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GLOBAL IMBALANCES AND POLICY WARS AT THE ZERO LOWER BOUND

Ricardo J. Caballero
Emmanuel Farhi
Pierre-Olivier Gourinchas

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ABSTRACT

This paper explores the consequences of extremely low real interest rates in a world with integrated but heterogenous capital markets, nominal rigidities and an effective lower bound (a ZLB for simplicity). We establish four main results: (i) At the ZLB, creditor countries export their recession abroad, which we illustrate with a new Metzler diagram in quantities; (ii) Beggar-thy-neighbor currency and trade wars provide stimulus to the undertaking country at the expense of other countries; (iii) (Safe) public debt issuances and increases in government spending anywhere are expansionary everywhere; (iv) When there is a scarcity of safe assets, net issuers of safe assets import the recession from abroad.

Ricardo J. Caballero
Department of Economics, E52-528
MIT
77 Massachusetts Avenue
Cambridge, MA 02139
and NBER
caball@mit.edu

Emmanuel Farhi
Harvard University
and CEPR

Pierre-Olivier Gourinchas
Department of Economics
University of California, Berkeley
530 Evans Hall #3880
Berkeley, CA 94720-3880
and CEPR
and also NBER
pog@econ.berkeley.edu

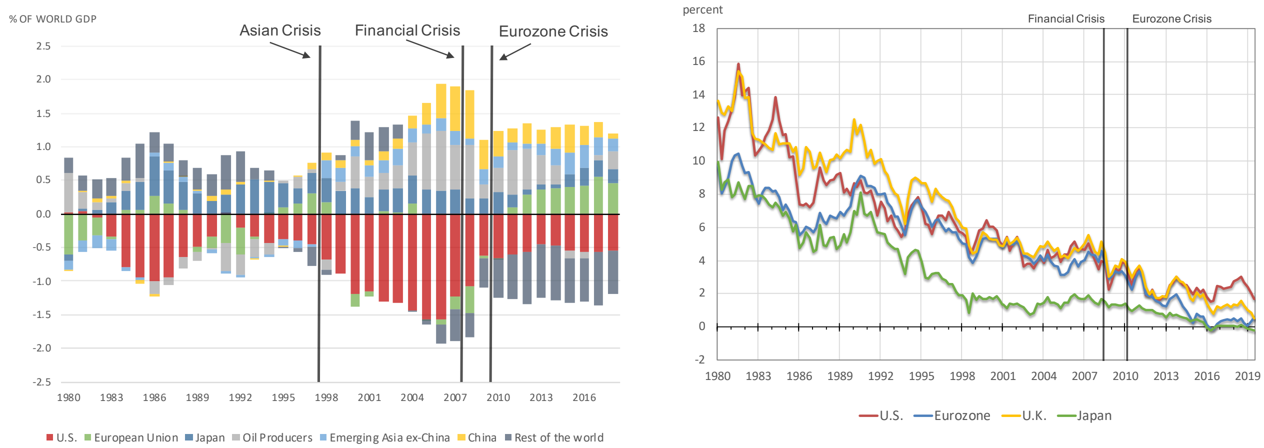
1 Introduction

Caballero, Farhi and Gourinchas (2008a,b) modeled global imbalances as the result of global differences in the capacity to produce assets, and the decline in potential growth in the developed world. The steady decline in interest rates was a natural outcome of this process. Fig. 1 illustrate these trends. The world has changed since then: interest rates have reached extremely low levels and there is limited space for further downward adjustment. While recognizing that nominal rates may become negative, for convenience we denote this downward rigidity in policy rates the 'Zero Lower Bound' (ZLB). How are global imbalances resolved in this ZLB context? How do policies in one country spill over to others in this environment? And how do local policymakers' incentives change at the ZLB?

We build a stylized model to address these questions. Our basic framework is a two-country perpetual-youth model with nominal rigidities, designed to highlight the heterogeneous relative demand for, and supply of, financial assets across countries. In the first part of the paper, we study a stationary world in which all countries share the same preferences for domestic and foreign goods and financial markets are fully integrated. This is an *all-or-none* world: Either all countries experience a *permanent* 'liquidity trap'—characterized by an inefficiently low level of aggregate economic activity—or none do. Within this model, we establish three main results: (i) the current account plays a key role in spreading liquidity traps from surplus countries to deficit ones; (ii) local governments have an incentive to engage in zero-sum currency and trade wars; and (iii) fiscal deficits (even if fiscally neutral) and public debt issuance generate positive global spillovers. In the second part, we expand the model to consider aggregate risk and heterogeneity in the net supply of *safe* assets across countries. We show that the overall scarcity of *safe* assets can tip the global economy into a global 'safety trap' wherein (iv) net issuers of safe assets import the recession from abroad.

The ZLB emerges as a natural tipping point. Away from the ZLB, real interest rates clear global asset markets: A shock that creates an *asset shortage* at the prevailing real interest rate results in an endogenous reduction in global real interest rates that restores equilibrium in global asset markets. At the ZLB, real interest rates cannot play their equilibrating role. Global output must adjust to clear asset markets. As global output declines, so does net global asset demand, restoring equilibrium in global asset markets. The role of capital flows also changes at the ZLB: Away from the ZLB, current account surpluses propagate low interest rates from the origin country to the rest of the world. At the ZLB, current account surpluses propagate recessions.

We characterize global imbalances at the ZLB with a *Metzler diagram in quantities* that connects the size of the global recession and net foreign asset positions (and current accounts) to the recessions that would



(a) Global Imbalances

(b) 10-year yields

Note: Panel (a) shows current account balances as a fraction of world GDP from 1980 to 2018. We observe the build-up of global imbalances in the early 2000s, until the financial crisis of 2008. Since then, global imbalances have receded but not disappeared. Deficits subsided in the U.S., and surpluses emerged in Europe. Source: World Economic Outlook (Oct. 2018), and authors' calculations. Oil Producers: Bahrain, Canada, Iran, Iraq, Kuwait, Libya, Mexico, Nigeria, Norway, Oman, Russia, Saudi Arabia, United Arab Emirates, Venezuela; Emerging Asia ex-China: India, Indonesia, Korea, Malaysia, Philippines, Singapore, Taiwan, Thailand, Vietnam. Panel (b) shows the decline in global interest rates. Following the financial crisis, the developed world has remained at or near the Zero Lower Bound.reports nominal yields on 10-years government securities, 1980-2019. We use Germany's 10-year yield as a proxy for the Eurozone 10-year yield. Source: Global Financial Database and FRED

Figure 1: Global Imbalances and Global Interest Rates, 1980-2019

prevail in each country under financial autarky. This is analogous to the case away from the ZLB, where the world equilibrium real interest rate and net foreign asset (and current account) positions are connected to the equilibrium real interest rate that would prevail in each country under autarky. Our analysis shows that, other things equal, when a country's autarky recession is more (less) severe than the global recession, that country is also a net creditor (debtor) and runs a current account surplus (deficit) in the financially integrated environment, effectively exporting its recession abroad. In turn, a country experiences a more (less) severe autarky recession than the average recession when its autarky asset shortage is more (less) severe than the global asset shortage. In this environment, a large country with a severe autarky liquidity trap recession can pull the world economy into a global liquidity trap recession.

But other things need not be equal. In particular, our benchmark model exhibits a critical degree of indeterminacy at the ZLB. This indeterminacy is related to the seminal result by Kareken and Wallace (1981) that the nominal exchange rate is indeterminate in a world with pure interest rate targets. This is *de facto* the case when the economy is in a persistent global liquidity trap at the ZLB. However, in our framework and in contrast to the environments envisioned by Kareken and Wallace (1981), this indeterminacy has substantive real implications. In the presence of nominal rigidities, different values of the nominal exchange rate correspond to different values of the real exchange rate, and therefore to different output levels and current account balances across countries. In a global liquidity trap, global output needs to decline, but the exchange rate affects the *distribution* of recessions across countries. This creates fertile grounds for *zero-sum* beggar-thy-neighbor devaluations achieved by direct interventions in exchange rate markets, stimulating output and improving the current account in one country at the expense of others. Such beggar-thy-neighbor policies can lead to "currency wars" when countries are at the ZLB. By the same token, countries have an incentive to engage in "trade wars", hiking tariffs to divert global demand away from foreign goods and toward domestic goods.

In sharp contrast, policies that alleviate asset scarcity have positive spillovers. In particular, fiscal expansions by countries with sound fiscal accounts have powerful positive spillovers. A balanced budget expansion reduces the net demand for assets, while an unbalanced expansion has the additional virtue of directly expanding asset supply. Moreover, as the global liquidity trap becomes more persistent, fiscal capacity constraints become less relevant. The upshot is that public debt issuance and increases in government spending *anywhere* are expansionary *everywhere*.

The first part of the paper considers a general scarcity of stores of value. The second part of the paper introduces a distinction between *safe* and *risky* assets, along the lines of Caballero and Farhi (2017). This enriched model highlights that macroeconomic outcomes depend on whether there is a scarcity of *safe* assets, not on whether there is an overall scarcity of stores of value. When the return on safe assets reaches the ZLB, the economy enters a 'safety trap.' The scarcity of safe assets depresses the risk-free returns relative to the expected return on risky assets and risk premia increase with the size of the recession. As before, the financial account plays a key role in transmitting economic shocks at the ZLB. However, the most important dimension of the financial account is the net flow of safe assets. At the ZLB, countries that are net issuers of safe assets experience a worse recession than under autarky. However, they also receive a higher return on their (riskier) external assets relative to their (safer) external liabilities, a form of the 'exorbitant privilege.' Because of these opposing forces, within-country wealth inequality worsens for net safe asset issuers relative to financial autarky.

Qualitatively, these results line up well with the available empirical evidence. Panel (a) of Fig. 2 shows Duarte and Rosa (2015)'s estimate of the expected return to U.S. equities along with the yield on one-year Treasuries. The difference between the two lines (light blue area) represents an estimate of the one-year expected equity risk premium (ERP). The figure illustrates how the decline in safe interest rates has not been matched by a decline in expected equity return, i.e. the risk premium has increased dramatically, especially following the Great Recession. Panel (b) reports two estimates of corporate bond risk premia: the Baa and Aaa spread over a 20-year Treasury yield. This figure also indicates a gradual increase in bond risk premia, especially after the financial crisis of 2008, a point first made by Negro, Giannone, Giannoni and Tambalotti (2017).[1] Panel (c) reports a simple estimate of the Net Foreign Asset position in safe assets relative to world GDP from 1980 to 2015.[2] The figure shows that the net supply of safe assets originates largely with the U.S. and—to a smaller extent—the Eurozone. In 2015, the U.S. net supply of safe assets accounted for 11.5% of world GDP, up from 5% in 2000, while the Eurozone net supply accounted for 1.5% of world GDP. On the net demand side, we observe a large increase from China, mostly in the form of Official Reserves, from 0.7% of world GDP in 2000 to 4.9% in 2015; a large increase from oil producers, from 0.24% in 2000 to 2.70% in 2015; and a continued large absorption from Japan (around 2.7% of world GDP). This figure differs substantially from Fig. 1. It indicates that the U.S. external safe asset imbalances have been increasing over time, unlike global imbalances which have stabilized.[3]

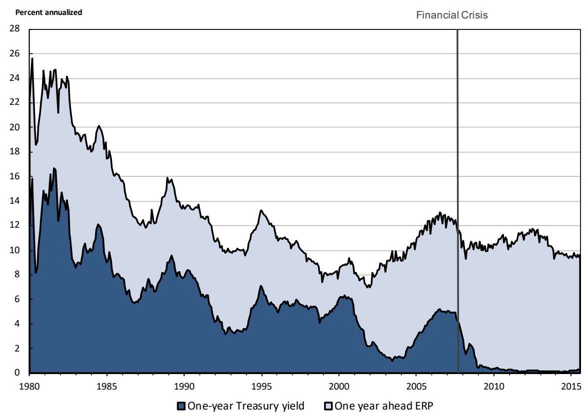
Section 2 presents our basic framework with a general scarcity of store of value. Section 3 explores the potential for negative spillovers -currency and trade wars- at the ZLB. Section 4 discusses positive spillover policies -public debt issuance and fiscal expansion- that directly address the shortage of stores of value that lies behind the ZLB. Section 5 introduces the distinction between safe and risky assets and discusses how macroeconomic outcomes depend on the scarcity of *safe assets* regardless of the overall scarcity of stores of value. We present several important extensions in the appendix. There, we allow for milder nominal rigidities (appendix A.2), introduce home bias (appendix A.3), relax some elasticity assumptions (appendix A.4), and consider a model with heterogeneity in the propensity to save both within and across countries (appendix A.6).

Related literature. Our paper is related to several strands of literature. Most closely related is the literature that identifies the shortage of assets, and especially the shortage of safe assets, as a key macroeconomic driver of global interest rates and capital flows (see e.g. Bernanke (2005), Caballero (2006, 2010); Caballero et al. (2008a,b), Caballero and Krishnamurthy (2009), Mendoza, Quadrini and Ríos-Rull (2009), Bernanke, Bertaut, DeMarco and Kamin (2011), Gourinchas, Rey and Govillot (2010), Maggiori (2012)

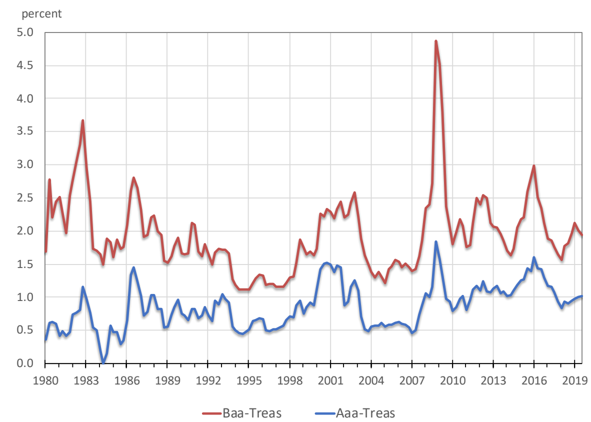
[1] Krishnamurthy (2019) further observes that bond risk premia are even more elevated when one takes into account the decline in volatility and default risk: because volatility has substantially decreased, default risk within a rating class, for example Baa, has decreased; a given credit spread for a given rating class is therefore indicative of a greater price of risk now than in the past. Fixing the riskiness of a bond instead of its rating would therefore result in more rapidly increasing spreads and risk premia over time.

[2] The Net Foreign Asset position in safe assets is constructed from Lane and Milesi-Ferretti (2018)'s update to their External Wealth of Nations dataset as the sum of Official Reserves (minus Gold holdings), Portfolio Debt and Other Assets, minus Portfolio Debt and Other Liabilities. This is a crude estimate of net safe asset positions since neither portfolio debt assets and liabilities nor other assets and liabilities (mostly cross border bank loans) need be safe. Nevertheless, these holdings can be considered *safer* than portfolio equity and direct investment.

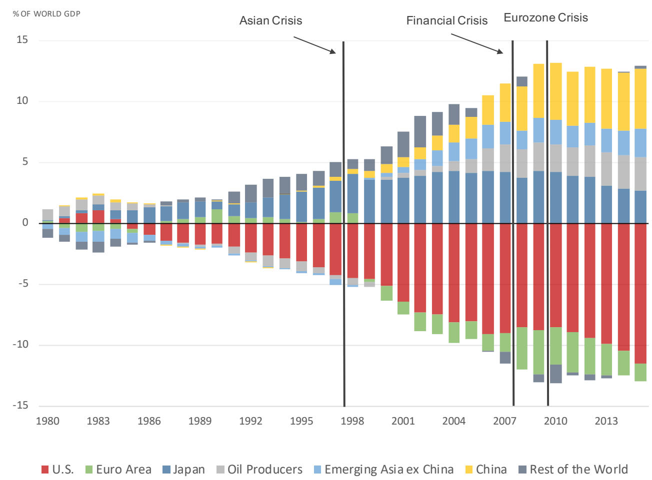
[3] Figure Fig. 1 reports flows (current accounts), while panel (c) of Fig. 2 reports stocks (net safe asset holdings). This is mostly because it is easier to construct estimates of the Net Foreign Asset position in safe assets than the corresponding Net Flows.



(a) Expected Equity Risk Premium



(b) Bond Premia



(c) Net Safe Asset Imbalances

Note: Panel (a) shows the one-year US Treasury yield (dark area) and the one-year expected risk premium (ERP) (grey area), calculated as the first principal component of 20 models of the one-year-ahead equity risk premium, 1980-2015. The figure shows that the equity risk premium has increased, especially since the Global Financial Crisis. Source: One-year Treasury yield: Federal Reserve H.15; ERP: Duarte and Rosa (2015). Panel (b) shows the spread between Moody's Aaa and Baa seasoned corporate bond yields and the 20-year constant maturity Treasury, 1980-2019. The Baa and Aaa spreads have increased since the 2008 financial crisis. Source: Fred. Panel(c) shows Net Safe positions as a fraction of world GDP, 1980-2015. Net Safe positions are defined as the sum of Official Reserves (minus Gold), Portfolio Debt and Other Assets, minus Portfolio Debt and Other Liabilities. The net supply of safe assets originates largely with the US and—to a smaller extent—the Eurozone. Source: Lane and Milesi-Ferretti (2018) update of the External Wealth of Nations. Oil Producers: Bahrain, Canada, Iran, Iraq, Kuwait, Libya, Mexico, Nigeria, Norway, Oman, Russia, Saudi Arabia, United Arab Emirates, Venezuela; Emerging Asia ex-China: India, Indonesia, Korea, Malaysia, Philippines, Singapore, Taiwan, Thailand, Vietnam.

Figure 2: Risk Premia and Net Safe Asset Positions

and Coeurdacier, Guibaud and Jin (2015)). In particular, Caballero et al. (2008a) develops the idea that global imbalances originated in the superior development of financial markets in developed economies (as well as in the decline in potential growth of Europe and Japan relative to the U.S.). This paper analyzes the implications of asset scarcity when the world economy experiences ultra-low natural real interest rates and is constrained by the Zero Lower Bound: The adjustment now occurs through quantities (output) rather than prices (interest rates), and exchange rates play an important role in allocating a global slump across countries.

Another strand of the literature emphasizes that public debt is safe because it is insensitive to information, mitigating the role of information asymmetries and discouraging investors from acquiring information (see for example Gorton (2010), Stein (2012), Moreira and Savov (2014), Gorton and Ordonez (2013, 2014), Dang, Gorton and Holmström (2015) and Greenwood, Hanson and Stein (2015)). A recent literature also considers relative degrees of safety and what makes some assets 'safe' in equilibrium when there are coordination problems (see for example He, Krishnamurthy and Milbradt (2015)). Our model offers a different interpretation, where the "specialness" of public debt and close substitutes arises from their safety in bad aggregate states (see also Gennaioli, Shleifer and Vishny (2012), Barro and Mollerus (2014), and Caballero and Farhi (2017)).

