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ABSTRACT

Endogenous diversification in a market with incomplete information generates a state-dependent premium for bearing idiosyncratic risk because time series variation in average idiosyncratic risk affects the disutility of under-diversification. This idea delivers a metric that maps the marginal disutility of under-diversification to the covariance of idiosyncratic risk with average idiosyncratic risk. The metric helps explain the cross-section of returns in the US between 1973 and 2014, especially in periods of low average idiosyncratic risk. In such periods, portfolios tests generate intercepts from factor models that are economically large, and present in both small and large capitalization stocks. We observe similarly large intercepts in markets outside the US, particularly in large stocks.

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1. Introduction

In canonical asset pricing models such as the Sharpe-Lintner-Mossin and Black CAPM [Sharpe (1964), Lintner (1965), Mossin (1966), Black (1972)], Ross's (1976) Arbitrage Pricing Theory, and Merton's (1973) ICAPM, there is no role for idiosyncratic risk. Although precedent belongs to Levy (1978), Merton (1987) abandons this frictionless market ideal and builds a model in which (by assumption) investors do not hold fully diversified portfolios. He justifies this assumption by appealing to incomplete information, though any multitude of frictions or behavioral biases could account for such behavior. This simple incomplete markets model generates a positive premium for idiosyncratic risk.

In this paper, we endogenize the representative agent's diversification decision in Merton's (1987) model with costly information acquisition. This serves two purposes.

First, it is economically plausible that agents diversify based on the marginal costs and benefits of diversification. This allows for a mapping from the marginal disutility of under-diversification, to the decision to diversify. Endogenous diversification results in diminishing marginal returns to incremental idiosyncratic risk: investors still require a positive premium for bearing higher levels of idiosyncratic risk, but endogenously optimal diversification reduces the required risk premium. As a result, the model introduces convexity into the compensation for bearing idiosyncratic risk, in sharp contrast to Merton's original model in which the idiosyncratic risk premium is constant. This difference is illustrated in Figure 1.

Second, as average idiosyncratic volatility changes over time, it affects the disutility of under-diversification, leading to a time varying premium. This can be seen in the cross-section of stocks via the covariance of a stock's idiosyncratic volatility with average idiosyncratic volatility. We refer to this covariance as the idiosyncratic risk premium sensitivity (IRPS), formally defined as the covariance of the idiosyncratic risk of a stock (σ_i^2) with one over the square root of the average idiosyncratic risk ($1/\sqrt{\overline{\sigma^2}}$). The intuition is best clarified by contrasting two states of the world, where average idiosyncratic volatility is either low or high. In periods where average idiosyncratic volatility is low, the marginal disutility of stocks with high idiosyncratic volatility relative to the average (i.e. high IRPS stocks) is higher, generating a high premium. In contrast, in periods where average idiosyncratic volatility is high, investors' (endogenously) diversify more, making them (relatively) less averse to stocks with high idiosyncratic risk. In other words, the marginal disutility of stocks with relatively high idiosyncratic volatility is lower, generating a lower (but still positive) premium. The implication is that the slope of regressions of expected returns on IRPS should be positive in periods where average idiosyncratic volatility is low, and less positive in periods where average idiosyncratic volatility is high.

Between 1973 and 2014, Fama-MacBeth regressions of monthly returns show a positive slope on IRPS, after controlling for firm size, book-to-market ratios and prior returns. This unconditional result, however, masks important conditional variation that is at the center of the model. We separate the sample into periods of high versus low average idiosyncratic volatility based on a trailing 10-year average. This separation captures the state dependence implied by the model in which the premium should be larger in low average idiosyncratic risk periods. The regressions show precisely that: in low average idiosyncratic volatility periods, the slope on IRPS is 40 percent larger with standard errors less than half of the coefficient. These positive slopes are present in both large and small stocks, quite unlike many cross-sectional return patterns that are confined to small stocks.

Time series tests allow for sharper inferences as well as a cleaner assessment of the magnitude of the premium. Portfolios sorted on firm size and IRPS show a monotonic relation between intercepts from three-factor models and IRPS. Over the entire sample period, a high-minus-low IRPS portfolio has an intercept of 0.28 percent per month in large stocks and 0.37 percent per month in small stocks. As with the Fama-MacBeth regressions, the more interesting results are in the low average idiosyncratic volatility periods where the model says that the premium associated with IRPS should be higher. In those periods, the intercepts on the high-minus-low IRPS portfolios rise to 0.69 percent per month for large stocks and 0.80 percent per month for small stocks.

We confront these portfolios with more complete models of expected returns, the Fama and French (2015a) five-factor comparative static model, and the Hou, Xue and Zhang (2014) q-theoretic model. Both models add factors related to investment and profitability, albeit in different forms. The addition of investment makes no difference to our basic results. Profitability or ROE, however, absorbs much of the variation in IRPS-based portfolio returns. In the full sample, the loadings on profitability or ROE monotonically increase across IRPS quintiles, and the intercepts are statistically indistinguishable from zero. This is good news for these factor models for at least two reasons. First, unlike finer sorts on size, book-to-market, profitability and investment, sorts on IRPS are independent (or, in the vernacular of Fama and French (2015b), they are not home games). Second, unlike sorts on well-known anomalies such as net issuance, accruals, momentum, and others on which there are scores of published papers, IRPS is hitherto unstudied. Data dredging cannot be an issue because the sorting variable arises directly from economic theory. However, the good news is tempered. While these models are powerful descriptors of average returns and span a variety of test assets (Hou, Xue and Zhang (2014), Fama and French (2015d), in low average idiosyncratic volatility periods, IRPS continues to generate excess returns. For example, using the Fama-French five-factor model, the high-minus-low IRPS portfolio has an intercept of 0.39 percent per month for large stocks and 0.51 percent per month for small stocks. And using the Hou, Xue and Zhang (2014) factors, the equivalent premiums are 0.38 and 0.53 percent per month.[1] The key is conditioning – unconditionally, portfolios generated by IRPS are spanned by these factor models, but conditioning on low average idiosyncratic volatility, the intercepts are reliably different from zero.

We conduct two sets of out-of-sample tests. First, we estimate factor models on size and IRPS sorted portfolios in 1931-1973. Profitability/ROE factors are unavailable in this pre-Compustat period so we can only estimate three-factor models. We continue to observe a monotonic relation between intercepts and IRPS, with high-minus-low spreads of between 0.35 and 0.41 percent per month. As in the later sample period, these spreads are attenuated in low average idiosyncratic volatility periods.

[1] As we show later in the paper, the average time series correlation between IRPS and idiosyncratic volatility is negative. Nonetheless, to alleviate concerns that the results are somehow generated by an inadvertent sort on idiosyncratic volatility, we also create portfolios sorted by size, idiosyncratic volatility and IRPS. Controlling for idiosyncratic volatility, we still observe economically large premiums in both large and small stocks.

Second, we build size and IRPS sorted portfolios with ex-US data for the 1990-2014 sample period. To maximize power and exploit the results in Fama and French (2012, 2015c), we build portfolios for four regions: North America, Europe, Japan and Asia Pacific (excluding Japan). In North America, Europe and Asia Pacific, the intercepts for high-minus-low IRPS portfolios from three-factor models are positive and large. Like the US, adding factors for profitability and investment shrinks the intercepts and renders them statistically insignificant. But again paralleling the US results, in low average idiosyncratic volatility periods, even intercepts from five-factor models are large and more than two standard errors from zero. In Japan, IRPS generates virtually no dispersion in returns using three- or five-factor models. This could be because of low power, or simply because Japan is somehow different; Fama and French (2015c) report that profitability and investment do not explain average returns in Japan, and it is well known that momentum portfolios also do not generate positive intercepts.

The limited time series (and hence power) for international data suggest that caution is warranted. Like Fama and French (2012), we can help the situation by further expanding the cross-section. We construct global portfolios, excluding the US in the spirit of true out-of-sample tests, and re-estimate factor models. The results are noticeably stronger. In the full sample period, for large stocks, the spread in intercepts from three- and five-factor models for high-minus-low IRPS portfolios is 0.62 and 0.70 percent per month, with t-statistics of 3.44 and 3.75 respectively.

It is useful to step back and view these results in the context of both the theoretical and empirical literature. As discussed earlier, classical theory denies any role for idiosyncratic volatility on asset prices. Empirically, Fama and MacBeth (1973), Bali and Cakici (2008), and others agree. Theories that acknowledge frictions or behavioral biases can generate a positive price of idiosyncratic risk [Levy (1978), Merton (1987), Barberis and Huang (2001)], and empirically, Malkiel and Xu (2002), Spiegel and Wang (2005), and Fu (2009) find some support. But Ang et al. (2006, 2009) find a negative relation between expected returns and lagged idiosyncratic volatility. Stambaugh, Yu and Yuan (2015) argue that a combination of arbitrage asymmetry and arbitrage risk generate this negative relation. We do not take a stand on these conflicting views and evidence. Our purpose is also not to document yet another anomaly devoid of theory. Rather our interest is in understanding the conditional pricing implication in a rational framework, driven by the simple economic intuition that diversification should matter to investors, even in incomplete markets.

It is also important to contrast our results with Herskovic, Kelly, Lustig and Nieuwerburgh (2015). They observe a common component in idiosyncratic volatility (CIV) and find that shocks to the common component are priced relative to the three-factor model. Motivated by these findings, they build a model in which consumption risk has the same factor structure as idiosyncratic volatility, so that high CIV-beta stocks serve a hedging purpose and have lower expected returns. CIV betas are obtained from regressions of returns on CIV innovations, quite different from a covariance between idiosyncratic volatility and average idiosyncratic volatility. Moreover, our point of embarkation is also quite different – endogenous diversification in incomplete markets, rather than the observation that there is a common factor in idiosyncratic volatility.

The remainder of the paper is organized as follows. In section 2, we sketch the model. Section 3 describes our sample and basic measurement approach. Regression and portfolio results for the US are in section 4. Section 5 contains out-of-sample tests using both US and international data. Section 6 concludes.

2. Theoretical framework

A formal model that endogenizes investor's diversification decisions in Merton's (1987) model is contained in the appendix. In this section, we provide an abbreviated discussion of the model that captures the economic intuition, and provides the foundation for the empirical tests.

2.2 A simplified model

Merton's model is motivated by the recognition that standard asset pricing models based on frictionless markets and complete information may be inadequate to capture the complexity of financial markets. The setup of our model follows his but with two additional assumptions. As in Merton (1987), investors risk averse, have identical preferences, are price-takers, have the same initial wealth, are mean variance optimizers, and have conditional homogenous beliefs. Investors are less than fully diversified as they only invest in a security if they "know" about that security.[2]

Our first point of departure is that we explicitly model costly information acquisition. An investor incurs a fixed cost I to learn about the security. Our second point of departure is our assumption that the fraction of all investors who know about a security is proportional to the market portfolio invested in that security. These plausible assumptions lead us to an asset pricing model with testable implications. The equilibrium expected return on security i in this model is:[3]

$$\overline{R_i} = R_f + b^2\delta + \frac{\delta}{\overline{Q^*}}\sigma_i^2 \quad (1)$$

where δ is the coefficient of risk aversion, R_f is the risk free rate, σ_i^2 is the idiosyncratic volatility of security i, $\overline{Q^*}$ is the average number of stocks held by an investor in equilibrium and $\overline{b}$ is the average beta of the investors' portfolio. As in Merton (1987), there is a positive premium for idiosyncratic volatility. The key deviation from Merton (1987), or for that matter, the intuition in Levy (1978)), is that the parameter $\overline{Q^*}$, representing portfolio diversification, is determined endogenously in equilibrium as follows:

$$\overline{Q^*} = \sqrt{\frac{\delta\overline{\sigma_i^2}}{2I}} \quad (2)$$

Notice that $\overline{Q^*}$ is determined by risk aversion, average idiosyncratic volatility $(\overline{\sigma_i^2})$ and the cost of information acquisition (I), which accords with our intuition. Combining equations (1) and (2), we can write the idiosyncratic risk premium (γ_{IVt}) in equation (1) as

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[2] In the sense that they know the mean and variance of its return distribution.

[3] See Appendix A for a detailed derivation.

$$\gamma_{IV} = \frac{\delta}{\overline{Q^*}} = \sqrt{\frac{2I\delta}{\overline{\sigma_i^2}}} \tag{3}$$

Equation (3) highlights several aspects of the model. With perfect information ($I = 0$), investors are fully diversified and the idiosyncratic risk premium disappears. Although the model regards I as the cost of information, to the extent that it influences $\overline{Q^*}$, one might also think of it as the cost of diversification. The cost of diversification has fallen over time, particularly with the advent of passive index funds and ETFs. This could potentially influence the premium for idiosyncratic risk, an issue we discuss later in the paper.

Equation (3) also highlights the role of average idiosyncratic volatility ($\overline{\sigma_i^2}$) in portfolio diversification. In this model, changes in average idiosyncratic volatility influence the disutility of under-diversification and therefore the idiosyncratic risk premium.[4] In the cross-section, when average idiosyncratic volatility is low, the marginal disutility of stocks with high idiosyncratic volatility is high relative to the average, generating a high premium. When average idiosyncratic volatility is high, there is still disutility associated with stocks that have relatively high idiosyncratic volatility, but the disutility is lower because of endogenous diversification. As a result, the premium for bearing idiosyncratic risk is still positive, but lower. The upshot is that the model delivers a state-dependent idiosyncratic risk premium. Because of this, in the spirit of Jagannathan and Wang (1996), we examine the empirical content of the model by taking unconditional expectations of a conventional factor model that subsumes equation (1). Consider the following factor model:

$$R_{it} = \gamma_{0t} + \sum_{f=1}^{F} \gamma_{ft} X_{fit} + \gamma_{IVt}\sigma_{IVit}^2 \tag{4}$$

where the factors f can arise from any other theoretical or empirically motivated factor model, and the idiosyncratic risk premium γ_{IVt} is allowed to be time varying. The

[4] Bekaert, Hodrick and Zhang (2012), Brown and Kapadia (2007) and others offer explanations for the source of time series variation in average idiosyncratic volatility, but for our purpose, it is exogenous and outside the model.

advantage of working with equation (4) is that it is very general, and empirically, allows us include any set of desired factors in the tests. Taking unconditional expectations of equation 4 introduces the following covariance term into the expected return relation.[5]

$$Cov(\sigma^2_{IVit}, \gamma_{IV}) = Cov(\sigma^2_{IVit}, \sqrt{\frac{2I\delta}{\sigma^2_t}}) \qquad (5)$$

Assuming constant (i.e. non-time-varying) risk aversion and cost of information acquisition, we can interpret this covariance as reflecting the sensitivity to the idiosyncratic risk premium (IRPS) and measure it as follows.

$$IRPS_{it} = Cov(\sigma^2_{IVit}, \sqrt{\frac{1}{\sigma^2_t}}) \qquad (6)$$

Our empirical tests, therefore, investigate whether this sensitivity to the idiosyncratic risk premium explains the cross-section of returns both unconditionally and conditionally.

3. Sample construction and measurement

3.1 US sample

Our sample of US stocks is derived from the CRSP-Compustat universe with CRSP share codes 10 or 11, and with exchange codes 1, 2 and 3. We eliminate stocks with a share price below $1 at the beginning of the month. Most tests are based on a sample period from July 1973 to 2014 because we need 5 years of data to calculate IRPS. A subset of tests go back to 1931, with the same sampling procedures. Daily MKT, SMB and HML factors starting from 1926 are obtained from Ken French's website. Monthly MKT, SMB and HML factors are also available from 1926 but profitability (RMW) and investment (CMA) factors start in 1964. The Hou, Xue and Zhang (2014) investment and ROE factors start in 1973.

