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EXOGENOUS IMPULSE OR ENDOGENOUS RESPONSE?

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ABSTRACT

Uncertainty about the future rises in recessions. But is uncertainty a source of business cycle fluctuations or an endogenous response to them, and does the type of uncertainty matter? Answer: we find that sharply higher uncertainty about real economic activity in recessions is fully an endogenous response to other shocks that cause business cycle fluctuations, while uncertainty about financial markets is a likely source of the fluctuations. Financial market uncertainty has quantitatively large negative consequences for several measures of real activity including employment, production, and orders. Such are the main conclusions drawn from estimation of three-variable structural vector autoregressions. To establish causal effects, we propose an iterative projection IV (IPIV) approach to construct external instruments that are valid under credible interpretations of the structural shocks.

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1 Introduction

A large literature in macroeconomics investigates the relationship between uncertainty and business cycle fluctuations. Interest in this topic has been spurred by a growing body of evidence that uncertainty rises sharply in recessions. This evidence is robust to the use of specific proxy variables such as stock market volatility and forecast dispersion as in Bloom (2009), or a broad-based measure of macroeconomic uncertainty, as in Jurado, Ludvigson, and Ng (2015) (JLN hereafter). But while this evidence substantiates a role for uncertainty in deep recessions, the question of whether uncertainty is an exogenous source of business cycle fluctuations or an endogenous response to economic fundamentals is not fully understood. Existing results are based on convenient but restrictive identifying assumptions and have no explicit role for financial markets, even though the uncertainty measures are correlated with financial variables. This paper considers a novel identification strategy to disentangle the causes and consequences of real and financial uncertainty.

The question of causality and the identification of exogenous variation in uncertainty is a long-standing challenge of the uncertainty literature. The challenge arises in part because there is no theoretical consensus on whether the uncertainty that accompanies deep recessions is primarily a cause or effect (or both) of declines in economic activity. Theories in which uncertainty is defined as the time varying volatility of a fundamental shock cannot address this question because, by design, there is no feedback response of uncertainty to other shocks if the volatility process is specified to evolve exogenously. And, obviously, models in which there is no exogenous variation in uncertainty cannot be used to analyze the direct effects of uncertainty shocks. It is therefore not surprising that many theories for which uncertainty plays a role in recessions reach contradictory conclusions on this question, as we survey below. It is clear that the body of theoretical work on uncertainty does not provide precise identifying restrictions for empirical work.

A separate challenge of the uncertainty literature pertains to the origins of uncertainty. Classic theories assert that uncertainty originates from economic fundamentals such as productivity, and that such real economic uncertainty, when interacted with market frictions, discourages real activity. But some researchers have argued that uncertainty dampens the economy through its influence on financial markets (e.g., Gilchrist, Sim, and Zakrajsek (2010)). Moreover, as surveyed by Ng and Wright (2013), all the post-1982 recessions have origins in financial markets, and these recessions have markedly different features from recessions where financial markets play a passive role. From this perspective, if financial shocks are subject to time-varying volatility, financial market uncertainty—as distinct from real economic uncertainty—could be a key player in recessions, both as a cause and as a propagating mechanism. The Great Recession of 2008, characterized by sharp swings in financial markets, hints at such a linkage. Yet so
far the literature has not disentangled the contributions of real versus financial uncertainty to business cycle fluctuations.

Econometric analyses aimed at understanding the role of uncertainty for business cycle fluctuations face their own challenges. Attempts to identify the “effects” of uncertainty shocks in existing empirical work are primarily based on recursive schemes within the framework of vector-autoregressions (VAR). But studies differ according to whether uncertainty is ordered ahead of or after real activity variables in the VAR. While a recursive structure is a reasonable starting point, any presumed ordering of the variables is hard to defend on theoretical grounds given the range of models in the literature. Contemporaneous changes in uncertainty can arise both as a cause of business cycle fluctuations and as a response to other shocks. Recursive structures explicitly rule out this possibility since they presume that some variables respond only with a lag to others.

It is with these challenges in mind that we return to the questions posed above: is uncertainty primarily a source of business cycle fluctuations or a consequence of them? And what is the relation of real versus financial uncertainty to business cycle fluctuations? The objective of this paper is to address these questions econometrically using a small-scale structural vector autoregression (SVAR). To confront the challenges just discussed, we take a two-pronged approach. First, our empirical analysis explicitly distinguishes macro uncertainty from financial uncertainty. The baseline SVAR we study describes the dynamic relationship between three variables: an index of macro uncertainty, $U_{Mt}$, a measure of real economic activity, $Y_t$ (e.g., production, employment), and a new financial uncertainty index introduced here, $U_{Ft}$. Second, rather than relying on timing assumptions for identification, we use a different identification scheme that is less restrictive, both because it allows for simultaneous feedback between uncertainty and real activity, and because it can be used to test whether a lower recursive structure is supported by the data.

Specifically, our identification scheme relies on the existence of two external instruments for uncertainty that are not part of the SVAR: a $Z_{1t}$ that is correlated with macro and financial uncertainty but contemporaneously uncorrelated with real activity, and a $Z_{2t}$ that is correlated with financial uncertainty but contemporaneously uncorrelated with both real activity and macro uncertainty. While such ideal instruments have no empirical counterparts, we propose an iterative projection IV (IPIV) approach to construct $Z_{1t}$ and $Z_{2t}$ with the desired properties from observables. The approach takes a variable $S_t$ that is not in the VAR system and uses projections to decompose it into two components, one that is correlated with a subset of the endogenous variables of interest, and one that is orthogonal to it. The orthogonal component is then used as an external instrumental variable (IV) for the remaining endogenous variables.

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1. See Bachmann, Elstner, and Sims (2013), Bloom (2009), Bloom (2014), Bekaert, Hoerova, and Duca (2013), Gilchrist, Sim, and Zakrajsek (2010), and JLN.
In the present context, the key is to find observables that are external to our SVAR, and are driven by a multitude of innovations including the uncertainty shocks that we are interested in. We argue that both theory and evidence suggest that aggregate stock market returns are such variables. Our IPIV approach therefore generates an instrument $Z_{1t}$ by purging the effects of real activity shocks from stock returns, and another instrument $Z_{2t}$ by further purges the effects of macro uncertainty shocks. Iteration ensures that the shocks used to generate the instruments are consistent with those identified by our SVAR. With this procedure, instrument exogeneity holds by construction and instrument relevance can be verified using the sample covariances and the estimated parameters. Details are given below.

The empirical exercise also requires that appropriate measures of macro and financial uncertainty are available. To this end, we exploit a data rich environment, working with 134 macro monthly time series and 147 financial variables. The construction of macro uncertainty follows JLN. The same approach is used to construct a broad-based measure of financial uncertainty that has never been used in the literature. Macro uncertainty is itself an aggregate of uncertainties in variables from three categories: real activity, price, and financial. To better understand the contributions of each of these categories, we also replace $U_{Mt}$ in the VAR with an uncertainty measure based on the sub-components, one at a time. Uncertainty about real activity is of special interest because classic uncertainty theories postulate that uncertainty shocks have their origins in economic fundamentals and hence should show up as uncertainty about real economic activity. We compare “short-run” uncertainty about outcomes over the next one month, with “longer horizon” uncertainty about outcomes a year hence.

Before summarizing our main results, it should be made clear that the structural shocks we identify do not in general correspond to primitive shocks in specific economic models. Real activity is endogenous and may respond to any number of primitive shocks (technology, monetary policy, preferences, wage or price markups, government expenditures, etc.). If a SVAR representation exists, our identified real activity shock would then be a composite of these primitive shocks, with the restriction that this composite be orthogonal to the other shocks in our system. The same could be said for either type of uncertainty, to the extent that these variables are endogenous. Our objective is not, therefore, to identify primitive shocks in specific models. Indeed, we argue that the questions raised above are ultimately empirical ones that call out for a model-free approach. (See the literature review below for further discussion.) What our approach offers, therefore, is something different: if there exists an SVAR in the system of interest, then under the assumptions stipulated below, IPIV can provide a less restrictive means of identifying dynamic causal effects when commonly used ordering or timing assumptions are difficult to defend.

Our main results can be stated as follows. First, positive shocks to financial uncertainty are found to cause a sharp decline in real activity that persists for many months, lending support
to the hypothesis that heightened uncertainty is an exogenous impulse that causes recessions. These effects are especially large for some measures of real activity, notably employment and orders. The finding that heightened uncertainty has negative consequences for real activity is qualitatively similar to that of preexisting empirical work that uses recursive identification schemes (e.g., Bloom (2009), JLN), but differs in that we trace the source of this result specifically to broad-based financial market uncertainty rather than to various uncertainty proxies or broad-based macro uncertainty. We also show that the converse is not supported by our evidence: exogenous shocks to real activity have little affect on financial uncertainty.

Second, the identification scheme used here reveals something new that is not possible to uncover under recursive schemes: macro and financial uncertainty have a very different dynamic relationship with real activity. Specifically, unlike financial uncertainty, sharply higher macro and real activity uncertainty in recessions is fully an endogenous response to business cycle fluctuations. That is, negative economic activity shocks are found to cause increases in both macro and real activity uncertainty, but there is no evidence that independent shocks to macro or real uncertainty cause lower economic activity. Indeed the opposite is true: exogenous shocks to both macro and real uncertainty are found to increase real activity, consistent with “growth options” theories discussed below.

Third, we investigate the timing of large adverse shocks in the SVAR systems. No matter which system we investigate, the Great Recession is a prominent example that is characterized by large negative real activity shocks and a large positive financial uncertainty shock but no corresponding large shock to real economic uncertainty, even though real economic uncertainty itself rose to unusual heights in this episode. This finding underscores the extent to which heightened uncertainty about real activity in recessions is more often an endogenous response to other shocks, rather than an exogenous impulse driving business cycles.

Our results are distinct from those obtained using recursive identification. Under any recursive ordering of the variables in our VAR, exogenous shocks that increase macro or real uncertainty appear to reduce real activity, in a manner that is qualitatively similar to financial uncertainty shocks. This result does not hold in the less restrictive SVAR studied here and appears to be an artifact of invalid timing assumptions under recursive identification. Further investigation reveals that the SVAR we study reflects a non-zero contemporaneous correlation between $U_{Mt}$ and $Y_t$, as well as between $U_{Mt}$ and $Y_t$, which is inconsistent with any recursive ordering. Tests of the validity of a recursive structure are easily rejected by the data.

The rest of this paper is organized as follows. Section 2 reviews related literature. Section 3 details the econometric framework and identification employed in our study, describes how our instruments are constructed, and discusses the data and empirical implementation. Section 4 presents empirical results using broad-based macro uncertainty $U_{Mt}$, while Section 5 reports results for systems that isolate sub-components of $U_{Mt}$ corresponding to real activity and price.
variables. Section 6 reports results pertaining to robustness and additional cases including
different subsamples, different uncertainty horizons, tests of recursive restrictions, different
external variables, and overidentifying restrictions. Section 7 summarizes and concludes. A
large number of additional results on the IPIV methodology are presented in (Ludvigson, Ma,
and Ng (2016)).

2 Related Literature

A large literature addresses the question of uncertainty and its relation to economic activ-
ity.\(^2\) Besides the evidence cited above for the U.S., Nakamura, Sergeyev, and Steinsson (2012)
estimate growth rate and volatility shocks for 16 developed countries and find that they are sub-
stantially negatively correlated. Theories for which uncertainty plays a key role differ widely
on the question of whether uncertainty is primarily a cause or a consequence of declines in
economic activity. In most cases, it is modeled either as a cause or an consequence, but not
both.

The first strand of the literature proposes uncertainty as a cause of lower economic growth.
This includes models of the real options effects of uncertainty (Bernanke (1983), McDonald
and Siegel (1986)), models in which uncertainty influences financing constraints (Gilchrist,
Sim, and Zakrajsek (2010), Arellano, Bai, and Kehoe (2011)), or precautionary saving (Basu
and Bundick (2012), Leduc and Liu (2012), Fernández-Villaverde, Pablo Guerrón-Quintana,
and Uribe (2011)). These theories almost always presume that uncertainty is an exogenous shock to
some economic fundamental. Some theories presume that higher uncertainty originates directly
in the process governing technological innovation, which subsequently causes a decline in real
activity (e.g., Bloom (2009), Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2012)).

A second strand of the literature postulates that higher uncertainty arises solely as a re-
sponse to lower economic growth, emphasizing a variety of mechanisms. Some of these theories
suggest that bad times incentivize risky behavior (Bachmann and Moscarini (2011), Fostel and
Geanakoplos (2012)), or reduce information and with it the forecastability of future outcomes
(Van Nieuwerburgh and Veldkamp (2006) Fajgelbaum, Schaal, and Taschereau-Dumouchel
(2014)), or provoke new and unfamiliar economic policies whose effects are highly uncertain
(Pástor and Veronesi (2013)), or create a greater misallocation of capital across sectors (Ai, Li,
and Yang (2015)), or generate endogenous countercyclical uncertainty in consumption growth
because investment is costly to reverse (Gomes and Schmid (2016)).

And yet a third literature has raised the possibility that some forms of uncertainty can
actually \textit{increase} economic activity. “Growth options” theories of uncertainty postulate that
a mean-preserving spread in risk generated from an unbounded upside coupled with a limited

\(^2\)This literature has become voluminous. See Bloom (2014) for a recent review of the literature.
downside can cause firms to invest and hire, since the increase in mean-preserving risk increases expected profits. Such theories were often used to explain the dot-com boom. Examples include Bar-Ilan and Strange (1996), Pastor and Veronesi (2006), Kraft, Schwartz, and Weiss (2013), Segal, Shaliastovich, and Yaron (2015).

This brief review reveals a rich literature with a wide range of predictions about the relationship between uncertainty and real economic activity. Yet the absence of a theoretical consensus on this matter, along with the sheer number of theories and limited body of evidence on the structural elements of specific models, underscores the extent to which the question of cause and effect is fundamentally an empirical matter that must be settled in an econometric framework with as little specific theoretical structure as possible, so that the various theoretical possibilities can be nested in empirical tests. Commonly used recursive identification schemes cannot achieve this objective, since by construction they rule out the possibility that uncertainty and real activity could influence one another within the period. Our econometric model nests any recursive identification scheme, so we can test whether such timing assumptions are plausible. We find they are rejected by the data.

Our construction of instruments for uncertainty builds on work in asset pricing emphasizing the idea that stock market variation is the result of several distinct (and orthogonal) sources of stochastic variation, some of which are likely to be uniquely suited as instruments for our uncertainty measures. For example, one quantitatively important component is attributable to acyclical risk premia variation, and more generally appears to be uncorrelated with most measures of real activity. This component is valuable for our objective because it is exogenous to real activity, but may still be relevant for both macro and financial uncertainty, as in our $Z_{1t}$. Yet another component could be attributable to fluctuations in factors like corporate leverage, or in the risk aversion or “sentiment” of market participants that may be correlated with the volatility of the stock market. In equilibrium asset pricing models, if leverage increases, volatility of the corporate sector’s equity return increases. Thus changes in factors like leverage (and possibly changes in risk aversion or sentiment) should be correlated with financial uncertainty, but have little to do with real economic uncertainty. This component is valuable for our objective because it is plausibly uncorrelated with both real activity and uncertainty about economic fundamentals, but may still be relevant for financial market uncertainty, as in our $Z_{2t}$. Consistent with the existence of this type of component, JLN document that there are many spikes in stock market uncertainty that do not coincide with an important movement in either real activity or macro uncertainty. These findings motivate our maintained hypothesis that measures of equity market activity are promising non-uncertainty variable comprised of

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several distinct sources of stochastic variation, two of which have the statistical characteristics of $Z_{1t}$ and $Z_{2t}$.