There is an extensive literature on liquidity traps (see e.g. Keynes (1936), Krugman (1998), Eggertsson and Woodford (2003), Christiano, Eichenbaum and Rebelo (2011), Guerrieri and Lorenzoni (2011), Eggertsson and Krugman (2012), Werning (2012), and Correia, Farhi, Nicolini and Teles (2013)). This literature emphasizes that the binding Zero Lower Bound on nominal interest rates presents an important challenge for macroeconomic stabilization. A subset of this literature considers the implications of a liquidity trap in the open economy (see e.g. Svensson (2003), Jeanne (2009), Farhi and Werning (2012), Cook and Devereux (2013a,b, 2014), Devereux and Yetman (2014), Benigno and Romei (2014),Erceg and Lindé (2014), and Fornaro and Romei (2019)). While many of these papers share similar themes, our paper makes three distinct contributions. First, we use our *Metzler diagram in quantities* to elucidate the link between the global recession and net foreign asset positions. We also allow for permanent liquidity traps and capital flows (global imbalances). Finally, we make a distinction between risky and safe assets.

A recent literature explores the link between rising inequality and imbalances (see for example Kumhof, Rancière and Winant (2015); Mian, Straub and Sufi (2020a,b) or Klein and Pettis (2020)). Many of these contributions emphasize the implications of rising income and wealth inequality for global asset returns and global imbalances. In contrast, our approach starts from asset shortages and explores the joint implications for global imbalances, asset returns and within-country inequality. Everything else equal, an increase in domestic income inequality that increases savings should tend to weakly improve external balances. Yet, the U.S. experienced both rising inequality and worsening external balances. Our approach predicts that net safe asset suppliers, like the U.S., experience external imbalances in a global safety trap, that these external imbalances imply a larger recession than under financial autarky, and that this larger recession is associated with increased wealth inequality.

Our paper is also related to the recent literature on secular stagnation (see e.g. Kocherlakota (2014), Eggertsson and Mehrotra (2014), Caballero and Farhi (2017)). Like us, these papers use an OLG structure with a zero lower bound and nominal rigidities, but in a closed economy. Our contribution is to explore the open economy dimension of the secular stagnation hypothesis. We study the propagation of liquidity traps from one country to another and the role of global imbalances and policy spillovers at the ZLB. From that perspective, the paper closest to ours is Eggertsson, Mehrotra, Singh and Summers (2015) which finds, like us, that exchange rates have powerful effects when the economy is in a global liquidity trap. Complementary to ours, their paper explores the role of market integration and capital controls. Our paper emphasizes other methodological and substantive dimensions, such as the Metzler diagram in quantities, the spillovers of safe public debt issuance, the role of capital flows in spreading liquidity traps and macroeconomic policies, and the role of safe vs. risky assets.

2 A Model of the Diffusion of Liquidity Traps

We begin our analysis with a model that considers a general scarcity of stores of value, but no risk dimension. Section 5 introduces macroeconomic risk and the distinction between safe and risky assets. We first lay out the assumptions of the model and characterize the world equilibrium. Throughout the paper, we focus on steady state balanced growth paths. To build the intuition, we consider first the benchmark of financial autarky, then move to the full model under financial integration. In each case, we characterize the equilibrium and discuss the relevant economic mechanisms both at and away from the Zero Lower Bound (ZLB).

2.1 Assumptions and Competitive Equilibrium

Time is continuous. There are two countries, *Home* and *Foreign*. Foreign variables are denoted with stars. We first describe *Home*, and then move on to *Foreign*.

Demographics. Population is constant and normalized to one. Agents are born and die at a constant hazard rate θ, independent across agents. Each dying agent is instantaneously replaced by a newborn. Therefore, in an interval dt, θdt agents die and θdt agents are born, leaving total population unchanged.

Preferences. Agents have a single opportunity to consume, c_t, at the time of death. Until they die, agents save and reinvest all their income.[4] Formally, we let τ_θ denote the stopping time for the idiosyncratic death process. Agents value home and foreign goods according to a Cobb-Douglas aggregate with an expenditure share on the home good $\gamma \in [0,1]$, are risk neutral over short time intervals, and do not discount the future. For a given stochastic consumption process -over the uncertain idiosyncratic time of death- of home and foreign goods $\{c_{H,t}, c_{F,t}\}$, which is measurable with respect to the information available at date t, we define the utility U_t of an agent alive at that date with the following stochastic differential equation:

$$\begin{aligned} U_t &= 1_{\{t-dt \leq \tau_\theta < t\}} c_t + 1_{\{t \leq \tau_\theta\}} \mathbb{E}_t[U_{t+dt}], \\ c_t &= c_{H,t}^{\gamma} c_{F,t}^{1-\gamma}, \end{aligned} \tag{1}$$

where we use the notation $\mathbb{E}_t[U_{t+dt}]$ to denote the expectation of U_{t+dt} conditional on the information available at date t.[5]

Nominal rigidities, potential output and actual output. In an interval dt, potential output of the home good is $\bar{Y}_t dt$, where $\bar{Y}_t$ grows at the exogenous rate g. Because of nominal rigidities, actual output Y_t is demand-determined and can be lower than potential output, $\bar{Y}_t$. We define $\xi_t \equiv Y_t/\bar{Y}_t \in [0,1]$, the ratio of output to potential output and, slightly abusing terminology, we refer to ξ_t as the *output gap*, with $\xi < 1$ when the economy is in a recession.

We assume that nominal rigidities take an extreme form: the prices of home goods are fully and permanently rigid in the home currency (Appendix A.2 relaxes this assumption). We normalize home prices to one, $P_{H,t} = 1$, and assume that the Law of One Price holds so that the price of home goods in the foreign currency is $P_{H,t}/E_t$ where E_t is the nominal exchange rate, defined as the home price of the foreign currency. With this definition, an increase in E_t represents a depreciation of the home currency. Home's consumer price index (CPI) satisfies $P_t = (1/\gamma)^\gamma (E_t/(1-\gamma))^{1-\gamma}$, and Home CPI inflation is $\pi_t = (1-\gamma)\dot{E}_t/E_t$. Appendix A.1 provides a micro-foundation in the New Keynesian tradition with monopolistic competition and rigid prices à la Blanchard and Kiyotaki (1987).

Private incomes, assets, and financial development. Domestic income has two components: the income of newborns and financial income. In the interval dt, newly born agents receive income $(1-\delta)\xi_t \bar{Y}_t dt$.

[4]This assumption allows us to focus on the store-of-value scarcity we wish to highlight, while removing non-central intertemporal substitution considerations. See Caballero et al. (2008a); Gourinchas and Rey (2014) for details.

[5]Note that the information at date t contains the information about the realization of the idiosyncratic shocks up to t, implying that $1_{\{t-dt \leq \tau_\theta < t\}}$ and $c_{H,t}$ and $c_{F,t}$ are known at date t. Similarly, the conditional expectation $\mathbb{E}_t$ is an expectation over idiosyncratic death shocks.

The remainder of income, $\delta\xi_t\bar{Y}_t dt$, is distributed as financial income. Specifically, we assume there is a mass $\bar{Y}_t$ of Lucas trees, each producing a claim to a dividend of $\delta\xi_t$ units of output in the interval dt. With independent and instantaneous probability ρ each tree dies and the corresponding stream of dividends is transferred to a new tree. The stock of trees grows at rate g to accommodate growth in potential output. All new trees are bestowed to newborns.

Financial development is controlled by two key parameters: ρ and δ. The assumption that trees die ($\rho > 0$) can be interpreted either as a consequence of creative-destruction, or as a form of weak property rights. Either way, this assumption reduces the share of future output that is capitalized into assets that are traded today: a higher ρ reduces the aggregate supply of assets.

The assumption that only a fraction of output can be capitalized into traded financial claims ($\delta < 1$) captures many factors behind the limited pledgeability of income, as in Caballero et al. (2008a). At the most basic level, one can interpret δ as the share of income paid to capital in production. But in reality only a fraction of this share can be committed to asset holders, as the government, managers, and other insiders can dilute and divert part of the profits. For this reason, we refer to δ as an index of financial development that captures how well-defined and tradable rights over earnings are in the home country's financial markets. A lower δ reduces asset supply and simultaneously increases asset demand, since newborns receive a higher share of total income, which they save.

Public debt and the provision of public liquidity. In addition to private assets, we assume that a home government issues short-term public debt D_t, which it services by levying taxes τ_t on the non-financial income $(1-\delta)\xi_t\bar{Y}_t$ of newborns. We let $d_t = D_t/\bar{Y}_t$ denote the ratio of home public debt to potential output and assume that the tax rate is adjusted to maintain the desired ratio of debt to potential output, d_t.

Public debt plays a critical role in our model. Since the environment is non-Ricardian, public debt does not fully crowd out private financial assets.[6] An increase in the ratio of public debt to potential output (d_t) increases the total supply of assets, while the concomitant increase in taxes decreases the demand for these assets, since it reduces the disposable income of newborns. Therefore, public debt provides 'public liquidity' in the sense of Holmström and Tirole (1998). By taxing the income of future (unborn) generations, the government capitalizes part of the economy's non-financial income into public debt.

[6] Our framework becomes Ricardian if all future financial income is capitalized into existing financial assets (i.e. if $\rho = g = 0$ so that there are no new trees) and if taxes fall entirely on financial income. This would occur despite the overlapping generations: public debt would crowd out private assets one-for-one. See Caballero et al. (2008a) and Gourinchas and Rey (2014) for a detailed discussion of the Non-Ricardian features of this type of model.

Monetary policy and the ZLB. Home monetary policy follows a truncated Taylor rule:

$$i_t = \max\{r_t^n + \psi_\pi \pi_{H,t} + \psi_\xi(\xi_t - 1), 0\}. \tag{2}$$

In this equation, i_t is the home nominal interest rates and r_t^n is the relevant *natural real interest rate* at Home, defined as the real interest rate that clears markets when we ignore the ZLB constraint. ψ_π and ψ_ξ denote, respectively, the Taylor rule coefficient on domestic price inflation $\pi_{H,t} = \dot{P}_{H,t}/P_{H,t}$ and on the output gap $(\xi_t - 1)$. Since prices are fully rigid, $\psi_\pi \pi_{H,t} = 0$ for any value of ψ_π, however large. For simplicity, we place ourselves in the limit of very reactive Taylor rule, with $\psi_\xi \to \infty$. This condition ensures that we have either $\xi_t = 1$ and $i_t = r_t^n > 0$ or $\xi_t \leq 1$ and $i_t = 0 > r_t^n$.

Foreign. Foreign differs from Home along five dimensions. First, potential output of the foreign good is given by $\bar{Y}_t^*$, which also grows at rate g, and the output gap is denoted $\xi_t^* = Y_t^*/\bar{Y}_t^*$. Second, we allow financial development to differ, with the financial capacity of the foreign country given by δ^*. Third, public debt in the foreign country is given by D_t^*, the debt to output ratio by d_t^*, and taxes by τ_t^*. Fourth, Foreign has its own currency and the prices of foreign goods are sticky in this currency. We normalize the price of the Foreign good to one in the foreign currency: $P_{F,t}^* = 1$. Fifth, foreign monetary policy follows a truncated Taylor rule similar to Home's:

$$i_t^* = \max\{r_t^{n*} + \psi_\pi \pi_{F,t} + \psi_\xi(\xi_t^* - 1), 0\}. \tag{3}$$

where r_t^{n*} is the relevant natural real rate in the foreign country and $\pi_{F,t} = \dot{P}_{F,t}/P_{F,t} = 0$ denotes foreign price inflation. As for the domestic Taylor rule, we consider the limit case where $\psi_\xi \to \infty$, so that either $\xi_t^* = 1$ and $i_t^* = r_t^{n*}$, or $\xi_t^* \leq 1$ and $i_t^* = 0 > r_t^{n*}$.

We assume that there is no home bias and that in both countries the share γ of home consumption is equal to the share of potential output of home goods in total output: $\gamma = y$, where $y \equiv \bar{Y}_t/(\bar{Y}_t + \bar{Y}_t^*)$ (Appendix A.3 relaxes the assumption of no home bias).

Competitive equilibrium. We denote by W_t and W_t^* the total wealth of home and foreign households in their respective currencies. V_t and V_t^* are the total value of home and foreign private assets in their respective currencies.

We start with the benchmark of *financial autarky* then move on to the full model with *financial integration*. Under financial autarky, agents are free to trade goods across countries, but they cannot trade financial claims. Under financial integration, agents can also trade claims to the Lucas trees and public debt across borders.

We now write the domestic and foreign wealth dynamics, asset pricing conditions, government constraints, and market clearing conditions, then define and characterize a competitive equilibrium of our economy in each environment.

First, at each instant aggregate nominal consumption expenditure satisfies $P_t c_t = \theta W_t$ and $P_t^* c_t^* = \theta W_t^*$, since a fraction θ of the population in each country dies every instant and consumes all its wealth.

Second, the evolution of Home and Foreign aggregate wealth follow:

$$\dot{W}_t = (1-\tau_t)(1-\delta)\xi_t \bar{Y}_t - \theta W_t + i_t W_t + (\rho+g)V_t, \tag{4a}$$

$$\dot{W}_t^* = (1-\tau_t^*)(1-\delta^*)\xi_t^* \bar{Y}_t^* - \theta W_t^* + i_t^* W_t^* + (\rho+g)V_t^*. \tag{4b}$$

The change in home aggregate wealth has three components: (i) the newborn's net of-tax-income $(1-\tau_t)(1-\delta)\xi_t \bar{Y}_t$ is earned and consumption expenditure θW_t from dying agents is subtracted; (ii) home wealth earns a return equal to the home nominal risk-free rate, i_t; (iii) new trees with aggregate value $(\rho+g)V_t$, accounting both for creative destruction and growth of potential output, are endowed to newborns. Foreign wealth follows similar dynamics with a return equal to the foreign nominal risk-free rate, i_t^*.

Third, since there is no aggregate risk, the return to private assets equals the nominal risk free rate in each country:

$$i_t V_t = \delta \xi_t \bar{Y}_t - \rho V_t + \dot{V}_t - g V_t, \tag{5a}$$

$$i_t^* V_t^* = \delta^* \xi_t^* \bar{Y}_t^* - \rho V_t^* + \dot{V}_t^* - g V_t^*. \tag{5b}$$

This return consists of three terms. First a dividend payment of $\delta \xi_t \bar{Y}_t$; second a capital loss equal to the fraction of trees that die, $-\rho V_t$; third a capital gain $\dot{V}_t - g V_t$ for surviving trees.[7]

In addition, under financial integration Uncovered Interest Parity (UIP) holds between Home and Foreign since agents are risk neutral:

$$i_t = i_t^* + \frac{\dot{E}_t}{E_t}. \tag{6}$$

Combined with the expression for domestic and foreign CPI inflation rates, UIP ensures that real returns are equalized under financial integration: $r_t = r_t^*$ where $r_t = i_t - \pi_t$ and $r_t^* = i_t^* - \pi_t^*$.

[7] The term $-gV_t$ is a correction for the fact that the number of trees is growing with potential output. To obtain this expression, observe that the value of a single home Lucas tree, v_t, defined as a claim to $\delta\xi_t$ units of output, satisfies $i_t v_t = \delta\xi_t - \rho v_t + \dot{v}_t$. The value of all home trees is $V_t = v_t \bar{Y}_t$.

Fourth, government debt dynamics can be expressed as

$$\dot{D}_t = i_t D_t - \tau_t \left(1-\delta\right) \xi_t \bar{Y}_t, \tag{7a}$$

$$\dot{D}^*_t = i^*_t D^*_t - \tau^*_t \left(1-\delta^*\right) \xi^*_t \bar{Y}^*_t, \tag{7b}$$

where the first term represents interest payments (at the risk free local interest rate) and the second term represents tax revenues on local non-financial income.

Fifth, market clearing conditions for home and foreign goods require

$$c_{H,t} + c^*_{H,t} = \gamma\theta(W_t + E_t W^*_t) = \xi_t \bar{Y}_t, \tag{8a}$$

$$E_t(c_{F,t} + c^*_{F,t}) = (1-\gamma)\theta(W_t + E_t W^*_t) = E_t \xi^*_t \bar{Y}^*_t. \tag{8b}$$

To understand the first expression, observe that home consumption expenditure on the home good (in home currency), $c_{H,t}$, represents a fraction γ of total home consumption expenditure θW_t, while foreign consumption expenditure on the home good (in foreign currency), $c^*_{H,t}/E_t$, represent the same fraction γ of total foreign consumption expenditure θW^*_t since there is no home bias in consumption. The second expression is derived in a similar way.

Finally, under financial integration asset market clearing requires that total asset demand equals total asset supply:

$$(V_t + D_t) + E_t(V^*_t + D^*_t) = W_t + EW^*_t, \tag{9}$$

while under financial autarky, asset demand must equal asset supply in each country:

$$V_t + D_t = W_t, \tag{10a}$$

$$V^*_t + D^*_t = W^*_t. \tag{10b}$$

We can now define a competitive equilibrium, both under financial integration, when home and foreign agents are free to trade financial claims, and under financial autarky, when they are restricted to trade financial assets within their country.

Definition 1. *(Competitive Equilibrium under Financial Integration and Financial Autarky)* *Given paths for the ratio of public debt to potential output, d_t and d^*_t, a competitive equilibrium consists of sequences for output gaps ξ_t and ξ^*_t, natural real rates r^n_t and r^{n*}_t, household wealth W_t and W^*_t, private financial assets V_t and V^*_t, taxes τ_t and τ^*_t, consumptions c_t and c^*_t, consumer prices P_t and P^*_t, policy rates i_t and i^*_t, and the nominal exchange rate E_t, such that (i) household consumption, wealth and private* *assets satisfy Eqs. (4) and (5); (ii) debt dynamics follow Eq. (7) with $D_t = d_t\bar{Y}_t$ and $D_t^* = d_t^*\bar{Y}_t^*$; (iii) policy rates are set according to Eqs. (2) and (3); and (iv) goods markets clear Eq. (8). Moreover:*

- *Under financial integration, global asset markets clear* (Eq. (9)) *and UIP holds* (Eq. (6))*;*
- *Under financial autarky, asset markets clear only locally* (Eq. (10))*.*

We specialize the model by focusing on steady state Balanced Growth Paths (BGP), where both economies grow at rate g and the ratio of debt to potential output in both countries, d and d^*, are constant. With some abuse of notation, we drop the time subscript. Along a BGP, the exchange rate E, prices P, P^*, output gaps ξ, ξ^*, policy rates i, i^* and taxes τ, τ^* are constant, while wealth W, W^*, private assets V, V^*, public debt D, D^* and consumption c, c^* grow at rate g.

First, we characterize the benchmark of financial autarky equilibrium both at and away from the ZLB, then we move to the full model with financial integration.

