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# EXOGENOUS IMPULSE OR ENDOGENOUS RESPONSE?

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Uncertainty and Business Cycles: Exogenous Impulse or Endogenous Response?
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**ABSTRACT**

Uncertainty about the future rises in recessions. But is uncertainty a source of business cycle fluctuations or an endogenous response to them, and does the type of uncertainty matter? Answer: we find that sharply higher uncertainty about real economic activity in recessions is fully an endogenous response to other shocks that cause business cycle fluctuations, while uncertainty about financial markets is a likely source of the fluctuations. Financial market uncertainty has quantitatively large negative consequences for several measures of real activity including employment, production, and orders. Such are the main conclusions drawn from estimation of three-variable structural vector autoregressions (SVAR). To establish dynamic causal effects, we propose an iterative projection external variable (IPEV) approach to identify the SVAR under credible interpretations of the structural shocks.

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# 1 Introduction

A large literature in macroeconomics investigates the relationship between uncertainty and business cycle fluctuations. Interest in this topic has been spurred by a growing body of evidence that uncertainty rises sharply in recessions. This evidence is robust to the use of specific proxy variables such as stock market volatility and forecast dispersion as in Bloom (2009), or a broad-based measure of macroeconomic uncertainty, as in Jurado, Ludvigson, and Ng (2015) (JLN hereafter). But while this evidence substantiates a role for uncertainty in deep recessions, the question of whether uncertainty is an exogenous source of business cycle fluctuations or an endogenous response to economic fundamentals is not fully understood. Existing results are based on convenient but restrictive identifying assumptions and have no explicit role for financial markets, even though the uncertainty measures are correlated with financial variables. This paper considers a novel identification strategy to disentangle the causes and consequences of real and financial uncertainty.

The question of causality and the identification of exogenous variation in uncertainty is a long-standing challenge of the uncertainty literature. The challenge arises in part because there is no theoretical consensus on whether the uncertainty that accompanies deep recessions is primarily a cause or effect (or both) of declines in economic activity. Theories in which uncertainty is defined as the time varying volatility of a fundamental shock cannot address this question because, by design, there is no feedback response of uncertainty to other shocks if the volatility process is specified to evolve exogenously. And, obviously, models in which there is no exogenous variation in uncertainty cannot be used to analyze the direct effects of uncertainty shocks. It is therefore not surprising that many theories for which uncertainty plays a role in recessions reach contradictory conclusions on this question, as we survey below. It is clear that the body of theoretical work on uncertainty does not provide precise identifying restrictions for empirical work.

A separate challenge of the uncertainty literature pertains to the origins of uncertainty. Classic theories assert that uncertainty originates from economic fundamentals such as productivity, and that such real economic uncertainty, when interacted with market frictions, discourages real activity. But some researchers have argued that uncertainty dampens the economy through its influence on financial markets (e.g., Gilchrist, Sim, and Zakrajsek (2010)). Moreover, as surveyed by Ng and Wright (2013), all the post-1982 recessions have origins in financial markets, and these recessions have markedly different features from recessions where financial markets play a passive role. From this perspective, if financial shocks are subject to time-varying volatility, financial market uncertainty–as distinct from real economic uncertainty–could be a key player in recessions, both as a cause and as a propagating mechanism. The Great Recession of 2008, characterized by sharp swings in financial markets, hints at such a linkage. Yet so far the literature has not disentangled the contributions of real versus financial uncertainty to business cycle fluctuations.

Econometric analyses aimed at understanding the role of uncertainty for business cycle fluctuations face their own challenges. Attempts to identify the "effects" of uncertainty shocks in existing empirical work are primarily based on recursive schemes within the framework of vector-autoregressions (VAR).[1] But studies differ according to whether uncertainty is ordered ahead of or after real activity variables in the VAR. While a recursive structure is a reasonable starting point, any presumed ordering of the variables is hard to defend on theoretical grounds given the range of models in the literature. Contemporaneous changes in uncertainty can arise both as a cause of business cycle fluctuations and as a response to other shocks. Recursive structures explicitly rule out this possibility since they presume that some variables respond only with a lag to others.

It is with these challenges in mind that we return to the questions posed above: is uncertainty primarily a source of business cycle fluctuations or a consequence of them? And what is the relation of real versus financial uncertainty to business cycle fluctuations? The objective of this paper is to address these questions econometrically using a small-scale structural vector autoregression (SVAR). To confront the challenges just discussed, we take a two-pronged approach. First, our empirical analysis explicitly distinguishes *macro* uncertainty from *financial* uncertainty. The baseline SVAR we study describes the dynamic relationship between three variables: an index of macro uncertainty, $U_{Mt}$, a measure of real economic activity, $Y_t$ (e.g., production, employment), and a new financial uncertainty index introduced here, $U_{Ft}$. Second, rather than relying on timing assumptions for identification, we use a different identification scheme that is less restrictive, both because it allows for simultaneous feedback between uncertainty and real activity, and because it can be used to test whether a lower recursive structure is supported by the data.

Specifically, our identification scheme makes use of the existence of two external variables that are presumed to be relevant for uncertainty shocks but are not part of the SVAR: a $Z_{1t}$ that is correlated with macro and financial uncertainty but contemporaneously uncorrelated (exogenous) with respect to real activity, and a $Z_{2t}$ that is correlated with financial uncertainty but contemporaneously uncorrelated with both real activity and macro uncertainty. While such external variables have no obvious empirical counterparts, we propose an *iterative projection external variable* (IPEV) approach to construct a $Z_{1t}$ and $Z_{2t}$ with the desired properties from observables. The approach takes a variable $S_t$ that is not in the VAR system and uses projections to decompose it into two components, one that is correlated with a subset of the endogenous variables of interest, and one that is orthogonal to it. We then use these components as part of

[1]See Bachmann, Elstner, and Sims (2013), Bloom (2009), Bloom (2014), Bekaert, Hoerova, and Duca (2013), Gilchrist, Sim, and Zakrajsek (2010), and JLN.

an identification strategy, a procedure that bears some analogies to the external instrumental variable (IV) approach in the SVAR literature but is distinct from it. The difference is that the external variables $S_t$ are not themselves presumed to be valid exogenous instruments. Instead, *components* of $S_t$ are presumed on economic grounds to exhibit a minimal degree of non-zero correlations with one or both uncertainty shocks but still satisfy the exogeneity properties of a $Z_{1t}$ and $Z_{2t}$. In the spirit of the IV literature, we refer to these constructed components as "synthetic instruments."

In the present context, the key is to find observables external to our SVAR that are driven by a multitude of innovations, including the uncertainty shocks we are interested in. We argue below that both theory and evidence suggest that aggregate stock market returns are natural candidates for such variables. Our maintained economic hypothesis is that components of stock market returns exist that have non-negligible correlations with both uncertainty shocks but still satisfy the exogeneity properties of $Z_{1t}$ and $Z_{2t}$. Our IPEV approach therefore generates a synthetic $Z_{1t}$ by purging the effects of real activity shocks from stock returns, and another synthetic $Z_{2t}$ by further purges the effects of macro uncertainty shocks. Iteration ensures that the shocks used to generate the synthetic instruments are consistent with those identified by our SVAR.

Identification is achieved by combining estimates of the synthetic $Z_{1t}$ and $Z_{2t}$ with economic restrictions. One economic restriction is that minimum thresholds for the correlations between the synthetic instruments and the relevant uncertainty shocks must be satisfied. Another economic restriction is that the identified shocks must be consistent with prior economic reasoning in a small number of extraordinary events, such as the 1987 stock market crash and the financial crisis/Great Recession of 2007-09. With these restrictions, the set of solutions to the SVAR identification problem can be substantially narrowed to reveal a well defined pattern of dynamic causal effects. Under a closely related set of restrictions, our approach can be fit into the classic simultaneous equations framework and interpreted as the output of a system estimation for a larger VAR that includes both $\mathbf{X}_t$ and $S_t$.

The empirical exercise also requires that appropriate measures of macro and financial uncertainty are available. To this end, we exploit a data rich environment, working with 134 macro monthly time series and 147 financial variables. The construction of macro uncertainty follows JLN. The same approach is used to construct a broad-based measure of financial uncertainty that has never been used in the literature. Macro uncertainty is itself an aggregate of uncertainties in variables from three categories: real activity, price, and financial. To better understand the contributions of each of these categories, we also replace $U_{Mt}$ in the VAR with an uncertainty measure based on the real activity sub-components. Uncertainty about real activity is of special interest because classic uncertainty theories postulate that uncertainty shocks have their origins in economic fundamentals and hence should show up as uncertainty about real economic activity.

Before summarizing our main results, it should be made clear that the structural shocks we identify do not in general correspond to primitive shocks in specific economic models. Real activity is endogenous and may respond to any number of primitive shocks (technology, monetary policy, preferences, wage or price markups, government expenditures, etc.). If a SVAR representation exists, our identified real activity shock would then be a composite of these primitive shocks, with the restriction that this composite be orthogonal to the other shocks in our system. The same could be said for either type of uncertainty, to the extent that these variables are endogenous. Our objective is not, therefore, to identify primitive shocks in specific models. Indeed, we argue that the questions raised above are ultimately empirical ones that call out for a model-free approach. (See the literature review below for further discussion.) What our approach offers, therefore, is something different: if there exists an SVAR in the system of interest, then under the assumptions stipulated below, IPEV can provide a less restrictive means of identifying dynamic causal effects when commonly used ordering or timing assumptions are difficult to defend.

Our main results can be stated as follows. First, positive shocks to financial uncertainty are found to cause a sharp decline in real activity that persists for many months, lending support to the hypothesis that heightened uncertainty is an exogenous impulse that causes recessions. These effects are especially large for some measures of real activity, notably employment and orders. The finding that heightened uncertainty has negative consequences for real activity is qualitatively similar to that of preexisting empirical work that uses recursive identification schemes (e.g., Bloom (2009), JLN), but differs in that we trace the source of this result specifically to broad-based financial market uncertainty rather than to various uncertainty proxies or broad-based macro uncertainty. We also show that the converse is not supported by our evidence: exogenous shocks to real activity have little affect on financial uncertainty.

Second, the identification scheme used here reveals something new that is not possible to uncover under recursive schemes: macro and financial uncertainty have a very different dynamic relationship with real activity. Specifically, unlike financial uncertainty, sharply higher macro and real activity uncertainty in recessions is fully an endogenous response to business cycle fluctuations. That is, negative economic activity shocks are found to cause increases in both macro and real activity uncertainty, but there is no evidence that independent shocks to macro or real uncertainty cause lower economic activity. Indeed the opposite is true: exogenous shocks to both macro and real uncertainty are found to *increase* real activity, consistent with "growth options" theories discussed below.

Third, our results are distinct from those obtained using recursive identification. Under any recursive ordering of the variables in our VAR, exogenous shocks that increase macro or real uncertainty appear to reduce real activity, in a manner that is qualitatively similar to financial uncertainty shocks. This result does not hold in the less restrictive SVAR studied here and appears to be an artifact of invalid timing assumptions under recursive identification. Further investigation reveals that the SVAR we study reflects a non-zero contemporaneous correlation between $U_{Ft}$ and $Y_t$, as well as between $U_{Mt}$ and $Y_t$, which is inconsistent with any recursive ordering. Tests of the validity of a recursive structure are easily rejected by the data.

The rest of this paper is organized as follows. Section 2 reviews related literature and provides motivation for our maintained economic hypothesis that there exists a component of stock returns that is correlated with macro and financial uncertainty shocks but contemporaneously uncorrelated with real activity, and another component that is correlated with financial uncertainty but contemporaneously uncorrelated with both real activity and macro uncertainty. Section 3 details the econometric framework and identification employed in our study, describes how our instruments are constructed, and discusses the data and empirical implementation. Section 4 presents empirical results using broad based macro uncertainty $U_{Mt}$, while Section 5 reports results for systems that isolate the sub-component of $U_{Mt}$ corresponding to real activity variables. Section 6 reports results pertaining to robustness and additional cases. In this section we consider an estimation where we take two observed external variables $S_{1t}$ and $S_{2t}$ and presume they are valid external instruments $Z_{1t}$ and $Z_{2t}$. This is compared to the case where the same variables are presumed not to be valid instruments and IPEV is used to construct synthetic instruments from $S_{1t}$ and $S_{2t}$. In this section we also show how, with some additional restrictions, our approach can interpreted as the output of a system estimation for a larger VAR that includes both $\mathbf{X}_t$ and $S_t$. Section 7 summarizes and concludes. A large number of additional results on the IPEV methodology are presented in (Ludvigson, Ma, and Ng (2016)).

## 2 Related Literature

A large literature addresses the question of uncertainty and its relation to economic activity.[2] Besides the evidence cited above for the U.S., Nakamura, Sergeyev, and Steinsson (2012) estimate growth rate and volatility shocks for 16 developed countries and find that they are substantially negatively correlated. Theories for which uncertainty plays a key role differ widely on the question of whether uncertainty is primarily a cause or a consequence of declines in economic activity. In most cases, it is modeled either as a cause or an consequence, but not both.

The first strand of the literature proposes uncertainty as a cause of lower economic growth. This includes models of the real options effects of uncertainty (Bernanke (1983), McDonald and Siegel (1986)), models in which uncertainty influences financing constraints (Gilchrist,

[2] This literature has become voluminous. See Bloom (2014) for a recent review of the literature.

Sim, and Zakrajsek (2010), Arellano, Bai, and Kehoe (2011)), or precautionary saving (Basu and Bundick (2012), Leduc and Liu (2012), Fernández-Villaverde, Pablo Guerrón-Quintana, and Uribe (2011)). These theories almost always presume that uncertainty is an exogenous shock to some economic fundamental. Some theories presume that higher uncertainty originates directly in the process governing technological innovation, which subsequently causes a decline in real activity (e.g., Bloom (2009), Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2012)).

A second strand of the literature postulates that higher uncertainty arises solely as a *response* to lower economic growth, emphasizing a variety of mechanisms. Some of these theories suggest that bad times incentivize risky behavior (Bachmann and Moscarini (2011), Fostel and Geanakoplos (2012)), or reduce information and with it the forecastability of future outcomes (Van Nieuwerburgh and Veldkamp (2006) Fajgelbaum, Schaal, and Taschereau-Dumouchel (2014)), or provoke new and unfamiliar economic policies whose effects are highly uncertain (Pástor and Veronesi (2013)), or create a greater misallocation of capital across sectors (Ai, Li, and Yang (2015)), or generate endogenous countercyclical uncertainty in consumption growth because investment is costly to reverse (Gomes and Schmid (2016)).

And yet a third literature has raised the possibility that some forms of uncertainty can actually *increase* economic activity. "Growth options" theories of uncertainty postulate that a mean-preserving spread in risk generated from an unbounded upside coupled with a limited downside can cause firms to invest and hire, since the increase in mean-preserving risk increases expected profits. Such theories were often used to explain the dot-com boom. Examples include Bar-Ilan and Strange (1996), Pastor and Veronesi (2006), Kraft, Schwartz, and Weiss (2013), Segal, Shaliastovich, and Yaron (2015).

This brief review reveals a rich literature with a wide range of predictions about the relationship between uncertainty and real economic activity. Yet the absence of a theoretical consensus on this matter, along with the sheer number of theories and limited body of evidence on the structural elements of specific models, underscores the extent to which the question of cause and effect is fundamentally an empirical matter that must be settled in an econometric framework with as little specific theoretical structure as possible, so that the various theoretical possibilities can be nested in empirical tests. Commonly used recursive identification schemes cannot achieve this objective, since by construction they rule out the possibility that uncertainty and real activity could influence one another within the period. Our econometric model nests any recursive identification scheme, so we can test whether such timing assumptions are plausible. We find they are rejected by the data.

Our construction of instruments for uncertainty builds on work in asset pricing emphasizing the idea that stock market variation is the result of several distinct (and orthogonal) sources of stochastic variation, some of which are likely to be uniquely suited as instruments for our uncertainty measures. For example, one quantitatively important component is attributable to *a*cyclical risk premia variation, and more generally appears to be uncorrelated with most measures of real activity.[3] This component is valuable for our objective because it is exogenous to real activity, but may still be relevant for both macro and financial uncertainty, as in our synthetic $Z_{1t}$. Yet another component could be attributable to fluctuations in factors like corporate leverage, or in the risk aversion or "sentiment" of market participants that may be correlated with the volatility of the stock market. In equilibrium asset pricing models, if leverage increases, volatility of the corporate sector's equity return increases. Thus changes in factors like leverage (and possibly changes in risk aversion or sentiment) should be correlated with financial uncertainty, but have little to do with real economic uncertainty. This component is valuable for our objective because it is plausibly uncorrelated with both real activity *and* uncertainty about economic fundamentals, but may still be relevant for financial market uncertainty, as in our synthetic $Z_{2t}$. Consistent with the existence of this type of component, JLN document that there are many spikes in stock market uncertainty that do not coincide with an important movement in either real activity or macro uncertainty. These findings motivate our maintained hypothesis that measures of equity market returns are promising non-uncertainty variables comprised of several distinct sources of stochastic variation, two of which have the statistical characteristics of a $Z_{1t}$ and $Z_{2t}$.

Our IPEV approach is related to a recent line of econometric research in SVARs that uses information contained in external instruments to identify structural dynamic causal effects.[4] Of these, Stock and Watson (2012) study uncertainty shocks, using a measure of stock market volatility and/or a news media measure of policy uncertainty from Baker, Bloom, and Davis (2013), as separate external instruments for identifying the effects of uncertainty shocks in a SVAR. Our study differs in some fundamental ways. First, our approach relies on a set of economic assumptions that is distinct from that of standard IV approach, hence the moment conditions used to identify the model parameters and shocks are not in general the same. Specifically, the identification strategy in Stock and Watson (2012) for uncertainty shocks presumes that the series themselves (i.e., stock market volatility, policy uncertainty) are valid instruments, correlated with the uncertainty shock of interest but not with the other shocks. By contrast, our approach explicitly views both the stock market and our uncertainty measures as partly endogenous, forcing us to confront the identification quandary. Our identification assumption is instead that aggregate stock market returns contain components that are non-negligibly correlated with both types of uncertainty shocks but still uncorrelated with real activity, or jointly

[3]For empirical evidence, see Lettau and Ludvigson (2013), Greenwald, Lettau, and Ludvigson (2014), Kozak and Santosh (2014), and Muir (2014). Theoretical examples include Greenwald, Lettau, and Ludvigson (2014); Bianchi, Ilut, and Schneider (2014); Gourio (2012);Wachter (2013); Brunnermeier and Sannikov (2012); Gabaix and Maggiori (2013); He and Krishnamurthy (2013).

[4]See for example Hamilton (2003), Kilian (2008), Mertens and Ravn (2013); Stock and Watson (2008), Stock and Watson (2012), and Olea, Stock, and Watson (2015).

uncorrelated with real activity and macro uncertainty, even while some of its variation is endogenous to these variables. Second, Stock and Watson (2012) focus exclusively on identifying the effects of uncertainty shocks and do not attempt to simultaneously identify the converse, namely the effects of real activity shocks on uncertainty.

Berger, Dew-Becker, and Giglio (2016) take a different approach. Using options data they find that bad times are associated with higher realized volatility but not higher expected volatility, a result that they interpret as consistent with the hypothesis that higher uncertainty is a consequence of negative economic shocks rather than a cause.

The study arguably closest in spirit to our identification approach is Baker and Bloom (2013), who use disaster-like events as instruments for stock market volatility with the aim of isolating exogenous variation in uncertainty. This has some similarities with our approach, in that it implicitly assumes that certain components of stock market fluctuations (those associated with "disasters") are exogenous. In contrast to our approach, exogenous events are chosen subjectively rather than constructed econometrically to satisfy specific orthogonality restrictions. It is of interest that we arrive at complementary conclusions, despite the differing methodologies for identifying exogenous variation.

# 3 Econometric Framework

This section outlines our econometric approach. Subsection 1 explains the identification strategy. Subsections 2 and 3 explain the construction of external instruments in the IPEV procedure and the uncertainty measures. This is followed by a discussion of the estimation procedure.

## 3.1 The SVAR and Identification

Our analysis is based on a structural vector autoregressive model (SVAR). Let $\mathbf{X}_t$ denote a $K \times 1$ time series. We suppose that the structural model has a $p$-th order vector autoregressive representation

$$\mathbf{X}_t = \mathbf{k} + \mathbb{A}_1 \mathbf{X}_{t-1} + \mathbb{A}_2 \mathbf{X}_{t-2} + \cdots + \mathbb{A}_p \mathbf{X}_{t-p} + \mathbf{H\Sigma e}_t. \tag{1}$$

$$\mathbf{e}_t \sim (0, \mathbf{I_K}), \qquad \mathbf{\Sigma} = \begin{pmatrix} \sigma_{11} & 0 & \cdot & 0 \\ 0 & \sigma_{22} & 0 & 0 \\ 0 & \cdot & \cdot & 0 \\ 0 & 0 & \cdot & \sigma_{KK} \end{pmatrix}, \quad \sigma_{jj} \geq 0 \; \forall j. \tag{2}$$

The structural shocks $\mathbf{e}_t$ are mean zero with unit variance, and are serially and mutually uncorrelated. The corresponding structural $MA(\infty)$ representation of $\mathbf{X}_t$ is

$$\mathbf{X}_t = \boldsymbol{\mu} + \mathbf{\Psi}(L) \mathbf{H\Sigma e}_t$$

where $\boldsymbol{\Psi}(L) = \boldsymbol{\Psi}_0 + \boldsymbol{\Psi}_1 L + \boldsymbol{\Psi}_2 L^2 + \ldots$ with $\boldsymbol{\Psi}_0 = \mathbf{I}$ is a polynomial in the lag operator $L$ of infinite order, $\boldsymbol{\Psi}_s$ is the $(n \times n)$ matrix of coefficients for the $s$th lag of $\boldsymbol{\Psi}(L)$. Note that $\boldsymbol{\Psi}(L) = \mathbb{A}(L)^{-1}$, where $\mathbb{A}(L) = \mathbf{I} - \mathbb{A}_1 L - \cdots - \mathbb{A}_p L^p$.

