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CURRENCY INCONTROVERTIBILITY,  
TRADE TAXES AND SMUGGLING

Jorge Braga de Macedo

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This paper is dedicated to the memory of Carlos Diaz-Alejandro, who encouraged me to explore the consequences of currency inconvertibility in my dissertation. Ten years after, the exploration continues. Earlier versions were presented at seminars at USAID/Khartoum, the University of Chicago, New York University, the New University of Lisbon, the University of Coimbra and the University of Maryland, College Park. Comments from participants and an anonymous referee are gratefully acknowledged. The research reported here is part of the NBER's research program in International Studies. Any opinions expressed are those of the author and not those of the National Bureau of Economic Research.

Currency Inconvertibility, Trade Taxes and Smuggling

ABSTRACT

In the classic analysis of smuggling importers choose the optimal mix of legal and illegal trade, given trade taxes and the technology of detection. This paper introduces an inconvertible currency in the framework, so that illegal trade is valued at a rate higher than the (fixed) official exchange rate. Sections 1 and 2 show how the smuggling ratio and the domestic price markup for the import and export good are simultaneously determined.

With balanced legal and illegal trade, changes in the (long-run) black market premium are a weighted average of changes in trade taxes, whereas changes in the smuggling ratios depend on the ratio of trade taxes. Thus, an import tariff and an export subsidy rising at the same rate would keep smuggling ratios constant but imply a rising black market premium (section 3 and 4).

To determine the quantity of exports and imports, a model of the economy is presented in section 5, featuring the production of exports and non-traded goods and the consumption of imports and non-traded goods, as well as a government confiscating the amounts of traded goods unsuccessfully smuggled. Then export production may fall, and welfare may rise, if trade taxes have a negative effect on the relative price of exports and imports stronger than the positive effect on smuggled exports and imports, which is always welfare-reducing.

Section 6 introduces the short-run determination of the black market premium via portfolio balance. In this case, rising trade taxes may be associated with a premium rising even faster if there is unreported capital flight and conversely.

Jorge Braga de Macedo  
UNL - New University of Lisbon  
Campo Grande, 185  
1700 Lisbon, Portugal

## Introduction

There has been a revival of interest in the consequences of currency inconvertibility and the existence of black markets for foreign exchange. Nevertheless, these widespread phenomena have been analyzed in isolation. In particular, their relationship with trade taxes and smuggling has been largely ignored in recent contributions. The model of Pitt (1984) hides the special risks involved in smuggling behind an arbitrary "smuggling function". This is recognized in the "crime-theoretic" smuggling model of Martin and Panagariya (1984) but it does not feature a black market for foreign exchange. This paper addresses the relationship between these two channels of illegal activity in a framework inspired by the classic analysis of Beccaria (1764).

Beccaria looked at a case where the probability of detection in import smuggling was exogenously given and he showed that importers would be indifferent between smuggling and legal trade when the tariff factor (one plus the tariff rate) equals the inverse of the probability of success. If this were one, for example, any tariff would induce smuggling. How much of the tariff revenue should be devoted to increased supervision, so as to lower the probability of success, was in fact Beccaria's main concern.

Here we focus on the choice of the importer or exporter, but it is useful to generalize slightly Beccaria's analysis as a motivation. Suppose the smuggled imports have to be paid for with black market foreign exchange (which for simplicity can be acquired at no risk), then a tariff lower than the premium of the black market over the official market would not induce smuggling even if the probability of success were one. The import tariff is

now the upper bound for the black market premium and the indifference point for the importer is defined by the equality of the tariff factor and the black market premium over the probability of success. Conversely, if the black market rate is lower than the official rate times the export tax factor, then it will not pay to smuggle exports even if the probability of success were one. The export tax is the lower bound of the black market premium.

Importers and exporters will therefore be indifferent between the two channels when the ratio of trade taxes times the product of the probability of success in import and export smuggling equals one. This may be thought of as a simple generalization from Beccaria's formula, based on a given probability of success in smuggling.

If the probability of success is endogenously determined, importers and exporters will take this into account in choosing their mix of smuggling and legal trade. Take the case where success is a negative function of the ratio of smuggling to legal trade. Then equating marginal revenue to marginal cost in both activities will determine the equilibrium price and smuggling ratio for importers and exporters, given trade taxes and the black market premium. Indeed, the ratio of domestic to free-trade prices will be a weighted average of trade taxes and the premium. The import price markup will lie between the tariff factor and the premium, and the export price markup will lie between the premium and the export tax.

The determination of the long-run black market premium follows from the identification of flow supply of and demand for black market foreign exchange. When the short-run premium is given by portfolio balance, then, we can trace the dynamics of the black market premium and analyze the relationship between

smuggling and the black market for foreign exchange.

The problem of importers and exporters is described as a choice of the relative magnitudes of legal trade and smuggling, much like the problem of choosing an optimal portfolio of assets. From the solutions presented in Sections 1 and 2, a flow model of the black market for foreign exchange is derived in Section 3 and the effect of trade taxes discussed in Section 4. This is embedded in Section 5 in a model of the rest of the economy in the spirit of Jones (1974). Section 6 reinterprets the flow model as the long-run solution of a portfolio model of the Kouri (1983) variety.

