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Fragility of Purely Real Macroeconomic Models

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ABSTRACT

Over the past thirty years, a great deal of business cycle research has been based on purely real models that abstract from the presence of nominal rigidities, and so (at least implicitly) assume that the Phillips curve is vertical. In this paper, I show that such models are fragile, in the sense that their implications change significantly when the Phillips curve is even slightly less than vertical. I consider a wide class of purely real macroeconomic models and perturb them by introducing a non-vertical Phillips curve. I show that in the perturbed models, if there is a lower bound on the nominal interest rate, then current outcomes necessarily depend on agents' beliefs about the long-run level of economic activity. The magnitude of this dependence becomes arbitrarily large as the slope of the Phillips curve becomes arbitrarily large in absolute value (closer to vertical). In contrast, the limiting purely real model ignores this form of monetary non-neutrality and macroeconomic instability. I conclude that purely real models are too incomplete to provide useful guides to questions about business cycles. I describe what elements should be added to such models in order to make them useful.

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1 Introduction

In the wake of the classic papers by Kydland and Prescott (1982) and Long and Plosser (1983), a great deal of macroeconomic research has focused on the implications of *purely real* models. By construction, such models abstract from nominal rigidities and so the adjustment of the price level to shocks is (at least implicitly) treated as instantaneous. Put another way, in these models, the Phillips curve is *vertical*, as the predicted level of economic activity is thought to be consistent with any level of inflation. Presumably, this class of models is intended to provide useful approximations to a more complicated real world, in which the price level adjusts (possibly very) rapidly but not instantaneously.

In this paper, I demonstrate that this presumption is wrong. I consider a wide class of purely real models. I perturb them by assuming that the adjustment of the price level is only *approximately* instantaneous, so that the Phillips curve is almost, but not exactly, vertical. I show that, if the nominal interest rate is pegged, then the perturbed models' predictions for current economic activity are radically different from those of the original purely real models. In particular, in the perturbed models, *current* economic outcomes are highly sensitive to agents' beliefs about the long run level of economic activity. The degree of this sensitivity becomes arbitrarily large as the slope of the Phillips curve gets arbitrarily large in absolute value, even though there is no such feedback from the future to the present in the limiting (original) model with a vertical Phillips curve. I conclude that the predictions of a purely real macroeconomic model are highly *fragile* to apparently small deviations in assumptions about price level adjustment.

This result is seemingly paradoxical. But the intuition behind it is simple. Consider a model in which the price level does adjust immediately and the same model perturbed so that the price level adjusts nearly instantaneously. Imagine that, at some point in time, there is a gap between the outcomes of the two models. With near-instantaneous

price level adjustment in the perturbed model, then the agents in that model will expect the gap to shrink rapidly. But this means that if agents expect there to be a gap - even a small one - in the long run, the current gap must be very large. It follows that in models with near-instantaneous adjustment of the price level, current outcomes are highly dependent on the agents' beliefs about long-run outcomes.

The above logic relies on the assumption that the nominal interest rate is pegged. The fragility result disappears if we restrict the central bank to use an *active* monetary policy in which the nominal interest rate changes more than one-for-one with changes in the current inflation rate. (The intuition changes because, under active monetary policies, a gap can only exist today if agents expect that gap to be growing, not shrinking, in the future.) However, active monetary policies are intrinsically unrealistic, because they assume that there is no lower bound on the nominal interest rate. I show that, if there is some lower bound on the nominal interest rate, the predictions of purely real models are again fragile, in the sense that current outcomes are highly sensitive to perceived *downside* risks to long-run economic activity.

I see the fragility result as being highly consequential for the construction of macroeconomic models. All models are necessarily approximations to reality. However, their fragility means that purely real models are simply too *incomplete* to be useful in interpreting the world. In the second part of the paper, I describe four elements that we need to add to a purely real model so that it will be sufficiently robust to be empirically useful. First, and perhaps most importantly, the model needs to have a Phillips curve that describes the connection between the level of real activity and inflation. Second, the model needs to have an explicit description of the central bank's reaction function. Third, the model needs to a description of how fiscal policy depends on the evolution of nominal variables. Finally, the model needs to include a description of how households form their expectations of long-run equilibrium outcomes, and how policy choices can

influence those expectations.¹

The class of models that I consider has two distinctive features relative to the models usually analyzed in the literature. First, I use a somewhat unusual formulation of the Phillips curve, in which the current marginal utility of consumption, averaged across agents, is related to the current rate of inflation.² In some special cases, this formulation is the same as more usual ones that connect output or employment to inflation and expectations of future inflation. More generally, though, the formulation in this paper allows for a clearer connection between goods and asset markets.

The second (and more important) distinctive feature is that the models are finite horizon. This feature matters a great deal for the results. Infinite horizon models typically impose tight implicit restrictions on long run outcomes. These implicit restrictions mean that the models are unable to capture the dependence of current outcomes on long run outcomes that occurs in models with long, but finite, horizons. This dependence lies at the core of my analysis.

Mathematically, my paper is related to recent work using New Keynesian models to study the impact of a zero lower bound on the nominal interest rate. Werning (2012) and Cochrane (2015) both consider the impact of price flexibility on equilibrium outcomes at the zero lower bound. As I do, Werning (2012) proves that the behavior of equilibrium outcomes is very different when prices adjust almost instantaneously, as opposed to literally instantaneously. Cochrane (2015) shows that this discontinuity result disappears if agents' post-zero lower bound expectations vary appropriately with the degree of price flexibility. Del Negro, Giannoni, and Patterson (2013), McKay, Nakamura, and Steinsson (2015) and Werning (2015) show that economic outcomes at the zero lower bound are highly sensitive to the central bank's forward guidance about interest rates in the distant future.

¹See Garcia-Schmidt and Woodford (2015) for one approach along these lines.

²In an appendix, I describe how the main results can be generalized to allow the current inflation rate to depend on expectations of future inflation.

The remainder of the paper is organized as follows. In the next section, I describe a large class of finite horizon real model economies. Then, I explain how to add a Phillips curve to these models. I next examine the properties of equilibrium outcomes when the Phillips curve is near-vertical. Finally, I discuss the implications of the results, in terms of what ingredients are needed in models if they are to be non-fragile.

2 Real Models

I consider a class of *real models* in which time is discrete and indexed $1, 2, \dots, T$. All agents in the economy live for T periods, and are indexed by ψ in a set Ψ . A real model has four components: (X, M, Γ, g) . The first two components (X, M) are sets, and the last two components are mappings between the sets:

$$\Gamma : M \rightarrow X \tag{1}$$

$$g : X \rightarrow M \tag{2}$$

I impose no restrictions on the set X . A typical element x in X is a description of economic characteristics. The flexibility in the specification of X means that it can include, for example, the laws of motion of the aggregate capital stock, the cross-sectional distribution of wealth, or labor market matches.