2.2 A Benchmark: Financial Autarky

It is useful to introduce the concepts of *financial autarky natural rates*, $r^{a,n}$ and $r^{a,n*}$ and *financial autarky natural output gaps* $\xi^{a,n}$ and $\xi^{a,n*}$:

$$r^{a,n} \equiv -\rho + \frac{\delta\theta}{1-\theta d} \quad ; \quad r^{a,n*} \equiv -\rho + \frac{\delta^*\theta}{1-\theta d^*} \tag{11a}$$

$$\xi^{a,n} \equiv \frac{\theta d}{1-\delta\theta/\rho} \quad , \quad \xi^{a,n*} \equiv \frac{\theta d^*}{1-\delta^*\theta/\rho}. \tag{11b}$$

The financial autarky natural rate is the real interest rate consistent with potential output under financial autarky, when we ignore the ZLB constraint. The financial autarky natural output gap is the level of output that obtains when the interest rate is set at the ZLB under financial autarky. We make the following assumption.

Assumption 1.

$$0 < \delta, \delta^* < \rho/\theta \quad ; \quad 0 < d, d^* \quad ; \quad 0 < \delta/(1-\theta d), \delta^*/(1-\theta d^*) \leq 1$$

We will discuss the role of Assumption 1 in detail after we state our first proposition, which characterizes the economy under financial autarky, both away from the ZLB and at the ZLB.

Proposition 1 (Financial Autarky Away from and At the Zero Lower Bound)**.** *Under financial autarky and Assumption 1, the competitive equilibrium is as follows:*

- *The home economy satisfies* $i^a = r^a = \max\{r^{a,n}, 0\}$ *and* $\xi^a = \min\{\xi^{a,n}, 1\}$.

  - *If* $r^{a,n} \geq 0$, *then* $\xi^{a,n} \geq 1$ *and the home economy is away from the Zero Lower Bound. There is a unique balanced growth path equilibrium with a positive interest rate,* $i^a = r^a = r^{a,n}$, *and output at its potential level,* $\xi^a = 1$.
  - *If* $r^{a,n} < 0$, *then* $\xi^{a,n} < 1$ *and the home economy is at the Zero Lower Bound. There is a unique balanced growth path equilibrium with* $i^a = r^a = 0 > r^{a,n}$, *and home output is below its potential level, with* $\xi^a = \xi^{a,n} < 1$.

- *Similarly, the foreign economy satisfies* $i^{a*} = r^{a*} = \max\{r^{a,n*}, 0\}$ *and* $\xi^{a*} = \min\{\xi^{a,n*}, 1\}$.

- *The autarky exchange rate satisfies:*
$$E^a = \frac{\xi^a}{\xi^{a*}}. \tag{12}$$

Proof. See text. □

To understand the economics behind this proposition, observe first that along a BGP the nominal exchange rate is constant, so all prices are constant and there is no inflation: $\pi^a = \pi^{a*} = 0$. It follows that nominal and real interest rates coincide, $i^a = r^a$ and $i^{a*} = r^{a*}$. From the goods market conditions Eqs. (8a) and (8b), the autarky exchange rate obtains immediately as the ratio of the output gaps, $E^a = \xi^a/\xi^{a*}$, which establishes the last part of the proposition.

Consider now a BGP financial autarky equilibrium with home output gap ξ and home real interest rate r. From Eq. (5a), total home asset supply along that BGP is given by

$$V + D = \frac{\delta}{r+\rho}\xi\bar{Y} + d\bar{Y}, \tag{13}$$

which is decreasing with the interest rate. From Eq. (4a), home asset demand along the BGP satisfies

$$W = \frac{\xi}{\theta}\bar{Y}, \tag{14}$$

which is invariant to the interest rate. The financial autarky natural rate $r^{a,n}$ given by Eq. (11a) equates asset demand with asset supply ($V + D = W$) when output is at its potential level, $\xi = 1$. Because of the ZLB, this is only possible if $r^{a,n} \geq 0$. Next, observe that we can rewrite $\xi^{a,n} = 1 + r^{a,n}(1-\theta d)/(\rho - \delta\theta)$, so that under Assumption 1, $\xi^{a,n} \geq 1$ if and only if $r^{a,n} \geq 0$. This establishes the first part of the proposition.

Suppose now that $r^{a,n} < 0$. Inspecting Eq. (11a), this occurs when δ is low or ρ is high (i.e. a low supply of private assets), when d is low (i.e. a low supply of public assets) or when θ is low (i.e. a high demand for stores of value). In this case, the ZLB constraint imposes $i^a = r^a = 0$. That is, a ZLB equilibrium arises when there is *a shortage of private or public assets that cannot be resolved by a decline in equilibrium real interest rates.*

Instead, at the ZLB an alternative (perverse) equilibrating mechanism endogenously arises in the form of a recession with $\xi^a < 1$. Under Assumption 1, at a fixed zero interest rate, the recession reduces asset demand (Eq. (14)) more than asset supply (Eq. (13)), which helps restore equilibrium in the global asset market. The size of the required home recession is given by Eq. (11b). This establishes the second part of the proposition. The last part of the proposition obtains by symmetry.[8]

We can now understand the role of Assumption 1. The conditions $\delta\theta - \rho < 0$ and $d > 0$ ensure that asset demand decreases faster than asset supply as ξ^a declines, and that the intersection satisfies $0 < \xi^{a,n} < 1$. The condition $\delta \leq 1 - \theta d$ ensures that the supply of public assets is not large enough to satisfy asset demand so that $\theta > r^{a,n}$.[9] The condition $d > 0$ is economically important: by appropriately adjusting the tax rate, the value of public debt is *not* affected by the size of the recession ξ. By contrast, along a balanced growth path, the supply of *private* financial assets is proportional to the level of aggregate activity: $V_t = \delta\xi/(r+\rho)\bar{Y}_t$. In that sense, public debt is a *'macroeconomic safe asset.'* We develop this point in more details in Section 5 where we introduce aggregate risk.

Even though agents from each country consume goods from both countries, ZLB recessions stay in their own economy. According to Proposition 1, whether a country is in a ZLB equilibrium depends only on its own financial autarky natural rate $r^{a,n}$. Under financial autarky, the nominal exchange rate adjusts to reflect the relative scarcity of goods according to Eq. (12). Countries with a more severe ZLB recession (a lower ξ^a) have a stronger currency (a lower E^a) that sustains their purchasing power for the foreign good and prevents the ZLB recession from spilling over to the other country. In other words, under financial autarky, domestic financial conditions determine the level of domestic output. The exchange rate simply adjusts to make sure that the corresponding equilibrium is consistent with an integrated goods market. This changes under financial integration.

[8] An equivalent interpretation of the ZLB equilibrium comes from the goods market. At every instant, the demand for goods arises from old households who die. At the ZLB, the aggregate purchasing power of these households in local currency (aggregate demand) is given by $\theta(V + D) = \theta(\xi^a\theta\delta/\rho + d)\bar{Y}$. The market value of domestic goods brought to the market in local currency (aggregate supply) is $\xi^a\bar{Y}$. Since trade is balanced under financial autarky, the two must be equal. When $r^{a,n} < 0$, aggregate supply exceeds aggregate demand at $\xi^a = 1$. In other words, old agents don't have enough purchasing power to buy all the goods supplied by the young. With nominally rigid prices, output is demand-determined, as in standard New Keynesian models. The recession simultaneously reduces aggregate supply and aggregate demand, but supply falls more than demand under the conditions of Assumption 1, helping to restore equilibrium in the goods market.

[9] The restrictions imposed by Assumption 1 are unlikely to be binding in practice. For instance, if we assume $\rho = 3\%$ and $\theta = 5\%$, Assumption 1 implies that $\delta, \delta^* < 3/5$ and $d < 8$. A reasonable estimate of δ is likely smaller than the capital share, often estimated around 1/3, while realistic debt-output ratios are significantly lower than 8.

2.3 The Full Model: Financial Integration

We now allow for financial integration. As in the benchmark case of financial autarky, the exchange rate is constant along a BGP. From Eq. (6), it follows that $i = i^* = i^w = r = r^* = r^w$, where i^w and r^w denote the world nominal and real interest rates. In sharp contrast with the autarky benchmark, this implies that either no country is trapped at the ZLB, $i^w = r^w > 0$, or all countries are, $i^w = r^w = 0$.

By analogy with the case of financial autarky, we define the *world natural interest rate* $r^{w,n}$ as

$$r^{w,n} = -\rho + \frac{\bar{\delta}\theta}{1 - \theta\bar{d}}, \tag{15}$$

where $\bar{\delta} = y\delta + (1-y)\delta^*$ is the world's financial capacity and $\bar{d} = yd + (1-y)d^*$ is the world's public debt to potential output ratio evaluated at $E = 1$. The world natural interest rate is the real rate consistent with global potential output when we ignore the ZLB constraint. It is similar to the autarky natural interest rate, Eq. (11a), but for the world as a whole. We further define two bounds on the nominal exchange rate:

$$\bar{E} = 1 - \frac{(1-\theta\bar{d})r^{w,n}}{(1-y)d^*\theta\rho} \quad ; \quad \underline{E} = \left(1 - \frac{(1-\theta\bar{d})r^{w,n}}{yd\theta\rho}\right)^{-1}. \tag{16}$$

The next proposition characterizes the BGP equilibrium away from the ZLB and at the ZLB.

Proposition 2 (Financial Integration). *Under financial integration and Assumption 1, competitive equilibria along a BGP are as follows:*

- *If $r^{w,n} \geq 0$, then the global economy is away from the Zero Lower Bound. There is a unique BGP with a positive interest rate, $i^w = r^w = r^{w,n}$, output is at its potential level, $\xi = \xi^* = 1$, and $E = 1$.*

- *If $r^{w,n} < 0$, then the global economy is at the ZLB. There is a continuum of BGP equilibria with $i^w = r^w = 0$, indexed by $E \in [\underline{E}, \bar{E}]$, where ξ and ξ^* satisfy*

$$\xi = \frac{\theta\bar{d}(E)}{1 - \bar{\delta}\theta/\rho} \quad ; \quad \xi^* = \frac{\theta\bar{d}(E)/E}{1 - \bar{\delta}\theta/\rho}, \tag{17}$$

  and $\bar{d}(E) = yd + (1-y)d^*E$ is the exchange-rate-adjusted ratio of global public debt to potential output.

- *In all cases, the exchange rate satisfies*

$$E = \frac{\xi}{\xi^*}. \tag{18}$$

Proof. See text. □

As in the financial autarky benchmark, the last part of the proposition obtains immediately from the goods market equilibrium conditions Eq. (8). Consider a BGP under financial integration away from the ZLB, $\xi = \xi^* = 1$ with global real rate r^w. It follows from Eq. (18) that the exchange rate is $E = 1$. Define world potential output $\bar{Y}^w = \bar{Y} + E\bar{Y}^*$, the world supply of private assets $V^w = V + EV^*$, the world supply of public assets $D^w = D + ED^*$, and world wealth $W^w = W + EW^*$, all in Home's currency. From Eqs. (5) and (7), total asset supply $V^w + D^w$ along the BGP is given by

$$V^w + D^w = \left(\frac{\bar{\delta}}{r^w + \rho} + \bar{d} \right) \bar{Y}^w, \tag{19}$$

which is decreasing in the global real rate r^w, while total asset demand along the BGP satisfies

$$W^w = \frac{\bar{Y}^w}{\theta}, \tag{20}$$

which is invariant to the interest rate. The natural rate $r^{w,n}$ given by Eq. (15) equates global asset demand and global asset supply, $W^w = V^w + D^w$. Because of the ZLB, this is only possible if $r^{w,n} \geq 0$, which establishes the first part of the proposition.

We can express the world natural rate $r^{w,n}$ as a weighted average of home and foreign financial autarky real rates:

$$r^{w,n} = y \frac{1 - \theta d}{1 - \theta \bar{d}} r^{a,n} + (1 - y) \frac{1 - \theta d^*}{1 - \theta \bar{d}} r^{a,n*}. \tag{21}$$

Under Assumption 1, the weights are positive and sum to one. It follows that the world natural rate always lies between the home and foreign financial autarky rates. Hence, the global economy may escape the ZLB ($r^{w,n} \geq 0$) even if Home (but not Foreign) finds itself at the ZLB under financial autarky, i.e. when $r^{a,n} < 0 \leq r^{w,n} < r^{a,n*}$. This occurs when the scarcity of assets in Home is offset by an abundance of assets in Foreign. In this case, financial integration pulls Home away from the ZLB.

Suppose now that $r^{w,n} < 0$. Inspecting Eq. (15), this occurs when $\bar{\delta}$ is low or ρ is high (i.e. a low global supply of private assets), when $\bar{d}$ is low (i.e. a low global supply of public assets) or when θ is low (i.e. a high global demand for stores of value). The ZLB constraint imposes $i^w = r^w = 0$. In other words, under financial integration a ZLB equilibrium arises when there is a *global* shortage of private or public assets that cannot be resolved by a decline in the global real rate.

A global ZLB can only arise if at least one country (e.g. Home) is at the ZLB under financial autarky. However, the global economy may be pushed against the ZLB ($r^{w,n} < 0$) even if Foreign would have remained away from the ZLB under financial autarky, i.e. when $r^{a,n} < r^{w,n} < 0 \leq r^{a,n*}$. This occurs when the scarcity of assets in Home is too large to be offset by asset supply in Foreign. Financial integration drags Foreign into a ZLB trap it would avoid under autarky.

Along a balanced growth path at the ZLB, total asset supply can be expressed as

$$V^w + D^w = \frac{\delta \xi \bar{Y} + \delta^* E \xi^* \bar{Y}^*}{\rho} + d\bar{Y} + E d^* \bar{Y}^*, \tag{22}$$

and global asset demand satisfies

$$W^w = \frac{\xi \bar{Y} + E \xi^* \bar{Y}^*}{\theta}. \tag{23}$$

In addition, both the asset market Eq. (9) and the goods market Eq. (18) need to clear.

This is a system of *four* equations Eqs. (9), (18), (22) and (23), in *five* unknowns V^w, W^w, ξ, ξ^*, and E. That is, there is a *degree of indeterminacy.* This indeterminacy is related to the seminal result by Kareken and Wallace (1981) that the exchange rate is indeterminate with pure interest rate targets, which is *de facto* the case when both countries are at the ZLB. However, unlike in Kareken and Wallace (1981), money is *not* neutral in our model: different exchange rates correspond to different levels of output at Home and in Foreign, as prescribed by Eq. (18). In other words, while global output needs to decline to restore equilibrium in asset markets, different combinations of domestic and foreign output—corresponding to different values of the exchange rate—are possible.[10,11]

Indexing these different solutions by the exchange rate, we can substitute $\xi^* E = \xi$ and equate world asset demand and world asset supply:

$$\frac{\xi}{\theta} = \frac{\bar{\delta}}{\rho}\xi + \bar{d}(E). \tag{24}$$

Solving for the home output gap yields Eq. (17). A similar derivation holds for Foreign. Any value of the exchange rate is possible as long as *both* countries are at the ZLB, i.e. $\xi \leq 1$ and $\xi^* \leq 1$. This determines a range $[\underline{E}, \bar{E}]$ with $\xi = 1$ for $E = \bar{E}$ and $\xi^* = 1$ for $E = \underline{E}$, where $\underline{E}$ and $\bar{E}$ are defined in Eq. (16). This establishes the second part of Proposition 2.

As in the benchmark case of financial autarky, under Assumption 1 a recession at Home or in Foreign

[10]Note that the indeterminacy we characterize is about the *level* of the exchange rate. Outside of a BGP, it might also be possible for the entire path $(E_t)_{t\geq 0}$ of the exchange rate to be indeterminate within $[\underline{E}, \bar{E}]$. We concentrate the analysis on BGPs.

[11]From a technical point of view, indeterminacy arises from the assumption that the liquidity trap is perceived as permanent. In Online Appendix B.2, we extend our model to consider the possibility of exit from the ZLB at some future stochastic time τ. Post-exit, the exchange rate is determinate. By the usual arbitrage arguments and backward induction, this pins down the exchange rate path pre-exit as well, removing the indeterminacy. There are, however, important reasons to be skeptical of the rational expectations backward-induction logic that pins the exchange rate today to its value after the economy exits the trap, especially when the trap may be very persistent, e.g. Farhi and Werning (2019). A natural practical interpretation is that the longer the liquidity trap is expected to last, the less anchored to fundamentals is the exchange rate rate today. Our model considers the limit case where exchange rate expectations are not anchored by long run outcomes or when long run outcomes themselves are constrained by the ZLB.

reduces total asset demand (the left hand side of Eq. (24)) faster than it reduces total asset supply (the right hand side of Eq. (24)). The novelty under financial integration is that a weaker currency (a higher E) increases the total supply of public assets in local currency $\bar{d}(E)$, while simultaneously reducing the total supply of public assets in foreign currency $\bar{d}(E)/E$. This results in a smaller recession at Home (higher ξ) and a larger recession in Foreign (lower ξ^*).

To summarize our findings, under financial integration global financial conditions (reflected in the determinants of the world natural rate $r^{w,n}$) determine whether the global economy is at the ZLB. Unlike under financial autarky, the exchange rate is not anchored by goods market fundamentals. Different values of the exchange rate affect local financial conditions by changing the relative supply of public assets. This affects relative demand and the allocation of output across countries.

We can illustrate the indeterminacy by considering the special case where both countries experience a liquidity trap under financial autarky (that is, when $r^{a,n} < 0$ and $r^{a,n*} < 0$). The equilibrium autarky exchange rate simplifies to

$$E^a = \frac{d}{d^*}\frac{\rho - \delta^*\theta}{\rho - \delta\theta}.$$

The country with worse asset scarcity (lower d or lower δ) has lower output and a stronger currency under financial autarky. Under financial integration, if $E = E^a$ the financial integration equilibrium coincides with the financial autarky equilibrium: $\xi = \xi^a$ and $\xi^* = \xi^{a*}$. For $E > E^a$, Eq. (17) implies that $\xi > \xi^a$ and $\xi^* < \xi^{a*}$, and vice-versa for $E < E^a$.

2.4 Net Foreign Assets, Current Accounts and the Metzler Diagram

We now characterize Net Foreign Asset positions and Current Accounts under financial integration, both away from the ZLB and at the ZLB. The next proposition considers the case where the global economy is away from the ZLB ($\xi = \xi^* = 1$).

