WAGE DISPERSION AND SEARCH BEHAVIOR

Robert E. Hall
Andreas I. Mueller

ABSTRACT

We use a rich new body of data on the experiences of unemployed job-seekers to determine the sources of wage dispersion and to create a search model consistent with the acceptance decisions the job-seekers made. From the data and the model, we identify the distributions of four key variables: offered wages, offered non-wage job values, the value of the job-seeker's non-work alternative, and the job-seeker's personal productivity. We find that, conditional on personal productivity, the dispersion of offered wages is moderate, accounting for 21 percent of the total variation in observed offered wages, whereas the dispersion of the non-wage component of offered job values is substantially larger. We relate our findings to an influential recent paper by Hornstein, Krusell, and Violante who called attention to the tension between the fairly high dispersion of the values job-seekers assign to their job offers–which suggest a high value to sampling from multiple offers–and the fact that the job-seekers often accept the first offer they receive.

Robert E. Hall
Hoover Institution
Stanford University
Stanford, CA 94305-6010
and NBER
rehall@gmail.com

Andreas I. Mueller
Columbia Business School
3022 Broadway
Uris Hall 824
New York, NY 10027
and NBER
amueller@columbia.edu

Search theory is firmly established as a useful way to think about unemployment and labor mobility. But it is well known that fitting search models to data on wages and labor flows is a challenge. Wages have huge dispersion among workers with similar observed characteristics. Traditional search theory hypothesized that job-seekers would keep considering wage offers until they found one high in the upper tail of the distribution of available opportunities. But the high rate of job-seekers' acceptance of job offers suggests that searchers are leaving money on the table by taking jobs long before it is likely that they have adequately sampled the upper tail. The addition of on-the-job search to the model relieves some of this tension, because job-seekers departing unemployment may do so by taking an interim job and continue to search for the dream job in the upper tail while employed in the interim job. Even then, models that calibrate the offer distribution to the distribution of wages across workers find that the exit rate from unemployment to jobs makes sense only if workers find unemployment virtually intolerable, else they would be more picky in their acceptance decisions. These points reflect the suspicions that search economists have harbored for some time. They recently came into sharp focus in an influential article, Hornstein, Krusell and Violante (2011) (HKV).

In this paper, we investigate a new data source with the aim of estimating the interpersonal dispersion of wages and values derived from jobs more broadly. That source is a novel survey of unemployed workers in 2009 and 2010 that Alan Krueger and Andreas Mueller (KM) carried out—see Krueger and Mueller (2011) and Krueger and Mueller (2015). The KM data permit a more refined measure of dispersion than do the data sources in earlier work—they liberate search theory from inferring the distribution of opportunities from the residuals of wage regressions. The data include prior wages collected from administrative sources and survey responses about reservation wages each week during a spell of unemployment, and the wages of job offers and of newly accepted jobs. Although earlier surveys have collected cross-section data on reservation wages, the KM survey is the first, as far as we know, that collects panel data on reservation wages. It is also the first U.S. source to match survey data and administrative data, we believe.

The KM data, together with a reasonable set of assumptions, permit a solution to a problem that has significantly impeded research on labor search behavior. HKV, Section II, discuss the challenges in detail with many references. The problem is the lack of information on individual wages relative to personal productivity. Conditional on measures of personal

characteristics available to the econometrician, wages have huge dispersion. As HKV observe, research that uses econometric residuals to measure wages relative to personal productivity has difficulty building a model that fits all the restrictions that seem reasonable.

Our identifying assumption is that the wage-related variables measured in the KM survey are proportional to personal productivity. Under the proportionality assumption for the reservation wage, both the offered wage and the wage in the prior job are proportional to personal productivity. The covariance of their logs reveals the dispersion of personal productivity. Then the difference between the log variances of the offered wage and the prior wage is the log variance of the offers facing a job-seeker with a given level of productivity.

An important extension of the standard search model adds a non-wage dimension to jobs. Our model is flexible in the sense that it allows the wage and non-wage components to be positively or negatively correlated. In the KM data, a job-seeker frequently accepts a job paying less than the previously stated reservation wage and, less frequently, rejects a job paying more than the reservation wage. Moreover, as revealed by direct questions in the survey, two thirds of rejections of job offers are motivated by non-wage related reasons. We use the observed relation between the acceptance probability and the difference between the offered and reservation wages to infer the correlation between the wage and non-wage value of job offers as well as the overall dispersion of the non-wage value.

Our model matches the means and standard deviations of the empirical distribution of the logs of offered wages and reservation wages, together with the observed frequency of acceptance of offers as a function of the amount by which the offered wage exceeds the reservation wage, and the fraction of rejections for non-wage reasons. We find robust estimates of dispersion—measured as the log standard deviation—for three of the four underlying unobserved variables. These are 0.24 for the offered wage, 0.09 for the non-work value (value of alternative non-market activity), and 0.43 for personal productivity. The bootstrap standard errors of these estimates are 0.02, 0.05 and 0.02, respectively. We estimate the log standard deviation of the component of the non-wage value of job offers that is independent of the offered wage to be 0.37 with a standard error of 0.09. The parameter determining the correlation between the wage and non-wage component of the wage tends to be close to 0, but with significant sampling error. Overall, these results imply that, conditional on personal productivity, the wage value of job offers has a moderate dispersion, accounting

for 21 percent of the total variation in observed offered wages. The dispersion in non-wage values is substantial and larger than the dispersion of the wage value of job offers.

HKV note that most empirical search models that appear to rationalize observed unemployment-to-employment flows imply an implausibly low flow value of unemployment. It is frequently negative. These models generally infer the value of job search from estimates of the dispersion of wage offers derived from cross-sectional data, where dispersion is high. Sampling from that distribution is highly valuable activity, which implies that people must truly hate unemployment to take the first job that comes along as frequently as they do in practice.

By design, the KM survey gathered information on the search decisions of only the unemployed. A re-employed worker can continue to search—job-to-job transitions account for about half of all hires in the U.S. economy. A reasonable strategy for the unemployed is to take an interim job and keep on searching for a more permanent job. Hall (1995) describes a labor market operating in this model. The job-ladder model, as in Burdett (1978), Burdett and Mortensen (1998), and Hagedorn and Manovskii (2013), formalizes the process. In that model, provided that the offer rate for employed workers does not fall short of the rate for unemployed job-seekers, the reservation wage for a job-seeker is just the flow value of unemployment. The reservation wage for a worker currently receiving a wage of w is w itself—a costless move to any job that beats the current wage is an improvement. If holding a job results in a lower offer rate or if the job-seeker incurs a cost upon changing jobs, the job-seeker will set a reservation wage above the flow value of unemployment to preserve the option value of unemployment.

In this paper, we consider a simple job-ladder model where unemployed and employed workers differ in the probability of sampling job offers and calibrate the model to the observed rate of the job-to-job transition in the data. We find that, given the dispersion of job values estimated in this paper, the implied flow value of unemployment is around 0.1, which is above but close to zero. The reason is that, while the wage value of job offers has a relatively modest dispersion, the dispersion of non-wage values is substantial and larger than the dispersion of the wage value of job offers. Taken together, this implies that job seekers' acceptance behavior can only be rationalized by a relatively low flow value of unemployment, as otherwise job seekers would reject most offers and wait for a job offer with high pay but mostly a high non-wage amenity to arrive.

4

1 Related Literature

HKV has an extensive discussion of the extensive literature on wage dispersion, with many cites, notably Mortensen (2003), Rogerson, Shimer and Wright (2005), Bontemps, Robin and Berg (2000), Postel-Vinay and Robin (2002), Jolivet, Postel-Vinay and Robin (2006), and Jolivet (2009). More recently, a few papers have estimated the extent of wage dispersion arising through search frictions. Hagedorn and Manovskii (2010), Low, Meghir and Pistaferri (2010), and Tjaden and Wellschmied (2014) infer the extent of wage dispersion arising from differences in match quality from the excess volatility of wage growth of those who switch jobs compared to those who stay on their jobs. With this approach, estimates of wage dispersion depend critically on how the process of on-the-job search is modeled. If the efficiency of on-the-job search is high, workers move up the job ladder relatively fast, and most job-to-job transitions are associated with small wage gains as workers continue to search for new jobs even when they are far up on the ladder. This process implies that, for a given observed variance of wage changes, the inferred dispersion in offered wages is increasing in the search efficiency of on-the-job search—see Tjaden and Wellschmied (2014). We estimate the dispersion in wages arising from search frictions with a different identification strategy from these papers. Our estimates of the dispersion in wage offers are closest to Low et al. (2010), who find a standard deviation of match-specific wage shocks of 0.23, but are substantially larger than the estimates in Tjaden and Wellschmied (2014) and Hagedorn and Manovskii (2010).

Our paper is also related to the literature on compensating differentials. In the presence of search frictions, a compensating differential does not necessarily arise, because firms do not need to pay the worker the marginal product—see Hwang, Mortensen and Reed (1998), Lang and Majumdar (2004), and Bonhomme and Jolivet (2009). In fact, wage and non-wage values may even be positively correlated if more productive firms compensate workers more highly with both a higher wage and a higher non-wage amenity. A recent paper by Sullivan and To (2014) infers the dispersion of non-wage amenities from the fraction of job-to-job transitions that result in a wage decrease, but they assume that wage and non-wage values are independent of each other. Their estimate of the dispersion of the non-wage value is similar to ours. Relative to their approach, ours is more flexible and allows for both positive or negative correlation of wage and non-wage values in job offers. Another recent paper

by Sorkin (2015) finds that non-wage characteristics are important for explaining inter-firm wage differentials, but he does not estimate the overall dispersion of non-wage values.

An important challenge for the papers discussed in this section is to distinguish job-to-job transitions that are value increasing—movements up the job ladder—from transitions that arise from layoffs or other involuntary separations. The conclusions emerging from this literature depend on whether one interprets wage decreases as compensated for by higher non-wage characteristics or as falling off the job ladder. This paper uses direct information on job acceptance decisions of unemployed workers, so the main parameters in our approach do not rest on assumptions about the process of on-the-job search.

2   The KM Survey

The KM survey enrolled roughly 6,000 job-seekers in New Jersey who were unemployed in September 2009 and collected weekly data from them for several months. The survey also incorporates data from administrative records for the respondents, notably their wages on the jobs they held just prior to becoming unemployed. We follow Krueger and Mueller (2011) and restrict the sample to survey participants of ages 20 to 65. For details on the KM survey, see Krueger and Mueller (2011) and Krueger and Mueller (2015).

2.1   Job offers

The KM survey asked respondents each week: "In the last 7 days, did you receive any job offers? If yes, how many?" The respondents in our sample received a total of 2,174 job offers in 37,609 reported weeks of job search. The ratio of the two, 0.058, is a reasonable estimate of the overall weekly rate of receipt of job offers.

For respondents who indicated that they received at least one job offer, the KM survey asked respondents: "What was the wage or salary offered (before deductions)? Is that per year, per month, bi-weekly, weekly or per hour?" In cases where respondents reported that they received more than one offer in a given week, the survey asked the offered wage only for the best wage offer. Among the individuals who reported at least one job offer, 86.3 percent reported that they received one offer in the last 7 days, 8.6 percent reported receiving two offers in the last 7 days, 2.4 percent received 3 offers and the remaining 2.7 percent received between 4 and 10 offers in the last 7 days.
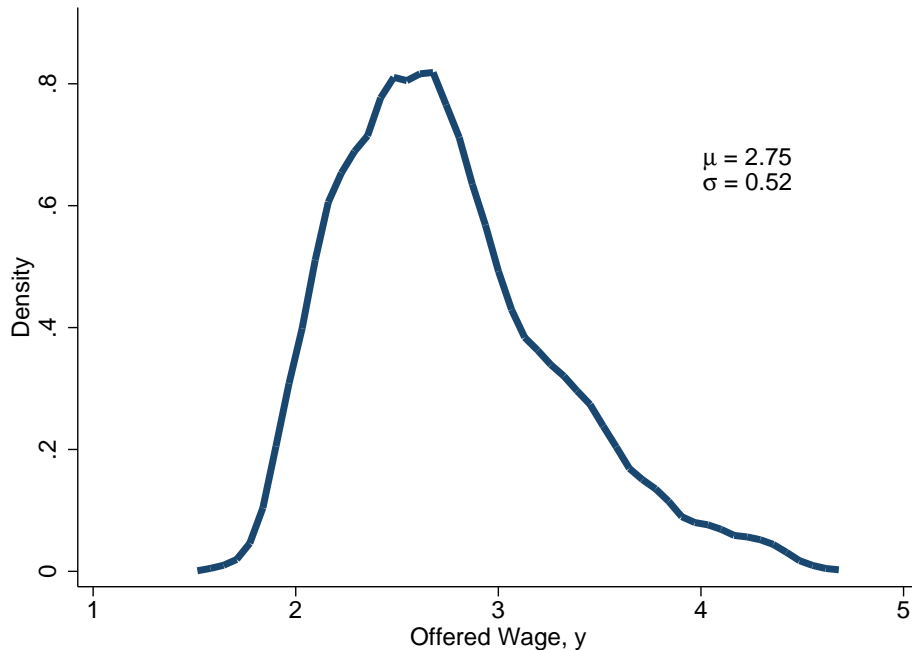
Figure 1: Kernel Density of the Log Hourly Offered Wage, y

Figure 1 reports the kernel density of the hourly offered wage for our sample of 1,153 job offers. The sample is restricted to cases where details of the offer (including the wage) and a reservation wage from a previous interview were available. We use the same sample below when we compute the acceptance frequency conditional on the difference between the log of the offered wage and the log of the reservation wage from a previous interview.

The model interprets this distribution as the mixture of the distribution of wage offers for a worker with standardized personal productivity and the distribution of productivity across workers—by mixture, we mean the weighted average of the offer distribution for given productivity, with the weights taken as the distribution of productivity.

