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UNCERTAINTY AND BUSINESS CYCLES:
EXOGENOUS IMPULSE OR ENDOGENOUS RESPONSE?

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ABSTRACT

Uncertainty about the future rises in recessions. But is uncertainty a source of business cycles or an endogenous response to them, and does the type of uncertainty matter? To address these questions, we propose a novel shock-restricted identification strategy. We find that sharply higher uncertainty about macroeconomic activity in recessions is often an endogenous response to output shocks, while uncertainty about financial markets is a likely source of output fluctuations. The findings point to the need for a better understanding of how uncertainty in financial markets is transmitted to the macroeconomy.

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1 Introduction

A large literature in macroeconomics investigates the relationship between uncertainty and business cycle fluctuations. Interest in this topic has been spurred by a growing body of evidence that uncertainty rises sharply in recessions. This evidence is robust to the use of specific proxy variables such as stock market volatility and forecast dispersion as in Bloom (2009), or a broad-based measure of macroeconomic uncertainty, as in Jurado, Ludvigson, and Ng (2015) (JLN hereafter). But while this evidence substantiates a role for uncertainty in deep recessions, the question of whether uncertainty is an exogenous source of business cycle fluctuations or an endogenous response to economic fundamentals is not fully understood. Existing results are based on convenient but restrictive identifying assumptions and have no explicit role for financial markets, even though uncertainty measures are strongly correlated with financial market variables. This paper considers a novel identification strategy to disentangle the causes and consequences of real and financial uncertainty.

The question of causality and the identification of exogenous variation in uncertainty is a long-standing challenge of the uncertainty literature. The challenge arises in part because there is no single uncertainty model, hence no theoretical consensus on whether the uncertainty that accompanies deep recessions is primarily a cause or effect (or both) of declines in economic activity. In fact, theory is even ambiguous about the sign of the effect, as we discuss below.

A separate challenge of the uncertainty literature pertains to the origins of uncertainty. Classic theories assert that uncertainty originates from economic fundamentals such as productivity, and that such real economic uncertainty, when interacted with market frictions, discourages real activity. But some researchers have argued that uncertainty dampens the economy through its influence on financial markets (e.g., Gilchrist, Sim, and Zakrajsek (2010)) or through sources of uncertainty specific to financial markets (e.g., Bollerslev, Tauchen, and Zhou (2009)). Moreover, as surveyed by Ng and Wright (2013), all the post-1982 recessions have origins in financial markets, and these recessions have markedly different features from recessions where financial markets play a passive role. From this perspective, if financial shocks are subject to time-varying volatility, financial market uncertainty–as distinct from real economic uncertainty–could be a key player in recessions, both as a cause and as a propagating mechanism. Yet so far the literature has not disentangled the contributions of real versus financial uncertainty to business cycle fluctuations.

Econometric analyses aimed at understanding the role of uncertainty for business cycle fluctuations face their own challenges. Attempts to identify the "effects" of uncertainty shocks in existing empirical work are primarily based on recursive schemes within the framework of vector-autoregressions (VAR).[1] While a recursive structure is a convenient starting point, it is

[1]See Bachmann, Elstner, and Sims (2013), Bloom (2009), Bloom (2014), Bekaert, Hoerova, and Duca (2013),

ultimately unsatisfactory as an identification strategy for a study on uncertainty and business cycles. Not only do the existing studies differ according to whether uncertainty is ordered ahead of or after real activity variables in the VAR, there is no compelling theoretical reason to restrict the timing of the relationship between uncertainty (a second moment variable) and real activity (a first moment variable). Uncertainty could comove contemporaneously with real activity both because it is an exogenous impulse driving business cycles and because it responds endogenously to first moment shocks. Recursive structures explicitly rule this out, since they presume that some variables respond only with a lag to others. Other commonly used VAR identification strategies, such as sign restrictions, long-run restrictions, and instrumental variables estimation, are likewise problematic, as we discuss further below.

It is with these challenges in mind that we return to the questions posed above: is uncertainty primarily a source of business cycle fluctuations or a consequence of them? And what is the relation of real versus financial uncertainty to business cycle fluctuations? The objective of this paper is to establish a set of stylized facts that addresses these questions econometrically. To do so, we take a two-pronged approach. First, we explicitly distinguish *macro* uncertainty U_{Mt}, from *financial* uncertainty U_{Ft}. These data are included in a structural vector autoregression (SVAR) along with a measure of real activity Y_t to evaluate their possibly distinct roles in business cycle fluctuations. Second, we propose a novel identification strategy that allows for simultaneous feedback between uncertainty and real activity using two types of *shock-based* restrictions. The first is a set of "event constraints" that require the identified financial uncertainty shocks to have defensible properties during the 1987 stock market crash and the 2007-09 financial crisis. The second is a set of "correlation constraints" that require the identified uncertainty shocks to exhibit a minimum absolute correlation with certain variables external to the VAR. While our shock-based restrictions do not permit point identification, the moment inequalities generated by these constraints (along with the standard reduced-form covariance restrictions), are able to achieve a substantial constriction of the set of model parameters consistent with the data so that, unambiguous conclusions can be drawn about most dynamic relationships in the system.

The empirical exercise additionally requires that appropriate measures of macro and financial uncertainty be available. Our measures of uncertainty quantify the magnitude of unpredictability about the future. As in JLN, macro uncertainty measures a common component in the time-varying volatilities of h-step ahead forecast errors across a large number of macroeconomic series. The same approach is used here to construct a broad-based index of financial uncertainty that has never been used in the literature. We also study the Baker, Bloom, and Davis (2016) economic policy uncertainty index, an alternative to the JLN macro uncertainty measure that is arguably relevant specifically for real activity uncertainty.

Gilchrist, Sim, and Zakrajsek (2010), and JLN.

Our main results may be stated as follows. First, positive shocks to financial uncertainty are found to cause a sharp and persistent decline in real activity, lending support to the hypothesis that heightened uncertainty is an exogenous impulse that causes recessions. In contrast to preexisting empirical work that uses recursive identification schemes (e.g., Bloom (2009), JLN), we trace the source of this result specifically to financial market uncertainty. However there is little evidence that negative shocks to real activity have adverse effects on financial uncertainty.

Second, the results suggest that sharply higher macro and policy uncertainty in recessions is best characterized as an endogenous response to business cycle fluctuations. That is, negative economic activity shocks are found to cause increases in both macro and policy uncertainty, but there is much less evidence that positive shocks to macro or policy uncertainty cause lower economic activity. Indeed, in some estimations the opposite is true: exogenous shocks to macro and policy uncertainty are found to *increase* real activity in the short-run, consistent with "growth options" theories discussed below.

Finally, an inspection of our identified solution sets shows that the admissible SVARs reflect a non-zero contemporaneous correlation between U_{Ft} and Y_t, as well as between U_{Mt} and Y_t, something that is inconsistent with any recursive ordering. Tests of the validity of a recursive structure are easily rejected by the data.

After a variety of robustness checks, we find that all three estimated shocks exhibit non-Gaussian features. This is of interest because structural economic modeling typically assumes Gaussian shocks. We find strong repercussions of financial uncertainty shocks for real activity but little evidence that macro uncertainty shocks drive down production. These findings call for a need to better understand the channel by which uncertainty in financial markets impacts the macroeconomy, and more generally, how changing expectations about second moments can have non-trivial effects on the level of economic variables.

The rest of this paper is organized as follows. Section 2 reviews related literature. Section 3 outlines the econometric framework and compares our approach to other methodologies. Section 4 discusses the data and implementation. Section 5 presents results for our baseline systems that use macro uncertainty, a measure of real activity, and financial uncertainty. Section 6 reports results for policy uncertainty Section 7 reports estimations of several additional cases and extensions. Section 8 summarizes and concludes. A large number of additional results are reported in the Online Appendix. Shock-based restrictions hold promise in other applications. A paper with greater detail on the methodology proposed here with additional applications can be found in Ludvigson, Ma, and Ng (2016).

2 Related Literature

A large literature addresses the question of uncertainty and its relation to economic activity.[2] Besides the evidence cited above for the U.S., Nakamura, Sergeyev, and Steinsson (2012) estimate growth rate and volatility shocks for 16 developed countries and find that they are substantially negatively correlated. Theories for which uncertainty plays a key role differ widely on the question of whether this correlation implies that uncertainty is primarily a cause or a consequence of declines in economic activity.

One strand of the literature proposes uncertainty as a cause of lower economic growth. This includes models of the real options effects of uncertainty (Bernanke (1983), McDonald and Siegel (1986)), models in which uncertainty influences financing constraints (Gilchrist, Sim, and Zakrajsek (2010), Arellano, Bai, and Kehoe (2011)), or precautionary saving (Basu and Bundick (2012), Leduc and Liu (2012), Fernández-Villaverde, Pablo Guerrón-Quintana, and Uribe (2011)). These theories almost always presume that uncertainty is an exogenous shock to the volatility of some economic fundamental. Some theories presume that higher uncertainty originates directly in the process governing technological innovation, which subsequently causes a decline in real activity (e.g., Bloom (2009), Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2012)). According to these theories, positive macro uncertainty shocks should cause declines in real economic activity. But while this theoretical literature has focused on uncertainty originating in economic fundamentals, the empirical literature has typically evaluated those theories using uncertainty proxies that are strongly correlated with financial market variables. This practice raises the question of whether it is real economic uncertainty or financial market uncertainty (or both) that is the driver of recessions, a question of interest to our investigation.

A second strand of the literature postulates that higher macro uncertainty arises solely as a *response* to lower economic growth. In these theories there is no exogenous uncertainty shock at all and all uncertainty variation is endogenous. Some theories presume that bad times incentivize risky behavior (Bachmann and Moscarini (2011), Fostel and Geanakoplos (2012)), or reduce information and with it the forecastability of future outcomes (Van Nieuwerburgh and Veldkamp (2006), Fajgelbaum, Schaal, and Taschereau-Dumouchel (2014), Ilut and Saijo (2016)), or provoke new and unfamiliar economic policies with uncertain effects (Pástor and Veronesi (2013)), or create a greater misallocation of capital across sectors (Ai, Li, and Yang (2015)), or generate endogenous countercyclical uncertainty in consumption growth because investment is costly to reverse (Gomes and Schmid (2016)).

And yet a third literature has raised the possibility that some forms of uncertainty can actually *increase* economic activity. "Growth options" theories of uncertainty postulate that

[2] This literature has become voluminous. See Bloom (2014) for a recent review of the literature.

a mean-preserving spread in risk generated from an unbounded upside coupled with a limited downside can cause firms to invest and hire, since the increase in mean-preserving risk increases expected profits. Such theories were often used to explain the dot-com boom. Examples originate in early work by Oi (1961), Hartman (1972), and Abel (1983), and more recently Bar-Ilan and Strange (1996), Pastor and Veronesi (2006), Kraft, Schwartz, and Weiss (2013), Segal, Shaliastovich, and Yaron (2015).

As this brief literature review makes plain, there is no single uncertainty theory or all-encompassing structural model that we can use to link with data. Put simply, the body of theoretical work does not provide precise identifying restrictions for empirical work. Instead, what the literature presents is a wide range of theoretical predictions about the relationship between uncertainty and real economic activity that are also ambiguous about the sign of the relationship. The absence of a theoretical consensus on this relationship, along with the sheer number of theories and limited body of evidence on the structural elements of specific models, underscores the extent to which the question of cause and effect is fundamentally an empirical matter.

Of course, all empirical studies of this nature require identifying assumptions. But commonly used SVAR identification schemes appear ill equipped to address the empirical questions at hand. Recursive identification schemes are inappropriate because, by construction, they rule out the possibility that uncertainty and real activity could influence one another within the period. Sign restrictions on impulse responses are inappropriate, since theory is ambiguous about the sign of the relationship. Zero-frequency restrictions are difficult to motivate as the long-run effects of uncertainty shocks have not been theorized. Instrumental variable analysis is challenging, since instruments that are credibly exogenous are difficult if not impossible to find for this application. All of these considerations motivate the alternative identification strategy proposed in this paper.

3 Econometric Framework

We consider a baseline system with $n = 3$ variables: $\mathbf{X}_t = (U_{Mt}, Y_t, U_{Ft})'$, where U_{Mt} denotes macro uncertainty, Y_t denotes a measure of real activity, and U_{Ft} denotes financial uncertainty. We suppose that $\mathbf{X}_t$ has a reduced-form finite-order autoregressive representation $\mathbf{X}_t = \sum_{j=1}^{p} \mathbf{A}_j \mathbf{X}_{t-j} + \boldsymbol{\eta}_t,\ \boldsymbol{\eta}_t \sim (0, \boldsymbol{\Omega}),\quad \boldsymbol{\Omega} = \mathbf{P}\mathbf{P}'$ where $\mathbf{P}$ is the unique lower-triangular Cholesky factor with non-negative diagonal elements. The reduced form parameters are collected into $\boldsymbol{\phi} = \big(\mathrm{vec}(\mathbf{A}_1)' \ \ \ldots \ \ \mathrm{vec}(\mathbf{A}_p)',\ \ \mathrm{vech}(\boldsymbol{\Omega})'\big)'$. The reduced form innovations $\boldsymbol{\eta}_t = (\eta_{Mt}, \eta_{Yt}, \eta_{Ft})'$ are related to the structural shocks $\mathbf{e}_t = (e_{Mt}, e_{Yt}, e_{Ft})'$ by an invertible matrix $\mathbf{H}$:

$$\boldsymbol{\eta}_t = \mathbf{H}\boldsymbol{\Sigma}\mathbf{e}_t \equiv \mathbf{B}\mathbf{e}_t, \qquad \mathbf{e}_t \sim (0, \mathbf{I_K}), \qquad \mathrm{diag}\,(\mathbf{H}) = 1,$$

where $\mathbf{B} \equiv \mathbf{H}\boldsymbol{\Sigma}$, and $\boldsymbol{\Sigma}$ is a diagonal matrix with variance of the shocks in the diagonal entries. The structural shocks $\mathbf{e}_t$ are mean zero with unit variance, serially and mutually uncorrelated. We adopt the unit effect normalization that $H_{jj} = 1$ for all j.

The goal of the exercise is analyze the dynamic effects of $\mathbf{e}_t$ on $\mathbf{X}_t$. Let "hats" denote estimated variables. Since the autoregressive parameters $\mathbf{A}_j$ can be consistently estimated under regularity conditions, the sample residuals $\hat{\boldsymbol{\eta}}_t(\hat{\boldsymbol{\phi}})$ are consistent estimates of $\boldsymbol{\eta}_t$. The empirical SVAR problem reduces to finding $\mathbf{B}$ from $\hat{\boldsymbol{\phi}}$. But there are nine parameters in $\mathbf{B}$ and the covariance structure only provides six restrictions in the form

$$\bar{g}_Z(\mathbf{B}) = \mathrm{vech}(\hat{\boldsymbol{\Omega}}) - \mathrm{vech}(\mathbf{B}\mathbf{B}') = \mathbf{0}.$$

The model is under-identified as there can be infinitely many solutions satisfying $\bar{g}_Z(\mathbf{B}) = \mathbf{0}$. Let such solutions be collected into the set $\hat{\mathcal{B}} = \{\mathbf{B} = \hat{\mathbf{P}}\mathbf{Q} : \mathbf{Q} \in \mathbb{O}_n, \ \mathrm{diag}(\mathbf{B}) \geq 0, \bar{g}_Z(\mathbf{B}) = \mathbf{0}\}$, where $\mathbb{O}_n$ is the set of $n \times n$ orthonormal matrices. To simply notation, the dependence of $\hat{\mathcal{B}}$ on $\mathbf{Q}$ and $\hat{\boldsymbol{\phi}}$ is suppressed. Narrowing this set requires restrictions beyond covariance restrictions on $\hat{\boldsymbol{\eta}}_t$.

Point identification requires restrictions to reduce $\hat{\mathcal{B}}$ to a singleton. This is in principle possible if we have a sufficient number of defensible restrictions on the elements of $\mathbf{B}$ and/or a sufficient number of exogenous and relevant external instrumental variables (IV). Hamilton (2003) was among the first to use external variables to identify SVARs. Recent work by Mertens and Ravn (2013), Stock and Watson (2008) have made the approach increasingly popular. An application relevant to our work is Stock and Watson (2012). Under the assumption that either stock market volatility or the EPU index of Baker, Bloom, and Davis (2016) are relevant and exogenous (hence valid instruments), Stock and Watson (2012) use these variables to identify the effects of uncertainty shocks only. By contrast, we are interested in the dynamic effects of all shocks in the model, not just uncertainty. Furthermore, we need more than one valid instrument since we have two types of uncertainty. IV analysis is unlikely to be appropriate for our investigation because our procedure explicitly recognizes that macro, policy and financial uncertainty are endogenous variables. Valid instruments are thus hard to find. As discussed above, the theories reviewed in previous section do not lend support to conventional identification schemes used in the literature. We pursue a new approach that restricts the behavior of the structural shocks.

3.1 Shock-Based Constraints

Let $\mathbf{e}_t(\mathbf{B}) = \mathbf{B}^{-1}\hat{\boldsymbol{\eta}}_t$ be the shocks implied by an arbitrary $\mathbf{B}$ for given $\hat{\boldsymbol{\eta}}_t$. Even though the stated goal of any SVAR exercise is to identify $\mathbf{e}_t$, it is somewhat surprising that little attention is paid to the shocks themselves. Our approach is to impose two types of shock-based constraints to shrink $\hat{\mathcal{B}}$.

A. Special Event Constraints A credible identification scheme should produce estimates of $\mathbf{e}_t$ with features that accord with our ex-post understanding of historical events, at least during episodes of special interest. We require that $\mathbf{e}_t(\mathbf{B})$ satisfies three event constraints parameterized by $\bar{\mathbf{k}} = \left(\bar{k}_1, \bar{k}_2, \bar{k}_3\right)'$ and $\bar{\boldsymbol{\tau}} = (\bar{\tau}_1, \bar{\tau}_2, \bar{\tau}_3)'$:

i $\bar{g}_{E1}(\mathbf{e}(\mathbf{B}); \bar{\tau}_1, \bar{k}_1)$: $e_{F\bar{\tau}_1} - \bar{k}_1 \geq 0$ for $\bar{\tau}_1$=1987:10.

ii $\bar{g}_{E2}(\mathbf{e}(\mathbf{B}); \bar{\tau}_2, \bar{k}_2)$: $e_{F\bar{\tau}_2} - \bar{k}_2 \geq 0$ for at least one $\bar{\tau}_2 \in$[2007:12, 2009:06].

iii $\bar{g}_{E3}(\mathbf{e}(\mathbf{B}); \bar{\tau}_3, \bar{k}_3)$: $\bar{k}_3 - e_{Y\bar{\tau}_3} \geq 0 \; \forall \; \bar{\tau}_3 \in$[2007:12,2009:06].