3.2 International sample

We obtain a time series of market and accounting information from Datastream. We start with an unconstrained universe of all firms in the following developed markets

[5] This covariance is analogous to the covariance between the conditional market risk premium and conditional beta in equation (4) of Jagannathan and Wang (1996).

between 1990 and 2014: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Hong Kong, Italy, Japan, Netherlands, New Zealand, Norway, Portugal, Singapore, Spain, Switzerland and the United Kingdom. The universe includes live as well as dead stocks. We apply the sequence of filters described in Goyal and Wahal (2015), requiring that stocks have data from both Datastream and Worldscope, retaining only equity issues from the primary exchange of the country, and ensuring that we only sample local (not cross-listed) stocks. US dollar returns are computed by converting local currency returns using the conversion function built into Datastream which uses spot rates. Market values are similarly converted to US dollar equivalents.

Our tests require monthly factors for the four regions (North America, Europe, Japan, and Asia Pacific ex Japan), as well as for the global ex US markets. Ken French's website provides monthly MKT, SMB and HML for these regions. We build RMW and CMA from our data following the procedures in Fama and French (2015c). To verify that our processes match theirs, we also build MKT, SMB and HML from our data. The average premiums are very similar to theirs, and the correlations between our factors and theirs are over 95 percent.[6]

3.3 Measuring IRPS

For each security-month, we estimate daily time series regressions of excess stock returns on MKT, SMB and HML. We calculate idiosyncratic volatility as the mean squared error of the residuals from these regression. Regressions are only estimated for stocks with at least 15 valid daily returns. To compute average idiosyncratic volatility for each month, we calculate value-weighted average idiosyncratic volatility for small and large stocks separately. We use the median market capitalization of NYSE stocks in June to separate small and large stocks. We then take a simple average of the idiosyncratic volatility of small and large stocks. This procedure, akin to Fama and French's (1993) construction of HML and other factors, avoids the trap of equal-weighted average idiosyncratic volatility being dominated by more numerous small firms with high

[6] Our data do not include Greece, Ireland and Sweden so we do not expect correlations to be perfect. However, the number of securities and aggregate market capitalization of these exclusions is quite small and does not significantly influence the factors.

volatilities, and that of value-weighted average idiosyncratic volatility being dominated by large (and less volatile) firms.[7]

As prescribed by equation (6), for each stock and month t, we compute IRPS as the covariance of stock-level idiosyncratic volatility with one over the square root of average idiosyncratic volatility over the prior 60 months (t-61 to t-1).

4. Results

4.1 Average idiosyncratic volatility and properties of IRPS

Figure 2 shows average monthly idiosyncratic volatility. We define high and low average idiosyncratic risk months as those for which the average idiosyncratic volatility is above or below the trailing 10-year mean. The series starts in July 1973 to enable the calculation of the trailing mean from July 1962. The identification of the low and high average idiosyncratic risk periods is very similar if we use the full sample (52-year) mean. We do not report results from the latter approach because it is subject to a look-ahead bias.

The graph shows considerable time-series variation in average idiosyncratic volatility. This is critical to our tests since average idiosyncratic volatility is the key state variable in the model. Between 1973 and 1997, the increase in average idiosyncratic volatility closely matches that seen in Campbell et al. (2001), despite the fact that our weighting scheme (value-weighted within small and large stocks) is different from theirs (value-weighted across all securities, holding industry betas to unity). After 1997, average idiosyncratic volatility shows a downward trend, except for spikes in the internet boom-and-bust period, and the financial crisis of 2008. With the benefit of an additional 17 years of data, it does not appear that individual stocks have become more volatile.

As in Campbell et al. (2001), the spikes in average idiosyncratic volatility are tied to periods of economic stress, particularly NBER recessions. High and low average

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[7] Herskovic et al. (2015) calculate average idiosyncratic volatility as the simple equal-weighted average of residuals. Our procedure is more conservative because we do not want results to be driven by numerous small stocks which represent less than 10 percent of the aggregate market capitalization. The results presented in the body of the paper are stronger across the board if we use equal-weights to calculate average idiosyncratic volatility and IRPS.

idiosyncratic risk periods are not concentrated in calendar time decades. The consequences for our empirical tests are manifold. Our model assumes that the cost of information (I), which one can also interpret as the cost of diversification, is constant over time. Empirically, the cost of diversification has likely declined over time, particularly with the advent of mutual funds and more recently, exchange traded funds. If average idiosyncratic risk contained a calendar time pattern, then IRPS could be conflated with systematic changes in the cost of diversification. The fact that there is no trend in average idiosyncratic volatility allows the tests to focus on the time-varying disutility of under-diversification, disconnecting IRPS from any drift in I. Second, roughly 45 percent of the months in our sample are classified as high average idiosyncratic risk months. The econometric benefit of this separation is that regression and portfolio tests in low and high idiosyncratic volatility periods do not have radically different levels of statistical power.

To provide some additional perspective, we calculate the cross-sectional average IRPS for each month. The time series mean of these cross-sectional averages over this sample period is -0.003, implying a positive correlation between stock-level idiosyncratic volatility and average idiosyncratic volatility. Although we do not present these results in a table, we observe considerable time series variation in these cross-sectional averages, with a sudden drop in average IRPS during the financial crisis as the common component of security returns increased.[8] The cross-sectional standard deviation of IRPS is relatively stable through most of the time series but perhaps unsurprisingly, also increases sharply during the financial crisis. By the end of 2013, however, it returns to historical norms.

4.2 Fama-MacBeth regressions

Table 1 contains Fama and MacBeth (1973) regressions of monthly returns on prior month's IRPS. As is standard, we control for market capitalization (ln(ME)), book-to-market ratios (ln(B/M)), momentum, measured as the return from two to 12 months prior ($R_{2,12}$), and the prior 1 month return ($R_{0,1}$). In a second specification, we also control for profitability (GP/A) defined as in Novy-Marx (2013), and investment defined as

[8] This is also visible from average R^2 of daily market model regressions. Such R^2 increase dramatically during the financial crisis (2008-2010) relative to prior years.

changes in assets (Fama and French (2015a). All independent variables are winsorized at the 1 and 99 percent level. Standard errors are based on Newey-West procedures using 8 lags.

The first two columns show results based on the full sample period from July 1973 to December 2014. Slopes and t-statistics on the control variables are comparable to those reported by many other authors and we do not dwell on them further. In the first regression, IRPS has a positive coefficient (0.64) with a modest t-statistic of 1.95. Novy-Marx (2014) shows that the underperformance of high volatility strategies is largely attributable to their tilts towards small growth, and particularly unprofitable stocks. Adding profitability and investment to the regression, the slope on IRPS is largely unaffected, dropping to 0.63 with a t-statistic of 1.91. If we extend the sample period back to July 1968 (the first month for which we can calculate IRPS), the slope on IRPS in the first specification rises to 0.76 (t-statistic = 2.09). Again, adding profitability and investment leaves the slope largely unchanged at 0.75 with a t-statistic of 2.01.

The model says that the coefficients on IRPS should be larger in low average idiosyncratic volatility months. The fourth and fifth columns of table 1 show regressions for such periods. The slope on IRPS rises to 1.06 for the first specification, and to 1.03 when including profitability and investment. The t-statistics on IRPS for the low average idiosyncratic volatility periods rise to 2.19 and 2.16 respectively. In contrast, in high average idiosyncratic risk periods, the slopes shrink to 0.16 and 0.15, and are statistically indistinguishable from zero (t-statistics are 0.33 and 0.32 respectively).

Table 2 shows the same sets of regressions separately for large and small firms. In the full sample period, the slope of IRPS for large stocks varies from 0.73 to 0.78 but with large standard errors. In the low average idiosyncratic volatility periods, however, the slopes are almost five times larger, varying from 3.28 to 3.53 with t-statistics of 1.99 and 2.04 respectively. In high average idiosyncratic volatility period, the slope coefficients are negative (-2.39 and -2.23) with t-statistics well below 2.00.

The behavior of small stocks differs from that of large stocks. Over the full sample period, the slopes on IRPS are positive and reliably different from zero. For instance, when including profitability and investment in the equation, the slope on IRPS is 1.04

with a t-statistic of 2.41. In the low average idiosyncratic volatility periods, the slope rises to 1.50 with a t-statistic of 2.56. In the high average idiosyncratic volatility periods, the slope is halved to 0.50 with a t-statistic of only 0.75.

Overall, the regressions contain some evidence that the covariance between stock-level idiosyncratic volatility and average idiosyncratic volatility helps explain the cross-section of returns. This is true for both large and small stocks. Much of the explanatory power of IRPS comes in low average idiosyncratic volatility periods, precisely as prescribed by the model.

4.3 Portfolio tests

4.3.1 Portfolio formation and characteristics

Fama and MacBeth (1973) regressions have a natural interpretation as zero investment portfolios but they are equal-weighted. Despite the fact that we eliminate stocks under $1, they can still be sensitive to small stocks. Since idiosyncratic volatility is substantially higher in small stocks, they could have a disproportionate influence on the economic magnitude of the effects we are interested it.

In this section, we build portfolios based on IRPS and estimate time series factor models. Univariate sorts on IRPS are disproportionately influenced by small stocks. Therefore, we sort all firms into two size portfolios (large and small) at the end of each June using NYSE breakpoints, and then within each size portfolio, into IRPS quintiles each month. (Updating size breaks monthly does not influence our results). We elect to use 2x5 sorts rather than 5x5 sorts to ensure that the portfolios are well diversified, especially early in the time series. All portfolios returns are value-weighted based on prior month market capitalization.

Panel A of table 3 shows the number of stocks in each portfolio and the percentage of the aggregate market capitalization. On average, IRPS quintiles in large stocks contain 156 securities and in small stocks contain 437 securities. Unsurprisingly, particularly given our data filters, large cap stocks account for over 90 percent of the aggregate market cap. Therefore, any evidence on the association of IRPS with a premium is only credible if we observe it in large stocks.

For each portfolio-month, we calculate the percentage of stocks in each book-to-market, profitability and investment group, and calculate time series averages. The breakpoints used for assigning stocks into groups are from Ken French's website. Panel B shows these averages. In large stocks, moving across increasing IRPS quintiles, there is a slight tilt away from growth stocks. For instance, 43.6 percent of the stocks in the low IRPS quintile are classified as growth, but only 37.5 percent of the stocks in the high IRPS quintile are growth. Similarly, in small stocks, the percentage of growth stocks goes from 28.3 in the low IRPS quintile to 23.3 in the high IRPS quintile. We do not consider any of these tilts to be particularly significant. Consider, for example, the low- versus high-IRPS portfolios within small stocks. The difference in the percentage of stocks that are small-growth between these portfolios is only 5 percent, not enough for the severe underperformance of small-growth to drive differences in IRPS portfolio returns (Fama and French (1993)). In addition, the decrease in growth stocks does not imply a tilt towards value. Rather the decline is taken up by a larger concentration of neutral stocks.

The tilts in the distribution of profitability are bigger. In large stocks, the percentage of stocks with weak profitability drops from 33.4 in the low IRPS quintile to 21.4 in the high profitability quintile. This decline is not taken up by an increasing fraction of robust profitability firms. Again, it is the percentage of firms in the neutral profitability tercile that rises across IRPS quintiles. In small stocks, the pattern is largely similar, but the differences in fraction of firms that have weak profitability are even larger. This basic pattern presages a key result: as in Novy-Marx (2014), some of the underperformance of the low IRPS quintiles is likely explained by the ability of profitability to explain expected returns.

Differences in the distribution of firms with aggressive, neutral and conservative investment fall somewhere between those for book-to-market and profitability. Moving from low to high IRPS quintiles, there is a small decline in the fraction of stocks that are classified as conservative. But as with book-to-market, differences in investment groups are not large enough to influence differences in IRPS portfolio returns.

4.3.2 Factor model tests

Table 4 shows three-factor models for size and IRPS sorted portfolios. Portfolio characteristics in table 3 do not necessarily equate to factor loadings, so we present intercepts as well as slopes. Panel A contains results for the full sample period, while Panels B and C are for low and high average idiosyncratic risk periods respectively.

In the full sample period, for large stocks, the intercept increases from -0.20 (t-statistic = 2.05) in the low IRPS quintile to 0.08 (*t*-statistic=1.35) in the high IRPS quintile. The high-minus-low spread portfolio has an intercept of 0.28 percent per month, with a *t*-statistic of 2.01. The slopes on HML are not large. The coefficient on SMB decreases from quintile 1 to quintile 5, reflecting the distribution of market capitalization in table 3. In small stocks, the difference in intercepts across IRPS quintiles are bigger. The high-minus-low IRPS portfolio has an intercept of 0.37 percent (*t*-statistic = 2.24). Again, the slope on SMB declines across IRPS quintiles but there is very little variation in HML.

The variation in intercepts between low and high average idiosyncratic risk periods is extremely large. In low average idiosyncratic risk periods, the high-minus-low IRPS portfolio for large stocks has an intercept of 0.69 percent with a *t*-statistic of 3.86. The spread is generated both in the short (-0.44 percent, *t*-statistic = 3.18)) and long leg of the portfolio (0.25 percent, *t*-statistic = 4.13). The intercepts are monotonically increasing, implying that the sorting variable (IRPS) has some power and that the effect is not just driven by extremes. In high average idiosyncratic volatility periods, the high-minus-low IRPS spread disappears (-0.17 percent with a *t*-statistic of 0.66). Similarly, in small stocks, the high-minus-low IRPS portfolio has an intercept of 0.80 percent (*t*-statistic = 4.03) in low average idiosyncratic risk periods.[9] Again, the intercepts increase monotonically across IRPS quintiles. In high average idiosyncratic volatility periods, however, this spread is 0.06 percent with a *t*-statistic of only 0.20.

The three factor model regressions echo the Fama and MacBeth (1973) regressions in the sense that the ability of IRPS to explain the cross-section of returns is substantially

[9] Most of this is due to the underperformance of the low IRPS quintile (-0.67 percent per month). The loading on HML is positive, indicating that this underperformance is not due to the inability of the three factor model to price small growth stocks.

different in low versus high average idiosyncratic volatility periods. Novy-Marx (2014) shows that the performance of low volatility strategies (or for that matter, low beta strategies) arises largely from tilts to small, unprofitable and growth firms. The portfolio characteristics in table 3 and the slopes in table 4 suggest that tilts towards small-growth cannot explain the return dispersion in IRPS portfolios. Tilts in profitability are another matter, however, because low IRPS portfolios do contain more unprofitable firms than high IRPS portfolios. Table 5 contains Fama-French (2015) five factor model regressions which add the profitability (RMW) and investment (CMA) to the existing three factor model. Table 6 contains similar regressions based on the Hou, Xue and Zhang (2014) factors which, in addition to the market and size factors, add an investment (I/A) and ROE factor.

In the full sample period, the influence of profitability or ROE is immediately apparent. In large stocks, for the low IRPS portfolio, the loadings on RMW and ROE are -0.37 and -0.27 respectively, with t-statistics of 5.83 and 5.42. These loadings rise across IRPS quintiles for both RMW and ROE. Profitability/ROE soaks up much of the variation in returns so that the low IRPS, high IRPS and high-minus-low IRPS portfolios all have intercepts that are less than 0.10 percent per month and statistically indistinguishable from zero. In small stocks, the story is largely the same – the variation in loadings on RMW and ROE across IRPS quintiles are systematic, driving the intercepts to zero. This is good news for these factor models on a number of fronts. Because the test assets (IRPS sorts) are independent of the factors, they constitute a clean examination of the ability of these models to span portfolio returns. On that basis, these factor models represent a clear improvement over the three-factor model.