The reduced form representation of $\mathbf{X}_t$ is a $p$-th order vector-autoregression (VAR) with corresponding reduced-form $MA(\infty)$ representation

$$\begin{aligned} \mathbf{X}_t &= \boldsymbol{\mu} + \boldsymbol{\Psi}(L)\,\boldsymbol{\eta}_t \\ \boldsymbol{\eta}_t &\sim (0, \boldsymbol{\Omega}), \quad \boldsymbol{\Omega} = \mathbb{E}\left(\boldsymbol{\eta}_t \boldsymbol{\eta}_t'\right). \end{aligned} \tag{3}$$

The structural shocks $\mathbf{e}_t$ are presumed to be related to the reduced form innovations by an invertible $K \times K$ matrix $\mathbf{H}$:

$$\boldsymbol{\eta}_t = \mathbf{H}\boldsymbol{\Sigma}\mathbf{e}_t \equiv \mathbf{B}\mathbf{e}_t,$$

where $\mathbf{B} \equiv \mathbf{H}\boldsymbol{\Sigma}$. We say that an SVAR for $\mathbf{X}_t$ exists if a rotation $\mathbf{H}^{-1}$ of the reduced form shocks $\boldsymbol{\eta}_t$ can be found such that its elements are serially and mutually uncorrelated.

A normalization is required to pin down the sign and scale of the shocks. We adopt the unit effect normalization

$$\mathrm{diag}(\mathbf{H}) = 1. \tag{4}$$

The objective of the exercise is to study the dynamic effects and the relative importance of the structural shocks. More precisely, the dynamic response to shock $j$ is summarized by the impulse response function (IRF):

$$\frac{\partial \mathbf{X}_{t+s}}{\partial e_{jt}} = \boldsymbol{\Psi}_s \mathbf{b}^j, \tag{5}$$

where $\mathbf{b}^j$ is the $j$th column of $\mathbf{B}$. The structural IRF $\boldsymbol{\Psi}_s \mathbf{b}^j$ gives the dynamic response of $\mathbf{X}_{t+s}$ to a one standard deviation shock. The quantitative importance of each shock is given by the fraction of $S$-step ahead forecast error variance of $\mathbf{X}_t$ that is attributable to each structural shock. The coefficient matrices of $\boldsymbol{\Psi}(L)^{-1}$ are identified from the projection of $\mathbf{X}_t$ onto its lags in the reduced form VAR (3). The SVAR identification problem therefore amounts to identifying the elements of $\mathbf{H}$ and $\boldsymbol{\Sigma}$, from which the structural IRFs are computed.

Let $Y_t$ denote a measure of real activity. Our objective is to study the impulse and propagating mechanism of uncertainty shocks, as well as how uncertainty reacts to shocks to $Y_t$, while explicitly distinguishing between macro and financial market uncertainty. Let $K = 3$. Hence our baseline SVAR is based on $\mathbf{X}_t = (U_{Mt}, Y_t, U_{Ft})'$, where $U_{Mt}$ denotes macro uncertainty, $U_{Ft}$ denotes financial uncertainty. The reduced form shocks $\boldsymbol{\eta}_t = (\eta_{Mt}, \eta_{Yt}, \eta_{Ft})'$ are linear combinations of the three structural form shocks $\mathbf{e}_t = (e_{Mt}, e_{Yt}, e_{Ft})'$ to macro uncertainty, real activity, and financial uncertainty, respectively.

$$\begin{aligned}
\eta_{Mt} &= B_{MM}e_{Mt} + B_{MY}e_{Yt} + B_{MF}e_{Ft} \\
\eta_{Yt} &= B_{YM}e_{Mt} + B_{YY}e_{Yt} + B_{YF}e_{Ft} \\
\eta_{Ft} &= B_{FM}e_{Mt} + B_{FY}e_{Yt} + B_{FF}e_{Ft},
\end{aligned}$$

where $B_{ij}$ is the element of $\mathbf{B}$ that gives the contemporaneous effect of the $j$th structural shock on the $i$th variable. The covariance structure of $\boldsymbol{\eta}_t$ provides $K(K+1)/2 = 6$ equations in $\mathbf{B}$:

$$\text{vech}(\boldsymbol{\Omega}) = \text{vech}(\mathbf{B}\mathbf{B}') \tag{6}$$

where vech($\boldsymbol{\Omega}$) stacks the unique elements of the symmetric matrix $\boldsymbol{\Omega}$. There are nine unknown elements in $\mathbf{B}$.

To motivate our IPEV procedure, it is helpful to begin by considering the classic IV approach where valid instruments are observed. To do so, suppose for the moment that we have measures of $Y_t$, $U_{Mt}$, $U_{Ft}$, and two external instruments, $Z_{1t}$ and $Z_{2t}$ satisfying the following:

**Assumption A:** Let $\mathbf{Z}_t = (Z_{1t}, Z_{2t})'$ be two instrumental variables such that

$$\begin{array}{llll}
(A.i) & \mathbb{E}[Z_{1t}e_{Mt}] \neq 0, & \mathbb{E}[Z_{1t}e_{Yt}] = 0, & E[Z_{1t}e_{Ft}] \neq 0 \\
(A.ii) & \mathbb{E}[Z_{2t}e_{Mt}] = 0, & \mathbb{E}[Z_{2t}e_{Yt}] = 0, & \mathbb{E}[Z_{2t}e_{Ft}] \neq 0.
\end{array}$$

Assumption A are conditions for instrument exogeneity and relevance. $Z_{1t}$ is an instrument that is correlated with both macro and financial uncertainty, but contemporaneously uncorrelated (exogenous) with respect to real activity. $Z_{2t}$ is an instrument that is correlated with financial uncertainty, but contemporaneously uncorrelated (exogenous) with respect to macro uncertainty and real activity.

Let $\mathbf{m}_{1t} = (\text{vech}(\boldsymbol{\eta}_t\boldsymbol{\eta}_t'), \text{vec}\,(\mathbf{Z}_t \otimes \boldsymbol{\eta}_t))'$ and $\boldsymbol{\beta} = \text{vec}(\mathbf{B})$. At the true value of $\boldsymbol{\beta}$, denoted $\boldsymbol{\beta}^0$, the model satisfies

$$0 = \mathbb{E}[\mathbf{g}_1(\mathbf{m}_{1t}; \boldsymbol{\beta}^0)], \tag{7}$$

written out in full as follows:

$$\begin{aligned}
0 &= var(\eta_M) - B_{MM}^2 + B_{MY}^2 + B_{MF}^2 \\
0 &= var(\eta_Y) - B_{YM}^2 + B_{YY}^2 + B_{YF}^2 \\
0 &= var\left(\eta_F\right) - B_{FM}^2 + B_{FY}^2 + B_{FF}^2 \\
0 &= cov(\eta_M, \eta_Y) - B_{MM}B_{YM} + B_{MY}B_{YY} + B_{MF}B_{YF} \\
0 &= cov(\eta_Y, \eta_F) - B_{YM}B_{FM} + B_{YY}B_{FY} + B_{FF}B_{YF} \\
0 &= cov\left(\eta_{M,}\eta_F\right) - B_{MM}B_{FM} + B_{MY}B_{FY} + B_{MF}B_{FF} \\
0 &= B_{MF}\mathbb{E}[Z_{2t}\eta_Y] - B_{YF}\mathbb{E}[Z_{2t}\eta_{Mt}] \\
0 &= B_{FF}\mathbb{E}[Z_{2t}\eta_{Yt}] - B_{YF}\mathbb{E}[Z_{2t}\eta_{Ft}] \\
0 &= (B_{MM}B_{FF} - B_{MF}B_{FM})\mathbb{E}[Z_{1t}\eta_{Yt}] - (B_{YF}B_{FM} - B_{YM}B_{FF})\mathbb{E}[Z_{1t}\eta_{Mt}] \\
&\quad -(B_{MM}B_{YF} - B_{MF}B_{YM})\mathbb{E}[Z_{1t}\eta_{Ft}].
\end{aligned}$$

The model has nine equations in nine unknowns. The first six are from the covariance structure. The next two equations are due to the three moments implied by Assumption $(A.ii)$. The final equation is due to the three moments implied by Assumption $(A.i)$.

**Proposition 1** *Under Assumption A with det$(B) > 0$, the normalization (4), and the restriction (2), $\boldsymbol{\beta}$ is identified.*

The Appendix gives a closed-form solution for $\mathbf{B}$, and shows that the covariance between the instruments and the structural shocks can be expressed as

$$\begin{aligned}
\mathbb{E}[Z_{2t}e_{Ft}]^2 &= \mathbb{E}[\boldsymbol{\eta}_t Z_{2t}]'\boldsymbol{\Omega}^{-1}\mathbb{E}[\boldsymbol{\eta}_t Z_{2t}] \\
\mathbb{E}[Z_{1t}e_{Mt}]^2 &= \left(\mathbb{E}[\boldsymbol{\eta}_t Z_{1t}] - \frac{\mathbb{E}[\boldsymbol{\eta}_t Z_{2t}]}{\mathbb{E}[Z_{2t}e_{Ft}]}\mathbb{E}[Z_{2t}e_{Ft}]\right)'\boldsymbol{\Omega}^{-1}\left(\mathbb{E}[\boldsymbol{\eta}_t Z_{1t}] - \frac{\mathbb{E}[\boldsymbol{\eta}_t Z_{2t}]}{\mathbb{E}[Z_{2t}e_{Ft}]}\mathbb{E}[Z_{2t}e_{Ft}]\right) \\
\mathbb{E}[Z_{2t}e_{Ft}]\mathbb{E}[Z_{1t}e_{Ft}] &= \mathbb{E}[\boldsymbol{\eta}_t Z_{2t}]'\boldsymbol{\Omega}^{-1}\mathbb{E}[\boldsymbol{\eta}_t Z_{1t}].
\end{aligned}$$

We verify that the closed-form solution is the the same as the unique numerical solution obtained with (4) and (2) imposed.

In essence, identification in this analysis is achieved by (i) using movements in $U_{Mt}$ and $U_{Ft}$ that are correlated with $Z_{1t}$ to identify the effects of uncertainty shocks and disentangle them from shocks to real activity, (ii) using movements in $U_{Ft}$ that are correlated with $Z_{2t}$ to identify the effects of $U_{Ft}$ shocks and disentangle them from macro uncertainty shocks, and (iii) using movements in $Y_t$ that are uncorrelated with both $Z_{1t}$ and $Z_{2t}$ to identify the effects of real activity shocks and disentangle them from uncertainty shocks.

We take the stand in this application that our uncertainty measures are potentially endogenous. It is then natural to ask why we do not simply find observable instruments. One answer is that credible valid instruments may be difficult or impossible to find and defend. We argue that existing uncertainty proxies are likely to be among those variables that fall into this category. JLN find that such measures, including options-based volatility indexes such as VIX or VXO, are less defensible measures of uncertainty than those employed here, so it makes little sense to instrument for the latter with the former. Options-based volatility indexes are doubly problematic for our purpose because they are known to contain a large component attributable to changes in the variance risk premium that are unrelated to common notions of uncertainty (e.g., Bollerslev, Tauchen, and Zhou (2009); Carr and Wu (2009)). On the other hand, options based indexes may be valuable in empirical contexts different from ours, such as those that seek to distinguish expected stock market volatility from realized stock market volatility (Berger, Dew-Becker, and Giglio (2016)). With these considerations in mind, the next subsection proposes a methodology that is in the spirit of classic IV. The approach constructs "synthetic instruments" from endogenous external variables that have properties analogous to those of Assumption A and can, under some additional economic assumptions, help identify dynamic causal effects in the SVAR for $\mathbf{X}_t$.

## 3.2 Construction of Instruments

Suppose that the external instruments $Z_{1t}$ and $Z_{2t}$ have no observable counterpart. The next step is to develop a methodology to construct synthetic proxies in the spirit of such variables. To motivate our method of IPEV, recall that two stage least squares uses projections to purge the endogenous variations from a relevant regressor. Our IPEV approach is similar in spirit except that we purge the endogenous variations from a observed variable that is not of first order relevance to our VAR system. The output of such a projection is a generated external "instrument."

In the present context, we make use of observables $S_t$ that are driven not only by our structural shocks $\mathbf{e}_t = (e_{Yt},\, e_{Mt}$ and $e_{Ft})'$, but also by other shocks collected into an $e_{St}$ that are uncorrelated with $\mathbf{e}_t$. A theoretical premise of the paper is that uncertainty shocks should be reflected in aggregate equity returns. Thus our choice of $S_t$ is a measure of stock market returns. Under these assumptions, we may represent $S_t$ as

$$S_t = \delta_0 + \delta_Y Y_t + \delta_M U_{Mt} + \delta_F U_{Ft} + \delta_S(L) S_{t-1} + \delta_X(L)'\mathbf{X}_{t-1} + e_{St} \tag{8}$$

where $\mathbf{X}_t = (Y_t, U_{Mt}, U_{Ft})'$. The residual $e_{St}$ could be driven by any number of shocks orthogonal to $\mathbf{e}_t$. One interpretation is risk premium shocks driven by factors orthogonal to uncertainty, such as a pure sentiment shock (one uncorrelated with uncertainty), but the precise interpretation is not important to what follows. Obviously, $S_t$ is an endogenous variable but it is external to the variable $\mathbf{X}_t$ system by assumption. (Below we show results from estimating a larger system that includes $S_t$ in which this exclusion restriction can be evaluated.) Omitting any component of $\mathbf{X}_t$ as an explanatory variable will yield inconsistent estimates of the parameters in (8). However, we are not interested in these parameters. Our objective in considering stock-market returns is solely to remove from it those variations due to the estimated $e_{Mt}$ and/or $e_{Yt}$. More precisely, (8) motivates two (non-structural) representations of $S_t$ (not necessarily the same variable):

$$S_{1t} = d_{10} + d_{1Y}e_{Yt} + d_{1S}(L)S_{1t-1} + Z_{1t} \tag{9a}$$

$$S_{2t} = d_{20} + d_{2M}e_{Mt} + d_{2Y}e_{Yt} + d_{2S}(L)S_{2t-1} + Z_{2t}, \tag{9b}$$

Equation (9a) forms an orthogonal decomposition of $S_{1t}$ into a component that is spanned by $e_{Yt}$ and a component $Z_{1t}$ that is orthogonal to $e_{Yt}$. Similarly, equation (9b) purges the effect of $e_{Yt}$ and $e_{Mt}$ from $S_{2t}$ to arrive at $Z_{2t}$. If $e_Y$ and $e_M$ were observed, these two $Z$ variables would satisfy Assumption A by construction. Note, however, that $Z_{1t}$ and $Z_{2t}$ include the effects of $\mathbf{X}_{t-1}$. Moreover, they are forecastable since both $U_{Mt}$ and $U_{Ft}$ can be serially correlated and their lagged values predict future excess stock market returns.

If $e_Y$ and $e_M$ were observed, then solving for the sample analog of (7) would produce estimates of $Z_{1t}$ and $Z_{2t}$ that satisfy Assumption A. Alternatively, if valid instruments $\mathbf{Z}_t$ were observed, Proposition 1 shows that we could identify $\mathbf{B}$, hence $\mathbf{e}_t$. Since both are unobserved, such regressions are infeasible. However, components of observed variables may have the correlation properties of $Z_{1t}$ and $Z_{2t}$ stipulated in Assumption A. Given the theory and evidence discussed above, our maintained hypothesis is that the stock market contains a component that is exogenous to real activity, but correlated with both uncertainty shocks, and another component that is exogenous to both real activity and macro uncertainty, but correlated with financial uncertainty. To the extent that we can identify such components by requiring that they satisfy the same nine equations described in (7), we interpret them as synthetic instruments. We denote these constructed components as $\mathbf{Z}(\boldsymbol{\beta})$ to emphasize the fact that they are functions of $\boldsymbol{\beta}$.

Unlike the classic IV case where $\mathbf{Z}$ is observed, the nine moment restrictions in (7) cannot by themselves serve to identify the SVAR parameters. Whereas $\mathbf{Z}'\boldsymbol{\eta}$ is fixed when $\mathbf{Z}$ is observed, this is no longer the case when $\mathbf{Z}$ is constructed because $\mathbf{Z}$ itself depends on $\boldsymbol{\beta}$. The problem that this creates is that if $\hat{\mathbf{Z}}_1'\hat{\mathbf{e}}_Y = 0$, $\hat{\mathbf{Z}}_2'\hat{\mathbf{e}}_Y = 0$, and $\hat{\mathbf{Z}}_2'\hat{\mathbf{e}}_M = 0$ for some $\hat{\boldsymbol{\beta}} = \text{vec}(\hat{\mathbf{B}})$, any orthonormal rotation of $\hat{\mathbf{B}}$ to $\tilde{\mathbf{B}} = \hat{\mathbf{B}}\mathbf{Q}'$ and $\hat{\mathbf{e}}$ to $\tilde{\mathbf{e}} = \mathbf{Q}\hat{\mathbf{e}}$ will have $\tilde{\mathbf{Z}}_1'\tilde{\mathbf{e}}_Y = 0$, $\tilde{\mathbf{Z}}_2'\tilde{\mathbf{e}}_Y = 0$, and $\tilde{\mathbf{Z}}_2'\tilde{\mathbf{e}}_M = 0$. This is because the three orthogonality conditions hold by construction; they are imposed to arrive at the nine equations. If we collect all the solutions that satisfy (7) into the set $\hat{\mathcal{B}}$, this set can be infinitely large.

To address this problem, we combine the nine moment restrictions in (7) using the synthetic $Z_{1t}$ and $Z_{2t}$ with economic restrictions. The first economic assumption we impose is that the there exist components of $S_t$ given by the constructed $\mathbf{Z}(\boldsymbol{\beta})$ that exhibit minimum non-zero correlations with the uncertainty shocks but still satisfy the exogeneity restrictions of Assumption A. This acts to shrink the unconstrained set because the observed uncertainty correlations $\mathbf{Z}_1'(\hat{\boldsymbol{\beta}})\hat{e}_M \neq 0$, $\mathbf{Z}_1'(\hat{\boldsymbol{\beta}})\hat{e}_F \neq 0$, $\mathbf{Z}_2'(\hat{\boldsymbol{\beta}})\hat{e}_F \neq 0$ are not invariant to orthonormal rotations, since the $\mathbf{Z}_t$ are constructed from data $S_t$, which is given. Hence $\hat{\mathbf{B}}$ and $\tilde{\mathbf{B}} = \hat{\mathbf{B}}\mathbf{Q}'$ will imply components $\mathbf{Z}(\boldsymbol{\beta})$ with different correlations. Second, while $\tilde{e}_t$ and $\hat{e}_t$ have the same mean and variance, $\tilde{e}_t \neq \hat{e}_t$ at any particular $t$. The second set of economic assumptions we impose requires that the identified shocks be consistent with prior economic reasoning in a small number of extraordinary events. Both sets of economic assumptions are used to dismiss solutions in $\hat{\mathcal{B}}$ to form a winnowed set of solutions $\overline{\mathcal{B}}(\bar{c}, \bar{C}, \bar{k})$, where $\bar{c}, \bar{C}$, and $\bar{k} \equiv (k_1, k_2, k_3)'$ are defined below.

**Winnowing Constraints** For any $\boldsymbol{\beta} \in \hat{\mathcal{B}}$ that satisfies the nine equations defined in (7) with $\mathbf{Z}(\boldsymbol{\beta})$ constructed as in (9a) and (9b), $\boldsymbol{\beta} \in \overline{\mathcal{B}}(\bar{c}, \bar{C}, \bar{k})$ only if all the following conditions are satisfied:

1 The $\mathbf{Z}(\boldsymbol{\beta})$ have non-negligible correlations with uncertainty shocks: Let $c_{kj}(\boldsymbol{\beta}) = \text{corr}(Z_{kt}(\boldsymbol{\beta}), e_{jt}(\boldsymbol{\beta})$ be the sample correlation between $\mathbf{Z}_k(\boldsymbol{\beta})$ and the shock in $\mathbf{e}_t(\boldsymbol{\beta}) = (e_{Mt}, e_{Yt}, e_{Ft})$ with label $j$.

   i $|c_{1M}(\boldsymbol{\beta})| > \bar{c}$, $|c_{1F}(\boldsymbol{\beta})| > \bar{c}$, and $|c_{2F}(\boldsymbol{\beta})| > \bar{c}$.

   ii For $c(\boldsymbol{\beta})= \big(c_{1M}(\boldsymbol{\beta}), c_{1F}(\boldsymbol{\beta}), c_{2F}(\boldsymbol{\beta})\big)'$, $\sqrt{c(\boldsymbol{\beta})'c(\boldsymbol{\beta})} > \bar{C}$.