The set M is more specific. Let (Ω, F, \Pr) be a probability space and $(F_t)_{t=1}^T$ be a filtration of F . The filtration is intended to represent the information encoded in the history of *aggregate shocks* that have affected the economy, including potential sunspots. Let M be a set of all m such that:

$$m : \Omega \rightarrow R_+^T \tag{3}$$

$$m_t \text{ is } F_t\text{-measurable}$$

(Here, R_+ represents the set of positive real numbers.) I view the typical element m in M as being an *average marginal utility stochastic process*. More particularly, let $MU_t(\psi)$ represent agent ψ 's marginal period 0 utility³ of additional units of a fixed consumption bundle \bar{c} in period t . This marginal utility $MU_t(\psi)$ is random and may not be the same for all ψ . Define m_t to be the average of $MU_t(\psi)$ over ψ . This cross-sectional average is assumed to be measurable with respect to F_t (the history of aggregate shocks).

An *equilibrium* to a real model (X, M, Γ, g) is defined to be (x^{REAL}, m^{REAL}) in $X \times M$ such that:

$$\Gamma(m^{REAL}) = x^{REAL} \quad (4)$$

$$g(x^{REAL}) = m^{REAL} \quad (5)$$

I focus on real models (X, M, Γ, g) in which an equilibrium exists and in which that equilibrium is unique.

Economically, I think of the average marginal utility process m^{REAL} as being a description of the evolution of aggregate (consumption) demand. (Aggregate demand is high in dates and states in which m^{REAL} is low, and vice versa.) In an equilibrium, an aggregate demand process m^{REAL} generates an economic outcome x^{REAL} that is, in turn, consistent with that same average marginal utility process m^{REAL} . The second equilibrium restriction is, essentially, Say's Law: the structure x^{REAL} of production and consumption in the economy generates sufficient aggregate demand m^{REAL} to clear markets.

The following example illustrates how this abstract formulation works. Note that, while the example assumes the existence of a representative agent, the overall framework makes no such assumption.

Example 1. Consider a T period representative agent model with capital accumulation.

³The definition of m_t implies that it equals $\delta^t u'(c_t)$, in models with a representative agent with time-additive preferences.

Agents have log utility with a discount factor δ . In this example, X contains stochastic processes for consumption $(c_t)_{t=1}^T$, capital $(k_t)_{t=1}^{T-1}$, and output $(y_t)_{t=1}^T$ that are adaptable to a filtration $(F_t)_{t=1}^T$. The initial level of capital is fixed at k_1^* . Output is produced using capital and labor as inputs; the efficient level of labor n_t is normalized to 1. The mapping Γ from stochastic processes for m_t to X is then defined (recursively) by:

$$c_t = \delta^t / m_t \quad (6)$$

$$k_t = \frac{\delta \alpha E(k_{t+1} c_{t-1} / c_t | F_{t-1}) + \delta \alpha c_{t-1}}{1 - \delta(1 - \alpha)(1 - \delta)E(c_{t-1} / c_t | F_{t-1})}, t = 2, \dots, T, \text{ where } k_{T+1} = 0 \quad (7)$$

$$y_t = (c_t + k_{t+1} - (1 - \delta)k_t), t = 1, \dots, T, \text{ where } k_1 = k_1^* \quad (8)$$

Note that this definition of Γ implies the usual first order conditions for a T period representative agent model with capital accumulation:

$$c_t + k_{t+1} = k_t(1 - \delta) + y_t \quad (9)$$

$$1/c_{t-1} = \delta E\left(\frac{(1 - \delta) + \alpha y_t / k_t}{c_t} | F_{t-1}\right), t = 2, \dots, T \quad (10)$$

$$k_{T+1} = 0 \quad (11)$$

$$k_1 = k_1^* \quad (12)$$

The mapping g is defined by setting output equal to its efficient level Ak_t^α in every date and state, as long as $k_t > 0$. More formally, if $\Pr(k_t \geq 0 \text{ for all } t = 1, \dots, T) = 1$, then:

$$g_t(c, k, y) = \frac{\delta^t}{(1 - \delta)k_t + Ak_t^\alpha - k_{t+1}}, t = 1, \dots, T, \text{ where } k_{T+1} = 0 \quad (13)$$

Otherwise, $g_t(c, k, y) = 0$ for any (t, c, k, y) (so that a negative value of k is inconsistent with equilibrium). \triangle

The following concepts will prove useful in the ensuing discussion. First, I define the period t (gross) *natural real interest rate* as:

$$r_t^{NAT} = \frac{m_t^{REAL}}{E\{m_{t+1}^{REAL}|F_t\}} \quad (14)$$

Note that r_t^{NAT} is F_t -measurable. As well, I define the *natural risk-neutral probability measure* Q^{NAT} to be:

$$Q^{NAT}(A) = E\{r_1^{NAT} \dots r_{T-1}^{NAT} \frac{m_T^{REAL}}{m_1^{REAL}} 1_A\} \quad (15)$$

where A is any element of the space F of events. I assume that Q^{NAT} is absolutely continuous with respect to the physical probability measure Pr . It is readily shown that, if z_{t+1} is F_{t+1} -measurable, then the Q^{NAT} -expectation of z_{t+1} , conditional on F_t , is given by:

$$E^{NAT}(z_{t+1}|F_t) = r_t^{NAT} E\left\{\frac{z_{t+1}m_{t+1}^{REAL}}{m_t^{REAL}}|F_t\right\} \quad (16)$$

3 Models with a Phillips Curve

I now describe a way to convert an arbitrary real model to a model with money and a simple Phillips curve, meant to capture the effects of sluggish price level adjustment.

3.1 Monetary Policy and Fiscal Policy

Each agent is given M dollars at the beginning of period 1. I let p_t denote the *price level* - that is, the price of the consumption bundle \bar{c} in terms of dollars. I let π_t denote the gross rate of inflation p_t/p_{t-1} . The gross rate of return on money is governed by an *interest rate rule* R that depends on the (gross) inflation rate in period t and on the natural real interest rate in period t . More specifically, money pays a gross rate of return $R(\pi_t, r_t^{NAT})$ (in dollars) in period $t = 2, \dots, T$.

Fiscal policy is quite simple. There is no initial government debt, so that the only initial outstanding government liability is money. In period T , agents pay a lump sum

tax equal to $M[\prod_{s=2}^T R(\pi_s, r_s^{NAT})]$. This lump sum tax extracts all of the outstanding money in the economy. It serves to give money its value in earlier periods. Note that this form of fiscal policy is Ricardian, in the sense that the government's budget constraint is satisfied for any sequence of equilibrium price levels.⁴

3.2 Aggregate Euler Equations

Money is an asset. If agents are to be marginally indifferent about further exchanges of \bar{c} for money, then the price level (the reciprocal of money's asset price) has to obey a set of *aggregate Euler equations*:

$$m_t/p_t = R(\pi_t, r_t^{NAT})E(m_{t+1}/p_{t+1}|F_t), t = 1, \dots, (T - 1) \quad (17)$$

This first order condition assumes that no agent faces a binding borrowing constraint with respect to money. The formulation of fiscal policy, which requires all agents to hold sufficient wealth at each date to know that they will be able to pay their taxes in the final period, implies that this assumption is valid in equilibrium.

