) utility times the rate at which those changes occur.¹⁴ The term $-V'(s)$ accounts for the steady passage of time: just

¹¹Suppose he instead bids price $p > b(s)$, and the second-highest bid is q . This results in the same payoff whenever $q \leq b(s)$, but yields negative surplus when $q \in (b(s), p]$. Similarly, if he bids $p < b(s)$, he has the same payoff whenever $q \leq p$, but misses out on positive surplus when $q \in (p, b(s))$.

¹²We consider seller optimization of the reserve price in Section B of the Supplemental Appendix, where we demonstrate that, provided that the search length T is reasonably short, $b(T)$ is in fact the optimal reserve price due to competition among potential sellers.

¹³Note that the optimal bid from (1) is applied in computing this expected utility. This HJB formulation is derived from a discrete-time Bellman equation in Section C of the Supplemental Appendix.

¹⁴If buyers received any enjoyment from the search process itself, it would appear as a positive constant on the right-hand side. This would have negligible effect on the solution other than lowering willingness to pay, since making a purchase would cut off this flow of enjoyment.

by remaining in the market for another unit of time, the buyer's state s decreases by 1 unit (hence the negative sign), so his utility changes by $-V'(s)$.

When an auction occurs and the individual participates in it — which occurs at a rate of $\tau\alpha$ auctions per unit of time — the expected payoff depends on the number n of other participants, which is Poisson distributed with mean λ . The buyer in state s will only win (have the highest bid $b(s)$) if all n other participants have more than s units of time remaining. This occurs with probability $(1 - F(s))^n$. When this occurs, the term $xe^{-\rho s} - V(s)$ depicts the change in utility due to winning.

The terms on the second line compute the expected cost of winning (*i.e.* the average second-highest bid times the probability of winning and thus paying it). If there are no other participants (which occurs with probability $e^{-\lambda}$), the bidder pays the starting price $b(T)$. Otherwise, inside the sum we find the probability of facing n opponents, while the integral computes the expected highest bid among those n opponents, which has a probability density of $n(1 - F(t))^{n-1}F'(t)$.

Buyers also have the option to purchase from the posted-price listings at any time, receiving utility $xe^{-\rho s} - z$. However, a buyer in state s can obtain a discounted expected utility of $(x - z)e^{-\rho s}$ by waiting until $s = 0$ to buy, which is strictly preferred. This delay strategy has even greater payoff due to the possibility of winning an auction in the meantime. Hence, the posted-price option is exercised if and only if $s = 0$, and the expected utility of a buyer who reaches his deadline is the consumer surplus from making the purchase:

$$V(0) = x - z. \tag{3}$$

2.1 Steady State Conditions

In most auction models, the distribution of willingness to pay is exogenously given as a primitive of the model (Milgrom and Weber, 1982; Athey and Haile, 2002). Here, the buyers' reservation prices $b(s)$ are endogenous, affected by the value of further search, $V(s)$, in addition to the underlying utility, x . The distribution $F(s)$ of buyer states is also endogenously determined by how likely a bidder is to win and thus exit the market at each state, which itself depends on the distribution of competitors he faces.

We require that the distribution of buyers remains constant over time. As buyers exit the market, they are exactly replaced by new customers; as one group of buyers get closer to their deadline, a proportional group follows behind. These steady state requirements are commonly used in equilibrium search theory to make models tractable. Rather than tracking the exact number of current buyers and sellers, which would change with each entry or exit and require a large state space, the aggregate state is always held at its average. This does not eliminate all uncertainty — for instance, the number of bidders in a given auction need not equal the average λ — but these shocks are transitory, as the number of bidders in the next auction is

independently drawn from a constant (though endogenous) Poisson distribution. Thus, steady state conditions smooth out the short-run fluctuations around the average, and capture the long-run average behavior in a market.

To begin the steady state analysis, consider the relative density of bidders over their time until deadline. For instance, consider a cohort in state $s > 0$. In each of the next Δ units of time, suppose on average that a fraction w of these buyers win an auction and exit. Then steady state requires that $F'(s - \Delta) = F'(s) - w\Delta F'(s)$. After rearranging and letting $\Delta \rightarrow 0$, we obtain $F''(s) = wF'(s)$. The steady-state law of motion is therefore:

$$F''(s) = F'(s)\tau\alpha \sum_{n=0}^{\infty} \frac{e^{-\lambda}\lambda^n}{n!} (1 - F(s))^n. \quad (4)$$

Here, $\tau\alpha$ is the rate at which buyers participate in an auction, and the summation indicates how likely they are to have the highest bid of n participants and thus win. Recall that all bidders enter the market in state $s = T$; at all other states, bidders only exit the market (by winning, or running out of time at $s = 0$). Thus the bidder density $F'(s)$ must decrease as s falls, which holds because the right-hand side of (4) is always positive.

Equation (4) defines the law of motion for the interior of the state space $s \in (0, T)$. The end points are defined by requiring $F(s)$ to be a continuous distribution:

$$\lim_{s \rightarrow 0} F(s) = F(0) = 0 \quad (5)$$

$$\lim_{s \rightarrow T} F(s) = F(T) = 1. \quad (6)$$

These two conditions prevent a discontinuous jump at either end of the distribution — that is, a positive mass (an *atom*) of buyers who share the same state, $s = 0$ or T . Atoms cannot occur in our environment because all buyers who reach state $s = 0$ immediately purchase from a posted-price listing and exit the market; hence, no stock of state 0 buyers can accumulate. Similarly, no stock of state T buyers can accumulate because as soon as they enter the market, their clock begins steadily counting down. Conveniently, a continuous distribution also ensures that no two bids will tie with positive probability.

Finally, we ensure that the total population of buyers remains steady. Since H is the stock of buyers in the market, $HF(s)$ depicts the mass of buyers with less than s units of time remaining, and $HF'(s)$ denotes the average flow of state s buyers over a unit of time. Thus, we can express the steady state requirement as:

$$\delta = H \cdot F'(T). \quad (7)$$

Recall that buyers exogenously enter the market at a rate of δ new buyers in one unit of time. Since all buyers enter the market in state T , this must equal $HF'(T)$, the average flow of state T buyers over one unit of time.

2.2 Buyer Equilibrium

The preceding optimization by buyers constitutes a dynamic game. We define a *buyer steady-state equilibrium* of this game as a bid function $b^* : [0, T] \rightarrow \mathbb{R}$, a distribution of buyers $F^* : [0, T] \rightarrow [0, 1]$, an average number of buyers $H^* \in \mathbb{R}^+$, and an average number of participants per auction $\lambda^* \in \mathbb{R}^+$, such that:

1. Bids b^* satisfy the HJB equations (1) through (3), taking F^* and λ^* as given.
2. The distribution F^* satisfies the Steady State equations (4) through (6).
3. The average mass of buyers in the market H^* satisfies Steady State equation (7).
4. The average number of participants per auction satisfies $\lambda^* = \tau H^*$.

The first requirement requires buyers to bid optimally; the last three require buyers' beliefs regarding the population of competitors to be consistent with steady state.

We now characterize the unique equilibrium of this sequential auctions game for a homogeneous retail good. Our equilibrium requirements can be translated into two second-order differential equations regarding $F(s)$ and $b(s)$. The equations themselves have a closed-form analytic solution, but one boundary condition does not. We solve for the equilibrium λ^* which implicitly solves the boundary condition. If $\phi(\lambda)$ is defined as:

$$\phi(\lambda) \equiv \delta - \alpha (1 - e^{-\lambda}) - \delta e^{\lambda - \tau T (\delta + \alpha e^{-\lambda})}, \quad (8)$$

then the boundary condition is equivalent to $\phi(\lambda^*) = 0$. Note that δ is the mass of buyers who enter the market over a unit of time, while $\alpha (1 - e^{-\lambda})$ is the mass of bidders who win an auction and thus exit over a unit of time. The last term in $\phi(\lambda)$ turns out to be $H \cdot F'(0)$ (derived below), which is the mass of buyers who exit because of hitting their deadline over a unit of time. Thus, $\phi(\lambda^*) = 0$ ensures that buyers newly entering the market exactly replace those who exit. The rest of the equilibrium solution is expressed in terms of λ^* .

First, the distribution of buyers over time remaining until deadline is:

$$F^*(s) = 1 - \frac{1}{\lambda^*} \ln \left(\frac{\delta + \alpha e^{-\lambda^*}}{\delta e^{\tau(s-T)(\delta + \alpha e^{-\lambda^*})} + \alpha e^{-\lambda^*}} \right), \quad (9)$$

and its associated density function is:

$$F'(s) = \frac{\delta}{\lambda^*} \cdot \frac{\tau (\delta + \alpha e^{-\lambda^*}) e^{\tau(s-T)(\delta + \alpha e^{-\lambda^*})}}{\delta e^{\tau(s-T)(\delta + \alpha e^{-\lambda^*})} + \alpha e^{-\lambda^*}}.$$

If the probability of winning an auction were constant throughout a buyer's search, then buyers would be exponentially distributed. Indeed, we see an exponential density function in the numerator of $F'(s)$, but it decays as if buyers exit more often than auctions even occur

(since $\tau(\delta + \alpha e^{-\lambda^*}) > \tau\alpha$ when $\phi(\lambda^*) = 0$). This exponential decay is slowed down by the fact that buyers only exit if they win an auction; this adjustment is reflected in the denominator of $F'(s)$.

Indeed, F' is always increasing in s but typically changes from convex to concave as s increases. This is because buyers rarely win at the beginning of their search, but increasingly do so as time passes and they increase their bids. Those near their deadline win quite frequently, but few of them remain in the population, so their rate of exit decreases.

The average number of buyers in the market is simply:

$$H^* = \frac{\lambda^*}{\tau}. \quad (10)$$

Equilibrium bids are expressed as a function of the buyer's state, s , as follows:

$$b^*(s) = ze^{-\rho s} \cdot \frac{\tau(\delta + \alpha e^{-\lambda^*})(\delta e^{\lambda^*} + \alpha e^{\rho(s-T)}) + \rho(\delta e^{\lambda^*} + \alpha e^{\tau(T-s)(\delta + \alpha e^{-\lambda^*})})}{\tau(\delta + \alpha e^{-\lambda^*})(\delta e^{\lambda^*} + \alpha e^{-\rho T}) + \rho(\delta e^{\lambda^*} + \alpha e^{\tau T(\delta + \alpha e^{-\lambda^*})})}. \quad (11)$$

Alternatively, we can express the bidding function more succinctly, and with easier interpretation, as follows:

$$b^*(s) = z \left(1 - \frac{\rho \int_0^s g(t) dt}{g(T) + \rho \int_0^T g(t) dt} \right), \quad \text{where} \quad g(t) \equiv \frac{\tau(\delta + \alpha e^{-\lambda^*}) e^{-\tau(\delta + \alpha e^{-\lambda^*})t}}{e^{-\lambda F(t)}} e^{-t\rho}.$$

To interpret the function $g(t)$, note that $e^{-\lambda F(t)}$ in the denominator is the probability of winning for a buyer who participates in the auction in state t . Thus, $1/(e^{-\lambda F(t)})$ is the average number of auctions in which a buyer in state t would need to participate before winning. The numerator of $g(t)$ is the density function describing the likelihood of the next auction occurring in exactly t units of time. Finally, a win in state t is discounted by $e^{-t\rho}$ because the item is not needed until time $s = 0$.

Thus, the integral $\int_0^T g(t) dt$ is the average (discounted) number of auction attempts required to win before the deadline. The term $g(T)$ in the denominator of $b(s)$ accounts for the possibility that the buyer does not win any auction and is forced to buy at the posted price. The integral $\int_0^s g(t) dt$ in the numerator of $b(s)$ is the portion of those auction attempts that are still possible. The ratio of these integrals indicates the fraction of opportunities remaining. Buyers are effectively shading their bid below the retail price of z in accordance with the likelihood of winning between s and the deadline. As that window closes, they expect to have fewer opportunities and they draw closer to bidding the retail price.

The next result shows that this proposed solution is both necessary and sufficient to satisfy the equilibrium requirements. In the proof (given in the Appendix), we translate the equilibrium conditions into first-order differential equations of $F(s)$ and $b(s)$. Our proposed

solution not only satisfies these differential equations, but is the unique solution to them.

Proposition 1. *Equations (9) through (11) satisfy equilibrium conditions 1 through 4, and this equilibrium solution is unique.*

As previously conjectured, one can readily show that $b'(s) < 0$; that is, bids increase as buyers approach their deadline. Moreover, this increase accelerates as the deadline approaches, since $b''(s) > 0$.

Proposition 2. *In equilibrium, $b'(s) < 0$ and $b''(s) > 0$.*

Discounting plays a critical role in creating dispersion among the bidder valuations, even separately from the deadline T . For instance, if buyers become extremely patient ($\rho \rightarrow 0$), the bidding function approaches $b(s) = z$ regardless of time until deadline.¹⁵ Discounting causes buyers to prefer postponing payment until closer to the time of consumption, and thereby creates some variation in willingness to pay. This makes it possible to win an auction at a discount relative to z , which further reduces the willingness to pay. But if discounting is eliminated, the variation disappears; everyone is willing to bid full price, so auctions do not offer a discount at all. Even so, the Proposition 2 prediction of bids increasing over time is not solely due to discounting; it also reflects reduced bid shading as buyers have fewer future opportunities to win an auction. We decompose these effects in Section 5.1.

The average time between auctions ($1/\alpha$) is of similar importance. This is the search friction that buyers face, as it prevents them from making unlimited attempts at winning an auction. In the extreme, if auctions almost never occurred ($\alpha \rightarrow 0$), the value of search $V(s)$ drops to zero, so a bidder's reservation price would simply equal his present value of the good: $b(s) = xe^{-\rho s}$.¹⁶ For larger values of α , a bidder would optimally reduce his bid well below this, as he is likely to have many opportunities to win a deal before his deadline.

2.3 Comparative Statics

We next examine how the equilibrium behavior reacts to changes in the underlying parameters. Although our equilibrium has no closed form solution, these comparative statics can be obtained by implicit differentiation of $\phi(k)$, which allows for analytic derivations reported in Section D of the Supplemental Appendix.

Table 1 reports the sign of the derivatives of four key statistics. The first and second are the average number of participants per auction, λ^* , which reflects how competitive the auction is among buyers, and the average mass of buyers in the market, H^* , which is always proportional to λ^* . Third is the flow of buyers who never win an auction and must use the posted-price listings; in the next section we will see that this crucially affects the profitability

¹⁵The fractional term of (11) approaches zero. Note that the equilibrium λ^* from (8) is unaffected, as is the distribution of bidders in (9).

¹⁶In this limit, the equilibrium $\lambda^* = \tau\delta T$, while the distribution of bidders is $F(s) = s/T$.

of the posted-price market. Fourth is the bid of new buyers in the market, indicating the effect on buyers' willingness to pay. This comparative static can be derived at any s and has a consistent effect, but the simplest computation occurs at $s = T$. This comparative static also captures price dispersion, both within auctions and between auctions and posted prices. The posted price z is fixed, so a lower $b^*(T)$ indicates greater dispersion.

Changes in α have an intuitive impact. With more frequent auctions (reduced search frictions) the value of continued search is greater as there are more opportunities to bid. The increase in auctions creates more winners, reducing the stock of bidders and the number of competitors per auction. Both of these effects lead bidders to lower reservation prices.

Changes in τ have nearly the reverse effect, though there are opposing forces at work. A higher likelihood of participating also reduces the search friction of a given bidder, as he will participate in more of the existing auctions. However, all other bidders are more likely to participate as well. The net result is typically higher bids, because the greater number of competitors dominates the increased auction participation to reduce the value of search. However, this does depend on parameter values; in particular, when τ or ρ are very close to zero, extra participation dominates extra competitors, leading to lower bids.

The discount rate has no impact on the number or distribution of bidders, as ρ does not enter into equations (8) through (10). Intuitively, this is because the rate at which bidders exit is a matter of how often auctions occur, which is exogenous here. Also, who exits is a matter of the ordinal ranking of their valuations, which does not change even if the cardinal values are altered. Indeed, the bids react as one would expect: buyers offer less when their utility from future consumption is more heavily discounted. Changes in ρ will play a greater role once we endogenize sellers in Section 3.

When buyers are given more time to search, one would intuitively expect this to work to their advantage. Indeed, bids are lower, but only because bidders now enter with more time and hence a lower starting bid, $b(T)$. Even so, the longer period of search allows for more bidders to accumulate (H^* increases), creating greater competition within each auction.

We can also consider the effect (not shown in Table 1) of the parameter change on the expected revenue generated in an auction, which we formally derive in the next section. For the first three parameters, revenue moves in the same direction as bids because the number of participants per auction is either constant or moves in the same direction. For instance, more auctions will reduce the bids and reduce the number of bidders; thus expected revenue must be lower. The intriguing exception is when the deadline changes; there, the more participants override the lower initial bid, driving up expected revenue.

3 Seller Behavior

We next examine optimization by sellers in this environment, allowing them to decide whether to enter the market and whether to sell their product via auctions or the posted-price listing.

We consider a continuum of sellers producing an identical good.¹⁷ Each has negligible effect on the market, taking the behavior of other sellers and the distribution and bidding strategy of buyers as given; yet collectively, their decisions determine the frequency with which auctions occur. In other words, by modeling seller choices we endogenously determine α from the preceding section.

Each seller can produce one unit of the good at a marginal cost of $c < z$, with fraction γ of this cost incurred at the time the good is sold (the *completion cost*), and $1 - \gamma$ incurred when seller first enters the market (the *initial production cost*).¹⁸ For either selling format, sellers also pay a listing fee of ℓ each unit of time from when the seller enters the market to when the good is sold. We assume that there are no barriers to entry for sellers. Upon entry, each seller must decide whether to join the auction or the posted-price market.

3.1 Auction Sellers

The advantage of the auction sector is that the sale occurs more quickly. Let η denote the exogenous Poisson rate of auction closing, so $1/\eta$ is the average time delay between the listing and closing of an auction.¹⁹ At its conclusion, the auction's expected revenue (conditional on at least one bidder participating) is denoted θ and computed as follows:

$$\theta \equiv \frac{1}{1 - e^{-\lambda}} \left(\lambda e^{-\lambda} b(T) + \sum_{n=2}^{\infty} \frac{e^{-\lambda} \lambda^n}{n!} \int_0^T b(s) n(n-1) F(s) (1 - F(s))^{n-2} F'(s) ds \right). \quad (12)$$

Inside the parentheses, the first term applies when only one bidder participates and therefore wins at the opening price of $b(T)$; this happens with probability $\lambda e^{-\lambda}$. The sum handles cases when there are $n \geq 2$ bidders, with the integral computing the expected bid $b(s)$ of the second-highest bidder. All of this must be divided by the probability that at least one bidder arrives, $1 - e^{-\lambda}$.

To determine the expected profit of the auction seller, we must also account for the costs of production and the expected time delay. Let Π_a denote the expected profit from the vantage of someone who has already incurred the initial production cost $(1 - \gamma)c$ and has just posted the listing. This profit can be computed in the following HJB equation:

$$\rho \Pi_a = -\ell + \eta (1 - e^{-\lambda}) (\theta - \gamma c - \Pi_a). \quad (13)$$

¹⁷While we refer to each seller as producing a single unit, one could also think of a seller offering multiple units so long as the production and listing costs scale up proportionately.