2 Big shock events: For $\mathbf{e}_t(\boldsymbol{\beta}) = \mathbf{B}^{-1}\boldsymbol{\eta}_t$,

   i $e_{Ft_1}(\boldsymbol{\beta}) > \bar{k}_1$ where $t_1$ is the period 1987:10 of the stock market crash.

   ii There exists a $t_2 \in$ [2007:12, 2009:06] such that $e_{Ft_2}(\boldsymbol{\beta}) > \bar{k}_2$.

   iii For all $t_2 \in [2007{:}12, 2009{:}06]$, $e_{Yt_2}(\boldsymbol{\beta}) < \bar{k}_3$

The first condition requires that each correlation $c_{1M}(\hat{\boldsymbol{\beta}}), c_{1F}(\hat{\boldsymbol{\beta}}), c_{2F}(\hat{\boldsymbol{\beta}})$ is individually high enough that it exceeds a pre-specified threshold $\bar{c}$, and that they are collectively high enough that the norm of $c(\boldsymbol{\beta})$ exceeds $\bar{C}$. If the thresholds are too low, many solutions will have correlations between the synthetic $Z$ and uncertainty shocks that are too low to identify these shocks and their dynamic effects well; in this case many $\hat{\boldsymbol{\beta}}$ in $\hat{\mathcal{B}}$ will also be in $\overline{\mathcal{B}}(\bar{c}, \bar{C}, \bar{k})$ and little progress is made. If the thresholds are too high, no solutions will exhibit such high correlations and $\overline{\mathcal{B}}(\bar{c}, \bar{C}, \bar{k})$ will be empty.

The restrictions on big shock events are used to ensure that the identified shocks at specific episodes have properties that agree with prior economic reasoning. In particular, we require that the financial uncertainty shocks identified in 1987 and during the 2007-2009 financial crisis be large and positive, and that the identified output shocks during the Great Recession not take on large positive values. The $t_2$ dates are set in accordance with NBER dating of the Great Recession, which coincides with the timing of the financial crisis.

It remains to discuss the construction of the unconstrained solution set $\hat{\mathcal{B}}$. To obtain a set of, say, $K$ solutions, we can solve $K$ GMM problems with different staring values. Each GMM problem consists of solving for nine unknowns from nine equations, with $\mathbf{Z}$ generated from (9a) and (9b). The guess of $\mathbf{Z}$ is updated each time $\boldsymbol{\beta}$ is updated during each iteration. We have found it more efficient to solve for $\boldsymbol{\beta}$ and $\mathbf{Z}$ iteratively, updating $\mathbf{Z}$ by projection only when a solution for $\boldsymbol{\beta}$ is obtained. The complete *iterative projection external variable* IPEV procedure is summarized as follows:

**Algorithm IPEV** For a given guess of $\boldsymbol{\beta}$ and therefore a guess of $\mathbf{e}_t = \mathbf{B}^{-1}\boldsymbol{\eta}_t$, the following steps are repeated until convergence:

i Put the guess of $(\mathbf{e_M}, \mathbf{e_Y})$ in (9a) and (9b) to construct $\mathbf{Z}_1$ and $\mathbf{Z}_2$.

ii Use $\mathbf{Z_1}$ and $\mathbf{Z_2}$ to solve the nine equations given in (7). This gives a new value for $\mathbf{B}$.

iii Construct new shocks $\mathbf{e}$ using the new estimate of $\mathbf{B}$.

iv If the difference between the new and old $\mathbf{e}$ exceeds a tolerance, return to (i). Else, put the solution in the unconstrained set $\hat{\mathcal{B}}$ if $\det(\mathbf{B}) > \bar{b}$.

v If the solution satisfies the winnowing constraints, put it in $\bar{\mathcal{B}}(\bar{c}, \bar{C}, \bar{k})$.

Several points about the implementation of this approach bear discussion.

First, note that the shocks are constructed as $\mathbf{e} = \mathbf{B}^{-1}\boldsymbol{\eta}$ and require $\mathbf{B}^{-1}$ to be well behaved. For this reason we keep only solutions that satisfy a minimum threshold for $\det(\mathbf{B}) \geq \bar{b}$. Second, we consider a large number of randomly chosen starting values, or guesses, for $\boldsymbol{\beta}$. Specifically, we initialize $\mathbf{B}$ to be the lower Cholesky factorization of $\boldsymbol{\Omega}$ for an arbitrary ordering of the variables (.e.g., $(U_{Mt}, Y_t, U_{Ft})'$). We then rotate it by 40,000 random orthogonal matrices to give 40,000 initial guesses on the shocks. Completely random starting values will always deliver some $\mathbf{Z}(\boldsymbol{\beta})$ and $e(\boldsymbol{\beta})$ with properties that are at odds with the data. Our winnowing constraints exclude such solutions from $\overline{\mathcal{B}}(\bar{c}, \bar{C}, \bar{k})$. We also estimate the model by GMM to verify that for a given initial guess, the solution agrees with the one obtained by IPEV estimation.

Second, the parameter values $\bar{c}$, $\overline{C}$, and $\bar{k}$ vary with the data under investigation. For some choice of $S$, it is entirely possible that there exists no solution satisfying non-negligible correlation thresholds $c_{kj}(\boldsymbol{\beta})$. To choose these thresholds for a particular DGP, we search over multiple discrete values for $\left(\bar{c}, \overline{C}\right)$ and store the number of survived solutions for each combination of $\left(\bar{c}, \overline{C}\right)$. Some combinations yield no solutions. For those that have at least one solution, we choose the set $\overline{\mathcal{B}}(\bar{c}, \bar{C}, \bar{k})$ that has the highest $\overline{C}$. If there is more than one set remaining with the same highest value for $\overline{C}$, the tie is broken by taking the set $\bar{\mathcal{B}}(\bar{c}, \bar{C}, \overline{k})$ with the highest $\bar{c}$. For the baseline case discussed below, this procedure yields $\bar{c} = 0.08$, $\overline{C} = 0.2430$. For the presentation of results below we focus on the single solution in the restricted set $\bar{\mathcal{B}}(\bar{c}, \bar{C}, \overline{k})$ with the highest $\sqrt{c(\boldsymbol{\beta})'c(\boldsymbol{\beta})}$:

$$\hat{\boldsymbol{\beta}} = \arg \max_{\boldsymbol{\beta} \in \overline{\mathcal{B}}(\bar{c}, \bar{C}, \overline{k})} \sqrt{c(\boldsymbol{\beta})'c(\boldsymbol{\beta})}.$$

However, we show that the set of winnowed solutions is not sensitive to the particular values $\bar{c} = 0.08$, $\overline{C} = 0.2430$ as long as they are not set too low.

Third, we set the parameters of the big shock event restrictions to $\overline{k}_1 = 4.0$, $\overline{k}_2 = 4.0$, and $\overline{k}_3 = 2$. The requirement that shocks to financial uncertainty during the 1987 stock market crash and 2007-09 financial crisis be at least four standard deviations larger than the mean is guided by Bloom (2009). In his work, uncertainty shocks are calibrated from innovations to the VXO stock market volatility index. Bloom (2009) studies the dynamic effects of four standard deviation shocks to uncertainty.

To summarize, identification is predicated on three economic assumptions. First, components of the external $S_t$ variables exist that satisfy a minimum degree of non-zero correlation with the relevant set of uncertainty shocks ($\{e_{Mt}, e_{Ft}\}$ or $\{e_{Ft}\}$) and are exogenous with respect to the remaining structural shocks in $\mathbf{e}_t$ that form the compliment of this set. Second, the identified shocks must be consistent with prior economic reasoning in a small number of extraordinary events. Third, idiosyncratic shocks to $S_t$ do not affect the variables in $\mathbf{X}_t$ either contemporaneously or with a lag, an assumption that is tantamount to assuming that $S_t$ can be excluded from the VAR. Below we show how this last assumption can be empirically evaluated in the section on system estimation. It is worth noting that the big shock constraints eliminate 99% of the solutions in $\hat{\mathcal{B}}$. When combined with minimum thresholds for the correlations between the $S_t$ components and the uncertainty shocks, the qualitative nature of the solutions to the IPEV and GMM estimation problem is not found to be sensitive to starting values.

To have confidence in this implementation, Ludvigson, Ma, and Ng (2016) use Monte Carlo experiments to study the properties of the estimator. In general, the degree of synthetic instrument correlation strength and required for precise identification varies with the data generating process (DGP). But the results for a DGP calibrated to the empirical application here suggest that the procedure can quite accurately recover the true structural shocks and $\mathbf{B}$ matrix when the procedure is initialized with the starting values employed in this application, when the estimated instruments have properties consistent with observed values of instrument strength $c(\boldsymbol{\beta})$, and when finite samples are set to be within range of the size used in this study.

### 3.3 Measuring Uncertainty and Stock Market Returns

In our estimation we work with several different aggregate measures of uncertainty, which are indexes constructed over individual uncertainties for a large number of observable time-series. A long-standing difficulty with empirical research on this topic has been the measurement of uncertainty. JLN find that common uncertainty proxies contain economically large components of their variability that do not appear to be generated by a movement in genuine uncertainty across the broader economy. This occurs both because these proxies over-weight certain series in the measurement of aggregate uncertainty, and because they erroneously attribute forecastable fluctuations to a movement in uncertainty. Equity market volatility, for example, contains a non-trivial component generated from forecastable variation in stock returns. The estimated macro uncertainty index constructed in JLN is designed to address these issues and improve the measurement of aggregate uncertainty. The methodology used here for constructing uncertainty indexes follows JLN and we refer the reader to that paper for details.

Let $y_{jt}^C \in Y_t^C = (y_{1t}^C, \ldots, y_{N_C t}^C)'$ be a variable in category $C$. Its $h$-period ahead uncertainty, denoted by $\mathcal{U}_{jt}^C(h)$, is defined to be the volatility of the purely unforecastable component of the future value of the series, conditional on all information available. Specifically,

$$\mathcal{U}_{jt}^C(h) \equiv \sqrt{\mathbb{E}\left[(y_{jt+h}^C - \mathbb{E}[y_{jt+h}^C|I_t])^2|I_t\right]} \tag{10}$$

where $I_t$ is information available. If the expectation today of the squared error in forecasting $y_{jt+h}$ rises, uncertainty in the variable increases. Uncertainty in category $C$ is an aggregate of individual uncertainty series in the category :

$$U_{Ct}(h) \equiv \text{plim}_{N_C \to \infty} \sum_{j=1}^{N_C} \frac{1}{N_C} \mathcal{U}_{jt}^C(h) \equiv \mathbb{E}_C[\mathcal{U}_{jt}^C(h)]. \tag{11}$$

As in JLN, the conditional expectation of squared forecast errors in (10) is computed from a stochastic volatility model, while the conditional expectation $\mathbb{E}[y_{jt+h}^C|I_t]$ is replaced by a diffusion index forecast, augmented to allow for nonlinearities. These are predictions of an autoregression augmented with a small number of common factors $q_t = (q_{1t}, \ldots, q_{rt})'$ estimated from a large number of economic time series $x_{it}$ each with factor representation $x_{it} = \Lambda_{it}' q_t + e_{\chi,it}$. The use of large datasets reduces the possibility of biases that arise when relevant predictive information is ignored. Let $Y_t^C = (y_{1t}^C, \ldots, y_{N_C t}^C)'$ generically denote the series that we wish to compute uncertainty in. In this paper, we consider four categories of uncertainty:

| Category ($C$) | $Y_t^C$ | $N_C$ |
|---|---|---|
| (M): Macro | all variables in $\chi^M$ | 134 |
| (F): Financial | all variables in $\chi^F$ | 147 |
| (R): Real activity | real activity variables in $\chi^M$ | 73 |

The uncertainty index $U_{Ct}$ for category $C$ is an equally-weighted average of the individual uncertainties in the category. We use two datasets covering the sample 1960:07-2015:04.[5] The first is a monthly *macro dataset*, $\mathcal{X}_t^M$, consisting of 134 mostly macroeconomic time series take from McCracken and Ng (2016). The second is a *financial dataset* $\mathcal{X}_t^F$ consisting of a 147 of monthly financial indicators, also used in Ludvigson and Ng (2007) and JLN, but updated to the longer sample. The real uncertainty index $U_{Rt}$ is an equally-weighted average of the individual uncertainties about 73 series in Groups 1 through 4 of $\mathcal{X}^M$. These include output and income variables, labor market measures, housing market indicators, and orders and inventories. Additional predictors for variables in $\mathcal{X}_{it}^M$ include factors formed from $\mathcal{X}_{it}^F$ and vice-versa, squares of the first factor of each, and factors in the squares of individual series, $\left(\mathcal{X}_{it}^M\right)^2$ and $\left(\mathcal{X}_{it}^F\right)^2$.

Our use of stock returns $S_t$ to generate instruments is grounded in the theoretical premise that both macro and financial uncertainty shocks should be reflected in stock market returns. There is no reason, however, that the regressands in (9a) and (9b) must be exactly the same measure of stock market activity. All measures of stock market activity are highly correlated because they contain a large common component (much of which is orthogonal to the rest of the economy). In order to introduce some additional independent variation in our two instruments, our base cases use different measures of aggregate stock market activity $S_{1t}$ and $S_{2t}$, although in practice we get very similar results if we use the same value-weighted stock market index return in (9a) and (9b). Specifically, we use the Standard and Poor 500 stock market index return, $S_{Pt}$, as $S_{1t}$, the regressand for (9b), and $S_{\alpha t} = \alpha_p \text{crsp}_t + (1-\alpha_p)\,\text{small}_t$, a portfolio weighted average of the return on the CRSP value-weighted stock index (in excess of the one-month Treasury bill rate) and the smallest decile stock market return in the NYSE as $S_{2t}$, the regressand for (9a).[6] We investigated a range of values for $\alpha_p$. Our choice of portfolio weight $\alpha_p$ is guided by empirical considerations. The small stock index is highly volatile, which can generate noise in the estimated SVAR parameters and large error bands for the impulse response functions. For the base case results presented below we set $\alpha_p = 0.94$ because it gives reasonably tight error bands. However, we also investigated a range of values for $\alpha_p \in [0,1]$ and found qualitatively similar point estimates and impulse responses, including setting $\alpha_p = 0$, which gives 100% of the weight to the small stock index. In our experience, wide error bands indicate difficulty identifying some element of the $\mathbf{B}$ matrix. We discuss this further below.

The parameters to be estimated include the reduced form VAR parameters in (3), from which we obtain $\hat{\boldsymbol{\eta}}_t$, the parameters in (9a) and (9b), from which we construct $Z_{1t}(\boldsymbol{\beta})$ and $Z_{2t}(\boldsymbol{\beta})$, the covariances between $Z_{1t}(\boldsymbol{\beta})$ and $Z_{2t}(\boldsymbol{\beta})$ and $\hat{\boldsymbol{\eta}}_t$, and the structural parameters

[5] A detailed description of the series is given in the Data Appendix of the online location where updated JLN uncertainty index data are posted: http://www.sydneyludvigson.com/s/jln_data_appendix_update.pdf

[6] The CRSP index is a value-weighted return of all stocks in NYSE, AMEX, and NASDAQ.

using results from the preceding three estimations. The sample moment conditions can be collected into one vector and Generalized Method of Moments (GMM, Hansen (1982)) applied to estimate all parameters, where the number of moments equals the number of parameters. Serial correlation and heteroskedasticity robust standard errors are constructed as in Newey and West (1987) wherever asymptotic standard errors are reported.

The next section presents empirical results. We begin by studying systems with macro uncertainty. We then move on to consider sub-indexes of $U_{Mt}$, including real uncertainty formed only over real activity variables $U_{Rt}$. Our final set of results report several additional cases pertaining to the plausibility of recursive identification schemes for our application, to different measures of $S_t$ and different treatments of $S_t$ as either valid known instruments where Assumption A applies or imperfect instruments where IPEV is used, and full system estimation where $S_t$ is included in a larger VAR with $\mathbf{X}_t$.

# 4 Results for $\mathbf{X}_t = (U_{Mt}, Y_t, U_{Ft})'$

Our first VAR is defined by $\mathbf{X}_t = (U_{Mt}, Y_t, U_{Ft})'$. We consider $h = 1$ (one-month uncertainty) and several measures of $Y_t$. The first two are the log of real industrial production, denoted $ip_t$, and the log of employment, denoted $emp_t$. While industrial production is a widely watched economic indicator of business cycles, it only captures goods-producing industries and has been a declining share of GDP. Employment only covers the labor market. Hence we also consider an additional measure of real activity: the cumulated sum of the first common factor estimated from the macro dataset $\chi^M$ (since the raw data used to form this factor $q_{1t}$ are transformed to stationary), which we denote $Q_{1t}$. Since our emphasis is on $h = 1$, we write $U_{Mt}$ instead of $U_{Mt}(1)$, and analogously for $U_{Ft}$, in order to simplify notation. We refer to the estimation using data on $\mathbf{X}_t = (U_{Mt}, ip_t, U_{Ft})'$ as our *base case.*

The top panel of Figure 1 plots the estimated macro uncertainty $U_{Mt}$ in standardized units along with the NBER recession dates. The horizontal bar corresponds to 1.65 standard deviation above unconditional mean of each series (which is standardized to zero). As is known from JLN, the macro uncertainty index is strongly countercyclical, and exhibits large spikes in the deepest recessions. The updated data $U_{Mt}$ series shows much the same. Though $U_{Mt}$ exceeds 1.65 standard deviations 48 times, they are clustered around the 1973-74 and 1981-82 recessions, as well as the Great Recession of 2007-09. Macroeconomic uncertainty is countercyclical and has a correlation of -0.65 with the 12-month moving-average of the growth in industrial production.

The bottom panel of Figure 1 plots the financial uncertainty series $U_{Ft}$ over time, which is new to this paper. $U_{Ft}$ is a broad-based measure of time varying financial uncertainty using data from the bond market, stock market portfolio returns, and commodity markets. Hence, it is smoother than proxies such as VIX or any particular bond index. As seen from Figure 1, $U_{Ft}$ is also countercyclical, though less so than $U_{Mt}$; the correlation with industrial production of -0.39. The series often exhibits spikes around the times when $U_{Mt}$ are high. However, $U_{Ft}$ is more volatile and spikes more frequently outside of recessions, the most notable being the 1987 stock market crash. Though observations on $U_{Ft}$ exceed the 1.65 standard deviation line 33 times, they are spread out in seven episodes, with the 2008 and 1987 episodes being the most pronounced.

As is clear from Figure 1, both indicators of macro and financial uncertainty are serially correlated and hence predictable. They have comovements but also have independent variations as the correlation between them is 0.58. However, this unconditional correlation cannot be given a structural interpretation. The heightened uncertainty measures can be endogenous responses to events that are expected to happen, but they can also be exogenous innovations. We use a VAR to capture the predictable variations, and then identify uncertainty shocks from the VAR residuals using the restrictions described in the previous section.

## 4.1 VAR Estimates and Uncertainty Shocks

Several features of the VAR estimates are qualitatively similar for all measures of $Y_t$. Table 1 highlights some of these results. As shown in Panel A, the sample correlation coefficient between $Z_{1t}$ and $\hat{e}_{Mt}$ and $\hat{e}_{Ft}$, and between $Z_{2t}$ and $\hat{e}_{Ft}$ are statistically significant and negative in each case, indicating that uncertainty shocks of both types are correlated with these instruments, as required, and tend to be high when these components of stock market returns are low. Notice that the magnitude of the instrument relevance correlations is modest for some systems, e.g. for $corr\,(Z_{1t}, \hat{e}_{Mt})$. However, simulations in Ludvigson, Ma, and Ng (2016) indicate that the degree of instrument relevance required for precise identification varies with the DGP, and results for a DGP calibrated to the empirical application here– including the size of the estimated instrument relevance correlations–show that the procedure can recover a close approximation of the true structural shocks and $\mathbf{B}$ matrix even with the correlations between $Z_{1t}$ and $\hat{e}_{Mt}$ found here. Panel A also shows that the correlation between $Z_{1t}$ and $\hat{e}_{Yt}$, and the correlation between $Z_{2t}$ and $\hat{e}_{Yt}$ and $\hat{e}_{Mt}$ are all zero as required, which is true by construction of the algorithm and solution for $\mathbf{B}$. Panel B shows that $\sigma_{MM}$, $\sigma_{YY}$, and $\sigma_{FF}$ are all strongly statistically significantly different from zero. This in turn indicates the presence of both macro and financial uncertainty shocks in the SVAR. Since both $U_{Mt}$ and $U_{Ft}$ are serially correlated, we should therefore find that $Z_{1t}$ is correlated with lags of $U_{Mt}$ and $U_{Ft}$, while $Z_{2t}$ is correlated with lags of $U_{Ft}$. Results not reported confirm this is the case.