In low average idiosyncratic risk periods (Panel B in tables 5 and 6), however, the picture becomes more complicated. The loadings on RMW and ROE still increase across all IRPS quintiles in both large and small stocks. But these loadings are not enough to drive the intercepts to zero. In large stocks, using the Fama-French five factor model, the low IRPS quintile has an intercept of -0.18 (t-statistic = 1.51) and the high IRPS quintile has an intercept of 0.21 (with a t-statistic of 3.14). Therefore, the spread in the high-minus-low IRPS portfolio is 0.39 percent (t-statistic = 2.44). If we use the Hou, Xue and Zhang (2014) factors, the high-minus-low IRPS portfolio has an identical spread (0.39 percent with a t-statistic of 2.26). In small stocks, the low IRPS quintile severely underperforms the five factor model (despite a small loading on HML and a large negative loading on RMW) to the tune of -0.45 percent (t-statistic = 3.57). This underperformance drives the high-minus-low IRPS spread portfolio's returns of 0.51 percent per month (t-statistic = 2.69). And again, the intercepts using the Hou, Xue and Zhang factors are very similar.

Factor specification does impact inferences in high average idiosyncratic volatility periods (Panel C in tables 5 and 6). Using the Hou, Xue and Zhang factors, we observe a negative spread generated by IRPS in both small and large stocks: the high-minus-low IRPS portfolio has an intercept of -0.59 percent ($t = 2.47$) in large stocks and -1.01 percent ($t = 3.21$) in small stocks. But we do not observe such premiums using the Fama-French five factor model where the t-statistics are well below 2.00. Even using the three-factor model (table 4), the intercepts in high average idiosyncratic volatility periods are not reliably different from zero.

4.3.3 Relation with idiosyncratic volatility

A natural concern is that IRPS is conflated with idiosyncratic volatility – in other words, that sorts on IRPS are essentially sorts on idiosyncratic volatility. As an empirical matter, we have two reasons to believe that this is not the case. First, for each security, we calculate the time series correlation between idiosyncratic volatility and IRPS. The average correlation across the entire cross-section is -0.09. These are average time series correlations; there could still be cross-sectional correlations. Therefore, we also build portfolios conditionally sorted on size, idiosyncratic volatility and IRPS. As before, we use two size groups (small and large). To ensure that portfolios remain well diversified we use terciles for both idiosyncratic volatility and IRPS (i.e. 2x3x3 dependent sorts). For each portfolio, we estimate three- and five-factor models. The results are in table 7. Each row in the table corresponds to low, medium and high idiosyncratic volatility terciles. The columns show the low, medium and high IRPS terciles, as well as the high-minus-low IRPS portfolio. Each cell contains the intercept from the appropriate factor model. Panels A and B show results for low and high average idiosyncratic risk periods respectively.

The core results in Ang et al. (2006) are easily visible. In Panel A, for instance, high idiosyncratic volatility stocks (High IVOL) generally have large negative intercepts relative to the three-factor model. This is true for both small and large capitalization stocks. When evaluated relative to the five factor model, however, this is only the case for small stocks.

Our interest is in whether there is a spread in portfolio returns across IRPS terciles, holding idiosyncratic volatility constant. In Panel A, the intercepts from three- and five factor models generally increase across IRPS quintiles for low, medium and high idiosyncratic volatility quintiles. In large stocks, for instance, the spread between high and low IRPS quintiles for the low IVOL group is 0.26 percent (t-statistic = 2.13) using a three-factor model, and 0.27 percent per month (t-statistic = 2.05) using a five-factor model. In medium idiosyncratic volatility terciles, the spreads rise to 0.64 and 0.51 percent per month, with still larger t-statistics. In high idiosyncratic volatility stocks, the spreads are 0.61 and 0.32 percent per month and remain statistically significant. In small stocks, where idiosyncratic volatility should play a greater role, the spreads in IRPS portfolio returns are smaller. However, with the exception of the low idiosyncratic volatility group, they are still positive. Finally, as in our earlier tests, the importance of IRPS is only evident in low average idiosyncratic volatility periods. In high average idiosyncratic volatility periods, there is no difference in intercepts.

4.3.4 Persistence in IRPS and the term structure of returns

IRPS is highly persistent. The average first-order autocorrelation in our sample is 0.85.[10] Some of that persistence is mechanical because it is calculated based on rolling 60-month covariances. High persistence implies that even monthly rebalanced portfolios are likely to have low turnover relative to strategies like momentum. In the 1973-2014 sample period, in large stocks, the monthly turnover rates for the low and high IRPS quintiles

[10] In contrast, idiosyncratic volatility is not very persistent. Fu (2009) reports that the average first order autocorrelation in his sample is 0.33. In our sample, the equivalent average is 0.23.

are 7.1 percent and 6.3 percent respectively, and average transition probabilities (computed as the percentage of stocks that remain in the portfolio over adjacent months) are 91.1 and 92.1 percent per month. In small stocks, the equivalent monthly turnover rates for low and high IRPS quintiles are 8.8 percent and 9.6 percent respectively (with equivalent average transition probabilities of 92.3 and 91.7 percent per month).

This persistence has two implications. First, quarterly or semi-annual rebalancing rules should not significantly degrade the return series. Second, the term structure of portfolio returns should be such that intercepts of spread portfolios should be positive even after the first month. To examine this, we use monthly rebalanced portfolios used in earlier tests, but estimate factor models using the n^{th} month after portfolio formation (up to the 6^{th} month). Figure 3 shows the term structure of portfolio returns using three- and five-factor models for the high-minus-low IRPS portfolios. In all four graphs, we fix the scale for the y-axis so as to allow visual comparisons. The top two graphs show results for the full sample period (1973-2014). Using the 3-factor model, intercepts remain high for at least 3 months after portfolio formation for both large and small stocks. Mirroring the results in table 5, intercepts from 5-factor models are indistinguishable from zero. But as before, the results for low average idiosyncratic volatility periods are quite different (the bottom two graphs). In these periods, intercepts from 3-factor models remain high several months after portfolio formation. Using 5-factor models, intercepts are high for two months after portfolio formation, decline somewhat in the third month, and rise again in months four and five. The term structure suggests that the returns of the IRPS portfolios do not simply disappear after the first month.

4.3.4 Robustness and loose ends

A key component to measuring IRPS is the measure of average idiosyncratic volatility that we use. Instead of taking a simple average of value-weighted idiosyncratic volatility in small and large stocks, one could equal-weight all stocks (as in Herskovic et al. (2015)). Our approach is more conservative: if we use equal-weights, the majority of intercepts in the factors models in tables 4-6 are larger, and the slopes on IRPS in the Fama and MacBeth (1973) regressions are larger (with lower standard errors).

IRPS is calculated using a 60-month rolling window. Volatility is known to be persistent, and its predictability is higher over shorter intervals. Therefore, using a shorter window to calculate IRPS should tighten the link between stock-level idiosyncratic volatility and average (persistent) idiosyncratic volatility. The "cost" is that reducing the number of observations in the rolling window reduces precision. We re-calculate IRPS using 36-month rolling windows and re-estimate our mains tests. Intercepts from factor models inevitably bounce around but the inferences remain the same.

Our main tests are based on quintile or tercile sorts to ensure that inferences are not driven by extremes. The results thus far show some monotonicity in intercepts across quintiles/terciles suggesting that extremes do not overtly influence our results. But since extremes are also interesting, we also separate stocks into positive and negative IRPS groups. Between 1973 and 2014, about 11 percent of all stocks (representing about 5 percent of aggregate market capitalization) have positive values of IRPS (i.e. a negative covariance between idiosyncratic volatility and average idiosyncratic volatility). Variation in the fraction of stocks with positive IRPS over time is quite large, reaching a high over 40 percent of stocks and 20 percent of aggregate market capitalization in June 1998. We build positive and negative IRPS portfolios across all stocks, and separately for large and small stocks. Across all stocks, the intercept from a five factor model for the positive IRPS portfolio in the 1973-2014 sample period is 0.44 percent per month (t-statistic = 2.31). The intercept of the positive IRPS minus negative IRPS portfolio is 0.47 percent per month (t-statistic = 2.42). The results in large stocks are similar: the intercept for the positive IRPS portfolio is 0.41 percent per month (t-statistic = 2.20), and the intercept for the positive IRPS minus negative IRPS portfolio is 0.45 percent per month (t-statistic = 2.28).

Finally, as we outline in the introduction, we do not see any obvious reason why our results are a reinvention of those in Herskovic, Kelly, Lustig and Nieuwerburgh (2015); their CIV betas are constructed from regressions of returns on changes in the common factor in idiosyncratic volatility. Nonetheless, we build high and low CIV beta portfolios following their methods for small and large stocks. We then compute the correlations of the high-minus-low CIV-beta portfolios with high-minus-low IRPS portfolios. The correlations are low: 0.21 and 0.24 for small and large stocks respectively. Moreover, triple sorts on size, CIV-betas and IRPS continue to generate spreads in intercepts with three-factor models.

5. Out-of-sample evidence

We conduct two sets of out-of-sample tests. The first extends the US sample back to 1931, the so-called "pre-Compustat" era. The second examines the evidence outside the US.

5.1 Early US evidence (1931-1973)

We compute idiosyncratic volatility from 1926 using the same procedures. Since we require 60 months of data to calculate covariances, IRPS estimates are available from 1931 onwards. As before, we build size and IRPS sorted portfolios. Although a sample period through 2014 would add power, we restrict our attention to 1931-1973 so that the analysis does not overlap with the results in tables 4-7. Profitability/ROE and investment factors are not available for this sample period. Therefore, we report loadings and intercepts from three factor models in table 8.

Panel A contains results for the 1931-1973 period. In both large and small stocks, intercepts increase systematically from low to high IRPS portfolios. In large stocks, the low IRPS portfolio underperforms the three factor model by 0.24 percent per month (t-statistic = 2.55) and the high IRPS portfolio outperforms by 0.10 percent per month (t-statistic = 2.30). The high-minus-low IRPS portfolio earns 0.35 percent per month in large stocks (t-statistic = 2.96) and 0.41 percent per month in small stocks (t-statistic = 3.09).

As in the later sample period, these differences are largely driven by low average idiosyncratic volatility periods. Panel B shows that in low average idiosyncratic volatility periods, the high-minus-low IRPS portfolios earn 0.34 percent per month in large stocks and 0.58 percent per month in small stocks. In high average idiosyncratic volatility periods (Panel C), the equivalent portfolio returns drop to 0.21 and 0.22 percent per month respectively and are statistically indistinguishable from zero.

5.2 International evidence

We examine the role of IRPS in the four regions studied by Fama and French (2012, 2015c), as well as global markets not including the US. The North America region includes Canada and US. Europe includes Austria, Belgium, Denmark, Finland, France, Germany, Italy, Netherlands, Norway, Portugal, Spain, Switzerland and the United Kingdom. Japan is examined separately, so that Asia Pacific excludes Japan, but includes Australia, Hong Kong, New Zealand and Singapore. Global includes all the above countries, except the US.

5.2.1 Modifications to estimation procedures

We make a number of modifications to the empirical procedures to accommodate the international data. We do not have daily SMB and HML factors for these regions for estimating idiosyncratic volatility. Therefore, we estimate simple market models but include four lags of the market return to account for non-synchronous trading. For each stock-month, we require a minimum of 15 valid daily returns. To calculate average idiosyncratic volatility, we need size breakpoints. We follow Fama and French (2012) and use the 90th percentile to separate large and small stocks in each region. As with the data US data, we calculate value-weighted average idiosyncratic volatility for large and small stocks, and then an equal-weighted average of the two. IRPS is calculated as before except that we require a minimum of 40 valid observations.

We use 2x3 (size x IRPS) sorts to construct portfolios rather than 2x5 sorts to ensure that portfolios remain well diversified. The sample period is 1990-2014. Because the time series is relatively short, we separate low versus high average idiosyncratic volatility periods by comparing average idiosyncratic volatility in each month to the full sample average rather than a rolling average.

5.2.2 International results

Table 9 contains intercepts from three factor models for size and IRPS portfolios in each region. In North America, Europe and Asia Pacific, intercepts for IRPS terciles increase monotonically in both large and small stocks. The resulting spreads in intercepts are large. For instance, in large stocks between 1990 and 2014, the high-minus-low portfolios have intercepts of 0.37, 0.36 and 0.64 percent per month, all more than two standard errors from zero. In low average idiosyncratic volatility periods, these intercepts are even bigger, and have larger t-statistics. The outlier seems to be Japan where IPRS portfolios generate no variation in returns. This is especially true in large stocks.[11] We are not the first to detect that the cross-section of returns in Japan does not behave in the same way as in other countries. Momentum appears to exist in most developed markets except Japan, and Fama and French (2015c) show that returns do not vary with profitability and investment.

The global portfolios in Panel E are the most diversified ex-US portfolios. In large stocks, each IRPS tercile contains over 270 securities and about 30 percent of the aggregate market capitalization. In small stocks, each tercile contains over 2,400 stocks. In large stocks, the intercept for the high-minus-low IRPS in the full sample period is 0.62 percent per month with a t-statistic of 3.44. As in our earlier tests, in high average idiosyncratic volatility periods, most of the intercepts are statistically indistinguishable from zero.

Table 10 contain five-factor model intercepts for the same regions and portfolios. For North America, Europe, Japan and Asia Pacific, intercepts for high-minus-low IRPS portfolios are about half the magnitude of their three-factor counterparts. In low average idiosyncratic volatility periods, the shrinkage in intercepts is noticeable smaller. And in these periods, even with five factor models, the intercepts of high-minus-low IRPS portfolios remain statistically significant (again, except Japan). In the global (ex US) portfolios, the monotonicity in intercepts from low to high IRPS portfolios remains. In the full sample period, the high-minus-low IRPS portfolio has an intercept of 0.48 percent per month (t-statistic = 2.55) for large stocks and 0.26 percent per month (t-statistic = 1.28) for small stocks. In the low average idiosyncratic volatility periods, the intercepts

[11] There is a hint of a spread in returns in small stocks in Japan. The high-minus-low IRPS portfolio in low average idiosyncratic volatility periods has an intercept of 0.30 percent per month (t-statistic = 2.12), but since we do not observe this in other specifications, we do not consider the evidence to be strong.

rise to 0.92 percent per month for large stocks and 0.56 percent per month for small stocks (with t-statistic of 4.60 and 2.94 respectively).

Pooling stocks in global markets gives us the luxury of a large number of securities so that we can also perform triple sorts on size, idiosyncratic volatility and IRPS. These 2x3x3 sorts, similar to those in table 7 for the US, are shown in table 11. Panels A and B contain three factor model intercepts for large and small stocks respectively. Panels C and D, contain parallel intercepts from five factor models. In both large and small stocks, we observe a spread in IRPS portfolios, controlling for idiosyncratic volatility. The high-minus-low IRPS portfolios have very similar intercepts in low, medium and high idiosyncratic volatility terciles, ranging from a low of 0.27 to a high of 0.85 percent per month.