### 1. The Importer

Consider a price-taking importer who chooses the amount of a good to be imported legally (denoted by  $L_m$ ) and the amount to be smuggled (denoted by  $S_m$ ). In the first case, he must pay an ad valorem tariff  $(t_m - 1) P_m^*$  but obtains the foreign exchange at the official exchange rate  $\tilde{e}$ . In the second case, he must buy the foreign exchange in the black market at a rate  $e$  and faces a probability  $(1-z)$  of being detected, in which case the amount  $S_m$  is confiscated. Denoting the domestic price by  $P_m$ , expected profits in domestic currency are a weighted average of profits in the two states of nature:

$$(1) \quad \rho_m = z [P_m (L_m + S_m) - \tilde{e} P_m^* t_m L_m - e P_m^* S_m] + (1-z) (P_m L_m - \tilde{e} P_m^* t_m L_m - e P_m^* S_m)$$

Now the probability of detection is an increasing function of the smuggling activity, which can be written as the probability of success,  $z$ , being a decreasing function of the smuggling ratio,  $s_m$ :

$$(2) \quad z = z(s_m); \quad z' < 0$$

$$s_m = S_m/L_m$$

$$z(0) = 1$$

If the  $z$  function is concave ( $z'' < 0$ ), the probability of success in smuggling decreases fast as some of it is undertaken and then more and more slowly as the smuggling ratio rises so that as  $s_m \rightarrow \infty$ ,  $z \rightarrow 0$ . Alternatively, if  $z'' \geq 0$ ,  $z$  reaches zero for some value  $\bar{s}_m$ . For example, if  $z = 1 - as_m$ , then  $\bar{s}_m = 1/a$ .

We write profits in terms of the import good valued at the official exchange rate. Defining the endogenous markup of the domestic price over the world price and the black market premium, we get:

$$(3) \quad \tilde{\rho}_m = \rho_m / \tilde{e}P_m^* = z\tilde{\beta}_m S_m + \tilde{\beta}_m L_m - t_m L_m - pS_m$$

where  $\tilde{\beta}_m = P_m / \tilde{e}P_m^*$ , the price markup; and  $p = e/\tilde{e}$ , the black market premium.

First-order conditions for profit maximization can be solved for the optimal combination of the import price markup and smuggling ratio, given the tariff and the black market premium:

$$(4) \quad \tilde{\beta}_m [1 - z's_m^2] = t_m$$

$$(5) \quad \tilde{\beta}_m z(1 - \zeta^m) = p$$

where  $\zeta^m = -z's_m/z$

According to (4), the domestic price is greater than the world price in domestic currency valued at the official rate but less than the tariff-inclusive price ( $1 < \tilde{\beta}_m < t_m$ ). This is what Pitt (1984) calls "price disparity."

It is clear from (5), furthermore, that the  $z$  function must be such that  $\zeta^m < 1$ . We assume that this condition is satisfied at the optimum, which implies in the earlier example that  $s_m < 1/2a$ .

The second-order condition for a maximum has the sign of:

$$(6) \quad B_m = z'(2 - \xi^m)$$

where  $\xi^m = -z''s_m/z'$ .

The condition  $B_m \leq 0$  is met when  $\xi^m < 2$ , which will always be true when  $z'' \leq 0$ . We assume that this is the case. Note that  $B_m(0) = 2z'(0)$  and that in the linear case,  $B_m = -2a$ .

At the optimum, profits are zero, as can be checked by substituting (4) and (5) into (1). Using the zero-profit condition to solve for the equilibrium domestic price, we get:

$$(7) \quad P_m = \frac{t_m + ps_m}{1 + zs_m} \tilde{e}P_m^*$$

According to (7), the domestic price markup is a weighted average of the tariff and the premium relative to the probability of success, with the weight on the tariff falling as the equilibrium smuggling ratio increases.

Eliminating  $\tilde{P}_m$  from (4) and (5), we can write a version of Beccaria's formula ( $zt_m = 1$ ) as:

$$(8) \quad zt_m - p = -z's_m(t_m + ps_m) > 0$$

An alternative interpretation of (8) is that it expresses the first-order condition for the minization of  $\tilde{P}_m$  in (7), which is equivalent to the profit-maximization carried out earlier, as noted by Pitt (1984).

The combination of the domestic price and the smuggling ratio consistent

with maximum profit for the importer can be presented graphically as the  $LL_m$  locus given by (4) and the  $SS_m$  locus given by (5). Their slopes are given by:

$$(9) \quad \left. \frac{d\tilde{P}_m}{ds_m} \right|_{LL_m} = \frac{B_m \tilde{P}_m s_m}{1 - z' s_m^2}$$

$$(10) \quad \left. \frac{d\tilde{P}_m}{ds_m} \right|_{SS_m} = \frac{-B_m \tilde{P}_m}{z + z' s_m}$$

We see from (9) that the  $LL$  locus is downward sloping. A higher smuggling ratio of imports implies a lower domestic price markup, because otherwise profits from legal trade would rise. Conversely, (10) shows that the  $SS$  locus is upward sloping. A higher smuggling ratio of imports implies a higher domestic price markup, because otherwise profits from smuggling would fall. The numerators of (9) and (10) have the inverse signs from the effect of an increase in the smuggling ratio of imports on marginal profits from legal trade and smuggling respectively. Denoting these by  $\rho_L$  and  $\rho_S$  respectively in Figure 1, right panel, we see that above  $SS_m(LL_m)$  profits from smuggling (legal trade) are rising and conversely. When  $s_m$  reaches the value for which  $\zeta^m = 1$ , say  $s_m$ , the slope of the  $SS_m$  curve becomes vertical. In the linear example, this will happen at  $s_m = \frac{1}{2}a$ .