¹⁸In the extreme, $\gamma = 1$ would indicate the ability to build-to-order or just-in-time inventories, while $\gamma = 0$ indicates a need to build in advance (like a spec home built without a committed buyer). Intermediate values could be taken literally as partial production, or as full initial production followed by additional expenses (such as shipping costs) at the time of sale. It could also reflect producing in advance but delaying full payment of the cost through the use of credit.

¹⁹Under the default setting at eBay, an auction concludes one week after creating the listing, which provides time for bidders to examine the listing. In our data, 43% of listings use this default setting.

The right-hand side of (13) indicates that the seller incurs the listing fee per unit of time. The listing closes at Poisson rate η , but if no bidders have arrived (which occurs with probability $e^{-\lambda}$) then the seller re-lists the item and continues waiting for the new auction to close. If at least one bidder participates, the seller's net gain is the realized benefit (revenue minus the completion cost) relative to the expected profit. Of course, from the perspective of a potential entrant as an auction seller, the expected profits from entry are net of the initial production cost: $\Pi_a - (1 - \gamma)c$.

3.2 Posted-Price Sellers

The posted-price listing will always sell the good at a higher price, since for all bids, $b(s) < z$; the disadvantage of this format is that sellers may wait a considerable time before being chosen by a buyer. Let ζ denote the rate of encountering a customer, so $1/\zeta$ is the average wait of a posted-price seller. Sellers take ζ as given, but it will be endogenously determined as described in the next subsection.

The discounted expected profit of posted-price sellers already in the market, denoted Π_p , is computed in the following HJB equation:

$$\rho\Pi_p = -\ell + \zeta(z - \gamma c - \Pi_p). \quad (14)$$

Like auction sellers, posted-price sellers incur the listing fee ℓ per unit of time that they await a buyer.²⁰ When they encounter a buyer (which they do at rate ζ), the purchase always occurs, with a net gain of $z - \gamma c$ relative to Π_p . For sellers contemplating entry into the posted-price market, their expected profit is $\Pi_p - (1 - \gamma)c$.

3.3 Steady State Conditions

As with the population of buyers, the stock and flow of sellers are also assumed to remain stable over time. In the aggregate, recall that δ buyers enter (and exit) the market over a unit of time; thus, we need an identical flow of δ sellers entering per unit of time so as to replenish the δ units sold.

In addition, the mass of sellers in each market must remain steady. Let σ be the fraction of newly-entered sellers joining the auction market, so that $\sigma\delta$ choose to list an auction over a unit of time. This must equal the mass of auctions that close with at least one bidder over the same unit of time:

$$\sigma\delta = \alpha(1 - e^{-\lambda}). \quad (15)$$

The remaining $(1 - \sigma)\delta$ sellers flow into the posted-price market over a unit of time. This

²⁰Very little changes if we allow the listing fee to differ across the two selling formats.

must equal the flow of purchases made by buyers that hit their deadline:

$$(1 - \sigma)\delta = HF'(0). \quad (16)$$

At any moment, both markets will have a stock of active listings — sellers who are waiting for a buyer to make a purchase or for their auction to close. Let A denote the measure of auction sellers with active listings, and P denote the same for posted-price sellers. From the perspective of the individual auction seller, her auction will close at rate η ; but with A sellers in the market at any instant, there will be ηA auctions that close over a unit of time. From the buyer's perspective, α auctions close over a unit of time; thus, these must equate in equilibrium:

$$\eta A = \alpha. \quad (17)$$

A similar condition applies to posted-price sellers. In aggregate, $HF'(0)$ purchases occur over a unit of time (sold to buyers who reach their deadline). From the individual seller's perspective, she can sell ζ units over one unit of time; collectively, these sellers expect to sell ζP units. In equilibrium, the expected sales must equal the expected purchases:

$$\zeta P = HF'(0). \quad (18)$$

3.4 Market Equilibrium

With the addition of the seller's problem, we augment the equilibrium definition with three conditions. A *market steady-state equilibrium* consists of a buyer equilibrium as well as expected revenue $\theta^* \in \mathbb{R}^+$, expected profits $\Pi_a^* \in \mathbb{R}^+$ and $\Pi_p^* \in \mathbb{R}^+$, arrival rates $\alpha^* \in \mathbb{R}^+$ and $\zeta^* \in \mathbb{R}^+$, seller stocks $A^* \in \mathbb{R}^+$ and $P^* \in \mathbb{R}^+$, and fraction of sellers who enter the auction sector, $\sigma^* \in [0, 1]$, such that:

1. Expected revenue θ^* is computed from Equation (12) using the bidding function $b^*(s)$ and distribution $F^*(s)$ derived from the buyer equilibrium, given α^* .
2. Prospective posted-price entrants earn zero expected profits: $\Pi_p^* = (1 - \gamma)c$, given ζ^* .
3. Prospective auction entrants earn zero expected profits: $\Pi_a^* = (1 - \gamma)c$ if $\alpha^* > 0$, or $\Pi_a^* \leq (1 - \gamma)c$ if $\alpha^* = 0$.
4. $\alpha^*, \zeta^*, \sigma^*, A^*$, and P^* satisfy the Steady State equations (15) through (18).

The first requirement simply imposes that buyers behave optimally as developed in Section 2, given the endogenously determined auction arrival rate. The fourth imposes the steady state conditions. The second and third requirements impose zero expected profits for both types of sellers, which is implied by the large, unrestricted pool of potential entrants. If either market offered positive profits, additional sellers would be attracted to that market and profits would

fall: more posted-price sellers P would reduce the rate of selling ζ , and more auction sellers A would increase the auction arrival rate and decrease expected revenue θ . Together, these two requirements also ensure that sellers are indifferent about which market they enter, allowing them to randomize according to the mixed strategy σ .

In the third requirement, we allow for the possibility that no auctions are offered, but this can only occur if the expected profit from an auction would be weakly less than that of a posted-price listing. A similar possibility could be added to the posted-price market, but that market would never shut down in equilibrium. Due to the search friction, a fraction of buyers will inevitably reach their deadline; as a consequence, the posted-price market can always break even if a sufficiently small stock of sellers serves these desperate buyers.

Since the posted price always exceeds the realized auction price, expected profits can only be equated between the two markets if auction listings are sold more quickly. In other words, if both types of listings are offered in equilibrium, then $\zeta^* < \eta$.

While the market equilibrium conditions simplify considerably, they do not admit an analytic solution and we must numerically solve for α^* and λ^* . All other equilibrium objects can be expressed in terms of these. The solution for these endogenous variables comes from equation (8) and the third market equilibrium requirement, which can be written:

$$\theta^* = c + \frac{\ell + \rho c(1 - \gamma)}{\eta(1 - e^{-\lambda^*})}. \quad (19)$$

This ensures that the expected revenue from each auction precisely covers the expected cost of listing and producing the good. The costs (on the right-hand side) are affected by λ because of the (small) chance that no bidders arrive, while expected revenue (on the left-hand side) is affected by both λ and α because of their influence on the bidding function and distribution of buyers. To compute θ^* , (12) must be evaluated using $b(s)$ and $F(s)$ from the buyer equilibrium; the resulting equation is cumbersome and is reported in the proof of Proposition 3 in the Appendix. At the same time, a buyer equilibrium requires that $\phi(\lambda^*) = 0$ from (8); here, we note that this equation involves both λ and α . Equilibrium is attained when both (8) and (19) simultaneously hold, which can only be solved numerically.

Once α^* and λ^* are found, the remaining equilibrium objects are easily solved as follows:

$$\Pi_a^* = (1 - \gamma)c \quad (20)$$

$$\Pi_p^* = (1 - \gamma)c \quad (21)$$

$$A^* = \frac{\alpha^*}{\eta} \quad (22)$$

$$P^* = \frac{(z - c)(\delta - \alpha^*(1 - e^{-\lambda^*}))}{\ell + \rho(1 - \gamma)c} \quad (23)$$

$$\zeta^* = \frac{\ell + \rho(1 - \gamma)c}{z - c} \quad (24)$$

$$\sigma^* = \frac{\alpha^* (1 - e^{-\lambda^*})}{\delta}. \quad (25)$$

It is apparent that $\sigma^* \geq 0$. To see that $\sigma^* < 1$, note that the equilibrium condition $\phi(\lambda^*) = 0$ requires that $\alpha (1 - e^{-\lambda}) < \delta$. This also ensures that $P^* > 0$.

The following proposition demonstrates that these solutions are necessary for any equilibrium in which auctions actually take place.

Proposition 3. *A market equilibrium with active auctions ($\alpha^* > 0$) must satisfy $\phi(\lambda^*) = 0$, equations (9) through (12), and equations (19) through (25).*

The solution described in Proposition 3 can be called a *dispersed equilibrium*, to use the language of equilibrium search theory, as we observe the homogeneous good being sold at a variety of prices. By contrast, in a *degenerate equilibrium*, the good is always sold at the same price. This only occurs if all goods are purchased via posted-price listings and no auctions are offered ($\alpha^* = \sigma^* = 0$). We can analytically solve for this degenerate equilibrium and for the conditions under which it exists, as described in the following proposition.

Proposition 4. *The degenerate market equilibrium, described by equations (9) through (11) and equations (21) through (25) with $\alpha^* = 0$ and $\lambda^* = \tau\delta T$, exists if and only if*

$$\frac{\tau\delta}{1 - e^{-\tau\delta T}} \cdot \frac{\tau\delta + (\rho T(\rho + \tau\delta) - \tau\delta) e^{-(\rho + \tau\delta)T}}{(\rho + \tau\delta)^2} z \leq c + \frac{\ell + \rho c(1 - \gamma)}{\eta(1 - e^{-\tau\delta T})}. \quad (26)$$

Moreover, if this condition fails, a dispersed market equilibrium will exist. Thus, an equilibrium always exists.

The left-hand side of (26) is the expression for θ when $\alpha = 0$; it calculates the expected revenue that a seller would earn by deviating from $\alpha = 0$, offering an auction when no one else does. For this equilibrium to exist, the expected revenue must be lower than the expected cost of entering the market (the right-hand side of (26)), so that not selling in the auction market is a best response. We can consider such a deviation because buyers still wait until their deadline before purchasing via the posted-price listing. Thus, in this equilibrium there would be $H^* = \delta T$ buyers in the market, uniformly distributed on $s \in [0, T]$, who would be available as bidders in the measure-zero event that an auction occurs and would bid their reservation price $b(s) = ze^{-\rho s}$.

Equation (26) provides insight on when a degenerate solution will occur. On the right-hand side, one can see that a high production cost or listing cost can make the auction market unprofitable. The posted-price market can compensate for these costs by keeping a low stock of sellers so that the item is sold very quickly. Long delays before closing the auction (a small η) also increase the cost of the auction. On the left-hand side, δ and τ have the largest impact of any of the parameters on expected revenue. With either a small flow of new buyers entering the market or a tiny fraction of them paying attention to a given auction, the number of

participants per auction will be low. Without much competition in the second-price auction, expected revenue will be too low to cover expected costs.

In equilibrium search models, a degenerate equilibrium often exists regardless of parameter values, essentially as a self-fulfilling prophecy. Buyers won't search if there is only one price offered, and sellers won't compete with differing prices if buyers don't search. Yet in our auction environment, the degenerate equilibrium does not always exist. This is because our buyers do not incur any cost to watch for auctions; even if no auctions are expected, buyers are still passively available should one occur.²¹ In that sense, they are always searching, giving sellers motivation to offer auctions, so long as (26) does not hold.

Proposition 4 proves that an equilibrium always exists; we further conjecture that the equilibrium is always unique. This claim would require that at most one dispersed equilibrium can occur, and that a dispersed equilibrium cannot occur when (26) holds — both of which are true if θ is a decreasing function of α (*i.e.*, more auctions always lead to lower revenue). The complicated expression for θ in the dispersed equilibrium precludes an analytic proof, but we have consistently observed this relationship between α and θ in numerous calculations across a wide variety of parameters.

3.5 Comparative Statics

We now present comparative statics for the market equilibrium. The computation of θ^* prevents analytic determination of the sign of the comparative statics, but numeric evaluation remains consistent over a large space of parameter values. We highlight a few of these, reported in Table 2, noting that even if these phenomena may not always occur, it is striking that they occur at all.

First, we note that for increases in τ under the market equilibrium, the bidding response is the opposite of in the buyer equilibrium. The difference is that when buyers are more attentive, sellers are willing to offer more auctions in the market equilibrium. Additional auctions will improve the continuation value of buyers, and thus reduce their bids. There will still be more participants per auction (which led to higher bids in the buyer equilibrium), but the effect of more auctions dominates to produce a net decline in bids and expected revenue in the market equilibrium.

In the buyer equilibrium, an increase in ρ reduced bids but had no effect on the distribution of buyers. In a market equilibrium, bids will still fall, but sellers offer fewer auctions. Surprisingly, this leads to higher revenue per auction, as it concentrates more buyers per auction. Changes in T behave similarly under either equilibrium definition.

For c , it is remarkable that even though increased production costs do not raise the retail price (by assumption), they still affect auctions in the distribution of buyers and their bids.

²¹This model feature of buyers watching for auctions can be thought of as capturing features of online retail platforms. For example, eBay allows users to create “Searches you follow,” in which the user enters search terms and is periodically informed of any newly listed items that match that search criteria.

Higher costs will shrink the margins in both markets, which the auction market responds to by reducing its flow of sellers. Fewer auctions necessarily mean that more buyers reach their deadline; and this increased demand for posted-price listings more than compensates for the smaller margin. That is, a larger stock of posted-price sellers is needed to return to normal profits. Also, with fewer available auctions, buyers have a lower continuation value from waiting for future auctions. This drives up bidders' reservation prices, but not enough to prevent a smaller flow of auction sellers.

A higher listing fee, ℓ , has a similar effect, but this is surprising because the listing fee applies to both markets and will be paid more often by posted-price sellers (who have to relist their good more times). This drives sellers away from the auction market, and any reduction in the rate of auctions will increase bids. This subtle response illustrates a potential hazard in auction design if buyer valuations are not fundamental but rather the endogenous results of deeper factors. A seemingly neutral change in the auction listing fee not only alters which market sellers use, but also warps the distribution of buyer valuations.

4 Empirical Evidence

4.1 Data and Descriptive Statistics

Our data consist of auctions and posted-price sales on eBay.com for the year from October 1st 2013 to September 30th 2014.²² As our model describes the sale of homogeneous goods, we restrict attention to brand new items which have been matched by the seller to an item in one of several commercially available catalogs; below we will refer to these catalogued products as simply a *product*, and a individual attempt to sell the product will be referred to as a *listing*. These products are narrowly defined, matching a product available at retail stores, such as: “Microsoft Xbox One, 500 GB Black Console,” “Chanel No.5 3.4oz, Women’s Eau de Parfum,” and “The Sopranos - The Complete Series (DVD, 2009).” We remove listings in which multiple quantities were offered for sale; listings with outlier prices (defined as bids in the top or bottom 1% of bids for auctioned items of that product, and similarly for posted-price sales); products with under 25 auction or posted-price sales; and products that went more than 30 days without an auction. The products in our final sample are principally electronics, media, or health/beauty products.

Table 3 presents descriptive statistics for the listings that ended in a sale. In all, there are over one million sales of 3,663 distinct products, split roughly evenly between auctions and posted prices. Means and standard deviations in panel A are computed by taking the mean and standard deviation over all transactions of a given product and then taking the

²²For a discussion of the eBay auction mechanism, see Lucking-Reiley et al. (2007). Not included in our analysis are hybrid formats, namely the *buy-it-now auction*, in which the seller simultaneously auctions the item and offers a posted price (Budish and Takeyama, 2001; Kirkegaard and Overgaard, 2008; Bauner, 2015), and posted-price sales which allow for buyer-seller bargaining (Backus et al., 2015).

mean of these product-level means and standard deviations. The average number of bidders per sold auction is 5.3. The average selling price is higher with posted prices than auctions (\$107 versus \$97). To adjust for differences across products, we follow Einav et al. (2016) and rescale all bids, dividing by the mean price of posted-price sales of that product. This rescaling is also consistent with our model above, in which bids scale multiplicatively with the posted price. The normalized revenue per auction sale is, on average, 85% of the posted price. Panel B demonstrates that both auctions and posted-price sales contain a large number of transactions per product, with a large number of distinct buyers and sellers involved in transactions of each product.

For the remainder of this section, we document several facts in the data that provide evidence in favor of the deadline model.

4.2 Bids Over Time

Our first fact requires following the same bidder across multiple auctions of the same product. Such bidders constitute an economically significant fraction of all bidding activity — 20% of bidders participate in more than one auction of some product, and collectively, these repeat bidders place 47% of all bids. We find that bidders tend to increase their bids from one auction to the next. To compute this, we keep each bidder’s highest bid in each auction in which he participates, yielding a sample of 4,077,410 bids. We order these bids in a chronological sequence for each bidder and product pair, ending when the bidder either wins an auction or does not participate in any more auctions in our sample. This yields 2,728,258 unique of bidding-sequence and product pairs.²³ We then compute the average of the normalized bids, separately for each sequence length and each step within the sequence.

Figure 1 displays the resulting trend across repeated auction participation. Each line corresponds to a different sequence length, and each point to the mean normalized bid for the corresponding auction in the sequence. Due to our normalization, the bids can be read as a percentage of the item’s retail price. For each sequence length, the average bid steadily rises over time from the first to the last auction in the sequence.²⁴ Averaging across all sequence lengths and auction numbers (or from the results of a regression), the bid increases by 2.0 percentage points in each successive auction. An additional pattern observable in Figure 1 consistent with the model is the surprising feature that the bid sequence lines never cross, as discussed further in Section 5.

²³Note that only 3.7% of bidders appear in multiple bidding sequences, meaning the bidder won an auction and was observed bidding again on another listing of the same product at a later point in time.

²⁴Figure A1 in the Supplemental Appendix contains similar results to those in Figure 1 in a regression framework, averaging over all auction sequence lengths. The increase in bids is not driven by the fact that the final bid in a sequence may be a winning bid, while by construction previous bids are not. Figure 1 shows that even before the final bid in a sequence, bids tend to increase, and Figure A2 in the Supplemental Appendix presents the same figure excluding winning bids. Nor is it due to selection in the product mix across the auction number variable, as the sequences are constructed at the bidder-by-product level, so conditional on sequence length the product mix is constant across auction number.

4.3 Price Dispersion

Across the repeated auctions of a given product, there is still substantial dispersion in transaction prices. The interquartile range of the normalized second-highest bid across auctions is 32 percentage points. Some of this dispersion is due to low price items, which sometimes sell for a relatively small or large percentage of their posted price. Restricting attention to products with a mean posted price of over \$100, there remains a good deal of price dispersion, with an interquartile range of 20 percentage points. This dispersion remains even after controlling for seller and product fixed-effects: the residuals from the regression of the normalized second-highest bid on seller and product fixed effects have a standard deviation and interquartile range of 9 and 5 percentage points.²⁵

4.4 Coexistence of Auctions and posted-price Sales

Most products are sold using a mix of auctions and posted-price listings. This finding necessarily holds in our preferred sample (which only includes products with at least 25 transactions of each type), and therefore to document evidence of the mix of sales mechanisms, we turn now to a larger sample nesting our preferred sample. This broader sample includes all products sold at least 50 times in our sample period without regard to listing method, yielding 12,368 products. For each product, we compute the fraction of listings sold via auction. The histogram in Figure 2 indicates the distribution of this fraction across products. Under 1% of products are sold exclusively by auction or by posted price. It is far more common to see a nontrivial fraction of sales through both formats: over 90% of products have between 10% and 90% of sales by auction. Averaging across products in this broader sample, the average rate of auction use is 47%.