3.3 Inverse Phillips Curves

I now add a set of short-term *inverse Phillips curves* to the model. They describe how deviations in the current and future inflation rates, from a benchmark gross inflation rate π^* , are associated with deviations of aggregate economic activity from the equilibrium level in the real model. I use the average marginal utility of \bar{c} as the measure of aggregate economic activity. More specifically, I define \hat{m}_t to be m_t/m_t^{REAL} and

⁴This model of monetary policy is motivated by the current situation of the Federal Reserve. As of this writing, banks hold a large amount of excess reserves and the Federal Reserve intends to affect financial market conditions by varying the interest rate on those reserves. It is traditional in macroeconomics to assume instead that the central bank influences the short-term interest rate by exchanging non-interest-bearing (relatively liquid) bank reserves for interest-bearing (relatively illiquid) short-term government debt. The results in the paper rely only on the specification of the aggregate Euler equation and so would generalize to this more traditional (but more complicated) model.

$\widehat{\pi}_t = \pi_t/\pi^*$ and define the inverse Phillips curves to be:

$$\widehat{m}_t = h(\widehat{\pi}_t; \phi), t = 1, \dots, T \quad (18)$$

where $\phi \geq 0$.

I impose the following restrictions on h . I assume that h is twice continuously differentiable with respect to ϕ , and that:

$$h(\widehat{\pi}; 0) = 1 \quad (19)$$

As we will see, this restriction corresponds to assuming that prices adjust instantaneously when $\phi = 0$. More substantively, I assume that, if inflation equals the benchmark π^* , then aggregate economic activity lines up with that in the real model.

$$h(1; \phi) = 1 \text{ for any } \phi \geq 0, t = 1, \dots, T \quad (20)$$

This restriction links the model with price rigidities back to the original real model.

The final assumptions about h deal with the association of inflation and economic activity. I assume that, if $\phi > 0$, the function h is strictly decreasing in $\widehat{\pi}$. This assumption implies that if inflation is above the benchmark level today, then aggregate economic activity is above its equilibrium level (so that \widehat{m} is low). I assume too that h satisfies Inada conditions:

$$\lim_{\widehat{\pi} \rightarrow 0} h(\widehat{\pi}; \phi) = \infty \text{ if } \phi > 0 \quad (21)$$

$$\lim_{\widehat{\pi} \rightarrow \infty} h(\widehat{\pi}; \phi) = 0 \text{ if } \phi > 0 \quad (22)$$

Intuitively, the parameter ϕ governs the elasticity of the Phillips curve (drawn with π on the vertical axis and m on the horizontal axis). Thus, when $\phi = 0$, m is independent

of π . This is a situation in which the inverse Phillips curve is horizontal, and the Phillips curve is vertical.. When ϕ is near zero, the differentiability assumption that I made about h ensures that the slope of the inverse Phillips curve is nearly zero, so that the Phillips curve is nearly vertical. The main point in this paper is that equilibrium outcomes are very different when the Phillips curve is nearly vertical as opposed to vertical.

3.4 Normalized Aggregate Euler Equations

It will be useful to rewrite the aggregate Euler equations in terms of the deviations \widehat{m} and $\widehat{\pi}$ from the original real equilibrium m^{REAL} and the benchmark gross inflation rate π^* . We can rewrite the aggregate Euler equations as follows:

$$\widehat{m}_t = \widehat{R}(\widehat{\pi}_t, r_t^{NAT}) E^{NAT} \left(\frac{\widehat{m}_{t+1}}{\widehat{\pi}_{t+1}} \middle| F_t \right) \quad (23)$$

where E_t^{NAT} is the risk-neutral conditional expectation described above. Here, I define a normalized interest rate rule \widehat{R} by:

$$\widehat{R}(\widehat{\pi}, r^{NAT}) = \frac{R(\widehat{\pi}\pi^*, r^{NAT})}{r^{NAT}\pi^*} \quad (24)$$

3.5 Summary of Equilibrium Conditions

Given the unique equilibrium (x^{REAL}, m^{REAL}) to a real model (X, M, Γ, g) , we can define an equilibrium (x^{MON}, m^{MON}, π) to a model $(X, M, \widehat{R}, \Gamma, \phi)$ with price rigidities to be any solution to the equations:

$$x^{MON} = \Gamma(m^{MON}) \quad (25)$$

$$m_t^{MON} = \widehat{m}_t m_t^{REAL}, t = 1, \dots, T \quad (26)$$

$$\pi_t = \widehat{\pi}_t \pi^*, t = 1, \dots, T \quad (27)$$

$$\widehat{m}_t = \widehat{R}(\widehat{\pi}_t, r_t^{NAT}) E^{NAT} \left(\frac{\widehat{m}_{t+1}}{\widehat{\pi}_{t+1}} | F_t \right), t = 1, \dots, T - 1 \quad (28)$$

$$\widehat{m}_t = h(\widehat{\pi}_t; \phi), t = 1, \dots, T \quad (29)$$

$$\widehat{\pi}_t \text{ is } F_t\text{-measurable} \quad (30)$$

where E^{NAT} , \widehat{R} , and r^{NAT} are defined as above.

This definition of an equilibrium assumes that the restrictions between x and m embedded in Γ are not altered by the addition of money. Verbally, this means that any *real* rigidities/frictions embedded in Γ are not affected by the introduction of money.

4 Basic Properties of Equilibrium

In this section, I describe the basic properties of equilibrium in a model with nominal rigidities. In models with a vertical Phillips curve, all equilibria have the same real allocation of resources as the original, real, economy. In models with a non-vertical Phillips curve, there is a continuum of equilibrium real allocations, indexed by the (random) rate of inflation in the final period T on which agents co-ordinate.

4.1 Vertical Phillips Curve: Unique Real Allocation

The first case that I consider is one in which the Phillips curve is vertical, so that $\phi = 0$.

Proposition 1. *Suppose (x^{MON}, m^{MON}, π) is an equilibrium to a model $(X, M, \widehat{R}, \Gamma, \phi)$ with price rigidities in which the parameter $\phi = 0$. Then, for all t , $m_t^{MON} = m_t^{REAL}$ with probability 1, and $x^{MON} = x^{REAL}$.*

Proof. The proof is trivial. Consider any equilibrium. If $\phi = 0$, then $h(\widehat{\pi}_t; \phi) = 1$ for any $\widehat{\pi}_t$, and so $\widehat{m}_t = 1$. Since $m_t^{MON} = \widehat{m}_t m_t^{REAL}$, it follows that $m_t^{MON} = m_t^{REAL}$. The equilibrium value of $x^{MON} = \Gamma(m^{MON}) = \Gamma(m^{REAL}) = x^{REAL}$. \square

Thus, if $\phi = 0$, prices adjust instantaneously, in the sense that the equilibrium marginal utility process is m^{REAL} .

4.2 Non-Vertical Phillips Curve

In this subsection, I describe the set of equilibrium real allocations in models with a non-vertical Phillips curve ($\phi > 0$). These allocations are indexed by the random inflation rate in the final period.

Proposition 2. *Consider a model $(X, M, \widehat{R}, \Gamma, \phi)$ with price rigidities in which $\phi > 0$.*

Pick an arbitrary F_T -measurable $\tilde{\pi}_T > 0$. Then, there exists an equilibrium (x^{MON}, m^{MON}, π) , in which $\pi_T = \tilde{\pi}_T$ with probability 1.