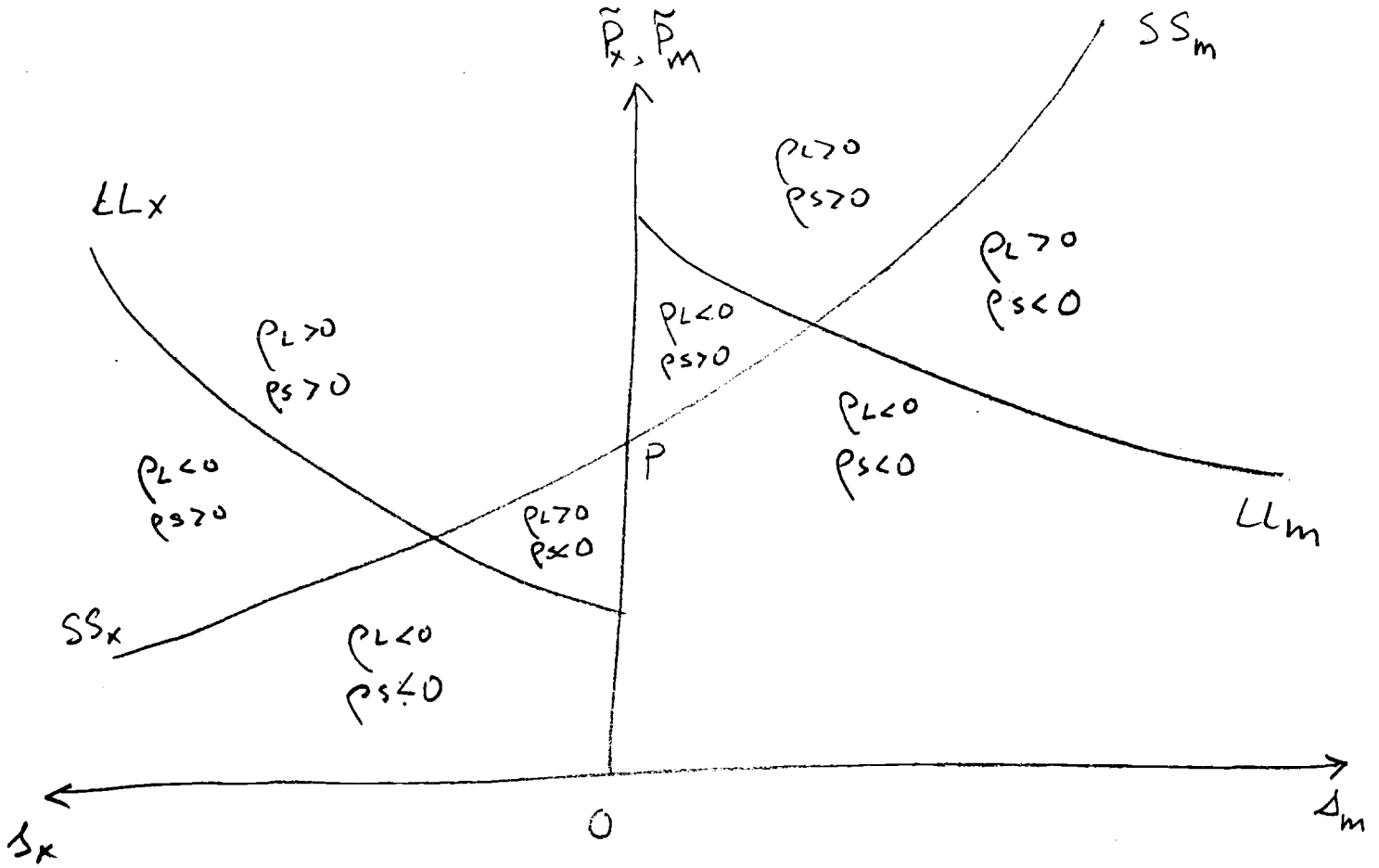
It is clear from Figure 1 that there always exists a solution if  $t_m > p$ . Note also that smuggling reduces the price markup but cannot eliminate it, even if  $p = 1$ .

An increase in the import tariff raises the domestic price markup and the smuggling ratio less than proportionately. On the other hand, an increase in the black market premium raises the domestic price of the imported good and



Figure 1

The Determination of Domestic Price Markups



lowers the smuggling ratio of imports.

## 2. The Exporter

The choice problem of the exporter can be analyzed in a similar set-up. Assuming the same detection technology, as captured by the  $z$  function, and indexing the variables with  $x$ , profits in domestic currency are given by

$$(11) \quad \rho_x = z S_x P_x^* e + P_x^* \tilde{e} t_x L_x - P_x (S_x + L_x)$$

where  $t_x = 1 - \tau_x$ , the export tax; and  $z = z(s_x)$ , with the properties indicated in (2) above.

Defining the international terms of trade  $\tau^*$ , we can express profits in units of the import good as:

$$(12) \quad \tilde{\rho}_x = [z S_x p + t_x L_x - \tilde{P}_x (S_x + L_x)] \tau^*$$

where  $\tau^* = P_x^*/P_m^*$ ; and  $\tilde{P}_x = P_x/\tilde{e}P_x^*$ .

The first-order conditions are again solved to obtain the optimal combination of the domestic price of exports and the share of legal exports, given the export tax and the premium:

$$(13) \quad \tilde{P}_x = t_x - pz's_x^2$$

$$(14) \quad \tilde{P}_x = pz(1-\zeta^X)$$

The second-order condition is still given by (6) above (with  $s_x$  replacing  $s_m$ ) and, at the optimum, exporters' profits are also zero, the domestic price being given by:

$$(15) \quad \tilde{P}_x = (t_x + zp s_x) / (1 + s_x)$$

Now  $\tilde{P}_x$  is again a weighted average of  $t_x$  and  $pz$ , with  $\tilde{P}_x = t_x / (1 + \bar{s}_x)$  when  $s_x$  reaches  $\bar{s}_x$ . Note, however, that, as before, the elasticity will become negative at a lower value,  $\underline{s}_x$ .

