LIQUIDITY RULES AND CREDIT BOOMS

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ABSTRACT

We show that stricter bank liquidity standards can trigger unintended credit booms when there is heterogeneity in interbank pricing power. Attempts to circumvent the regulation change the allocation of savings across institutions, eliciting strategic responses that also change the allocation of lending across markets. More credit is generated per unit of savings in the new equilibrium. A quantitative application to China illustrates the practical relevance of the mechanisms in our model.

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1 Introduction

A decade after the 2007-09 financial crisis, the debate about bank regulation remains unsettled. Politicians cite the crisis as prima facie evidence of under-regulation, central bankers are weary of the unintended consequences of over-regulation, and the academic literature has yet to agree on what would constitute optimal regulation. In the words of Stanley Fischer, former Vice Chairman of the U.S. Federal Reserve, a "tightening in regulation of the banking sector may push activity to other areas – and things happen." Exactly what happens, he argues, is difficult to predict as there is limited theoretical work on the interactions between regulated and unregulated institutions and the economic incentives that drive them.[1]

These gaps in our understanding seem especially pronounced when it comes to liquidity regulation. Diamond and Kashyap (2016) characterize the new liquidity rules agreed upon by the Basel Committee on Bank Supervision as "a situation where practice is ahead of both theory and measurement." Allen and Gale (2017) go even further in their survey of existing literature and conclude that "with liquidity regulation, we do not even know what to argue about." Understanding which features of the economic environment are important for shaping the aggregate effects of liquidity regulation would thus propel the literature forward. In this paper, we establish that heterogeneity in interbank pricing power is, to first order, one such feature.

Our model is one where banks engage in maturity transformation, borrowing short and lending long in the spirit of Diamond and Dybvig (1983). We fix the return on long-term lending and endogenize the interest rates that banks offer to attract short-term funds from savers. Maturity transformation leaves banks vulnerable to idiosyncratic withdrawal shocks, giving rise to an ex post interbank market where banks with insufficient liquidity can borrow from banks with surplus liquidity at an endogenously determined price. Such interbank markets exist in Bhattacharya and Gale (1987) and Allen and Gale (2004). We then add two ingredients to this environment.

The first ingredient is that banks differ in size. Without loss of generality for the main results, we model a large bank and a continuum of individually small banks. By definition, the large bank internalizes the effect of its choices on the rest of the economy. The individually small banks do not. Absent any other differences between large and small banks, our model predicts that the large bank chooses a lower ratio of long-term lending to short-term borrowing than the small banks. The large bank is more liquid because, when choosing the composition of its assets, it internalizes that its demand for liquidity will increase the price of

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liquidity on the interbank market. The small banks, in contrast, are interbank price-takers. If the government were more likely to rescue a large bank with liquidity problems than a small one, the price of liquidity may not increase in response to demand by the large bank, distorting the large bank's incentive to maintain liquidity. Without this distortion, however, small banks will be endogenously more constrained by the introduction of a liquidity regulation that caps the ratio of long-term lending to short-term borrowing at each bank.

The second ingredient is that each bank can choose how much maturity transformation to conduct in the regulated sector and how much to conduct outside the perimeter of regulation. We show that, in response to liquidity regulation, small banks find it optimal to offer a new savings instrument and manage the funds raised by this instrument on a balance sheet that is not subject to the regulation (e.g., the funds are managed in an off-balance-sheet vehicle that can make the loans the bank cannot make on its balance sheet without violating the liquidity minimum). This constitutes shadow banking: it achieves the same type of credit intermediation as a regular bank without appearing on a regulated balance sheet. It also achieves the same type of maturity transformation as a regular bank, with long-term assets financed by short-term liabilities.

As small banks push to attract savings into off-balance-sheet instruments, our model predicts that they raise the interest rates on these instruments above the interest rates on traditional deposits. On the margin, the premium that small banks are willing to pay for off-balance-sheet funding is exactly equal to the tax implicitly imposed on their deposits by a binding liquidity minimum.

All else constant, the emergence of a savings instrument that pays a premium relative to traditional deposits poaches some deposits away from the large bank. We demonstrate that the large bank responds to this loss of funding in two ways. First, it issues its own high-return savings instruments, competing directly with the small banks. Second, it decreases the amount of liquidity it makes available to the interbank market. The second response involves a more subtle form of competition, wherein the large bank uses its price impact on the interbank market to change the incentives of the small banks. Naturally, the incentive to evade a liquidity minimum is weaker when liquidity is expected to be expensive. Therefore, by tightening the interbank market and making liquidity more expensive, the large bank can compel the small banks to behave less aggressively in their quest for off-balance-sheet business and thus lessen the extent to which they poach deposits. The large bank is also increasing the price of liquidity for itself should it need to borrow in the interbank market but it does so in exchange for less competition for funding from the small banks.

The new equilibrium is characterized by an unintended credit boom, with more credit per unit of savings relative to the pre-regulation equilibrium. There are two channels behind this

result. Together, these channels more than offset the decline in on-balance-sheet lending by small banks triggered by the regulation. First, the migration of some savings from deposits at the large bank to the higher-return off-balance-sheet instruments of the small banks increases credit because the small banks, as interbank price-takers, make more long-term loans per unit of funding than the large bank. Second, the strategic response of the large bank on the interbank market contributes directly to the credit expansion: rather than sitting idly on the liquidity that it intends to withhold from the interbank market, the large bank lends more to non-financial borrowers.

We call the increase in credit that culminates from these two channels a supply-side credit boom because it originates from the banks themselves. These channels would not operate if off-balance-sheet vehicles were precluded, as small banks would have to mechanically switch from loans to more liquid assets in order to comply with the liquidity minimum. These channels also would not operate if the banking sector were only populated by a continuum of ex ante identical small banks, regardless of access to off-balance-sheet vehicles. To this point, we show that liquidity regulation still generates shadow banking but has the desired effect of decreasing credit in our model when there is no large bank.

The result that an aggregate credit boom can be born out of stricter liquidity regulation is surprising. However, our model generates it under a minimal set of assumptions, namely heterogeneity in interbank pricing power and accounting standards that do not outlaw off-balance-sheet business. The second assumption is satisfied around the world because of the discretion available in accounting rules.[2] The first assumption is a joint statement about the size distribution of banks and the conduct of monetary policy. Most countries have at least some banks that are large enough to shift the demand for liquidity relative to the supply, leading to sudden changes in the price that clears the interbank market. If the central bank does not automatically offset all such changes by targeting the interbank rate, large banks will have greater price impact than small banks and the assumption of heterogeneity in interbank pricing power will be satisfied. Effectively, the large bank's influence over the interbank market is decreasing in the responsiveness of the central bank's liquidity injections to the interbank rate and, in the quantitative analysis described below, we find that liquidity regulation leads to smaller credit booms when the central bank is more responsive to the interbank rate than when it is less responsive. This implication of our model is important because there are several settings where an automatic offset by the central bank does not

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[2]In particular, the legality of off-balance-sheet vehicles reflects the discretion available in accounting rules (e.g., U.S. GAAP, IFRS standards, etc). Banks can capitalize on this discretion, changing the form of an activity for reporting purposes without changing the true economic substance. For this reason, accounting assets and liabilities can differ from economic assets and liabilities.

exist. For example, central banks in countries with managed exchange rates are bounded in their ability to lean against fluctuations in the interbank market. The Federal Reserve's history also includes long periods of time where a short-term policy rate was not targeted

Our next contribution is to use the model to explore recent developments in China's economy. We choose China for the following reasons. Between 2007 and 2014, the ratio of debt to GDP in China exploded from 110% to 200%. The ratio of private credit to private savings, sometimes a more conservative gauge, also rose markedly from 65% to 75% over the same period. This credit boom appears to have occurred on the heels of stricter liquidity regulation. Around 2008, Chinese regulators began enforcing an old but hitherto neglected loan-to-deposit cap which forbade banks from lending more than 75% of their deposits to non-financial borrowers. Our model predicts that some credit booms are caused by stricter liquidity regulation so we are interested to know whether stricter liquidity regulation can account for at least part of the Chinese experience.

We use a rich, transaction-level dataset to establish that there is a high degree of heterogeneity in interbank pricing power among China's commercial banks. We then calibrate the model to Chinese data. The calibrated version of our model shows that loan-to-deposit enforcement alone generates one-third of the increase in China's aggregate credit-to-savings ratio between 2007 and 2014. We then pursue a quantitative extension that allows for multiple shocks to the Chinese economy: shocks to liquidity regulation, shocks to loan demand stemming from the fiscal stimulus package announced by China's State Council in late 2008, and money supply shocks. We find that loan demand shocks and money supply shocks produce counterfactual correlations between key market-determined interest rates, specifically interbank interest rates and spreads on the high-return savings instruments offered by small versus big banks. Allowing for all three shocks simultaneously, the quantitative extension matches a broad set of empirical moments almost perfectly, while still assigning a dominant role to variation in loan-to-deposit rules.

1.1 Related Literature

The literature on bank regulation is concerned with unintended consequences. Within this literature, there are many papers on capital requirements, largely because capital regulation was used more widely than liquidity regulation before the 2007-09 financial crisis. In the models of Harris, Opp, and Opp (2014), Plantin (2015), and Huang (2015), higher capital requirements lead to shadow banking, with various channels through which financial stability is affected. See also Acharya, Schnabl, and Suarez (2013), Demyanyk and Loutskina (2016), and Buchak, Matvos, Piskorski, and Seru (2018) for empirical evidence of regulatory arbi-

trage in the context of capital-related regulations. In the model of Begenau (2018), higher capital requirements make deposits scarce, lowering overall bank funding costs by enough to increase lending. A related model that allows for both traditional and shadow banks is studied in Begenau and Landvoigt (2017).

The liquidity problems experienced during the crisis and the subsequent introduction of global liquidity standards are now shifting attention towards liquidity regulation. Allen and Gale (2017) provide an excellent survey of this literature. Other, more recent contributions include Gorton and Muir (2016) who examine arbitrage during the U.S. National Banking Era to evaluate the merits of liquidity coverage ratios; Van den Heuvel (2016) who compares the welfare costs of liquidity and capital requirements; Adrian and Boyarchenko (2018) who present a dynamic model where liquidity requirements are preferable to capital requirements as a prudential policy tool; Banerjee and Mio (2018) who find no evidence that bank lending fell after the U.K. tightened liquidity regulation in 2010; Jin and Xiong (2018) who argue that macroprudential reserve requirements will unintentionally push banks to choose greater currency mismatch; and Davis, Korenok, Lightle, and Prescott (2018) who use experimental methods to explore whether liquidity regulations will improve interbank trade in response to shocks. An earlier contribution to which our paper most closely relates is Farhi, Golosov, and Tsyvinski (2007, 2009) who theoretically analyze the effect of liquidity regulation on market interest rates in a broad set of specifications.

We contribute to the literature on bank regulation by linking the effect of liquidity standards to interbank market structure and by developing a theory of unintended credit booms. Research on past financial crises demonstrates the importance of understanding interbank markets. Mitchener and Richardson (2016) show how a pyramid structure in U.S. interbank deposits propagated shocks during the Great Depression; Gorton and Tallman (2016) show how cooperation among members of the New York Clearinghouse helped end pre-Fed banking panics; and Frydman, Hilt, and Zhou (2015) show how a lack of cooperation with and between New York's trust companies contributed to the Panic of 1907. Relative to these papers, we focus on liquidity regulation and how it can be endogenously undermined when large banks have pricing power over small banks on the interbank market. Drechsler, Savov, and Schnabl (2017) also study the implications of non-competitive markets in banking but focus instead on the market power of banks over depositors in concentrated markets and how this can explain the transmission of monetary policy in the U.S. since the mid-1990s.

Our paper also relates to new work on the implications of size asymmetries in banking. Corbae and D'Erasmo (2013, 2014) study the relationship between bank entry, exit, and risk-taking in a quantitative model where large banks make lending decisions before small ones. Dávila and Walther (2018) instead focus on bank leverage and government bailouts when

large banks internalize that their decisions directly affect the government's bailout policy. They show that large banks choose more leveraged positions than small banks in this case, which would be consistent with large banks in the U.S. being more constrained by capital requirements than small banks. We show that large banks choose more liquid positions than small banks in an equilibrium model with endogenous interbank pricing, which is consistent with large banks in China being less constrained by liquidity requirements than small banks. See Hachem (2018) for further discussion, including a comparison between shadow banking in the U.S. and China.

Finally, our quantitative application is related to a rapidly growing literature on China's financial system. See Hachem (2018) and Song and Xiong (2018) for surveys. Chen, Ren, and Zha (2018) argue that an additional form of shadow banking emerged in China as an unintended consequence of contractionary monetary policy. The analysis is based on a different shock and a different set of financial products but their main finding echoes one of several findings original to our paper: policy tightenings in China have been undermined by the shadow banking products they triggered.

The rest of our paper is organized as follows. Sections 2 and 3 present the model. To help isolate the effect of interbank market structure, Section 2 lays out a benchmark model with only small banks and studies the equilibrium properties. Section 3 extends the benchmark to include a large bank, studies how the equilibrium properties are affected, and presents the main analytical results. All proofs are in Appendix A. Section 4 then applies the model to China, presenting the calibration results along with a structural estimation to evaluate the importance of various shocks. Section 5 concludes.

2   Benchmark Model

There are three periods, $t \in \{0, 1, 2\}$, and a unit mass of risk neutral banks, $j \in [0, 1]$. Let X_j denote the funding obtained by bank j at $t = 0$. Each bank can invest in a project which returns $(1 + i_A)^2$ per unit invested. Projects are long-term, meaning that they run from $t = 0$ to $t = 2$ without the possibility of liquidation at $t = 1$. To introduce a tradeoff between investing and not investing, banks are also subject to short-term idiosyncratic liquidity shocks which must be paid off at $t = 1$. More precisely, bank j must pay $\theta_j X_j$ at $t = 1$ in order to continue operation. The exact value of θ_j is drawn from a two-point distribution:

$$\theta_j = \begin{cases} \theta_\ell & prob. \ \pi \\ \theta_h & prob. \ 1 - \pi \end{cases}$$

where $0 < \theta_\ell < \theta_h < 1$ and $\pi \in (0, 1)$. Each bank learns the realization of its shock in $t = 1$. Prior to that, only the distribution is known.

2.1 Bank Liabilities

The liquidity shocks just described can be fleshed out using Diamond and Dybvig (1983). Specifically, the economy has an aggregate endowment $X > 0$ at $t = 0$ and banks attract funding by offering liquidity services to the owners of this endowment (ex ante identical households). The liquidity service offers households more than the long-term project if liquidated at $t = 1$ but less than this project if held until $t = 2$.

The traditional liquidity service is a deposit. To set notation, a dollar deposited at $t = 0$ becomes $1 + i_B$ if withdrawn at $t = 1$ and $(1 + i_D)^2$ if withdrawn at $t = 2$. In Diamond and Dybvig (1983), banks choose i_B and i_D to achieve optimal risk-sharing for households. In Diamond and Kashyap (2016), banks take i_B and i_D as given and, with $i_B = i_D = 0$, the traditional deposit is equivalent to storage.

In our model, each bank j can offer a liquidity service which delivers storage plus a return ξ_j. To ease the exposition, suppose ξ_j accrues at $t = 2$. As we will explain in Section 2.4, bank j chooses ξ_j at $t = 0$ to maximize its expected profit subject to household demand for liquidity services. If bank j chooses $\xi_j = 0$, then it is content offering storage. If bank j chooses $\xi_j > 0$, then it is choosing to offer more than storage.[3]

In practice, banks may have more elaborate liability structures, where they pay different prices for different units of funding. What matters for the analysis are the spreads so it suffices to fix characteristics and price for one type of funding and let the rest vary relative to it. To this end, we allow each bank j to simultaneously offer storage and another liquidity service that pays an endogenously chosen $\xi_j \geq 0$. We refer to the other liquidity service as a deposit-like product (DLP), with a choice of $\xi_j = 0$ implying that no DLPs are offered. The shock θ_j represents the fraction of households that withdraw funding from bank j at $t = 1$.

Let us now specify how households allocate their endowment at $t = 0$ conditional on interest rates. Denote by D_j the funding attracted by bank j in the form of storage. The funding attracted in the form of DLPs is denoted by W_j, with $X_j \equiv D_j + W_j$ and $\int X_j dj = X$. Appendix B sketches a simple household optimization problem with transactions costs which motivates the following functional forms:

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[3]Offering $\xi_j < 0$ would create an incentive for early withdrawals and cannot be an equilibrium outcome. We will be focusing on parameters such that a choice of $\xi_j = 0$ indicates a desire by bank j to offer exactly $\xi_j = 0$, as opposed to indicating that bank j would offer $\xi_j < 0$ if there were no concern about creating incentives for early withdrawals.

$$W_j = \omega \xi_j \tag{1}$$

$$D_j = X - (\omega - \rho)\, \xi_j - \rho \overline{\xi} \tag{2}$$

where ω and ρ are non-negative constants and $\overline{\xi}$ denotes the average DLP return offered by other banks. Intuitively, ω captures the substitutability between liquidity services within a bank while ρ governs the intensity of competition among banks. To see this, sum equations (1) and (2) to write bank j's funding share as:

$$X_j = X + \rho \left(\xi_j - \overline{\xi} \right) \tag{3}$$

If $\rho = 0$, then bank j perceives its funding share as fixed, shutting down competition. If $\rho > 0$, then bank j perceives a positive relationship between its funding share and the DLP return it offers relative to other banks.

Each individual bank will take $\overline{\xi}$ as given when making decisions. In a symmetric equilibrium, $\overline{\xi}$ will be such that the profit-maximizing choice of ξ_j equals $\overline{\xi}$ for all j.

2.2 Bank Assets and the Interbank Market

We now elaborate on how banks allocate their funding. The maturity mismatch between investment projects and liquidity shocks introduces a role for reserves (i.e., savings which can be used to pay realized liquidity shocks). As we will explain in Section 2.4, the division of X_j into investment and reserves is chosen at $t = 0$ to maximize expected profit.

Let $R_j \in [0, X_j]$ denote the reserve holdings of bank j at $t = 0$. If $\theta_j < \frac{R_j}{X_j}$, then bank j has a reserve surplus at $t = 1$. If $\theta_j > \frac{R_j}{X_j}$, then bank j has a reserve shortage at $t = 1$. An interbank market exists at $t = 1$ to redistribute reserves across banks. A market in which banks can share risk and obtain liquidity also exists in Bhattacharya and Gale (1987) and Allen and Gale (2004).

The interbank interest rate in our benchmark is denoted by i_L. Banks in the continuum are atomistic so they take i_L as given when making decisions. However, i_L is endogenous and adjusts to clear the interbank market. Interbank lenders (borrowers) are banks with reserve surpluses (shortages) at $t = 1$. In practice, central banks also serve as lenders of last resort so we introduce a supply of external funds, $\Psi(i_L) \equiv \psi i_L$, where $\psi > 0$.

We will focus on symmetric equilibrium, in which case R_j and ξ_j are the same across the unit mass of banks. Notice that symmetry of ξ_j in equation (3) implies $X_j = X$. The

condition for interbank market clearing is then:

$$R_j + \psi i_L = \bar{\theta} X \tag{4}$$

where $\bar{\theta} \equiv \pi \theta_\ell + (1 - \pi) \theta_h$ is the average liquidity shock. The left-hand side of (4) captures the supply of liquidity at $t = 1$ while the right-hand side captures the demand. Total credit in this economy is the total amount of funding invested in projects, $X - R_j$.

2.3   Liquidity Regulation and Possible Arbitrage

We now allow for the possibility of a government-imposed loan limit on each bank. This limit can also be viewed as a liquidity rule which says the ratio of reserves to funding must be at least $\alpha \in (0, 1)$. Given the structure of our model, reserves are meant to be used at $t = 1$ so enforcement of the liquidity rule is confined to $t = 0$. If the government does not enforce a liquidity rule, then $\alpha = 0$.

Importantly, the liquidity rule only applies to activities that the bank reports on its balance sheet. To model this, we allow banks to choose where to manage DLPs and the projects financed by those DLPs. If fraction $\tau_j \in [0, 1]$ is managed in an off-balance-sheet vehicle, then bank j's reserve holdings only need to satisfy:

$$R_j \geq \alpha \left(X_j - \tau_j W_j \right) \tag{5}$$

Off-balance-sheet vehicles can be viewed as accounting maneuvers that legally shift activities away from regulation without changing the nature of those activities. Such maneuvers capitalize on the discretion available in accounting rules and constitute regulatory arbitrage.[4]

Notice that bank j does not need to use off-balance-sheet vehicles if just attempting to change its funding share in equation (3). This is because ξ_j and τ_j are separate decisions. If bank j chooses $\xi_j > 0$ and $\tau_j = 0$, then it is simply offering a deposit with a competitive interest rate to boost its funding share. If it chooses $\xi_j > 0$ and $\tau_j > 0$, then it is offering this product to lessen the burden of the liquidity rule and hence engaging in regulatory arbitrage. The value of τ_j thus reveals the source of any spread between DLP returns and storage.

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[4]Adrian, Ashcraft, and Cetorelli (2013) define regulatory arbitrage as "a change in structure of activity which does not change the risk profile of that activity, but increases the net cash flows to the sponsor by reducing the costs of regulation." In principle, we could introduce a small cost to pursuing the accounting maneuvers that permit regulatory arbitrage. We do not do this here as it would clutter the exposition without producing much additional insight.