6. Conclusion

Understanding the nature of equilibrium in capital markets under the frictionless ideal is foundational to the development of asset pricing models. Levy (1978), Merton (1987), and others relax that frictionless ideal and investigate its implications for asset prices. We endogenize the diversification decision in Merton's (1987) model, allowing the investor to decide an appropriate level of diversification herself, while appreciating the costs and benefits of diversification. This adjustment delivers a state-dependent premium for idiosyncratic risk that reflects the marginal disutility to under-diversification.

We take this simple idea to the data. Because state-dependence is empirically measurable as the covariance between idiosyncratic volatility of a stock and average idiosyncratic volatility, it affords some power to the tests. In the US, we find that this covariance leaves a footprint in the cross-section of returns in 1973-2014. Some of this explanatory power is subsumed by extant factor models, but in low average idiosyncratic volatility periods (when the model predicts that this covariance should matter most), even the most complete factor models have intercepts that are reliably different from zero. We perform a battery of tests to ensure that our results are not driven by spurious relations, especially ruling out the possibility that idiosyncratic volatility itself is driving the results. The fact that we observe similar results in earlier time periods in the US (1931-1973), and in markets outside the US, offers an indication that the covariance of idiosyncratic volatility with average idiosyncratic volatility matters.

It is useful to return to the theme of endogenous diversification. The cost of diversification (I) has fallen over time with the advent of delegated portfolio management through mutual funds, ETFs and other investment vehicles. On the surface, this exogenous decline suggests that theory should be most applicable in earlier time periods or markets in which investors are restricted from using diversified funds. However, the risk premium depends on the cost/benefit of diversification as well as average idiosyncratic risk.[12] The latter is far from constant. One might also ask about the risk aversion parameter (δ) that we assume to be constant. It may very well be true that there is a positive correlation between risk aversion and average idiosyncratic risk. But empirically, to pin down one of the three primitive drivers (I, $\delta, \overline{\sigma_i^2}$), we are forced to assume that the other two remain constant. Identification without such an assumption is intriguing but remains a challenge.

[12] As we point out earlier in the paper, cross-sectional correlations across securities varies considerably in the time series, rising sharply during the financial crisis. Of course, this does not imply that diversification in not valuable during those times – quite the contrary – but that the benefits are time-varying.

Appendix A: Model Derivation

Our model closely follows Merton (1987). However, we make two additional assumptions. Our first point of departure is that we explicitly model costly information acquisition. An investor incurs a fixed cost I to learn about a security. Our second point of departure is our assumption that the fraction of all investors who know about a security is proportional to the market portfolio invested in that security. These plausible assumptions lead us to a model with testable implications (equation 1 in the paper). For the convenience of the reader and continuity of analysis we reproduce the first part of the derivation in Merton (1987)[13].

The economy has N firms, $N \gg 1$. The return from investing in firm i is:

$$\tilde{R}_i = \bar{R}_i + b_i \tilde{Y} + \sigma_i \tilde{\varepsilon}_i, \quad i = 1,\ldots,N \tag{A1}$$

where $\tilde{Y}$ is a common factor with $E\left(\tilde{Y}\right) = 0$, $E\left(\tilde{Y}^2\right) = 1$, b_i is the factor loading of security, $\tilde{\varepsilon}_i$ is a firm-specific random variable with

$$E\left(\tilde{\varepsilon}_i\right) = E\left(\tilde{\varepsilon}_i \mid \tilde{\varepsilon}_1,\ldots,\tilde{\varepsilon}_{i-1},\tilde{\varepsilon}_{i+1},\ldots,\tilde{\varepsilon}_N, Y\right) = 0, \quad i = 1,\ldots,N \tag{A2}$$

$E\left(\tilde{\varepsilon}_i^2\right) = 1$, σ_i^2 is the idiosyncratic volatility of security i, and $\overline{\sigma^2}$ is the value weighted average idiosyncratic volatility across the N securities. $\bar{R}_M$ denotes the value weighted expected return of the N securities.

In addition to the N securities issued by firms, the economy has two "inside" securities with zero net supply:

(a) a $\left(N+1\right)th$ security with return, $\tilde{R}_{N+1} = \bar{R}_{N+1} + \tilde{Y}$ and

(b) a riskless security with return R_f

The economy has K investors, $K \gg N$. Investors are risk averse, with identical mean-variance preferences:

$$U_k = E\left[\tilde{R}^k\right] - \frac{\delta}{2} Var\left[\tilde{R}^k\right], \quad k = 1,\ldots,K \tag{A3}$$

$\tilde{R}^k$ denotes the portfolio return, and δ is the coefficient of risk aversion. Investors are price takers and assumed to have identical initial wealth W_o.

[13] The reader familiar with Merton (1987) will note that equation (A22) below corresponds directly to equation (15) in his paper.

An investor only includes security i in his portfolio if he is "informed" in the sense that he knows $\left(\overline{R}_i, b_i, \sigma_i^2\right)$. Information is costly and can be acquired at a cost I per security. As a consequence investor k selects only a subset of the N available securities to include in his portfolio.[14] We assume that the securities he selects Q_k are much smaller than N ($Q_k \ll N$), and that the probability of selecting a firm is proportional to its value relative to the market portfolio. Θ_k is the set of integers that index the Q_k firms selected by investor k.[15]

In addition to firm-specific knowledge, each investor's information set contains common knowledge: $\left(R_f, \overline{R}_{N+1}, \overline{R}_M, \overline{\sigma^2}, I\right)$.

Equilibrium in capital markets is characterized as follows:

(a) Given the set of securities selected, each investor chooses an optimal portfolio.

(b) Markets clear.

(c) Investors have no incentive to increase their holdings Q_k.

The optimal portfolio holdings for any investor k is determined as follows:

From (A1) and (A3), an investor's portfolio return can be specified as:

$$\tilde{R}^k = \overline{R}^k + b^k \tilde{Y} + \sigma^k \tilde{\varepsilon}^k \tag{A4}$$

where:

$$b^k = \sum_{i \in \Theta_k} w_i^k b_i + w_{N+1}^k \tag{A5}$$

$$\left(\sigma^k\right)^2 = \sum_{i \in \Theta_k} \left(w_i^k\right)^2 \sigma_i^2 \tag{A6}$$

w_i^k and w_{N+1}^k denote the fraction of investor k's wealth allocated to security i and $N+1$. The expected portfolio return and variance are:

$$E\left(\tilde{R}^k\right) = R_f + b^k\left(\overline{R}_{N+1} - R_f\right) + \sum_{i \in \Theta_k} w_i^k \Delta_i \tag{A7}$$

$$Var\left(\tilde{R}^k\right) = \left(b^k\right)^2 + \sum_{i \in \Theta_k} \left(w_i^k\right)^2 \sigma_i^2 \tag{A8}$$

where:

$$\Delta_i = \left(\overline{R}_i - R_f\right) - b_i\left(\overline{R}_{N+1} - R_f\right), \; i \in \Theta_k \tag{A9}$$

[14] These subsets will in general differ across the K investors.

[15] They are a subset of the first N natural numbers.

The investor's optimal portfolio choice is the solution to the following problem:

$$\max_{\{b^k, w_i^k\}} \left[E\left(\tilde{R}^k\right) - \frac{\delta}{2} Var\left(\tilde{R}^k\right) \right], \quad i \in \Theta_k \tag{A10}$$

$$\text{Subject to } \sum_{i \in \Theta_k} w_i^k + w_{N+1}^k + w_f^k = 1$$

From (A7) (A8), the first-order conditions for (A10) are:

$$\bar{R}_{N+1} - R_f - b^k \delta = 0 \tag{A11}$$

$$\Delta_i - w_i^k \sigma_i^2 \delta = 0, \quad i \in \Theta_k \tag{A12}$$

From (A5) (A11) (A12), the investor's optimal portfolio solution is:

$$b^k = \frac{\left(\bar{R}_{N+1} - R_f\right)}{\delta} \tag{A13}$$

$$w_i^k = \frac{\Delta_i}{\sigma_i^2 \delta}, \quad i \in \Theta_k \tag{A14}$$

$$w_{N+1}^k = b^k - \sum_{i \in \Theta_k} w_i^k b_i \tag{A15}$$

$$w_f^k = 1 - b^k + \sum_{i \in \Theta_k} w_i^k \left(b_i - 1\right) \tag{A16}$$

We aggregate to determine equilibrium expected returns. From (A13), all investors choose the same b^k. Let $b^k = B,\ k = 1, \ldots, K$. Thus, from (A13), we have:

$$\bar{R}_{N+1} = R_f + B\delta \tag{A17}$$

From (A14), the aggregate demand for security i is:

$$D_i = \sum_{k=1}^{K_i} W_o w_i^k = \sum_{k=1}^{K_i} W_o \frac{\Delta_i}{\sigma_i^2 \delta} \tag{A18}$$

In the equation above, K_i is the number of investors who know about the firm i. From (A15) (A16), the aggregate demand for "inside" securities are:

$$D_{N+1} = \sum_{k=1}^{K} W_o w_{N+1}^k = \sum_{k=1}^{K} W_o B - \sum_{i=1}^{N} b_i D_i \tag{A19}$$

$$D_f = \sum_{k=1}^{K} W_o w_f^k = \sum_{k=1}^{K} W_o - \sum_{i=1}^{N+1} D_i \tag{A20}$$

Inside securities have zero demand in equilibrium: $D_{N+1} = D_f = 0$. Thus, from (A19) (A20), we have:

$$B = \frac{\sum_{i=1}^{N} b_i D_i}{\sum_{k=1}^{K} W_o} = \sum_{i=1}^{N} x_i b_i = \overline{b} \tag{A21}$$

where x_i is the fraction of investors' total wealth allocated to security i. Using (A21), we can rewrite (A17) as:

$$\overline{R}_{N+1} = R_f + \overline{b}\,\delta \tag{A22}$$

Let V_i denotes the equilibrium value of firm i, then

$$x_i = \frac{V_i}{\sum_{k=1}^{K} W_o} \tag{A23}$$

is the fraction of investors' total wealth invested in firm i. From the market clearing condition, $V_i = D_i$, and from the equation (A18), we have:

$$x_i = \frac{V_i}{\sum_{k=1}^{K} W_o} = \frac{D_i}{\sum_{k=1}^{K} W_o} = q_i \frac{\Delta_i}{\sigma_i^2 \delta} \tag{A24}$$

In equation (A24), which corresponds to equation 15 in Merton (1987),

$$q_i = \sum_{k=1}^{K_i} W_o \Big/ \sum_{k=1}^{K} W_o = K_i / K \tag{A25}$$

is the fraction of investors who invest in firm i.

The fraction of all investors who know about a security is proportional to the weight of the security in the market portfolio. That is, we assume that q_i is proportional to x_i

$$x_i = \phi q_i \tag{A26}$$

Using (A14), (A24) and (A26),

$$w_i^k = \frac{x_i}{q_i} = \phi \;, \quad i \in \Theta_k \tag{A27}$$

Since

$$\sum_{k=1}^{K} \left[\sum_{i \in \Theta_k} w_i^k + w_{N+1}^k + w_f^k \right] = K \tag{A28}$$

using (A27) we get

$$K = \sum_{k=1}^{K} \sum_{i \in \Theta_k} \phi + \sum_{k=1}^{K} (w_{N+1}^k + w_f^k) = \sum_{k=1}^{K} \phi Q_k = \phi \sum_{k=1}^{K} Q_k \tag{A29}$$

Here we have used the observation that the number of firms in Θ_k is Q_k and that the holdings of security $N+1$ and the risk-free asset sum to zero across all investors.

Hence

$$\phi = 1 / \frac{1}{K}\sum_{k=1}^{K} Q_k = 1 / \bar{Q} \tag{A30}$$

where $\bar{Q} = \frac{1}{K}\sum_{k=1}^{K} Q_k$ is the average number of securities in a portfolio.

From (A27) and (A30), we have:

$$w_i^k = \frac{1}{\bar{Q}} \tag{A31}$$

$$w_{N+1}^k = \bar{b} - \sum_{i\in\Theta_k} \frac{b_i}{\bar{Q}} \tag{A32}$$

$$w_f^k = 1 - \bar{b} + \sum_{i\in\Theta_k} \frac{b_i - 1}{\bar{Q}} \tag{A33}$$

As noted in (A31), w_i^k is the same for each investor in firm i, while w_{N+1}^k, w_f^k can be different across investors.

From (A7) (A22-26) (A30), expected security returns are linear in idiosyncratic volatility:

$$\bar{R}_i = R_f + b_i \bar{b}\delta + \frac{\sigma_i^2 \delta}{\bar{Q}}, \quad i = 1, \ldots, N \tag{A34}$$

From (A34), the expected market return $\bar{R}_M = \sum_{i=1}^{N} x_i \bar{R}_i$ is:

$$\bar{R}_M = R_f + \bar{b}^2\delta + \frac{\delta\overline{\sigma^2}}{\bar{Q}} \tag{A35}$$

where $\overline{\sigma^2} = \sum_{i=1}^{N} x_i \sigma_i^2$.

From (A7-9) (A31-35), the expected portfolio return and portfolio variance are:

$$E\left(\tilde{R}^k\right) = R_f + \bar{b}^2\delta + \frac{\delta}{\bar{Q}^2}\sum_{i\in\Theta_k} \sigma_i^2 \tag{A36}$$

$$Var\left(\tilde{R}^k\right) = \bar{b}^2 + \frac{1}{\bar{Q}^2}\sum_{i\in\Theta_k} \sigma_i^2 \tag{A37}$$

Thus, the utility of investor k is:

$$U_k = E\left(\tilde{R}^k\right) - \frac{\delta}{2} Var\left(\tilde{R}^k\right) = R_f + \frac{\overline{b}^2 \delta}{2} + \frac{\delta}{2\overline{Q}^2} \sum_{i \in \Theta_k} \sigma_i^2 \tag{A38}$$

Finally, given the expected returns in (A22) and (A34), we ensure that investor k has no incentive to acquire information about an additional security at cost I. We choose the information cost I so that the net-of-acquisition-cost I expected increase in the investor's marginal utility from knowing one additional firm is non-positive.