Eliminating  $\tilde{P}_x$  from (16) and (17), we get:

$$(16) \quad zp - t_x = -pz' s_x (1 + s_x) > 0$$

Again, (16) is simply the first-order condition for the maximization of  $\tilde{P}_x$  in (15). Now, since  $p$  and  $t_x$  are both positive, we have a stronger condition on the elasticity of the  $z$  function, namely that  $\tilde{\zeta}^x < 1$ , where  $\tilde{\zeta}^x = \zeta^x (1 + s_x)$  can be interpreted as the ratio of the elasticity to the weight of the export tax in (15), which is larger than one when there is export smuggling. We assume that this condition is met at the optimum, so that  $s_x < \underline{s}_x$ , where  $\tilde{\zeta}^x(\underline{s}_x) = 0$ . In the linear case  $\underline{s}_x = \sqrt{1 + 1/a} - 1$ .

For the exporter, the  $LL_x$  locus slopes upward and the  $SS_x$  locus slopes downward, as shown in Figure 1, left panel. Once again, above  $LL_x(SS_x)$  profits from legal trade (smuggling) are positive and conversely. From a point on  $LL_x$ , an increase in the smuggling ratio for exports requires a rise in the domestic price markup, otherwise profits from legal trade would fall. From a point on  $SS_x$ , a fall in the domestic price markup is required, otherwise profits from smuggling would rise. Again, there always exists a solution if  $p > 1 > t_x$ . Depending on the equilibrium smuggling, the price markup may be larger or smaller than one. When  $\tilde{P}_x > 1$ , the domestic price of the export good is larger than the world price despite export taxation.

An increase in the export tax raises the price and lowers the smuggling

ratio for exports, whereas an increase in the premium raises both.

### 3. The (Flow) Determination of the Black Market Premium

Identifying smuggled imports from many identical importers as (flow) demand for black market foreign exchange and undetected smuggled exports from many identical exports as (flow) supply of black market foreign exchange, the first-order conditions can be used to determine the black market premium. We then have from (4) and (5) that:

$$(17) \quad p = t_m m(s_m)$$

$$\text{where } m = \frac{z(1-\zeta^m)}{1-z's_m^2}; \text{ sgn } m' = \text{sgn } B_m < 0$$

Given the tariff, when the premium rises, demand for black market foreign exchange falls. Note that  $m \leq 1$  as long as  $\zeta_m < 1$  and that  $m(0) = 1$ .

Similarly, from (13) and (14), we get:

$$(18) \quad p = t_x x(s_x)$$

$$\text{where } x = 1/z(1-\zeta^x); \text{ sgn } x' = -\text{sgn } B_x > 0.$$

Note that  $s_x$  cannot be so large as to make  $x$  negative because of the condition that  $\zeta^x < 1$ . Also  $x = 1$  when  $s_x = 0$ . Given the export tax, when the premium rises, supply of black market foreign exchange rises (the share of legal exports falls).

In equilibrium, legal exports (in foreign currency) equal legal imports (in foreign currency) and undetected smuggled exports equal total smuggled imports. Using the definition of the terms of trade, we get

$$(19) \quad \tau^* L_X = L_m$$

$$(20) \quad \tau^* z(s_X) S_X = S_m$$

Dividing (20) by (19), we obtain a relationship between  $s_m$  and  $s_X$ :

$$(21) \quad s_m = z(s_X) S_X$$

The rise in the export smuggling ratio is always larger than the rise in the import smuggling ratio. This is because detected smuggled exports are confiscated before they can be exchanged for smuggled imports, so that a less than one probability of success in export smuggling is equivalent to the real cost of smuggling featured in Pitt (1984) and Martin and Panagariya (1984).

