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ABSTRACT

Uncertainty about the future rises in recessions. But is uncertainty a source of business cycle fluctuations or an endogenous response to them, and does the type of uncertainty matter? Answer: we find that sharply higher uncertainty about real economic activity in recessions is fully an endogenous response to other shocks that cause business cycle fluctuations, while uncertainty about financial markets is a likely source of the fluctuations. Financial market uncertainty has quantitatively large negative consequences for several measures of real activity including employment, production, and orders. Such are the main conclusions drawn from estimation of three-variable structural vector autoregressions. To establish causal effects, we propose an iterative projection IV (IPIV) approach to construct external instruments that are valid under credible interpretations of the structural shocks.

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1 Introduction

A large literature in macroeconomics investigates the relationship between uncertainty and business cycle fluctuations. Interest in this topic has been spurred by a growing body of evidence that uncertainty rises sharply in recessions. This evidence is robust to the use of specific proxy variables such as stock market volatility and forecast dispersion as in Bloom (2009), or a broad-based measure of macroeconomic uncertainty, as in Jurado, Ludvigson, and Ng (2015) (JLN hereafter). But while this evidence substantiates a role for uncertainty in deep recessions, the question of whether uncertainty is an exogenous source of business cycle fluctuations or an endogenous response to economic fundamentals is not fully understood. Existing results are based on convenient but restrictive identifying assumptions and have no explicit role for financial markets, even though the uncertainty measures are correlated with financial variables. This paper considers a novel identification strategy to disentangle the causes and consequences of real and financial uncertainty.

The question of causality and the identification of exogenous variation in uncertainty is a long-standing challenge of the uncertainty literature. The challenge arises in part because there is no theoretical consensus on whether the uncertainty that accompanies deep recessions is primarily a cause or effect (or both) of declines in economic activity. Theories in which uncertainty is defined as the time varying volatility of a fundamental shock cannot address this question because, by design, there is no feedback response of uncertainty to other shocks if the volatility process is specified to evolve exogenously. And, obviously, models in which there is no exogenous variation in uncertainty cannot be used to analyze the direct effects of uncertainty shocks. It is therefore not surprising that many theories for which uncertainty plays a role in recessions reach contradictory conclusions on this question, as we survey below. It is clear that the body of theoretical work on uncertainty does not provide precise identifying restrictions for empirical work.

A separate challenge of the uncertainty literature pertains to the origins of uncertainty. Classic theories assert that uncertainty originates from economic fundamentals such as productivity, and that such real economic uncertainty, when interacted with market frictions, discourages real activity. But some researchers have argued that uncertainty dampens the economy through its influence on financial markets (e.g., Gilchrist, Sim, and Zakrajsek (2010)). Moreover, as surveyed by Ng and Wright (2013), all the post-1982 recessions have origins in financial markets, and these recessions have markedly different features from recessions where financial markets play a passive role. From this perspective, if financial shocks are subject to time-varying volatility, financial market uncertainty–as distinct from real economic uncertainty–could be a key player in recessions, both as a cause and as a propagating mechanism. The Great Recession of 2008, characterized by sharp swings in financial markets, hints at such a linkage. Yet so far the literature has not disentangled the contributions of real versus financial uncertainty to business cycle fluctuations.

Econometric analyses aimed at understanding the role of uncertainty for business cycle fluctuations face their own challenges. Attempts to identify the "effects" of uncertainty shocks in existing empirical work are primarily based on recursive schemes within the framework of vector-autoregressions (VAR).[1] But studies differ according to whether uncertainty is ordered ahead of or after real activity variables in the VAR. While a recursive structure is a reasonable starting point, any presumed ordering of the variables is hard to defend on theoretical grounds given the range of models in the literature. Contemporaneous changes in uncertainty can arise both as a cause of business cycle fluctuations and as a response to other shocks. Recursive structures explicitly rule out this possibility since they presume that some variables respond only with a lag to others.

It is with these challenges in mind that we return to the questions posed above: is uncertainty primarily a source of business cycle fluctuations or a consequence of them? And what is the relation of real versus financial uncertainty to business cycle fluctuations? The objective of this paper is to address these questions econometrically using a small-scale structural vector autoregression (SVAR). To confront the challenges just discussed, we take a two-pronged approach. First, our empirical analysis explicitly distinguishes *macro* uncertainty from *financial* uncertainty. The baseline SVAR we study describes the dynamic relationship between three variables: an index of macro uncertainty, U_{Mt}, a measure of real economic activity, Y_t (e.g., production, employment), and a new financial uncertainty index introduced here, U_{Ft}. Second, rather than relying on timing assumptions for identification, we use a different identification scheme that is less restrictive, both because it allows for simultaneous feedback between uncertainty and real activity, and because it can be used to test whether a lower recursive structure is supported by the data.

Specifically, our identification scheme relies on the existence of two external instruments for uncertainty that are not part of the SVAR: a Z_{1t} that is correlated with macro and financial uncertainty but contemporaneously uncorrelated with real activity, and a Z_{2t} that is correlated with financial uncertainty but contemporaneously uncorrelated with both real activity and macro uncertainty. While such ideal instruments have no empirical counterparts, we propose an *iterative projection IV* (IPIV) approach to construct Z_{1t} and Z_{2t} with the desired properties from observables. The approach takes a variable S_t that is not in the VAR system and uses projections to decompose it into two components, one that is correlated with a subset of the endogenous variables of interest, and one that is orthogonal to it. The orthogonal component is then used as an external instrumental variable (IV) for the remaining endogenous variables.

[1] See Bachmann, Elstner, and Sims (2013), Bloom (2009), Bloom (2014), Bekaert, Hoerova, and Duca (2013), Gilchrist, Sim, and Zakrajsek (2010), and JLN.

In the present context, the key is to find observables that are external to our SVAR, and are driven by a multitude of innovations including the uncertainty shocks that we are interested in. We argue that both theory and evidence suggest that aggregate stock market returns are such variables. Our IPIV approach therefore generates an instrument Z_{1t} by purging the effects of real activity shocks from stock returns, and another instrument Z_{2t} by further purges the effects of macro uncertainty shocks. Iteration ensures that the shocks used to generate the instruments are consistent with those identified by our SVAR. With this procedure, instrument exogeneity holds by construction and instrument relevance can be verified using the sample covariances and the estimated parameters. Details are given below.

The empirical exercise also requires that appropriate measures of macro and financial uncertainty are available. To this end, we exploit a data rich environment, working with 134 macro monthly time series and 147 financial variables. The construction of macro uncertainty follows JLN. The same approach is used to construct a broad-based measure of financial uncertainty that has never been used in the literature. Macro uncertainty is itself an aggregate of uncertainties in variables from three categories: real activity, price, and financial. To better understand the contributions of each of these categories, we also replace U_{Mt} in the VAR with an uncertainty measure based on the sub-components, one at a time. Uncertainty about real activity is of special interest because classic uncertainty theories postulate that uncertainty shocks have their origins in economic fundamentals and hence should show up as uncertainty about real economic activity. We compare "short-run" uncertainty about outcomes over the next one month, with "longer horizon " uncertainty about outcomes a year hence.

Before summarizing our main results, it should be made clear that the structural shocks we identify do not in general correspond to primitive shocks in specific economic models. Real activity is endogenous and may respond to any number of primitive shocks (technology, monetary policy, preferences, wage or price markups, government expenditures, etc.). If a SVAR representation exists, our identified real activity shock would then be a composite of these primitive shocks, with the restriction that this composite be orthogonal to the other shocks in our system. The same could be said for either type of uncertainty, to the extent that these variables are endogenous. Our objective is not, therefore, to identify primitive shocks in specific models. Indeed, we argue that the questions raised above are ultimately empirical ones that call out for a model-free approach. (See the literature review below for further discussion.) What our approach offers, therefore, is something different: if there exists an SVAR in the system of interest, then under the assumptions stipulated below, IPIV can provide a less restrictive means of identifying dynamic causal effects when commonly used ordering or timing assumptions are difficult to defend.

Our main results can be stated as follows. First, positive shocks to financial uncertainty are found to cause a sharp decline in real activity that persists for many months, lending support to the hypothesis that heightened uncertainty is an exogenous impulse that causes recessions. These effects are especially large for some measures of real activity, notably employment and orders. The finding that heightened uncertainty has negative consequences for real activity is qualitatively similar to that of preexisting empirical work that uses recursive identification schemes (e.g., Bloom (2009), JLN), but differs in that we trace the source of this result specifically to broad-based financial market uncertainty rather than to various uncertainty proxies or broad-based macro uncertainty. We also show that the converse is not supported by our evidence: exogenous shocks to real activity have little affect on financial uncertainty.

Second, the identification scheme used here reveals something new that is not possible to uncover under recursive schemes: macro and financial uncertainty have a very different dynamic relationship with real activity. Specifically, unlike financial uncertainty, sharply higher macro and real activity uncertainty in recessions is fully an endogenous response to business cycle fluctuations. That is, negative economic activity shocks are found to cause increases in both macro and real activity uncertainty, but there is no evidence that independent shocks to macro or real uncertainty cause lower economic activity. Indeed the opposite is true: exogenous shocks to both macro and real uncertainty are found to *increase* real activity, consistent with "growth options" theories discussed below.

Third, we investigate the timing of large adverse shocks in the SVAR systems. No matter which system we investigate, the Great Recession is a prominent example that is characterized by large negative real activity shocks and a large positive financial uncertainty shock but no corresponding large shock to real economic uncertainty, even though real economic uncertainty itself rose to unusual heights in this episode. This finding underscores the extent to which heightened uncertainty about real activity in recessions is more often an endogenous response to other shocks, rather than an exogenous impulse driving business cycles.

Our results are distinct from those obtained using recursive identification. Under any recursive ordering of the variables in our VAR, exogenous shocks that increase macro or real uncertainty appear to reduce real activity, in a manner that is qualitatively similar to financial uncertainty shocks. This result does not hold in the less restrictive SVAR studied here and appears to be an artifact of invalid timing assumptions under recursive identification. Further investigation reveals that the SVAR we study reflects a non-zero contemporaneous correlation between U_{Ft} and Y_t, as well as between U_{Mt} and Y_t, which is inconsistent with any recursive ordering. Tests of the validity of a recursive structure are easily rejected by the data.

The rest of this paper is organized as follows. Section 2 reviews related literature. Section 3 details the econometric framework and identification employed in our study, describes how our instruments are constructed, and discusses the data and empirical implementation. Section 4 presents empirical results using broad-based macro uncertainty U_{Mt}, while Section 5 reports results for systems that isolate sub-components of U_{Mt} corresponding to real activity and price variables. Section 6 reports results pertaining to robustness and additional cases. Section 7 summarizes and concludes.

2 Related Literature

A large literature addresses the question of uncertainty and its relation to economic activity.[2] Theories for which uncertainty plays a key role differ widely on the question of whether uncertainty is primarily a cause or a consequence of declines in economic activity. In most cases, it is modeled either as a cause or an consequence, but not both.

The first strand of the literature proposes uncertainty as a cause of lower economic growth. This includes models of the real options effects of uncertainty (Bernanke (1983), McDonald and Siegel (1986)), models in which uncertainty influences financing constraints (Gilchrist, Sim, and Zakrajsek (2010), Arellano, Bai, and Kehoe (2011)), or precautionary saving (Basu and Bundick (2012), Leduc and Liu (2012), Fernández-Villaverde, Pablo Guerrón-Quintana, and Uribe (2011)). These theories almost always presume that uncertainty is an exogenous shock to some economic fundamental. Some theories presume that higher uncertainty originates directly in the process governing technological innovation, which subsequently causes a decline in real activity (e.g., Bloom (2009), Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2012)).

A second strand of the literature postulates that higher uncertainty arises solely as a *response* to lower economic growth, emphasizing a variety of mechanisms. Some of these theories suggest that bad times incentivize risky behavior (Bachmann and Moscarini (2011), Fostel and Geanakoplos (2012)), or reduce information and with it the forecastability of future outcomes (Van Nieuwerburgh and Veldkamp (2006) Fajgelbaum, Schaal, and Taschereau-Dumouchel (2014)), or provoke new and unfamiliar economic policies whose effects are highly uncertain (Pástor and Veronesi (2013)), or create a greater misallocation of capital across sectors (Ai, Li, and Yang (2015)), or generate endogenous countercyclical uncertainty in consumption growth because investment is costly to reverse (Gomes and Schmid (2016)).

And yet a third literature has raised the possibility that some forms of uncertainty can actually *increase* economic activity. "Growth options" theories of uncertainty postulate that a mean-preserving spread in risk generated from an unbounded upside coupled with a limited downside can cause firms to invest and hire, since the increase in mean-preserving risk increases expected profits. Such theories were often used to explain the dot-com boom. Examples include Bar-Ilan and Strange (1996), Pastor and Veronesi (2006), Kraft, Schwartz, and Weiss (2013), Segal, Shaliastovich, and Yaron (2015).

This brief review reveals a rich literature with a wide range of predictions about the relationship between uncertainty and real economic activity. Yet the absence of a theoretical

[2] This literature has become voluminous. See Bloom (2014) for a recent review of the literature.

consensus on this matter, along with the sheer number of theories and limited body of evidence on the structural elements of specific models, underscores the extent to which the question of cause and effect is fundamentally an empirical matter that must be settled in an econometric framework with as little specific theoretical structure as possible, so that the various theoretical possibilities can be nested in empirical tests. Commonly used recursive identification schemes cannot achieve this objective, since by construction they rule out the possibility that uncertainty and real activity could influence one another within the period. Our econometric model nests any recursive identification scheme, so we can test whether such timing assumptions are plausible. We find they are rejected by the data.

Our construction of instruments for uncertainty builds on work in asset pricing emphasizing the idea that stock market variation is the result of several distinct (and orthogonal) sources of stochastic variation, some of which are likely to be uniquely suited as instruments for our uncertainty measures. For example, one quantitatively important component is attributable to *a*cyclical risk premia variation, and more generally appears to be uncorrelated with most measures of real activity.[3] This component is valuable for our objective because it is exogenous to real activity, but may still be relevant for both macro and financial uncertainty, as in our Z_{1t}. Yet another component could be attributable to fluctuations in factors like corporate leverage, or in the risk aversion or "sentiment" of market participants that may be correlated with the volatility of the stock market. In equilibrium asset pricing models, if leverage increases, volatility of the corporate sector's equity return increases. Thus changes in factors like leverage (and possibly changes in risk aversion or sentiment) should be correlated with financial uncertainty, but have little to do with real economic uncertainty. This component is valuable for our objective because it is plausibly uncorrelated with both real activity *and* uncertainty about economic fundamentals, but may still be relevant for financial market uncertainty, as in our Z_{2t}. Consistent with the existence of this type of component, JLN document that there are many spikes in stock market uncertainty that do not coincide with an important movement in either real activity or macro uncertainty. These findings motivate our maintained hypothesis that measures of equity market activity are promising non-uncertainty variable comprised of several distinct sources of stochastic variation, two of which have the statistical characteristics of Z_{1t} and Z_{2t}.

Our IPIV approach is related to a recent line of econometric research in SVARs that uses information contained in external instruments to identify structural dynamic causal effects.[4]

[3]For empirical evidence, see Lettau and Ludvigson (2013), Greenwald, Lettau, and Ludvigson (2014), Kozak and Santosh (2014), and Muir (2014). Theoretical examples include Greenwald, Lettau, and Ludvigson (2014); Bianchi, Ilut, and Schneider (2014); Gourio (2012);Wachter (2013); Brunnermeier and Sannikov (2012); Gabaix and Maggiori (2013); He and Krishnamurthy (2013).

[4]See for example Hamilton (2003), Kilian (2008), Mertens and Ravn (2013); Stock and Watson (2008), Stock and Watson (2012), and Olea, Stock, and Watson (2015).

Of these, Stock and Watson (2012) study uncertainty shocks, using a measure of stock market volatility and/or a news media measure of policy uncertainty from Baker, Bloom, and Davis (2013), as separate external instruments for identifying the effects of uncertainty shocks in a SVAR. Our study differs in some fundamental ways. First, Stock and Watson (2012) focus exclusively on identifying the effects of uncertainty shocks and do not attempt to simultaneously identify the converse, namely the effects of real activity shocks on uncertainty. Second, the identification strategy in Stock and Watson (2012) for uncertainty shocks presumes that the series themselves (i.e., stock market volatility, policy uncertainty) are valid instruments, correlated with the uncertainty shock of interest but not with the other shocks. By contrast, our approach explicitly views both the stock market and our uncertainty measures as partly endogenous, forcing us to confront the identification quandary. Our identification assumption is instead that the aggregate stock market return contains components that satisfy population exogeneity restrictions, even while some of its variation is endogenous.

The study arguably closest in spirit to our identification approach is Baker and Bloom (2013), who use disaster-like events as instruments for stock market volatility with the aim of isolating exogenous variation in uncertainty. This has some similarities with our approach, in that it implicitly assumes that certain components of stock market fluctuations (those associated with "disasters") are exogenous. In contrast to our approach, exogenous events are chosen subjectively rather than constructed econometrically to satisfy specific orthogonality restrictions. It is of interest that we arrive at complementary conclusions, despite the differing methodologies for identifying exogenous variation.

3 Econometric Framework

This section explains our econometric approach. Subsection 1 explains the identification strategy. Subsections 2 and 3 explain the construction of external instruments and the uncertainty measures. This is followed by a discussion of the estimation procedure.

3.1 The SVAR and Identification

Our analysis is based on a structural vector autoregressive model (SVAR). Let $\mathbf{X}_t$ denote a $K \times 1$ time series. We suppose that the structural model has a p-th order vector autoregressive representation

$$\mathbf{X}_t = \mathbf{k} + \mathbb{A}_1\mathbf{X}_{t-1} + \mathbb{A}_2\mathbf{X}_{t-2} + \cdots + \mathbb{A}_p\mathbf{X}_{t-p} + \mathbf{H\Sigma e}_t. \tag{1}$$

$$\mathbf{e}_t \sim (0, \mathbf{I_K}), \qquad \mathbf{\Sigma} = \begin{pmatrix} \sigma_{11} & 0 & \cdot & 0 \\ 0 & \sigma_{22} & 0 & 0 \\ 0 & \cdot & \cdot & 0 \\ 0 & 0 & \cdot & \sigma_{KK} \end{pmatrix}.$$

The structural shocks $\mathbf{e}_t$ are mean zero with unit variance, and are serially and mutually uncorrelated. The corresponding structural $MA(\infty)$ representation of $\mathbf{X}_t$ is

$$\mathbf{X}_t = \boldsymbol{\mu} + \boldsymbol{\Psi}(L)\,\mathbf{H}\boldsymbol{\Sigma}\mathbf{e}_t,$$

where $\boldsymbol{\Psi}(L) = \boldsymbol{\Psi}_0 + \boldsymbol{\Psi}_1 L + \boldsymbol{\Psi}_2 L^2 + \ldots$ with $\boldsymbol{\Psi}_0 = \mathbf{I}$ is a polynomial in the lag operator L of infinite order, $\boldsymbol{\Psi}_s$ is the $(n \times n)$ matrix of coefficients for the sth lag of $\boldsymbol{\Psi}(L)$. Note that $\boldsymbol{\Psi}(L) = \mathbb{A}(L)^{-1}$, where $\mathbb{A}(L) = \mathbf{I} - \mathbb{A}_1 L - \cdots - \mathbb{A}_p L^p$.

The reduced form representation of $\mathbf{X}_t$ is a p-th order vector-autoregression (VAR) with corresponding reduced-form $MA(\infty)$ representation

$$\begin{aligned} \mathbf{X}_t &= \boldsymbol{\mu} + \boldsymbol{\Psi}(L)\,\boldsymbol{\eta}_t \\ \boldsymbol{\eta}_t &\sim (0, \boldsymbol{\Omega}), \quad \boldsymbol{\Omega} = \mathbb{E}(\boldsymbol{\eta}_t \boldsymbol{\eta}_t'). \end{aligned} \tag{2}$$

The structural shocks $\mathbf{e}_t$ are presumed to be related to the reduced form innovations by an invertible $K \times K$ matrix $\mathbf{H}$:

$$\boldsymbol{\eta}_t = \mathbf{H}\boldsymbol{\Sigma}\mathbf{e}_t \equiv \mathbf{B}\mathbf{e}_t,$$

where $\mathbf{B} \equiv \mathbf{H}\boldsymbol{\Sigma}$. We say that an SVAR for $\mathbf{X}_t$ exists if a rotation $\mathbf{H}^{-1}$ of the reduced form shocks $\boldsymbol{\eta}_t$ can be found such that its elements are serially and mutually uncorrelated.

A normalization is required to pin down the sign and scale of the shocks. We adopt the unit effect normalization

$$diag(\mathbf{H}) = 1. \tag{3}$$

Throughout, we restrict the admissible parameter space such that

$$\sigma_{jj} \geq 0 \tag{4}$$

for all j.

The objective of the exercise is to study the dynamic effects and the relative importance of the structural shocks. More precisely, the dynamic response to shock j is summarized by the impulse response function (IRF):

$$\frac{\partial \mathbf{X}_{t+s}}{\partial e_{jt}} = \boldsymbol{\Psi}_s \mathbf{b}^j, \tag{5}$$

where $\mathbf{b}^j$ is the jth column of $\mathbf{B}$. The structural IRF $\boldsymbol{\Psi}_s \mathbf{b}^j$ gives the dynamic response of $\mathbf{X}_{t+s}$ to a one standard deviation shock. The quantitative importance of each shock is given by the fraction of S-step ahead forecast error variance of $\mathbf{X}_t$ that is attributable to each structural shock. The coefficient matrices of $\boldsymbol{\Psi}(L)^{-1}$ are identified from the projection of $\mathbf{X}_t$ onto its lags in the reduced form VAR (2). The SVAR identification problem therefore amounts to identifying the elements of $\mathbf{H}$ and $\boldsymbol{\Sigma}$, from which the structural IRFs are computed.

Let Y_t denote a measure of real activity. Our objective is to study the impulse and propagating mechanism of uncertainty shocks, as well as how uncertainty reacts to shocks to Y_t, while explicitly distinguishing between macro and financial market uncertainty. Let $K = 3$. Hence our baseline SVAR is based on $\mathbf{X}_t = (U_{Mt}, Y_t, U_{Ft})'$, where U_{Mt} denotes macro uncertainty, U_{Ft} denotes financial uncertainty. The reduced form shocks $\boldsymbol{\eta}_t = (\eta_{Mt}, \eta_{Yt}, \eta_{Ft})'$ are linear combinations of the three structural form shocks $\mathbf{e}_t = (e_{Mt}, e_{Yt}, e_{Ft})'$ to macro uncertainty, real activity, and financial uncertainty, respectively.

$$\begin{aligned}
\eta_{Mt} &= B_{MM}e_{Mt} + B_{MY}e_{Yt} + B_{MF}e_{Ft} \\
\eta_{Yt} &= B_{YM}e_{Mt} + B_{YY}e_{Yt} + B_{YF}e_{Ft} \\
\eta_{Ft} &= B_{FM}e_{Mt} + B_{FY}e_{Yt} + B_{FF}e_{Ft},
\end{aligned}$$

where B_{ij} is the element of $\mathbf{B}$ that gives the contemporaneous effect of the jth structural shock on the ith variable. The covariance structure of $\boldsymbol{\eta}_t$ provides $K(K+1)/2 = 6$ equations in $\mathbf{B}$:

$$\text{vech}(\boldsymbol{\Omega}) = \text{vech}(\mathbf{B}\mathbf{B}') \tag{6}$$

where vech($\boldsymbol{\Omega}$) stacks the unique elements of the symmetric matrix $\boldsymbol{\Omega}$. Since there are nine unknown elements in $\mathbf{B}$, we need three more conditions for exact identification.

To identify these elements, we use two external instruments, denoted $Z_t = (Z_{1t}, Z_{2t})'$. For now, suppose that we have measures of Y_t, U_{Mt}, U_{Ft}, and two generic instruments, Z_{1t} and Z_{2t}.

Assumption A: For $K = 3$, let Z_{1t} and Z_{2t} be two instrumental variables such that

$$\begin{array}{llll}
(A.i) & \mathbb{E}[Z_{1t}e_{Mt}] = \phi_{1M}, & \mathbb{E}[Z_{1t}e_{Yt}] = 0, & E[Z_{1t}e_{Ft}] = \phi_{1F} \\
(A.ii) & \mathbb{E}[Z_{2t}e_{Mt}] = 0, & \mathbb{E}[Z_{2t}e_{Yt}] = 0, & \mathbb{E}[Z_{2t}e_{Ft}] = \phi_{2F}.
\end{array}$$

Assumption A are conditions for instrument exogeneity and relevance. Z_{1t} is an instrument that is correlated with both macro and financial uncertainty, but contemporaneously uncorrelated with real activity. By contrast, Z_{2t} is an instrument that is correlated with financial uncertainty, but contemporaneously uncorrelated with macro uncertainty and real activity.

Let $\mathbf{m}_{1t} = (\text{vech}(\eta_t\eta_t'), \text{vec}\,(Z_t \otimes \eta_t))'$ and $\boldsymbol{\beta}_1 = \text{vec}(\mathbf{B})$. At the true value of $\boldsymbol{\beta}_1$, denoted $\boldsymbol{\beta}_1^0$, the model satisfies

$$0 = \mathbb{E}[\mathbf{g}_1(\mathbf{m}_{1t}; \boldsymbol{\beta}_1^0)], \tag{7}$$

written out in full as follows:

$$
\begin{aligned}
0 &= var(\eta_M) - B_{MM}^2 + B_{MY}^2 + B_{MF}^2 \\
0 &= var(\eta_Y) - B_{YM}^2 + B_{YY}^2 + B_{YF}^2 \\
0 &= var\left(\eta_F\right) - B_{FM}^2 + B_{FY}^2 + B_{FF}^2 \\
0 &= cov(\eta_M, \eta_Y) - B_{MM}B_{YM} + B_{MY}B_{YY} + B_{MF}B_{YF} \\
0 &= cov(\eta_Y, \eta_F) - B_{YM}B_{FM} + B_{YY}B_{FY} + B_{FF}B_{YF} \\
0 &= cov\left(\eta_{M,}\eta_F\right) - B_{MM}B_{FM} + B_{MY}B_{FY} + B_{MF}B_{FF} \\
0 &= B_{MF}\mathbb{E}[Z_{2t}\eta_Y] - B_{YF}\mathbb{E}[Z_{2t}\eta_{Mt}] \\
0 &= B_{FF}\mathbb{E}[Z_{2t}\eta_{Yt}] - B_{YF}\mathbb{E}[Z_{2t}\eta_{Ft}] \\
0 &= (B_{MM}B_{FF} - B_{MF}B_{FM})\mathbb{E}[Z_{1t}\eta_{Yt}] - (B_{YF}B_{FM} - B_{YM}B_{FF})\mathbb{E}[Z_{1t}\eta_{Mt}] \\
&\quad -(B_{MM}B_{YF} - B_{MF}B_{YM})\mathbb{E}[Z_{1t}\eta_{Ft}].
\end{aligned}
$$

The model has nine equations in nine unknowns. The first six are from the covariance structure. The next two equations are due to the three moments implied by Assumption $(A.ii)$. The final equation is due to the three moments implied by Assumption $(A.i)$.