Figure 2 presents the time series of the standardized shocks $(e_M, e_{ip}, e_F)$ identified from the system with $Y_t = ip_t$. All shocks display strong departures from normality with excess skewness and/or excess kurtosis. The largest of the $e_{ip}$ shocks is recorded in 2008:09, followed by 1974:11, and 2005:09. There also appears to be a moderation in the volatility of the $ip$ shocks in the post-1983 period. The largest macro uncertainty shock is in 1970:12, followed by the shock in 2008:10. The largest financial uncertainty shock is recorded in 1987:10 (Black Monday), followed by the shock in 2008:09 during the financial crisis. For $e_F$, the extreme but transitory nature of the 1987 stock market crash leads to a very large spike upward in $e_F$ in the month of the crash, followed by a very large spike downward in the month following the crash as the market recovered strongly and quickly. While this episode magnifies the spike in $e_F$ in 1987, it is largely orthogonal to real activity and macro uncertainty.

Observe that the large $ip$ shock in 2005:09 is not associated with a contemporaneous spike in uncertainty, while there are several spikes in both types of uncertainty that do not coincide with spikes in $e_{ip}$. The next subsection uses impulse response functions to better understand the dynamic causal effects and propagating mechanisms of these shocks.

## 4.2 The Dynamic Effects of Uncertainty Shocks

Impulse response functions (IRFs) trace out the effects of counterfactual increases in the shocks. The estimated IRFs are presented with 90% bootstrapped confidence bands as vertical bars. All plots show responses to one standard deviation changes in $\epsilon_{jt}$ in the direction that leads to an *increase* in its own variable $X_{jt}$.

Figure 3 shows the dynamic responses of each variable in the SVAR to each structural shock. The figure displays the IRFs for systems with $Y_t = ip_t$, $emp_t$, and $Q_{1t}$, the real activity factor. We see that positive shocks to *financial* uncertainty $e_F$ lead to a sharp declines in all three measures of real activity that persists for many months (center plot, bottom row). Positive perturbations to $e_{Ft}$ also cause $U_{Mt}$ to increase. However, there is less evidence that shocks to macro uncertainty have effects on financial uncertainty: the impact response of $U_{Ft}$ to an increase in $e_{Mt}$ is not statistically different from zero for the system with $Y = ip$, though it is for the other two. These results lend support to the hypothesis that heightened financial uncertainty is an exogenous impulse that causes declines in real activity. Note, however, there is no evidence that high financial uncertainty is a consequence of lower economic activity. Instead, exogenous (positive) shocks to real activity either *increase* financial uncertainty or have a statistically insignificant effect on it.

While we find no evidence that high financial uncertainty is a consequence of lower economic activity, the results for macro uncertainty are quite different. Figure 3 (second row, first column) shows that macro uncertainty falls sharply in response to positive real activity shocks. Alternatively stated, negative real activity shocks increase macro uncertainty sharply. These effects persist for well over a year after the real activity shock. This result is strongly statistically significant, suggesting that higher macro uncertainty in recessions is a direct endogenous response to lower economic activity. However, there is no evidence that the negative correlation between macro uncertainty and real activity is driven by causality running in the opposite direction. Indeed, the top middle panel shows that exogenous increases in $e_{Mt}$ actually *increase* real activity in the short run, consistent with growth options theories discussed above.

The results in Figure 3 display are for the single solution in the restricted set $\bar{\mathcal{B}}(\bar{c}, \bar{C}, \overline{k})$ with the highest $\sqrt{c(\boldsymbol{\beta})'c(\boldsymbol{\beta})}$. Figure 4 shows the *set* of all solutions for the system with $Y_t = ip_t$ that satisfy the same big shock restrictions with $\overline{k}$ set as discussed above, but satisfy lower thresholds for correlations between the synthetic $Z$ and the uncertainty shocks equal to $\bar{c} = 0.04$, $\overline{C} = 0.240$. Since these thresholds for the correlations are considerably lower, there are many more solutions that satisfy them, 76 to be exact. Figure 4 shows that the pattern of dynamic responses of the set is very similar to the baseline solutions, indicating that the results is not sensitive to these particular values for the thresholds $\left(\bar{c}, \overline{C}\right)$ as long as they are not set too low. On the other hand, if $\bar{c}$ is set much lower than 0.04, many solutions will have estimated correlations that are too small to identify the uncertainty shocks and their dynamic effects well.

## 4.3 The Structural Shocks and Decomposition of Variance

In Figure 1 presented earlier, we find 1973-74, 1981-82, and 2007-2009 to be the three episodes of heightened macroeconomic uncertainty, defined as the periods when $U_{Mt}$ is 1.65 standard deviations above its unconditional mean. We now look for the "large adverse" shocks in the systems $(U_{Mt}, Y_t, U_{Ft})'$, with $Y_t = ip_t$, $emp_t$ , $Q_{1t}$. More precisely, we consider large positive uncertainty shocks and large negative real activity shocks.

Figure 5 displays the date and size of shocks that are at least two standard deviations above the mean, estimated using the three different measures of $Y_t$. In view of the non-normality of the shocks, the figure also plots horizontal lines corresponding to three standard deviation of the unit shocks, which is used as the reference point for 'large'. The lowest panel shows that, irrespective of the definition of $Y_t$, all SVARs identify big financial uncertainty shocks in 1987 and 2008. The middle panel shows that large negative real activity shocks are in alignment with all post-war recessions with one exception: the negative real activity shock in 2005 is not immediately associated with a recession, but it could be the seed of the Great Recession that followed. It is known that the housing market led the 2007-2009 recession (e.g., see Favilukis, Ludvigson, and Van Nieuwerburgh (2015) for a discussion). Indeed, all 10 housing series in $\mathcal{X}^M$ (most pertaining to housing starts and permits series) exhibit sharp declines starting in September 2005 and continuing through 2006, thereby leading the Great Recession. This suggests that the negative spike in real activity in 2005 were at least in part driven by the housing sector.

The top panel of Figure 5 shows that the dates of large increases in $e_M$ are less clustered. They generally coincide with, or occur shortly after, the big real activity shocks and the financial uncertainty shocks. Observe that large macro uncertainty shocks occurred more frequently in the pre-1983 than the post 1983 sample.

To give a sense of the historical importance of these shocks, we perform a decomposition of variance, which is the fraction of $s$-step-ahead forecast error variance attributable to each structural shock $\epsilon_{Mt}$, $\epsilon_{Yt}$, and $\epsilon_{Ft}$ for $s = 1$, $s = 12$, $s = \infty$. We also report the maximum fraction of forecast error variance over all VAR forecast horizons $s$ that is attributable to each shock, denoted $s = s_{max}$ in the table. Table 2 reports results for the system with $Y_t = ip_t$ (left column), $Y_t = emp_t$ (middle column), and $Y_t = Q_{1t}$ (right column).

According to the top row, all three real activity shocks $e_{ip}$, $e_{emp}$, and $e_{Q_1}$ have sizable effects on macroeconomic uncertainty $U_M$. But according to the bottom row, these same shocks have small effects on financial uncertainty $U_F$. At the same time, positive macro uncertainty shocks $e_M$, which increase rather than decrease real activity, explain a surprisingly large fraction of production (up to 539%), employment (up to 44%) and the real activity index (up to 30%). On the other hand, financial uncertainty shocks $e_F$ have a small contribution to the one-step-ahead forecast error variance of $ip$, but their relative importance increases over time. These $e_F$ shocks make larger contributions to the forecast error variance of $emp$ and $Q_1$. Financial uncertainty shocks explain up to 69% of the forecast error variance in employment and up to 49% of the forecast error variance in the real activity index, compared to 31% for production. Financial uncertainty shocks $e_F$ feedback into $U_M$, and macroeconomic uncertainty shocks $e_M$ also feedback into $U_F$.

Regardless of which measure of real activity is used, we find that financial uncertainty is unlike macro uncertainty or real activity in that its variation is far more dominated by its own shocks. For example, in the system with $ip$, $e_F$ shocks explain 95% of the $s = 1$ step-ahead forecast error variance in $U_{Ft}$, and 84% of the $s = \infty$ step-ahead forecast error variance. In the systems with $emp$ and $Q_1$, $e_F$ shocks explain 68% and89%, respectively, of the $s = 1$ step-ahead forecast error variance in $U_{Ft}$, and 46% and 66%, respectively, of the $s = \infty$ step-ahead forecast error variance.

To summarize, in all three systems, real activity shocks $e_Y$ have quantitatively large persistent negative effects on macro uncertainty $U_M$. In turn, macro uncertainty shocks $e_M$ have large positive impact effects on real activity measures $Y$. Financial uncertainty shocks $e_F$ have smaller impact effects but larger long run effects that dampen real activity $Y$. Across all systems, the forecast error variance of financial uncertainty is the least affected by shocks other than its own, suggesting that $U_F$ is quantitatively the most important exogenous impulse in the system.

# 5 Uncertainty in Real Activity

The results discussed above suggest that the dynamic relationship between macro uncertainty and real activity can be quite different from the relation between financial uncertainty and real activity. However, given the composition of our data $\chi^M$, macroeconomic uncertainty itself can be due to uncertainty in real activity variables such as output and unemployment, to price variables, and to financial market variables. The theoretical uncertainty literature has focused on modeling exogenous uncertainty shocks that arise specifically in measures of real economic fundamentals, rather than in prices or financial markets. To better evaluate the implications of these theoretical models, it is therefore of interest to know how systems defined by sub-components of broad-based macro uncertainty behave. To this end we consider systems that isolate uncertainty about real activity using the $U_{Rt}$ sub-index that more closely corresponds to the theoretical literature.

## 5.1 System $\mathbf{X}_t = (U_{Rt}, Y_t, U_{Ft})'$

We isolate the real activity components of macro uncertainty by aggregating the individual uncertainty estimates over the 73 real activity variables in the macro dataset $\mathcal{X}^M$. The one-period ahead uncertainty in real activity, denoted $U_{Rt}$, is show in Figure 6. This series, like $U_{Mt}$, is countercyclical though somewhat less so, having a correlation of -0.50 with industrial production (as compared to -0.66 for $U_{Mt}$). At first glance, $U_{Rt}$ appears to fluctuate in a manner similar to macroeconomic uncertainty $U_{Mt}$. The two series have a correlation of 0.71 and exhibit some overlapping spikes. But $U_{Rt}$ and $U_{Mt}$ also display notable independent variation. Figure 6 shows that there are 43 observations of $U_{Rt}$ that are at least 1.65 standard deviations above its mean. These can be organized into five episodes: 1965, 1970, 1975, 1982-83, and 2007. By contrast, $U_{Mt}$ in Figure 1 only exhibits three such episodes. Observe that the $U_{Rt}$ series exhibits several spikes before 1970 that are not accompanied by spikes in $U_{Mt}$.

Given the distinctive patterns in the time series behavior of $U_{Rt}$ and $U_{Mt}$, one might expect to find different dynamic relationships with the other variables in our systems when $U_{Mt}$ is replaced by $U_{Rt}$. Surprisingly, the impulse responses functions are in most cases qualitatively similar to systems studied above that use broad-based macro uncertainty. There are, however, differences in sampling error. These responses are displayed in Figure 7. We see that (i) positive shocks to real activity measures cause sharp declines in $U_{Rt}$ so that negative shocks cause sharp increases in real economic uncertainty; (ii) positive real activity uncertainty shocks $e_{Rt}$ do not cause statistically significant declines in real activity measures; instead the effect is statistically insignificant or, in the system using $Y = ip$, the opposite is true, though the wide error bands for this case indicate that the estimation for that VAR does not identify the $e_{Rt}$ shock well; (iii) positive financial uncertainty shocks $e_{Ft}$ lead to sharp declines in real activity measures that are strongly statistically significant, and (iv) there is little evidence that high (low) financial uncertainty is caused by negative (positive) real activity shocks.

Figure 8 plots the large adverse structural shocks identified from the systems $(U_{Rt}, Y_t, U_{Ft})'$ for $Y_t = ip_t, emp_t, Q_{1t}$ analogous to Figure 2. The top panel shows that the real uncertainty shock $e_{Rt}$ exhibits spikes in excess of three standard deviations during the Great Recession for two measures of real activity, $Y_t = emp_t$ and $Y_t = ip_t$. However, given that the $e_{Rt}$ shock seems poorly identified in the VAR with $Y_t = ip_t$, the result for that case should be viewed with caution. Ruling out this case, only the system that uses $Y_t = emp_t$ exhibits a spike in excess of two standard deviations. In particular, the system using the broadest measure of real activity, $Q_{1t}$, only exhibits a two standard deviation shock during this episode, despite the fact that $U_{Rt}$ itself exhibits a large spike (see Figure 6).

These episodes serve to reinforce the conclusion that the heightened macro and real economic uncertainty in recessions is more often an endogenous response to other shocks, rather than an exogenous impulse. Even though there were many large spikes in real uncertainty shocks $e_{Rt}$ pre-1983, there have fewer large adverse shocks to real economic uncertainty since 1983, a period that coincides with the so-called Great Moderation.

To complete the analysis, we present variance decompositions for the system $(U_{Rt}, Y_t, U_{Ft})'$, with three measures of real activity $Y_t = ip_t, emp_t, Q_{1t}$. These results, presented in Table 3, share some similarities with the systems that use macro uncertainty $U_{Mt}$ shown in Table 2, but there are at least two distinctions. First, financial uncertainty shocks decrease real activity and explain larger fractions of the forecast error variance in two measures of real activity at long horizons. At the longest $s = \infty$ VAR horizon, financial uncertainty shocks explain 66% of forecast error variance in employment and 41% of the forecast error variance in the real activity index. These results suggest that financial uncertainty has quantitatively large negative consequences for at least some measures of real activity.

Second, compared to systems that use $U_{Mt}$, larger fractions of the forecast error variance in $U_{Rt}$ are explained by its own shocks, while smaller fractions are explained by the real activity shocks for the systems using $Y_t = emp_t$ and $Q_{1t}$, though the opposite is true for the system using $Y_t = ip_t$.

To summarize, countercyclical increases in real uncertainty $U_{Rt}$, like macro uncertainty $U_{Mt}$, are found to be best characterized as endogenous responses to declines in real activity. At the same time, $U_{Rt}$ exhibits more variation than $U_{Mt}$ that is independent of fluctuations in real activity especially early in the sample, explaining why it is less countercyclical.

## 6 Robustness and Additional Cases

This section presents results for a number of additional cases.

First, we test whether restrictions implied by recursive identification are supported by the data. Second, we consider an estimation where we presume two observed external variables are valid external instruments $Z_{1t}$ and $Z_{2t}$, even though they may in fact contain an endogenous component. This is compared to the case where the same variables are presumed not to be valid instruments and IPEV is used to construct such instruments from $S_{1t}$ and $S_{2t}$. Finally, we show how the foregoing analysis can be related to a subsystem of a larger VAR in $\mathbf{X}_t$ and $S_t$ with explicit restrictions on the structure of $S_t$.

## 6.1 Tests of Recursive Identification Restrictions

The econometric model permits us to test whether a recursive structure is supported by the data. Specifically, Assumption A does not rule out the possibility of a recursive structure. Given that $\sqrt{T}(\hat{\boldsymbol{\beta}}_1 - \boldsymbol{\beta}_1^0)$ is asymptotically $N(0, \boldsymbol{\Sigma}^2_{\hat{\boldsymbol{\beta}}_1})$, the null hypothesis of a recursive structure is a test that the three components of $\boldsymbol{\beta}_1$ corresponding to the off-diagonal entries of $\mathbf{A}_0^{-1}$ are jointly zero. Hence it is chi-square distributed with three degrees of freedom. We first confirm that the test has the correct size in Monte Carlo simulations. Our estimates based on historical data strongly reject a lower triangular $\mathbf{A}_0^{-1}$ for *any* possible ordering of the variables. Table 4 shows results from Wald tests with $Y_t = ip_t$, and either using $U_{Mt}$ (first column) or $U_{Rt}$ (second column). What happens to the dynamic responses when we nevertheless impose restrictions based on recursive identification (and freely estimate the rest of the parameters)?

Figure 9 shows one case: dynamic responses for the system $\mathbf{X}_t = (U_{Ft}, U_{Mt}, ip_t)'$ with that ordering. Although there are many possible recursive orderings, and the estimated IRFs differ in some ways across these cases, the dynamic responses under recursive identification have one common feature that is invariant to the ordering and that provides the sharpest contrast with the results generated by the SVARs identified with external instruments studied here. Specifically, with recursive identification, macro uncertainty shocks–no matter which ordering–appear to cause a sharp decline in real activity, while real activity shocks have little effect on macro uncertainty in the short run and if anything increase it in the long run. This result, evident in Figure 9, gives precisely the opposite finding from what is reported above and appears to be an artifact of invalid timing assumptions under recursive identification. Further investigation reveals that the SVARs we study display non-zero contemporaneous correlations between $U_{Ft}$ and $Y_t$, as well as between $U_{Mt}$ and $Y_t$, a finding that is inconsistent with any recursive ordering. Imposing a structure that prohibits contemporaneous feedback spuriously suggests that macro uncertainty shocks are a cause of declines in real activity, rather than an endogenous response. This result is robust across any of the six possible recursive orderings and underscores the challenges of relying on convenient timing assumptions to sort out cause and effect in the relationship between uncertainty and real activity.[7]

## 6.2 Observed Instruments Case

The traditional approach to identification when the variables are simultaneously determined relies on the existence of valid instruments $Z$ that are exogenous and relevant. In many applications, few if any plausible instruments exist that satisfy these restrictions, motivating our use of IPEV. We've argued above that applications that seek to identify the empirical effects of uncertainty on real activity (and vice versa) are likely to be among those for which valid observed instruments are hard to find or identify. It is nevertheless of interest to consider an estimation in which two observed external variables $S_{1t}$ and $S_{2t}$ are *presumed* to be valid external instruments $Z_{1t}$ and $Z_{2t}$ and treated as such, even though they may in fact contain an endogenous component. This approach to estimation may be compared to the IPEV approach, where these same external variables are suspected to be possibly invalid (or imperfect) instruments because they may contain an endogenous component. For this exercise, we take $Z_{1t}$ and $S_{1t}$ to be the $S\&P$ 500 stock market index return, denoted $U_{SPXt}$, and $Z_{2t}$ and $S_{2t}$ to be CRSP value-weighted excess stock market return, denoted $r_{CRSPt}$.

To motivate the use of these two external variables as reasonable choices for the presumed valid instruments, we first employ the external variables $S_{1t} = U_{SPXt}$ and $S_{2t} = r_{CRSPt}$ to estimate the system $X_t = (U_{Mt}, ip_t, U_{Ft})'$ using IPEV following the method described above. We use the same winnowing constraints (with identical values for $\bar{c}$, $\bar{k}_1$, $\bar{k}_2$, and $\bar{k}_3$) as used for our base case. We then verify that the estimated slope coefficient in a regression of $S_{1t}$ on $\hat{e}_{ipt}$ is not statistically different from zero, and the estimated slope coefficients in a multivariate regression of $S_{2t}$ on $\hat{e}_{ipt}$ and $\hat{e}_{Mt}$ are also not statistically different from zero.[8] The statistically insignificant coefficients suggest that the external variables $S_{1t} = U_{SPXt}$ and $S_{2t} = r_{CRSPt}$ can credibly serve as at least approximately valid external instruments $Z_{1t}$ and $Z_{2t}$.

When we presume two external variables are valid instruments, we directly apply Assumption A setting $Z_{1t} = S_{1t}$ and $Z_{2t} = S_{2t}$. The solution for $\mathbf{B}$ then follows from Proposition 1 and can be obtained in closed form. No winnowing constraints are imposed and no projections are performed. We refer to this as a "presumed valid" IV case. This estimation may be directly compared to the analogous estimation where we do not make such a presumption, and instead employ IPEV using $S_{1t} = U_{SPXt}$ and $S_{2t} = r_{CRSPt}$ following the method described above. Figure 10 shows both sets of dynamic responses for the base case system $X_t = (U_{Mt}, ip_t, U_{Ft})'$. The figure shows that, qualitatively, most dynamic responses are similar to those obtained above for the base case, and to each other. However, the bootstrap error bands tend to be wider for the presumed valid IV case than the IPEV case, especially for the responses to macro uncertainty

[7] The figures for these cases are omitted to conserve space but are available upon request.

[8] Both regressions control for one lag of the dependent variable.

and real activity shocks. This happens because some of the GMM parameter estimates exhibit more sampling error in the presumed valid IV case than the IPEV estimation.

In our experience, the bootstrap standard error bands tend to be wide when the external variables produce instruments that only weakly identify some elements of $\mathbf{B}$. The case of presumed valid instruments requires that the observed $Z_{1t}$ and $Z_{2t}$ satisfy $\mathbb{E}(Z_{2t}e_{Mt}) \neq 0$ to identify the column that gives the effects of $e_{Mt}$ shocks. In cases when the GMM estimates of $\mathbb{E}(Z_{1t}\eta_{Yt})$ and $\mathbb{E}(Z_{1t}\eta_{Ft})$ are imprecise, we find $B_{MY}$ and $B_{YM}$ are poorly identified and the bootstrap error bands for the dynamic responses of $U_{Mt}$ to $e_{ipt}$ and for $ip_t$ to $e_{Mt}$ are then wide. An inspection of the closed-form solutions for $\mathbf{B}$ shows why. The $B_{MY}$ and $B_{YM}$ parameters are highly nonlinear functions of $\mathbb{E}(Z_{1t}\eta_{Yt})$ and $\mathbb{E}(Z_{1t}\eta_{Ft})$, so that small changes in the latter lead to large differences in the solution for $B_{MY}$ and $B_{YM}$. Since the bootstrap repeatedly makes draws from the distribution of the GMM estimates it depends on the variance of the point estimates. The bootstrap standard errors are correspondingly large whenever the point estimates of the variance of $\mathbb{E}(Z_{1t}\eta_{Yt})$ and $\mathbb{E}(Z_{1t}\eta_{Ft})$ and the other parameters are imprecise. By contrast, the IPEV case displays much narrower the bootstrap error bands for most IRFs. This happens because the winnowing constraints, especially the economic constraints, are imposed both in the IPEV estimation and in the bootstrap procedure. The imposition of these constraints in IPEV brings more information to bear, thereby improving the efficiency of the estimates. This points to a potential advantage of IPEV over traditional IV, when plausible prior economic reasoning can be imposed to improve efficiency.