Proposition 3 (Net Foreign Assets and Current Accounts Away from the ZLB)**.** *Under Assumption 1, if $r^{w,n} > 0$ then along a BGP:*

- *The world interest rate is a weighted average of the home and foreign autarky rates $r^{a,n}$ and $r^{a,n*}$, as in Eq. (21).*
- *Home is a net creditor and runs a current account surplus if and only if the world interest rate is higher than the autarky interest rate: $r^{a,n} < r^{w,n} < r^{a,n*}$.*
- *Home's Net Foreign Asset position (NFA), Current Account (CA) and Trade Balance (TB) are*

given by:

$$\frac{NFA}{\bar{Y}} = \frac{(1-\theta d)(r^w - r^{a,n})}{(g+\theta-r^w)(\rho+r^w)} \quad , \quad \frac{CA}{\bar{Y}} = g\frac{NFA}{\bar{Y}} \quad , \quad \frac{TB}{\bar{Y}} = (g-r^w)\frac{NFA}{\bar{Y}}. \tag{25}$$

Proof. See text. □

We have already established that away from the ZLB, the world interest rate is a weighted average of the financial autarky rates in both countries. Next, note that along a BGP and for a given world interest rate r^w, we can express asset demand W and private asset supply V from Eqs. (4a), (5a) and (7a) as

$$V = \frac{\delta}{r^w+\rho}\bar{Y}, \tag{26a}$$

$$W = \frac{(1-\delta)-(r^w-g)d+(\rho+g)\frac{\delta}{r^w+\rho}}{g+\theta-r^w}\bar{Y}. \tag{26b}$$

The net foreign asset position is defined as $NFA = W-(V+D)$, and the current account is the change in the net foreign asset position: $CA = N\dot{F}A = gNFA$ along the BGP. The trade balance obtains from the definition of the current account: $CA = TB + r^w NFA$. Substituting, we obtain Eq. (25), which says that the home Net Foreign Asset position increases with global interest rates r^w.

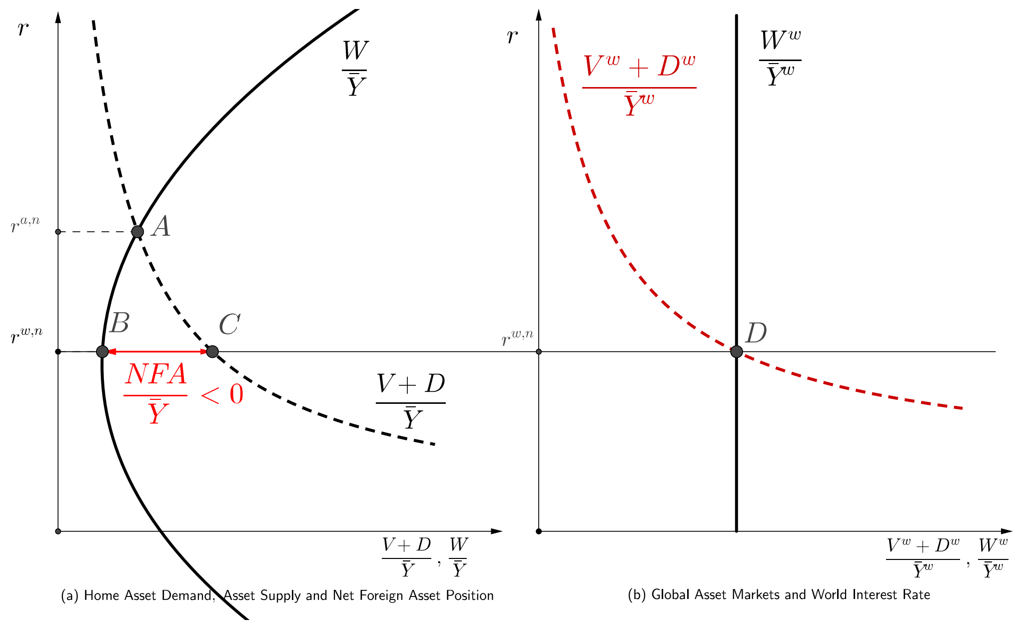
Similar equations hold for Foreign, which together with equilibrium in the world asset market allow us to characterize the world interest rate r^w in a conventional Metzler diagram (Fig. 3).

Panel (a) of Fig. 3 reports home asset supply $V+D$ (dashed line) and home asset demand W (solid line), scaled by Home potential output $\bar{Y}$, as functions of the world interest rate r^w.[12] The two curves intersect at the financial autarky natural interest rate $r^{a,n}$—assumed positive—where the country is neither a debtor nor a creditor (point A). For lower values of the world interest rate, Home is a net debtor: $NFA/\bar{Y} < 0$. For higher values, it is a net creditor. Panel (b) reports global asset supply $V^w + D^w$ (red dashed line) and global asset demand W^w (solid line), scaled by global potential output $\bar{Y}^w$, as a function of the global interest rate r^w (Eqs. (19) and (20)). Global asset supply decreases with the world interest rate, while global asset demand is constant. The two curves intersect at the world natural interest rate $r^{w,n}$, assumed positive. The figure assumes $r^{a,n*} < r^{a,n}$, hence $r^{w,n} < r^{a,n}$ and Home runs a current account deficit.[13]

Away from the ZLB, *Foreign's Current Account surplus helps propagate its asset shortage*, increasing

[12] Asset supply $(V+D)/\bar{Y}$ is monotonically decreasing in the world interest rate r^w. Asset demand $W/\bar{Y}$ is non-monotonic because of two competing effects. First, higher interest rates imply that wealth accumulates faster. But higher interest rates also reduce the value of the new trees endowed to the newborns and increase the tax burden required to pay the higher interest on public debt. For high levels of the interest rate and low levels of debt, the first effect dominates and asset demand increases with r^w. For low levels of the interest rate, the second effect dominates and asset demand decreases with r^w. Regardless of the shape of $W/\bar{Y}$, under Assumption 1, $g+\theta > r^w$ and $NFA/\bar{Y}$ is increasing in the interest rate.

[13] According to Eq. (25) Home also runs a trade deficit if the global rate r^w is lower than the growth rate g.



Panel (a) reports asset demand $W/\bar{Y}$ (solid line) and asset supply $(V + D)/\bar{Y}$ (dashed line) in Home, scaled by Home potential output. The two lines intersect at the autarky natural interest rate $r^{a,n}$ (point A). Panel (b) reports world asset demand $W^w/\bar{Y}^w$ (solid line) and world asset supply $(V^w + D^w)/\bar{Y}^w$ (dashed red line). The two lines intersect at the world natural interest rate $r^{w,n}$ (point D). When the world interest rate is below the autarky rate ($0 < r^{w,n} < r^{a,n}$) the country is a net debtor and runs a current account deficit.

Figure 3: World Interest Rates and Net Foreign Asset Positions: the Metzler Diagram

the foreign interest rate above its autarky level ($r^{a,n*} < r^{w,n}$), while reducing the home interest rate below autarky ($r^{w,n} < r^{a,n}$).

Consider the case where the global economy is at the ZLB ($r^{w,n} < 0$, $\xi \leq 1$ and $\xi^* \leq 1$). The next proposition characterizes global imbalances.

Proposition 4 (Net Foreign Assets and Current Accounts at the ZLB)**.** *Under Assumption 1, if $r^{w,n} < 0$, then given an exchange rate $E \in [\underline{E}, \bar{E}]$:*

- *Domestic output ξ is a weighted average of home and exchange-rate-weighted foreign financial autarky outputs, $\xi^{a,n}$ and $E\xi^{a,n*}$, according to*

$$\xi = y \frac{1 - \frac{\delta\theta}{\rho}}{1 - \frac{\bar{\delta}\theta}{\rho}} \xi^{a,n} + (1 - y) \frac{1 - \frac{\delta^*\theta}{\rho}}{1 - \frac{\bar{\delta}\theta}{\rho}} E\xi^{a,n*}. \tag{27}$$

- *Home is a net creditor and runs a current account surplus if and only if home output ξ exceeds its financial autarky level: $\xi^{a,n} < \xi$. Along the BGP, Home's Net Foreign Asset Position, Current*

Account and Trade Balance are given by

$$\frac{NFA}{\bar{Y}} = \frac{(1-\frac{\delta\theta}{\rho})(\xi - \xi^{a,n})}{g+\theta} \quad , \quad \frac{CA}{\bar{Y}} = \frac{TB}{\bar{Y}} = g\frac{NFA}{\bar{Y}}. \tag{28}$$

Proof. See text. □

The first part of the proposition obtains directly by manipulating Eq. (24), using the definition of $\xi^{a,n}$ and $\xi^{a,n*}$ in Proposition 1. Under Assumption 1, the weights are positive and sum to one.

Assume that $r^{w,n} < 0$ and fix a nominal exchange rate $E \in [\underline{E}, \bar{E}]$. We can rewrite wealth accumulation Eq. (5a) and home asset pricing Eq. (4a) along the BGP as a function of the domestic output level ξ:

$$V = \frac{\delta\xi}{\rho}\bar{Y}, \tag{29a}$$

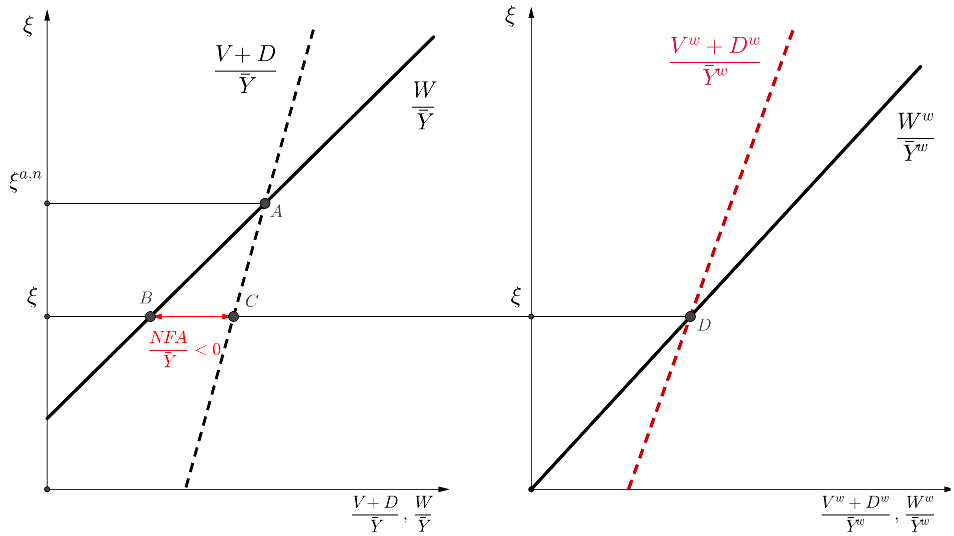
$$W = \frac{\xi + gd + g\frac{\delta\xi}{\rho}}{g+\theta}\bar{Y}, \tag{29b}$$

which immediately implies Eq. (28). This establishes the last part of the proposition.

Since $\xi \leq 1$, Home always runs a Current Account deficit when $\xi^{a,n} > 1$, i.e. when Home would escape the liquidity trap under financial autarky. A similar equation holds for Foreign, which together with equilibrium in the world asset market allows us to characterize the equilibrium Home recession ξ for a given exchange rate E in a *modified Metzler diagram in quantities* (Fig. 4).

Panel (a) of Fig. 4 reports home asset supply $V + D$ (dashed line) and home asset demand W (solid line) scaled by home potential output $\bar{Y}$, as functions of domestic output ξ, for a given exchange rate E (Eqs. (29a) and (29b)). Both asset demand and asset supply are increasing in output, but supply increases faster than demand. The two curves intersect at the financial autarky output $\xi^{a,n}$ (point A). For lower values of output, Home is a net debtor: $NFA/\bar{Y} < 0$. For higher values, it is a net creditor: $NFA/\bar{Y} > 0$. Panel (b) reports global asset supply $V^w + D^w$ (red dashed line) and global asset demand W^w (solid line) scaled by global potential output $\bar{Y}^w$, as a function of the home recession ξ (Eqs. (22) and (23)). Both global asset demand and supply are increasing in output, but supply increases faster than demand. The two curves intersect at the equilibrium level of home output $0 < \xi < 1$. The figure assumes $E\xi^{a,n*} < \xi^{a,n}$ or equivalently $E < E^a$.

Replacing ξ and $\xi^{a,n}$ from Eq. (17) and Eq. (11b) respectively, we can rewrite the home Net Foreign



(a) Home Asset Demand, Asset Supply and Net Foreign Asset Position

(b) Global Asset Markets and Home Recession

Panel (a) reports Home asset demand $W/\bar{Y}$ (solid line) and asset supply $(V+D)/\bar{Y}$ (dashed line) as functions of home output ξ. The two lines intersect at the autarky level of output $\xi^{a,n}$ (point A). Panel (b) reports global asset demand W^w (solid line) and asset supply V^w+D^w (red dashed line) scaled by world potential output $\bar{Y}^w$ as a function of home output ξ, for a given exchange rate $E < E^a$, when $\xi^{a,n} < 1$ and $\xi^{a,n*} < 1$. The two lines intersect at the home level of output ξ (point D). Home experiences a worse recession, $\xi < \xi^a$, when it is a net debtor, $NFA/\bar{Y} < 0$ and runs a Current Account deficit ($CA/\bar{Y} < 0$).

Figure 4: Recessions and Net Foreign Asset Positions in a Global Liquidity Trap: the Metzler Diagram in Quantities

Asset position and Current Account in Eq. (28) as

$$\frac{NFA}{\bar{Y}} = \frac{(1-\frac{\delta\theta}{\rho})}{g+\theta}\left[\frac{\theta\bar{d}(E)}{1-\frac{\bar{\delta}\theta}{\rho}} - \frac{\theta d}{1-\frac{\delta\theta}{\rho}}\right] \quad , \quad \frac{CA}{\bar{Y}} = g\frac{NFA}{\bar{Y}}. \tag{30}$$

A cheaper home currency implies a larger home Net Foreign Asset position and hence a larger Current Account, allowing Home to export more of its recession abroad. Depending on the value of the exchange rate E, Home can be a surplus country or a deficit country.

When both countries are in a liquidity trap under financial autarky, we can express the Net Foreign Asset position and Current Account directly as a function of the exchange rate E, relative to the autarky exchange rate E^a. Substituting the expression for the exchange rate-adjusted financial capacity, we obtain

$$\frac{NFA}{\bar{Y}} = \frac{1-\frac{\delta\theta}{\rho}}{1-\frac{\bar{\delta}\theta}{\rho}}\frac{(1-y)\theta d^*(E-E^a)}{g+\theta} \quad , \quad \frac{CA}{\bar{Y}} = g\frac{NFA}{\bar{Y}}. \tag{31}$$

We can now connect our results to the case of financial autarky. Under financial autarky, the exchange rate is determinate precisely because the capital account is closed. If both countries are in a liquidity trap under financial autarky, $\xi^a = \xi^{a,n} < 1$, $\xi^{a*} = \xi^{a,n*} < 1$ and $E^a = \xi^{a,n}/\xi^{a,n*}$. Then for $E = E^a$, the financial integration equilibrium coincides with the financial autarky equilibrium and there are no current account imbalances. For $E > E^a$, we have $\xi > \xi^a$, $\xi^* < \xi^{a*}$ and $NFA/\bar{Y} > 0$, and vice versa for $E < E^a$. By depreciating its exchange rate and running a Current Account surplus, Home can reduce the size of its recession.

In the ZLB equilibrium, *Home's current account surplus helps propagate recessions*, increasing Home's output and reducing Foreign's output. The ZLB is a *'tipping point'* for global imbalances, where the economy transitions from *benign* (current account surpluses propagating low interest rates) to *malign* (current account surpluses propagating recessions).

Our results are robust to several important extensions which we present in the appendix. There, we allow for milder nominal rigidities (appendix A.2), introduce home bias (appendix A.3), relax the unit elasticity assumption between Home and Foreign goods (appendix A.4), and consider a model with heterogeneity in the propensity to save both within and across countries (appendix A.6).

3 Negative Policy Spillovers: Currency and Trade Wars

The adverse impact of current account surpluses on foreign output in the ZLB equilibrium is a symptom of a more general increased policy interdependence. At the global ZLB, some policies have large positive spillovers; others have large negative spillovers. This section focuses on the negative spillovers. In particular, we consider currency wars and trade wars. Each of these policies affects the global equilibrium by reallocating demand towards the home country and away from the foreign country, without addressing the underlying cause of global stagnation.

As is well-known, a country may wish to manipulate its terms-of-trade to exploit monopoly power, either by adjusting its exchange rate, or by erecting trade barriers. This is true both outside and at the ZLB. By contrast, the reallocation of global demand that is our focus arises only at the ZLB. In order to isolate the latter, we consider a version of the model without terms-of-trade effects.

3.1 Currency Wars

Our model provides a way of thinking about "currency wars", i.e. the incentive for one country to manipulate its currency at the expense of its trading partners. Outside the global liquidity trap, the exchange rate is pinned down ($E = 1$), output in each country is at its potential level ($\xi = \xi^* = 1$) and the real interest rate is equal to its Wicksellian natural counterpart ($r = r^* = r^{w,n}$). In the absence of terms-of-trade effects, there are no gains from manipulating the exchange rate.

In the global liquidity trap, by contrast, Proposition 4 establishes that the distribution of the global recession across countries is mediated by the exchange rate and global imbalances. Even though the exchange rate is indeterminate in this global liquidity trap regime, it is in principle possible for the home monetary authority to peg the exchange rate at any level E in the indeterminacy region $[\underline{E}, \bar{E}]$, by simply standing ready to buy and sell the home currency for the foreign currency at the exchange rate E, as long as the other country remains passive.

By choosing a sufficiently depreciated exchange rate, Home is able to partly export its recession abroad by running a Current Account surplus (Proposition 2 and Proposition 4). That is, once interest rates are at the ZLB, our model indicates that exchange rate policies generate powerful beggar-thy-neighbor effects. This zero-sum logic resonates with concerns regarding "currency wars": in the global stagnation equilibrium, attempts to depreciate one's currency affect relative output one-for-one, according to Eq. (18).

Of course, if both countries attempt to simultaneously depreciate their currencies, these efforts cancel out, and the exchange rate remains a pure matter of coordination. Moreover, if agents coordinate on an equilibrium where the home exchange rate is appreciated, as could be the case if the home currency were perceived to be a "reserve currency," then this would worsen the recession at Home. In other words, while the reserve currency status may be beneficial outside a liquidity trap, it exacerbates the domestic recession in a global liquidity trap. This *"paradox of the reserve currency"* captures a dimension of the appreciation struggles of countries like Switzerland during the European turmoil in 2015, and of Japan before the implementation of Abenomics in 2012.

We develop these insights further by extending our baseline model. In order to neutralize terms-of-trade effects, we move away from the assumption of a unitary elasticity of substitution between home and foreign goods and consider instead the limit where goods become perfect substitutes. Appendix A.4 presents this extension, allowing for an arbitrary elasticity of substitution σ between home and foreign goods and taking the limit as $\sigma \to \infty$. When goods are perfect substitutes, there is no room to manipulate the terms-of-trade.