2.2   Reservation wage

Each week, the respondents in the KM survey answered a question about their reservation wages: "Suppose someone offered you a job today. What is the lowest wage or salary you would accept (before deductions) for the type of work you are looking for?" We only use the first reservation wage observation available for each person in the survey so that the sample is representative of the cross-section of unemployed workers. We apply the same sample restrictions as Krueger and Mueller (2011)—we exclude survey participants who reported
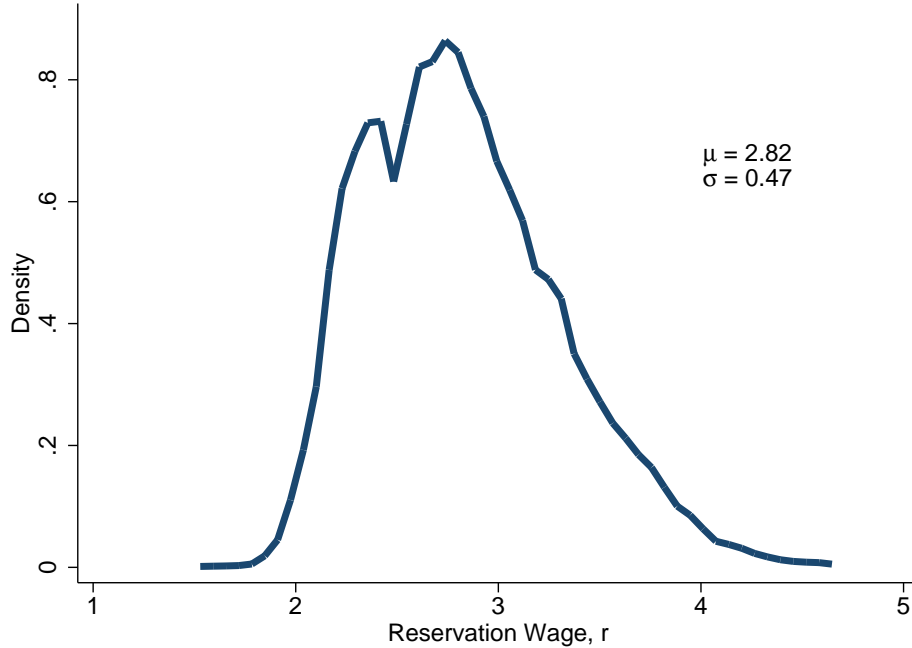
Figure 2: Kernel Density of the Log Hourly Reservation Wage, r

working in the last seven days or already accepted a job offer at the time of the interview. Figure 2 shows the kernel density of the hourly reservation wage for our sample of 4,138 unemployed workers.

The model interprets the reservation wage of an unemployed job-seeker as the value of the non-work option. The survey reveals the distribution of the non-work value among all respondents. The model interprets this distribution as the mixture of the distribution of non-work value for a worker with standardized personal productivity and the distribution of productivity across workers.

To avoid possible bias from cognitive dissonance among the respondents, we exploit the longitudinal structure of the survey and use the reservation wage value reported in the week *prior* to the receipt of a job offer, in our study of the probability of acceptance of an offer.

2.3 Acceptance

Many respondents accept job offers that pay less than the respondent's previously reported reservation wage. Some do the reverse, rejecting an offer that pays more than the reservation wage. Our model posits that jobs have non-wage values, to explain why the offered wage does not control the acceptance decision—job-seekers accept jobs paying less than the reservation

wage because these jobs have positive non-wage values that offset the low wage. The model accounts for the bias toward acceptance by treating the reported reservation wage as referring to a job with below-normal non-wage value.

We study the acceptance probability as a function of the difference between the log of the offered wage and the log of the reservation wage. Krueger and Mueller (2015) gives a detailed analysis of the acceptance frequency in the survey. The job acceptance frequency rises with $d = y - r$. The average frequency of job acceptance in our sample is 71.9 percent. In 20.9 percent of the cases, respondents indicated that they had not yet decided whether to accept the job offer or not.

To deal with the problem of missing data for acceptance of some job offers, we make use of administrative data on exit from unemployment insurance. UI exit is a potentially useful but imperfect indicator of acceptance, for four reasons: (1) A delay occurs between job acceptance and UI exit. (2) An exit from the UI system may relate to a different offer from the one reported in the survey. (3) UI exit data are censored at the point of UI exhaustion, as the data do not track recipients after they exhaust benefits. (4) An unemployed worker may perform limited part-time work while receiving benefits and thus acceptances of such offers will not be reflected in an exit from the UI system. Krueger and Mueller (2015) show that the rate of UI exit for those who were undecided was almost exactly half way between the rate of UI exit for those who accepted the offer and the rate of UI exit for those who rejected the offer. We believe that this estimate is the best available. Notwithstanding the imperfect relation between exits and acceptances of offers, we believe it is the best way to handle the problem of missing data, so we create an indicator variable A that takes on the value zero for a rejected offer, 0.5 for an offer for which the respondent was undecided, and 1 for an accepted offer.

Figure 3 shows the acceptance frequency smoothed in two ways: (1) as the fitted values from a regression of A on a 6th-order polynomial in $y - r$ and (2) as the fitted values from a locally weighted regression (LOWESS) with bandwidth 0.3. The figure runs from first-percentile value of d to the 99th percentile value. Values outside that range are inherently unreliable for any smoothing method.

The survey also asked a question about reasons for rejecting a job offer: 32.3 percent indicated that they rejected because of "inadequate pay/benefits" and the remaining 67.7 percent indicated another reason for rejecting such as unsuitable working conditions, in-
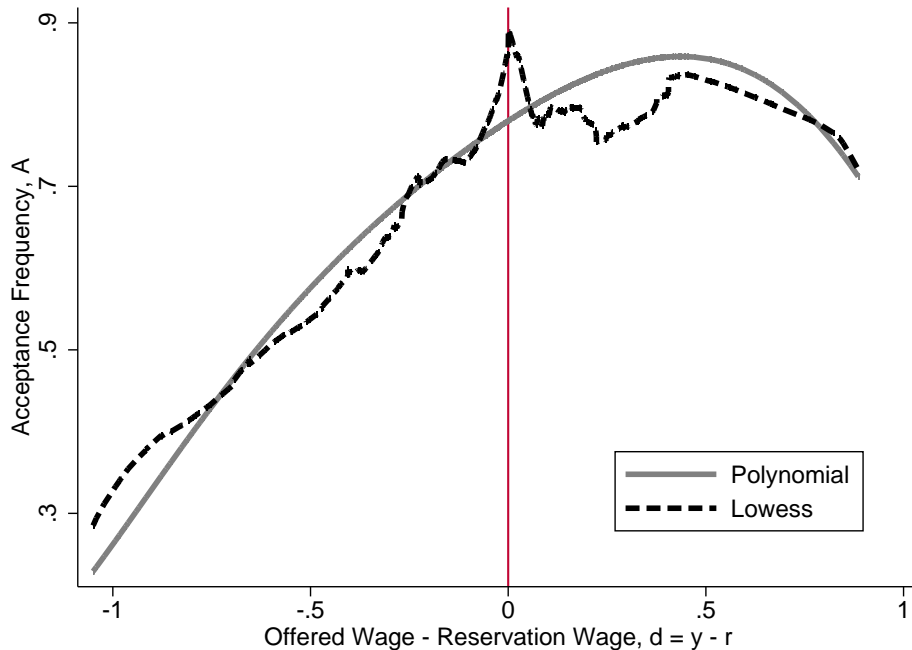
Figure 3: Smoothed Acceptance Frequency, A, as a Function of the Difference between the Log Hourly Offered Wage and the Log Hourly Reservation Wage, $d = y - r$

sufficient hours/too many hours, transportation issues, insufficient use of skills/experience. Because our approach to estimation does not allow for simultaneous job offers, we exclude from the sample the 5.0 percent of offers that respondents rejected because they accepted another job offer. Unfortunately, the survey did not distinguish between inadequate pay and inadequate benefits, but in response to a similar question in the National Longitudinal Survey of Youth (NLSY) in 1986-87, 36.8 percent of respondents mentioned "inadequate pay" as the reason for rejecting a job offer, indicating that the inadequate pay is the most common reason for rejecting the job offer, not inadequate benefits. Moreover, as reported in Krueger and Mueller (2015), 40 percent of offers below the reservation wage were rejected for inadequate pay or benefits, whereas only 1 percent of offers above the reservation wage was rejected for the same reason. This evidence suggests that either benefits are not an important factor in the acceptance-rejection decision or that benefits are quite positively correlated with the offered wage, as otherwise we would expect at least some rejections for the reason of inadequate benefits for job offers with wages above the reservation wage. As explained further below later in the paper, our model allows for correlation between wage offers and non-wage amenities.

In our approach to estimation, the shape of the acceptance function and the fraction of rejections for non-wage reasons together identify the dispersion of the non-wage value and the correlation of wages and non-wage values. The fact that many jobs are accepted that pay well below the reported reservation shows that fairly large positive non-wage values are common. We characterize the function by the acceptance rate at two values of d. Together with the fraction of offers rejected for non-wage reasons, these moments are sufficient to identify the mean and standard deviation of the log of the non-wage job value, as well as the correlation of wages and non-wage values in job offers.

2.4 Prior wage

Our model views the prior wage as based on the job-ladder model. A respondent searched during an earlier spell of unemployment and accepted the first job offered that exceeded the reservation job value (combining wage and non-wage components). While employed, the worker received offers, and accepted the ones that exceeded the job value of the prior job. The distribution of the observed wage on the job the respondent held just before the current spell of unemployment is the stationary distribution of the process of starting up the job ladder, making successive improvements, and occasionally suffering job loss and dropping back to the bottom of the ladder.

Figure 4 shows the kernel density of the hourly wage on the prior job. The wage is computed from administrative data on weekly earnings during the base year, which typically consist of the first four of the five quarters before the date of the UI claim, and survey data on weekly hours for the previous employment. Hours on the previous job might not perfectly overlap with the period of the base year. Moreover, roughly 15 percent of the respondents answered that hours varied on their previous jobs and we imputed their hours based on demographic characteristics as in Krueger and Mueller (2011). For these reasons, the hourly previous wage includes some measurement error despite the fact that weekly earnings are taken from administrative data.

In the model, the distribution of the prior wage depends on all four unobserved distributions. We carry out a rather complicated calculation of the distribution and match it to the observed one. We update the wage by 3.1 percent to adjust for the time elapsed between the measurement of the respondents' earnings in March 2008 to the median survey month,
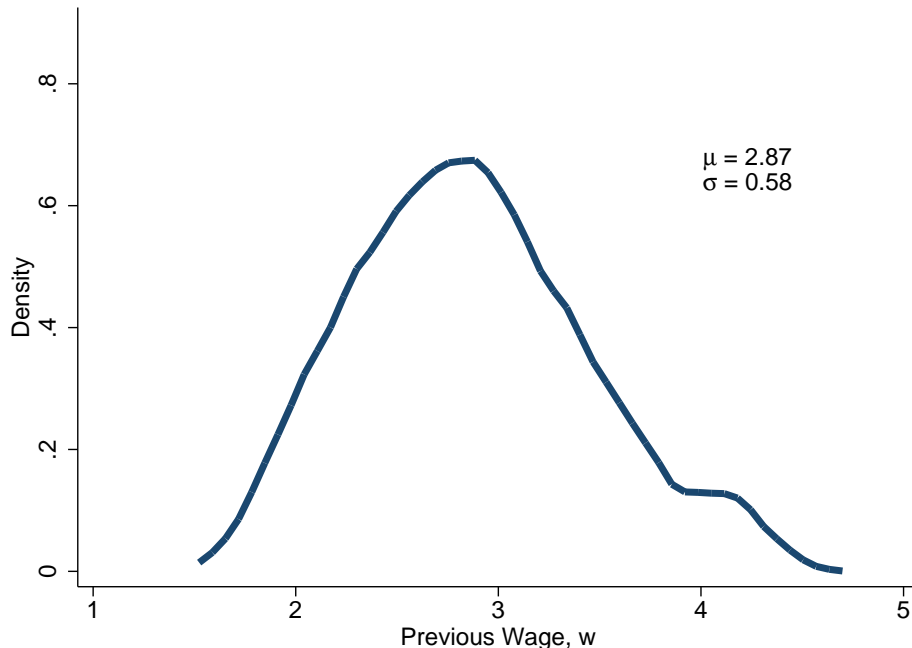
Figure 4: Kernel Density of the Log Hourly Wage on the prior Job, w

November 2009, based on the Bureau of Labor Statistics Employment Cost Index, which is adjusted for changes in the composition of employment.

2.5 Moments

Table 1 shows the moments of the data that are the targets for matching with the model. The moments for the acceptance frequency are taken from the predicted values of the polynomial of degree 6 evaluated at two values of d.

3 Model

We formulate the model in terms of the logs of the variables. We let x be the log of the personal productivity of a worker and assume it is known to employers and to the worker. When we refer to standardization for productivity, we mean the population with $x = 0$. Much of our discussion involves standardized variables. Where necessary to avoid confusion, we use a hat (ˆ) when we are referring to variables or their distributions in the entire population, not standardized for productivity. For the standardized variables, we denote the log of the offered wage as y, the log of the reported reservation wage as r, the log of the prior wage as w,

Moment	Symbol	Value
Mean offered wage	$m_{\hat{y}}$	2.75
Mean reservation wage	$m_{\hat{r}}$	2.82
Mean previous wage	$m_{\hat{w}}$	2.87
Standard deviation of offered wage	$s_{\hat{y}}$	0.525
Standard deviation of reservation wage	$s_{\hat{r}}$	0.474
Standard deviation of previous wage	$s_{\hat{w}}$	0.583
Covariance of offered wage and reservation wage	$c_{\hat{y},\hat{r}}$	0.183
Covariance of offered wage and previous wage	$c_{\hat{y},\hat{w}}$	0.183
Covariance of reservation wage and previous wage	$c_{\hat{r},\hat{w}}$	0.199
Acceptance frequency at $d_1 = -1$	$\hat{A}_1$	0.262
Acceptance frequency at $d_2 = 0.5$	$\hat{A}_2$	0.856
Fraction of rejections for non-wage reasons	$\hat{J}$	0.677

Table 1: Target Moments

and the log of the non-wage value as n. We proceed under the *proportionality-to-productivity hypothesis*:

> The distributions of $\hat{y} - x$, $\hat{r} - x$, and $\hat{w} - x$ in the population with personal productivity x are the same as the distributions of y, r, and w in the sub-population with $x = 0$.

The most controversial aspect of this hypothesis is that non-market productivity is higher by the entire amount of market productivity in the population with higher values of x. Low-x populations are not systematically more choosy about taking jobs than are high-x populations. While this assumption obviously fails if applied across the entire population including those out of the labor force, it appears reasonable in a sample of workers eligible for unemployment compensation. Moreover, we find that the average acceptance rates do not differ systematically across different levels of educational attainment, and toward the end of the paper, we report robustness checks that suggest that the non-proportionality in our sample is not important.

