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STOCK MARKET PRICES DO NOT FOLLOW RANDOM WALKS: EVIDENCE FROM A SIMPLE SPECIFICATION TEST

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ABSTRACT

In this paper, we test the random walk hypothesis for weekly stock market returns by comparing variance estimators derived from data sampled at different frequencies. The random walk model is strongly rejected for the entire sample period (1962-1985) and for all sub-periods for a variety of aggregate returns indexes and size-sorted portfolios. Although the rejections are largely due to the behavior of small stocks, they cannot be ascribed to either the effects of infrequent trading or time-varying volatilities. Moreover, the rejection of the random walk cannot be interpreted as supporting a mean-reverting stationary model of asset prices, but is more consistent with a specific nonstationary alternative hypothesis.

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I. INTRODUCTION.

Since Keynes' now famous pronouncement in his General Theory that most investors' decisions "can only be taken as a result of animal spirits--of a spontaneous urge to action rather than inaction, and not as the outcome of a weighted average of benefits multiplied by quantitative probabilities,"[1] a great deal of research has been devoted to examining the efficiency of stock market price formation.[2] In Fama's (1970) survey, the vast majority of those studies were unable to reject the "efficient markets" hypothesis for common stocks. Although several seemingly anomalous departures from market efficiency have been well-documented,[3] many financial economists would agree with Jensen's (1978) belief that "there is no other proposition in economics which has more solid empirical evidence supporting it than the Efficient Markets Hypothesis."

Although the precise formulation of an empirically refutable efficient markets hypothesis is obviously model specific, historically the majority of such tests have focused on the forecastability of common stock returns. Within this paradigm, which has been broadly categorized as the "random walk" theory of stock prices, few studies have been able to statistically reject the random walk model. However, several recent papers have uncovered new empirical evidence which suggests that stock returns contain stationary or mean-reverting components. For example, Keim and Stambaugh (1986) find statistically significant predictable components in stock prices using forecasts based upon certain predetermined variables. In addition, Fama and French (1986) show that long holding-period returns are significantly negatively serially correlated, implying that 25 to 45 percent of the variation of longer-horizon returns is predictable from past returns.

In this paper, we provide further evidence that stock prices do not follow random walks by using a simple specification test based upon variance estimators. Our empirical results indicate that the random walk model is generally not consistent with the stochastic behavior of weekly returns, especially for the smaller capitalization stocks. However, in contrast to the negative serial correlation which Fama and French (1986) find for longer horizon returns, we find significant positive serial correlation for weekly and monthly holding-period returns. For example, using 1216 weekly observations from September 6, 1962 to December 26, 1985 we compute the weekly first-order autocorrelation coefficient of the equal-weighted CRSP index to be 30 percent! This empirical puzzle becomes even more striking when we show that it cannot possibly be attributed to either the effects of infrequent trading or heteroscedasticity.

Of course, these results do not necessarily imply that the stock market is inefficient or that prices are not rational assessments of 'fundamental' values. As Leroy (1973) and Lucas (1978) have shown, rational expectations equilibrium prices need not even form a martingale sequence, of which the random walk is a special case. Therefore, without a more explicit economic model of the price-generating mechanism, a rejection of the random walk hypothesis has few implications for the efficiency of market price formation. Although our test results may be interpreted as a rejection of <u>some</u> economic model of efficient price formation, there may exist other plausible models which are consistent with the empirical findings. Our more modest goal in this study is to employ a test which is capable of distinguishing among an interesting set of alternative stochastic price processes. In particular, our test exploits the fact the variance of the increments of a random walk is linear in the sampling interval. Therefore, if stock prices are generated by a random walk (possibly with drift) then, for example, the variance of monthly-sampled log-price relatives must be four times as large as the variance of a weekly sample. Comparing the (per unit time) variance estimates obtained from weekly and monthly prices may then yield some indication of the plausibility of the random walk theory.[4] Such a comparison is formed quantitatively along the lines of the Hausman (1978) specification test and is particularly simple to implement.

In Section 2 we derive our specification test for both homoscedastic and heteroscedastic random walks. As a guide to interpreting the empirical results, we present in Section 3 the results of simulation experiments which give the power of our test against two specific alternative hypotheses: a mean-reverting stationary process, and a more empirically plausible nonstationary process. The main results of the paper are given in Section 4, where rejections of the random walk are extensively documented for weekly returns indexes and size-sorted portfolios. Section 5 contains a simple model which demonstrates that infrequent trading cannot possibly account for the magnitude of the estimated autocorrelations of weekly stock returns. We summarize briefly and conclude in Section 6.

2. THE SPECIFICATION TEST.

Denote by P_t the stock price at time t and define $X_t \equiv \ln P_t$ as the log-price process. Our maintained hypothesis is given by the recursive relation:

$$X_t = \mu + X_{t-1} + \varepsilon_t$$

We assume throughout that for all t, $E[\varepsilon_t] = 0$. In the next section we develop our test under the null hypothesis that the ε_t's are independently and identically distributed with variance σ_0^2. However, because there is mounting evidence that financial time series often possess time-varying volatilities, we construct a test statistic which is robust to such heteroscedasticity in Section 2.2.

2.1 HOMOSCEDASTIC INCREMENTS.

We begin with the null hypothesis that the disturbances ε_t are independently and identically distributed normal random variables with variance σ_o^2 thus:

$$H: \quad \varepsilon_t \text{ i.i.d. } N(0, \sigma_o^2) \tag{1}$$

Note that, in addition to homoscedasticity, we have made the assumption of independent Gaussian increments. An example of such a specification is the exact discrete-time process X_t obtained by sampling the following well-known continuous-time process at equally spaced intervals:

$$dX(t) = \mu dt + \sigma_0 dW \ . \tag{2}$$

This Itô process corresponds to the popular lognormal diffusion price process often used in contingent claims analysis.

Suppose we obtain 2n+1 observations $X_0, X_1, \ldots, X_{2n}$ of X_t at equally spaced intervals. Consider the following estimators for the unknown parameters μ and σ_o^2:

$$\hat{\mu} \equiv \frac{1}{2n} \sum_{k=1}^{2n} (X_k - X_{k-1}) = \frac{1}{2n} (X_{2n} - X_0) \tag{3a}$$

$$\hat{\sigma}_a^2 \equiv \frac{1}{2n} \sum_{k=1}^{2n} [X_k - X_{k-1} - \hat{\mu}]^2 = \frac{1}{2n} \sum_{k=1}^{2n} (X_k - X_{k-1})^2 - \hat{\mu}^2 \tag{3b}$$

$$\hat{\sigma}_b^2 \equiv \frac{1}{2n} \sum_{k=1}^{n} [X_{2k} - X_{2k-2} - 2\hat{\mu}]^2 = \frac{1}{2n} \sum_{k=1}^{n} (X_{2k} - X_{2k-2})^2 - 2\hat{\mu}^2 \ . \tag{3c}$$

The estimators $\hat{\mu}$ and $\hat{\sigma}_a^2$ correspond to the maximum-likelihood estimators of the

μ and σ_0^2 parameters, whereas $\hat{\sigma}_b^2$ is also an estimator of σ_0^2 but uses only the subset of n+1 observations $X_0, X_2, X_4, \ldots, X_{2n}$. Observe that under standard asymptotic theory, all three estimators are strongly consistent. That is, holding all other parameters constant, as the number of observations 2n increases without bound the estimators converge almost surely to their population values. In addition it may readily be shown that both $\hat{\sigma}_a^2$ and $\hat{\sigma}_b^2$ possess the following Gaussian limiting distributions:

$$\sqrt{2n}\ (\hat{\sigma}_a^2 - \sigma_0^2) \overset{a}{\sim} N(0,\ 2\sigma_0^4) \tag{4a}$$

$$\sqrt{2n}\ (\hat{\sigma}_b^2 - \sigma_0^2) \overset{a}{\sim} N(0,\ 4\sigma_0^4)\ . \tag{4b}$$

Since the estimator $\hat{\sigma}_a^2$ is asymptotically efficient under the null hypothesis (1), we may form the usual Hausman-type specification test by considering the difference J_d of the two estimators where:

$$J_d \equiv \hat{\sigma}_b^2 - \hat{\sigma}_a^2 \tag{5}$$

for which the asymptotic variance is simply the difference of the asymptotic variances of the two respective estimators under the null hypothesis (1), i.e.:[5]

$$\sqrt{2n}\ J_d \overset{a}{\sim} N(0,\ 2\sigma_0^4)\ . \tag{6}$$

Using any consistent estimator of the asymptotic variance of J_d, a standard significance test may be performed. A more convenient alternative test statistic is given by the ratio of the variances J_r:[6]

$$J_r \equiv \frac{\hat{\sigma}_b^2}{\hat{\sigma}_a^2} - 1, \qquad \sqrt{2n}\ J_r \overset{a}{\sim} N(0,\ 2)\ . \tag{7}$$

Although the variance estimator $\hat{\sigma}_b^2$ is based upon differences of every other observation X_{2k}, alternative variance estimators may be obtained by using differences of every q-th observation. Specifically, suppose that we obtain nq+1 observations $X_0, X_1, \ldots, X_{nq}$ where q is any integer greater than 1. Define the estimators:

$$\hat{\mu} \equiv \frac{1}{nq} \sum_{k=1}^{nq} (X_k - X_{k-1}) = \frac{1}{nq} (X_{nq} - X_0) \tag{8a}$$

$$\hat{\sigma}_a^2 \equiv \frac{1}{nq} \sum_{k=1}^{nq} [X_k - X_{k-1} - \hat{\mu}]^2 = \frac{1}{nq} \sum_{k=1}^{nq} (X_k - X_{k-1})^2 - \hat{\mu}^2 \tag{8b}$$

$$\hat{\sigma}_b^2 \equiv \frac{1}{nq} \sum_{k=1}^{n} [X_{qk} - X_{qk-q} - q\hat{\mu}]^2 = \frac{1}{nq} \sum_{k=1}^{n} (X_{qk} - X_{qk-q})^2 - q\hat{\mu}^2 \tag{8c}$$

$$J_d(q) \equiv \hat{\sigma}_b^2 - \hat{\sigma}_a^2 \qquad\qquad J_r(q) \equiv \frac{\hat{\sigma}_b^2}{\hat{\sigma}_a^2} - 1 \,. \tag{8d}$$