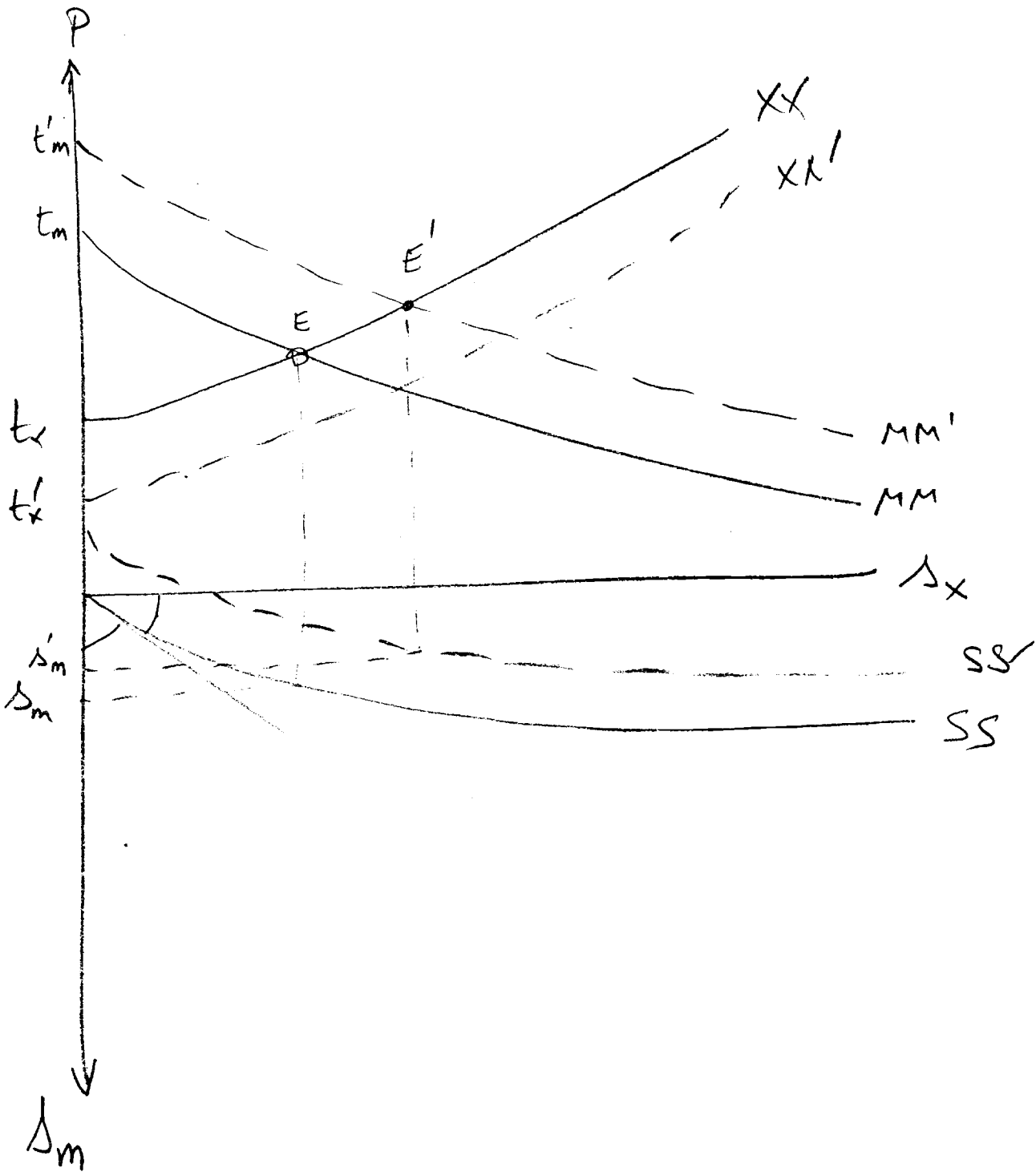
Thus, when smuggling is virtually non-existent, then  $s_X = s_m$  but, as the share of smuggled exports rises, we get  $s_m < s_X$  and the gap keeps growing as  $s_X$  increases. Note that  $\bar{s}_m$  must be such that  $z(s_X) S_X$  is non-zero.

From (17), (18), and (21), we can simultaneously determine the black market premium, the share of smuggled exports and the share of smuggled imports. In Figure 2, top panel, the XX locus slopes upward from  $t_X$  and the MM locus slopes downward from  $t_m$ . They intersect at E. An increase in the tariff increases the demand for black market foreign exchange which results in a rise in the premium and in the smuggling ratios (SS locus in bottom panel). An increase in the export tax increases the supply of black market foreign exchange: the premium falls but both smuggling ratios still rise. Thus higher trade taxes induce smuggling, but their effect on the black market premium depends on whether they affect supply or demand. Note that the SS schedule becomes flat when  $s_X = \bar{s}_X$ , where  $\bar{s}_X > \underline{s}_X$ , such that  $\bar{s}_m = z(\underline{s}_X) \underline{s}_X$

The effects of capital flight or of a legal trade deficit are

Figure 2

The Black Market Premium in Long-Run Equilibrium



straightforward to analyze. Thus, an undetected capital outflow  $k = K/L_m$  raises  $m(s_m+k)$  and thus the MM locus shifts up in Figure 2. This raises  $p$  and  $s_x$  (from  $s_x^0$  to  $s_x^k$ ). In (21), though, we now have  $s_m = zs_x - k$ , so that the SS locus also shifts up and  $s_m$  falls from  $s_m^0$  to  $s_m^k$ .

By totally differentiating (17), (18) and (21), denoting proportional rates of change by hats and the (positive) elasticities of the  $x$  and  $m$  functions by  $\epsilon^i$  ( $i = x, m$ ), we obtain the black market premium as a weighted average of  $t_m$  and  $t_x$ :

$$(22) \quad \hat{p} = (1-\alpha)\hat{t}_m + \alpha\hat{t}_x$$

where  $\alpha = \epsilon^m/(\epsilon^m + \epsilon^x)$ ;  $\epsilon^x = \tilde{\zeta}^x(2-\xi^x)/(1-\tilde{\zeta}^x)(1-\zeta^x)$ ; and

$$\epsilon^m = \zeta^m(1+ms_m)(2-\xi^m)/(1-\zeta^m).$$

Note that  $\epsilon^x > \epsilon^m$  with equality holding when  $s_m = s_x = 0$ . A premium in this limiting case requires different detection technologies for imports and exports. Otherwise, the initial situation may be  $t_m = t_x = p = 1$  and then  $\epsilon^m = \epsilon^x = -2z'(0)$ .

The smuggling ratios only depend on the ratio of one plus the trade tax rates,  $t = t_m/t_x$ . Note that changes both in the export tax and in the import tariff have the same effect on  $t$  (when  $\tau_x = \tau_m = 0$  initially), as required by Lerner's symmetry theorem:  $dt = (1-\tau_x)d\tau_m + (1+\tau_m)d\tau_x$ . We write:

$$(23) \quad \hat{s}_m = \frac{\hat{t}}{\epsilon^x + \epsilon^m}$$

$$(24) \quad \hat{s}_x = \frac{\hat{t}}{(\epsilon^x + \epsilon^m)(1-\zeta^x)}$$

Note that the analysis is applicable when instead of an export tax, we have an export subsidy, as in Branson and Macedo (1986). In that case,

taxes and subsidies rising at the same rate would keep  $t$  and thus  $s_m$  and  $s_x$  constant but would nevertheless imply a rising black market premium from (22).