Proposition 1 *Under Assumption A with $\phi_{1M} \neq 0, \phi_{1F} \neq 0, \phi_{2F} \neq 0$, the normalization (3), and the restriction (4), $\boldsymbol{\beta}_1$ is identified.*

The Appendix gives a proof of identification using a closed-form solution for $\mathbf{B}$, and we show that the covariance between the instruments and the structural shocks can be expressed as

$$
\begin{aligned}
\mathbb{E}[Z_{2t}e_{Ft}]^2 &= \mathbb{E}[\eta_t Z_{2t}]'^{-1}\mathbb{E}[\eta_t Z_{2t}] \\
\mathbb{E}[Z_{1t}e_{Mt}]^2 &= \left(\mathbb{E}[\eta_t Z_{1t}] - \frac{\mathbb{E}[\eta_t Z_{2t}]}{\mathbb{E}[Z_{2t}e_{Ft}]}\right)'\Omega^{-1}\left(\mathbb{E}[\eta_t Z_{1t}] - \frac{\mathbb{E}[\eta_t Z_{2t}]}{\mathbb{E}[Z_{2t}e_{Ft}]}\right) \\
\mathbb{E}[Z_{2t}e_{Ft}]\mathbb{E}[Z_{1t}e_{Ft}] &= \mathbb{E}[\eta_t Z_{2t}]'\Omega^{-1}\mathbb{E}[\eta_t Z_{1t}].
\end{aligned}
$$

We verify that the closed-form solution is the the same as the unique numerical solution obtained with (3) and (4) imposed.

In essence, identification in our analysis is achieved by (i) using movements in U_{Mt} and U_{Ft} that are correlated with Z_{1t} to identify the effects of uncertainty shocks and disentangle them from shocks to real activity, (ii) using movements in U_{Ft} that are correlated with Z_{2t} to identify the effects of U_{Ft} shocks and disentangle them from macro uncertainty shocks, and (iii) using movements in Y_t that are uncorrelated with both Z_{1t} and Z_{2t} to identify the effects of real activity shocks and disentangle them from uncertainty shocks.

We take the stand in this application that our uncertainty measures are potentially endogenous. It is then natural to ask why we do not simply find observable instruments. We avoid instrumenting one measure uncertainty with an uncertainty proxy (e.g., stock market volatility). JLN find that such measures, including the options-based volatility index VXO, are less defensible measures of uncertainty than those employed here, so it makes little sense to instrument for the latter with the former. Options-based measures of stock market volatility are doubly problematic because they are known to contain a large component attributable to changes in the variance risk premium (akin to movements in risk aversion), much of it that is *orthogonal* to realized volatility (e.g., Bollerslev, Tauchen, and Zhou (2009); Carr and Wu (2009)). Thus options-based volatility indexes such as the VXO or VIX are widely viewed to be "fear indexes" and are therefore less likely than stock market returns to be relevant for the uncertainty shocks of interest. Moreover, to the extent that some of this risk aversion component moves with our uncertainty measures, any uncertainty shock identified from movements in the VIX or VXO could be more reflective of countercyclical time-varying risk aversion rather than an exogenous movement in our uncertainty indexes. With these considerations in mind, the next subsection proposes a methodology for constructing the desired instruments.

3.2 Construction of Instruments

The external instruments Z_{1t} and Z_{2t} play an important role in our analysis but they have no observable counterpart. The next step is to develop a methodology to construct these variables. To motivate our method of IPIV, recall that two stage least squares uses projections to purge the endogenous variations from a relevant regressor. Our IPIV approach is similar in spirit except that we purge the endogenous variations from a observed variable that is not of first order relevance to our VAR system. The output of such a projection is a generated external instrument.

In the present context, we make use of observables S_t that are driven not only by our structural shocks $\mathbf{e}_t = (e_{Yt},\, e_{Mt}$ and $e_{Ft})'$, but also by other shocks collected into an e_{St} that are uncorrelated with $\mathbf{e}_t$. A theoretical premise of the paper is that uncertainty shocks should be reflected in aggregate equity returns. Thus our choice of S_t is a measure of stock market returns. Under these assumptions, we may represent S_t as

$$S_t = c_0 + c_Y Y_t + c_M U_{Mt} + c_F U_{Ft} + c_S(L) S_{t-1} + c_X(L)' \mathbf{X}_{t-1} + e_{St} \tag{8}$$

where $\mathbf{X}_t = (Y_t, U_{Mt}, U_{Ft})'$. The residual e_{St} could be driven by any number of shocks orthogonal to $\mathbf{e}_t$. One interpretation is risk premium shocks driven by factors orthogonal to uncertainty such as a pure sentiment shock (one not correlated with uncertainty), but the precise interpretation is not important to what follows. Obviously, S_t is an endogenous variable but it is external to the variable $\mathbf{X}_t$ system by assumption. From a regression point of view, omitting any component of $\mathbf{X}_t$ as an explanatory variable will yield inconsistently estimates of the parameters in (8). However, we are not interested in these parameters. Our objective in considering stock-market returns is solely to remove from it those variations due to e_{Mt} and/or e_{Yt}. More precisely, (8) motivates two (non-structural) representations of S_t (not necessarily the same variable):

$$S_t = \beta_{21} + \beta_{22} e_{Yt} + \beta_{20}(L) S_{t-1} + Z_{1t} \tag{9a}$$

$$S_t = \beta_{24} + \beta_{25} e_{Yt} + \beta_{26} e_{Mt} + \beta_{23}(L) S_{t-1} + Z_{2t}, \tag{9b}$$

Given the theory and evidence discussed above, our maintained hypothesis is that the stock market contains a component that is exogenous to real activity, but correlated with both uncertainty shocks, and another component that is exogenous to both real activity and macro uncertainty, but correlated with financial uncertainty. Equation (9a) forms an orthogonal decomposition of S_t into a component that is spanned by e_{Yt} and a component Z_{1t} that is orthogonal to e_{Yt}. Similarly, equation (9b) purges the effect of e_{Yt} and e_{Mt} from S_t to arrive at Z_{2t}. The two variables Z_{1t} and Z_{2t} are our desired instruments because they satisfy Assumption A by construction. Note, however, that Z_{1t} and Z_{2t} are forecastable since both U_{Mt} and U_{Ft} can be serially correlated.

Although written as such, the S_t variable on the left-hand-side of (9a) and (9b) need not be the same. Indeed, in our application we will use two different measures of stock market returns in (9a) and (9b), denoted S_{1t} and S_{2t}, respectively. Alternatively, one of these left-hand-side variables could be a nonstock market variable, as long as it is theoretically related to uncertainty shocks, and has a component unrelated to real activity. We discuss the data used for the regressands in (9a) and (9b) below.

Let $\mathbf{m}_{2t} = (1, S_t, S_{t-1}, e_{Yt}, e_{Mt})'$ and collect the projection coefficients in (9a) and (9b) into $\boldsymbol{\beta}_2$ whose population value is $\boldsymbol{\beta}_2^0$. The orthogonality conditions of the two projections can be compactly summarized by

$$0 = \mathbb{E}[\mathbf{g}_2(\mathbf{m}_{2t}; \boldsymbol{\beta}_2^0)]. \tag{10}$$

If e_Y and e_M were observed, then solving for the sample analog of (10) would produce estimates of Z_1 and Z_2 that satisfy Assumption A. However, these projections are infeasible because e_Y and e_M are not observed. In fact, the objective of this paper is to recover these shocks. We therefore propose a procedure to generate Z_1 and Z_2 using an iterative approach to jointly solve for shocks and instruments that satisfy the required exogeneity restrictions.

Let the $T \times 1$ vectors $\mathbf{e_M}^{(0)}$, $\mathbf{e_Y}^{(0)}$ be initial guesses and $i = 0$. The following steps are repeated until convergence:

i Replace $\mathbf{e_M}$ and $\mathbf{e_Y}$ in (9a) and (9b) by $\mathbf{e_M}^{(i)}$ and $\mathbf{e_Y}^{(i)}$. The projections give $\mathbf{Z}_1^{(i)}$ and $\mathbf{Z}_2^{(i)}$.

ii Use $\mathbf{Z_1}^{(i)}$ and $\mathbf{Z_2}^{(i)}$ to solve $\boldsymbol{\beta}_1$ using the nine equations defined by (7). Let $\boldsymbol{\beta}_1^{(i)}$ be the parameter estimates. Form $\mathbf{B}^{(i)}$ from $\boldsymbol{\beta}_1^{(i)}$.

iii Update the shocks to $\mathbf{e}^{(i+1)} = (\mathbf{e_M}^{(i+1)}, \mathbf{e_Y}^{(i+1)}, \mathbf{e_F}^{(i+1)}) = \left(\mathbf{B}^{(i)}\right)^{-1} \hat{\boldsymbol{\eta}}$.

iv If $\|\mathbf{e_M}^{(i+1)} - \mathbf{e_M}^{(i)}\| \leq$ tol and $\|\mathbf{e_Y}^{(i+1)} - \mathbf{e_Y}^{(i)}\| <$ tol, stop and let $\mathbf{e} = \mathbf{e}^{(i)}, \boldsymbol{\beta}_1 = \boldsymbol{\beta}_1^{(i)}$. Else, set $i = i + 1$ and return to (i).

Several points about the implementation of this approach bear discussion. First, there remains the question of choosing starting values. We initialize $e_{Yt}^{(0)} = Y_t$ (where the Y_t series used for this $e_{Yt}^{(0)}$ is discussed below) and $e_{Mt}^{(0)} = U_{Mt}$. These initial values are a sensible starting place because they remove from $Z_{1t}^{(0)}$the variation in e_{Yt} attributable to direct effects of the shocks on Y_t (which is observed), and similarly remove from $Z_{2t}^{(0)}$ the variation in e_{Yt} attributable to direct effects on Y_t and the variation in e_{Mt} attributable to direct effects on U_{Mt} (also observed). Iteration purges any remaining indirect effects.

Second, shocks are eventually identified by estimates of $\mathbf{B}$ (since $\mathbf{e} = \mathbf{B}^{-1}\boldsymbol{\eta}$ by definition). Thus the procedure requires $\mathbf{B}^{-1}$ to exist. The solution for $\mathbf{B}$ is unique, conditional on a converged estimate of Z and $\mathbf{e}$. We search for a unique value of $\mathbf{B}$ within a compact space around the initialized values $e_{Yt}^{(0)}$ and $e_{Mt}^{(0)}$, resulting in a converged $\mathbf{B}$ and shocks $\mathbf{e}$. Starting values should be chosen thoughtfully, so that the converged shocks are plausible. Below we study the estimated shocks in detail and check that the magnitudes and signs are sensible (e.g., positive rather than negative shocks to financial uncertainty during the 1987 stock market crash and in the financial crisis of 2007-2009). The Appendix presents the details and output of a Monte Carlo study in which we verify that, if the true data generating process followed a form such as (1), while stock returns are generated from a process such as (8), the iterative procedure identifies the true $\mathbf{B}$.

Third, the iterative algorithm as described forces the exogeneity restrictions of Assumption A to be satisfied by construction, but does nothing to enforce the instrument relevance conditions, which can only be checked ex-post. Hence we place additional restrictions on the algorithm, discarding solutions generated by starting values that lead any of the covariances ϕ_{1M}, $\phi_{1F,}$ or ϕ_{2F} to be negligible or too small to identify our $\mathbf{B}$ well in the finite sample available. We focus on the starting values chosen because they deliver the highest average (absolute value of) the corresponding correlations $\rho(Z_{1t}, e_{Mt})$, $\rho(Z_{1t}, e_{Ft})$ and $\rho(Z_{2t}, e_{Ft})$, while maintaining that each must be non-zero individually.

Fourth, it is known that stock returns are predicted by stock market volatility, and volatility in the stock market carries a positive risk premium. The risk premium component is likely correlated with uncertainty shocks, which we want to identify. Thus we are careful to avoid starting values that purge Z_2 of the risk premium component. This includes initializing with residuals from an autoregression in U_{Mt} because lags of U_{Mt} are likely to be correlated with the risk premium component.

Finally, this procedure employs variables that are external to those in the VAR to construct instruments for identification. This inevitably imposes certain restrictions on a larger VAR that includes S_t. Below we make these restrictions precise and consider its robustness.

3.3 Measuring Uncertainty and Stock Market Returns

In our estimation we work with several different aggregate measures of uncertainty, which are indexes constructed over individual uncertainties for a large number of observable time-series. A long-standing difficulty with empirical research on this topic has been the measurement of uncertainty. JLN find that common uncertainty proxies contain economically large components of their variability that do not appear to be generated by a movement in genuine uncertainty across the broader economy. This occurs both because these proxies over-weight certain series in the measurement of aggregate uncertainty, and because they erroneously attribute forecastable fluctuations to a movement in uncertainty. Equity market volatility, for example, contains a non-trivial component generated from forecastable variation in stock returns. The estimated macro uncertainty index constructed in JLN is designed to address these issues and improve the measurement of aggregate uncertainty. The methodology used here for constructing uncertainty indexes follows JLN and we refer the reader to that paper for details.

Let $y_{jt}^C \in Y_t^C = (y_{1t}^C, \ldots, y_{N_C t}^C)'$ be a variable in category C. Its h-period ahead uncertainty, denoted by $\mathcal{U}_{jt}^C(h)$, is defined to be the volatility of the purely unforecastable component of the future value of the series, conditional on all information available. Specifically,

$$\mathcal{U}_{jt}^C(h) \equiv \sqrt{\mathbb{E}\left[(y_{jt+h}^C - \mathbb{E}[y_{jt+h}^C|I_t])^2|I_t\right]} \tag{11}$$

where I_t is information available. If the expectation today of the squared error in forecasting y_{jt+h} rises, uncertainty in the variable increases. Uncertainty in category C is an aggregate of individual uncertainty series in the category :

$$U_{Ct}(h) \equiv \text{plim}_{N_C \to \infty} \sum_{j=1}^{N_C} \frac{1}{N_C} \mathcal{U}_{jt}^C(h) \equiv \mathbb{E}_C[\mathcal{U}_{jt}^C(h)]. \tag{12}$$

As in JLN, the conditional expectation of squared forecast errors in (11) is computed from a stochastic volatility model, while the conditional expectation $\mathbb{E}[y_{jt+h}^C|I_t]$ is replaced by a diffusion index forecast, augmented to allow for nonlinearities. These are predictions of an autoregression augmented with a small number of common factors $q_t = (q_{1t}, \ldots, q_{rt})'$ estimated from a large number of economic time series x_{it} each with factor representation $x_{it} = \Lambda_{it}' q_t + e_{\chi,it}$. The use of large datasets reduces the possibility of biases that arise when relevant predictive information is ignored. Let $Y_t^C = (y_{1t}^C, \ldots, y_{N_C t}^C)'$ generically denote the series that we wish to compute uncertainty in. In this paper, we consider four categories of uncertainty:

Category (C)	Y_t^C	N_C
(M): Macro	all variables in χ^M	134
(F): Financial	all variables in χ^F	147
(R): Real activity	real activity variables in χ^M	73
(π): Price	price variables in χ^M	21

The uncertainty index U_{Ct} for category C is an equally-weighted average of the individual uncertainties in the category. We use two datasets covering the sample 1960:07-2015:04.[5] The first is a monthly *macro dataset*, $\mathcal{X}_t^M$, consisting of 134 mostly macroeconomic time series take from McCracken and Ng (2016). The second is a *financial dataset* $\mathcal{X}_t^F$ consisting of a 147 of monthly financial indicators, also used in Ludvigson and Ng (2007) and JLN, but updated to the longer sample. The real uncertainty index U_{Rt} is an equally-weighted average of the individual uncertainties about 73 series in Groups 1 through 4 of $\mathcal{X}^M$. These include output and income variables, labor market measures, housing market indicators, and orders and inventories. A second subindex is constructed using only measures of consumer and producer prices as well as oil prices, commodity prices and crude materials prices. We call this index *price uncertainty*, $U_{\pi t}$, which averages over the individual uncertainties of the 21 price series in Group 7 of $\mathcal{X}^M$. Additional predictors for variables in $\mathcal{X}_{it}^M$ include factors formed from $\mathcal{X}_{it}^F$ and vice-versa, squares of the first factor of each, and factors in the squares of individual series, $\left(\mathcal{X}_{it}^M\right)^2$ and $\left(\mathcal{X}_{it}^F\right)^2$.

Our estimation considers different VARs with different Y_t. In principle, we could initialize our e_{Yt} shock in the algorithm above with a different measure of Y_t, depending on the VAR system being estimated. For simplicity, we instead set $e_{Yt}^{(0)} = q_{1t}$ for this purpose, where q_{1t} is the first common factor estimated from the macro dataset no matter what measure of Y_t we use in the VAR. In fact, this turns out to often work better for identifying shocks and instruments that satisfy the instrument relevance conditions of Assumption A. This common factor has long been understood to be a "real activity factor" that loads heavily on measures of employment and production such as employees on nonfarm payrolls and manufacturing output, as well as measures of capacity utilization and new manufacturing orders in all vintages of χ^M used in this study, see McCracken and Ng (2016). It loads very little if at all on consumer and producer inflation measures, and financial market variables.

Our use of stock returns S_t to generate instruments is grounded in the theoretical premise that both macro and financial uncertainty shocks should be reflected in stock market returns. There is no reason, however, that the regressands in (9a) and (9b) must be exactly the same measure of stock market activity. All measures of stock market activity are highly correlated because they contain a large common component (much of which is orthogonal to the rest of the economy). In order to introduce some additional independent variation in our two instruments,

[5] A detailed description of the series is given in the Data Appendix of the online location where updated JLN uncertainty index data are posted: http://www.sydneyludvigson.com/s/jln_data_appendix_update.pdf

our base cases use different measures of aggregate stock market activity to generate Z_{1t} and Z_{2t}, although in practice we get very similar results if we use the same value-weighted stock market index return in (9a) and (9b). Specifically, we use the Standard and Poor 500 stock market index return, S_{Pt}, as the regressand for (9b), and $S_{\alpha t} = \alpha_p \text{crsp}_t + (1 - \alpha_p)\,\text{small}_t$, a portfolio weighted average of the return on the CRSP value-weighted stock index (in excess of the one-month Treasury bill rate) and the smallest decile stock market return in the NYSE as the regressand for (9a).[6] Our choice of portfolio weight α_p is guided by empirical considerations. The small stock index is highly volatile, which generates noise in the estimated SVAR parameters and large error bands for the impulse response functions. To facilitate more precise statistical inference, we set $\alpha_p = 0.94$ for the base case results presented below. Results not reported indicate the the dynamic responses are qualitatively similar if we set portfolio weights to give greater weight to the small stock index, including $\alpha_p = 0$, but the impulse response error bands are wider.

It is reasonable to ask if variables other than stock market returns could serve as regressands in (9a) and (9b). Asset returns other than those for the stock market come to mind, such as those for corporate bonds. Since bonds must return a fixed stream of payments to claimholders (a legal requirement set in the bond covenant), bonds are like stocks without the dividend risk. Our prior is that high frequency macro and financial uncertainty shocks are likely to be more closely related to earnings and dividend payouts than default events, so they should be more relevant for stock returns than bond returns. But bonds that have some nontrivial probability of defaulting might also be affected by uncertainty, at least to some degree. We consider this possibility in the Robustness and Additional Cases section below, where we present results for one estimation in which we generate Z_{1t} from the return on a portfolio of Baa rated corporate bonds.

The parameters to be estimated include the reduced form VAR parameters in (2), from which we obtain $\hat{\boldsymbol{\eta}}_t$, the parameters in (10), from which we construct Z_{1t} and Z_{2t}, and the structural parameters using results from the preceding two estimations. The sample moment conditions in the three-step estimation can be collected into $\bar{\mathbf{g}}(\mathbf{m}_t; \boldsymbol{\beta})$ where $\boldsymbol{\beta}$ are parameters to be estimated. The Generalized Method of Moments (GMM, Hansen (1982)) estimator is $\hat{\boldsymbol{\beta}} = \text{argmin}_\beta \bar{\mathbf{g}}(\mathbf{m}_t; \boldsymbol{\beta})' \bar{\mathbf{g}}(\mathbf{m}_t; \boldsymbol{\beta})$. Under regularity conditions, the GMM estimator of Hansen (1982) is $\sqrt{T}$ consistent for $\boldsymbol{\beta}^0$ and asymptotically normal with asymptotic variance $\boldsymbol{\Sigma}^2_{\hat{\beta}}$. This variance matrix is block lower triangular as in Newey (1984) since estimation of $\boldsymbol{\beta}_2$ is not affected by estimation of $\boldsymbol{\beta}_1$ or of the VAR. Serial correlation and heteroskedasticity robust standard errors are constructed as in Newey and West (1987).

The next section presents empirical results. We begin by studying systems with macro uncertainty. We then move on to consider sub-indexes of U_{Mt}, including real uncertainty formed only over real activity variables U_{Rt} and price uncertainty $U_{\pi t}$. Our final set of results re-

[6]The CRSP index is a value-weighted return of all stocks in NYSE, AMEX, and NASDAQ.

port several additional cases pertaining to different measures of real activity, different samples, different uncertainty horizons, and to using recursive identification schemes.

4 Results for $\mathbf{X}_t = (U_{Mt}, Y_t, U_{Ft})'$

Our first VAR is defined by $\mathbf{X}_t = (U_{Mt}(h), Y_t, U_{Ft}(h))'$. For the base case, we consider $h = 1$ (one-month uncertainty) and several measures of Y_t: the log of real industrial production, denoted ip_t, and the log of employment, denoted emp_t. While industrial production is a widely watched economic indicator of business cycles, it only captures goods-producing industries and has been a declining share of GDP. Employment only covers the labor market. Hence we also consider two additional measures of real activity: the log NAPM new orders index, which we denote noi, and the cumulated sum of the first common factor estimated from the macro dataset χ^M (since the raw data used to form q_{1t} are transformed to stationary), which we denote Q_{1t}. We linearly detrend each real activity series before estimation. Results using the first three of these measures of real activity are presented in this section. Results using the real activity index Q_{1t} and longer uncertainty horizons ($h = 12$) are discussed in Section 6 below. Since our emphasis is on $h = 1$, we write U_{Mt} instead of $U_{Mt}(1)$, and analogously for U_{Ft}, in order to simplify notation.

The top panel of Figure 1 plots the estimated macro uncertainty U_{Mt} in standardized units along with the NBER recession dates. The horizontal bar corresponds to 1.65 standard deviation above unconditional mean of each series (which is standardized to zero). As is known from JLN, the macro uncertainty index is strongly countercyclical, and exhibits large spikes in the deepest recessions. The updated data U_{Mt} series shows much the same. Though U_{Mt} exceeds 1.65 standard deviations 48 times, they are clustered around the 1973-74 and 1981-82 recessions, as well as the Great Recession of 2007-09. Macroeconomic uncertainty is countercyclical and has a correlation of -0.65 with the 12-month moving-average of the growth in industrial production.

The bottom panel of Figure 1 plots the financial uncertainty series U_{Ft} over time, which is new to this paper. U_{Ft} is a broad-based measure of time varying financial uncertainty using data from the bond market, stock market portfolio returns, and commodity markets. Hence, it is smoother than proxies such as VIX or any particular bond index. As seen from Figure 1, U_{Ft} is also countercyclical, though less so than U_{Mt}; the correlation with industrial production of -0.39. The series often exhibits spikes around the times when U_{Mt} are high. However, U_{Ft} is more volatile and spikes more frequently outside of recessions, the most notable being the 1987 stock market crash. Though observations on U_{Ft} exceed the 1.65 standard deviation line 33 times, they are spread out in seven episodes, with the 2008 and 1997 episodes being the most pronounced.

As is clear from Figure 1, both indicators of macro and financial uncertainty are serially correlated and hence predictable. They have comovements but also have independent variations as the correlation between them is 0.58. However, this unconditional correlation cannot be given a structural interpretation. The heightened uncertainty measures can be endogenous responses to events that are expected to happen, but they can also be exogenous innovations. We use a VAR to capture the predictable variations, and then identify uncertainty shocks from the VAR residuals using the restrictions described in the previous section.

4.1 VAR Estimates and Uncertainty Shocks

Several features of the VAR estimates are qualitatively similar for all measures of Y_t. Table 1 highlights some of these results. As shown in panel A, the sample correlation coefficient between Z_{1t} and $\hat{e}_{Mt}$ and $\hat{e}_{Ft}$, and between Z_{2t} and $\hat{e}_{Ft}$ are statistically significant and negative in each case, indicating that uncertainty shocks of both types are correlated with these instruments, as required, and tend to be high when these components of stock market returns are low. Panel A also shows that the correlation between Z_{1t} and $\hat{e}_{Yt}$, and the correlation between Z_{2t} and $\hat{e}_{Yt}$ and $\hat{e}_{Mt}$ are all zero as required, which is true by construction of the algorithm and solution for $\mathbf{B}$. Panel B shows that σ_{MM}, σ_{YY}, and σ_{FF} are all strongly statistically significantly different from zero. This in turn indicates the presence of both macro and financial uncertainty shocks in the SVAR. Since both U_{Mt} and U_{Ft} are serially correlated, we should therefore find that Z_{1t} is correlated with lags of U_{Mt} and U_{Ft}, while Z_{2t} is correlated with lags of U_{Ft}. Results not reported confirm this is the case.