2.4 Optimization Problem of Representative Bank

The expected profit of bank j at $t = 0$ is:

$$\Upsilon_j \equiv (1 + i_A)^2 (X_j - R_j) + (1 + i_L) R_j - \left[i_L \bar{\theta} X_j + X_j + (1 - \bar{\theta}) \xi_j W_j \right] - \tfrac{\phi}{2} X_j^2 \qquad (6)$$

where W_j and X_j are given by (1) and (3) respectively. The first term in (6) is revenue from investment. The second term is revenue from lending reserves on the interbank market. The third term is the bank's expected funding cost, namely the expected cost of borrowing reserves on the interbank market and the expected payments to households. The fourth term is a general operating cost (with $\phi > 0$) which is quadratic in the bank's funding share. Operating costs will play a minimal role until Section 3.

The representative bank chooses the attractiveness of its DLPs ξ_j, the intensity of its off-balance-sheet activities $\tau_j \in [0, 1]$, and its reserve holdings R_j to maximize Υ_j subject to the liquidity rule in (5). The Lagrange multiplier on (5) is the shadow cost of holding reserves. We denote it by μ_j. The multipliers on $\tau_j \geq 0$ and $\tau_j \leq 1$ are denoted by η_j^0 and η_j^1 respectively. The first order conditions with respect to R_j, τ_j, and ξ_j are then:

$$\mu_j = (1 + i_A)^2 - (1 + i_L) \qquad (7)$$

$$\eta_j^1 = \eta_j^0 + \alpha \mu_j W_j \qquad (8)$$

$$\xi_j = \underbrace{\frac{(1 - \bar{\theta}) i_L + (1 - \alpha) \mu_j - \phi X_j}{2 (1 - \bar{\theta})} \times \frac{\rho}{\omega}}_{\text{competitive motive}} + \underbrace{\frac{\alpha \mu_j}{2 (1 - \bar{\theta})} \times \tau_j}_{\text{reg. arbitrage motive}} \qquad (9)$$

The first term on the right-hand side of equation (9) captures what we will call the competitive motive for DLP issuance. If this term is positive, then bank j wants to offer higher DLP returns in order to attract more funding. Recall that bank j's total funding, X_j, is given by equation (3). Each bank takes $\bar{\xi}$ as given so increasing ξ_j relative to $\bar{\xi}$ increases X_j. The second term on the right-hand side of equation (9) captures what we will call the regulatory arbitrage motive for DLP issuance. In the absence of a liquidity rule ($\alpha = 0$), there is no regulatory arbitrage motive. There is also no such motive when the interbank rate is high enough to make the shadow cost of holding reserves (μ_j) zero.

2.5 Results for Benchmark Model

We now study the equilibrium properties of the benchmark model.

We start by establishing the existence of an equilibrium where banks are content offering only storage (i.e., $\xi_j^* = 0$, where asterisks denote equilibrium values). We have already established that there is no regulatory arbitrage motive for DLP issuance without liquidity regulation ($\alpha = 0$). The following proposition establishes the conditions under which there is also no competitive motive:

Proposition 1 *Suppose $\phi \leq \overline{\phi}$ where $\overline{\phi}$ is a positive threshold that depends on parameters other than α and ρ. If $\alpha = 0$ and $\phi < \overline{\phi}$, then $\xi_j^* = 0$ if and only if $\rho = 0$. If $\alpha = 0$ and $\rho > 0$, then $\xi_j^* = 0$ if and only if $\phi = \overline{\phi}$.*

With $\rho = 0$, there is no competitive motive for DLP issuance because each bank perceives its funding share as fixed. With $\rho > 0$ and a high operating cost (i.e., $\phi = \overline{\phi}$), there is also no competitive motive because banks do not want to get bigger. Therefore, $\alpha = 0$ with either one of these parameterizations delivers an equilibrium where only storage is offered.

Suppose the economy starts in such an equilibrium. Proposition 2 shows that increasing α above a threshold value $\widetilde{\alpha}$ triggers the issuance of off-balance-sheet DLPs. The benchmark model thus delivers a shadow banking sector after the introduction of a sufficiently strict liquidity rule:

Proposition 2 *Suppose $\rho = 0$. There is a unique $\widetilde{\alpha} \in [0, \overline{\theta})$ such that $\xi_j^* = 0$ if $\alpha \leq \widetilde{\alpha}$ and $\xi_j^* > 0$ with $\tau_j^* = 1$ otherwise.*

The incentive to issue DLPs in Proposition 2 does not come from competition since $\rho = 0$ eliminates the competitive motive. Instead, DLPs are issued because they can be booked off-balance-sheet, away from the binding liquidity rule. Similar intuition can be delivered with $\rho > 0$ and ϕ sufficiently high.

Consider now the aggregate effects. Proposition 3 shows that introducing a liquidity minimum into the benchmark model lowers both the interbank rate and total credit in equilibrium:

Proposition 3 *Fix all parameters except for α. For any $\rho \geq 0$, the equilibrium under $\alpha = 0$ has a higher interbank rate and more total credit than the equilibrium under any $\alpha > 0$. It can also be shown, fixing all parameters except for α and ρ, that the equilibrium under $\alpha = 0$ and $\rho = 0$ has a higher interbank rate and more total credit than the equilibrium under any combination of $\alpha > 0$ and $\rho > 0$.*

Proposition 3 is effectively the market mechanism at work. The interbank market in the benchmark model is competitive so all banks are price-takers. Suppose there is no government intervention ($\alpha = 0$). At low interbank rates, none of these price-taking banks will

find it profitable to hold reserves. Instead, they will all want to invest heavily in the long-term project to earn a return, relying on the interbank market for cheap liquidity to pay off liquidity shocks. Liquidity demand at $t = 1$ will then exceed liquidity supply, which cannot be an equilibrium. The equilibrium interbank rate must therefore be high to incent banks to hold reserves when $\alpha = 0$. The introduction of a liquidity minimum by the government ($\alpha > 0$) substitutes somewhat for this market-based discipline and the equilibrium interbank rate falls.[5] The result on total credit then follows immediately from equation (4), given that total credit equals $X - R_j$.

3 Full Model: Heterogeneity in Bank Size

We now extend the benchmark model to include a big bank. By definition of being big, this bank will internalize how all of its choices affect the equilibrium.

We keep the continuum of small banks, $j \in [0, 1]$, and index the big bank by k. DLP demands are $W_j = \omega \xi_j$ and $W_k = \omega \xi_k$, similar to equation (1). The funding attracted by each bank is an augmented version of equation (3), namely:

$$X_j = 1 - \delta_0 + \delta_1 \left(\overline{\xi_j} - \xi_k \right) + \delta_2 \left(\xi_j - \overline{\xi_j} \right) \tag{10}$$

$$X_k = \delta_0 + \delta_1 \left(\xi_k - \overline{\xi_j} \right) \tag{11}$$

where total funding in the economy has been normalized to $X = 1$ and $\overline{\xi_j}$ is the average return on small bank DLPs. Here, δ_1 is the competition parameter between the big and small banks while δ_2 affects the competition among small banks.[6] Small banks take $\overline{\xi_j}$ and ξ_k as given, along with being interbank price-takers. In a symmetric equilibrium, the profit-maximizing choice of ξ_j equals $\overline{\xi_j}$.

The big bank does not take $\overline{\xi_j}$ as given. It is also not an interbank price-taker. As a result, the interbank rate will depend on the big bank's realized liquidity shock. This makes the big bank's shock an aggregate shock so, in Appendix C, we show that adding aggregate

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[5]Another way to think about this is as follows: the government intervention makes reserves more scarce, on the margin, which drives down their yield. See Farhi, Golosov, and Tsyvinski (2009) for a different environment in which a liquidity minimum decreases interest rates.

[6]The intuition behind equations (10) and (11) is easy to derive from a household problem with nested preferences over banks. For example, if households have CES preferences over big versus small banks and, within small banks, CES preferences over the different small banks, then X_j will be increasing in (i) the spread between the average small bank return and the big bank return and (ii) the spread between bank j's return and the average small bank return, provided both elasticities of substitution are greater than one. Equation (10) is a linear representation of this logic. If the elasticity of substitution between small banks equals one, then bank j's own return relative to the average small bank return will not matter, which is what $\delta_2 = 0$ would capture.

shocks to the benchmark model with only small banks does not change Proposition 3. It is therefore the strategic nature of the big bank's decision-making that will drive the substantive differences between the results of the full model being considered here and the results of the benchmark model considered earlier.

Let i_L^s denote the interbank rate when the big bank realizes θ_s at $t = 1$, where $s \in \{\ell, h\}$. The interbank market clearing condition for $s = h$ is:

$$R_j + R_k + \psi i_L^h = \overline{\theta} X_j + \theta_h X_k \tag{12}$$

The left-hand side of (12) captures the supply of liquidity while the right-hand side captures the demand for liquidity, in an equilibrium where small banks are symmetric. All decisions are made at $t = 0$ so it will be enough for the big bank to affect the expected interbank rate, $i_L^e \equiv \pi i_L^\ell + (1 - \pi) i_L^h$. We can therefore simplify the exposition by fixing $i_L^\ell = 0$ and letting i_L^e move with i_L^h, where i_L^h is determined as per equation (12). It will be verified in the proposition proofs that $i_L^\ell = 0$ does not result in a liquidity shortage when the big bank realizes $\theta_\ell < \theta_h$ in this class of equilibria.

3.1   Optimization Problem of Big Bank

At $t = 0$, the big bank's expected profit is:

$$\Upsilon_k \equiv (1 + i_A)^2 (X_k - R_k) + \left[1 + (1 - \pi) i_L^h\right] R_k - \left[(1 - \pi) i_L^h \theta_h X_k + X_k + \left(1 - \overline{\theta}\right) \omega \xi_k^2\right] - \tfrac{\phi}{2} X_k^2$$

The interpretation is similar to equation (6): the first term is revenue from investment, the second term is the potential expected revenue from lending reserves, the third term is the big bank's expected funding cost, and the fourth term is an operating cost.

The big bank chooses R_k, τ_k, and ξ_k to maximize Υ_k subject to three sets of constraints. First are the aggregate constraints, namely funding shares as per (10) and (11) and market clearing as per (12). The market clearing equation connects R_k and i_L^h so saying that the big bank chooses R_k with i_L^h determined by (12) is equivalent to saying that it chooses i_L^h with R_k determined by (12). This is the sense in which the big bank is a price-setter on the interbank market.

The second set of constraints are the first order conditions of small banks. The representative small bank solves essentially the same problem as before. Its objective function is still given by (6) but with $(1 - \pi) i_L^h$ as the interbank rate and X_j as per equation (10). It is also still subject to the liquidity rule in (5) with $\tau_j \in [0, 1]$. The results in Section 2.5 on which we want to build involved $\xi_j \geq 0$ so we will add this as an explicit condition here.

14

The last set of constraints on the big bank's problem are inequality constraints, namely a liquidity rule for the big bank and non-negativity conditions:

$$R_k \geq \alpha \left(X_k - \tau_k W_k \right)$$

$$\tau_k \in [0, 1]$$

$$\xi_k \geq 0$$

$$\mu_j \geq 0$$

where μ_j is the shadow cost of reserves or, equivalently, the Lagrange multiplier on the liquidity rule in the small bank problem. Each inequality constraint listed above can be either binding or slack.

3.2 Results for Full Model

An equilibrium in the full model is characterized by the first order conditions from the small bank problem, the first order conditions from the big bank problem, and interbank market clearing.

Following Section 2.5, we first discuss the equilibrium where all banks offer only storage. We know from our analysis of the benchmark model that small banks will have a competitive motive for DLP issuance if (i) they do not take their funding shares as given and (ii) operating costs are low enough that they want to expand. Notice from equation (10) that small banks will not take their funding shares as given if $\delta_2 > 0$.

If instead $\delta_2 = 0$, with $\delta_1 \geq 0$, then equation (10) simplifies to:

$$X_j = 1 - \delta_0 + \delta_1 \left(\overline{\xi_j} - \xi_k \right)$$

In a symmetric equilibrium, $\xi_j = \overline{\xi_j}$ so there is still an indirect effect of ξ_j on the funding share X_j. However, small banks are not setting ξ_j to exploit this effect. Instead, small banks take their funding shares as given and the first order conditions from their optimization problem deliver:

$$\mu_j \left[R_j - \alpha \left(X_j - \omega \xi_j \right) \right] = 0 \text{ with complementary slackness} \tag{13}$$

$$\mu_j = (1 + i_A)^2 - \left[1 + (1 - \pi) i_L^h \right] \tag{14}$$

$$\xi_j = \frac{\alpha \mu_j}{2 \left(1 - \overline{\theta} \right)} \tag{15}$$

15

with $\tau_j = 1$ for the reasons discussed at the beginning of the proof of Proposition 2. In words, these first order conditions say that small banks are content offering only storage unless there is a liquidity rule ($\alpha > 0$) and a shadow cost to holding reserves ($\mu_j > 0$). With $\alpha > 0$ and $\mu_j > 0$, small banks would also offer off-balance-sheet DLPs, which is the same regulatory arbitrage motive for DLP issuance seen in equations (8) and (9) of the benchmark model.

Clearly, $\alpha = 0$ will be enough to deliver an initial equilibrium without regulatory arbitrage so that small banks do indeed offer only storage at the combination of $\alpha = 0$ and $\delta_2 = 0$. To simplify the analytical exposition and develop clear intuition, this section will study a move from $\alpha = 0$ to $\alpha > 0$, assuming $\delta_2 = 0$. In Section 4.2, we will calibrate the starting and ending values of α to data and allow $\delta_2 > 0$. We will then calibrate an operating cost parameter for small banks (ϕ_j) that is consistent with minimal DLP issuance in the initial steady state.[7]

The property that the big bank offers only storage when $\alpha = 0$ can also be delivered in one of two ways. The first approach is to set $\delta_1 = 0$ in equation (11) so that the big bank's funding share is fixed at $X_k = \delta_0$. The second approach is to keep the funding share endogenous ($\delta_1 > 0$) and set a sufficiently high operating cost parameter which eliminates any incentive for the big bank to increase its funding share (and hence issue DLPs) at the configuration of parameters in the initial equilibrium. We will present analytical results for both approaches to isolate how, if at all, an endogenous funding share affects the big bank's decision-making. When considering the second approach in the analytical results below, we will set ϕ so that, in the initial equilibrium, ξ_k is exactly zero as opposed to being constrained by zero. The quantitative analysis in Section 4.2 will also follow the second approach and allow $\delta_1 > 0$. We will then calibrate a ϕ_k for the big bank to distinguish it from the ϕ_j for the small banks mentioned above.

Having explained the defining features of the initial equilibrium, let us consider the distribution of reserves between big and small banks in this equilibrium. The distribution of reserves across banks was not a consideration in the benchmark model because all banks were ex ante identical price-takers. Now, however, we have a big bank who is a price-setter so its reserve choice may differ from that of small banks.

Proposition 4 *Suppose $\alpha = 0$. Consider $\delta_2 = 0$ and either $\delta_1 > 0$ with ϕ sufficiently positive or $\delta_1 = 0$ so that the initial equilibrium has $\xi_j^* = \xi_k^* = 0$. If i_A lies within an intermediate range, then the initial equilibrium also involves $\mu_j^* > 0$, $R_j^* = 0$, and $R_k^* > 0$.*

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[7]With $\delta_2 = 0$, small banks never have a competitive motive for DLP issuance. With $\delta_2 > 0$ and ϕ_j sufficiently high, they have no such motive at the initial equilibrium. The second approach imposes weaker conditions than the first but the main qualitative results do not depend on which approach is used.

Proposition 4 says that reserves in the initial equilibrium are held disproportionately by the big bank when the returns to investment (i_A) are moderate. The big bank's willingness to hold liquidity reflects its status as an interbank price-setter. In particular, the big bank understands that not holding enough liquidity will increase its funding costs should it experience a high liquidity shock. In contrast, the price-taking small banks invest all their funding in projects and rely on the interbank market, which now includes the big bank, to honor short-term obligations.

We saw in Section 2.5 that introducing a liquidity minimum into the benchmark model with only small banks decreased both the interbank rate and the total amount of credit. In other words, regulation had the intended effect. We want to see whether this is still the case in the full model with big and small banks or whether there are conditions under which the result is reversed.

To fix ideas, suppose the government moves from $\alpha = 0$ to $\alpha = \bar{\theta}$. As shown next, introducing a liquidity minimum into the full model can lead to an increase in the interbank rate, in sharp contrast to the benchmark prediction:

Proposition 5 *Keep $\delta_2 = 0$ as in Proposition 4. The following are sufficient for the equilibrium under $\alpha = \bar{\theta}$ to have a higher interbank rate i_L^{h*} than the equilibrium under $\alpha = 0$, while preserving slackness of the big bank's liquidity rule $(R_k^* > \alpha X_k^*)$, bindingness of the small bank liquidity rule $(\mu_j^* > 0)$, and feasibility of $i_L^{\ell*} = 0$:*

1. *Suppose $\delta_1 = 0$ so that the big bank's funding share is fixed. The sufficient conditions are: π sufficiently high, θ_ℓ and $\frac{\psi}{\omega}$ sufficiently low, and i_A within an intermediate range.*

2. *Suppose $\delta_1 = \omega > 0$ so that the big bank's funding share is endogenous. Also set ϕ so that ξ_k^* is exactly zero at $\alpha = 0$ for the reasons discussed earlier in this section. The sufficient conditions are: π sufficiently high, θ_ℓ and $\frac{\psi}{\omega}$ sufficiently low, and i_A and δ_0 within intermediate ranges.*

There is a non-empty set of parameters satisfying the sufficient conditions in both 1 and 2. All else constant, the model with an endogenous funding share generates a larger increase in the interbank rate than the model with a fixed funding share.

We devote Section 3.2.1 to explaining the interbank rate results just established. Section 3.2.2 will then establish several other results that distinguish the full model from the benchmark, including the effect of liquidity regulation on total credit. Note that the parameter restrictions in Proposition 5 are sufficient conditions. In the quantitative analysis of Section 4, we will explore the size of the parameter space that supports the results.

3.2.1 Intuition for Interbank Rate Response

To explain Proposition 5, it will be useful to summarize all the forces behind the big bank's choice of i_L^h. Differentiating the big bank's objective function with respect to i_L^h, we get:

$$\frac{\partial \Upsilon_k}{\partial i_L^h} \propto \underbrace{R_k - \theta_h X_k}_{\text{direct motive}} - \underbrace{\left[\frac{(1+i_A)^2 - 1}{1-\pi} - i_L^h\right] \frac{\partial R_k}{\partial i_L^h}}_{\text{reallocation motive}} + \underbrace{\left[\frac{(1+i_A)^2 - 1 - \phi X_k}{1-\pi} - \theta_h i_L^h\right] \frac{\partial X_k}{\partial i_L^h}}_{\text{funding share motive}}$$

(16)

The equilibrium i_L^h solves $\frac{\partial \Upsilon_k}{\partial i_L^h} = 0$ when the relevant inequality constraints in the big bank's problem are slack. This is the appropriate case given the statement of Proposition 5. We will start by explaining the three motives identified in (16). We will then explain how the strength of each motive varies with α in order to understand why moving from $\alpha = 0$ to $\alpha = \bar{\theta}$ generates a higher interbank rate.

First is the direct motive. The big bank has reserves R_k and a funding share X_k. Its net reserve position when hit by a high liquidity shock is therefore $R_k - \theta_h X_k$. Each unit of reserves is valued at an interest rate of i_L^h when the big bank's shock is high so, on the margin, an increase in i_L^h changes the big bank's profits by $R_k - \theta_h X_k$.

Second is the reallocation motive. The idea is that changes in i_L^h also affect how many reserves the big bank needs to hold in a market clearing equilibrium. If $\frac{\partial R_k}{\partial i_L^h} < 0$, then an increase in i_L^h elicits enough liquidity from other sources to let the big bank reallocate funding from reserves to investment. On the margin, the value of this reallocation is the shadow cost of reserves, hence the coefficient on $\frac{\partial R_k}{\partial i_L^h}$ in (16).

Third is the funding share motive. The idea is that changes in i_L^h also affect how much funding the big bank attracts when funding shares are endogenous. If $\frac{\partial X_k}{\partial i_L^h} > 0$, then an increase in i_L^h decreases the shadow cost of reserves and curtails the DLP offerings of small banks by enough to boost the big bank's funding share. The coefficient on $\frac{\partial X_k}{\partial i_L^h}$ in (16) captures the marginal value of a higher funding share for the big bank. We will discuss this coefficient in more detail below.

To gain some insight into how changes in α will affect the solution to $\frac{\partial \Upsilon_k}{\partial i_L^h} = 0$ through each motive, we start with the case of fixed funding shares ($\delta_1 = 0$). Consider first the direct motive. Using the market clearing condition:

$$R_k - \theta_h X_k \stackrel{\delta_1 = 0}{=} \bar{\theta}(1 - \delta_0) - \psi i_L^h - \underbrace{\alpha\left(1 - \delta_0 - \frac{\alpha\omega(1-\pi)}{2(1-\bar{\theta})}\left[\frac{(1+i_A)^2 - 1}{1-\pi} - i_L^h\right]\right)}_{R_j \text{ as per small bank FOCs in (13) to (15)}}$$

(17)

18

For a given value of i_L^h, the magnitude of the direct motive in (17) depends on α through the reserve holdings of small banks. There are two competing effects. On one hand, higher α forces small banks to hold more reserves per unit of on-balance-sheet funding. On the other hand, higher α can compel small banks to engage in regulatory arbitrage, decreasing their on-balance-sheet funding as they offer off-balance-sheet DLPs ($\xi_j > 0$ with $\tau_j = 1$). The net effect is ambiguous so we must look beyond the direct motive to fully understand Proposition 5.