The analysis outside the ZLB under financial integration, is identical to the case $\sigma = 1$. In particular, $\xi = \xi^* = 1$ and $E = P = 1$ when the global natural rate is positive, $r^{w,n} \geq 0$, under Proposition 2. Consider now the case of a global liquidity trap under financial integration. In the limit $\sigma \to \infty$ it is still the case that $E = P = 1$. Yet there remains a degree of indeterminacy, indexed by a renormalization of the exchange rate $\hat{E} \equiv E^\sigma = \xi/\xi^*$. Changes in $\hat{E}$ can be interpreted as infinitesimal attempts to manipulate the exchange rate. We show in Appendix A.4 that, when $\sigma \to \infty$, home and foreign output satisfy

$$\xi(\hat{E}) = \frac{\theta \hat{E} \bar{d}}{y(1-\frac{\delta\theta}{\rho})\hat{E} + (1-y)(1-\frac{\delta^*\theta}{\rho})}, \tag{32a}$$

$$\xi^*(\hat{E}) = \frac{\theta \bar{d}}{y(1-\frac{\delta\theta}{\rho})\hat{E} + (1-y)(1-\frac{\delta^*\theta}{\rho})}. \tag{32b}$$

Intuitively, both the relative value of home vs. foreign output $\xi/(E\xi^*)$ and the real consumption of home agents $\theta W/P$ relative to the real consumption of foreign agents $\theta W^*/P^*$ increase with $\hat{E}$. By choosing a more depreciated $\hat{E}$, Home can stimulate domestic output and consumption at the expense of Foreign.[14]

To develop this idea further, assume that the central bank at Home can take some 'non-conventional' action $a \geq 1$, while the central bank in Foreign can take some action $a^* \geq 1$. These actions can be interpreted as non-conventional monetary policies such as Large-Scale Asset Purchases, foreign exchange interventions or any other (costly) communication by central banks. We rule out policies with a 'fiscal dimension', for instance non-conventional monetary policies that expand the supply of public or quasi-public debt, since these would have positive spillovers (see Section 4). These actions are assumed to impact the renormalized exchange rate, with $\hat{E} = \mathcal{E}(a, a^*) \equiv (a/a^*)^{\mathfrak{n}}$ denoting how the exchange rate responds to the actions of both central banks, and $0 < \mathfrak{n} < 1$. A stronger action by the Home (resp. Foreign) central bank depreciates (resp. appreciates) the currency, at a decreasing rate.

These actions come at a *non-pecuniary* cost $\mathcal{C}(a) \geq 0$ and $\mathcal{C}(a^*) \geq 0$ per unit of output. These costs could be interpreted as the political-economy cost for the central bank of deviating from a narrow interest rate policy. Alternatively, as in Caballero and Simsek (2020), we can interpret the cost of Large Scale Asset Purchases or foreign exchange interventions as resulting from the increased risk-exposure of the consolidated public sector balance sheet. We assume that the function $\mathcal{C}(a)$ is twice continuously differentiable and convex in a, with $\mathcal{C}(1) = \mathcal{C}'(1) = 0$ and let $\eta_c = a\mathcal{C}''(a)/\mathcal{C}'(a) > 0$ denote the elasticity of the marginal cost.

In the limit $\sigma \to \infty$, there is no incentive to manipulate the exchange rate outside the ZLB since output is already at its potential level ($\xi = 1$) and the goods are perfect substitutes. It follows that : $a = a^* = 1$ and $\hat{E} = 1$.

Consider now what happens in the global liquidity trap when the central bank aims to maximize domestic consumption $c = \theta W/P$, net of the non-pecuniary cost $\mathcal{C}(a)$, given foreign action a^*. Using

[14] In the general $\sigma > 1$ case, a depreciation of the exchange rate E has two effects on real consumption $c = \theta W/P$: it stimulates output ξ, which increases wealth and consumption, but also leads to an increase in the price level P, which reduces real consumption.

Eqs. (29b) and (32a), Home's optimal non-conventional action a satisfies

$$\xi\left(\mathcal{E}(a,a^*)\right)\frac{(1-y)(1-\frac{\delta^*\theta}{\rho})}{y(1-\frac{\delta\theta}{\rho})\mathcal{E}(a,a^*)+(1-y)(1-\frac{\delta^*\theta}{\rho})}\frac{\theta(1+\frac{g\delta}{\rho})}{g+\theta}\frac{\mathfrak{n}}{a}=\mathcal{C}'(a), \tag{33}$$

which defines implicitly a best-response function $a=\mathcal{A}(a^*)$.By symmetry, the foreign central bank aims to maximize foreign consumption $c^*=\theta W^*/P^*$, net of the non-pecuniary cost $\mathcal{C}(a^*)$, given home action a. Using Eq. (32b), Foreign's optimal action a^* satisfies

$$\xi^*\left(\mathcal{E}(a,a^*)\right)\frac{y(1-\frac{\delta\theta}{\rho})\mathcal{E}(a,a^*)}{y(1-\frac{\delta\theta}{\rho})\mathcal{E}(a,a^*)+(1-y)(1-\frac{\delta^*\theta}{\rho})}\frac{\theta(1+\frac{g\delta^*}{\rho})}{g+\theta}\frac{\mathfrak{n}}{a^*}=\mathcal{C}'\left(a^*\right), \tag{34}$$

which defines implicitly a best-response function $a^*=\mathcal{A}^*(a)$. A Nash equilibrium of the Currency War game obtains when $a=\mathcal{A}(a^*)$ and $a^*=\mathcal{A}^*(a)$ hold simultaneously. Under some restrictions on the parameters (described in appendix A.4), a Nash equilibrium exists, is unique, and is asymptotically stable. This equilibrium features $a>1$ and $a^*>1$: both countries have an incentive to depreciate their currency. As usual, this is generically inefficient since the efforts of each country are undone by the other, while each country bears the full cost of its action, $\mathcal{C}(a)$ and $\mathcal{C}(a^*)$.

Furthermore, each country's optimal action is increasing in the amount of public debt $\bar{d}$: $\partial a/\partial\bar{d}>0$ and $\partial a^*/\partial\bar{d}>0$. The intuition is straightforward: from Eq. (32) an increase in public debt $\bar{d}$ increases output in both countries for any level of the exchange rate $\hat{E}$, since it alleviates the shortage of stores of values. From Eqs. (33) and (34), it follows that the marginal benefit of non-conventional action a and a^* (the left hand side) increases, regardless of the action of the other country. In equilibrium, both a and a^* increase. It follows that, if *one* country issues more public debt, *all* countries attempt to depreciate their currency more aggressively, in a largely futile effort.

We summarize these results in the following proposition.

Proposition 5 (Currency Wars)**.** *Under financial integration, in the limit of perfectly substitutable goods ($\sigma\to\infty$) and under the parameter restriction described in appendix A.4, the Nash equilibria of the Currency War game where each central bank tries to maximize consumption c,c^* by choosing actions (a^N,a^{N*}), are as follows:*

- *If $r^{w,n}\geq 0$, the global economy is away from the Zero Lower Bound. There is a unique balanced growth path Nash equilibrium with positive interest rate $i^w=r^w=r^{w,n}$, output is at its potential, $\xi=\xi^*=1$, and there is no incentive to manipulate the exchange rate: $E=1$, $a^N=a^{N*}=1$.*
- *If $r^{w,n}<0$, the global economy is at the ZLB: $i^w=r^w=0$. There is a unique asymptotically stable*

balanced growth path Nash equilibrium with $a^N > 1$, $a^{N*} > 1$ characterized by Eqs. (33) and (34); the normalized exchange rate satisfies $\hat{E}^N = \mathcal{E}(a^N, a^{N*})$ and $\xi^N \leq 1$, $\xi^{N*} \leq 1$ satisfy Eq. (32).

- *At the ZLB Nash equilibrium, the more public debt a country issues, the more each country tries to depreciate its currency: $\partial a^N / \partial \bar{d} > 0$, $\partial a^{*N} / \partial \bar{d} > 0$.*

Proof. See text and appendix A.4. □

3.2 Trade Wars

Our model also provides a way of thinking about "trade wars," i.e. the incentive for one country to erect trade barriers at the expense of its trading partners. We introduce the possibility of asymmetric tariffs into our baseline model and show that trade wars share the mechanisms and negative spillovers of currency wars at the ZLB.

We start with the full model of Section 2, except that we now allow Home to impose an ad-valorem tariff λ on imports from Foreign, and conversely allow Foreign to impose an ad-valorem tariff λ^* on imports from Home. Under the law of one price, households in Home now face import prices $P_{F,t} = E_t P^*_{F,t}(1+\lambda)$, while households in Foreign now face import prices $P^*_{H,t} = P_{H,t}(1+\lambda^*)/E_t$. As before, we assume that prices are fully rigid in their home market and normalize: $P_{H,t} = P^*_{F,t} = 1$. We also assume that each country instantaneously rebates tariff revenues to the consuming households. With Cobb-Douglas preferences, aggregate expenditure shares become (see Appendix A.5):

$$c_H = \frac{\gamma(1+\lambda)}{1+\gamma\lambda}\theta W \quad , \quad c_F = \frac{(1-\gamma)}{1+\gamma\lambda}\frac{\theta W}{E} \tag{35a}$$

$$c^*_H = \frac{\gamma}{1+\lambda^*(1-\gamma)}\theta E W^* \quad , \quad c^*_F = \frac{(1-\gamma)(1+\lambda^*)}{1+\lambda^*(1-\gamma)}\theta W^*. \tag{35b}$$

Everything else equal, tariffs shift households' expenditure shares towards domestic goods: as Home increases its tariffs on Foreign goods, demand for Foreign goods by Home households decreases by a factor $(1+\gamma\lambda)^{-1} < 1$. Further, since tariff revenues are rebated lump sum to consumers, demand for Home goods by Home households increases by a factor $(1+\lambda)/(1+\gamma\lambda) > 1$. The same effect holds for the tariffs imposed by Foreign.

Substituting Eq. (35) into the goods market clearing conditions, Eq. (8) becomes

$$\theta\left(\frac{y(1+\lambda)}{1+y\lambda}W + \frac{y}{1+\lambda^*(1-y)}EW^*\right) = \xi\bar{Y}, \tag{36a}$$

$$\theta\left(\frac{1-y}{1+y\lambda}W + \frac{(1-y)(1+\lambda^*)}{1+\lambda^*(1-y)}EW^*\right) = E\xi^*\bar{Y}^*. \tag{36b}$$

Because tariff revenues are rebated lump sum to households, all remaining equilibrium conditions are unchanged: wealth accumulation Eq. (4), asset pricing Eq. (5) and government debt dynamics Eq. (7).

Manipulating the equilibrium conditions, we can show that under financial autarky the natural rate $r^{a,n}$, the natural output gap $\xi^{a,n}$, and the equilibrium allocations, are the same as in Proposition 1, regardless of the tariffs λ and λ^*. The autarky rate and output gap are determined by: $r^a = \max\{r^{a,n}, 0\}$ and $\xi^a = \min\{\xi^{a,n}, 1\}$. It follows immediately that the only effect of the tariffs is to force an adjustment in the autarky exchange rate, now equal to

$$E^a = \frac{\xi^a}{\xi^{a,*}} \frac{1 + \lambda^*(1-y)}{1+\lambda y}. \tag{37}$$

This result is an illustration of Lerner symmetry (Lerner, 1936). Under autarky, the natural rate $r^{a,n}$ is determined in asset markets. Since asset market conditions are not changed by the tariffs, the natural rate is unchanged: whether the economy is away from or at the ZLB. It follows that wealth (in domestic currency) is also independent of the tariffs. Consequently, the exchange rate must adjust to counteract the shift in relative demand induced by the tariffs in Eq. (35). Since the root of the ZLB equilibrium lies in the financial sphere, reallocating demand between Home and Foreign goods cannot resolve this problem: an increase in tariffs in Home simply appreciates the currency, leaving the domestic economy just as depressed. Tariffs, however, could increase welfare by raising the relative price of domestic goods, through the usual terms-of-trade argument.

Lerner symmetry breaks down under financial integration, both outside and at the ZLB. First, consider what happens away from the ZLB ($\xi = \xi^* = 1$). A change in tariffs requires an adjustment in exchange rates. As the exchange rate varies, so does the global supply of assets relative to global asset demand, hence global interest rates need to adjust. This can be illustrated most directly by combining the asset supply and asset demand conditions Eqs. (4), (5) and (7) with the fact that global wealth spent must equal global output, $\theta W^w = \bar{Y} + E\bar{Y}^*$. This yields an expression for the world risk free rate as a weighted average of the home and foreign natural autarky rates, where the weights are a function of the exchange rate:

$$r^w = \frac{y(1-\theta d)}{y(1-\theta d) + E(1-y)(1-\theta d^*)} r^{a,n} + \frac{(1-y)E(1-\theta d^*)}{y(1-\theta d) + E(1-y)(1-\theta d^*)} r^{a,n*}. \tag{38}$$

An appreciation of the exchange rate shifts the global interest rate towards the Home country's autarky natural rate as it increases Home asset supply relative to Foreign.

Given a global interest rate r^w, Home and Foreign asset demands satisfy Eq. (26). Substituting this into the goods market equilibrium conditions Eq. (35) yields an expression for the exchange rate needed to clear the goods markets, given a global interest rate r^w:

$$(g+\theta-r^w) = \frac{\theta y(1+\lambda)}{1+y\lambda}\left[1+(g-r^w)\left(\frac{\delta}{\rho+r^w}+d\right)\right] \\ + E\frac{\theta(1-y)}{1+\lambda^*(1-y)}\left[1+(g-r^w)\left(\frac{\delta^*}{\rho+r^w}+d^*\right)\right] \tag{39}$$

As before, for a given world interest rate (and therefore asset demands), an increase in domestic tariffs requires an appreciation of the domestic currency to clear the goods markets. But this movement in the exchange rate now affects world interest rates according to Eq. (38).

As long as this system admits a solution $r^{w,n}$ with $r^{w,n} > 0$, the economy escapes the ZLB. Output in each country is unaffected by tariffs, whose effect is absorbed by a combination of exchange rate and global interest rate adjustments.

While tariffs leave output unchanged, they do affect global imbalances: Home's net foreign asset position along the BGP is still given by Eq. (25) from Proposition 3, reproduced here:

$$\frac{NFA}{\bar{Y}} = \frac{(1-\theta d)(r^w - r^{a,n})}{(g+\theta-r^w)(\rho+r^w)} \quad , \quad \frac{CA}{\bar{Y}} = g\frac{NFA}{\bar{Y}} \quad , \quad \frac{TB}{\bar{Y}} = (g-r^w)\frac{NFA}{\bar{Y}}.$$

An increase in tariffs at Home, which appreciates the currency, *reduces* global imbalances at Home (relative to its output) as it reduces the gap between the world interest rate and Home's autarky rate. This is true regardless of whether the country is a creditor or a debtor: following an increase in its tariffs, a creditor country runs a smaller current account surplus; a debtor country runs a smaller current account deficit.[15] This illustrates that global imbalances are not driven by the expenditure switching effect due to the tariffs or to the exchange rate appreciation, for otherwise the current account would always either improve or deteriorate regardless of its initial position. Instead, global imbalances are determined in global financial markets and reflect the tension between the local and global supply and demand of assets.

Consider now what happens when the natural rate $r^{w,n}$ becomes negative and the economy experiences a global liquidity trap. As in Proposition 2, given tariff policies λ and λ^*, $r^w = 0$ and there is a continuum of balanced growth path equilibria indexed by the exchange rate E within a range $[\underline{E}, \bar{E}]$. Combining asset demand, supply and goods market conditions Eqs. (4), (5), (7) and (35) for a given exchange rate E, the

[15] The same expression implies that Foreign's external imbalances, as a fraction of foreign output, must become larger when Home tariffs increase. This is consistent with $NFA + E\,NFA^* = 0$ since the exchange rate appreciates. Note also that the impact on the trade balance depends on the change in global interest rate.

output gaps satisfy the following system:

$$\frac{\theta+g}{\theta}\xi = \frac{y(1+\lambda)}{1+\lambda y}\left[gd + \left(1+\frac{g\delta}{\rho}\right)\xi\right] + \frac{1-y}{1+\lambda^*(1-y)}E\left[gd^* + \left(1+\frac{g\delta^*}{\rho}\right)\xi^*\right] \tag{40a}$$

$$\frac{\theta+g}{\theta}E\xi^* = \frac{y}{1+\lambda y}\left[gd + \left(1+\frac{g\delta}{\rho}\right)\xi\right] + \frac{(1-y)(1+\lambda^*)}{1+\lambda^*(1-y)}E\left[gd^* + \left(1+\frac{g\delta^*}{\rho}\right)\xi^*\right]. \tag{40b}$$

This system boils down to Eq. (17) in the absence of tariffs, $\lambda = \lambda^* = 0$.[16] Conditional on an exchange rate E, an increase in tariffs in Home increases Home's output, i.e. $\partial\xi/\partial\lambda > 0$, while decreasing Foreign's output: $\partial\xi^*/\partial\lambda^* < 0$. The intuition is simple: at the global ZLB asset prices and the exchange rate are fixed. Hence Home tariffs, which tilt global demand towards the Home good, must reduce home slack at the expense of foreign slack. In that sense, trade wars, like currency wars, simply reallocate a global deficiency of aggregate demand without addressing the underlying cause, which lies in global financial markets.

Next, observe that global imbalances still satisfy Eq. (28) from Proposition 4:

$$\frac{NFA}{\bar{Y}} = \frac{(1-\frac{\delta\theta}{\rho})(\xi - \xi^{a,n})}{g+\theta} \quad , \quad \frac{CA}{\bar{Y}} = \frac{TB}{\bar{Y}} = g\frac{NFA}{\bar{Y}}.$$

Consequently, at the ZLB tariffs always increase a country's net foreign asset position and current account, regardless of its autarky position, and deteriorate the net foreign position and current account of the rest of the world. This is intuitive, since at the ZLB the effect of tariffs on aggregate output operates entirely via the reallocation of demand, with no effect on the underlying global financial conditions.

We now analyze the Nash game where each country chooses its tariffs non-cooperatively to maximize its own real consumption along the BGP, taking as given the other country's tariff. A full characterization of the Nash equilibria of this Trade War game is possible but complicated by the presence of the usual terms-of-trade effects. As in the case of currency wars, we obtain a sharp characterization by moving away from the assumption of unitary elasticity of substitution between home and foreign goods ($\sigma = 1$) and considering instead the limit where goods become perfect substitutes ($\sigma \to \infty$). Appendix A.5 presents this extension.

In the limit of perfectly substitutable goods, there is no monopoly rent to be extracted by manipulating terms-of-trade. It follows immediately that, under financial autarky or under financial integration but outside the ZLB, the optimal tariffs are $\lambda = \lambda^* = 0$, with $E = P = 1$.