Our model is exactly identified and does not permit a test for the validity of the restrictions. Nonetheless, we can test the if a lower recursive structure is supported by the data. Specifically, Assumption A does not rule out the possibility of a recursive structure. Given that $\sqrt{T}(\hat{\boldsymbol{\beta}}_1 - \boldsymbol{\beta}_1^0)$ is asymptotically $N(0, \mathbf{\Sigma}^2_{\hat{\beta}_1})$, the null hypothesis of a recursive structure is a test that the three components of $\boldsymbol{\beta}_1$ corresponding to the off-diagonal entries of $\mathbf{A}_0^{-1}$ are jointly zero. Hence it is chi-square distributed with three degrees of freedom. We first confirm that the test has the correct size in Monte Carlo simulations. Our estimates based on historical data strongly reject a lower triangular $\mathbf{A}_0^{-1}$ for *any* possible ordering of the variables. Table 2 shows results from Wald tests with $Y_t = ip_t$ and $Y_t = emp_t$, for $h = 1$ and $h = 12$. Results not reported find that the $\mathbf{A}$ matrix reflects a non-zero contemporaneous correlation between U_{Ft} and Y_t, as well as between U_{Mt} and Y_t; no recursive ordering is consistent with such a correlation. In Section 6 below, we discuss how estimates of the dynamic relationships are affected by imposing recursive identification.

Figure 2 presents the time series of the standardized shocks (e_M, e_{ip}, e_F) identified from the system with $Y_t = ip_t$. All shocks display strong departures from normality with excess skewness and/or excess kurtosis. The largest of the e_{ip} shocks is recorded in 2008:09, followed by 1974:11, and 1980:04. There also appears to be a moderation in the volatility of the ip shocks in the post-1983 period. The largest macro uncertainty shock is in 1970:12, followed by the shock in 2008:10. The largest financial uncertainty shock is recorded in 1987:10, followed by the shock in 2008:09. For e_F, the 1987 stock market crash evidently dwarfs all other spikes. Because of the extreme but transitory nature of the crash, there is a very large spike downward in e_F in the month following the crash, as the market recovered strongly. While this episode magnifies the spike in e_F in 1987, it is largely orthogonal to real activity and macro uncertainty and we have verified that none of our results are materially affected by dummying out the episode in the VAR. Appendix Figure A1 shows a representative set of impulse responses from one of our benchmark systems in which we dummy out 1987:10 and 1987:11. These responses are remarkably similar to those without the dummies, as shown below.

Observe that the large ip shock in 2005:09 is not associated with a contemporaneous spike in uncertainty (we discuss this episode further below), while there are several spikes in both types of uncertainty that do not coincide with spikes in e_{ip}. The next subsection uses impulse response functions to better understand the dynamic causal effects and propagating mechanisms of these shocks.

4.2 The Dynamic Effects of Uncertainty Shocks

Impulse response functions (IRFs) trace out the effects of counterfactual increases in the shocks. The estimated IRFs are presented with 90% bootstrapped confidence bands. All plots show responses to one standard deviation changes in ϵ_{jt} in the direction that leads to an *increase* in its own variable X_{jt}.

Figure 3 shows the dynamic responses of each variable in the SVAR to each structural shock for our baseline system with $Y_t = ip_t$. The responses show that positive shocks to *financial* uncertainty e_F lead to a sharp decline in real production that persists for many months (center plot, bottom row). Positive perturbations to e_{Ft} also cause U_{Mt} to increase. However, there is less evidence that shocks to macro uncertainty have effects on financial uncertainty: the impact response of U_{Ft} to an increase in e_{Mt} is not statistically different from zero. Overall, these results lend support to the hypothesis that heightened financial uncertainty is an exogenous impulse that causes declines in real activity. Note that the converse relationship is not supported by our evidence: exogenous (positive) shocks to ip have statistically insignificant effects on financial uncertainty. If anything, perturbations to e_{ip} modestly increase financial uncertainty in the long-run.

While we find no evidence that high financial uncertainty is a consequence of lower economic activity, the results for macro uncertainty are quite different. Figure 3 (second row, first column) shows that macro uncertainty falls sharply in response to positive shocks to industrial production, e_{ip}. Alternatively stated, negative ip shocks increase macro uncertainty sharply. These effects persist for well over a year after the ip shock. This result is strongly statistically significant, suggesting that higher macro uncertainty in recessions is a direct endogenous response to lower economic activity. However, there is no evidence that the negative correlation between macro uncertainty and real activity is driven by causality running in the opposite direction. Indeed, the top middle panel shows that exogenous increases in e_{Mt} actually *increase* real activity, consistent with growth options theories discussed above.

The standard error bands for this case with $Y_t = ip_t$ are wide, indicating considerable sampling uncertainty as to the magnitude of these effects. However, the systems that use $Y_t = ip_t$ appear to be unusual in this respect. The impulse responses are more precisely estimated when we use any number of alternative measures of real activity Y_t. Impulse responses using $Y_t = emp_t$ and $Y_t = noi_t$ are displayed in Figures 4 and 5, respectively. These systems tell the same story regarding the dynamic causal influences in the system, but here the responses have tighter standard error bands. A positive shock to emp_t or noi_t causes a sharp decline in macro uncertainty, whereas there is again no evidence that positive shocks to macro uncertainty cause declines either measure of real activity; indeed the opposite occurs. But positive shocks to financial uncertainty cause declines in both emp_t or noi_t. In contrast to the responses in systems using ip_t, these effects are strongly statistically significant in the systems using emp_t and noi_t. We find that this same result also holds for the responses using Q_{1t}, as discussed in Section 6 below.

4.3 The Structural Shocks and Decomposition of Variance

In Figure 1 presented earlier, we find 1973-74, 1981-82, and 2007-2009 to be the three episodes of heightened macroeconomic uncertainty, defined as the periods when U_{Mt} is 1.65 standard deviations above its unconditional mean. We now look for the "large adverse" shocks in the systems $(U_{Mt}, Y_t, U_{Ft})'$, with $Y_t = ip_t$, emp_t, noi_t , Q_{1t}. More precisely, we consider large positive uncertainty shocks and large negative real activity shocks.

Figure 6 displays the date and size of shocks that are at least two standard deviations above the mean, estimated using the four different measures of Y_t. In view of the non-normality of the shocks, the figure also plots horizontal lines corresponding to three standard deviation of the unit shocks, which is used as the reference point for 'large'. The lowest panel shows that, irrespective of the definition of Y_t, all SVARs identify big financial uncertainty shocks in 1987 and 2008. The middle panel shows that large negative real activity shocks are in alignment with all post-war recessions with one exception: the negative real activity shock in 2005 is not immediately associated with a recession, but it could be the seed of the Great Recession that followed. It is known that the housing market led the 2007-2009 recession (e.g., see Favilukis, Ludvigson, and Van Nieuwerburgh (2015) for a discussion). We confirm that all 10 housing series in $\mathcal{X}^M$ (most pertaining to housing starts and permits series) exhibit sharp declines starting in September 2005 and continuing through 2006, when almost all of the total decrease in these series through 2009 occurred.

The top panel of Figure 6 shows that the dates of large increases in e_M are less clustered. They generally coincide with, or occur shortly after, the big real activity shocks and the financial uncertainty shocks. Observe that large macro uncertainty shocks occurred more frequently in the pre-1983 than the post 1983 sample. An exception is the Great Recession, where there were large e_M shocks in some systems but not others: large macroeconomic uncertainty shocks are found when $Y = ip$ and when $Y = noi$ but are not found in the SVAR with emp_t or Q_{1t}. Thus the finding that there are simultaneous occurrences of big shocks to U_{Mt}, U_{Ft}, and Y_t during the Great Recession seems to depend on which measure of real activity Y_t is used. We return to this issue when sub-indexes of U_{Mt} are considered.

To give a sense of the historical importance of these shocks, we perform a decomposition of variance, which is the fraction of s-step-ahead forecast error variance attributable to each structural shock ϵ_{Mt}, ϵ_{Yt}, and ϵ_{Ft} for $s = 1$, $s = 12$, $s = \infty$. We also report the maximum fraction of forecast error variance over all VAR forecast horizons s that is attributable to each shock, denoted $s = s_{max}$ in the table. Table 3 reports results for the system with $Y_t = ip_t$ (left column), $Y_t = emp_t$ (middle column), and $Y_t = noi_t$ (right column).

According to the top row, all three real activity shocks e_{ip}, e_{emp}, and e_{noi} have sizable effects on macroeconomic uncertainty U_M. But according to the bottom row, these same shocks have small effects on financial uncertainty U_F. At the same time, positive macro uncertainty shocks e_M, which increase rather than decrease real activity, explain a surprisingly large fraction of production (up to 42%), employment (up to 37%) and orders (up to 28%), though their relative importance declines as the forecast horizon increases. On the other hand, financial uncertainty shocks e_F have a small contribution to the one-step-ahead forecast error variance of ip, but their relative importance increases over time. These e_F shocks make much larger contributions to the forecast error variance of emp and noi. Financial uncertainty shocks explain up to 59% of the forecast error variance in employment and up to 51% of the forecast error variance in orders, compared to 27% for production. Financial uncertainty shocks e_F feedback into U_M, and macroeconomic uncertainty shocks e_M also feedback into U_F.

Regardless of which measure of real activity is used, we find that financial uncertainty is unlike macro uncertainty or real activity in that its variation is far more dominated by its own shocks. For example, in the system with ip, e_F shocks explain 95% of the $s = 1$ step-ahead forecast error variance in U_{Ft}, and 75% of the $s = \infty$ step-ahead forecast error variance. In the systems with emp and noi, e_F shocks explain 74 and 72%, respectively, of the $s = 1$ step-ahead forecast error variance in U_{Ft}, and 53 and 57% of the $s = \infty$ step-ahead forecast error variance.

To summarize, in all three systems, real activity shocks e_Y have quantitatively large persistent negative effects on macro uncertainty U_M. In turn, macro uncertainty shocks e_M have large positive impact effects on real activity measures Y. Financial uncertainty shocks e_F have smaller impact effects but larger long run effects that dampen real activity Y. Across all systems, the forecast error variance of financial uncertainty is the least affected by shocks other than its own, suggesting that U_F is quantitatively the most exogenous variable in the system.

5 Uncertainty in Real Activity and Inflation

The results discussed above suggest that the dynamic relationship between macro uncertainty and real activity can be quite different from the relation between financial uncertainty and real activity. However, given the composition of our data χ^M, macroeconomic uncertainty itself can be due to uncertainty in real activity variables such as output and unemployment, to price variables, and to financial market variables. The theoretical uncertainty literature has focused on modeling exogenous uncertainty shocks that arise specifically in measures of real economic fundamentals, rather than in prices or financial markets. To better evaluate the implications of these theoretical models, it is therefore of interest to know how systems defined by sub-components of broad-based macro uncertainty behave. We first consider systems that isolate uncertainty about real activity using the U_{Rt} sub-index that more closely corresponds to the theoretical literature. We then move on to study systems that use a sub-index of macro uncertainty focused on price variables, $U_{\pi t}$, which has not been the focus on the uncertainty literature but may be of independent interest.

5.1 System $\mathbf{X}_t = (U_{Rt}, Y_t, U_{Ft})'$

We isolate the real activity components of macro uncertainty by aggregating the individual uncertainty estimates over the 73 real activity variables in the macro dataset $\mathcal{X}^M$. The one-period ahead uncertainty in real activity, denoted U_{Rt}, is show in Figure 7. This series, like U_{Mt}, is countercyclical though somewhat less so, having a correlation of -0.50 with industrial production (as compared to -0.66 for U_{Mt}). At first glance, U_{Rt} appears to fluctuate in a manner similar to macroeconomic uncertainty U_{Mt}. The two series have a correlation of 0.71 and exhibit some overlapping spikes. But U_{Rt} and U_{Mt} also display notable independent variation. Figure 7 shows that there are 43 observations of U_{Rt} that are at least 1.65 standard deviations above its mean. These can be organized into five episodes: 1965, 1970, 1975, 1982-83, and 2007. By contrast, U_{Mt} in Figure 1 only exhibits three such episodes. Observe that the U_{Rt} series exhibits several spikes before 1970 that are not accompanied by spikes in U_{Mt}.

Given the distinctive patterns in the time series behavior of U_{Rt} and U_{Mt}, one might expect to find different dynamic relationships with the other variables in our systems when U_{Mt} is replaced by U_{Rt}. Surprisingly, the impulse responses functions are qualitatively similar to systems studied above that use broad-based macro uncertainty. Since the responses are qualitatively similar using all measures of real activity, we only present one representative example in Figure 8, for the system $(U_{Rt}, emp_t, U_{Mt})'$. We see that (i) positive shocks to employment cause sharp declines in U_{Rt} so that negative shocks cause sharp increases in real economic uncertainty; (ii) positive real activity shocks e_{Rt} do not cause declines in emp_t; instead the opposite is true; (iii) positive financial uncertainty shocks e_{Ft} lead to sharp declines in employment that are strongly statistically significant, and (iv) there is no evidence that financial uncertainty is significantly affected by real activity shocks.

But while these counterfactual dynamic responses are similar to those reported for the base case when U_{Mt} is used, the realized shocks that are uncovered from the historical data are different. Figure 9 plots the large adverse structural shocks identified from the systems $(U_{Rt}, Y_t, U_{Ft})'$ for $Y_t = ip_t, emp_t, noi_t, Q_{1t}$ analogous to Figure 2. The top panel shows that the real uncertainty shock e_{Rt} exhibits no spike in excess of three standard deviations during the Great Recession for any measure of real activity, despite the fact that U_{Rt} itself exhibits a large spike (see Figure 7). This is in contrast to the behavior of e_{Mt} and especially e_{Ft} in Figure 2, both of which show much larger spikes during this episode. This pattern occurs in other recessions as well. In the 1973-75 recession, the real uncertainty shocks e_{Rt} show a large spike only for the system using orders, but not for the systems using production, employment or Q_{1t}, though all measures of real activity shocks e_{ip}, e_{emp}, e_{noi}, and $e_{Q_{1t}}$ exhibited large spikes downward. Likewise, both the 1980 recession and the 1982-1983 recession were characterized by large negative real activity shocks that met or exceeded three standard deviations from the mean, while real uncertainty shocks e_R were comparatively muted and if anything spiked after the recession was over.

These episodes serve to reinforce the conclusion that the heightened real economic uncertainty in recessions is more often an endogenous response to other shocks, rather than an exogenous impulse. Even though there were many large spikes in real uncertainty shocks e_{Rt} pre-1983, there have not been much in the way of large adverse shocks to real economic uncertainty since 1983, a period that coincides with the so-called Great Moderation. Large real uncertainty shocks are also absent from the Great Recession. This is an episode characterized by a large negative e_{Yt} and a large increase in e_{Ft}. Both adverse shocks are sufficiently large to drive U_{Rt} upward without a large exogenous increase e_{Rt}.

One might ask why we find large macro uncertainty shocks e_M in the Great Recession, at least for some measures of real activity, while the corresponding real activity uncertainty shocks e_R are much smaller. Recall that our U_M is a broad-based measure of uncertainty and, as such, contains some 25 financial variables. These are also the most volatile variables in the large macro dataset used to construct U_{Mt}. Hence U_M picks up a fair amount of its movement from financial variables, which were especially large in this episode. By isolating uncertainty attributable only to real variables, we can see more clearly the role of uncertainty about real activity variables in this episode. By the same reasoning, once we control explicitly for financial uncertainty, it makes little difference whether we use U_{Mt} or U_{Rt} in the SVAR. The impulse responses are similar, as can be seen from a comparison of the base case IRFs and those in Figure 8. Controlling for U_{Ft} is thus important as it removes the variation in U_{Mt} attributable to financial variable uncertainty. Whether we directly or indirectly control for uncertainty from financial variables, the main finding is that macroeconomic uncertainty rises in recessions primarily in response to real activity shocks, while financial uncertainty shocks are exogenous impulses that have significant negative effects on real activity.

To complete the analysis, we present variance decompositions for the system $(U_{Rt}, Y_t, U_{Ft})'$, with three measures of real activity $Y_t = ip_t, emp_t, noi_t$. These results, presented in Table 4, share some similarities with the systems that use macro uncertainty U_{Mt} shown in Table 3, but there are at least two important distinctions. First, financial uncertainty shocks decrease real activity and explain larger fractions of the forecast error variance in two measures of real activity. At the longest $s = \infty$ VAR horizon, financial uncertainty shocks explain 85% of forecast error variance in employment and 63% of the forecast error variance in orders. These results suggest that financial uncertainty has quantitatively large negative consequences for at least some measures of real activity.

Second, compared to systems that use U_{Mt}, smaller fractions of the forecast error variance in U_{Rt} are explained by its own shocks, while larger fractions are explained by the financial uncertainty shocks. Real activity shocks still have non-trivial consequences for U_{Rt}. For example, shocks to industrial production e_{ipt} still explain 41% of the one-step-ahead forecast error variance in U_{Rt}, though smaller than the 53% found earlier using U_{Mt}.

To summarize, countercyclical increases in real uncertainty U_{Rt}, like macro uncertainty U_{Mt}, are found to be fully an endogenous response to declines in real activity. Indeed, the most striking episode of heightened uncertainty in the post-war period, the Great Recession, was characterized by large negative real activity e_Y shocks and a large positive financial uncertainty e_F shock, but no corresponding large shock to real uncertainty e_R. These results underscore the extent to which the countercyclical variation in U_{Rt} is often an endogenous response to other shocks. At the same time, U_{Rt} exhibits more variation than U_{Mt} that is independent of fluctuations in real activity especially early in the sample, explaining why it is less countercyclical.

5.2 System $\mathbf{X}_t = (U_{\pi_t}, Y_t, U_{Ft})'$

The preceding subsection investigates the real activity component of macroeconomic uncertainty and its interaction with Y_t and U_{Ft}. This subsection studies the price component of macroeconomic uncertainty U_π which aggregates the 21 uncertainty indicators in the price block of χ^M. This block includes consumer and producer prices that tend to be more stable, as well as the price of oil, commodities, and raw materials that tend to be more volatile. With the exception of the NAPM commodity price index, the price data are second differenced after log transformation. Hence, the uncertainty indicators pertain to the *change* in monthly inflation. We refer to this measure simply as "price uncertainty."

The top panel of Figure 10 plots this measure of price uncertainty over our sample. It is countercyclical and has a correlation with industrial production is -0.51. There are 40 observations that are 1.65 standard deviations above the unconditional mean. These are clustered into three episodes: 1974-75, 2006-07, and 2008-09. There is a large spike upward in $U_{\pi t}$ visible during the Great Recession. This spike actually occurs over four months, from 2008:10-2009:01, during which $U_{\pi t}$ was unusually high. Also plotted in Figure 10 is a $U^x_{\pi,t}$ uncertainty index that removes from $U_{\pi,t}$ five of the most volatile price uncertainty series, namely PPI intermediate materials, PPI crude materials, oil, PPI metals and metal products, and CPI transportation. The more volatile price series apparently did not contribute to noticeable changes to aggregate price uncertainty.

Further investigation reveals that the increase in price uncertainty around the Great Recession was broad based, as 13 of the 21 series in the price group had uncertainty risen by at least three standard deviations above its mean in 2008:11, the peak of the spike. Results not reported show that these series all exhibited large negative forecast errors in 2008:10-2008:12, and then a large positive error in 2009:01. The change in inflation across many price series appears to have been volatile and difficult to predict at the peak of the Great Recession. Thus the Great Recession was hit by the rare occurrence of simultaneous adverse shocks to financial uncertainty, to real activity, and to price uncertainty.

The bottom panel of 10 plots the large adverse shocks for the systems $\mathbf{X}_t = (U_{\pi t}, Y_t, U_{Ft})'$ with $Y_t = ip_t, emp_t, noi_t, Q_{1t}$, and for an alternative set of systems $\mathbf{X}_t = (U^x_{\pi t}, Y_t, U_{Ft})'$. Notably, most of the spikes are concentrated in the years before 1983. Nonetheless, the price uncertainty spike in 2008 is evident both e_π and e^x_π. Together with the results reported earlier, the broad based nature of the surge in uncertainty in 2008 is unprecedented.

We estimate an SVAR for $\mathbf{X}_t = (U_{\pi t}, Y_t, U_{Ft})'$. The responses are again similar for all measures of Y_t so we conserve space by showing just one. Figure 11 shows the dynamic responses with $Y_t = emp_t$. As before, it is exogenous shocks to financial uncertainty that drive real activity endogenously lower. By contrast, positive shocks to price uncertainty do not decrease real activity, indeed the opposite is true. We see also that positive shocks to price uncertainty $e_{\pi t}$ lead to a sharp increase in financial uncertainty U_{Ft}. Financial uncertainty shocks, on the other hand, have no effect on price uncertainty $U_{\pi t}$.

Figure 11 also shows that employment shocks e_{emp} impact price uncertainty in a manner that is qualitatively similar to how they impact macro and real economic uncertainty. Positive (negative) shocks to real activity cause sharp decreases (increases) in price uncertainty, but have little effect on financial uncertainty. Thus a boom in real activity appears to reduce macroeconomic uncertainty broadly across many indicators, including uncertainty about price variables, though not about financial markets.

On the whole, these findings reinforce the notion that financial uncertainty is primarily an exogenous impulse acting on real activity, while countercyclical uncertainty about other macroeconomic activity, be it real activity or prices, is primarily an endogenous response to real activity. But price uncertainty increases financial uncertainty, a finding that is theoretically consistent with evidence that inflation uncertainty is correlated with higher risk spreads in bond markets (e.g., Wright (2011)). An interesting direction for future research is to investigate the dynamic linkages between inflation uncertainty, financial market uncertainty, and term premia.

6 Robustness and Additional Cases

This section presents results for a number of additional cases.

6.1 Different Measures of Real Activity and Different Sample

Rather than using specific real activity measures such as production, employment and orders, we now use a more broad-based measure of real activity that we construct, namely the real activity index Q_{1t}. Figure 12 presents impulse responses for $\mathbf{X}_t = (U_{Mt}, Q_{1t}, U_{Ft})'$. The responses are quite similar to those using $Y_t = ip_t$ with the main difference being that the standard error bands are narrower especially for the response of Q_{1t} to U_{Ft} shock. Financial uncertainty shocks lead to large, statistically significant declines in the index of real activity. Moreover, as for the systems using other measures of real activity, high macro uncertainty in bad times if fully an endogenous response to declines in real activity, as measured by Q_{1t}.

Given the importance of the Great Recession for the uncertainty series, we asked whether our main results were affected by stopping the sample at the end of 2007:12. A representative set of impulse response functions is shown in Figure (A2) for the system $\mathbf{X}_t = (U_{Mt}, emp_t, U_{Ft})'$ (the other systems show similar responses). The figure shows that the qualitative nature of all the responses, including standard error bands, is quite similar to the comparable case for the full sample (Figure 4). This implies that main findings above are robust to this sample that excludes the Great Recession and the concomitant financial crisis. Further inspection indicates that the main difference created by using different samples is evident in the variance decompositions (not shown): somewhat less of the forecast error variance in U_F in the pre-2008 sample is attributable to its own shocks than in the full sample, while correspondingly more of the forecast error variance in U_F is attributable to real activity shocks. For example, in the full sample, 95% of the one-step-ahead forecast error variance in U_F is attributable to its own shocks in the system with $Y_t = ip_t$, whereas this estimate is 82% for the pre-2008 sample. At the same time, the variance decompositions pertaining to the impact of financial uncertainty on real activity are little effected by removing the post 2008 part of the sample. This shows that the negative impact of financial uncertainty shocks for real activity does not hinge on one episode, and that many episodes prior to 2008 that were characterized by more modest financial uncertainty shocks also had consequences for real activity.

6.2 One year Uncertainty

So far we have been considering uncertainty about events one-month ahead. To consider a longer horizon uncertainty, we estimate systems using uncertainty about events 12 months ahead, denoted $U_{Mt}(12)$ and $U_{Ft}(12)$. For the dynamic responses, the findings are qualitatively similar to the benchmark cases with $h = 1$ period ahead uncertainty. Figure 13 presents a representative example for the system: $\mathbf{X}_t = (U_{Mt}(12), emp_t, U_{Ft}(12))$. But an inspection of the variance decompositions suggests some notable differences from the $h = 1$ uncertainty systems. Table 5 shows variance decompositions for the systems $\mathbf{X}_t = (U_{Mt}(12), Y_t, U_{Ft}(12))$ with $Y_t = ip_t, emp_t, noi_t$. One-year financial uncertainty shocks explain smaller fractions of the variation in all measures of real activity than do one-month uncertainty shocks, especially over the longer VAR horizons for which their impact is non-trivial. For example, 12-month-ahead financial uncertainty e_{Ft} shocks explain just 10% of the long-run forecast error variance in ip_t. In contrast Table 3 above showed that one-month-ahead financial uncertainty e_{Ft} shocks explain 23% of the long-run forecast error variance in production. Similar comparisons hold for the other two measures of real activity, emp_t and noi_t. $U_{Ft}(12)$ shocks also explain smaller fractions of the forecast error variance in macro uncertainty U_{Mt} than do $U_{Ft}(1)$ shocks. This result occurs in part because long-run uncertainty is simply much less volatile than short-run uncertainty. While the level of uncertainty increases with h (on average), the variability of uncertainty decreases because the forecast tends to the unconditional mean as the forecast horizon tends to infinity. On the other hand, the impact of macro uncertainty shocks on the other variables in the system is less affected by the uncertainty horizon h. For example, the effects of e_{Mt} shocks on all measures of real activity are about the same for systems using $U_{Mt}(12)$ as they are for the systems studied above that use $U_{Mt}(1)$.

6.3 Imposing Recursive Identification Restrictions

The SVARs studied here nest any recursive structure so that by imposing additional restrictions we can recover any such structure. We can also test the validity of these restrictions. The results above show that these restrictions are rejected in the data. We now show what happens to the dynamic responses when we nevertheless impose restrictions consistent with recursive identification (and freely estimate the rest of the parameters). Figure 14 shows one case: dynamic responses for the system $\mathbf{X}_t = (U_{Ft}, U_{Mt}, ip_t)'$ with that ordering. Although there are many possible recursive orderings, and the estimated IRFs differ in some ways across these cases, the dynamic responses under recursive identification have one common feature that is invariant to the ordering and that provides the sharpest contrast with the results generated by the SVARs identified with external instruments studied here. Specifically, with recursive identification, macro uncertainty shocks–no matter which ordering–appear to cause a sharp decline in real activity, while real activity shocks have little effect on macro uncertainty in the short run and if anything increase it in the long run. This result, evident in Figure 14, is precisely the opposite of what is reported above and appears to be an artifact of invalid timing assumptions under recursive identification. Further investigation reveals that the SVARs we study display non-zero contemporaneous correlations between U_{Ft} and Y_t, as well as between U_{Mt} and Y_t, which is inconsistent with any recursive ordering. Imposing a structure that prohibits contemporaneous feedback spuriously suggests that macro uncertainty shocks are a cause of declines in real activity, rather than an endogenous response. This result is robust across any of the six possible recursive orderings and underscores the challenges of relying on convenient timing assumptions to sort out cause and effect in the relationship between uncertainty and real activity.[7]

6.4 Different External Variables

We reestimate the model using a corporate bond return as the regressand in (9a) to generate Z_{1t}. We generate Z_{2t} in (9b) using the monthly CRSP value-weighted excess stock market return crsp_t. The bond yield measure is the yield on a portfolio of Baa Moodys seasoned corporate bonds, where Baa represents a credit score on the border of the investment and junk categories. Because the Baa yield is highly serially correlated, we use the first difference of the yield. The estimation procedure in all other ways is the same as above.