With fixed funding shares, the only other motive is the reallocation motive, where:

$$\left.\frac{\partial R_k}{\partial i_L^h}\right|_{\delta_1 = 0} = -\psi - \frac{\alpha^2 \omega (1 - \pi)}{2 (1 - \overline{\theta})} < 0 \qquad (18)$$

This expression is negative for two reasons. First, and as captured by the first term in (18), a higher interbank rate will attract more external liquidity, allowing the big bank to hold fewer reserves. Second, and as captured by the second term in (18), small banks will increase their reserves when the interbank rate increases, also allowing the big bank to hold fewer reserves. The effect of i_L^h on R_j that underlies the second term in (18) works through the regulatory arbitrage motive of small banks: there is less incentive to circumvent a liquidity minimum when liquidity is expected to be expensive. We can also infer from the second term in (18) that the effect of i_L^h on R_j strengthens with α. This is both because R_j is more responsive to changes in ξ_j at high α (see equation (13)) and because ξ_j is more responsive to changes in i_L^h at high α (see equations (14) and (15)).

This discussion helps explain the first bullet in Proposition 5: when funding shares are fixed, high α makes it easier for the big bank to use high interbank rates to incent small banks to share the burden of keeping the system liquid.

Does the same intuition extend to the case of endogenous funding shares? No, because:

$$\left.\frac{\partial R_k}{\partial i_L^h}\right|_{\delta_1 = \omega} = -\psi + \frac{\alpha \omega \pi (\theta_h - \theta_\ell)(1 - \pi)}{2 (1 - \overline{\theta})} \qquad (19)$$

An increase in i_L^h still decreases ξ_j but now a decrease in ξ_j also decreases how much funding X_j small banks attract in equilibrium, which then decreases how many reserves they hold. This effect is strong enough to make the second term in (19) positive, in contrast to the second term in (18) which was negative.

We must therefore turn to the funding share motive to fully understand why introducing a liquidity minimum can increase the equilibrium interbank rate when funding shares are endogenous. Recall from (16) the expression for the funding share motive and note:

$$\left.\frac{\partial X_k}{\partial i_L^h}\right|_{\delta_1 = \omega} = \frac{\alpha\omega(1 - \pi)}{2(1 - \bar{\theta})} > 0 \tag{20}$$

We already know from the discussion of (19) that an increase in i_L^h decreases ξ_j which then decreases how much funding X_j small banks attract in equilibrium. Total funding is normalized to one so the decrease in X_j implies an increase in the big bank funding share X_k. The expression in (20) is therefore positive. The magnitude of this expression increases with α because ξ_j is more responsive to changes in i_L^h at high α (see again equations (14) and (15)). It is therefore easier for the big bank to increase its funding share by increasing i_L^h when α is high.

There is, of course, a difference between the ability to increase funding share and the desire to do so. To complete the intuition, let us reconcile the big bank's desire to increase its funding share when α is high with the existence of convex operating costs. Return to the coefficient on $\frac{\partial X_k}{\partial i_L^h}$ in (16). All else constant, moving from $\alpha = 0$ to $\alpha = \bar{\theta}$ will trigger regulatory arbitrage by small banks ($\xi_j > 0$ with $\tau_j = 1$). The presence of $\xi_j > 0$ will then erode the big bank's funding share X_k, lowering its marginal operating cost ϕX_k.

We can now understand the second bullet in Proposition 5 as follows: when funding shares are endogenous, high α makes it easier for the big bank to use high interbank rates to stop small banks from encroaching on its funding share. The last part of Proposition 5 establishes that sizeable increases in the interbank rate are most consistent with this sort of asymmetric competition, wherein the big bank uses its price impact on the interbank market to fend off competition from small banks and their off-balance-sheet activities.

3.2.2   Credit Boom and Cross-Sectional Predictions

We have now explained how the full model can deliver an increase in the equilibrium interbank rate when a liquidity minimum is introduced. Next, we establish how the introduction of this regulation changes the liquidity ratios of big and small banks, the liquidity services they provide, and the total amount of credit generated in equilibrium:

Proposition 6 *Invoke the parameter conditions from Proposition 5. The equilibrium under $\alpha = \bar{\theta}$ has higher total credit $(1 - R_j^* - R_k^*)$ and a smaller gap between the on-balance-sheet liquidity ratios of big and small banks at $t = 0$ than the equilibrium under $\alpha = 0$. Moreover, $\xi_j^* > \xi_k^*$ at $\alpha = \bar{\theta}$, with $\xi_k^* > 0$ if and only if funding share is endogenous. This is in contrast to $\xi_j^* = \xi_k^* = 0$ at $\alpha = 0$.*

In sharp contrast to the benchmark model with only small banks, Proposition 6 shows that introducing a liquidity minimum into the full model increases total credit. The on-balance-

sheet lending of small banks falls but there are several offsetting channels and, as we will see below, all rely on the ability of the big bank to affect the interbank market through its reserve holdings.

As was the case in the benchmark model, small banks move into off-balance-sheet DLPs after the introduction of a sufficiently strict liquidity minimum. As they push to attract funding into these products, the small banks offer interest rates that exceed the rates on traditional deposits (storage). Effectively, the tightening of liquidity rules implies a higher regulatory burden for on-balance-sheet activities relative to off-balance-sheet activities and, when the rule is strict enough to constrain the small banks, they are willing to pay higher interest rates for off-balance-sheet DLPs relative to storage.

Under the parameter conditions in Proposition 5, which are also the parameter conditions in Proposition 6, the liquidity minimum is strict enough to constrain the small banks but not strict enough to constrain the big bank. The big bank can unilaterally affect the interbank market so it internalizes the impact of its reserve holdings on the expected price of interbank liquidity. Compared to the small banks, then, the big bank always undertakes less long-term investment per unit of funding attracted. In other words, the big bank has a higher liquidity ratio than the small banks at $t = 0$. This is why a liquidity minimum can introduce a binding constraint on small banks without also introducing one on the big bank.

The big bank thus has no incentive to offer off-balance-sheet DLPs after the liquidity minimum in Propositions 5 and 6 is introduced. If its funding share is fixed, the big bank also has no incentive to offer on-balance-sheet DLPs, hence the statement in Proposition 6 that $\xi_k^* > 0$ if and only if funding share is endogenous. However, as discussed in Section 3.2.1, tougher liquidity regulation makes the interbank rate a more powerful tool for getting the small, price-taking banks to share the burden of keeping the system liquid. All else constant, the interbank market at $t = 1$ will be less liquid, and the expected interbank rate will rise, if the big bank holds fewer reserves at $t = 0$. Proposition 6 shows that the gap between the on-balance-sheet liquidity ratios of big and small banks narrows after the liquidity minimum is introduced. The liquidity ratio of small banks, as measured *on* balance sheet, must rise to comply with the regulation. The liquidity ratio of the big bank, however, falls as the big bank shifts from reserves to investment to tighten the interbank market. On net, total liquidity falls and total credit rises on the heels of the big bank's strategy.

Consider now the more general case where the big bank's funding share is endogenous. All else constant, some funding will migrate from the big bank to the small banks, as the latter begin offering off-balance-sheet DLPs that pay higher interest rates than storage. We have already explained that the big bank internalizes the impact of its reserve holdings on the expected price of interbank liquidity and hence has a higher liquidity ratio than the small

banks. Therefore, the reallocation of funding from the big bank to the small banks, as the latter poach from the former, decreases total liquidity and increases total credit. This is one of two channels for the credit boom when funding shares are endogenous.

The big bank can respond to its loss of funding by offering its own DLPs with high interest rates. Naturally, this is costly because of the high rates. Proposition 6 shows that the big bank engages in some of this activity ($\xi_k^* > 0$), but not to the same extent as the small banks ($\xi_k^* < \xi_j^*$). Moreover, unlike the small banks who are constrained by the liquidity minimum and therefore issue all of their DLPs off-balance-sheet ($\tau_j^* = 1$), the big bank is not constrained and is therefore indifferent between any $\tau_k^* \in [0, 1]$.

The big bank can also respond to its loss of funding by using its price impact on the interbank market. We discussed this motive and its implications for the interbank rate in Section 3.2.1. Small banks have less incentive to skirt the liquidity minimum if they expect liquidity to be expensive. The big bank therefore tightens the interbank market to make small banks scale back their issuance of DLPs. The gap between the on-balance-sheet liquidity ratios of big and small banks again narrows but, unlike the case with fixed funding shares, the big bank is now using its price impact on the interbank market to fight the competitive pressures that arise as the small banks engage in regulatory arbitrage. While this strategy by the big bank curbs *some* of the increase in total credit from the first channel, it also boosts credit directly because the big bank is shifting from reserves to investment to tighten the interbank market. This is the second channel for the credit boom when funding shares are endogenous.

4   Quantitative Analysis

We have focused so far on qualitative predictions of the theory. We now want to study quantitative implications. We choose China as the setting for our quantitative analysis. In addition to being one of the world's largest economies, China has experienced a near doubling of its debt-to-GDP ratio over the past decade, along with unprecedented growth in its ratio of private credit to private savings. Our model predicts that some credit booms are caused by stricter liquidity regulation so we are interested to know whether stricter liquidity regulation can account for at least part of the Chinese experience.

Liquidity rules in China involve reserve requirements and, until late 2015, a loan-to-deposit cap. The loan-to-deposit cap was introduced in 1995 to prevent banks from lending more than 75% of the value of their deposits to non-financial borrowers. The remaining 25% had to be kept liquid, with reserve requirements dictating how this liquidity was to be

divided between pure reserves and other liquid assets. In practice, enforcement of the 75%
loan-to-deposit cap was lax until 2008, when the China Banking Regulatory Commission
(CBRC) announced a tougher stance and began increasing the frequency of its loan-to-
deposit monitoring. The enforcement action began with CBRC monitoring the end-of-year
loan-to-deposit ratios of all banks more carefully. CBRC then switched to monitoring end-
of-quarter ratios in late 2009, end-of-month ratios in late 2010, and average daily ratios
in mid-2011. The increasing frequency of CBRC's loan-to-deposit enforcement was also
complemented by a rapid increase in the reserve requirements set by the central bank. We
refer the reader to Hachem (2018) and Song and Xiong (2018) for more on China's regulatory
environment and financial institutions.

Heterogeneity in interbank pricing power was central to our theory of unintended credit
booms in Section 3. Credit did not increase after the introduction of a liquidity minimum
in the benchmark model with a competitive interbank market. We would therefore like to
establish that large commercial banks in China can impact the interbank market to a much
greater extent than small commercial banks before applying the model to China. This is
done in Section 4.1. We then calibrate the model to Chinese data in Section 4.2. We use the
calibrated model to study how large a credit boom our model can produce (Section 4.3) and
present a structural estimation to evaluate the importance of various shocks (Section 4.4).

4.1 Interbank Market Structure in China

The Chinese economy is served by both big and small banks. The small banks include
twelve joint-stock commercial banks (JSCBs) which operate nationally, as well as over two
hundred city banks operating in specific regions. Many rural banks have also emerged.
The JSCBs are typically larger than the city and rural banks but all of these banks are still
individually small when compared to China's big banks (the Big Four). The Big Four are the
four commercial banks established by the central government after the Cultural Revolution.
Market-oriented reforms initiated in the 1990s made the Big Four almost entirely profit-
driven and removed government involvement from day-to-day operations. However, a legacy
of minimal competition between these four banks remains. China's banking sector is therefore
well approximated by a model with one big bank and many small banks.

Importantly, this big bank, as represented by the Big Four, is large enough to impact
prices on the interbank market. China has both an interbank repo market and an uncollat-
eralized money market. We will focus on the repo market since it is vastly larger. To better
understand the market structure and the relative importance of the Big Four, we obtained
anonymized data on each individual trade that took place in China's interbank repo market

during June 2013. The majority of transactions had either an overnight or a seven-day maturity and there was not much variation in collateral or haircuts so we can focus on interest rates and loan amounts.

The main sample for the analysis excludes June 20 and 21. There was a dramatic spike in interbank interest rates on June 20, which many observers characterized as either a market liquidity crisis or a failure by the government to respond. In Appendix D, we conduct a detailed analysis of China's interbank repo market around this spike and demonstrate that the traditional narrative is incorrect: agents of the government provided generous amounts of liquidity but interbank rates did not fall because the funds were absorbed by the Big Four and re-intermediated at much higher interest rates. This is a concrete example of price-setting by the Big Four and we will refer back to it in what follows.

Figure 1 graphs the interbank network for the main sample. Each node represents a group of banks. In addition to the Big Four, the JSCBs, and other smaller players, China has three policy banks which participate in the interbank repo market. The policy banks are the agents of the government referred to above. They are not commercial banks. Instead, they raise money on bond markets and take directives from the central government about where to invest. The flow of funds between the nodes in Figure 1 is indicated by the direction of the arrows, with thicker arrows signifying more trade.

Eigenvector centrality is one way to put numbers on the approximate importance of each of the nodes in Figure 1. It is based on the idea that a central node is connected to other central nodes. We only need to specify an adjacency matrix A that summarizes the connections between the nodes. The centrality of node i is then the ith element of the eigenvector associated with the largest eigenvalue of A. The first column in Table 1 reports the results when the connection from node i to node s in the adjacency matrix is based on average daily lending from i to s. The second column reports the results when the connection from i to s is based on average daily borrowing by i from s. It is clear from these two columns that the policy banks and the Big Four are the central lending nodes in the main sample.

The third and fourth columns of Table 1 repeat the eigenvector centrality analysis with adjacency matrices constructed using data from June 20, as opposed to the main sample. We know from the analysis in Appendix D that the spike in interbank rates on June 20 was driven by the Big Four. The results in Table 1 show minimal change in the centrality of the policy banks on June 20 relative to the main sample. In contrast, the Big Four became much less central on the lending side and much more central on the borrowing side. Therefore, the lending and borrowing decisions of the Big Four have a dramatic effect on the tightness of the interbank market, even if the policy banks remain a central lending node.

We can also compare the ability of each node in Figure 1 to impact interbank conditions

by calculating the elasticity of total lending by the interbank market with respect to the money that each of these nodes brings into the market. The procedure for computing the elasticities is described in Appendix E and the results using the main sample are reported in the last column of Table 1. An elasticity of 0.29 for the Big Four means that, on an average trading day in the main sample, a 1 percent increase in the amount of money brought into the interbank market by the Big Four leads to a 0.29 percent increase in total lending by this market. This is 3.7 times the elasticity for the JSCBs and 0.5 times the elasticity for the policy banks, which is substantial given the quantity adjustments that the Big Four can make. The scale of these adjustments was apparent on June 20. Policy banks brought 72 percent more money into the interbank market than they did on an average trading day in the main sample. Total lending by the interbank market should have then increased by 41 percent, given the elasticity of 0.57 in Table 1. However, the Big Four brought 183 percent less money into the interbank market than they did on an average trading day in the main sample and, with an elasticity of 0.29, this leads to a 53 percent decrease in total lending by the interbank market, more than enough to offset the efforts of the policy banks.

4.2 Calibration

We calibrate the model to data from 2014. Our primary dataset is the Wind Financial Terminal, supplemented by data from bank annual reports.

We take the time from $t = 0$ to $t = 2$ to be a quarter, with all interest rates quoted on an annualized basis. China's central bank (PBOC) set benchmark interest rates for traditional deposits in China until late 2015. Recall from Section 2.1 that our model has a normalized liquidity service called storage with $i_B = i_D = 0$. In the calibration, we will re-normalize storage to be a traditional deposit that has $i_B > 0$ and $i_D > 0$ as set by the PBOC. Any DLPs offered in equilibrium will pay an additional return relative to these positive rates.

We set $(1 + i_D)^2 = 1.026$ to match the average benchmark interest rate of 2.6% for 3-month deposits in China. We set $(1 + i_B)^2 = 1.004$ to match the average benchmark interest rate of 0.4% for demand deposits. The central bank's benchmark interest rate for loans with a maturity of less than six months averaged 5.6%. We set $(1 + i_A)^2 = 1.05$ since banks can offer a discount of up to 10% on the benchmark loan rate.[8]

In Propositions 5 and 6, we considered a tightening of liquidity regulation from $\alpha = 0$ to $\alpha = \bar{\theta}$. We relax this in the calibration. The starting α is not restricted to be zero and there is no presumed relationship between the new regulation and $\bar{\theta}$. Instead, we set the new

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[8]We are assuming the same return i_A for all banks. In practice, different banks may invest in different sectors but, adjusting for political risk, the returns are roughly comparable in China. Some anecdotal evidence can be found in Dobson and Kashyap (2006).

liquidity regulation to $\alpha = 0.25$ since CBRC was strictly enforcing the 75% loan-to-deposit cap in 2014. We then calibrate the average liquidity shock, $\bar{\theta} \equiv \pi \theta_\ell + (1 - \pi) \theta_h$, to get an average interbank rate of 3.6% when $\alpha = 0.25$. The 3.6% target is the weighted average seven-day interbank repo rate in 2014. The seven-day rate is the longest maturity for which there is significant trading volume. It is difficult to target the level of shorter-term (e.g., overnight) repo rates since there are two model periods and each period must be long enough to match reasonable data on the level of loan returns (i_A). This is merely a level effect: the correlation between the overnight and seven-day repo rates is around 0.95. We normalize the low liquidity shock to $\theta_\ell = 0$ and set its probability to $\pi = 0.75$. The calibration of $\bar{\theta}$ then pins down the high liquidity shock θ_h.

To set the external liquidity parameter (ψ), we look at data on monetary injections by the PBOC over a sufficiently long horizon, namely 2002 to 2014. A 1 percentage point increase in the weighted average interbank repo rate predicts that the PBOC will inject liquidity on the order of 0.5% of total savings. We therefore set the external liquidity parameter to $\psi = 0.5$. We will allow $i_L^\ell = i_B > 0$ in the calibration since surplus reserves can earn a small interest rate from the central bank. We then redefine $\Psi(i_L) \equiv \psi(i_L - i_B)$ to preserve $\Psi(i_L^\ell) = 0$.

The competition parameters (δ_1 and δ_2) and the DLP demand parameter (ω) are calibrated to match funding outcomes in 2014. Propositions 5 and 6 used $\delta_1 \in \{0, \omega\}$ and $\delta_2 = 0$ to fix ideas. We relax these restrictions here. The DLPs in our model are well approximated by wealth management products (WMPs) in China. In 2005, the Chinese government expanded the range of financial services banks could provide. This led to the advent of WMPs which represent a liquidity service provided by banks at endogenous interest rates. Banks can also choose where to report their WMPs by choosing whether or not to provide an explicit principal guarantee. Any WMPs issued with an explicit principal guarantee must be reported on-balance-sheet. Absent such a guarantee, the WMP and the assets it invests in do not have to be consolidated into the bank's balance sheet. These unconsolidated WMPs are instead invested off-balance-sheet. The lack of explicit guarantees on off-balance-sheet WMPs is only for accounting purposes though: there is a general perception that all WMPs are at least implicitly guaranteed by traditional banks (Elliott, Kroeber, and Qiao (2015)).

We target a big bank funding share of $X_k = 0.45$ when $\alpha = 0.25$ since roughly 45% of total savings in China (i.e., traditional deposits plus WMPs) were held at the Big Four in 2014. We also target DLP issuance of $W_j = 0.10$ and $W_k = 0.05$ for small and big banks respectively when $\alpha = 0.25$. WMPs represented 15% of total savings in China at the end of 2014. Small banks accounted for roughly two-thirds of WMPs issued and were also much more involved in off-balance-sheet issuance than the Big Four. We present empirical evidence

on this point in Appendix F.

To calibrate δ_0, we target an aggregate credit-to-savings ratio $(1 - R_j - R_k)$ of 75% when $\alpha = 0.25$. We get this target from the data as follows. Commercial banks in China for which Bankscope has complete data collectively added RMB 40 trillion of new loans between 2007 and 2014. As a result, the ratio of traditional lending to GDP increased by 20 percentage points. The ratio of off-balance-sheet WMPs to GDP increased by 15 percentage points over the same period, which accounts for the majority of the growth in broader measures of shadow banking that can be constructed using data from China's National Bureau of Statistics (Hachem (2018)). Adding the growth of the traditional and shadow sectors, we get a 35 percentage point increase in the ratio of total credit to GDP from 2007 to 2014, which translates into a roughly 10 percentage point increase in China's credit-to-savings ratio. The ratio of private credit to private savings was 65% in 2007. This is easy to calculate since WMP issuance was minimal prior to 2008 and all the relevant information was therefore reported on bank balance sheets. It then follows that the credit-to-savings ratio in China was roughly 75% in 2014.