Consider now the case of a global liquidity trap under financial integration. In that limit, it is still the case that $E = P = 1$ and $\lambda = \lambda^* = 0$. There remains, as in the previous section, a degree of indeterminacy,

[16] The range of indeterminacy $[\underline{E}, \bar{E}]$ is also constrained by tariff policy. $\bar{E}$ is defined such that $\xi = 1$ in Eq. (40a), while $\underline{E}$ is defined such that $\xi^* = 1$ in Eq. (40b). As tariffs increase, this range shrinks and converges to E^a.

indexed by $\hat{E} = E^{\sigma}$. Further, for a given value of $\hat{E}$, we can consider 'infinitesimal' attempts to erect unilateral trade barriers through the choice of $\hat{\lambda} = \exp(\lambda\sigma) \geq 1$ and $\hat{\lambda}^* = \exp(\lambda^*\sigma) \geq 1$, where $\hat{\lambda} = 1$ corresponds to $\lambda = 0$. In the limit $\sigma \to \infty$, home and foreign output satisfy the following generalization of Eq. (32):[17]

$$\xi = \frac{1}{\kappa\kappa^*(1-\tilde{\delta})}\left(y\frac{\kappa^*\hat{\lambda}\hat{E}+\alpha^*}{y\hat{\lambda}\hat{E}+1-y}(1-\frac{\theta\delta}{\rho})\xi^a + (1-y)\frac{\kappa^*\hat{E}+\alpha^*\hat{\lambda}^*}{y\hat{E}+(1-y)\hat{\lambda}^*}(1-\frac{\theta\delta^*}{\rho})\xi^{a*}\right) \quad (41a)$$

$$\xi^* = \frac{1}{\kappa\kappa^*(1-\tilde{\delta})}\left(y\frac{\kappa+\alpha\hat{\lambda}\hat{E}}{y\hat{\lambda}\hat{E}+1-y}(1-\frac{\theta\delta}{\rho})\xi^a + (1-y)\frac{\kappa\hat{\lambda}^*+\alpha\hat{E}}{y\hat{E}+(1-y)\hat{\lambda}^*}(1-\frac{\theta\delta^*}{\rho})\xi^{a*}\right), \quad (41b)$$

where

$$\alpha = \frac{y}{y\hat{E}\hat{\lambda}+1-y}\left(\frac{\theta}{g}+\frac{\theta\delta}{\rho}\right) \quad ; \quad \alpha^* = \frac{(1-y)\hat{E}}{y\hat{E}+(1-y)\hat{\lambda}^*}\left(\frac{\theta}{g}+\frac{\theta\delta^*}{\rho}\right)$$

$$\kappa = \frac{(1-y)(1+\frac{\theta}{g})+y\hat{E}\hat{\lambda}(1-\frac{\theta\delta}{\rho})}{y\hat{E}\hat{\lambda}+1-y} \quad ; \quad \kappa^* = \frac{y\hat{E}(1+\frac{\theta}{g})+(1-y)\hat{\lambda}^*(1-\frac{\theta\delta^*}{\rho})}{y\hat{E}+(1-y)\hat{\lambda}^*}$$

$$\tilde{\delta} = \alpha\alpha^*/\kappa\kappa^*.$$

Output in Home and Foreign is a combination of the autarky natural output rates, ξ^a and ξ^{a*} with weights that depend on the exchange rate and on tariffs. We show in Appendix A.5 that output at Home increases with tariffs, $\partial\xi/\partial\hat{\lambda} > 0$ while output in Foreign decreases, $\partial\xi^*/\partial\hat{\lambda} < 0$. The intuition is similar to the unitary elasticity case: with a given exchange rate, tariffs reallocate global demand towards the Home good, reducing home slack and expanding foreign slack.

In the limit of perfectly substitutable goods, real consumption in Home is only a function of domestic output: $c = \theta((1+\frac{g\delta}{\rho})\xi + gd)/(g+\theta)\bar{Y}$. It follows immediately that, for a given exchange rate $\hat{E}$ and a given tariff level set by their neighbor, countries want to increase their own tariff as long as they are not at potential output. In other words, countries have strong incentives to engage in 'trade wars' to mitigate their own recession at the expense of their neighbors. This is a direct parallel to the currency wars result presented in Section 3.1. Moreover, as tariffs increase in both countries, the world converge to financial autarky as the only Nash equilibrium. The following proposition parallels Proposition 5 for the case of Trade Wars.

Proposition 6 (Trade Wars). *Under financial integration, in the limit $\sigma \to \infty$ and under Assumption 1, the Nash equilibria of the Trade War game where policymakers try to maximize consumption c, c^* by choosing unilateral tariffs $(\hat{\lambda}^N, \hat{\lambda}^{*N})$ are as follows:*

[17] The range of indeterminacy on $\hat{E}$ is defined as before by the conditions that $\xi^* = 1$ and $\xi = 1$ respectively.

- *If $r^{w,n} \geq 0$, the global economy is away from the Zero Lower Bound. There is a unique BGP Nash equilibrium with positive interest rate $i^w = r^w = r^{w,n}$, output is at its potential, $\xi = \xi^* = 1$, $E = 1$ and there is no incentive to impose tariffs: $\hat{\lambda}^N = \hat{\lambda}^{*N} = 1$.*

- *If $r^{w,n} < 0$, the global economy is at the ZLB: $i^w = r^w = 0$. There is a unique asymptotically stable balanced growth path Nash equilibrium with $E = 1$ and $\hat{\lambda}^N \to \infty$ and $\hat{\lambda}^{*N} \to \infty$. The economy reverts to financial autarky: $\xi \to \xi^{a,n}$, $\xi^* \to \xi^{*a,n}$. Consequently $NFA \to 0$ and $NFA^* \to 0$.*

Proof. See text and Appendix A.5. □

4 Positive Policy Spillovers: Public Debt and Balanced-Budget Fiscal Expansions

Currency depreciations and tariffs are zero-sum (at best) because they do not address the key shortage of stores of value, which lies behind the global liquidity trap. In contrast, public debt issuances and balanced-budget fiscal expansions have the potential to generate positive spillovers. These two methods of expansionary fiscal policy reduce the net supply of safe assets via distinct channels: public debt issuances increase the supply of assets while balanced-budget fiscal expansions reduce the net demand for assets. Both have the potential to stimulate the economy by alleviating the global excess demand for financial assets and the corresponding global excess supply of goods in the ZLB equilibrium.

4.1 Public Debt: The Net Asset Creation Channel

We first focus on public debt (deficits), assuming that there is no change in government spending. At the ZLB, public debt issuances can be financed without levying any extra taxes. Public debt is essentially a rational bubble. Public debt issuances increase the global (safe) asset supply, and stimulate global output, thereby generating positive spillovers.

In the interest of space, we only consider the case of a global liquidity trap. From Proposition 2, Eq. (17), it is immediate that for a given exchange rate E, an increase in public debt at Home (D) or in Foreign (D^*) increases world net asset supply and reduces the world asset shortage. As a result, home and foreign outputs ξ and ξ^* increase proportionately. It does not matter whether the increase in public debt originates at Home or at Foreign: an increase in public debt anywhere is expansionary everywhere. From Eq. (30) an increase in debt at Home (D) decreases the Home Net Foreign Asset position and pushes the Home Current Account toward a deficit.

It is important to note that in a liquidity trap we have $r^w \leq g$ since $r^w = 0$ and $g \geq 0$. This implies that the government does not need to levy taxes to sustain debt, and in fact can afford to rebate some tax revenues to households. Fiscal capacity is therefore not a constraint on the use of debt as an instrument to stimulate the economy. This conclusion rests on the assumption that the trap is permanent.[18]

Note also that at the ZLB, public debt and money are (at the margin) perfect substitute zero interest rate government liabilities. As a result, issuing government bonds and issuing money as a helicopter drop are equivalent at the ZLB. Hence all the results regarding the issuance of public debt at the ZLB apply *identically* to the issuance of money.[19] Through the lens of the model, helicopter drops anywhere are expansionary everywhere.

Remark 1. *We conclude this subsection with a negative observation: there is a perverse interaction between currency wars and public debt issuance. Proposition 5 shows that both a and a^* increase in $\bar{d}$: public debt issuances increase the efforts of each country to engage in a currency war to depreciate its currency. This in turn discourages each country from issuing more public debt. That is, the possibility of a currency war reduces the domestic benefits from issuing public debt. Recall that—at a constant exchange rate—issuing more public debt in one country raises output in all countries. This creates room for some coordination, as countries will not take into account the impact of their debt issuance on foreign output. However, there is a more pernicious effect: issuing more public debt in one country increases the incentives for foreign exchange intervention in the other country. The resulting appreciation of the Home currency reduces the expansion in home output, in favor of foreign. Hence, the possibility of a future currency war dilutes the domestic benefits from an expansion of public debt.*

4.2 Balanced-Budget Fiscal Expansions: the Net Asset Demand Channel

We now focus on a specific case of fiscal policy: balanced-budget increases in government spending. Because of this assumption, we shut down the asset creation mechanism of public debt (deficit) increases. Budget-balanced government spending has two components: contemporaneous government consumption spending and taxes on private income (which would have been partly saved). Thus, on net budget-balanced government spending reduces desired global savings and global asset demand. At the ZLB, this reduction in net asset demand stimulates output in all countries, thereby generating positive spillovers. This can

[18] If the trap were only temporary, as in the model with exit in Online Appendix B.2, the results could be different, depending on whether the post-exit economy is dynamically efficient (i.e. $r^w < g$) or not. If the post-exit economy is dynamically inefficient, then the conclusion holds. But if the post-exit economy is dynamically efficient, then fiscal capacity is eventually required to service debt, constraining the use of debt issuance as a stimulus tool during the liquidity trap.

[19] In both cases, if the liquidity trap were temporary, and if the economy were dynamically efficient after exiting the trap, fiscal capacity would be needed either to soak up the extra money that was issued at the ZLB, or to service the government bonds that were issued at the ZLB.

also be seen in the goods market, where the reduction in net asset demand directly increases the demand for the goods that are consumed by the government, and indirectly increases the demand for all goods by increasing income via a Keynesian multiplier.

At Home, government spending on domestic goods $\gamma_G \bar{Y}$ is financed by increasing the tax τ on the income of newborns, $(1-\delta)\,\xi\bar{Y}$, for a constant level of public debt $D/\bar{Y}$. The same applies to Foreign where government spending $\gamma_G^* \bar{Y}^*$ is financed by increasing the tax τ^* on the income of newborns, $(1-\delta^*)\,\xi^*\bar{Y}^*$, for a constant level of public debt $D^*/\bar{Y}^*$. In the interest of space, we only consider the case of a global liquidity trap. Following the same steps as in the baseline model, the BGP equilibrium satisfies:

$$
\begin{aligned}
E &= \frac{\xi-\gamma_G}{\xi-\gamma_G^*}, \\
\xi &= \gamma_G + \frac{\theta\bar{d}(E) + \frac{\overline{\delta\gamma_G}(E)\theta}{\rho}}{1-\frac{\bar{\delta}\theta}{\rho}} \quad , \quad \xi = \gamma_G^* + \frac{1}{E}\frac{\theta\bar{d}(E) + \frac{\overline{\delta\gamma_G}(E)\theta}{\rho}}{1-\frac{\bar{\delta}\theta}{\rho}}, \\
\frac{NFA}{\bar{Y}} &= \frac{\xi(1-\frac{\delta\theta}{\rho}) - \gamma_G - \theta d}{g+\theta} = \frac{(1-\frac{\delta\theta}{\rho})}{g+\theta}\left[\frac{\theta\bar{d}(E) + \frac{\overline{\delta\gamma_G}(E)\theta}{\rho}}{1-\frac{\bar{\delta}\theta}{\rho}} - \frac{\frac{\delta\gamma_G\theta}{\rho}+\theta d}{1-\frac{\delta\theta}{\rho}}\right], \\
\frac{CA}{\bar{Y}} &= g\frac{NFA}{\bar{Y}},
\end{aligned}
$$

where $\overline{\delta\gamma_G}(E) \equiv y\delta\gamma_G + (1-y)\delta^*\gamma_G^* E$. These equations show that, given the exchange rate E, home government spending stimulates home output more than one-for-one, i.e. with a Keynesian government spending multiplier:

$$\frac{\partial\xi}{\partial\gamma_G} = 1 + \frac{y\delta\theta}{\rho-\bar{\delta}\theta} > 1,$$

while it stimulates foreign output but less so, with a Keynesian government spending multiplier of

$$\frac{\partial\xi^*}{\partial\gamma_G} = \frac{1}{E}\frac{y\delta\theta}{\rho-\bar{\delta}\theta} > 0.$$

These two effects are intuitive given that government spending not only increases the demand for home goods and reduces the asset demand arising from Home households, but also indirectly increases asset supply by stimulating home output. This explains why the domestic government spending multiplier is greater than one, and why the effect on foreign output is positive. Moreover, and for the same reason, Home government spending reduces the home Net Foreign Asset position and pushes the Home Current Account toward a deficit. Similar effects apply for Foreign government spending.

All in all, fiscal policy—be it in the form of public debt issuances, helicopter drops of money, or budget-balanced increases in government spending—is a *positive-sum* remedy in a ZLB environment.

5 A Model of the Diffusion of Safety Traps

In this section, we introduce macroeconomic risk and risk aversion, with the purpose of drawing a distinction between risky and safe assets. This extension is conceptually important for four reasons. First, what matters in our extended environment is not the overall scarcity of stores of value, but whether there is a scarcity of *safe assets*. When the return on safe assets reaches the ZLB, the economy enters a 'safety trap.' Second, this scarcity of safe assets depresses the risk free return relative to the expected return on risky assets. In the safety trap, risk premia increase with the size of the recession. Third, as before, the financial account plays a key role in transmitting economic shocks at the ZLB. However, the dimension of the financial account that matters is the *net flow of safe assets*. At the ZLB, countries that are net issuers of safe assets experience a worse recession than under financial autarky. Fourth, net issuers of safe assets also enjoy an 'exorbitant privilege' i.e. a high return on their (riskier) gross external assets relative to their (safer) gross external liabilities, as in Gourinchas and Rey (2007). This exorbitant privilege improves net safe asset issuers' Net Foreign Asset Position, while allowing them to run a larger trade deficit. The combined effects of the recession and high excess returns on net foreign assets worsens wealth inequality for net safe asset issuers, relative to financial autarky.

The model in this section builds on Caballero and Farhi (2017). Aggregate risk arises from the (vanishingly) small possibility of a disaster shock, modeled as a permanent drop in output in both countries. After the realization of this Poisson shock, all uncertainty is resolved: risk disappears and the economy behaves as in Section 2.

Prior to the shock, a fraction of (locally) infinitely risk-averse investors only hold safe assets, which, for simplicity, we assume consists only of public debt.[20] All remaining assets are held by risk neutral investors. The infinitely risk-averse investors capture a central notion of "fear," i.e. an extreme reluctance to hold any risk in the face of uncertainty. When the supply of safe assets is insufficient to meet the demand from these risk averse investors, the market segments and the expected return on safe assets drops relative to that on risky assets (a safety premium). As the demand for safe assets grows, or as the capacity of the global economy to produce them shrinks, this safety premium can become so large that it pushes the global economy into what Caballero and Farhi (2017) label a 'safety trap.'

The safety trap of this section and the liquidity trap of Section 2 are intimately related: they both arise when interest rates cannot fall far enough to restore equilibrium in asset markets. There are, however, two important differences. First, exiting a *safety* trap requires an increase in the net supply of *safe* assets,

[20]The working paper version of this paper, Caballero, Farhi and Gourinchas (2015), allows for the *private* provision of safe assets, synthesized from private risky assets. The distinction between private and public safe assets is not essential for the results we present here. What matters is the total (private and public) supply of safe assets.

regardless of the overall net supply of *total* assets. By contrast, exiting a liquidity trap only requires an increase in the net supply of assets, regardless of their risk characteristics. Second, unlike liquidity traps, safety traps are characterized by an increase in risk-premia, required to clear the market for risky assets.

5.1 Assumptions and Constrained Symmetric Competitive Equilibrium

We modify the model of Section 2 along the following lines.

Risk. A Poisson shock occurs with instantaneous probability ι. When the Poisson shock occurs, output in both countries drops instantaneously and permanently by a factor $\mu < 1$. For simplicity, we consider a scenario where, after the Poisson shock, all uncertainty is resolved and the natural interest rate $r^{w,n}$ is positive. In other words, the world after the Poisson shock is characterized by the baseline model of Section 2, outside the ZLB. However, and crucially, the possibility of an adverse future shock depresses the world natural interest rate *before* the Poisson shock. It follows that the economy may be at the ZLB before the Poisson shock, but never after it. Since our focus is on the period before the Poisson shock, we simplify the equations by considering the limit $\iota \to 0$. Since some agents are infinitely risk averse, the Poisson shock matters even in the limit where its intensity becomes vanishingly small.

Heterogeneity in Risk Appetite: Neutrals and Knightians. We allow a fraction α of savers in each country to be 'Knightians,' i.e. infinitely locally risk-averse agents. The remaining fraction $1-\alpha$ of savers are 'Neutrals,' i.e. risk-neutral agents as in the benchmark model. Knightians exhibit full home bias: they only consume the goods of their own country. This implies that domestic Knightians only value financial assets whose payoffs have no downside risk in terms of the home good numeraire.[21] By contrast, Neutrals have no home bias.

Formally, the preferences of Home Knightians and Neutrals are given by the following stochastic differential equations, where τ_θ denotes the stopping time for the idiosyncratic death process:

$$\begin{aligned} U_t^K &= 1_{\{t-dt\le\tau_\theta<t\}} c_{H,t} + 1_{\{t\le\tau_\theta\}} \min{}_t\{U_{t+dt}^K\}, \\ U_t^N &= 1_{\{t-dt\le\tau_\theta<t\}} c_{H,t}^{\gamma} c_{F,t}^{1-\gamma} + 1_{\{t\le\tau_\theta\}} \mathbb{E}_t[U_{t+dt}^N]. \end{aligned}$$

Home Neutral and Knightian savers receive the same pre-tax income $(1-\delta)\,\xi_t\bar{Y}_t$ at birth, and save it until the time of death by investing in different portfolios. Foreign has a similar setup with a fraction of

[21] Without full home bias, Knightians would value financial assets whose payoff has no downside risk in terms of their consumption basket. This would complicate the analysis without delivering significant additional insights. We observe that the currency home bias that results from our assumption is strongly present in the data. See Maggiori, Neiman and Schreger (2020).

safe asset savers α^*.

We make three further assumptions that simplify the analysis but do not matter for our main results. First, Neutral newborns pay all taxes and receive all new trees. Second, we assume that ability to pledge future output into current assets is the same in both countries: $\delta = \delta^*$. Third, trees live forever: $\rho = 0$.