#### 4. Trade Taxes and Relative Prices

Having obtained  $s_m$ ,  $s_x$  and  $p$ , we obtain prices from the zero profit conditions in (7) and (15) above. Log differentiating those, we get:

$$(7'') \quad \hat{\tilde{P}}_m = \hat{t}_m + \alpha\beta_m \hat{t}$$

$$(15'') \quad \hat{\tilde{P}}_x = \hat{t}_x + (1-\alpha)\beta_x \hat{t}$$

where  $\beta_m = ps_m/(t_m+ps_m)$  and  $\beta_x = ps_xz/(t_x+ps_xz)$ .

Trade taxes have a direct (and proportional) effect on price markups since these markups are tax-inclusive. But we see in (7'') and (15'') that there is also an indirect effect through the changes in smuggling ratios induced by changes in relative trade taxes. An increase in the import smuggling ratio decreases the tax-exclusive import price markup, because the lower probability of success lowers demand for black market foreign exchange and thus requires a lower black market premium for profits not to fall. Conversely, an increase in the export smuggling ratio increases the tax-exclusive export price markup, because the lower probability of success lowers supply of black market foreign exchange and thus requires a higher black market premium for profits not to fall. On both counts, therefore, trade taxes increase the export relative to the import markup, net of taxes.



The effect of smuggling ratios on the relative price  $\pi = P_x/P_m$  can be seen from the difference between (7'') and (15''):

$$(25) \quad \hat{\pi} = \hat{\tau}^* - [\omega/(\epsilon^x + \epsilon^m)] \hat{t}$$

where  $\omega = \epsilon^x(1-\beta_x) + \epsilon^m(1+\beta^m)$ .

Since  $t_m > t_x$  and therefore  $\beta_x > \beta_m$  and since  $\epsilon_x > \epsilon_m$  as well,  $0 < \omega < \epsilon^x + \epsilon^m$  and the effect of  $t$  on  $\pi$  is less than proportional. If  $s_x = s_m = 0$  initially, however, it will be proportional.

##### 5. A Model of the Economy

Having obtained the equilibrium values of  $p$ ,  $s_x$ ,  $s_m$  and  $\pi$ , we need a model of the economy to determine total imports, exports, production, consumption and welfare. Defining a production possibilities frontier in terms of importables and exportables (net of enforcement costs), an homothetic social welfare function in terms of the consumption of the two goods, setting it equal to production plus imports and less exports respectively, and making total imports equal to total exports at world prices yields the total amount imported as a function of the relative price,  $\pi$ . This exercise is carried out in Martin and Panagariya (1984) and, for this purpose, the model of the black market presented above could be grafted onto their model of the economy.

We present here a slightly different model, which allows for the existence of a non-traded good. As in Jones (1974) we assume that there is neither domestic production of the import good nor domestic consumption of the export good. As a consequence, the production possibilities frontier and the social welfare function are written as:

$$(26) \quad Q_{nt} = f(Q_x); f' < 0$$

$$(27) \quad U = U(C_m, C_{nt})$$

We neglect the distributional consequences of trade taxes and assume that revenues from detected smuggling are consumed by the government. Then consumption equals production in the market for the non-traded good and any decrease in export production will raise welfare via the extra consumption of the non-traded good:

$$(28) \quad Q_{nt} = C_{nt}$$

Efficiency in production implies that the price of exports in terms of non-traded goods equals the marginal rate of transformation:

$$(29) \quad q = -f'(Q_x); f'' < 0$$

where  $q = P_x/P_{nt}$ .

Efficiency in consumption implies that the price of imports in terms of non-traded goods (given by  $q/\pi$ ) is a negative function, denoted by  $g$ , of the consumption ratio (assuming unitary income elasticities, so that the marginal and average propensities to import are the same): eliminating  $q$  from (29) and using the trade balance equilibrium, we obtain an equation in  $L_m$ ,  $\pi$ ,  $s_m$  and  $s_x$ :

$$(30) \quad -f'(Q_x) = \pi g[C_m/f(Q_x)]$$

where  $C_m = L_m(1+zs_m)$ , and  $Q_x = L_m(1+s_x)/\tau^*$ .

Log differentiating (30), we see that the domestic terms of trade increases with import consumption and export production:

$$(30') \quad (\eta^f + \epsilon^g \epsilon^f) \hat{Q}_X + \epsilon^g \hat{C}_m = \hat{\pi}$$

where  $\epsilon^g = -g' C_m / fg$ , the inverse of the aggregate elasticity of substitution in demand,  $\eta^f = f'' Q_X / f'$ , the inverse of the elasticity of production (similarly  $\eta_n^f$  for on-traded goods), and  $\epsilon^f = -f' Q_X / f = \eta^f / \eta_n^f$ .