Auctions typically yield a completed transaction faster than a posted-price listing. The seller explicitly chooses the auction length for either 1, 3, 5, 7 or 10 days. In contrast, posted-price listings are available until a buyer purchases it, and can be renewed if not purchased after 30 days. Figure 3 graphs the cumulative fraction of listings sold against the number of days after listing the item for sale, for auctions and posted-price listings. Auctions are about as likely to sell as posted-price listings within a day (21% vs 24%), and are over twice as likely to sell within 10 days (83% vs 41%). This is a reason why sellers might be willing to sell through auctions or posted prices: the former mechanism sells considerably faster but at a lower price than the latter.

4.5 Winners, Losers, and Shipping Speed

Here we document several other patterns relating time and bidder behavior that are consistent with our model. First, the bidder in an auction with the longest observed time on the market

²⁵The corresponding numbers for all products, rather than only those with a mean posted price over \$100, are 17 and 13 percentage points.

(i.e the time since the bidder’s first observed bid) is frequently the winner, occurring in 40% of auctions. In Section 5.1, we find that this is consistent with our model’s predictions. In contrast, if elapsed time and likelihood of winning were orthogonal, the longest duration bidder would only have a 7.7% chance of winning, since each of the bidders are equally likely to win.²⁶ Moreover, elapsed time and likelihood of winning would be inversely correlated if valuations were constant over time, since high valuation buyers would win shortly after entering the market while low valuations bidders will require many repeated attempts to get lucky.

Second, buyers who lose in an auction and then purchase the same good from one of eBay’s posted-price listings tend to do so shortly after their last observed auction attempt: 52% do so within one day of their last losing attempt. Furthermore, the cumulative probability of switching to a posted-price listing is concave in the time elapsed since the last losing attempt (see Figure A3 in the Supplemental Appendix). This is as the model predicts: the buyer’s last observed auction attempt must be close to the buyer’s deadline, or else the buyer would have attempted more auctions, and thus a posted-price purchase is most likely to occur close to the last auction attempt.

Third, after repeated losses, buyers are increasingly more likely to participate in auctions where expedited shipping is available, also consistent with the time sensitivity we model. The overall fraction of buyers bidding in auctions with fast shipping available is 0.45, and this fraction increases on average by 0.6 percentage points per auction attempt (see Figure A4 of the Supplemental Appendix).

4.6 Bidder Learning and Alternative Explanations

Deadlines provide a single explanation for multiple data patterns, one of which is the robust pattern observed in our data of bidders increasing their bids over time. Another possible explanation for that particular fact might involve bidder learning.²⁷ Consider the case where bidders are uncertain about the degree of competition they face, and form different estimates of its intensity. A bidder who underestimates the number of competitors or the bids of competitors will overestimate his likelihood of winning in future auctions; this raises his continuation value and causes him to shade his bid lower. Such a bidder will gradually revise his estimates upwards as he fails to win auctions, and thus tend to bid more over time. Bidders who overestimate the amount of competition will bid more aggressively than those who underestimate. However, their initial aggressive bidding will tend to result in their winning auctions early on; they may not remain in the market for long enough to learn their way to lower bids. Thus, in principle, bidder learning could also explain the pattern of bidders increasing their bids over

²⁶As explained in Section 5, the average number of participants per auction is 13.

²⁷We note that learning does not necessarily imply increases in bids across auctions. In Jeitschko (1998), bidders can learn their opponents types from their bids in the first auction, but in equilibrium, they reach the same expected price in the second auction. The model in Iyer et al. (2014) generates bids that rise on average, but there learning occurs only for the auction winner, who needs to experience the good to refine his information about its value.

time. In practice, there are several reasons why bidder learning is unlikely to be the key driver of increasing bids.

First, users can easily learn prices and bid histories for current and past listings by selecting the “Sold Listings” checkbox on the search results page; this is far more informative and quicker than auction participation. Second, experienced bidders should have more familiarity with the auction environment and with alternative means for gathering information, so learning by participation should not affect them. Yet the same pattern of increasing bids appears among experienced bidders, as shown in the left panel of Figure A5 in the Supplemental Appendix for bidders who participated in at least 50 auctions prior to the current auction and in the right panel for bidders who, over the past year, participated at least 10 auctions in the same product grouping as the reference auction.²⁸ Third, learning by participation is more costly with expensive products, due to the danger of bidding too high and winning when initially uninformed. We would expect buyers to be more cautious with such products and use alternative methods of learning. Yet Figure A6 in the Supplemental Appendix shows the same increasing bid pattern for products with a median price over \$100.²⁹

We emphasize here that we do not attempt to entirely rule out the possibility of bidder learning or other alternative explanations for individual pieces of the patterns we observe. However, the appeal of our model of time-sensitive buyers is that it provides a single, unified explanation of all of these facts together. For example, one alternative explanation for the increase in bids at the end of bidding sequences in Figure 1 is that from one auction to the next bidders receive random shocks to their valuations and that the increase at the end of the sequence is caused by bidders winning and exiting after a positive shock. However, a story of random valuation shocks would fail to explain the pattern of increasing bids *prior* to the final auction in the sequence, nor would it speak to the coexistence of auctions and posted-price sales or the relationships discussed above between time and bidder behavior. While these alternative explanations could play a role, the bulk of the evidence also seems to indicate a role for time-sensitivity.

5 Parameter Estimation and Counterfactuals

The preceding section documented that the data facts are qualitatively similar to the model predictions. Here, we illustrate the equilibrium behavior under estimated parameter values. In most dimensions, we find that the model closely matches the data facts. Also, to assess the importance of model selection, we compare demand estimates made using common models if

²⁸Examples of product groupings are DVDs, video games, and cell phones.

²⁹One might be tempted to test the learning story by looking for positive correlation between the amount by which a bidder loses an auction and the amount he increases his bid in subsequent auctions. Yet such a positive correlation is also consistent with the deadlines model, because low bidders (early in their search) will typically lose by the largest margins. As discussed in the next section, these buyers are rarely observed until much later in the search process when their bid has increased substantially.

the data were generated in our environment. Finally, the estimated parameters allow us to compute the welfare impact of eliminating retail auctions.

Fitting our model to the data is relatively straightforward, as each parameter either corresponds directly to a sample moment, or to a known transformation of sample moments. The moments we match and the corresponding model parameter estimates for the buyer equilibrium are shown in the upper half of Table 4, while additional moments and parameters for the market equilibrium are shown in the lower half. Specifics of the estimation process are detailed in Section E of the Supplemental Appendix.

Several of the moments used in this procedure relate to bidders' participation in auctions, and these participation estimates require an adjustment to match the data and theory. In our model, auctions proceed by sealed bid, yet eBay conducts ascending auctions. As pointed out by Song (2004), this can prevent a willing participant from submitting a bid if the auction's standing price passes his valuation before his arrival. While this does not affect the eventual winner or final price, it will cause the observed number of bidders to underestimate the true number of participants. We adjust for this bias using the approach proposed in Platt (2015). Under the assumption that participants in an auction arrive in random order,³⁰ he derives the probability that a random participant places a bid as:

$$P(\lambda) \equiv \frac{1}{\lambda} \left(2 (\ln(\lambda) + \Gamma(1) - \Gamma(0, \lambda)) - 1 + e^{-\lambda} \right), \quad (27)$$

where $\Gamma(1) \approx 0.57721$ is Euler's constant and $\Gamma(0, \lambda) \equiv \int_{\lambda}^{\infty} \frac{e^{-t}}{t} dt$ is the incomplete gamma function. Thus, the expected number of bidders per auction is a strictly increasing function $\lambda \cdot P(\lambda)$, yielding a one-to-one relation between the average number of observed bidders and the average number of would-be auction participants. Also, $\tau \alpha P(\lambda)$ is the average rate at which buyers in the market place bids.

We can also compute the number of bids a given buyer might place over the full duration of his search. This is given by:

$$D \equiv \tau \alpha \int_0^T \left(\frac{2 (1 - e^{-\lambda F(s)})}{\lambda F(s)} - e^{-\lambda F(s)} \right) ds. \quad (28)$$

The integrand is the probability that a participant in state s can place a bid (Platt, 2015).

With these adjustments, estimation of the model parameters is straightforward, as described in Section E of the Supplemental Appendix. In computing these moments, we normalize the posted price as $z = 1$. As discussed in Section 4.1, this has no effect on the distribution; $F(s)$, and $b(s)$ will scale proportionally. Therefore we similarly transform bid-

³⁰This assumption does not affect our model, since the payoff of losing participants is the same whether or not they were able to actually place a losing bid. The assumption implies that buyers with low willingness to pay will only be observed placing bids when the buyer happens to have arrived early compared to other bidders in that auction, as in Hendricks and Sorensen (2015).

ding data: for each item, we compute the average price (including shipping costs) among all sold posted-price listings (the analog of z), then divide all bids (including shipping costs) for that item by the average. This rescaling is equivalent to “homogenizing” bids (Haile et al., 2003). We consider one unit of time to be a month, which is also a normalization that merely adjusts the interpretation of T and ρ .

5.1 Equilibrium Behavior in the Fitted Model

We use the estimated parameters to illustrate the equilibrium behavior and to compare the predicted outcomes to the facts reported in Section 4. The solid line in the right panel of Figure 4 depicts equilibrium bids as a function of time remaining. Since $z = 1$, these can be read as the factor by which bidders shade their bids below the posted price. Note that bids are dispersed across a range equal to 47% of the posted price; this dispersion becomes larger with higher discounting or longer deadlines. Initially (for s near T), the price path is more or less linear, but as the deadline approaches, greater curvature is introduced. The dashed line in the right panel of Figure 4 indicates the utility that the buyer gets if he purchases at time s , which increases as the deadline approaches purely due to discounting. The gap between the lines indicates shading relative to the bidder’s current utility. This also highlights the fact that the increasing bids pattern predicted by the model is not solely due to time discounting, but also reflects the reduced option value of future auction opportunities.

On average, a buyer increases his bid at a rate of 4.3 percentage points per month. As the average bidder participates in 1.18 auctions per month, that translates to an increase of 3.6 percentage points between each auction of a given item. As reported in Section 4.2, in the data we see an increase of 2.0 percentage point between each auction attempt. Bear in mind that in estimating the model’s parameters we did not exploit any details about average bids over time.

Our model also explains why bidders with longer sequences start with a lower initial bid and why bidding sequences of different lengths should not cross, seen in the data in Figure 1. Recall that auction participation is stochastic, so some unlucky bidders will wait longer than others to place their first bid. These unlucky bidders will place a higher first bid (because their s is smaller), but they will also have less time until their deadline, and thus expect to participate in fewer additional auctions. Indeed, if we simulate the model under the estimated parameters and then summarize the data similarly, the resulting Figure 5 is qualitatively similar.

Next, consider the distribution of bids. The left panel of Figure 4 illustrates the equilibrium density of bidders. Note that $F'(s)$ is nearly constant from $s = 4$ to 10. With an average of $\lambda = 13$ participants per auction, those with lower valuations (hence longer time remaining) are highly unlikely to win. Yet the relative density cuts in half between $s = 4$ and $s = 1$, and then nearly does so again before $s = 0$. Those closest to their deadline are far more likely to

win and exit.

Figure 6 provides another perspective on the realized bids in the auction. The dot-dashed line plots the density of a randomly selected bid in the typical auction. That is, for any price p on the x-axis, the y value indicates the relative likelihood of that price being placed as a bid. Effectively, this is $F'(b^{-1}(p)) \cdot (b^{-1})'(p)$, obtained via a parametric plot since $b^{-1}(p)$ cannot be analytically derived. This is contrasted with the dotted line, which plots the density of the highest bid in each auction. Note that this is concentrated more towards higher prices, even though bidders with those valuations are somewhat scarce. This is because, with an average of $\lambda = 13$ bidders per auction, the highest bid tends to be closer to the top of available bids. Of course, only the second-highest price is actually paid; this density is depicted with the solid line. Despite the uniform nature of the auctioned goods, closing prices are significantly dispersed, with an interquartile range of 5.9 percentage points. As reported in Section 4.3, in the data the interquartile range of closing prices, after controlling for seller and product fixed effects, is 5 percentage points, serving as further evidence of the model’s fit given that this price dispersion was not exploited in the estimation.

Our model predicts that the auction participant with the most time in the market will always win; however, in auction data, we do not observe when the participant entered the market, but only the first time he placed a bid. Indeed, the winner may have been in the market the longest but never have placed a prior bid, giving him the shortest *observed* duration. We therefore simulate the model, but measure search duration as the elapsed time since the buyer’s first observed bid. Under that metric, the winner has the longest search duration in 45.7% of the simulated auctions, which is remarkably close to the 40% reported in Section 4.5 for our data.

The notable departure of the fitted model from the data is in the distribution of bid sequences. In our data, 85% of bidders only bid once on a product, while the model predicts that only 12.5% place a single bid. However, after conditioning on bidders who participated in three or more auctions, the model prediction is much closer to the observed distribution of bidding sequence lengths (see Figure A7 in the Supplemental Appendix).

Under the parameter estimates for the full market equilibrium, it is interesting to note that the parameterization indicates that nearly all the costs of production are incurred at the time of sale, with $\gamma = 0.984$. Also, a sale in the posted-price market generates $z - \theta = 14.6\%$ more revenue, but this is offset in that the sale occurs after 1.8 month on average, which is 11 times longer than the average auction. Also, Section 4.4 reports that 47% of sales occur via auction, while our model predicts a somewhat higher $\sigma^* = 60\%$.

5.2 Interpretation of Bidding Data

Our model considers bidders with repeat opportunities to bid while facing a deadline; but to what extent do these features matter in empirical uses of auction data? We explore this

question by considering how data generated in our setting would be interpreted using two standard auction models, specifically in the estimation of demand and consumer surplus.

Second-price auctions are empirically convenient because bids truthfully reveal the bidders' valuation, so the observed distribution of bids traces a demand curve for the product. Truthful revelation also allows a precise measurement of consumer surplus, since the first price truthfully reveals the winner's valuation, while the second price indicates what winner actually paid. Yet these interpretations of bidding data are only correct in a *static* second-price auction, with private valuations independently drawn from an exogenous distribution; estimates of demand or consumer surplus can be wildly skewed if the same interpretation is applied to bidding data generated in a setting of dynamic bidding by time-sensitive buyers.

In our setting, the buyer's value, $xe^{-\rho s}$, is no longer the same as willingness to pay, $b(s) = xe^{-\rho s} - V(s)$. Buyers are truthful about their willingness to pay, but they do not bid their full value because tomorrow's bidding opportunities provide positive expected surplus. They would rather lose and continue their search than pay a price above $b(s)$, even if they would have gotten positive surplus. Thus, the bid only reveals how much the bidder values *the current auction*, not the product itself. Thus, the static interpretation of our data will underestimate demand — on average by 5.3% of the retail price using our estimated parameter values, as illustrated in the dotted line in Figure 7. Similarly, auction winners in our model enjoy a consumer surplus equal to 8.7% of the retail price; yet the static interpretation would estimate a surplus of 5.4%, underestimating by 39%.

Of course, other models with an infinite sequence of auctions (Zeithammer, 2006; Ingster, 2009; Said, 2011; Backus and Lewis, 2012; Bodoh-Creed, 2012) can make a similar critique, since the option to participate in future auctions reduces willingness to bid in those models as well. Yet the non-stationary search process in our model still leads to significantly different interpretations of bid data. For illustration, we compare our model to a stationary dynamic auction environment in which a bidder of type s gets utility $x(s)$ from purchase, but this is constant throughout his search. Types are still distributed according to $F(s)$, and bidders still participate at rate $\tau\alpha$ with an average of λ bidders per auction. For low bids, this model ascribes practically no shading, while high bidders shade aggressively (by 60%). Thus, if the data were generated by in our non-stationary environment, but then interpreted using the stationary dynamic model, demand would be overstated by an average of 2.5% of the retail price (the dashed line in Figure 7), while consumer surplus would be highly overestimated at a value of 52.4% of the retail price.

Incorrect estimates of the demand curve or consumer surplus could easily distort calculations needed for profit maximization, price discrimination, regulation, and many other applications. Section F of the Supplemental Appendix provides the details of demand and consumer surplus calculations under all three models.

5.3 Welfare Consequences of Retail Auctions

For standardized retail goods, auctions are not serving the traditional role of price discovery, and appear to be superfluous given that posted-price markets are readily available.³¹ Here, we consider whether retail auctions enhance or detract from total expected welfare, comparing welfare in the dispersed (auctions and posted prices) versus degenerate (posted prices only) equilibria. For this exercise, we ignore whether sellers prefer posted prices over auctions (Eq. 26), since the latter could be inefficient even if sellers find them to be individually rational; thus, the counterfactual exercise asks whether welfare would increase if auctions are forbidden.

Total expected welfare is derived from three sources in our environments: buyers, sellers, and the selling platform (*e.g.* eBay). A newly-entering buyer expects utility of $V(T)$ by participating in the market (measured in terms of dollars, and net of any payments to sellers). We multiply this expected utility by $e^{\rho T}$ to measure it as of the time of consumption of the good, rather than the time of entry in the market. A newly-entered seller earns zero expected profit due to free entry into the market. As for the selling platform, revenue equals the listing fees collected from the average item, but we have not specified platform costs before this point. Here we consider two stark cases.

In the first setting, the listing fee exactly covers the average cost of each listing; thus the selling platform also earns zero profit. If so, total welfare is simply equal to $V(T)e^{\rho T}$, and auctions are socially beneficial, as shown below.

Proposition 5. *If the selling platform earns zero expected profit, then a dispersed equilibrium produces total welfare $V(T)e^{\rho T}$, which strictly exceeds the total welfare $x - z$ in a degenerate equilibrium.*

To appreciate this result, one should note that there are two avenues through which sellers are being driven to zero profit: one is to receive less revenue through auctions, and the other is to incur more costs by waiting longer for buyers in the posted-price market. But the latter is completely inefficient in this setting because it burns up real resources. Auctions also create some inefficiency as buyers pay for the product earlier than they need it, but most of the dissipated seller profit is transferred to buyers rather than destroyed. In this zero-platform profit scenario, under our estimated parameters, the existence of auctions increase welfare compared to a market with only posted prices by 1.8% of the product's retail price.

In the second setting, the platform can host listings costlessly, so all listing fees are profit and contribute to social welfare. If a product is listed for t periods, then the present value of those listing fees is $(1 - e^{-\rho t}) \ell / \rho$. With a posted-price listing, the duration is exponentially distributed with parameter ζ , whereas with an auction, the parameter is $\eta(1 - e^{-\lambda})$. Recall that fraction σ of products are listed as auctions, so the present value of expected platform

³¹ Auctions also have the potential to bring in additional customers who value the good at less than the retail price z , though this is beyond the scope of our model.

profit on the average item is:

$$\sigma \int_0^\infty \eta (1 - e^{-\lambda}) e^{-\eta(1-e^{-\lambda})t} \frac{(1 - e^{-\rho t}) \ell}{\rho} dt + (1 - \sigma) \int_0^\infty \zeta e^{-\zeta t} \frac{(1 - e^{-\rho t}) \ell}{\rho} dt. \quad (29)$$

After evaluating the integrals and substituting for the equilibrium values of σ and ζ , this becomes:

$$\frac{\ell}{\delta} \left(\frac{(z - c)(\kappa - \alpha)}{\rho(z - c\gamma) + \ell} + \frac{\alpha(1 - e^{-\lambda})}{\eta(1 - e^{-\lambda}) + \rho} \right) \quad (30)$$

In the degenerate equilibrium, $\alpha = 0$, so the expected profit becomes: $\frac{(z-c)\ell}{\rho(z-c\gamma)+\ell}$.