Our IPIV approach is related to a recent line of econometric research in SVARs that uses information contained in external instruments to identify structural dynamic causal effects. Of these, Stock and Watson (2012) study uncertainty shocks, using a measure of stock market volatility and/or a news media measure of policy uncertainty from Baker, Bloom, and Davis (2013), as separate external instruments for identifying the effects of uncertainty shocks in a SVAR. Our study differs in some fundamental ways. First, Stock and Watson (2012) focus exclusively on identifying the effects of uncertainty shocks and do not attempt to simultaneously identify the converse, namely the effects of real activity shocks on uncertainty. Second, the identification strategy in Stock and Watson (2012) for uncertainty shocks presumes that the series themselves (i.e., stock market volatility, policy uncertainty) are valid instruments, correlated with the uncertainty shock of interest but not with the other shocks. By contrast, our approach explicitly views both the stock market and our uncertainty measures as partly endogenous, forcing us to confront the identification quandary. Our identification assumption is instead that the aggregate stock market return contains components that satisfy population exogeneity restrictions, even while some of its variation is endogenous.

Berger, Dew-Becker, and Giglio (2016) take a different approach. Using options data they find that bad times are associated with higher realized volatility but not higher expected volatility, a result that they interpret as consistent with the hypothesis that higher uncertainty is a consequence of negative economic shocks rather than a cause.

The study arguably closest in spirit to our identification approach is Baker and Bloom (2013), who use disaster-like events as instruments for stock market volatility with the aim of isolating exogenous variation in uncertainty. This has some similarities with our approach, in that it implicitly assumes that certain components of stock market fluctuations (those associated with “disasters”) are exogenous. In contrast to our approach, exogenous events are chosen subjectively rather than constructed econometrically to satisfy specific orthogonality restrictions. It is of interest that we arrive at complementary conclusions, despite the differing methodologies for identifying exogenous variation.

3 Econometric Framework

This section outlines our econometric approach. Subsection 1 explains the identification strategy. Subsections 2 and 3 explain the construction of external instruments in the IPIV procedure and the uncertainty measures. This is followed by a discussion of the estimation procedure.

4 See for example Hamilton (2003), Kilian (2008), Mertens and Ravn (2013); Stock and Watson (2008), Stock and Watson (2012), and Olea, Stock, and Watson (2015).
Details on the IPIV methodology are provided in Ludvigson, Ma, and Ng (2016).

3.1 The SVAR and Identification

Our analysis is based on a structural vector autoregressive model (SVAR). Let $X_t$ denote a $K \times 1$ time series. We suppose that the structural model has a $p$-th order vector autoregressive representation

$$X_t = k + A_1 X_{t-1} + A_2 X_{t-2} + \cdots + A_p X_{t-p} + H \Sigma e_t.$$  \hfill (1)

$$e_t \sim (0, I_K), \quad \Sigma = \begin{pmatrix} \sigma_{11} & 0 & 0 \\ 0 & \sigma_{22} & 0 \\ 0 & 0 & \sigma_{KK} \end{pmatrix}.$$  

The structural shocks $e_t$ are mean zero with unit variance, and are serially and mutually uncorrelated. The corresponding structural $MA(\infty)$ representation of $X_t$ is

$$X_t = \mu + \Psi(L) H \Sigma e_t,$$  

where $\Psi(L) = \Psi_0 + \Psi_1 L + \Psi_2 L^2 + \cdots$ with $\Psi_0 = I$ is a polynomial in the lag operator $L$ of infinite order, $\Psi_s$ is the $(n \times n)$ matrix of coefficients for the $s$th lag of $\Psi(L)$. Note that $\Psi(L) = A_s(L)^{-1}$, where $A_s(L) = I - A_1 L - \cdots - A_p L^p$.

The reduced form representation of $X_t$ is a $p$-th order vector-autoregression (VAR) with corresponding reduced-form $MA(\infty)$ representation

$$X_t = \mu + \Psi(L) \eta_t \quad (2)$$

$$\eta_t \sim (0, \Omega), \quad \Omega = E(\eta_t, \eta_t').$$

The structural shocks $e_t$ are presumed to be related to the reduced form innovations by an invertible $K \times K$ matrix $H$:

$$\eta_t = H \Sigma e_t \equiv B e_t,$$  

where $B \equiv H \Sigma$. We say that an SVAR for $X_t$ exists if a rotation $H^{-1}$ of the reduced form shocks $\eta_t$ can be found such that its elements are serially and mutually uncorrelated.

A normalization is required to pin down the sign and scale of the shocks. We adopt the unit effect normalization

$$\text{diag}(H) = 1.$$  \hfill (3)

Throughout, we restrict the admissible parameter space such that

$$\sigma_{jj} \geq 0.$$  \hfill (4)

for all $j$.  

The objective of the exercise is to study the dynamic effects and the relative importance of the structural shocks. More precisely, the dynamic response to shock \( j \) is summarized by the impulse response function (IRF):

\[
\frac{\partial X_{t+s}}{\partial e_{jt}} = \Psi_s b^j,
\]

where \( b^j \) is the \( j \)th column of \( B \). The structural IRF \( \Psi_s b^j \) gives the dynamic response of \( X_{t+s} \) to a one standard deviation shock. The quantitative importance of each shock is given by the fraction of \( S \)-step ahead forecast error variance of \( X_t \) that is attributable to each structural shock. The coefficient matrices of \( (\Lambda) \) are identified from the projection of \( X_t \) onto its lags in the reduced form VAR (2). The SVAR identification problem therefore amounts to identifying the elements of \( H \) and \( \Sigma \), from which the structural IRFs are computed.

Let \( Y_t \) denote a measure of real activity. Our objective is to study the impulse and propagating mechanism of uncertainty shocks, as well as how uncertainty reacts to shocks to \( Y_t \), while explicitly distinguishing between macro and financial market uncertainty. Let \( K = 3 \). Hence our baseline SVAR is based on \( X_t = (U_{Mt}, Y_t, U_{Ft})' \), where \( U_{Mt} \) denotes macro uncertainty, \( U_{Ft} \) denotes financial uncertainty. The reduced form shocks \( \eta_t = (\eta_{Mt}, \eta_{Yt}, \eta_{Ft})' \) are linear combinations of the three structural form shocks \( e_t = (e_{Mt}, e_{Yt}, e_{Ft})' \) to macro uncertainty, real activity, and financial uncertainty, respectively.

\[
\eta_{Mt} = B_{MM} e_{Mt} + B_{MY} e_{Yt} + B_{MF} e_{Ft} \\
\eta_{Yt} = B_{YM} e_{Mt} + B_{YY} e_{Yt} + B_{YF} e_{Ft} \\
\eta_{Ft} = B_{FM} e_{Mt} + B_{FY} e_{Yt} + B_{FF} e_{Ft},
\]

where \( B_{ij} \) is the element of \( B \) that gives the contemporaneous effect of the \( j \)th structural shock on the \( i \)th variable. The covariance structure of \( \eta_t \) provides \( K(K+1)/2 = 6 \) equations in \( B \):

\[
\text{vech}(\Omega) = \text{vech}(BB')
\]

where \( \text{vech}(\Omega) \) stacks the unique elements of the symmetric matrix \( \Omega \). Since there are nine unknown elements in \( B \), we need three more conditions for identification.

To identify these elements, we use two external instruments, denoted \( Z_t = (Z_{1t}, Z_{2t})' \). For now, suppose that we have measures of \( Y_t, U_{Mt}, U_{Ft} \), and two generic instruments, \( Z_{1t} \) and \( Z_{2t} \).

**Assumption A:** For \( K = 3 \), let \( Z_{1t} \) and \( Z_{2t} \) be two instrumental variables such that with \( \phi_{1M} \neq 0, \phi_{1F} \neq 0, \phi_{2F} \neq 0 \),

\[
(A.i) \quad E[Z_{1t} e_{Mt}] = \phi_{1M}, \quad E[Z_{1t} e_{Yt}] = 0, \quad E[Z_{1t} e_{Ft}] = \phi_{1F} \\
(A.ii) \quad E[Z_{2t} e_{Mt}] = 0, \quad E[Z_{2t} e_{Yt}] = 0, \quad E[Z_{2t} e_{Ft}] = \phi_{2F}.
\]

Assumption A are conditions for instrument exogeneity and relevance. \( Z_{1t} \) is an instrument that is correlated with both macro and financial uncertainty, but contemporaneously uncorrelated
with real activity. By contrast, $Z_{2t}$ is an instrument that is correlated with financial uncertainty, but contemporaneously uncorrelated with macro uncertainty and real activity.

Let $m_{it} = \text{vech}(\eta_{i}^{\prime} \eta_{i}), \text{vec}(Z_{i} \otimes \eta_{i})^{\prime}$ and $\beta_{1} = \text{vec}(B)$. At the true value of $\beta_{1}$, denoted $\beta_{1}^{0}$, the model satisfies

$$0 = \mathbb{E}[g_{i}(m_{i}; \beta_{1}^{0})],$$

written out in full as follows:

\[
\begin{align*}
0 &= \var(\eta_{M}) - B_{MM}^{2} + B_{MY}^{2} + B_{MF}^{2} \\
0 &= \var(\eta_{Y}) - B_{YM}^{2} + B_{YY}^{2} + B_{YF}^{2} \\
0 &= \var(\eta_{F}) - B_{FM}^{2} + B_{FY}^{2} + B_{FF}^{2} \\
0 &= \text{cov}(\eta_{M}, \eta_{Y}) - B_{MM}B_{YM} + B_{MY}B_{YY} + B_{MF}B_{YF} \\
0 &= \text{cov}(\eta_{Y}, \eta_{F}) - B_{YM}B_{FM} + B_{FY}B_{YF} + B_{FF}B_{YF} \\
0 &= \text{cov}(\eta_{M}, \eta_{F}) - B_{MM}B_{FM} + B_{MY}B_{FY} + B_{MF}B_{FF} \\
0 &= B_{MF}\mathbb{E}[Z_{2t}\eta_{Y}] - B_{YF}\mathbb{E}[Z_{2t}\eta_{M}] \\
0 &= B_{FF}\mathbb{E}[Z_{2t}\eta_{Y}] - B_{YF}\mathbb{E}[Z_{2t}\eta_{F}] \\
0 &= (B_{MM}B_{FF} - B_{MF}B_{FM})\mathbb{E}[Z_{1t}\eta_{Y}] - (B_{YF}B_{FM} - B_{YM}B_{FF})\mathbb{E}[Z_{1t}\eta_{M}] \\
&\quad - (B_{MM}B_{YF} - B_{MF}B_{YM})\mathbb{E}[Z_{1t}\eta_{F}].
\end{align*}
\]

The model has nine equations in nine unknowns. The first six are from the covariance structure. The next two equations are due to the three moments implied by Assumption (A.ii). The final equation is due to the three moments implied by Assumption (A.i).

**Proposition 1** Under Assumption A with det($B$) > 0, the normalization (3), and the restriction (4), $\beta_{1}$ is identified.

The Appendix gives a proof of identification using a closed-form solution for $B$, and we show that the covariance between the instruments and the structural shocks can be expressed as

\[
\begin{align*}
\mathbb{E}[Z_{2t}e_{Fj}]^{2} &= \mathbb{E}[\eta_{i}Z_{2t}]^{\prime}\Omega^{-1}\mathbb{E}[\eta_{i}Z_{2t}] \\
\mathbb{E}[Z_{1t}e_{Mj}]^{2} &= \left(\mathbb{E}[\eta_{i}Z_{1t}] - \frac{\mathbb{E}[\eta_{i}Z_{2t}]}{\mathbb{E}[Z_{2t}e_{Fj}]}\mathbb{E}[Z_{2t}e_{Fj}]\right)^{\prime}\Omega^{-1}\left(\mathbb{E}[\eta_{i}Z_{1t}] - \frac{\mathbb{E}[\eta_{i}Z_{2t}]}{\mathbb{E}[Z_{2t}e_{Fj}]\mathbb{E}[Z_{2t}e_{Fj}]}\right) \\
\mathbb{E}[Z_{2t}e_{Fj}]\mathbb{E}[Z_{1t}e_{Fj}] &= \mathbb{E}[\eta_{i}Z_{2t}]^{\prime}\Omega^{-1}\mathbb{E}[\eta_{i}Z_{1t}].
\end{align*}
\]

We verify that the closed-form solution is the same as the unique numerical solution obtained with (3) and (4) imposed. Although these conditions exactly identify the $B$ and shocks $e_{i}$, the complete econometric model imposes overidentifying restrictions, as we discuss below.
In essence, identification in our analysis is achieved by (i) using movements in $U_{Mt}$ and $U_{Ft}$ that are correlated with $Z_{1t}$ to identify the effects of uncertainty shocks and disentangle them from shocks to real activity, (ii) using movements in $U_{Ft}$ that are correlated with $Z_{2t}$ to identify the effects of $U_{Ft}$ shocks and disentangle them from macro uncertainty shocks, and (iii) using movements in $Y_t$ that are uncorrelated with both $Z_{1t}$ and $Z_{2t}$ to identify the effects of real activity shocks and disentangle them from uncertainty shocks.

We take the stand in this application that our uncertainty measures are potentially endogenous. It is then natural to ask why we do not simply find observable instruments. We avoid instrumenting one measure uncertainty with an uncertainty proxy (e.g., stock market volatility). JLN find that such measures, including options-based volatility indexes such as VIX or VXO are less defensible measures of uncertainty than those employed here, so it makes little sense to instrument for the latter with the former. Options-based volatility indexes are doubly problematic for our purpose because they are known to contain a large component attributable to changes in the variance risk premium that are unrelated to common notions of uncertainty (e.g., Bollerslev, Tauchen, and Zhou (2009); Carr and Wu (2009)). On the other hand, options based indexes may be valuable in empirical contexts different from ours, such as those that seek to distinguish expected stock market volatility from realized stock market volatility (Berger, Dew-Becker, and Giglio (2016)). With these considerations in mind, the next subsection proposes a methodology for constructing the desired instruments.

### 3.2 Construction of Instruments

The external instruments $Z_{1t}$ and $Z_{2t}$ play an important role in our analysis but they have no observable counterpart. The next step is to develop a methodology to construct these variables. To motivate our method of IPIV, recall that two stage least squares uses projections to purge the endogenous variations from a relevant regressor. Our IPIV approach is similar in spirit except that we purge the endogenous variations from an observed variable that is not of first order relevance to our VAR system. The output of such a projection is a generated external instrument.

In the present context, we make use of observables $S_t$ that are driven not only by our structural shocks $e_t = (e_{Yt}, e_{Mt}, e_{Ft})'$, but also by other shocks collected into an $e_{St}$ that are uncorrelated with $e_t$. A theoretical premise of the paper is that uncertainty shocks should be reflected in aggregate equity returns. Thus our choice of $S_t$ is a measure of stock market returns. Under these assumptions, we may represent $S_t$ as

$$
S_t = d_0 + d_Y Y_t + d_M U_{M_t} + d_F U_{F_t} + d_S(L)S_{t-1} + d_X(L)'X_{t-1} + e_{St} \tag{8}
$$

where $X_t = (Y_t, U_{Mt}, U_{Ft})'$. The residual $e_{St}$ could be driven by any number of shocks orthogonal to $e_t$. One interpretation is risk premium shocks driven by factors orthogonal to uncertainty such
as a pure sentiment shock (one not correlated with uncertainty), but the precise interpretation is not important to what follows. Obviously, $S_t$ is an endogenous variable but it is external to the variable $X_t$ system by assumption. Omitting any component of $X_t$ as an explanatory variable will yield inconsistent estimates of the parameters in (8). However, we are not interested in these parameters. Our objective in considering stock-market returns is solely to remove from it those variations due to $e_{Mt}$ and/or $e_{Yt}$. More precisely, (8) motivates two (non-structural) representations of $S_t$ (not necessarily the same variable):

\begin{align}
S_{1t} &= d_{10} + d_{12} e_{Yt} + d_{14} (L) S_{1t-1} + Z_{1t} \\ S_{2t} &= d_{20} + d_{21} e_{Mt} + d_{22} e_{Yt} + d_{24} (L) S_{2t-1} + Z_{2t},
\end{align}

Equation (9a) forms an orthogonal decomposition of $S_{1t}$ into a component that is spanned by $e_{Yt}$ and a component $Z_{1t}$ that is orthogonal to $e_{Yt}$. Similarly, equation (9b) purges the effect of $e_{Yt}$ and $e_{Mt}$ from $S_{2t}$ to arrive at $Z_{2t}$. These two $Z$ variables are our desired instruments because they satisfy Assumption A by construction. Note, however, that $Z_{1t}$ and $Z_{2t}$ include the effects of $X_{t-1}$. Moreover, in this application they are forecastable since both $U_{Mt}$ and $U_{Ft}$ can be serially correlated and their lagged values predict future excess stock market returns.