The specification test may then be performed using the $J_d(q)$ and $J_r(q)$ statistics for which the asymptotic distributions are given by:[7]

$$\sqrt{nq}\; J_d(q) \overset{a}{\sim} N(0,\; 2(q - 1)\sigma_0^4) \tag{9a}$$

$$\sqrt{nq}\; J_r(q) \overset{a}{\sim} N(0,\; 2(q - 1)) \,. \tag{9b}$$

For practical purposes, two further refinements of the statistics J_d and J_r result in more desirable finite sample properties. The first is to use <u>overlapping</u> q-th differences of X_t in estimating the variances. Specifically, we define the following estimator of σ_o^2:

$$\hat{\sigma}_c^2 = \frac{1}{nq^2} \sum_{k=q}^{nq} [X_k - X_{k-q} - q\hat{\mu}]^2 \,. \tag{10}$$

Note that this differs from the estimator $\hat{\sigma}_b^2$ in that this sum contains nq - q + 1 terms whereas the estimator $\hat{\sigma}_b^2$ contains only n terms. By using overlapping q-th increments, we hope to obtain a more efficient estimator and hence a more powerful test. Using $\hat{\sigma}_c^2$ in our variance ratio test, we define the corresponding difference and ratio test statistics as:

$$M_d(q) \equiv \hat{\sigma}_c^2 - \hat{\sigma}_a^2 \qquad M_r(q) \equiv \frac{\hat{\sigma}_c^2}{\hat{\sigma}_a^2} - 1 . \tag{11}$$

The second and final refinement involves using unbiased variance estimators in the calculation of the M statistics. Although this does not yield an unbiased variance ratio, simulation experiments show that the finite-sample properties of the test statistics are closer to their asymptotic counterparts when this bias adjustment is made.[8] Indeed, according to the results of Monte Carlo experiments in Lo and MacKinlay (1987), the behavior of the bias-adjusted M statistics (which we denote as $\bar{M}_d(q)$ and $\bar{M}_r(q)$) does not depart significantly from that of their asymptotic limits even for small sample sizes. As a result, our empirical results are based only upon the $\bar{M}_r(q)$ statistic. Since it may be shown that the $\bar{M}_r(q)$ statistics have the following limiting distribution:

$$\sqrt{nq}\ \bar{M}_r(q) \overset{a}{\sim} N\left(0, \frac{2(2q-1)(q-1)}{3q}\right) \tag{12a}$$

we base our tests upon the standardized test statistic z where:

$$z \equiv \sqrt{nq}\ \bar{M}_r(q) \cdot \left(\frac{2(2q-1)(q-1)}{3q}\right)^{-\frac{1}{2}} \overset{a}{\sim} N(0, 1) . \tag{12b}$$

In order to develop some intuition for these variance ratios, observe that for an aggregation value q of 2, the $M_r(q)$ statistic may be re-expressed as:

$$M_r(2) = \hat{\rho}(1) - \frac{1}{4nh\hat{\sigma}_a^2}\left[(X_1 - X_0 - \hat{\mu}h)^2 + (X_{2n} - X_{2n-1} - \hat{\mu}h)^2\right] \tag{13}$$

hence for q = 2 the $M_r(q)$ statistic is approximately the first-order autocorrelation coefficient estimator $\hat{\rho}(1)$ of the differences. More generally, it may be shown that:

$$M_r(q) = \frac{2(q-1)}{q}\hat{\rho}(1) + \frac{2(q-2)}{q}\hat{\rho}(2) + \ldots + \frac{2}{q}\hat{\rho}(q-1) + o_p(n^{-\frac{1}{2}}) \qquad (14)$$

where $o_p(n^{-\frac{1}{2}})$ denotes terms which are of order smaller than $n^{-\frac{1}{2}}$ in probability and $\hat{\rho}(k)$ denotes the k-th order autocorrelation coefficient estimator of the first-differences of X. Equation (14) provides a simple interpretation for the variance ratios: they are particular linear combinations of the autocorrelation coefficients (plus or minus some asymptotically negligible terms) of first-differences. Specifically, variance ratios computed with an aggregation value q are (approximately) linear combinations of the first q-1 autocorrelation coefficients estimators of the first differences with arithmetically declining weights.[9]

2.2 HETEROSCEDASTIC INCREMENTS.

Since there is already a growing consensus among financial economists that volatilities do change over time,[10] a rejection of the random walk hypothesis due to heteroscedasticity would not be of much interest. We therefore wish to derive a version of our specification test of the random walk model which is robust to changing variances. Now it is evident that, as long as the increments are uncorrelated, even in the presence of heteroscedasticity the variance ratio must still approach unity as the number of observations increase without bound. This is simply due to the fact that the variance of the sum of uncorrelated increments must still equal the sum of the variances despite heteroscedastic increments.[11] However, the asymptotic variance of the variance ratios (or M statistics) will clearly depend upon the type and degree of heteroscedasticity present. One possible approach is to assume some specific form of heteroscedasticity (such as Engle's (1982) ARCH process), and then calculate the asymptotic variance of $\bar{M}_r(q)$ under this null hypothesis. However, in order to allow for more general forms of heteroscedasticity, we employ an approach due to White (1980) and White and Domowitz (1984). Specifically, in addition to some technical regularity conditions,[12] we assume:

$$H^*: \quad E[\varepsilon_t \varepsilon_s] = 0 \text{ for all } t \neq s \ . \tag{15}$$

This null hypothesis requires that X_t possess uncorrelated increments, but allows for quite general forms of heteroscedasticity, including deterministic changes in the variance (due, for example, to seasonal factors) and ARCH processes (in which the conditional variance depends upon past information).

Since $\bar{M}_r(q)$ still approaches 0 under H^*, we need only compute its asymptotic variance (call it θ) in order to perform the standard inferences. We do this in two steps. First, recall that the following equality obtains asymptotically:

$$\bar{M}_r(q) \overset{a}{=} \sum_{j=1}^{q-1} \frac{2(q-j)}{q} \cdot \hat{\rho}(j) \ . \tag{16}$$

Second, note that under H^* the autocorrelation coefficients $\hat{\rho}(j)$ are asymptotically uncorrelated.[13] Therefore, if we can obtain asymptotic variances $\delta(j)$ for each of the $\hat{\rho}(j)$ under H^*, we may readily calculate the asymptotic variance of $\bar{M}_r(q)$ as the weighted sum of the $\delta(j)$ where the weights are simply the weights in (16) squared. Using the results of White (1980) and White and Domowitz (1984), consistent estimators for $\delta(j)$ are given by:[14]

$$\hat{\delta}(j) = \frac{\sum_{k=j+1}^{nq} (X_k - X_{k-1} - \hat{\mu})^2 \cdot (X_{k-j} - X_{k-j-1} - \hat{\mu})^2}{\left[\sum_{k=1}^{nq} (X_k - X_{k-1} - \hat{\mu})^2\right]^2} . \tag{17}$$

Therefore, a consistent estimator $\hat{\theta}$ for the asymptotic variance of $\bar{M}_r(q)$ is then given by:

$$\hat{\theta} \equiv \sum_{j=1}^{q-1} \left[\frac{2(q - j)}{q}\right]^2 \cdot \hat{\delta}(j) . \tag{18}$$

We thus conclude that, even in the presence of general heteroscedasticity, the test statistic $z^* \equiv \sqrt{nq}\ \bar{M}_r(q)/\sqrt{\hat{\theta}}$ is still asymptotically standard normal. In Section 4 we use both the z and z* statistics to empirically test for random walks in weekly stock returns data.

3. POWER AGAINST FADS ALTERNATIVES.

Before applying our test to the data, we wish to examine its power against two specific alternative hypotheses. The first alternative is a mean-reverting process for prices which has been advanced by several authors as a model of stock market 'fads'.[15] For contrast, a second alternative price process with essentially opposite autocorrelation patterns is also explored. Since our test statistic involves variances of data sampled at different frequencies, it is natural to formulate the alternative hypotheses in continuous time. To simplify matters, both alternatives are homoscedastic hence we only consider the power of the z statistic and do not study the power of the heteroscedasticity-consistent statistic z*.

3.2 POWER AGAINST PRICE FADS.

As an alternative to the random walk model for asset prices, several recent studies have examined what Shiller (1981) describes as a 'fads' model; market prices fluctuate according to investors' fads which have exponentially decaying influence. In discrete time, this hypothesis has been implemented by supposing that deviations from the rational expectations of the present value of future earnings are autocorrelated as, for example, in Fama and French (1986) and Summers (1986). In continuous time, one representation of the fads model is given by the Ornstein-Uhlenbeck (O.U.) process for log-prices $X(t)$:

$$K_1: \qquad dX(t) = -\gamma_p\left[X(t) - \alpha_p\right]dt + \sigma_p dW(t) \qquad \gamma_p > 0 \tag{19}$$

which Shiller and Perron (1985) consider. To differentiate this process from our second alternative, we call K_1 the 'price fads' model. In order to develop some intuition for the empirical implications of this alternative, we report its first two population moments (all conditional upon $X(0) = X_0$):

$$E_0\left[X(t)\right] = \alpha_p + (X_0 - \alpha_p)e^{-\gamma_p t} \tag{20a}$$

$$VAR_0\left[X(t)\right] = \frac{\sigma_p^2}{2\gamma_p}(1 - e^{-2\gamma_p t}) \tag{20b}$$

$$COV_0\left[X(s),\ X(t)\right] = \frac{\sigma_p^2}{2\gamma_p}(1 - e^{-2\gamma_p s})\cdot e^{-\gamma_p (t-s)} \qquad s \le t\ . \tag{20c}$$

From (20a) we see that for large t the mean log-price $E\left[X(t)\right]$ tends to its steady state value of α_p, and the speed with which $E[X]$ reverts to this mean depends upon the parameter γ_p. It is well-known that the process of K_1 has the following exact discrete-time representation:

$$X_k = \alpha_p + \psi(h)\cdot(X_{k-1} - \alpha_p) + \zeta_k \tag{21}$$

where $X_k \equiv X(kh)$, $\psi(h) \equiv e^{-\gamma_p h}$, h is the (fixed) sampling interval, and ζ_k are independently and identically distributed Gaussian disturbances with expectation 0 and variance $\frac{\sigma_p^2}{2\gamma_p}(1 - e^{-2\gamma_p h})$. Observe that for more finely sampled data (smaller h) the autoregressive coefficient $\psi(h)$ becomes closer to unity. Such a process is precisely the continuous-time analogue of Phillips' (1986) discrete-time 'near-integrated' time series.