Comparing these, the change in expected profit from shutting down auctions is:

$$\frac{\ell\alpha(1 - e^{-\lambda})}{\delta} \left(\frac{z - c}{\rho(z - c\gamma) + \ell} - \frac{\theta - c}{\rho(\theta - c\gamma) + \ell} \right), \quad (31)$$

after substituting for η using (19). Since $z > \theta$, this expression is positive, so the platform always collects more revenue without auctions.

To determine the welfare effect of shutting down auctions, one must weigh the loss of expected utility to consumers against the increase in profits to the platform. This requires numeric evaluation due to the endogenous α , λ , and θ involved; however, it is typically the case that auctions are inefficient under a costless platform. Under our estimated parameters (and assuming $x = z$), the platform's profit falls from 14.4% to 6.5% of the retail price if auctions are allowed; even with the 5.6% benefit to buyers, auctions creates an overall welfare loss of 2.3% of the retail price (with total welfare being 12.1% of the retail price).

This is essentially a competition between two forms of inefficiency: in auctions, buyers are paying for the item earlier than they need it, but in posted-price listings, sellers are entering the market earlier than needed. The latter inefficiencies tend to be smaller; exceptions only occur when the listing fees ℓ are quite small or the flow of buyers δ is large.

Even so, the posted-price listings do create inefficiency, falling short of a first-best solution. For instance, if buyers could produce their own unit, total welfare would be $x - c$, which is 16.0% of the retail price under our estimated parameters. To see this formally, we compare this first-best welfare to expected welfare in the costless-platform degenerate equilibrium:

$$x - c > x - z + \frac{(z - c)\ell}{\rho(z - c\gamma) + \ell} \iff \frac{\rho(z - c\gamma)(z - c)}{\rho(z - c\gamma) + \ell} > 0.$$

Intermediation through the platform requires sellers to wait for buyers to arrive. This delay is socially costly, but unavoidable in this search environment.

In sum, in the zero-profit extreme, auctions are welfare improving even with homogeneous goods that are readily available elsewhere. In the pure-profit extreme, auctions are often wasteful. In reality, one would expect that a platform such as eBay can sustain some profits

(for instance, network externalities make it beneficial for sellers and buyers to rely on a single platform), but it is also unlikely to be costless to facilitate the listings. An intermediate level of platform profits could result in auctions being welfare neutral.

In Tables A1 through A3 of the Supplemental Appendix, we examine whether the welfare implications of offering auctions for these new-in-box goods have changed over the period from 2010–2014. Einav et al. (2016), studying an earlier time period, document that posted-price listings grew in popularity, overtaking auctions by 2009.³² We find that this rising use of posted prices relative to auctions has continued post-2010 among new goods. One possible explanation for this shift could be increased time sensitivity of buyers. Using the same sample restrictions as in our main data sample, we recompute the model’s parameters and the corresponding welfare measures in other years. This analysis suggests that the flow of buyers δ has substantially declined, despite minimal change in auction prices. Moreover, auction participants have not become more time sensitive over the last five years, as the search time has grown longer and buyers are more patient (larger T and smaller ρ).³³ In addition, γ has risen dramatically, consistent with a shift toward “just-in-time” production, which erodes some of the advantages of the auction mechanism. Assuming a costless platform, we find that allowing auctions in 2010 increased welfare by 0.6% of the retail price; however, by 2014, auctions reduced welfare by 2.1%. Section G of the Supplemental Appendix provides further details.

6 Conclusion

This work reexamines the auction environment as a venue for selling retail goods, operating parallel to a posted-price market. Our analysis leverages methods frequently used in search theory, which provide analytic tractability and plausibly match the online retail setting. Standard auction theory ascribes all variation in bids as generated by exogenous differences in valuations; but this seems less compelling when there is a readily-available outside option for the same item. In our model, buyers’ differences arise endogenously, depending on how soon they must acquire the item. This produces an increasing and accelerating path for an individual’s bidding over time, and a rich continuous distribution of closing prices across auctions.

Our model’s predictions are borne out in a new dataset of new-in-box items sold through both mechanisms on eBay. In particular, we document that repeat bidders tend to raise their bids in each successive auctions. Despite the redundancy of auctions in a market that readily offers posted-price listings of the same good, auctions are unambiguously welfare enhancing

³²Einav et al. (2016) do not limit to new-in-box items as we do, as they study the period from 2003–2009 and eBay’s categorization of new vs. used items is only available from 2010 onward.

³³Even so, this could be driven by a changing composition of buyers: the most time-sensitive buyers could be entirely avoiding online auctions more than in the past and hence the remaining participants would look less time sensitive. Similarly, the remaining buyers in the market appear to be paying greater attention (τ) to each auction.

if listing fees are exactly covering platform costs. However, if a listing imposes zero marginal cost, auctions are typically inefficient.

We also show that interpreting data from our setting using standard models will lead to substantially biased estimates of demand. Nor should one expect the distribution of valuations to be invariant to policy interventions or changes in auction rules or even related markets. This is because bids are shaded below the eventual utility from the good, and will change in response to market design, including those directed only at the posted-price market.

While we have focused on new, homogenous goods, the same lessons are equally applicable for impatient repeat bidders on imperfectly interchangeable items. Indeed, we anticipate similar results from other sales mechanisms where buyers must make repeated attempts, such as bargaining or discount shopping: time sensitive buyers will adjust their strategy as they approach their deadlines and eventually resign themselves to the posted-price market. As with auctions, this could generate diverse behavior in otherwise homogenous markets.

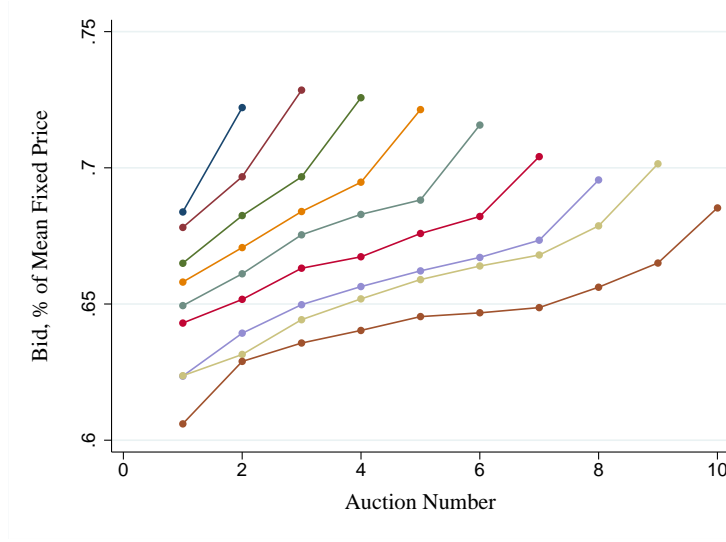
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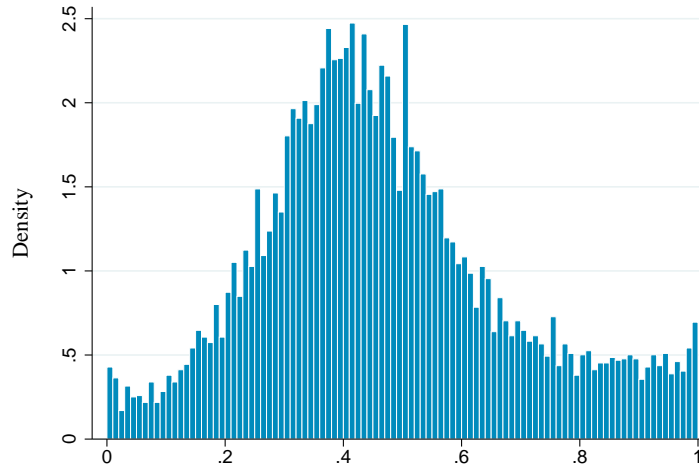
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Figure 1: Bids Over Time



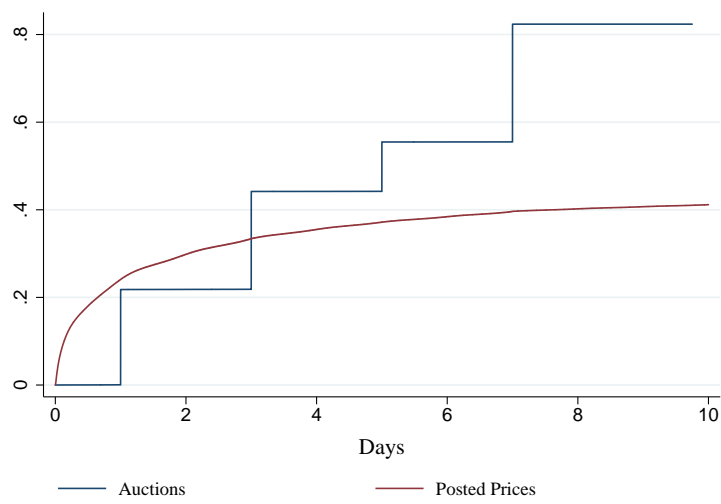
Notes: In the figure, a given line with n points corresponds to bidders who bid in n auctions total for a given product without winning in the first $n - 1$ auctions. Horizontal axis represents auction number within the sequence (from 1 to n) and vertical axis represents the average normalized bid.

Figure 2: Distribution of Fraction of Sales By Auction Across Items



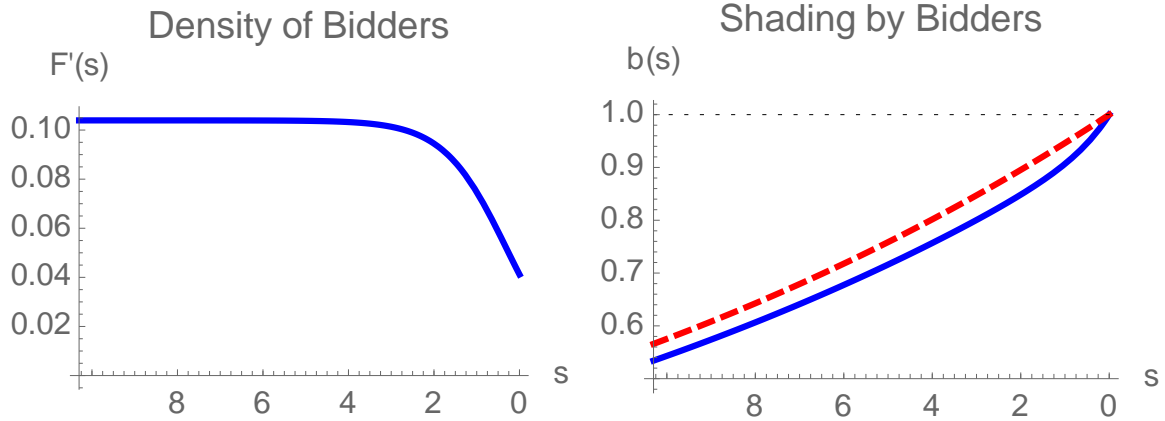
Notes: Figure shows a histogram at the product level of the fraction of listings sold by auction (rather than posted price) for a given product. The sample used to generate this figure is a superset of our main sample, containing the 12,368 distinct products with at least 50 transactions observed in the sample period without regards to listing method.

Figure 3: Cumulative Fraction Sold by Days Since Listing



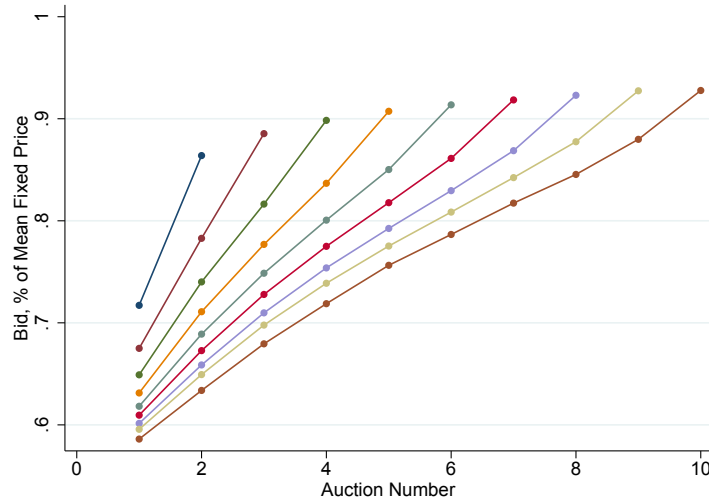
Notes: Figure displays the cumulative fraction of listings sold (vertical axis) against the number of days since the listing was posted (horizontal axis) for auctions and posted-price listings.

Figure 4: Bidding under Fitted Parameters



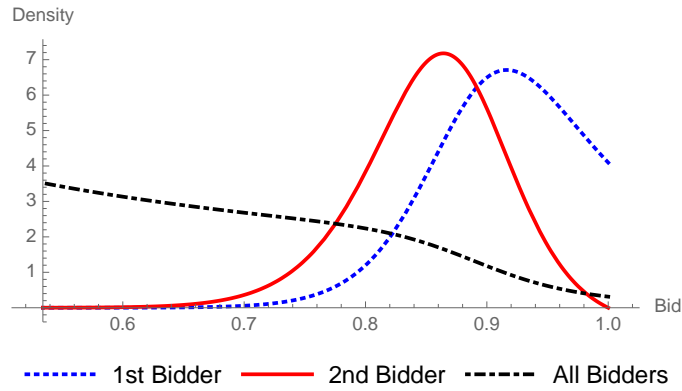
Notes: The left panel displays the density of bidders with s time remaining until deadline. The right panel indicates the bids (solid line) and utility (dashed line) as a function of time remaining s . Since $z = 1$, these may be read as percentages relative to the retail price.

Figure 5: Bids Over Time, Simulated Data



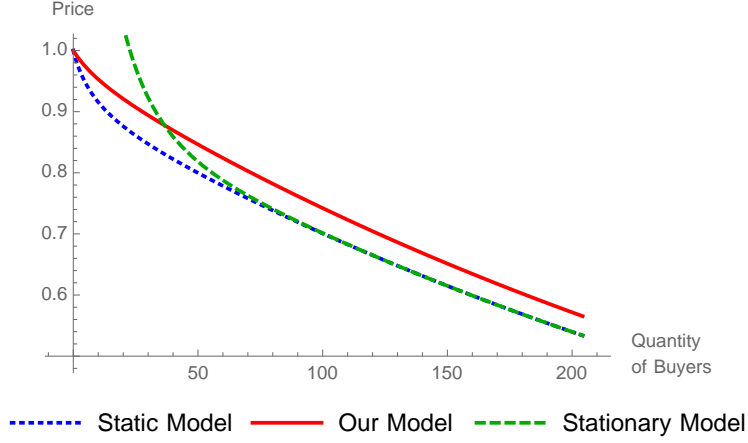
Notes: Figure displays average bids at each auction number for different auction sequence lengths, simulated from the model at the estimated parameter values. In the figure, a given line with n points corresponds to bidders who bid in n auctions total for a given product without winning in the first $n - 1$ auctions. Horizontal axis represents auction number within the sequence (from 1 to n) and vertical axis represents the average normalized bid.

Figure 6: Price Density under Fitted Parameters



Notes: Figure displays the equilibrium density of the highest bid in an auction (dotted), the second highest bid (solid), and all bids (dot-dashed), simulated from the model at the estimated parameter values.

Figure 7: Auction Demand Curve



Notes: Figure shows inferred auction demand curve using the deadlines model (solid red line) vs. treating the data as though it came from a static model (dotted blue line) or a stationary dynamic model (dashed green line). The dashed line is truncated, but would intersect the vertical axis at a price of 2.5.

Table 1: Comparative Statics on Key Statistics: Buyer Equilibrium

		$\partial/\partial\alpha$	$\partial/\partial\tau$	$\partial/\partial\rho$	$\partial/\partial T$
Participants per Auction	λ^*	–	+	0	+
Number of Buyers	H^*	–	+	0	+
% of Buyers using Posted Price	$F'(0)$	–	–	0	–
Lowest Bid	$b^*(T)$	–	**	–	–

Note: Derivations are provided in Section D of the Supplemental Appendix. ** indicates that the sign depends on parameter values. Sufficient conditions for a positive sign are $\delta\tau T > 1$ and $\tau(\kappa - \alpha) > \rho > \tau(2\kappa - \alpha)\sqrt{\tau\kappa T}e^{-\lambda}$. An exact condition is provided in the Online Appendix.

Table 2: Comparative Statics on Key Statistics: Market Equilibrium

		$\partial/\partial\tau$	$\partial/\partial\rho$	$\partial/\partial T$	$\partial/\partial c$	$\partial/\partial\ell$	$\partial/\partial\gamma$
Auction Rate	α^*	+	−	−	−	−	+
Participants per Auction	λ^*	+	+	+	+	+	−
% Buying via Posted Price	$F'(0)H^*$	−	+	+	+	+	−
Stock of Posted-Price Sellers	P^*	−	+	−	+	+	+
Lowest Bid	$b^*(T)$	−	−	−	+	+	−
Expected Revenue	θ^*	−	+	−	+	+	−

Note: Reported signs are for numeric computations on the example.

Table 3: Descriptive Statistics

A. Transaction level	Posted Price		Auctions	
	<u>Mean</u>	<u>Std. dev.</u>	<u>Mean</u>	<u>Std. dev.</u>
Bidders per transaction	1	–	5.30	2.20
Revenue per transaction	106.82	21.74	97.27	16.60
Revenue per transaction, normalized by avg posted price	1	–	0.85	0.17
Number of transactions	494,448		560,861	
B. Product level	Posted Price		Auctions	
	<u>Mean</u>	<u>Std. dev.</u>	<u>Mean</u>	<u>Std. dev.</u>
Number of transactions per product	134.98	220.82	153.12	343.63
Unique sellers per product	82.70	137.84	68.53	201.30
Unique buyers per product	129.03	208.02	606.08	1,365.60
Number of products	3,663			

Notes: Table displays descriptive statistics for primary data sample: transactions from October 1, 2013 through September 30, 2014 meeting the sample restrictions described in the text. All values are computed for completed (sold) listings. In panel A, values reported are means of product-level means and means of product-level standard deviations. Normalized revenue is computed by first dividing auction price by product-level average of posted-price sales. Panel B reports average and standard deviation, taken across all products, of the number transactions of a given product, the number of unique sellers selling a given product, and the number of unique buyers bidding in an auction for a given product or purchasing the product through a posted-price listing.