3.1 Job acceptance

We use the term "offer" to describe a job-seeker's encounter with a definite opportunity to take a job. Nothing in this paper requires that employers make firm job offers and that job-seekers then make up-or-down decisions. The job-seeker's decision problem, upon finding a job opportunity, is the same whether the employer is making a single firm offer, or they engage in alternating-offer bargaining. That said, the survey included a question about the nature of the job offer and in the majority of cases, the employer did make a firm offer.

3.1.1 Relation between the reservation job value and the reservation wage

An unemployed job-seeker decides about accepting a job offer by comparing the *job value* $v = y + n$ to a reservation value, r_v. The KM survey asks about a reservation wage, not a reservation job value. We make the *reference non-wage value hypothesis*:

> The reported reservation wage r is the reservation wage applicable to an offer with a reference value, $\bar{n}$, of the non-wage value, n.

We take the reference non-wage value to be $\bar{n} = 0$. This choice is only a normalization, because we estimate the mean of the distribution of n, μ_n. Because acceptance choices

conditional on reservation wages are the only evidence we have about non-wage values, we cannot distinguish between the mean of non-wage values and the reference level that respondents use in answering the question about the reservation wage. The fact that it is more common for an unemployed job-seeker to accept an offer below the reservation wage than reject one above the reservation wage is equally well explained by two views: (1) the distribution of non-wage values has a positive mean, or (2) the respondents use a high reservation wage on account of answering the question with respect to a hypothetical offer with a job value well below average.

With the normalization $\bar{n} = 0$, the reservation job value is the same as the reservation wage:

$$r_v = r. \tag{1}$$

3.1.2 Correlation between the offered wage and the non-wage value

The principle of compensating wage differentials suggests that the correlation between wage offers y and non-wage values n should be negative—employers offer lower wages for jobs with favorable non-wage values. The correlation is not perfect, however, because there is a personal dimension to the non-wage value that the firm may ignore, under a posted-wage policy, or respond to only partially, in a bargained-wage policy. For example, commuting cost varies across individual workers. For this reason, we assume that the non-wage value n comprises (1) a component η that is uncorrelated with the other fundamentals and (2) a component that is the negative of a fraction κ of the offered wage minus its mean:

$$n = \eta - \kappa(y - \mu_y). \tag{2}$$

3.1.3 Relation between the reservation wage and acceptance probability

Many job-seekers accept wage offers below the reservation wage and a smaller fraction reject offers above the reservation wage. The main way we account for the first group is that the distribution of non-wage values has a positive mean but respondents use zero job value for the hypothetical job that lies behind the survey's question about the reservation wage. Our acceptance model accounts for the acceptances and rejections that appear contrary to the reservation wage in two ways. First, we invoke a non-wage value that is imperfectly correlated with the offered wage. Second, we attribute measurement errors to the reported

values of the offered wage and the reservation wage. We assume that $\hat{y}$ and $\hat{r}$ are:

$$\hat{y} = y + x + \epsilon_y \tag{3}$$

$$\hat{r} = r + x + \epsilon_r \tag{4}$$

where the measurement errors $\epsilon_y \sim N(0, \sigma_{\epsilon_y})$ and $\epsilon_r \sim N(0, \sigma_{\epsilon_r})$, and are independent.

Let $d = \hat{y} - \hat{r}$ be the difference between the offered wage and the reservation wage. Also let $m = v - r$ (recall that $v = w + n$, the job value). We write the acceptance probability A as a function of d:

$$\begin{aligned} A(d) &= \text{Prob}[m \geq 0|d] \\ &= 1 - \text{Prob}[0 \geq m|d] \\ &= 1 - F_m(0|d). \end{aligned} \tag{5}$$

In the case where $\kappa = 0$ and there is no measurement error, the probability of acceptance of a job offer is

$$A(d) = \text{Prob}[y + n \geq r] = \text{Prob}[y - r \geq -n] = \text{Prob}[n \geq -d] \tag{6}$$

Then we can write

$$A(d) = 1 - F_n(-d) \tag{7}$$

Thus we can calculate F_n directly from the acceptance function:

$$F_n(n) = 1 - A(-n). \tag{8}$$

In this case, F_n, a theoretical distribution referring to individuals with productivity $x = 0$, turns out to be equal to the observed function $A(y - r)$ relating the acceptance probability to the gap between the offered wage y and the reservation wage r, both observed. This identification rests on the proportionality-to-productivity hypothesis and our assumption about what respondents mean by their reservation wages. Notice that $A(y - r)$ is the same for all values of x, because subtracting x from both y and r leaves the difference unchanged.

3.1.4 Wage and non-wage reasons for rejection

The shape of the acceptance function does not separately identify the dispersion of the idiosyncratic part of non-wage values, σ_η, and the compensating differential parameter κ, as higher values of either parameter imply a flatter acceptance function. For this reason, we use

information on the fraction of rejections for non-wage reasons to identify these parameters. We adopt the *preponderant reason for rejection hypothesis*:

> Respondents report that they rejected a job offer for a non-wage reason if the deviation from the mean is more negative for the non-wage value than for the wage value: $n - \mu_n < y - \mu_y$.

Let $p = (\eta - \mu_\eta) - (y - \mu_y)(1 + \kappa)$. The fraction of rejections for non-wage reasons for a person with reservation wage r, denoted J_r, is:

$$
\begin{aligned}
J_r &= P(\text{non-wage preponderates} \mid \text{offer rejected}) \\
&= P(n - \mu_n < y - \mu_y \mid v < r) \\
&= \frac{P(p < 0 \text{ and } v < r)}{P(v < r)} \\
&= \frac{\int_{-\infty}^{v=r} P(p < 0 \mid v) dF_v(v)}{P(v < r)} \\
&= \int_{-\infty}^{v=r} \frac{F_p(0 \mid v)}{F_v(r)} dF_v(v),
\end{aligned}
$$

and integrating over the distribution of r, we get:

$$
J = \int_{-\infty}^{\infty} \int_{-\infty}^{v=r} \frac{F_p(0 \mid v)}{F_v(r)} dF_v(v) dF_r(r). \tag{9}
$$

4   Estimation

We estimate the parameters of the distributions of the four variables y, r, η, and x, and the compensating-difference parameter κ. Later, we extend the analysis to include a calibrated job-ladder model to compute the distribution of the value of non-market activities and to examine how well the job-ladder model matches the observed moments of the distribution of the prior wage.

We take the distributions of the four variables, y, r, η, and x to be log-normal and independent. We normalize the mean of x to zero. The other three means, μ_y, μ_r and μ_η; the standard deviations, σ_y, σ_r, σ_η, and σ_x; and κ (the relation of the non-wage value n to the offered wage y,), are parameters to estimate, for a total of 8. We match the following 8 data moments: the means $m_{\hat{y}}$ and $m_{\hat{r}}$, standard deviations $s_{\hat{y}}$ and $s_{\hat{r}}$ of the two directly observed variables, the covariance $c_{\hat{y},\hat{r}}$, the two values $\hat{A}_1$ and $\hat{A}_2$ of the acceptance frequency, and the fraction of rejections for non-wage reasons, $\hat{J}$. We infer κ and the moments of η by

picking two values d_1 and d_2 and solving equation (5) and equation (9) to match the fraction of rejections for non-wage reasons in the data.

We allow for measurement error in the reservation wage and the offered wage, by assuming that 13 percent of the total variation in offered wages and reservation wages is due to measurement error, which corresponds to the estimate in measurement error in Bound and Krueger (1991) who compared survey data to administrative data.

To sum up, the model has 8 parameters to estimate: μ_y, μ_r, σ_y, σ_r, σ_x, μ_η, σ_η, and κ. The observed moments and their counterparts in the model are:

$$m_{\hat{y}} = \mu_y \tag{10}$$

$$m_{\hat{r}} = \mu_r \tag{11}$$

$$s_{\hat{y}} = \sqrt{\sigma_y^2 + \sigma_x^2 + \sigma_{\epsilon_{\hat{y}}}^2} \tag{12}$$

$$s_{\hat{r}} = \sqrt{\sigma_r^2 + \sigma_x^2 + \sigma_{\epsilon_{\hat{r}}}^2} \tag{13}$$

$$c_{\hat{y},\hat{r}} = \sigma_x^2 \tag{14}$$

$$A(d_i) = 1 - \Phi(0, \mu_{m|d_i}, \sigma_{m|d_i}), i = 1, 2 \tag{15}$$

$$J = \int_{-\infty}^{\infty} \int_{-\infty}^{r} \frac{\Phi(0, \mu_{p|v}, \sigma_{p|v})}{\Phi(r, \mu_v, \sigma_v)} \phi(v, \mu_v, \sigma_v) dv \phi(r, \mu_r, \sigma_r) dr. \tag{16}$$

Here $\Phi(x, \mu, \sigma)$ is the normal cdf and $\phi(x, \mu, \sigma)$ is the normal pdf. Note that the functions $\mu_{m|d}$, $\sigma_{m|d}$, $\mu_{p|v}$, and $\sigma_{p|v}$; and the values μ_v, and σ_v are functions of the 8 parameters to be estimated—see Appendix A for details.

To measure sampling variation, we calculate the bootstrap distribution of the estimates. In our actual estimation procedure, we compute our moments from two different samples: We take the moments $m_{\hat{r}}$ and $s_{\hat{r}}$ from the first interview for all unemployed workers in the survey who were not working or had not yet accepted a job offer, whereas we take $m_{\hat{y}}$, $s_{\hat{y}}$, $c_{\hat{y},\hat{r}}$, $\hat{A}_1$ and $\hat{A}_2$ from the sample of 1,153 job offers with information on the offered wage and on the lagged reservation wage. The standard bootstrap strategy applies to single samples. Accordingly, we use only the smaller sample. This smaller sample appears not to be biased, as $m_{\hat{r}} = 2.83$ and $s_{\hat{r}} = 0.47$, which are almost identical to the estimates in the bigger sample. In the smaller sample, $m_{\hat{w}} = 2.86$, which is also very close to the estimate in the bigger sample, and $s_{\hat{w}} = 0.61$, which is a little higher than in the bigger sample. For the bootstrap, we thus sample with replacement from the 1,153 job offers, and compute the moments in the data and in the model for 100 draws. The resulting bootstrap distribution provides an upper bound on the dispersion of our actual sampling distribution.

| Parameter | Explanation | The extent of measurement error: | | | |
| | | Baseline | | Large | |
		Estimate	(s.e.)	Estimate	(s.e.)
μ_y	Mean of wage offers	2.75	(0.03)	2.75	(0.03)
μ_r	Mean of reservation wages	2.83	(0.03)	2.83	(0.03)
μ_η	Mean of the independent component of non-wage value of wage offer	0.25	(0.07)	0.24	(0.06)
κ	Compensating differential	0.10	(0.34)	-0.32	(0.38)
σ_y	Standard deviation of the offered wage	0.24	(0.02)	0.20	(0.02)
σ_r	Standard deviation of the reservation wage	0.09	(0.05)	0.00	(0.04)
σ_η	Standard deviation of the independent component of non-wage value of wage offer	0.37	(0.09)	0.31	(0.08)
σ_x	Standard deviation of personal productivity	0.43	(0.02)	0.43	(0.02)
σ_v	Standard deviation of offered job values ($v = y + n$)	0.43	(0.11)	0.41	(0.08)

Table 2: Parameter Estimates

4.1 Estimation Results

Table 2 shows the estimation results. Our main findings are:

1. The dispersion in the offered wage among people with the same personal productivity is moderate but not small: $\sigma_y = 0.24$.

2. The dispersion in the reservation wage among people with the same personal productivity is small: $\sigma_r = 0.09$.

3. The dispersion of the independent component of the non-wage job value is substantial: $\sigma_\eta = 0.37$.

4. The dispersion of personal productivity is substantial: $\sigma_x = 0.43$.

5. There is a moderate amount of compensating wage differentials: $\kappa = 0.10$.

19

6. The mean value of the non-wage value of a job offer is 0.25 log points: $\mu_n = \mu_\eta = 0.25$.

The variance of observed offered wages decomposes as

$$s_{\hat{y}}^2 = 0.28 = \sigma_y^2 + \sigma_x^2 + 0.13s_{\hat{y}}^2 = 0.06 + 0.18 + 0.04. \tag{17}$$

Thus $0.18/0.28 = 66$ percent of the cross-sectional variance in offered wages is explained by dispersion in personal productivity x, and only $0.06/0.28 = 21$ percent is explained by differences in wage offers among workers with the same productivity, y. The remaining 13 percent is explained by measurement error. Our results, however, also show that there is substantial dispersion in the non-wage job values, with the dispersion of non-wage job values being larger than the dispersion in offered wages. Our estimates imply that the standard deviation of job values $v = y + n$ is 0.43, which is much larger than the standard deviation for offered wages y alone.

Our data do not identify the amount of measurement error—we rely on extrinsic evidence from Bound and Krueger (1991) about measurement errors in actual wages. Measurement error in reservation wages is potentially higher than measurement error in actual wages, if unemployed workers do not understand the intended meaning of the reservation-wage question or have different reference levels in mind when they express the reservation wage. Our identification strategy results in an estimate of the variance of personal productivity x of $\sigma_x^2 = 0.43^2 = 0.18$, which corresponds to 82 percent of the total variance of reservation wages ($s_{\hat{r}}^2 = 0.474^2 = 0.22$). The remaining 18 percent is thus a natural upper bound for measurement error in the reservation wage, so we report alternative estimation results in the second column of Table 2 under the assumption of measurement error equaling 18 percent of the total variation of wages. The table shows that our main results are not sensitive to the assumptions regarding measurement error. As expected, the standard deviation of offered wages y and reservation wages r are now somewhat smaller—a direct result of the assumption of the larger amount of measurement error. The estimated dispersion in non-wage values of job offers η is also somewhat smaller compared to the baseline case, but the estimate of κ becomes negative, indicating that the wage and non-wage values are positively correlated in job offers. The standard deviation of job values v, however, is almost the same as in the baseline case, as the changes in the estimated parameters κ and σ_η have offsetting effects on the estimate of σ_v. This finding suggests that one should be careful in interpreting the estimate of the parameter κ as evidence of compensating differentials, as it appears to

be a substitute for measurement error in explaining the shape of the acceptance function. However, our main parameters of interest, which are the dispersion of offered wages y and offered job values v, are little affected by the different assumptions about the extent of measurement error.