Estimates of these cases indicate that the correlation between the resulting Z_{1t} and both uncertainty shocks is now positive. Thus high uncertainty of both types is associated with rising yields on risky corporate debt. For the $(U_{Mt}, ip_t, U_{Ft})'$ system, the correlations with Z_{1t} are $\rho(Z_{1t}, \hat{e}_{Mt}) = 0.1988$, $\rho(Z_{1t}, \hat{e}_{Ft}) = 0.1219$, while the correlation of Z_{2t} with $\hat{e}_{Ft}$ remains similar

[7] The figures for these cases are omitted to conserve space but are available upon request.

to the base cases, with $\rho(Z_{2t}, \hat{e}_{Ft}) = -0.1617$. The correlation between Z_{1t} and Z_{2t} is -0.2 in this case. Figure (15) presents the dynamic responses for the system $(U_{Mt}, ip_t, U_{Ft})'$. The pattern of responses is qualitatively similar to the base cases presented above. But the SVAR parameter estimates exhibit more sampling error. This leads to error bands for the dynamic responses of U_{Mt} to e_{ipt} and for ip_t to e_{Mt} to be wider than in the corresponding base case for the same system.

In our experience, the bootstrap standard error bands tend to be wide when the external variables produce instruments that only weakly identify some elements of $\mathbf{B}$. Our analysis requires $\mathbb{E}(Z_{2t}e_{Mt}) \neq 0$ to identify the column that gives the effects of e_{Mt} shocks. In cases when the GMM estimates of $\mathbb{E}(Z_{1t}\eta_{Yt})$ and $\mathbb{E}(Z_{1t}\eta_{Ft})$ are imprecise, we find B_{MY} and B_{YM} are poorly identified and the bootstrap error bands for the dynamic responses of U_{Mt} to e_{ipt} and for ip_t to e_{Mt} are then wide. An inspection of the closed-form solutions for $\mathbf{B}$ shows why. The B_{MY} and B_{YM} parameters are highly nonlinear functions of $\mathbb{E}(Z_{1t}\eta_{Yt})$ and $\mathbb{E}(Z_{1t}\eta_{Ft})$, so that small changes in the latter lead to large differences in the solution for B_{MY} and B_{YM}. Since the bootstrap repeatedly makes draws from the distribution of the GMM estimates it depends on the variance of the point estimates. The bootstrap standard errors are correspondingly large when the point estimates of the variance of $\mathbb{E}(Z_{1t}\eta_{Yt})$ and $\mathbb{E}(Z_{1t}\eta_{Ft})$ and the other parameters are imprecise. Thus, while our approach provides a new way to estimate the SVAR, the methodology requires Assumption A to be satisfied.

An appeal of our estimation strategy is that the estimates provide some guide to the validity of Assumption A for various external instruments used. As an example, consider the system $\mathbf{X}_t = (U_{Mt}, ip_t, U_{Ft})'$. Our analysis requires $\mathbb{E}(Z_{2t}e_{Ft}) \neq 0$ to identify the column that gives the effects of e_{Ft} shocks. When we set S_{2t} equal to the Baa-fed funds rate *spread* (rather than the Baa rate itself), while keeping S_{1t} the same as in our baseline case, the resulting Z_{2t} becomes weakly correlated with $\hat{e}_{Ft}$, so the financial uncertainty shock is poorly identified. The same finding arises when S_{2t} is set equal to the growth in the spot market oil price.

The standard errors are also large when we use the Baa-fed funds rate spread as S_{1t}. With this choice of S_{1t}, the resulting Z_{1t} is weakly correlated with $\hat{e}_{Mt}$ and so the macro uncertainty shock is poorly identified. When S_{1t} or alternatively S_{2t} is set equal to Δnoi_t, the estimated B_{YY} element is close to zero, indicating that the real activity shock is poorly identified. This can be understood by recalling that the ip_t shock is identified off of movements in real activity that are *un*correlated with the instruments, which are components of S_{1t} and S_{2t}. If S_{1t} or S_{2t} are themselves some measure of real activity (such as orders), there may be little uncorrelated variation left to identify the ip_t shock.

A third example is when S_{1t} is the small stock index return, then B_{MY} and B_{YM} are poorly identified in our sample, for the same reasons as described above: the bootstrap leads to large variation in the estimates of B_{MY} and B_{YM} if the sample variance of these parameters is large.

Indeed, compared to our baseline cases, the sample variance of $\mathbb{E}(Z_{1t}\eta_{Yt})$ is two times larger when S_{1t} is the small stock index return, while the sample variance of $\mathbb{E}(Z_{1t}\eta_{Ft})$ is three times larger.

6.5 Alternative Assumptions on S_t

The identification strategy relies on the use of external variables $\mathbf{S}_t$ that are not part of the SVAR we seek to identify. This inevitably imposes certain restrictions on a larger VAR that includes S_t. We now make these restrictions precise and consider its robustness.

Let $\mathbf{X}_t = (U_{Mt}, Y_t, U_{Ft})'$ be the endogenous variables of interest and $\mathbf{S}_t = (S_{1t}, S_{2t})'$ be the two variables used to construct the external instruments. We express (1) using the equivalent structural econometric model representation:

$$\mathbf{A}_0\mathbf{X}_t = \mathbf{k} + \mathbf{A}_1\mathbf{X}_{t-1} + \mathbf{A}_2\mathbf{X}_{t-2} + \cdots + \mathbf{A}_p\mathbf{X}_{t-p} + \boldsymbol{\Sigma}\mathbf{e}, \tag{13}$$

where $\mathbf{A}_0 = \mathbf{H}^{-1}$, and $\mathbf{A}_j = \mathbf{H}^{-1}\mathbb{A}_j$. A five variable VAR(1) in $(\mathbf{X}_t', \mathbf{S}_t')'$ can therefore be written

$$\begin{pmatrix} \mathbf{A}_{XX,0} & \mathbf{A}_{XS,0} \\ \mathbf{A}_{SX,0} & \mathbf{A}_{SS,0} \end{pmatrix} \begin{pmatrix} \mathbf{X}_t \\ \mathbf{S}_t \end{pmatrix} = \begin{pmatrix} \mathbf{A}_{XX,1} & \mathbf{A}_{XS,1} \\ \mathbf{A}_{SX,1} & \mathbf{A}_{SS,1} \end{pmatrix} \begin{pmatrix} \mathbf{X}_{t-1} \\ \mathbf{S}_{t-1} \end{pmatrix} + \begin{pmatrix} \boldsymbol{\Sigma}_X & 0 \\ 0 & \boldsymbol{\Sigma}_S \end{pmatrix} \begin{pmatrix} \mathbf{e}_{Xt} \\ \mathbf{e}_{St} \end{pmatrix}.$$

The relation between the reduced form and the structural shocks is now

$$\begin{pmatrix} \boldsymbol{\eta}_{Xt} \\ \boldsymbol{\eta}_{St} \end{pmatrix} = \begin{pmatrix} \mathbf{A}_{XX,0} & \mathbf{A}_{XS,0} \\ \mathbf{A}_{SX,0} & \mathbf{A}_{SS,0} \end{pmatrix}^{-1} \begin{pmatrix} \boldsymbol{\Sigma}_X\mathbf{e}_{Xt} \\ \boldsymbol{\Sigma}_S\mathbf{e}_{St} \end{pmatrix} = \begin{pmatrix} \mathbf{B}_{XX} & \mathbf{B}_{XS} \\ \mathbf{B}_{SX} & \mathbf{B}_{SS} \end{pmatrix} \begin{pmatrix} \mathbf{e}_{Xt} \\ \mathbf{e}_{St} \end{pmatrix}. \tag{14}$$

By substituting out S_t, it is straightforward to show that

$$\Big[(\mathbf{A}_{XX,0} - \mathbf{A}_{XX,1}L) + (\mathbf{A}_{XS,0} - \mathbf{A}_{XS,1}L)\mathbf{C}_{SX}(L)\Big] X_t = -(\mathbf{A}_{XS,0} - \mathbf{A}_{XS,1}L)\mathbf{C}_{SS}(L)\boldsymbol{\Sigma}_S\mathbf{e}_{St} + \boldsymbol{\Sigma}_X\mathbf{e}_{Xt}$$

where $\mathbf{C}_{SX}(L) = \mathbf{C}_{SS}(L)(\mathbf{A}_{SX,0} - \mathbf{A}_{SX,1}L)$, $\mathbf{C}_{SS}(L) = -(\mathbf{A}_{SS,0} - \mathbf{A}_{SS,1}L)^{-1}$. Without further restrictions, X_t is a VARMA(1,1) driven by a combination of shocks to $\mathbf{X}_t$ as well as $\mathbf{S}_t$.

Our maintained assumption in the base case studied above is $\mathbf{A}_{XS,0} = \mathbf{A}_{XS,1} = \mathbf{0}_{3\times 2}$. Under this assumption, the terms that multiply into $\mathbf{C}_{SX}(L)$ and $\mathbf{C}_{SS}(L)$ drop out, giving

$$\mathbf{A}_{XX,0}\mathbf{X}_t = \mathbf{A}_{XX,1}X_{t-1} + \boldsymbol{\Sigma}_X\mathbf{e}_{Xt}$$

which is our base case VAR with $p = 1$. For arbitrary $p \geq 1$, the assumptions $\mathbf{A}_{XS,j} = 0$ for all $j \geq 0$ effectively restricts the five variable system to be block recursive, with the three variables in $\mathbf{X}_t$ ordered ahead of the two variables in $\mathbf{S}_t$. Since the dynamic responses of $\mathbf{S}_t$ are not of direct interest, the block recursive assumption permits us to analyze the smaller VAR for $\mathbf{X}_t$.

However, though the assumption that $\mathbf{A}_{XS,j} = 0$ for all j is necessary to justify the smaller three variable VAR, it is stronger than is necessary for the identification of $\mathbf{e}_{Xt}$. The reason is that, provided $\mathbf{A}_{XS,0} = 0$, $\mathbf{B}_{XS}$ will be zero. The lower block triangularity of $\mathbf{B}$ implies that $\mathbf{A}_{XX,0}$ can be identified by Assumption A along with the covariance structure of $\boldsymbol{\eta}_{Xt}$ associated with the five variable system. In other words, we can in principle leave $\mathbf{A}_{XS,j}$ for $j \geq 1$ unconstrained to allow the effects of e_{Xt} to feedback to $\mathbf{X}_t$ through lags of $\mathbf{S}_t$.

To entertain this possibility, we estimate a five variable SVAR in $(U_{Mt}, Y_t, U_{Ft}, S_{1t}, S_{2t})'$ as well as a four variable system in $(U_{Mt}, Y_t, U_{Ft}, S_t)'$, both imposing $\mathbf{A}_{XS,0} = \mathbf{0}$. The results for the four and five variable systems are similar and we report results for the four variable case. In this four variable VAR, S_t is the return on the CRSP value-weighted stock market index. The vector of reduced-form innovations is $\boldsymbol{\eta}_t = (\eta_{Mt}, \eta_{Yt}, \eta_{Ft}, \eta_{St})'$. As just discussed, we can identify $\mathbf{B}_{XX}$ from the first three equations of this VAR alone using IPIV. The only difference from the base case is that for $j = M, Y, F$, $\hat{\boldsymbol{\eta}}_j$ is a vector of residuals from a regression of $\mathbf{X}_j$ on lags U_M, Y, U_M *and* lags of S. Since $\mathbf{A}_{XS,0} = \mathbf{0}$ by assumption, it holds that $\mathbf{B}_{XS} = \mathbf{0}$. It only remains to identify $\mathbf{B}_{SX}$ and $\mathbf{B}_{SS}$. These can be recovered by least squares regression of $\hat{\eta}_{St}$ on $\hat{e}_{Mt}, \hat{e}_{Yt}, \hat{e}_{Ft}$ to give a fitted residual

$$\hat{e}_{St} = \eta_{St} - \hat{\mathbf{B}}_{SM}\hat{e}_{Mt} - \hat{\mathbf{B}}_{SY}\hat{e}_{Yt} - \hat{\mathbf{B}}_{SF}\hat{e}_{Ft}$$

where $\hat{\mathbf{B}}_{SX} = (\hat{\mathbf{B}}_{SM}, \hat{\mathbf{B}}_{SY}, \hat{\mathbf{B}}_{SF})'$ are the OLS estimates, and $\hat{\mathbf{B}}_{SS}$ is the standard deviation of $\hat{e}_{St}$. The SVAR estimates are then used to compute impulse responses for the four variable system.

Figure 16 compares the impulse responses for $\mathbf{X}_t = (U_{Mt}, Y_t, U_{Ft})'$ to shocks $\mathbf{e}_X$ from the four and three variable VARs. The responses are little different. The data thus appear consistent with the assumption that $\mathbf{A}_{XS,j} = 0$ for $j \geq 1$.

7 Conclusion

A growing body of research establishes uncertainty as a feature of deep recessions but leaves open two key questions: is uncertainty primarily a source of business cycle fluctuations or an endogenous response to them? And where does uncertainty originate? There is no theoretical consensus on the question of whether uncertainty is primarily a cause or a consequence of declines in economic activity. In most theories, it is modeled either as a cause or an effect, but not both, underscoring the extent to which this question is fundamentally an empirical matter.

The objective of this paper is to address both questions econometrically using small-scale structural VARs that are general enough to nest the range of theoretical possibilities in empirical tests. Commonly used recursive identification schemes cannot achieve this objective, since by construction they rule out the possibility that uncertainty and real activity could influence one another contemporaneously. The econometric model employed in this paper nests the recursive identification scheme, and we find that it is strongly rejected by the data. An empirical model in which uncertainty and real activity simultaneously influence each other fits the data far better than one in which these relationships are restricted by timing assumptions that prohibit contemporaneous feedback.

To identify dynamic causal effects, this paper takes an alternative identification approach by using external instruments that we construct in a novel way to be valid under credible interpretations of the structural shocks. We call this approach iterative projection IV (IPIV). In addition, our empirical analysis explicitly distinguishes macro uncertainty and uncertainty about real activity from financial uncertainty, thereby allowing us to shed light on the origins of uncertainty shocks that drive real activity lower, to the extent that any of them do. The econometric framework allows uncertainty to be an exogenous source of business cycle fluctuations, or an endogenous response to them, or any combination of the two, without restricting the timing of these relationships. Underlying our approach is a maintained theoretical assumption that variables such as stock market returns, while endogenous, are nevertheless driven by distinct sources of stochastic variation, some of which satisfy exogeneity restrictions required to identify independent structural shocks.

Estimates of the econometric model are used to inform the nature of these dynamic relationships in U.S. data. The results from these estimations show that sharply higher uncertainty about real economic activity in recessions is fully an endogenous response to business cycle fluctuations, while uncertainty about financial markets is a likely source of them. Exogenous declines in economic activity have quantitatively large effects that drive real economic uncertainty endogenously higher. Financial uncertainty, by contrast, is dominated by its own shocks, implying that it is primarily an exogenous impulse vis-a-vis real activity and macro uncertainty. These results reinforce the hypothesis laid out in much of theoretical uncertainty literature, namely that uncertainty shocks are a source of business cycle fluctuations. But they also stand in contrast to this literature, which has emphasized the role of uncertainty fluctuations in productivity and other real economic fundamentals. The findings here imply that the uncertainty shocks that drive real activity lower appear to have their have origins, not in measures of real activity, but in financial markets.

Appendix

Closed-Form Solution for B

Lemma 2 *The solution to the system (7) exists and is unique if* $\mathbb{E}\left[e_{Ft}Z_2\right] \neq 0$ *and* $\mathbb{E}\left[e_{Mt}Z_1\right] \neq 0$.

Proof. To facilitate the presentation throughout the proof, let

$$\begin{aligned} \boldsymbol{\eta}_t &= \mathbf{B}\mathbf{e}_t \\ \mathbf{B} &= \left[\underset{3\times 1}{\mathbf{B}_M}, \underset{3\times 1}{\mathbf{B}_Y}, \underset{3\times 1}{\mathbf{B}_F}\right] \\ \boldsymbol{\Omega} &= \mathbb{E}\left(\boldsymbol{\eta}_t\boldsymbol{\eta}_t'\right) \end{aligned}$$

and we have two external instruments (Z_1, Z_2) satisfying

$$\begin{aligned} \mathbb{E}\left[e_{Ft}Z_1\right] &\equiv \phi_{1F} \neq 0,\ \mathbb{E}\left[e_{Mt}Z_1\right] \equiv \phi_{1M} \neq 0 \text{ and } \mathbb{E}\left[e_{Yt}Z_1\right] = 0 \\ \mathbb{E}\left[e_{Ft}Z_2\right] &\equiv \phi_{2F} \neq 0 \text{ and } \mathbb{E}\left[e_{Mt}Z_2\right] = \mathbb{E}\left[e_{Yt}Z_2\right] = 0 \end{aligned}$$

Then

$$\mathbb{E}\left[\boldsymbol{\eta}_t Z_2\right] = \mathbb{E}\left[\mathbf{B}\mathbf{e}_t Z_2\right] = \mathbf{B}\begin{bmatrix} 0 \\ 0 \\ \phi_{2F} \end{bmatrix} = \phi_{2F}\mathbf{B}_F \tag{A.1}$$

Thus $\mathbf{B}_F$ exists if $\phi_{2F} \neq 0$. Observe that, since

$$\boldsymbol{\Omega} = \mathbb{E}\left[\boldsymbol{\eta}_t\boldsymbol{\eta}_t'\right] = \mathbf{B}\mathbf{B}'$$

we have

$$\mathbf{B}'\boldsymbol{\Omega}^{-1}\mathbf{B} = \mathbf{I}$$

hence, $\forall i, j = M, Y, F$

$$\mathbf{B}_j'\boldsymbol{\Omega}^{-1/2}\boldsymbol{\Omega}^{-1/2}\mathbf{B}_i = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases}.$$

Therefore,

$$\mathbb{E}\left[\boldsymbol{\eta}_t Z_2\right]' \Omega^{-1}\mathbb{E}\left[\boldsymbol{\eta}_t Z_2\right] = \left(\phi_{2F}\mathbf{B}_F\right)' \boldsymbol{\Omega}^{-\frac{1}{2}}\boldsymbol{\Omega}^{-\frac{1}{2}}\left(\phi_{2F}\mathbf{B}_F\right) = \phi_{2F}^2$$

This implies that the scale ϕ_{2F} is identified up to a sign by

$$\phi_{2F} = \pm\sqrt{\mathbb{E}\left[\boldsymbol{\eta}_t Z_2\right]\boldsymbol{\Omega}^{-1}\mathbb{E}\left[\boldsymbol{\eta}_t Z_2\right]}. \tag{A.2}$$

Next,

$$\mathbb{E}\left[\boldsymbol{\eta}_t Z_1\right] = \mathbb{E}\left[\mathbf{B}\mathbf{e}_t Z_1\right] = \mathbf{B}\begin{bmatrix} \phi_{1M} \\ 0 \\ \phi_{1F} \end{bmatrix} = \phi_{1M}\mathbf{B}_M + \phi_{1F}\mathbf{B}_F$$

But note that

$$\begin{aligned}\mathbb{E}[\boldsymbol{\eta}_t Z_2]\,\boldsymbol{\Omega}^{-1}\mathbb{E}[\boldsymbol{\eta}_t Z_1] &= \phi_{2F}\mathbf{B}_{F'}\boldsymbol{\Omega}^{-1}(\phi_{1M}\mathbf{B}_M+\phi_{1F}\mathbf{B}_F)\\ &= \phi_{2F}\mathbf{B}_{F'}(\mathbf{B}\mathbf{B}')^{-1}(\phi_{1M}\mathbf{B}_M+\phi_{1F}\mathbf{B}_F)\\ &= \phi_{2F}\phi_{1F}\end{aligned}$$

This implies that ϕ_{1F} is identified as

$$\phi_{1F}=\frac{\mathbb{E}[\boldsymbol{\eta}_t Z_2]\,\boldsymbol{\Omega}^{-1}\mathbb{E}[\boldsymbol{\eta}_t Z_1]}{\phi_{2F}}$$

which in turn implies

$$\phi_{1M}\mathbf{B}_M=\mathbb{E}[\boldsymbol{\eta}_t Z_1]-\frac{\mathbb{E}[\boldsymbol{\eta}_t Z_2]}{\phi_{2F}}\phi_{1F}. \tag{A.3}$$

Thus solution to $\mathbf{B}_M$ exists if $\phi_{1M}\neq 0$. Furthermore, note that

$$\begin{aligned}&\left(\mathbb{E}[\boldsymbol{\eta}_t Z_1]-\frac{\mathbb{E}[\boldsymbol{\eta}_t Z_2]}{\phi_{2F}^2}\phi_{1F}\right)'\Omega^{-1}\left(\mathbb{E}[\boldsymbol{\eta}_t Z_1]-\frac{\mathbb{E}[\boldsymbol{\eta}_t Z_2]}{\phi_{2F}^2}\phi_{1F}\right)\\ =\;&\boldsymbol{\Omega}^{-\frac{1}{2}}\mathbf{B}_M\phi_{1M}^2\mathbf{B}_M'\boldsymbol{\Omega}^{-\frac{1}{2}}=\phi_{1M}^2\end{aligned}$$

This implies that the parameter ϕ_{1M} is identified up to a sign as

$$\phi_{1M}^2=\left(\mathbb{E}[\boldsymbol{\eta}_t Z_1]-\frac{\mathbb{E}[\boldsymbol{\eta}_t Z_2]}{\phi_{2F}^2}\phi_{1F}\right)'\boldsymbol{\Omega}^{-1}\left(\mathbb{E}[\boldsymbol{\eta}_t Z_1]-\frac{\mathbb{E}[\boldsymbol{\eta}_t Z_2]}{\phi_{2F}^2}\phi_{1F}\right). \tag{A.4}$$

It only remains to identify $\mathbf{B}_Y$. $\mathbf{B}_Y$ must satisfy

$$\begin{aligned}\mathbf{B}_Y'\boldsymbol{\Omega}^{-1/2}\boldsymbol{\Omega}^{-1/2}\mathbf{B}_Y&=1\\ \mathbf{B}_Y'\boldsymbol{\Omega}^{-1/2}\boldsymbol{\Omega}^{-1/2}\mathbf{B}_M&=0\\ \mathbf{B}_Y'\boldsymbol{\Omega}^{-1/2}\boldsymbol{\Omega}^{-1/2}\mathbf{B}_F&=0\end{aligned} \tag{A.5}$$

$\mathbf{B}_Y$ can be solved analytically using (**A.5**) provided that $\mathbf{B}_F$ and $\mathbf{B}_Y$ are identified. In addition, since the equation (**A.5**) is quadratic in $\mathbf{B}_Y$, $\mathbf{B}_Y$ is unique up to sign. It follows that there exists a τ such that

$$\mathbf{B}_Y=\tau\tilde{\mathbf{B}}_Y \tag{A.6}$$

where $\tilde{\mathbf{B}}_Y$ is unique conditional on ϕ_{2F} and ϕ_{1M}, but the scalar τ is unique up to sign.

This shows that the solution to the system (7) exists and is unique up to sign if $\phi_{2F}\neq 0$, $\phi_{1M}\neq 0$. Combined with unit effect normalization (3) and the restriction on the admissible parameter space (4), $\mathbf{B}$ can be uniquely identified. The unit effect normalization implies

$$\begin{aligned}\begin{pmatrix}B_{MM}&B_{MY}&B_{MF}\\B_{YM}&B_{YY}&B_{YF}\\B_{FM}&B_{FY}&B_{FF}\end{pmatrix}&=\begin{pmatrix}1&H_{MY}&H_{MF}\\H_{YM}&1&H_{YF}\\H_{FM}&H_{FY}&1\end{pmatrix}\begin{pmatrix}\sigma_{MM}&0&0\\0&\sigma_{YY}&0\\0&0&\sigma_{FF}\end{pmatrix}\\ &=\begin{pmatrix}\sigma_{MM}&H_{MY}\sigma_{YY}&H_{MF}\sigma_{FF}\\H_{YM}\sigma_{MM}&\sigma_{YY}&H_{YF}\sigma_{FF}\\H_{FM}\sigma_{MM}&H_{FY}\sigma_{YY}&\sigma_{FF}\end{pmatrix}\end{aligned} \tag{A.7}$$

Equation ($\mathbf{A.7}$) combined with the restriction $\sigma_{jj} > 0$ for all $j = M, Y, F$, implies $B_{jj} > 0$ for all $j = M, Y, F$. From equation ($\mathbf{A.1}$), $B_{FF} > 0$ pins down the sign of ϕ_{2F} conditional $\mathbf{Z}_t$. Since the sign of ϕ_{2F} is pinned down, the signs of B_{MF} and B_{YF} are also pinned down by the same restriction. From equation ($\mathbf{A.3}$), $B_{MM} > 0$ pins down the sign of ϕ_{1M} conditional $\mathbf{Z}_t$ and therefore the signs of B_{YM} and B_{FM} are pinned down by the same restriction. It only remains to show the uniqueness of $\mathbf{B}_Y$. Provided that $\mathbf{B}_F$ and $\mathbf{B}_Y$ are identified and given the closed-form solution ($\mathbf{A.5}$) that is quadratic in $\mathbf{B}_Y$, then $B_{YY} > 0$ pins down the sign of τ conditional $\mathbf{Z}_t$ and hence the sign of B_{MY} and B_{FY} are also pinned down by the same restriction. ■

The system of equations defining $\mathbf{B}$ is

$$0 = \mathbb{E}[\mathbf{g}_1(\mathbf{m}_{1t}; \boldsymbol{\beta}_1)] \equiv \overline{\mathbf{g}}_1.$$

The rank condition is satisfied when $\mathbf{J} \equiv \partial \mathbb{E}_T[\mathbf{g}_1]/\partial \boldsymbol{\beta}_1'$ is full column rank. We check that the rank condition is satisfied by evaluating $\mathbf{J}$ at the estimated parameter values for each case.