Table 4: Key Data Moments and Matching Parameter Values

	Observed Value in Data	Theoretical Equivalent	Fitted Parameter
Bidders per completed auc- tion	5.30	$\frac{\lambda \cdot P(\lambda)}{1 - e^{-\lambda}}$	$\lambda = 13.10$ (0.243)
Completed auctions per month	12.76	$\alpha (1 - e^{-\lambda})$	$\alpha = 12.76$ (0.525)
Auctions a bidder tries per month	1.18	$\frac{\tau \alpha P(\lambda)}{1 - e^{-\tau \alpha P(\lambda)}}$	$\tau = 0.064$ (0.002)
New repeat bidders per month who never win	7.41	$(\delta - \alpha) (1 - (1 + D)e^{-D})$	$\delta = 21.13$ (1.163)
—	—	Eq. (8)	$T = 10.30$ (0.255)
Average revenue per com- pleted auction	0.853	θ	$\rho = 0.055$ (0.003)
Average listing fee paid	0.087	ℓ	$\ell = 0.087$ (0.0003)
Average duration of an auc- tion listing (months)	0.156	$1/\eta$	$\eta = 6.39$ (0.028)
Average % of posted-price listing sold in 30 days	48.1%	$1 - e^{-\frac{\ell + \rho(1-\gamma)c}{z-c}}$	$\gamma = 0.985$ (0.039)
—	—	Eq. (19)	$c = 0.840$ (0.003)

Notes: Table displays observed moments in data and corresponding theoretical equivalent for additional market equilibrium parameters. Standard errors, from 200 bootstrap replications at the product level, are contained in parentheses. Data moments are averaged for each product (and month, where noted), then averaged across these.

Appendix: Proofs

Proof of Proposition 1. First, we note that the infinite sums in equations (2) and (4) can be readily simplified. In the case of the latter, it becomes:

$$F''(s) = \alpha\tau F'(s)e^{-\lambda F(s)}. \quad (32)$$

This differential equation has the following unique solution, with two constants of integration k and m :

$$F(s) = \frac{1}{\lambda} \ln \left(\frac{\alpha\tau - e^{\lambda k(s+m)}}{\lambda k} \right). \quad (33)$$

The constants are determined by our two boundary conditions. Applying (6), we obtain $m = \frac{1}{\lambda k} \ln(\alpha\tau - \lambda k e^\lambda) - T$. By substituting this into (33), one obtains:

$$F(s) = \frac{1}{\lambda} \ln \left(\frac{\alpha\tau - e^{\lambda k(s-T)} (\alpha\tau - \lambda k e^\lambda)}{\lambda k} \right). \quad (34)$$

The other boundary condition, (5), requires that k satisfy:

$$\alpha\tau (1 - e^{-\lambda T k}) - \lambda k (1 - e^{\lambda - \lambda T k}) = 0. \quad (35)$$

From (7), we know that $H = \delta/F'(T)$, and using the solution for F in (34), this yields $H = \delta\lambda/(\lambda k - \alpha e^{-\lambda}\tau)$. We then substitute this into the fourth equilibrium requirement, $\lambda = \tau H$, and solve for k to obtain:

$$k = \frac{\tau}{\lambda} (\delta + \alpha e^{-\lambda}). \quad (36)$$

When we substitute this for k in (34), we obtain the equilibrium solution for F^* depicted in (9). Also, (36) is used to replace k in the boundary condition in (35), we obtain the formula for ϕ in (8) which implicitly solves for λ^* .

We now show that a solution always exists to $\phi(\lambda^*) = 0$ and is unique. Note that as $\lambda \rightarrow +\infty$, $\phi(\lambda) \rightarrow +\infty$. Also, $\phi(0) = -\delta(1 - e^{-\tau(\alpha+\delta)T}) < 0$. Since ϕ is a continuous function, there exists a $\lambda^* \in (0, +\infty)$ such that $\phi(\lambda^*) = 0$.

We next turn to uniqueness. The derivative of ϕ w.r.t. λ is always positive:

$$\phi'(\lambda) = \alpha e^{-\lambda} + \delta(e^\lambda + \alpha\tau T)e^{-\tau(\alpha e^{-\lambda} + \delta)T} > 0.$$

Thus, as an increasing function, $\phi(\lambda)$, crosses zero only one time, at λ^* .

We finally turn to the solution for the bidding function. Again, we start by simplifying the infinite sums in (2). The first sum is similar to that in (4). For the second, we first change the order of operation, to evaluate the sum inside the integral. This is permissible by the

monotone convergence theorem, because $F(s)$ is monotone and $\sum \frac{e^{-\lambda} \lambda^n}{n!} b(t) n (1 - F(t))^{n-1}$ converges uniformly on $t \in [0, T]$. After evaluating both sums, we obtain:

$$\rho V(s) = -V'(s) + \alpha \tau \left(e^{-\lambda F(s)} (x e^{-\rho s} - V(s)) - e^{-\lambda} b(T) - \int_s^T \lambda e^{-\lambda F(t)} b(t) F'(t) dt \right).$$

Next, by taking the derivative of $b(s) = x e^{-\rho s} - V(s)$ in (1), we obtain $b'(s) = -\rho x e^{-\rho s} - V'(s)$. We use these two equations to substitute for $V(s)$ and $V'(s)$, obtaining:

$$(\rho + \alpha \tau e^{-\lambda F(s)}) b(s) + b'(s) = \alpha \tau \left(e^{-\lambda} b(T) + \int_s^T \lambda e^{-\lambda F(t)} b(t) F'(t) dt \right). \quad (37)$$

This equation holds only if its derivative with respect to s also holds, which is:

$$(\rho + \alpha \tau e^{-\lambda F(s)}) b'(s) + b''(s) = 0. \quad (38)$$

After substituting for $F(s)$ solved above, this differential equation has the following unique solution, with two constants of integration a_1 and a_2 :

$$b(s) = a_1 \cdot \left(\frac{\delta e^{\lambda^* - \tau T (\delta + \alpha e^{-\lambda^*})}}{\rho} + \frac{\alpha e^{-\tau s (\delta + \alpha e^{-\lambda^*})}}{\rho + \tau (\delta + \alpha e^{-\lambda^*})} \right) e^{-s \rho} + a_2. \quad (39)$$

This solves the differential equation, but to satisfy (37), a particular constant of integration must be used. We substitute for $b(s)$ in (37) using (39), and solve for a_2 . This can be done at any $s \in [0, T]$ with equivalent results, but is least complicated at $s = T$ since the integral disappears: $(\rho + \alpha \tau e^{-\lambda F(T)}) b(T) + b'(T) = \alpha \tau e^{-\lambda} b(T)$. After substituting $b(T)$, $b'(T)$, and $F(T)$, solving for a_2 yields:

$$a_2 = a_1 \frac{\alpha \tau (\delta + \alpha e^{-\lambda^*})}{\rho (\rho + \delta \tau + \alpha \tau e^{-\lambda^*})} e^{-\rho T - \tau T (\delta + \alpha e^{-\lambda^*})}. \quad (40)$$

The other constant of integration is determined by boundary condition (3). If we translate this in terms of $b(s)$ as we did for the interior of the HJB equation, we get $b(0) = z$. We then substitute for $b(0)$ using (39) evaluated at 0, and substitute for a_2 using (40), then solve for a_1 :

$$a_1 = \frac{\rho z (\rho + \delta \tau + \alpha \tau e^{-\lambda^*}) e^{\tau T (\delta + \alpha e^{-\lambda^*})}}{\tau (\delta + \alpha e^{-\lambda^*}) (\delta e^{\lambda^*} + \alpha e^{-\rho T}) + \rho (\delta e^{\lambda^*} + \alpha e^{\tau T (\delta + \alpha e^{-\lambda^*})})}.$$

If the solutions for a_1 and a_2 are both substituted into (39), one obtains (11) with minor simplification. \square

Proof of Proposition 2. The first derivative of $b^*(s)$ is:

$$b'(s) = - \frac{\rho z (\rho + \delta\tau + \alpha\tau e^{-\lambda^*}) \left(\delta e^{\lambda^*} + \alpha e^{\tau(T-s)(\delta + \alpha e^{-\lambda^*})} \right)}{\tau (\delta + \alpha e^{-\lambda^*}) (\delta e^{\lambda^*} + \alpha e^{-\rho T}) + \rho \left(\delta e^{\lambda^*} + \alpha e^{\tau T(\delta + \alpha e^{-\lambda^*})} \right)}.$$

Each of the parenthetical terms is strictly positive, thus the negative in front ensures that the derivative is negative.

The second derivative is:

$$b''(s) = \frac{\rho z (\rho + \delta\tau + \alpha\tau e^{-\lambda^*}) \left(\delta \rho e^{\lambda^*} + \alpha (\rho + \delta\tau + \alpha\tau e^{-\lambda^*}) e^{\tau(T-s)(\delta + \alpha e^{-\lambda^*})} \right)}{\tau (\delta + \alpha e^{-\lambda^*}) (\delta e^{\lambda^*} + \alpha e^{-\rho T}) + \rho \left(\delta e^{\lambda^*} + \alpha e^{\tau T(\delta + \alpha e^{-\lambda^*})} \right)} e^{-s\rho}.$$

Again, each parenthetical term is positive. Hence $b''(s) > 0$. \square

Proof of Proposition 3. By Proposition 1, equations (9) through (11) and $\phi(\lambda^*) = 0$ must be satisfied in order to be a buyer equilibrium, as required in the first condition.

The solutions to A^* and σ^* are simply restatements of (17) and (15), respectively.

The profits stated in (20) and (21) are required by the third and second equilibrium conditions, respectively. From (14), profit solves as: $\Pi_p = \frac{\eta(z-\gamma c)-\ell}{\rho+\zeta}$, so for this to equal $(1-\gamma)c$, we require $\zeta^* = \frac{\ell+\rho c(1-\gamma)}{z-c}$ as in equation (24). With this, (16) readily yields P^* as listed in (23).

The only remaining element regards expected auction profit. Equation (13) solves as: $\Pi_a = \frac{\eta(1-e^{-\lambda})(\theta-c\gamma)-\ell}{\eta(1-e^{-\lambda})+\rho}$. By setting this equal to $(1-\gamma)c$ and solving for θ , we obtain (19).

To evaluate the integrals in (12), we first note that by interchanging the sum and integral and evaluating the sum, expected revenue simplifies to:

$$\theta = \frac{\lambda}{1-e^{-\lambda}} \left(e^{-\lambda} b(T) + \lambda \int_0^T b(s) F(s) F'(s) e^{-\lambda F(s)} ds \right). \quad (41)$$

After substituting for $b(s)$ and $F(s)$ from the buyer equilibrium, this evaluates to:

$$\theta = \frac{z}{1-e^{-\lambda}} \cdot \left(1 + \frac{1}{(\rho + \kappa\tau)(\rho\delta + \tau(\kappa - \alpha)(\delta + \alpha e^{-\lambda-\rho T}))} \cdot \left((\alpha - \kappa)e^{-\lambda-\rho T} (\kappa\tau(\kappa\tau - \lambda\rho) - \lambda\rho^2) - \delta\rho(2\kappa\tau + \rho) + \kappa\rho\tau \left(\delta\Psi\left(1 - \frac{\kappa}{\alpha}\right) + (\alpha - \kappa)e^{-\lambda-\rho T}\Psi\left(1 - \frac{\kappa e^{\lambda}}{\alpha}\right) \right) \right) \right),$$

where $\kappa \equiv \delta + \alpha e^{-\lambda}$ and $\Psi(q)$ is Gauss's hypergeometric function with parameters $a = 1$,

$b = -1 - (\rho/\tau\kappa)$, $c = -\rho/\tau\kappa$, evaluated at q . Under these parameters, the hypergeometric function is equivalent to the integral:

$$\Psi(q) \equiv - \left(1 + \frac{\rho}{\tau\kappa}\right) \int_0^1 \frac{t^{-2-\frac{\rho}{\tau\kappa}}}{1-qt} dt.$$

While not analytically solvable for these parameters, Ψ is readily computed numerically. \square

Proof of Proposition 4. The proposed Buyer and Market Equilibria still apply when $\alpha^* = 0$, bearing in mind that as $\alpha \rightarrow 0$, the solution to $\phi(\lambda^*) = 0$ approaches $\lambda^* = \tau\delta T$. In the absence of auctions, the distribution of bidders is uniformly distributed across $[0, T]$, since none of them exit early; so $F^*(s) = s/T$ and $H^* = \delta T$. Moreover, the buyer's willingness to bid (if an auction unexpectedly occurred) reduces to: $b(s) = ze^{-\rho s}$.

For $\alpha^* = 0$ to be a market equilibrium, we need $\Pi_a^* \leq \Pi_p^*$. To prevent further entry, $\Pi_p^* = (1 - \gamma)c$ is still required. From (13), if an auction were unexpectedly offered, the seller would generate $\Pi_a^* = \frac{\eta(\theta - \gamma c) - \ell}{\rho + \eta(1 - e^{-\tau\delta T})}$. Thus, the expected profit comparison simplifies to: $\theta \leq c + \frac{\ell + \rho c(1 - \gamma)}{\eta(1 - e^{-\tau\delta T})}$. This is equivalent to (26), where the left-hand side is evaluated from (41):

$$\begin{aligned} \theta &= \frac{\tau\delta T}{1 - e^{-\tau\delta T}} \left(e^{-\tau\delta T} b(T) + \int_0^T b(s) F(s) F'(s) e^{-\tau\delta T F(s)} ds \right) \\ &= \frac{\tau\delta T}{1 - e^{-\tau\delta T}} \left(e^{-\tau\delta T} z e^{-\rho T} + \int_0^T z e^{-\rho s} \frac{s}{T^2} e^{-\tau\delta s} ds \right) \\ &= \frac{\tau\delta}{1 - e^{-\tau\delta T}} \cdot \frac{\tau\delta + (\rho T(\rho + \tau\delta) - \tau\delta) e^{-(\rho + \tau\delta)T}}{(\rho + \tau\delta)^2} z. \end{aligned}$$

Thus, if (26) holds, then the profit from offering an auction is never greater than continuing to offer a posted-price listing, making $\alpha^* = 0$ an equilibrium. If (26) fails to hold, then $\alpha^* = 0$ cannot be an equilibrium, since some firms will earn greater profit by deviating and offering an auction.

To prove the last claim, first note that in a buyer equilibrium, $\lambda \rightarrow 0$ as $\alpha \rightarrow \infty$. In addition, $b(s) \rightarrow 0$ for all $s > 0$, because auctions occur every instant, in which the buyer faces no competition. Thus, expected revenue is 0 in the limit, yielding profit $\Pi_a < 0$ for $\alpha \rightarrow \infty$. At the same time, the violation of (26) is equivalent to $\Pi_a > 0$ for $\alpha = 0$. Since expected revenue is continuous in α , by the intermediate value theorem there must exist an $\alpha^* > 0$ such that $\Pi_a(\alpha^*) = 0$, which will constitute a dispersed equilibrium. \square

Proof of Proposition 5. Total expected welfare is simply the expected utility of the new entrant, measured at the time of consumption, $V(T)e^{\rho T}$. This is compared to the welfare that would occur if all buyers were forced to use the posted price, $x - z$. The former will be greater

if:

$$\begin{aligned}
x - \frac{z\kappa(\tau\kappa + \rho)}{\delta(\tau\kappa + \rho) + (\rho e^{\tau\kappa T} + \tau\kappa e^{-\rho T})\alpha e^{-\lambda}} &> x - z &\iff \\
\kappa(\tau\kappa + \rho) < \delta(\tau\kappa + \rho) + (\rho e^{\tau\kappa T} + \tau\kappa e^{-\rho T})\alpha e^{-\lambda} &⇔ \\
\alpha e^{-\lambda}(\tau\kappa + \rho) < (\rho e^{\tau\kappa T} + \tau\kappa e^{-\rho T})\alpha e^{-\lambda} &\iff \\
\rho(e^{\tau\kappa T} - 1) - \tau\kappa(1 - e^{-\rho T}) &> 0
\end{aligned}$$

The l.h.s. of the last line is strictly increasing in T , with derivative: $\tau\kappa\rho(e^{(\tau\kappa+\rho)T} - 1)e^{-\rho T}$. Moreover, at $T = 0$, it evaluates to 0. Therefore, the expression is always greater than 0 for $T > 0$, and expected welfare is strictly greater with auctions than without.

□

Supplemental Appendix to “A Theory of Bidding Dynamics and Deadlines in Online Retail”

Dominic Coey Bradley Larsen Brennan C. Platt³⁴

A Model with Immediate Consumption

In the paper, we assumed that the good is only needed at the time of the deadline, as in the purchase of a birthday gift, travel arrangements, or seasonal sports equipment. Suppose instead that utility x is split between the immediate value from purchase, βx , and the value realized only at the deadline, $(1 - \beta)x$, where the latter is discounted at rate ρ . The extreme of $\beta = 0$ indicates that the good is literally of no use until the date of the deadline, while $\beta = 1$ indicates that it starts providing the same flow of value regardless of when it is purchased. The intermediate case seems reasonable for many deadlines: for instance, a gift is not needed until the birthday, but the giver may enjoy some peace of mind of having it secured early. A spare automobile part provides similar insurance even if it is not literally needed until the failure of the part it replaces. Thus, if the good is purchased with s units of time remaining until the buyer’s deadline, his realized utility is $(\beta + (1 - \beta)e^{-\rho s})x$ minus the purchase price.

We assume throughout that $\beta x < z$; otherwise, buyers would strictly prefer using the posted-price option in equilibrium, as the possible savings from winning at auction would not compensate for delaying the immediate benefits of an outright purchase.³⁵

The optimal behavior is still to bid one’s reservation value, setting:

$$b(s) = (\beta + (1 - \beta)e^{-\rho s})x - V(s). \quad (42)$$

In light of this, a buyer’s HJB equation is revised to be:

$$\begin{aligned} \rho V(s) = & -V'(s) + \tau\alpha \left(\sum_{n=0}^{\infty} \frac{e^{-\lambda}\lambda^n}{n!} (1 - F(s))^n ((\beta + (1 - \beta)e^{-\rho s})x - V(s)) \right. \\ & \left. - e^{-\lambda}b(T) - \sum_{n=1}^{\infty} \frac{e^{-\lambda}\lambda^n}{n!} \int_s^T b(t)n(1 - F(t))^{n-1}F'(t)dt \right). \end{aligned} \quad (43)$$

Buyers also have the option to purchase from the posted-price listings at any time, receiving utility $(\beta + (1 - \beta)e^{-\rho s})x - z$. However, a buyer in state s can obtain a discounted expected

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³⁵To see this in our equilibrium solution, note that buyers are willing to pay more than z (that is, $b^*(s) > z$ for $s > 0$) if and only if $\beta x \geq z$.

utility of $e^{-\rho s}(x - z)$ by waiting until $s = 0$ to make the purchase, and this is strictly preferred assuming that $\beta x < z$. Hence, the posted-price option is exercised if and only if $s = 0$, and the expected utility of a buyer who reaches his deadline is simply the consumer surplus from making the purchase:

$$V(0) = x - z. \quad (44)$$

Note that the steady state conditions and equilibrium definitions are unaffected by the immediate consumption of the good by auction winners. Indeed, in the equilibrium solution, we obtain the same distribution of buyers, $F(s)$ and H . The only alteration is to the equilibrium bidding function:

$$b^*(s) = z - (z - \beta x) \frac{\delta(\kappa\tau + \rho)e^{\lambda^*}(1 - e^{-\rho s}) + \alpha\rho e^{\tau\kappa T}(1 - e^{-s(\rho + \tau\kappa)})}{\delta\rho e^{\lambda^*} + \tau\kappa(\delta e^{\lambda^*} + \alpha e^{-\rho T}) + \alpha\rho e^{\tau\kappa T}}, \quad (45)$$

where $\kappa \equiv \delta + \alpha e^{-\lambda^*}$.