For an additional security a, where a is an element of $\{N\} \setminus \Theta_k$, the investor's new optimal portfolio choice is again the solution to the maximization problem:

$$\max_{\left\{b^k, w_i^k\right\}} \left[E\left(\tilde{R}^k\right) - \frac{\delta}{2} Var\left(\tilde{R}^k\right) \right], \quad i \in \Theta_k \cup \{a\} \tag{A39}$$

The above problem is similar to that of (A10). As in (A11-16), we apply the first-order conditions to equation (A39).

$$b^k = \frac{\left(\overline{R}_{N+1} - R_f\right)}{\delta} \tag{A40}$$

$$w_i^k = \frac{\Delta_i}{\sigma_i^2 \delta}, \quad i \in \Theta_k \cup \{a\} \tag{A41}$$

Because the expected returns of all securities are unchanged from (A22) (A34), thus, from (A22) (A40), b^k is unchanged:

$$b^k = \overline{b} \tag{A42}$$

From (A9) (A24-30), Δ_i is unchanged:

$$\Delta_i = \frac{\sigma_i^2 \delta}{\overline{Q}}, \quad i = 1, \ldots, N \tag{A43}$$

Then from (A41), we have:

$$w_i^k = \frac{1}{\overline{Q}}, \quad i \in \Theta_k \cup \{a\} \tag{A44}$$

From (A7-8) (A44), the expected portfolio return and variance *conditional* on selecting security a is:

$$E\left(\tilde{R}_k \mid a\right) = R_f + \overline{b}^2 \delta + \frac{\delta}{\overline{Q}^2} \left(\sum_{i \in \Theta_k} \sigma_i^2 + \sigma_a^2 \right) \tag{A45}$$

$$Var\left(\tilde{R}^k \mid a\right) = \overline{b}^2 + \frac{1}{\overline{Q}^2} \left(\sum_{i \in \Theta_k} \sigma_i^2 + \sigma_a^2 \right) \tag{A46}$$

Since the probability of selecting an additional security is proportional to its market capitalization, the expected idiosyncratic volatility of the additional security (for an uninformed investor) is the value weighted average idiosyncratic volatility across the firms he doesn't know:

$$E\left[\sigma_a^2\right] = \sum_{i \in \{N\} \setminus \Theta_k} x_i \sigma_i^2 \tag{A47}$$

where $\{N\}$ is the set of integers $1,\ldots,N$. Since $N \gg Q_k$, the investor only knows a fraction of all securities. Hence, from (A47), we have:

$$E\left[\sigma_a^2\right] \cong \sum_{i=1}^{N} x_i \sigma_i^2 = \overline{\sigma^2} \tag{A48}$$

Using (A48), we rewrite the *unconditional* expected portfolio return and variance in (A45) and (A46) as:

$$E\left(\tilde{R}^k\right) = R_f + \overline{b}^2 \delta + \frac{\delta}{\overline{Q}^2}\left(\sum_{i \in \Theta_k} \sigma_i^2 + \overline{\sigma^2}\right) \tag{A49}$$

$$Var\left(\tilde{R}^k\right) = \overline{b}^2 + \frac{1}{\overline{Q}^2}\left(\sum_{i \in \Theta_k} \sigma_i^2 + \overline{\sigma^2}\right) \tag{A50}$$

The expected utility of investor k is:

$$U_k^{'} = E\left(\tilde{R}^k\right) - \frac{\delta}{2} Var\left(\tilde{R}^k\right) = R_f + \frac{\overline{b}^2 \delta}{2} + \frac{\delta}{2\overline{Q}^2}\left(\sum_{i \in \Theta_k} \sigma_i^2 + \overline{\sigma^2}\right) \tag{A51}$$

Comparing (A38) with (A51), the expected increase in marginal utility is:

$$\Delta U_k = U_k^{'} - U_k = \frac{\delta}{2\overline{Q}^2} \overline{\sigma^2} \tag{A52}$$

As shown in (A52), ΔU_k is same for all investors. Investor k has no incentive to learn about more firms as long as ΔU_k is no greater than the disutility of the information cost *I*:[16]

$$\Delta U_k - I \leq 0 \tag{A53}$$

Therefore, from (A52) (A53), in equilibrium, we have:

$$\frac{\delta}{2\overline{Q}^{*2}} \overline{\sigma^2} = I \tag{A54}$$

[16] In this framework *U (I)* = a constant x *I*. We have normalized the constant to be 1 as it does not affect the subsequent analysis.

In the above equation, $\bar{Q}^*$ is the average number of stocks held by investor k in equilibrium.

From (A54), we have:

$$\bar{Q}^* = \sqrt{\frac{\delta\overline{\sigma^2}}{2I}} \tag{A55}$$

Since $\bar{Q}^*$ is proportional to average idiosyncratic risk $\sqrt{\overline{\sigma^2}}$, portfolio diversification is determined endogenously, and is proportional to average idiosyncratic risk .
From (A34) (A54), expected returns are given by:

$$\bar{R}_i = R_f + \bar{b}^2\delta + \frac{\sigma_i^2\delta}{\bar{Q}^*} \tag{A56}$$

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Table 1

Fama-MacBeth Regressions for All Stocks, 1973-2014

IRPS is calculated as the covariance between stock-level idiosyncratic volatility and one over the square root of average idiosyncratic volatility over the prior 60 months. The table contains slopes of Fama and MacBeth (1973) regressions of monthly stock returns on prior month IRPS. Regressions include controls for the log of firms market capitalization (ln(ME)), the log of book-to-market ratios (ln(B/M)), prior returns measured over the prior 11 month period after skipping the prior month ($R_{2,12}$), gross profitability defined as gross profits scaled by assets, and investment defined as percentage change in assets. The sample covers July 1973 through 2014. Low and high risk periods are defined as months in which the average idiosyncratic volatility is below or above the 10 year trailing average. All coefficients except IRPS are multiplied by 100. T-statistics are based on Newey-West standard errors with 8 lags.

	Full Sample (1973-2014)		Low Avg Idio. Volatility Periods		High Avg Idio. Volatility Periods	
ln(ME)	-0.05	-0.04	-0.05	-0.04	-0.05	-0.04
	(-1.63)	(-0.60)	(-1.16)	(-0.89)	(-0.97)	(-0.66)
ln(B/M)	0.27	0.31	0.31	0.34	0.22	0.27
	(3.56)	(4.17)	(3.28)	(3.68)	(1.84)	(2.37)
$R_{2,12}$	0.53	0.49	0.65	0.62	0.40	0.34
	(3.02)	(2.77)	(3.90)	(3.74)	(1.36)	(1.17)
$R_{0,1}$	-4.51	-4.62	-3.62	-3.73	-5.53	-5.66
	(-9.17)	(-9.36)	(-5.78)	(5.99)	(-7.50)	(-5.62)
GP/A	-	0.69	-	0.63	-	0.77
		(4.30)		(3.36)		(3.08)
Inv.	-	-0.26	-	-0.23	-	-0.29
		(-4.05)		(-2.50)		(-3.14)
IRPS	0.64	0.63	1.06	1.03	0.16	0.15
	(1.95)	(1.91)	(2.19)	(2.16)	(0.33)	(0.32)

Table 2

Fama-MacBeth Regressions for Large and Small Stocks, 1973-2014

IRPS is calculated as the covariance between stock-level idiosyncratic volatility and one over the square root of average idiosyncratic volatility over the prior 60 months. The table contains slopes of Fama and MacBeth (1973) regressions of monthly stock returns on prior month IRPS separately for large and small stocks. We use the median NYSE breakpoint for separating firms into small and large stocks. Regressions include controls for the log of firms market capitalization (ln(ME)), the log of book-to-market ratios (ln(B/M)), prior returns measured over the prior 11 month period after skipping the prior month ($R_{2,12}$), gross profitability defined as gross profits scaled by assets, and investment defined as percentage change in assets. The sample covers July 1973 through 2014. Low and high risk periods are defined as months in which the average idiosyncratic volatility is below or above the 10 year trailing average. All coefficients except IRPS are multiplied by 100. T-statistics are based on Newey-West standard errors with 8 lags.

	Full Sample		Low Avg Idio. Volatility Periods		High Avg Idio. Volatility Periods	
Panel A: Large Stocks						
ln(ME)	-0.09	-0.09	-0.09	-0.09	-0.09	-0.10
	(-2.65)	(-2.76)	(-2.26)	(-2.27)	(-1.54)	(-1.65)
ln(B/M)	0.16	0.22	0.18	0.23	0.12	0.21
	(2.10)	(2.98)	(1.99)	(2.37)	(1.04)	(1.87)
$R_{2,12}$	0.50	0.47	0.65	0.62	0.31	0.29
	(2.09)	(1.98)	(2.36)	(2.28)	(0.75)	(0.70)
$R_{0,1}$	-2.75	-2.97	-1.89	-2.01	-3.74	-4.08
	(-4.53)	(-4.87)	(-2.45)	(2.60)	(-4.28)	(-4.08)
GP/A	-	0.41	-	0.29	-	0.51
		(2.26)		(1.29)		(1.72)
Inv.	-	-0.20	-	-0.10	-	-0.30
		(-2.00)		(-1.10)		(-1.63)
IRPS	0.78	0.73	3.52	3.28	-2.39	-2.23
	(0.58)	(0.55)	(2.04)	(1.99)	(1.16)	(-1.10)
Panel B: Small Stocks						
ln(ME)	0.01	0.02	-0.02	-0.02	0.05	0.08
	(0.27)	(0.70)	(-0.53)	(-0.38)	(0.82)	(1.25)
ln(B/M)	0.21	0.25	0.25	0.28	0.17	0.22
	(2.52)	(3.06)	(2.37)	(2.69)	(1.25)	(1.69)
$R_{2,12}$	0.45	0.42	0.54	0.53	0.34	0.29
	(2.34)	(2.20)	(2.97)	(2.91)	(1.04)	(0.90)
$R_{0,1}$	-3.44	-3.58	-2.87	-2.99	-4.09	-4.27
	(-6.79)	(-7.03)	(-5.10)	(-5.29)	(-4.77)	(-4.97)
GP/A	-	0.56	-	0.43	-	0.71
		(3.39)		(2.38)		(2.66)
Inv.	-	-0.30	-	-0.20	-	-0.40
		(-3.18)		(-1.48)		(3.24)
IRPS	1.08	1.04	1.52	1.50	0.57	0.50
	(2.52)	(2.41)	(2.56)	(2.56)	(0.85)	(0.75)

Table 3

Portfolio Characteristics for Sorts on Size and IRPS, 1973-2014

Stocks are sorted into two size portfolios based on NYSE median market capitalization cutoffs and within size portfolios, into quintiles based on IRPS. Portfolios are rebalanced each month. Each portfolio characteristic is computed as time series averages of monthly statistics. Panel A shows the average number of stocks in each portfolio and the percentage of aggregate market capitalization. Stocks are placed in book-to-market, profitability and investment terciles based on breakpoints from Ken French's website. Panel B shows the percentage of stocks in each portfolio that fall into these terciles. The full sample period is July 1973 to December 2014.

	Large Stocks					Small Stocks				
	Low IRPS	2	3	4	High IRPS	Low IRPS	2	3	4	High IRPS
Panel A: Number of Stocks and Distribution of Aggregate Market Capitalization										
Number of stocks	156	156	156	156	156	437	437	437	437	437
% of Market Cap	9.30	14.20	18.10	25.20	25.70	0.80	1.30	1.80	2.10	1.50
Panel B: Percentage of Portfolio Stocks in Book-to-Market, Profitability and Investment Terciles										
Book-to-Market										
Growth	43.6	40.4	39.1	38.8	37.5	28.3	24.6	21.6	19.3	23.8
Neutral	33.9	41.0	41.8	42.3	40.5	26.7	32.5	37.0	40.8	37.7
Value	22.5	18.6	19.0	18.9	22.0	45.0	42.9	41.5	39.8	38.5
Profitability										
Robust	31.0	33.3	35.1	36.6	32.2	14.6	16.2	19.8	20.6	19.2
Neutral	35.6	42.4	43.7	44.7	46.3	21.1	30.0	37.2	42.6	36.4
Weak	33.4	24.2	21.1	18.7	21.4	64.3	52.9	43.0	36.8	44.3
Investment										
Aggressive	41.6	36.0	32.6	29.5	29.7	29.3	31.7	30.8	29.2	30.0
Neutral	31.9	41.8	46.7	51.2	50.6	23.1	30.0	35.5	40.0	35.8
Conservative	26.4	22.2	20.7	19.3	19.8	47.6	38.3	33.6	30.8	34.2

Table 4

Intercepts and Slopes from Three-Factor Model Regressions for Size and IRPS Portfolios, 1973-2014

IRPS is calculated as the covariance between stock-level idiosyncratic volatility and one over the square root of average idiosyncratic volatility over the prior 60 months. All stocks in a month are sorted into two size portfolios (small and large) based on NYSE median market capitalization cutoffs. Within each size portfolio, stocks are sorted into quintiles based on IRPS. The table shows intercept and slopes from three-factor models for these portfolio returns. The full sample period is July 1973 to December 2014. The sample of low and high average idiosyncratic volatility periods are defined as months in which the average idiosyncratic volatility is below or above the 10 year trailing average.

	Large Stocks						Small Stocks					
IRPS Quint.	Low	2	3	4	High	5-1	Low	2	3	4	High	5-1
						Panel A: Full Sample Period						
α	-0.20	-0.06	0.04	0.08	0.08	0.28	-0.33	-0.08	0.07	0.13	0.04	0.37
	(-2.05)	(-0.76)	(0.71)	(1.56)	(1.35)	(2.01)	(-2.66)	(-1.14)	(0.91)	(1.67)	(0.49)	(2.24)
Mkt	1.31	1.16	1.02	0.92	0.86	-0.45	1.29	1.22	1.11	0.95	0.87	-0.42
	(39.10)	(49.10)	(76.30)	(38.40)	(48.10)	(-9.81)	(21.00)	(37.60)	(39.10)	(36.90)	(37.30)	(-5.88)
SMB	0.30	0.01	-0.15	-0.24	-0.19	-0.50	1.48	1.04	0.77	0.64	0.71	-0.77
	(7.60)	(0.19)	(-4.70)	(-10.5)	(-7.40)	(-8.54)	(21.80)	(15.60)	(8.20)	(6.94)	(13.3)	(-7.15)
HML	-0.02	0.07	0.08	0.08	-0.03	-0.01	0.15	0.31	0.51	0.51	0.31	0.16
	(-0.27)	(1.86)	(2.20)	(2.13)	(-0.99)	(-0.14)	(1.17)	(5.01)	(6.60)	(6.43)	(5.62)	(0.99)
						Panel B: Low Average Idiosyncratic Volatility Periods						
α	-0.44	-0.22	0.03	0.10	0.25	0.69	-0.67	-0.16	0.07	0.14	0.13	0.80
	(-3.18)	(-2.72)	(0.49)	(1.39)	(4.13)	(3.86)	(-4.91)	(-2.04)	(1.02)	(1.76)	(1.55)	(4.03)
Mkt	1.32	1.19	1.02	0.90	0.85	-0.48	1.38	1.25	1.05	0.91	0.86	-0.53
	(33.48)	(47.82)	(66.90)	(38.58)	(33.30)	(-7.91)	(23.05)	(34.52)	(38.74)	(30.85)	(26.76)	(-6.55)
SMB	0.30	-0.04	-0.08	-0.20	-0.21	-0.51	1.22	1.02	0.92	0.74	0.76	-0.46
	(4.84)	(-1.18)	(-2.57)	(-6.45)	(-6.34)	(-6.16)	(17.83)	(26.13)	(32.42)	(21.03)	(16.96)	(-4.98)
HML	0.03	0.12	0.01	0.02	-0.05	-0.08	0.18	0.18	0.29	0.29	0.13	-0.05
	(0.34)	(2.52)	(0.32)	(0.51)	(-1.39)	(-0.71)	(1.83)	(2.91)	(6.61)	(6.42)	(2.12)	(-0.33)
						Panel C: High Average Idiosyncratic Volatility Periods						
α	0.10	0.11	0.06	0.10	-0.07	-0.17	0.02	0.15	0.18	0.21	0.08	0.06
	(0.58)	(0.90)	(0.69)	(1.24)	(-0.60)	(-0.66)	(0.09)	(1.00)	(1.40)	(1.52)	(0.58)	(0.20)
Mkt	1.29	1.15	1.03	0.93	0.86	-0.43	1.26	1.23	1.13	0.98	0.87	-0.39
	(24.87)	(31.64)	(54.06)	(24.46)	(32.83)	(-6.12)	(13.43)	(31.80)	(34.67)	(31.83)	(26.86)	(-3.66)
SMB	0.31	0.02	-0.17	-0.25	-0.19	-0.49	1.60	1.04	0.71	0.59	0.71	-0.89
	(5.86)	(0.44)	(-4.34)	(-8.39)	(-5.50)	(-6.51)	(21.33)	(11.28)	(6.13)	(5.23)	(10.13)	(-7.44)
HML	-0.04	0.07	0.12	0.10	-0.02	0.03	0.16	0.37	0.61	0.62	0.38	0.22
	(-0.51)	(1.15)	(2.53)	(1.90)	(-0.41)	(0.25)	(0.88)	(4.47)	(7.18)	(7.03)	(6.04)	(0.99)

Table 5

Intercepts and Slopes from Fama-French Five-Factor Model Regressions for Size and IRPS Portfolios, 1973-2014
IRPS is the covariance between stock-level idiosyncratic volatility and one over the square root of average idiosyncratic volatility over the prior 60 months. All stocks in a month are sorted into two size portfolios (small and large) based on NYSE median market capitalization cutoffs. Within each size portfolio, stocks are sorted into quintiles based on IRPS. The table shows intercept and slopes from Fama and French (2015) five-factor models for these portfolio returns. The sample of low and high average idiosyncratic volatility periods are defined as months in which the average idiosyncratic volatility is below or above the 10 year trailing average.