Sampling uncertainty aside, it is notable that the qualitative nature of the responses in Figure 10 for both the valid IV and IPEV estimations are similar to that obtained for the base case above: positive financial uncertainty shocks drive down production sharply and persistently, while positive production shocks endogenously decrease macro uncertainty but not financial uncertainty. Likewise, there is no evidence that positive macro uncertainty shocks drive down production.

Beyond this particular estimation, the findings of this section suggest that IPEV can be employed as an ex-post validity check in any application where specific external variables are presumed to be valid instruments $Z$ that satisfy the required exogeneity and relevance conditions. Not only can the two estimations be directly compared, since IPEV provides an estimate of SVAR without requiring the external variables $S$ to be exogenous, IPEV estimates can be used ex-post to verify that the presumed valid instruments $Z = S$ actually satisfy the required exogeneity restrictions vis-a-vis the estimated IPEV structural shocks $\hat{\mathbf{e}}_t$.

### 6.3 System Estimation

In this section, we relate the foregoing analysis to a subsystem of a larger VAR for $(\mathbf{X}_t, S_t)'$ with explicit restrictions on the structure of $S_t$. For this purpose, we consider a single $S_t$.

The reduced form errors of the larger VAR are $\boldsymbol{\eta}_t = (\eta'_{Xt}, \eta_{St})'$. The structural shocks are $\begin{pmatrix}\mathbf{e}'_{Xt} & e_{St}\end{pmatrix}'$ with $\boldsymbol{\eta}_t = \mathbf{B}\mathbf{e}_t$. The $\mathbf{B}$ matrix has 16 parameters and the covariance structure gives 10 pieces of information. But the assumption that the shocks $e_{St}$ do not contemporaneously affect $\mathbf{X}_t$ gives three restrictions. This is weaker than assuming that $S_t$ is exogenous for $\mathbf{X}_t$, which would also have constrained the lags of $S_t$ from affecting $\mathbf{X}_t$. The three restrictions imply that

$$\begin{pmatrix} \eta_{Yt} \\ \eta_{Mt} \\ \eta_{Ft} \\ \eta_{St} \end{pmatrix} = \begin{pmatrix} B_{YY} & B_{YM} & B_{YF} & 0 \\ B_{MY} & B_{MM} & B_{MF} & 0 \\ B_{FY} & B_{FM} & B_{FF} & 0 \\ B_{SY} & B_{SM} & B_{SF} & B_{SS} \end{pmatrix} \begin{pmatrix} e_{Yt} \\ e_{Mt} \\ e_{Ft} \\ e_{St} \end{pmatrix}. \tag{12}$$

As in the earlier analysis, we also make use of Assumption A, but with the synthetic $Z_t$ implicitly defined as

$$\begin{aligned} Z_{1t} &= \eta_{St} - B_{SY}e_{Yt} = B_{SM}e_{Mt} + B_{SF}e_{Ft} + B_{SS}e_{St} \\ Z_{2t} &= Z_{1t} - B_{SM}e_{Mt} = B_{SF}e_{Ft} + B_{SS}e_{St}. \end{aligned}$$

Unlike the previous analysis of the VAR for $\mathbf{X}_t$ with $\mathbf{Z}_t$ was treated as a residual from a projection, the synthetic $\mathbf{Z}_t$ here has a structural interpretation. The synthetic $Z$ interpretation motivates the exogeneity restrictions of Assumption A, $E[Z_{1t}e_{Yt}] = E[Z_{2t}e_{Yt}] = E[Z_{2t}e_{Mt}] = 0$, which gives three additional restrictions. These are exactly the same three restrictions used to solve for $\mathbf{B}$ in the previous subsystem analysis, except that the residual $\eta_{St}$ is used to construct $\mathbf{Z}_t$ in place of $S_t$. Estimation of the larger system $(\mathbf{X}_t, S_t)'$ therefore proceeds exactly as in GMM estimation of the previous analysis of the VAR for $\mathbf{X}_t$. Given the block structure of $\mathbf{B}$, we can also use IPEV to solve the $\mathbf{X}$ subsystem and the $S$ equation iteratively.

But as is clear from the last three equations of (7), Assumption A does not provide restrictions for $B_{SY}, B_{SM}, B_{SF}$. There can be a large number of solutions consistent with the covariance structure of $\boldsymbol{\eta}_t$ and yet satisfy Assumption $A$. We again use synthetic instrument-uncertainty correlations to help tie down these parameters. Specifically, in the full system,

$$\begin{aligned} c_{1M}(\boldsymbol{\beta}) &= \frac{\text{corr}(Z_{1t}, e_{Mt})}{\sigma_{Z_1}} = \frac{B_{SM}}{\sqrt{B_{SM}^2 + B_{SF}^2 + B_{SS}^2}} \\ c_{1F}(\boldsymbol{\beta}) &= \frac{\text{corr}(Z_{1t}, e_{Ft})}{\sigma_{Z_1}} = \frac{B_{SF}}{\sqrt{B_{SM}^2 + B_{SF}^2 + B_{SS}^2}} \\ c_{2F}(\boldsymbol{\beta}) &= \frac{\text{corr}(Z_{2t}, e_{Ft})}{\sigma_{Z_2}} = \frac{B_{SF}}{\sqrt{B_{SF}^2 + B_{SS}^2}}, \end{aligned}$$

where the second equalities follow by recalling that $e_{Mt}$ and $e_{Ft}$ have unit standard deviations. Evidently, these correlations explicitly depend on the parameters of the $S$ equation. It is not invariant to orthonormal rotation of $\mathbf{e}_X$ and the parameters of the subsystem. Requiring that $c_{1M}(\boldsymbol{\beta}) > \bar{c}, c_{1F}(\boldsymbol{\beta}) > \bar{c}, c_{2F} > \bar{c}$ may still not be enough to point identify $\mathbf{B}$. But the number of admissible solutions can be reduced by increasing $\bar{c}$, and additionally requiring that the shocks have the expected properties during the stock market crash of 1987 and the Great Recession/financial crisis of 2008-09, as presented above. The final GMM estimate $\hat{\boldsymbol{\beta}}$ is the one that maximizes $\sqrt{c(\boldsymbol{\beta})'c(\boldsymbol{\beta})}$ in the set of possible solutions that satisfy these winnowing constraints.

The system estimation is in some ways more restrictive than the subsystem approach. In the subsystem analysis, the process that generates $S_t$ is left unspecified. As such, it can be a function of any variables other than $\mathbf{X}_t$, both contemporaneously, and at lags. The full system approach specifies the process for $S_t$. Any misspecification in one equation can affect all equations in the system. On the other hand, the subsystem approach excludes not just the current, but also the past values of $S_t$ from the equations for $\mathbf{X}_t$ even though such a restriction is not needed for identification. Such overidentifying restrictions can be tested. A simple way to evaluate these restrictions is to compare the impulse response functions estimated for the three variable system $\mathbf{X}_t = (U_{Mt}, Y_t, U_{Ft})'$ with those from a larger system that includes $S_t$ but does not restrict the coefficients of $S_{t-j}$ in the equations for $X_t$ to zero, for $j \geq 1$. Denote these coefficients by $\mathbf{A}_{XS,j}$.

We estimate a four variable system in $(\mathbf{X}_t, S_t)'$, imposing $\mathbf{A}_{XS,0} = \mathbf{0}$, but without imposing $\mathbf{A}_{XS,j} = \mathbf{0}$ for $j \geq 1$. We report results for the four variable case where $S_t$ is measured as the return on the CRSP value-weighted stock market index and using $ip_t$ as $Y_t$, as in our base case. Figure 11 presents these two sets of impulse responses for the systems. The responses are little different. Indeed, the coefficients on lags of $S_t$ appear to be close to zero for all three variables. The data thus appear at least qualitatively consistent with the restrictions that $\mathbf{A}_{XS,j} = \mathbf{0}$ for $j \geq 1$ and therefore the assumption that stock returns can be excluded from the VAR.

In summary, even though a VAR that directly incorporates $S$ is possible, the system estimation approach restricts $S$ to be explained only by lags of $S$ and $\mathbf{X}$ which could in general be restrictive. Our the three variable approach is more robust to such misspecification that could affect the entire system. On the other hand, the system estimation allows lags of $S_t$ to feed back into future $\mathbf{X}_t$ whereas in the three variable approach they are restricted to have no impact. These form part of the exclusion restriction on $S_t$. Estimation of the four variable system that includes $S_t$ suggests that these exclusion restrictions are qualitatively consistent with the data.

# 7 Conclusion

A growing body of research establishes uncertainty as a feature of deep recessions but leaves open two key questions: is uncertainty primarily a source of business cycle fluctuations or an endogenous response to them? And where does uncertainty originate? There is no theoretical consensus on the question of whether uncertainty is primarily a cause or a consequence of declines in economic activity. In most theories, it is modeled either as a cause or an effect, but not both, underscoring the extent to which this question is fundamentally an empirical matter.

The objective of this paper is to address both questions econometrically using small-scale structural VARs that are general enough to nest the range of theoretical possibilities in empirical tests. Commonly used recursive identification schemes cannot achieve this objective, since by construction they rule out the possibility that uncertainty and real activity could influence one another contemporaneously. The econometric model employed in this paper nests the recursive identification scheme, and we find that it is strongly rejected by the data. An empirical model in which uncertainty and real activity simultaneously influence each other fits the data far better than one in which these relationships are restricted by timing assumptions that prohibit contemporaneous feedback.

To identify dynamic causal effects, this paper takes an alternative identification approach by using external instruments that we construct in a novel way to be valid under credible interpretations of the structural shocks. We call this approach iterative projection external variable (IPEV). In addition, our empirical analysis explicitly distinguishes macro uncertainty and uncertainty about real activity from financial uncertainty, thereby allowing us to shed light on the origins of uncertainty shocks that drive real activity lower, to the extent that any of them do. The econometric framework allows uncertainty to be an exogenous source of business cycle fluctuations, or an endogenous response to them, or any combination of the two, without restricting the timing of these relationships. Underlying our approach is a maintained theoretical assumption that variables such as stock market returns, while endogenous, are nevertheless driven by distinct sources of stochastic variation, some of which satisfy exogeneity restrictions required to identify independent structural shocks.

Estimates of the econometric model are used to inform the nature of these dynamic relationships in U.S. data. The results from these estimations show that sharply higher uncertainty about real economic activity in recessions is fully an endogenous response to business cycle fluctuations, while uncertainty about financial markets is a likely source of them. Exogenous declines in economic activity have quantitatively large effects that drive real economic uncertainty endogenously higher. Financial uncertainty, by contrast, is dominated by its own shocks, implying that it is primarily an exogenous impulse vis-a-vis real activity and macro uncertainty. These results reinforce the hypothesis laid out in much of theoretical uncertainty literature, namely that uncertainty shocks are a source of business cycle fluctuations. But they also stand in contrast to this literature, which has emphasized the role of uncertainty fluctuations in productivity and other real economic fundamentals. The findings here imply that the uncertainty shocks that drive real activity lower appear to have their have origins, not in measures of real activity, but in financial markets.

# Appendix

## Closed-Form Solution for B when $Z$ is observed

**Lemma 2** *There exists a unique solution to the system (7) if* $\mathbb{E}[e_{Ft}Z_2] \neq 0$ *and* $\mathbb{E}[e_{Mt}Z_1] \neq 0$.

**Proof.** To facilitate the presentation throughout the proof, let

$$\begin{aligned}
\boldsymbol{\eta}_t &= \mathbf{B}\mathbf{e}_t \\
\mathbf{B} &= \Big[\underset{3\times1}{\mathbf{B}_M}, \underset{3\times1}{\mathbf{B}_Y}, \underset{3\times1}{\mathbf{B}_F}\Big] \\
\boldsymbol{\Omega} &= \mathbb{E}\left(\boldsymbol{\eta}_t\boldsymbol{\eta}_t'\right).
\end{aligned}$$

Let $\phi_{1F} = c_{1F}\sigma_{Z1}$, $\phi_{2F} = c_{2F}\sigma_{Z2}$, $\phi_{1M} = c_{1M}\sigma_{Z1}$. We have two external instruments $(Z_1, Z_2)$ satisfying

$$\begin{aligned}
\mathbb{E}[e_{Ft}Z_1] &\equiv \phi_{1F} \neq 0, \mathbb{E}[e_{Mt}Z_1] \equiv \phi_{1M} \neq 0 \text{ and } \mathbb{E}[e_{Yt}Z_1] = 0 \\
\mathbb{E}[e_{Ft}Z_2] &\equiv \phi_{2F} \neq 0 \text{ and } \mathbb{E}[e_{Mt}Z_2] = \mathbb{E}[e_{Yt}Z_2] = 0
\end{aligned}$$

Then

$$\mathbb{E}[\boldsymbol{\eta}_t Z_2] = \mathbb{E}[\mathbf{B}\mathbf{e}_t Z_2] = \mathbf{B}\begin{bmatrix} 0 \\ 0 \\ \phi_{2F} \end{bmatrix} = \phi_{2F}\mathbf{B}_F \tag{A.1}$$

Thus $\mathbf{B}_F$ exists if $\phi_{2F} \neq 0$. Observe that, since

$$\boldsymbol{\Omega} = \mathbb{E}[\boldsymbol{\eta}_t\boldsymbol{\eta}_t'] = \mathbf{B}\mathbf{B}'$$

we have

$$\mathbf{B}'\boldsymbol{\Omega}^{-1}\mathbf{B} = \mathbf{I}$$

hence, $\forall i, j = M, Y, F$

$$\mathbf{B}_j'\boldsymbol{\Omega}^{-1/2}\boldsymbol{\Omega}^{-1/2}\mathbf{B}_i = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases}.$$

Therefore,

$$\mathbb{E}[\boldsymbol{\eta}_t Z_2]'\,\Omega^{-1}\mathbb{E}[\boldsymbol{\eta}_t Z_2] = (\phi_{2F}\mathbf{B}_F)'\,\boldsymbol{\Omega}^{-\frac{1}{2}}\boldsymbol{\Omega}^{-\frac{1}{2}}(\phi_{2F}\mathbf{B}_F) = \phi_{2F}^2$$

This implies that the scale $\phi_{2F}$ is identified up to a sign by

$$\phi_{2F} = \pm\sqrt{\mathbb{E}[\boldsymbol{\eta}_t Z_2]\,\boldsymbol{\Omega}^{-1}\mathbb{E}[\boldsymbol{\eta}_t Z_2]}. \tag{A.2}$$

Next,

$$\mathbb{E}[\boldsymbol{\eta}_t Z_1] = \mathbb{E}[\mathbf{B}\mathbf{e}_t Z_1] = \mathbf{B}\begin{bmatrix} \phi_{1M} \\ 0 \\ \phi_{1F} \end{bmatrix} = \phi_{1M}\mathbf{B}_M + \phi_{1F}\mathbf{B}_F$$

But note that

$$\begin{aligned}\mathbb{E}\left[\boldsymbol{\eta}_t Z_2\right]\boldsymbol{\Omega}^{-1}\mathbb{E}\left[\boldsymbol{\eta}_t Z_1\right] &= \phi_{2F}\mathbf{B}_{F'}\boldsymbol{\Omega}^{-1}\left(\phi_{1M}\mathbf{B}_M+\phi_{1F}\mathbf{B}_F\right)\\ &= \phi_{2F}\mathbf{B}_{F'}\left(\mathbf{B}\mathbf{B}'\right)^{-1}\left(\phi_{1M}\mathbf{B}_M+\phi_{1F}\mathbf{B}_F\right)\\ &= \phi_{2F}\phi_{1F}\end{aligned}$$

This implies that $\phi_{1F}$ is identified as

$$\phi_{1F}=\frac{\mathbb{E}\left[\boldsymbol{\eta}_t Z_2\right]\boldsymbol{\Omega}^{-1}\mathbb{E}\left[\boldsymbol{\eta}_t Z_1\right]}{\phi_{2F}}$$

which in turn implies

$$\phi_{1M}\mathbf{B}_M=\mathbb{E}\left[\boldsymbol{\eta}_t Z_1\right]-\frac{\mathbb{E}\left[\boldsymbol{\eta}_t Z_2\right]}{\phi_{2F}}c_{1F}. \tag{A.3}$$

Thus solution to $\mathbf{B}_M$ exists if $\phi_{1M}\neq 0$. Furthermore, note that

$$\begin{aligned}&\left(\mathbb{E}\left[\boldsymbol{\eta}_t Z_1\right]-\frac{\mathbb{E}\left[\boldsymbol{\eta}_t Z_2\right]}{\phi_{2F}^2}\phi_{1F}\right)'\Omega^{-1}\left(\mathbb{E}\left[\boldsymbol{\eta}_t Z_1\right]-\frac{\mathbb{E}\left[\boldsymbol{\eta}_t Z_2\right]}{\phi_{2F}^2}c_{1F}\right)\\ = \;&\boldsymbol{\Omega}^{-\frac{1}{2}}\mathbf{B}_M\phi_{1M}^2\mathbf{B}_M'\boldsymbol{\Omega}^{-\frac{1}{2}}=\phi_{1M}^2\end{aligned}$$

This implies that the parameter $\phi_{1M}$ is identified up to a sign as

$$\phi_{1M}^2=\left(\mathbb{E}\left[\boldsymbol{\eta}_t Z_1\right]-\frac{\mathbb{E}\left[\boldsymbol{\eta}_t Z_2\right]}{\phi_{2F}^2}c_{1F}\right)'\boldsymbol{\Omega}^{-1}\left(\mathbb{E}\left[\boldsymbol{\eta}_t Z_1\right]-\frac{\mathbb{E}\left[\boldsymbol{\eta}_t Z_2\right]}{\phi_{2F}^2}\phi_{1F}\right). \tag{A.4}$$

It only remains to identify $\mathbf{B}_Y$. $\mathbf{B}_Y$ must satisfy

$$\begin{aligned}\mathbf{B}_Y'\boldsymbol{\Omega}^{-1/2}\boldsymbol{\Omega}^{-1/2}\mathbf{B}_Y&=1\\ \mathbf{B}_Y'\boldsymbol{\Omega}^{-1/2}\boldsymbol{\Omega}^{-1/2}\mathbf{B}_M&=0\\ \mathbf{B}_Y'\boldsymbol{\Omega}^{-1/2}\boldsymbol{\Omega}^{-1/2}\mathbf{B}_F&=0\end{aligned} \tag{A.5}$$

$\mathbf{B}_Y$ can be solved analytically using (**A.5**) provided that $\mathbf{B}_F$ and $\mathbf{B}_Y$ are identified. In addition, since the equation (**A.5**) is quadratic in $\mathbf{B}_Y$, $\mathbf{B}_Y$ is unique up to sign. It follows that there exists a $\tau$ such that

$$\mathbf{B}_Y=\tau\tilde{\mathbf{B}}_Y \tag{A.6}$$

where $\tilde{\mathbf{B}}_Y$ is unique conditional on $\phi_{2F}$ and $\phi_{1M}$, but the scalar $\tau$ is unique up to sign.

This shows that the solution to the system (7) exists and is unique up to sign if $\phi_{2F}\neq 0$, $\phi_{1M}\neq 0$. Combined with unit effect normalization (4) and the restriction on the admissible parameter space (2), $\mathbf{B}$ can be uniquely identified. The unit effect normalization implies

$$\begin{aligned}\begin{pmatrix}B_{MM} & B_{MY} & B_{MF}\\ B_{YM} & B_{YY} & B_{YF}\\ B_{FM} & B_{FY} & B_{FF}\end{pmatrix} &= \begin{pmatrix}1 & H_{MY} & H_{MF}\\ H_{YM} & 1 & H_{YF}\\ H_{FM} & H_{FY} & 1\end{pmatrix}\begin{pmatrix}\sigma_{MM} & 0 & 0\\ 0 & \sigma_{YY} & 0\\ 0 & 0 & \sigma_{FF}\end{pmatrix}\\ &= \begin{pmatrix}\sigma_{MM} & H_{MY}\sigma_{YY} & H_{MF}\sigma_{FF}\\ H_{YM}\sigma_{MM} & \sigma_{YY} & H_{YF}\sigma_{FF}\\ H_{FM}\sigma_{MM} & H_{FY}\sigma_{YY} & \sigma_{FF}\end{pmatrix}\end{aligned}$$

Combined with the restriction $\sigma_{jj} > 0$ for all $j = M, Y, F$, implies $B_{jj} > 0$ for all $j = M, Y, F$. From equation (**A.1**), $B_{FF} > 0$ pins down the sign of $\phi_{2F}$ conditional $\mathbf{Z}_t$. Since the sign of $\phi_{2F}$ is pinned down, the signs of $B_{MF}$ and $B_{YF}$ are also pinned down by the same restriction. From equation (**A.3**), $B_{MM} > 0$ pins down the sign of $\phi_{1M}$ conditional $\mathbf{Z}_t$ and therefore the signs of $B_{YM}$ and $B_{FM}$ are pinned down by the same restriction. It only remains to show the uniqueness of $\mathbf{B}_Y$. Provided that $\mathbf{B}_F$ and $\mathbf{B}_Y$ are identified and given the closed-form solution (**A.5**) that is quadratic in $\mathbf{B}_Y$, then $B_{YY} > 0$ pins down the sign of $\tau$ conditional $\mathbf{Z}_t$ and hence the sign of $B_{MY}$ and $B_{FY}$ are also pinned down by the same restriction. ■

The system of equations defining $\mathbf{B}$ is

$$0 = \mathbb{E}[\mathbf{g}_1(\mathbf{m}_{1t}; \boldsymbol{\beta}_1)] \equiv \overline{\mathbf{g}}_1.$$

The rank condition is satisfied when $\mathbf{J} \equiv \partial \mathbb{E}_T[\mathbf{g}_1]/\partial \boldsymbol{\beta}_1'$ is full column rank. We check that the rank condition is satisfied by evaluating $\mathbf{J}$ at the estimated parameter values for each case.