4.2 Robustness

In this section, we test the sensitivity of our results to a number of alternative identification assumptions and estimation procedures. In particular, an important identification assumption is that y, r and x are independently distributed and thus it is important to assess whether these assumptions are plausible and to test the robustness of the main results to deviations from these assumptions.

4.2.1 Independence of y and r

One important assumption in our estimation strategy is that—conditional on personal productivity x—offered wages and reservation wages are uncorrelated, that is, $\text{cov}(\hat{y}, \hat{r}|x) = 0$, as it implies that $\text{cov}(\hat{y}, \hat{r}) = \sigma_x^2$. One possible concern with this assumption is that it may not hold if the employer knows the outside option of the job-seeker and thus tailors the job offer accordingly. Evidence against this is that 76 percent of the survey respondents indicated that the offer was a take-it-or leave-it offer as opposed to 24 percent who said that some bargaining was involved over pay. In any case, our estimate of σ_y changed little when we restricted the sample to take-it-or leave-it offers only— $\sigma_y = 0.21$ as opposed to 0.24 in the baseline case.

A model where the employer knows the reservation wage of the job applicant also implies that $\text{cov}(\hat{y}, \hat{r}) > \text{cov}(\hat{y}, \hat{w})$, as the correlation between wages and the values of non-market activities will be dissipated through the process of on-the-job search and job-to-job transitions. The reason is that, while for an unemployed job seeker the value of non-market activities may, through bargaining, directly influence the final wage offered, for an employed job seeker the value of non-market activities is less relevant for the bargaining outcome as the employed worker's outside option is the value of the current job (it still matters to the extent that the value of non-market activities affected the current wage, but less so). However, as Table 1 shows, $\text{cov}(\hat{y}, \hat{r})$ and $\text{cov}(\hat{y}, \hat{w})$ are the same in the data.

Finally, in Appendix C.1, we study a model with Nash bargaining and find that our main results do not change in this case. The main reason for this result is that the variance of r is small, so it would require a high correlation of y and r to have a meaningful impact on the overall covariance of $\hat{y}$ and $\hat{r}$. In other words, as long as the worker's bargaining share α is not too close to 1, the estimate of σ_r will be small and thus the estimate of σ_x large, as in our baseline model.

A related concern with our estimation strategy may be that measurement error in y and r are correlated, which would also violate our assumption that $\text{cov}(\hat{y}, \hat{r}|x) = 0$. Recall that we exploit the longitudinal structure of the survey and use the reservation wage value reported in a week *prior* to the receipt of the job offer. In addition, in the presence of correlated measurement error, we would expect this correlation to be much larger for the pair (y, r) than for the pair (y, w). The reason is that the prior hourly wage is computed from administrative data on weekly wages and hours on last job reported in the first week of the survey. Thus, we gain confidence from the finding that $\text{cov}(\hat{y}, \hat{r}) = \text{cov}(\hat{y}, \hat{w}) = 0.183$.

4.2.2 Proportionality-to-productivity

As explained earlier, we make the assumption that the distributions of $\hat{y} - x$ and $\hat{r} - x$ in the population with personal productivity x are the same as the distributions of y and r. The most controversial aspect of this hypothesis is that non-market productivity is higher by the entire amount of market productivity in the population with higher values of x. One can test for the presence of non-proportionality in reservation wages by looking at the acceptance rates of job offers across different education levels. Under the proportionality-to-productivity assumption, the average acceptance rate should be the same across workers with characteristics associated with different market productivity x, as these workers should all be equally picky about accepting a job offer. We find, however, that the average acceptance rates do not differ systematically across different levels of educational attainment: The acceptance rate for those with a high-school degree or less is 72.6 percent, for those with some college education is 67.4 percent and for those with a college degree is 74.9 percent, and the differences are not statistically significant. These results are not consistent with a major deviation from the proportionality-to-productivity assumption.

In addition, we estimated the model with a set of moments based on deviations from a model relating wages to their determinants instead of the moments reported in Table 1 based on the wages themselves. More precisely, we ran a Mincer-type regression of the log

reservation and offered wage on years of schooling, potential experience, potential experience squared, and dummies for gender, marital status, race and ethnicity, and used the residuals of these regressions to compute the same moments as in Table 1 (except of course for the means, which we left unchanged from Table 1). One would expect the estimation results to change if the proportionality-to-productivity assumption does not hold in the data. To see this, consider the extreme case where the observable characteristics capture all the variance in productivity x. In this case, the proportionality-to-productivity assumption is not necessary for identification as the residualized moments of $\hat{y}$ and $\hat{r}$ are independent of x and thus directly capture the moments of interest (plus some measurement error). The results in Appendix Table 5, however, show that all estimated parameters are similar to the results in Table 2 except for the variance of x, which, as expected, is estimated to be substantially smaller, and the compensating differential parameter κ. Appendix Table 5 also shows sub-sample results for those with some college education and less as well as those with a college degree. The mean of the job offer distribution is 38 log points higher for those with a college degree compared to those with some college education or less, whereas the mean of the reservation wage is 41 log points higher (the difference of 38 log points is within sampling variation). The standard deviation of offered wages y is also similar across the two groups, though there is a big difference in terms of the compensating differential parameters κ. The reason is that the sample used to estimate the shape of the acceptance function is quite small and thus, the estimated parameters κ, μ_η and σ_η, which are identified of the shape of the acceptance function, have to be taken with caution in the sub-sample analysis. Overall, these results suggest that proportionality-to-productivity is a reasonable assumption.

Finally, we extend the model by allowing for non-proportionality in the reservation wage variable. This enables us to analyse whether deviations from the assumption of proportionality have an impact on the estimation results. We assume that $\hat{r} = (1 + \kappa_r)x + r + \epsilon_{\hat{r}}$ and use the same moment conditions to re-estimate the model (see Appendix C.3 for details) for different values of κ_r. The sub-sample analysis by education group gives some indication of the potential magnitude of the non-proportionality parameter κ_r. The point estimates of μ_y and μ_r for the two education groups imply that $\kappa_r = 0.087$, because the difference in μ_r is 0.413, which is slightly larger than the difference in μ_y of 0.38. The results shown in the Appendix show that the non-proportionality tends to raise the dispersions of y and η slightly, but the differences from the estimates of the baseline model where $\kappa_r = 0$ are small.

4.2.3 Identification of σ_η and κ

As discussed earlier, the shape of the acceptance function, $A(\hat{y} - \hat{r})$, does not separately identify σ_η and κ. The reason is that both parameters increase the likelihood that a high-wage offer is associated with a low non-wage value and thus both parameters increase the probability that a high-wage offer is rejected. σ_η raises the probability of rejection of a high-wage offer because it increases the variance of the non-wage values n, whereas κ raises the probability of rejection of a high-wage offer mainly because positive values lead to a negative correlation between the wage value y and the non-wage value n. For these reasons, we use the fraction of rejections for non-wage reasons, $\hat{J}$, as an additional moment to estimate the model in our base specification. To make sure that the model is identified, for a given σ_η, we estimated the 7 parameters μ_y, μ_r, σ_x, σ_y, σ_r, μ_η and κ by using the first 7 moment conditions above but not the moment condition for J. In Appendix Figure 7, we plot the fraction of rejections for non-wage reasons, J, for various values of the parameter σ_η. The figure shows that the value of J is strongly increasing in σ_η, demonstrating that the 8 parameters of the model are fully identified with this additional moment. The main reason that the fraction of rejections for non-wage reasons adds valuable information for separately identifying σ_η and κ is that, while higher values of both σ_η and κ make the acceptance functions flatter, the fraction of rejections for non-wage reasons depends mainly on σ_η, because it depends strongly on the relative importance of the idiosyncratic variance of y and n, but is not much affected by the correlation between y and n (and thus κ).

4.2.4 The acceptance function

We have investigated the sensitivity of the estimation results to the targeted moments for the acceptance frequency. For the baseline estimation of the model, we target the acceptance frequency at $d = -1$ and $d = 0.5$, where $d = \hat{y} - \hat{r}$. In additional results reported in Appendix Table 5, we target the acceptance frequency at four points ($d = -1$, $d = -0.5$, $d = 0.0$ and $d = 0.5$) instead of the two and minimize the sum of squared differences of the acceptance frequency at these points. We find that the estimated parameters are very similar to the ones in the baseline estimation.

4.2.5 Non-stationarity

In our baseline model, we assume a stationary environment for the unemployed job-seeker and thus abstract from forces that lead to changes in the reservation wages over the spell of unemployment. The limited duration of unemployment benefits, declining savings, or changes in the wage offer distribution throughout the spell of unemployment all could lead to declining reservation wages over the spell of unemployment. However, as documented in detail in Krueger and Mueller (2015), reservation wages for a given unemployed worker tend to decline little over the spell of unemployment, with point estimates ranging from 1.4 to 3.4 percent over a 25 week period. Moreover, a tendency for the flow value of non-work to change over the spell of unemployment should be reflected in the dispersion of non-work values, but our estimates show little dispersion in non-work values and thus are consistent with close to constant reservation wages over unemployment spells.

4.2.6 Flow versus stock sampling

Our sample is representative of the *stock* of unemployed workers in New Jersey in 2009, but it may be preferable to estimate the model on a sample representative of the *inflow* of unemployed individuals, as those with low reservation wages or characteristics associated with higher job-offer rates find jobs and thus leave the sample more quickly than those with high reservation wages and low job offer rates. To assess this issue, we divided our sample into short- and long-term unemployed individuals, using a cutoff duration of unemployment of 26 weeks at the start of the survey. While the short-term unemployed tend to be individuals with higher personal productivity, we find that the point estimates of our main parameters of interest are similar across the two groups and the differences are statistically ambiguous. We find that $\kappa = 0.27$ for both groups, σ_y is 0.23 for the short-term unemployed and 0.25 for the long-term unemployed, σ_r is 0.07 for the short-term unemployed and 0.12 for the long-term unemployed, and σ_η is 0.32 for the short-term unemployed and 0.41 for the long-term unemployed. Appendix Table 5 provides the details. An alternative way to investigate this issue would be to reweight the sample based on observable demographic characteristics, to make it representative of the inflow, but this would not account for the role of selection based on unobservable characteristics and, in any event, the sub-sample results provided here suggest that reweighting would make little difference.

5 The Job-Ladder Model

In this section, we extend the analysis to include the distribution of values in non-market activities and the distribution of the prior wage. To this purpose, we develop a job-ladder model where employed workers continue to search for better jobs, but search on the job is less effective than while unemployed. With this factor, the decision to take a job offer while unemployed involves a real-option element.

We denote the value of non-market activities as h. We keep this in dollars per hour rather than taking logs, because the log-normal distribution would not make sense—there is no reason to exclude negative values. We extend the proportionality-to-personal-productivity hypothesis to include $\frac{\hat{h}}{e^x}$.

Under the assumption of proportionality, the value functions of employed workers are proportional to personal productivity. Our next step is to derive the Bellman equations for an individual with $x = 0$. As before, those for individuals with other values of x scale in proportion. The Bellman equation for an unemployed person with non-work value h and offer rate λ_u adjusts the reservation job value r_v to include the lost option value associated with accepting a job offer while unemployed:

$$\rho U_h = h + \max_{r_v} \lambda_u \int_{r_v} (W_h(\tilde{v}) - U_h) dF_v(\tilde{v}). \tag{18}$$

On the left is the value of being unemployed, stated as the continuous-time discount rate ρ multiplied by the asset value U_h associated with being unemployed. On the right, the individual receives the non-work flow value h and finds the best reservation job value to maximize the flow value arising from the capital gain that occurs upon accepting a job. A higher r_v raises the capital gain but lowers the probability of receiving it.

The Bellman equation for a worker with non-work value h and offer rate λ_e is

$$\rho W_h(v) = e^v + \lambda_e \int_v (W_h(\tilde{v}) - W_h(v)) dF_v(\tilde{v}) - s(W_h(v) - U_h). \tag{19}$$

The worker automatically accepts any job with a value greater than the current job value, v, because there is no loss of option value. There is a flow value from the probability of finding a better job with capital gain $W_h(\tilde{v}) - W_h(v)$. There is also a flow probability s, the separation rate, of suffering the capital loss $W_h(v) - U_h$.

26

5.1 The distribution of values in non-market activities

The reservation value condition $U_h = W_h(r)$, defines a function $h = H(r)$ that relates the value of non-market activities h to the reported reservation wage r—see Appendix D for details. The cdf of the distribution of values in non-market activities, $F_h(h)$, satisfies

$$F_r(r) = F_h(H(r)), \tag{20}$$

so, from the estimated parameters of the distribution of reported reservation wage values, $F(r)$, and the function $H(r)$, we can compute the implied distribution of values in non-market activities, $F_h(h)$. Note that in the case where search on the job is equally effective as when unemployed, $\lambda_e = \lambda_u$, the model simplifies to $H(r) = e^r$ and thus $F_r(r) = F_h(e^r)$.

5.2 The stationary distribution of wages

We let $F_w(w)$ be the cdf of wages among workers with $x = 0$. An individual draws a non-work value h at the outset, associated with a reservation wage r through $h = H(r)$. A personal state variable records whether the individual is unemployed or employed. The flow value of the current job, $v = w + n$, is a second personal state variable for the employed. Jobs end because of the arrival of a better offer or through exogenous separation and a drop to the bottom of the ladder. The latter occurs with fixed probability s and sends the worker into unemployment at the bottom of the ladder.

Define

$$F_v(v) = \int f_{y,\eta} \left(\frac{v - \eta - \kappa \bar{y}}{1 - \kappa}, \eta \right) d\eta, \tag{21}$$

the cdf of a job offer with value v. Here $f_{y,\eta}(y, \eta)$ is the joint density of y and η. The probability in one period that an unemployed worker with a reservation value r will remain unemployed in the next period is

$$T_{uu}(r) = 1 - \lambda_u(1 - F_v(r)). \tag{22}$$

The probability that an unemployed individual will be at work in the succeeding period with a job value not greater than v' is

$$T_{ue}(v'|r) = \lambda_u(F_v(v') - F_v(r)). \tag{23}$$

The probability that an employed worker will be unemployed in the next period is

$$T_{eu} = s. \tag{24}$$

27

The probability that an employed individual will remain employed at the same job value with value v is

$$T_{ee}(v|v) = (1-s)[1 - \lambda_e(1 - F_v(v))]. \tag{25}$$

The probability that an employed individual will move to a better job with value $v' > v$ is

$$T_{ee}(v'|v) = (1-s)\lambda_e(F_v(v') - F_v(v)). \tag{26}$$

Let q be the compound state variable combining a binary indicator for unemployment/ employment and the job value v and let $T(q'|q,r)$ be its transition cdf derived above. The stationary distribution of q, $F_q(q|r)$ satisfies the invariance condition,

$$F_q(q'|r) = \int T(q'|q,r)dF_q(q|r). \tag{27}$$

Throughout, an integral without limits of integration is over the support of the integrand. The ergodic distribution of the job value for employed workers, $F_v(v|r)$, is the conditional distribution of v for values of q for employed workers.