Given the theory and evidence discussed above, our maintained hypothesis is that the stock market contains a component that is exogenous to real activity, but correlated with both uncertainty shocks, and another component that is exogenous to both real activity and macro uncertainty, but correlated with financial uncertainty. In our application we will use two different measures of stock market returns. We further discuss the choice of the regressands in (9a) and (9b) below.

Let $m_{2t} = (1, S_t, S_{t-1}, e_{Yt}, e_{Mt})'$ and collect the projection coefficients in (9a) and (9b) into $\beta_2$ whose population value is $\beta_2^0$. The orthogonality conditions of the two projections can be compactly summarized by

$$0 = \mathbb{E}[g_2(m_{2t}; \beta_2^0)].$$

If $e_Y$ and $e_M$ were observed, then solving for the sample analog of (10) would produce estimates of $Z_1$ and $Z_2$ that satisfy Assumption A. Our proposed approach is to jointly solve for shocks and instruments from the eleven conditions: the nine equations described in (7) when $Z_t$ were observed, along with the two equations described in (10) that are needed to construct $Z_t$ when they are not observed.

But whereas the moment matrices $Z'\eta$ are fixed given $\eta$ and data for $Z$, now $Z(\beta)'\eta$ depends on $\beta_1$ which needs to be estimated. Many such $Z$s can in principle be constructed from different values of $\beta_1$. Furthermore, this component can be ill-behaved during iterations especially when starting values are poor. Both problems can result in a multiplicity of solutions which we indeed encountered in unconstrained estimation. To deal with these problems, we make use of
the fact that instrument relevance is needed for identification even when \( Z \) is observed. Hence we explicitly impose Assumption A in estimation. Precisely, we remove values of \( \beta_1 \) from the parameter space if they imply instruments that are individually or collectively too weak to be consistent with Assumption A. While there may be a set of \( \beta_1 \) consistent with Assumption A, this set can be narrowed by successively raising the bar for instrument strength. This leads to the following algorithm.

**Algorithm IPIV** Let the \( T \times 1 \) vectors \( e_M^{(0)k}, e_Y^{(0)k} \) be the \( k^{th} \) initial guess in a compact set \( \mathcal{K} \). Initialize \( j = 0 \). The following steps are repeated until convergence:

i Replace \((e_M, e_Y)\) in (9a) and (9b) by \((e_M^{(j)k}, e_Y^{(j)k})\). The projections give \( Z_1^{(j)k} \) and \( Z_2^{(j)k} \).

ii Use \( Z_1^{(j)k} \) and \( Z_2^{(j)k} \) to solve \( 0 = E[g_1(m_1t; \beta_1^{(0)})] \) for \( \beta_1^{(j)k} = \text{vec}(B^{(j)k}) \). Form \( B^{(j)k} \) from \( \beta_1^{(j)k} \).

iii Update the shocks to \( e^{(j+1)k} = (e_M^{(j+1)k}, e_Y^{(j+1)k}, e_F^{(j+1)k}) = (B^{(j)k})^{-1} \hat{\eta} \).

iv If \( \|e_M^{(j+1)k} - e_M^{(j)k}\| \leq \text{tol} \) and \( \|e_Y^{(j+1)k} - e_Y^{(j)k}\| < \text{tol} \), let \( e^K = e^{(j)}, \beta_1^K = \beta_1^{(j)} \). Else, set \( j = j + 1 \) and return to (i).

v Store \( \hat{c}_1 = \left| \text{corr}(Z_{1t}(\beta_1^K), e_{Mt}^K) \right|, \hat{c}_2 = \left| \text{corr}(Z_{1t}(\beta_1^K), e_{Ft}^K) \right|, \hat{c}_3 = \left| \text{corr}(Z_{2t}(\beta_1^K), e_{Ft}^K) \right|, C(\beta_1^K) = \frac{1}{3}(\left| c_1 \right| + \left| c_2 \right| + \left| c_3 \right|) \). Keep solutions \( \beta_1^K \) from the set generated by different starting values in \( \mathcal{K} \) that satisfy (a) \( C(\beta_1^K) \geq \bar{C} \), (b), each \( \hat{c}_i \geq \bar{c} \), and (c) \( \det(B^{(j)}) \geq \bar{b} \).

Several points about the implementation of this approach bear discussion.

First, note that the shocks are eventually identified by estimates of \( B \) (since \( e = B^{-1} \eta \) by definition). Thus the procedure requires \( B^{-1} \) to exist. For this reason we keep only solutions that satisfy a minimum threshold for \( \det(B^{(j)}) \geq \bar{b} \). Second, the IPIV estimation collapses to a one-step constrained nonlinear GMM problem subject to the three constraints defined by step (v) above. IPIV is typically implemented with greater speed by undertaking the iterative procedure described above. A unique feature of IPIV is that both the instruments and the structural parameters are taken as objects of the estimation. We also estimate the model by GMM to verify that the solution agrees with the one obtained by IPIV estimation.

Third, we keep only the converged solutions obtained from different starting values that satisfy the three conditions in step (v). Thus the instrument relevance conditions of Assumption A are used to pick the best set of solutions among those that emerge from the iterative algorithm using different starting values. As a practical matter, our base case estimates are obtained with the initial guesses \( e_{Yt}^{(0)} = q_{1t} \) for all \( Y_t \) considered, and \( e_{Mt}^{(0)} = U_{Mt} \). These are equivalently initial
guesses on instruments $Z_{1t}^{(0)}$ and $Z_{2t}^{(0)}$ (see 9a and 9b).\textsuperscript{5} To further check the sensitivity of our results, we consider additional random starting values in a bounded space around a starting point based on observable data. Specifically, we implemented the algorithm above over a grid of 500 additional randomly chosen starting values \{\(Z_{1t}^{(0)k}, Z_{2t}^{(0)k}\)\}_{k=1}^{500} given by $Z_{1t}^{(0)k} = Z_{1t}^{(0)*} + \sigma_1 \varepsilon_t$ and $Z_{2t}^{(0)k} = Z_{2t}^{(0)*} + \sigma_1 \varepsilon_t$, where $Z_{1t}^{(0)*}$ and $Z_{2t}^{(0)*}$ are the instruments obtained using our base case initial guesses and where \(\varepsilon \sim N(0, I)\) are 500 draws for the $T \times 1$ shock sequence $\varepsilon$ from a normal i.i.d. distribution with size $\sigma_1$ scaled to match the standard deviation of $Z_{1t}^{(0)*}$ and $Z_{2t}^{(0)*}$, which are both close to 0.04.

Fourth, while these restrictions on instrument strength significantly narrow the set of solutions, a random search over starting values will always deliver some estimated instruments and shocks that would be very unlikely to occur in our sample. Thus we further winnow the set of solutions with prior economic reasoning. Specifically, we study the estimated shocks in detail and check that the magnitudes and signs are sensible. For this application, we argue that the 1987 stock market crash and the 2007-09 financial crisis should be identified as big positive financial uncertainty shocks, while the Great Recession should not be identified with a big positive real activity shock. Solutions that deliver such shock series are discarded. This further winnowing narrows the outcomes to a handful of credible solutions that include our base case. There are around six such solutions for most of the estimations and therefore six sets of instruments. This is analogous to the observed instrument case, in that there is almost always more than one valid instrument that can be used. The shocks implied by each of these solutions have correlations well above 98%. And since all solutions in this set tell the same economic story, we report results only for the base case. In summary, the qualitative nature of the solution to the IPIV and GMM estimation problem is not found to be sensitive to starting values once the above restrictions and economic considerations are imposed, even though a formal proof for global identification in the restricted parameter space is not available.

To have confidence in this implementation, Ludvigson, Ma, and Ng (2016) use Monte Carlo experiments to study the properties of the estimator. The simulations confirm the importance of Step (v) for the IPIV estimates, which is not surprising given that the condition is important even if $Z$ were observed. In general, the degree of instrument strength and relevance required for precise identification varies with the data generating process (DGP). But the results for a DGP calibrated to the empirical application here suggest that the procedure can quite accurately recover the true structural shocks and $B$ matrix when the procedure is initialized with the starting values employed in this application, when the estimated instruments have properties consistent with observed values of $c_1$, $c_2$ and $c_3$, and when finite samples are set to be within

\textsuperscript{5}These were obtained after an initial investigation analyzing the results over a grid of different observed starting values, such as those using AR(1) residuals for $Y_t$ and $U_{Mt}$, or $Z_{it}^{(0)} = S_{it}$, or those that remove current and lagged values of $Y_t$ and/or $U_{Mt}$ from the initial $Z_{it}^{(0)k}$.
range of the size used in this study.

3.3 Measuring Uncertainty and Stock Market Returns

In our estimation we work with several different aggregate measures of uncertainty, which are indexes constructed over individual uncertainties for a large number of observable time-series. A long-standing difficulty with empirical research on this topic has been the measurement of uncertainty. JLN find that common uncertainty proxies contain economically large components of their variability that do not appear to be generated by a movement in genuine uncertainty across the broader economy. This occurs both because these proxies over-weight certain series in the measurement of aggregate uncertainty, and because they erroneously attribute forecastable fluctuations to a movement in uncertainty. Equity market volatility, for example, contains a non-trivial component generated from forecastable variation in stock returns. The estimated macro uncertainty index constructed in JLN is designed to address these issues and improve the measurement of aggregate uncertainty. The methodology used here for constructing uncertainty indexes follows JLN and we refer the reader to that paper for details.

Let \( y_{jt}^C \in Y_t^C = (y_{1t}^C, \ldots, y_{N_t}^C) \)' be a variable in category \( C \). Its \( h \)-period ahead uncertainty, denoted by \( U_{jt}^C(h) \), is defined to be the volatility of the purely unforecastable component of the future value of the series, conditional on all information available. Specifically,

\[
U_{jt}^C(h) \equiv \sqrt{\mathbb{E} \left[ (y_{jt+h}^C - \mathbb{E}[y_{jt+h}^C | I_t])^2 | I_t \right]} \tag{11}
\]

where \( I_t \) is information available. If the expectation today of the squared error in forecasting \( y_{jt+h} \) rises, uncertainty in the variable increases. Uncertainty in category \( C \) is an aggregate of individual uncertainty series in the category:

\[
U_{Ct}(h) \equiv \text{plim}_{N_C \to \infty} \sum_{j=1}^{N_C} \frac{1}{N_C} U_{jt}^C(h) \equiv \mathbb{E}_C[U_{jt}^C(h)]. \tag{12}
\]

As in JLN, the conditional expectation of squared forecast errors in (11) is computed from a stochastic volatility model, while the conditional expectation \( \mathbb{E}[y_{jt+h}^C | I_t] \) is replaced by a diffusion index forecast, augmented to allow for nonlinearities. These are predictions of an autoregression augmented with a small number of common factors \( q_t = (q_{1t}, \ldots, q_{rt}) \)' estimated from a large number of economic time series \( x_{it} \) each with factor representation \( x_{it} = \Lambda_{it} q_t + e_{x,it} \).

The use of large datasets reduces the possibility of biases that arise when relevant predictive information is ignored. Let \( Y_t^C = (y_{1t}^C, \ldots, y_{N_t}^C) \)' generically denote the series that we wish to compute uncertainty in. In this paper, we consider four categories of uncertainty:
The uncertainty index $U_{Ct}$ for category $C$ is an equally-weighted average of the individual uncertainties in the category. We use two datasets covering the sample 1960:07-2015:04. The first is a monthly macro dataset, $X^M_t$, consisting of 134 mostly macroeconomic time series taken from McCracken and Ng (2016). The second is a financial dataset $X^F_t$ consisting of a 147 of monthly financial indicators, also used in Ludvigson and Ng (2007) and JLN, but updated to the longer sample. The real uncertainty index $U_{Rt}$ is an equally-weighted average of the individual uncertainties about 73 series in Groups 1 through 4 of $X^M_t$. These include output and income variables, labor market measures, housing market indicators, and orders and inventories. A second subindex is constructed using only measures of consumer and producer prices as well as oil prices, commodity prices and crude materials prices. We call this index price uncertainty, $U_{\pi t}$, which averages over the individual uncertainties of the 21 price series in Group 7 of $X^M_t$. Additional predictors for variables in $X^M_{it}$ include factors formed from $X^F_{it}$ and vice-versa, squares of the first factor of each, and factors in the squares of individual series, $(X^M_{it})^2$ and $(X^F_{it})^2$.

Our estimation considers different VARs with different $Y_t$. In principle, we could initialize our $e_{Yt}$ shock in the algorithm above with a different measure of $Y_t$, depending on the VAR system being estimated. For simplicity, we instead set $e_{Yt}^{(0)} = q_{it}$ for this purpose, where $q_{it}$ is the first common factor estimated from the macro dataset no matter what measure of $Y_t$ we use in the VAR. In fact, this turns out to often work better for identifying shocks and instruments that satisfy the instrument relevance conditions of Assumption A. This common factor has long been understood to be a “real activity factor” that loads heavily on measures of employment and production such as employees on nonfarm payrolls and manufacturing output, as well as measures of capacity utilization and new manufacturing orders in all vintages of $X^M_t$ used in this study, see McCracken and Ng (2016). It loads very little if at all on consumer and producer inflation measures, and financial market variables.

Our use of stock returns $S_t$ to generate instruments is grounded in the theoretical premise that both macro and financial uncertainty shocks should be reflected in stock market returns. There is no reason, however, that the regressands in (9a) and (9b) must be exactly the same measure of stock market activity. All measures of stock market activity are highly correlated because they contain a large common component (much of which is orthogonal to the rest of the economy). In order to introduce some additional independent variation in our two instruments,

<table>
<thead>
<tr>
<th>Category $(C)$</th>
<th>$Y^C_t$</th>
<th>$N_C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(M): Macro</td>
<td>all variables in $X^M_t$</td>
<td>134</td>
</tr>
<tr>
<td>(F): Financial</td>
<td>all variables in $X^F_t$</td>
<td>147</td>
</tr>
<tr>
<td>(R): Real activity</td>
<td>real activity variables in $X^M_t$</td>
<td>73</td>
</tr>
<tr>
<td>(π): Price</td>
<td>price variables in $X^M_t$</td>
<td>21</td>
</tr>
</tbody>
</table>

---

6A detailed description of the series is given in the Data Appendix of the online location where updated JLN uncertainty index data are posted: http://www.sydneyludvigson.com/s/jln_data_appendix_update.pdf
our base cases use different measures of aggregate stock market activity to generate \(Z_{1t}\) and \(Z_{2t}\), although in practice we get very similar results if we use the same value-weighted stock market index return in (9a) and (9b). Specifically, we use the Standard and Poor 500 stock market index return, \(S_{Pt}\), as the regressand for (9b), and \(S_{at} = \alpha_p \text{crsp}_t + (1 - \alpha_p) \text{small}_t\), a portfolio weighted average of the return on the CRSP value-weighted stock index (in excess of the one-month Treasury bill rate) and the smallest decile stock market return in the NYSE as the regressand for (9a).\(^7\) We investigated a range of values for \(\alpha_p\). Our choice of portfolio weight \(\alpha_p\) is guided by empirical considerations. The small stock index is highly volatile, which generates noise in the estimated SVAR parameters and large error bands for the impulse response functions. For the base case results presented below we set \(\alpha_p = 0.94\) because it gives reasonably tight error bands. However, we also investigated a range of values for \(\alpha_p \in [0, 1]\) and found qualitatively similar results, including setting \(\alpha_p = 0\), which gives 100\% of the weight to the small stock index. With this value for \(\alpha_p\) however, the impulse response error bands are wider. In our experience, wide error bands indicate difficulty identifying some element of the \(B\) matrix. We discuss this further below.

It is reasonable to ask if variables other than stock market returns could serve as regressands in (9a) and (9b). Asset returns other than those for the stock market come to mind, such as those for corporate bonds. Since bonds must return a fixed stream of payments to claimholders (a legal requirement set in the bond covenant), bonds are like stocks without the dividend risk. Our prior is that high frequency macro and financial uncertainty shocks are likely to be more closely related to earnings and dividend payouts than default events, so they should be more relevant for stock returns than bond returns. But bonds that have some nontrivial probability of defaulting might also be affected by uncertainty, at least to some degree. We consider this possibility in the Robustness and Additional Cases section below, where we present results for one estimation in which we generate \(Z_{1t}\) from the return on a portfolio of Baa rated corporate bonds.