Procedure for Bootstrap

The bootstrap follows Krinsky and Robb (1986). We sample repeatedly from the joint distribution $N\left(\hat{\boldsymbol{\beta}}, \hat{\Theta}/T\right)$, where $\hat{\Theta}$ is the estimated GMM variance-covariance matrix to obtain $\mathcal{B}$ new sets of parameters $\hat{\boldsymbol{\beta}}^{(1)}....\hat{\boldsymbol{\beta}}^{(B)}$ and calculate the impulse response function values at each draw, $\boldsymbol{\Upsilon}_{s,j}^{(1)}, ..., \boldsymbol{\Upsilon}_{s,j}^{(B)}$, where s indexes the VAR horizon and j the variable being shocked, and where $\boldsymbol{\Upsilon}_{s,j}^{(b)} = \boldsymbol{\Upsilon}_{s,j}\left(\hat{\boldsymbol{\beta}}^{(b)}\right)$. The confidence intervals are ranges for $\boldsymbol{\Upsilon}_{s,j}^{(b)}$ created by trimming $\alpha/2$ from each tail of the resulting distribution of the function values. The parameter $\mathcal{B}$ is set to 10,000.

Monte Carlo

This section presents details of a Monte Carlo investigation of the projection IV application of this paper. The Monte Carlo procedure is as follows.

1. For each MC iteration $i = 1, ..., I$, draw $T \times 1$ vectors $\mathbf{e}_F^{(i)}, \mathbf{e}_Y^{(i)}, \mathbf{e}_M^{(i)}$ independently from $N(0, 1)$.

2. Generate true data for $\left(U_M^{(i)}, Y^{(i)}, U_F^{(i)}\right)$ from the trivariate VAR

$$\underbrace{\begin{pmatrix} A_{MM}(0) & A_{MY}(0) & A_{MF}(0) \\ A_{YM}(0) & A_{YY}(0) & A_{YF}(0) \\ A_{FM}(0) & A_{FY}(0) & A_{FF}(0) \end{pmatrix}}_{\mathbf{A}_0} \begin{pmatrix} U_{Mt}^{(i)} \\ Y_t^{(i)} \\ U_{Ft}^{(i)} \end{pmatrix} = \underbrace{\begin{pmatrix} A_{MM}(1) & A_{MY}(1) & A_{MF}(1) \\ A_{YM}(1) & A_{YY}(1) & A_{YF}(1) \\ A_{FM}(1) & A_{FY}(1) & A_{FF}(1) \end{pmatrix}}_{\mathbf{A}_1} \begin{pmatrix} U_{Mt-1}^{(i)} \\ Y_{t-1}^{(i)} \\ U_{Ft-1}^{(i)} \end{pmatrix} + \begin{pmatrix} e_{Mt}^{(i)} \\ e_{Yt}^{(i)} \\ e_{Ft}^{(i)} \end{pmatrix} \qquad (\mathbf{A.5})$$

The $\mathbf{A}_0$ and $\mathbf{A}_1$ are estimated from a VAR(1) on the historical data and the precise values are given in the tables below.

3. Generate data for S_{1t} and S_{2t} by drawing $T \times 1$ vectors $e_{S1t}^{(i)}$, $e_{S2t}^{(i)}$ independently from $N(0,1)$ distributions, where

$$\begin{aligned} S_{1t}^{(i)} &= \quad d_{10} + d_{11}S_{1t-1}^{(i)} + d_{12}e_{Mt}^{(i)} + d_{13}e_{Yt}^{(i)} + d_{14}e_{Ft}^{(i)} + d_{15}e_{S1t}^{(i)} + d_{16}e_{S2t}^{(i)} \\ S_{2t}^{(i)} &= \quad d_{20} + d_{21}S_{2t-1}^{(i)} + d_{22}e_{Mt}^{(i)} + d_{23}e_{Yt}^{(i)} + d_{24}e_{Ft}^{(i)} + d_{25}e_{S1t}^{(i)} \end{aligned} \tag{A.1}$$

   The calibration for the parameters of (A.1) is designed to roughly match the historical correlations $\mathbb{E}(Z_1\hat{e}_{Mt})$, $\mathbb{E}(Z_1\hat{e}_{Ft})$, $\mathbb{E}(Z_2\hat{e}_{Ft})$, as well as the correlation between Z_{1t} and Z_{2t}. The precise values are given in the tables below.

4. Taking $\left(U_M^{(i)}, Y^{(i)}, U_F^{(i)}, S_{1t}^{(i)}, S_{2t}^{(i)}\right)'$ as data, estimate a reduced form VAR counterpart to (**A.5**) and obtain the reduced form shocks $\hat{\boldsymbol{\eta}}_t^{(i)}$. Initialize $j = 0$ and $\left(\hat{\mathbf{e}}_Y^{(i),[0]}, \hat{\mathbf{e}}_M^{(i),[0]}\right)' = \left(Y^{(i)}, U_M^{(i)}\right)'$.

   4.1 Given $\left(\hat{\mathbf{e}}_Y^{(i),[j]}, \hat{\mathbf{e}}_M^{(i),[j]}\right)$, calculate the $\mathbf{Z}$ by running the following regressions.

$$\begin{aligned} S_{1t}^{(i)} &= \beta_1' x_{1t}^{(i),[j]} + Z_{1t}^{(i),[j]} \\ S_{2t}^{(i)} &= \beta_2' x_{2t}^{(i),[j]} + Z_{2t}^{(i),[j]}, \end{aligned}$$

   where $x_{1t}^{(i)} = (1, S_{1t-1}^{(i)}, e_Y^{(i),[j]})'$ and $x_{2t}^{(i)} = (1, S_{2t-1}^{(i)}, e_Y^{(i),[j]}, e_M^{(i),[j]})'$,

   4.2 Use $Z_1^{(i),[j]}$ and $Z_2^{(i),[j]}$ and estimates $\mathrm{vech}\left(\hat{\boldsymbol{\eta}}_t^{(i)}\hat{\boldsymbol{\eta}}_t^{(i)\prime}\right)$ and $\mathrm{vec}\left(Z_t^{(i),[j]} \otimes \hat{\boldsymbol{\eta}}_t^{(i)}\right)$ to impose Assumption A of the paper and solve for $\mathbf{B}$. We obtain $\hat{e}_Y^{(i),[j+1]}$, $\hat{e}_M^{(i),[j+1]}$, $\hat{e}_F^{(i),[j+1]}$ from $\hat{\mathbf{e}}^{(i),[j+1]} = \left(\mathbf{B}^{(i),[j]}\right)^{-1}\hat{\boldsymbol{\eta}}_t^{(i)}$

   4.3 If $\left\|\hat{\mathbf{e}}^{(i),[j+1]} - \hat{\mathbf{e}}^{(i),[j]}\right\| < \epsilon$ (where ϵ is an arbitrarily small number), then set $\hat{\mathbf{e}}^{(i)} = \hat{\mathbf{e}}^{(i),[j]}$. Otherwise, set $j = j + 1$ and return to step 4.1.

For each i, we store $\hat{\mathbf{e}}^{(i)}$ and record the series correlations between the estimated and true shocks for each $j = Y, M, F$,

$$\rho_j^{(i)} = corr\left(e_{jt}^{(i)}, \hat{e}_{jt}^{(i)}\right) = \frac{1}{T}\sum_{t=1}^{T}\left(e_{jt}^{(i)}\right)\left(\hat{e}_{jt}^{(i)}\right)$$

We report the average of the series correlations $\rho_Y^{(i)}$, $\rho_M^{(i)}$, $\rho_F^{(i)}$ across I iterations. We also calculate the correlations, for each t and each $j = Y, M, F$,

$$\rho_{jt} = \frac{\frac{1}{I}\sum_{i=1}^{I}\left(e_{jt}^{(i)}\right)\left(\hat{e}_{jt}^{(i)}\right)}{\sqrt{\frac{1}{I}\sum_{i=1}^{I}\left(e_{jt}^{(i)}\right)^2}\sqrt{\frac{1}{I}\sum_{i=1}^{I}\left(\hat{e}_{jt}^{(i)}\right)^2}}.$$

We report $\sum_{t=1}^{T}\rho_t$. Finally, we report the cross-iteration average of $\hat{\mathbf{B}}^{(i)}$ and $\hat{\mathbf{A}}_0^{(i)}$.

We report results for $T = 500$. The data generating process S_{1t} and S_{2t} is set to mimic our estimates using observed stock market data, which has a large idiosyncratic component e_{st}. The results are reported in Table (A2). Under these parameterizations we generate instruments Z_{1t} and Z_{2t} that have the empirically relevant correlations with the estimated e_{Mt} and e_{Ft}. These correlations are $corr\left(Z_{1t}, e_{Mt}\right) = -0.07$, $corr\left(Z_{1t}, e_{Ft}\right) = -0.13$, and $corr\left(Z_{2t}, e_{Ft}\right) = -0.17$. In addition, the calibration leads Z_{1t} and Z_{2t} to be highly correlated (98%), very close to the historical correlation attributable to the large common component in stock returns. The results show that the procedure recovers a close approximation of the true structural shocks (and therefore $\mathbf{B}$ matrix) when the instruments have the observed degree of relevance for the uncertainty shocks, and when finite samples are set to be within range of the size used in this study.

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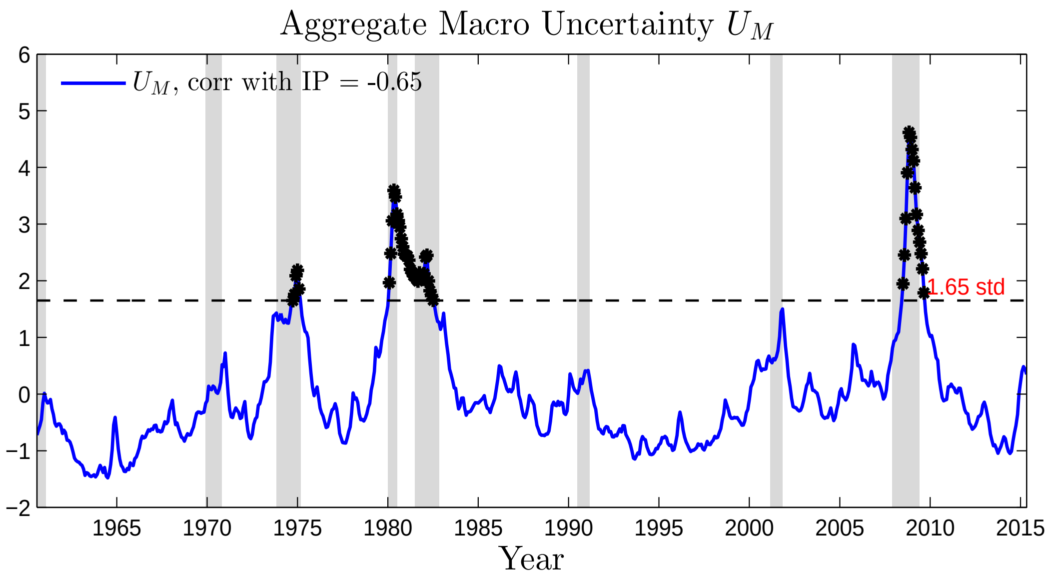
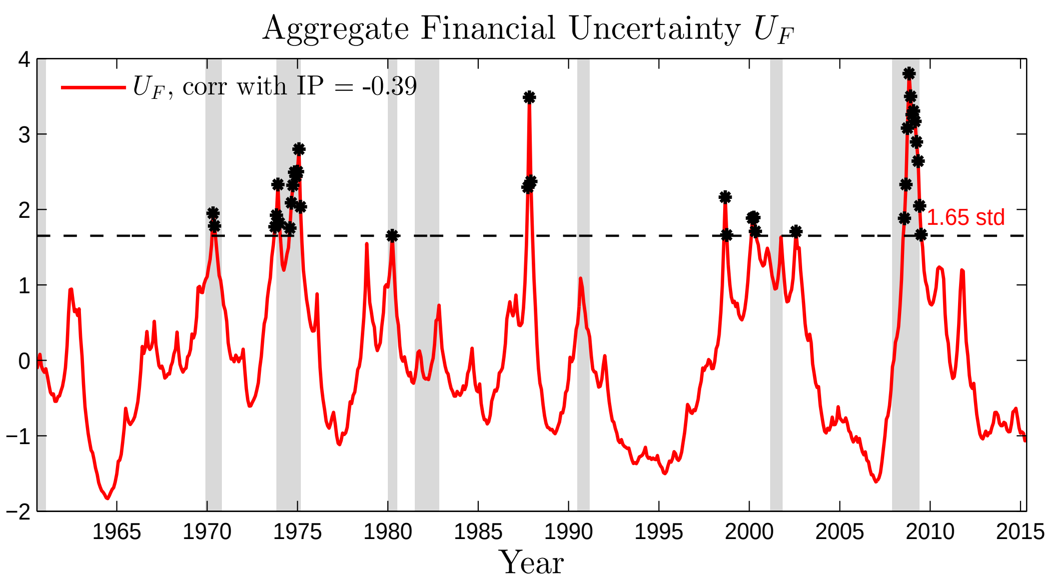
8 Figures and Tables

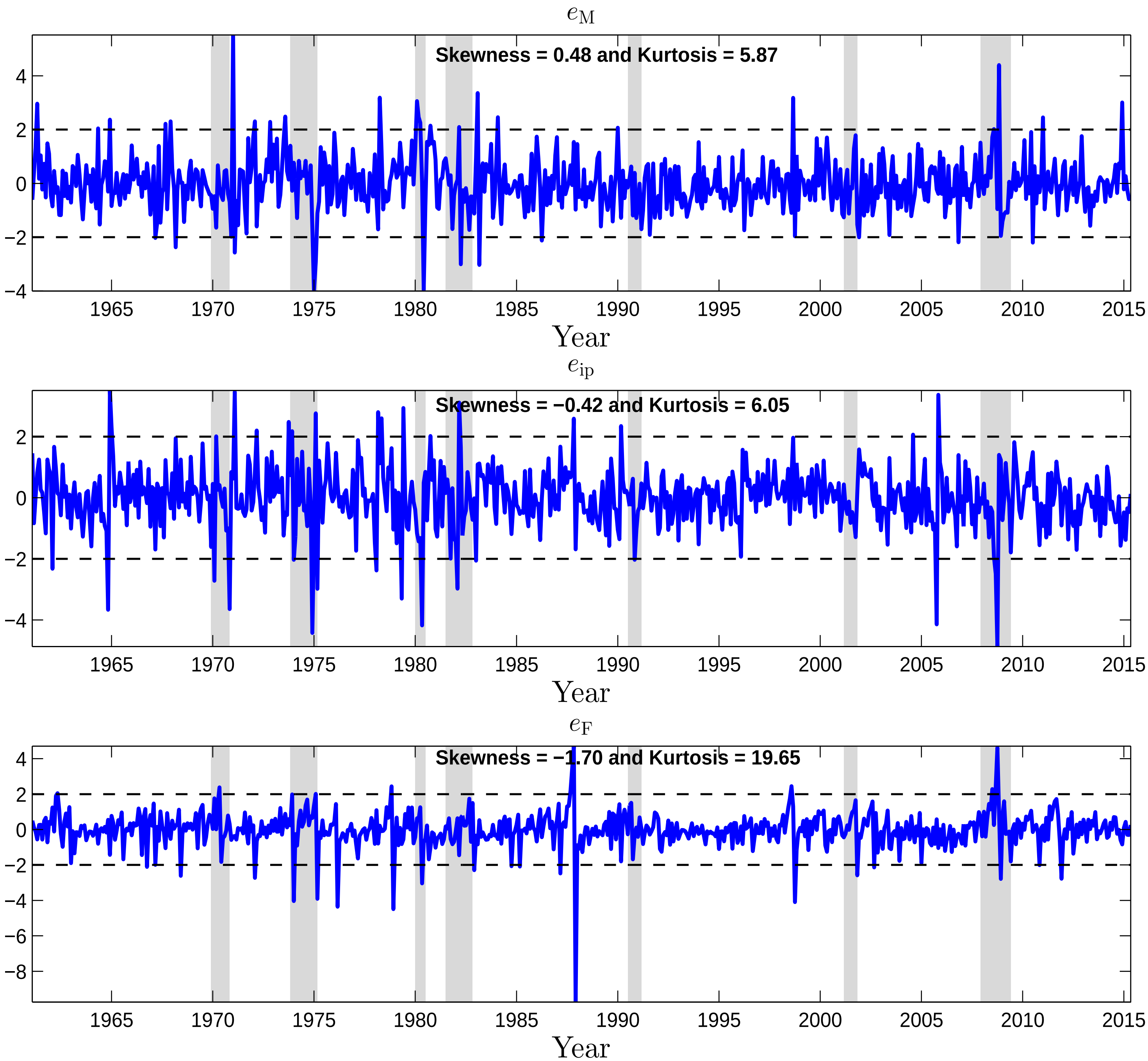
Figure 1: Macro and Financial Uncertainty Over Time





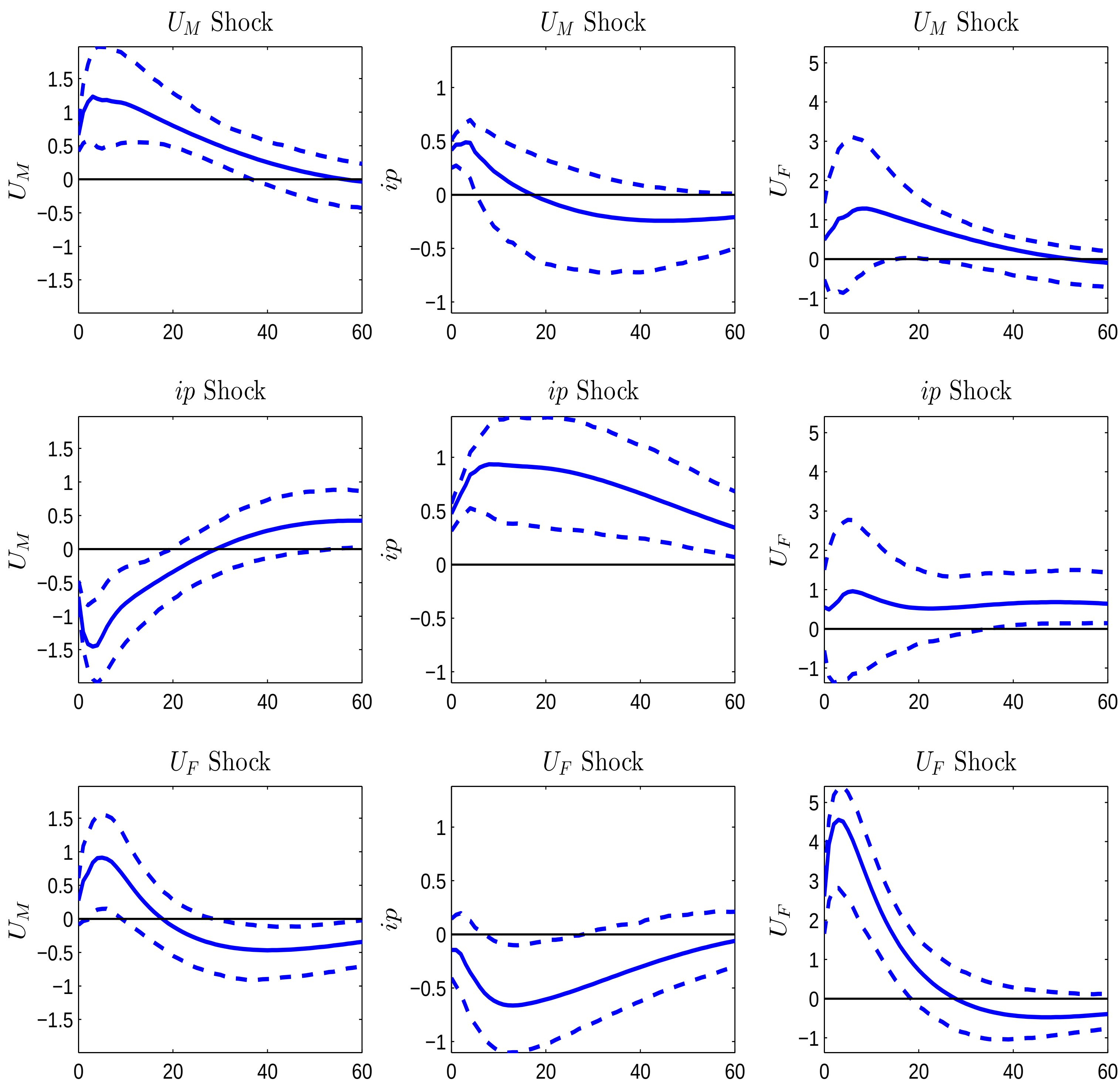
The upper panel plots the time series of the macro uncertainty U_M, expressed in standardized units. The lower panel shows the time series of financial uncertainty U_F expressed in standardized units. The vertical lines correspond to the NBER recession dates. The horizontal line corresponds to 1.65 standard deviations above the unconditional mean of each series (which has been normalized to zero). Correlations with the 12-month moving average of IP growth are reported. The black dots represent months when uncertainty is 1.65 standard deviations above its unconditional mean. The data are monthly and span the period 1960:07 to 2015:04.

Figure 2: Time Series of e Shock from SVAR System $(U_M, ip, U_F)'$



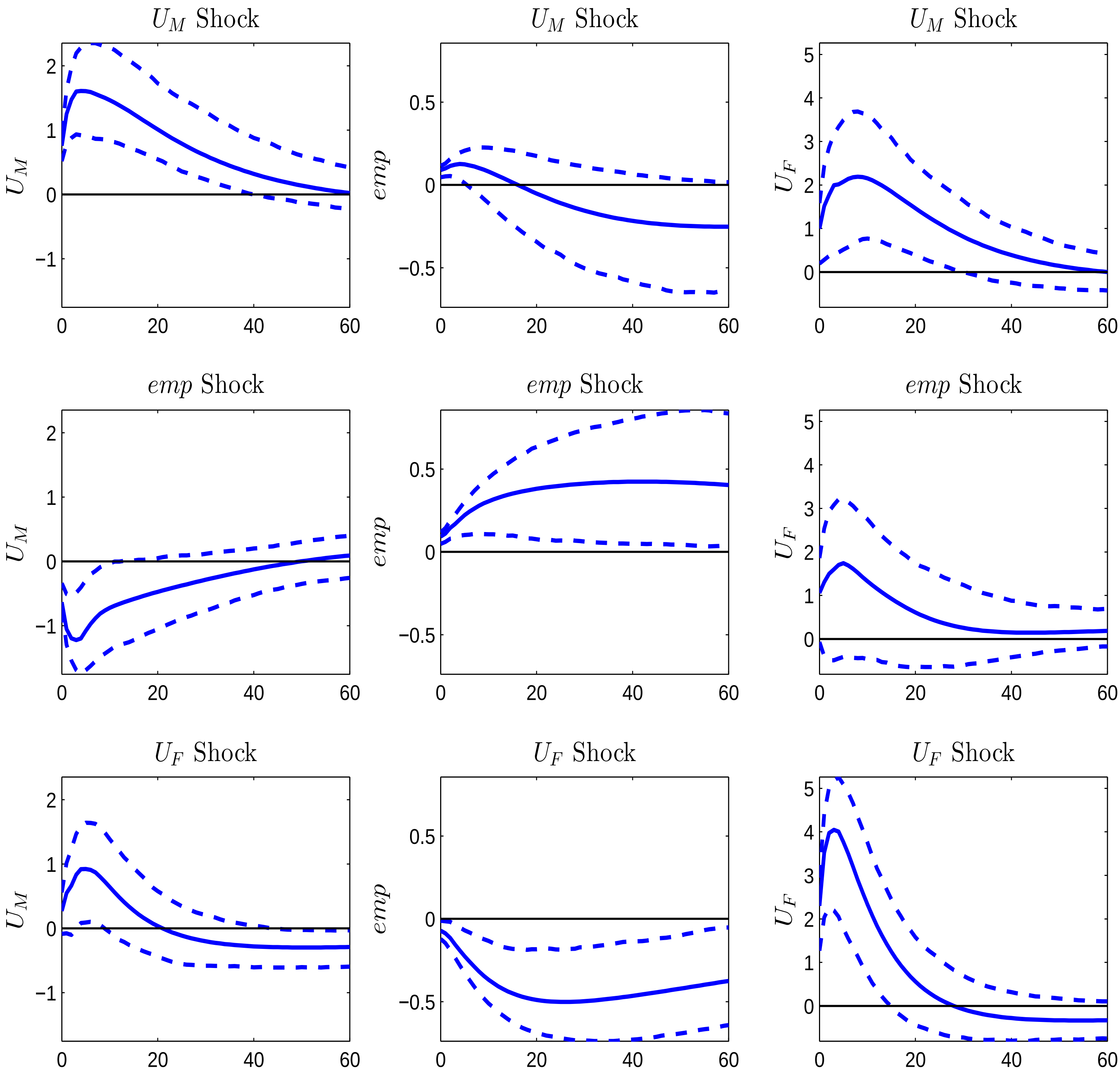
The horizontal line corresponds to 3 standard deviations above/below the unconditional mean of each series. The shocks $e = B^{-1}\eta_t$ are reported, where η_t is the residual from VAR(6) of $(U_M, ip, U_F)'$ and $B = A^{-1}\Sigma^{\frac{1}{2}}$. Skewness is defined as $s = \frac{\sum_t^T (e_t - \bar{e})^3/T}{Var(e)}$. Kurtosis is defined as $\kappa = \frac{\sum_t^T (e_t - \bar{e})^4/T}{[Var(e)]^2}$. The sample spans the period 1960:07 to 2015:04.