Proof. Define the function $\Psi : \mathbb{R}^2 \rightarrow \mathbb{R}$ to be the solution to the equation:

$$\frac{h(\Psi(w, r^{NAT}; \phi); \phi)}{\widehat{R}(\Psi(w, r^{NAT}), r^{NAT})} = w \quad (31)$$

Ψ is well-defined and strictly decreasing in w because of the Inada conditions on h , because h is strictly decreasing and because \widehat{R} is strictly increasing. Then, recursively define a stochastic process for (normalized) inflation by:

$$\widehat{\pi}_t = \Psi(E^{NAT}(h(\widehat{\pi}_{t+1}; \phi)\widehat{\pi}_{t+1}^{-1} | F_t), r_t^{NAT}; \phi), t = 2, \dots, (T - 1) \quad (32)$$

where:

$$\widehat{\pi}_T = \frac{\tilde{\pi}_T}{\pi^*} \quad (33)$$

Define period t (normalized) marginal utility to be:

$$\widehat{m}_t = h(\widehat{\pi}_t; \phi), t = 1, \dots, T \quad (34)$$

For any t , the random variable $\widehat{\pi}_t$ is F_t -measurable, and $(\widehat{m}, \widehat{\pi})$ satisfies the asset pricing

equation:

$$\widehat{m}_{t-1} = \widehat{R}(\widehat{\pi}_{t-1}, r_{t-1}^{NAT}) E^{NAT}(\widehat{m}_t \widehat{\pi}_t^{-1} | F_{t-1}) \quad (35)$$

We can complete the construction of the equilibrium by defining:

$$\pi = \widehat{\pi}_t \pi^* \quad (36)$$

$$m^{MON} = \widehat{m}_t m^{REAL} \quad (37)$$

$$x^{MON} = \Gamma(m^{MON}) \quad (38)$$

□

Regardless of whether the Phillips curve is vertical or not, there is a continuum of equilibria. The set of equilibria is indexed by the (random) final period inflation rate on which agents co-ordinate. When the Phillips curve is vertical, real allocations are the same in all of these equilibria. When the Phillips curve is non-vertical, the different inflation rate processes translate into different real allocation processes.

5 Equilibria Under Different Interest Rate Rules

In this section, I show how the nature of equilibrium depends on the nature of the interest rate rule used by the central bank.

5.1 Interest Rate Peg

I first consider an interest rate peg, in which the nominal interest rate evolves so as to be equal to the sum of the inflation target π^* and the natural real interest rate r_t^{NAT} . (I refer to this policy rule as a peg, because the evolution of the nominal interest rate R over time is independent of any variables affected by monetary policy.) This specification of monetary policy implies that if households believe that inflation equals

π^* in the long run, then the real outcomes in any model with price rigidities are same as those in the real model.

Proposition 3. *Consider a model $(X, M, \widehat{R}, \Gamma, \phi)$ with price rigidities in which $\phi > 0$ and $\widehat{R}(\widehat{\pi}, r^{NAT}) = 1$ for all $\widehat{\pi}$ and all r^{NAT} . There exists an equilibrium (x^{MON}, m^{MON}, π) , in which $\pi_T = \pi^*$ with probability 1. In that equilibrium:*

$$m_t^{MON} = m_t^{REAL} \text{ w.p. 1 for all } t \quad (39)$$

$$\pi_t = \pi^* \text{ for all } t \quad (40)$$

$$x^{MON} = x^{REAL} \quad (41)$$

Proof. The existence of such an equilibrium is a direct consequence of proposition 2. We can then establish the results about (m^{MON}, π) using reverse induction. Suppose that $\pi_{t+1} = \pi^*$ with probability one and $m_{t+1}^{MON} = m_{t+1}^{REAL}$ with probability one. The asset pricing equation says that:

$$\widehat{m}_t = E(\widehat{m}_{t+1} \widehat{\pi}_{t+1}^{-1} | F_t) \quad (42)$$

Because $\widehat{m}_t = 1$ and $\widehat{\pi}_t = 1$, the equation implies that $m_t^{MON} = m_t^{REAL}$ with probability one. Since the Phillips curve is strictly decreasing in $\widehat{\pi}$, we can conclude that $\widehat{\pi}_t = 1$ with probability one. This establishes that $m_t^{MON} = m_t^{REAL}$ and $\pi_t = \pi^*$ with probability one for all t . As well:

$$x^{MON} = \Gamma(m^{MON}) \quad (43)$$

$$= \Gamma(m^{REAL}) \quad (44)$$

$$= x^{REAL} \quad (45)$$

which proves the theorem. □

More generally, though, there is no reason why agents should believe that inflation will in fact equal π^* in the long run. The next proposition shows that current economic outcomes are highly sensitive to beliefs about long-run economic activity when the Phillips curve is nearly vertical. In reading the proposition, recall that h is the inverse Phillips curve. The hypothesis of the proposition imposes an upper bound on the absolute value of the elasticity of h and this imposes a lower bound on the absolute value of the elasticity of the Phillips curve itself.

Proposition 4. *Suppose that the inverse Phillips curve h satisfies the restrictions:*

$$h(\hat{\pi}; \phi) \geq \hat{\pi}^{-\phi} \text{ for all } \hat{\pi} > 1$$

$$h(\hat{\pi}; \phi) \leq \hat{\pi}^{-\phi} \text{ for all } \hat{\pi} < 1$$

Consider a model $(X, M, \hat{R}, \Gamma, \phi)$ with price rigidities in which $\phi > 0$ and $\hat{R}(\hat{\pi}, r^{NAT}) = 1$ for all $\hat{\pi}$ and all r^{NAT} . There exists an equilibrium (x^{MON}, m^{MON}, π) in which there is a positive probability event A in F_t such that:

$$(\ln m_T^{MON}(\omega) - \ln m_T^{REAL}(\omega)) > B \text{ for all } \omega \text{ in } A \quad (46)$$

where B is a positive constant. In any such equilibrium:

$$(\ln m_t^{MON}(\omega) - \ln m_t^{REAL}(\omega)) > B(1 + 1/\phi)^{T-t} \text{ for all } \omega \text{ in } A \quad (47)$$

There is also an equilibrium (x^{MON}, m^{MON}, π) in which there is a positive probability event A in F_t such that:

$$(\ln m_T^{MON}(\omega) - \ln m_T^{REAL}(\omega)) < -B \text{ for all } \omega \text{ in } A \quad (48)$$

where B is a positive constant. In any such equilibrium:

$$(\ln m_t^{MON}(\omega) - \ln m_t^{REAL}(\omega)) < -B(1 + 1/\phi)^{T-t} \text{ for all } \omega \text{ in } A \quad (49)$$

Proof. I will prove the lower bound result; the upper bound result can be proved similarly. Pick some $\tilde{\pi}_T$ such that $h(\tilde{\pi}_T(\omega)/\pi^*; \phi) > \exp(B)$ for almost all ω in an event $A \in F_t$. From Proposition 2, there exists an equilibrium (m^{MON}, x^{MON}, π) in which $\pi_T = \tilde{\pi}_T$. In that equilibrium, the inverse Phillips curve implies that:

$$(\ln m_T^{MON}(\omega) - \ln m_T^{REAL}(\omega)) > B \text{ for almost all } \omega \text{ in } A \quad (50)$$