The parameters to be estimated include the reduced form VAR parameters in (2), from which we obtain \(\hat{\eta}_t\), the parameters in (10), from which we construct \(Z_{1t}\) and \(Z_{2t}\), and the structural parameters using results from the preceding two estimations. The sample moment conditions in the three-step estimation can be collected into \(\bar{g}(m_t; \beta)\) where \(\beta\) are parameters to be estimated. The Generalized Method of Moments (GMM, Hansen (1982)) estimator is \(\hat{\beta} = \arg \min_{\beta} \bar{g}(m_t; \beta)' \bar{g}(m_t; \beta)\). Under regularity conditions, the GMM estimator of Hansen (1982) is \(\sqrt{T}\) consistent for \(\beta^0\) and asymptotically normal with asymptotic variance \(\Sigma_{\beta}^2\). This variance matrix is block lower triangular as in Newey (1984) since estimation of \(\beta_2\) is not affected by estimation of \(\beta_1\) or of the VAR. Serial correlation and heteroskedasticity robust standard errors are constructed as in Newey and West (1987).

\(^7\)The CRSP index is a value-weighted return of all stocks in NYSE, AMEX, and NASDAQ.
The next section presents empirical results. We begin by studying systems with macro uncertainty. We then move on to consider sub-indexes of $U_{Mt}$, including real uncertainty formed only over real activity variables $U_{Rt}$ and price uncertainty $U_{\pi t}$. Our final set of results report several additional cases pertaining to different measures of real activity, different samples, different uncertainty horizons, and to using recursive identification schemes.

4 Results for $X_t = (U_{Mt}, Y_t, U_{Ft})'$

Our first VAR is defined by $X_t = (U_{Mt}(h), Y_t, U_{Ft}(h))'$. For the base case, we consider $h = 1$ (one-month uncertainty) and several measures of $Y_t$: the log of real industrial production, denoted $ip_t$, and the log of employment, denoted $emp_t$. While industrial production is a widely watched economic indicator of business cycles, it only captures goods-producing industries and has been a declining share of GDP. Employment only covers the labor market. Hence we also consider an additional measure of real activity: the cumulated sum of the first common factor estimated from the macro dataset $\chi^M$ (since the raw data used to form $q_{1t}$ are transformed to stationary), which we denote $Q_{1t}$. We linearly detrend each real activity series before estimation. Since our emphasis is on $h = 1$, we write $U_{Mt}$ instead of $U_{Mt}(1)$, and analogously for $U_{Ft}$, in order to simplify notation.

The top panel of Figure 1 plots the estimated macro uncertainty $U_{Mt}$ in standardized units along with the NBER recession dates. The horizontal bar corresponds to 1.65 standard deviation above unconditional mean of each series (which is standardized to zero). As is known from JLN, the macro uncertainty index is strongly countercyclical, and exhibits large spikes in the deepest recessions. The updated data $U_{Mt}$ series shows much the same. Though $U_{Mt}$ exceeds 1.65 standard deviations 48 times, they are clustered around the 1973-74 and 1981-82 recessions, as well as the Great Recession of 2007-09. Macroeconomic uncertainty is countercyclical and has a correlation of -0.65 with the 12-month moving-average of the growth in industrial production.

The bottom panel of Figure 1 plots the financial uncertainty series $U_{Ft}$ over time, which is new to this paper. $U_{Ft}$ is a broad-based measure of time varying financial uncertainty using data from the bond market, stock market portfolio returns, and commodity markets. Hence, it is smoother than proxies such as VIX or any particular bond index. As seen from Figure 1, $U_{Ft}$ is also countercyclical, though less so than $U_{Mt}$; the correlation with industrial production of -0.39. The series often exhibits spikes around the times when $U_{Mt}$ are high. However, $U_{Ft}$ is more volatile and spikes more frequently outside of recessions, the most notable being the 1987 stock market crash. Though observations on $U_{Ft}$ exceed the 1.65 standard deviation line 33 times, they are spread out in seven episodes, with the 2008 and 1997 episodes being the most pronounced.
As is clear from Figure 1, both indicators of macro and financial uncertainty are serially correlated and hence predictable. They have comovements but also have independent variations as the correlation between them is 0.58. However, this unconditional correlation cannot be given a structural interpretation. The heightened uncertainty measures can be endogenous responses to events that are expected to happen, but they can also be exogenous innovations. We use a VAR to capture the predictable variations, and then identify uncertainty shocks from the VAR residuals using the restrictions described in the previous section.

4.1 VAR Estimates and Uncertainty Shocks

Several features of the VAR estimates are qualitatively similar for all measures of $Y_t$. Table 1 highlights some of these results. As shown in panel A, the sample correlation coefficient between $Z_{1t}$ and $\hat{e}_{Mt}$ and $\hat{e}_{Ft}$, and between $Z_{2t}$ and $\hat{e}_{Ft}$ are statistically significant and negative in each case, indicating that uncertainty shocks of both types are correlated with these instruments, as required, and tend to be high when these components of stock market returns are low. Notice that the magnitude of the instrument relevance correlations is quite modest, especially for $\text{corr}(Z_{1t}, \hat{e}_{Mt})$, which is on the order of -0.07, suggesting that the converged instrument could be weak for macro uncertainty. However, simulations in Ludvigson, Ma, and Ng (2016) indicate that the degree of instrument relevance required for precise identification varies with the DGP, and results for a DGP calibrated to the empirical application here– including the size of the estimated instrument relevance correlations–show that the procedure can recover a close approximation of the true structural shocks and $B$ matrix even with the low correlations between $Z_{1t}$ and $\hat{e}_{Mt}$ found here. Panel A also shows that the correlation between $Z_{1t}$ and $\hat{e}_{Yt}$, and the correlation between $Z_{2t}$ and $\hat{e}_{Yt}$ and $\hat{e}_{Mt}$ are all zero as required, which is true by construction of the algorithm and solution for $B$. Panel B shows that $\sigma_{MM}$, $\sigma_{YY}$, and $\sigma_{FF}$ are all strongly statistically significantly different from zero. This in turn indicates the presence of both macro and financial uncertainty shocks in the SVAR. Since both $U_{Mt}$ and $U_{Ft}$ are serially correlated, we should therefore find that $Z_{1t}$ is correlated with lags of $U_{Mt}$ and $U_{Ft}$, while $Z_{2t}$ is correlated with lags of $U_{Ft}$. Results not reported confirm this is the case.

Figure 2 presents the time series of the standardized shocks ($e_M, e_{ip}, e_F$) identified from the system with $Y_t = ip_t$. All shocks display strong departures from normality with excess skewness and/or excess kurtosis. The largest of the $e_{ip}$ shocks is recorded in 2008:09, followed by 1974:11, and 1980:04. There also appears to be a moderation in the volatility of the $ip$ shocks in the post-1983 period. The largest macro uncertainty shock is in 1970:12, followed by the shock in 2008:10. The largest financial uncertainty shock is recorded in 1987:10, followed by the shock in 2008:09. For $e_F$, the 1987 stock market crash evidently dwarfs all other spikes. Because of the extreme but transitory nature of the crash, there is a very large spike downward in $e_F$ in
the month following the crash, as the market recovered strongly. While this episode magnifies
the spike in $e_F$ in 1987, it is largely orthogonal to real activity and macro uncertainty.

Observe that the large $ip$ shock in 2005:09 is not associated with a contemporaneous spike
in uncertainty, while there are several spikes in both types of uncertainty that do not coincide
with spikes in $e_{ip}$. The next subsection uses impulse response functions to better understand
the dynamic causal effects and propagating mechanisms of these shocks.

4.2 The Dynamic Effects of Uncertainty Shocks

Impulse response functions (IRFs) trace out the effects of counterfactual increases in the shocks.
The estimated IRFs are presented with 90% bootstrapped confidence bands as vertical bars.
All plots show responses to one standard deviation changes in $\epsilon_{jt}$ in the direction that leads to
an increase in its own variable $X_{jt}$.

Figure 3 shows the dynamic responses of each variable in the SVAR to each structural shock.
The figure displays the IRFs for systems with $Y_t = ip_t$, $emp_t$, and $Q_{1t}$, the real activity factor.
Considering first the dynamic responses of production, we see that positive shocks to financial
uncertainty $e_F$ lead to a sharp decline in real production that persists for many months (center
plot, bottom row). Positive perturbations to $e_{Ft}$ also cause $U_{Mt}$ to increase. However, there is
less evidence that shocks to macro uncertainty have effects on financial uncertainty: the impact
response of $U_{Ft}$ to an increase in $e_{Mt}$ is not statistically different from zero. These results lend
support to the hypothesis that heightened financial uncertainty is an exogenous impulse that
causes declines in real activity. Note that the converse relationship is not supported by our
evidence: exogenous (positive) shocks to $ip$ have statistically insignificant effects on financial
uncertainty. If anything, perturbations to $e_{ip}$ modestly increase financial uncertainty in the
long-run.

While we find no evidence that high financial uncertainty is a consequence of lower eco-
nomic activity, the results for macro uncertainty are quite different. Figure 3 (second row, first
column) shows that macro uncertainty falls sharply in response to positive shocks to industrial
production, $e_{ip}$. Alternatively stated, negative $ip$ shocks increase macro uncertainty sharply.
These effects persist for well over a year after the $ip$ shock. This result is strongly statistically
significant, suggesting that higher macro uncertainty in recessions is a direct endogenous re-
sponse to lower economic activity. However, there is no evidence that the negative correlation
between macro uncertainty and real activity is driven by causality running in the opposite di-
rection. Indeed, the top middle panel shows that exogenous increases in $e_{Mt}$ actually increase
real activity, consistent with growth options theories discussed above.

The standard error bands for this case with $Y_t = ip_t$ are wide, indicating considerable
sampling uncertainty as to the magnitude of these effects. However, the systems that use $Y_t =$
$i_p_t$ appear to be unusual in this respect. The impulse responses are more precisely estimated when we use alternative measures of real activity $Y_t$. Impulse responses using $Y_t = emp_t$ and $Y_t = Q_{1t}$ are displayed as separate lines in Figure 3. These systems tell the same story regarding the dynamic causal influences in the system, but the responses have tighter standard error bands. A positive shock to $emp_t$ or $Q_{1t}$ causes a sharp decline in macro uncertainty, whereas there is again no evidence that positive shocks to macro uncertainty cause declines either measure of real activity; indeed the opposite occurs. But positive shocks to financial uncertainty cause declines in both $emp_t$ or $Q_{1t}$. In contrast to the responses in systems using $i_p_t$, these effects are strongly statistically significant in the systems using $emp_t$ and $Q_{1t}$.

### 4.3 The Structural Shocks and Decomposition of Variance

In Figure 1 presented earlier, we find 1973-74, 1981-82, and 2007-2009 to be the three episodes of heightened macroeconomic uncertainty, defined as the periods when $U_{Mt}$ is 1.65 standard deviations above its unconditional mean. We now look for the “large adverse” shocks in the systems $(U_{Mt}, Y_t, U_{Ft})^t$, with $Y_t = ip_t, emp_t, Q_{1t}$. More precisely, we consider large positive uncertainty shocks and large negative real activity shocks.

Figure 4 displays the date and size of shocks that are at least two standard deviations above the mean, estimated using the four different measures of $Y_t$. In view of the non-normality of the shocks, the figure also plots horizontal lines corresponding to three standard deviation of the unit shocks, which is used as the reference point for ‘large’. The lowest panel shows that, irrespective of the definition of $Y_t$, all SVARs identify big financial uncertainty shocks in 1987 and 2008. The middle panel shows that large negative real activity shocks are in alignment with all post-war recessions with one exception: the negative real activity shock in 2005 is not immediately associated with a recession, but it could be the seed of the Great Recession that followed. It is known that the housing market led the 2007-2009 recession (e.g., see Favilukis, Ludvigson, and Van Nieuwerburgh (2015) for a discussion). Indeed, all 10 housing series in $X^M$ (most pertaining to housing starts and permits series) exhibit sharp declines starting in September 2005 and continuing through 2006, thereby leading the Great Recession. This suggests that the negative spike in real activity in 2005 were at least in part driven by the housing sector.

The top panel of Figure 4 shows that the dates of large increases in $e_{Mt}$ are less clustered. They generally coincide with, or occur shortly after, the big real activity shocks and the financial uncertainty shocks. Observe that large macro uncertainty shocks occurred more frequently in the pre-1983 than the post 1983 sample.

To give a sense of the historical importance of these shocks, we perform a decomposition of variance, which is the fraction of $s$-step-ahead forecast error variance attributable to each structural shock $e_{Mt}$, $e_{Yt}$, and $e_{Ft}$ for $s = 1, s = 12, s = \infty$. We also report the maximum
fraction of forecast error variance over all VAR forecast horizons $s$ that is attributable to each shock, denoted $s = s_{\text{max}}$ in the table. Table 2 reports results for the system with $Y_t = ip_t$ (left column), $Y_t = emp_t$ (middle column), and $Y_t = Q_t$ (right column).

According to the top row, all three real activity shocks $e_{ip}$, $e_{emp}$, and $e_{Q_1}$ have sizable effects on macroeconomic uncertainty $U_M$. But according to the bottom row, these same shocks have small effects on financial uncertainty $U_F$. At the same time, positive macro uncertainty shocks $e_M$, which increase rather than decrease real activity, explain a surprisingly large fraction of production (up to 53%), employment (up to 38%) and the real activity index (up to 50%), though their relative importance declines as the forecast horizon increases. On the other hand, financial uncertainty shocks $e_F$ have a small contribution to the one-step-ahead forecast error variance of $ip$, but their relative importance increases over time. These $e_F$ shocks make much larger contributions to the forecast error variance of $emp$ and $Q_1$. Financial uncertainty shocks explain up to 59% of the forecast error variance in employment and up to 36% of the forecast error variance in the real activity index, compared to 27% for production. Financial uncertainty shocks $e_F$ feedback into $U_M$, and macroeconomic uncertainty shocks $e_M$ also feedback into $U_F$.

Regardless of which measure of real activity is used, we find that financial uncertainty is unlike macro uncertainty or real activity in that its variation is far more dominated by its own shocks. For example, in the system with $ip$, $e_F$ shocks explain 95% of the $s = 1$ step-ahead forecast error variance in $U_{Ft}$, and 75% of the $s = \infty$ step-ahead forecast error variance. In the systems with $emp$ and $Q_1$, $e_F$ shocks explain 74% and 96%, respectively, of the $s = 1$ step-ahead forecast error variance in $U_{Ft}$, and 53% and 96% of the $s = \infty$ step-ahead forecast error variance.

To summarize, in all three systems, real activity shocks $e_Y$ have quantitatively large persistent negative effects on macro uncertainty $U_M$. In turn, macro uncertainty shocks $e_M$ have large positive impact effects on real activity measures $Y$. Financial uncertainty shocks $e_F$ have smaller impact effects but larger long run effects that dampen real activity $Y$. Across all systems, the forecast error variance of financial uncertainty is the least affected by shocks other than its own, suggesting that $U_F$ is quantitatively the most important exogenous impulse in the system.

5 Uncertainty in Real Activity and Inflation

The results discussed above suggest that the dynamic relationship between macro uncertainty and real activity can be quite different from the relation between financial uncertainty and real activity. However, given the composition of our data $\chi^M$, macroeconomic uncertainty itself can be due to uncertainty in real activity variables such as output and unemployment, to price variables, and to financial market variables. The theoretical uncertainty literature
has focused on modeling exogenous uncertainty shocks that arise specifically in measures of real economic fundamentals, rather than in prices or financial markets. To better evaluate the implications of these theoretical models, it is therefore of interest to know how systems defined by sub-components of broad-based macro uncertainty behave. We first consider systems that isolate uncertainty about real activity using the $U_{Rt}$ sub-index that more closely corresponds to the theoretical literature. We then move on to study systems that use a sub-index of macro uncertainty focused on price variables, $U_{Pt}$, which has not been the focus on the uncertainty literature but may be of independent interest.