The cdf of the wage, w, conditional on the job value v, is

$$F_w(w|v) = \frac{\int^w f_{y,\eta}(y, v - y(1-\kappa) - \kappa\bar{y})dy}{\int f_{y,\eta}(y, v - y(1-\kappa) - \kappa\bar{y})dy}. \tag{28}$$

The implied ergodic distribution for the wage is

$$F_w(w|r) = \int F_w(w|v)dF_v(v|r). \tag{29}$$

Finally, the distribution in the population with $x = 0$ is the mixture,

$$F_w(w) = \int F_w(w|r)dF_r(r) \tag{30}$$

and the distribution in the overall population is the mixture,

$$F_{\hat{w}}(\hat{w}) = \int F_w(\hat{w} - x)dF_x(x). \tag{31}$$

5.3   Calibration

We take the weekly offer arrival rate to be $\lambda_u = 0.058$ from the survey. We calculate the entry rate to unemployment, s, as

$$s = \frac{u}{1-u}\lambda_u a = 0.0041 \text{ per week}, \tag{32}$$

the weekly rate consistent in stationary stochastic equilibrium with an unemployment rate of $u = 0.09$ and the observed job-finding rate. This calculation omits job-finding from out-of-the-labor force and exits from unemployment and employment to out-of-the-labor force. We posit that $\lambda_e = 0.5\lambda_u$. While we do not have a direct estimate of the job offer rate while employed, this calibration matches the rate of job-to-job transitions in the data. To compute the monthly job-to-job transition rate in the data, we use the CPS monthly files for the years 2009 and 2010, and estimated the fraction of those who reported to work at a different employer than in the previous month, as in Fallick and Fleischman (2004). We adjusted the moments from the model for time aggregation. To make the weekly job-to-job transition rates in the model comparable to the monthly job-to-job transition rates in the CPS data, we aggregated the weekly job-to-job transition rates to monthly rates, taking into account that short unemployment spells of duration less than a month may be misleadingly counted as job-to-job transitions. We set the weekly discount rate $\rho = 0.001$, equivalent to an annual discount factor of 0.949.

5.4   Results

The job-ladder model has no new estimated parameters. We solve it with the estimated parameters reported in Table 2 and the calibrated values of λ_u, λ_e, s and ρ. We ask, what are the estimates of the distribution of the value of non-market activities h, and how well does the calibrated model match the additional moments in Table 1 including the prior wage? Table 3 answers this question:

1. The model exactly matches the mean of the wage on the previous job, $m_{\hat{w}}$, in the case of the moderate amount of measurement error.

2. The model is not capable of matching the standard deviation of the prior wage, $s_{\hat{w}}$. The fitted value is about 0.06 log points below the actual value of the moment for both values of measurement error. The job-ladder model implies that the dispersion of offered wages is larger than the dispersion of wages on the prior job, which is violated in the data. We abstract here from any other sources of wage dispersion that may arise during an employment spell, such as heterogeneous job tenure effects or variation in wages due to changes in job- and firm-specific productivity, which may account for the shortfall.

29

Parameter	Explanation	Actual values	The extent of measurement error:			
			Baseline		Large	
			Estimate	(s.e.)	Estimate	(s.e.)
μ_h	Mean of non-work values		$2.80	($4.97)	$3.71	($3.72)
σ_h	Standard deviation of non-work values		$2.62	($1.48)	$0.00	($1.16)
$m_{\hat{w}}$	Mean previous wage, adjusted for intervening wage growth	2.90	2.90	(0.06)	2.92	(0.05)
$\sigma_{\hat{w}}$	Standard deviation of previous wage	0.58	0.52	(0.02)	0.52	(0.02)
$\sigma_{\hat{y},\hat{w}}$	Covariance of offered wage and prior wage	0.183	0.183	(0.014)	0.183	(0.014)
$\sigma_{\hat{r},\hat{w}}$	Covariance of reservation wage and prior wage	0.199	0.183	(0.014)	0.183	(0.014)
T_{ee}	Monthly job-to-job transition rate (adjusted for time aggregation)	0.019	0.020	(0.002)	0.020	(0.001)

Table 3: Actual and Fitted Values of the Job-Ladder Model

3. The fitted value of the covariance of the offered and prior wages, $c_{\hat{y},\hat{w}}$ fits the observed value perfectly.

4. The fitted value of the covariance of the reservation and prior wages, $c_{\hat{r},\hat{w}}$ fits the observed value reasonably closely.

5. The bootstrap dispersion of the fitted values is quite small in all cases.

The model does also well in matching the job-to-job transition rates in the CPS data in 2009 and 2010. The mean of non-work values is positive but relatively small. Recall that it is stated in dollars per hour, not log points. Figure 5 shows the pdf of h implied by our calibrated job ladder model for our baseline calibration with a moderate amount of measurement error. While the dispersion in h is rather small, there is a substantial fraction of h's with negative values, supporting our choice to express the non-work values in dollars rather than logs.
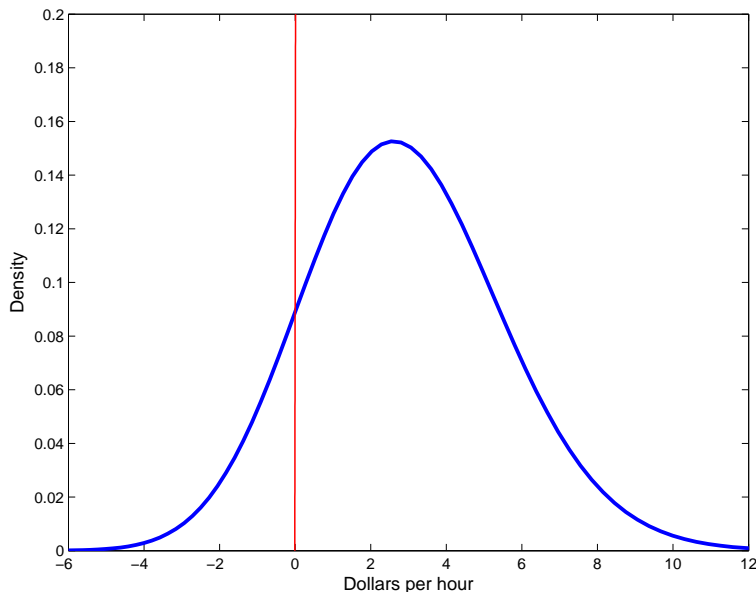
Figure 5: The Density of h

6 The Flow Value of Non-Work

Hornstein et al. (2011) take earlier authors to task for failing to observe that search models imply an extremely low, even negative, value of non-work. The essential point is that the dispersion of offered wages is high enough to justify sampling a large number of offers before picking the best, so that the observed time to acceptance only makes sense if waiting to go to work is painful. They note that the problem remains, though less acute, with on-the-job search.

In the search-and-matching literature, whose canon is Mortensen and Pissarides (1994), a variable often called z describes the relation between the flow value of remaining out of the labor market and the flow value of participating in the market. z is often taken as a parameter in these models. It is the ratio of the flow value of non-work to the mean of the marginal product of labor.

6.1 The implied value of z

In a model that recognizes that jobs have non-wage values, there is a potentially large divergence between the marginal product and the flow value of work depending on the specification of the employment bargain. On the one hand, if n is not included in the bargaining—for example, if n represents commuting distance or preferences over other given

31

job characteristics—then for a typical calibration of an MP-type model, as in Hall and Milgrom (2008), the ratio of the wage to the marginal product is 0.985. On the other hand, the flow value of work is potentially larger than the marginal product given our finding that μ_n is positive. If all relevant aspects of the job are included in the bargain, the ratio of the flow value of work—including non-wage amenities—to the marginal product is 0.985 for a typical calibration.

Table 4 shows the calculation of z for both versions of the model. Line 1 shows the value of non-work as estimated in that table, expressed in dollars per hour at the median of the distribution of h. Line 2a shows the median wage, whereas line 2b shows the median flow value of work. Note that there is a substantial difference between the two, suggesting that unemployed and employed workers select into jobs with higher non-wage values. This is not surprising given that the estimated variance of the idiosyncratic part of non-wage values is more than twice as large as the variance of offered wages. Line 3 gives an estimate of the marginal product, which is computed by dividing the estimates in lines 2a and 2b by 0.985. Line 4 reports the resulting value of z, the ratio of the value of non-work to the marginal product. The values are robustly positive, but considerable smaller than in the Hall-Milgrom calibration. Note that the results relying on earnings in Columns 1 and 3 depend on our normalization that $\bar{n} = 0$, whereas the results in Columns 2 and 4 don't, as higher assumed values of $\bar{n}$ translate into higher estimated values of μ_η.

Outside information about the value of z is scant. Chodorow-Reich and Karabarbounis (2015), a deep investigation of the time-series properties of z, is agnostic about its level. Hall and Milgrom (2008) finds a value of 0.71 based on an assumed functional form that satisfies certain elasticity conditions, but the resulting formula for z depends on differences in utility between non-workers and workers, which are not pinned down by the slopes they consider. If the Frisch constant-marginal-utility-of-consumption labor supply function is not a smooth curve in the hours-wage space, but has zero hours until the wage nears a reservation level and then shoots up, the value of z is much lower than Hall and Milgrom calculated.

Another important consideration is that the formula for z in Hall-Milgrom and Chodorow-Reich-Karabarbounis includes the replacement rate for unemployment insurance with a coefficient of one. Our sample is drawn from workers who receive benefits, so the replacement rate is likely to be higher than the 25 percent that Hall and Milgrom assume. The corresponding value of z is much higher— about equal to the median wage—with the 50-percent replace-

Step	Explanation	The extent of measurement error:			
		Baseline		Large	
1	Value of non-work at median for x=0, μ_h	2.80	2.80	3.71	3.71
2a	Earnings while employed, median for x=0, $\exp(m_w)$	18.52		18.52	
2b	Job value while employed, median for x=0, $\exp(m_v)$		34.90		34.90
3	Implied marginal product	18.80	35.43	18.80	35.43
4	Ratio of value of non-work to marginal product	0.15	0.08	0.20	0.10

Table 4: Ratio of the Flow Value of Non-Work to the Marginal Product of Labor

ment rate we believe is more realistic. We do not believe that z could possibly be that high, because it would imply, in the context of our model, that almost half the employed workers were earning less than their non-work values. Rather, it shows that the calibration does not give reasonable results with a higher replacement rate. This observation supports the proposition that Hall-Milgrom probably overstated z by choosing an unrealistic functional form for the Frisch supply function.

To the extent that our estimate that z in the range of 0.1 to 0.2 coincides or falls short of values derived from preferences, our assumption of the simple job-ladder model, with no option value to remaining unemployed, receives support.

As discussed in detail in HKV, the crucial parameter for the estimate of z is the offer rate while employed, λ_e, as it determines the option value of remaining unemployed in the event of receiving a job offer. For example, if we calibrated $\lambda_e = 0.7\lambda_u$, our estimate of z lies in the range of 0.28 to 0.53 instead of 0.08 to 0.15, while yielding a job-to-job transition rate of 2.4 percent, which is somewhat larger than in the CPS data at the time of the survey. See the Figure 6, which shows the job-to-job transition rates and values of z for values of $\frac{\lambda_e}{\lambda_u}$ ranging from 0.1 to 1.
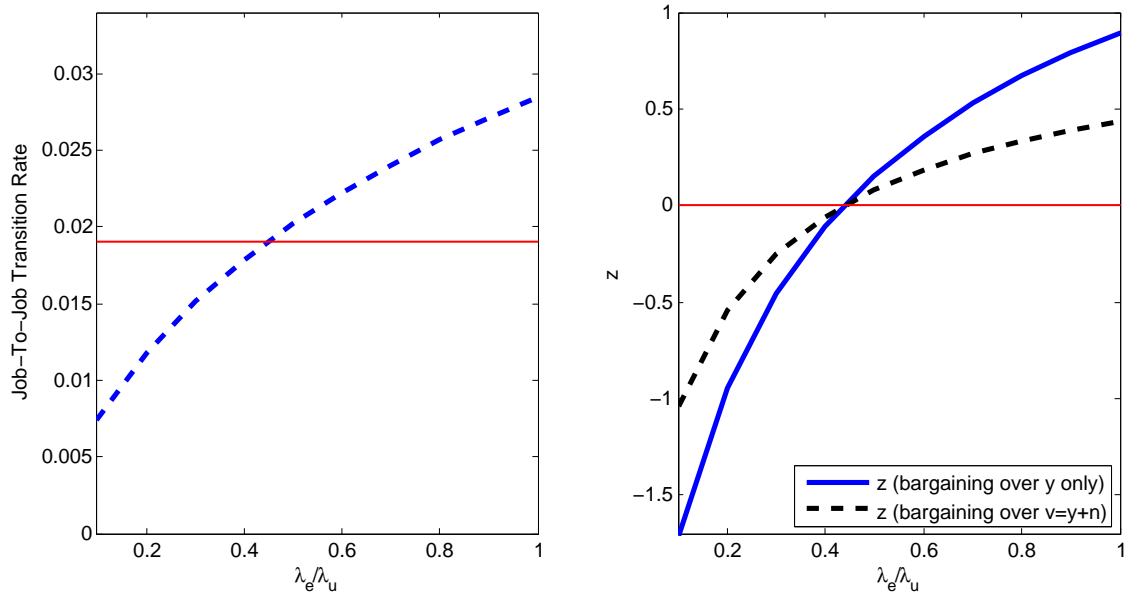
Figure 6: The Efficiency of Search On-the-Job and the Flow Value of Non-Market Time

6.2   Re-employment wages

Job-ladder models focus on employment spells—chains of jobs linked by job-to-job transitions. One feature that is common to most job-ladder models is that the combination of high wage dispersion and high offer rates while employed leads to substantial wage growth during an employment spell, as employed workers transition from low to high paying jobs. This feature leads to the prediction of a substantial drop in the wage when a worker falls off the job ladder and resumes employment at the bottom of the ladder after an unemployment spell. Our data do not support this property, as the mean accepted log wage is just 2.81 compared to the mean log wage on the prior job of 2.90, adjusted for real wage growth as in Table 3. Our model, on the other hand, perfectly matches the mean wage on the prior job as wages do not grow much during a spell of employment despite the job-to-job transitions. The reason is that the dispersion in the idiosyncratic part of non-wage values is larger than the dispersion in offered wages alone, and thus non-wage values tend to dominate wages in the acceptance decision. In other words, employed workers in our model transition frequently from one job to the next, but mostly because new jobs offer higher non-wage values rather than higher wages, and while there is little growth in wages over the course of an employment spell, non-wage values grow substantially, as can be seen from comparing lines 2a and 2b in Table 4. As emphasized earlier in the paper, we think of non-wage values not only

as comprising employee benefits such as health insurance, but also preferences over other characteristics of the job, such as commuting distance, relationships with co-workers, and the flexibility of the work schedule. What we label as *non-wage* values may also capture differences in the chances of promotion and pay raises at a future date within the same firm, as in the model of Cahuc, Postel-Vinay and Robin (2006).