Now consider any equilibrium in which:

$$(\ln m_T^{MON}(\omega) - \ln m_T^{REAL}(\omega)) > B \quad (51)$$

for almost all ω in A . I proceed by reverse induction. Let $\hat{m}_s = m_s^{MON}/m_s^{REAL}$ and suppose that $\ln \hat{m}_{s+1}(\omega) > B(1 + 1/\phi)^{T-s-1}$ for any $(s + 1) > t$ and almost all ω in A . Note that since $\hat{m}_{s+1}(\omega) > 1$ for almost all ω in A , $\hat{\pi}_{s+1}(\omega) < 1$ for almost all ω in A . We know from the asset pricing equation and the restrictions on h that for almost all ω in A :

$$\hat{m}_s(\omega) = E^{NAT}(\hat{m}_{s+1}\hat{\pi}_{s+1}^{-1}|F_s)(\omega) \quad (52)$$

$$\geq E^{NAT}(\hat{m}_{s+1}^{1+1/\phi}|F_s)(\omega) \quad (53)$$

Hence:

$$\hat{m}_s(\omega) > \exp(B(1 + 1/\phi)^{T-s}) \quad (54)$$

for almost all ω in A . □

The assumption about h says that the slope of the Phillips curve (as drawn with

$\ln \hat{\pi}_t$ on the vertical axis and $\ln \hat{m}_t$ on the horizontal axis) is more negative than $-1/\phi$, so that the Phillips curve is nearly vertical if ϕ is small. In that case, agents expect that any current marginal utility gap ($\ln \hat{m}_t$) will vanish rapidly. But the contrapositive of this statement is that, if people believe that there will still be a marginal utility gap in many periods, the gap today must be very large. The size of the current gap is highly sensitive to variations in agents' beliefs about the long-run marginal utility gap. The sensitivity is decreasing in ϕ - that is, the current marginal utility gap is more sensitive with respect to the long run if the Phillips curve is close to vertical.

Proposition 4 stresses a lack of continuity with respect to ϕ . However, the persistence of marginal utility gaps *is* continuous with respect to ϕ . When ϕ is positive, but near zero, agents expect a marginal utility gap at date t to shrink over time, with a rate of contraction that is decreasing in ϕ . When ϕ is exactly zero, then any gap disappears instantaneously. It is this kind of continuity that, for example, Golosov and Lucas (2007) have in mind when they argue that money is nearly neutral for plausible estimates of menu costs.

The point of Proposition 4 (and this paper) is that we cannot gauge the neutrality of money only through forward-looking metrics like the persistence of marginal utility gaps. Indeed, it is exactly when money is nearly neutral in a forward-looking sense that it is highly non-neutral in a backward-looking sense, so that current marginal utility gaps are in fact highly sensitive to long-run beliefs about real outcomes. Using purely real models will miss this latter backward-looking non-neutrality.

Following Atkeson and Ohanian (2001), a number of authors have obtained estimates indicating that, in aggregate data, the Phillips curve is nearly horizontal. If the inverse Phillips curve h is log-linear, then this characterization corresponds to setting the power ϕ to be very large. The proof of Proposition 4 demonstrates that in this case, current marginal utility gaps still depend on agents' beliefs about long-run gaps under an interest rate peg. However, the degree of dependence is typically much smaller when

the Phillips curve is near-horizontal. For example, if $\widehat{R}(\widehat{\pi}, r^{NAT})$ equals one (as assumed in Proposition 4) and $1/\phi$ is set equal to zero, then the marginal utility gap process is a martingale with respect to the measure defined by Q^{NAT} . Current marginal utility gaps respond only one-for-one to changes in beliefs about long-run gaps.

5.2 Active Interest Rate Rule

A nominal interest rate peg like that studied above is a well-known example of a passive monetary policy rule. The next proposition shows that the fragility result does not apply if monetary policy is active, in the sense that the nominal interest rate adjusts more than one-for-one with changes in the inflation rate.

Proposition 5. *Suppose that the inverse Phillips curve $h(\widehat{\pi}; \phi) = \widehat{\pi}^{-\phi}$. Consider a model $(X, M, \widehat{R}, \Gamma, \phi)$ with price rigidities in which $\phi > 0$ and $\widehat{R}(\widehat{\pi}, r^{NAT}) = \widehat{\pi}^\gamma$ for all $\widehat{\pi}$ and all r^{NAT} for some $\gamma > 1$. There exists an equilibrium (x^{MON}, m^{MON}, π) in which there are two constants $b < 0$ and $B > 0$ such that:*

$$\Pr(b \leq \ln(m_T^{MON}) - \ln(m_T^{REAL}) \leq B) = 1 \quad (55)$$

In any such equilibrium, at any date t :

$$\Pr(b[(\phi + 1)/(\phi + \gamma)]^{T-t} \leq \ln(m_t^{MON}) - \ln(m_t^{REAL}) \leq B[(\phi + 1)/(\phi + \gamma)]^{T-t}) = 1 \quad (56)$$

Proof. The existence of such an equilibrium follows trivially from Proposition 2. To establish the rest of the proposition, we can again proceed by reverse induction. The claim is true for $t = T$. Suppose it is true for $t + 1$, and let $\widehat{m}_s = m_s^{MON}/m_s^{REAL}$ for all s . Then, the aggregate Euler equation and the definition of h implies:

$$\widehat{m}_t^{1+\gamma/\phi} = E^{NAT}(\widehat{m}_{t+1}^{1+1/\phi} | F_t) \quad (57)$$

$$\leq \exp(B(1/\phi + 1)[(\phi + 1)/(\phi + \gamma)]^{T-t-1}) \text{ w.p. } 1 \quad (58)$$

and so:

$$\widehat{m}_t \leq \exp(B[(\phi + 1)/(\phi + \gamma)]^{T-t}) \text{ w.p. } 1 \quad (59)$$

We can readily prove the lower bound result in the same fashion. \square

With an active policy rule, both the upper and lower bounds on $\ln(\widehat{m}_t)$ converge to zero as $(T - t)$ converges to infinity. The key is that, with an active monetary policy rule, a period t gap \widehat{m}_t can only survive in equilibrium if agents expect an even larger gap next period. The current gap is consequently not all that sensitive to the size of the long-run gap.

Note that this limiting behavior with respect to $(T - t)$ is independent of the slope of the Phillips curve $(-1/\phi)$. However, the rate of convergence with respect to $(T - t)$ is faster when ϕ is near zero - that is, when the Phillips curve is closer to vertical. If ϕ is near infinity (as suggested by the empirical estimates of the Phillips curve mentioned earlier), then the rate of convergence is relatively slow. In this “nearly flat” Phillips curve case, the dependence of \widehat{m}_t on long-run economic activity can be substantial, unless $(T - t)$ is very large.

This proposition underscores the potential importance of monetary policy in stabilizing the macro-economy. As we have seen, under an interest rate peg, the marginal utility gap \widehat{m}_t fluctuates in response to shocks to agents’ beliefs about the long-run gap \widehat{m}_T . This problem is actually more pronounced when prices are more flexible. Active monetary policy can eliminate the influence of the distant future on current outcomes. It is actually more effective in doing so when prices are more flexible.