5.1 System $X_t = (U_{Rt}, Y_t, U_{Ft})'$

We isolate the real activity components of macro uncertainty by aggregating the individual uncertainty estimates over the 73 real activity variables in the macro dataset $X^M$. The one-period ahead uncertainty in real activity, denoted $U_{Rt}$, is show in Figure 5. This series, like $U_{Mt}$, is countercyclical though somewhat less so, having a correlation of -0.50 with industrial production (as compared to -0.66 for $U_{Mt}$). At first glance, $U_{Rt}$ appears to fluctuate in a manner similar to macroeconomic uncertainty $U_{Mt}$. The two series have a correlation of 0.71 and exhibit some overlapping spikes. But $U_{Rt}$ and $U_{Mt}$ also display notable independent variation. Figure 5 shows that there are 43 observations of $U_{Rt}$ that are at least 1.65 standard deviations above its mean. These can be organized into five episodes: 1965, 1970, 1975, 1982-83, and 2007. By contrast, $U_{Mt}$ in Figure 1 only exhibits three such episodes. Observe that the $U_{Rt}$ series exhibits several spikes before 1970 that are not accompanied by spikes in $U_{Mt}$.

Given the distinctive patterns in the time series behavior of $U_{Rt}$ and $U_{Mt}$, one might expect to find different dynamic relationships with the other variables in our systems when $U_{Mt}$ is replaced by $U_{Rt}$. Surprisingly, the impulse responses functions are qualitatively similar to systems studied above that use broad-based macro uncertainty. These responses are displayed in Figure 6. We see that (i) positive shocks to real activity measures cause sharp declines in $U_{Rt}$ so that negative shocks cause sharp increases in real economic uncertainty; (ii) positive real activity uncertainty shocks $e_{Rt}$ do not cause declines in real activity measures; instead the opposite is true; (iii) positive financial uncertainty shocks $e_{Ft}$ lead to sharp declines in real activity measures that are strongly statistically significant, and (iv) there is little evidence that financial uncertainty is statistically significantly affected by real activity shocks (the error bars are wide at all horizons, even those not shown).

But while these dynamic responses are similar to those reported for the base case when $U_{Mt}$ is used, the realized shocks that are uncovered from the historical data are different. Figure 7 plots the large adverse structural shocks identified from the systems $(U_{Rt}, Y_t, U_{Ft})'$ for $Y_t = ip_t, emp_t, Q_{1t}$ analogous to Figure 2. The top panel shows that the real uncertainty shock
$e_{Rt}$ exhibits no spike in excess of three standard deviations during the Great Recession for any measure of real activity, despite the fact that $U_{Rt}$ itself exhibits a large spike (see Figure 5). Only the system that uses $Y_t = emp_t$ exhibits a spike in excess of two standard deviations. This is in contrast to the behavior of $e_{Mt}$ and especially $e_{Ft}$ in Figure 2, both of which show much larger spikes during this episode. This pattern occurs in other recessions as well. In the 1973-75 recession, the real uncertainty shocks $e_{Rt}$ show no large spikes, though all measures of real activity shocks $e_{ip}, e_{emp},$ and $e_{Q_{1t}}$ exhibited large spikes downward. Likewise, both the 1980 recession and the 1982-1983 recession were characterized by large negative real activity shocks that met or exceeded three standard deviations from the mean, while real uncertainty shocks $e_R$ were comparatively muted and if anything spiked after the recession was over.

These episodes serve to reinforce the conclusion that the heightened real economic uncertainty in recessions is more often an endogenous response to other shocks, rather than an exogenous impulse. Even though there were many large spikes in real uncertainty shocks $e_{Rt}$ pre-1983, there have not been much in the way of large adverse shocks to real economic uncertainty since 1983, a period that coincides with the so-called Great Moderation. Large real uncertainty shocks are also absent from the Great Recession. This is an episode characterized by a large negative $e_{Yt}$ and a large increase in $e_{Ft}$. Both adverse shocks are sufficiently large to drive $U_{Rt}$ upward without a large exogenous increase $e_{Rt}$.

One might ask why we find large macro uncertainty shocks $e_M$ in the Great Recession, at least for some measures of real activity, while the corresponding real activity uncertainty shocks $e_R$ are much smaller. Recall that our $U_M$ is a broad-based measure of uncertainty and, as such, contains some 25 financial variables. These are also the most volatile variables in the large macro dataset used to construct $U_{Mt}$. Hence $U_M$ picks up a fair amount of its movement from financial variables, which were especially large in this episode. By isolating uncertainty attributable only to real variables, we can see more clearly the role of uncertainty about real activity variables in this episode. By the same reasoning, once we control explicitly for financial uncertainty, it makes little difference whether we use $U_{Mt}$ or $U_{Rt}$ in the SVAR. The impulse responses are similar, as can be seen from a comparison of the base case IRFs and those in Figure 3. Controlling for $U_{Ft}$ is thus important as it removes the variation in $U_{Mt}$ attributable to financial variable uncertainty. Whether we directly or indirectly control for uncertainty from financial variables, the main finding is that macroeconomic uncertainty rises in recessions primarily in response to real activity shocks, while financial uncertainty shocks are exogenous impulses that have significant negative effects on real activity.

To complete the analysis, we present variance decompositions for the system $(U_{Rt}, Y_t, U_{Ft})'$, with three measures of real activity $Y_t = ip_t, emp_t, Q_{1t}$. These results, presented in Table 3, share some similarities with the systems that use macro uncertainty $U_{Mt}$ shown in Table 2, but there are at least two important distinctions. First, financial uncertainty shocks decrease real activity
and explain larger fractions of the forecast error variance in two measures of real activity. At
the longest $s = \infty$ VAR horizon, financial uncertainty shocks explain 85% of forecast error
variance in employment and 50% of the forecast error variance in the real activity index. These
results suggest that financial uncertainty has quantitatively large negative consequences for at
least some measures of real activity.

Second, compared to systems that use $U_{Mt}$, smaller fractions of the forecast error variance
in $U_{Rt}$ are explained by its own shocks, while larger fractions are explained by the financial
uncertainty shocks. Real activity shocks still have non-trivial consequences for $U_{Rt}$. For example,
shocks to industrial production $e_{ipt}$ still explain 41% of the one-step-ahead forecast error
variance in $U_{Rt}$, though smaller than the 53% found earlier using $U_{Mt}$.

To summarize, countercyclical increases in real uncertainty $U_{Rt}$, like macro uncertainty
$U_{Mt}$, are found to be fully an endogenous response to declines in real activity. Indeed, the most
striking episode of heightened uncertainty in the post-war period, the Great Recession, was
characterized by large negative real activity $e_Y$ shocks and a large positive financial uncertainty
$e_F$ shock, but no corresponding large shock to real uncertainty $e_R$. These results underscore the
extent to which the countercyclical variation in $U_{Rt}$ is often an endogenous response to other
shocks. At the same time, $U_{Rt}$ exhibits more variation than $U_{Mt}$ that is independent of fluctua-
tions in real activity especially early in the sample, explaining why it is less countercyclical.

5.2 System $X_t = (U_{\pi t}, Y_t, U_{Ft})'$

The preceding subsection investigates the real activity component of macroeconomic uncer-
tainty and its interaction with $Y_t$ and $U_{Ft}$. This subsection studies the price component of
macroeconomic uncertainty $U_{\pi}$ which aggregates the 21 uncertainty indicators in the price
block of $\chi^M$. This block includes consumer and producer prices that tend to be more stable, as
well as the price of oil, commodities, and raw materials that tend to be more volatile. With the
exception of the NAPM commodity price index, the price data are second differenced after log
transformation. Hence, the uncertainty indicators pertain to the change in monthly inflation.
We refer to this measure simply as “price uncertainty.”

The top panel of Figure 8 plots this measure of price uncertainty over our sample. It is
countercyclical and has a correlation with industrial production is -0.51. There are 40 observa-
tions that are 1.65 standard deviations above the unconditional mean. These are clustered
into three episodes: 1974-75, 2006-07, and 2008-09. There is a large spike upward in $U_{\pi t}$ visible
during the Great Recession. This spike actually occurs over four months, from 2008:10-2009:01,
during which $U_{\pi t}$ was unusually high. Also plotted in Figure 8 is a $U_{\pi,t}^{x}$ uncertainty index that
removes from $U_{\pi,t}$ five of the most volatile price uncertainty series, namely PPI intermediate
materials, PPI crude materials, oil, PPI metals and metal products, and CPI transportation.
The more volatile price series apparently did not contribute to noticeable changes to aggregate price uncertainty.

Further investigation reveals that the increase in price uncertainty around the Great Recession was broad based, as 13 of the 21 series in the price group had uncertainty risen by at least three standard deviations above its mean in 2008:11, the peak of the spike. Results not reported show that these series all exhibited large negative forecast errors in 2008:10-2008:12, and then a large positive error in 2009:01. The change in inflation across many price series appears to have been volatile and difficult to predict at the peak of the Great Recession. Thus the Great Recession was hit by the rare occurrence of simultaneous adverse shocks to financial uncertainty, to real activity, and to price uncertainty.

The bottom panel of 8 plots the large adverse shocks for the systems $\mathbf{X}_t = (\pi_t, Y_t, U_{Ft})'$ with $Y_t = ip_t, emp_t, Q_{1t}$, and for an alternative set of systems $\mathbf{X}_t = (\pi_t^e, Y_t, U_{Ft})'$. Notably, most of the spikes are concentrated in the years before 1983. Nonetheless, the price uncertainty spike in 2008 is evident both $e_\pi$ and $e_\pi^e$. Together with the results reported earlier, the broad based nature of the surge in uncertainty in 2008 is unprecedented.

We estimate an SVAR for $\mathbf{X}_t = (\pi_t, Y_t, U_{Ft})'$. The responses are again similar for all measures of $Y_t$ so we conserve space by showing just one. Figure 9 shows the dynamic responses with $Y_t = emp_t$. As before, it is exogenous shocks to financial uncertainty that drive real activity endogenously lower. By contrast, positive shocks to price uncertainty do not decrease real activity, indeed the opposite is true. We see also that positive shocks to price uncertainty $e_\pi$ lead to a sharp increase in financial uncertainty $U_{Ft}$. Financial uncertainty shocks, on the other hand, have no effect on price uncertainty $U_{\pi t}$.

Figure 9 also shows that employment shocks $e_{emp}$ impact price uncertainty in a manner that is qualitatively similar to how they impact macro and real economic uncertainty. Positive (negative) shocks to real activity cause sharp decreases (increases) in price uncertainty, but have little effect on financial uncertainty. Thus a boom in real activity appears to reduce macroeconomic uncertainty broadly across many indicators, including uncertainty about price variables, though not about financial markets.

On the whole, these findings reinforce the notion that financial uncertainty is primarily an exogenous impulse acting on real activity, while countercyclical uncertainty about other macroeconomic activity, be it real activity or prices, is primarily an endogenous response to real activity. But price uncertainty increases financial uncertainty, a finding that is theoretically consistent with evidence that inflation uncertainty is correlated with higher risk spreads in bond markets (e.g., Wright (2011)). An interesting direction for future research is to investigate the dynamic linkages between inflation uncertainty, financial market uncertainty, and term premia.
6 Robustness and Additional Cases

This section presents results for a number of additional cases.

6.1 Different Sample

We asked whether our main results were affected by stopping the sample at the end of 2007:12 or dummying out the 1987 crash. Note however, that we maintain the full sample as part of the solution selection process, as described above, since the financial crisis/Great Recession and 1987 crash episodes are valuable for applying prior economic reasoning to narrow the set of solutions. Conditional on these solutions, we analyze subsamples.

A representative set of impulse response functions is shown in Figure A1 for the system $X_t = (U_{Mt}, emp_t, U_{Ft})'$ for the case where we stop the sample in 2007:12. (The other systems show similar responses.) The figure shows that the qualitative nature of all the responses, including standard error bands, is quite similar to the comparable case for the full sample (Figure 3). This implies that main findings above are robust to this sample that excludes the Great Recession and the concomitant financial crisis. Further inspection indicates that the main difference created by using different samples is evident in the variance decompositions (not shown): somewhat less of the forecast error variance in $U_F$ in the pre-2008 sample is attributable to its own shocks than in the full sample, while correspondingly more of the forecast error variance in $U_F$ is attributable to real activity shocks. For example, in the full sample, 95% of the one-step-ahead forecast error variance in $U_F$ is attributable to its own shocks in the system with $Y_t = ip_t$, whereas this estimate is 82% for the pre-2008 sample. At the same time, the variance decompositions pertaining to the impact of financial uncertainty on real activity are little affected by removing the post 2008 part of the sample. This shows that the negative impact of financial uncertainty shocks for real activity does not hinge on one episode, and that many episodes prior to 2008 that were characterized by more modest financial uncertainty shocks also had consequences for real activity.

We also asked whether the results were affected by the 1987 stock market crash. As expected, the 1987 stock market crash generates a positive spike in $e_F$ in 1987, however it is largely orthogonal to real activity and macro uncertainty. This episode is widely understood as being one largely confined to the stock market with little if any lasting impact on the rest of the economy. Prior economic reasoning therefore implies that the episode should show up as a positive financial uncertainty shock, as we find, but perhaps not an episode with important consequences for real activity or macro uncertainty. We have verified that none of our results are materially affected by dummying out the episode in the VAR. Appendix Figure A2 shows a representative set of impulse responses from one of our benchmark systems in which we dummy out 1987:10 and 1987:11. These responses are remarkably similar to those without the dummies,
as shown below.

6.2 One year Uncertainty

So far we have been considering uncertainty about events one-month ahead. To consider a longer horizon uncertainty, we estimate systems using uncertainty about events 12 months ahead, denoted $U_{Mt}(12)$ and $U_{Ft}(12)$. For the dynamic responses, the findings are qualitatively similar to the benchmark cases with $h = 1$ period ahead uncertainty. Figure 10 presents a representative example for the system: $X_t = (U_{Mt}(12), emp_t, U_{Ft}(12))$. But an inspection of the variance decompositions suggests some notable differences from the $h = 1$ uncertainty systems. Table 4 shows variance decompositions for the systems $X_t = (U_{Mt}(12), Y_t, U_{Ft}(12))$ with $Y_t = ip_t, emp_t, noi_t$. One-year financial uncertainty shocks explain smaller fractions of the variation in all measures of real activity than do one-month uncertainty shocks, especially over the longer VAR horizons for which their impact is non-trivial. For example, 12-month-ahead financial uncertainty $e_{Ft}$ shocks explain just 10% of the long-run forecast error variance in $ip_t$. In contrast Table 2 above showed that one-month-ahead financial uncertainty $e_{Ft}$ shocks explain 23% of the long-run forecast error variance in production. Similar comparisons hold for the other two measures of real activity, $emp_t$ and $noi_t$. $U_{Ft}(12)$ shocks also explain smaller fractions of the forecast error variance in macro uncertainty $U_{Mt}$ than do $U_{Ft}(12)$ shocks. This result occurs in part because long-run uncertainty is simply much less volatile than short-run uncertainty. While the level of uncertainty increases with $h$ (on average), the variability of uncertainty decreases because the forecast tends to the unconditional mean as the forecast horizon tends to infinity. On the other hand, the impact of macro uncertainty shocks on the other variables in the system is less affected by the uncertainty horizon $h$. For example, the effects of $e_{Mt}$ shocks on all measures of real activity are about the same for systems using $U_{Mt}(12)$ as they are for the systems studied above that use $U_{Mt}(1)$.  

6.3 Tests of Recursive Identification Restrictions

The econometric model permits us to test whether a recursive structure is supported by the data. Specifically, Assumption A does not rule out the possibility of a recursive structure. Given that $\sqrt{T}(\hat{\beta}_1 - \beta_1^0)$ is asymptotically $N(0, \Sigma_{\beta_1}^2)$, the null hypothesis of a recursive structure is a test that the three components of $\beta_1$ corresponding to the off-diagonal entries of $A_0^{-1}$ are jointly zero. Hence it is chi-square distributed with three degrees of freedom. We first confirm that the test has the correct size in Monte Carlo simulations. Our estimates based on historical data strongly reject a lower triangular $A_0^{-1}$ for any possible ordering of the variables. Table 5 shows results from Wald tests with $Y_t = ip_t$ and $Y_t = emp_t$, for $h = 1$ and $h = 12$. Results not reported find that the $A$ matrix reflects a non-zero contemporaneous correlation between
U_{Ft} and Y_t, as well as between U_{Mt} and Y_t; no recursive ordering is consistent with such a correlation. What happens to the dynamic responses when we nevertheless impose restrictions based on recursive identification (and freely estimate the rest of the parameters)?