Proof. This closely follows the proof of Proposition 1. The HJB equation simplifies to:

$$\rho V(s) = -V'(s) + \alpha\tau \left(e^{-\lambda F(s)} ((\beta + (1 - \beta)e^{-\rho s})x - V(s)) - e^{-\lambda}b(T) - \int_s^T \lambda e^{-\lambda F(t)}b(t)F'(t)dt \right).$$

Next, by taking the derivative of the bidding function $b(s) = (\beta + (1 - \beta)e^{-\rho s})x - V(s)$, we obtain $b'(s) = -\rho(1 - \beta)xe^{-\rho s} - V'(s)$. We use these two equations to substitute for $V(s)$ and $V'(s)$, obtaining:

$$(\rho + \alpha\tau e^{-\lambda F(s)})b(s) + b'(s) = \rho\beta x + \alpha\tau \left(e^{-\lambda}b(T) + \int_s^T \lambda e^{-\lambda F(t)}b(t)F'(t)dt \right). \quad (46)$$

This equation holds only if its derivative with respect to s also holds. Yet this delivers the same differential equation as when $\beta = 0$:

$$(\rho + \alpha\tau e^{-\lambda F(s)})b'(s) + b''(s) = 0. \quad (47)$$

As in the paper, this differential equation has the following unique solution, with two constants of integration a_1 and a_2 :

$$b(s) = a_1 \cdot \left(\frac{\delta e^{\lambda^* - \tau T(\delta + \alpha e^{-\lambda^*})}}{\rho} + \frac{\alpha e^{-\tau s(\delta + \alpha e^{-\lambda^*})}}{\rho + \tau(\delta + \alpha e^{-\lambda^*})} \right) e^{-s\rho} + a_2. \quad (48)$$

What changes are the constants of integration. We substitute for $b(s)$ in (46) using (48)

and evaluate it at $s = T$: $(\rho + \alpha\tau e^{-\lambda F(T)})b(T) + b'(T) = \rho\beta x + \alpha\tau e^{-\lambda}b(T)$. After substituting $b(T)$, $b'(T)$, and $F(T)$, solving for a_2 yields:

$$a_2 = \beta x + a_1 \frac{\alpha\tau (\delta + \alpha e^{-\lambda^*})}{\rho (\rho + \delta\tau + \alpha\tau e^{-\lambda^*})} e^{-\rho T - \tau T (\delta + \alpha e^{-\lambda^*})}. \quad (49)$$

The other constant of integration is determined by boundary condition (3), which is still $b(0) = z$. We then substitute for $b(0)$ using (48) evaluated at 0, and substitute for a_2 using (49), then solve for a_1 :

$$a_1 = \frac{\rho(z - \beta x) (\rho + \delta\tau + \alpha\tau e^{-\lambda^*}) e^{\tau T (\delta + \alpha e^{-\lambda^*})}}{\delta\rho e^{\lambda^*} + \tau (\delta + \alpha e^{-\lambda^*}) (\delta e^{\lambda^*} + \alpha e^{-\rho T}) + \alpha\rho e^{\tau T (\delta + \alpha e^{-\lambda^*})}}.$$

If the solutions for a_1 and a_2 are both substituted into (48), one obtains (45) with minor simplification. \square

Qualitatively, the model with immediate consumption behaves quite similarly to the model in the paper. As β increases, the bid $b(s)$ rises, all else equal. This will flatten the bidding function (since $b(0) = z$ as before), so that the bid increases by a smaller amount each unit of time. It also compresses the range of bids and reduces the variance of expected revenue. Moreover, immediate consumption will reduce auction inefficiencies as only a portion of the utility is delayed.

One difficulty in including immediate consumption in the calibration exercise is that β is not separately identifiable from x , as these two parameters always appear multiplied together in any of our equilibrium conditions. However, one could exogenously set $x = z$, and then calibrate for β to match another moment of the distribution of auction revenue. We attempted this with median as a target. When $\beta = 0$, the model predicts a median revenue that is 85.4% of the posted price, while the data reports a median revenue of 83.2%. However, any increase in β would increase the predicted median. Thus, the corner solution of $\beta x = 0$ is the closest fit possible. The same occurs if the variance of revenue is targeted instead.

B Optimal Reserve Prices

We now relax the assumption that auction sellers always set their reserve price equal to $b(T)$, the lowest bid any buyer might make in equilibrium. There is clearly no incentive to reduce the reserve price below that point: doing so would not bring in any additional bidders, but would decrease revenue in those instances where only one bidder participates.

Now consider a seller who contemplates raising the reserve price to $R > b(T)$, taking the behavior of all others in the market as given. This will only affect the seller when a single bidder arrives or the second highest bid is less than R . With this higher reserve price, the

seller closes the auction without sale in these situations and re-lists the good, a strategy that has a present discounted value of Π_a . Of course, the seller gives up the immediate revenue and completion cost, which is no greater than $R - \gamma c$.

Since $\Pi_a = (1 - \gamma)c$ in equilibrium, deviating to the reserve price R is certain to be unprofitable if $R - \gamma c < (1 - \gamma)c$, or rearranged, $R < c$. In words, the optimal seller reserve price should equal the total cost of production. Thus, in our context, $b(T)$ is the optimal seller reserve price so long as $b^*(T) \geq c$.

If $b^*(T) < c$, then the seller would prefer to set a reserve price of c . One can still analyze this optimal reserve price in our model by endogenizing the buyer deadline, T . For instance, suppose that buyers who enter six months before their deadline are only willing to bid below the cost of production. By raising the reserve price, these bidders are effectively excluded from all auctions; it is as if they do not exist. They only begin to participate once they reach time S such that $b^*(S) = c$. In other words, it is as if all buyers enter the market with S units of time until their deadline. To express this in terms of our model, we make T endogenous, requiring $b^*(T^*) = c$ in equilibrium. All else will proceed as before.

Even with optimal reserve prices, the entry and exit of sellers will ensure that expected profits from entering the market are zero. Any gains from raising the reserve price are dissipated as more auctions are listed. To consider the absence of this competitive response, consider if one seller had monopoly control of both markets. The optimal choice would be to shut down the auction market, forcing all buyers to purchase at the highest price z . When there are numerous independent sellers, however, they cannot sustain this degenerate equilibrium (at least when Proposition 4 in the paper does not hold). There is always an advantage to offering an auction if all other sellers offer posted-price listings: the product sells faster, even if at a slightly lower price.

C Discrete Time Derivation of Bellman Equations

Each of the continuous-time Bellman equations— (2), (13), and (14)—in the model can be derived from a discrete-time formulation as follows. First, consider the expected profit of a seller in the auction market, Π_a . Let Δ be the length of a period of time, which we assume to be sufficiently short such that $\eta\Delta < 1$; this can then be interpreted as the probability of the auction closing during that period of time. The discrete time Bellman equation is thus:

$$\Pi_a = -\ell\Delta + \frac{1}{1 + \rho\Delta} \left(\eta\Delta \left(1 - e^{-\lambda} \right) (\theta - \gamma c) + \left(1 - \eta\Delta \left(1 - e^{-\lambda} \right) \right) \Pi_a \right). \quad (50)$$

The term $\ell\Delta$ is the listing fee incurred during the period of time. The term in parentheses computes the expected outcome in the next period of time: either the auction closes with at least one bidder, earning $\theta - \gamma c$, or it does not close or attracts no bidders, so the seller enters the next period with the same expected payoffs as the current period. These future payoffs

are discounted by the factor $1/(1 + \rho\Delta)$.

By moving $\Pi_a/(1 + \rho\Delta)$ to the left-hand side, then dividing by Δ , this becomes:

$$\frac{\rho}{1 + \rho\Delta} \Pi_a = -\ell + \frac{\eta(1 - e^{-\lambda})}{1 + \rho\Delta} (\theta - \gamma c - \Pi_a),$$

and taking the limit as $\Delta \rightarrow 0$, we obtain (13).

The expected profit for posted-price sellers is derived similarly. Again, we assume that a period is short enough that $\zeta\Delta < 1$.

$$\Pi_p = -\ell\Delta + \frac{1}{1 + \rho\Delta} (\zeta\Delta(z - \gamma c) + (1 - \zeta\Delta) \Pi_p). \quad (51)$$

Like auction sellers, posted-price sellers incur the listing fee $\ell\Delta$. With probability $\zeta\Delta$, they encounter a buyer in the next period and earn $z - \gamma c$; otherwise they continue waiting. This rearranges as:

$$\frac{\rho}{1 + \rho\Delta} \Pi_p = -\ell + \frac{\zeta}{1 + \rho\Delta} (z - \gamma c - \Pi_p),$$

and taking the limit as $\Delta \rightarrow 0$, we obtain (14).

The derivation for the buyer's expected utility is similar, only with more sources of uncertainty when an auction occurs. Let the period length Δ be sufficiently short that $\tau\alpha\Delta < 1$. This can then be interpreted as the probability that an auction occurs and the buyer participates during the unit of time. A buyer's expected utility in state s can be expressed as follows:

$$\begin{aligned} V(s) = \frac{1}{1 + \rho\Delta} & \left[\left(1 - \tau\alpha\Delta \sum_{n=0}^{\infty} \frac{e^{-\lambda}\lambda^n}{n!} (1 - F(s))^n \right) V(s - \Delta) \right. \\ & + \tau\alpha\Delta \left(\sum_{n=0}^{\infty} \frac{e^{-\lambda}\lambda^n}{n!} (1 - F(s))^n x e^{-\rho s} \right. \\ & \left. \left. - e^{-\lambda} b(T) - \sum_{n=1}^{\infty} \frac{e^{-\lambda}\lambda^n}{n!} \int_s^T b(t) n (1 - F(t))^{n-1} F'(t) dt \right) \right]. \end{aligned} \quad (52)$$

On the right-hand side, all utility is discounted by factor $1/(1 + \rho\Delta)$, meaning that the buyer does not receive any utility during the current period. By the next period, one of two outcomes could occur: either the buyer wins an auction and exits (second and third lines of (52)), or he continues his search (first line, due to losing or not participating).

Specifically, the second line computes the probability of the individual participating in an auction ($\tau\alpha\Delta$) and winning (the first two terms of the summation), times the utility enjoyed from winning ($x e^{-\rho s}$). The third line compute the expected second-highest bid times the probability of winning and thus paying it. The first line considers when the buyer does not win or does not participate (the probability in parentheses), in which case the buyer will

continue waiting for future auction opportunities. Yet, he will do so with less time remaining before his deadline, reflected in his state changing to $s - \Delta$.

To transform this to a continuous-time Hamilton-Jacobi-Bellman equation, we first multiply both sides by $(1 + \rho\Delta)/\Delta$, then subtract $V(s)/\Delta$ from both sides, obtaining:

$$\begin{aligned} \rho V(s) = \frac{V(s - \Delta) - V(s)}{\Delta} + \tau\alpha \left(\sum_{n=0}^{\infty} \frac{e^{-\lambda}\lambda^n}{n!} (1 - F(s))^n (xe^{-\rho s} - V(s - \Delta)) \right. \\ \left. - e^{-\lambda}b(T) - \sum_{n=1}^{\infty} \frac{e^{-\lambda}\lambda^n}{n!} \int_s^T b(t)n(1 - F(t))^{n-1}F'(t)dt \right). \end{aligned}$$

Then, by letting $\Delta \rightarrow 0$, we obtain (2).

D Proofs of Comparative Statics

Here we derive the signs of comparative statics that are reported in Table 1 of the paper. Because we do not have a closed form solution for the endogenous number of participants per auction, we use implicit differentiation of $\phi(\lambda^*) = 0$ from (8) to determine the effect of the exogenous parameters on λ^* . In preparation for this, we initially note that $\phi'(\lambda) < 0$ for all λ :

$$\frac{\partial \phi}{\partial \lambda} = -\alpha e^{-\lambda} - (\tau T \alpha + e^{\lambda}) \delta e^{-\tau T \kappa} < 0, \quad (53)$$

where $\kappa \equiv \delta + \alpha e^{-\lambda}$ is used for notational convenience, though we treat κ as a function of α and λ when taking derivatives.

Also note that $H = \frac{\lambda^*}{\tau}$ and $F'(0) = \kappa - \alpha$, while the lowest bid is:

$$b(T) = ze^{-\rho T} \cdot \frac{\kappa(\tau\kappa + \rho)e^{\lambda^*}}{\tau\kappa(\delta e^{\lambda^*} + \alpha e^{-\rho T}) + \rho(\delta e^{\lambda^*} + \alpha e^{\tau T \kappa})}. \quad (54)$$

Because this is always evaluated at the equilibrium λ^* , we can substitute for e^{λ^*} using $\phi(\lambda^*) = 0$, which is $\delta e^{\lambda^*} = (\kappa - \alpha)e^{\tau T \kappa}$, thus obtaining:

$$b(T) = \frac{ze^{-\rho T}}{\delta} \cdot \frac{(\tau\kappa + \rho)(\kappa - \alpha)}{\tau(\kappa - \alpha + \alpha e^{-(\rho - \tau\kappa)T}) + \rho}. \quad (55)$$

D.1 Auction Rate, α

Using implicit differentiation, we compute the effect of α on λ^* .

$$\frac{\partial \phi}{\partial \alpha} = -1 + e^{-\lambda} + \tau T \delta e^{-\tau T \kappa} \quad (56)$$

$$= -1 + e^{-\lambda} \left(1 + \left(\frac{\delta + \alpha e^{-\lambda} - \alpha}{\delta + \alpha e^{-\lambda}} \right) \ln \left(\frac{\delta e^{\lambda}}{\delta + \alpha e^{-\lambda} - \alpha} \right) \right). \quad (57)$$

The second equality comes from substituting for T using a rearrangement of $\phi(\lambda^*) = 0$, which is $T = \frac{1}{\tau\kappa} \ln\left(\frac{\delta e^\lambda}{\kappa - \alpha}\right)$.

By rearrangement, $\frac{\partial\phi}{\partial\alpha} \leq 0$ if and only if:

$$\ln\left(\frac{\delta e^\lambda}{\delta + \alpha e^{-\lambda} - \alpha}\right) - (e^\lambda - 1) \frac{\delta + \alpha e^{-\lambda}}{\delta + \alpha e^{-\lambda} - \alpha} \leq 0 \quad (58)$$

As $\lambda \rightarrow 0$, the left-hand side approaches 0. If we take the derivative of the left-hand side w.r.t. λ , we obtain:

$$-\frac{(e^\lambda - 1)(\alpha + \delta e^\lambda)(2\alpha + e^\lambda(\delta - \alpha))}{(\alpha + (\delta - \alpha)e^\lambda)^2} \quad (59)$$

Each parenthetical term is strictly positive for all $\lambda > 0$, so the left-hand side of (58) is strictly decreasing in λ . Thus, (58) strictly holds for any $\lambda > 0$, including the equilibrium λ^* . Therefore, $\frac{\partial\phi}{\partial\alpha} < 0$, and $\frac{\partial\lambda}{\partial\alpha} = -\left(\frac{\partial\phi}{\partial\alpha}\right) / \left(\frac{\partial\phi}{\partial\lambda}\right) < 0$. Specifically,

$$\frac{\partial\lambda}{\partial\alpha} = -\frac{1 - (1 + \tau T(\kappa - \alpha))e^{-\lambda}}{\kappa - \alpha + (1 + \tau T(\kappa - \alpha))\alpha e^{-\lambda}}. \quad (60)$$

Next, consider the impact on the fraction purchasing from posted-price listings, which is affected both directly by α and indirectly through λ :

$$\frac{\partial F'(0)}{\partial\alpha} = e^{-\lambda} - 1 + \alpha \cdot \frac{\partial\lambda}{\partial\alpha}. \quad (61)$$

This is strictly negative because $e^{-\lambda} < 1$ and $\frac{\partial\lambda}{\partial\alpha} < 0$.

To demonstrate the effect to α on the bidding function, we use the alternate depiction in terms of the function $g(t)$:

$$b(T) = \frac{g(T)}{g(T) + \rho \int_0^T g(t) dt},$$

recalling that

$$g(t) \equiv \tau e^{-\rho t} (\kappa - \alpha (1 - e^{-t\tau\kappa})).$$

Of course, $g(t)$ is a function of α (including its effect on κ), so let $g_\alpha(t)$ denote its derivative with respect to α . Thus,

$$g_\alpha(t) = \tau e^{-\rho t} \left(e^{-\tau\kappa t} + \frac{\kappa (1 - \alpha\tau t e^{-\tau\kappa t})}{\alpha + (\kappa - \alpha)(e^\lambda + \alpha\tau T)} - 1 \right).$$

When we take the derivative of $b(T)$ w.r.t. α , we obtain:

$$\frac{\partial b(T)}{\partial\alpha} = z\rho \frac{\int_0^T (g(t)g_\alpha(T) - g(T)g_\alpha(t)) dt}{\left(g(T) + \rho \int_0^T g(t) dt\right)^2}.$$

The denominator is clearly positive. The numerator is always negative; in particular, at each $t \in [0, T]$, the integrand is negative. This integrand simplifies to:

$$-\frac{\kappa\tau^2 e^{-(t+T)(\kappa\tau+\rho)} \left(\alpha^2\tau(T-t) + (\kappa-\alpha) \left(\alpha\tau(T-t)e^{\kappa\tau T} + e^\lambda (e^{\kappa\tau T} - e^{\kappa t\tau}) \right) \right)}{\alpha + (\kappa-\alpha)(e^\lambda + \alpha\tau T)} < 0.$$

The inequality holds that because $T \geq t$ and $\kappa > \alpha$, making each parenthetical term in the expression positive.

D.2 Attention, τ

Using implicit differentiation, we compute the effect of τ on λ^* .

$$\frac{\partial\phi}{\partial\tau} = \delta\kappa T e^{-\tau T\kappa} > 0. \quad (62)$$

All of these terms are strictly positive. Since $\frac{\partial\phi}{\partial\lambda} < 0$, then by implicit differentiation, $\frac{\partial\lambda}{\partial\tau} = -\left(\frac{\partial\phi}{\partial\tau}\right) / \left(\frac{\partial\phi}{\partial\lambda}\right) > 0$. Specifically,

$$\frac{\partial\lambda}{\partial\tau} = \frac{\delta\kappa T e^\lambda}{\alpha e^{\tau T\kappa} + \delta e^\lambda (e^\lambda + \alpha\tau T)}. \quad (63)$$

Next, consider the impact on the fraction purchasing from posted-price listings. The probability of participation τ has no direct effect on $F'(0)$, but affects it only through λ :

$$\frac{\partial F'(0)}{\partial\tau} = \frac{\partial F'(0)}{\partial\lambda} \cdot \frac{\partial\lambda}{\partial\tau} = -\alpha e^{-\lambda} \cdot \frac{\partial\lambda}{\partial\tau} \quad (64)$$

which is always negative.