5.3 Interest Rate Rule with a Lower Bound

We have seen that, if the interest rate rule is active, then the current equilibrium level of economic activity is essentially independent of agents’ beliefs about long-run gaps. However, any central bank that faces a lower bound on the nominal interest rate is

forced to follow a rule that is in some sense passive. In this subsection, I examine the effects of this kind of passivity on economic outcomes.

I prove two propositions about interest rate rules with a lower bound. The first proposition uses a specific equilibrium selection to rationalize the claim made by some⁵ that the long-run level of inflation is *increasing* in the size of the lower bound.

Proposition 6. *Suppose that the inverse Phillips curve $h(\hat{\pi}; \phi) = \hat{\pi}^{-\phi}$. Consider a model $(X, M, \hat{R}, \Gamma, \phi)$ with price rigidities in which $\phi > 0$ and the interest rate rule \hat{R} takes the following form:*

$$\hat{R}(\hat{\pi}, r_{nat}) = \hat{R}_{LB} \text{ if } \hat{\pi} \leq \pi_+ \quad (60)$$

$$\hat{R}(\hat{\pi}, r_{nat}) = (\hat{\pi}/\pi_+)^{\gamma} \hat{R}_{LB}, \text{ if } \hat{\pi} > \pi_+, \text{ where } \gamma \geq 1 \quad (61)$$

where $\hat{R}_{LB} < \pi_+ < 1$. There exists an equilibrium (x^{MON}, m^{MON}, π) in which $\hat{\pi}_t = \hat{R}_{LB}$ and $\hat{m}_t > 1$ with probability one for all t .

Proof. Set:

$$\hat{\pi}_t = \hat{R}_{LB} \text{ with probability one for all } t$$

$$\hat{m}_t = h(\hat{R}_{LB}; \phi) \text{ with probability one for all } t$$

The specification of $\hat{\pi}_t$ implies that:

$$\hat{R}(\hat{\pi}_t, r_{nat}) = \hat{R}_{LB}$$

with probability one for all t . It is readily verified that $(\hat{\pi}, \hat{m})$ satisfy the equilibrium conditions (28)-(30). We can use the other equilibrium conditions (25)-(27) to define (x^{MON}, m^{MON}, π) . Since $\hat{\pi}_t < 1$ for all t with probability one, we know that $\hat{m}_t > 1$ for all t with probability one. \square

⁵See, among others, Schmitt-Grohe and Uribe (2013) and Cochrane (2015).

In this equilibrium, inflation is stuck at an unduly low level (because $\hat{\pi}_t = \hat{R}_{LB} < 1$ with probability one.). Raising \hat{R}_{LB} would raise $\hat{\pi}$ and lower \hat{m} closer to one. In this sense, it appears desirable to raise the lower bound \hat{R}_{LB} .

But this proposition relies on a particular equilibrium selection (imposing the requirement that equilibrium inflation is constant over dates and states). As the next proposition shows, there is in fact a continuum of equilibria. This set of equilibrium outcomes responds in an intuitive fashion to an increase in \hat{R}_{LB} . As well, within any one of these equilibria, equilibrium outcomes are arbitrarily sensitive to beliefs about downside long-run outcomes when the Phillips curve is arbitrarily close to vertical.

Proposition 7. *Suppose that the inverse Phillips curve h satisfies the restrictions:*

$$h(\hat{\pi}; \phi) \geq \hat{\pi}^{-\phi} \text{ for all } \hat{\pi} > 1$$

$$h(\hat{\pi}; \phi) \leq \hat{\pi}^{-\phi} \text{ for all } \hat{\pi} < 1$$

Consider a model $(X, M, \hat{R}, \Gamma, \phi)$ with price rigidities in which $\phi > 0$ and there exists a constant $\hat{R}_{LB} < 1$ such that $\hat{R}(\hat{\pi}, r^{NAT}) \geq \hat{R}_{LB}$ for all $\hat{\pi}$ and r^{NAT} . There is an equilibrium (x^{MON}, m^{MON}, π) in which there is a positive probability event A in F_t such that for almost all ω in A :

$$(\ln m_T^{MON}(\omega) - \ln m_T^{REAL}(\omega)) > B \tag{62}$$

where B satisfies $(B + \phi \ln \hat{R}^{LB}) > 0$. For any such equilibrium, for almost all ω in A :

$$\ln m_t^{MON}(\omega) - \ln m_t^{REAL}(\omega) > (1 + \phi^{-1})^{T-t} [B + \phi \ln \hat{R}^{LB}] - \phi \ln \hat{R}^{LB} \tag{63}$$

Proof. Let $\tilde{\pi}_T$ be an F_T -measurable random variable that satisfies the restriction $h(\tilde{\pi}_T(\omega)/\pi^*; \phi) > \exp(B)$ for almost all ω in A , where $A \in F_t$. Proposition 2 implies that there exists an equilibrium (x^{MON}, m^{MON}, π) such that $\pi_T = \tilde{\pi}_T$ with probability one. In that

equilibrium, for almost all ω in A :

$$\ln m_T^{MON}(\omega) - \ln m_T^{REAL}(\omega) > B \quad (64)$$

Now consider any equilibrium of this kind, and define $\hat{m}_s = m_s^{MON}/m_s^{REAL}$. I proceed inductively. Assume that $\hat{\pi}_s(\omega) < 1$ and:

$$\ln(\hat{m}_s(\omega)) > (1 + 1/\phi)^{T-s}[B + \phi \ln \hat{R}^{LB}] - \phi \ln \hat{R}^{LB} > 0 \quad (65)$$

for all $s \geq \tau + 1$, where $\tau \geq t$, and almost all ω in A . Note that A is in F_τ . Hence, the asset pricing equation and the restrictions on the inverse Phillips curve imply that for almost all ω in A :

$$\begin{aligned} \hat{m}_\tau(\omega) &= \hat{R}_{LB} E^{NAT}(\hat{\pi}_{\tau+1}^{-1} \hat{m}_{\tau+1} | F_\tau)(\omega) \\ &\geq \hat{R}_{LB} E^{NAT}(\hat{m}_{\tau+1}^{1+1/\phi} | F_\tau)(\omega) \\ \ln(\hat{m}_\tau(\omega)) &\geq \ln \hat{R}_{LB} + (1 + 1/\phi) E^{NAT}(\ln \hat{m}_{\tau+1} | F_\tau)(\omega) \end{aligned}$$

where the last step uses Jensen's inequality. Substituting in the lower bound on $\ln(\hat{m}_{\tau+1})$, we get:

$$\ln \hat{m}_\tau(\omega) \geq \ln \hat{R}_{LB} + (1 + 1/\phi)^{T-\tau}(B + \phi \ln \hat{R}^{LB}) - \phi(1 + 1/\phi) \ln \hat{R}^{LB} \quad (66)$$

$$= (1 + 1/\phi)^{T-\tau}(B + \phi \ln \hat{R}^{LB}) - \phi \ln \hat{R}^{LB} \quad (67)$$

$$(68)$$

for almost all ω in A . This implies that $\ln \hat{m}_\tau(\omega) > 0$ and that $\hat{\pi}_\tau(\omega) < 1$ for almost all ω in A , which completes the inductive step. \square

For $t < T$, there is a lower bound on $\ln \hat{m}_t$ which is *increasing* in $\ln \hat{R}^{LB}$ and in beliefs about highly adverse (that is, positive) realizations of the long-run marginal gap $\ln \hat{m}_T$.