In performing our simulations, we choose parameter values $(\alpha_p, \gamma_p, \sigma_p)$ so that the resulting magnitudes of the above statistics correspond roughly to reasonable empirical values. Specifically, we assume that the weekly first-order autocorrelation coefficient for log-prices is 0.95, implying a weekly (steady-state) first-order autocorrelation coefficient of -0.025 for weekly <u>returns</u>. We also assume that the unconditional variance of weekly returns is 0.0004. These assumptions yield the parameter values $\gamma_p = 0.051$ and $\sigma_p^2 = 4.10 \times 10^{-4}$ for h = 1 week. Since the value of α_p does not affect our test statistics, we set it to zero without loss of generality. To develop further intuition for the implications of our parameter values, we report in Table 1a the implied steady-state first-order autocorrelation coefficients of <u>returns</u> under K_1 for a variety of holding-periods. The first row gives the serial correlation of weekly returns whereas the fourth row reports the serial correlation of monthly returns, etc. Note that the serial correlations are negative for all holding periods, as it must be under K_1. In addition, the absolute value of the autocorrelations <u>increases</u> monotonically with the holding period, so that a weekly serial correlation of -2.5 percent becomes -9.3 percent for monthly returns.

Using the above parameter values, we generate one realization of the price series according to (21) for a specific sample size, compute the test statistic z corresponding to the statistic $\bar{M}_r(q)$, repeat this procedure 5000 times, and examine the resulting frequency distribution of z. Table 2a reports the results for a variety of sample sizes, time spans, and aggregation values q. Panel A reports the test's power for data sets spanning 1216 weeks whereas Panels B and C report power results for data sets of 608 and 304 week time spans respectively. Within Panel A, the three subsections correspond to tests based upon different base intervals for the fixed time span of 1216 weeks. For example, the first sub-section reports results for base intervals of one week. The first row shows that comparing one-week variances to (one-half) two-week variances using a 5 percent test has 14.3 percent power. The first row of Panel A's second subsection shows that a 5 percent test based upon a two-week versus four-week variance comparison has 21.2 percent power.

An interesting pattern emerges from Table 2a. Within any fixed time span, the power of the test increases as the aggregation value q increases. For example, although the power of a 5 percent tests is 14.3 percent for $q = 2$ using a 1216 week time span, when q is 16 the power increases to 71.5 percent, almost quintupling its value at $q = 2$! Using the same time span with a two-week base period ($h = 2$), Panel A's second subsection reports that the same test ($q = 16$) has 90.8 percent power. Even with only 304 base observations (monthly, or $h = 4$) in the 1216 week time span, a 5 percent test with $q = 16$ has 97.6 percent power! Decreasing the time span decreases the power of the test as Panels B and C attest. Indeed, within a 304 week time span, the highest power which the 5 percent test ever achieves is 10.4 percent ($h = 4$, $q = 4$).

The explanation for this pattern of power increasing with q lies in the pattern of serial correlations of K_1 given in Table 1a. As we noted, the price fads model more closely resembles a random walk as the observation interval h decreases. This is confirmed by the serial correlations of Table 1a which grow farther away from the value 0 (implied by a random walk) as the holding period increases. Therefore, under this alternative it becomes easiest to detect departures from the random walk by comparing the most coarsely-sampled data to the finest. This corresponds to using larger aggregation values q.

3.2 POWER AGAINST RETURNS FADS.

One of the implications of K_1 is that continuously-compounded returns over any two non-overlapping holding periods are always <u>negatively</u> correlated and that the autocorrelation increases in absolute value for longer holding periods. Because it will become evident (see Section 4) that this pattern is inconsistent with the data, we consider another related alternative hypothesis which is empirically more relevant. Heuristically, this consists of modelling instantaneous returns as an O.U. process and deriving the log-price process by integration. More formally, let R(t) denote the instantaneous return of a security at time t with price P(t). Then we have:

$$R(t) = \frac{\dot{P}(t)}{P(t)} + \frac{D(t)}{P(t)} = \dot{X}(t) + \frac{D(t)}{P(t)} \tag{22}$$

where D(t) is the dividend flow of the security at time t. For simplicity, we assume that D(t) = 0 for all t so that the return consists solely of capital appreciation.[16] Observe that if the log-price process X(t) were any type of diffusion, the instantaneous return R(t) is no longer well-defined since the sample paths of P(t) are nowhere differentiable. However, if we begin by first specifying the dynamics of R(t), then equation (22) may be used to define the log-price process X(t). Specifically, we have:

$$dR(t) = -\gamma_r[R(t) - \alpha_r]dt + \sigma_r dW(t) \qquad \gamma_r > 0 \tag{23a}$$

$$K_2: \qquad X(t) \equiv X_0 + \int_0^t R(s)ds \ . \tag{23b}$$

We refer to this alternative as the 'returns fads' model of prices. Since R(t) is an O.U. process, its population moments are analogous to those in (20). However, note that in contrast to the price fads model K_1, under this alternative log-prices are explosive. Furthermore, the log-price process is qualitatively different under the two alternatives since, under the returns fads model the price process is (mean-square) differentiable whereas it is of unbounded variation under the price fads alternative.[17] The moments of the log-price process under the returns fads model K_2 are given by:

$$E_0[X(t)] = X_0 + \alpha_r t - \frac{1}{\gamma_r}[R(0) - \alpha_r]\cdot[1 - e^{-\gamma_r t}] \tag{24a}$$

$$VAR_0[X(t)] = \frac{\sigma_r^2}{\gamma_r^2} t + \frac{\sigma_r^2}{2\gamma_r^3}[1 - e^{-\gamma_r t}]\cdot[e^{-\gamma_r t} - 3] \tag{24b}$$

$$COV_0[X(s), X(t)] = \frac{\sigma_r^2}{\gamma_r^2} t + \frac{\sigma_r^2}{2\gamma_r^3}[2e^{-\gamma_r s} + 2e^{-\gamma_r t} - 2 - e^{-\gamma_r (t-s)} - e^{-\gamma_r (t+s)}] \qquad s \le t \ . \tag{24c}$$

Moreover, it may readily be shown that under this alternative all non-overlapping finite holding-period returns are positively correlated and that the autocorrelations decline as the holding-period increases. This is precisely opposite the pattern of the price fads autocorrelations.

As in the price fads case, we chose values of the parameters $(\alpha_r, \gamma_r, \sigma_r)$ which correspond roughly to reasonable empirical values. Specifically, we set $(\alpha_r, \gamma_r, \sigma_r) = (0.0040, 2.0, 0.040)$ for h = 1 week. This implies a steady-state mean of 0.004 for weekly returns, and a weekly variance of 0.0004. Table 1b reports first-order autocorrelation coefficients for various holding period returns. As we noted above, all the autocorrelations are positive in contrast to those of the price fads model. Moreover, they decline rapidly as the holding period increases, so that a weekly autocorrelation of 30.3 percent becomes a mere 7.1 percent with monthly returns.

Table 2b reports the power of our test against the returns fads alternative. Note that even with 304 weekly observations, using aggregation value q of 2 yields a 5 percent test with 99.1 percent power. However, in contrast to the price fads alternative, Table 2b shows that larger aggregation values general yields less powerful tests. For example, the power of a 5 percent test using 1216 weekly observations is 100 percent when q = 2, but declines to 64 percent when q increases to 64. This pattern is understandable in view of Table 1b's holding-period autocorrelations since they imply that, unlike the price fads model, the returns fads process behaves more like a random walk with coarser sampling. Therefore, a comparison of variance estimators based on coarser to finer data is less likely to reveal a returns fads. Indeed, the results in Table 2b imply that the simple first-order serial correlation coefficient of the returns (using the finest-sampled data) would yield a more powerful test than using any of the variance ratios with aggregation values larger than 2.

Of course, the power levels reported in Tables 2a and b are obviously parameter specific. Therefore, caution must be exercised in using these simulation results to compare the power of our test with those of the literature. This comparison is performed explicitly in Lo and MacKinlay (1987). In this paper, we present the power results for the two alternatives in order to highlight their qualitative differences. Whereas the absolute magnitude of the power depends critically upon the values chosen for γ_p and γ_r, the patterns of increasing or decreasing power in q do not depend upon any parameter values but are general properties of the alternative hypotheses. These patterns will prove to be important in interpreting the empirical results of the next section.