Figure 3: Dynamic Responses in SVAR $(U_M, ip, U_F)'$



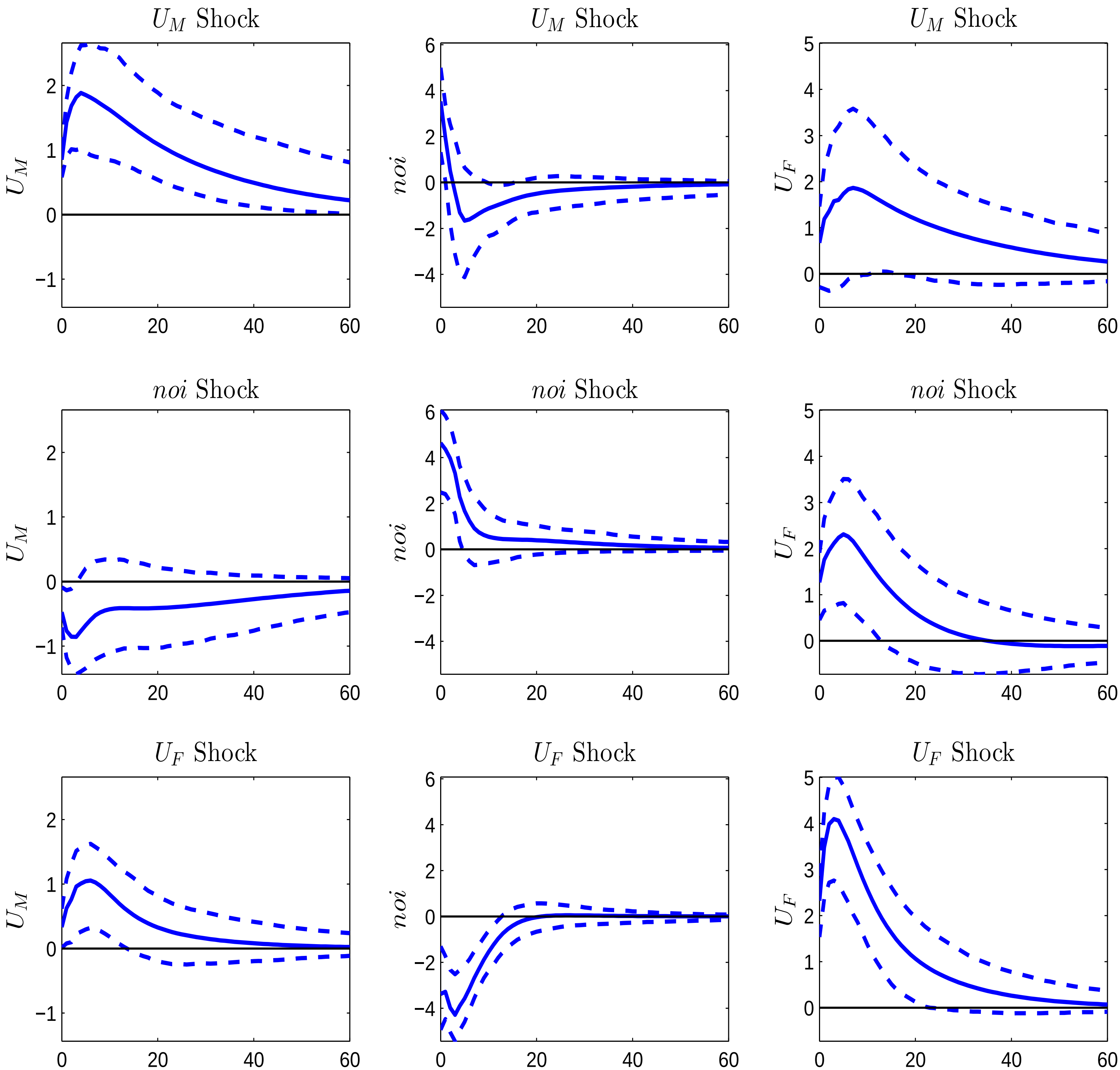
Bootstrapped 90% error bands appear as dashed lines. Response units are reported in percentage points. The sample spans the period 1960:07 to 2015:04.

Figure 4: Dynamic Responses in SVAR $(U_M, emp, U_F)'$



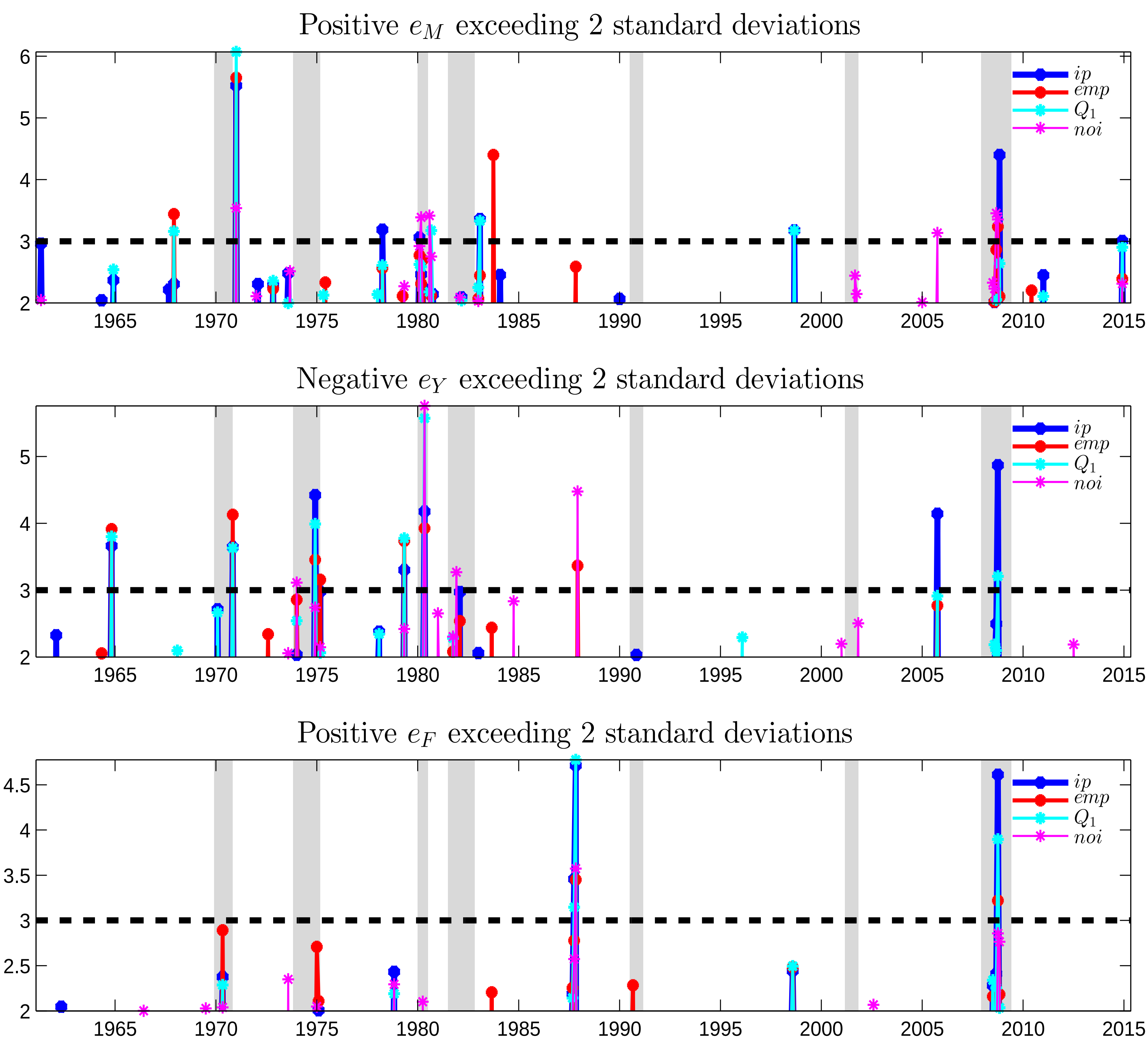
Bootstrapped 90% error bands appear as dashed lines. Response units are reported in percentage points. The sample spans the period 1960:07 to 2015:04.

Figure 5: Dynamic Responses in SVAR$(U_M, noi, U_F)'$



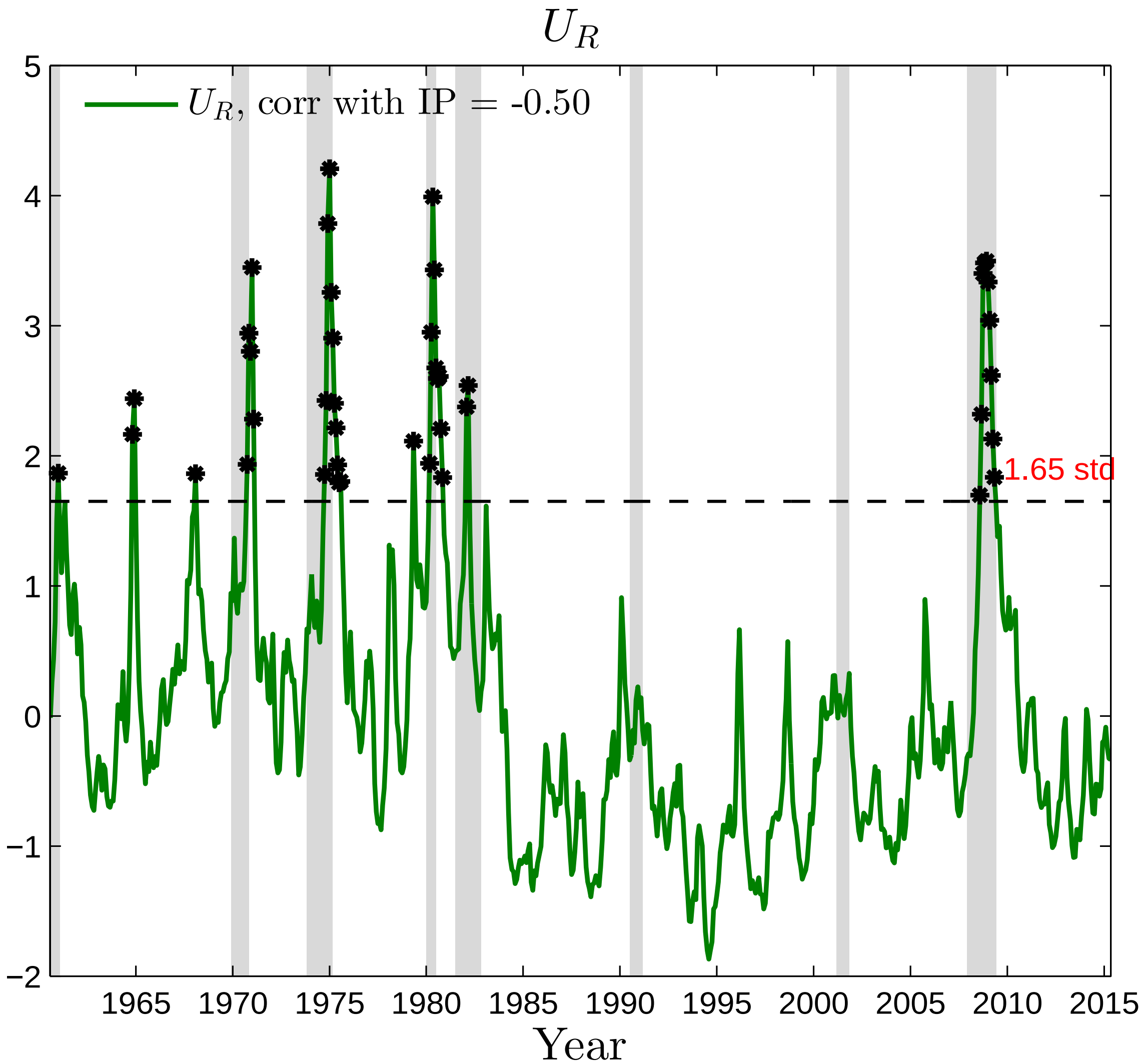
Bootstrapped 90% error bands appear as dashed lines. Response units are reported in percentage points. The sample spans the period 1960:07 to 2015:04.

Figure 6: Large Shock Episodes in SVAR$(U_M, Y, U_F)'$



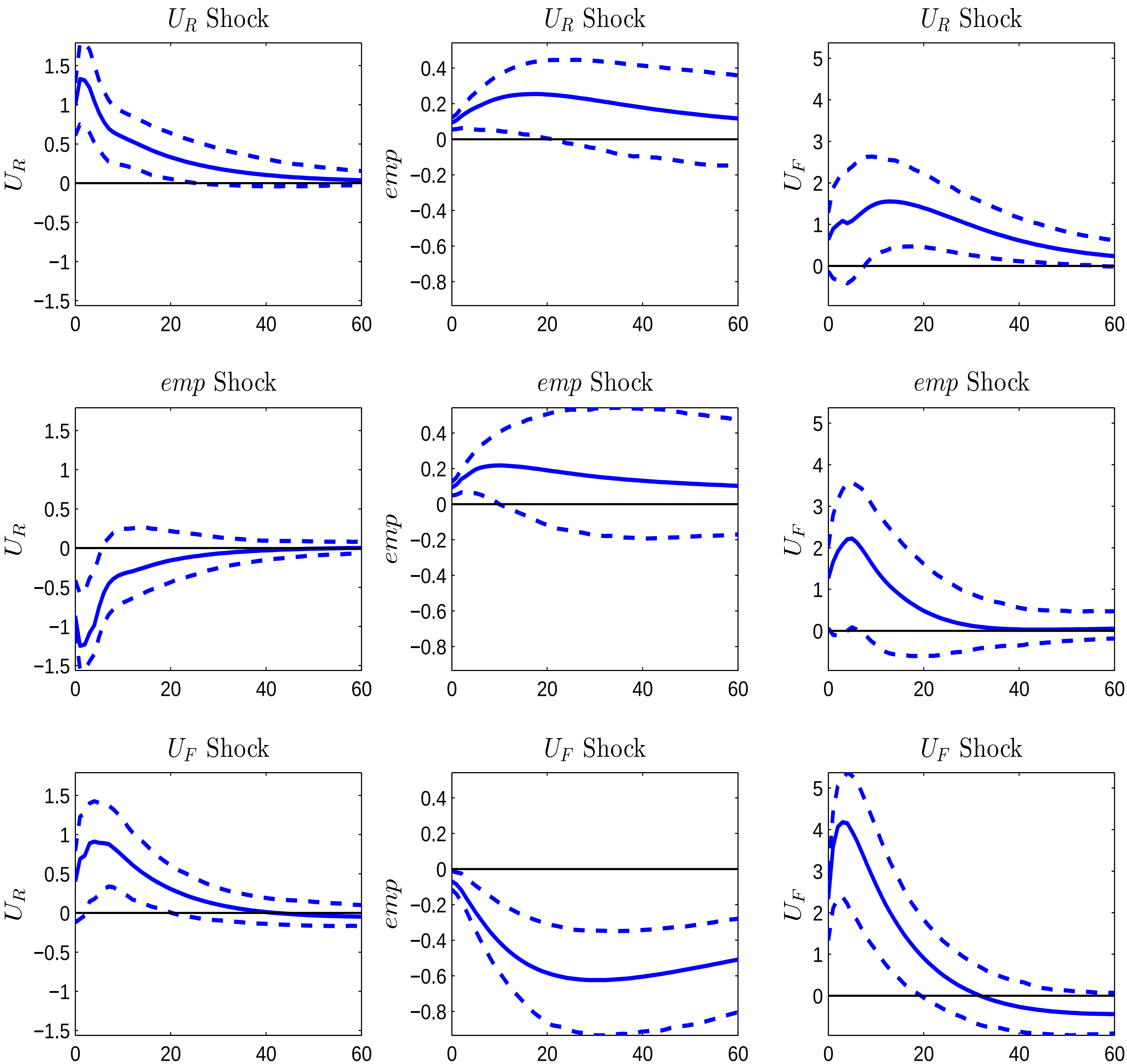
The figure exhibits shocks that are at least 2 standard deviations above the unconditional mean for e_M and e_F and below for e_Y for three cases where $Y = ip, emp, Q_1$. The shocks $e = B^{-1}\eta_t$ are reported, where η_t is the residual from VAR(6) and $B = A^{-1}\Sigma^{\frac{1}{2}}$. The horizontal line corresponds to 3 standard deviations shocks. The sample spans the period 1960:07 to 2015:04.

Figure 7: Real Uncertainty Over Time



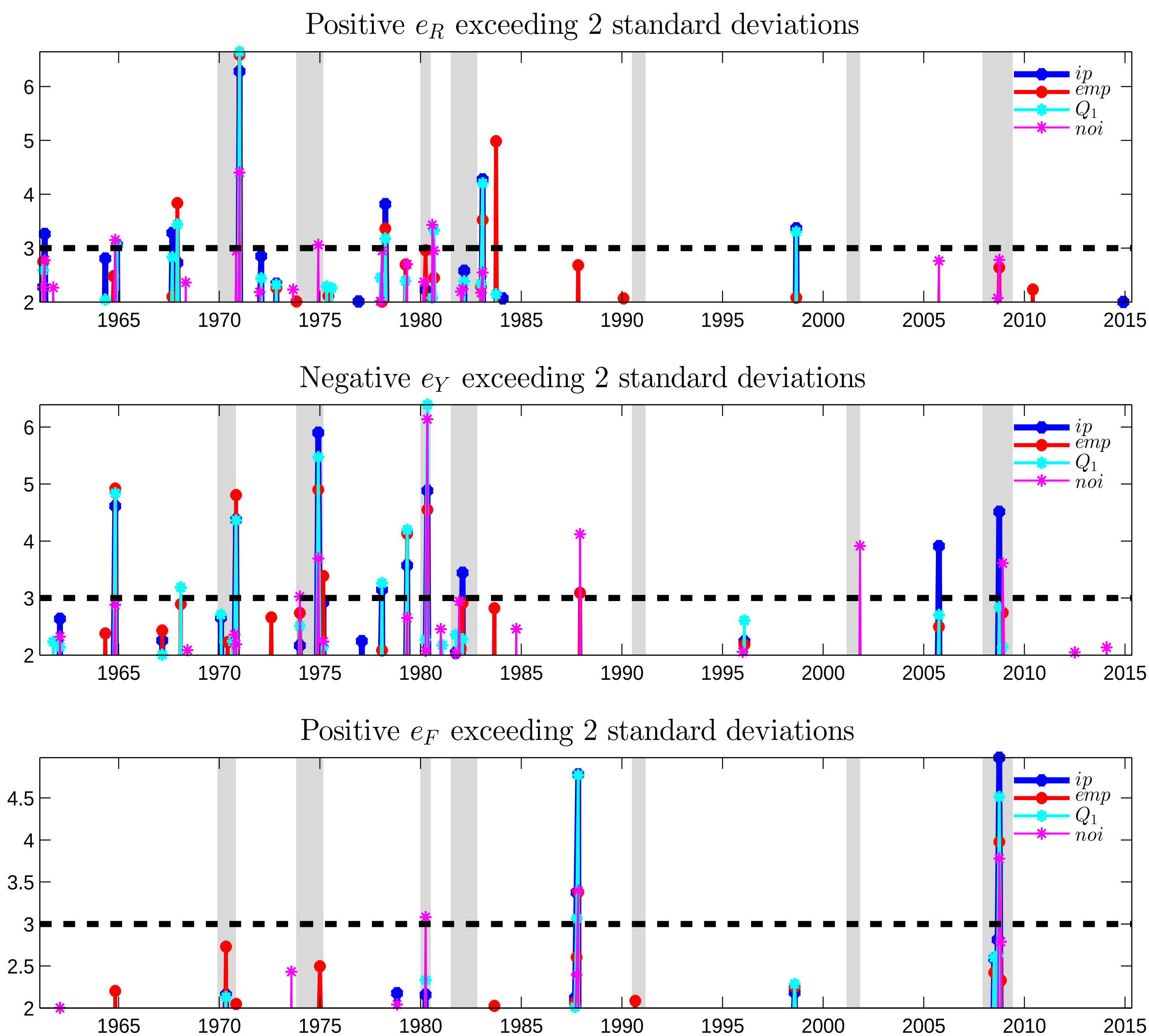
This plot shows time series of U_R, expressed in standardized units. The vertical lines correspond to the NBER recession dates. The horizontal line corresponds to 1.65 standard deviations above the unconditional mean of each series (which has been normalized to zero). Correlations with the 12-month moving average of IP growth are reported. The black dots represent months when U_R is 1.65 standard deviations above its unconditional mean. The data are monthly and span the period 1960:07 to 2015:04.

Figure 8: Dynamic Responses in SVAR $(U_R, emp, U_F)'$



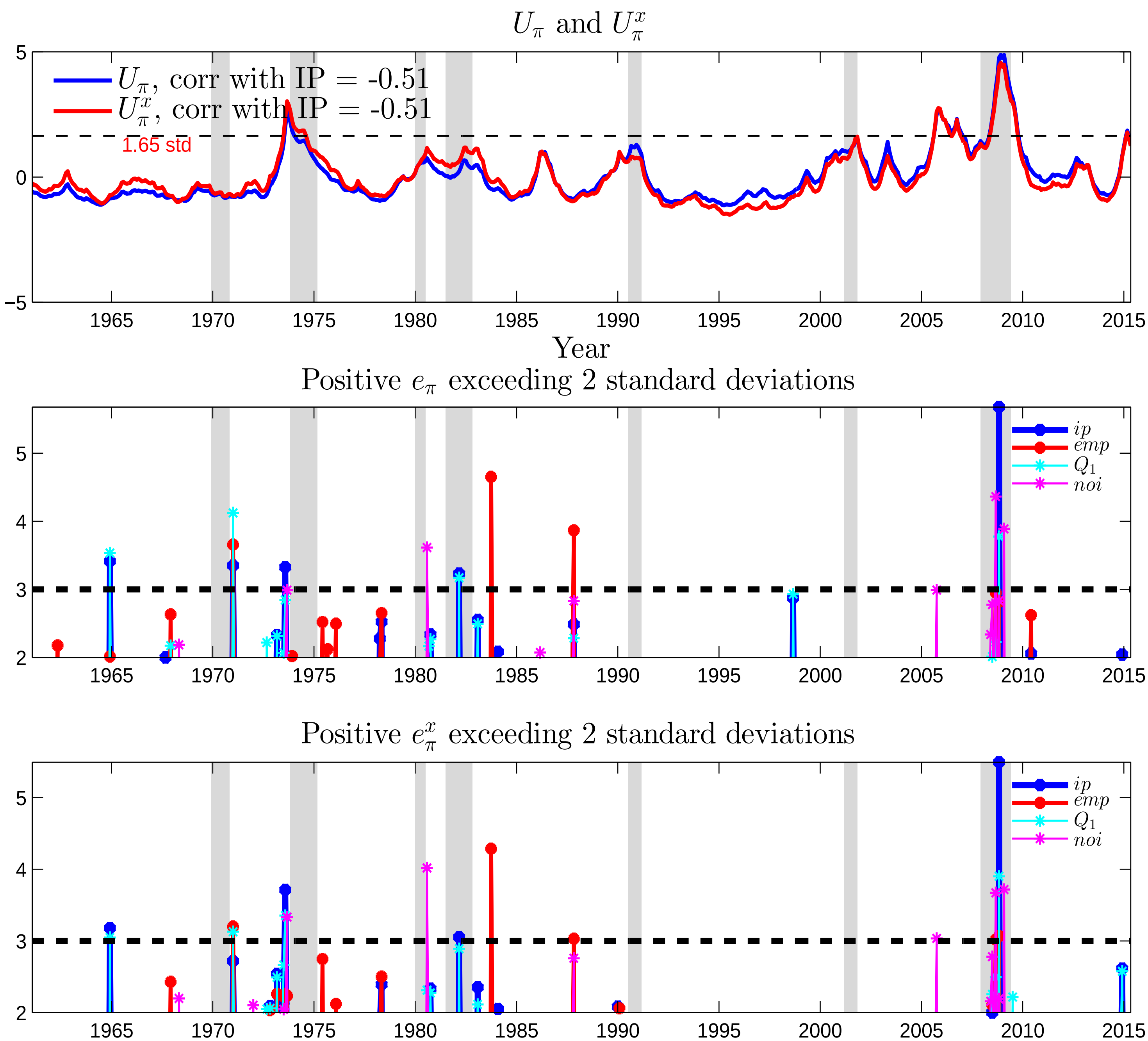
Bootstrapped 90% error bands appear as dashed lines. Response units are reported in percentage points. The sample spans the period 1960:07 to 2015:04.

Figure 9: Large Shock Episodes in SVAR$(U_R, Y, U_F)'$



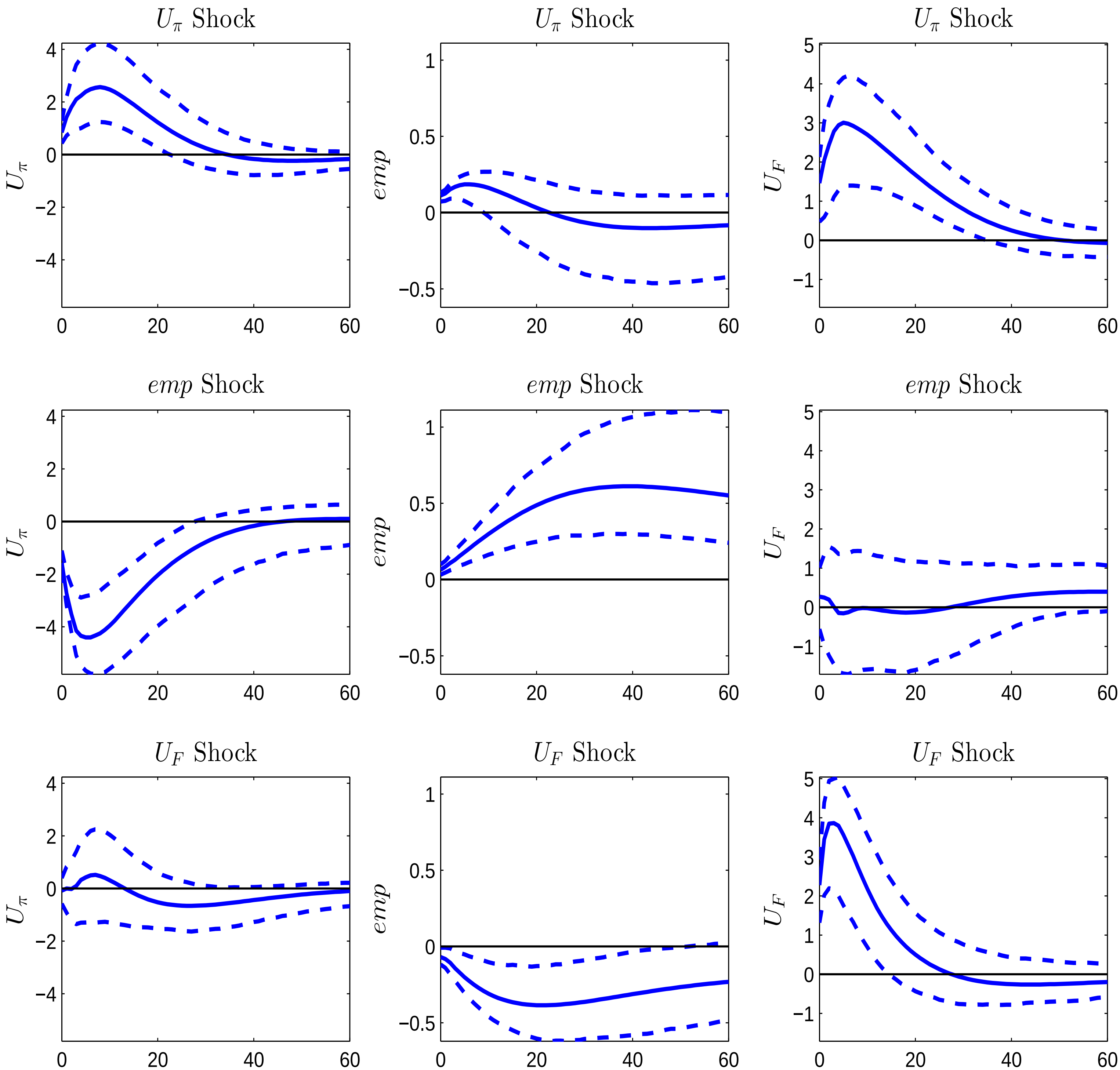
The figure exhibits shocks that are at least 2 standard deviations above the unconditional mean for e_R and e_F and below for e_Y for four cases where $Y = ip, emp, Q_1, noi$. The shocks $e = B^{-1}\eta_t$ are reported, where η_t is the residual from VAR(6) and $B = A^{-1}\Sigma^{\frac{1}{2}}$. The horizontal line corresponds to 3 standard deviations shocks. The sample spans the period 1960:07 to 2015:04.