Event constraints put restrictions on the sign and the magnitude of $\mathbf{e}(\mathbf{B})$ rather than on the impulse responses, as is standard in the SVAR literature. Specifically, $\bar{g}_{E1}$ requires that the financial uncertainty shocks found in October 1987 (black Monday) be large; $\bar{g}_{E2}$ requires that there is at least one month during the 2007-2009 financial crisis during which the financial uncertainty shock is large and positive. Finally, $\bar{g}_{E3}$ requires that the real activity shocks found during the Great Recession not to take on unusually large positive values.[3] Those $\mathbf{B}$s generating shocks that fail any of the three constraints are dismissed on grounds that it is hard to defend any solution that implies favorable financial uncertainty and output shocks during these two special episodes. The three event constraints can be summarized by a system of inequalities

$$\bar{g}_E(\mathbf{e}_t(\mathbf{B}); \bar{\boldsymbol{\tau}}, \bar{\mathbf{k}}) \geq 0.$$

Special events turn out to be valuable for identification because, although two feasible structural models $\mathbf{B}$ and $\tilde{\mathbf{B}}$, will generate shocks $\{\mathbf{e}_t\}_{t=1}^T$ and $\{\tilde{\mathbf{e}}_t\}_{t=1}^T$ with equivalent first and second moments, $\mathbf{e}_t$ and $\tilde{\mathbf{e}}_t$ are not necessarily the same at any given t. It is not hard to see that if $\mathbf{e}_t = \mathbf{B}^{-1}\hat{\boldsymbol{\eta}}_t = \mathbf{Q}'\mathbf{P}^{-1}\hat{\boldsymbol{\eta}}_t$ and $\tilde{\mathbf{e}}_t = \tilde{\mathbf{Q}}'\mathbf{P}^{-1}\hat{\boldsymbol{\eta}}_t = \tilde{\mathbf{Q}}\mathbf{e}_t$, then $\tilde{\mathbf{e}}_t \neq \mathbf{e}_t$ at any given t when $\tilde{\mathbf{Q}} \neq \mathbf{Q}$.[4] Put differently, two series with equivalent properties "on average"can still have distinguishable features in certain subperiods.

Why Large Financial Uncertainty Shocks? The event constraints on financial uncertainty, $\bar{g}_{E1}$ and $\bar{g}_{E2}$, warrant further discussion to clarify what the constraints do and do not assume. In the sample considered here, the two episodes of most extreme volatility in financial uncertainty occur in the month of the 1987 stock market crash and the 2007-09 financial crisis. This can be observed in Figure 1 discussed below. The restriction stipulated in the event constraints above is that *at least some* of the forecast error variance of U_F in these episodes of

[3]The NBER recession dates 2007:12-2009:06 are taken to be coincident with the financial crisis.

[4]Consider the $n = 2$ case: $\begin{pmatrix}\eta_{1t}\\ \eta_{2t}\end{pmatrix} = \begin{pmatrix}B_{11} & B_{12}\\ B_{21} & B_{22}\end{pmatrix}\begin{pmatrix}e_{1t}\\ e_{2t}\end{pmatrix}$. Solving for e_{1t} gives $e_{1t} = |\mathbf{B}|^{-1}(B_{22}\eta_{1t} - B_{12}\eta_{2t})$, where $|\mathbf{B}| = B_{11}B_{22} - B_{12}B_{21}$ is the determinant of $\mathbf{B}$. The values of η_{1t} and η_{2t} are given by the data. Hence, a restriction on the behavior of e_{1t_1} at specific time t_1 is a non-linear restriction on $\mathbf{B}$, or equivalently, on $\mathbf{Q}$.

most extreme financial uncertainty is attributable to large shocks that originated in financial markets, modeled here by our e_F. The restrictions do not require that all or even most of the variation in these episodes be attributable to shocks that originated in financial markets. In particular, they do not rule out large adverse roles for the other shocks, e_M and e_Y, something discussed further below.

Imposing that there be at least some role for large U_F shocks in these episodes is a maintained assumption, but one that we argue is grounded in a broad historical reading of the times. On Monday October 19, 1987, the Dow Jones Industrial Average dropped 22.6 percent, the largest one-day stock market decline in history. Popular explanations include the rapidly rising globalization of financial markets and financial innovations associated with index futures and portfolio insurance. A belief that such financial innovations played an important role in the crash was sufficiently widespread that new regulations for exchange trading, such as "circuit breakers," and an overhaul of trade clearing protocols were developed in the aftermath.[5] On the basis of these facts, we argue that it reasonable to presume at least part of the high financial uncertainty in this episode was attributable to forces that originated in financial markets.

In October of 2008, the Dow Jones Industrial average began a pronounced decline and subsequently fell more than 50% over a period of 17 months. The collapse in the market over this period has been associated with a broad-based financial crisis that is often cited as a "trigger" of the Great Recession.[6] Many possible contributors to the crisis have been noted, including problems with subprime lending and a proceeding housing boom. But at least some of the variation in financial uncertainty appears to have its origins in securities markets. Financial intermediaries played a large role in the crisis, primarily because they hold vast portfolios of financial securities. Speculative trading activities by large financial institutions such as AIG, Lehman Brothers, and Bear Stearns, possibly spurred by a mistaken pricing of risk, have been placed at the center of the crisis by some analyses (e.g., Glaeser, Santos, and Weyl (2017)). Several highly leveraged financial institutions (BNP Paribas, Northern Rock) experienced a total collapse in liquidity that began August of 2007, preceding the recession. And uncertainty about the value of new products of financial innovation have been cited as pertinent to the financial crisis, including the securitization of mortgages and other debt obligations, and the rapid growth in credit default swaps.[7] This historical understanding of events suggests that factors originating in financial markets contributed to the extreme volatility in those markets during the financial crisis. Of course, this episode is also plausibly characterized by concomitant large adverse shocks in the other variables of our system. Since the event constraints do not rule out such a concurrence, our results can be used to evaluate the extent to which this is so.

[5] See for example, https://www.federalreservehistory.org/essays/stock_market_crash_of_1987

[6] https://en.wikipedia.org/wiki/Financial_crisis_of_2007-2008

[7] "FT Martin Wolf – Reform of Regulation and Incentives". Financial Times. June 23, 2009.

B. Correlation Constraints Theory or economic reasoning often imply that certain variables external to the VAR should be informative about the shocks of interest. Let $\mathbf{S}_t$ be variables that encode information about uncertainty shocks with random processes determined outside of our three variable SVAR. We argue that the aggregate stock market return is one such variable. Our baseline specification uses restrictions on its correlation with uncertainty shocks to generate additional inequality restrictions.[8]

These correlation constraints are grounded in the many macro and finance theories that imply uncertainty shocks are correlated with stock market returns because they drive risk premium variation. A leading example is the Capital Asset Pricing Model (CAPM) of Sharpe (1964) and Lintner (1965) which implies that fluctuations in the stock market risk premium are perfectly correlated with shocks to financial uncertainty. The consumption-CAPM of Breeden (1979) implies that the risk premium is perfectly correlated with shocks to consumption uncertainty. Variants of the long-run risk paradigm of Bansal and Yaron (2004) also imply that stock market risk premia are perfectly correlated with shocks to real economic uncertainty, which endogenously feed into financial uncertainty. Bollerslev, Tauchen, and Zhou (2009) (BTZ) and Campbell, Giglio, Polk, and Turley (2012) suggest that the volatility of volatility in financial market returns introduces an additional source of uncertainty specific to financial markets. Other theories suggest that changes in factors like leverage, intermediary risk-bearing capacity, and in risk aversion or sentiment, can also be relevant for both stock market risk premia and financial market uncertainty.

In short, there are many theoretical reasons why equity market returns should contain valuable information about the parameters of the model. 'Valuable' is defined in terms of the correlation between stock market returns and the uncertainty shocks. To formalize this notion, let S_t be a measure of the aggregate stock market return and $\hat{u}_{St}$ be the first order autoregressive residual for S_t. Being a reduced form residual, $\hat{u}_{St}$ is a combination of primitive shocks from multiple sources, including the three shocks in our system. Let $(c_M(\mathbf{B}), c_Y(\mathbf{B}), c_F(\mathbf{B}))$ be the sample correlation between $\hat{u}_{St}$, and the shocks $(e_{Mt}(\mathbf{B}), e_{Yt}(\mathbf{B}), e_{Ft}(\mathbf{B}))$ respectively. We impose the following restrictions:

i $\bar{g}_{C1}(\mathbf{e}(\mathbf{B}); \bar{\lambda}_1 < 0, ,\mathbf{S})$: $\begin{pmatrix} \bar{\lambda}_1 - c_M(\mathbf{B}) \\ \bar{\lambda}_1 - c_F(\mathbf{B}) \end{pmatrix} \geq 0$;

ii $\bar{g}_{C2}(\mathbf{e}(\mathbf{B}); \bar{\lambda}_2 \geq 1, \mathbf{S})$: $|c_F(\mathbf{B})| - \bar{\lambda}_2 |c_M(\mathbf{B})| \geq 0$

iii $\bar{g}_{C3}(\mathbf{e}(\mathbf{B}); \bar{\lambda}_3, \mathbf{S})$: $c_{MF} - \bar{\lambda}_3 \geq 0$, $c_{MF}^2 = c_M(\mathbf{B})^2 + c_F(\mathbf{B})^2$.

[8] Below we use the real price of gold as an alternative external variable. Other researchers have used information in special variables to identify certain effects of uncertainty. Berger, Dew-Becker, and Giglio (2016) use options data and find that bad times are associated with higher realized volatility but not higher expected volatility, a result that they interpret as consistent with the hypothesis that higher uncertainty is a consequence of negative economic shocks rather than a cause. This interpretation is not intended to provide an explicit identification of uncertainty shocks, however.

Constraints (i) and (iii) require that e_M and e_F are negatively correlated with S_t. Specifically, we require each individual correlation exceeds $\bar{\lambda}_1$ in absolute terms and collectively exceed $\bar{\lambda}_3$. Constraint (ii) requires that financial uncertainty shocks be more highly correlated with $\hat{u}_{St}$ than macro uncertainty shocks, according to a magnitude dictated by the lower bound $\bar{\lambda}_2$. Let $\bar{\boldsymbol{\lambda}} = (\bar{\lambda}_1, \bar{\lambda}_2, \bar{\lambda}_3)$. The correlation constraints can be summarized by a system of inequalities:

$$\bar{g}_C(\mathbf{e}(\mathbf{B}); \bar{\boldsymbol{\lambda}}, \mathbf{S}) \geq 0.$$

The correlation constraints provide cross-equation restrictions on the parameters in $\mathbf{B}$. To see this, note first that u_{St} is a reduced form error that, by assumption, is a function of the shocks we seek to recover. Hence the error could be modeled as $u_{St} = d_Y e_{Yt} + d_M e_{Mt} + d_F e_{Ft} + e_{St}$, where e_{St} is orthogonal to (e_{Yt}, e_{Mt}, e_{Ft}). But $c_M(\mathbf{B}) = \text{corr}(u_{St}, e_{Mt})$ depends among other things on the volatility of $e_{Mt}(\mathbf{B})$. Requiring that $c_M(\mathbf{B}) \geq \bar{\lambda}_1$ is thus implicitly a non-linear constraint on the parameters of the model. An important aspect is that the correlations are not invariant to orthonormal rotations. That is to say, correlations generated by $\mathbf{B}$ will in general be different from those generated by $\tilde{\mathbf{B}} = \mathbf{B}\mathbf{Q}'$.

3.2 Comparison With Other Methodologies

Estimates of $\mathbf{B}$ that satisfy the covariance structure restrictions, event constraints, and correlation constraints together give the *identified* solution set defined by

$$\begin{aligned} \bar{\mathcal{B}}(\mathbf{B}; \bar{\mathbf{k}}, \bar{\boldsymbol{\tau}}, \bar{\lambda}, \mathbf{S}) \quad = \quad & \{\mathbf{B} = \hat{\mathbf{P}}\mathbf{Q} : \mathbf{Q} \in \mathbb{O}_n, \quad \text{diag}(\mathbf{B}) > 0; \\ & \bar{g}_Z(\mathbf{B}) = 0,\ \bar{g}_E(\mathbf{B}; \bar{\tau}, \bar{\mathbf{k}}) \geq 0,\ \bar{g}_C(\mathbf{B}; \mathbf{S}, \bar{\boldsymbol{\lambda}}) \geq 0\}. \end{aligned}$$

To simplify notation, we simply write $\bar{\mathcal{B}}(\mathbf{B}; \bar{\mathbf{k}}, \bar{\boldsymbol{\tau}}, \bar{\lambda}, \mathbf{S})$ as $\bar{\mathcal{B}}$. A particular solution can be in both $\hat{\mathcal{B}}$ and $\overline{\mathcal{B}}$ only if all the event and correlation restrictions are satisfied. Though $\bar{\mathcal{B}}$ is still a set, it should be smaller than $\hat{\mathcal{B}}$, which is based on the covariance restrictions alone. Though no one solution in $\bar{\mathcal{B}}$ is any more likely than another, we sometimes use what will be referred to as the 'maxG' solution as reference point:

$$\mathbf{B}^{\max G} \equiv \arg\max_{\mathbf{B} \in \overline{\mathcal{B}}} \sqrt{\bar{g}(\mathbf{B})'\bar{g}(\mathbf{B})}, \quad \text{where} \quad \bar{g}(\mathbf{B}; \bar{\mathbf{k}}, \bar{\boldsymbol{\tau}}, \bar{\boldsymbol{\lambda}}, \mathbf{S}) = \begin{pmatrix} \bar{g}_Z(\mathbf{B})' \\ \bar{g}_E(\mathbf{B}; \bar{\boldsymbol{\tau}}, \bar{\mathbf{k}})' \\ \bar{g}_C(\mathbf{B}; \bar{\boldsymbol{\lambda}})' \end{pmatrix}'. \tag{1}$$

This is the solution at which the value of the inequalities are jointly maximized. In this application, the individual inequalities are large when financial uncertainty shocks in 1987 and in the financial crisis are most extremely positive, when real activity shocks in the Great Recession are most negative, and when stock returns and uncertainty shocks have the highest absolute correlation, jointly and collectively. If a high correlation between stock returns and uncertainty shocks delivers a high risk premium, and if a "bad" economic state is characterized by a higher stock market risk premium, higher financial uncertainty, and lower production, then the maxG solution has the economically interesting interpretation the "worst-case" solution from the perspective of an agent who fears bad outcomes. We now relate our identification to related work in the literature.

The idea of using specific events and/or external variables to identify shocks is not new. Many important studies have used the narrative approach to construct shock series from historical readings of political and economic events. The resulting oil price shocks based on timing of wars, tax shocks from fiscal policy announcements, and monetary policy shocks from a reading of FOMC meetings are typically used as though they were exogenous and accurately measured. But as noted in Ramey (2016), both assumptions are questionable. To deal with possible measurement errors, Mertens and Ravn (2014) uses the narrative tax changes as an external instrument. Similarly, Baker and Bloom (2013) use disaster-like events as instruments for stock market volatility with the aim of isolating exogenous variation in uncertainty. More generally, a prominent literature proposes using variables external to the VAR as instrumental variables to identify SVARs, discussed above. In all of these papers, point identification is achieved by assuming that the instruments have a zero correlation with some shocks and a non-zero correlation with others. By contrast, our approach makes no such exogeneity assumption. We only assert that the events and external variables be driven at least in part by one or more of the shocks, thereby allowing us to narrow the set of solutions but not achieve point identification.

Our event constraints differ from the narrative approach in other ways. First, they are data driven rather than being based on a narrative reading of history. We use features of the shocks during selected episodes to determine whether a possible solution is admissible. This is tantamount to creating dummy variables from the timing of specific events, and then putting restrictions on their correlation with the identified shocks. Second, the same SVAR is used to identify all shocks simultaneously; it is not a two-step procedure that identifies some shocks ahead of others.

It is worth contrasting the non-Bayesian approach taken here with recent work on sign-restricted SVARs in Bayesian contexts. Rubio Ramírez, Waggoner, and Zha (2010) point out that choosing $\mathbf{Q}$ according to the $\mathbf{QR}$ decomposition amounts to drawing $\mathbf{Q}$ from a uniform distribution over the space of orthogonal matrices. Baumeister and Hamilton (2015) note that an uninformative prior over $\mathbf{Q}$ can be informative for the posterior over the structural impact matrix and impulse responses in sign-restricted SVARs. We differ from these papers in at least two ways. First, these papers focus specifically on restrictions placed on the sign of impulse response functions, whereas our restrictions are on timing, magnitude, and correlation, of the shocks. Second, our approach is frequentist in the spirit of the moment inequality framework of Andrews and Soares (2010), with moment conditions given by the inequalities from the event and correlation constraints, and equalities provided by the covariance structure. We use the

QR decomposition merely to generate candidate values of $\mathbf{B}$, and check if the resulting $\mathbf{e}_t(\mathbf{B})$ satisfies the constraints.

Since an earlier version of this paper was circulated, we became aware of contemporaneous work by Antolin-Diaz and Rubio Ramírez (2016) who suggest using restrictions on the shocks (such as restrictions on the signs of the shocks) during certain episodes of history to help identification. This is similar in spirit to our event constraints, though there are several differences. They impose a class of restrictions that assumes a particular shock is either the most (or least) important contributor, or the overwhelming (or negligible) contributor to the unexpected change in certain variables during a certain period. These constraints, which play up the role of some shocks while simultaneously playing down the role of others, differ from the event constraints proposed here because they restrict the relative importance of different types of shocks in the episodes. By contrast, the event constraints we propose restrict only the absolute importance of certain shocks in certain episodes. For example, our restrictions require large financial uncertainty shocks in 1987:10 and in at least one month of the financial crisis, but they do not rule out equally large roles for the other shocks during these episodes (see below). Other differences are that Antolin-Diaz and Rubio Ramírez (2016) do not use external variables at all, and their focus is on methodology in a Bayesian context at a general level.

An additional point about the procedure is worth mentioning. The structural shocks we identify do not necessarily correspond to primitive shocks of any particular model, as this is not our goal. Our real activity shocks are 'first moment' shocks that could originate from technology, monetary policy, preferences, or government expenditure innovations. Financial uncertainty, a type of 'second moment' shock, could arise because of expected volatility in financial markets such as fear of a bank run or fear of bankruptcy. Another type of second moment shock, macro uncertainty, could arise because of expected volatility in the macro economy, such as an expectation of greater difficulty in predicting future productivity, future monetary policy or future fiscal policy. An objective of this study is to disentangle whether it is shifts to first or second moments (or both) that drive economic fluctuations. Disentangling the two types of uncertainty is a worthy exercise because the theoretical macro literature on uncertainty has focused on exogenous changes in real activity induced (macro) uncertainty, while the empirical literature has used proxies for macro uncertainty that are highly correlated with volatility in financial markets.

To have confidence in this implementation, we use a simulation study to take into account sampling error and study the properties of the estimator. In the Online Appendix, we show results from a Monte Carlo simulation that bootstraps from the $\mathbf{e}_t\left(\mathbf{B}^{\max G}\right)$ shocks of the maxG solution for the $\mathbf{X}_t$ system. We find that the procedure produces solution sets that are substantially narrowed by applying the event and correlation constraints described above. It should be noted that while our focus here is to use event and correlation constraints to help understand the role macro or financial uncertainty in the macro economy, the use of shock-based restrictions is not limited to this particular application.