## Procedure for Bootstrap

The bootstrap follows Krinsky and Robb (1986). Let $\hat{\boldsymbol{\beta}}$ and $\hat{\Theta}$ be the estimated GMM parameters and covariance of parameters for each case. We sample repeatedly from the joint distribution $N\left(\hat{\boldsymbol{\beta}}, \hat{\Theta}/T\right)$, where $\hat{\Theta}$ is the estimated GMM variance-covariance matrix to obtain $\mathcal{B}$ new sets of parameters $\hat{\boldsymbol{\beta}}^{(1)} .... \hat{\boldsymbol{\beta}}^{(B)}$. For each $\hat{\boldsymbol{\beta}}^{(i)}$ we infer the $\mathbf{e}^{(i)}$ for that draw and check that the winnowing constraints are satisfied. If they are, we keep the draw. If not, we redraw. We continue until the number of kept draws $\mathcal{B}$=10,000. From these $\mathcal{B}$ saved draws, we calculate the impulse response function values at each draw, $\boldsymbol{\Upsilon}_{s,j}^{(1)}, ..., \boldsymbol{\Upsilon}_{s,j}^{(B)}$, where $s$ indexes the VAR horizon and $j$ the variable being shocked, and where $\boldsymbol{\Upsilon}_{s,j}^{(b)} = \boldsymbol{\Upsilon}_{s,j}\left(\hat{\boldsymbol{\beta}}^{(b)}\right)$. The confidence intervals are ranges for $\boldsymbol{\Upsilon}_{s,j}^{(b)}$ created by trimming $\alpha/2$ from each tail of the resulting distribution of the function values.

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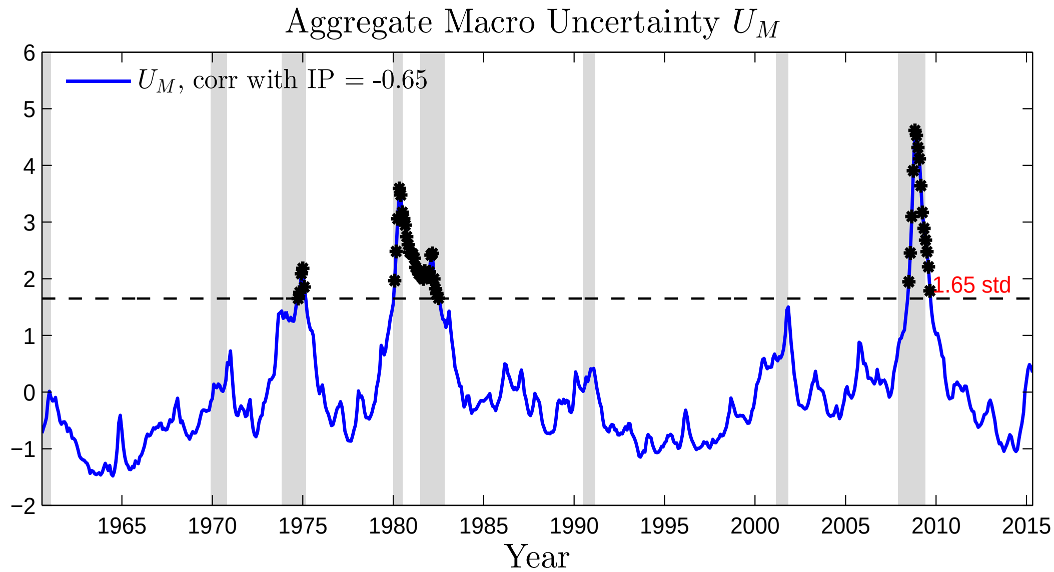
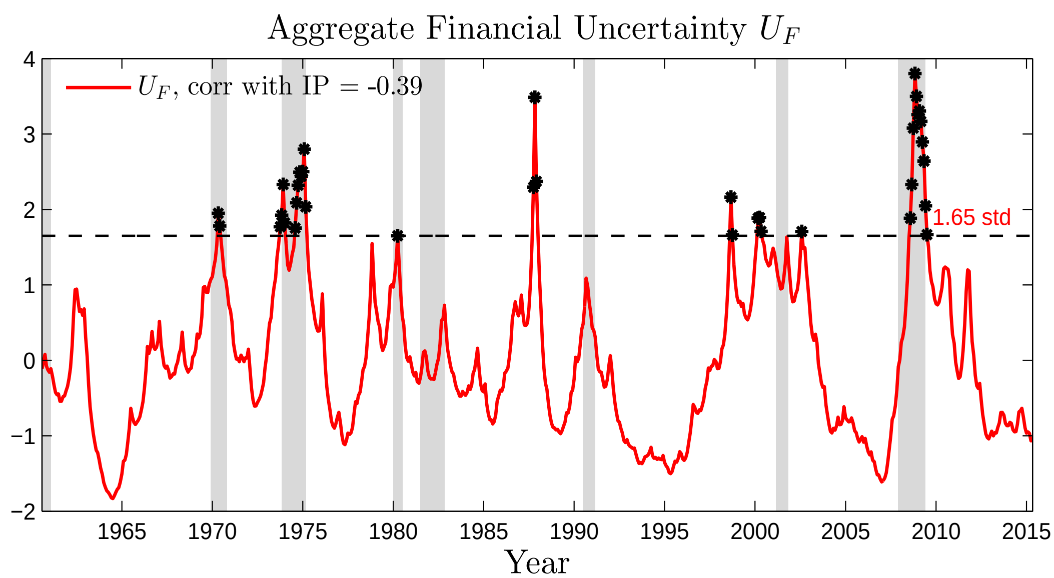
# 8 Figures and Tables

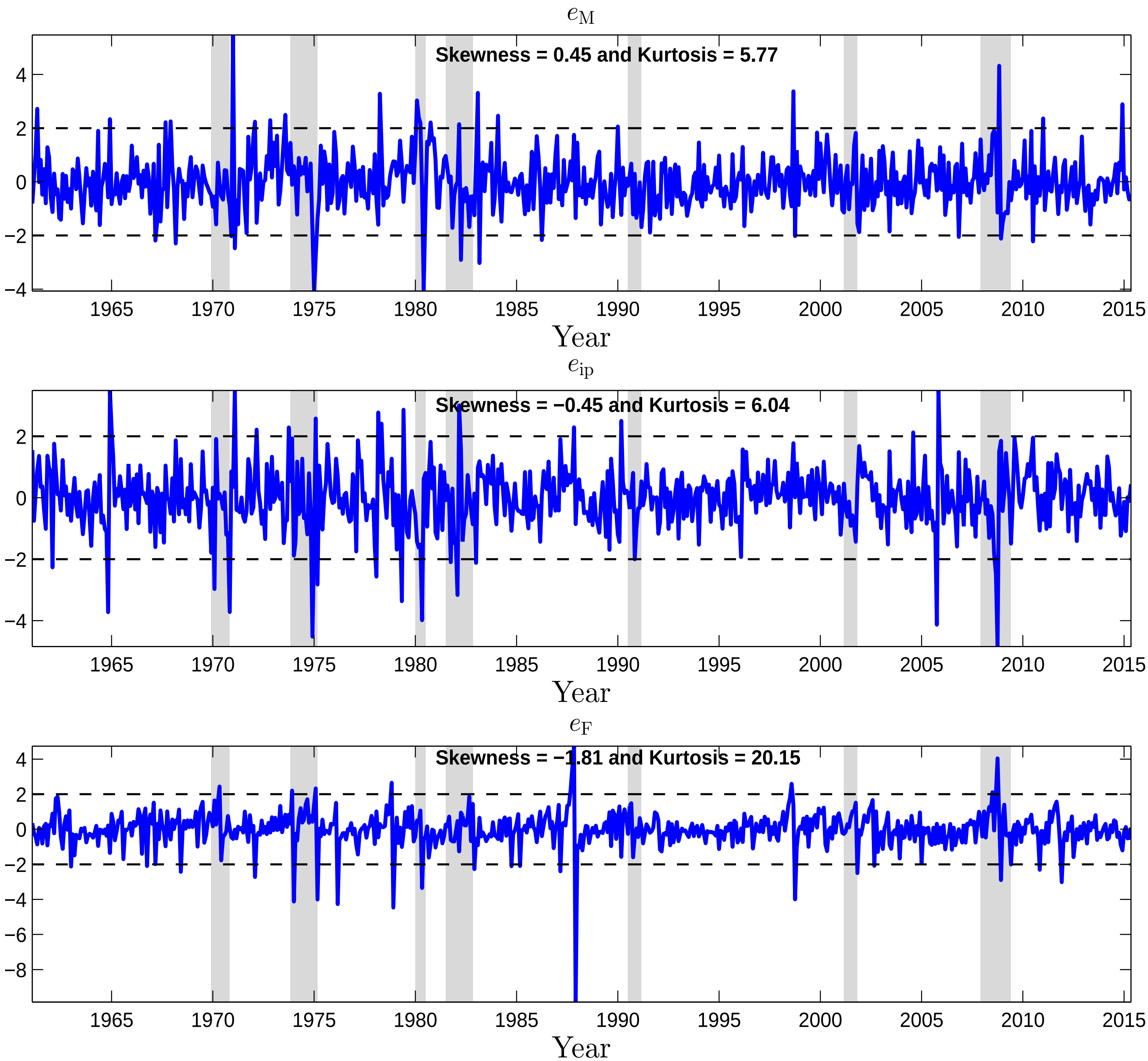
**Figure 1: Macro and Financial Uncertainty Over Time**



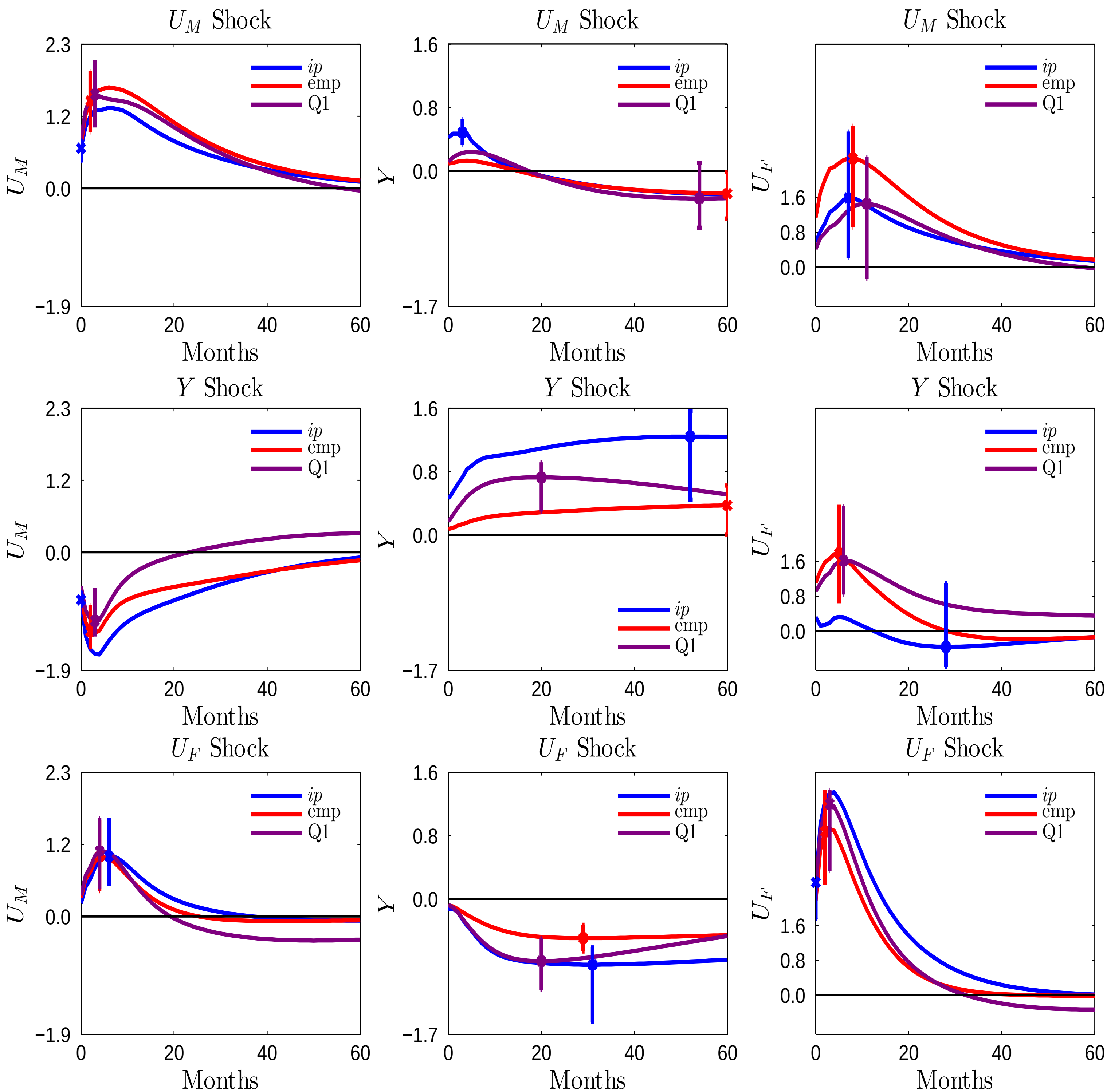

The upper panel plots the time series of the macro uncertainty $U_M$, expressed in standardized units. The lower panel shows the time series of financial uncertainty $U_F$ expressed in standardized units. The shaded areas correspond to the NBER recession dates. The horizontal line corresponds to 1.65 standard deviations above the unconditional mean of each series (which has been normalized to zero). Correlations with the 12-month moving average of IP growth are reported. The black dots represent months when uncertainty is 1.65 standard deviations above its unconditional mean. The data are monthly and span the period 1960:07 to 2015:04.

**Figure 2: Time Series of $e$ Shock from SVAR System $(U_M, ip, U_F)'$**



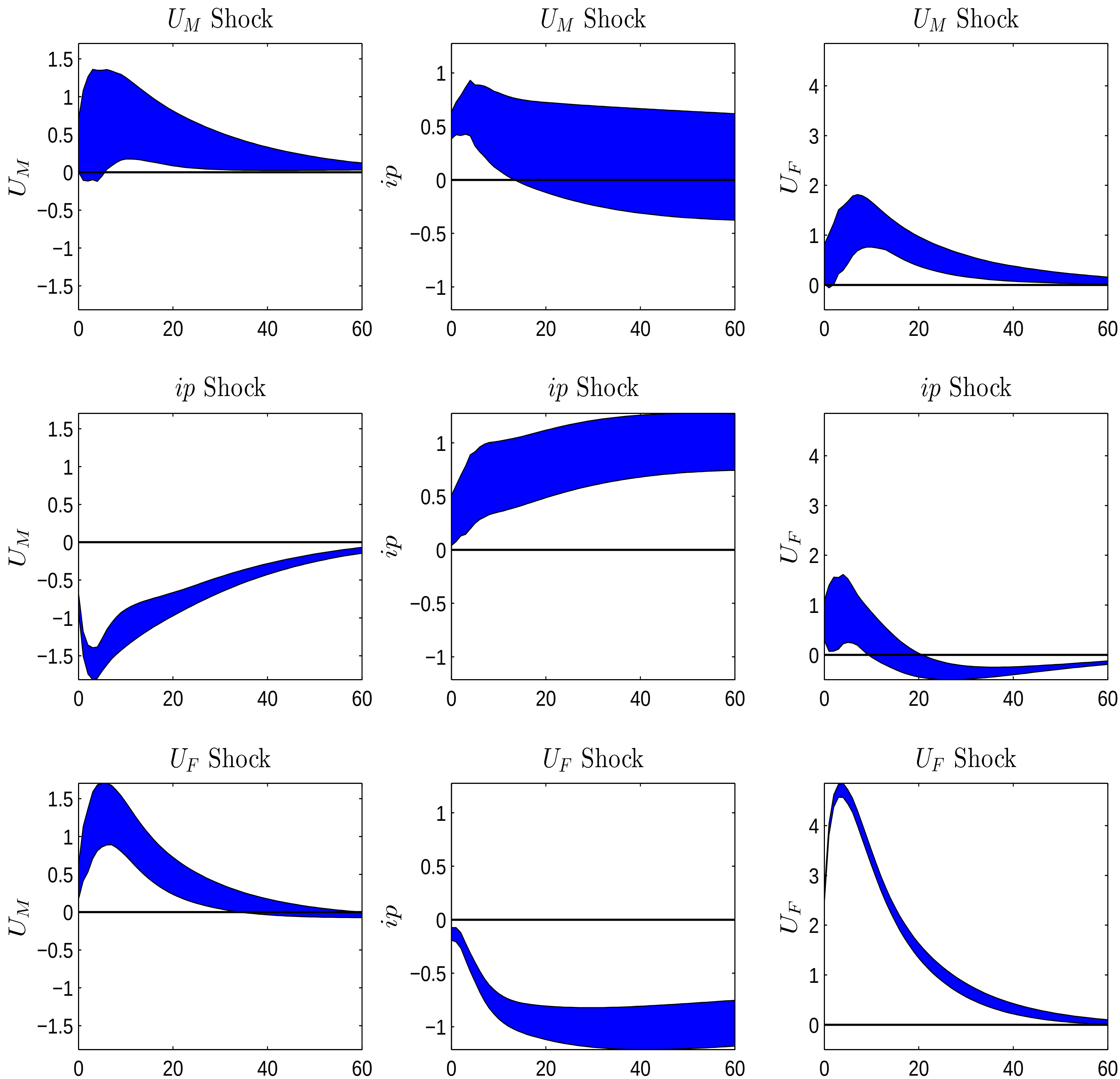

The horizontal line corresponds to 3 standard deviations above/below the unconditional mean of each series. The shocks $e = B^{-1}\eta_t$ are reported, where $\eta_t$ is the residual from VAR(6) of $(U_M, ip, U_F)'$ and $B = A^{-1}\Sigma^{\frac{1}{2}}$. Skewness is defined as $s = \frac{\sum_t^T (e_t - \bar{e})^3/T}{Var(e)}$. Kurtosis is defined as $\kappa = \frac{\sum_t^T (e_t - \bar{e})^4/T}{[Var(e)]^2}$. The sample spans the period 1960:07 to 2015:04.