Figure 11 shows one case: dynamic responses for the system \( \mathbf{X}_t = (U_{Ft}, U_{Mt}, i_{Pt})' \) with that ordering. Although there are many possible recursive orderings, and the estimated IRFs differ in some ways across these cases, the dynamic responses under recursive identification have one common feature that is invariant to the ordering and that provides the sharpest contrast with the results generated by the SVARs identified with external instruments studied here. Specifically, with recursive identification, macro uncertainty shocks—no matter which ordering—appear to cause a sharp decline in real activity, while real activity shocks have little effect on macro uncertainty in the short run and if anything increase it in the long run. This result, evident in Figure 11, gives precisely the opposite finding from what is reported above and appears to be an artifact of invalid timing assumptions under recursive identification. Further investigation reveals that the SVARs we study display non-zero contemporaneous correlations between \( U_{Ft} \) and \( Y_t \), as well as between \( U_{Mt} \) and \( Y_t \), which is inconsistent with any recursive ordering. Imposing a structure that prohibits contemporaneous feedback spuriously suggests that macro uncertainty shocks are a cause of declines in real activity, rather than an endogenous response. This result is robust across any of the six possible recursive orderings and underscores the challenges of relying on convenient timing assumptions to sort out cause and effect in the relationship between uncertainty and real activity.\(^8\)

### 6.4 Different External Variables

We reestimate the model using a corporate bond return as the regressand in (9a) to generate \( Z_{1t} \). We generate \( Z_{2t} \) in (9b) using the monthly CRSP value-weighted excess stock market return \( \text{crsp}_t \). The bond yield measure is the yield on a portfolio of Baa Moodys seasoned corporate bonds, where Baa represents a credit score on the border of the investment and junk categories. Because the Baa yield is highly serially correlated, we use the first difference of the yield. The estimation procedure in all other ways is the same as above.

Estimates of these cases indicate that the correlation between the resulting \( Z_{1t} \) and both uncertainty shocks is now positive. Thus high uncertainty of both types is associated with rising yields on risky corporate debt. For the \( (U_{Mt}, i_{Pt}, U_{Ft})' \) system, the correlations with \( Z_{1t} \) are \( \rho(Z_{1t}, \hat{e}_{Mt}) = 0.1988 \), \( \rho(Z_{1t}, \hat{e}_{Ft}) = 0.1219 \), while the correlation of \( Z_{2t} \) with \( \hat{e}_{Ft} \) remains similar to the base cases, with \( \rho(Z_{2t}, \hat{e}_{Ft}) = -0.1617 \). The correlation between \( Z_{1t} \) and \( Z_{2t} \) is -0.2 in this case. Figure 12 presents the dynamic responses for the system \( (U_{Mt}, i_{Pt}, U_{Ft})' \). The pattern of responses is qualitatively similar to the base cases presented above. But the SVAR

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\(^8\)The figures for these cases are omitted to conserve space but are available upon request.
parameter estimates exhibit more sampling error. This leads to error bands for the dynamic responses of $U_{Mt}$ to $e_{ipt}$ and for $ip_t$ to $e_{Mt}$ to be wider than in the corresponding base case for the same system.

In our experience, the bootstrap standard error bands tend to be wide when the external variables produce instruments that only weakly identify some elements of $B$. Our analysis requires $\mathbb{E}(Z_2\epsilon_{Mt}) \neq 0$ to identify the column that gives the effects of $\epsilon_{Mt}$ shocks. In cases when the GMM estimates of $\mathbb{E}(Z_1\eta_{Yt})$ and $\mathbb{E}(Z_1\eta_{Ft})$ are imprecise, we find $B_{MY}$ and $B_{YM}$ are poorly identified and the bootstrap error bands for the dynamic responses of $U_{Mt}$ to $e_{ipt}$ and for $ip_t$ to $e_{Mt}$ are then wide. An inspection of the closed-form solutions for $B$ shows why. The $B_{MY}$ and $B_{YM}$ parameters are highly nonlinear functions of $\mathbb{E}(Z_1\eta_{Yt})$ and $\mathbb{E}(Z_1\eta_{Ft})$, so that small changes in the latter lead to large differences in the solution for $B_{MY}$ and $B_{YM}$. Since the bootstrap repeatedly makes draws from the distribution of the GMM estimates it depends on the variance of the point estimates. The bootstrap standard errors are correspondingly large when the point estimates of the variance of $\mathbb{E}(Z_1\eta_{Yt})$ and $\mathbb{E}(Z_1\eta_{Ft})$ and the other parameters are imprecise. Thus, while our approach provides a new way to estimate the SVAR, the methodology requires Assumption A to be satisfied.

An appeal of our estimation strategy is that the estimates provide some guide to the validity of Assumption A for various external instruments used. As an example, consider the system $X_t = (U_{Mt}, ip_t, U_{Ft})'$. Our analysis requires $\mathbb{E}(Z_2\epsilon_{Ft}) \neq 0$ to identify the column that gives the effects of $\epsilon_{Ft}$ shocks. When we set $S_2t$ equal to the Baa-fed funds rate spread (rather than the Baa rate itself), while keeping $S_1t$ the same as in our baseline case, the resulting $Z_2t$ becomes weakly correlated with $\hat{\epsilon}_{Ft}$, so the financial uncertainty shock is poorly identified. The same finding arises when $S_2t$ is set equal to the growth in the spot market oil price.

The standard errors are also large when we use the Baa-fed funds rate spread as $S_1t$. With this choice of $S_1t$, the resulting $Z_1t$ is weakly correlated with $\hat{\epsilon}_{Mt}$ and so the macro uncertainty shock is poorly identified. When $S_1t$ or alternatively $S_2t$ is set equal to $\Delta noit$, the estimated $B_{YY}$ element is close to zero, indicating that the real activity shock is poorly identified. This can be understood by recalling that the $ip_t$ shock is identified off of movements in real activity that are uncorrelated with the instruments, which are components of $S_1t$ and $S_2t$. If $S_1t$ or $S_2t$ are themselves some measure of real activity (such as orders), there may be little uncorrelated variation left to identify the $ip_t$ shock.

A third example is when $S_1t$ is the small stock index return, then $B_{MY}$ and $B_{YM}$ are poorly identified in our sample, for the same reasons as described above: the bootstrap leads to large variation in the estimates of $B_{MY}$ and $B_{YM}$ if the sample variance of these parameters is large. Indeed, compared to our baseline cases, the sample variance of $\mathbb{E}(Z_1\eta_{Yt})$ is two times larger when $S_1t$ is the small stock index return, while the sample variance of $\mathbb{E}(Z_1\eta_{Ft})$ is three times larger.
6.5 Overidentifying Restrictions

Although the restrictions in Assumption A are sufficient to imply that the order condition for identifying $B$ is exactly satisfied, the larger econometric model imposes overidentifying restrictions. There are two sources of overidentification. The first arises from the restrictions stated in Assumption A, and the second arises from the key identifying assumption that it is valid to exclude $S_t$ from the SVAR.

For the first, note that, as shown in the Appendix, the proof of Proposition 1 requires that $E[Z_{1t}e_{Mt}] = \phi_{1M} \neq 0$ and $E[Z_{2t}e_{Ft}] = \phi_{2F} \neq 0$, but there is no analogous requirement that $E[Z_{1t}e_{Ft}] = \phi_{1F}$ be nonzero. Indeed, $\phi_{1F}$ could be zero and identification of $\beta_1^*$ still follows. However, the theoretical assumptions stated above for $S_t$ in (9a) and (9b) impose additional restrictions on the econometric model. Since both $S_{1t}$ and $S_{2t}$ are both aggregate stock market returns, $Z_{1t}$ and $Z_{2t}$ should both be functions of the financial uncertainty shock. Thus $Z_{1t}$ should be correlated with $e_{Ft}$ if $Z_{2t}$ is. This imposes the overidentifying restriction that $\phi_{1F} \neq 0$, a testable restriction that Table 1 shows is not rejected.

For the second, note that conventional IV analysis assumes that the valid instrument $Z_t$ is excluded from the equation of interest. Here, the invalid instrument $S_t$ is excluded from the VAR system of interest. This is tantamount to imposing exclusion restrictions on a larger VAR that includes $S_t$. We now make these exclusion restrictions precise in the context of the three variable VAR and show that they impose overidentifying restrictions on a larger VAR that includes $S_t$.

To do so, we rewrite the SVAR (1) in the canonical structural economic model formulation:

$$A_0X_t = A_1X_{t-1} + A_2X_{t-2} + \cdots + A_pX_{t-p} + \Sigma e_X$$

where $A_j$ are $3 \times 3$ matrices which are related to the SVAR (1) by $A_0 = H^{-1}$, $A_j \equiv H^{-1}A_j$. A five variable VAR(1) in $(X_t', S_t')'$ can be written

$$
\begin{pmatrix}
A_{XX,0} & A_{XS,0} \\
A_{SX,0} & A_{SS,0}
\end{pmatrix}
\begin{pmatrix}
X_t \\
S_t
\end{pmatrix} =
\begin{pmatrix}
A_{XX,1} & A_{XS,1} \\
A_{SX,1} & A_{SS,1}
\end{pmatrix}
\begin{pmatrix}
X_{t-1} \\
S_{t-1}
\end{pmatrix} +
\begin{pmatrix}
\Sigma_X & 0 \\
0 & \Sigma_S
\end{pmatrix}
\begin{pmatrix}
e_{Xt} \\
e_{St}
\end{pmatrix}.
\]

The relation between the reduced form and the structural shocks is now

$$
\begin{pmatrix}
\eta_{Xt} \\
\eta_{St}
\end{pmatrix} =
\begin{pmatrix}
A_{XX,0} & A_{XS,0} \\
A_{SX,0} & A_{SS,0}
\end{pmatrix}^{-1}
\begin{pmatrix}
\Sigma_X e_{Xt} \\
\Sigma_S e_{St}
\end{pmatrix} =
\begin{pmatrix}
B_{XX} & B_{XS} \\
B_{SX} & B_{SS}
\end{pmatrix}
\begin{pmatrix}
e_{Xt} \\
e_{St}
\end{pmatrix}.
\]

By substituting out $S_t$, it is straightforward to show that

$$
\left[ (A_{XX,0} - A_{XX,1}L) + (A_{XS,0} - A_{XS,1}L)C_{SX}(L) \right] X_t = -(A_{XS,0} - A_{XS,1}L)C_{SS}(L)\Sigma_S e_{St} + \Sigma_X e_{Xt}
$$

where $C_{SX}(L) = C_{SS}(L)/(A_{SX,0} - A_{SX,1}L)$, $C_{SS}(L) = -(A_{SS,0} - A_{SS,1}L)^{-1}$. Without further restrictions, $X_t$ is a VARMA(1,1) driven by a combination of shocks to $X_t$ as well as $S_t$.  

31
Under our maintained assumption that $S_t$ is excluded from the VAR, $A_{XS,0} = A_{XS,1} = 0_{3 \times 2}$ and the terms that multiply into $C_{SX}(L)$ and $C_{SS}(L)$ drop out, giving
\[
A_{XX,0}X_t = A_{XX,1}X_{t-1} + \Sigma_X e_{Xt}
\]
which is our assumed SVAR with $p = 1$. For arbitrary $p \geq 1$, the assumptions $A_{XS,j} = 0$ for all $j \geq 0$ effectively restricts the five variable system to be block recursive, with the three variables in $X_t$ ordered ahead of the two variables in $S_t$. Since the dynamic responses of $S_t$ are not of direct interest, the block recursive assumption permits us to analyze the smaller VAR for $X_t$.

Though the assumption that $A_{XS,j} = 0$ for all $j$ is necessary to justify the smaller three variable VAR, it is stronger than is necessary for the identification of $e_{Xt}$. The reason is that, provided $A_{XS,0} = 0$, $B_{XS}$ will be zero. The lower block triangularity of $B$ implies that $A_{XX,0}$ can be identified by Assumption LMN along with the covariance structure of $\eta_{Xt}$ associated with the five variable system. In other words, we can in principle leave $A_{XS,j}$ for $j \geq 1$ unconstrained to allow the effects of $e_{Xt}$ to feedback to $X_t$ through lags of $S_t$. Thus while $A_{XS,0} = 0$ is required for exact identification and cannot be tested, the $B_{XX}$ submatrix for the three variable system $X_t = (U_{Mt}, Y_t, U_{Ft})'$ can still be estimated using IPIV when $X_t$ is part of a larger VAR that includes $S_t$, without imposing the additional restrictions that $A_{XS,j} = 0$ for $j \geq 1$. The restrictions $A_{XS,j} = 0$ for $j \geq 1$ that are part of the exclusion assumption therefore impose overidentifying restrictions. A simple way to evaluate their validity is to compare the impulse response functions estimated above for the three variable system $X_t = (U_{Mt}, Y_t, U_{Ft})'$ with those from a larger system that includes $S_t$ but does not restrict $A_{XS,j} = 0$ for $j \geq 1$.

To do so, we estimate a four variable system in $(U_{Mt}, Y_t, U_{Ft}, S_t)'$, imposing $A_{XS,0} = 0$, but without imposing $A_{XS,j} = 0$ for $j \geq 1$. We report results for the four variable case where $S_t$ is measured as the return on the CRSP value-weighted stock market index. We can identify $B_{XX}$ from the first three equations of this VAR alone using IPIV. The only difference from the base case is that $\eta_{Xt}$ is a vector of residuals from a regression of $X_{jt}$ on lags of $X_t$ and lags of $S_t$. Since $A_{XS,0} = 0$ by assumption, it holds that $B_{XS} = 0$. It only remains to identify $B_{SS}$ and $B_{SX}$. These can be recovered by least squares regression of $\hat{\eta}_{St}$ on $\hat{e}_{Xt}$ to give a fitted residual
\[
\hat{e}_{St} = \eta_{St} - \hat{B}'_{SX} \hat{e}_{t}
\]
where $\hat{B}'_{SX}$ are the OLS coefficient estimates, and $\hat{B}_{SS}$ is the standard deviation of $\hat{e}_{St}$. The SVAR estimates may then be used to compute impulse responses for the four variable system. The validity of the overidentifying exclusion restrictions for $S_t$ can be evaluated by comparing the impulse responses to shocks $e_X$ from the three variable VAR in $X_t$ with those for the four variable VAR in $(X_t', S_t)'$ that do not impose $A_{XS,j} = 0$ for $j \geq 1$.

Figure 13 presents these two sets of impulse responses for the system with $Y_t = ip_t$. The responses are little different. Indeed, the coefficients on lags of $S_t$ appear to be close to zero for
all three variables. The data thus appear consistent with the restrictions that $A_{X,S,j} = 0$ for $j \geq 1$ and therefore the assumption that stock returns can be excluded from the VAR. Even though a VAR that directly incorporates $S$ is possible, the system restricts $S$ to be explained only by lags of $S$ and $X$ which could be restrictive. Our the three variable approach is more robust to such misspecification that could the entire system.

7 Conclusion

A growing body of research establishes uncertainty as a feature of deep recessions but leaves open two key questions: is uncertainty primarily a source of business cycle fluctuations or an endogenous response to them? And where does uncertainty originate? There is no theoretical consensus on the question of whether uncertainty is primarily a cause or a consequence of declines in economic activity. In most theories, it is modeled either as a cause or an effect, but not both, underscoring the extent to which this question is fundamentally an empirical matter.

The objective of this paper is to address both questions econometrically using small-scale structural VARs that are general enough to nest the range of theoretical possibilities in empirical tests. Commonly used recursive identification schemes cannot achieve this objective, since by construction they rule out the possibility that uncertainty and real activity could influence one another contemporaneously. The econometric model employed in this paper nests the recursive identification scheme, and we find that it is strongly rejected by the data. An empirical model in which uncertainty and real activity simultaneously influence each other fits the data far better than one in which these relationships are restricted by timing assumptions that prohibit contemporaneous feedback.