4. THE RANDOM WALK HYPOTHESIS FOR WEEKLY RETURNS.

To test for random walks in stock market returns, we focus on the 1216-week time span from September 6, 1962 to December 26, 1985. Our choice of a weekly observation interval was determined by several considerations. Since our sampling theory is wholly based upon asymptotic approximations, a large number of observations are required in order to obtain reasonably accurate inferences. Therefore, using monthly returns data even from the period 1926 to 1985 would only yield 720 observations of what most economists would consider a highly volatile time series. However, we also decided against the use of daily returns even though such a data set clearly contains many more observations. With a daily observation interval, the biases associated with non-trading, the bid-ask spread, asynchronous prices, etc. may become statistically significant. Without any formal model of the market micro-structure, it would be virtually impossible to obtain reliable inferences using daily (or finer-sampled) observations. Therefore, a weekly sampling interval seemed to be the ideal compromise, yielding a large number of observations but minimizing the effects of micro-structure biases.

The weekly stock returns are derived from the CRSP daily returns file. The weekly return of each security is computed as the return from Wednesday's closing price to the following Wednesday's close. If the following Wednesday's price is missing, then Thursday's price (or Tuesday's if Thursday's is missing) is used. If both Tuesday's and Thursday's prices are also missing, the return for that week is reported as missing.

In Section 4.1 we perform our test upon both equal and value-weighted CRSP indexes for the entire 1216-week period as well as 608-week and 304-week sub-periods using aggregation values q ranging from 2 to 64. Section 4.2 reports corresponding test results for size-sorted portfolios.

4.1 RESULTS FOR MARKET INDEXES.

Tables 3 and 4 report the variance ratios and test statistics z and z* for return indexes and size-sorted portfolios. Tables 3a and 3b display the results of the $\bar{M}_r(q)$ tests for CRSP NYSE-AMEX market indexes. Table 3a presents the results for a one-week base observation period (h = 1 week) and Table 3b contains corresponding results for a four-week base observation period (h = 4 weeks). Tables 4a and 4b report results of the variance ratio test for size sorted portfolios also with base observation periods of one and four weeks respectively. The values reported in the main rows are the actual variance ratios $(\bar{M}_r(q) + 1)$. The values enclosed in parentheses immediately below the main rows are corresponding z statistics and the second set of parenthetical entries below the first are the z* statistics which are robust to heteroscedasticity.

Consider Panel A of Table 3a which displays the results for the CRSP equal-weighted index. The first row presents the variance ratios and test statistics for the entire 1216 week sample period, the next two give the results for the two 608 week sub-periods, and the last four are the results for the four 304 week sub-periods. It is clear from Panel A that the random walk null hypothesis may be rejected at all the usual significance levels for the entire time period and all sub-periods. Moreover, the rejections are not due to heteroscedasticity since the z^* statistics also reject the random walk model. Also, note that the estimates of the variance ratio are larger than 1.0 for all cases. Specifically, consider the entries in the first column of Panel A. These correspond to variance ratios with an aggregration value q of 2. In view of equation (13), we may regard one minus this ratio as a consistent estimate of the first-order serial correlation coefficient of weekly returns. The entry in the first row, 1.30, implies that the first-order autocorrelation for weekly returns is approximately 30 percent. Since the 1.30 ratio is based upon 1216 observations, the standard test of the first-order autocorrelation coefficient (based upon the standard error $1/\sqrt{1216} = 0.03$) easily rejects the random walk hypothesis at any significance level.

Although the variance ratios increase with q , note that the magnitude of the z and z^* statistics do not. Indeed, the test statistics seem to decline with q hence the significance of the rejections becomes weaker as coarser-sample variances are compared to weekly variances. This pattern is inconsistent with the price fads alternative K_1 under which the power is an increasing function of q. If price fads were indeed present in the data, we should observe more significant rejections for larger q. Moreover, since price fads imply negative serial correlation of returns, we should also observe variance ratios less than 1.0. However, the results of Table 3a are inconsistent with these implications and support those of the returns fads alternative: positive serial correlation which declines for longer holding-periods, implying variance ratios greater than 1.0 and weaker rejections of the random walk model as q increases.

Although the rejection of the random walk hypothesis is much weaker for the value-weighted index as Panel B indicates, nevertheless the general patterns persist. The variance ratios also exceed 1.0, and the z and z^* statistics decline as q increases. Note that the rejections for the value-weighted index are primarily due to the first 304 weeks of the sample period.

Table 3b presents the variance ratios using a base observation period of 4 weeks hence the first entry of the first row, 1.15, is the variance ratio of eight-week returns to four-week returns, etc. Note that with a base interval of a month, we generally do not reject the random walk model even for the equal-weighted index. This result lends further support to the returns fads model since, as Table 1b shows, the weekly-sampled process can deviate considerably from a random walk whereas the monthly-sampled increments may be very close to white noise.

Finally, although the test statistics in Tables 1-3 are based upon nominal stock returns, it is apparent that virtually the same results would obtain with real or excess returns. Since the volatility of weekly nominal returns is so much larger than that of the inflation and T-bill rates, it should be obvious that the use of nominal, real, or excess returns in a volatility-based test must yield practically identical inferences.

4.2 RESULTS FOR SIZE BASED PORTFOLIOS.

Researchers have recently argued that one can construct portfolios with constant return characteristics by using the market value of equity as a portfolio classification variable.[18] Also, an implication of the work of Keim and Stambaugh (1986) is that, conditional on stock and bond market variables, the logarithm of wealth relatives of portfolios of smaller stocks do not follow random walks. For portfolios of larger stocks their results are less conclusive. Consequently, it is of interest to explore what evidence our tests provide for the random walk hypothesis for the logarithm of size based portfolio wealth relatives.

We compute weekly returns for five size based portfolios from the NYSE-AMEX universe on the CRSP daily return file. Stocks with returns for any given week are assigned to portfolios based on which quintile their market value of equity is in. The portfolios are equal weighted and have a continually changing composition.[19] The number of stocks included in the portfolios varies from 2036 to 2720. Tables 4a and 4b report the $\bar{M}_r(q)$ test results for the size-based portfolios.

Table 4a displays the results using a base observation interval of one week. Panel A reports the results for the portfolio of small firms (first quintile), Panel B, reports the results for the portfolios of medium-sized firms (third quintile), and Panel C reports the results for the portfolio of large firms (fifth quintile). Evidence against the random walk hypothesis for small firms is strong for all time periods considered. In Panel A all the z and z* statistics are well above 2.0, ranging from z* = 3.52 to 11.92 and z = 4.00 to 18.06. As we proceed through the Panels to the results for the portfolio of large firms the z and z* statistics become smaller, but even for the large firms portfolio the evidence against the null hypothesis is strong. In the 304 week subperiods several of the z* statistics are high. For example, when the aggregation value equals 4 the z* statistics are 2.85, 2.78, 1.60, and 1.17 across the four subperiods. As in the case of the returns indexes, we may obtain estimates of the first-order autocorrelation coefficient for returns on these size-sorted portfolios simply by subtracting 1.0 from entries in the q = 2 column. The values in Table 4a indicate that portfolio returns for the smallest quintile have a 42 percent weekly autocorrelation over the entire sample period! Moreover, this autocorrelation reaches 51 percent in sub-period 3 (May 2, 1974 to December 19, 1979) and its lowest sub-period value (July 4, 1968 to May 1, 1974) is 32 percent. In addition, although the serial correlation for the portfolio returns of the largest quintile is much smaller (14 percent for the entire sample period), it is nevertheless still statistically significant.

Table 4b reports the results using a base observation interval of four weeks. In Panel A, the results for the smallest firms are also inconsistent with the random walk hypothesis. For example, the ratio estimate for an aggregation value of 8 for the overall period is 1.41 with a z^* statistic of 2.04. In this panel all the ratio estimates are greater than 1.0 and many of the z^* statistics are greater than 2.0. Moreover, the implied first-order autocorrelation for monthly portfolio returns of the smallest quintile is still quite significant (23 percent). Proceeding through the table we see that the evidence against the random walk hypothesis disappears so that, in Panel C, the results for the large firms are all consistent with the random walk hypothesis. Several of the ratio estimates are below 1.0 and all of the z^* statistics are between -2.0 and 2.0.

The results for size-based portfolios are generally consistent with those for the market indexes. The patterns of variance ratios increasing in q and significance of rejections decreasing in q which we observed for the indexes also obtain for these portfolios. The evidence against the random walk hypothesis for the logarithm of wealth relatives of small-firms portfolios is strong in all cases considered. For larger firms and a one week base observation interval, the evidence is also inconsistent with the random walk. However, as the base observation interval is increased to four weeks, our test does not reject the random walk model for larger firms.

5. SPURIOUS AUTOCORRELATION INDUCED BY NON-TRADING.

Although we have based our empirical results upon weekly data to minimize the biases associated with market micro-structure issues, this alone does not insure against their possibly substantial influences. In this section, we consider explicitly the conjecture that infrequent trading may induce significant spurious correlation in stock returns.[20] The common intuition for the cause of such artificial serial correlation is that, for whatever reasons, small capitalization stocks are less liquid than larger stocks. Therefore, new information is impounded first into large-capitalization stock prices and then into smaller-stock prices with a lag. This lag induces a positive serial correlation in, for example, an equally-weighted index of stock returns. Of course, this induced positive serial correlation would be less pronounced in a value-weighted index. Since our rejections of the random walk hypothesis are most resounding for the equal-weighted index, they may very well be the result of this non-trading phenomenon. In order to investigate this possibility, we consider the following simple model of nontrading.

Suppose our universe of stocks consists of N securities indexed by i, each with the return generating process given by:

$$R_{it} = R_{Mt} + \epsilon_{it} \qquad i = 1, \ldots, N \tag{25}$$

R_{Mt} represents a factor common to all returns (e.g., the market) and is assumed to be an independently and identically distributed random variable with mean μ_M and variance σ^2_M. The ϵ_{it} term represents the idiosyncratic component of security i's return and it is also assumed to be i.i.d. (over <u>both</u> i and t), with mean 0 and variance σ^2_M. The return-generating process may thus be identified with N securities each with a unit beta such that the theoretical R^2 of a market model regression for each security is 0.50.