**Figure 3: IRFs of SVAR** $(U_M, Y, U_F)'$



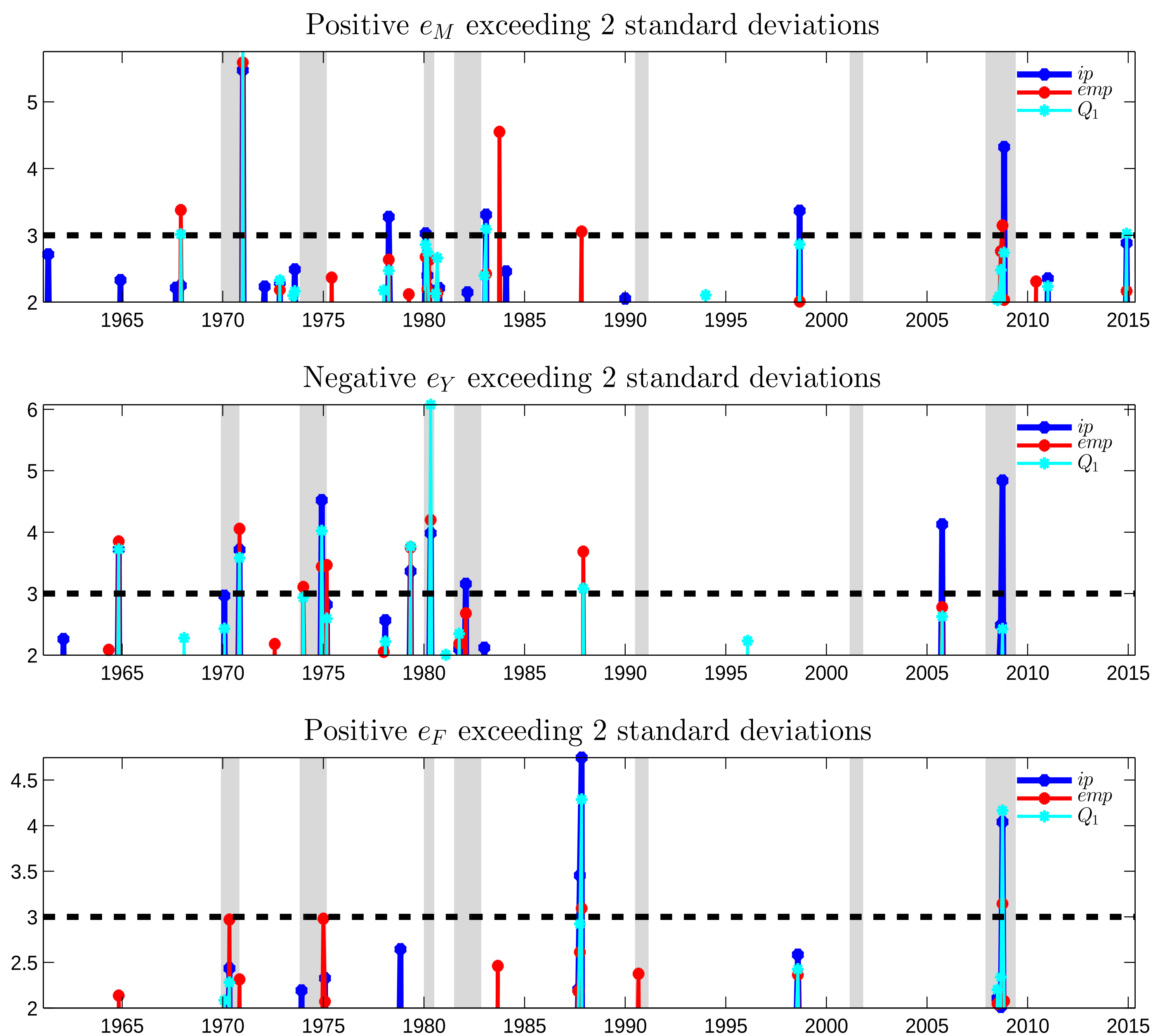

The figure displays impulse responses to one standard deviation shocks. Response units are reported in percentage points. Bootstrapped 90% error bands appear as vertical lines. The sample spans the period 1960:07 to 2015:04

**Figure 4: Set of solutions using $\bar{\mathbf{c}} = 0.04$, $\bar{\mathbf{C}} = 0.24$**



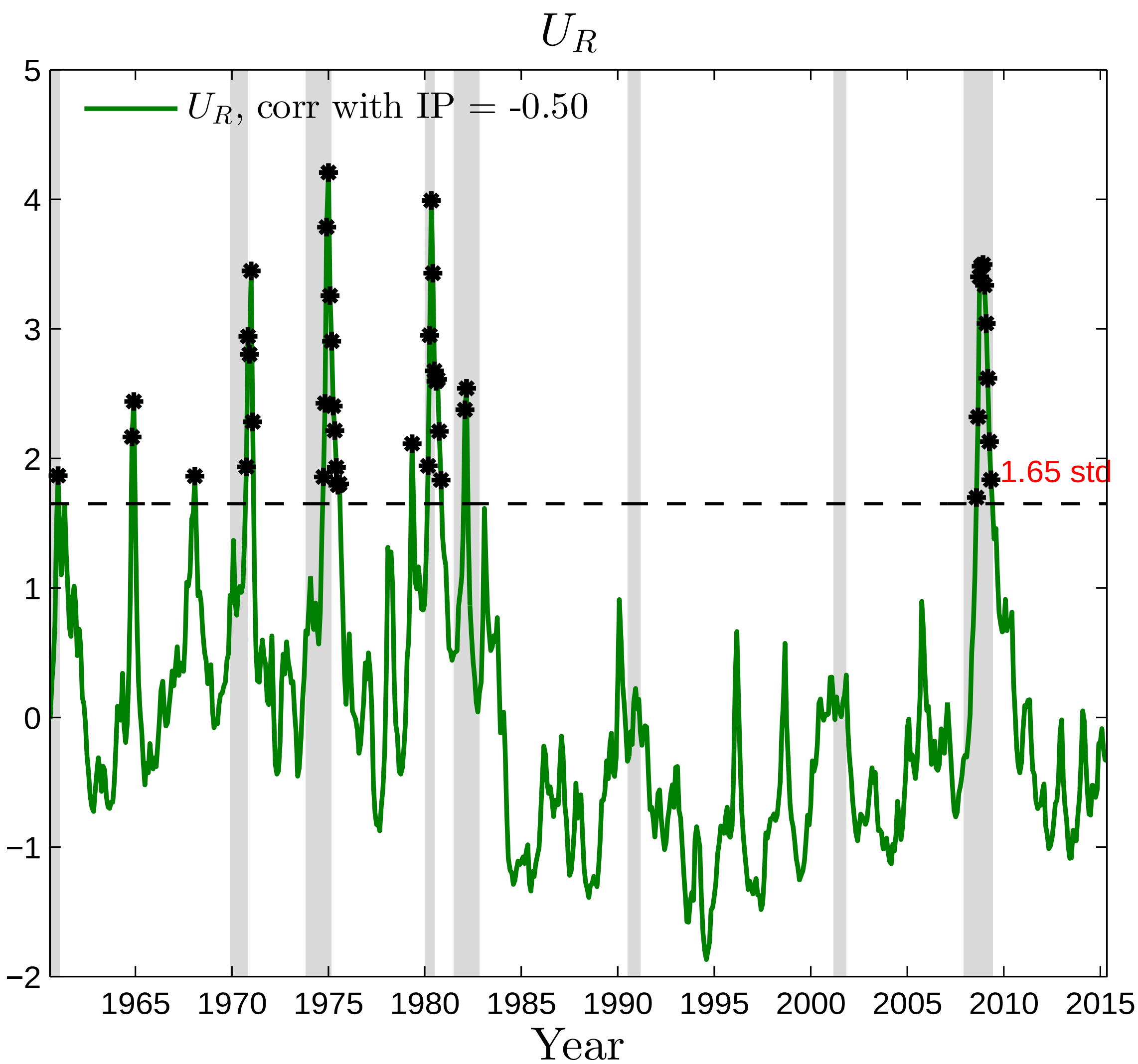

The figure displays impulse responses to one standard deviation shocks. Response units are reported in percentage points. The sample spans the period 1960:07 to 2015:04

**Figure 5: Large Shock Episodes in SVAR$(U_M, Y, U_F)'$**



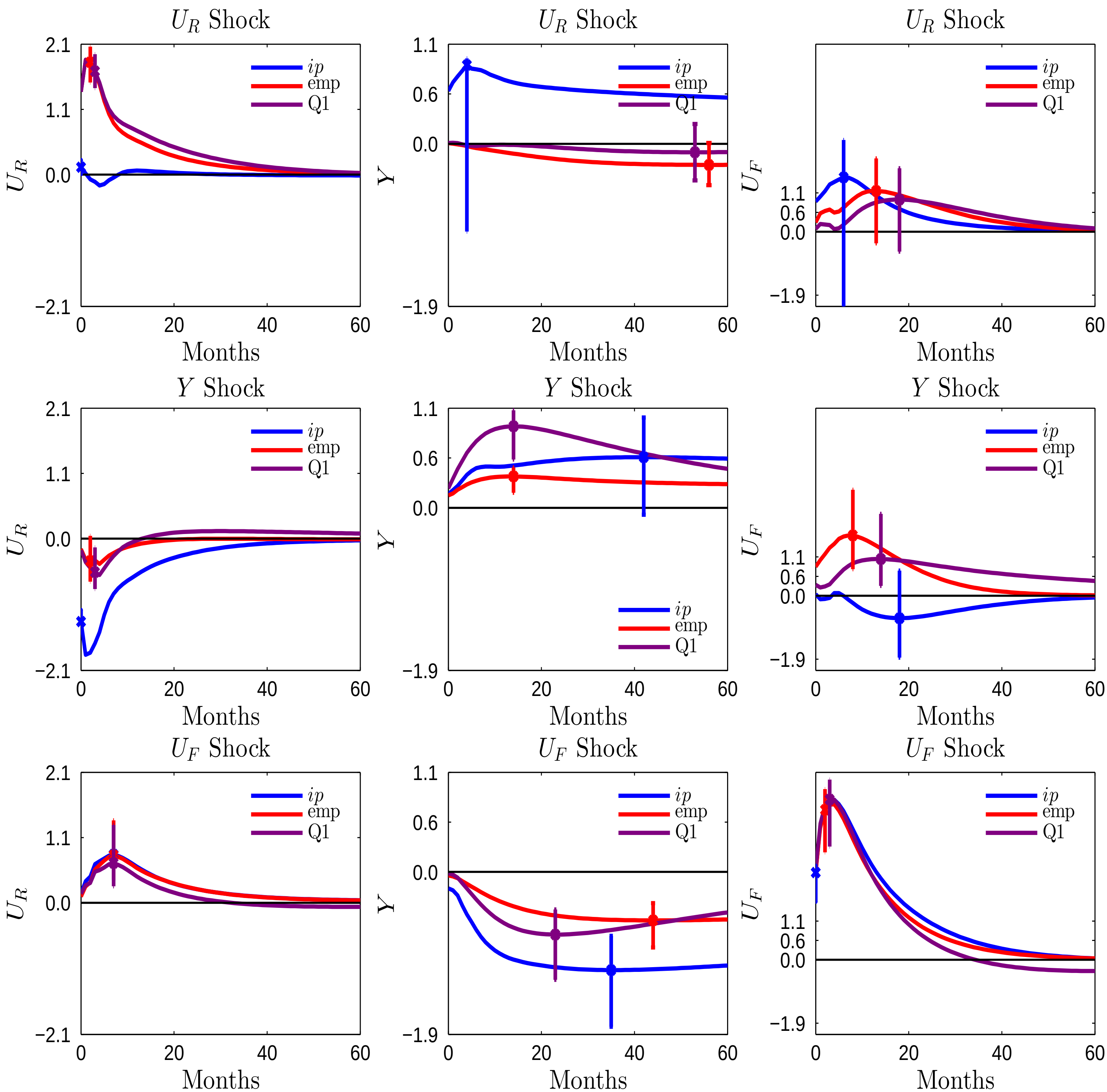

The figure exhibits shocks that are at least 2 standard deviations above the unconditional mean for $e_M$ and $e_F$ and below for $e_Y$ for three cases where $Y = ip, emp, Q_1$. The shocks $\mathbf{e}_t = \mathbf{B}^{-1}\boldsymbol{\eta}_t$ are reported, where $\boldsymbol{\eta}_t$ is the residual from VAR(6) and $\mathbf{B} = \mathbf{A}^{-1}\boldsymbol{\Sigma}^{\frac{1}{2}}$. The horizontal line corresponds to 3 standard deviations shocks. The sample spans the period 1960:07 to 2015:04.

**Figure 6: Real Uncertainty Over Time**



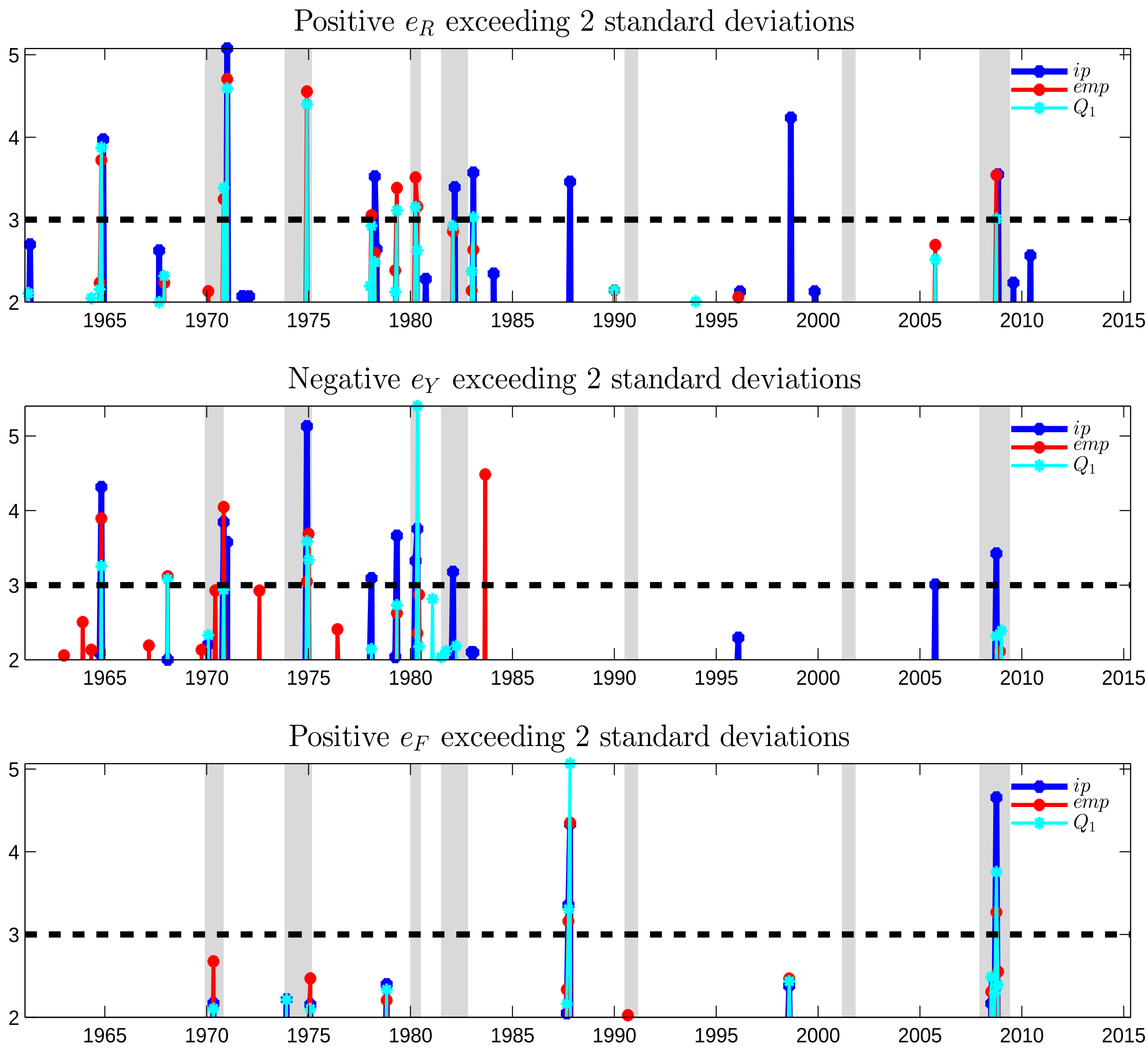

This plot shows time series of $U_R$, expressed in standardized units. The shaded areas correspond to the NBER recession dates. The horizontal line corresponds to 1.65 standard deviations above the unconditional mean of each series (which has been normalized to zero). Correlations with the 12-month moving average of IP growth are reported. The black dots represent months when $U_R$ is 1.65 standard deviations above its unconditional mean. The data are monthly and span the period 1960:07 to 2015:04.

**Figure 7: IRFs of SVAR** $(U_R, Y, U_F)'$



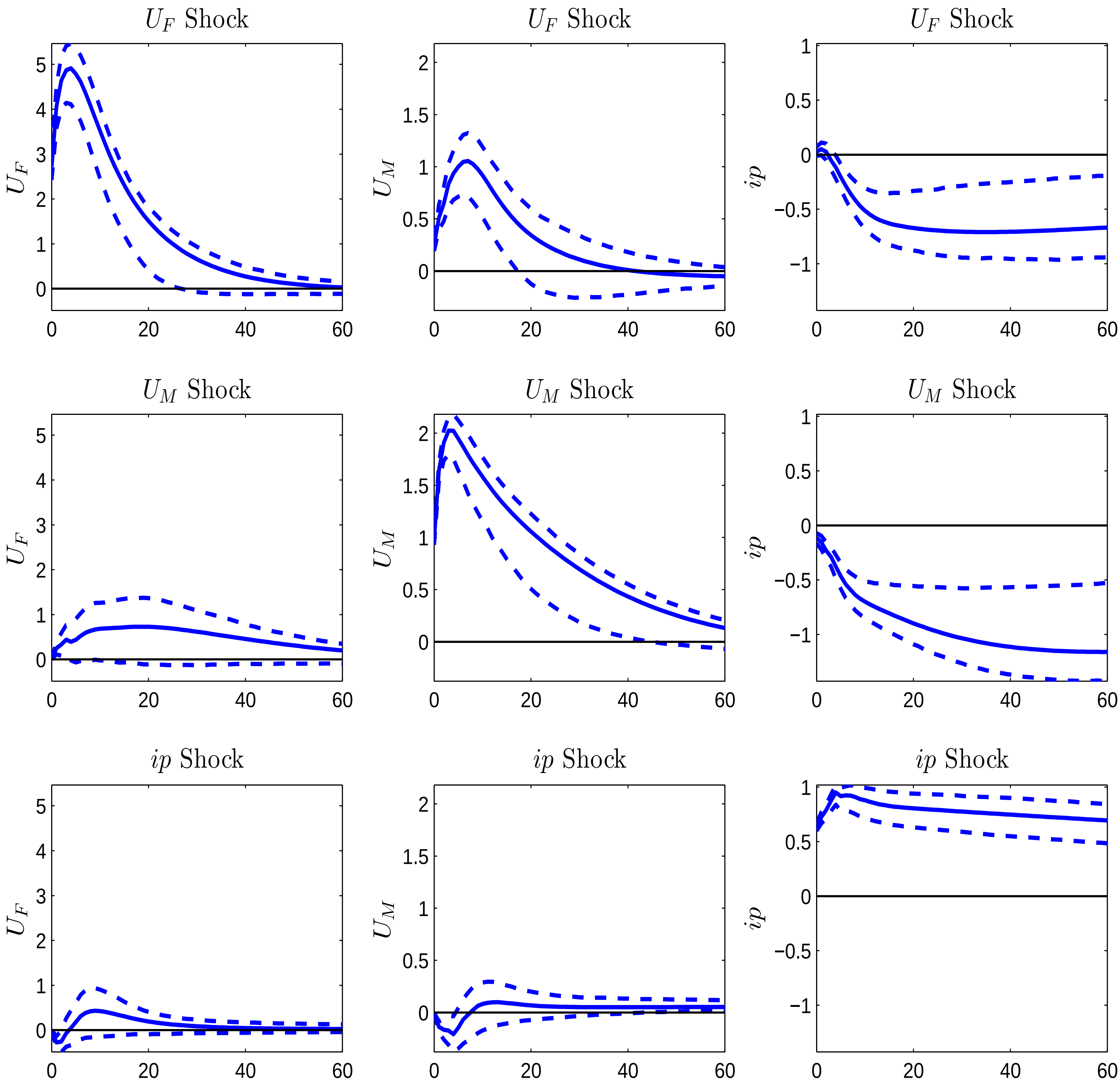

The figure displays impulse responses to one standard deviation shocks. Response units are reported in percentage points. Bootstrapped 90% error bands appear as vertical lines. The sample spans the period 1960:07 to 2015:04

**Figure 8: Large Shock Episodes in SVAR**$(U_R, Y, U_F)'$



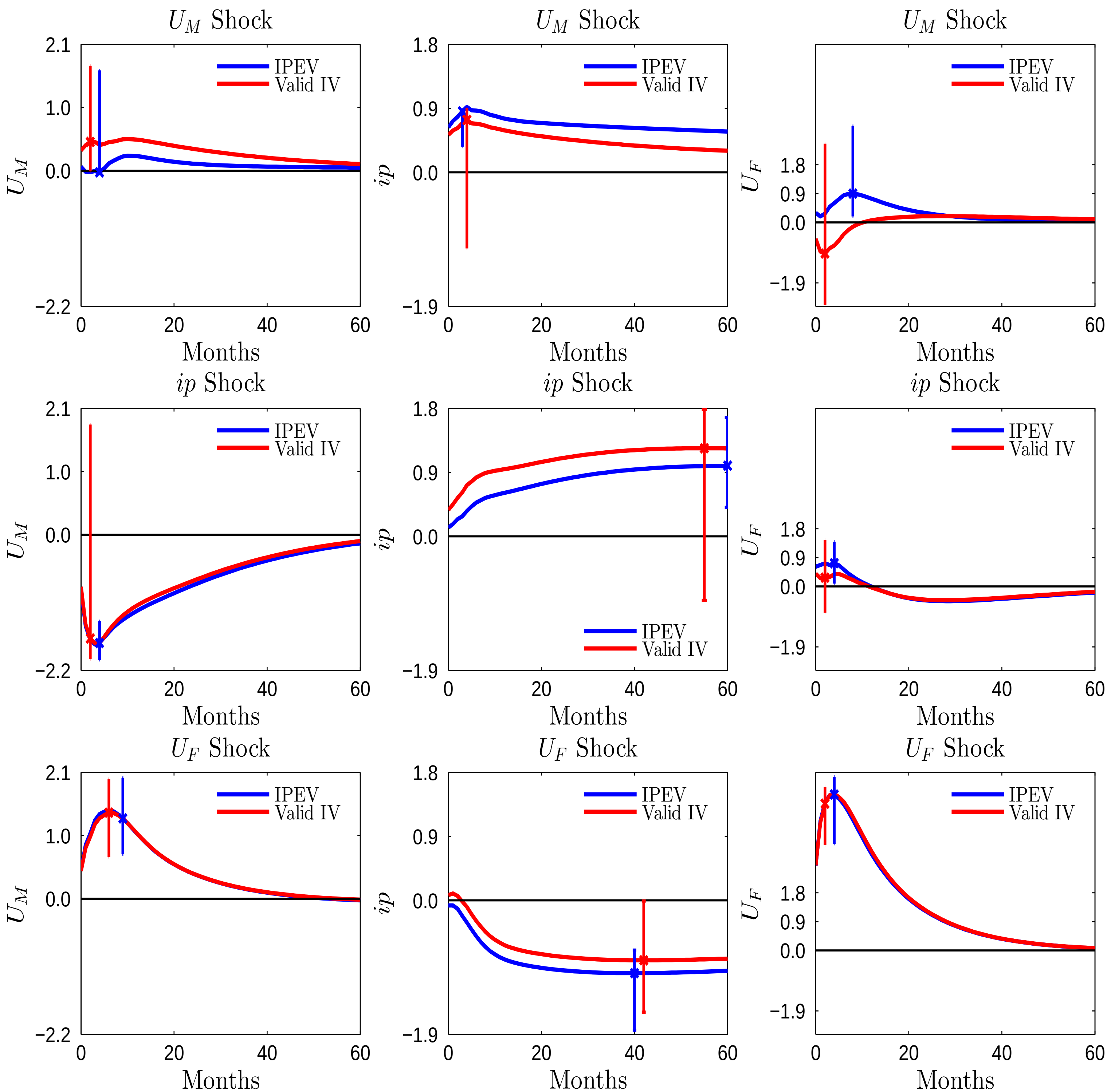

The figure exhibits shocks that are at least 2 standard deviations above the unconditional mean for $e_R$ and $e_F$ and below for $e_Y$ for three cases where $Y = ip, emp, Q_1$. The shocks $\mathbf{e}_t = \mathbf{B}^{-1}\boldsymbol{\eta}_t$ are reported, where $\boldsymbol{\eta}_t$ is the residual from VAR(6) and $\mathbf{B} = \mathbf{A}^{-1}\boldsymbol{\Sigma}^{\frac{1}{2}}$. The horizontal line corresponds to 3 standard deviations shocks. The sample spans the period 1960:07 to 2015:04.