We note from the definitions of  $C_m$  and  $Q_X$  in (30) that (realized) import demand excludes the amount confiscated by the government, whereas production of exports includes the amount confiscated by the government. The effect of trade taxes given  $L_m$  is thus to raise  $C_m$  and  $Q_X$ , but the latter by more:

$$(31) \quad \hat{C}_m = \hat{L}_m + [\beta_m / (\epsilon^X + \epsilon^m)] \hat{t}$$

$$(32) \quad \hat{Q}_X = \hat{L}_m + [\beta_X / (\epsilon^X + \epsilon^m)] \hat{t} - \hat{\tau}^*$$

Now substituting in (31) for  $\hat{C}_m$ ,  $\hat{Q}_X$  and  $\hat{\pi}$  from (32), (32') and (25), respectively, we can solve for the legal amount of imports as a function of the international terms of trade and trade taxes:

$$(33) \quad [(1 + \epsilon^f) \epsilon^g + \eta^f] \hat{L}_m = (1 + \epsilon^f \epsilon^g + \eta^f) \hat{\tau}^* - \frac{1}{\epsilon^X + \epsilon^m} [\omega + \beta_X (\epsilon^f \epsilon^g + \eta^f) + \beta_m \epsilon^g] \hat{t}.$$

The term in square brackets includes the price effect  $\omega$  as well as a quantity effect. Both are negative and less than proportional if  $\beta_X < (\epsilon^X + \epsilon^m)$ . If  $s_X = s_m = 0$  initially, the total effect will be one-to-one (in absolute value).

A deterioration in the terms of trade lowers legal imports (as well as legal exports). When utility is Cobb-Douglas, ( $\epsilon^g = 1$ ), they fall in proportion.

Using (33) in (32) and (32'), we obtain the effect of trade taxes on

import consumption and export production. Leaving out the terms of trade ( $\hat{\tau}^* = 0$ ), we have:

$$(31') \quad E(\epsilon^X + \epsilon^M) \hat{C}_m = -[\omega + (\beta_X - \beta_m)(\epsilon^f \epsilon^g \eta^f)] \hat{t}$$

$$(32') \quad E(\epsilon^X + \epsilon^M) \hat{Q}_X = -[\omega - (\beta_X - \beta_m) \epsilon^g] \hat{t}$$

where  $E = (1 + \epsilon^f) \epsilon^g + \eta^f$ .

We see from (31') that import consumption falls by less than legal imports, and from (32') that trade taxes need not depress export production and shift resources to the production of non-traded goods. This requires a large  $\epsilon^g$ , or a small aggregate elasticity of substitution in demand.

From (29), the relative price of exports and non-traded goods is positively related to  $Q_X$ . The effect of the terms of trade is given by  $\eta^f(1 - \epsilon^g)$ . Thus, if the aggregate elasticity of substitution in demand is greater than one, then  $\epsilon^g < 1$  and a terms of trade deterioration raises the domestic price of exports relative to non-traded goods. There is no effect in the Cobb-Douglas case. The effect of trade taxes will tend to be negative so that the price of exports falls relative to the price of non-traded goods. We see from (32'), however, that the effect could be positive if  $\epsilon^g$  is large enough.

Coming to the welfare effects, we differentiate (27) and choose units so that  $\partial U / \partial C_m = 1$ :

$$(34) \quad \frac{C_{nt}}{y} dU = \alpha_m \hat{C}_m + (1 - \alpha_m) \hat{C}_{nt}$$

where  $\alpha_m = P_m C_m / P_{nt} y$ , the share of imports in consumption; and  $y = \frac{P_m}{P_{nt}} C_m + C_{nt}$ , real income.

Since a terms of trade improvement reduces  $Q_X$  and thus increases  $C_{nt}$  while raising  $L_m$  from (33), its effect on welfare is unambiguously

positive. The effect of trade taxes is ambiguous, though, because of the ambiguity of the effect on  $Q_x$ . If  $Q_x$  falls,  $C_{nt}$  rises and so does welfare unless  $C_m$  falls by more. We can express the condition for trade taxes to be welfare-improving as:

$$(35) \quad \eta^f [1 - \alpha_m - (\alpha_m \eta_n^f + \epsilon^g)(\beta_x - \beta_m)/\omega] > \alpha_m \eta_n^f.$$

This reduces to the condition that the term in square brackets in (32') be positive when  $\alpha_m = 0$ . Otherwise, it is more likely to happen when  $\eta^f$  is large and  $\eta_n^f$  and  $\epsilon^g$  are small. Thus the relative magnitude of the supply and demand elasticities and the consumption share of imports are what introduces this ambiguity, absent from the model of Martin and Panagariya (1984), where tariffs are welfare-reducing because of the ex ante deficit in the total trade balance. While, in the present model, it is also the case that  $L_x + S_x < L_m + S_m$ , allowing for the existence of a non-traded good introduces the possibility of a positive effect, at least under the extreme assumption that government activities are unproductive and that the authorities react passively. Instead of introducing the choice of the government with respect to trade taxes and enforcement, however, we allow asset stocks to determine the black market premium at each instant of time.

## 6. A Portfolio Model of the Black Market for Foreign Exchange

In the previous section, we neglected the existence of the stock demand for foreign assets which was introduced in Macedo (1982). It can be written as:

$$(36) \quad pF = A^d$$

where  $F$  is the given stock of foreign assets (in foreign currency) and  $A^d$  is the given demand for foreign assets valued at the official exchange rate.

The stock of foreign assets changes with the unreported current account. Denoting rates of change by dots:

$$(37) \quad \dot{F} = B = (\tau^* z s_x - s_m) P_m^*$$

Dividing by the amounts of legal trade and assuming for the moment that the reported trade balance is zero, we get

$$(38) \quad B = P_m^* L_m \{ \tau^* z (s_x)^{-1} [p/t_x] - m^{-1} [p/t_m] \}$$

Even though a rise in  $p$  raises  $s_x$  and therefore lowers  $z$ , the direct effect always dominates since the elasticity is less than one, so that we can rewrite (37) and (38) as:

$$(39) \quad \dot{F} = B(\dot{p}, \dot{\tau}^*, \tau_x^+, \tau_m^-, L_m^+)$$

The sign of the first term makes the system stable and retrieves the "acceleration hypothesis" of Kouri (1983) in the analysis of the black market for foreign exchange:

$$(40) \quad \hat{p} = -B/F$$

The premium will rise when the unreported current account is in deficit ( $B < 0$ ) and conversely. A rise in tariffs will generate a deficit (and increase the premium as before), whereas a rise in export taxes will generate a surplus.