	Large Stocks						Small Stocks					
IRPS	Low	2	3	4	High	5-1	Low	2	3	4	High	5-1
						Panel A: Full Sample Period						
α	-0.03	-0.03	-0.04	-0.04	0.06	0.10	-0.04	-0.00	-0.03	-0.03	-0.02	0.02
	(-0.35)	(-0.41)	(-0.69)	(-0.75)	(0.97)	(0.71)	(-0.30)	(-0.04)	(-0.41	(-0.21)	(-0.280	(0.13)
Mkt	1.28	1.16	1.04	0.95	0.86	-0.42	1.24	1.21	1.12	0.97	0.87	-0.37
	(49.60)	(57.80)	(79.60)	(47.70)	(55.80)	(-12.2)	(26.90)	(39.70)	(49.80)	(54.10)	(44.00)	(-6.81)
SMB	0.20	-0.01	-0.10	-0.20	-0.17	-0.37	1.29	1.02	0.87	0.76	0.77	-0.52
	(4.98)	(-0.34)	(-4.11)	(-9.11)	(-6.77)	(-6.78)	(22.70)	(20.40)	(21.80)	(21.80)	(22.10)	(-7.04)
HML	0.02	0.09	0.08	0.03	0.01	-0.01	0.04	0.22	0.39	0.39	0.19	0.15
	(0.30)	(1.49)	(2.17)	(0.75)	(0.33)	(-0.09)	(0.31)	(2.68)	(6.02)	(7.02)	(4.14)	(1.02)
RMW	-0.37	-0.06	0.17	0.21	0.08	0.44	-0.69	-0.12	0.31	0.39	0.16	0.84
	(-5.83)	(-1.24)	(4.29)	(4.93)	(2.24)	(5.37)	(-7.66)	(-1.46)	(3.80)	(5.80)	(3.21)	(7.01)
CMA	-0.14	-0.03	0.05	0.19	-0.04	0.11	-0.13	-0.09	0.02	0.06	0.06	0.19
	(-1.59)	(-0.44)	(1.04)	(3.42)	(-0.70)	(0.88)	(-0.82)	(-0.82)	(0.34)	(1.38)	(1.17)	(1.04)
						Panel B: Low Average Idiosyncratic Volatility Periods						
α	-0.18	-0.21	0.01	-0.00	0.21	0.39	-0.45	-0.06	0.01	0.011	0.06	0.51
	(-1.51)	(-2.56)	(0.19)	(-0.03)	(3.14)	(2.44)	(-3.57)	(-0.70)	(0.19)	(0.15)	(0.73)	(2.69)
Mkt	1.26	1.19	1.02	0.93	0.86	-0.39	1.31	1.21	1.06	0.94	0.87	-0.44
	(33.38)	(45.50)	(59.41)	(49.90)	(36.32)	(-7.31)	(29.28)	(44.06)	(41.25)	(39.59)	(30.54)	(-7.17)
SMB	0.21	-0.05	-0.07	-0.18	-0.20	-0.41	1.13	0.97	0.93	0.78	0.79	-0.34
	(4.17)	(-1.41)	(-2.30)	(-6.74)	(-6.16)	(-5.69)	(16.61)	(25.47)	(37.17)	(22.79)	(15.07)	(-3.20)
HML	0.03	0.13	0.05	-0.02	-0.05	-0.08	0.027	0.05	0.21	0.242	0.10	0.08
	(0.31)	(2.22)	(1.58)	(-0.41)	(-0.88)	(-0.59)	(0.26)	(0.89)	(5.61)	(5.75)	(1.59)	(0.51)
RMW	-0.49	-0.02	0.05	0.18	0.08	0.58	-0.47	-0.24	0.10	0.25	0.13	0.60
	(-6.34)	(-0.32)	(1.26)	(3.34)	(1.64)	(5.64)	(-4.21)	(-4.11)	(2.60)	(5.44)	(1.99)	(3.74)
CMA	-0.17	-0.02	-0.06	0.17	0.05	0.21	0.05	0.07	0.06	0.07	-0.01	-0.06
	(-1.59)	(-0.24)	(-1.79)	(2.92)	(0.67)	(1.37)	(0.37)	(1.20)	(0.97)	(1.05)	(-0.13)	(-0.34)
						Panel C: High Average Idiosyncratic Volatility Periods						
α	0.28	0.17	-0.05	-0.04	-0.08	-0.36	0.38	0.28	0.12	0.010	0.018	-0.36
	(1.67)	(1.58)	(-0.61)	(-0.47)	(-0.60)	(-1.37)	(1.34)	(1.60)	(1.02)	(1.06)	(0.15)	(-1.15)
Mkt	1.26	1.14	1.04	0.96	0.86	-0.40	1.22	1.19	1.11	0.97	0.87	-0.35
	(30.68)	(41.26)	(50.55)	(28.72)	(37.08)	(-7.40)	(17.43)	(38.17)	(48.91)	(50.08)	(34.44)	(-4.63)
SMB	0.19	0.01	-0.09	-0.19	-0.18	-0.37	1.32	1.02	0.85	0.75	0.78	-0.54
	(3.39)	(0.02)	(-2.66)	(-5.98)	(-4.93)	(-4.84)	(16.38)	(15.62)	(15.46)	(16.13)	(16.44)	(-5.56)
HML	0.04	0.10	0.06	0.04	0.02	-0.02	0.13	0.31	0.47	0.46	0.21	0.08
	(0.49)	(1.30)	(1.25)	(0.65)	(0.38)	(-0.19)	(0.71)	(2.86)	(6.59)	(8.08)	(3.83)	(0.43)
RMW	-0.34	-0.08	0.25	0.23	0.06	0.39	-0.81	-0.08	0.36	0.45	0.18	0.98
	(-4.06)	(-1.16)	(5.12)	(3.75)	(1.24)	(3.44)	(-6.45)	(-0.89)	(4.27)	(6.31)	(2.96)	(6.30)
CMA	-0.19	-0.08	0.11	0.18	-0.03	0.16	-0.24	-0.23	-0.07	-0.02	0.07	0.31
	(-1.41)	(-0.68)	(1.20)	(1.85)	(-0.38)	(0.87)	(-0.85)	(-1.30)	(-0.70)	(-0.26)	(0.81)	(0.97)

Table 6

Intercepts and Slopes from Hou, Xue and Zhang Four-Factor Model Regressions for Size and IRPS Portfolios, 1973-2014
IRPS is calculated as the covariance between stock-level idiosyncratic volatility and one over the square root of average idiosyncratic volatility over the prior 60 months. All stocks in a month are sorted into two size portfolios (small and large) based on NYSE median market capitalization cutoffs. Within each size portfolio, stocks are sorted into quintiles based on IRPS. The table shows intercept and slopes from Hou, Xue and Zhang (2014) factor models for these portfolio returns. The full sample period is July 1973 to December 2014. The sample of low and high average idiosyncratic volatility periods are defined as months in which the average idiosyncratic volatility is below or above the 10 year trailing average.

	Large Stocks						Small Stocks					
IRPS Quint.	Low	2	3	4	High	5-1	Low	2	3	4	High	5-1
						Panel A: Full Sample Period						
α	0.05	-0.02	-0.06	-0.08	0.06	0.01	0.23	0.20	0.04	0.00	0.03	-0.20
	(0.55)	(-0.36)	(-0.96)	(-1.24)	(0.93)	(0.06)	(1.32)	(1.96)	(0.42)	(0.02)	(0.32)	(-0.93)
Mkt	1.28	1.15	1.03	0.94	0.86	-0.42	1.24	1.19	1.07	0.93	0.84	-0.39
	(47.20)	(54.00)	(63.50)	(41.70)	(43.30)	(-10.8)	(31.00)	(40.70)	(30.50)	(29.60)	(34.10)	(-8.16)
ME	0.19	-0.01	-0.11	-0.17	-0.17	-0.36	1.14	0.87	0.76	0.65	0.71	-0.43
	(4.51)	(-0.32)	(-3.41)	(-6.86)	(-6.34)	(-5.69)	(20.90)	(11.90)	(7.04)	(6.19)	(10.70)	(-4.16)
I/A	-0.17	0.07	0.17	0.23	-0.00	0.17	-0.16	0.10	0.41	0.46	0.22	0.38
	(-2.50)	(1.52)	(2.96)	(4.43)	(-0.07)	(1.82)	(-0.98)	(1.19)	(3.86)	(4.24)	(3.24)	(1.90)
ROE	-0.27	-0.03	0.11	0.18	0.09	0.36	-0.84	-0.38	-0.00	0.09	0.01	0.85
	(-5.42)	(-0.98)	(3.84)	(4.56)	(2.25)	(4.58)	(-6.32)	(-5.56)	(-0.02)	(1.30)	(0.28)	(5.59)
						Panel B: Low Average Idiosyncratic Volatility Periods						
α	-0.22	-0.14	0.00	-0.03	0.17	0.39	-0.36	0.03	0.15	0.17	0.17	0.53
	(-1.76)	(-1.72)	(0.07)	(-0.45)	(2.43)	(2.26)	(-2.89)	(0.41)	(2.01)	(1.82)	(1.97)	(2.87)
Mkt	1.29	1.17	1.02	0.92	0.87	-0.42	1.30	1.19	1.01	0.87	0.83	-0.47
	(34.28)	(50.65)	(63.15)	(54.11)	(33.82)	(-7.24)	(34.08)	(55.87)	(41.33)	(33.22)	(26.45)	(-8.59)
ME	0.23	-0.06	-0.07	-0.18	-0.20	-0.43	1.08	0.93	0.88	0.71	0.74	-0.34
	(4.27)	(-1.54)	(-2.57)	(-6.40)	(-5.70)	(-5.34)	(17.31)	(24.92)	(27.53)	(16.91)	(15.53)	(-3.55)
I/A	-0.09	0.09	0.015	0.18	0.06	0.15	0.09	0.11	0.16	0.21	0.02	-0.07
	(-0.92)	(2.04)	(0.44)	(3.03)	(1.32)	(1.17)	(0.76)	(1.98)	(2.52)	(2.91)	(0.34)	(-0.46)
ROE	-0.27	-0.04	0.058	0.16	0.09	0.36	-0.53	-0.33	-0.06	0.03	-0.03	0.50
	(-3.13)	(-1.04)	(2.02)	(4.48)	(2.14)	(3.22)	(-5.75)	-(6.04)	(-1.66)	(0.61)	(-0.62)	(3.64)
						Panel C: High Average Idiosyncratic Volatility Periods						
α	0.49	0.15	-0.11	-0.13	-0.10	-0.59	0.90	0.47	-0.02	-0.10	-0.10	-1.01
	(3.59)	(1.22)	(-0.84)	(-1.27)	(-0.77)	(-2.47)	(3.23)	(2.42)	(-0.13)	(-0.52)	(-0.70)	(-3.21)
Mkt	1.24	1.14	1.04	0.96	0.85	-0.40	1.20	1.18	1.11	0.98	0.87	-0.33
	(31.98)	(32.61)	(40.58)	(26.91)	(31.02)	(-7.01)	(17.28)	(29.78)	(23.51)	(23.02)	(26.05)	(-4.56)
ME	0.15	0.01	-0.11	-0.17	-0.15	-0.30	1.10	0.82	0.71	0.64	0.72	-0.38
	(2.64)	(0.34)	(-2.48)	(-4.78)	(-4.11)	(-3.53)	(15.3)	(8.03)	(5.10)	(4.52)	(8.19)	(-3.12)
I/A	-0.28	0.05	0.26	0.28	-0.04	0.24	-0.26	0.07	0.54	0.62	0.33	0.60
	(-2.85)	(0.62)	(2.79)	(3.91)	(-0.62)	(1.71)	(-1.12)	(0.61)	(3.94)	(4.51)	(4.12)	(2.34)
ROE	-0.29	-0.02	0.13	0.19	0.12	0.41	-1.03	-0.43)	0.02	0.11	0.03	1.07
	(-4.44)	(-0.63)	(3.04)	(3.38)	(2.09)	(3.84)	(-5.75)	(-4.73)	(0.16)	(1.18)	(0.82)	(5.53)

Table 7

Intercepts from Factor Models on Portfolios Sorted by Size, Idiosyncratic Volatility and IRPS, 1973-2014

IRPS is calculated as the covariance between stock-level idiosyncratic volatility and one over the square root of average idiosyncratic volatility over the prior 60 months. Stocks in each month are sorted into two size portfolios (large and small stocks) based on NYSE median market capitalization cutoffs. Each size portfolio is then sorted into terciles based on idiosyncratic volatility over the prior month (low, med., and high IVOL). These 2x3 portfolios are further sorted into terciles based on IRPS, resulting in 2x3x3 sorts. All portfolio returns are value weighted. The table shows intercepts from three- and five-factor models for these portfolios returns. The sample of low and high average idiosyncratic volatility periods are defined as months in which the average idiosyncratic volatility is below or above the trailing average.