To summarize, set identification is predicated on three core economic assumptions. First, the shocks to stock returns must be correlated with the uncertainty shocks, as specified by the correlation constraints. Second, the identified shocks must be consistent with a priori economic reasoning in a small number of extraordinary events whose interpretation is relatively incontrovertible. Third, a maintained assumption of the analysis is that the dynamic responses of interest can be captured without explicitly modeling the random processes behind the external variables. Below we consider an alternative specification in which S_t is explicitly modeled as part of the VAR.

4 Data

We study VAR systems for three systems of data. Our main system is $\mathbf{X}_t = (U_{Mt}, ip_t, U_{Ft})'$, where U_{Mt} and U_{Ft} are statistical uncertainty indices constructed using the methodology of JLN. Financial uncertainty U_{Ft} is new to this paper. In all cases, we use the log of real industrial production, denoted ip_t, to measure Y_t. Industrial production is a widely watched economic indicator of business cycles. A subsequent section considers two additional systems that use policy uncertainty indices in place of U_{Mt}. For S_t we use the Center for Research in Securities Prices (CRSP) value-weighted stock market index return.[9]

Our statistical measures of uncertainty follows the framework of JLN which aggregates over a large number of estimated uncertainties constructed from a large panel of data. Let $y_{jt}^C \in Y_t^C = (y_{1t}^C, \ldots, y_{N_C t}^C)'$ be a variable in category C. Its h-period ahead uncertainty, denoted by $\mathcal{U}_{jt}^C(h)$, is defined to be the volatility of the purely unforecastable component of the future value of the series, conditional on all information available. Specifically,

$$\mathcal{U}_{jt}^C(h) \equiv \sqrt{\mathbb{E}\Big[(y_{jt+h}^C - \mathbb{E}[y_{jt+h}^C|I_t])^2|I_t\Big]} \tag{2}$$

where I_t denotes the information available. Uncertainty in category C is an aggregate of individual uncertainty series in the category:

$$U_{Ct}(h) \equiv \text{plim}_{N_C\to\infty} \sum_{j=1}^{N_C} \frac{1}{N_C} \mathcal{U}_{jt}^C(h) \equiv \mathbb{E}_C[\mathcal{U}_{jt}^C(h)]. \tag{3}$$

If the expectation today of the squared error in forecasting y_{jt+h} rises, uncertainty in the variable increases. As in JLN, the conditional expectation of squared forecast errors in (2) is computed from a stochastic volatility model, while the conditional expectation $\mathbb{E}[y_{jt+h}^C|I_t]$

[9]The CRSP index is a value-weighted return of all stocks in NYSE, AMEX, and NASDAQ.

is replaced by a diffusion index forecast, augmented to allow for nonlinearities. These are predictions of an autoregression augmented with a small number of common factors estimated from a large number of economic time series x_{it} assumed to have factor structure. Nonlinearities are accommodated by including polynomial terms in the factors, and factors estimated from squares of the raw data. The use of large datasets reduces the possibility of biases that arise when relevant predictive information is ignored.

In this paper, we consider two categories of uncertainty, macro M and financial F. Hence there are two datasets, both covering the sample 1960:07-2015:04. For macro uncertainty U_{Mt}, we use a monthly *macro dataset*, $\mathcal{X}_t^M$, consisting of 134 mostly macroeconomic time series take from McCracken and Ng (2016). For financial uncertainty U_{Ft}, we use a *financial dataset* $\mathcal{X}_t^F$ consisting of 148 measures of monthly financial indicators.[10] We also use two measures of policy uncertainty taken from Baker, Bloom, and Davis (2016) in lieu of the statistical measure of macro uncertainty U_{Mt}.

The 134 macro series in X^m are selected to represent broad categories of macroeconomic time series. The majority of these are real activity measures: real output and income, employment and hours, real retail, manufacturing and trade sales, consumer spending, housing starts, inventories and inventory sales ratios, orders and unfilled orders, compensation and labor costs, and capacity utilization measures. The dataset also includes commodity and price indexes and a handful of bond and stock market indexes, and foreign exchange measures. The financial dataset X^f is an updated monthly version of the of 148 purely financial time series used in Ludvigson and Ng (2007). These data include valuation ratios such as the dividend-price ratio and earnings-price ratio, growth rates of aggregate dividends and prices, default and term spreads, yields on corporate bonds of different ratings grades, yields on Treasuries and yield spreads, and a broad cross-section of industry, size, book-market, and momentum portfolio equity returns.[11] The indexes U_{Mt} and U_{Ft} lend themselves to different interpretations because they are constructed from different variables.

The top panel of Figure 1 plots the estimated macro uncertainty U_{Mt} in standardized units along with the NBER recession dates. The horizontal bar corresponds to 1.65 standard deviation above unconditional mean of each series (which is standardized to zero). As is known from JLN, the macro uncertainty index is strongly countercyclical, and exhibits large spikes in the deepest recessions. The updated data U_{Mt} series shows much the same. Though U_{Mt} exceeds 1.65 standard deviations 48 times, they are clustered around the 1973-74 and 1981-82 recessions, as well as the Great Recession of 2007-09. Macroeconomic uncertainty has a correlation of -0.65 with the 12-month moving-average of the growth in industrial production.

[10]Both datasets were previously used in Ludvigson and Ng (2007) and JLN, but they are updated to the longer sample.

[11]A detailed description of the series is given in the Data Appendix of the online supplementary file at www.sydneyludvigson.com/s/ucc_data_appendix.pdf

The middle panel of Figure 1 plots the financial uncertainty series U_{Ft} over time, which is new to this paper. U_{Ft} is a broad-based measure of time varying financial uncertainty using data from the bond market, stock market portfolio returns, and commodity markets. Hence, it is smoother than proxies such as VIX or any particular bond index. As seen from Figure 1, U_{Ft} is also countercyclical, though less so than U_{Mt}; the correlation with industrial production is -0.39. The series often exhibits spikes around the times when U_{Mt} is high. However, U_{Ft} is more volatile and spikes more frequently outside of recessions, the most notable being the 1987 stock market crash. Though observations on U_{Ft} exceed the 1.65 standard deviation line 33 times, they are spread out in seven episodes, with the 2008 and 1987 episodes being the most pronounced.

As is clear from Figure 1, both indicators of macro and financial uncertainty are serially correlated and hence predictable. They have comovements but also have independent variations as the correlation between them is only 0.58. However, this unconditional correlation cannot be given a structural interpretation. To the extent that our uncertainty variables measure expectations about future volatility, the heightened uncertainty measures can respond endogenously to events that are expected to happen, but they can also be exogenous changes to expected volatility. We use a VAR to capture the predictable variations, and then identify uncertainty shocks from the VAR residuals using the restrictions described above. We now turn to the implementation issues.

5 Implementation and Base Case Results

We focus our main analysis on one-month uncertainty $h = 1$ and discuss results for long-horizon uncertainty in the Additional Cases section below. We use $p = 6$ lags in the VARs, nothing that using 12 lags makes no difference to the results.

An important part of our exercise is to construct the unconstrained solution set $\hat{\mathcal{B}}$ and the identified set $\bar{\mathcal{B}}$. The possible solutions in $\hat{\mathcal{B}}$ are obtained by initializing $\mathbf{B}$ to be the unique lower-triangular Cholesky factor of $\hat{\mathbf{\Omega}}$ with non-negative diagonal elements, $\hat{\mathbf{P}}$, and then rotating it by $K = 1.5$ million random orthogonal matrices $\mathbf{Q}$. Each rotation begins by drawing an $n \times n$ matrix $\mathbf{M}$ of NID(0,1) random variables. Then $\mathbf{Q}$ is taken to be the orthonormal matrix in the $\mathbf{QR}$ decomposition of $\mathbf{M}$. Since $\mathbf{B} = \hat{\mathbf{P}}\mathbf{Q}$, the procedure imposes the covariance restrictions vech($\mathbf{\Omega}$) =vech($\mathbf{BB}'$) by construction. A solution in the unconstrained set $\hat{\mathcal{B}}$ is also in the constrained set $\bar{\mathcal{B}}$ only if the event and correlation constraints are all satisfied.

Construction of the identified solution necessitates choice of $\bar{\boldsymbol{\lambda}}$, $\bar{\boldsymbol{\tau}}$, and $\bar{\mathbf{k}}$. If the values for the bounds are overly restrictive, the identified solution set will be empty. If they are too unrestrictive, the constraints will have no identifying power. Moreover, shock-based restrictions are not invariant to the system being analyzed because the data may have different variability.

Even though the fact that correlations are between zero and one facilitates the calibration, the bounds for one system of data could be too restrictive for another. We use a combination of theory, empirical analysis, and prior economic reasoning to set the bounds for the system $\mathbf{X}_t = (U_{Mt}, ip_t, U_{Ft})'$. We then adapt these bounds to other systems. Specific values will be made precise in the section where results for the specific system are presented. Before turning to the analysis, we close this section with some general remarks on the choice of $\bar{\mathbf{k}}$, $\bar{\boldsymbol{\lambda}}$, and $\bar{\boldsymbol{\tau}}$.

The correlation constraints are predicated on an asset pricing literature which states that uncertainty shocks should be correlated with stock market returns because they are the drivers of stock market risk premium variation. BTZ find that shocks to the volatility of volatility account for a larger fraction of stock market risk premium variation than realized volatility. Since the volatility of volatility shocks in their model affect financial but not macro uncertainty, financial uncertainty should be more correlated with stock returns than macro uncertainty, a prediction for which they find empirical support. Hence we restrict $\bar{\lambda}_2$ to be bigger than 1. Note that without taking a stand on a specific asset pricing model and parameterization, there can be many ways to divide the collective correlation into components attributable to e_{Mt} and e_{Ft}. We therefore place no additional restrictions on how the collective correlation is divided between the individual correlations beyond the requirement that $\left(|c_F(\mathbf{B})| - \bar{\lambda}_2 |c_M(\mathbf{B})|\right) \geq 0$.

The special event constraints require us to take a stand on when in the sample large financial uncertainty shocks must occur (during the 1987 crash or the financial crisis), or when the real activity shocks must not be unusually favorable (the Great Recession). Since $\mathbf{e}_t = \mathbf{B}^{-1}\hat{\boldsymbol{\eta}}_{\mathbf{t}}$, we check that the shocks implied by each draw of $\mathbf{B} = \hat{\mathbf{P}}\mathbf{Q}$ at particular episodes satisfy the event constraints parameterized by $\bar{\mathbf{k}}$.

5.1 System $\mathbf{X}_t = (U_{Mt}, ip_t, U_{Ft})'$

We first consider a system $\mathbf{X}_t = (U_{Mt}, ip_t, U_{Ft})'$. For this system we set $\bar{\lambda}_1 = -0.05$, $\bar{\lambda}_2 = 2$, and collective correlation bound $\bar{\lambda}_3 = 0.18$. This parameterization sets a lower bound of 5% for the absolute correlation between S_t and each uncertainty shock individually, and targets a collective correlation between the two uncertainty shocks and S_t that is at least 0.18. According to the BTZ model, the correlation between macro and financial uncertainty is approximately equal to the correlation between stock returns and discount rate news $(\mathbb{E}_{t+1} - \mathbb{E}_t)\left[\sum_{j=1}^{\infty} \rho^j S_{t+1+j}\right]$. In the data, the latter correlation is around 0.18 in absolute value, which we use for $\bar{\lambda}_3$.[12]

Our choice of the event constraint bounds $\bar{k}_1$ and $\bar{k}_2$ is partly guided by Bloom (2009). In his work, uncertainty shocks are calibrated from innovations to the VXO stock market

[12] This statement is based on estimates $\hat{\eta}_{dt+1}$ from a forecasting VAR for S_t. The parameter $\bar{\lambda}_3$ is a lower bound on the required collective correlation. Results imply that the correlation between $\hat{\eta}_{dt+1}$ and S_{t+1} depends to on the sample and on the forecasting variables in the VAR, but correlations of at least 0.18 in absolute value constitute a plausible lower bound.

volatility index. Bloom (2009) then studies the dynamic effects of four standard deviation shocks to uncertainty. Likewise in our data we find that the largest shocks to U_{Ft} for the system $\mathbf{X}_t = (U_{Mt}, ip_t, U_{Ft})'$ using $h = 1$ month uncertainty are typically close to four standard deviations. We therefore set $\bar{k}_1$ and $\bar{k}_2$ equal to 4. If shocks were Gaussian, the probability of a shock of this magnitude is 1.3e-4. But as we show below, the identified shocks here are non-Gaussian and exhibit excess skewness and leptokurtosis. We set $\bar{k}_3 = 2$ to dismiss real activity shocks that are greater than two standard deviations above its sample mean during the Great Recession. The $\mathbf{X}_t$ system with the above values for $(\bar{\mathbf{k}}, \bar{\boldsymbol{\tau}}, \bar{\boldsymbol{\lambda}})$ will be referred to as our baseline system.

It is worth noting that the 1987 event constraint alone eliminates 72% of the solutions, while the two events in the 2008 Recession eliminate 90%. The three event constraints together eliminate 99% of the solutions in $\hat{\mathcal{B}}$. Of course one percent of 1.5 million draws is still a non-trivial number. But when the event constraints are combined with the correlation constraints, we are left with 1,110 accepted draws, which is 1.7 times the sample size. Results under alternative choices for these parameters will be explored below.

5.2 Uncertainty Shocks

To get a sense of the behavior of the shocks, Figure 2 presents the time series of the standardized shocks (e_M, e_{ip}, e_F) for the maxG solution. All shocks display strong departures from normality with excess skewness and/or excess kurtosis. The largest of the positive e_{ip} shocks is recorded in 1975:01 followed by 1971:01, while the largest of the negative e_{ip} shocks is recorded in 1980:04, followed by 1979:04. There also appears to be a moderation in the volatility of the ip shocks in the post-1983 period. The largest positive e_M shock is in 1970:12, followed by the shock in 2008:10. The largest positive e_F shock is recorded in 2008:09 during the financial crisis followed by 1987:10 (Black Monday). For e_F, the extreme but transitory nature of the 1987 stock market crash leads to a very large spike upward in e_F in the month of the crash, followed by a very large spike downward in the month following the crash as the market recovered strongly and quickly. While this episode magnifies the spike in e_F in 1987, it is largely orthogonal to real activity and macro uncertainty. Observe that the large ip shock in 2005:09 is not associated with a contemporaneous spike in uncertainty, while there are several spikes in both types of uncertainty that do not coincide with spikes in e_{ip}.

In Figure 1 presented earlier, we find 1973-74, 1981-82, and 2007-2009 to be the three episodes of heightened macroeconomic uncertainty, defined as the periods when U_{Mt} is at least 1.65 standard deviations above its unconditional mean. We now focus on large "adverse" shocks, namely large positive uncertainty shocks and large negative real activity shocks recovered by the econometric methodology. Figure 3 displays the date and size of e_M and e_F shocks that are at least two standard deviations above the mean and negative e_{ip} shocks exceeding two standard deviations for all solutions in the identified set. In view of the non-normality of the shocks, the figure also plots horizontal lines corresponding to three standard deviation of the unit shocks, which is used as the reference point for 'large'.

The bottom panel shows that the solutions identify big financial uncertainty shocks in October 1987 and in one or more months of 2008. Such solutions are selected as part of the identification scheme. The middle panel shows that large negative real activity shocks are in alignment with all post-war recessions with one exception: the negative real activity shock in 2005 is not immediately associated with a recession, but it could be the seed of the Great Recession that followed. It's clear that parts of the real economy were showing signs of deterioration prior to the onset of the recession as dated by the NBER. For example, it is known that the housing market led the 2007-2009 recession (e.g., see Favilukis, Ludvigson, and Van Nieuwerburgh (2015) for a discussion). Indeed, all 10 housing series in $\mathcal{X}^M$ (most pertaining to housing starts and permits series) exhibit sharp declines starting in September 2005 and continuing through 2006, thereby leading the Great Recession.

Figure 3 shows that the dates of large increases in e_M are less clustered. They generally coincide with, or occur shortly after, the big real activity shocks and the financial uncertainty shocks. Large macro uncertainty shocks occurred more frequently in the pre-1983 than the post 1983 sample, consistent with a Great Moderation occurring over the period ending in the Great Recession.

Although our event restrictions require that large financial uncertainty shocks play an important role in the 1987 crash and 2007-09 financial crisis, they by no means rule out large adverse roles for the other shocks. In particular, our restrictions do not require that all or even most of the variation in these episodes be attributable to shocks that originated in financial markets. Figure 3 shows many large adverse values of e_M and e_Y in these episodes. Indeed, all of the solutions in the identified set under the baseline bounds have an e_M greater than three standard deviations above the mean in the 2007-09 financial crisis, and 60% of the solutions have an e_Y three standard deviations or more below the mean in this period. Thus, the results imply that there were big shocks everywhere in the Great Recession/financial crisis. It would be desirable for dynamic equilibrium models that purport to study the effects of uncertainty to incorporate shocks with such non-Gaussian features.

5.3 Impulse Response Functions

We now use impulse response functions to better understand the dynamic causal effects and propagating mechanisms of the shocks. Figure 4 shows in shaded areas the identified set of dynamic responses of each variable in the SVAR to a standard deviation *increase* in each of structural shocks. These are responses for all solutions in the identified set $\bar{\mathcal{B}}$. The dotted line shows the maxG solution. Several results stand out.

First, positive shocks to financial uncertainty e_F (center plot, bottom row) lead to a sharp decline in production that persists for many months. All solutions that satisfy the identification restrictions have this pattern and the identified set of responses is bounded well away from zero as the horizon increases. Positive perturbations to e_{Ft} also cause U_{Mt} to increase sharply (third row). These results lend support to the hypothesis that heightened financial uncertainty is an exogenous impulse that causes declines in real activity. However, there is little evidence that heightened financial uncertainty is a *result* of lower economic activity. Instead, positive shocks to production increase financial uncertainty at least initially (second row, third column).

Second, while we find no evidence that high financial uncertainty is a consequence of lower economic activity, the results for macro uncertainty are quite different. Macro uncertainty falls sharply in response to positive ip shocks. Alternatively stated, negative ip shocks cause macro uncertainty to increase sharply. These endogenous movements in macro uncertainty persist for about five years after the real activity shock, a result that is strongly apparent in all the solutions of the identified set.

Third, there is little evidence that the observed negative correlation between macro uncertainty and real activity is the result of positive macro uncertainty shocks that drive down production. The top middle panel shows that all solutions in the identified set imply that positive macro uncertainty shocks *increase* real activity in the short run, consistent with growth options theories discussed above. Many solutions in the identified set have this implication even in the long-run, though the finding that not all of them do indicates that the long-run effects of a macro uncertainty shock are less well identified. Whether short- or long-run, there is little basis for concluding that positive U_M shocks reduce real activity. The findings suggest instead that higher macro uncertainty in recessions is a response to lower economic activity rather than a causal factor in recessions.