Figure 10: Price uncertainty



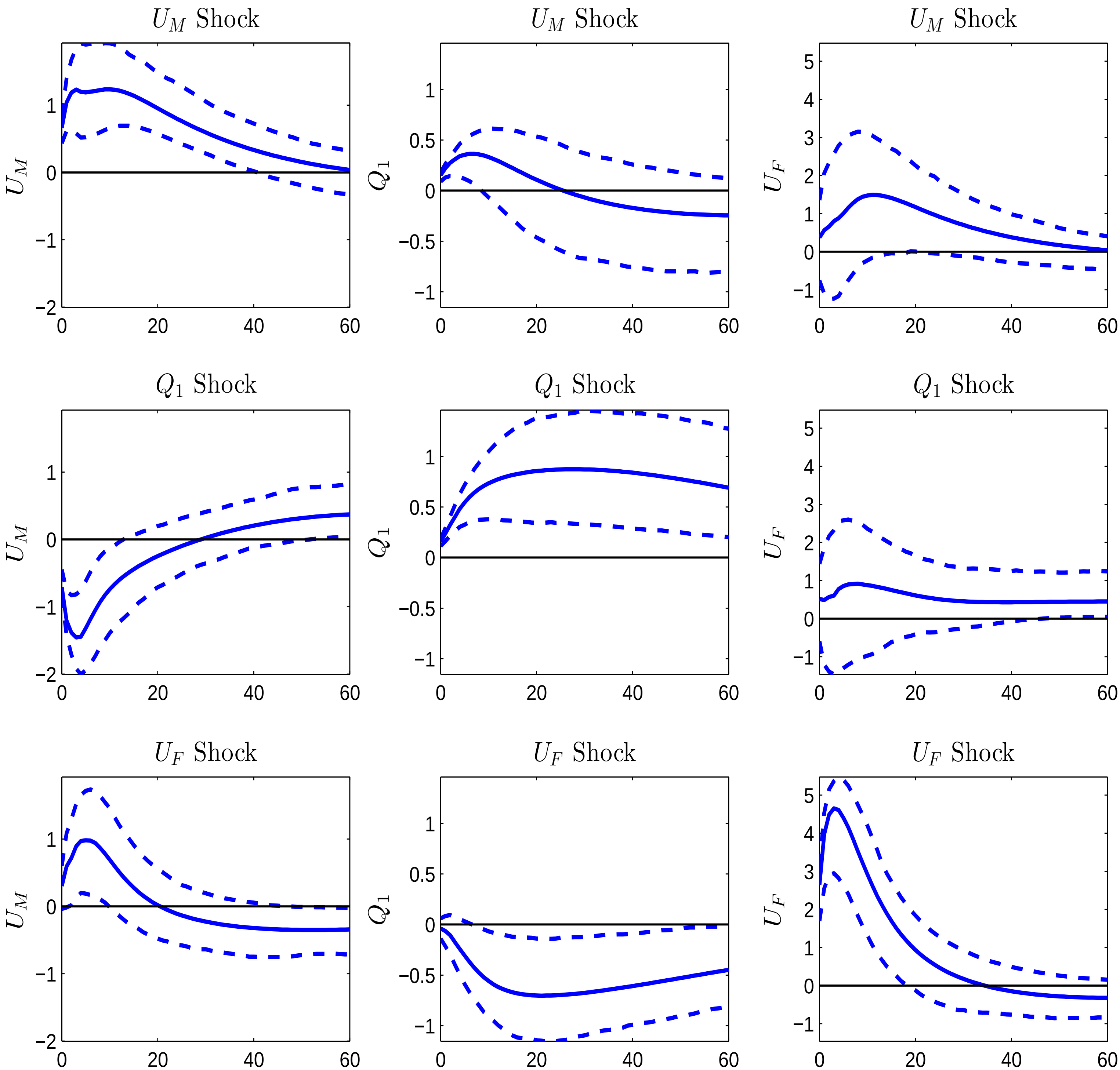
The upper panel plots U_π and U_π^x where the latter excludes uncertainties for five volatile sub-series defined in the text, expressed in standardized units. The five series are: PPI intermediate materials, PPI crude materials, oil, PPI metals and metal products, and CPI transportation. The middle and lower panel exhibit shocks that are at least 2 standard deviations above the unconditional mean for U_π and U_π^x. The shaded vertical bars correspond to the NBER recession dates. Correlations with the 12-month moving average of IP growth are reported. The data are monthly and span the period 1960:07 to 2015:04.

Figure 11: Dynamic Responses in SVAR $(U_\pi, emp, U_F)'$



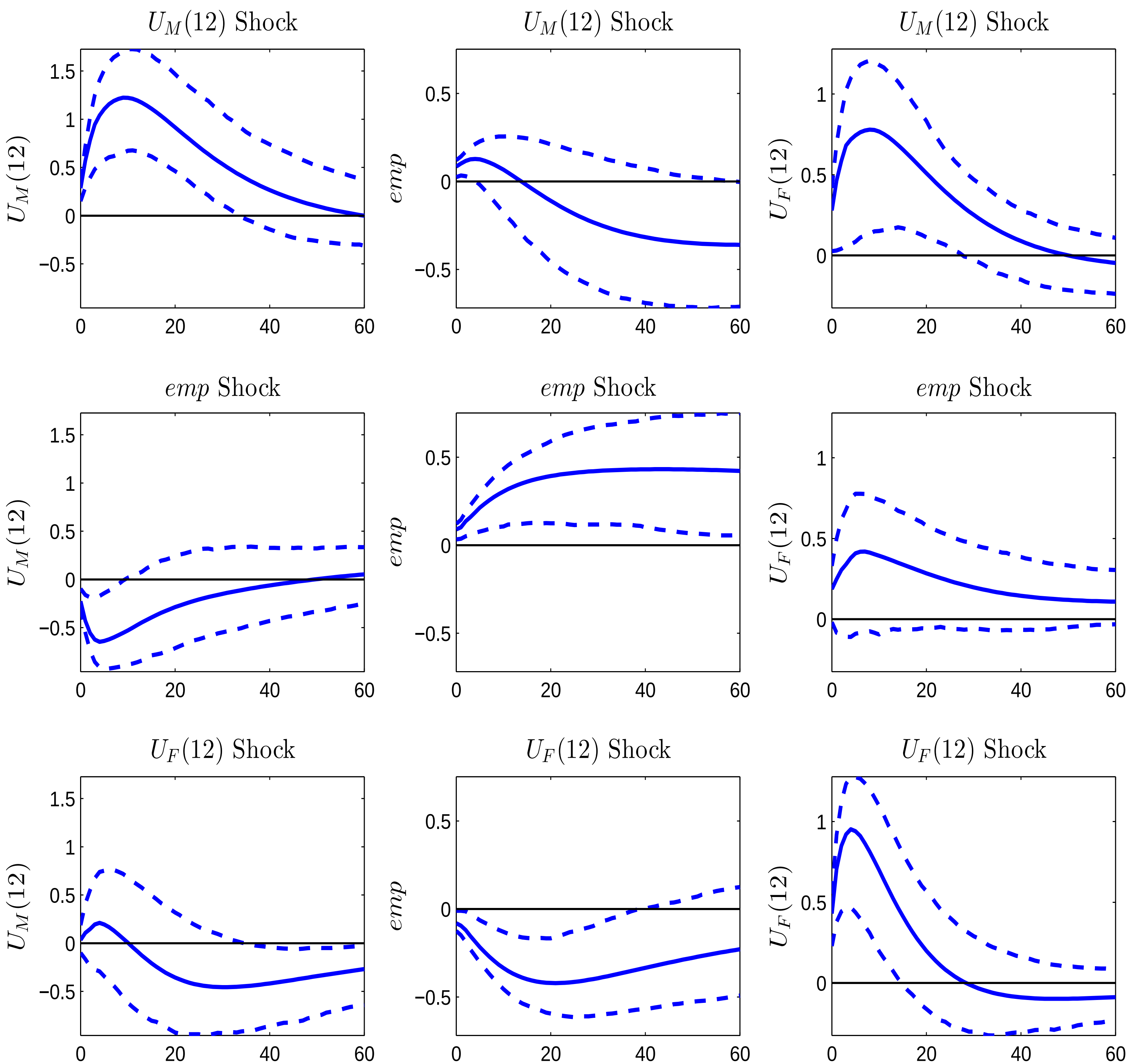
Bootstrapped 90% error bands appear as dashed lines. Response units are reported in percentage points. The sample spans the period 1960:07 to 2015:04.

Figure 12: Dynamic Responses in SVAR $(U_M, Q_1, U_F)'$



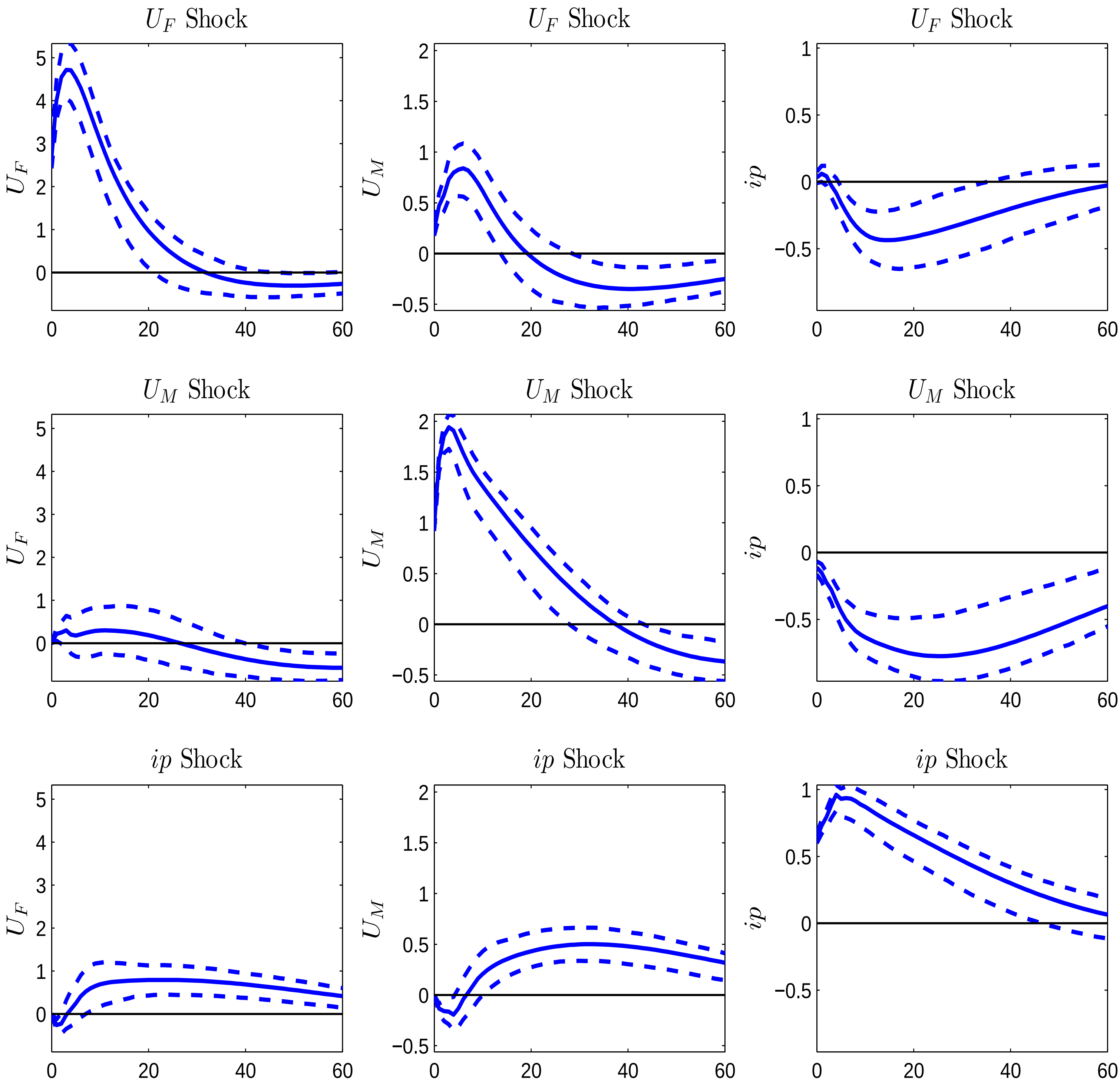
Bootstrapped 90% error bands appear as dashed lines. Response units are reported in percentage points. The sample spans the period 1960:07 to 2015:04.

Figure 13: Dynamic Responses in SVAR$(U_M(12), emp, U_F(12))'$



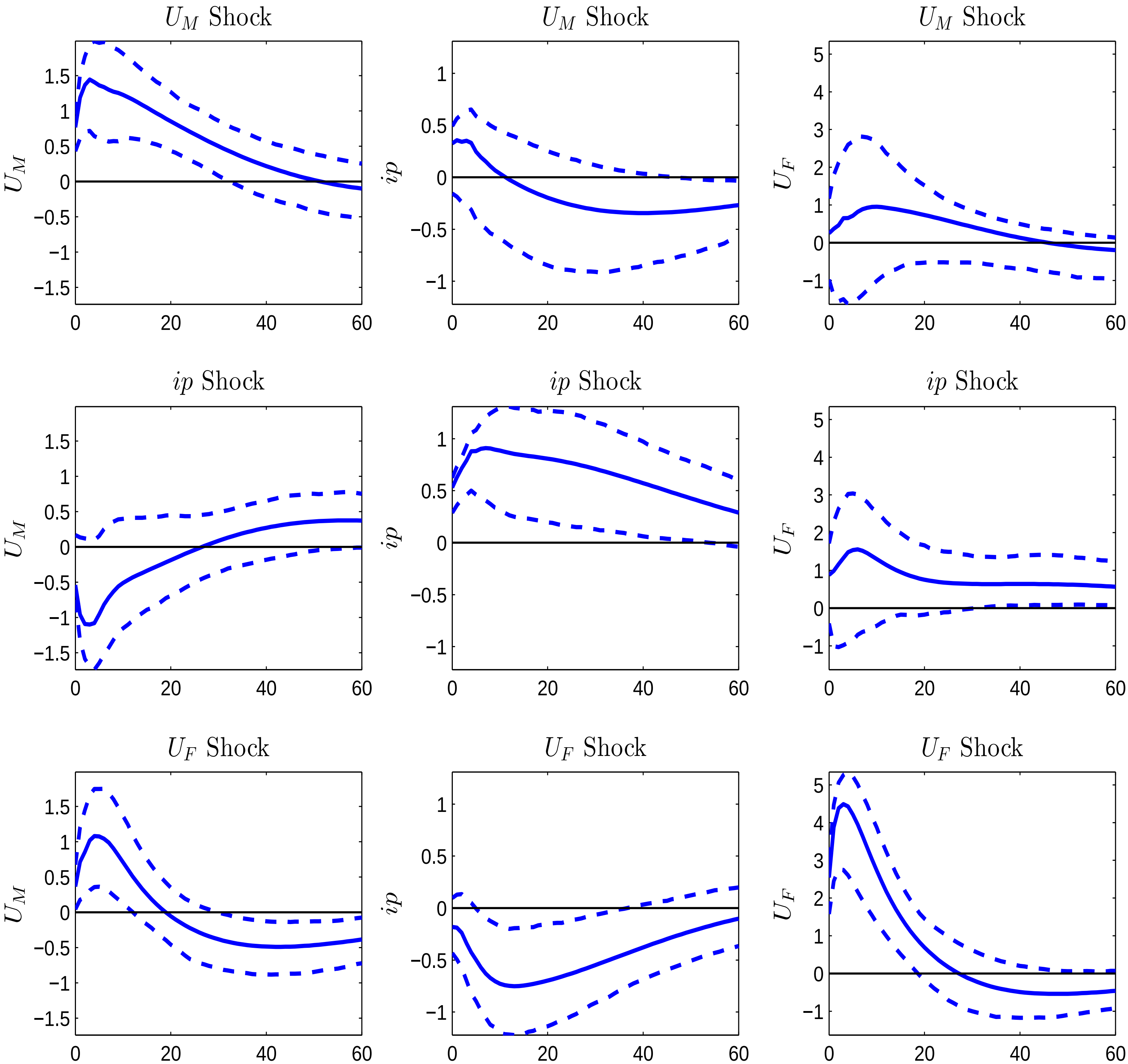
Bootstrapped 90% error bands appear as dashed lines. Response units are reported in percentage points. The sample spans the period 1960:07 to 2015:04.

Figure 14: Dynamic Responses using Recursive Identification with Order $(U_F, U_M, ip)'$



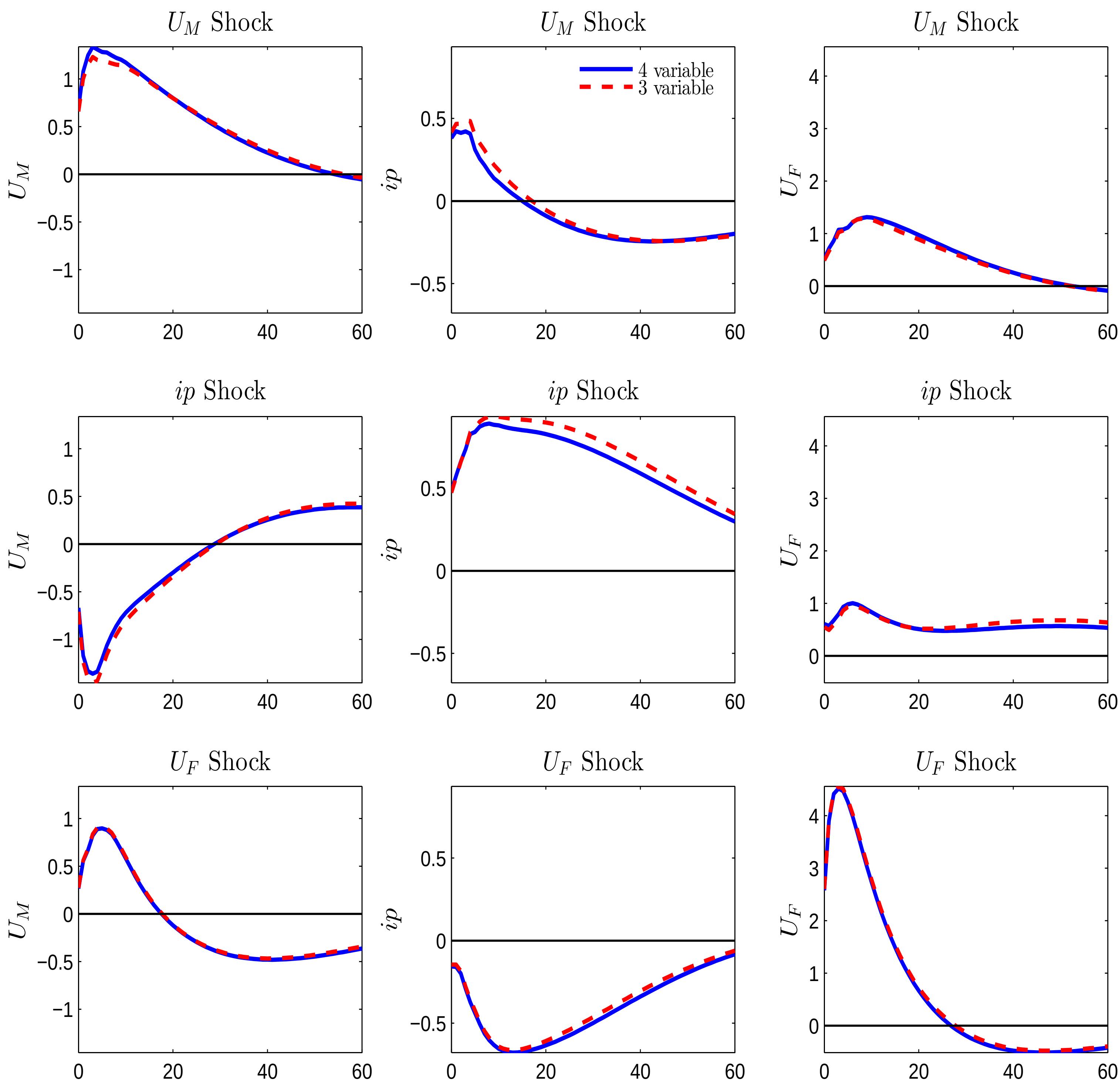
Bootstrapped 90% error bands appear as dashed lines. Response units are reported in percentage points. The sample spans the period 1960:07 to 2015:04.

Figure 15: Dynamic Responses in SVAR $(U_M, ip, U_F)'$



Moody's Seasoned Baa corporate bond yield Baa_t is used to construct Z_1 and the CRSP excess return to construct Z_2. Bootstrapped 90% error bands appear as dashed lines. Response units are reported in percentage points. The sample spans the period 1960:07 to 2015:04.

Figure 16: Dynamic Responses in SVAR $(U_M, ip, U_F, S_t)'$ v.s. $(U_M, ip, U_F)'$



S_t is the CRSP value weighted average returns. Response units are reported in percentage points. The sample spans the period 1960:07 to 2015:04.

Table 1: Sample Statistics

Panel A: Correlations between Instruments and Shocks				
$SVAR$	$(U_M, ip, U_F)'$	$(U_M, emp, U_F)'$	$(U_M, Q_1, U_F)'$	$(U_M, noi, U_F)'$
$\rho\left(Z_{1t}, \hat{e}_{Mt}\right)$	$-\mathbf{0.0744}$	$-\mathbf{0.0680}$	$-\mathbf{0.0708}$	$-\mathbf{0.0509}$
	(0.0041)	(0.0037)	(0.0039)	(0.0028)
$\rho\left(Z_{1t}, \hat{e}_{Ft}\right)$	$-\mathbf{0.1701}$	$-\mathbf{0.1812}$	$-\mathbf{0.1742}$	$-\mathbf{0.1913}$
	(0.0093)	(0.0099)	(0.0095)	(0.0105)
$\rho\left(Z_{2t}, \hat{e}_{Ft}\right)$	$-\mathbf{0.1557}$	$-\mathbf{0.1687}$	$-\mathbf{0.1594}$	$-\mathbf{0.1752}$
	(0.0093)	(0.0101)	(0.0096)	(0.0106)
$\rho\left(Z_{1t}, \hat{e}_{Yt}\right)$	0.0000	0.0000	0.0000	0.0000
$\rho\left(Z_{2t}, \hat{e}_{Yt}\right)$	0.0000	0.0000	0.0000	0.0000
$\rho\left(Z_{2t}, \hat{e}_{Mt}\right)$	0.0000	0.0000	0.0000	0.0000
Panel B: Estimates of $\mathbf{\Sigma}$				
σ_{MM}	**0.0066**	**0.0076**	**0.0066**	**0.0085**
	(0.0010)	(0.0009)	(0.0010)	(0.0011)
	$[0.041, 0.086]$	$[0.005, 0.010]$	$[0.004, 0.009]$	$[0.006, 0.010]$
σ_{YY}	**0.0047**	**0.0009**	**0.0016**	**0.0464**
	(0.0007)	(0.0002)	(0.0002)	(0.0092)
	$[0.003, 0.006]$	$[0.001, 0.001]$	$[0.001, 0.002]$	$[0.025, 0.060]$
σ_{FF}	**0.0260**	**0.0230**	**0.0264**	**0.0232**
	(0.0029)	(0.0025)	(0.0030)	(0.0025)
	$[0.017, 0.031]$	$[0.013, 0.029]$	$[0.017, 0.031]$	$[0.014, 0.029]$

Panel A reports the correlation between the estimated uncertainty shocks and the instruments. Panel B reports estimates of $\mathbf{\Sigma}$ that give the standard deviation of each structural shock. Asymptotic standard errors are reported in brackets and bootstrapped 90 percent confidence intervals are reported in parentheses. Bolded numbers indicate statistical significance at 10 percent level. The data are monthly and span the period 1960:07 to 2015:04.