Since the black market premium is determined by (36) at each moment in time, (17) and (18) above are now solved for  $s_m$  and  $s_x$ :

$$(41) \quad \hat{s}_m = -(1/\epsilon^m)(\hat{p}-\hat{t}_m)$$

$$(42) \quad \hat{s}_x = (1/\epsilon^x)(\hat{p}-\hat{t}_x)$$

Using (41) and (42), we can rewrite (38) as:

$$(38') \quad B \approx P_m^* S_m [(\epsilon^m + \epsilon^x)/\epsilon^x][\hat{p} - (1-\alpha)\hat{t}_x - \alpha\hat{t}_m]$$

Comparing to (22) above, when the premium is above the long-run value given by the  $\alpha$ -weighted average of trade taxes, the reported trade balance is in surplus and conversely.

To relax the assumption made earlier that the reported trade balance is always zero, and therefore allow for changes in domestic money, consider the following extension. Portfolio balance is now written as

$$(36') \quad pF = h\tilde{H}$$

where  $\tilde{H} = H/\tilde{e}$  is the stock of domestic money valued at the official exchange rate

and  $h$  is the given currency ratio (obtained from the asset demand function).

While domestic credit creation and a given rate of crawl for the official exchange rate are easy to incorporate, see Macedo (1985), we neglect them in order to identify the reported trade balance with the rate of increase of domestic money:

$$(43) \quad \dot{\tilde{H}} = \tilde{B} = P_X^* L_X - P_M^* L_M$$

Using (37) and (43) we see that the premium changes with the difference between the normalized unreported and reported trade balances:

$$(44) \quad \hat{p} = -(B/F) + (\tilde{B}/\tilde{H})$$

The premium will change if the unreported trade deficit is different from the reported trade surplus weighted by the currency ratio valued at the official exchange rate  $\tilde{h} = \tilde{e}F/H$ , which will always be smaller than the currency ratio relevant for portfolio choice since  $h = p\tilde{h}$ .

Using now (41) and (42) to solve for relative price disparities, we get, instead of (25):

$$(45) \quad \hat{\pi} = \hat{\tau}^* + (E_X + E_M)\hat{p} + (1 - E_X)\hat{t}_X - (1 + E_M)\hat{t}_M$$

where  $E_i = \epsilon^{\pi i} / \epsilon^i$ ,  $i = x, m < 1$ .

Since trade is not balanced, however, we simultaneously determine  $L_M$  and  $L_X$  in terms of  $\tau^*$ ,  $p$ ,  $t_m$ ,  $t_x$  and the reported surplus:

$$(46) \quad \begin{aligned} \Delta \hat{L}_M &= P_X^* L_X [1 + \eta^f + (1 + \epsilon^f) \epsilon^g] \hat{\tau}^* - P_X^* L_X (\tilde{E}_X + \tilde{E}_M) \hat{p} \\ &\quad + P_X^* L_X (1 - \tilde{E}_X) \hat{t}_X + P_X^* L_X (1 + \tilde{E}_M) \hat{t}_M - [\eta^f + (1 + \epsilon^f) \epsilon^g] d\tilde{B} \\ \Delta \hat{L}_X &= \Delta \hat{L}_M / \tilde{D} + \{[(\tilde{D} + \epsilon^f) \epsilon^g + \eta^f] / \tilde{D}\} d\tilde{B} \end{aligned}$$

where  $\tilde{E}_M = E_M (1 + \epsilon^g \epsilon^c)$

$\tilde{E}_X = E_X (1 + [(1 + \epsilon^f) \epsilon^g + \eta^f] \epsilon^Q)$



$$\tilde{D} = P_{X^*}^* L_{X^*} / P_{m^*}^* L_m$$

and 
$$\Delta = P_{m^*}^* L_m [(1 + \epsilon^f + \tilde{D}) \epsilon^g + \eta^f]$$

Note that when  $\tilde{D} = 1$ , we get back to the original expression. An increase in the premium will decrease legal trade and conversely. Increases in trade taxes unambiguously reduce legal trade, and an increase in the trade surplus  $\tilde{B}$  increases exports and lowers imports. A devaluation of the official rate, by lowering the premium, will increase both exports and imports, such that the net effect is zero. Nevertheless, the effect of the official exchange rate on the reported trade balance should be explicitly introduced.

### Conclusion

This model can easily be extended to allow for wealth effects, along the lines of Macedo (1985). Despite its simplicity, it captures the basic relationship between currency inconvertibility, trade taxes and smuggling. In particular it shows that, even when trade is balanced, the existence of a black market premium and non-traded goods allows for the possibility of trade taxes not reducing welfare. The model did not make the usual assumption that there were real costs to smuggling activity, except for the possibility of detection. It shows that the black market premium is a weighted average of trade taxes, thus providing current account transactions as the fundamental determinants of the long-run free exchange rate.

The model can also be enriched by specifying the choice of the government with respect to the enforcement of exchange controls and the detection of tax

evasion. Most important, however, would be the explicit consideration of monetary and exchange rate policy, which is pushed to the background by assuming that legal trade is balanced at the prevailing official exchange rate. Some of these extensions are in our current research agenda.

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