Finally, consider the effect on the lowest bid. Here, the sign of the derivative will depend on parameter values, so it is more convenient to take comparatives on (55) rather than examining it in terms of $g(t)$. Since $\kappa'(\tau) = \alpha e^{-\lambda}\lambda'(\tau)$, the comparative static on $b(T)$ works out to:

$$\frac{\partial b(T)}{\partial\tau} = \frac{z\alpha e^\lambda \psi}{(\kappa-\alpha) \left(\tau\alpha + (\tau(\kappa-\alpha) + \rho)e^{T(\rho+\tau\kappa)} \right)^2 (\alpha + (\kappa-\alpha)(\tau\alpha T + e^\lambda))}. \quad (65)$$

where

$$\begin{aligned} \psi \equiv & e^\lambda (\kappa-\alpha)^2 \left(\rho(\tau\delta T - 1) + \delta\kappa\tau^2 T - \frac{\alpha e^{-\lambda}\rho}{\kappa-\alpha} \right) \\ & + \delta e^{\lambda+\rho T} \left(\rho \left(e^\lambda (\kappa-\alpha) + \alpha \right) - T(\kappa\tau + \rho) (\tau(\kappa-\alpha)^2 + \kappa\rho) \right). \end{aligned}$$

The lowest bid is increasing in τ if and only if $\psi > 0$, since the remaining terms in $\frac{\partial b(T)}{\partial\tau}$ are always positive.

To verify the sufficient conditions listed under Table 1 in the paper, note that $\tau\delta T > 1$ ensures that the first term in the first line is positive. For the remaining terms of the first line, note that $\delta\kappa\tau^2 T > \kappa\tau$ by the same assumption. Moreover, since $\kappa > \alpha$ and $1 > e^{-\lambda}$, then $\delta\kappa\tau^2 T > \alpha\tau e^{-\lambda}$. Thus, the sufficient condition $\tau(\kappa - \alpha) > \rho$ ensures that $\delta\kappa\tau^2 T > \frac{\alpha e^{-\lambda} \rho}{\kappa - \alpha}$.

For the second line, we note that by omitting the first and last α in the first step, then applying the second sufficient condition twice in the second, we get:

$$\begin{aligned} \rho \left(e^\lambda (\kappa - \alpha) + \alpha \right) - T(\kappa\tau + \rho) (\tau(\kappa - \alpha)^2 + \kappa\rho) &> \rho e^\lambda (\kappa - \alpha) - T(\tau\kappa + \rho)^2 \kappa \\ &> \frac{\rho^2 e^\lambda}{\tau} - T(\tau(2\kappa - \alpha))^2 \kappa. \end{aligned}$$

The third sufficient condition, $\rho > \tau(2\kappa - \alpha)\sqrt{\tau\kappa T e^{-\lambda}}$, ensures that this last term is positive.

D.3 Impatience, ρ

The discount rate ρ does not enter into ϕ , so therefore $\frac{\partial \phi}{\partial \rho} = 0$ and $\frac{\partial \lambda}{\partial \rho} = 0$. Similarly, ρ has no direct effect on $F'(0)$ or indirect effect through λ .

To demonstrate the effect to ρ on the bidding function, we use the alternate depiction in terms of the function $g(t)$:

$$b(T) = \frac{g(T)}{g(T) + \rho \int_0^T g(t) dt},$$

recalling that

$$g(t) \equiv \tau e^{-\rho t} \left(\delta + \alpha \left(e^{-\lambda} + e^{-t\tau(\delta + \alpha e^{-\lambda})} - 1 \right) \right).$$

Of course, $g(t)$ is a function of ρ , so let $g_\rho(t)$ denote its derivative with respect to ρ . Thus,

$$g_\rho(t) = -t\tau e^{-\rho t} \left(\delta + \alpha \left(e^{-\lambda} + e^{-t\tau(\delta + \alpha e^{-\lambda})} - 1 \right) \right).$$

Therefore, when we take the derivative of $b(T)$ w.r.t. ρ , we obtain:

$$\frac{\partial b(T)}{\partial \rho} = z \frac{\int_0^T (\rho g(t) g_\rho(T) - \rho g(T) g_\rho(t) - g(t) g(T)) dt}{\left(g(T) + \rho \int_0^T g(t) dt \right)^2}.$$

The denominator is necessarily positive. We will show that the integrand is negative for all t , implying that $\frac{\partial b(T)}{\partial \rho} < 0$. The integrand simplifies to:

$$\frac{\tau^2(\rho(t - T) - 1)}{e^{(t+T)(\tau(\alpha e^{-\lambda} + \delta) + \rho)}} \cdot \left((\alpha(1 - e^{-\lambda}) - \delta) e^{\tau t(\alpha e^{-\lambda} + \delta)} - \alpha \right) \cdot \left((\alpha(1 - e^{-\lambda}) - \delta) e^{\tau T(\alpha e^{-\lambda} + \delta)} - \alpha \right).$$

Since $t \leq T$, the numerator is always negative, and the exponential term in the denominator is always positive. Finally, we note that $\alpha(1 - e^{-\lambda}) - \delta < 0$ because $\delta - \alpha(1 - e^{-\lambda}) -$

$\delta e^{\lambda - \tau T (\delta + \alpha e^{-\lambda})} = 0$ in equilibrium. This ensures that second and third parenthetical terms are negative.

D.4 Deadline, T

Using implicit differentiation, we compute the effect of T on λ^* .

$$\frac{\partial \phi}{\partial T} = \delta \kappa \tau e^{\lambda^*} e^{-\tau T \kappa}, \quad (66)$$

which is clearly positive. Then by implicit differentiation, $\frac{\partial \lambda}{\partial T} = - \left(\frac{\partial \phi}{\partial T} \right) / \left(\frac{\partial \phi}{\partial \lambda} \right) > 0$. Specifically,

$$\frac{\partial \lambda}{\partial T} = \frac{\delta \tau \kappa}{\delta (1 + \tau T \alpha e^{-\lambda^*}) + \alpha e^{\tau T \kappa - 2\lambda^*}} \quad (67)$$

Moreover, the number of buyers H^* is not directly affected by T , so it increases only because λ^* increases.

Next, consider the impact on the fraction purchasing from posted-price listings. The deadline T has no direct effect on $F'(0)$, but affects it only through λ :

$$\frac{\partial F'(0)}{\partial T} = \frac{\partial F'(0)}{\partial \lambda} \cdot \frac{\partial \lambda}{\partial T} = -\alpha e^{-\lambda} \cdot \frac{\partial \lambda}{\partial T} \quad (68)$$

which is always negative.

To demonstrate the effect of T on the bidding function, we again use the definition of $b(T)$ in terms of $g(t)$, but to distinguish between an intermediate time t and the initial time T , we write it as:

$$b(T) = \frac{g(T, T)}{g(T, T) + \rho \int_0^T g(t, T) dt},$$

where

$$g(t, T) \equiv \tau e^{-\rho t} (\kappa - \alpha (1 - e^{-t\tau\kappa})),$$

where T only affects the expression by changing λ and hence changing κ .

The derivative of $b(T)$ w.r.t. T is thus:

$$\frac{\partial b(T)}{\partial T} = - \frac{z\rho \left(\int_0^T \left(\frac{g(T, T)^2}{T} - g(t, T) g_t(T, T) \right) dt + \int_0^T \left(g(T, T) g_T(t, T) - g(t, T) g_T(T, T) \right) dt \right)}{\left(g(T) + \rho \int_0^T g(t) dt \right)^2},$$

where g_t and g_T are derivatives with respect to the first and second terms, respectively. Specifically, these evaluate to:

$$g_t(T, T) = (\rho\tau(\alpha - \kappa) - \alpha\tau(\kappa\tau + \rho)e^{-\tau\kappa T}) e^{-T\rho}$$

and

$$g_T(t, T) = \alpha\tau (\alpha t\tau - e^{\kappa t\tau}) e^{-\lambda - t(\kappa\tau + \rho)} \lambda'(T).$$

Because $\kappa > \alpha$, we know that $g_t(T, T) < 0$ and $g(t, T) > 0$ for all t . Thus, the first integral in the numerator is always positive.

The integrand of the second integral simplifies to $\mu(t)\alpha^2\tau^2\lambda'(T)e^{-\lambda - (t+T)(\tau\kappa + \rho)}$, where:

$$\mu(t) \equiv e^{\tau\kappa t}(\tau T(\alpha - \kappa) - 1) + e^{\tau\kappa T}(t\tau(\kappa - \alpha) + 1) + \alpha\tau(t - T).$$

We have already shown that $\lambda'(T) > 0$; thus, to show that the integral is positive, we only need to show that $\mu(t) \geq 0$ for all t . First note that $\mu(T) = 0$ and $\mu(0) = e^{\tau\kappa T} - \tau\kappa T - 1 > 0$. To see the latter inequality, note that this has the form $e^x - x - 1$, which is equal to 0 at $x = 0$ and has a positive derivative $e^x - 1 \geq 0$ for all x .

Next, note that $\mu''(t) = -(1 + \tau T(\kappa - \alpha))\tau^2\kappa^2 e^{\tau\kappa t} < 0$ for all $t \in [0, T]$. Since $\mu(0) > \mu(T) = 0$ and $\mu''(t) < 0$, then $\mu(t) > 0$ for all $t \in [0, T]$.

Thus, the integrand of the second integral is always positive. Thus $\frac{\partial b(T)}{\partial T} < 0$.

E Model Estimation Details

We first discuss the moments used to identify the parameters of the buyer equilibrium and then discuss those used for estimating the full market equilibrium. The buyer equilibrium depends on one endogenous variable (λ) and five parameters (α , δ , τ , T , and ρ); the market equilibrium depends on the additional parameters ℓ , η , γ , and c .

The first moment in Table 4 is the number of observed bidders per completed auction. The theoretical equivalent of this is $\lambda P(\lambda)$ bidders per auction, since $P(\lambda)$ is the probability that a participant is able to place a bid and be observed, as described in Section 5. In the data, we restrict ourselves to auctions that resulted in a sale, meaning that at least one bidder arrived. In the model, this occurs with probability $1 - e^{-\lambda}$, so we divide $\lambda P(\lambda)$ by this to get the average number of observed bidders per *completed* auctions.

The second moment is the number of completed auctions per month. The model predicts α auctions per month, but this includes auctions that close without a bidder. Thus, $(1 - e^{-\lambda})\alpha$ provides the average number of completed auctions per month.

In the third moment, we compute the number of auctions per month that a buyer places a bid. Again, this is measured conditional on having at least one bid by that buyer during the month. The number of observed bids per month is Poisson distributed with parameter $\tau\alpha P(\lambda)$, since opportunities arise for the bidder at rate α and are used with probability τ , but will only register as a bid with probability $P(\lambda)$. To obtain the predicted conditional average, we divide this by $1 - e^{-\tau\alpha P(\lambda)}$.

To get a measure of the market size, we measure the number of new bidders per month. We focus this by conditioning on bidding at least twice during the span of the data; this is

done through the last term, since D is the Poisson rate of bids per buyer over the duration of his search (as described in Section 5). However, D is computed for a complete search spell (*i.e.* all T periods); thus, we multiply by $\delta - \alpha$ rather than δ to get the average flow per month of newly observed repeat bidders who never win, and similarly condition the data moment on never winning.

The deadline length T is not directly observable, since we cannot observe when a buyer entered or exited the market, but only the first and last times they bid on an item. Instead, we select T so that the equilibrium condition $\phi(\lambda) = 0$ holds. Effectively, we are choosing the deadline T such that the observed number of bidders per auction (which is endogenously determined) will occur in equilibrium.

The discount rate is determined by matching the mean of auction revenue θ , whose formula is reported in the Appendix with the proof of Proposition 3. Note that θ is already conditional on having at least one bid, consistent with the computed data moment.

The parameters in the full market equilibrium can also be computed using simple empirical averages or functions of these averages. First, we can directly observe the average listing fee paid (yielding an estimate of ℓ) and the average time for which an auction is listed (yielding an estimate of $1/\eta$).

We also observe the fraction of posted-price listing that sell within the 30 day window of their listing. Since the model predicts that posted-price sellers exit at Poisson rate ζ , $1 - e^{-\zeta}$ will have exited within one month; the table simply inserts the equilibrium value for ζ . We also note that α , which was computed in the second row of Table 4 for the buyer equilibrium, is now endogenous. In the market equilibrium, we can use the equilibrium condition for α , equation (19), to determine the underlying cost of production. The last two conditions are solved jointly to recover γ and c .

We now summarize what sources of variation in the data leads to the most change in the estimate of each parameter. Variation in many of the parameters (λ , α , τ , δ , ℓ , and η) maps directly back to variation in the data moments found in the corresponding rows of Table 4. The effect is less obvious for those involving the equilibrium conditions (T , ρ , γ , and c), which we now consider in turn.

The parameter T relies on the requirement for a buyer equilibrium found in (8), which effectively balances buyer entry and exit in steady state. It is most sensitive to increases in the number of bidders per auction (the first data moment in Table 4). If the number of bidders per auction increases, buyers are more likely to hit their deadline without winning, causing too much exit relative to entry; but the estimation restores balance by keeping buyers in the market longer, giving them more chances to win. While α , τ , and δ also appear in (8), the effect of their corresponding moments on the estimated T are somewhat muted.

The parameter ρ is directly determined by revenue per auction, holding the five preceding parameters fixed. Higher revenue occurs if buyers are more patient, since they bid closer to the posted price. The number of bidders per auction also indirectly affects ρ because, as

explained above, an increase in the number of bidders per auction leads to an increase in the inferred T , and as T increases, the average bidder will be further from his deadline and bid less.

The parameters γ and c are jointly determined in the last two rows of Table 4. Recall that the market equilibrium requirement in (19) determines the endogenous number of auctions so that both mechanisms will be equally profitable. As a consequence, c is closely tied to average auction revenue, keeping ex-post profits low. Thus, γ is left to reconcile the time it takes to sell a posted-price listing and the number of auctions listings. The latter has a particularly large impact, as seen in Table A3. While the auction revenue and speed of selling have hardly changed, the number of auctions declined precipitously; this can be justified if γ increased, because posted-price sellers will have fewer up-front costs before the long wait for a buyer.

F Alternative Models for Interpreting Auction Data

Here, we briefly describe the static and the stationary dynamic models that could be used for the interpretation of bidding data. We highlight how product demand and consumer surplus would be determined in each.

For the static model, assume that bidder types s determine their valuation $x(s)$, which is a decreasing function of s . Types are independently drawn from an exogenous distribution $F(s)$. Each bidder has only one opportunity to bid. In such a model, the optimal bid will be $b(s) = x(s)$, so that bids precisely reveal the underlying utility of bidders.

For the stationary dynamic model, bidder types s still determine their valuation $x(s)$, and these valuations are persistent throughout their search. Types in a given auction are distributed by $F(s)$, which could be endogenously determined. Bidders participate in auctions at rate $\tau\alpha$ with an average of λ bidders per auction. In this dynamic environment, the continuation value of a bidder is:

$$\rho V(s) = \alpha\tau \left(e^{-\lambda F(s)} (x(s) - V(s)) - e^{-\lambda} b(T) - \int_s^T \lambda e^{-\lambda F(t)} b(t) F'(t) dt \right).$$

The optimal bid is $b(s) = x(s) - V(s)$; so after substituting this into the HJB equation, it simplifies to:

$$x(s) \equiv b(s) + \frac{\tau\alpha}{\rho} \left(e^{-\lambda F(s)} b(s) - e^{-\lambda} \left(b(T) + e^\lambda \int_s^T b(t) \lambda e^{-\lambda F(t)} F'(t) dt \right) \right). \quad (69)$$

That is, by observing all bids, $b(s)$, and their distribution, $F(s)$, one can infer the underlying utility of the bidders.

To estimate demand in the static model, the econometrician inverts the empirical CDF of bids, with bids on the vertical axis and number of bidders on the horizontal. Thus, we

employed a parametric plot of $(H \cdot F(s), b(s))$ to create the dashed line in Figure 7. In the stationary dynamic model, one would use the empirical CDF of bids in (69) to determine the valuation associated with each $b(s)$. The demand curve plots $(H \cdot F(s), x(s))$. Finally, in our deadline model, one would estimate the parameter values as indicated in Section 5, with the demand curve plotting $(H \cdot F(s), xe^{-\rho s})$. Note that x is not identifiable from the model, as it drops out of the equilibrium solution; here, we set $x = z$, which creates the smallest difference between the static model and ours.

Next, for each model we provide the theoretical expressions for consumer surplus in the average auction, conditional on having any participants. These can then be computed using the estimates from the preceding paragraph. In our deadline model, this would be:

$$CS_{\text{model}} = \frac{\int_0^T xe^{-\rho s} \lambda e^{-\lambda F(s)} F'(s) ds - \int_0^T b(s) \lambda^2 e^{-\lambda F(s)} F(s) F'(s) ds - e^{-\lambda} b(T)}{1 - e^{-\lambda}}. \quad (70)$$

In the first integral, the remaining time until deadline, s , of the highest bidder is distributed according to $\lambda e^{-\lambda F(s)} F'(s)$ (after evaluating the sum over the Poisson distribution of bidders in the auction), and the utility enjoyed by the highest bidder is $xe^{-\rho s}$. In the second integral, we determine the average of the second highest bid, which is distributed according to $\lambda^2 e^{-\lambda F(t)} F(t) F'(t)$. The final term accounts for when only one bidder arrives and thus wins at price $b(T)$.

In the static model, consumer surplus is the difference between the first price (which truthfully reveals the winner's valuation) and the second price (which the winner actually pays). Therefore, we replace $xe^{-\rho s}$ in (70) with $b(s)$, which simplifies to:

$$CS_{\text{static}} = \frac{\lambda}{1 - e^{-\lambda}} \left(\int_0^T b(s) e^{-\lambda F(s)} (1 - \lambda F(s)) F'(s) ds - e^{-\lambda} b(T) \right). \quad (71)$$

In the stationary dynamic model, the expected consumer surplus replaces $xe^{-\rho s}$ in (70) with $x(s)$:

$$CS_{\text{stationary}} = \frac{\int_0^T x(s) \lambda e^{-\lambda F(s)} F'(s) ds - \int_0^T b(s) \lambda^2 e^{-\lambda F(s)} F(s) F'(s) ds - e^{-\lambda} b(T)}{1 - e^{-\lambda}}. \quad (72)$$

When the static model is applied to deadline data, it consistently underestimates demand and consumer surplus. This is because it ignores the continuation value of further search, which causes the observed bid to be less than the true value. When the stationary dynamic model is applied to deadline data, it rotates the true demand curve, estimating it to be steeper than it really is. This is because buyers with a low s are almost certain to win in any auction, and thus can drastically shade their bid in hopes of a good deal. As s increases, the bid still falls, but the bid shading becomes less extreme; those with a very high s hardly shade at all because they are so unlikely to win that $V(s)$ is nearly zero. In other words, the stationary

model interprets the true valuations as being nearly the same as the lowest bids, but rising much steeper for those who bid more. The overestimates of consumer surplus on high bids far outweigh the underestimates on low bids.

G Estimation Across Years

This section considers how eBay retail markets have evolved over a five-year period, and examines how this affects our estimated parameters. Using one-year intervals, we apply the same sample restriction as in the paper from October 1st, 2010 through September 30th, 2015. Table A1 reports the descriptive statistics from each year, while Table A2 summarizes the data moments used to estimate our parameters. Note that the number of auction transactions has fallen dramatically. The number of unique auction buyers has also fallen significantly.

We then recompute the model’s parameters and the corresponding welfare measures for each year, as reported in Table A3. In reviewing the estimated parameters and data moments, it is noteworthy that some have changed very little. In particular, the average auction revenue has held fairly steady at $\theta \approx 85\%$ of the retail price (see the fifth row of Table A2). In Table A3, the production cost remains about 1.4% below expected revenue. Thus, over this time frame, it is not that auctions have become less profitable. Even so, the flow of buyers into the market (δ) has decreased by nearly half, and similarly for the rate at which auctions are offered (α).