The degree of sensitivity of the lower bound to both $\ln \widehat{R}^{LB}$ and to long-run downside risks becomes more pronounced if ϕ is near zero (that is, the Phillips curve is near-vertical). The key to this result lies in the behavior of expected inflation. Definitionally, a near-vertical Phillips curve implies that even tiny expected future marginal utility gaps are associated with very high rates of anticipated deflation (very negative inflation). In this way, small long-run marginal utility gaps are associated with high real interest rates, and large current marginal utility gaps.

5.4 Summary

When the interest rate rule takes the form of a peg, current equilibrium outcomes in models with a near-vertical Phillips curve are highly sensitive to changes in agents' beliefs about long-run outcomes. The degree of sensitivity is, in fact, increasing as the Phillips curve becomes closer to vertical. An active interest rate rule can essentially eliminate this dependence on the long run. However, if the interest rate rule is bounded from below, then current equilibrium outcomes are highly sensitive to downside shocks to beliefs about long-run economic activity. The degree of sensitivity is, again, increasing as the Phillips curve becomes closer to vertical.

It is common in the New Keynesian literature to restrict attention to monetary policy rules that imply that equilibrium is determinate. The above analysis suggests that, under this restriction, current outcomes are essentially unaffected by beliefs about the long run. As a result, real equilibrium outcomes in a model with a near-vertical Phillips curve will be similar to those in a model with a vertical Phillips curve. However, this restriction on monetary policy rules seems unrealistic. Clarida, Gali, and Gertler (2000) argue that the Federal Reserve followed a passive monetary policy rule during 1970s. As well, as long as there is an effective lower bound on nominal interest rates, the central bank is forced to follow a rule that is, in some sense, passive.

The above analysis hinges on the aggregate Euler equation, which provides a tight

connection between the distant future and current outcomes. As Del Negro, Giannoni and Patterson (2013) and MacKay, Nakamura, and Steinsson (2015) have emphasized, this tight connection implies that forward guidance is a surprisingly powerful monetary policy tool. The former authors suggest relaxing this connection by assuming that agents' lifetimes are, on average, shorter than the life of the economy. The latter authors suggest relaxing this connection via binding borrowing constraints. These suggestions lead both sets of authors to use models in which the log-linearized Euler equation features discounting, so that aggregate marginal utilities satisfy a collection of normalized Euler equations of the form:

$$\widehat{m}_t = \widehat{R}(\widehat{\pi}_t, r_t^{NAT}) E^{NAT} \left(\frac{\widehat{m}_{t+1}^\delta}{\widehat{\pi}_{t+1}} | F_t \right), 0 < \delta < 1 \quad (69)$$

We can readily extend the proofs of the key fragility results (Propositions 4 and 7) to cover the case in which the Euler equations take this discounted form. It is true that the discounting in (69) weakens the connection between current outcomes and beliefs about the future. Nonetheless, households still expect any current marginal utility gap to disappear exponentially quickly when the Phillips curve is near-vertical and the interest rate rule is passive. This expectation means, again, that current marginal utility gaps are highly sensitive to beliefs about long-run gaps.⁶

⁶In a recent paper, Kaplan, Moll, and Violante (KMV) (2015) explore the properties of what they term HANK (Heterogenous Agent New Keynesian) models. In these models, many households have zero liquid assets. As a result (and as KMV emphasize), the standard intertemporal substitution forces are much weaker. Nonetheless, I conjecture that the main fragility results in this paper (Propositions 4 and 7) would generalize to KMV's framework. In particular, it is still true in HANK models that when the Phillips curve is arbitrarily near-vertical (so that ε/θ is near infinity in KMV's notation), expectations of tiny future marginal gaps are associated with expectations of arbitrarily negative inflation. The resulting infinitely high real interest rates will have a crushing effect on consumption demand and economic activity.

6 Building Useful Macroeconomic Models

The previous section shows that, for particular monetary policy regimes, the predictions of a purely real model change dramatically if we add a slight amount of nominal frictions to the model. These results imply that purely real models are too *incomplete* to provide robust predictions about the macroeconomy. In this section, I describe four modeling ingredients that we need to add to purely real models if they are to deliver robust predictions: nominal rigidities, the central bank reaction function, fiscal policy, and beliefs about long-run outcomes.

6.1 Nominal Rigidities

A model with a near-vertical Phillips curve, and a fixed interest rate, has a near-zero impulse response from current gaps to future outcomes. This kind of relationship between the present and future is well-approximated by purely real models with a vertical Phillips curve. However, as we have seen, a model with a near-vertical Phillips curve predicts that current gaps are highly sensitive to information about future gaps. Purely real models zero out this sensitivity. Hence, we can only usefully approximate the feedback of the future to the present by using models that include an explicit treatment of nominal rigidities.

It is probably worth emphasizing that the magnitude of this feedback is especially dependent on the degree of nominal rigidity, as captured by the slope of the Phillips curve, when prices are highly flexible. In this sense, the behavior of the macroeconomy may be highly sensitive to the frequency with which firms adjust their prices.⁷

⁷Bils and Klenow (2004) initiated a large literature that provides microeconomic estimates of the frequency of price changes by firms. See Klenow and Malin (2011) for a survey of that literature.

6.2 Central Bank Reaction Function

The above argument relies on the central bank's reaction function being a peg. With an active reaction function, in which the interest rate moves more than one-for-one with the inflation rate, a gap can only exist today if agents expect an even larger future gap. In this case, as we have seen in Proposition 5, the current gap displays relatively little sensitivity to information about the size of gaps in the distant future. Hence, accurate models of how information about the future affect current outcomes need to include an accurate model of the central bank's reaction function. Recently, for example, some central banks have successfully lowered their target interest rates below zero. Relaxing these constraints may matter for how downside long-run risk affects current outcomes.

6.3 Fiscal Policy

I model fiscal policy as being Ricardian, so that the government's budget constraint is satisfied for any price level process. This kind of fiscal policy combined with an interest rate peg implies that there is a large number of equilibria. As Woodford (1995) argues, other kinds of fiscal policy can be used to winnow down the number of equilibria.⁸ Suppose, for example, that the government commits to exchange consumption bundles \bar{c} for money at a fixed price p_t in period t , where $p_t = \pi^* p_{t-1}$. This commitment would pin down the inflation rate at π^* and eliminates the indeterminacy at the heart of this paper. Hence, an accurate model of the feedback from the future to the present requires an accurate model of how fiscal policy responds to the evolution of the price level.

6.4 Beliefs about Long-Run Macroeconomic Outcomes

In the class of models that I study, expectations are fully rational. Because fiscal policy is Ricardian, there are many possible equilibria in the final period. Agents co-ordinate on an aggregate sunspot to determine which of these equilibria get played. Over time,

⁸See also Kocherlakota and Phelan (1999).

agents receive information about the realization of the sunspot. It is this information that generates variation in their beliefs about long-run outcomes. As we have seen, in models with near-vertical Phillips curves, the changes in these beliefs can have a large influence on the evolution of equilibrium outcomes. To be useful, macroeconomic frameworks of this kind should include accurate models of the evolution of these beliefs, and how policy choices (or communications) might be able to affect that evolution.⁹

Infinite horizon models are unlikely to be useful in this context. Users of infinite horizon models focus on equilibria in which logged outcomes are finite in absolute value. But under an interest rate peg, logged gaps are expected to shrink exponentially over time (in absolute value). In an infinite horizon model, logged equilibrium outcomes can only be finite in absolute value if the long-run gap is always zero.¹⁰ Focusing on infinite horizon models, and on finite equilibria, eliminates any variation in beliefs about long-run gaps.