Table 2: Tests of Validity of Recursive Restriction in System $(U_M, Y, U_F)'$

Ordering:	$(U_M, ip, U_F)'$	$(U_M(12), ip, U_F(12))'$
H_0: $B_{RY} = B_{RF} = B_{YF} = 0$	**239.54**	**127.75**
	[110.79]	[38.60]
H_0: $B_{YR} = B_{YF} = B_{RF} = 0$	**25.96**	**275.35**
	[65.89]	[47.22]
H_0: $B_{RY} = B_{RF} = B_{FY} = 0$	**225.18**	**123.08**
	[113.74]	[43.26]
$\chi^2_{5\%}(3)$	7.81	7.81
	$(U_M, emp, U_F)'$	$(U_M(12), emp, U_F(12))'$
H_0: $B_{RY} = B_{RF} = B_{YF} = 0$	**236.29**	**113.63**
	[79.12]	[47.42]
H_0: $B_{YR} = B_{YF} = B_{RF} = 0$	**70.73**	**229.54**
	[53.61]	[69.62]
H_0: $B_{RY} = B_{RF} = B_{FY} = 0$	**228.85**	**116.15**
	[88.95]	[63.02]
$\chi^2_{5\%}(3)$	7.81	7.81

The table reports the Wald test statistic for testing the null hypothesis given in the column. The bold indicates that Wald test rejects the null at 95 percent level according to $\chi^2(3)$ distribution. The SVAR system is solved using GMM and delta method is used for computing the standard error. Estimates of $\mathbf{B}$ are based on the SVAR identified with external instruments described in the text. The mean of bootstrap Wald statistics is reported in parenthesis. The sample size spans 1960:07 to 2015:04.

Table 3: Variance Decomposition for SVARs in System $(U_M, Y, U_F)'$

	SVAR $(U_M, ip, U_F)'$			SVAR $(U_M, emp, U_F)'$			SVAR $(U_M, noi, U_F)'$		
	Fraction variation in U_M			Fraction variation in U_M			Fraction variation in U_M		
s	U_M Shock	ip Shock	U_F Shock	U_M Shock	emp Shock	U_F Shock	U_M Shock	noi Shock	U_F Shock
1	0.371	0.527	0.102	0.531	0.376	0.093	0.679	0.198	0.123
12	0.419	0.409	0.172	0.601	0.249	0.150	0.706	0.098	0.196
∞	0.420	0.368	0.212	0.619	0.220	0.161	0.739	0.116	0.145
$s_{\max}$	0.511	0.528	0.215	0.664	0.384	0.161	0.740	0.212	0.199
	$[0.25, 0.79]$	$[0.22, 0.71]$	$[0.05, 0.57]$	$[0.34, 0.87]$	$[0.15, 0.59]$	$[0.06, 0.46]$	$[0.37, 0.94]$	$[0.04, 0.50]$	$[0.03, 0.53]$
	Fraction variation in ip			Fraction variation in emp			Fraction variation in noi		
s	U_M Shock	ip Shock	U_F Shock	U_M Shock	emp Shock	U_F Shock	U_M Shock	noi Shock	U_F Shock
1	0.401	0.556	0.043	0.352	0.402	0.246	0.206	0.513	0.280
12	0.121	0.659	0.220	0.075	0.406	0.519	0.144	0.350	0.507
∞	0.082	0.691	0.227	0.124	0.424	0.453	0.162	0.348	0.489
$s_{\max}$	0.415	0.696	0.272	0.373	0.424	0.587	0.275	0.513	0.507
	$[0.19, 0.61]$	$[0.34, 0.94]$	$[0.04, 0.73]$	$[0.21, 0.63]$	$[0.16, 0.85]$	$[0.16, 0.92]$	$[0.13, 0.58]$	$[0.20, 0.82]$	$[0.24, 0.75]$
	Fraction variation in U_F			Fraction variation in U_F			Fraction variation in U_F		
s	U_M Shock	ip Shock	U_F Shock	U_M Shock	emp Shock	U_F Shock	U_M Shock	noi Shock	U_F Shock
1	0.029	0.023	0.948	0.140	0.119	0.743	0.078	0.195	0.728
12	0.080	0.041	0.878	0.243	0.133	0.624	0.153	0.218	0.630
∞	0.121	0.131	0.748	0.332	0.138	0.530	0.233	0.197	0.570
$s_{\max}$	0.128	0.131	0.950	0.339	0.152	0.744	0.233	0.218	0.732
	$[0.03, 0.47]$	$[0.05, 0.52]$	$[0.53, 0.99]$	$[0.08, 0.64]$	$[0.03, 0.58]$	$[0.33, 0.95]$	$[0.02, 0.60]$	$[0.05, 0.63]$	$[0.36, 0.93]$

Each panel shows the fraction of s-step-ahead forecast-error variance of the variable given in the panel title that is explained by the shock named in the column heading. The row denoted "$s = s_{max}$" reports the maximum fraction (across all VAR forecast horizons m) of forecast error variance explained by the shock listed in the column heading. The numbers in parentheses represent the 5th and 95th percentiles of these statistics from bootstrapped samples using the procedure described in the Appendix. The data are monthly and span the period 1960:07 to 2015:04.

Table 4: Variance Decomposition for SVARs in System $(U_R, Y, U_F)'$

	SVAR $(U_R, ip, U_F)'$		
	Fraction variation in U_R		
s	U_R Shock	ip Shock	U_F Shock
$s=1$	0.359	0.513	0.128
$s=12$	0.253	0.463	0.285
$s=\infty$	0.302	0.407	0.291
$s=s_{\max}$	0.302	0.407	0.291
	$[0.16, 0.72]$	$[0.18, 0.80]$	$[0.07, 0.63]$
	Fraction variation in ip		
s	U_R Shock	ip Shock	U_F Shock
$s=1$	0.391	0.577	0.032
$s=12$	0.295	0.456	0.249
$s=\infty$	0.211	0.326	0.463
$s=s_{\max}$	0.397	0.580	0.463
	$[0.10, 0.73]$	$[0.22, 0.89]$	$[0.08, 0.84]$
	Fraction variation in U_F		
s	U_R Shock	ip Shock	U_F Shock
$s=1$	0.010	0.059	0.941
$s=12$	0.011	0.083	0.906
$s=\infty$	0.117	0.093	0.790
$s=s_{\max}$	0.117	0.093	0.943
	$[0.04, 0.35]$	$[0.03, 0.52]$	$[0.56, 0.99]$

	SVAR $(U_R, emp, U_F)'$		
	Fraction variation in U_R		
s	U_R Shock	emp Shock	U_F Shock
$s=1$	0.483	0.405	0.112
$s=12$	0.409	0.292	0.299
$s=\infty$	0.419	0.263	0.318
$s=s_{\max}$	0.519	0.405	0.318
	$[0.23, 0.80]$	$[0.13, 0.69]$	$[0.07, 0.62]$
	Fraction variation in emp		
s	U_R Shock	emp Shock	U_F Shock
$s=1$	0.378	0.392	0.230
$s=12$	0.220	0.217	0.563
$s=\infty$	0.092	0.064	0.845
$s=s_{\max}$	0.392	0.395	0.845
	$[0.13, 0.68]$	$[0.14, 0.74]$	$[0.32, 0.96]$
	Fraction variation in U_F		
s	U_R Shock	emp Shock	U_F Shock
$s=1$	0.050	0.182	0.768
$s=12$	0.094	0.200	0.707
$s=\infty$	0.214	0.167	0.619
$s=s_{\max}$	0.217	0.216	0.774
	$[0.06, 0.49]$	$[0.04, 0.64]$	$[0.37, 0.97]$

	SVAR $(U_R, noi, U_F)'$		
	Fraction variation in U_R		
s	U_R Shock	noi Shock	U_F Shock
$s=1$	0.587	0.281	0.132
$s=12$	0.445	0.200	0.355
$s=\infty$	0.420	0.180	0.401
$s=s_{\max}$	0.601	0.285	0.401
	$[0.18, 0.90]$	$[0.05, 0.68]$	$[0.10, 0.67]$
	Fraction variation in noi		
s	U_R Shock	noi Shock	U_F Shock
$s=1$	0.268	0.437	0.295
$s=12$	0.114	0.259	0.627
$s=\infty$	0.112	0.256	0.632
$s=s_{\max}$	0.342	0.437	0.632
	$[0.07, 0.71]$	$[0.09, 0.80]$	$[0.29, 0.80]$
	Fraction variation in U_F		
s	U_R Shock	noi Shock	U_F Shock
$s=1$	0.030	0.249	0.721
$s=12$	0.047	0.285	0.668
$s=\infty$	0.083	0.255	0.662
$s=s_{\max}$	0.083	0.286	0.730
	$[0.01, 0.36]$	$[0.06, 0.63]$	$[0.40, 0.92]$

Each panel shows the fraction of s-step-ahead forecast-error variance of the variable given in the panel title that is explained by the shock named in the column heading. The row denoted "$s = s_{max}$" reports the maximum fraction (across all VAR forecast horizons m) of forecast error variance explained by the shock listed in the column heading. The numbers in parentheses represent the 5th and 95th percentiles of these statistics from bootstrapped samples using the procedure described in the Appendix. The data are monthly and span the period 1960:07 to 2015:04.

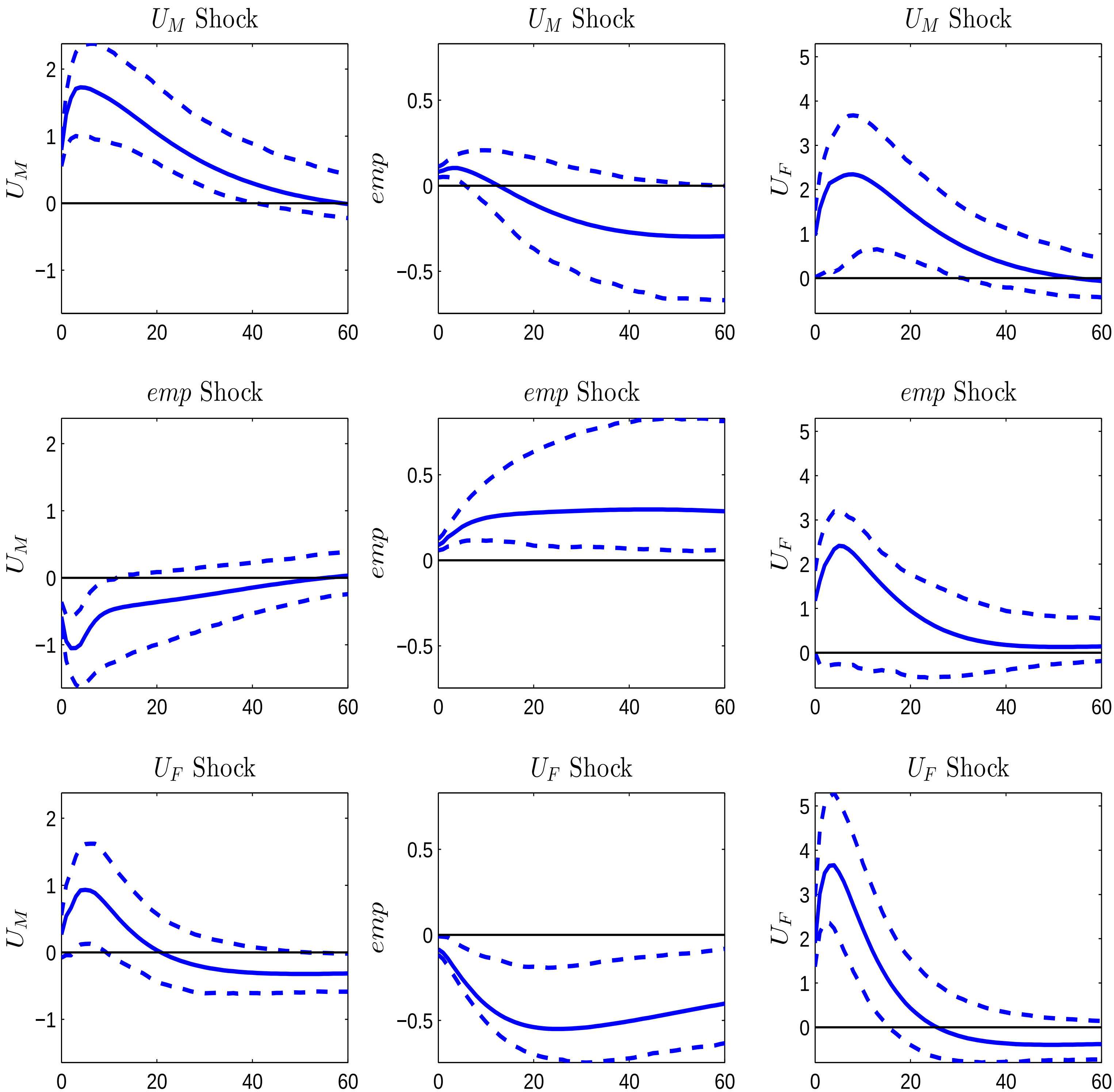
Table 5: Variance Decomposition for SVARs in System $(U_M(12), Y, U_F(12))'$

	SVAR $(U_M(12), ip, U_F(12))'$			SVAR $(U_M(12), emp, U_F(12))'$			SVAR $(U_M(12), noi, U_F(12))'$		
	Fraction variation in $U_M(12)$			Fraction variation in $U_M(12)$			Fraction variation in $U_M(12)$		
s	$U_M(12)$ Shock	ip Shock	$U_F(12)$ Shock	$U_M(12)$ Shock	emp Shock	$U_F(12)$ Shock	$U_M(12)$ Shock	noi Shock	$U_F(12)$ Shock
1	0.548	0.432	0.020	0.621	0.360	0.019	0.707	0.276	0.017
12	0.763	0.219	0.018	0.776	0.212	0.012	0.903	0.091	0.006
∞	0.635	0.206	0.159	0.682	0.135	0.183	0.824	0.081	0.087
$s_{\max}$	0.813	0.432	0.165	0.682	0.135	0.183	0.908	0.328	0.087
	$[0.48, 0.94]$	$[0.17, 0.66]$	$[0.06, 0.51]$	$[0.37, 0.96]$	$[0.10, 0.62]$	$[0.09, 0.52]$	$[0.55, 0.99]$	$[0.07, 0.64]$	$[0.04, 0.40]$
	Fraction variation in ip			Fraction variation in emp			Fraction variation in noi		
s	$U_M(12)$ Shock	ip Shock	$U_F(12)$ Shock	$U_M(12)$ Shock	emp Shock	$U_F(12)$ Shock	$U_M(12)$ Shock	noi Shock	$U_F(12)$ Shock
1	0.379	0.591	0.030	0.342	0.355	0.303	0.223	0.406	0.361
12	0.124	0.757	0.119	0.076	0.433	0.491	0.264	0.345	0.391
∞	0.202	0.697	0.101	0.269	0.482	0.250	0.309	0.321	0.371
$s_{\max}$	0.382	0.772	0.145	0.342	0.482	0.519	0.309	0.445	0.429
	$[0.20, 0.71]$	$[0.42, 0.93]$	$[0.04, 0.59]$	$[0.23, 0.76]$	$[0.17, 0.86]$	$[0.18, 0.88]$	$[0.21, 0.64]$	$[0.16, 0.80]$	$[0.19, 0.76]$
	Fraction variation in $U_F(12)$			Fraction variation in $U_F(12)$			Fraction variation in $U_F(12)$		
s	$U_M(12)$ Shock	ip Shock	$U_F(12)$ Shock	$U_M(12)$ Shock	emp Shock	$U_F(12)$ Shock	$U_M(12)$ Shock	noi Shock	$U_F(12)$ Shock
1	0.091	0.002	0.907	0.273	0.090	0.637	0.291	0.147	0.562
12	0.165	0.017	0.819	0.389	0.108	0.503	0.423	0.168	0.409
∞	0.200	0.162	0.638	0.448	0.165	0.387	0.519	0.151	0.330
$s_{\max}$	0.206	0.162	0.907	0.464	0.165	0.637	0.519	0.170	0.584
	$[0.04, 0.71]$	$[0.05, 0.46]$	$[0.37, 0.99]$	$[0.09, 0.76]$	$[0.04, 0.59]$	$[0.20, 0.94]$	$[0.08, 0.81]$	$[0.04, 0.60]$	$[0.20, 0.89]$

Each panel shows the fraction of s-step-ahead forecast-error variance of the variable given in the panel title that is explained by the shock named in the column heading. The row denoted "$s = s_{max}$" reports the maximum fraction (across all VAR forecast horizons m) of forecast error variance explained by the shock listed in the column heading. The numbers in parentheses represent the 5th and 95th percentiles of these statistics from bootstrapped samples using the procedure described in the Appendix. The data are monthly and span the period 1960:07 to 2015:04.

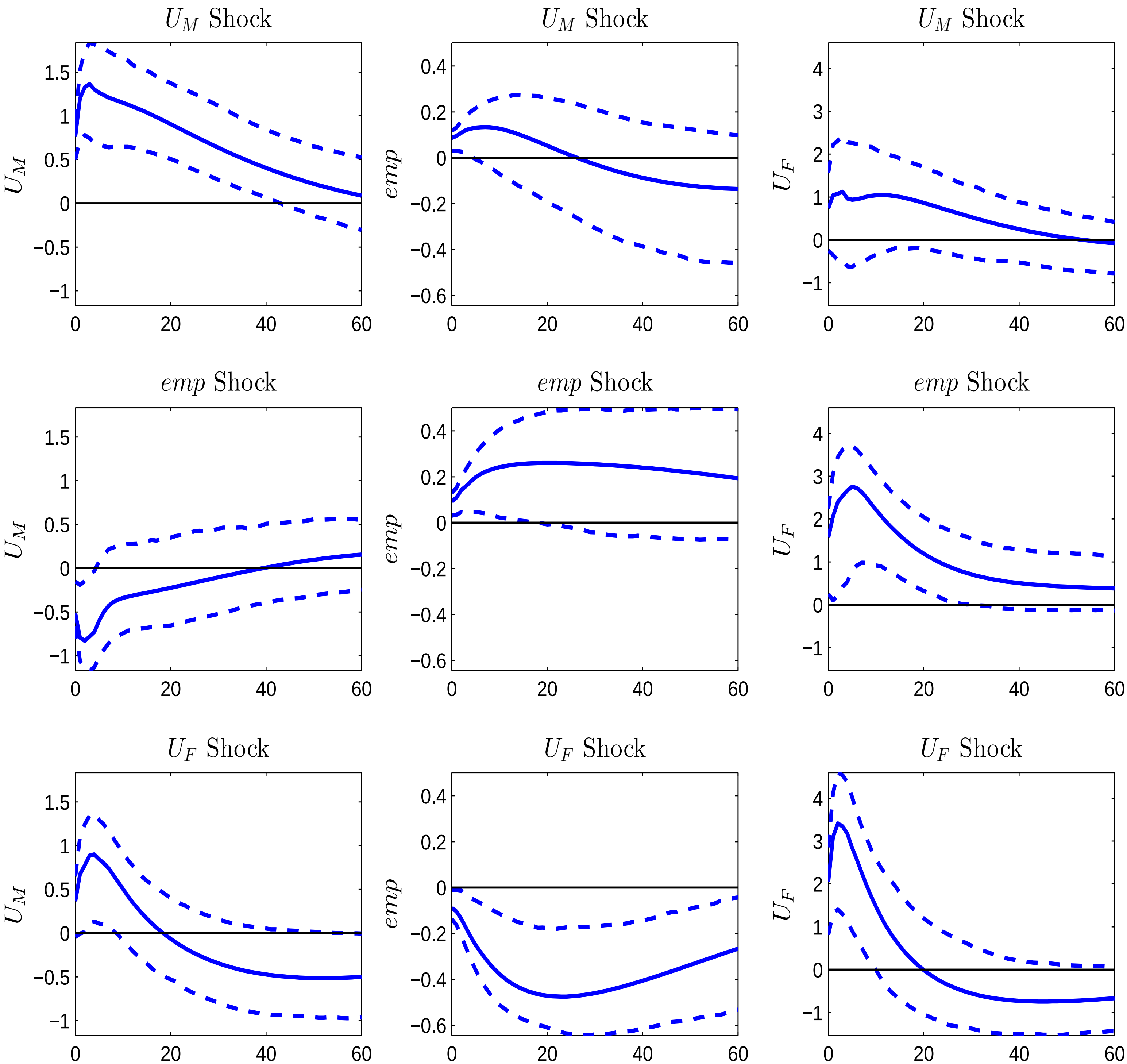
Appendix Figures and Tables

Figure A1: Dynamic Responses using 1987 Crash Dummies in SVAR$(U_M, emp, U_F)'$



Bootstrapped 90% error bands appear as dashed lines. Dummies for 1987:10 and 1989:11 are included in VAR estimation. Response units are reported in percentage points. The sample spans the period 1960:07 to 2015:04.

Figure A2: Pre-2008 Dynamic Responses in SVAR $(U_M, emp, U_F)'$



Bootstrapped 90% error bands appear as dashed lines. Response units are reported in percentage points. The sample spans the period 1960:07 to 2007:12.

Table A1: Tests of Validity of Recursive Restriction in System $(U_R, Y, U_F)'$

Ordering:	$(U_R, ip, U_F)'$	$(U_R(12), ip, U_F(12))'$
H_0: $B_{RY} = B_{RF} = B_{YF} = 0$	**133.69**	**303.24**
	[71.23]	[77.88]
H_0: $B_{YR} = B_{YF} = B_{RF} = 0$	**29.11**	**167.57**
	[35.83]	[52.54]
H_0: $B_{RY} = B_{RF} = B_{FY} = 0$	**130.41**	**306.34**
	[77.34]	[72.79]
$\chi^2_{5\%}(3)$	7.81	7.81
	$(U_R, emp, U_F)'$	$(U_R(12), emp, U_F(12))'$
H_0: $B_{RY} = B_{RF} = B_{YF} = 0$	**178.68**	**327.91**
	[62.11]	[76.35]
H_0: $B_{YR} = B_{YF} = B_{RF} = 0$	**85.58**	**244.85**
	[46.43]	[67.50]
H_0: $B_{RY} = B_{RF} = B_{FY} = 0$	**154.76**	**310.66**
	[76.22]	[78.04]
$\chi^2_{5\%}(3)$	7.81	7.81

The table reports the Wald test statistic for testing the null hypothesis given in the column. The bold indicates that Wald test rejects the null at 95 percent level according to $\chi^2(3)$ distribution. The SVAR system is solved using GMM and delta method is used for computing the standard error. Estimates of **B** are based on the SVAR identified with external instruments described in the text. The mean of bootstrap Wald statistics is reported in parenthesis. The sample size spans 1960:07 to 2015:04.

Table A2: Monte Carlo

Data Generating Process:

$$S_{1t}^{(i)} = 0.0281 + 0.212 S_{1t-1}^{(i)} - 0.0033 e_{Mt}^{(i)} + 0.00005 e_{Yt}^{(i)} - 0.005 e_{Ft}^{(i)} + 0.0425 e_{S2t}^{(i)} + 0.007 e_{S1t}^{(i)}$$

$$S_{2t}^{(i)} = 0.0110 + 0.212 S_{2t-1}^{(i)} - 0.0002 e_{Mt}^{(i)} - 0.00007 e_{Yt}^{(i)} - 0.007 e_{Ft}^{(i)} + 0.0425 e_{S2t}^{(i)}$$

Panel A: Correlations between True and Generated Structural Shocks

Average over Iterations		Average over Iterations and t	
$\frac{1}{I}\sum_{i=1}^{I}\left\|\rho_M^{(i)}\right\|$	$= 0.9770$	$\frac{1}{T}\sum_{t=1}^{T}\left\|\rho_{Mt}\right\|$	$= 0.9768$
$\frac{1}{I}\sum_{i=1}^{I}\left\|\rho_Y^{(i)}\right\|$	$= 0.9795$	$\frac{1}{T}\sum_{t=1}^{T}\left\|\rho_{Yt}\right\|$	$= 0.9792$
$\frac{1}{I}\sum_{i=1}^{I}\left\|\rho_F^{(i)}\right\|$	$= 0.9741$	$\frac{1}{T}\sum_{t=1}^{T}\left\|\rho_{Ft}\right\|$	$= 0.9740$
$\frac{1}{I}\sum_{i=1}^{I}\left\|corr\left(Z_{1t}^{(i)}, Z_{2t}^{(i)}\right)\right\|$	$= 0.9823$		

Panel B: Correlations between Instruments and Shocks: Average over Iterations

	$\hat{e}_M^{(i)}$	$\hat{e}_Y^{(i)}$	$\hat{e}_F^{(i)}$	$e_M^{(i)}$	$e_Y^{(i)}$	$e_F^{(i)}$
$Z_1^{(i)}$	−0.0714	0.0000	−0.1287	−0.0751	0.0063	−0.1214
$Z_2^{(i)}$	−0.0000	0.0000	−0.1733	−0.0045	0.0030	−0.1692

Panel C: True and Estimated Parameters

$$\mathbf{B} = \begin{pmatrix} 0.0066 & -0.0071 & 0.0027 \\ 0.0042 & 0.0047 & -0.0014 \\ 0.0049 & 0.0050 & 0.0260 \end{pmatrix} \qquad \frac{1}{I}\sum_{i=1}^{I}\mathbf{B}^{(i)} = \begin{pmatrix} 0.0068 & -0.0065 & 0.0030 \\ 0.0039 & 0.0048 & -0.0014 \\ 0.0033 & 0.0058 & 0.0254 \end{pmatrix}$$

$$\mathbf{\Sigma} = \begin{pmatrix} 0.0066 & 0 & 0 \\ 0 & 0.0047 & 0 \\ 0 & 0 & 0.0260 \end{pmatrix} \qquad \frac{1}{I}\sum_{i=1}^{I}\mathbf{\Sigma}^{(i)} = \begin{pmatrix} 0.0068 & 0 & 0 \\ 0 & 0.0048 & 0 \\ 0 & 0 & 0.0254 \end{pmatrix}$$

$$\mathbf{A}_1 = \begin{pmatrix} 0.4906 & 0.7834 & -0.0071 \\ -0.3378 & 0.4309 & 0.0527 \\ -0.0502 & -1.0000 & 0.9272 \end{pmatrix} \qquad \frac{1}{I}\sum_{i=1}^{I}\mathbf{A}_1^{(i)} = \begin{pmatrix} 0.5298 & 0.7490 & -0.0207 \\ -0.3173 & 0.4691 & 0.0575 \\ 0.0491 & -0.8935 & 0.8780 \end{pmatrix}$$

The sample size $T = 500$, number of iteration $I = 100$, inner loop tolerance is set to be 10^{-7}. $e^{(i)}$ is the true value generated from DGP and $\hat{e}^{(i)}$ is the corresponding converged value in iteration i, $\rho^{(i)} = corr\left(e^{(i)}, \hat{e}^{(i)}\right)$. The initial guess for the inner loop is set to be $(e_Y^{(i),[0]}, e_M^{(i),[0]}) = (Y^{(i)}, U_M^{(i)})$. The DGP coefficient $\mathbf{A}_0$ and $\mathbf{A}_1$ are set to be the ones from reduced form VAR(1) of system $(U_M, ip, U_F)'$.