Monetary policy and ZLB. By analogy with Section 2, Home monetary policy follows a truncated Taylor rule that can be summarized as follows:

$$\xi_t = 1 \text{ and } i_t = r_t^{K,n} > 0, \text{ or } \xi_t \leq 1 \text{ and } r_t^{K,n} < i_t = 0, \tag{43}$$

where i_t is the home nominal interest rate and $r_t^{K,n}$ is the relevant *natural real risk-free interest rate* at Home, defined as the risk-free real interest rate that clears markets when ignoring the ZLB constraint.

Constrained Symmetric Competitive Equilibrium under Financial Integration. We focus throughout on constrained symmetric stochastic BGP under financial integration. Along that BGP, interest rates will be constant. We denote by r^K and r^{K*} the risk-free interest rates in the home and foreign numeraires and by r^w the risky rate of return, which is the same in the home and foreign numeraires.

Under our assumptions, the natural rate after the Poisson shock is: $r^{w,n} = \theta\delta/(1-\theta\bar{d}/\mu) > 0$ and the economy escapes the ZLB.[22] From Proposition 2, the exchange rate must satisfy $E = 1$ after the shock. The symmetry assumption imposes $E = 1$ and $\xi = \xi^*$ prior to the shock. It follows that the exchange rate is constant along the BGP and assets that are safe in one numeraire are safe in the other. This implies that safe interest rates in the home and foreign numeraires are equal $r^K = r^{K*}$.

We further denote by W_t^K and W_t^N the wealth of Home Knightians and Neutrals in the Home currency; and by V_t the value of Home private assets prior to the Poisson shock in the Home currency. Similar definitions hold for Foreign.

By analogy with Eq. (4), we can write the evolution of wealth along a BGP prior to the Poisson shock, for the two groups of Home savers (with similar equations for Foreign):

$$gW_t^K = \dot{W}_t^K = -\theta W_t^K + \alpha\,(1-\delta)\,\xi\bar{Y}_t + r^K W_t^K, \tag{44a}$$

$$gW_t^N = \dot{W}_t^N = -\theta W_t^N + (1-\tau)\,(1-\alpha)\,(1-\delta)\,\xi\bar{Y}_t + r^w W_t^N + gV_t. \tag{44b}$$

The evolution of wealth for Neutrals Eq. (44b) differs from that of Knightians Eq. (44a) because they earn the risky return r^w instead of the riskless return r^K, pay taxes and receive the new trees.

[22] To see this, observe that asset demand equals $\mu\bar{Y}^w/\theta$ after the Poisson shock while asset supply equals $(\delta\mu/r^{w,n} + \bar{d})\bar{Y}^w$.

Second, home private asset value satisfies (with a similar equation for Foreign)

$$V_t = \frac{\delta \xi}{r^w} \bar{Y}_t. \tag{45}$$

To understand Eq. (45), observe that private assets constitute a claim to a future stream of dividends $\delta \xi \bar{Y}_t$ until the Poisson shock and $\delta \mu \bar{Y}_t$ afterwards. They are held exclusively by Neutrals and in the limit $\iota \to 0$, they are worth $\delta \xi \bar{Y}_t / r^w$.

Third, along the BGP, government debt dynamics satisfy the equivalent of Eq. (7):

$$\left(r_t^K - g\right) d = \tau_t \left(1-\alpha\right)\left(1-\delta\right) \xi_t, \tag{46a}$$

$$\left(r_t^{K*} - g\right) d^* = \tau_t^* \left(1-\alpha^*\right)\left(1-\delta\right) \xi_t^*. \tag{46b}$$

Fourth, market clearing conditions for home and foreign goods require (recall that $E = 1$)

$$\theta \left(W_t^K + y\left(W_t^N + W_t^{N*}\right)\right) = \xi \bar{Y}_t, \tag{47a}$$

$$\theta \left(W_t^{K*} + (1-y)\left(W_t^N + W_t^{N*}\right)\right) = \xi^* \bar{Y}_t^*. \tag{47b}$$

To understand the first expression, recall that Home Knightians have full home bias, while Home and Foreign Neutrals spend a share y of expenditures on the Home good. The second expression has a similar interpretation for the foreign good.

Finally, asset market clearing requires

$$W_t^w = W_t^K + W_t^N + W_t^{K*} + W_t^{N*} = V_t + V_t^* + D_t + D_t^* = V_t^w + D_t^w, \tag{48a}$$

$$W_t^K + W_t^{K*} \leq d\bar{Y}_t + d^* \bar{Y}_t^*. \tag{48b}$$

The first equation states that global asset demand equals global asset supply. The second equation states that Knightians' wealth must be smaller than the payoff from safe (i.e. public) assets in the event of a Poisson shock. There are different regimes depending on whether Eq. (48b) holds as a strict inequality or as an equality. If Eq. (48b) holds as a strict inequality, the marginal holder of safe assets is a Neutral investor and there are no risk premia: $r^K = r^w$ (unconstrained regime). In this regime, there is no scarcity of safe assets. In the second case, the marginal holder of safe assets is a Knightian, Eq. (48b) holds with equality and there is a risk premium: $r^w > r^K$ (constrained regime).

We assume throughout that we are in the constrained regime, which occurs when $\bar{\alpha}$ is large enough (i.e. the total demand for safe assets is high enough), or when $\bar{d}$ is small enough (i.e. the total supply of safe assets is small). Next, we define a constrained symmetric competitive equilibrium under financial integration.

Definition 2. *Constrained Symmetric Competitive Equilibrium under Financial Integration*
Given paths for the ratio of public debt to potential output, d_t and d_t^*, a constrained competitive symmetric equilibrium under financial integration prior to the Poisson process consists of sequences for output gaps ξ_t and ξ_t^*, risk-free rate r_t^K and r_t^{K*}, household wealth for Knightians and Neutrals W_t^K, W_t^N, W_t^{K*} and W_t^{N*}, private financial assets V_t and V_t^*, taxes τ_t and τ_t^*, policy rates i_t and i_t^*, risky return r_t^w, and nominal exchange rate E_t, such that (i) household wealth and private assets satisfy equations Eq. (44) and Eq. (45); (ii) debt dynamics follow Eq. (46); (iii) policy rates are set according to Eq. (43); (iv) goods markets clear Eq. (47); (iv) global asset markets clear Eq. (48a); (v) the equilibrium is constrained, so that Eq. (48b) holds as an equality; and (vi) the equilibrium is symmetric, so that $E_t = 1$, $\xi_t = \xi_t^* \equiv \xi^w$, $r_t^K = r_t^{K*} \equiv r^{K,w}$.

5.2 The Diffusion of Safety Traps

We can now characterize the constrained symmetric competitive equilibria. We make the following assumption on the parameters of the problem.

Assumption 2. *The parameters satisfy*

$$0 < \theta\bar{d} < \mu(1-\delta) \quad ; \quad \frac{\bar{\alpha}(1-\delta)}{g+\theta-\theta\delta/(1-\theta\bar{d})} > \bar{d}.$$

The first part of Assumption 2 corresponds to $\theta d < 1-\delta$ in Assumption 1. It ensures that there is a positive demand for private assets after the Poisson shock. The second part ensures that the competitive equilibrium is constrained. The left part of the inequality represents the demand for safe assets from Knightians, $W^{K,w}/\bar{Y}^w$, when the equilibrium is unconstrained. It increases with the global share of Knightians, $\bar{\alpha}$. The right hand side of the inequality represents the global supply of safe assets, $\bar{d}$.

Safety Trap Equilibrium. The constrained safe asset market condition Eq. (48b) implies $W^{K,w} = \bar{d}\bar{Y}^w$. From the wealth accumulation equation for Knightians Eq. (44a), safe asset demand satisfies

$$W^{K,w} = \frac{\bar{\alpha}\,(1-\delta)\,\xi^w}{g+\theta-r^{K,w}}\bar{Y}^w. \tag{49}$$

The second part of Assumption 2 obtains when Eq. (48b) binds, i.e. safe asset demand Eq. (49) exceeds safe asset supply at the unconstrained natural rate $r^{u,w} \equiv \delta\theta/(1-\theta\bar{d}) > 0$. Combining these two expressions, we obtain expressions for the natural risk free rate $r^{K,w,n}$ and the natural output gap $\xi^{w,n}$:

$$r^{K,w,n} = g + \theta - (1-\delta)\frac{\bar{\alpha}}{\bar{d}} < \frac{\theta\delta}{1-\theta\bar{d}} = r^{u,w}, \tag{50a}$$

$$\xi^{w,n} = \frac{g+\theta}{1-\delta}\frac{\bar{d}}{\bar{\alpha}} = \frac{g+\theta}{g+\theta-r^{K,w,n}}. \tag{50b}$$

Under Assumption 2, the risk free rate is lower than the natural rate of the unconstrained equilibrium, $r^{u,w}$. When $r^{K,w,n} \geq 0$, the economy avoids the safety trap, so that $r^{K,w} = r^{K,w,n}$ and $\xi^w = 1$. Instead when the natural risk free rate becomes negative, $r^{K,w} = 0$ and the global economy experiences a recession, $\xi^w = \xi^{w,n} < 1$. Eq. (50a) reveals that whether a safety trap occurs is determined entirely by the net demand for *safe assets*. The tighter the scarcity of safe assets as defined in Assumption 2, the lower the risk-free rate. A safety trap arises when this scarcity is so acute that the risk free rate reaches the ZLB. This may occur even though *the overall supply of assets is always sufficient to avoid a liquidity trap when the regime is unconstrained*, i.e. $r^{u,w} > 0$. Summarizing, the global economy satisfies $r^{K,w} = \max\{r^{K,w,n}, 0\}$ and $\xi^w = \min\{\xi^{w,n}, 1\}$.

Risk Premium. Next, we solve for the risky return r^w. We first use the market clearing conditions Eq. (47) and the global asset market equilibrium Eq. (48a) to express total asset demand W^w:

$$W^w = \xi^w \frac{\bar{Y}^w}{\theta} = V^w + \bar{d}\bar{Y}^w. \tag{51}$$

Replacing V^w in Eq. (51) using Eq. (45), we obtain

$$\xi^w \left(1 - \frac{\delta\theta}{r^w}\right) = \theta\bar{d}. \tag{52}$$

Outside the ZLB ($r^{K,w,n} \geq 0$), $\xi^w = 1$ and the risky return satisfies $r^w = r^{u,w} = \theta\delta/(1-\theta\bar{d})$, i.e. the natural rate of the unconstrained equilibrium. At the ZLB, ($r^{K,w,n} < 0$), $\xi^w < 1$ and the risky return rises strictly above the natural rate of the unconstrained equilibrium: $r^w = \theta\delta/(1-\theta\bar{d}/\xi^w) > r^{u,w}$. From Eq. (52), we see that the risk premium is decreasing in ξ^w at the ZLB: a deeper safety trap is associated with a higher risk premium.

We can understand the root cause of the safety trap as follows: Outside of the safety trap, the excess demand for safe assets pushes down the risk-free rate $r^{K,w}$ so as to lower the wealth of Knightian investors and clear the market for safe assets. Once the ZLB is reached, the risk-free rate cannot fall sufficiently to clear the safe asset market anymore. The global recession in the safety trap reduces safe asset demand by lowering Knightians' wealth, $W^{K,w}$, according to Eq. (49). The global recession, however, also reduces the wealth of Neutral agents, according to Eq. (44b). To clear the market for risky assets, the risky return r^w needs to *increase*. This lowers the value of risky asset supply, V^w, according to Eq. (45), and increases the wealth of Neutral's $W^{N,w}$ so that Eq. (52) holds.

Net Foreign Assets, Current Accounts, Trade Balance and 'Exorbitant Privilege'. We can express Home's Net Foreign Asset position independently of whether the global economy is in a safety trap equilibrium. Combining Eqs. (44) to (46) for Home and Foreign, using $NFA = W^K + W^N - V - D$ and the condition that $NFA + NFA^* = 0$, we obtain:

$$\frac{NFA}{\bar{Y}} = -\frac{\theta}{g+\theta-r^w}\left(d-\bar{d}\right) + \frac{(r^w - r^{K,w})}{g+\theta-r^w}\left((d-\bar{d}) - \frac{(1-\delta)\xi^w}{g+\theta-r^{K,w}}(\alpha-\bar{\alpha})\right), \tag{53a}$$

$$\frac{CA}{\bar{Y}} = g\frac{NFA}{\bar{Y}}. \tag{53b}$$

Home's Net Foreign Asset position contains two terms. The first term, $-\theta(d-\bar{d})/(g+\theta-r^w)$, obtains in the unconstrained equilibrium and is identical to that in the baseline model outside the ZLB (Proposition 3). In the unconstrained equilibrium, Home is a net debtor if it issues more public debt ($d > \bar{d}$). This is a familiar feature of Non-Ricardian models: issuing public debt increases total asset supply, $V + D$, more than it increases total asset demand, W, since public debt is partly repaid by future unborn generations. The second term arises when the equilibrium is constrained, $r^w > r^{K,w}$. In that case, the country that issues more public debt ($d > \bar{d}$) or demands less safe assets ($\alpha < \bar{\alpha}$) enjoys an 'exorbitant privilege' since it pays a lower interest rate on its (safe) external liabilities than it receives on its (riskier) external assets. This increases Home's wealth and improves its Net Foreign Asset position.

Next, we can express the Trade Balance independently of whether the global economy is in a safety trap equilibrium. Using Eq. (44), the definition of the Home Trade Balance $TB = \xi^w\bar{Y} - \theta(W^K + W^N)$ and the condition $TB + TB^* = 0$, we obtain:

$$\frac{TB}{\bar{Y}} = \frac{(r^w-g)}{g+\theta-r^w}\theta\left(d-\bar{d}\right) - \frac{\theta(r^w - r^{K,w})}{g+\theta-r^w}\left((d-\bar{d}) - \frac{(1-\delta)\xi^w}{g+\theta-r^{K,w}}(\alpha-\bar{\alpha})\right). \tag{54}$$

Home's Trade Balance contains two terms. The first term, $(r^w-g)\theta/(g+\theta-r^w)(d-\bar{d})$ is identical to that in the baseline model outside the ZLB (Proposition 3). In the unconstrained equilibrium, whether Home runs a Trade surplus or deficit depends on the signs of $r^w - g$ and $d - \bar{d}$. When $r^w - g > 0$, a Trade surplus is needed when the country is a Net Debtor, $d > \bar{d}$. The second term arises in the constrained equilibrium: issuing more public debt ($d > \bar{d}$) or demanding less safe assets ($\alpha < \bar{\alpha}$) increases Home's consumption and wealth, worsening its Trade Balance.

When $d > \bar{d}$ or $\alpha < \bar{\alpha}$, Home is able to run a larger Trade deficit without a deterioration in its Net Foreign Asset position because it receives larger net financial payments from abroad. Specifically, Home's Net Factor Payments, defined as $NFP = CA - TB$, satisfy:

$$\frac{NFP}{\bar{Y}} = \frac{-\theta r^w}{g+\theta-r^w}(d-\bar{d}) + \frac{(g+\theta)(r^w - r^{K,w})}{g+\theta-r^w}\left((d-\bar{d}) - \frac{(1-\delta)\xi^w}{g+\theta-r^{K,w}}(\alpha-\bar{\alpha})\right). \tag{55}$$

Home's Net Factor Payment contains two terms. The first term, $-r^w\theta(d-\bar{d})/(g+\theta-r^w)$ obtains in the unconstrained equilibrium. Net Factor Payments are negative when $d > \bar{d}$ since Home is a Net Debtor. The second term arises in the constrained equilibrium. It is positive when Home issues more public debt ($d > \bar{d}$) or demand less safe assets ($\alpha < \bar{\alpha}$) and reflects Home's *exorbitant privilege*: net financial income offsets the larger Trade deficit and improves the Net Foreign Asset position.

Gross Capital Flows and Metzler Diagram in Safe Assets. Both outside of a safety trap (when $r^{K,w,n} \geq 0$) and in a symmetric safety trap (when $r^{K,w,n} < 0$), we can represent the equilibrium determination of the safe interest rate $r^{K,w}$ and of the recession ξ^w through a *Metzler diagram in safe assets.* The key is to focus on the safe asset component of the Net Foreign Asset Position $NFA^K = W^K - D$ and the corresponding safe asset component of the Current Account CA^K:

$$\begin{aligned}
\frac{NFA^K}{\bar{Y}} &= \frac{\alpha(1-\delta)\xi^w}{\theta+g-r^{K,w}} - d = \frac{\alpha(1-\delta)\xi^w}{\theta+g-r^{K,w}} - \frac{\alpha(1-\delta)\xi^a}{\theta+g-r^{K,a}}, \\
\frac{CA^K}{\bar{Y}} &= g\frac{NFA^K}{\bar{Y}},
\end{aligned}$$

where $r^{K,a}$ and ξ^a are the financial autarky risk-free rate and output gap, obtained by imposing $NFA^K = 0$ in the above expression and $r^{K,a} \geq 0$. Outside the ZLB, $\xi^w = 1$ and the Metzler diagram in safe assets connects imbalances in safe assets to the risk-free rate $r^{K,w}$. When the global risk-free rate $r^{K,w}$ is lower than the autarky risk-free rate $r^{K,a}$, Home is a net debtor in safe asset: $NFA^K < 0$. At the ZLB, $r^{K,w} = 0$ and the Metzler diagram in safe assets connects imbalances in safe assets to the recession ξ^w: when the global recession is larger than the autarky recession ($\xi^w < \xi^a$), Home is a net debtor in safe assets $NFA^K < 0$. At the ZLB, *countries that are net suppliers of safe assets experience a larger recession than under financial autarky.*

Safety Trap and Within-Country Inequality. From Eq. (44), along a BGP with recession ξ, safe asset return r^K and risky return r, the wealth of Knightians is $W^K/\bar{Y} = \alpha(1-\delta)\xi/(g+\theta-r^K)$, while that of Neutrals is $W^N/\bar{Y} = (((1-\alpha)(1-\delta) + g\delta/r)\xi - (r^K - g)d)/(g+\theta-r)$. It is easy to check that the income of Knightians relative to Neutrals, $(W^K/\alpha)/(W^N/(1-\alpha)$, increases with output ξ and the risk free rate r^K, and decreases with the risky return r. Provided the safe rate r^K is not too high, Knightians are poorer than Neutrals: $W^N/\alpha < W^N/(1-\alpha)$.

Two main implications follow. First, within-country inequality must increase in each country along the BGP when the economy is in the global safety trap, since this is associated with a recession ($\xi^w < 1$), a lower risk free return ($r^{K,w} = 0$), and a higher return on risky assets ($r^w > r^{u,w}$). Intuitively, the safety trap recession restores equilibrium in asset markets by reducing the wealth of Knightians relative to that of Neutrals in all countries, worsening wealth inequality. In our model, increased within-country inequality between holders of safe and risky assets is a consequence of the safety trap equilibrium, rather than its cause.