5.4 Decomposition of Variance

To give a sense of the historical importance of these shocks, we perform a decomposition of variance for each solution in the identified set. We report the fraction of s-step-ahead forecast error variance attributable to each structural shock e_{Mt}, e_{ipt}, and e_{Ft} for $s = 1$, $s = 12$, $s = \infty$, and s_{max}, where s_{max} is the horizon at which the fraction of forecast error variance is maximized. Because we have a set of solutions, we have a range of forecast error variances for each s. The left panel of Table 1 reports the range of values for the $\mathbf{X}_t$ system. The right panel of Table 1 are results for an alternative measure of uncertainty and will be discussed below.

According to the top row, real activity shocks e_{ipt} have sizable effects on macroeconomic uncertainty U_M, with the fraction of forecast error variance ranging from 0.48 to 0.81 at the s_{max} horizon. But according to the bottom row, these same shocks have small effects on financial uncertainty U_{Ft}, with a range of forecast error variance from 0.02 to 0.10 at horizon s_{max}. The middle row shows that positive macro uncertainty shocks e_M, which increase rather than decrease real activity, explain a surprisingly large fraction of production, with effects at s_{max} horizon ranging from 0.34 to 0.96.

Though financial uncertainty shocks e_{Ft} have a small contribution to the one-step-ahead forecast error variance of ip_t, their relative importance increases over time so that they account for 0.30 to 0.61 of the forecast error variance in ip at the s_{max} horizon. Financial uncertainty is unlike macro uncertainty or real activity in that its variation is far more dominated by its own shocks. As seen from Table 1, e_{Ft} shocks explain between 0.72 and 0.97 of the $s = 1$ step-ahead forecast error variance in U_{Ft}, and between 0.44 and 0.80 at the $s = \infty$ horizon. At the s_{max} horizon, the range of forecast error variance is 0.75 to 0.97.

To summarize, positive real activity shocks have quantitatively large persistent and negative effects on macro uncertainty U_{Mt}. In turn, positive macro uncertainty shocks e_{Mt} have positive effects on production, especially in the short-run. By contrast, positive financial uncertainty shocks e_F have large negative effects on production, especially in the long run. Across all VAR forecast horizons, the forecast error variance of financial uncertainty is the least affected by shocks other than its own, implying that U_{Ft} is quantitatively the most important exogenous impulse in the system.

6 Policy Uncertainty

The results above suggest that the dynamic relationship between macro uncertainty and real activity may be quite different from the relation between financial uncertainty and real activity. However, given the composition of our macro data, macroeconomic uncertainty itself can be due to uncertainty in real activity variables such as output and unemployment, to price variables, and to financial market variables. The theoretical uncertainty literature has focused on modeling exogenous uncertainty shocks that arise specifically in measures of real economic fundamentals, rather than in prices or financial markets. To better evaluate the implications of these theoretical models and to examine robustness to alternative measures of macro or real economic uncertainty, we repeat our analysis using the economic policy uncertainty (EPU) indices of Baker, Bloom, and Davis (2016) (BBD). BBD find that firms with greater exposure to government expenditures reduce investment and employment growth when policy uncertainty rises, suggesting that the EPU indices are well characterized as measures of real economic uncertainty.

BBD compute two EPU indices, a "baseline" EPU index that has three components, and a news-only index that is a subindex and one component of the baseline EPU index. We denote these the EPU and EPN index, respectively. The left panel of Figure 5 shows the two indices from 1987:01 to 2017:06. We observe that the two largest spikes up in the baseline index are in and just after the debt ceiling crisis resolution, which correspond to the dates 2011:07 and 2011:08. For news index, there is one additional spike upward that rivals these in size: that for September 11, 2001. We hereafter assume that the debt-ceiling crisis of 2011 and, in the case of the EPN index, the September 11th, 2001 terrorist attacks are plausible large historical policy uncertainty events.

We repeat the analysis for two systems: $\mathbf{X}_t^{EPU} = (EPU_t, ip_t, U_{Ft})'$, and $\mathbf{X}_t^{EPN}(EPN_t, ip_t, U_{Ft})'$. The constraints $\bar{g}_{E1}, \bar{g}_{E2}, \bar{g}_{E3}$ used above on e_{Ft} and e_{ipt} are maintained in these systems, but we keep only a single constraint on the correlation between e_{Ft} with the stock market, requiring $c_F(\mathbf{B}) - \bar{\lambda}_4 \geq 0$. No correlation constraint is imposed on the policy shocks $e_{EPUt}(\mathbf{B})$ and $e_{EPNt}(\mathbf{B})$, but we use a new set of event constraints for these shocks set as follows:

Shocks	$\mathbf{X}_t^{EPU}$	$\mathbf{X}_t^{EPN}$	τ
$\bar{g}_{E4}$	$e_{EPU\overline{\tau}_4}(\mathbf{B}) - 2 \geq 0$	$e_{EPN\overline{\tau}_4}(\mathbf{B}) - 2 \geq 0$	for $\overline{\tau}_4 =$ [2011:08, 2011:09]
$\bar{g}_{E5}$	–	$e_{EPN\overline{\tau}_5}(\mathbf{B}) - 2 \geq 0$	for $\overline{\tau}_5 =$ 2001:9

The above constraints restrict the policy shocks to be at least 2 standard deviations above the mean in the months of the debt ceiling crisis in both systems and, in the case of the $\mathbf{X}_t^{EPN}$ system, in the month of the 2001 terrorist attacks. We normalize EPU_t and EPN_t to have the same mean and standard deviation as U_{Mt} and set bounds for the event constraints on $e_{ipt}(\mathbf{B})$ and $e_{Ft}(\mathbf{B})$ are to be the same as in the baseline parametrization for the $\mathbf{X}_t$ system. Note that since we now have only a single correlation restriction for $e_{Ft}(\mathbf{B})$, the previous collective and individual correlation constraints coincide. We set $\bar{\lambda}_4 = 0.12$, which is in between the individual bound $\bar{\lambda}_1$ and the collective bound $\bar{\lambda}_3$ in the $\mathbf{X}_t$ system.

The right panel of Figure 5 shows the dynamic responses for the $\mathbf{X}_t^{EPU}$ and $\mathbf{X}_t^{EPN}$ systems. The character of the responses is similar to those for the systems based on the JLN uncertainty measures. Policy uncertainty falls sharply in response to positive production shock. Alternatively stated, negative shocks to production increase policy uncertainty sharply. These endogenous movements in policy uncertainty are more transient than those to macro uncertainty, however, and are eliminated in about two years. Financial uncertainty shocks in this system continue to be a driving force for real activity, with positive shocks driving down ip_t sharply and persistently. But there is no evidence that positive shocks to ip_t drive down financial uncertainty; in fact such shocks drive financial uncertainty persistently upward. There is no evidence based on the either system that positive policy uncertainty shocks drive down real activity; the opposite is found, with positive shocks to policy uncertainty driving up production even more persistently than in the $\mathbf{X}_t$ system. These findings reinforce the previous results that countercyclical increases in real economic uncertainty are often well characterized as endogenous responses to declines in real activity, rather than exogenous impulses driving real activity downward, while the opposite is true for financial uncertainty. Interestingly, positive shocks to policy uncertainty drive financial uncertainty down, suggesting that markets may view times of high policy uncertainty as coincident with upside rather than downside risk.

To complete the analysis, we present variance decompositions for the $\mathbf{X}_t^{EPU}$ system (the results for the system $\mathbf{X}_t^{EPN}$ are similar). These results, presented in the right panel of Table 1, share some similarities with the $\mathbf{X}_t$ system shown in the left panel, but there are at least two distinctions. First, financial uncertainty shocks that decrease real activity in both systems explain a smaller fraction of the forecast error variance in production in the $\mathbf{X}_t^{EPU}$ system at all but the $s = 1$ forecast horizon. The ranges for these numbers at the $s = s_{max}$ horizon across all solutions in the identified set are $[0.17, 0.34]$ in the $\mathbf{X}_t^{EPU}$ system compared to $[0.30, 0.61]$ in the $\mathbf{X}_t$ system. Second, compared to the $\mathbf{X}_t$ system, greater fractions of the forecast error variance in U_{Ft} are explained by ip shocks. That is likely because positive shocks to production have more persistent effects on financial uncertainty in the $\mathbf{X}_t^{EPU}$ system.[13]

7 Additional Cases

This section considers different bounds, different external variables, different samples, longer-horizon uncertainty, the validity of recursive identification restrictions, and a bigger SVAR system that includes S_t. Detailed results are reported in an online Appendix. The main findings are summarized below.

7.1 Alternative Bounds

To give a sense of which constraints are most important for identification, in this section we present results under alternative choices for parameterization of the bounds in the correlation and event constraints.

The left panel of Figure 6 shows the results when the individual correlation bound $\bar{\lambda}_1$ is strengthened from -0.05 to -0.07, with all other bounds are held fixed at their baseline values. Thus uncertainty shocks are now required to be individually more correlated with stock market returns than in the baseline bounds previously reported. The set of solutions is relatively insensitive to the individual correlation bound.

The right panel of Figure 6 shows the baseline systems impulse response functions (IRFs) when the parameters $\overline{\mathbf{k}}$ governing the event constraints are slackened from (4,4,2) to (2,2,2).

[13]It is worth noting that the results for the EPU systems are very similar even if no correlation constraints with S_t are imposed. For these systems, the event constraints alone appear to be sufficient for identifying the dynamic relationships in the system.

Thus, a "big" financial uncertainty shock is now defined as just two standard deviations above the mean. This leaves about twice more solutions than the baseline parameterization (2098 compared to 1,110). Eight of the nine responses are qualitatively similar to those under baseline bounds. In particular, all solutions in the identified set imply that a positive U_F shock drives down ip eventually; all solutions in the identified set show that U_M endogenously falls sharply and persistently in response to positive ip shocks, and all solutions imply that there is much less evidence that U_F endogenously declines in response to positive ip shocks, at least in the short-run. The one response that is not well identified at any horizon when we define a large U_F shock as 2 standard deviations is the effect of e_M on production. Even in this case, there is little basis for concluding that positive U_M shocks reduce real activity, while there remains strong evidence that negative real activity shocks increase U_M.

The left panel of Figure 7 shows the IRFs when the parameter $\bar{\lambda}_2$ governing the relative correlations c_M and c_F is varied between 1.5 and 2.5. The identified sets with $\bar{\lambda}_2 = 2.5$ are similar to the baseline bound with $\bar{\lambda}_2 = 2$. With $\bar{\lambda}_2 = 1.5$, the sets are wider since this parameterization slackens the constraint relative to the baseline. Eight of the nine responses are again qualitatively similar to those under baseline bounds. Financial uncertainty shocks drive ip_t down eventually, though for some solutions the effect is small; all solutions show that positive production shocks drive U_{Mt} down sharply and persistently, while there is much less evidence that U_F endogenously declines in response to positive ip shocks, at least in the short-run. The effects of e_M on ip_t are again inconclusive in this case.

Finally, the right plot of Figure 7 shows the results when the collective correlation bound $\bar{\lambda}_3$ is reduced from 0.18 to 0.15, so that the uncertainty shocks are allowed to be less correlated collectively with stock market returns. Weakening the collective bound leads to wider sets and inconclusive dynamic effects for many IRFs as we now have approximately four times more solutions in the identified set (4,054 compared to 1,110 under the baseline parameterization). This constraint is especially nonlinear in the parameter values in this system, so seemingly small changes in the bounds can have large effects on the identified set. A noticeable difference compared to the baseline case concerns the effects of financial uncertainty shocks on real activity, which are no longer well determined when uncertainty shocks are permitted to have weaker collective correlation with the stock market. Further inspection shows that this occurs because a slackening of the collective constraint admits solutions into the identified set for which financial uncertainty shocks have a weaker absolute correlation with the stock market than under the baseline parameterization. Solutions for which financial uncertainty shocks have a low correlation with the stock market are unlikely to signify an important role for financial uncertainty in business cycle downturns.

There are additional results pertinent to this latter finding. First, the character of the dynamic responses is less sensitive to all the correlation constraint bounds $\bar{\boldsymbol{\lambda}}$ if we use an alternative external variable (the real price of gold) in place of stock market returns–these results are presented below–and they are not sensitive at all if we combine the two types of correlation constraints. Second, results not reported indicate that the dynamic responses in the two EPU systems are unaffected by a slackening of the correlation restriction bounds $\bar{\boldsymbol{\lambda}}$ because for that system the constraints are non-binding for all solutions in the identified set. All of these cases have the same flavor or results as obtained for the system $\mathbf{X}_t = (U_{Mt}, ip_t, U_{Ft})'$ under baseline bounds.

Taken together, the results in Figure 7 demonstrate that both correlation and event constraints contribute to shrinking the unconstrained solution set. More generally, our investigations of what happens under departures from the baseline bounds may be summarized as falling into one of three categories: (i) inconclusive results where the identified set of some responses is wide, (ii) results incompatible with the data where the identified set is empty, or (iii) conclusive results where the character of the responses is similar to what is displayed in Figures 4 and 5. We do not find alternative bounds for our constraints in which clear conclusions of an entirely different nature can be drawn. No matter what the bounds, there is little basis for concluding that positive macro uncertainty shocks drive down production. It is noteworthy that many of the additional solutions retained when the event and correlation constraints are slackened either fail to produce large negative financial uncertainty shocks in the financial crisis, and/or they suggest weak or negligible correlations between financial uncertainty shocks and stock market returns. Such solutions are ruled out as implausible under the baseline parameterization.

7.2 Alternative External Variable

We considered other external variables that could be informative about uncertainty shocks, namely the real price of gold, with the idea that it is a quintessential safe-haven asset whose value should rise when uncertainty rises. Piffer and Podstawski (2016) have suggested using at variation in the real price of gold around specific events as an instrumental variable to identify uncertainty shocks. We instead use it as an alternative external variable in the manner used above, without requiring it to be a valid instrument. The gold price level is deflated using the Consumer Price Index (CPI) with Jan. 2018 as the base month.[14]

The event constraints $\bar{g}_{E1}$, $\bar{g}_{E2}$, and $\bar{g}_{E3}$ remain the same as above. But we replace the previous correlation constraints $\bar{g}_{C1} - \bar{g}_{C3}$ with two individual correlation constraints using the real gold price, denoted G_t, that require $\text{corr}(e_j, G_t) \geq 0.07$ for $j = M, F$. Unlike our use of stock market returns, for the gold price we impose no constraint on the relative correlations across the two types of uncertainty shocks (i.e., there is no $\bar{g}_{C2}$ constraint). Moreover, we drop the collective correlation restriction $\bar{g}_{C3}$ since it is redundant given the symmetry of the

[14] The data source is http://www.macrotrends.net/.

individual correlation constraints.

Figure 8 shows the results for the system $\mathbf{X} = (U_M, ip, U_F)'$. All solutions in the identified set imply that a positive U_F shock eventually drives down ip; all solutions in the identified set show that U_M endogenously falls sharply and persistently in response to positive ip shocks, and all solutions imply that there is little evidence that financial uncertainty endogenously declines in response to positive ip shocks. In addition, all solutions imply that positive macro uncertainty shocks increase real activity in the short run, with the majority of solutions in this system also displaying this result in the long-run. Using the weaker correlation bound of 5% produces similar results.

7.3 Pre-Crisis Sample

We have used the Great Recession/financial crisis as one of our special events to help identify the transmission of uncertainty and real activity shocks. To give a sense of how much identifying power is attributable to this episode, we repeated our analysis on the sample of data up through the month just prior to the recession, 1960:07 to 2007:11. In the process we loose all of the identifying power of the event restrictions associated with the Great Recession. We maintain the event constraint for the 1987:10 stock market crash, as well as the correlation constraints, where the latter now apply to the shorter sample.

Figure A1 (reported in the Online Appendix to conserve space) shows the dynamic responses for the baseline system $\mathbf{X}_t = (U_M, ip_t, U_{Ft})'$ estimated the sample 1960:07 to 2007:11. It is necessary to slacken slightly the collective correlation constraint bound $\bar{\lambda}_3$ so that the identified set is non-empty, but otherwise the bounds are the same as for the full sample. Not surprisingly given the loss of identifying restrictions in the truncated sample, the dynamic responses to many shocks are now inconclusive. The set of IRFs for the effects of e_{Mt} shocks on ip_t and e_{ipt} shocks on U_{Mt} is much wider and zero is in the range. These sets nevertheless continue to show that financial uncertainty shocks e_{Ft} drive down ip_t sharply and persistently, at least eventually. A premise of this paper is that the 2007-09 financial crisis was an important rare event that can help distinguish the transmission of financial versus real uncertainty shocks. This maintained assumption appears supported by the subsample analysis.

7.4 Longer-Horizon Uncertainty

The baseline analysis uses one-month-ahead uncertainty. We repeated our analysis using six-month-ahead uncertainty. The Online Appendix reports the IRFs for this system in Figure A2. Six-month-ahead uncertainty is less volatile than one-month-ahead uncertainty.[15] Hence

[15] While the level of uncertainty increases with the uncertainty horizon h (on average), the variability of uncertainty decreases because the forecast converges to the unconditional mean as the forecast horizon tends to infinity.

the baseline bounds need to be altered and we adjust them as stated in the figure notes to stay as close as possible to the baseline bounds while still ensuring that the identified set is non-empty.[16] The results in Figure A2 exhibit similar patterns as our baseline case, but there is one difference. While positive shocks to six-month-ahead macro uncertainty are found, like one-month-ahead uncertainty, to increase production in the short run, unlike the results for one-month-uncertainty, all solutions in the identified set indicate that production declines in the long-term. These findings suggest that longer-term macro uncertainty may be more detrimental to real activity than shorter-term macro uncertainty.

7.5 Validity of Recursive Identification Restrictions

The econometric model permits us to test whether a recursive structure is supported by the data. Specifically, the assumptions in our event and correlation constraints do not rule out the possibility of a recursive structure, so that if such a structure is consistent with the data, our identifying restrictions are free to recover it. With three variables in the SVAR, there are six possible recursive orderings corresponding to six different 3×1 vectors of elements of $\mathbf{B}$ that must be jointly zero. It is straightforward to assess whether our identified solutions are consistent with a recursive structure by examining the distribution of solutions in the constrained set for four elements of the $\mathbf{B}$ matrix: $\hat{B}_{FY}, \hat{B}_{YM}, \hat{B}_{MY}$, and $\hat{B}_{MF}$. None of the distributions contain any values near zero. The minimum absolute values in each case are 0.003, 0.004, 0.007, and 0.002, respectively, which are all bounded away from zero. The implication is that the recursive structure is inconsistent with any recursive ordering across all solutions in the identified set.