7   Concluding Remarks

The KM data provide a novel view of unemployed workers' search behavior and the dispersion in potential wage offers they face when looking for a job. The data are unique—they contain direct information on reservation wages, job offers, and job acceptance decisions. The data on reservation wages permit identification of the variation in job offers that is due to differences in personal productivity. We use the job seeker's acceptance decisions to infer the dispersion in non-wage values and to account for the asymmetry in acceptance frequencies of offers above and below the previously reported reservation wage.

We find that the dispersion of the wage offer distribution is moderate, but larger than what HKV associate with the search model without on-the-job search. We find that the dispersion of the non-wage value in job offers is at least as large as the dispersion of wages. The implied overall dispersion in job values for a job seeker relative to the job-seeker's productivity is substantial. A related finding is that the implied value of non-market time, though not negative, is quite low—around 10 percent of a worker's productivity. We believe that this finding does not contradict other evidence about labor supply. We study an alternative specification of the job ladder model with lower job-finding efficiency among employed searchers, but find that the specification implies even lower values of non-work. We think these findings point in the direction of equal job-finding efficiency for on-the-job search. The pronounced tendency for job seekers to accept the first job offer they receive is inconsistent with the sacrifice of option value that occurs when a worker takes a job that interferes with subsequent on-the-job search. Evidence from audit studies that find higher call-back rates for employed job-seekers is consistent with our findings.

HKV noted that job-ladder models with sequential auctions, such as in Cahuc et al. (2006) weaken the link between the offer rate while employed and the estimate of z, as in these models firms may make counter-offers if a worker receives an outside offer. Outside offers lead to job-to-job transitions only if the outside offer comes from a more productive

firm, which can outbid the employee's current firm. See also Papp (2013) who provides a detailed analysis of this issue. Similarly, Christensen, Lentz, Mortensen, Neumann and Werwatz's (2005) model with endogenous search effort implies that workers further up the wage ladder search less and thus transition less frequently to other jobs. Therefore, these models can accommodate larger dispersion in wage offers with more reasonable values of z, as the data on job-to-job transitions do not imply a large option value of unemployment in these models.

We believe that our assumption that the distributions of key observed and latent variables are log-normal or normal is reasonable as a starting point for research on the multiple dimensions of wage dispersion, but the methods of this paper could be extended to other more flexible parametric distributions, such as mixtures of log-normal distributions. We also believe that our finding of high dispersion in non-wage job values shows the potential value of new surveys that collect data on the non-wage characteristics of job offers such as benefits, commuting time, hours, flexibility, job security, firm size, and promotion prospects.

References

Bonhomme, Stéphane and Grégory Jolivet, "The pervasive absence of compensating differentials," *Journal of Applied Econometrics*, 2009, *24* (5), 763–795.

Bontemps, Christian, Jean-Marc Robin, and Gerard J. van den Berg, "Equilibrium Search with Continuous Productivity Dispersion: Theory and Nonparametric Estimation," *International Economic Review*, 2000, *41* (2), 305–358.

Bound, John and Alan B. Krueger, "The Extent of Measurement Error in Longitudinal Earnings Data: Do Two Wrongs Make a Right?," *Journal of Labor Economics*, January 1991, *9* (1), 1–24.

Burdett, Kenneth, "A Theory of Employee Job Search and Quit Rates," *American Economic Review*, 1978, *68* (1), 212–220.

___ and Dale T. Mortensen, "Wage Differentials, Employer Size,and Unemployment," *International Economic Review*, May 1998, *39* (2), 257–273.

Cahuc, Pierre, Fabien Postel-Vinay, and Jean-Marc Robin, "Wage Bargaining with On-the-Job Search: Theory and Evidence," *Econometrica*, March 2006, *74* (2), 323–364.

Chodorow-Reich, Gabriel and Loukas Karabarbounis, "The Cyclicality of the Opportunity Cost of Employment," *Journal of Political Economy*, 2015. Forthcoming.

Christensen, Bent Jesper, Rasmus Lentz, Dale T. Mortensen, George R. Neumann, and Axel Werwatz, "On-the-Job Search and the Wage Distribution," *Journal of Labor Economics*, 2005, *23* (1), 31–58.

Fallick, Bruce and Charles A. Fleischman, "Employer-to-employer flows in the U.S. labor market: the complete picture of gross worker flow," 2004. Finance and Economics Discussion Series 2004-34.

Hagedorn, Marcus and Iourii Manovskii, "Search Frictions and Wage Dispersion," Manuscript, University of Zurich and University of Pennsylvania October 2010.

___ and ___ , "Job Selection and Wages over the Business Cycle," *American Econmic Review*, 2013. Forthcoming.

Hall, Robert E., "Lost Jobs," *Brookings Papers on Economic Activity*, 1995, (1), 221–273.

___ and Paul R. Milgrom, "The Limited Influence of Unemployment on the Wage Bargain," *American Economic Review*, September 2008, *98* (4), 1653–1674.

Hornstein, Andreas, Per Krusell, and Giovanni L. Violante, "Frictional Wage Dispersion in Search Models: A Quantitative Assessment," *American Economic Review*, 2011, *101* (7), 2873–98.

Hwang, Hae Shin, Dale T. Mortensen, and W. Robert Reed, "Hedonic Wages and Labor Market Search," *Journal of Labor Economics*, 1998, *16* (4), 815–847.

Jolivet, Grégory, "A Longitudinal Analysis of Search Frictions and Matching in the U.S. labor market," *Labour Economics*, 2009, *16* (2), 121–134.

___ , Fabien Postel-Vinay, and Jean-Marc Robin, "The Empirical Content of the Job Search Model: Labor Mobility and Wage Distributions in Europe and the US," *European Economic Review*, 2006, *50* (4), 877–907.

Krueger, Alan B. and Andreas I. Mueller, "Job Search, Emotional Well-Being, and Job Finding in a Period of Mass Unemployment: Evidence from High-Frequency Longitudinal Data," *Brookings Papers on Economic Activity*, Spring 2011, (1), 1–70.

___ and ___ , "A Contribution to the Empirics of Reservation Wages," *American Economic Journal: Economic Policy*, 2015. Forthcoming.

Lang, Kevin and Sumon Majumdar, "The Pricing of Job Characteristics When Markets Do Not Clear: Theory and Implications," *International Economic Review*, 2004, *45* (4), 1111–1128.

Low, Hamish, Costas Meghir, and Luigi Pistaferri, "Wage Risk and Employment Risk over the Life Cycle," *American Economic Review*, 2010, *100* (4), 1432–67.

Mortensen, Dale T., *Wage Dispersion: Why Are Similar Workers Paid Differently?*, Cambridge, Massachusetts: MIT Press: Zeuthen Lecture Book Series, 2003.

___ and Christopher Pissarides, "Job Creation and Job Destruction in the Theory of Unemployment," *Review of Economic Studies*, 1994, *61*, 397–415.

Papp, Tamas, "Frictional wage dispersion with Bertrand competition: an assessment," *Review of Economic Dynamics*, July 2013, *16* (3), 540–552.

Postel-Vinay, Fabien and Jean-Marc Robin, "Equilibrium Wage Dispersion with Worker and Employer Heterogeneity," *Econometrica*, 2002, *70* (6), 2295–2350.

Rogerson, Richard, Robert Shimer, and Randall Wright, "Search-Theoretic Models of the Labor Market: A Survey," *Journal of Economic Literature*, December 2005, *43*, 959–988.

Sorkin, Isaac, "Ranking Firms Using Revealed Preference," Manuscript, University of Michigan October 2015.

Sullivan, Paul and Ted To, "Search and Nonwage Job Characteristics," *The Journal of Human Resources*, 2014, *49* (2), 472–507.

Tjaden, Volker and Felix Wellschmied, "Quantifying the Contribution of Search to Wage Inequality," *American Economic Journal: Macroeconomics*, 2014, *6* (1), 134–161.

Appendixes

A Details on the Moment Conditions

The observed moments and their counterparts in the model are:

$$m_{\hat{y}} = \mu_y \tag{33}$$

$$m_{\hat{r}} = \mu_r \tag{34}$$

$$s_{\hat{y}} = \sqrt{\sigma_y^2 + \sigma_x^2 + \sigma_{\epsilon_{\hat{y}}}^2} \tag{35}$$

$$s_{\hat{r}} = \sqrt{\sigma_r^2 + \sigma_x^2 + \sigma_{\epsilon_{\hat{r}}}^2} \tag{36}$$

$$c_{\hat{y},\hat{r}} = \sigma_x^2 \tag{37}$$

$$A(d_i) = 1 - \Phi(0, \mu_{m|d_i}, \sigma_{m|d_i}), i = 1, 2 \tag{38}$$

$$J = \int \int^r \left[\Phi(0, \mu_{p|v}, \sigma_{p|v}) \frac{\phi(v, \mu_v, \sigma_v)}{\Phi(r, \mu_v, \sigma_v)} dv \right] \phi(r, \mu_r, \sigma_r) dr. \tag{39}$$

where $m = v - r$, $d = \hat{y} - \hat{r}$ and $p = (\eta - \mu_\eta) - (y - \mu_y)(1 + \kappa)$. The functions $\mu_{m|d}$, $\sigma_{m|d}$, $\mu_{p|v}$ and $\sigma_{p|v}$ are determined by the parameters μ_y, μ_r, σ_y, σ_r, μ_η, σ_η, κ, $\sigma_{\epsilon_{\hat{y}}}^2$ and $\sigma_{\epsilon_{\hat{r}}}^2$, as follows:

$$\mu_{m|d} = \mu_m + \frac{\sigma_{m,d}}{\sigma_d^2}(d - \mu_d) \tag{40}$$

$$\sigma_{m|d}^2 = \sigma_m^2 - \frac{\sigma_{m,d}^2}{\sigma_d^2} \tag{41}$$

$$\mu_{p|v} = \frac{\sigma_\eta^2 - (1 - \kappa)(1 + \kappa)\sigma_y^2}{\sigma_v^2}(v - \mu_v) \tag{42}$$

$$\sigma_{p|v}^2 = \sigma_y^2(1 + \kappa)^2 + \sigma_\eta^2 - \frac{(\sigma_\eta^2 - (1 - \kappa)(1 + \kappa)\sigma_y^2)^2}{\sigma_v^2}. \tag{43}$$

where

$$\mu_d = \mu_y - \mu_r \tag{44}$$

$$\mu_m = \mu_y + \mu_\eta - \mu_r \tag{45}$$

$$\sigma_d^2 = \sigma_y^2 + \sigma_{\epsilon_{\hat{y}}}^2 + \sigma_r^2 + \sigma_{\epsilon_{\hat{r}}}^2 \tag{46}$$

$$\sigma_m^2 = (1-\kappa)^2\sigma_y^2 + \sigma_\eta^2 + \sigma_r^2 \tag{47}$$

$$\sigma_{m,d} = (1-\kappa)\sigma_y^2 + \sigma_r^2 \tag{48}$$

$$\mu_v = \mu_y + \mu_\eta \tag{49}$$

$$\sigma_v^2 = (1-\kappa)^2\sigma_y^2 + \sigma_\eta^2. \tag{50}$$

B Additional Results

Table 5 and Figure 7 below show additional results:

1. Column 2 shows the estimation results using residualized data instead of the raw data
   for the moment conditions. To be precise, we ran Mincer-type wage regressions of
   the log hourly offered wage $\hat{y}$ and the log hourly reservation wage $\hat{r}$ on observable
   characteristics (years of schooling, potential experience, potential experience squared,
   and dummies for gender, marital status, race and ethinicity) and used the residuals of
   these regressions to compute the same moments as in Table 1 (except of course for the
   means, which we left unchanged from Table 1). One would expect the estimation results
   to change if the proportionality-to-productivity assumption does not hold in the data.
   To see this, consider the extreme case where the observable characteristics capture
   all the variance in productivity x. In this case, the proportionality-to-productivity
   assumption is not necessary for identification as the residualized moments of $\hat{y}$ and
   $\hat{r}$ are independent of x and thus directly capture the moments of interest (plus some
   measurement error). The results in Appendix Table 5, however, show that all estimated
   parameters are similar to the results in Table 2 except for the variance of x, which
   as expected is estimated to be substantially lower, and the compensating differential
   parameter κ, which is estimated with a lot of sampling error. Overall, these results
   suggest that proportionality-to-productivity is a reasonable assumption.

2. The results in columns 3 and 4 show the sub-sample results for those with some college
   education or less and for those with a college degree or more. The standard deviation

of offered wages is similar across the two samples, whereas the mean is about 38 log points higher (or 46 percent higher). The standard deviation of the reservation wage is small in both sub-samples, and the differences in the means of the reservation wage is 41.3 log points (or 51 percent). The remaining parameters differ more substantially across the two samples. In particular, the parameter κ differs between the two sub-samples, but sampling error was big even in the full sample and thus the sub-sample results with respect to this parameter are noisy.

3. The results in columns 5 and 6 show the sub-sample results for short-term and long-term unemployed workers, where short-term unemployed was defined as any individual who was unemployed for less than 26 weeks at the start of the survey. The results show that the results are very similar across the two samples.

4. The results in column 7 show the results where we target the acceptance frequency at 4 points ($d = -1$, $d = -0.5$, $d = 0.0$ and $d = 0.5$) instead of two. We then minimize the squared differences of the moments related to the job-acceptance decision. The results are similar to the baseline estimation results where we targeted the acceptance frequency at $d = -1$ and $d = 0.5$ only.