Finally, we allow big and small banks to have different operating cost parameters, ϕ_k and ϕ_j respectively. China has around 200 commercial banks so, with a funding share of 45% for the Big Four in 2014, a big bank was on average 40 times as large as a small one (i.e., $\frac{0.45}{4} / \frac{0.55}{196} \approx 40$). In the context of our model, this size difference implies that the big bank faces ϕ_k below ϕ_j. To match the observed size difference in 2014, we set $\phi_j = 40\phi_k$ so that marginal operating costs are the same across banks.[9] We then calibrate ϕ_k to match a loan-to-deposit ratio of 0.70 for the Big Four when $\alpha = 0.25$, which is the loan-to-deposit ratio observed in 2014 data. We will see below that the resulting operating cost parameters are high enough to deliver minimal WMP issuance in 2007, consistent with the initial equilibrium considered in the theoretical analysis.[10]

4.3   Policy Experiment

We now use the calibrated model to predict what would have happened in 2007 had the only difference between 2007 and 2014 been the strength of CBRC's loan-to-deposit enforcement. Recall that 2007 is just prior to China's adoption of stricter liquidity rules. Comparing the predicted change in the aggregate credit-to-savings ratio between 2007 and 2014 to the actual change observed in the data, we get an estimate of the quantitative importance of stricter

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[9]Differences in ϕ can be interpreted as differences in retail networks that stem from exogenous social or political forces. In robustness checks, we found that cutting the $\frac{\phi_j}{\phi_k}$ ratio to five (based on the size difference between the Big Four and only the JSCBs) and re-calibrating the model generates very similar results.

[10]The calibrated parameters are $\omega = 126$, $\delta_0 = 0.55$, $\delta_1 = 303$, $\delta_2 = 0.41$, $\phi_k = 0.03$, $\bar{\theta} = 0.135$.

liquidity rules.

The results are summarized in Table 2. Recall that the calibration targeted the 2014 values of all the variables in this table. To obtain the predictions for 2007, we decreased the liquidity rule from $\alpha = 0.25$ to $\alpha = 0.14$, keeping all other parameters unchanged. We chose $\alpha = 0.14$ because the loan-to-deposit ratio of small banks in China was 86% in 2007, suggesting that CBRC was willing to tolerate a ratio of 86% in 2007 despite the 75% cap having existed since 1995. In contrast, the loan-to-deposit ratio of small banks in China was just under 75% in 2014, consistent with $\alpha = 0.25$ after CBRC's decision to begin strict enforcement of the cap. All loan-to-deposit ratios reported here are calculated using the average balances of loans and deposits during the year, not the year-end balances, because the ultimate target of CBRC's enforcement action was the average loan book of each bank. See Appendix F for more on the importance of using average balance data.

Table 2 shows that our model generates most of the rise in WMPs in China between 2007 and 2014. It also delivers a 7 percentage point decrease in the Big Four's funding share, which is most of the 10 percentage point decrease observed in the data.

We also obtain a large increase in the Big Four's loan-to-deposit ratio, from 58% in 2007 to the targeted 70% in 2014. This is slightly bigger than the increase from 62% to 70% in the data, but the general pattern is clearly consistent. Also notice the large difference between the 2007 loan-to-deposit ratios of big and small banks in China: 62% for the big versus 86% for the small. Stricter enforcement of the 75% cap starting in 2008 therefore introduced a binding constraint on China's small banks but not on the Big Four. This is exactly the type of liquidity regulation considered in Propositions 5 and 6.

Table 2 also shows that our model generates a 25 basis point increase in the interbank interest rate between 2007 and 2014. This is half of the 50 basis point increase in the average seven-day interbank repo rate observed in the data. Since yearly averages can mask some of the most severe events, it is also useful to consider the peak interbank rates observed in daily data before and after CBRC's enforcement action. The peak rate before the enforcement began was 10.1% while the peak rate after was 11.6%. Of this 150 basis point increase in the data, our model delivers 90 basis points.

Finally, we obtain a sizeable 3.2 percentage point increase in the aggregate credit-to-savings ratio between 2007 and 2014. The credit boom in the data is roughly 10 percentage points, as explained earlier. The calibrated version of our model therefore generates one-third of China's overall credit boom as the outcome of stricter liquidity regulation. The forces in favor of an increase in credit in Table 2 are the increase in shadow lending by small banks (which rises from 3% of total savings to 10%) and the increase in traditional lending

by large banks (which rises from 0.52*58%=30.2% of total savings to 0.45*70%=31.5%).[11] These forces more than offset the decrease in traditional lending by small banks (which falls from (72.1%-3%-30.2%)=38.9% of total savings to (75.3%-10%-31.5%)=33.8%) to produce the overall increase in credit from 72.1% of total savings to 75.3% in Table 2.

The calibrated model can also be used to conduct a sensitivity analysis with respect to some key parameters. We find that the competition parameters (δ_1 and δ_2) and the DLP demand parameter (ω) could each be either doubled or halved from their calibrated values, holding all other parameters constant, without changing the qualitative conclusion that the effect of higher α on aggregate credit is positive. Anything in between doubling and halving would also work, and the range of double to half is itself just chosen for convenience. The sufficient conditions used to fix ideas in Section 3.2 are therefore part of a large parameter space that delivers a credit boom after a tightening of liquidity rules.

Another important parameter is ψ, which captures the responsiveness of external liquidity to the interbank rate. The higher is ψ, the less control the big bank has over the interbank rate (i.e., a high rate will prompt a larger liquidity injection by the central bank, bringing down the rate that prevails in equilibrium). We find that our qualitative conclusions are robust to halving ψ but not to doubling it. Instead, ψ can be increased by at most 50% given the other calibrated parameters before the big bank has sufficiently little control over the interbank rate that it joins the small banks in being constrained by the liquidity regulation. Our results are thus strongest for low values of ψ because low values of ψ afford the big bank the most interbank pricing power.

4.4 Simulation Results

We now subject the calibrated model to various shocks to see how well it matches empirical moments not targeted in the calibration. We are interested in (i) the overall ability to match these moments and (ii) the relative importance of each shock in doing so.

Table 3 reports observed correlations between the interbank repo rate and the returns to WMPs issued by small and big banks. These are the key market-determined interest rates in China and their correlations were not targeted in the calibration.

The correlations in Table 3 are calculated using monthly data from January 2008 to December 2014. The time series for i_L is the average interbank repo rate weighted by transaction volume. The time series for ξ_j and ξ_k are the average returns promised by small and big banks respectively on 3-month WMPs. Since Wind has only partial data on

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[11]The Big Four are not constrained by liquidity regulation and hence do not need to book any activities off the balance sheet so we classify all of their lending as traditional.

the amount of funding raised by each WMP, ξ_j and ξ_k are unweighted averages. We will introduce error terms to absorb imperfections in the measurement of ξ_j and ξ_k.

Table 3 shows that i_L is positively correlated with each of ξ_j, ξ_k, and $\xi_j - \xi_k$. It also shows that ξ_j is positively correlated with each of ξ_k and $\xi_j - \xi_k$ while the correlation between ξ_k and $\xi_j - \xi_k$ is negative but not highly significant. We would like to know the extent to which our calibrated model can replicate the correlations in Table 3. We start by considering three shocks separately: shocks to liquidity regulation, shocks to loan demand, and money supply shocks. We then simulate the model allowing for all three shocks simultaneously.

4.4.1 Shocks to Liquidity Regulation

We allow α, the parameter governing liquidity regulation, to be drawn from a normal distribution:

$$\alpha = \bar{\alpha} + \varepsilon_\alpha \tag{21}$$

where ε_α is normally distributed with mean 0 and variance σ_α^2. We set $\bar{\alpha} = 0.195$, which is the midpoint between the α that generates the loan-to-deposit ratio of small banks in 2007 ($\alpha = 0.14$) and the α that generates the regulated ratio ($\alpha = 0.25$).

We draw values of α using equation (21) and simulate the model for each value to generate the average interbank rate, $\pi i_L^\ell + (1 - \pi) i_L^h$, the WMP returns offered by small banks, $\xi_j + \varepsilon_{\xi_j}$, and the WMP returns offered by big banks, $\xi_k + \varepsilon_{\xi_k}$. Here, ε_{ξ_j} and ε_{ξ_k} denote measurement errors which are drawn from two independent normal distributions with mean 0 and variances $\sigma_{\xi_j}^2$ and $\sigma_{\xi_k}^2$ respectively.[12] We then use Simulated Method of Moments to estimate the three unknown parameters σ_α, σ_{ξ_j}, and σ_{ξ_k}. Appendix G describes the estimation procedure in more detail.

The first column of Table 4 reports the estimated parameter values (Panel A) and predicted correlations (Panel B). The observed correlations from Table 3 appear in the last column of Panel B. Notice that σ_α is quantitatively large and highly significant. Also notice that the estimated model matches very well the observed correlations between i_L and each of ξ_j, ξ_k, and $\xi_j - \xi_k$. Shocks to α are therefore important for generating the right correlations between the interbank rate and WMP returns. At the same time though, the estimated model matches less well the magnitudes of the pairwise correlations among WMP returns. It will thus be useful to also allow for other shocks, as is done next.

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[12] All distributions are truncated to avoid abnormal values of α, ξ_j, and ξ_k.

4.4.2 Loan Demand Shocks

Shocks to loan demand are introduced by allowing i_A to exceed a floor $\bar{\imath}_A$. Specifically:

$$i_A = \bar{\imath}_A + |\varepsilon_{i_A}|$$

where ε_{i_A} is normally distributed with mean 0 and variance $\sigma_{i_A}^2$. The floor represents the benchmark loan rate after the highest permissible discount is applied. Loan demand shocks have their own importance in China given that fiscal stimulus was undertaken in 2009 and 2010. The stimulus package sought to combat negative spillover from the global financial crisis by providing a direct boost to aggregate demand. To the extent that stimulus increased loan demand, it did so at all banks in a largely uniform way (Bai, Hsieh, and Song (2016)). An increase in i_A relative to $\bar{\imath}_A$ captures this.

We simulate the model for different values of i_A while holding $\alpha = \bar{\alpha}$. The results are reported in the second column of Table 4. The estimated value of σ_{i_A} in Panel A is not significantly different from zero and the overall fit in Panel B is much worse than the model with only variations in α.

Intuitively, banks will want more funding when investment opportunities become more attractive, as is the case when higher loan demand raises i_A. Funding shares are given by equations (10) and (11) so, all else constant, small banks will increase ξ_j and the big bank will increase ξ_k following an increase in i_A. However, when $\delta_2 > 0$ as allowed in the calibrated model, the big bank understands that an increase in ξ_k will push small banks to increase ξ_j even further. All else constant, higher ξ_k lowers the small bank funding share X_j in (10). The first order condition for ξ_j in equation (15) was derived under $\delta_2 = 0$ so, to understand the response of ξ_j to X_j when $\delta_2 > 0$, we can just go back to equation (9) when $\rho > 0$. There, we easily see that a decrease in X_j elicits an increase in ξ_j through the competitive motive for DLP issuance. Therefore, the big bank internalizes that an increase in ξ_k elicits an increase in ξ_j, forcing the big bank to increase ξ_k by even more in order to change its funding share in (11). Each individual small bank takes the actions of other banks as given so there is no similar ratchet effect when the small banks choose ξ_j. This makes the response of ξ_k to i_A more dramatic than the response of ξ_j to i_A. As a result, the correlation between i_L and ξ_k is stronger than the correlation between i_L and ξ_j in the model with only shocks to i_A. The correlation between i_L and $\xi_j - \xi_k$ in the second column of Table 4 is then negative, contradicting the positive correlation in the data.

Shocks to liquidity regulation generated a positive correlation between i_L and $\xi_j - \xi_k$ in Section 4.4.1. The response of ξ_k to α was less dramatic than the response of ξ_j to α. The

difference relative to i_A arises because the small banks, as interbank price-takers, want lower liquidity ratios than the big bank and are therefore endogenously more constrained than the big bank following an increase in α. Accordingly, they respond more than the big bank, even though they do not internalize any ratchet effects when choosing ξ_j.

4.4.3 Money Supply Shocks

Money supply shocks are introduced by allowing for exogenous variation in external liquidity:

$$\Psi\left(i_L\right) = \psi\left(i_L - i_B\right) + \varepsilon_\Psi$$

where ε_Ψ is normally distributed with mean 0 and variance σ_Ψ^2. We simulate the model for different draws of ε_Ψ while holding $\alpha = \bar{\alpha}$ and $i_A = \bar{i}_A$. Note that i_L is endogenously determined for each draw.

The results are reported in the third column of Table 4. As was the case with only loan demand shocks, the estimated value of σ_Ψ in Panel A is not significantly different from zero and the overall fit in Panel B is much worse than the model with only variations in α.

All else constant, a decrease in external liquidity increases i_L but reduces both ξ_j and ξ_k. Intuitively, the increase in i_L reflects the fact that the central bank is tightening the interbank market by removing liquidity, the decrease in ξ_j reflects the fact that small banks have less of a regulatory arbitrage motive when the interbank rate is high, and the decrease in ξ_k reflects the fact that the big bank is competing against less aggressive products by the small banks. Money supply shocks thus generate negative correlations between the interbank rate and WMP returns, contradicting the positive correlations in the data.

Shocks to liquidity regulation generated these positive correlations in Section 4.4.1. All else constant, an increase in α increases the regulatory arbitrage motive of small banks so ξ_j (where $\tau_j = 1$) goes up. The big bank responds to the resulting loss in its funding share by increasing ξ_k and i_L. The increase in i_L tempers the increase in ξ_j, but ξ_j still increases on net because of the increase in α.

4.4.4 Multiple, Simultaneous Shocks

Now consider a version of the quantitative model which has shocks to liquidity regulation, shocks to loan demand, and money supply shocks, all at the same time. The shocks (ε_α, ε_{i_A}, and ε_Ψ) and measurement errors (ε_{ξ_j} and ε_{ξ_k}) are drawn from the relevant distributions, all of which are assumed to be independent of each other. We are able to separately identify σ_α, σ_{i_A}, and σ_Ψ since shocks to liquidity regulation, loan demand, and external liquidity imply

different correlations between i_L, ξ_j, and ξ_k, as discussed above.

The results are reported in the fourth column of Table 4. The quantitative model with three shocks matches the six empirical correlations almost perfectly. Moreover, σ_α, σ_{i_A}, and σ_Ψ are all statistically significant, indicating that all three shocks are relevant.[13] However, as we saw when we considered each shock separately, shocks to liquidity regulation play a much more important role than shocks to either loan demand or external liquidity when it comes to getting the right signs for the correlations.

To this point, we also find that variations in α explain 46% of the variance of i_L in the data while variations in i_A and the intercept of $\Psi\left(\cdot\right)$ explain only 21% and 33% respectively. This complements our finding in Section 4.3 that changes in liquidity regulation can explain about half of the increase in the interbank repo rate between 2007 and 2014, along with explaining one-third of the increase in the aggregate credit-to-savings ratio.

5   Conclusion

This paper has developed a theoretical framework to study the endogenous response of the banking sector to liquidity regulation and the implications for the aggregate economy. We showed that stricter liquidity standards can generate unintended credit booms when there is heterogeneity in interbank pricing power. Liquidity minimums are endogenously more binding on a small bank than on a large one in this environment. In response, small banks find it optimal to offer a new savings instrument and manage the funds raised by this instrument in an off-balance-sheet vehicle that is not subject to liquidity regulation. As small banks push to attract savings into off-balance-sheet instruments, they raise the interest rates on these instruments above the rates on traditional deposits and poach funding from the big bank. The big bank responds to this competitive threat both by issuing its own high-return savings instruments and by tightening the interbank market for emergency liquidity against small banks. The new equilibrium is characterized by more credit as savings are reallocated across banks and lending is reallocated across markets.

Applying our framework to China, we found that a regulatory push to increase bank liquidity and cap loan-to-deposit ratios in the late 2000s accounts for one-third of China's unprecedented credit boom between 2007 and 2014. A quantitative extension that allowed for other, non-regulatory shocks also identified variation in liquidity rules as the dominant force behind observed co-movements in market-determined interest rates.

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[13]This can also be seen from estimated measurement errors: σ_{ξ_j} becomes statistically insignificant and the magnitude of σ_{ξ_k} is less than a quarter of the previous estimates.

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Figure 1

Interbank Network in China, Net Flows
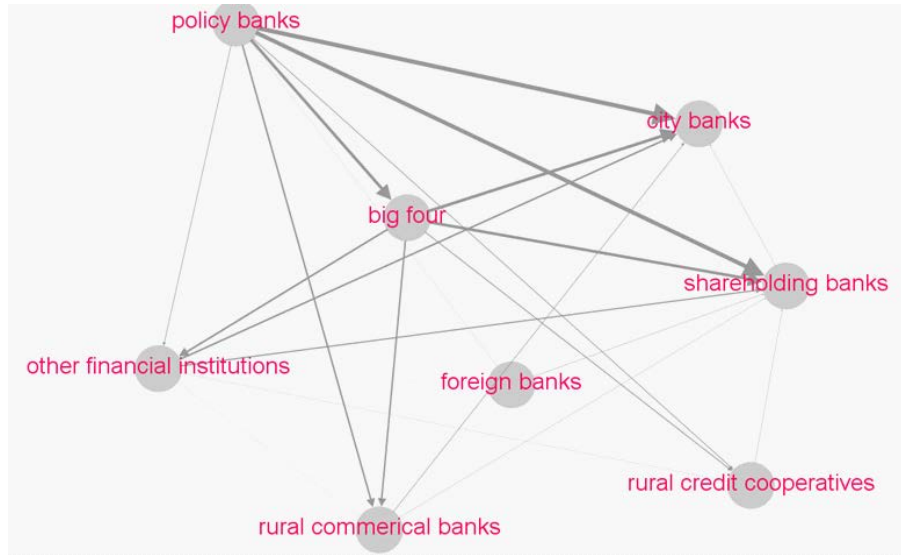
Notes: Based on main sample. Shareholding banks are the JSCBs.

Table 1

Measures of Bank Importance on Interbank Market

| | Eigen-Centrality | | | | Elasticity |
| | Main Sample | | June 20 | | |
	Out	In	Out	In	
Policy Banks	1.00	0.01	1.00	0.07	0.572
Big Four	0.97	0.23	0.56	0.54	0.287
JSCBs	0.67	0.71	0.47	1.00	0.078
City Banks	0.77	1.00	0.33	0.95	0.037
Rural Banks	0.37	0.34	0.20	0.29	0.018
Rural Co-ops	0.18	0.20	0.11	0.12	0.002
Foreign Banks	0.08	0.04	0.02	0.06	0.006
Other	0.97	0.73	0.93	0.73	0.000

Notes: Out is based on lending. In is based on borrowing. Last column is elasticity of total lending by interbank market with respect to money brought into market by node.

37

Table 2
Calibration Results

	(1)	(2)	(3)	(4)
	Model	Data	Model	Data
	$\alpha = 0.14$	2007	$\alpha = 0.25$	2014
Average Interbank Rate $\left(\pi i_L^{\ell} + (1-\pi) i_L^h\right)$	3.35%	3.1%	3.6%	3.6%
Small Bank WMPs (W_j)	0.03	NA	0.10	0.10
Big Bank WMPs (W_k)	0.01	NA	0.05	0.05
Big Bank Funding Share (X_k)	0.52	0.55	0.45	0.45
Big Bank Loan-to-Deposit Ratio $(1 - \frac{R_k}{X_k})$	58%	62%	70%	70%
Credit-to-Savings Ratio $(1 - R_j - R_k)$	72.1%	65%	75.3%	75%

Notes: We target the 2014 values of all variables in this table. The 2007 values in (1) are generated by the calibrated model keeping all parameters except α unchanged. NA denotes negligible issuance.

Table 3
Pairwise Correlations

	i_L	ξ_j	ξ_k
ξ_j	0.456	-	-
	(0.077)		
ξ_k	0.329	0.736	-
	(0.095)	(0.052)	
$\xi_j - \xi_k$	0.259	0.550	-0.152
	(0.093)	(0.088)	(0.147)

Notes: Bootstrapped standard errors are in parentheses.

Table 4

Estimation Results

	Panel A: Parameter Values			
	Model with only σ_α	Model with only σ_{i_A}	Model with only σ_Ψ	Model with σ_α, σ_{i_A}, σ_Ψ
σ_α	0.0680	-	-	0.0281
	(3.60)			(7.40)
σ_{i_A}	-	0.0551	-	0.0004
		(1.44)		(10.40)
σ_Ψ	-	-	0.0006	0.0052
			(1.33)	(11.96)
$\sigma_{\xi_j} \times 10^4$	3.12	6.12	6.36	0.0013
	(1.91)	(1.17)	(1.76)	(0.15)
$\sigma_{\xi_k} \times 10^4$	2.48	5.68	2.38	0.4436
	(1.65)	(2.68)	(1.55)	(12.32)

	Panel B: Pairwise Correlations				
	Model with only σ_α	Model with only σ_{i_A}	Model with only σ_Ψ	Model with σ_α, σ_{i_A}, σ_Ψ	Data
$corr\left(i_L, \xi_j\right)$	0.475	0.115	-0.008	0.458	0.456
$corr\left(i_L, \xi_k\right)$	0.318	0.411	-0.002	0.331	0.329
$corr\left(i_L, \xi_j - \xi_k\right)$	0.237	-0.227	-0.006	0.263	0.259
$corr\left(\xi_j, \xi_k\right)$	0.141	0.051	-0.004	0.730	0.736
$corr\left(\xi_j, \xi_j - \xi_k\right)$	0.811	0.662	0.932	0.565	0.550
$corr\left(\xi_k, \xi_j - \xi_k\right)$	-0.465	-0.714	-0.367	-0.151	-0.152

Notes: Panel A reports the estimated parameter values. Bootstrapped t-statistics are in parentheses. Columns 1 to 4 in Panel B report the simulated correlations using the estimated parameter values in each model. Column 5 in Panel B reports the correlations in the data as per Table 3.