What happens to the dynamic responses when we nevertheless impose restrictions based on recursive identification (and freely estimate the rest of the parameters)? With these recursive restrictions the SVAR is point-identified so no winnowing constraints are needed. Of course, there are many possible recursive orderings, and inevitably, the estimated IRFs differ in some ways across these cases. However, the dynamic responses under recursive identification have one common feature that is invariant to the ordering. Results available on request show that, no matter which ordering is assumed in the recursive structure, macro uncertainty shocks appear to cause a sharp decline in real activity, much like financial uncertainty shocks, while positive real activity shocks have little effect on macro uncertainty in the short run and if anything increase it in the long run, as shown in the figure. This is in stark contrast to the results from our identification scheme, which is capable of recovering a recursive structure if it were true. But we fail to find such a structure. These results show that imposing a structure that prohibits contemporaneous feedback may spuriously suggest that macro uncertainty shocks are a cause of declines in real activity, rather than an endogenous response. The finding underscores

[16] Similar results are obtained if we alternatively adjust the bounds to target an accepted-draws to sample-size ratio 1.7, the same as for the system using one-month uncertainty.

the challenges of relying on convenient timing assumptions to sort out cause and effect in the relationship between uncertainty and real activity.

7.6 Including S_t in the VAR

A maintained assumption of the analysis is that the parameters of the system $\mathbf{X}_t = (U_{Mt}, ip_t, U_{Ft})'$ can be consistently estimated without explicitly modeling the random processes behind the external variables. This assumption can be relaxed. Specifically, we consider a systems approach that appends S_t to our three variable SVAR for $\mathbf{X}_t$. A difficulty is that the system is even more under-identified than before. The impact matrix $\mathbf{B}$ now has 16 rather than 9 unknown parameters. Clearly additional identifying restrictions are required to achieve informative identified sets. We restrict idiosyncratic stock market shocks e_{St} (those orthogonal to both types of uncertainty shocks and real activity shocks) to affect $\mathbf{X}_t$ with a one-period lag. With these restrictions and the same event and correlation constraints used above, Figure A3 of the online Appendix shows that the IRFs are very close to the those for $\mathbf{X}_t$ under the baseline bounds. The results imply that, under these restrictions, idiosyncratic stock market shocks are not highly important for the dynamics of $\mathbf{X}_t$.

8 Conclusion

A growing body of research establishes uncertainty as a feature of deep recessions but leaves open two key questions: is uncertainty primarily a source of business cycle fluctuations or an endogenous response to them? And does the type of uncertainty matter? The objective of this paper is to address both questions econometrically using small-scale structural VARs capable of nesting a range of theoretical possibilities.

The macro literature on uncertainty has focused on real activity induced macro uncertainty as a driver of economic fluctuations. Using a novel identification approach that imposes economic assumptions on the behavior of the shocks, we find from a variety of parameterizations and specifications that macro uncertainty rises endogenously in response to real activity shocks, contributing to strongly its countercyclical behavior. It is shocks to financial uncertainty, rather than macro uncertainty, that are found to be a driver of economic fluctuations. An implication of our findings is that dynamic equilibrium models should allow for broad-based macro uncertainty to respond endogenously to a variety of shocks, while entertaining the notion that occasional large shocks to uncertainty originating in financial markets may be a source of deep recessions.

Our findings call for a need to better understand how uncertainty in financial markets is transmitted to the macroeconomy, and why the two types of uncertainty have a distinct relationship with economic activity. A burgeoning business cycle literature has begun to postulate theoretical linkages between financial market uncertainty, real/macro uncertainty, and real activity.[17] Although these models are currently too stylized to be confronted with actual data, they appear capable of generating implications that are consistent at least qualitatively with our finding that positive shocks to financial uncertainty are a driving force of declines in productive activity, while real uncertainty is often an endogenous response to such declines.

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[17] For example. Benhabib, Liu, and Wang (2017) studies self-fulfilling surges in financial and real uncertainty in a model of informational interdependence and mutual learning; Adrian and Boyarchenko (2012), Brunnermeier and Sannikov (2014), and He and Krishnamurthy (2012) study production economies with financial intermediaries that give rise to time-varying GDP vulnerabilities (downside real risk) as a function of time-varying financial frictions; hence financial uncertainty drives both GDP and its volatility.

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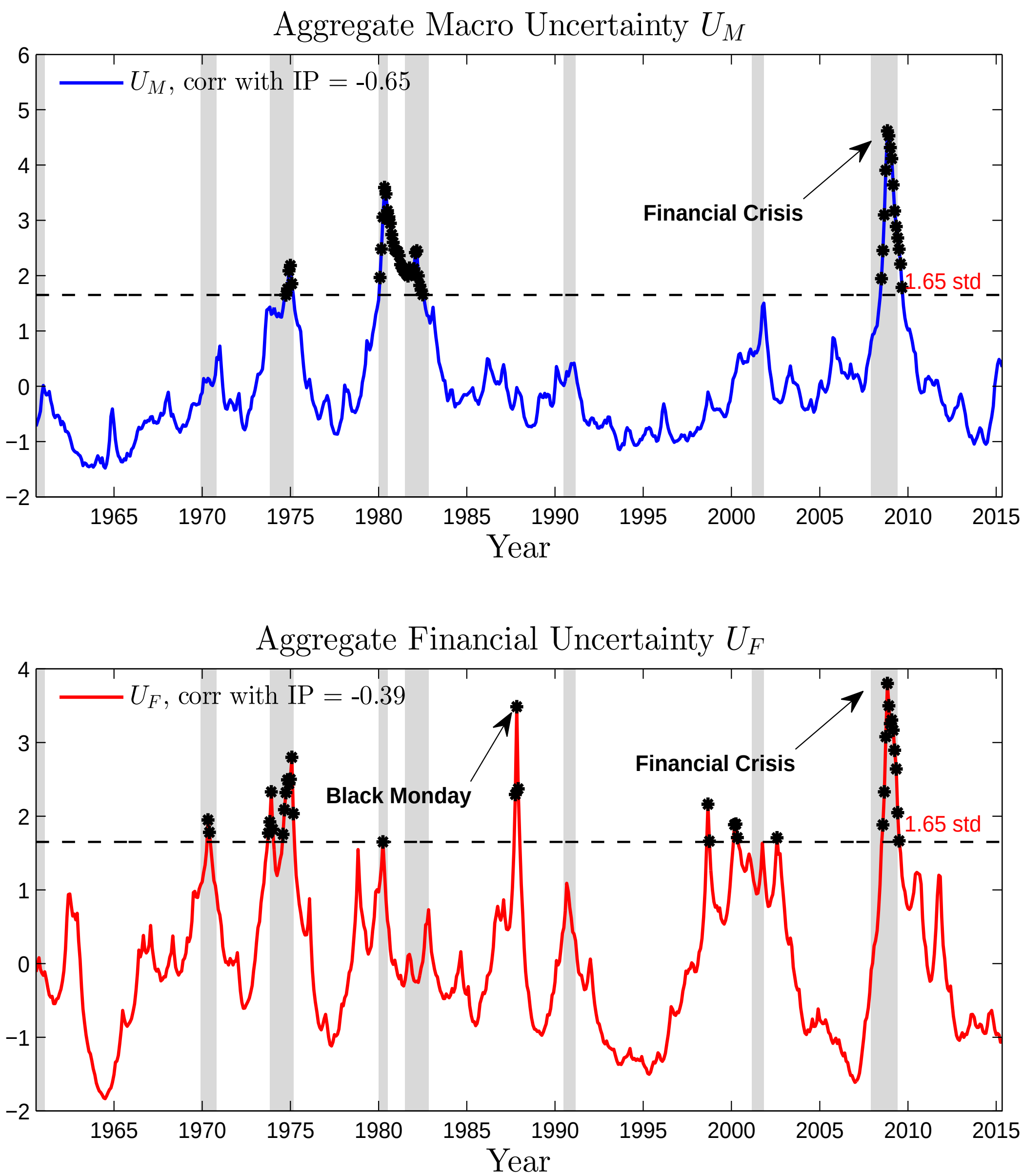
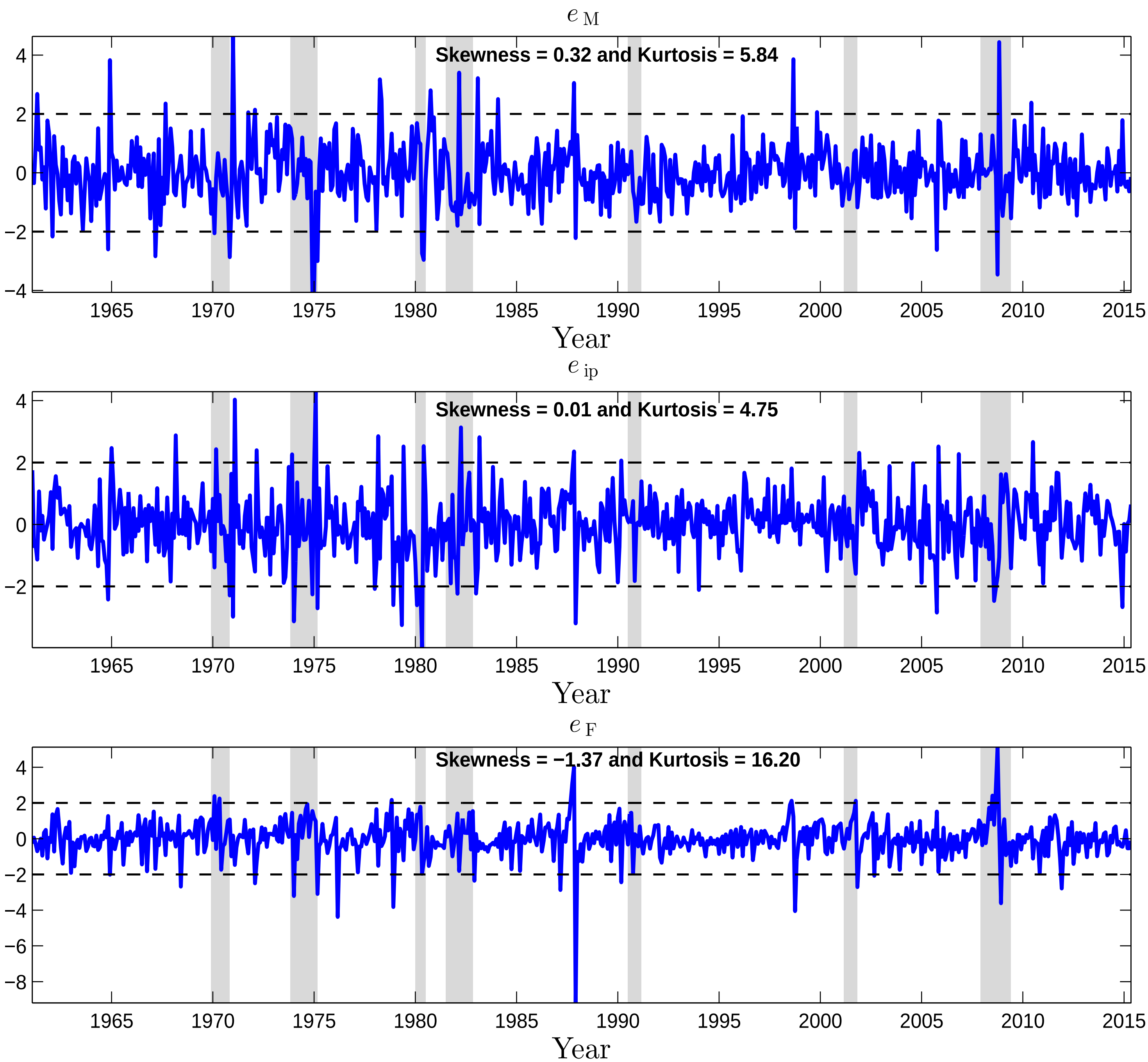
Figure 1: Macro and Financial Uncertainty Over Time



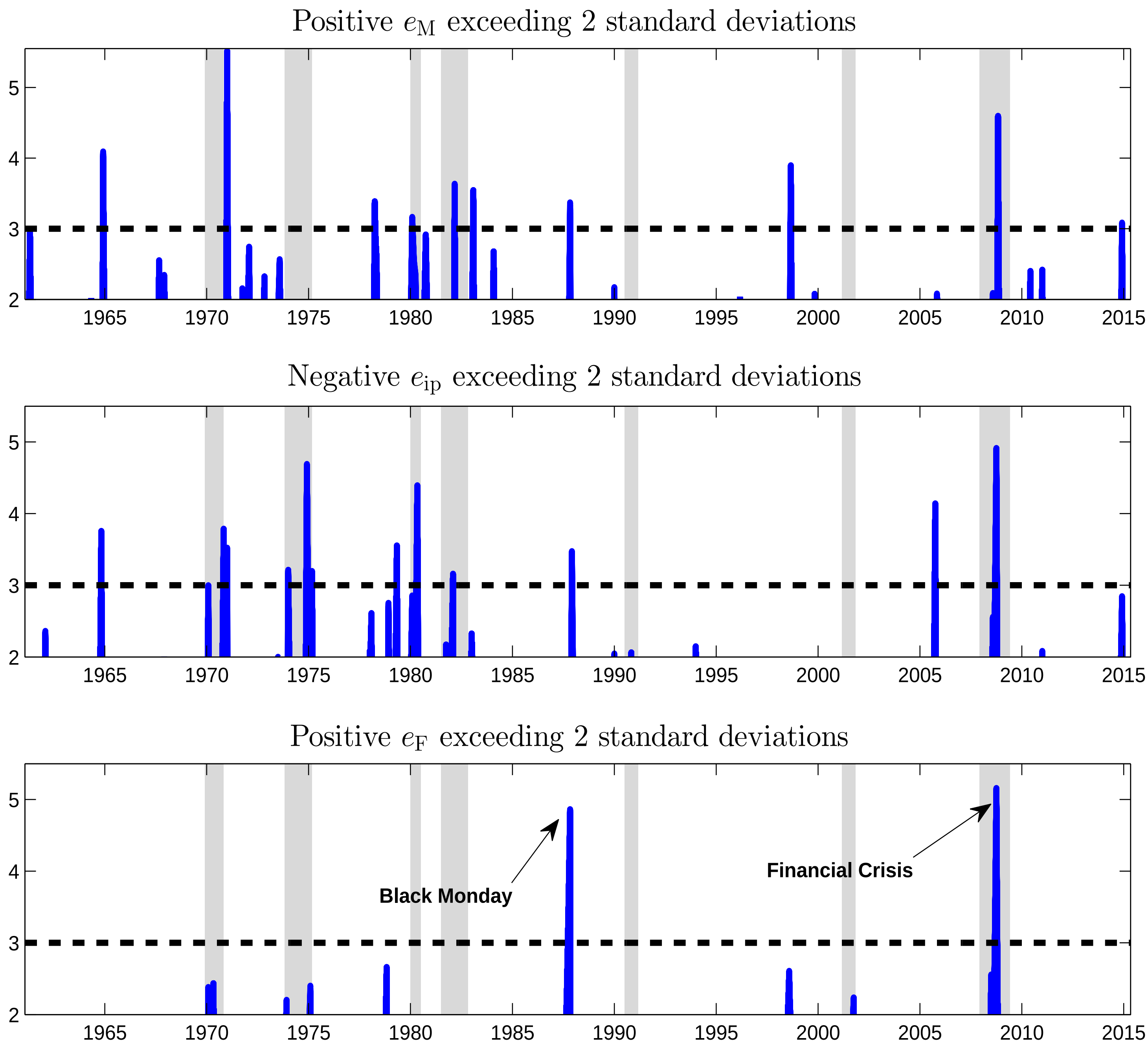
The panels plot the time series of macro uncertainty U_M and financial uncertainty U_F expressed in standardized units. Shaded areas correspond to NBER recession dates. The horizontal line corresponds to 1.65 standard deviations above the unconditional mean of each series (which has been normalized to zero); the black dots are months when uncertainty is at least 1.65 standard deviations above the mean. Correlations with the 12-month moving average of IP growth are reported. The data span the period 1960:07 to 2015:04.

Figure 2: Time Series of e Shock from SVAR $(U_M, ip, U_F)'$



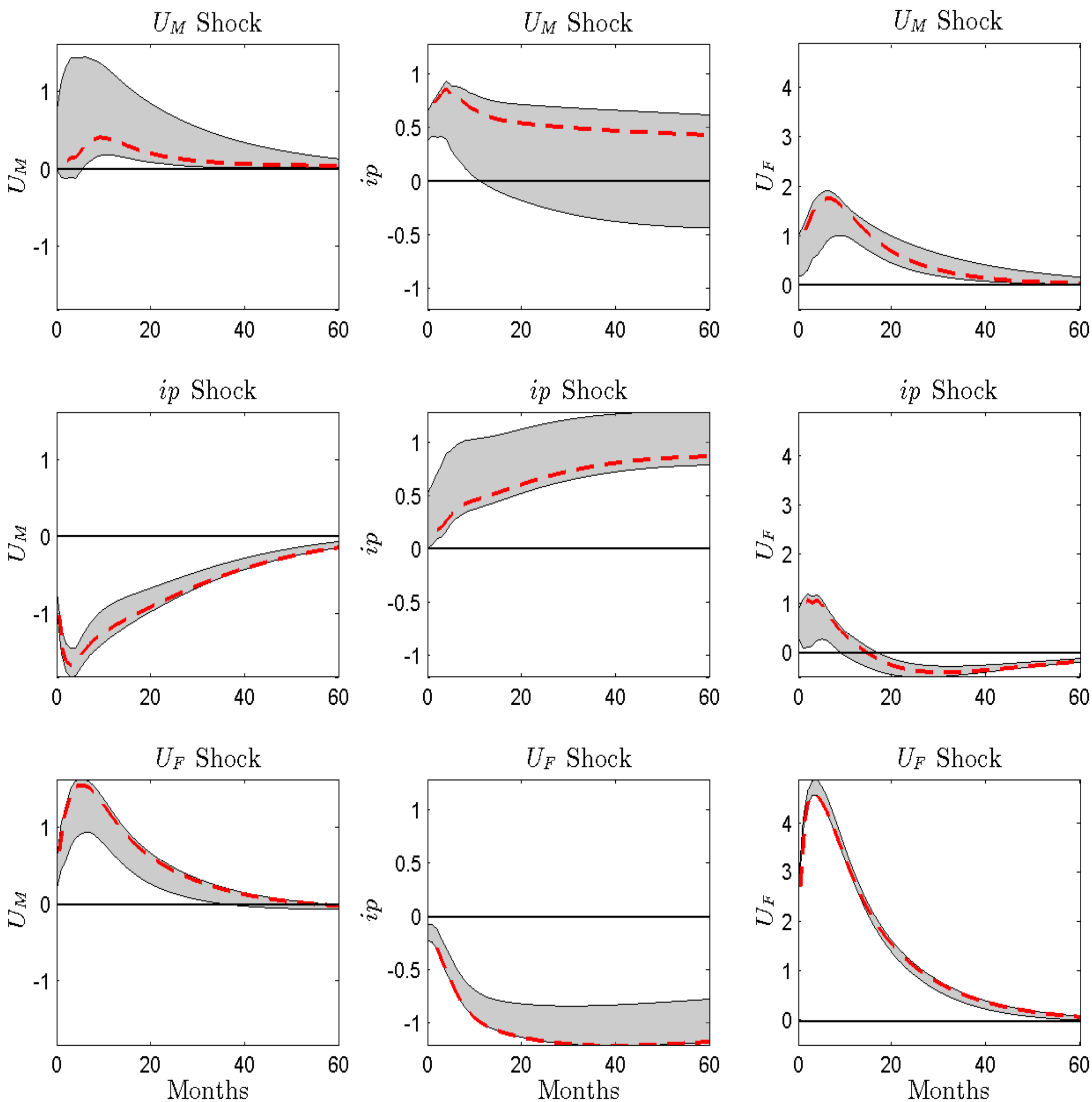
The horizontal line corresponds to 3 standard deviations above/below the unconditional mean of each series. The shocks $e = B^{-1}\eta_t$ for maxG solution are reported, where η_t is the residual from VAR(6) of $(U_M, ip, U_F)'$. The bounds are $\bar{\lambda}_1 = -0.05$, $\bar{\lambda}_2 = 2$, $\bar{\lambda}_3 = 0.18$, $\bar{k}_1 = 4$, $\bar{k}_2 = 4$, $\bar{k}_3 = 2$. The sample spans the period 1960:07 to 2015:04.