**Figure 9: IRFs using Recursive Identification with Order $(U_F, U_M, ip)'$**



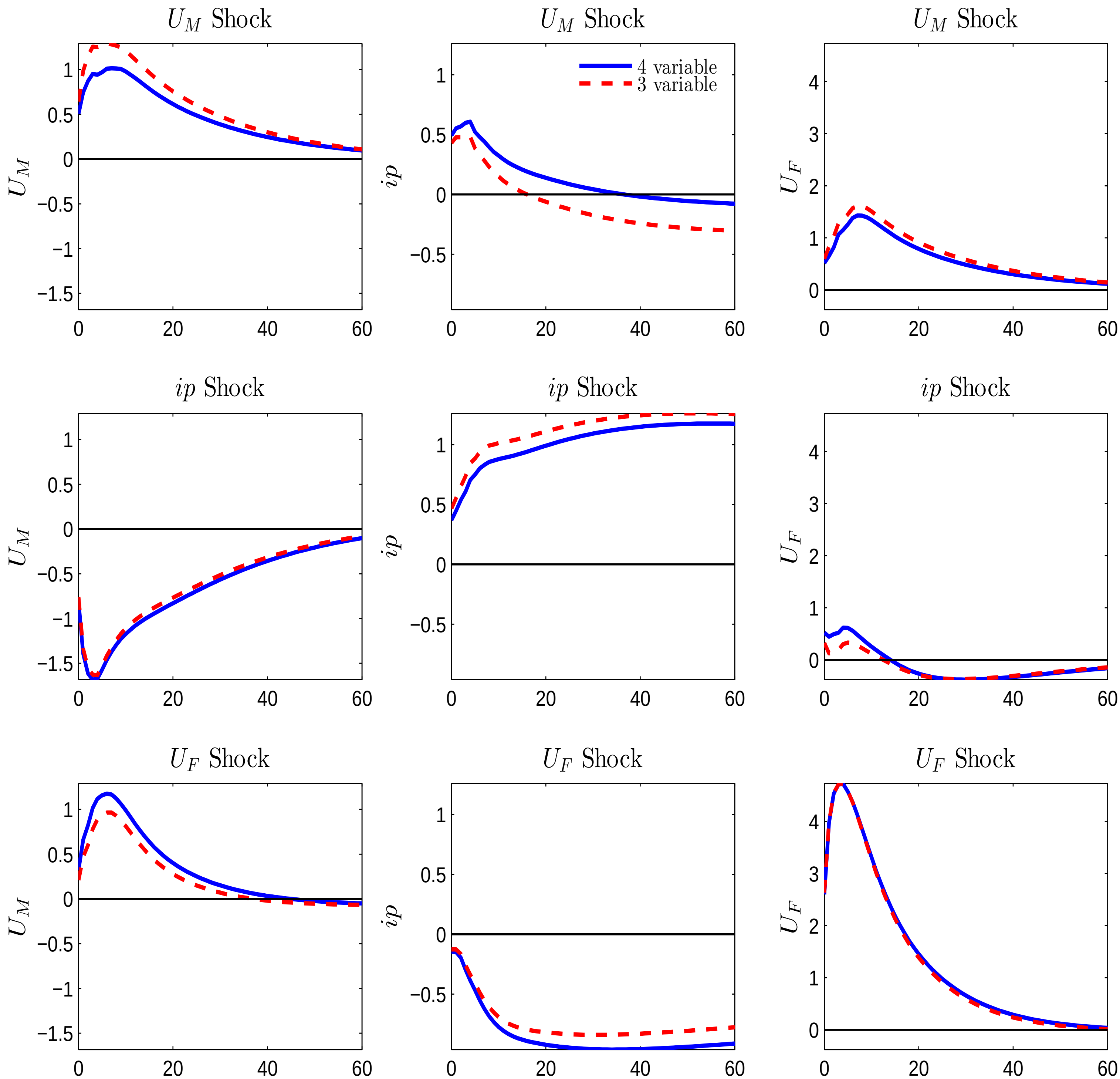

Bootstrapped 90% error bands appear as dashed lines. Response units are reported in percentage points. The sample spans the period 1960:07 to 2015:04.

**Figure 10: IRFs of SVAR $(U_M, ip, U_F)'$, presumed valid IV v.s. IPEV**




The figure displays impulse responses to one standard deviation shocks. Response units are reported in percentage points. Bootstrapped 90% error bands appear as vertical lines. Presumed valid IV uses $Z_1 = U_{\text{SPX}}$ and $Z_2 = r_{\text{CRSP}}$. IPEV uses $S_1 = U_{\text{SPX}}$ and $S_2 = r_{\text{CRSP}}$. The sample spans the period 1960:07 to 2015:04

**Figure 11: IRFs of SVAR** $(U_M, ip, U_F, r_{\text{CRSP}})'$ v.s. $(U_M, ip, U_F)'$




The figure displays impulse responses to one standard deviation shocks. Response units are reported in percentage points. The sample spans the period 1960:07 to 2015:04

**Table 1: Sample Statistics**

| Panel A: Correlations between Instruments and Shocks | | | |
|---|---|---|---|
| $SVAR$ | $(U_M, ip, U_F)'$ | $(U_M, emp, U_F)'$ | $(U_M, Q_1, U_F)'$ |
| $\rho\left(Z_{1t}(\boldsymbol{\beta}), \hat{e}_{Mt}\right)$ | $\mathbf{-0.0804}$ | $\mathbf{-0.0736}$ | $\mathbf{-0.0764}$ |
| | (0.0044) | (0.0040) | (0.0042) |
| $\rho\left(Z_{1t}(\boldsymbol{\beta}), \hat{e}_{Ft}\right)$ | $\mathbf{-0.1698}$ | $\mathbf{-0.1782}$ | $\mathbf{-0.1655}$ |
| | (0.0093) | (0.0097) | (0.0090) |
| $\rho\left(Z_{2t}(\boldsymbol{\beta}), \hat{e}_{Ft}\right)$ | $\mathbf{-0.1547}$ | $\mathbf{-0.1635}$ | $\mathbf{-0.1491}$ |
| | (0.0093) | (0.0098) | (0.0089) |
| $\rho\left(Z_{1t}(\boldsymbol{\beta}), \hat{e}_{Yt}\right)$ | 0.0000 | 0.0000 | 0.0000 |
| $\rho\left(Z_{2t}(\boldsymbol{\beta}), \hat{e}_{Yt}\right)$ | 0.0000 | 0.0000 | 0.0000 |
| $\rho\left(Z_{2t}(\boldsymbol{\beta}), \hat{e}_{Mt}\right)$ | 0.0000 | 0.0000 | 0.0000 |
| Panel B: Estimates of $\boldsymbol{\Sigma}$ | | | |
| $\sigma_{MM}$ | **0.0064** | **0.0071** | **0.0079** |
| | (0.0008) | (0.0009) | (0.0005) |
| | [0.004, 0.008] | [0.005, 0.009] | [0.001, 0.010] |
| $\sigma_{YY}$ | **0.0047** | **0.0008** | **0.0017** |
| | (0.0006) | (0.0002) | (0.0001) |
| | [0.003, 0.005] | [0.001, 0.001] | [0.002, 0.002] |
| $\sigma_{FF}$ | **0.0263** | **0.0219** | **0.0252** |
| | (0.0030) | (0.0025) | (0.0025) |
| | [0.018, 0.027] | [0.015, 0.027] | [0.016, 0.026] |

Panel A reports the correlation between the estimated uncertainty shocks and the instruments. Panel B reports estimates of $\boldsymbol{\Sigma}$ that give the standard deviation of each structural shock. Asymptotic standard errors are reported in brackets and bootstrapped 90 percent confidence intervals are reported in parentheses. Bold numbers indicate statistical significance at 10 percent level. The data are monthly and span the period 1960:07 to 2015:04.

## Table 2: Variance Decomposition for SVARs in System $(U_M, Y, U_F)'$

| | SVAR $(U_M, ip, U_F)'$ | | | SVAR $(U_M, emp, U_F)'$ | | | SVAR $(U_M, Q_1, U_F)'$ | | |
|---|---|---|---|---|---|---|---|---|---|
| | Fraction variation in $U_M$ | | | Fraction variation in $U_M$ | | | Fraction variation in $U_M$ | | |
| $s$ | $U_M$ Shock | $ip$ Shock | $U_F$ Shock | $U_M$ Shock | $emp$ Shock | $U_F$ Shock | $U_M$ Shock | $Q_1$ Shock | $U_F$ Shock |
| 1 | 0.344 | 0.590 | 0.066 | 0.466 | 0.424 | 0.109 | 0.565 | 0.295 | 0.141 |
| 12 | 0.371 | 0.464 | 0.166 | 0.586 | 0.262 | 0.152 | 0.598 | 0.188 | 0.214 |
| $\infty$ | 0.403 | 0.463 | 0.134 | 0.641 | 0.256 | 0.103 | 0.591 | 0.177 | 0.232 |
| $s_{\max}$ | 0.404 | 0.590 | 0.166 | 0.647 | 0.440 | 0.165 | 0.700 | 0.295 | 0.232 |
| | $[0.22, 0.65]$ | $[0.44, 0.70]$ | $[0.05, 0.49]$ | $[0.38, 0.87]$ | $[0.24, 0.63]$ | $[0.04, 0.40]$ | $[0.51, 0.88]$ | $[0.13, 0.46]$ | $[0.11, 0.50]$ |
| | Fraction variation in $ip$ | | | Fraction variation in $emp$ | | | Fraction variation in $Q_1$ | | |
| $s$ | $U_M$ Shock | $ip$ Shock | $U_F$ Shock | $U_M$ Shock | $emp$ Shock | $U_F$ Shock | $U_M$ Shock | $Q_1$ Shock | $U_F$ Shock |
| 1 | 0.427 | 0.540 | 0.033 | 0.405 | 0.288 | 0.307 | 0.288 | 0.606 | 0.106 |
| 12 | 0.107 | 0.670 | 0.223 | 0.089 | 0.299 | 0.612 | 0.071 | 0.529 | 0.400 |
| $\infty$ | 0.038 | 0.683 | 0.278 | 0.155 | 0.315 | 0.530 | 0.108 | 0.458 | 0.433 |
| $s_{\max}$ | 0.442 | 0.683 | 0.310 | 0.425 | 0.315 | 0.688 | 0.298 | 0.616 | 0.490 |
| | $[0.30, 0.65]$ | $[0.34, 0.78]$ | $[0.25, 0.81]$ | $[0.29, 0.68]$ | $[0.21, 0.66]$ | $[0.37, 0.90]$ | $[0.17, 0.48]$ | $[0.45, 0.81]$ | $[0.30, 0.83]$ |
| | Fraction variation in $U_F$ | | | Fraction variation in $U_F$ | | | Fraction variation in $U_F$ | | |
| $s$ | $U_M$ Shock | $ip$ Shock | $U_F$ Shock | $U_M$ Shock | $emp$ Shock | $U_F$ Shock | $U_M$ Shock | $Q_1$ Shock | $U_F$ Shock |
| 1 | 0.044 | 0.005 | 0.951 | 0.183 | 0.136 | 0.681 | 0.027 | 0.088 | 0.885 |
| 12 | 0.107 | 0.003 | 0.890 | 0.313 | 0.134 | 0.553 | 0.087 | 0.132 | 0.781 |
| $\infty$ | 0.148 | 0.016 | 0.836 | 0.422 | 0.120 | 0.458 | 0.152 | 0.188 | 0.660 |
| $s_{\max}$ | 0.148 | 0.016 | 0.951 | 0.422 | 0.170 | 0.682 | 0.155 | 0.188 | 0.888 |
| | $[0.02, 0.45]$ | $[0.02, 0.26]$ | $[0.70, 0.98]$ | $[0.09, 0.59]$ | $[0.04, 0.51]$ | $[0.52, 0.93]$ | $[0.02, 0.39]$ | $[0.11, 0.59]$ | $[0.66, 0.97]$ |

Each panel shows the fraction of $s$-step-ahead forecast-error variance of the variable given in the panel title that is explained by the shock named in the column heading. The row denoted "$s = s_{max}$" reports the maximum fraction (across all VAR forecast horizons $m$) of forecast error variance explained by the shock listed in the column heading. The numbers in parentheses represent the 5th and 95th percentiles of these statistics from bootstrapped samples using the procedure described in the Appendix. The data are monthly and span the period 1960:07 to 2015:04.

## Table 3: Variance Decomposition for SVARs in System $(U_R, Y, U_F)'$

| | SVAR $(U_R, ip, U_F)'$ | | | SVAR $(U_R, emp, U_F)'$ | | | SVAR $(U_R, Q_1, U_F)'$ | | |
|---|---|---|---|---|---|---|---|---|---|
| | Fraction variation in $U_R$ | | | Fraction variation in $U_R$ | | | Fraction variation in $U_R$ | | |
| $s$ | $U_R$ Shock | $ip$ Shock | $U_F$ Shock | $U_R$ Shock | $emp$ Shock | $U_F$ Shock | $U_R$ Shock | $Q_1$ Shock | $U_F$ Shock |
| 1 | 0.003 | 0.970 | 0.027 | 0.960 | 0.026 | 0.015 | 0.949 | 0.032 | 0.018 |
| 12 | 0.004 | 0.787 | 0.208 | 0.770 | 0.037 | 0.193 | 0.808 | 0.062 | 0.130 |
| $\infty$ | 0.005 | 0.743 | 0.252 | 0.728 | 0.033 | 0.239 | 0.793 | 0.072 | 0.135 |
| $s_{\max}$ | 0.008 | 0.978 | 0.252 | 0.980 | 0.041 | 0.239 | 0.969 | 0.074 | 0.137 |
| | $[0.01, 0.11]$ | $[0.84, 1.00]$ | $[0.08, 0.55]$ | $[0.83, 1.00]$ | $[0.01, 0.18]$ | $[0.09, 0.53]$ | $[0.84, 0.99]$ | $[0.03, 0.20]$ | $[0.04, 0.44]$ |
| | Fraction variation in $ip$ | | | Fraction variation in $emp$ | | | Fraction variation in $Q_1$ | | |
| $s$ | $U_R$ Shock | $ip$ Shock | $U_F$ Shock | $U_R$ Shock | $emp$ Shock | $U_F$ Shock | $U_R$ Shock | $Q_1$ Shock | $U_F$ Shock |
| 1 | 0.854 | 0.066 | 0.080 | 0.001 | 0.916 | 0.083 | 0.001 | 0.990 | 0.008 |
| 12 | 0.506 | 0.130 | 0.364 | 0.023 | 0.617 | 0.360 | 0.000 | 0.799 | 0.200 |
| $\infty$ | 0.209 | 0.169 | 0.622 | 0.116 | 0.222 | 0.662 | 0.009 | 0.577 | 0.414 |
| $s_{\max}$ | 0.857 | 0.169 | 0.622 | 0.116 | 0.924 | 0.662 | 0.009 | 0.993 | 0.414 |
| | $[0.71, 0.97]$ | $[0.01, 0.39]$ | $[0.40, 0.93]$ | $[0.02, 0.31]$ | $[0.72, 0.99]$ | $[0.33, 0.94]$ | $[0.01, 0.16]$ | $[0.79, 1.00]$ | $[0.19, 0.83]$ |
| | Fraction variation in $U_F$ | | | Fraction variation in $U_F$ | | | Fraction variation in $U_F$ | | |
| $s$ | $U_R$ Shock | $ip$ Shock | $U_F$ Shock | $U_R$ Shock | $emp$ Shock | $U_F$ Shock | $U_R$ Shock | $Q_1$ Shock | $U_F$ Shock |
| 1 | 0.078 | 0.001 | 0.922 | 0.016 | 0.076 | 0.908 | 0.003 | 0.007 | 0.991 |
| 12 | 0.108 | 0.004 | 0.889 | 0.042 | 0.140 | 0.819 | 0.013 | 0.038 | 0.950 |
| $\infty$ | 0.108 | 0.030 | 0.862 | 0.094 | 0.162 | 0.744 | 0.073 | 0.140 | 0.787 |
| $s_{\max}$ | 0.110 | 0.030 | 0.928 | 0.094 | 0.162 | 0.910 | 0.075 | 0.140 | 0.993 |
| | $[0.03, 0.38]$ | $[0.01, 0.25]$ | $[0.71, 0.98]$ | $[0.01, 0.30]$ | $[0.04, 0.51]$ | $[0.68, 0.99]$ | $[0.01, 0.29]$ | $[0.07, 0.44]$ | $[0.80, 1.00]$ |

Each panel shows the fraction of $s$-step-ahead forecast-error variance of the variable given in the panel title that is explained by the shock named in the column heading. The row denoted "$s = s_{max}$" reports the maximum fraction (across all VAR forecast horizons $m$) of forecast error variance explained by the shock listed in the column heading. The numbers in parentheses represent the 5th and 95th percentiles of these statistics from bootstrapped samples using the procedure described in the Appendix. The data are monthly and span the period 1960:07 to 2015:04.

**Table 4: Tests of Validity of Recursive Restriction in System $(U_M, Y, U_F)'$**

| Ordering: | $(U_M, ip, U_F)'$ | $(U_R, ip, U_F)'$ |
|---|---|---|
| $H_0$: $B_{RY} = B_{RF} = B_{YF} = 0$ | **265.64** | **337.54** |
| | [147.78] | [83.54] |
| $H_0$: $B_{YR} = B_{YF} = B_{RF} = 0$ | **383.28** | **457.95** |
| | [108.25] | [146.74] |
| $H_0$: $B_{RY} = B_{RF} = B_{FY} = 0$ | **265.49** | **227.58** |
| | [164.29] | [95.82] |
| $\chi^2_{5\%}(3)$ | 7.81 | 7.81 |
| | $(U_M, emp, U_F)'$ | $(U_R, emp, U_F)'$ |
| $H_0$: $B_{RY} = B_{RF} = B_{YF} = 0$ | **316.22** | **9.89** |
| | [120.73] | [8.48] |
| $H_0$: $B_{YR} = B_{YF} = B_{RF} = 0$ | **223.98** | **11.03** |
| | [66.29] | [8.35] |
| $H_0$: $B_{RY} = B_{RF} = B_{FY} = 0$ | **318.61** | **8.64** |
| | [121.75] | [9.65] |
| $\chi^2_{5\%}(3)$ | 7.81 | 7.81 |

The table reports the Wald test statistic for testing the null hypothesis given in the column. The bold indicates that Wald test rejects the null at 95 percent level according to $\chi^2(3)$ distribution. The SVAR system is solved using GMM and delta method is used for computing the standard error. Estimates of $\mathbf{B}$ are based on the SVAR identified with external instruments described in the text. The mean of bootstrap Wald statistics is reported in parenthesis. The sample size spans 1960:07 to 2015:04.