5. As explained in the main text, the shape of the acceptance function does not separately identify σ_η and κ, and thus we use the fraction of rejections for non-wage reasons, $\hat{J}$, as an additional moment. To make sure that the model is identified, for a given σ_η, we estimate the 7 parameters μ_y, μ_r, σ_x, σ_y, σ_r, μ_η and κ by targeting the first 7 moments in the data (the moment conditions 33-38 above). In Figure 7 we plot the fraction of rejections for non-wage reasons, J, for various values of the parameter σ_η. The figure shows that the value of J is strongly increasing in σ_η, demonstrating that the 8 parameters of the model are fully identified with this additional moment. The main reason that the fraction of rejections for non-wage reasons adds valuable information for identifying σ_η and κ separately is the following: On the one hand, higher values of both σ_η and κ make the acceptance functions flatter, because they both increase the likelihood that a high-wage offer is associated with a low non-wage value. On the other hand, the fraction of rejections for non-wage reasons mainly depends on σ_η, because it strongly depends on the relative importance of the idiosyncratic variance of y and n, but is not much affected by the correlation between y and n (and thus κ).
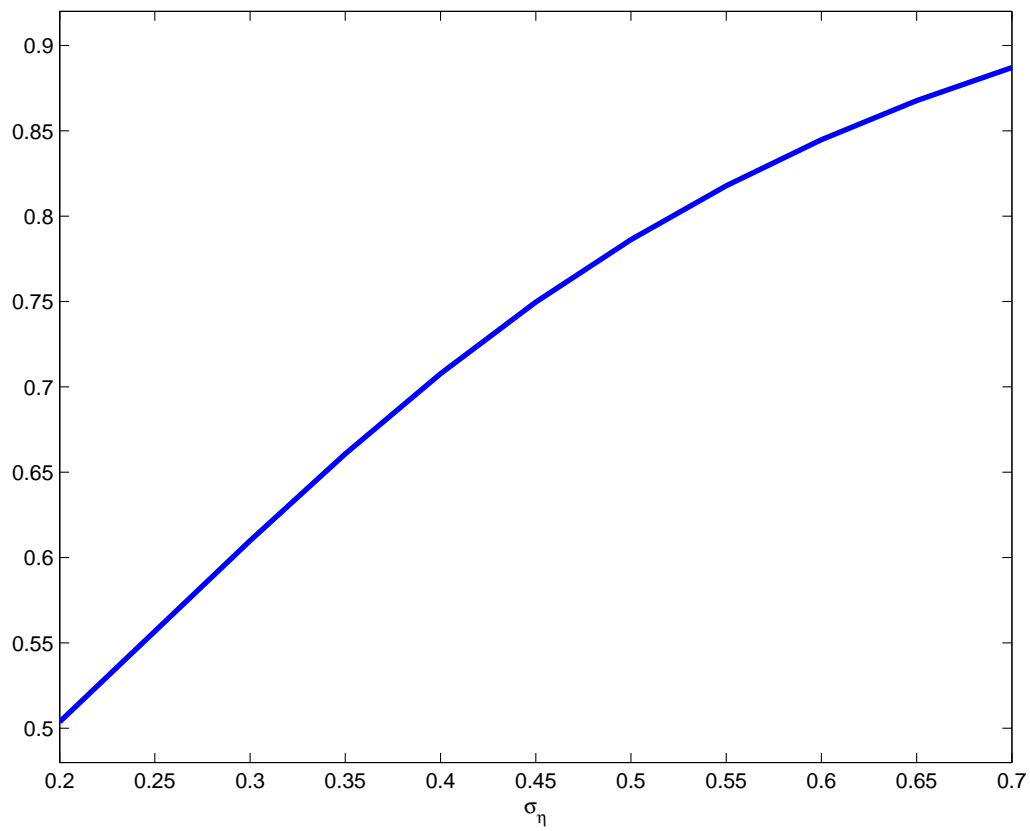
Figure 7: The Fraction of Rejections for Non-Wage Reasons for a given σ_η

Parameter	Explanation	Baseline	Residualized data	Some college education or less	College degree or more	Short-term unemployed	Long-term unemployed	$A(d_i)$ evaluated at 4 different points
μ_y	Mean of wage offers	2.75	2.75	2.60	2.98	2.85	2.62	2.75
μ_r	Mean of reservation wages	2.83	2.83	2.67	3.09	2.94	2.72	2.83
μ_η	Mean of the independent component of non-wage value of wage offer	0.25	0.23	0.26	0.29	0.25	0.24	0.33
κ	Compensating differential	0.10	0.42	1.03	-0.09	0.27	0.27	0.00
σ_y	Standard deviation of the offered wage	0.24	0.22	0.25	0.26	0.23	0.25	0.24
σ_r	Standard deviation of the reservation wage	0.09	0.12	0.13	0.00	0.07	0.12	0.09
σ_η	Standard deviation of the independent component of non-wage value of wage offer	0.37	0.32	0.35	0.25	0.32	0.41	0.35
σ_x	Standard deviation of personal productivity	0.43	0.32	0.33	0.46	0.44	0.38	0.43
σ_v	Standard deviation of offered job values	0.43	0.35	0.35	0.38	0.36	0.45	0.42

Table 5: Additional Estimation Results

C   Extensions

C.1   Nash bargaining with observable values of non-market activities

Our main model assumes that the value of non-market activities h is not known to the firm when making the wage offer and thus wage offers are independent of the reservation wage r. This assumption does not hold in the standard search-and-matching framework with Nash bargaining *if* the value of non-market activities h is known to the employer. In that case, the Nash bargaining solution implies that:

$$e^y = \alpha e^{p_f} + (1-\alpha)e^r, \tag{51}$$

where all variables are expressed in natural logarithms, p_f is the firm- or match-specific productivity, and α is the worker's bargaining share, assumed to be equal to 0.5. It is difficult to model rejections of offers in this environment, but we assume here that firms make wage offers even if $p_f < r$ and thus the offer is going to be rejected by the worker. Note also that we start here with a model where we assume that there are no non-wage amenities and thus only match the first five moments in equations 33-37 (see the next section for Nash bargaining with non-wage amenities). We also assume that there is no on-the-job search. In this model, the moment conditions are:

$$
\begin{align}
m_{\hat{y}} &= E(y(p_f, r)) \tag{52}\\
m_{\hat{r}} &= \mu_r \tag{53}\\
s_{\hat{y}} &= \sqrt{Var(y(p_f, r)) + \sigma_x^2 + \sigma_{\epsilon_{\hat{y}}}^2} \tag{54}\\
s_{\hat{r}} &= \sqrt{\sigma_r^2 + \sigma_x^2 + \sigma_{\epsilon_{\hat{r}}}^2} \tag{55}\\
c_{\hat{y}, \hat{r}} &= \sigma_x^2 + \text{cov}(y(p_f, r), r). \tag{56}
\end{align}
$$

There are two new parameters to be estimated in this model (μ_{p_f} and σ_{p_f}, instead of μ_y and σ_y in the baseline model). The most important change relative to the estimation of the baseline model is that now the covariance of $\hat{y}$ and $\hat{r}$ not only depends on the variance of x but also on the covariance of the bargained wage y and the reservation wage r. The estimation of the model yields a value of $\sigma_x = 0.42$, which is only slightly below the baseline estimate, and thus the remaining parameter estimates of the model are affected only to a minor degree. The main reason for this result is that the variance of r is small, thus it would

require a high correlation of y and r to have a meaningful impact on the overall covariance of $\hat{y}$ and $\hat{r}$. More precisely, one can reformulate the moment conditions such that

$$
\begin{aligned}
s_{\hat{r}}^2 - c_{\hat{y},\hat{r}} - \sigma_{\epsilon_{\hat{r}}}^2 &= \sigma_r^2 - \text{cov}(y(p_f, r), r) & (57) \\
&= \sigma_r^2(1 - \rho_{y,r}\frac{\sigma_y}{\sigma_r}). & (58)
\end{aligned}
$$

Given that the right hand side is relatively small and as long as the correlation coefficient $\rho_{y,r}$ (which is determined mainly by the worker's bargaining share α) is not too close to 1, the estimate of σ_r will be small and thus the estimate of σ_x large, as in our baseline model.

C.2 Nash bargaining with non-wage amenities

This sub-section extends the Nash-bargaining to a model with non-wage amenities in the total compensation package v. The Nash-bargained compensation package satisfies the following equation:

$$
e^v = \alpha e^{p_f} + (1-\alpha)e^r, \tag{59}
$$

where we imposed the normalization that $\bar{n} = 0$. Note that the Nash-bargain outcome v does not provide any guidance into whether y and n are positively or negative correlated. On the one hand, predetermined aspects of n would lead y and n to be negatively correlated (as the offered wage should compensate for non-wage values), whereas more productive employers may offer more of both and thus y and n may be positively correlated. We let χ be the predetermined part of the non-wage value and ψ be the part of the non-wage value that is determined in the Nash bargain (note that the notation here deviates from the main text; the variance of χ and ψ is captured in our baseline model by the parameters κ and σ_η). We further assume that employers use the following simple rule $y = \gamma_y(v - \psi)$ and $\chi = (1 - \gamma_y)(v - \psi)$ such that $v = y + \chi + \psi$. In this case:

$$
y = \gamma_y(ln(\alpha e^{p_f} + (1-\alpha)e^r) - \psi). \tag{60}
$$

As mentioned, the model's parameters γ_y and ψ do not directly map into the parameters of the baseline model and thus we would have to derive slightly different moment conditions for the moments involving acceptance and rejection. As a short cut, we calibrate the dispersion of non-wage values and investigate how the remaining parameters of the model are affected by the presence of non-wage amenities. To do this, we estimate the five parameters of the

model μ_{p_f}, σ_{p_f}, μ_r, σ_r and σ_x, for a given γ_y and a given σ_ψ, matching the moment conditions 52-56 above. Our main estimation results are:

1. For $\gamma_y = 1$ and $\sigma_\psi = 0$ (i.e., the baseline from Section C.1 above), we obtain the values $\mu_{p_f} = 2.56$, $\sigma_{p_f} = 0.58$, $\mu_r = 2.83$, $\sigma_r = 0.13$ and $\sigma_x = 0.42$.

2. For $\gamma_y = 0.5$ and $\sigma_\psi = 0$, we obtain the values $\mu_{p_f} = 6.15$, $\sigma_{p_f} = 0.53$, $\mu_r = 2.83$, $\sigma_r = 0.09$ and $\sigma_x = 0.43$.

3. For $\gamma_y = 1$ and $\sigma_\psi = 0.2$, we obtain the values $\mu_{p_f} = 2.61$, $\sigma_{p_f} = 0.37$, $\mu_r = 2.83$, $\sigma_r = 0.13$ and $\sigma_x = 0.42$.

4. For $\gamma_y = 0.5$ and $\sigma_\psi = 0.2$, we obtain the values $\mu_{p_f} = 6.15$, $\sigma_{p_f} = 0.48$, $\mu_r = 2.83$, $\sigma_r = 0.09$ and $\sigma_x = 0.43$.

These results indicate that adding non-wage amenities to the model in Section C.1 leaves our conclusion unchanged that Nash bargaining has little effect on the estimated level of σ_x. The reason is that a higher variance of non-wage amenities will lead to a lower estimated variance of p_f but the total variance of the offered wage $y(p_f, r, \psi)$ as well as the covariance of $y(p_f, r, \psi)$ and r is hardly affected.

C.3 Non-proportionality

In this Appendix, we drop the assumption that reservation wages are fully proportional to personal productivity x. Instead, we assume that

$$\hat{r} = x + r(x) + \epsilon_{\hat{r}} \tag{61}$$
$$r(x) = \kappa_r x + r. \tag{62}$$

The moment conditions are:

$$m_{\hat{y}} = \mu_y \tag{63}$$

$$m_{\hat{r}} = \mu_r \tag{64}$$

$$s_{\hat{y}} = \sqrt{\sigma_y^2 + \sigma_x^2 + \sigma_{\epsilon_{\hat{y}}}^2} \tag{65}$$

$$s_{\hat{r}} = \sqrt{\sigma_r^2 + (1+\kappa_r)^2\sigma_x^2 + \sigma_{\epsilon_{\hat{r}}}^2} \tag{66}$$

$$c_{\hat{y},\hat{r}} = (1+\kappa_r)\sigma_x^2 \tag{67}$$

$$A(d_i) = 1 - \Phi(0, \mu_{m|d_i}, \sigma_{m|d_i}), i = 1,2 \tag{68}$$

$$J = \int\int^r \left[\Phi(0, \mu_{p|v}, \sigma_{p|v})\frac{\phi(v, \mu_v, \sigma_v)}{\Phi(r, \mu_v, \sigma_v)}dv\right]\phi(r, \mu_r, \sqrt{\sigma_r^2 + \kappa_r^2\sigma_x^2})dr, \tag{69}$$

where $\Phi(x, \mu, \sigma)$ and $\phi(x, \mu, \sigma)$ stand for the cdf resp. the pdf of the normal distribution with mean μ and standard deviation σ, evaluated at x, and where $m = v - r(x)$, $d = \hat{y} - \hat{r} = y - r(x) + \epsilon_{\hat{y}} - \epsilon_{\hat{r}}$ and $p = (\eta - \mu_\eta) - (y - \mu_y)(1 + \kappa)$. The parameters $\mu_{m|d}$, $\sigma_{m|d}$, $\mu_{p|v}$ and $\sigma_{p|v}$ are determined by the parameters μ_y, μ_r, σ_y, σ_r, μ_η, σ_η, κ, κ_r, $\sigma_{\epsilon_{\hat{y}}}^2$ and $\sigma_{\epsilon_{\hat{r}}}^2$, as follows:

$$\mu_{m|d} = \mu_m + \frac{\sigma_{m,d}}{\sigma_d^2}(d - \mu_d) \tag{70}$$

$$\sigma_{m|d}^2 = \sigma_m^2 - \frac{\sigma_{m,d}^2}{\sigma_d^2} \tag{71}$$

$$\mu_{p|v} = \frac{\sigma_\eta^2 - (1-\kappa)(1+\kappa)\sigma_y^2}{\sigma_v^2}(v - \mu_v) \tag{72}$$

$$\sigma_{p|v}^2 = \sigma_y^2(1+\kappa)^2 + \sigma_\eta^2 - \frac{(\sigma_\eta^2 - (1-\kappa)(1+\kappa)\sigma_y^2)^2}{\sigma_v^2}, \tag{73}$$

where

$$\mu_d = \mu_y - \mu_r \tag{74}$$

$$\mu_m = \mu_y + \mu_\eta - \mu_r \tag{75}$$

$$\sigma_d^2 = \sigma_y^2 + \sigma_{\epsilon_{\hat{y}}}^2 + \sigma_r^2 + \sigma_{\epsilon_{\hat{r}}}^2 + \kappa_r^2\sigma_x^2 \tag{76}$$

$$\sigma_m^2 = (1-\kappa)^2\sigma_y^2 + \sigma_\eta^2 + \sigma_r^2 + \kappa_r^2\sigma_x^2 \tag{77}$$

$$\sigma_{m,d} = (1-\kappa)\sigma_y^2 + \sigma_r^2 + \kappa_r^2\sigma_x^2 \tag{78}$$

$$\mu_v = \mu_y + \mu_\eta \tag{79}$$

$$\sigma_v^2 = (1-\kappa)^2\sigma_y^2 + \sigma_\eta^2. \tag{80}$$

We estimate the model for different values of κ_r. The sub-sample analysis by education group gives some indication of the potential magnitude of the non-proportionality parameter. The point estimates in Appendix Table 5 of μ_y and μ_r for the two education groups imply that $\kappa_r = 0.087$, since the difference in μ_r between the two education groups is 0.413, which is slightly larger than the difference in μ_y of 0.38. The results for the model with $\kappa_r = 0.087$ in Table 6 show that the standard deviation of y and η are somewhat larger than in the baseline model with $\kappa_r = 0$ but the differences in estimates are relatively small.