	Three-Factor Model Intercepts				Five-Factor Model Intercepts			
IRPS Tercile	1	2	3	3-1	1	2	3	3-1
Panel A: Low Average Idiosyncratic Volatility Periods								
Large Stocks								
Low IVOL	0.01	0.08	0.27	0.26	-0.06	-0.03	0.21	0.27
	(0.09)	(1.16)	(3.51)	(2.13)	(-0.61)	(-0.41)	(2.82)	(2.05)
Med. IVOL	-0.42	-0.01	0.22	0.64	-0.39	-0.02	0.12	0.51
	(-3.12)	(-0.08)	(2.39)	(3.27)	(-3.02)	(-0.29)	(1.3)	(2.72)
High IVOL	-0.53	-0.25	0.08	0.61	-0.16	-0.12	0.15	0.32
	(-3.18)	(-2.22)	(0.81)	(3.20)	(-1.13)	(-1.14)	(1.39)	(1.95)
Small Stocks								
Low IVOL	0.35	0.26	0.36	0.01	0.25	0.10	0.26	-0.01
	(4.79)	(2.93)	(4.01)	(0.11)	(2.83)	(1.31)	(2.96)	(-0.02)
Med. IVOL	-0.27	0.01	0.14	0.41	-0.17	-0.03	0.25	0.20
	(-2.10)	(0.18)	(1.71)	(2.59)	(-1.48)	(-0.38)	(0.38)	(1.44)
High IVOL	-1.27	-0.58	-0.64	0.63	-0.91	-0.29	-0.50	0.41
	(-6.24)	(-3.91)	(-4.98)	(3.12)	(-4.75)	(-1.75)	(-3.78)	(2.01)
Panel B: High Average Idiosyncratic Volatility Periods								
Large Stocks								
Low IVOL	0.16	0.10	0.07	0.08	-0.04	-0.14	-0.06	-0.02
	(1.53)	(0.70)	(0.45)	(-0.38)	(-0.28)	(1.20)	(-0.29)	(0.08)
Med. IVOL	0.17	0.05	0.19	0.02	0.16	-0.07	0.19	0.03
	(1.31)	(0.42)	(1.52)	(0.12)	(1.20)	(-0.63)	(1.23)	(0.14)
High IVOL	0.01	0.14	-0.22	-0.23	0.27	0.34	-0.15	-0.43
	(0.04)	(0.81)	(-1.38)	(-0.93)	(1.10)	(1.87)	(0.91)	(1.53)
Small Stocks								
Low IVOL	0.21	0.21	0.19	-0.02	0.10	0.10	0.06	-0.04
	(1.78)	(1.65)	(1.24)	(-0.15)	(1.06)	(1.26)	(0.56)	(-0.26)
Med. IVOL	0.36	0.19	0.32	-0.04	0.50	0.12	0.29	-0.21
	(1.88)	(1.27)	(2.52)	(-0.21)	(2.19)	(0.89)	(2.62)	(-1.01)
High IVOL	-0.49	-0.04	-0.43	0.06	0.03	0.27	-0.37	-0.39
	(-1.46)	(-0.20)	(-2.00)	(0.17)	(0.08)	(0.91)	(-1.57)	(1.38)

Table 8

Intercepts and Slopes from Three-Factor Model Regressions for Size and IRPS Portfolios, 1931-1973

IRPS is calculated as the covariance between stock-level idiosyncratic volatility and one over the square root of average idiosyncratic volatility over the prior 60 months. All stocks in a month are sorted into two size portfolios (small and large) based on NYSE median market capitalization cutoffs. Within each size portfolio, stocks are sorted into quintiles based on IRPS. The table shows intercept and slopes from three-factor models for these portfolio returns. The full sample period is July 1931 to June 1973. The sample of low and high average idiosyncratic volatility periods are defined as months in which the average idiosyncratic volatility is below or above the 10 year trailing average.

	Large Stocks						Small Stocks					
IRPS Quint.	Low	2	3	4	High	5-1	Low	2	3	4	High	5-1
						Panel A: Full Sample Period						
α	-0.24	-0.23	-0.15	0.06	0.10	0.35	-0.32	-0.14	-0.16	-0.01	0.09	0.41
	(-2.55)	(-3.70)	(-2.23)	(1.20)	(2.30)	(2.96)	(-3.10)	(-1.70)	(-1.93)	(-0.17)	(1.18)	(3.09)
Mkt	1.10	1.19	1.17	1.01	0.91	-0.19	1.08	1.07	1.09	1.07	1.03	-0.05
	(20.79)	(52.79)	(31.51)	(39.32)	(48.71)	(-3.65)	(17.53)	(27.59)	(27.97)	(29.94)	(22.80)	(-0.58)
SMB	0.33	0.11	-0.04	-0.14	-0.13	-0.47	1.52	1.22	1.10	0.76	0.67	-0.84
	(3.06)	(3.30)	(-1.08)	(-5.77)	(-4.85)	(-4.03)	(22.32)	(10.37)	(7.04)	(15.07)	(17.11)	(-10.3)
HML	0.34	0.29	0.22	0.11	-0.09	-0.42	0.58	0.41	0.46	0.54	0.33	-0.24
	(2.44)	(4.12)	(3.73)	(4.22)	(-5.80)	(-2.93)	(5.89)	(7.01)	(9.95)	(9.40)	(7.39)	(-1.84)
						Panel B: Low Average Idiosyncratic Volatility Periods						
α	-0.26	-0.15	-0.12	0.07	0.08	0.34	-0.38	-0.25	-0.11	0.05	0.21	0.58
	(-3.18)	(-1.95)	(-1.67)	(1.45)	(2.13)	(3.16)	(-2.82)	(-2.69)	(-1.14)	(0.63)	(2.86)	(3.33)
Mkt	1.19	1.13	1.11	0.99	0.92	-0.26	1.19	1.17	1.12	1.02	0.96	-0.22
	(37.59)	(42.93)	(24.07)	(64.93)	(50.92)	(-6.16)	(20.88)	(35.81)	(28.28)	(15.28)	(32.13)	(-4.86)
SMB	0.38	0.21	0.09	-0.04	-0.14	-0.52	1.24	0.92	1.00	0.99	0.86	-0.38
	(6.40)	(4.11)	(1.87)	(-1.20)	(-5.10)	(-7.39)	(21.46)	(13.59)	(17.34)	(15.81)	(19.62)	(-4.74)
HML	0.21	0.12	0.03	0.10	-0.07	-0.28	0.50	0.32	0.22	0.26	0.22	-0.27
	(3.46)	(3.32)	(0.59)	(2.67)	(-3.45)	(-3.71)	(6.59)	(6.55)	(4.32)	(3.09)	(3.54)	(-2.55)
						Panel C: High Average Idiosyncratic Volatility Periods						
α	-0.14	-0.07	-0.01	0.04	0.08	0.21	-0.21	-0.24	-0.03	-0.01	0.01	0.22
	(-1.00)	(-0.97)	(-0.09)	(0.54)	(0.78)	(1.06)	(-1.65)	(-2.68)	(-0.38)	(-0.05)	(0.14)	(1.21)
Mkt	1.18	1.10	1.01	0.93	0.96	-0.21	1.15	1.13	1.08	0.94	0.86	-0.28
	(24.89)	(50.65)	(65.85)	(60.67)	(44.13)	(-3.39)	(23.11)	(50.16)	(32.86)	(52.29)	(32.17)	(-4.99)
SMB	0.46	0.08	-0.09	-0.21	-0.23	-0.69	1.55	1.11	0.79	0.63	0.67	-0.87
	(5.74)	(2.27)	(-3.28)	(-6.84)	(-6.88)	(-6.64)	(18.63)	(17.87)	(21.80)	(13.30)	(13.61)	(-8.24)
HML	0.01	0.07	-0.01	0.09	-0.07	-0.07	0.36	0.37	0.38	0.35	0.40	0.03
	(0.08)	(2.09)	(-0.22)	(3.21)	(-1.78)	(-0.80)	(6.18)	(8.04)	(7.16)	(11.75)	(7.51)	(0.41)

Table 9

Intercepts from Three-Factor Model Regressions for Size and IRPS Portfolios for North America, Europe, Japan, Asia Pacific (excluding Japan) and Global (excluding US)

Idiosyncratic volatility is calculated as the mean squared error of the residuals from daily market model regressions for each stock month. The market model includes four lags of the market return. We require a minimum of 15 valid returns per month to calculate idiosyncratic volatility. Average idiosyncratic volatility for each region is the simple average of the value-weighted idiosyncratic volatility in large and small cap stocks. Large and small stocks are based on the 90th percentile of market capitalization in each region-month. IRPS is the covariance between idiosyncratic volatility and one over the square root of average idiosyncratic volatility over the prior 60 months, requiring at least 40 valid observations. Within each size portfolio, stocks are sorted into terciles based on IRPS. The table shows intercept and slopes from three-factor models for these portfolio returns. North America includes Canada and the US. Europe includes Austria, Belgium, Denmark, Finland, France, Germany, Italy, the Netherlands, Norway, Portugal, Spain, Switzerland, and the United Kingdom. Asia Pacific includes Australia, Hong Kong, New Zealand and Singapore. Global ex US includes all countries in these regions but not the US. The sample period is 1990-2014. T-statistics appear in parentheses.

	Full Sample Period				Low Average Idiosyncratic Volatility Periods				High Average Idiosyncratic Volatility Periods			
IRPS Tercile	1	2	3	3-1	1	2	3	3-1	1	2	3	3-1
Panel A: North America												
Large	-0.23	0.01	0.14	0.37	-0.40	-0.01	0.23	0.63	0.09	0.10	-0.08	-0.16
	(-2.24)	(0.22)	(1.91)	(2.35)	(-4.18)	(-0.20)	(3.40)	(4.23)	(0.35)	(0.67)	(-0.43)	(-0.43)
Small	-0.12	-0.02	0.16	0.28	-0.45	-0.12	0.20	0.65	0.66	0.26	0.13	-0.53
	(-0.83)	(-0.83)	(2.24)	(1.53)	(-3.51)	(-1.87)	(3.41)	(4.02)	(2.01)	(1.46)	(0.80)	(-1.27)
Panel B: Europe												
Large	-0.25	0.14	0.10	0.36	-0.43	0.16	0.14	0.57	0.29	-0.00	-0.12	-0.41
	(-2.11)	(2.15)	(1.28)	(1.99)	(-3.62)	(2.62)	(1.81)	(3.24)	(1.07)	(-0.01)	(-0.64)	(-1.01)
Small	-0.18	0.07	0.03	0.21	-0.24	0.08	-0.09	0.14	0.08	0.03	0.11	0.03
	(-1.94)	(1.22)	(0.34)	(1.46)	(-2.57)	(1.31)	(-1.22)	(1.03)	(0.39)	(0.26)	(0.63)	(0.09)
Panel C: Japan												
Large	-0.01	0.05	0.04	0.06	-0.19	0.05	0.00	0.19	0.09	0.02	0.01	-0.08
	(-0.09)	(0.49)	(0.40)	(0.31)	(-1.47)	(0.51)	(0.02)	(1.17)	(0.32)	(0.08)	(0.07)	(-0.20)
Small	-0.20	0.10	0.08	0.28	-0.23	-0.01	0.07	0.30	-0.11	0.31	0.23	0.34
	(-1.34)	(0.96)	(0.83)	(1.65)	(-1.90)	(-0.09)	(0.83)	(2.12)	(-0.35)	(1.53)	(1.17)	(0.96)
Panel D: Asia Pacific (ex Japan)												
Large	-0.36	-0.02	0.28	0.64	-0.29	-0.15	0.36	0.64	-0.41	0.16	0.06	0.47
	(-2.56)	(-0.18)	(2.41)	(3.05)	(-1.80)	(-1.33)	(2.97)	(2.82)	(-1.53)	(0.73)	(0.23)	(1.12)
Small	-0.85	0.00	0.34	1.19	-0.10	-0.10	0.38	1.35	-0.54	0.32	0.25	0.79
	(-4.02)	(0.03)	(3.06)	(4.31)	(-0.97)	(-0.96)	(3.60)	(6.00)	(-1.16)	(1.50)	(1.00)	(1.25)
Panel E: Global (ex US)												
Large	-0.35	-0.05	0.28	0.62	-0.49	-0.07	0.47	0.96	-0.15	0.02	0.04	0.18
	(-3.12)	(-0.75)	(3.30)	(3.44)	(-4.14)	(-0.99)	(4.55)	(4.60)	(-0.64)	(0.18)	(0.24)	(0.52)
Small	-0.31	-0.02	0.08	0.39	-0.45	-0.06	0.12	0.57	0.18	0.15	0.13	-0.05
	(-2.15)	(0.18)	(0.95)	(2.03)	(-3.33)	(-0.75)	(1.41)	(2.94)	(0.55)	(0.75)	(0.70)	(-0.13)

Table 10

Intercepts from Five-Factor Model Regressions for Size and IRPS Portfolios for North America, Europe, Japan, Asia Pacific (ex Japan) and Global (ex US)

Idiosyncratic volatility is calculated as the mean squared error of the residuals from daily market model regressions for each stock month. The market model includes four lags of the market return. We require a minimum of 15 valid returns per month to calculate idiosyncratic volatility. Average idiosyncratic volatility for each region is the simple average of the value-weighted idiosyncratic volatility in large and small cap stocks. Large and small stocks are based on the 90th percentile of market capitalization in each region-month. IRPS is the covariance between idiosyncratic volatility and one over the square root of average idiosyncratic volatility over the prior 60 months, requiring at least 40 valid observations. Within each size portfolio, stocks are sorted into terciles based on IRPS. The table shows intercept and slopes from three-factor models for these portfolio returns. North America includes Canada and the US. Europe includes Austria, Belgium, Denmark, Finland, France, Germany, Italy, the Netherlands, Norway, Portugal, Spain, Switzerland, and the United Kingdom. Asia Pacific includes Australia, Hong Kong, New Zealand and Singapore. Global ex US includes all countries in these regions but not the US. The sample period is 1990-2014. T-statistics are in parentheses.

	Full Sample Period				Low Average Idiosyncratic Volatility Periods				High Average Idiosyncratic Volatility Periods			
IRPS Tercile	1	2	3	3-1	1	2	3	3-1	1	2	3	3-1
					Panel A: North America							
Large	-0.15	-0.01	0.14	0.29	-0.34	0.01	0.22	0.55	0.19	0.07	-0.12	-0.31
	(-1.45)	(-0.11)	(1.95)	(1.81)	(-3.53)	(0.23)	(3.32)	(3.70)	(0.74)	(0.48)	(-0.70)	(-0.79)
Small	-0.10	-0.03	0.17	0.27	-0.37	-0.08	0.23	0.60	0.53	0.26	0.16	-0.37
	(-0.62)	(-0.50)	(2.59)	(1.35)	(-2.93)	(-1.51)	(4.07)	(3.72)	(1.28)	(1.76)	(1.05)	(-0.75)
					Panel B: Europe							
Large	-0.18	0.09	-0.02	0.16	-0.29	0.18	0.16	0.45	0.32	-0.01	-0.29	-0.61
	(-1.25)	(1.17)	(-0.20)	(0.86)	(-1.92)	(2.40)	(1.79)	(2.34)	(1.03)	(-0.03)	(-1.40)	(-1.48)
Small	-0.05	0.16	0.13	0.18	-0.14	0.17	0.09	0.23	0.24	0.15	0.17	-0.07
	(-0.61)	(2.91)	(1.60)	(1.26)	(-1.74)	(3.80)	(1.03)	(1.63)	(1.18)	(1.11)	(0.94)	(-0.21)
					Panel C: Japan							
Large	-0.06	-0.04	-0.04	0.02	-0.19	0.02	-0.01	0.18	0.03	-0.09	-0.08	-0.11
	(-0.47)	(-0.57)	(-0.54)	(0.09)	(-1.69)	(0.29)	(-0.13)	(1.06)	(0.11)	(-0.67)	(-0.49)	(0.28)
Small	-0.12	0.11	0.08	0.20	-0.13	0.05	0.12	0.25	-0.15	0.22	0.13	0.28
	(-0.93)	(1.67)	(1.14)	(1.18)	(-1.33)	(0.91)	(1.87)	(1.87)	(-0.52)	(1.52)	(0.97)	(0.76)
					Panel D: Asia Pacific (ex Japan)							
Large	-0.07	0.14	0.22	0.29	0.04	0.06	0.50	0.46	-0.18	0.20	-0.23	-0.05
	(-0.60)	(1.34)	(1.62)	(1.45)	(-0.34)	(0.62)	(3.70)	(2.26)	(0.75)	(0.91)	(-0.82)	(-0.12)
Small	-0.35	0.23	0.21	0.55	-0.75	0.09	0.43	1.18	0.30	0.68	-0.06	-0.37
	(-1.59)	(2.51)	(1.96)	(2.08)	(-3.88)	(1.06)	(4.70)	(5.27)	(0.63)	(3.78)	(-0.26)	(-0.61)
					Panel E: Global (ex US)							
Large	-0.28	-0.04	0.21	0.48	-0.37	-0.04	0.55	0.92	-0.24	0.04	-0.02	0.23
	(-2.32)	(-0.58)	(1.89)	(2.55)	(-2.98)	(-0.50)	(4.49)	(4.40)	(-0.97)	(0.27)	(-0.08)	(0.64)
Small	-0.06	0.08	0.20	0.26	-0.22	0.13	0.34	0.56	0.24	0.10	0.15	-0.09
	(-0.43)	(1.42)	(2.15)	(1.28)	(-2.01)	(2.31)	(3.21)	(2.92)	(0.74)	(0.88)	(0.81)	(-0.20)

Table 11

Intercepts from Three- and Five-Factor Model Regressions for Size and IRPS Portfolios for Global (ex US)

All stocks in the following developed markets are pooled: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Hong Kong, Italy, Japan, Netherlands, New Zealand, Norway, Portugal, Singapore, Spain, Switzerland, and the United Kingdom. Idiosyncratic volatility is calculated as the mean squared error of the residuals from daily market model regressions for each stock month. Average idiosyncratic volatility is the simple average of the value-weighted idiosyncratic volatility in large and small cap stocks (based on the 90^{th} percentile of market capitalization). IRPS is the covariance between idiosyncratic volatility and one over the square root of average idiosyncratic volatility over the prior 60 months, requiring at least 40 valid observations. Stocks are sorted into two size groups, then into terciles by idiosyncratic volatility, and finally into terciles by IRPS. The sample period is 1990-2014. T-statistics are in parentheses.