Appendix A – Proofs

Proof of Proposition 1

By contradiction. Suppose $\rho > 0$. If $\mu_j > 0$, then $R_j = 0$ so (4) implies $i_L = \frac{\bar{\theta}X}{\psi}$. Substituting into (9) then implies $\xi_j > 0$ if and only if $\phi < \frac{(1+i_A)^2 - 1}{X} - \frac{\bar{\theta}^2}{\psi} \equiv \overline{\phi}_1$ (where we have used $X_j = X$ in symmetric equilibrium). If instead $\mu_j = 0$, then (7) implies $i_L = (1 + i_A)^2 - 1$. Substituting into (9) then implies $\xi_j > 0$ if and only if $\phi < \frac{1 - \bar{\theta}}{X} \left[(1 + i_A)^2 - 1 \right] \equiv \overline{\phi}_2$. The condition for $\overline{\phi}_1 > \overline{\phi}_2$ is also the condition for $\mu_j > 0$. Defining $\overline{\phi} \equiv \max \left\{ \overline{\phi}_1, \overline{\phi}_2 \right\}$ completes the proof. ∎

Proof of Proposition 2

With $\rho = 0$, the equilibrium is characterized by (4), (7), and:

$$\xi_j = \frac{\alpha \mu_j}{2 \left(1 - \bar{\theta} \right)}$$

$$\mu_j \left[R_j - \alpha \left(X - \omega \xi_j \right) \right] = 0 \text{ with complementary slackness}$$

There is an implicit refinement here since we are writing $\xi_j = \frac{\alpha \mu_j}{2(1 - \bar{\theta})}$ instead of $\xi_j = \frac{\alpha \mu_j \tau_j}{2(1 - \bar{\theta})}$. Both produce $\xi_j = 0$ if $\alpha \mu_j = 0$ so the refinement only applies if $\alpha \mu_j > 0$. Return to equations (8) and (9) with $\rho = 0$ and $\alpha \mu_j > 0$. If $\xi_j > 0$, then $\eta_j^1 > 0$. This implies $\tau_j = 1$ which confirms $\xi_j > 0$. If $\xi_j = 0$, then $\eta_j^1 = \eta_j^0$. This implies $\tau_j \in [0, 1]$. However, any $\tau_j \in (0, 1]$ would return $\xi_j > 0$, violating $\xi_j = 0$. We thus eliminate $\xi_j = 0$ by refinement. Instead, $\alpha \mu_j > 0$ is associated with $\xi_j > 0$ and thus $\tau_j = 1$. For this reason, we write $\xi_j = \frac{\alpha \mu_j}{2(1 - \bar{\theta})}$. We can now proceed with the rest of the proof. There are two cases:

1. If $\mu_j = 0$, then $\xi_j = 0$ and $1 + i_L = (1 + i_A)^2$. Equation (4) then pins down R_j. To ensure that $R_j \geq \alpha \left(X - \omega \xi_j \right)$ is satisfied, we need $\alpha \leq \bar{\theta} - \frac{\psi \left[(1+i_A)^2 - 1 \right]}{X} \equiv \tilde{\alpha}_1$. We have now established $\xi_j = 0$ if $\alpha \leq \tilde{\alpha}_1$.

2. If $\mu_j > 0$, then complementary slackness implies $R_j = \alpha \left(X - \omega \xi_j \right)$. Combining with the other equilibrium conditions, we find that $\mu_j > 0$ delivers:

$$i_L = \frac{\alpha^2 \omega \left[(1 + i_A)^2 - 1 \right] - 2 \left(1 - \bar{\theta} \right) \left(\alpha - \bar{\theta} \right) X}{\alpha^2 \omega + 2\psi \left(1 - \bar{\theta} \right)} \tag{22}$$

   Verifying $\mu_j > 0$ is equivalent to verifying $1 + i_L < (1 + i_A)^2$. This reduces to $\alpha > \tilde{\alpha}_1$. If $\tilde{\alpha}_1 \geq 0$, then we have established $\xi_j > 0$ with $\tau_j = 1$ for any $\alpha > \tilde{\alpha}_1$.

Defining $\widetilde{\alpha} = \max\{\widetilde{\alpha}_1, 0\}$ completes the proof. ∎

Proof of Proposition 3

Consider $\alpha = 0$. If $\mu_j = 0$, then (7) implies $i_L = (1 + i_A)^2 - 1$ which is the highest feasible interbank rate. If instead $\mu_j > 0$, then the liquidity rule binds. In particular, $R_j = \alpha(X_j - \tau_j W_j)$ which is just $R_j = 0$ when $\alpha = 0$. We can then conclude $i_L = \frac{\bar{\theta}X}{\psi}$ from equation (4). Note that $\mu_j > 0$ is verified if and only if $\frac{\bar{\theta}X}{\psi} < (1 + i_A)^2 - 1$.

Based on the results so far, we can see that the interbank rate at $\alpha = 0$ is independent of ρ. Let i_{L0} denote the interbank rate at $\alpha = 0$. Let $i_{L1}(\rho)$ denote the interbank rate at some $\alpha > 0$, allowing for any $\rho \geq 0$. From (4), we know $i_{L1}(\rho) = \frac{\bar{\theta}X}{\psi} - \frac{R_{j1}(\rho)}{\psi}$, where $R_{j1}(\rho)$ is reserve holdings at the $\alpha > 0$ being considered. The rest of the proof proceeds by contradiction. In particular, suppose $i_{L1}(\rho) > i_{L0}$. Then $\alpha = 0$ must be associated with $\mu_j > 0$, otherwise i_{L0} would be the highest feasible interbank rate and the supposition would be incorrect. We can thus write $i_{L0} = \frac{\bar{\theta}X}{\psi}$ and $i_{L1}(\rho) = i_{L0} - \frac{R_{j1}(\rho)}{\psi}$. The only way to get $i_{L1}(\rho) > i_{L0}$ is then $R_{j1}(\rho) < 0$ which is impossible. We can now conclude $i_{L0} > i_{L1}(\rho)$.

The result on total credit follows immediately. Total credit equals $X - R_j$, with R_j as per (4). Therefore, under $\psi > 0$, combinations of α and ρ that deliver higher i_L in equilibrium also deliver higher total credit. If instead $\psi = 0$, then total credit is constant and independent of either α or ρ. Either way then, total credit does not increase. ∎

Proof of Proposition 4

Start with general α. The derivatives of the big bank's objective function are:

$$\frac{\partial \Upsilon_k}{\partial \xi_k} \propto -\frac{2\omega(1 - \bar{\theta})}{1 - \pi}\xi_k - \left[\frac{(1 + i_A)^2 - 1}{1 - \pi} - i_L^h\right]\frac{\partial R_k}{\partial \xi_k} + \left[\frac{(1 + i_A)^2 - 1 - \phi X_k}{1 - \pi} - \theta_h i_L^h\right]\frac{\partial X_k}{\partial \xi_k}$$

$$\frac{\partial \Upsilon_k}{\partial i_L^h} \propto R_k - \theta_h X_k - \left[\frac{(1 + i_A)^2 - 1}{1 - \pi} - i_L^h\right]\frac{\partial R_k}{\partial i_L^h} + \left[\frac{(1 + i_A)^2 - 1 - \phi X_k}{1 - \pi} - \theta_h i_L^h\right]\frac{\partial X_k}{\partial i_L^h}$$

It will be convenient to reduce these derivatives to a core set of variables (ξ_j, ξ_k, and i_L^h). If $\mu_j > 0$, then the complementary slackness in equation (13) implies:

$$R_j = \alpha(X_j - \omega\xi_j) \tag{23}$$

With $\delta_2 = 0$ and $\overline{\xi_j} = \xi_j$, equations (10) and (11) are:

$$X_j = 1 - \delta_0 + \delta_1 \left(\xi_j - \xi_k \right) \tag{24}$$

$$X_k = \delta_0 + \delta_1 \left(\xi_k - \xi_j \right) \tag{25}$$

Substitute (23) to (25) into equation (12) to write:

$$R_k = \delta_0 \theta_h + (1 - \delta_0) \left(\overline{\theta} - \alpha \right) + \delta_1 \left(\theta_h - \overline{\theta} + \alpha \right) \left(\xi_k - \xi_j \right) + \alpha \omega \xi_j - \psi i_L^h \tag{26}$$

Finally, combine equations (14) and (15) to get:

$$\xi_j = \frac{\alpha (1 - \pi)}{2 (1 - \overline{\theta})} \left[\frac{(1 + i_A)^2 - 1}{1 - \pi} - i_L^h \right] \tag{27}$$

We can now write $\frac{\partial \Upsilon_k}{\partial \xi_k} = 0$ as:

$$\xi_k = \frac{\delta_1 \left[\left(1 - \theta_h + \overline{\theta} - \alpha \right) \left[(1 + i_A)^2 - 1 \right] - \phi \delta_0 + \phi \delta_1 \xi_j - \left(\overline{\theta} - \alpha \right) (1 - \pi) i_L^h \right]}{2 \omega \left(1 - \overline{\theta} \right) + \phi \delta_1^2} \tag{28}$$

We can also write $\frac{\partial \Upsilon_k}{\partial i_L^h} = 0$ as:

$$\begin{aligned}
i_L^h &= \frac{\left[\frac{\psi}{1 - \pi} + \frac{\alpha \left[\alpha \omega + \delta_1 \left(1 - \theta_h + \overline{\theta} - \alpha \right) \right]}{2 (1 - \overline{\theta})} \right] \left[(1 + i_A)^2 - 1 \right]}{2\psi + \frac{\alpha (1 - \pi)}{2 (1 - \overline{\theta})} \left[\alpha \omega + \delta_1 \left(\overline{\theta} - \alpha \right) \right]} \\
&+ \frac{(1 - \delta_0) \left(\overline{\theta} - \alpha \right) - \frac{\alpha \phi \delta_0 \delta_1}{2 (1 - \overline{\theta})} + \alpha \omega \xi_j - \delta_1 \left[\overline{\theta} - \alpha + \frac{\alpha \phi \delta_1}{2 (1 - \overline{\theta})} \right] \left(\xi_k - \xi_j \right)}{2\psi + \frac{\alpha (1 - \pi)}{2 (1 - \overline{\theta})} \left[\alpha \omega + \delta_1 \left(\overline{\theta} - \alpha \right) \right]}
\end{aligned} \tag{29}$$

Note that the second order conditions are:

$$\frac{\partial^2 \Upsilon_k}{\partial \left(\xi_k \right)^2} = -\phi \left(\frac{\partial X_k}{\partial \xi_k} \right)^2 - 2 \left(1 - \overline{\theta} \right) \omega < 0$$

$$\frac{\partial^2 \Upsilon_k}{\partial \left(i_L^h \right)^2} = - \left[2 (1 - \pi) \theta_h + \phi \frac{\partial X_k}{\partial i_L^h} \right] \frac{\partial X_k}{\partial i_L^h} + 2 (1 - \pi) \frac{\partial R_k}{\partial i_L^h}$$

For $\frac{\partial^2 \Upsilon_k}{\partial \left(i_L^h \right)^2} < 0$, we need:

$$\frac{\partial R_k}{\partial i_L^h} < \left[\theta_h + \frac{\phi}{2 (1 - \pi)} \frac{\partial X_k}{\partial i_L^h} \right] \frac{\partial X_k}{\partial i_L^h}$$

or, equivalently:

$$\overline{\theta}\delta_1 + \alpha\left(\omega - \delta_1\right) + \frac{2\psi\left(1 - \overline{\theta}\right)}{\alpha\left(1 - \pi\right)} > -\frac{\alpha\phi\delta_1^2}{4\left(1 - \overline{\theta}\right)}$$

This is certainly true for $\delta_1 = 0$. It is also true for $\delta_1 = \omega$ which is the other case we will take up in Proposition 5.

Remark 1 *If the big bank's inequality constraints are non-binding, the equilibrium is a triple* $\left\{\xi_j, \xi_k, i_L^h\right\}$ *that solves (27), (28), and (29). It must then be verified that the solution to these equations satisfies* $\xi_k \geq 0$ *along with* $R_k > \alpha X_k$ *and* $\mu_j > 0$. *The big bank is technically indifferent between any* $\tau_k \in [0, 1]$ *if its liquidity rule is slack so, for analytical convenience, consider* $\tau_k = 0$. *We also need to check* $W_j \leq X_j$ *and* $W_k \leq X_k$ *so that deposits are non-negative. Finally, we want to check that* $i_L^\ell = 0$ *does not result in a liquidity shortage when the big bank realizes* θ_ℓ *at* $t = 1$.

The rest of this proof focuses on $\alpha = 0$. Notice $\xi_j = 0$ from (27). As discussed in the main text, we also want $\xi_k = 0$. Subbing $\alpha = 0$ and $\xi_j = \xi_k = 0$ into (28) and (29) yields:

$$\delta_1 \left[\frac{\left(1 - \theta_h + \overline{\theta}\right)\left[\left(1 + i_A\right)^2 - 1\right] - \phi\delta_0}{\overline{\theta}\left(1 - \pi\right)} - i_L^h\right] = 0 \tag{30}$$

$$i_L^h = \frac{\left(1 + i_A\right)^2 - 1}{2\left(1 - \pi\right)} + \frac{\overline{\theta}\left(1 - \delta_0\right)}{2\psi} \tag{31}$$

To verify $\xi_k = 0$, we must verify that (30) holds when i_L^h is given by (31). This requires either $\delta_1 = 0$ or:

$$\phi = \frac{1}{\delta_0}\left[1 - \theta_h + \frac{\overline{\theta}}{2}\right]\left[\left(1 + i_A\right)^2 - 1\right] - \frac{\overline{\theta}^2\left(1 - \pi\right)\left(1 - \delta_0\right)}{2\psi\delta_0} \equiv \phi^* \tag{32}$$

In other words, we can use either $\delta_1 = 0$ or the combination of $\delta_1 > 0$ and $\phi = \phi^*$ to get ξ_k exactly zero at $\alpha = 0$. Note that $W_j \leq X_j$ and $W_k \leq X_k$ are trivially true with $\xi_j = \xi_k = 0$. We now need to check $R_k > \alpha X_k$ and $\mu_j > 0$. Using (14) and (31), rewrite $\mu_j > 0$ as:

$$\frac{\left(1 + i_A\right)^2 - 1}{1 - \pi} > \frac{\overline{\theta}\left(1 - \delta_0\right)}{\psi} \tag{33}$$

Note that condition (33) is also sufficient for $\phi^* > 0$. With $\mu_j > 0$ verified, we can substitute $\alpha = 0$ into equation (23) to get $R_j = 0$. The next step is to check $R_k > \alpha X_k$ which is simply $R_k > 0$ at $\alpha = 0$. Recall that R_k is given by equation (26). Use $\alpha = 0$ and $\xi_j = \xi_k = 0$ along with i_L^h as per (31) to rewrite equation (26) as:

$$R_k = \theta_h \delta_0 + \frac{\overline{\theta}(1 - \delta_0)}{2} - \psi \frac{(1 + i_A)^2 - 1}{2(1 - \pi)} \tag{34}$$

The condition for $R_k > 0$ is therefore:

$$\frac{(1 + i_A)^2 - 1}{1 - \pi} < \frac{\overline{\theta}(1 - \delta_0)}{\psi} + \frac{2\delta_0 \theta_h}{\psi} \tag{35}$$

The last step is to check that there is sufficient liquidity at $t = 1$ when the big bank's liquidity shock is low. The demand for liquidity in this case will be $\overline{\theta} X_j + \theta_\ell X_k$. The supply of liquidity will be $R_j + R_k$ since we have fixed $i_L^\ell = 0$. We already know $\xi_j = \xi_k = 0$ at $\alpha = 0$. Therefore, $X_j = 1 - \delta_0$ and $X_k = \delta_0$. We also know $R_j = 0$ and R_k as per (34). Therefore, $R_j + R_k \geq \overline{\theta} X_j + \theta_\ell X_k$ can be rewritten as:

$$\frac{(1 + i_A)^2 - 1}{1 - \pi} \leq \frac{2\delta_0 (\theta_h - \theta_\ell)}{\psi} - \frac{\overline{\theta}(1 - \delta_0)}{\psi} \tag{36}$$

Condition (36) is stricter than (35) so we can drop (35). We now just need to make sure that conditions (33) and (36) are not mutually exclusive. Using $\overline{\theta} \equiv \pi \theta_\ell + (1 - \pi) \theta_h$, this requires:

$$\theta_\ell < \left[1 - \frac{1 - \delta_0}{\delta_0 + \pi(1 - \delta_0)} \right] \theta_h \tag{37}$$

The right-hand side of (37) is positive if and only if:

$$\pi > \frac{1 - 2\delta_0}{1 - \delta_0} \tag{38}$$

Therefore, with θ_ℓ sufficiently low and π sufficiently high, conditions (33) and (36) define a non-empty interval for i_A, completing the proof. ∎

Proof of Proposition 5

Fixed Funding Share Impose $\alpha = \overline{\theta}$ and $\delta_1 = 0$ on equations (27), (28), and (29). The resulting system can be written as $\xi_k = 0$ and:

$$\xi_j = \frac{\left[(1 + i_A)^2 - 1 \right] \frac{\overline{\theta}\psi}{2}}{2\psi(1 - \overline{\theta}) + \omega \overline{\theta}^2 (1 - \pi)} \tag{39}$$

$$i_L^h = \frac{(1 + i_A)^2 - 1}{2\psi(1 - \overline{\theta}) + \omega \overline{\theta}^2 (1 - \pi)} \left[\frac{\psi(1 - \overline{\theta})}{1 - \pi} + \omega \overline{\theta}^2 \right] \tag{40}$$

With $\delta_1 = 0$ in equations (24) and (25), the funding shares are $X_j = 1 - \delta_0$ and $X_k = \delta_0$. Impose along with $\alpha = \bar{\theta}$ on equations (23) and (26) to get:

$$R_k = \theta_h \delta_0 + \omega \bar{\theta} \xi_j - \psi i_L^h$$

$$R_j + R_k = \bar{\theta} \left(1 - \delta_0 \right) + \theta_h \delta_0 - \psi i_L^h$$

where ξ_j and i_L^h are given by (39) and (40) respectively. We now need to go through all the steps in Remark 1 to establish the equilibrium for $\alpha = \bar{\theta}$ and fixed funding shares. Using equations (14) and (40), we can see that $\mu_j > 0$ is trivially true. Using $\xi_k = 0$ and $X_k = \delta_0$, we can also see that $W_k \leq X_k$ is trivially true. The condition for $W_j \leq X_j$ is:

$$\frac{\left(1 + i_A \right)^2 - 1}{1 - \pi} \leq \frac{2 \left(1 - \delta_0 \right)}{\psi} \left[\bar{\theta} + \frac{2 \psi \left(1 - \bar{\theta} \right)}{\omega \bar{\theta} \left(1 - \pi \right)} \right] \tag{41}$$

The conditions for $R_k > \bar{\theta} X_k$ and $R_j + R_k \geq \bar{\theta} X_j + \theta_\ell X_k$ are respectively:

$$\frac{\left(1 + i_A \right)^2 - 1}{1 - \pi} < \frac{2 \pi \left(\theta_h - \theta_\ell \right) \delta_0}{\psi} \tag{42}$$

$$\frac{\left(1 + i_A \right)^2 - 1}{1 - \pi} \leq \frac{2 \psi \left(1 - \bar{\theta} \right) + \omega \bar{\theta}^2 \left(1 - \pi \right)}{\psi \left(1 - \bar{\theta} \right) + \omega \bar{\theta}^2 \left(1 - \pi \right)} \frac{\left(\theta_h - \theta_\ell \right) \delta_0}{\psi} \tag{43}$$

Now, for the interbank rate to increase when moving from $\alpha = 0$ to $\alpha = \bar{\theta}$, we need (40) to exceed (31). Equivalently, we need:

$$\frac{\left(1 + i_A \right)^2 - 1}{1 - \pi} > \frac{\bar{\theta} \left(1 - \delta_0 \right)}{\psi} \left[1 + \frac{2 \psi \left(1 - \bar{\theta} \right)}{\omega \bar{\theta}^2 \left(1 - \pi \right)} \right] \tag{44}$$

We must now collect all the conditions involved in the $\alpha = 0$ and $\alpha = \bar{\theta}$ equilibria and make sure they are mutually consistent. There are two lowerbounds on i_A, namely (33) and (44). Condition (44) is clearly stricter so it is the relevant lowerbound. There are also four upperbounds on i_A, namely (36), (41), (42), and (43). For the lowerbound in (44) to not violate any of these upperbounds, we need:

$$\frac{\psi \left(1 - \bar{\theta} \right)}{\omega \left(1 - \pi \right)} < \bar{\theta}^2 \min \left\{ \frac{\pi \left(\theta_h - \theta_\ell \right) \delta_0}{\bar{\theta} \left(1 - \delta_0 \right)} - \frac{1}{2}, \frac{\left(\theta_h - \theta_\ell \right) \delta_0}{\bar{\theta} \left(1 - \delta_0 \right)} - 1 \right\}$$

This inequality is only possible if the right-hand side is positive. Therefore, we need:

$$\theta_\ell < \left[1 - \frac{1 - \delta_0}{\min\left\{ \delta_0 + \pi\left(1 - \delta_0\right), \pi\left(1 + \delta_0\right) \right\}} \right] \theta_h \tag{45}$$

Once again, the right-hand side must be positive so we need:

$$\pi > \max\left\{ \frac{1 - 2\delta_0}{1 - \delta_0}, \frac{1 - \delta_0}{1 + \delta_0} \right\} \tag{46}$$

Notice that (45) and (46) are just refinements of (37) and (38). We can now conclude that the model with fixed funding shares generates the desired results under the following conditions: π sufficiently high, θ_ℓ and $\frac{\psi}{\omega}$ sufficiently low, and i_A within an intermediate range. $\square$

Endogenous Funding Share Return to equations (27), (28), and (29). Impose $\alpha = \overline{\theta}$ and $\delta_1 = \omega$ with $\phi = \phi^*$ as per (32). Combine to get:

$$i_L^h = \frac{\left(1 + i_A\right)^2 - 1}{1 - \pi} - \frac{\left[\frac{2\psi}{1-\pi} + \frac{\omega\overline{\theta}^2}{2\left(1-\overline{\theta}\right)+\phi^*\omega} \right] \frac{\left(1+i_A\right)^2 - 1}{2(1-\pi)} - \frac{\omega\overline{\theta}^3\left(1-\delta_0\right)}{2\psi\left[2\left(1-\overline{\theta}\right)+\phi^*\omega\right]}}{\frac{2\psi}{1-\pi} + \frac{\omega\overline{\theta}^2}{2\left(1-\overline{\theta}\right)}\left[2 + \frac{\phi^*\omega}{2\left(1-\overline{\theta}\right)+\phi^*\omega} \right]} \tag{47}$$

$$\xi_k = \frac{\frac{\overline{\theta}(1-\pi)}{2}\left[\frac{\overline{\theta}(1-\delta_0)}{\psi} + \left(\frac{\phi^*\omega}{1-\overline{\theta}} - 1 \right) \frac{\left(1+i_A\right)^2 - 1}{1-\pi} - \frac{\phi^*\omega}{1-\overline{\theta}} i_L^h \right]}{2\left(1 - \overline{\theta}\right) + \phi^*\omega} \tag{48}$$

We now need to go through the steps in Remark 1 to establish the equilibrium for $\alpha = \overline{\theta}$ and endogenous funding shares. The expressions here are more complicated so we proceed by finding one value of i_A that satisfies all the steps in Remark 1. A continuity argument will then allow us to conclude that all the steps are satisfied for a non-empty range of i_A.