Now suppose that, in each period t, there is some chance that security i does not trade. One simple approach to modelling this phenomenon is to distinguish between the observed returns process and the virtual returns process. For example, suppose security i has traded in period t-1; consider its behavior in period t. If security i does not trade in period t, we define its virtual return as R_{it} (which is given by (25)), whereas its observed return R^o_{it} is 0. If security i then trades at t+1, its observed return R^o_{it+1} is defined to be the sum of its virtual returns R_{it} and R_{it+1}, hence non-trading is assumed to cause returns to cumulate. The cumulation of returns over periods of non-trading captures the essence of spuriously induced correlations due to the non-trading lag.

In order to calculate the magnitude of the positive serial correlation induced by non-trading, we must specify the probability law governing the non-trading event. For simplicity, we assume that whether or not a security trades may be modelled by a Bernoulli trial, so that in each period and for each security there is a probability p that it trades and a probability 1 - p that it does not. Moreover, it is assumed that these Bernoulli trials are i.i.d. across securities and, for each security, they are i.i.d. over time. Now consider the observed return R^o_t at time t of an equally-weighted portfolio:

$$R^o_t \equiv \frac{1}{N} \sum_i^N R^o_{it} \; . \tag{26}$$

But the observed return R^o_{it} for security i may be expressed as:

$$R^o_{it} = X_{it}(0) \cdot R_{it} + X_{it}(1) \cdot R_{it-1} + X_{it}(2) \cdot R_{it-2} + \cdots \tag{27}$$

where $X_{it}(j)$, j = 1, 2, 3, ... are random variables defined as:

$$X_{it}(0) \equiv \begin{cases} 1 & \text{If } i \text{ trades at } t. \\ 0 & \text{Otherwise.} \end{cases} \tag{28a}$$

$$X_{it}(1) \equiv \begin{cases} 1 & \text{If } i \text{ does not trade at } t-1 \text{ and } i \text{ trades at } t. \\ 0 & \text{Otherwise.} \end{cases} \tag{28b}$$

$$X_{it}(2) \equiv \begin{cases} 1 & \text{If } i \text{ trades at } t \text{ and does not trade at } t-1 \text{ and } t-2. \\ 0 & \text{Otherwise.} \end{cases} \tag{28c}$$

$$\vdots$$

The $X_{it}(j)$ variables are merely indicators of the number of <u>consecutive</u> periods before t in which security j has not traded. Using this relation, we have:

$$R_t^o = \frac{1}{N}\sum_i^N X_{it}(0)\cdot R_{it} + \frac{1}{N}\sum_i^N X_{it}(1)\cdot R_{it-1} + \frac{1}{N}\sum_i^N X_{it}(2)\cdot R_{it-2} + \cdot\cdot\cdot \tag{29}$$

For large N, it may readily be shown that because the ϵ_{it} component of each security's return is idiosyncratic and has zero expectation, the following approximation obtains:

$$R_t^o \cong \frac{1}{N}\sum_i^N X_{it}(0)\cdot R_{Mt} + \frac{1}{N}\sum_i^N X_{it}(1)\cdot R_{Mt-1} + \frac{1}{N}\sum_i^N X_{it}(2)\cdot R_{Mt-2} + \cdot\cdot\cdot \tag{30}$$

Moreover, it is also apparent that the averages $\frac{1}{N}\sum_i^N X_{it}(j)$ become arbitrarily close, again for large N, to the probability of j consecutive no-trades followed by a trade, i.e.:

$$\underset{N}{\text{plim}}\ \frac{1}{N}\sum_i^N X_{it}(j) = p\cdot(1-p)^j\ . \tag{31}$$

The observed equal-weighted return is then given by the approximation:

$$R_t^o \cong p \cdot R_{Mt} + p \cdot (1 - p) \cdot R_{Mt-1} + p \cdot (1 - p)^2 \cdot R_{Mt-2} + \cdots \tag{32}$$

Using this expression, the general j^{th}-order autocorrelation coefficient $\rho(j)$ may be readily computed as:

$$\rho(j) \equiv \frac{\mathrm{COV}[R_t^o, R_{t-j}^o]}{\mathrm{VAR}[R_t^o]} = (1 - p)^j . \tag{33}$$

Assuming that the implicit time interval corresponding to our single period is one trading day, we may also compute the weekly (five-day) first-order autocorrelation coefficient of R_t^o as:

$$\rho^W(1) = \frac{\rho(1) + 2\rho(2) + \cdots + 5\rho(5)}{5 + 4\rho(1) + \cdots + \rho(4)} . \tag{34}$$

By specifying reasonable values for the probability of non-trading, we may calculate the induced autocorrelation using equation (34). In order to develop some intuition for the parameter p, observe that the total number of securities which trade in any given period t is given by the sum $\sum_i^N X_{it}(0)$. Under our assumptions, this random variable has a binomial distribution with parameters (N, p) hence its expected value and variance are given by Np and Np(1-p) respectively. Therefore, the probability p may be interpreted as the fraction of the total number of N securities which trades on average in any given period. A value of 0.90 implies that, on average, 10 percent of the securities do not trade in a single period.

Table 5 presents the theoretical daily and weekly autocorrelations induced by non-trading for non-trading probabilities of 10 to 50 percent. The first row shows that when (on average) 10 percent of the stocks do not trade each day, this induces a weekly autocorrelation of only 2.3 percent!

Moreover, even when the probability of non-trading is increased to 50 percent, the induced weekly autocorrelation is 22 percent, which is still considerably lower than the estimated 30 percent autocorrelation in the weekly equal-weighted CRSP index. Furthermore, it should be emphasized that the 22 percent spurious autocorrelation requires that on average half the stocks do not trade on any given day. Clearly, this is unrealistically high hence a 22 percent autocorrelation is a very conservative upper bound.[21] A daily non-trading probability of 10 percent is empirically more relevant, from which we conclude that our rejection of the random walk hypothesis cannot plausibly be attributed to infrequent trading.[22]

6. CONCLUSION.

In this paper, we have rejected the random walk hypothesis for weekly stock market returns using a simple volatility-based specification test. These rejections cannot be ascribed to infrequent trading or to time-varying volatilities. Moreover, the pattern of rejections indicate that a mean-reverting price fads model of Shiller and Perron (1985) and Summers (1986) cannot account for the departures from the random walk.

As we stated in the introduction, the rejection of the random walk model does not necessarily imply the inefficiency of stock price formation. Our results do, however, impose restrictions upon the set of plausible economic models for asset pricing; any structural paradigm of rational price formation must now be able to explain this pattern of serial correlation present in weekly data. As purely descriptive tools for examining the stochastic evolution of prices through time, specification tests of price processes also serve a useful purpose. This is especially true in cases where a 'reduced form' model of the price process is of more importance than a structural framework within which those prices are determined in equilibrium. For example, the pricing of complex financial claims often depend critically upon the specific stochastic process driving underlying asset returns whereas, since such models are usually based upon arbitrage considerations, the particular economic equilibrium which generates prices may be of less consequence. In particular, one implication of our empirical findings is that the standard Black-Scholes pricing formula for stock index options may be misspecified.

Although our variance-based test may be used as a diagnostic check for the random walk specification, it is a more difficult task to determine precisely which single process best fits the data. Our empirical results suggest that a returns fads model may provide a more likely explanation for the stochastic properties of short-run returns. A direct parameter estimation of this process may shed more light on the behavior of market prices and will be pursued in future research. However, this alternative also has its limitations. In particular, it implies that all non-overlapping holding-period returns are positively correlated, whereas Fama and French (1986) have shown that long holding-period returns (3 to 5 years) are negatively serially correlated. Furthermore, the results of French and Roll (1986) for return variances when markets are open versus when they are closed adds yet another dimension to this empirical puzzle. Whether or not it is possible to construct a single stochastic process which exhibits this rich pattern of autocorrelations for various holding periods is an intriguing problem which merits further investigation.

FOOTNOTES

[1]See Keynes (1936) Chapter 12, Section VII.

[2]See Fama's (1970) survey and, more recently, Chapter 5 of Fama (1976) for a sample of this vast and still growing literature.

[3]See, for example, the studies in Jensen's (1978) volume on anomalous evidence regarding market efficiency.

[4]Other studies which have also employed variance ratios are Campbell and Mankiw (1986), Cochrane (1986), Fama and French (1986), and French and Roll (1986). There is also a sizable literature concerning the testing of unit roots, of which the random walk process is a special case. See Lo and MacKinlay (1987) for a discussion of how the proposed variance ratio test is related to the more general unit root tests.

[5]Briefly, Hausman (1978) exploits the fact that any asymptotically efficient estimator of a parameter θ, say $\hat{\theta}_e$, must possess the property that it is asymptotically uncorrelated with the difference $\hat{\theta}_a - \hat{\theta}_e$ where $\hat{\theta}_a$ is any other estimator of θ. If not, then there exists a linear combination of $\hat{\theta}_e$ and $\hat{\theta}_a - \hat{\theta}_e$ which is more efficient than $\hat{\theta}_e$, contradicting the assumed efficiency of $\hat{\theta}_e$. The result follows directly then since:

$$\text{AVAR}[\hat{\theta}_a] = \text{AVAR}[\hat{\theta}_e + \hat{\theta}_a - \hat{\theta}_e] = \text{AVAR}[\hat{\theta}_e] + \text{AVAR}[\hat{\theta}_a - \hat{\theta}_e]$$

$$\Rightarrow \quad \text{AVAR}[\hat{\theta}_a - \hat{\theta}_e] = \text{AVAR}[\hat{\theta}_a] - \text{AVAR}[\hat{\theta}_e]$$

where AVAR[$\cdot$] denotes the asymptotic variance operator.