To identify dynamic causal effects, this paper takes an alternative identification approach by using external instruments that we construct in a novel way to be valid under credible interpretations of the structural shocks. We call this approach iterative projection IV (IPIV). In addition, our empirical analysis explicitly distinguishes macro uncertainty and uncertainty about real activity from financial uncertainty, thereby allowing us to shed light on the origins of uncertainty shocks that drive real activity lower, to the extent that any of them do. The econometric framework allows uncertainty to be an exogenous source of business cycle fluctuations, or an endogenous response to them, or any combination of the two, without restricting the timing of these relationships. Underlying our approach is a maintained theoretical assumption that variables such as stock market returns, while endogenous, are nevertheless driven by distinct sources of stochastic variation, some of which satisfy exogeneity restrictions required to identify independent structural shocks.

Estimates of the econometric model are used to inform the nature of these dynamic relationships in U.S. data. The results from these estimations show that sharply higher uncertainty
about real economic activity in recessions is fully an endogenous response to business cycle fluctuations, while uncertainty about financial markets is a likely source of them. Exogenous declines in economic activity have quantitatively large effects that drive real economic uncertainty endogenously higher. Financial uncertainty, by contrast, is dominated by its own shocks, implying that it is primarily an exogenous impulse vis-a-vis real activity and macro uncertainty. These results reinforce the hypothesis laid out in much of theoretical uncertainty literature, namely that uncertainty shocks are a source of business cycle fluctuations. But they also stand in contrast to this literature, which has emphasized the role of uncertainty fluctuations in productivity and other real economic fundamentals. The findings here imply that the uncertainty shocks that drive real activity lower appear to have their origins, not in measures of real activity, but in financial markets.
Appendix

Closed-Form Solution for B

Lemma 2 The solution to the system (7) exists and is unique if \( \mathbb{E}[e_{F1}Z_2] \neq 0 \) and \( \mathbb{E}[e_{M1}Z_1] \neq 0 \).

Proof. To facilitate the presentation throughout the proof, let

\[
\eta_t = B e_t \\
B = \begin{bmatrix} B_M, B_Y, B_F \end{bmatrix} \\
\Omega = \mathbb{E}(\eta, \eta')
\]

and we have two external instruments \((Z_1, Z_2)\) satisfying

\[
\mathbb{E}[e_{F1}Z_1] \equiv \phi_{1F} \neq 0, \quad \mathbb{E}[e_{M1}Z_1] \equiv \phi_{1M} \neq 0 \text{ and } \mathbb{E}[e_{Y1}Z_1] = 0 \\
\mathbb{E}[e_{F1}Z_2] \equiv \phi_{2F} \neq 0 \text{ and } \mathbb{E}[e_{M1}Z_2] = \mathbb{E}[e_{Y1}Z_2] = 0
\]

Then

\[
\mathbb{E}[\eta_t Z_2] = \mathbb{E}[Be_t Z_2] = B \begin{bmatrix} 0 \\ 0 \\ \phi_{2F} \end{bmatrix} = \phi_{2F} B_F 
\]

(A.1)

Thus \( B_F \) exists if \( \phi_{2F} \neq 0 \). Observe that, since

\[
\Omega = \mathbb{E}[\eta, \eta'] = BB'
\]

we have

\[
B'\Omega^{-1}B = I
\]

hence, \( \forall i, j = M, Y, F \)

\[
B'_j \Omega^{-1/2} \Omega^{-1/2} B_i = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases}
\]

Therefore,

\[
\mathbb{E}[\eta_t Z_2]' \Omega^{-1} \mathbb{E}[\eta_t Z_2] = (\phi_{2F} B_F)' \Omega^{-1/2} \Omega^{-1/2} (\phi_{2F} B_F) = \phi_{2F}^2
\]

This implies that the scale \( \phi_{2F} \) is identified up to a sign by

\[
\phi_{2F} = \pm \sqrt{\mathbb{E}[\eta_t Z_2]' \Omega^{-1} \mathbb{E}[\eta_t Z_2]}.
\]

(A.2)

Next,

\[
\mathbb{E}[\eta_t Z_1] = \mathbb{E}[Be_t Z_1] = B \begin{bmatrix} \phi_{1M} \\ 0 \\ \phi_{1F} \end{bmatrix} = \phi_{1M} B_M + \phi_{1F} B_F
\]
But note that
\[
\mathbb{E} [\eta_i Z_2] \Omega^{-1} \mathbb{E} [\eta_i Z_1] = \phi_{2F} B_F \Omega^{-1} (\phi_{1M} B_M + \phi_{1F} B_F)
\]
\[
= \phi_{2F} B_F (BB')^{-1} (\phi_{1M} B_M + \phi_{1F} B_F)
\]
\[
= \phi_{2F} \phi_{1F}
\]
This implies that $\phi_{1F}$ is identified as
\[
\phi_{1F} = \frac{\mathbb{E} [\eta_i Z_2] \Omega^{-1} \mathbb{E} [\eta_i Z_1]}{\phi_{2F}}
\]
which in turn implies
\[
\phi_{1M} B_M = \mathbb{E} [\eta_i Z_1] - \frac{\mathbb{E} [\eta_i Z_2]}{\phi_{2F}} \phi_{1F}.
\] (A.3)

Thus solution to $B_M$ exists if $\phi_{1M} \neq 0$. Furthermore, note that
\[
\left( \mathbb{E} [\eta_i Z_1] - \frac{\mathbb{E} [\eta_i Z_2]}{\phi_{2F}} \phi_{1F} \right)' \Omega^{-1} \left( \mathbb{E} [\eta_i Z_1] - \frac{\mathbb{E} [\eta_i Z_2]}{\phi_{2F}} \phi_{1F} \right)
= \Omega^{-\frac{1}{2}} B_M \phi_{1M}^2 B_M \Omega^{-\frac{1}{2}} = \phi_{1M}^2
\]
This implies that the parameter $\phi_{1M}$ is identified up to a sign as
\[
\phi_{1M}^2 = \left( \mathbb{E} [\eta_i Z_1] - \frac{\mathbb{E} [\eta_i Z_2]}{\phi_{2F}} \phi_{1F} \right)' \Omega^{-1} \left( \mathbb{E} [\eta_i Z_1] - \frac{\mathbb{E} [\eta_i Z_2]}{\phi_{2F}} \phi_{1F} \right).
\] (A.4)

It only remains to identify $B_Y$. $B_Y$ must satisfy
\[
B'_Y \Omega^{-\frac{1}{2}} \Omega^{-\frac{1}{2}} B_Y = 1
\]
\[
B'_Y \Omega^{-\frac{1}{2}} \Omega^{-\frac{1}{2}} B_M = 0
\] (A.5)
\[
B'_Y \Omega^{-\frac{1}{2}} \Omega^{-\frac{1}{2}} B_F = 0
\]

$B_Y$ can be solved analytically using (A.5) provided that $B_F$ and $B_Y$ are identified. In addition, since the equation (A.5) is quadratic in $B_Y$, $B_Y$ is unique up to sign. It follows that there exists a $\tau$ such that
\[
B_Y = \tau \tilde{B}_Y
\] (A.6)

where $\tilde{B}_Y$ is unique conditional on $\phi_{2F}$ and $\phi_{1M}$, but the scalar $\tau$ is unique up to sign.

This shows that the solution to the system (7) exists and is unique up to sign if $\phi_{2F} \neq 0$, $\phi_{1M} \neq 0$. Combined with unit effect normalization (3) and the restriction on the admissible parameter space (4), $B$ can be uniquely identified. The unit effect normalization implies
\[
\begin{pmatrix}
B_{MM} & B_{MY} & B_{MF} \\
B_{YM} & B_{YY} & B_{YF} \\
B_{FM} & B_{FY} & B_{FF}
\end{pmatrix}
= \begin{pmatrix}
1 & H_{MY} & H_{MF} \\
H_{YM} & 1 & H_{YF} \\
H_{FM} & H_{FY} & 1
\end{pmatrix}
\begin{pmatrix}
\sigma_{MM} & 0 & 0 \\
0 & \sigma_{YY} & 0 \\
0 & 0 & \sigma_{FF}
\end{pmatrix}
= \begin{pmatrix}
\sigma_{MM} & H_{MY} \sigma_{YY} & H_{MF} \sigma_{FF} \\
H_{YM} \sigma_{MM} & \sigma_{YY} & H_{YF} \sigma_{FF} \\
H_{FM} \sigma_{MM} & H_{FY} \sigma_{YY} & \sigma_{FF}
\end{pmatrix}
\]
Combined with the restriction \( \sigma_{jj} > 0 \) for all \( j = M, Y, F \), implies \( B_{jj} > 0 \) for all \( j = M, Y, F \). From equation (A.1), \( B_{FF} > 0 \) pins down the sign of \( \phi_{2F} \) conditional \( Z_t \). Since the sign of \( \phi_{2F} \) is pinned down, the signs of \( B_{MF} \) and \( B_{YF} \) are also pinned down by the same restriction. From equation (A.3), \( B_{MM} > 0 \) pins down the sign of \( \phi_{1M} \) conditional \( Z_t \) and therefore the signs of \( B_{YM} \) and \( B_{FM} \) are pinned down by the same restriction. It only remains to show the uniqueness of \( B_Y \). Provided that \( B_F \) and \( B_Y \) are identified and given the closed-form solution (A.5) that is quadratic in \( B_Y \), then \( B_{YY} > 0 \) pins down the sign of \( \tau \) conditional \( Z_t \) and hence the sign of \( B_{MY} \) and \( B_{FY} \) are also pinned down by the same restriction. ■

The system of equations defining \( B \) is

\[
0 = \mathbb{E}[g_1(m_{1t}; \beta_1)] \equiv \bar{g}_1.
\]

The rank condition is satisfied when \( J \equiv \partial \mathbb{E}_t[g_1]/\partial \beta_i \) is full column rank. We check that the rank condition is satisfied by evaluating \( J \) at the estimated parameter values for each case.

**Procedure for Bootstrap**

The bootstrap follows Krinsky and Robb (1986). We sample repeatedly from the joint distribution \( N(\hat{\beta}, \hat{\Theta}/T) \), where \( \hat{\Theta} \) is the estimated GMM variance-covariance matrix to obtain \( B \) new sets of parameters \( \hat{\beta}^{(1)}, \ldots, \hat{\beta}^{(B)} \) and calculate the impulse response function values at each draw, \( \Upsilon_{s,j}^{(1)}, \ldots, \Upsilon_{s,j}^{(B)} \), where \( s \) indexes the VAR horizon and \( j \) the variable being shocked, and where \( \Upsilon_{s,j}^{(b)} = \Upsilon_{s,j}(\hat{\beta}^{(b)}) \). The confidence intervals are ranges for \( \Upsilon_{s,j}^{(b)} \) created by trimming \( \alpha/2 \) from each tail of the resulting distribution of the function values. The parameter \( B \) is set to 10,000.
References


The upper panel plots the time series of the macro uncertainty $U_M$, expressed in standardized units. The lower panel shows the time series of financial uncertainty $U_F$ expressed in standardized units. The vertical lines correspond to the NBER recession dates. The horizontal line corresponds to 1.65 standard deviations above the unconditional mean of each series (which has been normalized to zero). Correlations with the 12-month moving average of IP growth are reported. The black dots represent months when uncertainty is 1.65 standard deviations above its unconditional mean. The data are monthly and span the period 1960:07 to 2015:04.
Figure 2: Time Series of $e$ Shock from SVAR System $(U_M, ip, U_F)'$

The horizontal line corresponds to 3 standard deviations above/below the unconditional mean of each series. The shocks $e = B^{-1} \eta_t$ are reported, where $\eta_t$ is the residual from VAR(6) of $(U_M, ip, U_F)'$ and $B = A^{-1} \Sigma^\frac{1}{2}$. Skewness is defined as $s = \frac{\sum_t (e_t - \bar{e})^3}{\text{Var}(e)} T$. Kurtosis is defined as $\kappa = \frac{\sum_t (e_t - \bar{e})^4}{[\text{Var}(e)]^2} T$. The sample spans the period 1960:07 to 2015:04.
Figure 3: Dynamic Responses in SVAR $(U_M, Y, U_F)'$

The figure displays impulse responses to one standard deviation shocks. Response units are reported in percentage points. Bootstrapped 90% error bands appear as vertical lines. The sample spans the period 1960:07 to 2015:04.
The figure exhibits shocks that are at least 2 standard deviations above the unconditional mean for $e_M$ and $e_F$ and below for $e_Y$ for three cases where $Y = ip, emp, Q_1$. The shocks $e = B^{-1} \eta_t$ are reported, where $\eta_t$ is the residual from VAR(6) and $B = A^{-1} \Sigma^{1/2}$. The horizontal line corresponds to 3 standard deviations shocks. The sample spans the period 1960:07 to 2015:04.
This plot shows time series of $U_R$, expressed in standardized units. The vertical lines correspond to the NBER recession dates. The horizontal line corresponds to 1.65 standard deviations above the unconditional mean of each series (which has been normalized to zero). Correlations with the 12-month moving average of IP growth are reported. The black dots represent months when $U_R$ is 1.65 standard deviations above its unconditional mean. The data are monthly and span the period 1960:07 to 2015:04.
The figure displays impulse responses to one standard deviation shocks. Response units are reported in percentage points. Bootstrapped 90% error bands appear as vertical lines. The sample spans the period 1960:07 to 2015:04
The figure exhibits shocks that are at least 2 standard deviations above the unconditional mean for $e_R$ and $e_F$ and below for $e_Y$ for three cases where $Y = \text{ip}, \text{emp}, Q_1$. The shocks $e = B^{-1}\eta_t$ are reported, where $\eta_t$ is the residual from VAR(6) and $B = A^{-1}\Sigma^{1/2}$. The horizontal line corresponds to 3 standard deviations shocks. The sample spans the period 1960:07 to 2015:04.
The upper panel plots $U_\pi$ and $U_{\pi}^x$ where the latter excludes uncertainties for five volatile sub-series defined in the text, expressed in standardized units. The five series are: PPI intermediate materials, PPI crude materials, oil, PPI metals and metal products, and CPI transportation. The middle and lower panel exhibit shocks that are at least 2 standard deviations above the unconditional mean for $U_\pi$ and $U_{\pi}^x$. The shaded vertical bars correspond to the NBER recession dates. Correlations with the 12-month moving average of IP growth are reported. The data are monthly and span the period 1960:07 to 2015:04.
Figure 9: Dynamic Responses in SVAR $(U_\pi, emp, U_F)'$

Bootstrapped 90% error bands appear as dashed lines. Response units are reported in percentage points. The sample spans the period 1960:07 to 2015:04.
Figure 10: Dynamic Responses in SVAR\((U_M(12), emp, U_F(12))'\)

Bootstrapped 90% error bands appear as dashed lines. Response units are reported in percentage points. The sample spans the period 1960:07 to 2015:04.
Figure 11: Dynamic Responses using Recursive Identification with Order $(U_F, U_M, ip)'$

Bootstrapped 90% error bands appear as dashed lines. Response units are reported in percentage points. The sample spans the period 1960:07 to 2015:04.
Moody’s Seasoned Baa corporate bond yield $Baa_t$ is used to construct $Z_1$ and the CRSP excess return to construct $Z_2$. Bootstrapped 90% error bands appear as dashed lines. Response units are reported in percentage points. The sample spans the period 1960:07 to 2015:04.
Figure 13: Dynamic Responses in SVAR \((U_M, ip, U_F, S_t)^t\) v.s. \((U_M, ip, U_F)^t\)

\(S_t\) is the CRSP value weighted average returns. Response units are reported in percentage points. The sample spans the period 1960:07 to 2015:04.
Panel A reports the correlation between the estimated uncertainty shocks and the instruments. Panel B reports estimates of $\Sigma$ that give the standard deviation of each structural shock. Asymptotic standard errors are reported in brackets and bootstrapped 90 percent confidence intervals are reported in parentheses. Bold numbers indicate statistical significance at 10 percent level. The data are monthly and span the period 1960:07 to 2015:04.
Table 2: Variance Decomposition for SVARs in System $(U_M, Y, U_F)'$