Consider i_A such that:

$$\frac{\left(1 + i_A\right)^2 - 1}{1 - \pi} = \frac{\overline{\theta}}{\psi} \tag{49}$$

Substituting into (32) then pins down ϕ^* as:

$$\phi^* = \frac{\overline{\theta}\left(1 - \pi\right)}{\psi} \left[\frac{1 - \theta_h}{\delta_0} + \frac{\overline{\theta}}{2} \right] \tag{50}$$

From the proof of Proposition 4, we already have (33) and (36) as restrictions on i_A. We also have (37) as an upperbound on θ_ℓ and (38) as a lowerbound on π. It is easy to see that i_A as defined in (49) satisfies (33). For (49) to also satisfy (36), we need:

$$\theta_\ell < \left[1 - \frac{2 - \delta_0}{2\delta_0 + \pi\left(2 - \delta_0\right)}\right]\theta_h \tag{51}$$

$$\pi > \frac{2 - 3\delta_0}{2 - \delta_0} \tag{52}$$

Conditions (51) and (52) are stricter than (37) and (38). We can thus drop (37) and (38).

The first step is to verify $\mu_j > 0$. Use (14) and (47) to write $\mu_j > 0$ as:

$$\frac{\left(1 + i_A\right)^2 - 1}{1 - \pi}\left[1 + \frac{2\psi\left[2\left(1 - \overline{\theta}\right) + \phi^*\omega\right]}{\omega\overline{\theta}^2\left(1 - \pi\right)}\right] > \frac{\overline{\theta}\left(1 - \delta_0\right)}{\psi}$$

This is true by condition (33).

The second step is to verify $\xi_k > 0$. Substituting (47) into (48), we see that we need:

$$\frac{\left(1 + i_A\right)^2 - 1}{1 - \pi}\left[1 - \frac{\phi^*}{\frac{2\left(1 - \overline{\theta}\right)}{\omega} + \frac{\overline{\theta}^2\left(1 - \pi\right)}{\psi}}\right] < \frac{\overline{\theta}\left(1 - \delta_0\right)}{\psi} \tag{53}$$

Using i_A as per (49) and ϕ^* as per (50):

$$\frac{\psi\left(1 - \overline{\theta}\right)}{\omega\left(1 - \pi\right)} < \frac{\overline{\theta}}{2\delta_0^2}\underbrace{\left[1 - \theta_h - \overline{\theta}\delta_0\left(\delta_0 - \frac{1}{2}\right)\right]}_{\text{call this } Z_1} \tag{54}$$

If $Z_1 > 0$, then (54) requires $\frac{\psi}{\omega}$ sufficiently low. Note that $Z_1 > 0$ can be made true for any $\delta_0 \in (0, 1)$ by assuming $\overline{\theta} < 2\left(1 - \theta_h\right)$ or, equivalently, $\theta_\ell < \frac{2 - (3 - \pi)\theta_h}{\pi}$. This is another positive ceiling on θ_ℓ provided $\pi > 3 - \frac{2}{\theta_h}$.

The third step is to verify $R_k > \overline{\theta}X_k$. Use $\alpha = \overline{\theta}$ and $\delta_1 = \omega$ to rewrite (25) and (26) as:

$$X_k = \delta_0 + \omega\left(\xi_k - \xi_j\right) \tag{55}$$

$$R_k = \delta_0\theta_h + \omega\theta_h\xi_k - \omega\left(\theta_h - \overline{\theta}\right)\xi_j - \psi i_L^h \tag{56}$$

Therefore, $R_k > \overline{\theta}X_k$ requires:

$$i_L^h < \frac{\delta_0\left(\theta_h - \overline{\theta}\right)}{\psi} + \frac{\omega\left(\theta_h - \overline{\theta}\right)}{\psi}\left(\xi_k - \xi_j\right) + \frac{\omega\overline{\theta}}{\psi}\xi_j$$

Use (48) to replace ξ_k and (27) with $\alpha = \overline{\theta}$ to replace ξ_j:

$$\left[1 + \frac{\omega\overline{\theta}(1-\pi)}{2\psi(1-\overline{\theta})}\left[\overline{\theta} - \frac{2(1-\overline{\theta})(\theta_h - \overline{\theta})}{2(1-\overline{\theta}) + \phi^*\omega}\right]\right]i_L^h$$

$$< \frac{\theta_h - \overline{\theta}}{\psi}\left[\delta_0 + \frac{\omega\overline{\theta}^2(1-\pi)(1-\delta_0)}{2\psi[2(1-\overline{\theta}) + \phi^*\omega]}\right] - \frac{\omega\overline{\theta}\left[(1+i_A)^2 - 1\right]}{2\psi}\left[\frac{3(\theta_h - \overline{\theta})}{2(1-\overline{\theta}) + \phi^*\omega} - \frac{\overline{\theta}}{1-\overline{\theta}}\right]$$

Now use (47) to replace i_L^h and rearrange to isolate i_A:

$$\frac{(1+i_A)^2 - 1}{1-\pi}\left[2\theta_h - \frac{3\overline{\theta}}{2} + \frac{2\psi(1-\overline{\theta})}{\omega\overline{\theta}(1-\pi)} + \frac{\omega\overline{\theta}^2(1-\pi)}{4\psi(1-\overline{\theta})}(3\theta_h - 4\overline{\theta}) + \frac{\phi^*}{\overline{\theta}}\left[\frac{\omega\overline{\theta}^2}{1-\overline{\theta}} + \frac{\psi}{1-\pi}\right]\right]$$

$$< \left[\frac{2\delta_0(\theta_h - \overline{\theta})}{1-\pi}\frac{2(1-\overline{\theta}) + \phi^*\omega}{\omega\overline{\theta}} - \frac{\overline{\theta}^2(1-\delta_0)}{2\psi}\right]\left[1 + \frac{\overline{\theta}^2}{2}\frac{\omega(1-\pi)}{\psi(1-\overline{\theta})}\right]$$

$$+ (\theta_h - \overline{\theta})\frac{\overline{\theta}}{\psi}\left[\frac{\omega\phi^*\delta_0}{2(1-\overline{\theta})} + (1-\delta_0)\left[1 + \frac{3\overline{\theta}^2}{4}\frac{\omega(1-\pi)}{\psi(1-\overline{\theta})}\right]\right]$$

We can simplify a bit further by using (32) to replace all instances of $\phi^*\delta_0$ then grouping like terms:

$$\frac{(1+i_A)^2 - 1}{1-\pi}\left[\begin{array}{c}\theta_h - \frac{\overline{\theta}}{2} + \frac{2\psi(1-\overline{\theta})}{\omega\overline{\theta}(1-\pi)} - \frac{\omega\overline{\theta}^3(1-\pi)}{4\psi(1-\overline{\theta})} + \frac{\phi^*}{\overline{\theta}}\left[\frac{\omega\overline{\theta}^2}{1-\overline{\theta}} + \frac{\psi}{1-\pi}\right] \\ - (\theta_h - \overline{\theta})(1-\theta_h)\left[\frac{2}{\overline{\theta}} + \frac{3\overline{\theta}}{2}\frac{\omega(1-\pi)}{\psi(1-\overline{\theta})}\right]\end{array}\right]$$

$$< \left[\frac{4\delta_0(1-\overline{\theta})(\theta_h - \overline{\theta})}{\omega\overline{\theta}(1-\pi)} - \frac{\overline{\theta}^2(1-\delta_0)}{2\psi}\right]\left[1 + \frac{\overline{\theta}^2}{2}\frac{\omega(1-\pi)}{\psi(1-\overline{\theta})}\right]$$

Substitute i_A as per (49) and ϕ^* as per (50) then rearrange:

$$\frac{\psi(1-\overline{\theta})}{\omega(1-\pi)}\left[\theta_h - \frac{\overline{\theta}(1+\delta_0)}{2} + \left[\overline{\theta} + \frac{1-\theta_h}{\delta_0} + \frac{2}{\overline{\theta}}\frac{\psi(1-\overline{\theta})}{\omega(1-\pi)}\right]\left[1 - \frac{2\delta_0(\theta_h - \overline{\theta})}{\overline{\theta}}\right]\right] \quad (57)$$

$$< \frac{\overline{\theta}}{\delta_0}\underbrace{\left[\frac{\overline{\theta}^2\delta_0^2}{4} + \frac{\delta_0}{2}\left[3(\theta_h - \overline{\theta})(1-\theta_h) - \overline{\theta}^2\right] - \overline{\theta}(1-\theta_h)\right]}_{\text{call this } Z_2}$$

Condition (57) will be true for $\frac{\psi}{\omega}$ sufficiently low if $Z_2 > 0$. Use $\overline{\theta} \equiv \pi\theta_\ell + (1-\pi)\theta_h$ to rewrite $Z_2 > 0$ as:

$$\pi^2 \left(\theta_h - \theta_\ell\right)^2 - 2\left[\theta_h + \frac{\left(2 + 3\delta_0\right)\left(1 - \theta_h\right)}{\delta_0 \left(2 - \delta_0\right)}\right]\pi\left(\theta_h - \theta_\ell\right) + \theta_h\left[\theta_h + \frac{4\left(1 - \theta_h\right)}{\delta_0 \left(2 - \delta_0\right)}\right] < 0$$

Based on the roots of this quadratic, we can conclude that $Z_2 > 0$ requires:

$$\pi\left(\theta_h - \theta_\ell\right) > \theta_h + \frac{\left(2 + 3\delta_0\right)\left(1 - \theta_h\right)}{\delta_0 \left(2 - \delta_0\right)} - \sqrt{\frac{1 - \theta_h}{2 - \delta_0}\left(6\theta_h + \frac{\left(2 + 3\delta_0\right)^2 \left(1 - \theta_h\right)}{\delta_0^2 \left(2 - \delta_0\right)}\right)} \qquad (58)$$

Condition (58) is satisfied by $\theta_\ell = 0$ and $\pi = 1$. The left-hand side is decreasing in θ_ℓ and increasing in π so it follows that $Z_2 > 0$ requires θ_ℓ sufficiently low and π sufficiently high.

The fourth step is to verify $W_j \leq X_j$. Use $W_j = \omega\xi_j$ and (24) with $\delta_1 = \omega$ to rewrite $W_j \leq X_j$ as:

$$\xi_k \leq \frac{1 - \delta_0}{\omega}$$

Now use (48) with i_L^h as per (47) to replace ξ_k. Substitute i_A as per (49) and ϕ^* as per (50). Rearrange to isolate all terms with $\frac{\psi\left(1 - \overline{\theta}\right)}{\omega\left(1 - \pi\right)}$ on one side. The condition for $W_j \leq X_j$ becomes:

$$\frac{\psi\left(1 - \overline{\theta}\right)}{\omega\left(1 - \pi\right)}\left[\frac{\overline{\theta}^2}{2} + \left(1 - \delta_0\right)\left[\overline{\theta}^2 + \frac{\overline{\theta}\left(1 - \theta_h\right)}{\delta_0} + 2\frac{\psi\left(1 - \overline{\theta}\right)}{\omega\left(1 - \pi\right)}\right]\right] \qquad (59)$$

$$\geq \underbrace{\frac{\overline{\theta}^3}{4}\left[\left(1 - \theta_h\right)\left(3 - \frac{2}{\delta_0}\right) - \overline{\theta}\left(1 - \frac{\delta_0}{2}\right)\right]}_{\text{call this } Z_3}$$

A sufficient condition for $Z_3 < 0$, and hence $W_j \leq X_j$, is $\delta_0 \leq \frac{2}{3}$.

The fifth step is to verify $W_k \leq X_k$. Use $W_k = \omega\xi_k$ and (55) to rewrite $W_k \leq X_k$ as:

$$\xi_j \leq \frac{\delta_0}{\omega}$$

Now use (27) with $\alpha = \overline{\theta}$ and i_L^h as per (47) to replace ξ_j. Substitute i_A as per (49) and ϕ^* as per (50). Rearrange to isolate all terms with $\frac{\psi\left(1 - \overline{\theta}\right)}{\omega\left(1 - \pi\right)}$ on one side. The condition for $W_k \leq X_k$ becomes:

$$\frac{\psi\left(1-\overline{\theta}\right)}{\omega\left(1-\pi\right)}\left[1-3\delta_0-\frac{2\delta_0}{\overline{\theta}}\left[\frac{1-\theta_h}{\delta_0}+\frac{2}{\overline{\theta}}\frac{\psi\left(1-\overline{\theta}\right)}{\omega\left(1-\pi\right)}\right]\right] \tag{60}$$

$$\leq \underbrace{\frac{\overline{\theta}}{2}\left[\left(1-\theta_h\right)\left(3-\frac{1}{\delta_0}\right)-\overline{\theta}\left(\frac{1}{2}-\delta_0\right)\right]}_{\text{call this } Z_4}$$

Condition (60) will be true for $\frac{\psi}{\omega}$ sufficiently low if $Z_4 > 0$. Use the definition of $\overline{\theta}$ to rewrite $Z_4 > 0$ as:

$$\pi\left(\theta_h-\theta_\ell\right)\delta_0\left(1-2\delta_0\right) > \theta_h\delta_0\left(1-2\delta_0\right)-2\left(1-\theta_h\right)\left(3\delta_0-1\right) \tag{61}$$

If $\delta_0 \geq \frac{1}{2}$, then (61) is always true. If $\delta_0 < \frac{1}{2}$, then (61) reduces to:

$$\theta_\ell < \frac{1}{\pi}\left[\frac{2\left(1-\theta_h\right)\left(3\delta_0-1\right)}{\delta_0\left(1-2\delta_0\right)}-\theta_h\left(1-\pi\right)\right]$$

This is a positive ceiling on θ_ℓ provided $\pi > 1 - \frac{2(1-\theta_h)(3\delta_0-1)}{\theta_h\delta_0(1-2\delta_0)}$ with $\delta_0 > \frac{1}{3}$. Therefore, (61) is guaranteed by θ_ℓ sufficiently low, π sufficiently high, and $\delta_0 > \frac{1}{3}$.

The sixth step is to verify feasibility of $i_L^\ell = 0$. This requires $R_j + R_k \geq \overline{\theta}X_j + \theta_\ell X_k$. Use (23) with $\alpha = \overline{\theta}$ to replace R_j. The desired inequality becomes:

$$R_k \geq \theta_\ell X_k + \omega\overline{\theta}\xi_j$$

Substituting X_k and R_k as per equations (55) and (56):

$$i_L^h \leq \frac{\theta_h-\theta_\ell}{\psi}\left[\delta_0+\omega\left(\xi_k-\xi_j\right)\right]$$

Use (48) to replace ξ_k. Also use (27) with $\alpha = \overline{\theta}$ to replace ξ_j. Rearrange to isolate i_L^h then use (47) to replace i_L^h. Substitute i_A as per (49) and ϕ^* as per (50). Rearrange to isolate all terms with $\frac{\psi\left(1-\overline{\theta}\right)}{\omega\left(1-\pi\right)}$ on one side. The feasibility condition for $i_L^\ell = 0$ becomes:

$$\frac{\psi\left(1-\overline{\theta}\right)}{\omega\left(1-\pi\right)}\left[\begin{array}{l}\frac{\overline{\theta}(5-\delta_0)}{4}-\left(\theta_h-\theta_\ell\right)\left[\frac{1-\theta_h}{\overline{\theta}}+\frac{2\delta_0-1}{2}\right]\\[2mm]+\frac{1-\theta_h}{2\delta_0}+\left[1-\frac{2(\theta_h-\theta_\ell)\delta_0}{\overline{\theta}}\right]\frac{1}{\overline{\theta}}\frac{\psi\left(1-\overline{\theta}\right)}{\omega\left(1-\pi\right)}\end{array}\right] \leq \underbrace{\frac{3\overline{\theta}}{4}\left[\left(1-\theta_h\right)\left[\theta_h-\theta_\ell-\frac{\overline{\theta}}{\delta_0}\right]-\frac{\overline{\theta}^2}{2}\right]}_{\text{call this } Z_5} \tag{62}$$

Condition (62) will be true for $\frac{\psi}{\omega}$ sufficiently low if $Z_5 > 0$. Use the definition of $\overline{\theta}$ to rewrite $Z_5 > 0$ as:

50

$$\pi^2 (\theta_h - \theta_\ell)^2 - 2 \left[\pi\theta_h + \frac{(\pi + \delta_0)(1 - \theta_h)}{\delta_0} \right] (\theta_h - \theta_\ell) + \theta_h \left[\theta_h + \frac{2(1 - \theta_h)}{\delta_0} \right] < 0$$

Based on the roots of this quadratic, we can conclude that $Z_5 > 0$ requires:

$$\theta_\ell < \frac{1}{\pi^2} \left[\sqrt{2\pi\theta_h (1 - \theta_h) + \frac{(\pi + \delta_0)^2 (1 - \theta_h)^2}{\delta_0^2}} - \frac{(\pi + \delta_0)(1 - \theta_h)}{\delta_0} - \theta_h \pi (1 - \pi) \right]$$

This is a positive upperbound on θ_ℓ provided $\frac{\theta_h (1-\pi)^2}{2(1-\theta_h)} + \frac{1-\pi}{\delta_0} < 1$. Therefore, $Z_5 > 0$ requires θ_ℓ sufficiently low and π sufficiently high.

It now remains to check that the interbank rate increases when moving from $\alpha = 0$ to $\alpha = \overline{\theta}$. This requires (47) to exceed (31) or, equivalently:

$$\frac{(1 + i_A)^2 - 1}{1 - \pi} > (1 - \delta_0) \left[\frac{\overline{\theta}}{\psi} + \frac{4(1 - \overline{\theta})}{\omega\overline{\theta}(1 - \pi)} \frac{2(1 - \overline{\theta}) + \phi^*\omega}{2(1 - \overline{\theta}) + 3\phi^*\omega} \right]$$

Using i_A as per (49) and ϕ^* as per (50):

$$\frac{\psi(1 - \overline{\theta})}{\omega(1 - \pi)} \left[\frac{1 - \theta_h}{\delta_0} + \frac{\overline{\theta}(1 - 2\delta_0)}{2(1 - \delta_0)} + \frac{2}{\overline{\theta}} \frac{\psi(1 - \overline{\theta})}{\omega(1 - \pi)} \right] < \frac{3\overline{\theta}^2}{4(1 - \delta_0)} \left[1 - \theta_h + \frac{\overline{\theta}\delta_0}{2} \right] \qquad (63)$$

The right-hand side is positive so (63) will be true for $\frac{\psi}{\omega}$ sufficiently low.