[6]Note that if $(\hat{\sigma}_a^2)^2$ is used to estimate σ_0^4, then the standard 't-test' of $J_d = 0$ will yield inferences identical to those obtained from the corresponding test of $J_r = 0$ for the ratio since

$$\frac{J_d}{\sqrt{2\hat{\sigma}_a^4}} = \frac{\hat{\sigma}_b^2 - \hat{\sigma}_a^2}{\sqrt{2}\,\hat{\sigma}_a^2} = \frac{J_r}{\sqrt{2}} \sim N(0,\ 1) \ .$$

[7]See Lo and MacKinlay (1987).

[8]More formally, the unbiased variance estimators $\bar{\sigma}_a^2$, $\bar{\sigma}_b^2$, $\bar{\sigma}_c^2$ are given by:

$$\bar{\sigma}_a^2 = \frac{1}{nq - 1} \sum_{k=1}^{nq} [X_k - X_{k-1} - \hat{\mu}]^2$$

$$\bar{\sigma}_b^2 = \frac{1}{nq - q} \sum_{k=1}^{n} [X_{qk} - X_{qk-q} - q\hat{\mu}]^2$$

$$\bar{\sigma}_c^2 = \frac{1}{m} \sum_{k=q}^{nq} [X_k - X_{k-q} - q\hat{\mu}]^2 \qquad m \equiv q(nq - q + 1)\cdot(1 - \frac{q}{nq}) \ .$$

[9]Note the similarity between these variance ratios and the Box-Pierce Q-statistic which is a linear combination of squared autocorrelations with all the weights set identically equal to unity. Although we may expect the finite-sample behavior of the variance ratios to be comparable to that of the Q-statistic under the null hypothesis, they may have very different power properties under various alternatives. See Lo and MacKinlay (1987) for further details.

[10]See, for example, Merton (1980), Poterba and Summers (1985), and French, Schwert, and Stambaugh (1985).

[11]We must of course assume that the average variance converges asymptotically, otherwise no statistical inference is possible. In particular, Assumption A of White and Domowitz (1984) is assumed.

[12]In particular, we require Assumption A and the assumptions in Theorem 2.3 of White and Domowitz (1984), and the added condition that $E[\varepsilon_t \varepsilon_{t-j} \varepsilon_t \varepsilon_{t-k}] = 0$ for all t and for non-zero $j \neq k$. This last condition implies that the estimators of the increments' autocorrelation coefficients are asymptotically uncorrelated so that the estimator of the asymptotic variance of $\bar{M}_r(q)$ takes on the particularly simple form in (18). Although this restriction on the fourth cross-moments of ε_t may seem somewhat unintuitive, note that it is satisfied for any process with independent increments (irregardless of heterogeneity) and also for linear Gaussian ARCH processes. Moreover, at the expense of computational simplicity this assumption may be relaxed entirely, requiring the estimation of the asymptotic covariances of the autocorrelation estimators in order to estimate the limiting variance θ of $\bar{M}_r(q)$ via relation (16). Although the resulting estimator of θ would be more complicated than (18), it is conceptually straightforward and may be readily formed along the lines of Newey and West (1986).

[13]See footnote 12.

[14]An equivalent and somewhat more intuitive method of arriving at this same formula is to consider the regression of the increments on a constant and the j-th lagged increments. The estimated slope coefficient is then simply the j-th autocorrelation coefficient. It may then be shown that estimated variance of the slope coefficient given by White's (1980) heteroscedasticity-consistent covariance matrix estimator for this regression is numerically identical to $\hat{\delta}(j)$. Note that White (1980) requires independent disturbances

whereas White and Domowitz (1984) allow for weak dependence (of which uncorrelated errors is a special case). Taylor (1984) also obtains this result under the assumption that the multivariate distribution of the sequence of disturbances is symmetric.

[15]See, for example, Shiller and Perron (1985), Fama and French (1986), and Summers (1986).

[16]This, of course, entails no loss of generality if all dividends are re-invested in the security or if the dividend-price ratio is a nonstochastic function of time.

[17]The integrated O.U process has a long history in the physical sciences. The prime motivation for its development was the need to model the _velocity_ of a particle suspended in fluid. Because the Brownian motion model of a particle's position yields sample paths of unbounded variation, the particle's velocity cannot be defined. This problem was solved by modelling velocity itself as a Brownian motion and then integrating to obtain the particle's position. However, for purposes of modelling the stochastic behavior of an individual security, the integrated O.U. process still contains a serious limitation: it implies the existence of pure arbitrage opportunities in the context of frictionless markets in which continuous trading is possible. Specifically, Harrison, Pitbladdo, and Schaefer (1984) demonstrate that continuous time price processes in frictionless markets with continuous sample paths must be of unbounded variation to rule out arbitrage. This is clearly violated by the integrated O.U. process which, by construction, possesses a mean-square derivative. We therefore do not advocate the returns fads process as a reasonable alternative to the lognormal diffusion. Its use is merely for purposes of illustrating the power of our test statistics against an alternative under which returns are positively serially correlated. Note, however, that the integrated O.U. process may be appropriate as a model of the behavior of _aggregate_ wealth (e.g., in a single-good representative agent model).

[18]Huberman and Kandel (1985) is one example.

[19]We also performed our tests using value-weighted portfolios and obtained essentially the same results. The only difference appeared in the largest quintile of the value-weighted portfolio, for which the random walk hypothesis was generally not rejected. This, of course, is not surprising given that the largest value-weighted quintile is quite similar to the value-weighted market index.

[20]See, for example, Cohen, Hawawini, Maier, Schwartz, and Whitcomb (1983).

[21]Indeed, several other factors imply that the actual size of the spurious autocorrelations induced by infrequent trading are lower than those given in Table 5. For example, in calculating the induced correlations using equation (33), we have ignored the idiosyncratic components in returns due to diversification whereas, in practice, perfect diversification is never achieved. But note that any residual risk increases the denominator of (33) but does not necessarily increase the numerator (since the ε_{it}'s are cross-

sectionally uncorrelated). To see this explicitly, we simulated the returns for 1000 stocks over 5120 days, calculated the weekly autocorrelations for the virtual returns and for the observed returns, computed the difference of those autocorrelations, repeated this procedure 20 times and then averaged the differences. With a (daily) non-trading probability of 10 percent, the simulations yield a difference in weekly autocorrelations of 2.1 percent (lower than the theoretical 2.3 percent), 4.3 percent for a non-trading probability of 20 percent (theoretically 5.3 percent), and 7.6 percent for a non-trading probability of 30 percent (theoretically 9.3 percent).

Another factor which may reduce the spurious positive autocorrelation empirically is that, within the CRSP files, if a security does not trade its price is reported as the average of the bid-ask spread. Therefore, as long as the specialist adjusts the spread to reflect the new information, even if no trade occurs the reported CRSP will reflect the new information. Although there may still be some delay before the bid-ask spread is adjusted, it is presumably less than the lag between trades.

Also, if it is assumed that the probability of no-trades depends upon whether or not the security has traded recently, it is natural to suppose that the likelihood of a no-trade tomorrow is lower if there is a no-trade today. In that case, it may readily be shown that the induced autocorrelation is even lower than that computed in our i.i.d. framework.

Finally, the well-known bias induced by the bid-ask spread (which we attempt to minimize by using weekly data) also serves to reduce the estimated autocorrelation of returns.

[22] In fact, for the value-weighted CRSP index a non-trading probability of 10 percent is probably too high since the smaller stocks (which might possibly have a non-trading probability as high as 10 percent) are given almost no weight, whereas the stocks with significant weight such as IBM and GM have almost zero probability of a no-trade.

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TABLE 1a

Theoretical steady state first-order serial correlation **coefficients** of price fads returns for one to twelve week **holding**-periods when $\gamma_p = 0.051$.

Holding Period (Weeks)	First-Order Serial Correlation
1	-0.025
2	-0.049
3	-0.071
4	-0.093
5	-0.113
6	-0.132
7	-0.151
8	-0.168
9	-0.185
10	-0.201
11	-0.216
12	-0.230

TABLE 1b

Theoretical steady state first-order serial correlation **coefficients** of returns fads returns for one to twelve week holding-periods when $\gamma_r = 2.0$.

Holding Period (Weeks)	First-Order Serial Correlation
1	0.329
2	0.160
3	0.099
4	0.071
5	0.056
6	0.045
7	0.038
8	0.033
9	0.029
10	0.026
11	0.024
12	0.022

TABLE 2a

Power of the specification test $\bar{M}_r(q)$ against the price fads alternative with parameters $(\alpha_p, \gamma_p, \sigma_p) = (0.000, 0.051, 0.020$ when $h = 1$ for various time spans, aggregation values, and observation intervals. All simulations are based upon 5000 replications.

Sample Size	q	h	Power (10%)	Power (5%)	Power (1%)
A. Power Results for 1216 Week Time Span.					
1216	2	1	0.225	0.143	0.088
1216	4	1	0.373	0.251	0.152
1216	8	1	0.592	0.455	0.313
1216	16	1	0.837	0.715	0.541
1216	32	1	0.975	0.922	0.797
1216	64	1	0.999	0.985	0.906
608	2	2	0.323	0.212	0.131
608	4	2	0.547	0.404	0.281
608	8	2	0.821	0.697	0.535
608	16	2	0.968	0.908	0.784
608	32	2	0.999	0.987	0.910
304	2	4	0.464	0.334	0.230
304	4	4	0.766	0.637	0.485
304	8	4	0.952	0.883	0.750
304	16	4	0.998	0.976	0.888
B. Power Results for 608 Week Time Span.					
608	2	1	0.144	0.080	0.043
608	4	1	0.215	0.121	0.065
608	8	1	0.328	0.201	0.114
608	16	1	0.497	0.309	0.165
608	32	1	0.666	0.413	0.170
304	2	2	0.203	0.120	0.067
304	4	2	0.320	0.198	0.108
304	8	2	0.472	0.285	0.149
304	16	2	0.651	0.412	0.182
152	2	4	0.275	0.171	0.095
152	4	4	0.449	0.293	0.165
152	8	4	0.625	0.400	0.187
C. Power Results for 304 Week Time Span.					
304	2	1	0.125	0.065	0.033
304	4	1	0.140	0.074	0.036
304	8	1	0.168	0.078	0.029
304	16	1	0.201	0.074	0.020
152	2	2	0.146	0.078	0.042
152	4	2	0.171	0.085	0.038
152	8	2	0.214	0.099	0.028
76	2	4	0.170	0.092	0.045
76	4	4	0.211	0.104	0.042

TABLE 2b

Power of the specification test $\bar{M}_r(q)$ against the returns fads alternative with parameters $(\alpha_r, \gamma_r, \sigma_r)$ = (0.004, 2.0, 0.040) when h = 1 for various time spans, aggregation values, and observation intervals. All simulations are based upon 5000 replications.