It is important to note that there is a natural upper bound on κ_r as the moment conditions imply that

$$\kappa_r = \frac{s_{\hat{r}}^2 - \sigma_r^2 - \sigma_{\epsilon_{\hat{r}}}^2}{c_{\hat{y},\hat{r}}} - 1.$$

The upper bound occurs at $\sigma_r = 0$ and, for $\sigma_{\epsilon_{\hat{r}}}^2 = 0.13 s_{\hat{r}}^2$, we get an upper bound of

$$\kappa_r = \frac{0.87 s_{\hat{r}}^2}{c_{\hat{y},\hat{r}}} - 1 = 0.07.$$

For the calibrations in Table 6 where we assumed that $\kappa_r > 0.07$, we assumed that the measurement error was slightly smaller so as to meet the moment condition for $s_{\hat{r}}^2$.

Parameter	Explanation	Value of non-proportionality parameter				
		$\kappa_r = -0.05$	$\kappa_r = 0$	$\kappa_r = 0.05$	$\kappa_r = 0.087$	$\kappa_r = 0.10$
μ_y	Mean of wage offers	2.75	2.75	2.75	2.75	2.75
μ_r	Mean of reservation wages	2.83	2.83	2.83	2.83	2.83
μ_η	Mean of the independent component of non-wage value of wage offer	0.23	0.25	0.26	0.26	0.26
κ	Compensating differential	0.23	0.10	0.04	0.12	0.14
σ_y	Standard deviation of the offered wage	0.22	0.24	0.26	0.27	0.27
σ_r	Standard deviation of the reservation wage	0.13	0.09	0.00	0.00	0.00
σ_η	Standard deviation of the independent component of non-wage value of wage offer	0.33	0.37	0.40	0.41	0.42
σ_x	Standard deviation of personal productivity	0.44	0.43	0.42	0.41	0.41
σ_v	Standard deviation of offered job values	0.37	0.43	0.47	0.48	0.48

Table 6: Estimation results of the model with non-proportionality

D The Distribution of Values in Non-Market Activities

This section presents the details of the job-ladder model where search on the job is less effective than while unemployed, and derives in the context of this model the implied distribution of values in non-market activities h for a given distribution of reported reservation wages r. We start by defining the value functions of the unemployed and employed worker of type (h, x), where h stands for the flow value of non-work and x for personal productivity. The value functions in continuous time become:

$$\rho U_{h,x} = \hat{h}(h, x) + \max_{r_v} \lambda_u \int_{r_v} (W_{h,x}(k) - U_{h,x}) dF_v(k|x) \tag{81}$$

$$\rho W_{h,x}(v) = e^{\hat{v}(v,x)} + \lambda_e \int_v (W_{h,x}(k) - W_{h,x}(v)) dF_v(k|x) - \sigma[W_{h,x}(v) - U_{h,x}], \tag{82}$$

where ρ is the discount rate, λ_u the offer rate while unemployed, λ_e the offer rate while employed, $U_{h,x}$ the value of being unemployed for type (h, x), $W_{h,x}(v)$ the value of being employed with flow value $e^{\hat{v}(v,x)}$ for type (h, x). Note that $\hat{v}(v, x)$ is the log of the flow value during employment, whereas $\hat{h}(h, x)$ is the flow value during unemployment, which is expressed in absolute values in order to allow for negative values. Under the assumption of proportionality, $\hat{h}(h, x) = he^x$, $\hat{v}(v, x) = v + x$ and $dF_v(v|x) = dF_v(v)$, and thus:

$$\rho U_{h,x} = he^x + \max_{r_v} \lambda_u \int_{r_v} (W_{h,x}(k) - U_{h,x}) dF_v(k) \tag{83}$$

$$\rho W_{h,x}(v) = e^{v+x} + \lambda_e \int_v (W_{h,x}(k) - W_{h,x}(v)) dF_v(k) - \sigma[W_{h,x}(v) - U_{h,x}]. \tag{84}$$

The reservation value r_v satisfies $U_{h,x} = W_{h,x}(r_v(h, x))$ and thus:

$$\rho U_{h,x} = he^x + \lambda_u \int_{r_v(h,x)} (W_{h,x}(k) - W_{h,x}(r_v(h, x))) dF_v(k) \tag{85}$$

$$= e^{r_v(h,x)+x} + \lambda_e \int_{r_v(h,x)} (W_{h,x}(k) - W_{h,x}(r_v(h, x))) dF_v(k). \tag{86}$$

and solving for h, we get

$$h = e^{r_v(h,x)} - e^{-x}(\lambda_u - \lambda_e) \int_{r_v(h,x)} (W_{h,x}(k) - W_{h,x}(r_v(h, x))) dF_v(k). \tag{87}$$

We follow HKV and assume that there is a finite upper bound $\bar{v}$ to the offer distribution. Integrating by parts, as in the online Appendix of HKV, we get

$$h = e^{r_v(h,x)} - e^{-x}(\lambda_u - \lambda_e) \int_{r_v(h,x)}^{\bar{v}} (W_{h,x}(k) - W_{h,x}(r_v(h,x))))dF_v(k) \tag{88}$$

$$= e^{r_v(h,x)} - e^{-x}(\lambda_u - \lambda_e)[[(W_{h,x}(k) - W_{h,x}(r_v(h,x)))F_v(k)]_{r_v(h,x)}^{\bar{v}} - \int_{r_v(h,x)}^{\bar{v}} W'_{h,x}(k)dF_v(k)] \tag{89}$$

$$= e^{r_v(h,x)} - e^{-x}(\lambda_u - \lambda_e) \int_{r_v(h,x)}^{\bar{v}} W'_{h,x}(k)(1 - F_v(k))dk, \tag{90}$$

and differentiating $W_{h,x}(k)$, we get

$$W'_{h,x}(k) = \frac{e^{k+x}}{\rho + \sigma + \lambda_e(1 - F_v(k))},$$

and, therefore,

$$h = e^{r_v(h,x)} - (\lambda_u - \lambda_e) \int_{r_v(h,x)}^{\bar{v}} \left[\frac{1 - F_v(k)}{\rho + \sigma + \lambda_e(1 - F_v(k))} \right] e^k dk. \tag{91}$$

and thus $r_v(h,x) = r_v(h)$, which implies that one can define value functions U_h and $W_h(v)$ such that:

$$U_{h,x} = U_h e^x \tag{92}$$

$$W_{h,x}(v) = W_h(v)e^x. \tag{93}$$

As explained in the main text, we treat the reported reservation wage as the lowest wage a job-seeker will accept for a job with a reference level of its non-wage value of $\bar{n}$, and thus we can define a function $r(h)$, such that

$$r_v(h) = r(h) + \bar{n}. \tag{94}$$

The inverse function $H(r) = r^{-1}(h)$, will give the value of non-market activities implied by the job-ladder model for a given observed reservation wage r, and is defined by:

$$H(r) = e^{r+\bar{n}} - (\lambda_u - \lambda_e) \int_{r+\bar{n}}^{\bar{v}} \left[\frac{1 - F_v(k)}{\rho + \sigma + \lambda_e(1 - F_v(k))} \right] e^k dk, \tag{95}$$

where

$$H'(r) = e^{r+\bar{n}} \left(1 + (\lambda_u - \lambda_e) \frac{1 - F_v(r + \bar{n})}{\rho + \sigma + \lambda_e(1 - F_v(r + \bar{n}))} \right). \tag{96}$$

Therefore, given the distribution of reported reservation wages r, one can find the distribution of values of non-market activities h, by solving:

$$F_r(r) = F_h(H(r)), \tag{97}$$

$$f_r(r) = f_h(H(r))H'(r). \tag{98}$$

E The Distribution of Wages in the Job-Ladder Model

Let u be the fraction of the labor force unemployed and let $F_e(v|h)$ be the fraction of the labor force employed at a job value not higher than v. Note that $F_e(v|h)$ is not a cdf; rather, $F_e(\infty|h) = 1 - u(h)$, the fraction employed among those with non-work value h. The transition equation for the unemployment rate is

$$u(h)' = s(1 - u(h)) + [1 - \lambda_u(1 - F_v(r_v(h)))]u(h), \qquad (99)$$

so the ergodic unemployment rate is

$$u^*(h) = \frac{s}{s + \lambda_u(1 - F_v(r_v(h)))}. \qquad (100)$$

The transition equation for the value distribution is

$$F_e(v'|h) = \lambda_u(F_v(v') - F_v(r_v(h)))u + (1 - s)\int_{r_v(h)}^{v'} (1 - \lambda_e + \lambda_e F_v(v'))dF_e(v|h), \qquad (101)$$

The first term says that a fraction $\lambda_u(F_v(v') - F_v(r_v(h)))$ of the unemployed find jobs with values not greater than v'. The second term says that a fraction $1 - s$ of those currently employed at value no greater than v do not suffer an exogenous shock sending them into unemployment. Among the survivors, a fraction $1 - \lambda_e$ receive no offer and remain at value $v' = v$. A fraction $\lambda_e F_v(v)$ receive an offer no better than the current job and a fraction $\lambda_e(F_v(v') - F_v(v))$ take a better job with value no greater than v'. Then

$$F_e(v'|h) = \lambda_u(F_v(v') - F_v(h))u(h) + (1 - s)(1 - \lambda_e + \lambda_e F_v(v'))F_e(v'|h), \qquad (102)$$

because $F_e(r_v(h)|h) = 0$.

The ergodic distribution F_e satisfies

$$F_e(v|h) = \lambda_u(F_v(v) - F_v(r_v(h)))u^*(h) + (1 - s)(1 - \lambda_e + \lambda_e F_v(v))F_e(v|h). \qquad (103)$$

Integrating over h, we have

$$F_e(v) = \int^v \lambda_u(F_v(v) - F_v(r_v(h)))u^*(h)dF_h(h) + (1 - s)(1 - \lambda_e + \lambda_e F_v(v))F_e(v). \qquad (104)$$

where

$$F_e(v) = \int^v F_e(v|h)dF_h(h). \qquad (105)$$

Finally,

$$F_e(v) = \frac{\int^v \lambda_u(F_v(v) - F_v(r_v(h)))u^*(h)dF_h(h)}{1 - (1 - s)(1 - \lambda_e + \lambda_e F_v(v))}. \qquad (106)$$

E.1 F_v and dF_v

The distribution of job values among job offers is

$$F_v(v) = \int F_y(v - n)dF_n(n). \tag{107}$$

and its differential is

$$dF_v(v) = \int dF_y(v - n)dF_n(n). \tag{108}$$

E.2 F_e

The distribution of job values among the employed is

$$F_e(v) = \frac{N(v)}{D(v)}, \tag{109}$$

where

$$N(v) = \int^v \lambda_u(F_v(v) - F_v(r_v(h)))u^*(h)dF_h(h), \tag{110}$$

$$D(v) = 1 - (1 - s)(1 - \lambda_e + \lambda_e F_v(v)), \tag{111}$$

and

$$u^*(h) = \frac{s}{s + \lambda_u(1 - F_v(r_v(h)))}. \tag{112}$$

Then

$$dN(v) = \lambda_u dF_v(v) \int^v u^*(h)dF_h(h) \tag{113}$$

and

$$dD(v) = -(1 - s)\lambda_e dF_v(v). \tag{114}$$

Finally,

$$dF_e(v) = \frac{dN(v)}{D(v)} - \frac{N(v)dD(v)}{D(v)^2}. \tag{115}$$

E.3 $F_w(w)$

$$F_w(w) = \frac{1}{1 - u^*} \int \frac{\int_{-\infty}^w f_n(v - y)dF_y(y)}{\int_{-\infty}^\infty f_n(v - y)dF_y(y)} dF_e(v), \tag{116}$$

where $u^* = \int u^*(h)dF_h(h)$. Note that

$$F_w(w) = \frac{1}{1 - u^*} \int F_y(w|v)dF_e(v), \tag{117}$$

and thus with log-normal distributed variables, $F_y(y|v)$ is the cdf of a normal distribution with the following mean and variance:

$$\mu_{y|v} = \mu_y + \frac{(1-\kappa)\sigma_y^2}{(1-\kappa)^2\sigma_y^2 + \sigma_\eta^2}(v - \mu_y - \mu_\eta) \tag{118}$$

$$\sigma_{y|v}^2 = \frac{\sigma_y^2\sigma_\eta^2}{(1-\kappa)^2\sigma_y^2 + \sigma_\eta^2}. \tag{119}$$

E.4 $F_{\hat{r}}(\hat{r})$, $F_{\hat{y}}(\hat{y})$, $F_w(\hat{w})$

$$F_{\hat{r}}(\hat{r}) = \int F_h(H(\hat{r} - x))dF_x(x), \tag{120}$$

$$F_{\hat{y}}(\hat{y}) = \int F_y(\hat{y} - x)dF_x(x), \tag{121}$$

and

$$F_{\hat{w}}(\hat{w}) = \int F_w(\hat{w} - x)dF_x(x). \tag{122}$$