Putting everything together, we have shown that the model with endogenous funding shares generates the desired results under the following conditions: π sufficiently high, θ_ℓ and $\frac{\psi}{\omega}$ sufficiently low, $\delta_0 \in \left(\frac{1}{3}, \frac{2}{3} \right)$, and i_A as per (49). The results then extend to a non-empty range of i_A by continuity. $\square$

Comparison   We now compare the interbank rate increases in the fixed share and endogenous share models. Notice from the proof of Proposition 4 that the interbank rate at $\alpha = 0$ is the same in both models. Therefore, we just need to show that the interbank rate in the endogenous share model exceeds the interbank rate in the fixed share model at $\alpha = \overline{\theta}$. In other words, we need to show that (47) exceeds (40) for a given set of parameters. This reduces to:

$$\frac{(1+i_A)^2 - 1}{1 - \pi} \left[1 - \frac{\phi^*}{\frac{2(1-\bar{\theta})}{\omega} + \frac{\bar{\theta}^2(1-\pi)}{\psi}} \right] < \frac{\bar{\theta}(1 - \delta_0)}{\psi}$$

which is exactly (53), where (53) was the condition for $\xi_k > 0$ at $\alpha = \bar{\theta}$ in the endogenous share model. To complete the proof, we must now show that there are indeed parameters that satisfy the conditions in both models. For $\alpha = 0$, we imposed conditions (33) and (36) along with π sufficiently high and θ_ℓ sufficiently low. These conditions applied to both models. For $\alpha = \bar{\theta}$ in the fixed share model, we also imposed conditions (41), (42), (43), and (44) along with $\frac{\psi}{\omega}$ sufficiently low. For $\alpha = \bar{\theta}$ in the endogenous share model, we added $\delta_0 \in \left(\frac{1}{3}, \frac{2}{3}\right)$ and i_A in the neighborhood of (49). In (51) and (52), we showed that π sufficiently high and θ_ℓ sufficiently low make (49) satisfy condition (36). We have also shown that condition (44) is stricter than condition (33). Therefore, we just need to show that (49) satisfies conditions (41), (42), (43), and (44). Substituting i_A as per (49) into these conditions produces the following inequalities which we must check:

$$\frac{\psi(1 - \bar{\theta})}{\omega(1 - \pi)} > \frac{\bar{\theta}^2(2\delta_0 - 1)}{4(1 - \delta_0)} \tag{64}$$

$$\theta_\ell < \left[1 - \frac{1}{\pi(1 + 2\delta_0)} \right] \theta_h \tag{65}$$

$$\frac{\psi(1 - \bar{\theta})}{\omega(1 - \pi)} \left[1 - \frac{2(\theta_h - \theta_\ell)\delta_0}{\bar{\theta}} \right] < \bar{\theta}^2 \left[\frac{(\theta_h - \theta_\ell)\delta_0}{\bar{\theta}} - 1 \right] \tag{66}$$

$$\frac{\psi(1 - \bar{\theta})}{\omega(1 - \pi)} < \frac{\bar{\theta}^2 \delta_0}{2(1 - \delta_0)} \tag{67}$$

A sufficient condition for (64) is $\delta_0 \leq \frac{1}{2}$ which is still consistent with $\delta_0 \in \left(\frac{1}{3}, \frac{2}{3}\right)$. Condition (65) is just another positive upperbound on θ_ℓ provided $\pi > \frac{1}{1+2\delta_0}$. In other words, (65) is satisfied by θ_ℓ sufficiently low and π sufficiently high. Condition (66) will be true for $\frac{\psi}{\omega}$ sufficiently low if $(\theta_h - \theta_\ell)\delta_0 > \bar{\theta}$ or, equivalently, $\theta_\ell < \left[1 - \frac{1}{\delta_0 + \pi} \right] \theta_h$ with $\pi > 1 - \delta_0$ which again means θ_ℓ sufficiently low and π sufficiently high. Finally, condition (67) is clearly satisfied by $\frac{\psi}{\omega}$ sufficiently low. $\square$ ∎

Proof of Proposition 6

Evaluate (27) at $\alpha = \bar{\theta}$ then subtract (48) to get:

$$\xi_j - \xi_k \overset{sign}{=} \left[\frac{(1+i_A)^2 - 1}{1-\pi} - \frac{\overline{\theta}(1-\delta_0)}{\psi} \right] + 2 \left[\frac{(1+i_A)^2 - 1}{1-\pi} - i_L^h \right]$$

The expression in the first set of square brackets is positive by condition (33). The expression in the second set of square brackets is proportional to μ_j. The proof of Proposition 5 established $\mu_j > 0$. Therefore, $\xi_j > \xi_k$ at $\alpha = \overline{\theta}$.

Now consider total credit:

$$TC \equiv 1 - R_j - R_k$$

Use market clearing as per (12) to replace $R_j + R_k$:

$$TC = 1 - \overline{\theta} X_j - \theta_h X_k + \psi i_L^h$$

Use (24) and (25) to replace X_j and X_k:

$$TC = 1 - \overline{\theta} - \left(\theta_h - \overline{\theta} \right) \delta_0 + \delta_1 \left(\theta_h - \overline{\theta} \right) \left(\xi_j - \xi_k \right) + \psi i_L^h$$

Proposition 5 showed $i_L^h \big|_{\alpha=\overline{\theta}} > i_L^h \big|_{\alpha=0}$. We also know $\xi_j = \xi_k = 0$ at $\alpha = 0$ and $\xi_j > \xi_k$ at $\alpha = \overline{\theta}$. Therefore, we can conclude $TC \big|_{\alpha=\overline{\theta}} > TC \big|_{\alpha=0}$.

Finally, we want to show that the loan-to-deposit ratios of big and small banks converge. The equilibrium has $\tau_j = 1$, meaning that small banks move all DLPs (and the associated investments) off-balance-sheet. The loan-to-deposit ratio of the representative small bank is then $\lambda_j \equiv 1 - \frac{R_j}{X_j - W_j}$. The equilibrium also has $\tau_k = 0$, meaning that the big bank records everything on-balance-sheet. Its loan-to-deposit ratio is then $\lambda_k \equiv 1 - \frac{R_k}{X_k}$. Proposition 4 established $R_k > 0 = R_j$ at $\alpha = 0$ so it follows that $\lambda_k \big|_{\alpha=0} < 1 = \lambda_j \big|_{\alpha=0}$. To show convergence, we just need to show $\lambda_k \big|_{\alpha=\overline{\theta}} > \lambda_k \big|_{\alpha=0}$ since $\lambda_j \big|_{\alpha=\overline{\theta}} < \lambda_j \big|_{\alpha=0}$ follows immediately from equation (23). Use $X_j + X_k = 1$ along with the definition of λ_k to rewrite (12) as:

$$\psi i_L^h = \overline{\theta} + \left[\theta_h - \overline{\theta} - (1 - \lambda_k) \right] X_k - R_j$$

We know $i_L^h \big|_{\alpha=\overline{\theta}} > i_L^h \big|_{\alpha=0}$ so it must be the case that:

$$\left[\theta_h - \overline{\theta} - (1 - \lambda_k \big|_{\alpha=\overline{\theta}}) \right] X_k \big|_{\alpha=\overline{\theta}} - R_j \big|_{\alpha=\overline{\theta}} > \left[\theta_h - \overline{\theta} - (1 - \lambda_k \big|_{\alpha=0}) \right] X_k \big|_{\alpha=0}$$

Proposition 4 also established $\xi_j = \xi_k = 0$ at $\alpha = 0$. Substituting into equation (25) then implies $X_k = \delta_0$ at $\alpha = 0$ so:

$$\lambda_k|_{\alpha=\bar\theta} \frac{X_k|_{\alpha=\bar\theta}}{\delta_0} - \lambda_k|_{\alpha=0} > \underbrace{\frac{R_j|_{\alpha=\bar\theta}}{\delta_0} - [1 - \pi(\theta_h - \theta_\ell)]\left[1 - \frac{X_k|_{\alpha=\bar\theta}}{\delta_0}\right]}_{\text{call this } Z_6}$$

We have shown $\xi_j > \xi_k$ at $\alpha = \bar\theta$ so equation (25) also implies $\frac{X_k|_{\alpha=\bar\theta}}{\delta_0} \le 1$ for any $\delta_1 \ge 0$. Therefore, $Z_6 \ge 0$ will be sufficient for $\lambda_k|_{\alpha=\bar\theta} > \lambda_k|_{\alpha=0}$. If $\delta_1 = 0$, then $Z_6 \propto R_j|_{\alpha=\bar\theta} \ge 0$. If $\delta_1 = \omega$, then we can rewrite $Z_6 \ge 0$ as:

$$1 - \delta_0 - \omega\xi_k \ge \frac{1 - \pi(\theta_h - \theta_\ell)}{\bar\theta} \omega (\xi_j - \xi_k) \tag{68}$$

where ξ_j is given by (27) with $\alpha = \bar\theta$ and ξ_k is given by (48). Use these expressions to substitute out ξ_j and ξ_k then use equation (47) to substitute out i_L^h. Evaluate i_A at (49) and ϕ^* at (50) to rewrite (68) as:

$$\frac{4\psi(1 - \bar\theta)(1 - \delta_0)}{\omega\bar\theta(1 - \pi)} + \underbrace{\bar\theta(2 - 3\delta_0) + (1 - \theta_h)\left(\frac{2}{\delta_0} - 3 - \delta_0\right)}_{\text{call this } \Delta(\delta_0)}$$

$$\ge -\frac{\bar\theta^2}{4}\frac{\omega(1 - \pi)}{\psi(1 - \bar\theta)}\underbrace{\left[2\bar\theta(1 - 2\delta_0) + (1 - \theta_h)\left(\frac{4}{\delta_0} - 6 - 3\delta_0\right)\right]}_{\text{call this } \widetilde\Delta(\delta_0)}$$

A sufficient condition for this is $\min\left\{\Delta(\delta_0), \widetilde\Delta(\delta_0)\right\} \ge 0$. Notice $\Delta'(\cdot) < 0$ and $\widetilde\Delta'(\cdot) < 0$. Also notice $\min\left\{\Delta\left(\frac{1}{2}\right), \widetilde\Delta\left(\frac{1}{2}\right)\right\} > 0$ and $\min\left\{\Delta\left(\frac{2}{3}\right), \widetilde\Delta\left(\frac{2}{3}\right)\right\} < 0$. Therefore, there is a threshold $\overline{\delta_0} \in \left(\frac{1}{2}, \frac{2}{3}\right)$ such that $\delta_0 \le \overline{\delta_0}$ guarantees $Z_6 \ge 0$. $\blacksquare$

Appendix B – Deposit and DLP Demands

Here we sketch a simple household maximization problem which generates the demands in equations (1) and (2). There is a continuum of ex ante identical households indexed by $i \in [0,1]$. Each household is endowed with X units of funding. Let D_{ij} and W_{ij} denote the deposits and DLPs purchased by household i from bank j, where:

$$\sum_j \left(D_{ij} + W_{ij} \right) \leq X \tag{69}$$

Assume that buying W_{ij} entails a transaction cost of $\frac{1}{2\omega_0} W_{ij}^2$, where $\omega_0 > 0$.[14] As per the main text, the interest rate on the DLP is zero if withdrawn early and ξ_j otherwise. The interest rate on deposits is always zero and the average probability of early withdrawal is $\bar{\theta}$. The household requires subsistence consumption of X in each state, above which it is risk neutral. If the household were to bypass the banking system and invest in long-term projects directly, it would fall below subsistence in the state where it needs to liquidate early since long-term projects cannot be liquidated early. Therefore, the household does not invest directly. Instead, it chooses D_{ij} and W_{ij} for each j to maximize:

$$\sum_j \left(D_{ij} + \left[1 + \left(1 - \bar{\theta} \right) \xi_j \right] W_{ij} - \frac{W_{ij}^2}{2\omega_0} \right)$$

subject to (69) holding with equality.[15] The first order condition with respect to W_{ij} is:

$$W_{ij} = \left(1 - \bar{\theta} \right) \omega_0 \xi_j \tag{70}$$

Substituting (70) into (69) when the latter holds with equality gives the household's total deposit demand, $D_i \equiv \sum_j D_{ij}$. The household is indifferent about the allocation of D_i across banks so we assume that it simply allocates D_i uniformly. For J banks, this yields:

$$D_{ij} = \frac{X}{J} - \frac{\left(1 - \bar{\theta} \right) \omega_0}{J} \xi_j - \frac{\left(J - 1 \right) \left(1 - \bar{\theta} \right) \omega_0}{J} \frac{1}{J-1} \sum_{x \neq j} \xi_x \tag{71}$$

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[14]We interpret transactions costs broadly. They have been used in many literatures to parsimoniously model imperfect substitutability between goods.

[15]Here is how to recover the two-point distribution of idiosyncratic bank shocks in Section 2 from the household withdrawals. Each household has probability θ_ℓ of being hit by an idiosyncratic consumption shock at $t = 1$ and having to withdraw all of its funding early. This results in each bank losing fraction θ_ℓ of its deposits and DLPs at $t = 1$. Then $\theta_h - \theta_\ell$ of the remaining $1 - \theta_\ell$ households observe a sunspot and withdraw all of their funding from $1 - \pi$ banks at $t = 1$. The $\theta_h - \theta_\ell$ households and $1 - \pi$ banks involved in the sunspot are chosen at random. Note $\bar{\theta} \equiv \pi \theta_\ell + (1 - \pi) \theta_h$.

With a unit mass of ex ante identical households, $W_j = W_{ij}$ and $D_j = D_{ij}$. As J approaches a unit mass of equally-weighted banks, (70) and (71) belong to the family of functions specified by (1) and (2).

Appendix C – Benchmark with Aggregate Shock

Consider the benchmark model (only price-taking banks) in Section 2 but with an aggregate interbank shock. In particular, the interbank rate is i_L^ℓ with probability π and i_L^h with probability $1 - \pi$. The expected interbank rate is $i_L^e \equiv \pi i_L^\ell + (1 - \pi) i_L^h$. We will specify how i_L^ℓ and i_L^h are determined shortly. In the meantime, banks take both as given.

The objective function of the representative bank simplifies to:

$$\Upsilon_j = (1 + i_A)^2 (X_j - R_j) + (1 + i_L^e) R_j - \left[X_j + i_L^e \overline{\theta} X_j + \left(1 - \overline{\theta} \right) \xi_j W_j \right] - \tfrac{\phi}{2} X_j^2$$

This is identical to the benchmark model except with the expected interbank rate i_L^e instead of the deterministic i_L. Therefore, the first order conditions are still given by equations (7) to (9) but with i_L^e in place of i_L.

The goal is to show that i_L^e is always highest at $\alpha = 0$. The proof follows Proposition 3 but, to proceed, we must replace the deterministic market clearing condition (equation (4)) with conditions for each realization of the aggregate shock. We model the shock as a shock to the aggregate demand for liquidity at $t = 1$. In particular, aggregate liquidity demand is $\overline{\theta} X - \varepsilon$ with probability π and $\overline{\theta} X$ with probability $1 - \pi$, where $\varepsilon > 0$. The interbank rates are then i_L^ℓ and i_L^h respectively. To avoid liquidity shortages, we need these rates to satisfy:

$$R_j + \psi i_L^\ell \geq \overline{\theta} X - \varepsilon \tag{72}$$

$$R_j + \psi i_L^h \geq \overline{\theta} X \tag{73}$$

The equilibrium i_L^h solves (73) with equality. If $i_L^h \leq \frac{\varepsilon}{\psi}$, then we can set $i_L^\ell = 0$. Otherwise, the equilibrium i_L^ℓ solves (72) with equality.

Let i_{L0}^e denote the expected interbank rate at $\alpha = 0$ and let $i_{L1}^e (\rho)$ denote the expected interbank rate at some $\alpha > 0$. Using (72) and (73), we can write:

$$i_{L1}^e (\rho) = \frac{\overline{\theta} X}{\psi} - \frac{R_{j1} (\rho)}{\psi} - \frac{\pi}{\psi} \min \left\{ \overline{\theta} X - R_{j1} (\rho), \varepsilon \right\} \tag{74}$$

where $R_{j1} (\rho)$ is reserve holdings at the $\alpha > 0$ being considered. The proof of $i_{L1}^e (\rho) \leq i_{L0}^e$ proceeds by contradiction. In particular, suppose $i_{L1}^e (\rho) > i_{L0}^e$. Then (7) implies $\mu_j > 0$ at $\alpha = 0$. Complementary slackness then implies $R_j = 0$ at $\alpha = 0$ so we can write:

$$i_L^e = \frac{\overline{\theta}X}{\psi} - \frac{\pi}{\psi} \min\left\{\overline{\theta}X, \varepsilon\right\} \tag{75}$$

Subtract (75) from (74) to get:

$$i_{L1}^e(\rho) = i_{L0}^e - \frac{R_{j1}(\rho)}{\psi} + \frac{\pi}{\psi}\left[\min\left\{\overline{\theta}X, \varepsilon\right\} - \min\left\{\overline{\theta}X - R_{j1}(\rho), \varepsilon\right\}\right]$$

There are three cases. If $\varepsilon \leq \overline{\theta}X - R_{j1}(\rho)$, then:

$$i_{L1}^e(\rho) = i_{L0}^e - \frac{R_{j1}(\rho)}{\psi}$$

If $\overline{\theta}X - R_{j1}(\rho) < \varepsilon < \overline{\theta}X$, then:

$$i_{L1}^e(\rho) = i_{L0}^e - \frac{1 - \pi}{\psi}R_{j1}(\rho) - \frac{\pi}{\psi}\left(\overline{\theta}X - \varepsilon\right)$$

If $\overline{\theta}X \leq \varepsilon$, then:

$$i_{L1}^e(\rho) = i_{L0}^e - \frac{1 - \pi}{\psi}R_{j1}(\rho)$$

In each case, $i_{L1}^e(\rho) > i_{L0}^e$ would require $R_{j1}(\rho) < 0$ which is impossible. ∎

Appendix D – The June 20 Event in China

Here we study in more detail the dramatic spike in interbank interest rates that occurred in China on June 20, 2013. The weighted average interbank repo rate hit an unprecedented 11.6% on this date. For comparison, the average across all other trading days in June 2013 was 6.4%, the average in the prior month (May) was 3.0%, and the average in the following month (July) was 3.6%.

A common narrative in China is that interbank conditions tightened on June 20 because the government wanted to discipline the market, either deliberately or by not responding to some market pressures. An analysis of individual transactions will show whether or not this narrative is correct. Our identification strategy makes use of the fact that China's three policy banks participate in the interbank repo market. The policy banks are agents of the government so the price and quantity of the liquidity that they provide is easily controlled by the government. In contrast, China's big commercial banks have become much more independent since the market-oriented reforms discussed in Section 4.1. If China's interbank repo market tightened at the hands of the government, there should be at least some evidence of restrictive behavior by policy banks relative to other banks on June 20.

The transaction-level data show that this was not the case. The policy banks provided a lot of liquidity to the interbank market at fairly low interest rates, to the point that they became the largest net lenders on June 20. The Big Four, on the other hand, were extremely restrictive, amassing RMB 50 billion of net borrowing by the end of the trading day.
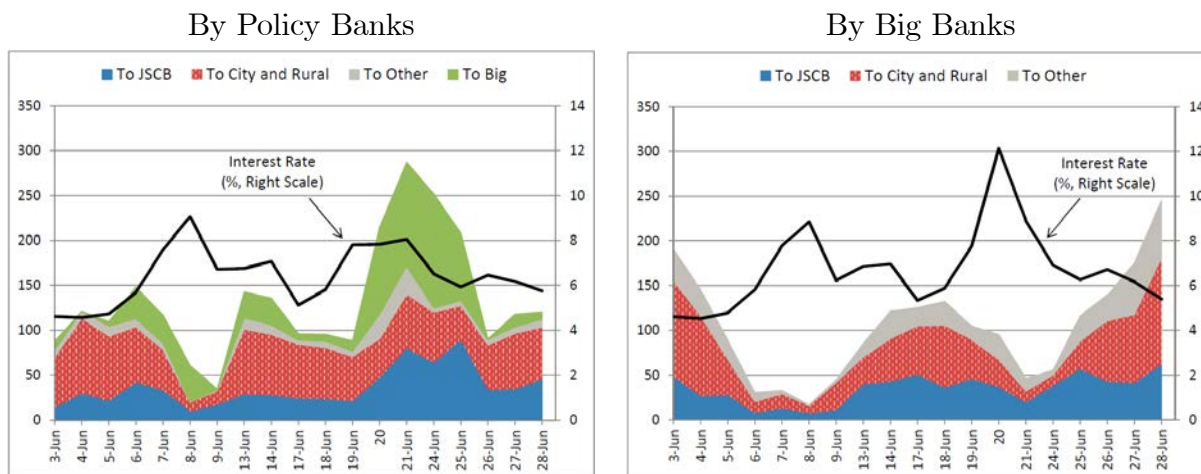
Figure D.1: Repo Lending (RMB Billions)

Figure D.1 illustrates the sharp difference between the Big Four and the policy banks in terms of both quantity and price of liquidity provision on June 20. Notice the sizeable increase in policy bank loans and the more moderate nature of policy bank interest rates.

Figure D.1 also reveals that much of the increase in policy bank lending on June 20 was absorbed by the Big Four, a fact also visible from the flow of funds depicted in Figure D.2.