On its face, time sensitivity does not seem to be the driving force, as the search time T has grown longer and buyers are more patient (smaller ρ). Indeed, the remaining buyers in the market appear to be paying greater attention (τ) to each auction. Even so, this could be driven by a changing composition of buyers: δ has fallen dramatically, and if those who no longer participate were the most time-sensitive buyers, those who remain would look less time sensitive.

One of the most dramatic changes over time is found in γ , the fraction of production costs that can be delayed until the time of sale. We estimate $\gamma = 35\%$ in 2010, but found it to be above 85% in the last three years of data. The primary feature of the data driving this estimated decline is that auction revenue and the selling speed of posted-price listings both stayed fairly constant over this period, implying that costs were fairly constant; yet the endogenous arrival rate of auctions decreased, which can be justified if production costs are incurred later.³⁶ This could be due to changes in the relative importance of shipping costs, changes in how sellers source their inventories, or other movements toward a “just-in-time” production setting. As this occurs, the long wait (for sellers) associated with posted-price listings becomes less costly and more beneficial for overall welfare.

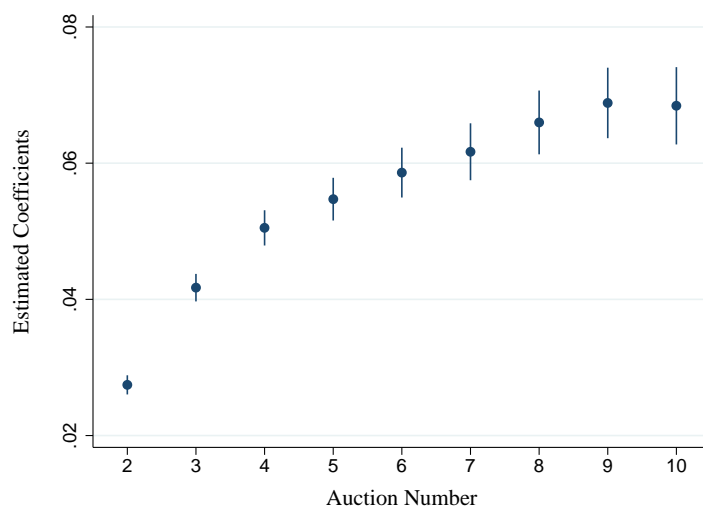
Indeed, these delayed costs have greatly improved market efficiency. Between 2010 and 2014, total welfare in the dispersed equilibrium has grown by 7% from 0.112 to 0.120 of the

³⁶The end of Section E examines the sensitivity of each parameter to specific data moments.

retail price. However, the welfare under a degenerate, posted-price-only market has grown even more (33%), from 0.106 to 0.141 of the retail price. This created a large enough shift to reverse the welfare comparison. In 2010, allowing auctions increased welfare 6% above what posted-prices alone could offer (0.112 vs. 0.106); by 2014, a market with auctions provides 15% less welfare than the posted-prices alone (0.120 vs. 0.141). This assumes that the platform incurs no costs, which is the least favorable scenario for judging auction efficiency.

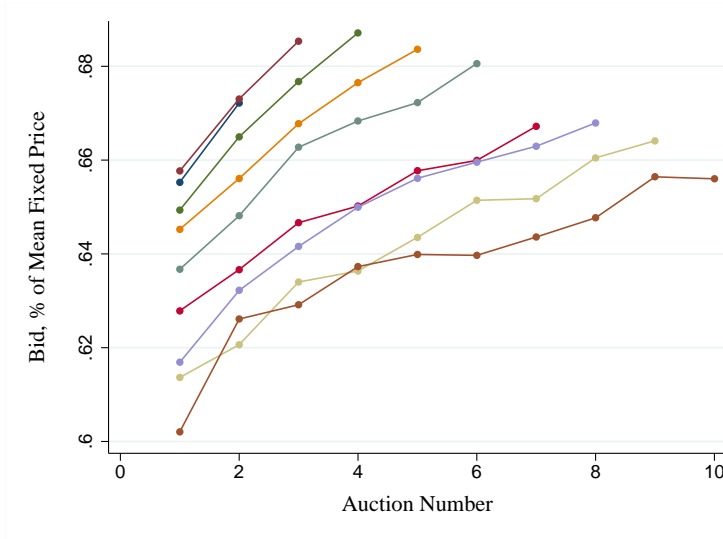
H Additional Figures and Tables

Figure A1: Bids Over Time, Regression Results



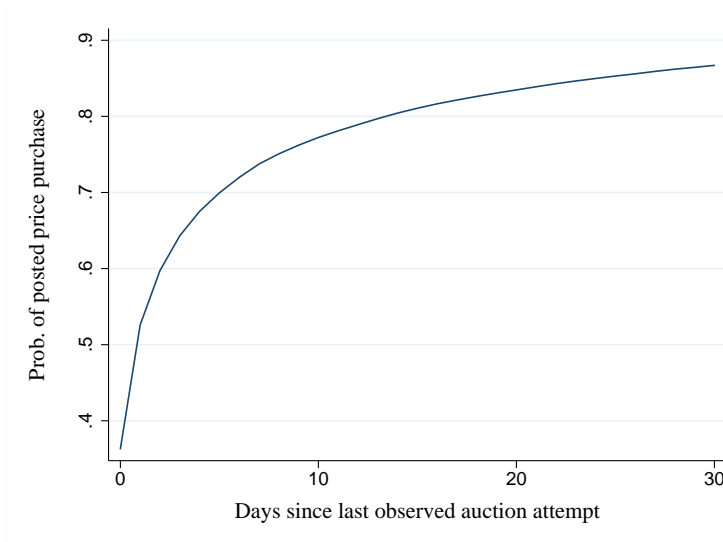
Notes: Figure displays estimated coefficients for dummy variables for each auction number (i.e. where the auction appears in the sequence) from a regression of normalized bid on these dummies and on dummies for the length of auction sequence. This regression is performed after removing outliers in the auction number variable (defined as the largest 1% of observations). 95% confidence intervals are displayed about each coefficient.

Figure A2: Bids Over Time, Excluding Winning Bids



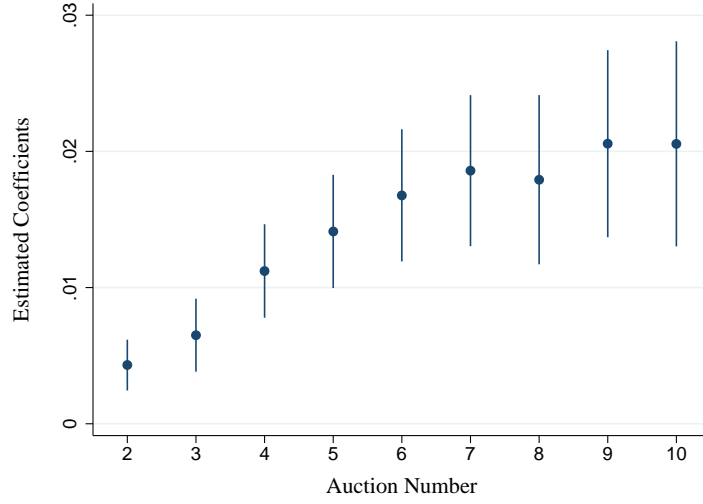
Notes: Figure displays the same analysis as in Figure 1 from the paper, but with winning bids excluded. In the figure, a given line with n points corresponds to bidders who bid in n auctions total for a given product without winning in the first $n - 1$ auctions. Horizontal axis represents auction number within the sequence (from 1 to n) and vertical axis represents the average normalized bid.

Figure A3: Time To Posted-Price Purchase Since Last Losing Auction



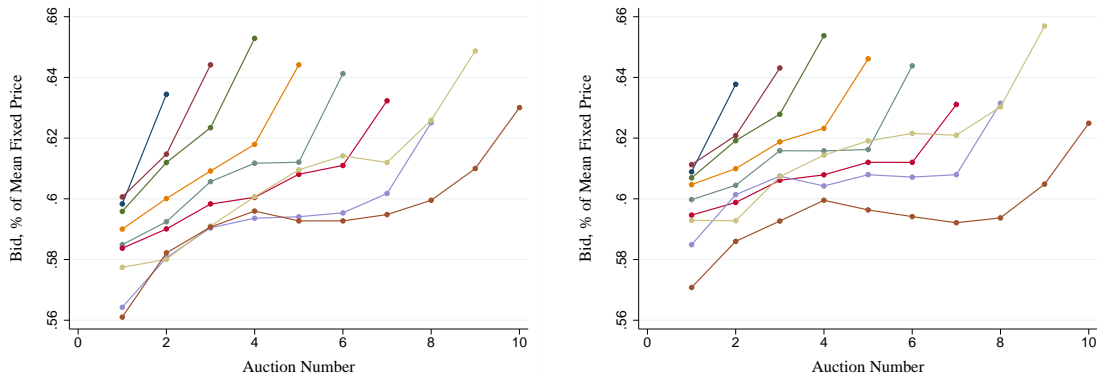
Notes: Figure displays cumulative density of the time difference between the last observed auction attempt and the posted-price purchase conditioning on bidders who attempted an auction and did not win and were later observed purchasing the good on an eBay posted-price listing.

Figure A4: Fraction Bidding on Listings with Fast Shipping



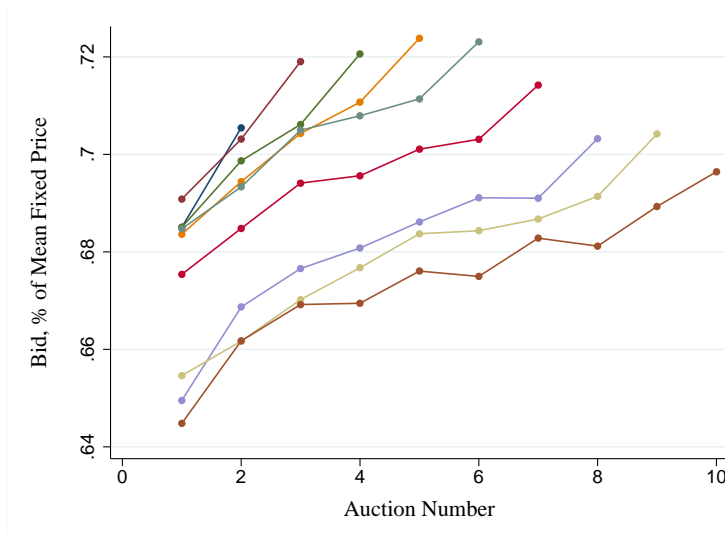
Notes: Figure displays estimated coefficients for dummy variables for each auction number (i.e. where the auction appears in the sequence) from a regression of a fast-shipping dummy (an indicator for whether the listing offered a shipping option guaranteed to arrive within 96 hours) on these auction number dummies and on dummies for the length of auction sequence. This regression is performed after removing outliers in the auction number variable (defined as the largest 1% of observations). 95% confidence intervals are displayed about each coefficient. The regression constant (overall mean of the fast shipping dummy) is 0.43. Averaging over all regression coefficients yields an estimate of 0.0028, or 0.6% of the overall mean.

Figure A5: Bids over Time, Experienced Bidders



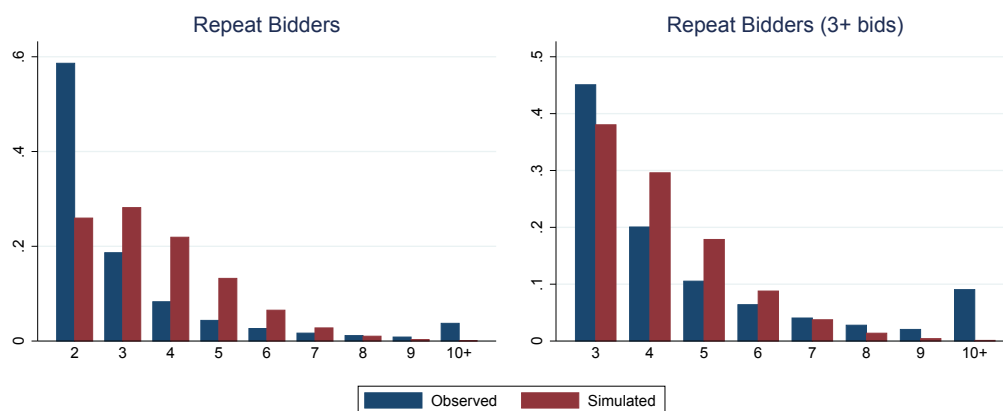
Notes: Figures limit to bidders who have bid in at least 50 auctions (left panel) or bidders who have bid in at least 10 auctions for products in the same product grouping (right panel) in the past year prior to the current auction. In the figures, a given line with n points corresponds to bidders who bid in n auctions total for a given product without winning in the first $n - 1$ auctions. Horizontal axis represents auction number within the sequence (from 1 to n) and vertical axis represents the average normalized bid.

Figure A6: Bids Over Time, Products With Average Transaction Price of At Least \$100



Notes: Figure limits to products with average transaction price $\geq \$100$. In the figure, a given line with n points corresponds to bidders who bid in n auctions total for a given product without winning in the first $n - 1$ auctions. Horizontal axis represents auction number within the sequence (from 1 to n) and vertical axis represents the average normalized bid.

Figure A7: Participation by Number of Auctions



Notes: Each histogram reports the fraction of bidders who participate in a given number of auctions, conditional on participating in at least two (left panel) or three (right panel) auctions, for observed data (blue) and simulated data (red).

Table A1: Descriptive Statistics From 2010–2014 Data Samples

	2010	2011	2012	2013	2014
Number of products	4,060	3,704	4,231	3,663	2,102
<i>Posted prices</i>					
# transactions	335,691	416,300	537,441	494,448	269,263
Revenue	104.55 (23.89)	115.54 (23.45)	108.24 (23.22)	106.82 (21.74)	127.08 (23.22)
# transactions per product	82.68 (116.63)	112.39 (149.47)	127.02 (166.57)	134.98 (220.82)	128.10 (142.06)
Unique sellers per product	45.47 (57.09)	66.62 (85.24)	75.86 (92.86)	82.70 (137.84)	78.49 (79.18)
Unique buyers per product	79.77 (108.52)	106.32 (138.21)	117.65 (149.02)	129.03 (208.02)	122.42 (130.46)
<i>Auctions</i>					
# transactions	914,219	795,699	818,223	560,861	268,001
Revenue	93.14 (16.40)	105.15 (16.86)	99.20 (16.71)	97.27 (16.60)	116.52 (19.09)
Normalized revenue	0.83 (0.17)	0.85 (0.14)	0.87 (0.17)	0.85 (0.17)	0.86 (0.18)
Bidders per transaction	4.99 (2.21)	5.27 (2.29)	5.41 (2.28)	5.30 (2.20)	5.41 (2.05)
# transactions per product	225.18 (383.68)	214.82 (331.60)	193.39 (320.73)	153.12 (343.63)	127.50 (177.81)
Unique sellers per product	102.36 (144.91)	95.40 (126.33)	82.97 (109.57)	68.53 (201.30)	60.49 (69.04)
Unique buyers per product	792.02 (1468.70)	783.63 (1299.21)	734.97 (1212.34)	622.18 (1394.80)	508.69 (632.59)

Notes: Table displays descriptive statistics computed as in Table 3 using data samples from 2010–2014. Data was constructed using same sample restrictions as in main sample. Each year label corresponds to one year of data beginning in Oct. of that year and continuing through Sep. of the following year (i.e. the 2013 column corresponds to the main data sample used in the body of the paper). Values in table are computed as described in notes of Table 3. In Revenue, Normalized revenue, and Bidders per transaction rows, values reported are means of product-level means, with means of product-level standard deviations in parentheses. In all rows specifying per-product measures, values reported are the average values across all products, with standard deviations across products in parentheses.

Table A2: Data Moments Used in Estimation From 2010–2014 Data Samples

	2010	2011	2012	2013	2014
Bidders per completed auc- tion	4.988	5.267	5.406	5.301	5.408
Completed auctions per month	18.765	17.902	16.116	12.760	10.625
Auctions a bidder tries per month	1.215	1.215	1.201	1.176	1.150
New repeat bidders per month who never win	10.635	10.522	9.552	7.406	5.261
Average revenue per com- pleted auction	0.835	0.852	0.868	0.853	0.855
Average listing fee paid	0.070	0.076	0.081	0.087	0.089
Average duration of an auc- tion listing (months)	0.154	0.154	0.157	0.156	0.159
Average % of posted-price listing sold in 30 days	0.512	0.556	0.521	0.481	0.515

Notes: Table displays observed data used for estimation (as in Table 4) for data samples from 2010–2014. Data was constructed using same sample restrictions as in main sample. Each year label corresponds to one year of data beginning in Oct. of that year and continuing through Sept. of the following year (i.e. the 2013 column corresponds to the main data sample used in the body of the paper). Displayed moments were used to obtain parameter estimates displayed in Table A3.

Table A3: Parameter Values and Welfare Estimates in 2010–2014

	2010	2011	2012	2013	2014
λ	11.196 (0.201)	12.877 (0.250)	13.807 (0.225)	13.101 (0.243)	13.817 (0.311)
α	18.765 (0.533)	17.902 (0.462)	16.116 (0.395)	12.760 (0.525)	10.625 (0.340)
τ	0.048 (0.001)	0.055 (0.001)	0.060 (0.001)	0.064 (0.002)	0.069 (0.002)
δ	31.033 (1.065)	29.862 (0.909)	26.879 (0.795)	21.131 (1.163)	16.357 (0.557)
T	8.125 (0.148)	8.394 (0.144)	9.154 (0.146)	10.301 (0.255)	13.233 (0.267)
ρ	0.069 (0.002)	0.068 (0.002)	0.058 (0.002)	0.055 (0.003)	0.041 (0.001)
ℓ	0.070 (0.000)	0.076 (0.000)	0.081 (0.000)	0.087 (0.000)	0.089 (0.000)
η	6.475 (0.023)	6.476 (0.026)	6.372 (0.023)	6.395 (0.028)	6.300 (0.033)
γ	0.351 (0.036)	0.463 (0.029)	0.889 (0.038)	0.985 (0.039)	0.855 (0.071)
c	0.818 (0.003)	0.836 (0.002)	0.855 (0.003)	0.840 (0.003)	0.841 (0.004)
Welfare comparison (costless platform)					
Degenerate equil.	0.106 (0.001)	0.106 (0.001)	0.124 (0.001)	0.144 (0.001)	0.141 (0.001)
Dispered equil.	0.112 (0.001)	0.105 (0.001)	0.107 (0.001)	0.121 (0.001)	0.120 (0.001)
First best	0.182 (0.003)	0.164 (0.002)	0.145 (0.003)	0.160 (0.003)	0.159 (0.004)

Notes: Table displays estimates, using data from 2010–2014, of parameter values and welfare estimates for the degenerate (posted prices only) equilibrium, dispersed (auctions and posted prices) equilibrium, and first best settings when the platform faces zero costs. Data was constructed using same sample restrictions as in main sample. Each year label corresponds to one year of data beginning in Oct. of that year and continuing through Sep. of the following year (i.e. the 2013 column corresponds to the main data sample used in the body of the paper). Units for parameters are as in Table 4 and units for welfare measures are fraction of retail (posted) price. Standard errors, from 200 bootstrap replications at the product level, are contained in parentheses.