7 Conclusions

George Box (1979) famously said that "All models are wrong but some are useful." Over the past thirty-plus years, economists have written literally thousands of papers that attempt to analyze macroeconomic data using purely real models. This paper is

⁹As I note in the introduction, the recent work by Garcia-Schmidt and Woodford (2015) could be potentially useful along these lines. However, it is important to supplement their purely theoretical approach with more empirical studies of how households actually form expectations.

¹⁰To proceed somewhat more formally: suppose that $\widehat{R} = 1$ and that the inverse Phillips curve $h(\widehat{\pi}; \phi) = \widehat{\pi}^{-\phi}$. Then the aggregate Euler equation implies that:

$$\infty > \ln \widehat{m}_t \geq \lim_{T \rightarrow \infty} (1 + 1/\phi)^{T-t} E^{NAT}(\ln \widehat{m}_{t+T} | F_t)$$

It follows that

$$\lim_{T \rightarrow \infty} E^{NAT}(\ln \widehat{m}_{t+T} | F_t) = 0$$

and so:

$$Var^{NAT}(\lim_{T \rightarrow \infty} E^{NAT}(\ln \widehat{m}_{t+T} | F_t)) = 0$$

There is no variability in expected long-run gaps, when the model horizon is infinite.

not about the (well-understood) *empirical* point that these models are *wrong*. This paper reaches a distinct *theoretical* conclusion: purely real models are too fragile to be *useful* as a means of answering questions of interest. To take but one famous example: Kydland and Prescott (1982) conclude that their purely real model explains about 2/3 of business cycle fluctuations. But adding even a slight amount of nominal rigidity, in the form of a slightly non-vertical Phillips curve, would make the model's implied variance of output highly dependent on agents' assessments of long-run risk. Like all other purely real models, Kydland and Prescott's framework ignores key sources of monetary non-neutrality and macroeconomic instability. It is not a useful model of output fluctuations.

How, then, do we go about building useful models? We need to take explicit account of nominal rigidities that will give rise to non-vertical Phillips curves. We need to capture the impact of shocks and policy choices on agents' beliefs about long-run economic activity. We need to take explicit account of the operating frameworks of central banks and fiscal authorities. To my knowledge, there has as yet been no research into the empirical properties of models that have all of these elements.

Appendix

In this appendix, I describe how to extend propositions 4 and 7 to the case of a forward-looking Phillips curve similar to that used in much of the New Keynesian literature.

Suppose that we change the normalized equilibrium condition (29) to take the form:

$$\begin{aligned}\hat{\pi}_t &= E_t^{NAT} H(\hat{m}_t, \hat{\pi}_{t+1}; \phi) \\ \hat{\pi}_T &= H_T(\hat{m}_T; \phi)\end{aligned}$$

where H is strictly decreasing in both arguments, H_T is strictly increasing, and $H(1, 1; \phi) = H_T(1) = 1$. (Note that H and H_T are Phillips curves, not inverse Phillips curves.) Sup-

pose too that H and H_T satisfy the restrictions:

$$H(\widehat{m}, 1; \phi) \leq \widehat{m}^{-1/\phi} \text{ if } \widehat{m} \geq 1 \quad (70)$$

$$H(\widehat{m}, 1; \phi) \geq \widehat{m}^{-1/\phi} \text{ if } \widehat{m} \leq 1 \quad (71)$$

$$H_T(\widehat{m}; \phi) \leq \widehat{m}^{-1/\phi} \text{ if } \widehat{m} \geq 1 \quad (72)$$

$$H_T(\widehat{m}; \phi) \geq \widehat{m}^{-1/\phi} \text{ if } \widehat{m} \leq 1 \quad (73)$$

These restrictions ensure that the short-term elasticity of the Phillips curve in each period is larger than $1/\phi$ in absolute value.

The above elasticity restrictions imply that the basic inductive logic underlying Proposition 7 is still valid. Suppose that $\widehat{m}_{t+1}(\omega) > \exp(B_{t+1})$, where $B_{t+1}/\phi + \ln \widehat{R}_{LB} > 0$, and that $\widehat{\pi}_{t+2}(\omega) > 1$ for almost all ω in a set A in F_t . We can show that:

$$\begin{aligned} \ln(\widehat{m}_t(\omega)) &\geq \ln \widehat{R}_{LB} + \ln E^{NAT}(\widehat{m}_{t+1} \widehat{\pi}_{t+1}^{-1} | F_t)(\omega) \\ &\geq \ln \widehat{R}_{LB} + \ln E^{NAT}(\widehat{m}_{t+1}^{1+1/\phi} | F_t)(\omega) \\ &\geq \ln \widehat{R}_{LB} + (1 + 1/\phi) E^{NAT}(\ln(\widehat{m}_{t+1}) | F_t)(\omega) \\ &\geq \ln \widehat{R}_{LB} + (1 + 1/\phi) B_{t+1} \\ &> 0 \end{aligned}$$

It follows that $\widehat{\pi}_{t+1}(\omega) = H(\widehat{m}_{t+1}(\omega), \widehat{\pi}_{t+2}(\omega); \phi) < 1$ and that $\widehat{m}_t(\omega) > \exp(B_t) \equiv \exp(\ln(\widehat{R}_{LB}) + B_{t+1}/\phi) \exp(B_{t+1}) > \exp(B_{t+1})$ for almost all ω in A . We can use this basic inductive argument to establish the validity of the lower bound on marginal utility gaps in Proposition 7.

We can use the same assumptions about H and H_T to prove an analog to Proposition

4. Thus, suppose that $\widehat{R}(\cdot, \cdot) = 1$ for all values of its arguments. Then, in equilibrium:

$$\widehat{m}_t = E^{NAT}(\widehat{m}_{t+1}/\widehat{\pi}_{t+1}|F_t)$$

Assume, inductively, that $\widehat{m}_{t+1}(\omega) > 1$ and $\widehat{\pi}_{t+2}(\omega) \leq 1$ for almost all ω in A , where A is in F_t . The restrictions on H imply that, for almost all ω in A :

$$\widehat{\pi}_{t+1}(\omega)^{-1} \geq \widehat{m}_{t+1}(\omega)^{1/\phi} \geq 1.$$

It follows that for almost all ω in A :

$$\ln(\widehat{m}_t(\omega)) \geq (1 + 1/\phi)\ln(\widehat{m}_{t+1}(\omega)),$$

and so $\widehat{m}_{t+1}(\omega) > 1$ and $\widehat{\pi}_{t+2}(\omega) \leq 1$ for almost all ω in A . We can use this inductive logic to derive the lower (and upper) bounds on marginal utility gaps established in Proposition 4.

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