Figure 3: Large Shocks



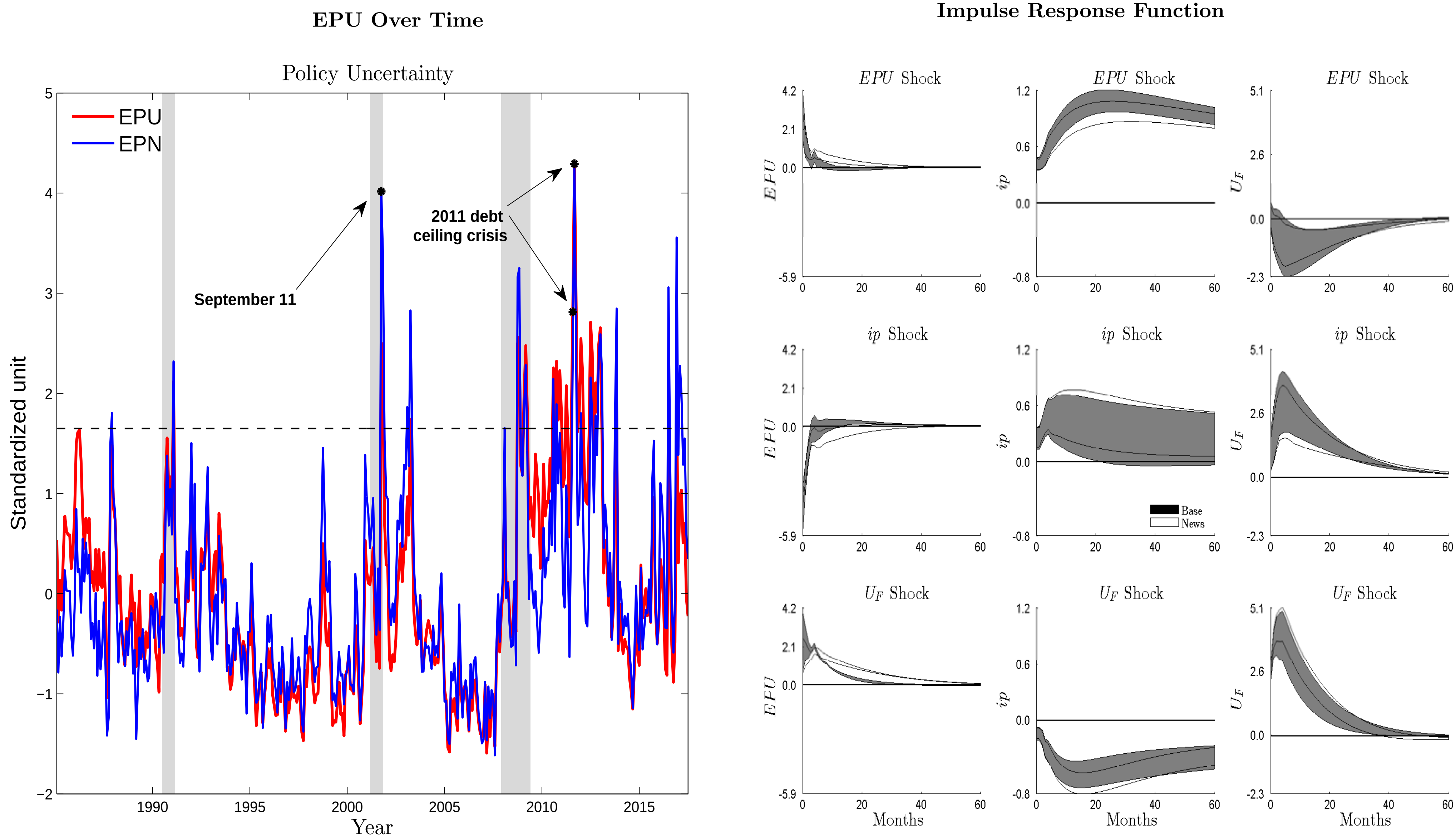
The figure exhibits all shocks in the identified set that are at least 2 standard deviations above the unconditional mean for e_M and e_F and at least 2 standard deviations below the mean for e_{ip}. The horizontal line corresponds to 3 standard deviations. The bounds are $\bar{\lambda}_1 = -0.05$, $\bar{\lambda}_2 = 2$, $\bar{\lambda}_3 = 0.18$, $\bar{k}_1 = 4$, $\bar{k}_2 = 4$, $\bar{k}_3 = 2$. The sample spans the period 1960:07 to 2015:04.

Figure 4: SVAR $(U_M, ip, U_F)'$



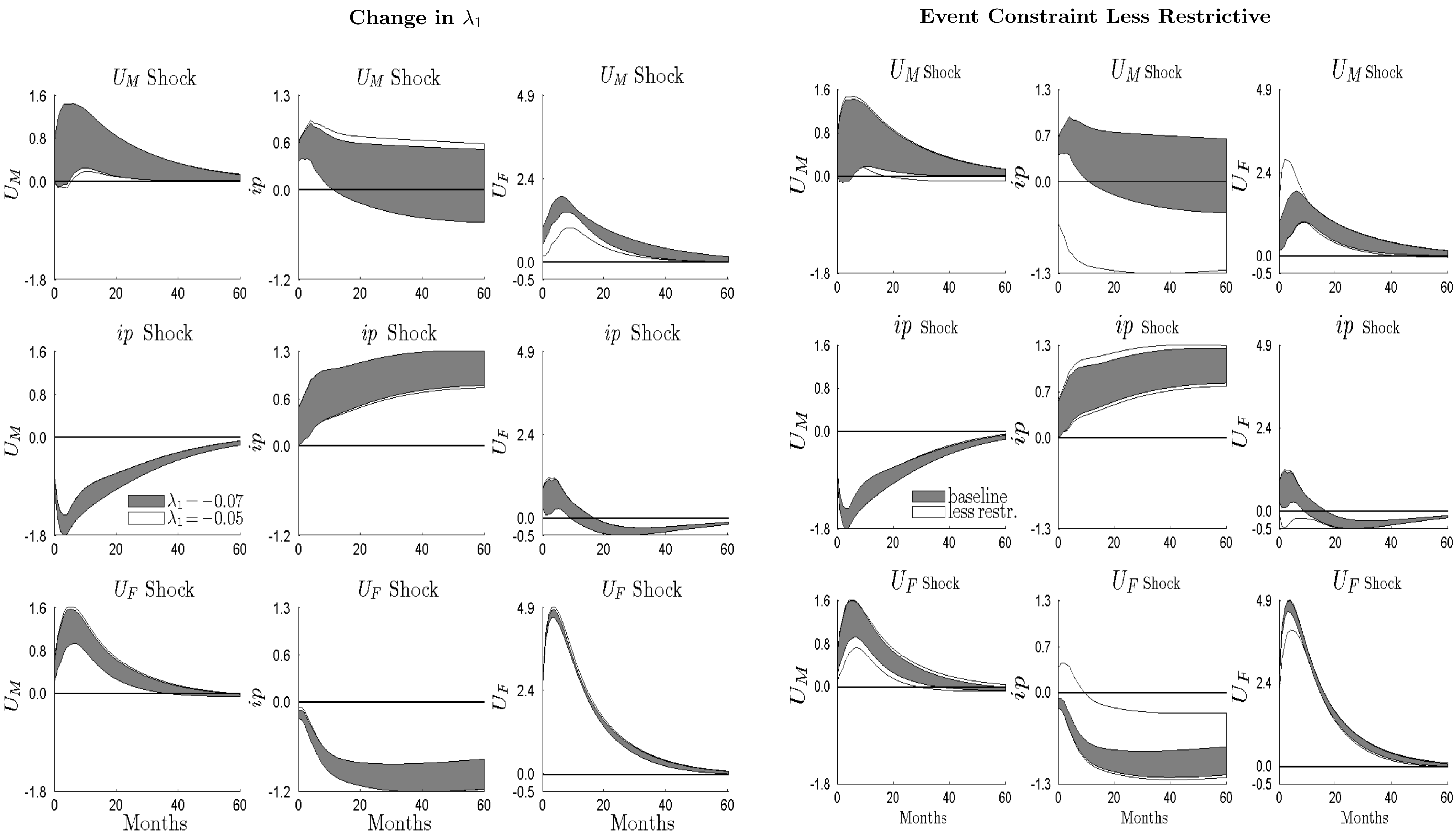
The figure reports the IRFs of SVAR $(U_M, ip, U_F)'$. The dashed line is the maxG solution. The shaded areas represent sets of solutions that satisfy the correlation and event constraints. Responses to positive one standard deviation shocks are reported in percentage points. The bounds are $\bar{\lambda}_1 = -0.05$, $\bar{\lambda}_2 = 2$, $\bar{\lambda}_3 = 0.18$, $\bar{k}_1 = 4$, $\bar{k}_2 = 4$, $\bar{k}_3 = 2$. The sample spans the period 1960:07 to 2015:04.

Figure 5: SVAR $(EPU, ip, U_F)'$



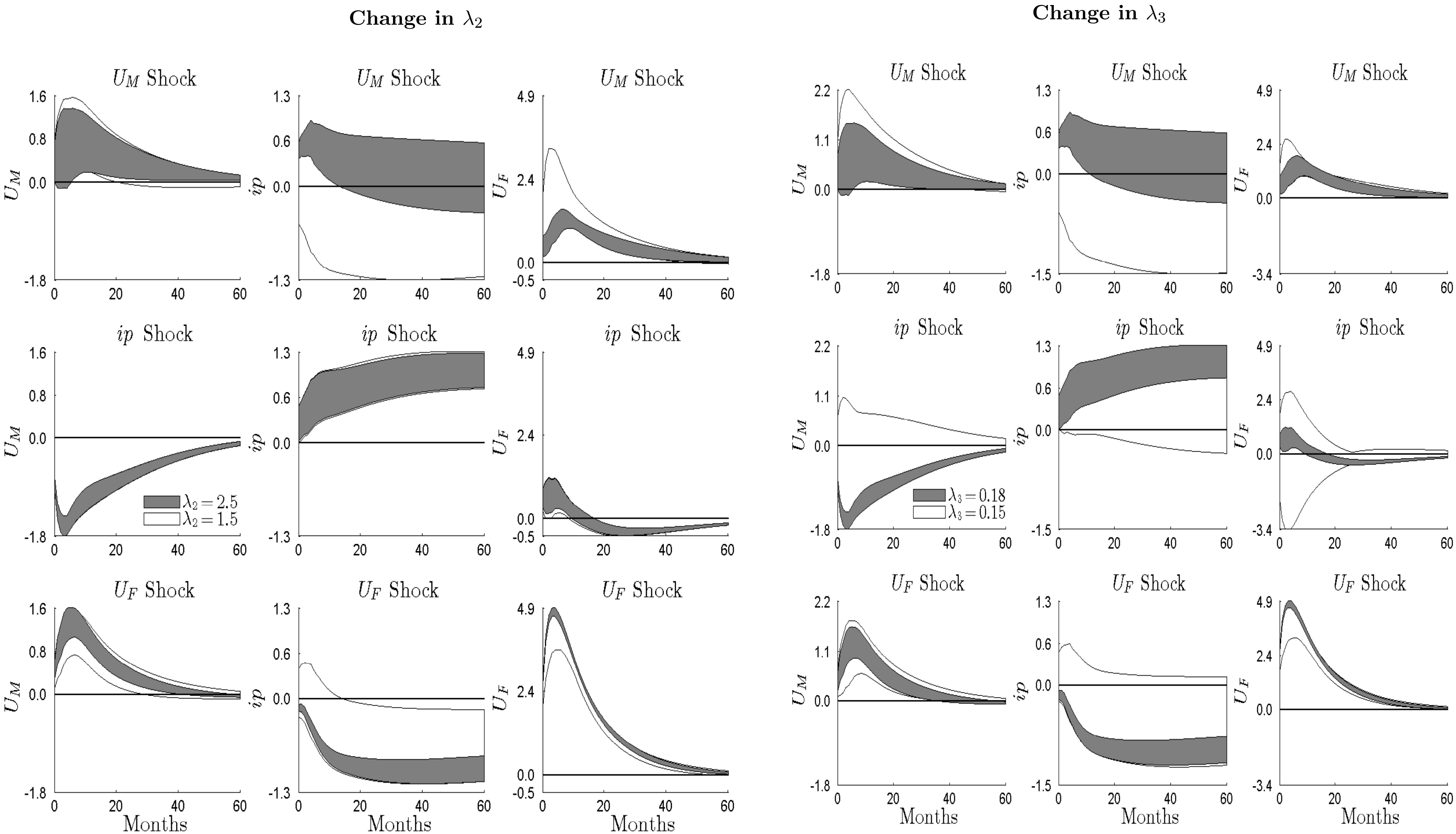
The left panel plots the time series of baseline policy uncertainty EPU and news-based EPN, expressed in standardized units. Shaded areas correspond to NBER recession dates. The horizontal line corresponds to 1.65 standard deviations above the unconditional mean. The right panel displays impulse responses to one standard deviation shocks. Response units are reported in percentage points. The bounds are $\bar{\lambda}_4 = 0.12$, $\bar{k}_3 = 2$, $\bar{k}_1 = \bar{k}_2 = 4$. Additional identifying restriction: for EPU, $e_{EPU,\, t_3} \geq \bar{k}_4 = 2$ for for all $t_3 \in \{2011{:}07, 2011{:}08\}$; for EPN, $e_{EPN,\, t_4} \geq \bar{k}_4 = 2$ for all $t_4 \in \{2001{:}09, 2011{:}07, 2011{:}08\}$. The sample spans the period 1987:01 to 2015:04.

Figure 6: IRFs of SVAR $(U_M, ip, U_F)'$ under Alternative Bounds



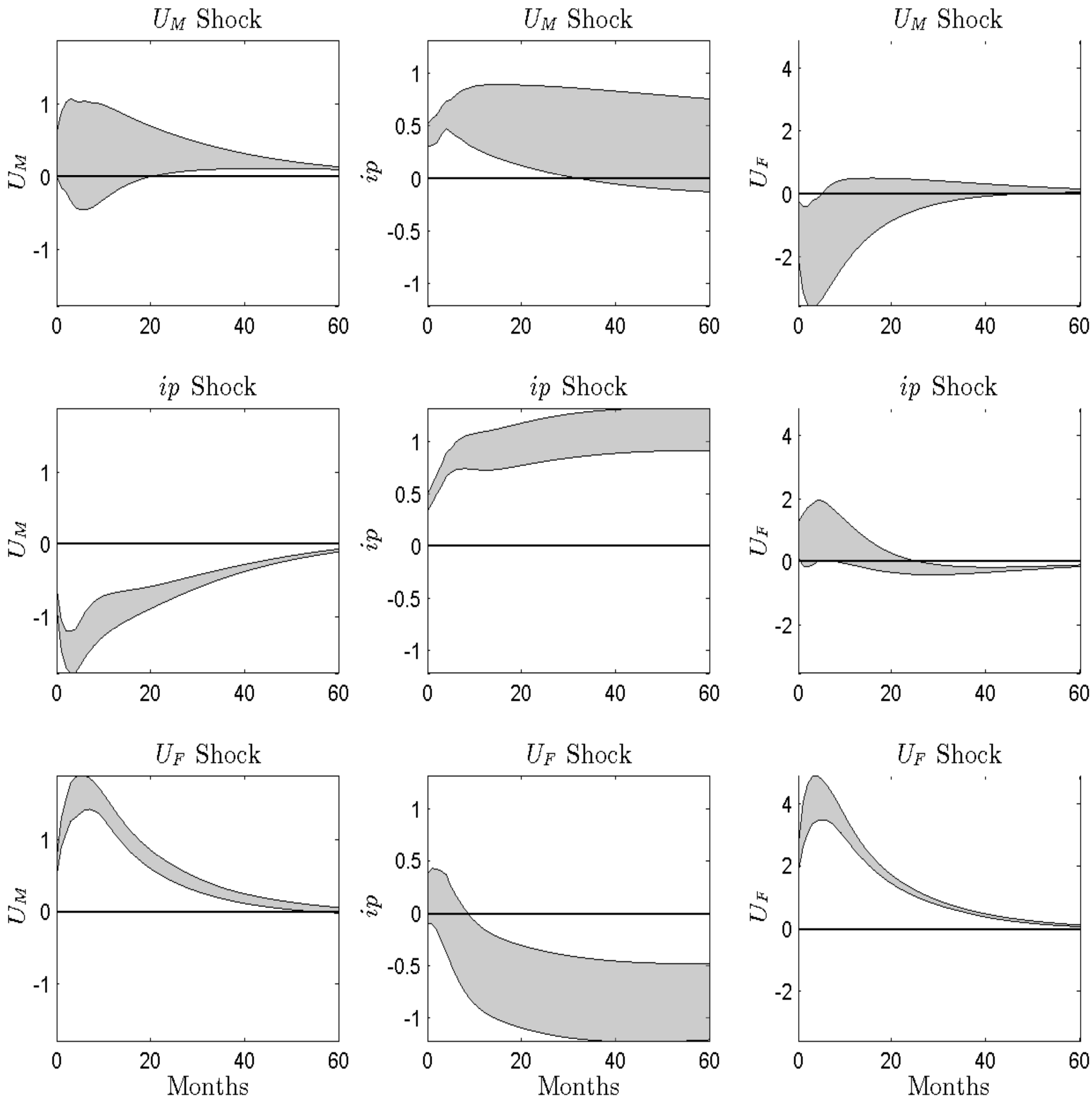
The left panel reports sets of solutions obtained when λ_1 is tightened to -0.07 from -0.05 with (λ_2, λ_3) and the event parameters $(\bar{k}_1, \bar{k}_2, \bar{k}_3)$ held fixed at their baseline values.The right panel reports sets of solutions obtained when the event parameters $(\bar{k}_1, \bar{k}_2, \bar{k}_3)$ are relaxed to $(2, 2, 2)$ from $(4, 4, 2)$ while λ_1, λ_2 and λ_3 are held fixed at their baseline values. The baseline bound values are $\bar{\lambda}_1 = -0.05$, $\bar{\lambda}_2 = 2$, $\bar{\lambda}_3 = 0.18$, $\bar{k}_1 = 4$, $\bar{k}_2 = 4$, $\bar{k}_3 = 2$. The sample spans the period 1960:07 to 2015:04.