<table>
<thead>
<tr>
<th></th>
<th>SVAR $(U_M, ip, U_F)'$</th>
<th>SVAR $(U_M, emp, U_F)'$</th>
<th>SVAR $(U_M, Q_1, U_F)'$</th>
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<td><strong>Fraction variation in $U_M$</strong></td>
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<tr>
<td>1</td>
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<td>0.420</td>
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<td>$s_{\text{max}}$</td>
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<td>0.528</td>
<td>0.215</td>
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<td>[0.22, 0.71]</td>
<td>[0.05, 0.57]</td>
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<tr>
<td><strong>Fraction variation in $ip$</strong></td>
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<td></td>
<td></td>
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</tbody>
</table>

Each panel shows the fraction of $s$-step-ahead forecast-error variance of the variable given in the panel title that is explained by the shock named in the column heading. The row denoted “$s = s_{\text{max}}$” reports the maximum fraction (across all VAR forecast horizons $m$) of forecast error variance explained by the shock listed in the column heading. The numbers in parentheses represent the 5th and 95th percentiles of these statistics from bootstrapped samples using the procedure described in the Appendix. The data are monthly and span the period 1960:07 to 2015:04.
### Table 3: Variance Decomposition for SVARs in System \((U_R, Y, U_F)\)'

<table>
<thead>
<tr>
<th></th>
<th>SVAR ((U_R, ip, U_F))'</th>
<th>SVAR ((U_R, emp, U_F))'</th>
<th>SVAR ((U_R, Q_1, U_F))'</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Fraction variation in (U_R)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(s)</td>
<td>(U_R) Shock</td>
<td>ip Shock</td>
<td>(U_F) Shock</td>
</tr>
<tr>
<td>(s = 1)</td>
<td>0.359</td>
<td>0.513</td>
<td>0.128</td>
</tr>
<tr>
<td>(s = 12)</td>
<td>0.253</td>
<td>0.463</td>
<td>0.285</td>
</tr>
<tr>
<td>(s = \infty)</td>
<td>0.302</td>
<td>0.407</td>
<td>0.291</td>
</tr>
<tr>
<td>(s = s_{\text{max}})</td>
<td>0.302</td>
<td>0.407</td>
<td>0.291</td>
</tr>
<tr>
<td></td>
<td>[0.16, 0.72]</td>
<td>[0.18, 0.80]</td>
<td>[0.70, 0.63]</td>
</tr>
<tr>
<td><strong>Fraction variation in (ip)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(s)</td>
<td>(U_R) Shock</td>
<td>ip Shock</td>
<td>(U_F) Shock</td>
</tr>
<tr>
<td>(s = 1)</td>
<td>0.391</td>
<td>0.577</td>
<td>0.032</td>
</tr>
<tr>
<td>(s = 12)</td>
<td>0.295</td>
<td>0.456</td>
<td>0.249</td>
</tr>
<tr>
<td>(s = \infty)</td>
<td>0.211</td>
<td>0.326</td>
<td>0.463</td>
</tr>
<tr>
<td>(s = s_{\text{max}})</td>
<td>0.397</td>
<td>0.580</td>
<td>0.463</td>
</tr>
<tr>
<td></td>
<td>[0.10, 0.73]</td>
<td>[0.22, 0.89]</td>
<td>[0.08, 0.84]</td>
</tr>
<tr>
<td><strong>Fraction variation in (U_F)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(s)</td>
<td>(U_R) Shock</td>
<td>ip Shock</td>
<td>(U_F) Shock</td>
</tr>
<tr>
<td>(s = 1)</td>
<td>0.010</td>
<td>0.059</td>
<td>0.941</td>
</tr>
<tr>
<td>(s = 12)</td>
<td>0.011</td>
<td>0.083</td>
<td>0.906</td>
</tr>
<tr>
<td>(s = \infty)</td>
<td>0.117</td>
<td>0.093</td>
<td>0.790</td>
</tr>
<tr>
<td>(s = s_{\text{max}})</td>
<td>0.117</td>
<td>0.093</td>
<td>0.943</td>
</tr>
<tr>
<td></td>
<td>[0.04, 0.35]</td>
<td>[0.03, 0.52]</td>
<td>[0.56, 0.99]</td>
</tr>
</tbody>
</table>

Each panel shows the fraction of \(s\)-step-ahead forecast-error variance of the variable given in the panel title that is explained by the shock named in the column heading. The row denoted “\(s = s_{\text{max}}\)” reports the maximum fraction (across all VAR forecast horizons \(m\)) of forecast error variance explained by the shock listed in the column heading. The numbers in parentheses represent the 5th and 95th percentiles of these statistics from bootstrapped samples using the procedure described in the Appendix. The data are monthly and span the period 1960:07 to 2015:04.
Table 4: Variance Decomposition for SVARs in System \((U_M(12), Y, U_F(12))'\)

<table>
<thead>
<tr>
<th></th>
<th>(U_M(12)) Shock</th>
<th>(ip) Shock</th>
<th>(U_F(12)) Shock</th>
<th>(U_M(12)) Shock</th>
<th>(emp) Shock</th>
<th>(U_F(12)) Shock</th>
<th>(U_M(12)) Shock</th>
<th>(Q_1) Shock</th>
<th>(U_F(12)) Shock</th>
</tr>
</thead>
<tbody>
<tr>
<td>(s)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.548</td>
<td>0.432</td>
<td>0.020</td>
<td>0.621</td>
<td>0.360</td>
<td>0.019</td>
<td>0.590</td>
<td>0.381</td>
<td>0.029</td>
</tr>
<tr>
<td>12</td>
<td>0.763</td>
<td>0.219</td>
<td>0.018</td>
<td>0.776</td>
<td>0.212</td>
<td>0.012</td>
<td>0.801</td>
<td>0.168</td>
<td>0.031</td>
</tr>
<tr>
<td>(\infty)</td>
<td>0.635</td>
<td>0.206</td>
<td>0.159</td>
<td>0.682</td>
<td>0.135</td>
<td>0.183</td>
<td>0.692</td>
<td>0.202</td>
<td>0.106</td>
</tr>
<tr>
<td>(s_{\text{max}})</td>
<td>0.813</td>
<td>0.432</td>
<td>0.165</td>
<td>0.682</td>
<td>0.135</td>
<td>0.183</td>
<td>0.868</td>
<td>0.388</td>
<td>0.107</td>
</tr>
<tr>
<td></td>
<td>[0.48, 0.94]</td>
<td>[0.17, 0.66]</td>
<td>[0.06, 0.51]</td>
<td>[0.37, 0.96]</td>
<td>[0.10, 0.62]</td>
<td>[0.09, 0.52]</td>
<td>[0.48, 0.95]</td>
<td>[0.17, 0.61]</td>
<td>[0.04, 0.49]</td>
</tr>
<tr>
<td>1</td>
<td>0.379</td>
<td>0.591</td>
<td>0.030</td>
<td>0.342</td>
<td>0.355</td>
<td>0.303</td>
<td>0.384</td>
<td>0.602</td>
<td>0.014</td>
</tr>
<tr>
<td>12</td>
<td>0.124</td>
<td>0.757</td>
<td>0.119</td>
<td>0.076</td>
<td>0.433</td>
<td>0.491</td>
<td>0.099</td>
<td>0.748</td>
<td>0.154</td>
</tr>
<tr>
<td>(\infty)</td>
<td>0.202</td>
<td>0.697</td>
<td>0.101</td>
<td>0.269</td>
<td>0.482</td>
<td>0.250</td>
<td>0.256</td>
<td>0.623</td>
<td>0.121</td>
</tr>
<tr>
<td>(s_{\text{max}})</td>
<td>0.382</td>
<td>0.772</td>
<td>0.145</td>
<td>0.342</td>
<td>0.482</td>
<td>0.519</td>
<td>0.388</td>
<td>0.751</td>
<td>0.210</td>
</tr>
<tr>
<td></td>
<td>[0.20, 0.71]</td>
<td>[0.42, 0.93]</td>
<td>[0.04, 0.59]</td>
<td>[0.23, 0.76]</td>
<td>[0.17, 0.86]</td>
<td>[0.18, 0.88]</td>
<td>[0.23, 0.75]</td>
<td>[0.41, 0.96]</td>
<td>[0.05, 0.66]</td>
</tr>
<tr>
<td>1</td>
<td>0.091</td>
<td>0.002</td>
<td>0.907</td>
<td>0.273</td>
<td>0.090</td>
<td>0.637</td>
<td>0.059</td>
<td>0.001</td>
<td>0.940</td>
</tr>
<tr>
<td>12</td>
<td>0.165</td>
<td>0.017</td>
<td>0.819</td>
<td>0.389</td>
<td>0.108</td>
<td>0.503</td>
<td>0.127</td>
<td>0.016</td>
<td>0.858</td>
</tr>
<tr>
<td>(\infty)</td>
<td>0.200</td>
<td>0.162</td>
<td>0.638</td>
<td>0.448</td>
<td>0.165</td>
<td>0.387</td>
<td>0.178</td>
<td>0.151</td>
<td>0.671</td>
</tr>
<tr>
<td>(s_{\text{max}})</td>
<td>0.206</td>
<td>0.162</td>
<td>0.907</td>
<td>0.464</td>
<td>0.165</td>
<td>0.637</td>
<td>0.178</td>
<td>0.151</td>
<td>0.945</td>
</tr>
<tr>
<td></td>
<td>[0.04, 0.71]</td>
<td>[0.05, 0.46]</td>
<td>[0.37, 0.99]</td>
<td>[0.09, 0.76]</td>
<td>[0.04, 0.59]</td>
<td>[0.20, 0.94]</td>
<td>[0.04, 0.69]</td>
<td>[0.05, 0.48]</td>
<td>[0.40, 0.99]</td>
</tr>
</tbody>
</table>

Each panel shows the fraction of \(s\)-step-ahead forecast-error variance of the variable given in the panel title that is explained by the shock named in the column heading. The row denoted “\(s = s_{\text{max}}\)” reports the maximum fraction (across all VAR forecast horizons \(m\)) of forecast error variance explained by the shock listed in the column heading. The numbers in parentheses represent the 5th and 95th percentiles of these statistics from bootstrapped samples using the procedure described in the Appendix. The data are monthly and span the period 1960:07 to 2015:04.
Table 5: Tests of Validity of Recursive Restriction in System $(U_M, Y, U_F)'$

<table>
<thead>
<tr>
<th>Ordering:</th>
<th>$(U_M, ip, U_F)'$</th>
<th>$(U_M (12), ip, U_F (12))'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_0$: $B_{RY} = B_{RF} = B_{YF} = 0$</td>
<td>$239.54$</td>
<td>$127.75$</td>
</tr>
<tr>
<td></td>
<td>[110.79]</td>
<td>[38.60]</td>
</tr>
<tr>
<td>$H_0$: $B_{YR} = B_{YF} = B_{RF} = 0$</td>
<td>$25.96$</td>
<td>$275.35$</td>
</tr>
<tr>
<td></td>
<td>[65.89]</td>
<td>[47.22]</td>
</tr>
<tr>
<td>$H_0$: $B_{RY} = B_{RF} = B_{FY} = 0$</td>
<td>$225.18$</td>
<td>$123.08$</td>
</tr>
<tr>
<td></td>
<td>[113.74]</td>
<td>[43.26]</td>
</tr>
<tr>
<td>$\chi^2_{95%} (3)$</td>
<td>$7.81$</td>
<td>$7.81$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Ordering:</th>
<th>$(U_M, emp, U_F)'$</th>
<th>$(U_M (12), emp, U_F (12))'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_0$: $B_{RY} = B_{RF} = B_{YF} = 0$</td>
<td>$236.29$</td>
<td>$113.63$</td>
</tr>
<tr>
<td></td>
<td>[79.12]</td>
<td>[47.42]</td>
</tr>
<tr>
<td>$H_0$: $B_{YR} = B_{YF} = B_{RF} = 0$</td>
<td>$70.73$</td>
<td>$229.54$</td>
</tr>
<tr>
<td></td>
<td>[53.61]</td>
<td>[69.62]</td>
</tr>
<tr>
<td>$H_0$: $B_{RY} = B_{RF} = B_{FY} = 0$</td>
<td>$228.85$</td>
<td>$116.15$</td>
</tr>
<tr>
<td></td>
<td>[88.95]</td>
<td>[63.02]</td>
</tr>
<tr>
<td>$\chi^2_{95%} (3)$</td>
<td>$7.81$</td>
<td>$7.81$</td>
</tr>
</tbody>
</table>

The table reports the Wald test statistic for testing the null hypothesis given in the column. The bold indicates that Wald test rejects the null at 95 percent level according to $\chi^2(3)$ distribution. The SVAR system is solved using GMM and delta method is used for computing the standard error. Estimates of $B$ are based on the SVAR identified with external instruments described in the text. The mean of bootstrap Wald statistics is reported in parenthesis. The sample size spans 1960:07 to 2015:04.
Appendix Figures and Tables

Figure A1: Pre-2008 Dynamic Responses in SVAR $(U_M, emp, U_F)'$

Bootstrapped 90% error bands appear as dashed lines. Response units are reported in percentage points. The sample spans the period 1960:07 to 2007:12.
Figure A2: Dynamic Responses using 1987 Crash Dummies in SVAR($U_M, emp, U_F$)

Bootstrapped 90% error bands appear as dashed lines. Dummies for 1987:10 and 1989:11 are included in VAR estimation. Response units are reported in percentage points. The sample spans the period 1960:07 to 2015:04.
<table>
<thead>
<tr>
<th>Ordering:</th>
<th>((U_R, Y, U_F))'</th>
<th>((U_R (12), ip, U_F (12))')</th>
</tr>
</thead>
<tbody>
<tr>
<td>(H_0: B_{RY} = B_{RF} = B_{YF} = 0)</td>
<td>133.69</td>
<td>303.24</td>
</tr>
<tr>
<td></td>
<td>[71.23]</td>
<td>[77.88]</td>
</tr>
<tr>
<td>(H_0: B_{YR} = B_{YF} = B_{RF} = 0)</td>
<td>29.11</td>
<td>167.57</td>
</tr>
<tr>
<td></td>
<td>[35.83]</td>
<td>[52.54]</td>
</tr>
<tr>
<td>(H_0: B_{RY} = B_{RF} = B_{FY} = 0)</td>
<td>130.41</td>
<td>306.34</td>
</tr>
<tr>
<td></td>
<td>[77.34]</td>
<td>[72.79]</td>
</tr>
<tr>
<td>(\chi^2_{5%}(3))</td>
<td>7.81</td>
<td>7.81</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Ordering:</th>
<th>((U_R, emp, U_F))'</th>
<th>((U_R (12), emp, U_F (12))')</th>
</tr>
</thead>
<tbody>
<tr>
<td>(H_0: B_{RY} = B_{RF} = B_{YF} = 0)</td>
<td>178.68</td>
<td>327.91</td>
</tr>
<tr>
<td></td>
<td>[62.11]</td>
<td>[76.35]</td>
</tr>
<tr>
<td>(H_0: B_{YR} = B_{YF} = B_{RF} = 0)</td>
<td>85.58</td>
<td>244.85</td>
</tr>
<tr>
<td></td>
<td>[46.43]</td>
<td>[67.50]</td>
</tr>
<tr>
<td>(H_0: B_{RY} = B_{RF} = B_{FY} = 0)</td>
<td>154.76</td>
<td>310.66</td>
</tr>
<tr>
<td></td>
<td>[76.22]</td>
<td>[78.04]</td>
</tr>
<tr>
<td>(\chi^2_{5%}(3))</td>
<td>7.81</td>
<td>7.81</td>
</tr>
</tbody>
</table>

The table reports the Wald test statistic for testing the null hypothesis given in the column. The bold indicates that Wald test rejects the null at 95 percent level according to \(\chi^2(3)\) distribution. The SVAR system is solved using GMM and delta method is used for computing the standard error. Estimates of \(B\) are based on the SVAR identified with external instruments described in the text. The mean of bootstrap Wald statistics is reported in parenthesis. The sample size spans 1960:07 to 2015:04.