Sample Size	q	h	Power (10%)	Power (5%)	Power (1%)
A. Power Results for 1216 Week Time Span.					
1216	2	1	1.000	1.000	1.000
1216	4	1	1.000	1.000	1.000
1216	8	1	1.000	1.000	1.000
1216	16	1	1.000	1.000	0.999
1216	32	1	0.993	0.971	0.916
1216	64	1	0.827	0.640	0.378
608	2	2	0.987	0.970	0.943
608	4	2	0.963	0.915	0.847
608	8	2	0.808	0.677	0.528
608	16	2	0.530	0.353	0.198
608	32	2	0.263	0.117	0.040
304	2	4	0.349	0.239	0.150
304	4	4	0.241	0.139	0.069
304	8	4	0.153	0.072	0.033
304	16	4	0.109	0.041	0.012
B. Power Results for 608 Week Time Span.					
608	2	1	1.000	1.000	1.000
608	4	1	1.000	1.000	1.000
608	8	1	1.000	1.000	0.998
608	16	1	0.985	0.958	0.898
608	32	1	0.819	0.633	0.378
304	2	2	0.864	0.775	0.679
304	4	2	0.750	0.614	0.468
304	8	2	0.511	0.342	0.195
304	16	2	0.268	0.118	0.037
152	2	4	0.216	0.138	0.083
152	4	4	0.167	0.082	0.037
152	8	4	0.120	0.047	0.016
C. Power Results for 304 Week Time Span.					
304	2	1	0.998	0.991	0.980
304	4	1	0.999	0.996	0.982
304	8	1	0.975	0.931	0.852
304	16	1	0.807	0.610	0.358
152	2	2	0.591	0.470	0.348
152	4	2	0.461	0.309	0.178
152	8	2	0.265	0.120	0.042
76	2	4	0.174	0.098	0.053
76	4	4	0.118	0.050	0.018

TABLE 3a

Variance ratio test $\bar{M}_r(q)$ of the random walk hypothesis for CRSP equal and value weighted indexes using a one-week base observation interval (h = 1 week) for the sample period September 6, 1962 to December 26, 1985 and sub-periods. The actual variance ratios are reported in the main rows, with the z and z* statistics given in parentheses in rows immediately below each main row.

*Indicates significance at the 5 percent level

Time period	Number nq of base observations	Number q of base observations aggregated to form variance ratio					
		2	4	8	16	32	64
A. Equal-Weighted CRSP NYSE-AMEX Index							
620906-851226	1216	1.30	1.64	1.94	2.05	2.22	2.23
		(10.29)*	(11.94)*	(11.08)*	(8.30)*	(6.66)*	(4.71)*
		(7.51)*	(8.87)*	(8.48)*	(6.59)*	(5.52)*	(4.05)*
620906-740501	608	1.31	1.62	1.92	2.09	2.37	
		(7.53)*	(8.23)*	(7.64)*	(6.10)*	(5.28)*	
		(5.38)*	(6.03)*	(5.76)*	(4.77)*	(4.32)*	
740502-851226	608	1.28	1.65	1.93	1.91	1.74	
		(7.02)*	(8.51)*	(7.75)*	(5.07)*	(2.84)*	
		(5.32)*	(6.52)*	(6.13)*	(4.17)*	(2.45)*	
620906-680703	304	1.32	1.68	1.92	2.07		
		(5.66)*	(6.29)*	(5.44)*	(4.26)*		
		(4.12)*	(4.77)*	(4.23)*	(3.45)*		
680704-740501	304	1.29	1.58	1.83	1.87		
		(4.99)*	(5.36)*	(4.90)*	(3.46)*		
		(4.03)*	(4.44)*	(4.18)*	(3.04)*		
740502-791219	304	1.29	1.71	2.01	1.91		
		(5.12)*	(6.58)*	(5.93)*	(3.60)*		
		(3.80)*	(5.02)*	(4.66)*	(2.93)*		
791220-851226	304	1.26	1.49	1.66	2.00		
		(4.61)*	(4.55)*	(3.91)*	(3.94)*		
		(3.99)*	(3.83)*	(3.46)*	(3.63)*		
B. Value-Weighted CRSP NYSE-AMEX Index							
620906-851226	1216	1.08	1.16	1.22	1.22	1.35	1.31
		(2.96)*	(2.94)*	(2.59)*	(1.71)	(1.94)	(1.17)
		(2.33)*	(2.31)*	(2.07)*	(1.38)	(1.60)	(1.00)
620906-740501	608	1.15	1.22	1.27	1.32	1.42	
		(3.66)*	(2.87)*	(2.22)*	(1.78)	(1.61)	
		(2.89)*	(2.28)*	(1.79)	(1.46)	(1.37)	
740502-851226	608	1.05	1.12	1.18	1.10	1.01	
		(1.13)	(1.57)	(1.50)	(0.56)	(0.06)	
		(0.92)	(1.28)	(1.24)	(0.46)	(0.05)	
620906-680703	304	1.20	1.29	1.32	1.29		
		(3.55)*	(2.71)*	(1.90)	(1.15)		
		(2.87)*	(2.19)*	(1.55)	(0.96)		
680704-740501	304	1.12	1.18	1.22	1.30		
		(2.12)*	(1.69)	(1.32)	(1.18)		
		(1.86)	(1.49)	(1.18)	(1.08)		
740502-791219	304	1.00	1.11	1.21	1.14		
		(-0.01)	(1.07)	(1.21)	(0.57)		
		(-0.01)	(0.87)	(0.99)	(0.47)		
791220-851226	304	1.10	1.09	1.08	1.12		
		(1.72)	(0.82)	(0.49)	(0.47)		
		(1.42)	(0.68)	(0.42)	(0.41)		

Note: Under the random walk null hypothesis the value of the variance ratio is 1.0 and the test statistic has a standard normal distribution (asymptotically).

TABLE 3b

Variance ratio test $\overline{M}_r(q)$ of the random walk hypothesis for CRSP equal and value weighted indexes using a four-week base observation interval (h = 4 weeks) for the sample period September 6, 1962 to December 26, 1985 and sub-periods. The actual variance ratios are reported in the main rows, with the z and z* statistics given in parentheses in rows immediately below each main row.

*Indicates significance at the 5 percent level.

Time period	Number nq of base observations	Number q of base observations aggregated to form variance ratio			
		2	4	8	16
A. Equal-Weighted CRSP NYSE-AMEX Index					
620906-851226	304	1.15	1.19	1.30	1.30
		(2.63)*	(1.80)	(1.74)	(1.20)
		(2.26)*	(1.54)	(1.52)	(1.07)
620906-740501	152	1.13	1.23	1.40	
		(1.64)	(1.54)	(1.67)	
		(1.39)	(1.32)	(1.46)	
740502-851226	152	1.15	1.11	1.02	
		(1.86)	(0.73)	(0.10)	
		(1.68)	(0.64)	(0.09)	
620906-680703	76	1.11	1.20		
		(0.92)	(0.95)		
		(0.80)	(0.87)		
680704-740501	76	1.12	1.15		
		(1.01)	(0.71)		
		(0.90)	(0.64)		
740502-791219	76	1.16	1.07		
		(1.43)	(0.30)		
		(1.23)	(0.27)		
791220-851226	76	1.02	1.21		
		(0.21)	(1.00)		
		(0.29)	(1.10)		
B. Value-Weighted CRSP NYSE-AMEX Index					
620906-851226	304	1.05	1.00	1.11	1.07
		(0.79)	(0.00)	(0.64)	(0.28)
		(0.75)	(0.00)	(0.57)	(0.26)
620906-740501	152	1.02	1.04	1.12	
		(0.27)	(0.29)	(0.50)	
		(0.26)	(0.26)	(0.46)	
740502-851226	152	1.05	0.95	0.89	
		(0.64)	(-0.34)	(-0.46)	
		(0.63)	(-0.31)	(-0.42)	
620906-680703	76	1.00	1.02		
		(0.02)	(0.08)		
		(0.02)	(0.08)		
680704-740501	76	1.02	1.05		
		(0.18)	(0.22)		
		(0.18)	(0.21)		
740502-791219	76	1.12	0.98		
		(1.07)	(-0.11)		
		(1.01)	(-0.10)		
791220-851226	76	0.90	0.95		
		(-0.89)	(-0.24)		
		(-0.95)	(-0.23)		

Note: Under the random walk null hypothesis the value of the variance ratio is 1.0 and the test statistic has a standard normal distribution (asymptotically).

TABLE 4a

Variance ratio test $\bar{M}_r(q)$ of the random walk hypothesis for size-sorted portfolios using a one-week base observation interval (h = 1 week) for the sample period September 6, 1962 to December 26, 1985 and sub-periods. The actual variance ratios are reported in the main rows, with the z and z* statistics given in parentheses in rows immediately below each main row.