Figure 7: IRFs of SVAR $(U_M, ip, U_F)'$ under Alternative Bounds



The left panel reports sets of solutions obtained when λ_2 is relaxed to 1.5 from 2.5, and (λ_1, λ_3) and the event parameters $(\bar{k}_1, \bar{k}_2, \bar{k}_3)$ are held fixed at their baseline values. The right panel reports sets of solutions obtained when λ_3 is varied from 0.18 to 0.15, and the parameters $(\bar{k}_1, \bar{k}_2, \bar{k}_3)$ and (λ_1, λ_2) are held fixed at their baseline values. The baseline bound values are $\bar{\lambda}_1 = -0.05$, $\bar{\lambda}_2 = 2$, $\bar{\lambda}_3 = 0.18$, $\bar{k}_1 = 4$, $\bar{k}_2 = 4$, $\bar{k}_3 = 2$. The sample spans the period 1960:07 to 2015:04.

Figure 8: IRFs of SVAR $(U_M, ip, U_F)'$, Using Gold as External Variable



The figure reports the IRFs of SVAR $(U_M, ip, U_F)'$. The shaded areas represent sets of solutions that satisfy the correlation and event constraints. Responses to positive one standard deviation shocks are reported in percentage points. The bounds are $\bar{\lambda}_1 = 0.07$, $\bar{k}_1 = 4$, $\bar{k}_2 = 4$, $\bar{k}_3 = 2$. The sample spans the period 1960:07 to 2015:04.

Table 1: Variance Decomposition

	SVAR $(U_M, ip, U_F)'$			SVAR $(EPU, ip, U_F)'$		
	Fraction variation in U_M			Fraction variation in EPU		
s	U_M Shock	ip Shock	U_F Shock	EPU Shock	ip Shock	U_F Shock
1	$[0.00, 0.44]$	$[0.48, 0.78]$	$[0.06, 0.36]$	$[0.08, 0.39]$	$[0.45, 0.87]$	$[0.04, 0.37]$
12	$[0.00, 0.46]$	$[0.36, 0.62]$	$[0.15, 0.51]$	$[0.07, 0.29]$	$[0.23, 0.59]$	$[0.33, 0.66]$
∞	$[0.01, 0.50]$	$[0.35, 0.67]$	$[0.12, 0.45]$	$[0.06, 0.26]$	$[0.19, 0.51]$	$[0.42, 0.72]$
$s_{\max}$	$[0.01, 0.50]$	$[0.48, 0.81]$	$[0.15, 0.51]$	$[0.09, 0.42]$	$[0.48, 0.88]$	$[0.42, 0.72]$
	Fraction variation in ip			Fraction variation in ip		
s	U_M Shock	ip Shock	U_F Shock	EPU Shock	ip Shock	U_F Shock
1	$[0.33, 0.96]$	$[0.00, 0.64]$	$[0.01, 0.11]$	$[0.46, 0.79]$	$[0.09, 0.47]$	$[0.03, 0.14]$
12	$[0.07, 0.59]$	$[0.07, 0.70]$	$[0.22, 0.44]$	$[0.32, 0.61]$	$[0.07, 0.45]$	$[0.15, 0.33]$
∞	$[0.02, 0.21]$	$[0.23, 0.69]$	$[0.27, 0.60]$	$[0.50, 0.75]$	$[0.01, 0.29]$	$[0.11, 0.26]$
$s_{\max}$	$[0.34, 0.96]$	$[0.23, 0.73]$	$[0.30, 0.61]$	$[0.50, 0.80]$	$[0.14, 0.57]$	$[0.17, 0.34]$
	Fraction variation in U_F			Fraction variation in U_F		
s	U_M Shock	ip Shock	U_F Shock	EPU Shock	ip Shock	U_F Shock
1	$[0.00, 0.10]$	$[0.00, 0.08]$	$[0.86, 0.98]$	$[0.01, 0.04]$	$[0.02, 0.29]$	$[0.69, 0.98]$
12	$[0.03, 0.15]$	$[0.00, 0.04]$	$[0.83, 0.96]$	$[0.01, 0.13]$	$[0.08, 0.46]$	$[0.48, 0.88]$
∞	$[0.05, 0.17]$	$[0.01, 0.05]$	$[0.81, 0.92]$	$[0.01, 0.17]$	$[0.11, 0.50]$	$[0.41, 0.82]$
$s_{\max}$	$[0.05, 0.17]$	$[0.02, 0.10]$	$[0.87, 0.99]$	$[0.02, 0.17]$	$[0.11, 0.50]$	$[0.72, 0.98]$

Each panel shows the fraction of s-step-ahead forecast-error variance of the variable given in the panel title that is explained by the shock named in the column heading. The row denoted "$s = s_{max}$" reports the maximum fraction of forecast error variance explained across all VAR forecast horizons s. The numbers in brackets represent the ranges for these numbers across all solutions in the identified set. The data are monthly and span the period 1960:07 to 2015:04.

Online Appendix

System Estimation

The estimation procedure used in the text is based on an SVAR for $\mathbf{X}_t$. While S_t plays a role in identification, it is excluded from the SVAR. We refer to the foregoing analysis as the *subsystem approach*. However, it is also possible to apply the event and correlation constraints to a larger VAR in $(\mathbf{X}_t, S_t)'$. We refer to this as the *full system approach.*

The full system VAR takes the same form as (3); the only difference is that S_t is now included in the VAR. The reduced form errors for the full system are $\boldsymbol{\eta}_t = (\eta'_{Xt}, \eta_{St})'$. The structural shocks are $(\mathbf{e}'_{Xt} \quad e_{St})'$ with $\boldsymbol{\eta}_t = \mathbf{B}\mathbf{e}_t$. The stock market shocks e_{St} are idiosyncratic to the stock market, something akin to a pure sentiment shock, in the sense that they are orthogonal to real activity, macro and financial uncertainty. The $\mathbf{B}$ matrix now has 16 parameters and the covariance structure gives 10 pieces of information. Because the larger system requires a greater number of parameters to be identified, additional identifying restrictions will be required to achieve a narrowing of the identified set that is comparable to the smaller system with fewer parameters. We assume that the idiosyncratic stock market shocks e_{St} do not *contemporaneously* affect $\mathbf{X}_t$, but allow them to do so with a lag. This means that the impact sub-vector giving the effects of e_{St} on $\mathbf{X}_t$, denoted $\mathbf{B}_{XS} = (B_{MS}, B_{YS}, B_{FS})'$, is zero. These three zero restrictions imply

$$\begin{pmatrix} \eta_{Mt} \\ \eta_{Yt} \\ \eta_{Ft} \\ \eta_{St} \end{pmatrix} = \begin{pmatrix} B_{MM} & B_{MY} & B_{MF} & 0 \\ B_{YM} & B_{YY} & B_{YF} & 0 \\ B_{FM} & B_{FY} & B_{FF} & 0 \\ B_{SM} & B_{SY} & B_{SF} & B_{SS} \end{pmatrix} \begin{pmatrix} e_{Mt} \\ e_{Yt} \\ e_{Ft} \\ e_{St} \end{pmatrix}. \tag{A1}$$

As in the case with the subsystem analysis, the model is still underidentified. We again use the event and correlation constraints to narrow the set of plausible parameters. Let $c_j = \text{corr}(\eta_{St}, e_{jt})$ be the sample correlation between η_{St}, and the shock in $\mathbf{e}_t = (e_{Mt}, e_{Yt}, e_{Ft})$ with label j. With this definition of c_j, the correlation constraints are again,

- $c_M \leq \lambda_1$ and $c_F \leq \lambda_1$
- $|c_F| - \lambda_2 |c_M| \geq 0, \lambda_2 \geq 1$
- For $c = (c_M, c_F)'$, $\sqrt{c'c} \geq \lambda_3$

As in the subsystem analysis, the correlations c are not invariant to orthonormal rotation of $\mathbf{e}_X$ and the parameters of the subsystem. The event constraints remains the same as in the main text.

It is of interest to compare the full and subsystem analyses. In the subsystem analysis, the process that generates S_t is left unspecified. As such, it can be a function of variables other than $\mathbf{X}_t$, both contemporaneously, and at lags. By contrast, the full system approach specifies the process for S_t. Any misspecification in one equation can affect all equations in the system. On the other hand, the full system merely constrains the contemporaneous effect of S_t on $\mathbf{X}_t$ to zero. This is a weaker than assuming that S_t is exogenous for $\mathbf{X}_t$, which additionally prevents the lags of S_t from affecting $\mathbf{X}_t$. Constraining the current and lagged values of S_t to zero amounts to the subsystem analysis of excluding S_t from the larger VAR altogether. It should however be noted that excluding the past values of S_t from the equations for $\mathbf{X}_t$ is not needed for the system analysis. Thus we can evaluate whether it is reasonable to exclude S_t from the VAR by comparing the impulse response functions estimated for the three variable system $\mathbf{X}_t = (U_{Mt}, Y_t, U_{Ft})'$ with those from a larger system that includes S_t but does not restrict the coefficients of S_{t-j} in the equations for $\mathbf{X}_t$ to zero, for $j \geq 1$.

We estimate a four variable system in $(\mathbf{X}_t, S_t)'$ where S_t is measured as the return on the CRSP value-weighted stock market index return. The bounds for the event and correlation constraints are the same as for the subsystem analysis for the same variables. We compare the impulse response functions estimated for the three variable subsystem for $\mathbf{X}_t = (U_{Mt}, Y_t, U_{Ft})'$, with those from the larger system that includes S_t but does not restrict the coefficients of S_{t-j} in the equations for $\mathbf{X}_t$ to zero, for $j \geq 1$. Figure A3 presents the sets of identified impulse responses that satisfy the constraints in each case, overlaid on one another. The identified sets lie almost on top of each other, indicating that the responses are little different. Indeed, the coefficients on lags of S_t appear to be close to zero in all three $\mathbf{X}_t$ equations. The data thus appear qualitatively consistent with the assumption that stock returns do not appreciatively affect the dynamics of $\mathbf{X}_t$ even as they serve as a valuable source of identifying information.

Sampling Simulation

In point-identified models, sampling uncertainty can be evaluated using frequentist confidence intervals or Bayesian credible regions, and they coincide asymptotically. Inference for set-identified SVARs is, however, more challenging because no consistent point estimate is available. As pointed out in Moon and Schorfheide (2012), the credible regions of Bayesian identified impulses responses will be distinctly different from the frequentist confidence sets, with the implication that Bayesian error bands cannot be interpreted as approximate frequentist error bands. Our analysis is frequentist, and while the two applications presented above illustrate how the dynamic responses vary across estimated models, where each model is evaluated at a solution in $\bar{\mathcal{B}}(\mathbf{B}; \bar{\mathbf{k}}, \bar{\boldsymbol{\tau}}, \bar{\lambda}, \mathbf{S}) \equiv \bar{\mathcal{B}}(\cdot)$, we still need a way to assess the robustness of our procedure, especially since it is new to the literature.

Unfortunately, few methods are available to evaluate the sampling uncertainty of partially identified SVARs from a frequentist perspective, and these tend to be specific to the imposition of particular identifying restrictions. Moon, Schorfheide, and Granziera (2013) suggest a projections based method within a moment-inequality setup, but it is designed to study SVARs that only impose restrictions on one set of impulse response functions. Furthermore, the method is computationally intense, requiring a simulation of critical value for each rotation matrix. Gafarov, Meier, and Olea (2015) suggest to collect parameters of the reduced form model in a $1-\alpha$ Wald ellipsoid but the approach is conservative. For the method to get an exact coverage of $1-\alpha$, the radius of the Wald-ellipsoid needs to be carefully calibrated. As discussed in Kilian and Lutkepohl (2016), even with these adjustments, existing frequentist confidence sets for set-identified models still tend to be too wide to be informative. It is fair to say that there exists no generally agreed upon method for conducting inference in set-identified SVARs. While we do not have a fully satisfactory solution to offer, our restrictions can further tighten the identified set, and by implication the confidence sets. We now explore this possibility in simulations.

We use a bootstrap/Monte Carlo experiment to assess the robustness of our inequality restrictions when S_t is a variable external to the three variable SVAR. In the simulation exercise, we simulate from a data generating process (DGP) calibrated to one particular solution in our identified set, the "maxG" solution that has the highest value for $\sqrt{\bar{g}(\mathbf{B})'\bar{g}(\mathbf{B})}$.The maxG solution is defined:

$$\mathbf{B}^{\max G}\equiv\arg\max_{\mathbf{B}\in\overline{\mathcal{B}}}\sqrt{\bar{g}(\mathbf{B})'\bar{g}(\mathbf{B})}, \quad \text{where} \quad \bar{g}(\mathbf{B};\bar{\mathbf{k}},\bar{\boldsymbol{\tau}},\bar{\boldsymbol{\lambda}},\mathbf{S})=\begin{pmatrix}\bar{g}_Z(\mathbf{B})' \\ \bar{g}_E(\mathbf{B};\bar{\boldsymbol{\tau}},\bar{\mathbf{k}})' \\ \bar{g}_C(\mathbf{B};\bar{\boldsymbol{\lambda}})'\end{pmatrix}'.$$

This is the solution at which the value of the inequalities are jointly maximized.

Let R be the number of replications in the repeated sampling experiment. To generate samples of the structural shocks from this solution in a way that ensures the events that appear in historical data also occur in our simulated samples, we draw randomly with replacement from the sample estimates of the shocks $\mathbf{e}_t^{\mathrm{maxG}}$ for the "maxG" solution, with the exception that we fix the values for these shocks in each replication in the periods $\overline{\tau}_1$ and $\overline{\tau}_2$ to be the observed ones, where $\overline{\tau}_1$ is the period 1987:10 of the stock market crash and $\overline{\tau}_2 \in$ [2007:12, 2009:06]. Each draw of the shocks $\mathbf{e}_t^{\mathrm{maxG}}$ is combined with the maxG estimates of the parameters in $\mathbf{A}(L)=\mathbf{I}-\mathbf{A}_1L-\cdots-\mathbf{A}_pL^p$ and $\mathbf{B}$ to generate $R=1,000$ samples of size T of $\boldsymbol{\eta}_t=\mathbf{B}\mathbf{e}_t^{\mathrm{maxG}}$ and $\mathbf{X}_t$ using the SVAR $\mathbf{X}_t=\sum_{j=1}^{p}\mathbf{A}_j\mathbf{X}_{t-j}+\mathbf{B}\mathbf{e}_t^{\mathrm{maxG}}$. From these samples of $\mathbf{X}_t$ regressed on p lags of itself we generate new sets of $\mathbf{B}^r=\mathbf{P}^r\mathbf{Q}$ and the dynamic responses to shock j summarized by the IRF:

$$\frac{\partial \mathbf{X}_{t+s}}{\partial e_{jt}}=\boldsymbol{\Psi}_s^r\mathbf{b}^{rj}, \tag{A2}$$

where $\mathbf{b}^{rj}$ is the jth column of $\mathbf{B}^r$ and the coefficient matrixes $\boldsymbol{\Psi}_s^r$ are given by $\boldsymbol{\Psi}^r(L)=\boldsymbol{\Psi}_0^r+\boldsymbol{\Psi}_1^rL+\boldsymbol{\Psi}_2^rL^2+\ldots=\mathbf{A}^r(L)^{-1}$.

To generate samples of S from this solution in a way that ensures that the correlations with the uncertainty shocks that appear in our historical data also appear in our simulated samples, we generate idiosyncratic stock market shocks e_{St} from

$$e_{St} = \underbrace{S_t - \hat{\rho}_s S_{t-1}}_{\hat{u}_t} - d_F e_{Ft}^{\max G} - d_Y e_{Yt}^{\max G} - d_M e_{Mt}^{\max G}, \tag{A3}$$

where $\hat{\rho}_s$ is the estimated first-order autocorrelation coefficient for S_t from historical data, and the $d = (d_F, d_Y, d_M)'$ parameters in (A3) are calibrated to target the observed correlations $c_M = -0.07$, and $c_F = -0.18$ for the maxG solution in historical data. Given the d parameters, observations on $\mathbf{e}_t^{\max G}$ and $\hat{u}_t$, we observe e_{St} on the left-hand-side of (A3). We generate 1,000 samples of S_t by drawing with replacement from these e_{St}, in the same manner described above for $\mathbf{e}_t^{\max G}$ and recursively iterating on (A3) using the first observations on S_1 in our historical sample as initial values.[18] Note that the auto-regressive residual for S_t, upon which our correlation constraints are based, is a function of the four shocks: $S_t - \hat{\rho}_s S_{t-1} = e_{St} + d_F e_{Ft}^{\max G} + d_M e_{Mt}^{\max G} + d_Y e_{Yt}^{\max G}$.

For each of these $R = 1,000$ replications, we construct an identified set of solutions $\bar{\mathcal{B}}(\cdot)$. In each replication r, $K = 1.5$ million possible solutions for $\mathbf{B}$ are generated by initializing $\mathbf{B}$ to be the lower Cholesky factorization of $\mathbf{\Omega}$ for an arbitrary ordering of the variables. These are then rotated by 1.5 million random orthogonal matrices $\mathbf{Q}$. We consider two approaches to constructing confidence sets.

The first approach constructs a confidence set for a set of solutions in repeated samples. In each replication, $K = 1.5$ million rotation matrices are entertained, but only $K_r \leq K$ rotations of $\mathbf{Q}$ will generate solutions that are admitted into the constrained set for that replication, $\bar{\mathcal{B}}^r(\phi, F)$. Let $\Theta_{i,j,s}^{r,k}$ be the s-period ahead response of the ith variable to a standard deviation change in shock j at the k-th rotation of replication r. Let $\underline{\Theta}_{i,j,s}^r = \min_{k \in [1,K_r]} \Theta_{i,j,s}^{r,k}$ and $\overline{\Theta}_{i,j,s}^r = \max_{k \in [1,K_r]} \Theta_{i,j,s}^{r,k}$. Each $(\underline{\Theta}_{i,j,s}^r, \overline{\Theta}_{i,j,s}^r)$ pair represents the extreme (highest and lowest) dynamic responses in replication r. From the quantiles of $\underline{\Theta}_{i,j,s}^r$, we can obtain the $\alpha/2$ critical point $\underline{\Theta}_{i,j,s}(\alpha/2)$. Similarly, from the quantiles of $\overline{\Theta}_{i,j,s}^r$, we have the $1 - \alpha/2$ critical point $\overline{\Theta}_{i,j,s}(1-\alpha/2)$. Eliminating the lowest and highest $\alpha/2$ percent of the samples gives a $(1-\alpha)\%$ percentile-based confidence interval defined by

$$CI_{\alpha,g} = \left[\underline{\Theta}_{i,j,s}(\alpha/2), \;\; \overline{\Theta}_{i,j,s}(1-\alpha/2)\right].$$

$CI_{\alpha,g}$ denotes the confidence intervals for sets of solutions that satisfy all constraints, including the event and correlation constraints: $\bar{g}_Z(\mathbf{B}) = 0$, $\bar{g}_E(\mathbf{B}; \bar{\tau}, \bar{\mathbf{k}}) \geq 0$, $\bar{g}_C(\mathbf{B}; \mathbf{S}, \bar{\boldsymbol{\lambda}}) \geq 0$. We use CI_α to denote the confidence intervals for sets of solutions that satisfy only the reduced form covariance restrictions $\bar{g}_Z(\mathbf{B}) = 0$.