	Full Sample Period				Low Average Idiosyncratic Volatility Periods				High Average Idiosyncratic Volatility Periods			
IRPS Tercile	1	2	3	3-1	1	2	3	3-1	1	2	3	3-1
					Panel A: Intercepts from three-factor models for large stocks							
Low IVOL	-0.30	0.00	0.37	0.67	-0.19	0.18	0.69	0.89	-0.43	-0.14	-0.03	0.40
	(-2.53)	(0.01)	(3.03)	(3.89)	(-1.49)	(1.42)	(4.98)	(4.27)	(-1.85)	(-0.82)	(-0.13)	(1.27)
Med IVOL	-0.28	-0.03	0.19	0.46	-0.46	-0.15	0.25	0.71	-0.04	0.13	0.06	0.10
	(-2.14)	(-0.34)	(1.97)	(2.50)	(-3.13)	(-1.35)	(2.31)	(3.36)	(-0.15)	(0.73)	(0.35)	(0.28)
High IVOL	-0.45	-0.07	0.25	0.70	-0.65	-0.22	0.14	0.79	-0.12	0.11	0.40	0.52
	(-2.07)	(-0.54)	(1.61)	(2.77)	(-3.19)	(-1.43)	(0.70)	(2.91)	(-0.24)	(0.40)	(1.54)	(0.97)
					Panel B: Intercepts from three-factor models for small stocks							
Low IVOL	-0.09	0.14	0.18	0.27	0.01	0.29	0.37	0.35	-0.21	-0.01	-0.03	0.18
	(-1.06)	(1.91)	(1.93)	(2.10)	(0.14)	(3.33)	(3.43)	(2.25)	(-1.30)	(-0.07)	(-0.18)	(0.75)
Med IVOL	-0.27	-0.09	0.06	0.33	-0.45	-0.17	-0.01	0.44	0.22	0.10	0.30	0.08
	(-1.78)	(-0.81)	(0.61)	(1.74)	(-2.90)	(-1.60)	(-0.06)	(2.23)	(0.70)	(0.38)	(1.38)	(0.20)
High IVOL	-0.59	-0.22	-0.20	0.39	-0.81	-0.48	-0.36	0.45	-0.05	0.45	0.36	0.41
	(-2.54)	(-1.15)	(-1.26)	(1.50)	(-3.79)	(-2.90)	(-1.90)	(1.67)	(-0.10)	(0.98)	(1.21)	(0.74)
					Panel C: Intercepts from five-factor models for large stocks							
Low IVOL	-0.38	-0.10	0.30	0.68	-0.22	0.14	0.72	0.94	-0.63	-0.19	-0.03	0.60
	(-2.99)	(-0.90)	(2.06)	(3.7)	(-1.49)	(1.31)	(4.64)	(4.49)	(-2.63)	(-0.86)	(-0.10)	(1.84)
Med IVOL	-0.20	-0.06	0.12	0.32	-0.38	-0.11	0.29	0.67	-0.02	0.05	0.06	0.09
	(-1.44)	(-0.64)	(1.06)	(2.61)	(-2.43)	(-1.00)	(2.42)	(3.07)	(-0.08)	(0.26)	(0.28)	(0.23)
High IVOL	-0.26	0.05	0.37	0.63	-0.36	-0.06	0.41	0.76	-0.10	0.14	0.46	0.57
	(-1.13)	(0.34)	(2.10)	(2.37)	(-1.78)	(-0.39)	(1.92)	(2.75)	(-0.19)	(0.49)	(1.46)	(0.99)
					Panel D: Intercepts from five-factor models for small stocks							
Low IVOL	-0.06	0.16	0.25	0.30	0.06	0.37	0.52	0.46	-0.30	-0.10	-0.08	0.22
	(-0.60)	(1.75)	(2.15)	(2.25)	(0.59)	(3.42)	(3.97)	(2.97)	(-1.72)	(-0.55)	(-0.36)	(0.87)
Med IVOL	-0.09	0.01	0.19	0.28	-0.23	0.02	0.24	0.47	0.12	0.08	0.34	0.22
	(-0.72)	(0.17)	(2.03)	(1.44)	(-1.90)	(0.21)	(2.41)	(2.40)	(0.42)	(0.49)	(1.79)	(0.52)
High IVOL	-0.12	0.15	0.13	0.24	-0.40	-0.12	0.08	0.47	0.18	0.75	0.43	0.24
	(-0.48)	(0.89)	(0.81)	(0.88)	(-1.91)	(-0.88)	(0.39)	(1.67)	(0.32)	(1.92)	(1.68)	(0.46)

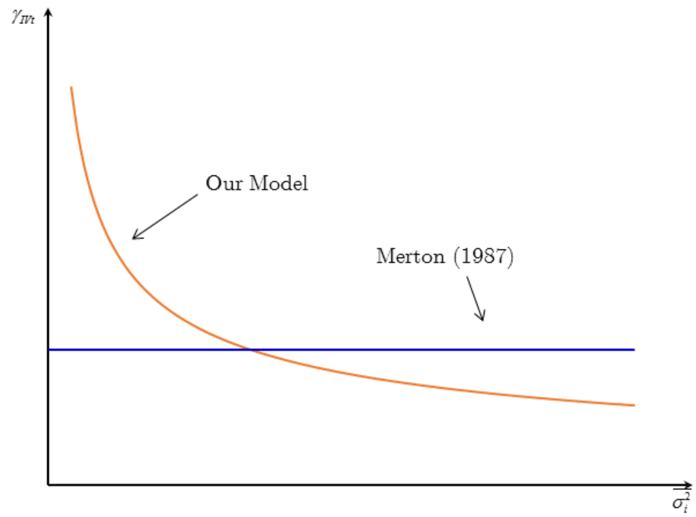

Figure 1: The x-axis shows average idiosyncratic volatility. The y-axis shows the premium associated with idiosyncratic volatility.

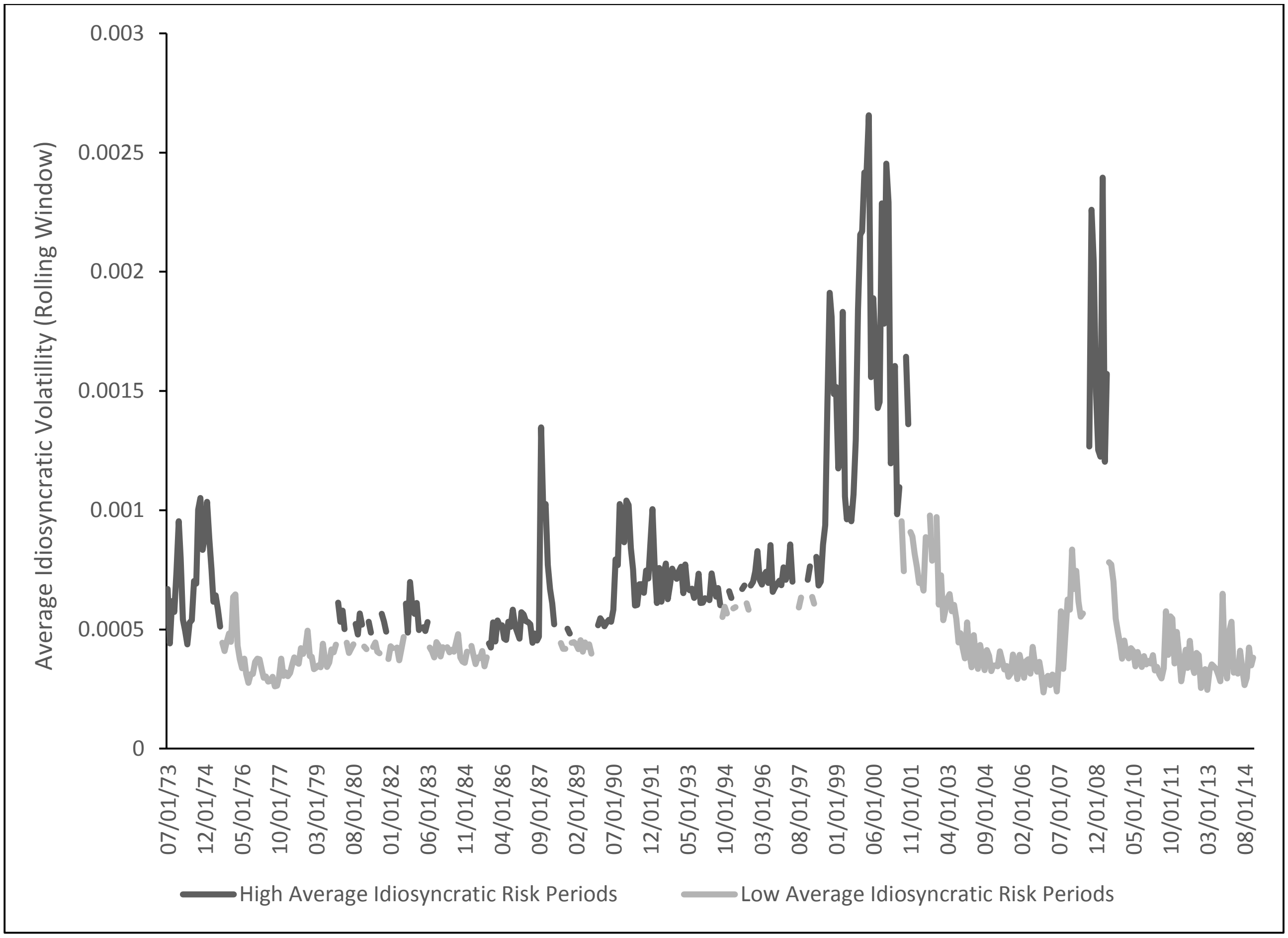

Figure 2. We compute the value-weighted average idiosyncratic volatility for small and large capitalization stocks, and then calculate a simple average of the two to obtain average idiosyncratic volatility for each month. We use NYSE median breaks to separate small and large cap stocks. Each month is classified as a low or high average idiosyncratic risk month if the month's average idiosyncratic volatility is above or below the trailing 10 year average.

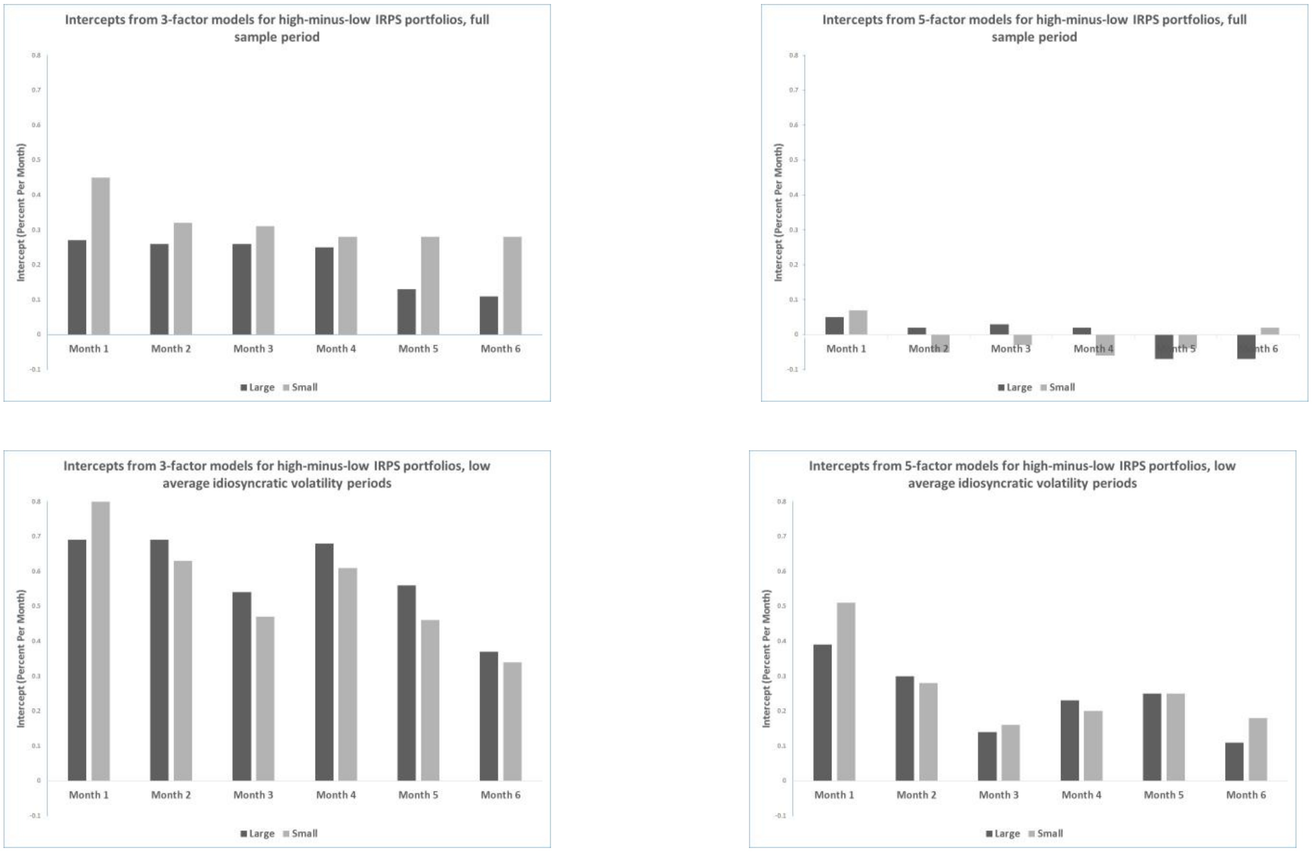

Figure 3. IRPS is calculated as the covariance between stock-level idiosyncratic volatility and one over the square root of average idiosyncratic volatility over the prior 60 months. All stocks in a month are sorted into two size portfolios (small and large) based on NYSE median market capitalization cutoffs. Within each size portfolio, stocks are sorted into quintiles based on IRPS. The figure shows intercepts from 3- and 5-factor models for the high-minus-low IPRS portfolios for month n after portfolio formation (i.e. the term structure of returns). The full sample period is July 1973 to December 2014. The sample of low average idiosyncratic volatility periods are defined as months in which the average idiosyncratic volatility is below or above the 10 year trailing average.