*Indicates significance at the 5 percent level

Time period	Number nq of base observations	Number q of base observations aggregated to form variance ratio					
		2	4	8	16	32	64
A. Portfolio of firms with market values in smallest NYSE-AMEX quintile							
620906-851226	1216	1.42	1.97	2.49	2.68	2.83	2.96
		(14.71)*	(18.06)*	(17.54)*	(13.28)*	(9.98)*	(7.50)*
		(8.81)*	(11.58)*	(11.92)*	(9.65)*	(7.84)*	(6.23)*
620906-740501	608	1.37	1.83	2.27	2.52	2.98	
		(9.05)*	(10.92)*	(10.60)*	(8.49)*	(7.64)*	
		(6.12)*	(7.83)*	(7.94)*	(6.68)*	(6.32)*	
740502-851226	608	1.49	2.14	2.76	2.87	2.56	
		(12.13)*	(15.08)*	(14.65)*	(10.48)*	(6.03)*	
		(6.44)*	(8.66)*	(9.06)*	(7.06)*	(4.53)*	
620906-680703	304	1.46	2.09	2.65	3.05		
		(8.01)*	(10.13)*	(9.74)*	(8.11)*		
		(5.86)*	(7.90)*	(7.91)*	(6.92)*		
680704-740501	304	1.32	1.70	2.04	2.01		
		(5.60)*	(6.54)*	(6.14)*	(4.00)*		
		(4.19)*	(5.21)*	(5.13)*	(3.52)*		
740502-791219	304	1.51	2.19	2.80	2.78		
		(8.91)*	(11.08)*	(10.63)*	(7.07)*		
		(4.77)*	(6.46)*	(6.68)*	(4.85)*		
791220-851226	304	1.45	1.99	2.50	3.19		
		(7.79)*	(9.25)*	(8.86)*	(8.68)*		
		(6.17)*	(7.42)*	(7.59)*	(7.97)*		
B. Portfolio of firms with market values in central NYSE-AMEX quintile							
620906-851226	1216	1.28	1.60	1.84	1.91	2.08	2.15
		(9.85)*	(11.10)*	(9.94)*	(7.21)*	(5.89)*	(4.39)*
		(7.38)*	(8.37)*	(7.70)*	(5.78)*	(4.92)*	(3.80)*
620906-740501	608	1.30	1.59	1.85	2.01	2.28	
		(7.39)*	(7.82)*	(7.08)*	(5.67)*	(4.93)*	
		(5.31)*	(5.73)*	(5.33)*	(4.42)*	(4.03)*	
740502-851226	608	1.27	1.59	1.80	1.69	1.49	
		(6.53)*	(7.73)*	(6.69)*	(3.87)*	(1.90)*	
		(5.31)*	(5.73)*	(5.33)*	(4.42)*	(4.03)*	
620906-680703	304	1.29	1.58	1.75	1.84		
		(5.10)*	(5.40)*	(4.43)*	(3.34)*		
		(3.81)*	(4.20)*	(3.52)*	(2.75)*		
680704-740501	304	1.29	1.57	1.80	1.85		
		(5.05)*	(5.28)*	(4.72)*	(3.38)*		
		(4.07)*	(4.34)*	(3.99)*	(2.94)*		
740502-791219	304	1.26	1.62	1.81	1.63		
		(4.61)*	(5.81)*	(4.80)*	(2.51)*		
		(3.63)*	(4.58)*	(3.88)*	(2.09)*		
791220-851226	304	1.26	1.46	1.61	1.84		
		(4.50)*	(4.26)*	(3.61)*	(3.34)*		
		(3.99)*	(3.64)*	(3.23)*	(3.12)*		

TABLE 4a (continued)

C. Portfolio of firms with market values in largest NYSE-AMEX quintile							
620906-851226	1216	1.14	1.27	1.36	1.34	1.44	1.31
		(4.74)*	(4.96)*	(4.24)*	(2.71)*	(2.43)*	(1.17)
		(3.82)*	(3.99)*	(3.45)*	(2.22)*	(2.03)*	(1.00)
620906-740501	608	1.21	1.36	1.45	1.44	1.46	
		(5.23)*	(4.75)*	(3.73)*	(2.48)*	(1.78)	
		(4.04)*	(3.70)*	(2.96)*	(2.02)*	(1.49)	
740502-851226	608	1.09	1.20	1.27	1.18	1.08	
		(2.11)*	(2.58)*	(2.28)*	(1.03)	(0.32)	
		(1.80)	(2.18)*	(1.95)	(0.87)	(0.28)	
620906-680703	304	1.26	1.39	1.46	1.40		
		(4.47)*	(3.65)*	(2.68)*	(1.60)		
		(3.51)*	(2.85)*	(2.12)*	(1.30)		
680704-740501	304	1.19	1.34	1.43	1.42		
		(3.33)*	(3.19)*	(2.52)*	(1.68)		
		(2.86)*	(2.78)*	(2.24)*	(1.53)		
740502-791219	304	1.05	1.20	1.31	1.24		
		(0.90)	(1.90)	(1.84)	(0.94)		
		(0.77)	(1.60)	(1.55)	(0.79)		
791220-851226	304	1.12	1.15	1.16	1.20		
		(2.11)*	(1.36)	(0.95)	(0.80)		
		(1.82)	(1.17)	(0.85)	(0.72)		

Note: Under the random walk null hypothesis the value of the variance ratio is 1.0 and the test statistic has a standard normal distribution (asymptotically).

TABLE 4b

Variance ratio test $\bar{M}_r(q)$ of the random walk hypothesis for size-sorted portfolios using a four-week base observation interval (h = 4 weeks) for the sample period September 6, 1962 to December 26, 1985 and sub-periods. The actual variance ratios are reported in the main rows, with the z and z* statistics given in parentheses in rows immediately below each main row.

*Indicates significance at the 5 percent level

Time period	Number nq of base observations	Number q of base observations aggregated to form variance ratio: 2	4	8	16
A. Portfolio of firms with market values in smallest NYSE-AMEX quintile					
620906-851226	304	1.23	1.32	1.41	1.47
		(4.01)*	(3.01)*	(2.39)*	(1.86)
		(3.09)*	(2.40)*	(2.04)*	(1.66)
620906-740501	152	1.20	1.34	1.59	
		(2.50)*	(2.22)*	(2.45)*	
		(2.06)*	(1.93)	(2.23)*	
740502-851226	152	1.26	1.31	1.19	
		(3.16)*	(2.04)*	(0.78)	
		(2.31)*	(1.52)	(0.63)	
620906-680703	76	1.21	1.39		
		(1.87)	(1.80)		
		(1.58)	(1.65)		
680704-740501	76	1.17	1.18		
		(1.49)	(0.83)		
		(1.23)	(0.73)		
740502-791219	76	1.26	1.25		
		(2.25)*	(1.16)		
		(1.67)	(0.89)		
791220-851226	76	1.18	1.48		
		(1.54)	(2.24)*		
		(1.77)	(2.46)*		
B. Portfolio of firms with market values in central NYSE-AMEX quintile					
620906-851226	304	1.13	1.14	1.24	1.29
		(2.20)*	(1.30)	(1.43)	(1.14)
		(1.94)	(1.12)	(1.24)	(1.01)
620906-740501	152	1.11	1.21	1.37	
		(1.38)	(1.37)	(1.56)	
		(1.16)	(1.15)	(1.32)	
740502-851226	152	1.12	1.02	0.91	
		(1.49)	(0.13)	(-0.36)	
		(1.44)	(0.12)	(-0.33)	
620906-680703	76	1.08	1.15		
		(0.72)	(0.68)		
		(0.65)	(0.63)		
680704-740501	76	1.09	1.14		
		(0.79)	(0.63)		
		(0.71)	(0.57)		
740502-791219	76	1.11	0.94		
		(0.93)	(-0.26)		
		(0.88)	(-0.24)		
791220-851226	76	1.03	1.16		
		(0.24)	(0.75)		
		(0.33)	(0.83)		

TABLE 4b (continued)

C. Portfolio of firms with market values in largest NYSE-AMEX quintile					
620906-851226	304	1.06 (1.12) (1.03)	1.01 (0.10) (0.09)	1.08 (0.46) (0.40)	0.98 (-0.09) (-0.08)
620906-740501	152	1.03 (0.40) (0.35)	1.02 (0.11) (0.10)	1.03 (0.12) (0.10)	
740502-851226	152	1.07 (0.87) (0.83)	0.96 (-0.27) (-0.24)	0.88 (-0.50) (-0.45)	
620906-680703	76	1.02 (0.15) (0.17)	1.03 (0.13) (0.13)		
680704-740501	76	1.03 (0.26) (0.24)	1.00 (0.02) (0.02)		
740502-791219	76	1.11 (0.97) (0.90)	0.96 (-0.19) (-0.17)		
791220-851226	76	0.94 (-0.50) (-0.55)	0.99 (-0.04) (-0.03)		

Note: Under the random walk null hypothesis the value of the variance ratio is 1.0 and the test statistic has a standard normal distribution (asymptotically).

TABLE 5

Magnitudes of spurious autocorrelations of returns induced by the non-trading phenomena for daily non-trading probabilities 1 - p of 10 to 50 percent. The theoretical values of daily j-th order autocorrelations $\rho(j)$ and weekly first-order autocorrelation $\rho^W(1)$ are all zero in the absence of the non-trading problem.

Probability of Non-trading 1 - p	$\rho(1)$	$\rho(2)$	$\rho(3)$	$\rho(4)$	$\rho(5)$	$\rho^W(1)$
0.10	0.1000	0.0100	0.0010	0.0001	0.0000	0.0227
0.20	0.2000	0.0400	0.0080	0.0016	0.0003	0.0525
0.30	0.3000	0.0900	0.0270	0.0081	0.0024	0.0927
0.40	0.4000	0.1600	0.0640	0.0256	0.0102	0.1473
0.50	0.5000	0.2500	0.1250	0.0625	0.0312	0.2209