The second approach constructs a confidence set for a particular solution in repeated samples. We consider the "maxG" solution. For replication r with K_r solutions in $\bar{\mathcal{B}}^r(\cdot)$, the

[18] S_t is the return on the CRSP value-weighted index.

"maxG" solution is defined in (1). Let $\breve{\Theta}^r_{i,j,s}$ be the dynamic response of variable i to shock j at horizon s associated with the "maxG" solution. Note that the same "maxG" solution is used to evaluate the dynamic responses at all (i, j, s). The critical points associated with the quantiles of $\breve{\Theta}^r_{i,j,s}$ define the $(1-\alpha)\%$ confidence interval

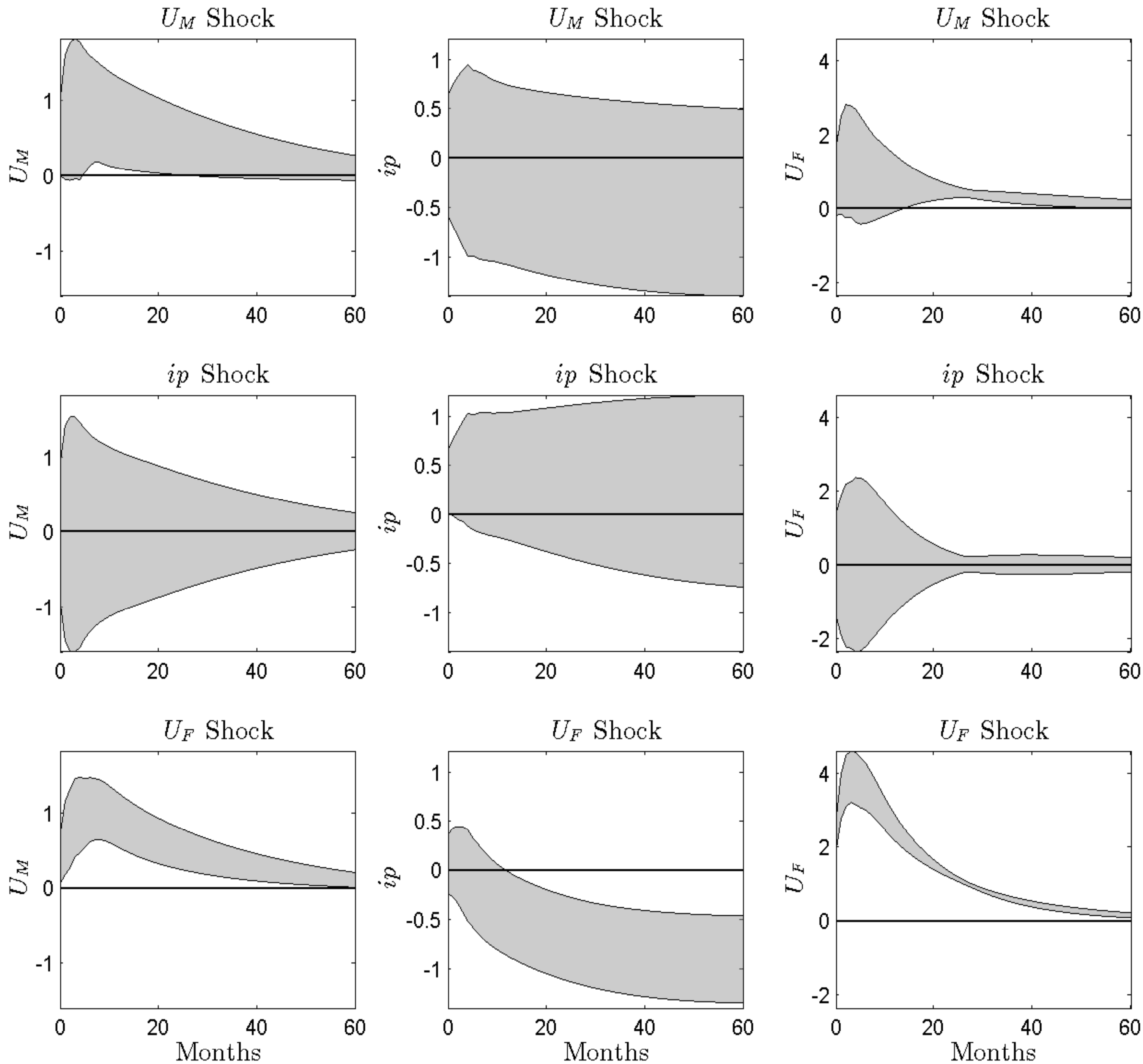
$$CI^{\text{maxG}}_{\alpha,g} = \left[\breve{\Theta}_{i,j,s}(\alpha/2), \;\; \breve{\Theta}_{i,j,s}(1-\alpha/2)\right].$$

Since the $CI_{\alpha,g}$ interval is formed from the tails of the distribution of solutions, it is conservative and can be expected to be wider than $CI^{\text{maxG}}_{\alpha,g}$.

The confidence intervals CI_α, $CI_{\alpha,g}$, $CI^{\text{maxG}}_{\alpha,g}$ for the IRFs are reported in Figure A4. The results show that the confidence intervals $CI_{\alpha,g}$, $CI^{\text{maxG}}_{\alpha,g}$ formed from estimations that impose the event and correlation constraints are noticeably narrower than CI_α formed from estimations that impose only covariance restrictions.

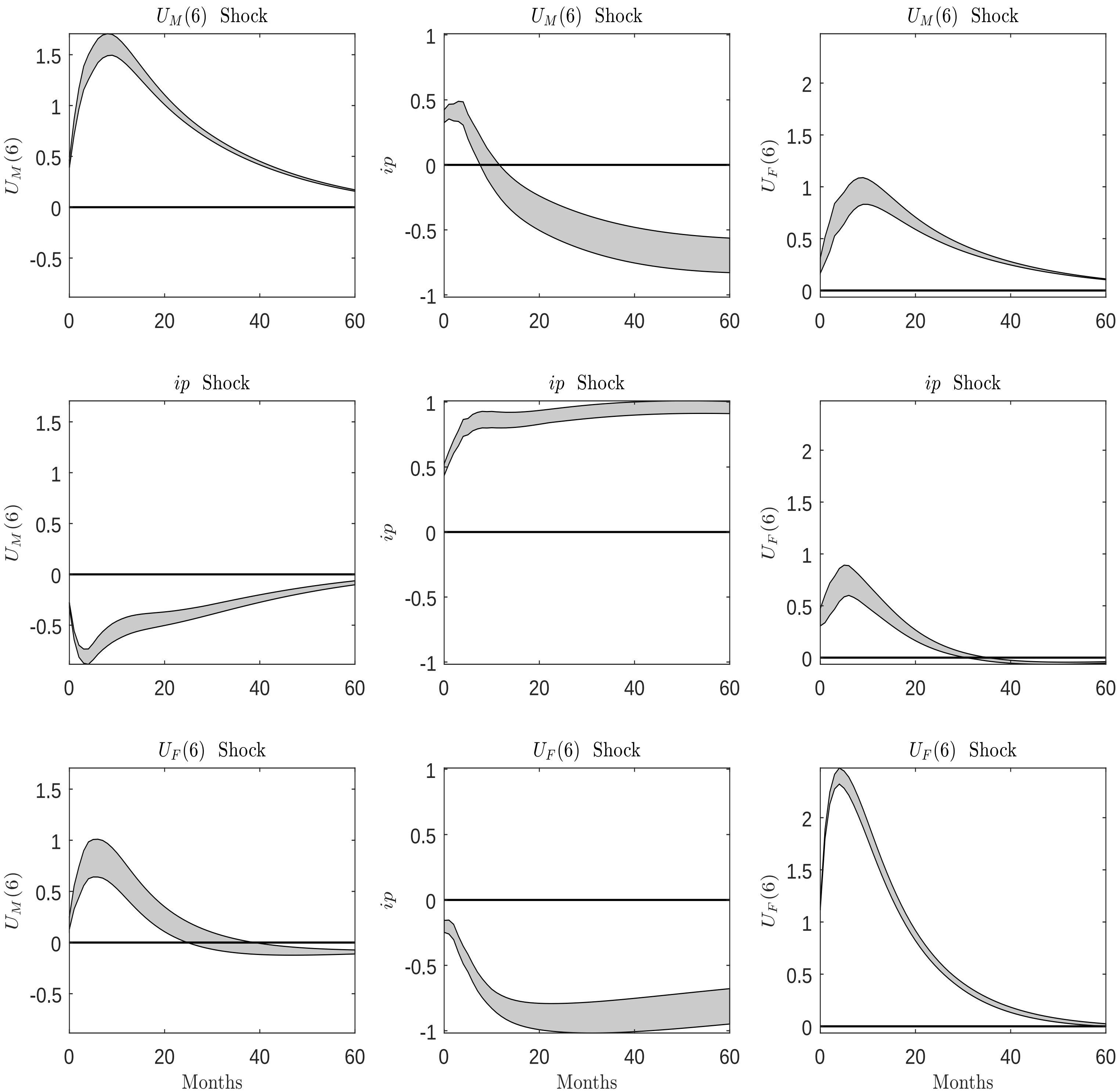
Appendix Tables and Figures

Figure A1: IRFs of SVAR $(U_M, ip, U_F)'$, Pre-crisis Sample



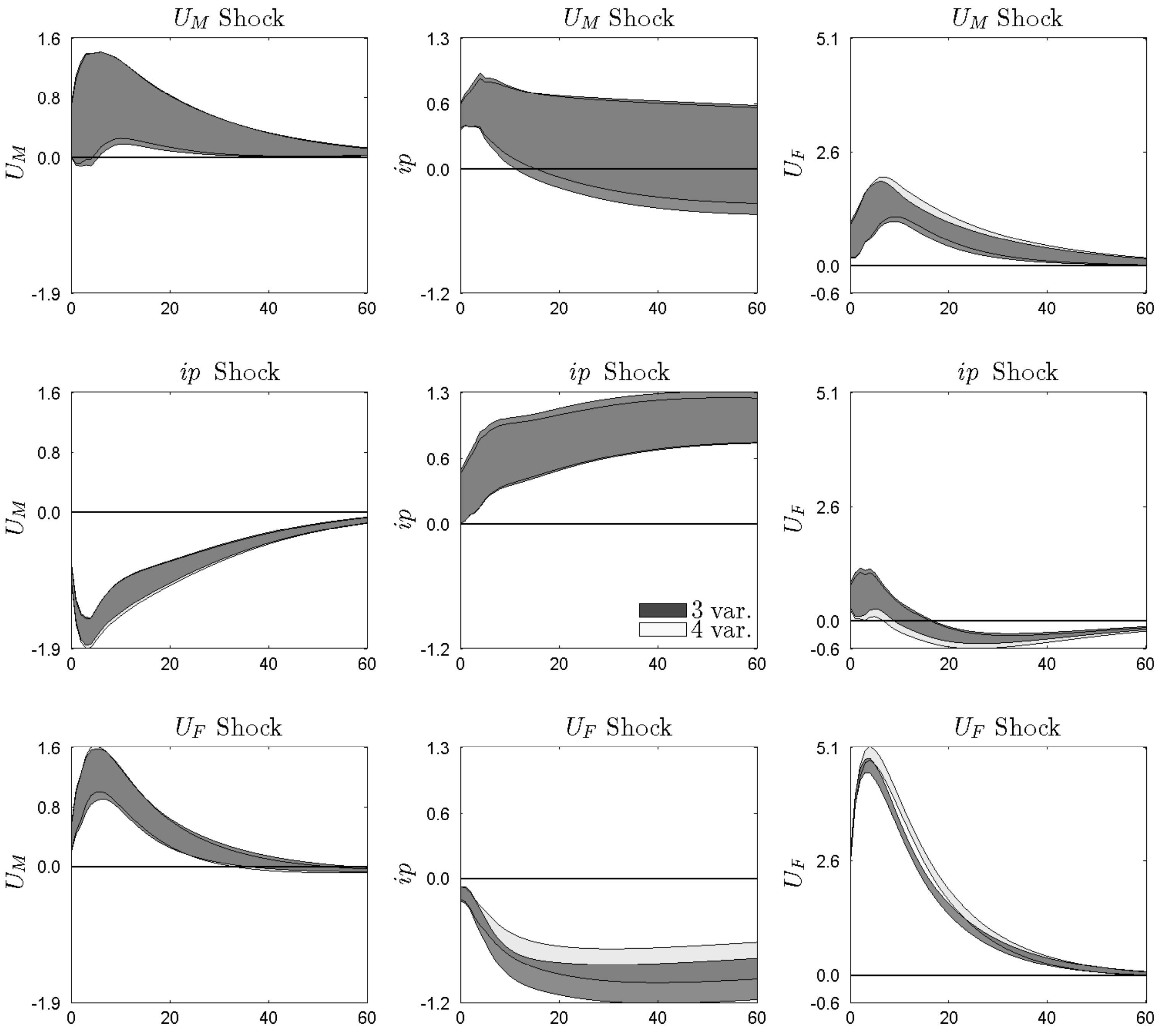
The figure displays impulse responses to one standard deviation shocks. Response units are reported in percentage points. The estimation eliminates the financial crisis/Great Recession as an identifying restriction. The constraint is $\bar{\lambda}_1 = -0.05$, $\bar{\lambda}_2 = 2$, $\bar{\lambda}_3 = 0.17$, $\bar{k}_1 = 4$. The sample spans the period 1960:07 to 2007:11

Figure A2: IRFs of SVAR $(U_M, ip, U_F)'$, 6 Month Uncertainty



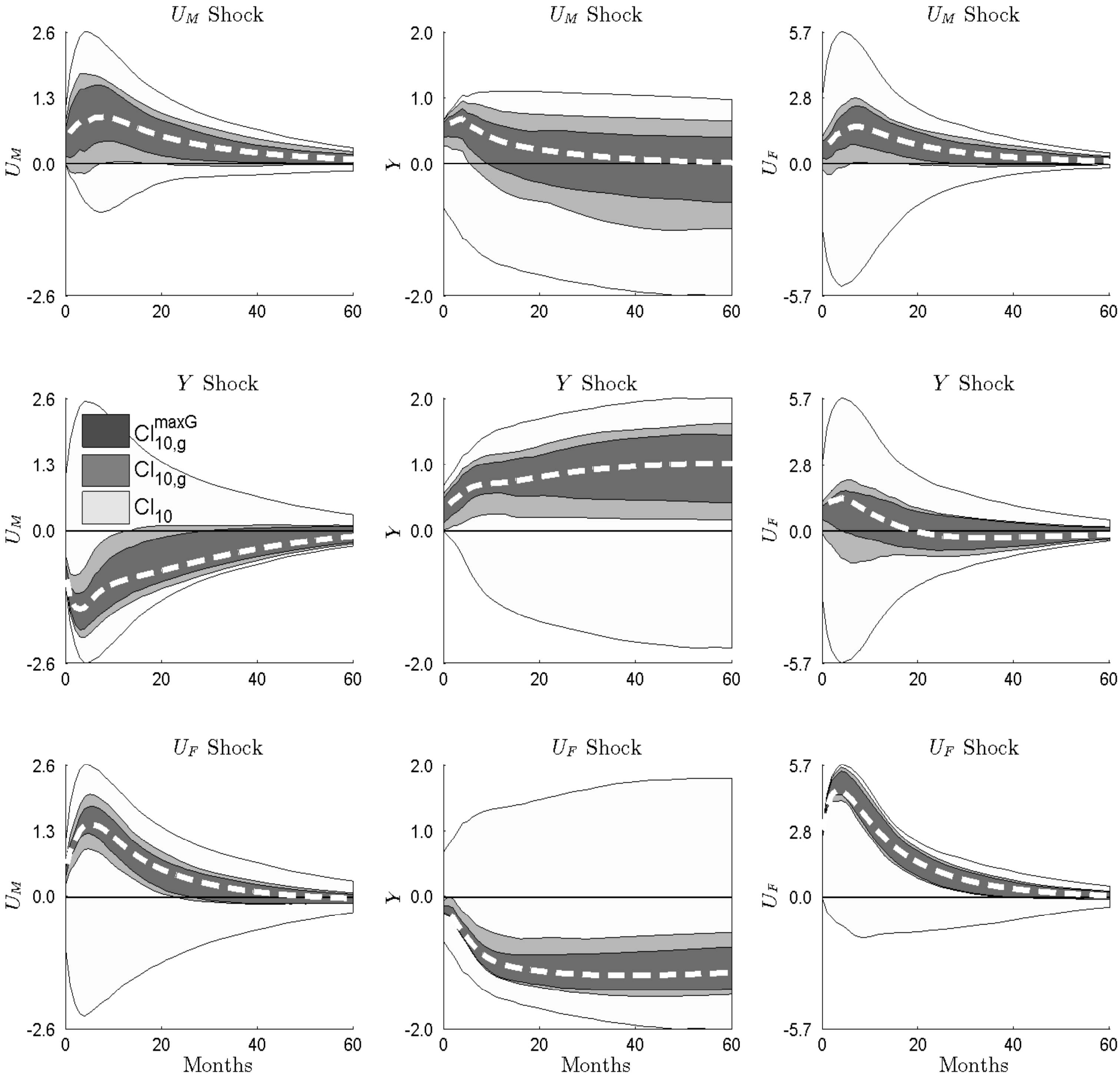
The figure displays impulse responses to one standard deviation shocks. Uncertainty indexes are for 6 months ahead. Response units are reported in percentage points. The solutions are obtained when the bound values are $\bar{\lambda}_1 = -0.05$, $\bar{\lambda}_2 = 2$, $\bar{\lambda}_3 = 0.15$, $\bar{k}_1 = 3$, $\bar{k}_2 = 3$, $\bar{k}_3 = 2$. The sample spans the period 1960:07 to 2015:04

Figure A3: SVAR $(U_M, Y, U_F, S)'$



The figure displays impulse responses to one standard deviation shocks. Response units are reported in percentage points. The constraint is $\bar{\lambda}_1 = -0.05$, $\bar{\lambda}_2 = 2$, $\bar{\lambda}_3 = 0.18$, $\bar{k}_1 = 4$, $\bar{k}_2 = 4$, $\bar{k}_3 = 2$. The sample spans the period 1960:07 to 2015:04.

Figure A4: Monte Carlo Simulation



The shaded area reports the 90 percent confidence (CI) interval across 1000 replications. Dotted line is the historical MaxG IRF. $CI^{\text{maxG}}_{10,g}$ is the CI with maxG solution. $CI_{10,g}$ is the CI with all constraints imposed. CI_{10} is the CI with only reduced form covariance restrictions imposed. The sample size is $T = 652$ and 1.5 millions random rotations are used for each replication.