Figure D.2: Interbank Network on June 20, Net Flows
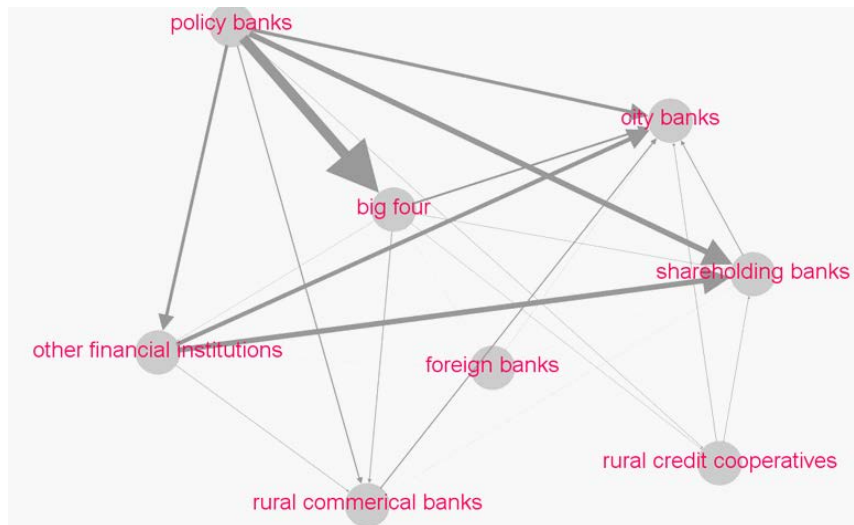
Were big banks borrowing because they really needed liquidity? Two pieces of evidence suggest no. First, the Big Four's ratio of gross lending to gross borrowing was 0.7 on June 20, with 71% of the loans directed towards small banks. If the Big Four were in dire need of liquidity, we would expect to see very little outflow. Second, the repo market activities of big banks on June 20 involved a maturity mismatch. Overnight trades accounted for 96% of big bank borrowing but only 83% of big bank lending to small banks. Roughly 80% of policy bank lending to small banks was also at the overnight maturity. If big banks really needed liquidity on June 20, we would expect the maturity of their lending to be closer to the maturity of their borrowing. Instead, it was closer to the maturity offered by policy banks to borrower groups that policy banks and big banks had in common.

Figure D.3: Interbank Market Spreads
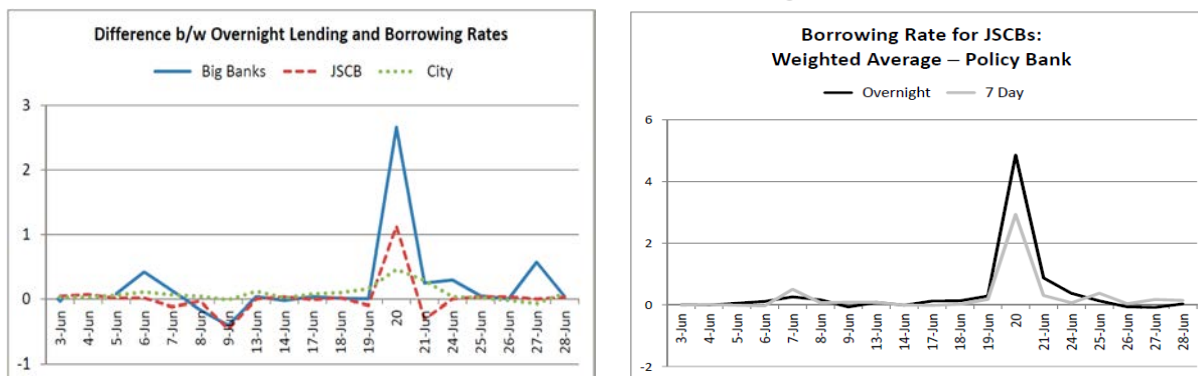
The left panel of Figure D.3 shows that big banks also commanded an abnormally high interest rate spread on June 20. In particular, their weighted average lending rate was 266 basis points above their weighted average borrowing rate. This is high relative to other banks: JSCBs and city banks commanded spreads of 113 and 46 basis points respectively. It is also high relative to other days in the sample: on any other day in June 2013, the spread commanded by big banks was between -40 and 58 basis points. Pricing among big banks was also much more uniform than pricing among small banks, both on June 20 and throughout our sample. To this point, we calculate the coefficient of variation (CV) of overnight lending rates offered by banks in different groups and find that the CV among big banks was 61% of the CV among JSCBs and 21% of the CV among city banks on June 20. Averaging over all trading days in June 2013 yields similar figures, namely 62% and 29% respectively.[16]

The right panel of Figure D.3 shows that JSCBs paid a lot more for non-policy bank loans on June 20 than they did for policy bank loans.[17] There were no major differences in the haircuts imposed by policy banks versus other lenders. It then stands to reason that JSCBs would have liked a higher share of policy bank lending. Instead, they received 20% of what policy banks lent on June 20, down from an average of 28% over the rest of the month. The situation was similar for city and rural banks: they faced large price differentials between policy and non-policy bank loans yet their share of policy bank lending on June 20 was 22%, well below an average of 47% over the rest of the month.

Taken together, the evidence presented in this appendix has identified a concrete example of price-setting by the Big Four. Specifically, the Big Four can and do change prices on China's interbank market, even controlling for government policy. China's policy banks provided a sizeable amount of liquidity on June 20 but interbank rates did not fall because the funds were absorbed by the Big Four and re-intermediated at much higher interest rates.

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[16]We exclude lending rates charged to policy banks given the proximity of policy banks to the government.

[17]For completeness, the overnight and 7 day maturities shown in the right panel of Figure D.3 were 94% of JSCB borrowing on June 20. They were also 100% of JSCB borrowing from policy banks on this date.

Appendix E – Supplement to Table 1

This appendix explains how we calculated the elasticities in Table 1 of the main text.

Consider the N nodes in Figure 1. Let ε_i^+ denote the money that node i brings into the interbank market and let ε_i^- denote the money that node i takes out of the interbank market. Also let $y_{i,s}$ denote the money that node i lends to node s on the interbank market. The adding-up constraint for each node i is therefore:

$$\sum_s y_{i,s} + \varepsilon_i^- = \sum_s y_{s,i} + \varepsilon_i^+ \tag{76}$$

It will be convenient to rewrite in matrix notation. Define $y_i \equiv \sum_s y_{i,s} + \varepsilon_i^-$ and $m_{i,s} \equiv \frac{y_{i,s}}{y_i}$. Also define an $N \times N$ matrix $M = (m_{i,s})$ and $N \times 1$ vectors $Y = (y_i)$ and $E^+ = (\varepsilon_i^+)$. The system of (76) for all i is just:

$$Y = M'Y + E^+$$

which can be rearranged to write:

$$Y = [I - M']^{-1} E^+ \tag{77}$$

where I is an $N \times N$ identity matrix. Suppose the matrix M and the vector E^+ are fixed. Then, for each node i, we can use (77) to calculate the elasticity of total lending by the interbank market, $\sum_s y_s$, to the money that i brings into the interbank market, ε_i^+.

To proceed, we need the matrix M. The $(i, s)^{th}$ element of M is $m_{i,s} \equiv \frac{y_{i,s}}{y_i}$, where $y_i \equiv \sum_s y_{i,s} + \varepsilon_i^-$. For $y_{i,s}$, we use the average daily lending from node i to node s in June 2013, excluding June 20 and 21. The policy banks and the Big Four are net lenders so we assume $\varepsilon_i^- = 0$ for each of them then use (76) to get their respective ε_i^+'s. For each of the other nodes, we assume that the money it brings into the interbank market (ε_i^+) as a fraction of what the Big Four brings equals the ratio of its deposits to the Big Four's deposits in 2013. We can then use (76) to get ε_i^- for each of these other nodes.

Appendix F – WMP Issuance in China

This appendix provides further background on the issuance of WMPs in China. As discussed in the main text, WMPs are an empirical counterpart to the DLPs in our theoretical model. Proposition 6 predicts that all banks will issue DLPs in response to tighter liquidity regulation but the DLPs issued by small banks will have higher interest rates than those issued by large banks and the issuance of off-balance-sheet DLPs will be dominated by small banks.

The first column in Table F.1 shows that the realized returns on WMPs issued by the Big Four were on average 24 basis points lower than the realized returns on WMPs issued by small and medium-sized banks (SMBs). The regression uses product-level data from 2008 to 2014 from Wind, includes time dummies, and controls for WMP maturity. The second and third columns of Table F.1 then show that realized returns relative to the expected floors and ceilings advertised at issuance were higher for Big Four WMPs than for the WMPs issued by SMBs. Taken together with the first column, this suggests that the Big Four were also more conservative in the range of returns they advertised to investors, with ceilings and floors respectively averaging 29 and 110 basis points below those advertised by SMBs.

The data also corroborate the more aggressive issuance of off-balance-sheet WMPs by SMBs. Between 2008 and 2014, the JSCBs accounted for 73% of all new WMP batches and issued 57% of their batches without a guarantee. The Big Four issued only 46% of their batches in this way. The gap in non-guaranteed intensity widens in the second half of the sample, with the JSCBs at 62% and the Big Four at 43%. These estimates are based on product counts since Wind does not yet have complete data on the total funds raised by each product. However, using data from CBRC and the annual reports of the Big Four, we estimate that SMBs (i.e., JSCBs and smaller) accounted for roughly 64% of non-guaranteed WMP balances outstanding at the end of 2013.[18] This conveys a consistent message with the batch statistics.

These differences between large and small banks arise because small banks are endogenously more constrained by liquidity regulation. Loan-to-deposit ratios based on *average balances during the year* in the early stages of CBRC's enforcement action provide a reasonable indicator of how constrained a bank would be by full implementation of the action. Average balance data is tabulated in the net interest analysis of bank annual reports, not the standard balance sheets that appear at the end of these reports.

Figure F.1 compares loan-to-deposit ratios based on average balance data (dashed lines)

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[18]The entire WMP balance in Bank of China's annual report is described in the notes as an unconsolidated balance yet the micro data in Wind includes several guaranteed batches for this bank that would not have matured by the end of 2013. We therefore remove Bank of China and rescale the other banks in the Big Four to back out our 64% estimate for small and medium-sized banks.

to those based on end-of-year data (solid lines). We plot ratios for the Big Four (blue) and the JSCBs (red) from 2005 to 2014. The shaded area is the interquartile range of the end-of-year ratios of the JSCBs. Data are from Bankscope and bank annual reports. Historical data for city and rural banks is spotty, especially when it comes to average balances, so these banks are excluded from the figure.[19]

Figure F.1: Loan-to-Deposit Ratios
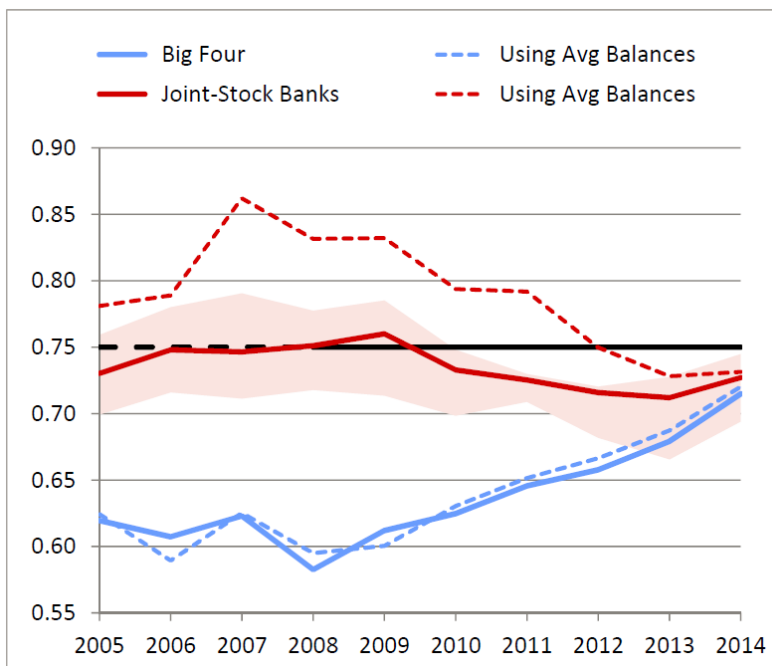
It is clear from Figure F.1 that the JSCBs would have been more constrained than the Big Four as CBRC transitioned towards monitoring average balance ratios. First, there has never been a sizeable difference between the average balance and year-end loan-to-deposit ratios of the Big Four. In contrast, the JSCBs had consistently higher average balance ratios than year-end ratios prior to 2012, the first full year of average balance monitoring by CBRC. Second, the Big Four had both ratios comfortably below 75% before CBRC heralded the era of stricter and more frequent loan-to-deposit enforcement in 2008. This was not the case for the JSCBs who, as a group, were well above 75% based on average balance data and very close to 75% based on year-end data.[20]

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[19]One JSCB (Evergrowing Bank) is also excluded for similar reasons.

[20]We make two comments here. First, banks whose loan-to-deposit ratios are materially lower at the end of the year than on an average day during the year are window-dressing their year-end balance sheets. Hachem (2018) discusses the practices used in China before 2008 and why these practices could not be used to window-dress average balance ratios. Second, Figure F.1 shows that the loan-to-deposit ratio of the Big Four has increased towards 75% since the beginning of the enforcement. In Section 4.3, we demonstrate that much of this increase can be explained as a strategic response to increased competition from shadow banking.

To provide formal evidence that off-balance-sheet issuance was driven by the bindingness of liquidity regulation, we run panel regressions in Table F.2. We use bank-level data for each bank in the Big Four and the JSCBs. The dependent variable is the log of non-guaranteed WMP batches issued by bank i in year t scaled by the average balance of deposits at the bank in that year. The main sample covers 2008 to 2010 inclusive. Recall that CBRC reached its final goal of average balance monitoring in mid-2011, making 2010 the last full year in which average balance ratios exhibit meaningful variation among constrained banks. In the first column of Table F.2, we regress the dependent variable on the loan-to-deposit ratio of bank i in year t, as measured using average balance data.[21] All columns include year fixed effects. We also control for the maturity of the non-guaranteed WMPs issued by bank i in year t. A bank that issues 3-month WMPs will issue twice as many WMP batches as a bank that issues 6-month WMPs to raise the same amount of funding over the course of a year. The bank with shorter-term WMPs will therefore have more batches, even if it is otherwise identical to the bank with longer-term WMPs. Including maturity as a regressor controls for this.

The results in the first column of Table F.2 confirm that banks with higher average balance ratios issued more non-guaranteed WMPs than banks with lower ratios. The second column shows that this finding is robust to controlling for the average return floor advertised by bank i when issuing non-guaranteed WMPs in year t. The third column shows that it is also robust to including bank fixed effects. In the fourth column, we extend the sample to 2014. The coefficient on the average balance ratio is still positive but its magnitude is roughly one-third of the magnitudes in the columns based on the main sample, and it is only statistically significant at the 10% level. The extended sample gives us more observations, and hence more degrees of freedom, to include the bank fixed effects. However, as noted earlier, the average balance ratio becomes a truncated indicator after 2010.

In the last two columns of Table F.2, we rerun the main sample regressions with the average balance ratio decomposed into two components: the regulated ratio of bank i in year t, as measured at the end of the year, and the degree of window-dressing by bank i in year t, as measured by the percent difference between the bank's average balance and regulated ratios.[22] The degree of window-dressing is the indication of constraint in this

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[21]One may worry about reverse causality here. Specifically, the average balance ratio will decrease as the bank issues non-guaranteed WMPs to move some business off-balance-sheet. However, this implies a negative relationship between the dependent variable and the average balance ratio, which will bias the regression against us.

[22]We have to focus on the main sample in these columns as this is the sample where window-dressing is an observable (i.e., the average balance ratio becomes the regulated ratio once CBRC begins monitoring average balance ratios).

decomposition. The results in the last two columns corroborate what we found earlier: banks more constrained by CBRC's impending monitoring of average balance ratios issued more non-guaranteed WMPs than banks less constrained.

As a final exercise, we conduct Granger causality tests on total WMP issuance in Table F.3 and find that the WMPs issued by the Big Four were a response to the WMP activities of small and medium-sized banks (SMBs). We use differenced monthly data on WMP batches between January 2008 and September 2014 to run the tests. The Akaike Information Criterion (AIC) selects a VAR with 21 lags. As shown in Table F.3, the null hypothesis that WMP issuance by SMBs does not Granger-cause WMP issuance by the Big Four is rejected at 1% significance. The opposite hypothesis that WMP issuance by the Big Four does not Granger-cause WMP issuance by SMBs cannot be rejected at any reasonable level of significance. Using the Bayesian Information Criterion (BIC) to select the number of lags yields similar results, as do Granger causality tests based on other orders. The impetus for WMP activity in China is therefore coming from the SMBs, who are also the constrained banks and the banks more heavily involved in non-guaranteed issuance.

Table F.1: WMP Returns

	realized	r_gap_h	r_gap_l
B4	-0.237***	0.049***	0.866***
	(0.012)	(0.013)	(0.025)
Maturity	0.092***	-0.044***	-0.021***
	(0.001)	(0.002)	(0.003)
Observations	54,294	52,090	34,876
Year Dummies	✓	✓	✓
Month Dummies	✓	✓	✓
R-squared	0.293	0.069	0.090

Notes: "realized" is the realized WMP return, "r_gap_h" is the realized return less the ceiling expected by the bank at issuance, "r_gap_l" is the realized return less the floor expected by the bank at issuance. B4 is a dummy for banks in the Big Four. Maturity is the maturity of the WMP. Standard errors are in parentheses. ***p<0.01, **p<0.05, *p<0.1

Table F.2: Non-Guaranteed WMP Issuance

	(1)	(2)	(3)	(4)	(5)	(6)
LDR	8.793***	9.764***	8.815**	2.720*		
	(1.719)	(2.623)	(3.283)	(1.381)		
Maturity	-0.145***	-0.138***	-0.181***	-0.045	-0.136***	-0.184***
	(0.016)	(0.018)	(0.042)	(0.045)	(0.022)	(0.052)
MinROR		-0.171	-0.135	-0.108	-0.171	-0.127
		(0.105)	(0.082)	(0.089)	(0.097)	(0.099)
WinDress					6.907*	6.179*
					(3.583)	(2.938)
RegRatio					10.676***	8.175
					(2.491)	(5.874)
Observations	41	31	31	79	31	31
Year Dummies	✓	✓	✓	✓	✓	✓
Bank Dummies	×	×	✓	✓	×	✓
R-squared	0.583	0.654	0.965	0.793	0.658	0.963

Notes: The dependent variable is the log of the total number of non-guaranteed WMPs issued by a bank in a year scaled by the average balance of deposits at the bank in that year. LDR is the loan-to-deposit ratio based on average balances of a bank in a year. Maturity and MinROR are respectively the average maturity and expected return floor on non-guaranteed WMPs issued by a bank in a year. WinDress is the percent difference between the average balance and year-end loan-to-deposit ratios of a bank in a year. RegRatio is the year-end ratio of a bank in a year. In all columns except (4), the sample period is 2008-2010. In column (4), the sample period is 2008-2014. Standard errors, clustered at the bank level, are in parentheses. ***p<0.01, **p<0.05, *p<0.1

Table F.3: Granger Causality Tests

H$_0$: SMB WMPs do not cause Big Four WMPs			
Criteria	Order	F-statistic	P-value
AIC	21	3.737	0.00
BIC	1	17.707	0.00
	3	15.276	0.00
	6	11.514	0.00
	9	2.295	0.02
H$_0$: Big Four WMPs do not cause SMB WMPs			
Criteria	Order	F-statistic	P-value
AIC	21	0.236	0.99
BIC	1	0.098	0.75
	3	0.966	0.41
	6	1.590	0.15
	9	0.492	0.88

Notes: We use monthly differenced data on WMP batches. AIC is the Akaike Information Criterion, which helps select the lag order of a VAR model for the Granger tests. BIC is the Bayesian Information Criterion. AIC usually over-estimates the order with positive probability, whereas BIC estimates the order consistently under fairly general conditions. Thus, BIC is typically used as the main selection criterion.

Appendix G – Estimation Procedure

Let $m = 1, ..., 6$ index the empirical moments to be matched. The six moments are the six correlations in Table 3.

1. Bootstrap: Let N denote the total number of random samples generated by bootstrap. We set $N = 500$. Denote by $g_{m,n}$ the mth moment in the nth sample. We will target $\frac{1}{N} \sum_{n}^{N} g_{m,n}$, the mth moment averaged across N samples.

2. Denote by Ω the vector of parameters to be estimated. Given Ω, we can simulate the model to generate the moments $g_m(\Omega)$. Denote by $\varepsilon_{m,n} = g_m(\Omega) - g_{m,n}$ the residual for moment m in sample n. Define the weighting matrix $(M \times M)$ as:

$$\mathcal{W} = \frac{1}{N} \sum_{n}^{N} \varepsilon_{m,n} \varepsilon_{m,n}^{T}$$

3. Minimizing the weighted sum of the distance between the empirical and simulated moments:

$$\widehat{\Omega} = \arg \min_{\Omega} h(\Omega)' \mathcal{W}^{-1} h(\Omega)$$

where $h(\Omega)$ is a vector with M elements and $h_m(\Omega) = g_m(\Omega) - \frac{1}{N} \sum_{n}^{N} g_{m,n}$.

4. We use two-step Simulated Method of Moments. We set $\mathcal{W}$ to the identity matrix in the first step and use the variance-covariance matrix of the residuals from the first-step as the weighting matrix for the second-step estimation.

5. Repeat the above exercise 100 times to calculate the standard errors of the estimated parameters.