Second, we have established that net suppliers of safe assets (with $\alpha < \bar{\alpha}$ or $d > \bar{d}$) experience a larger recession under financial integration than under financial autarky ($\xi^w < \xi^a$). Net suppliers of safe assets also experience a (weakly) larger return on risky assets under financial integration than under autarky.[23] It follows that net suppliers of safe assets experience a larger increase in wealth inequality between holders of safe and risky assets in the global safety trap than under financial autarky.

We summarize these results in the following proposition.

Proposition 7 (Constrained Symmetric Competitive Equilibrium, Safety Traps, Risk Premia, Net Foreign Assets and Current Accounts under Financial Integration)**.** *Under Assumption 2, the constrained symmetric competitive equilibrium is such that:*

- *The global economy satisfies $r^{K,w} = \max\{r^{K,w,n}, 0\}$, $\xi^w = min\{\xi^{w,n}, 1\}$, and $E = 1$.*
    - *If $r^{K,w,n} \geq 0$, then $\xi^{w,n} \geq 1$ and the global economy is outside the Safety Trap. There is a unique constrained symmetric BGP equilibrium with $r^{K,w} = r^{K,w,n} > 0$ and output at its potential, $\xi^w = 1$;*
    - *If $r^{K,w,n} < 0$, then $\xi^{w,n} < 1$ and the global economy is in a Safety Trap, there is a unique constrained symmetric BGP equilibrium with $r^{K,w} = 0$ and $\xi^w = \xi^{w,n} < 1$.*
- *Outside or inside the Safety Trap, there is a positive risk premium $r^w - r^{K,w}$, decreasing in the size of the global recession ξ^w.*
- *If Home has a larger capacity to produce safe assets ($d > \bar{d}$), it is a net debtor, runs a current account deficit, and enjoys a version of the 'exorbitant privilege', paying lower interest rates on its (safe) liabilities than on its (riskier) assets.*

[23]The return on risky assets at the ZLB under financial integration is $r^w = \delta\theta/(1-\theta(1-\delta)\bar{\alpha}/(g+\theta))$ while that under financial autarky is $r^a \equiv \delta\theta/(1-\theta(1-\delta)\alpha/(g+\theta))$.

- *At the ZLB equilibrium, countries that are net suppliers of safe assets experience a larger recession and a larger increase in wealth inequality between holders of safe and risky assets than under financial autarky.*

Proof. See text. □

We conclude this section by pointing out that while it is immediate that an increase in public debt d anywhere is useful everywhere in a global safety trap, this debt must be safe. Before the Poisson shock, fiscal capacity is irrelevant since $r^{K,w} - g = -g < 0$. It might therefore be tempting to think that any increase public debt d will help to escape the safety trap. However, fiscal capacity still matters after the Poisson shock if $r^{K,w} = r^w > g$, in which case taxes are needed to stabilize the debt. Imagine an initial situation where the foreign country is at its fiscal capacity after the Poisson shock, defined as the maximum level of debt achievable at the highest sustainable tax rate. Suppose that the foreign country nonetheless decides to increase its debt before the Poisson shock. This increase in debt is effectively risky: it does not increase its payoff after the Poisson shock. In turn, Knightians will not be willing to hold the increase in public debt: debt issuance would have no effect on ξ^w prior to the Poisson shock. It is the fiscal capacity of the country *after* it exits the safety trap that matters for the impact of public debt issuance during a safety trap.

6 Final Remarks

This paper proposes a model to characterize the behavior of a financially integrated global economy when interest rates are at the ZLB.

At the ZLB, the global asset market remains in disequilibrium when output is at its potential. The resulting global asset shortage cannot be resolved by lower world interest rates. It is instead alleviated by a world recession, which is propagated by global imbalances. Current Account surplus countries push world output down, exerting a negative effect on the world economy. Economic policy becomes more interconnected, with either negative or positive spillovers depending on the policy instrument. Countries can engage in Currency Wars, where each country aims to depreciate its exchange rate, at the expense of other countries. They can also engage in Trade Wars by unilaterally imposing tariffs on foreign goods. In contrast, safe public debt issuances, increases in government spending, and support for private securitization are positive-sum and stimulate output in all countries. Our Metzler Diagram in Quantities is a powerful new tool illustrating the economics of global imbalances and economic wars at the ZLB.

Once we introduce risk considerations, the model illustrates that the relevant dimension of global asset shortages at the ZLB is not an overall scarcity of stores of value, but a scarcity of safe assets whose return is constrained by the ZLB. That is, safe asset shortages push the global economy into a global safety trap. This depresses the return on safe assets relative to risky ones and increases risk premia. Recessions propagate through the safe component of the financial account. Reserve currency countries can be pushed into a larger safety trap than under financial autarky, although they benefit from an 'exorbitant privilege' on their Net Foreign Asset position. Within country inequality increases in all countries in a global safety trap, especially so for net suppliers of safe assets.

References

Barro, Robert J. and Andrew Mollerus, "Safe Assets," NBER Working Papers 20652, National Bureau of Economic Research, Inc October 2014.

Benigno, Pierpaolo and Federica Romei, "Debt deleveraging and the exchange rate," *Journal of International Economics*, 2014, *93* (1), 1–16.

Bernanke, Ben S., "The Global Saving Glut and the U.S. Current Account Deficit," Sandridge Lecture, Virginia Association of Economics, Richmond, Virginia, Federal Reserve Board March 2005.

_ , Carol C. Bertaut, Laurie DeMarco, and Steven B. Kamin, "International capital flows and the returns to safe assets in the United States 2003-2007," *Financial Stability Review*, February 2011, (15), 13–26.

Blanchard, Olivier J. and Nobuhiro Kiyotaki, "Monopolistic Competition, Aggregate Demand Externalities and Real Effects of Nominal Money," *American Economic Review*, February 1987, *77* (4), 647–666.

Caballero, Ricardo and Alp Simsek, "Asset Prices and Aggregate Demand in a" Covid-19" Shock: A Model of Endogenous Risk Intolerance and LSAPs," August 2020. CEPR Discussion Paper DP14627-5.

Caballero, Ricardo J., "On the Macroeconomics of Asset Shortages," in Andreas Beyer and Lucrezia Reichlin, eds., *The Role of Money: Money and Monetary Policy in the Twenty-First Century*, Vol. 4 of *European Central Banking Conference* 2006, pp. 272–283.

_ , "The 'Other' Imbalance and the Financial Crisis," NBER Working Papers 15636, National Bureau of Economic Research, Inc 2010.

_ and Arvind Krishnamurthy, "Global Imbalances and Financial Fragility," *American Economic Review*, May 2009, *99* (2), 584–88.

_ and Emmanuel Farhi, "The Safety Trap," *The Review of Economic Studies*, 2017, *85* (1), 223–274.

_ , _ , and Pierre-Olivier Gourinchas, "An Equilibrium Model of "Global Imbalances" and Low Interest Rates," *American Economic Review*, March 2008, *98* (1), 358–93.

_ , _ , and _ , "Financial Crash, Commodity Prices and Global Imbalances," *Brookings Papers on Economic Activity*, 2008, *2*, 1–55.

Caballero, Ricardo J, Emmanuel Farhi, and Pierre-Olivier Gourinchas, "Global Imbalances and Currency Wars at the ZLB.," Technical Report, National Bureau of Economic Research 2015.

Christiano, Lawrence, Martin Eichenbaum, and Sergio Rebelo, "When Is the Government Spending Multiplier Large?," *Journal of Political Economy*, 2011, *119* (1), 78 – 121.

Coeurdacier, Nicolas, Stéphane Guibaud, and Keyu Jin, "Credit Constraints and Growth in a Global Economy," *American Economic Review*, 2015, *105* (9), 2838–2881.

Cook, David and Michael B. Devereux, "Sharing the Burden: Monetary and Fiscal Responses to a World Liquidity Trap," *American Economic Journal: Macroeconomics*, July 2013, *5* (3), 190–228.

_ and _ , "The Optimal Currency Area in a Liquidity Trap," NBER Working Papers 19588, National Bureau of Economic Research, Inc October 2013.

— **and** — , "Exchange rate flexibility under the zero lower bound," Globalization and Monetary Policy Institute Working Paper 198, Federal Reserve Bank of Dallas 2014.

Correia, Isabel, Emmanuel Farhi, Juan Pablo Nicolini, and Pedro Teles, "Unconventional Fiscal Policy at the Zero Bound," *American Economic Review*, June 2013, *103* (4), 1172–1211.

Dang, Tri Vi, Gary Gorton, and Bengt Holmström, "The Information Sensitivity of a Security," MIT mimeo, MIT July 2015.

Devereux, Michael B. and James Yetman, "Capital Controls, Global Liquidity Traps, and the International Policy Trilemma," *Scandinavian Journal of Economics*, 01 2014, *116* (1), 158–189.

Duarte, Fernando M. and Carlo Rosa, "The equity risk premium: a review of models," Staff Reports 714, Federal Reserve Bank of New York February 2015.

Eggertsson, Gauti B. and Michael Woodford, "Optimal Monetary Policy in a Liquidity Trap," NBER Working Papers 9968, National Bureau of Economic Research September 2003.

— **and Neil R. Mehrotra**, "A Model of Secular Stagnation," NBER Working Papers 20574 October 2014.

— **and Paul Krugman**, "Debt, Deleveraging, and the Liquidity Trap: A Fisher-Minsky-Koo Approach," *The Quarterly Journal of Economics*, 2012, *127* (3), 1469–1513.

— **, Neil R. Mehrotra, Sanjay Singh, and Lawrence H. Summers**, "Contagious Malady? Global Dimensions of Secular Stagnation," Working paper, Brown University 2015.

Erceg, Christopher and Jesper Lindé, "Is There A Fiscal Free Lunch In A Liquidity Trap?," *Journal of the European Economic Association*, 02 2014, *12* (1), 73–107.

Farhi, Emmanuel and Iván Werning, "Fiscal Multipliers: Liquidity Traps and Currency Unions," NBER Working Papers 18381, National Bureau of Economic Research, Inc September 2012.

— **and Iván Werning**, "Monetary policy, bounded rationality, and incomplete markets," Working Paper No 23281, National Bureau of Economic Research February 2019.

Fornaro, Luca and Federica Romei, "The Paradox of Global Thrift," *American Economic Review*, 2019, *forthcoming.*

Gennaioli, Nicola, Andrei Shleifer, and Robert Vishny, "Neglected risks, financial innovation, and financial fragility," *Journal of Financial Economics*, 2012, *104* (3), 452–468.

Gorton, Gary, *Slapped by the Invisible Hand: The Panic of 2007* number 9780199734153. In 'OUP Catalogue.', Oxford University Press, July 2010.

— **and Guillermo Ordonez**, "The Supply and Demand for Safe Assets," NBER Working Papers 18732, National Bureau of Economic Research, Inc January 2013.

— **and** — , "Collateral Crises," *American Economic Review*, February 2014, *104* (2), 343–378.

Gourinchas, Pierre-Olivier and Hélène Rey, "From World Banker to World Venture Capitalist: US External Adjustment and the Exorbitant Privilege," in Richard Clarida, ed., *G-7 Current Account Imbalances: Sustainability and Adjustment*, University of Chicago Press, Chicago, 2007, pp. 11–55.

— **and Helene Rey**, "External Adjustment, Global Imbalances and Valuation Effects," in Gita Gopinath, Elhanan Helpman, and Ken Rogoff, eds., *Handbook of International Economics*, Vol. 4, Elsevier, 2014.

—, **Hélène Rey, and Nicolas Govillot**, "Exorbitant Privilege and Exorbitant Duty," mimeo, UC Berkeley 2010.

Greenwood, Robin, Samuel Hanson, and Jeremy C. Stein, "A Gap-Filling Theory of Corporate Debt Maturity Choice," *Journal of Finance*, 09 2015, *117* (3), 449–469.

Guerrieri, Veronica and Guido Lorenzoni, "Credit Crises, Precautionary Savings, and the Liquidity Trap," Working Paper 17583, National Bureau of Economic Research November 2011.

He, Zhiguo, Arvind Krishnamurthy, and Konstantin Milbradt, "A Model of the Reserve Asset," Technical Report, Stanford University, Graduate School of Business June 2015.

Holmström, Bengt and Jean Tirole, "Private and public supply of liquidity," *Journal of political Economy*, 1998, *106* (1), 1–40.

Jeanne, Olivier, "The Global Liquidity Trap," Working paper, John Hopkins University 2009.

Kareken, John and Neil Wallace, "On the Indeterminacy of Equilibrium Exchange Rates," *The Quarterly Journal of Economics*, 1981, *96* (2), 207–222.

Keynes, John Maynard, *The General Theory of Employment, Interest and Money*, Palgrave MacMillan, 1936.

Klein, Matthew C and Michael Pettis, *Trade wars are class wars: How rising inequality distorts the global economy and threatens international peace*, Yale University Press, 2020.

Kocherlakota, Narayana, "Low Real Interest Rates," 22nd Annual Hyman P. Minsky Conference, Levy Economics Institute of Bard College, Federal Reserve Bank of Minneapolis 2014.

Krishnamurthy, Arvind, "Comments on Rachel and Summers 'On Falling Neutral Real Rates, Fiscal Policy, and the Risk of Secular Stagnation'," in "Brookings Papers on Economic Activity," Vol. Spring, Brookings, 2019.

Krugman, Paul, "It's Baaack: Japan's Slump and the Return of the Liquidity Trap," *Brookings Papers on Economic Activity*, 1998, *29* (2), 137–206.

Kumhof, Michael, Romain Rancière, and Pablo Winant, "Inequality, leverage, and crises," *American Economic Review*, 2015, *105* (3), 1217–45.

Lane, Philip R. and Gian Maria Milesi-Ferretti, "The External Wealth of Nations Revisited: International Financial Integration in the Aftermath of the Global Financial Crisis," *IMF Economic Review*, Mar 2018, *66* (1), 189–222.

Lerner, Abba P, "The symmetry between import and export taxes," *Economica*, 1936, *3* (11), 306–313.

Maggiori, Matteo, "Financial Intermediation, International Risk Sharing, and Reserve Currencies," 2012 Meeting Papers 146, Society for Economic Dynamics 2012.

—, **Brent Neiman, and Jesse Schreger**, "International currencies and capital allocation," *Journal of Political Economy*, 2020, *128* (6), 2019–2066.

Mendoza, Enrique G., Vincenzo Quadrini, and José-Víctor Ríos-Rull, "Financial Integration, Financial Development, and Global Imbalances," *Journal of Political Economy*, June 2009, *117* (3), 371–416.

Mian, Atif R, Ludwig Straub, and Amir Sufi, "Indebted Demand," April 2020. NBER Working Paper No. 26940.

_ , _ , **and** _ , "The saving glut of the rich and the rise in household debt," April 2020. NBER Working Paper No. 26941.

Moreira, Alan and Alexi Savov, "The Macroeconomics of Shadow Banking," NBER Working Papers 20335, National Bureau of Economic Research, Inc July 2014.

Negro, Marco Del, Domenico Giannone, Marc P. Giannoni, and Andrea Tambalotti, "Safety, Liquidity, and the Natural Rate of Interest," *Brookings Papers on Economic Activity*, 2017, *48* (1 (Spring), 235–316.

Stein, Jeremy C., "Monetary Policy as Financial-Stability Regulation," Technical Report 1 2012.

Svensson, Lars E.O., "Escaping from a Liquidity Trap and Deflation: The Foolproof Way and Others," *Journal of Economic Perspectives*, Fall 2003, *17* (4), 145–166.

Werning, Iván, "Managing a Liquidity Trap: Monetary and Fiscal Policy," Technical Report 2012. NBER Working Papers.

A Appendix

A.1 New-Keynesian Microfoundations for Section 2.1

We provide one possible exact microfoundation for demand determined output in the presence of nominal rigidities in the model in Section 2.1. The microfoundation is in the New Keynesian tradition. We focus on the case of Home (the case of Foreign is identical). In a nutshell, domestic monopolistic firms produce imperfectly substitutable varieties of home intermediate goods using non-traded intermediates, and compete in prices. The firms' posted prices are rigid in the home currency, and they accommodate demand at the posted price. The different varieties of the home intermediate goods are combined into a home final good by a competitive sector according to a Dixit-Stiglitz aggregator. These assumptions are standard in the New Keynesian literature starting with Blanchard and Kiyotaki (1987).

Specifically, between t and $t+dt$, there is an endowment $\bar{Y}_t dt$ of each differentiated variety $i \in [0,1]$ of non-traded input. Each variety i of non-traded input can be transformed into one unit of variety i of home intermediate good using a one-to-one linear technology by a monopolistic firm indexed by i, which is owned and operated by the agents supplying variety i of the non-traded input.

The differentiated varieties of final home intermediate goods are then combined together into a home final good by a competitive sector according to a standard Dixit-Stiglitz aggregator

$$Y_t = \left(\int_0^1 Y_{H,i,t}^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}} dt,$$

where $Y_{H,i,t}dt$ is the quantity of variety i of the final good used for production of the final good. The price of the final home good is

$$P_{H,t} = \left(\int_0^1 p_{H,i,t}^{1-\varepsilon} di \right)^{\frac{1}{1-\varepsilon}},$$

where $p_{H,i,t}$ is the home currency price posted by monopolistic firm i for variety i of the home intermediate good. The resulting individual demand for each variety is given by

$$Y_{H,i,t}dt = \left(\frac{p_{H,i,t}}{P_{H,t}} \right)^{-\varepsilon} Y_t dt.$$

Prices set by monopolistic firms are perfectly rigid in the home currency, equal to each other. We normalize these prices in the home currency to one

$$p_{H,i,t} = P_{H,t} = 1.$$

All the varieties of intermediate goods are then produced in the same amount

$$Y_{H,i,t}dt = Y_t dt = \xi_t \bar{Y}_t dt.$$

Between t and $t+dt$, the varieties of non-traded inputs indexed by $i \in [\delta, 1]$, are distributed equally to the different agents who are born during that interval of time. The varieties of non-traded inputs indexed by $i \in [0, \delta]$ accrue equally as dividends on the different Lucas trees.

Real income (equal to real output) $Y_t dt$ is divided into an endowment $(1-\delta)Y_t dt$ distributed equally to agents who are born during that interval of time, and the dividend $\delta Y_t dt$ of the Lucas trees. This provides an exact microfoundation for the model presented in Section 2.1.

A.2 Inflation

So far, we have assumed that prices are fully rigid. In this section, we relax this assumption and allow for *some* price adjustment through a Phillips curve.

This extension gives us the opportunity to reiterate some well-known insights about the economics of liquidity traps, and to obtain some new ones. The former are that credibly higher inflation targets reduce the severity of a liquidity trap, that more (downward) price flexibility can exacerbate the severity of the trap as the economy may fall into a deflationary spiral. The less-known one is that in a *global* liquidity trap, it is the *more rigid* country that experiences the worst trap (note the contrast between the effect of relative rigidity and that of aggregate rigidity). Moreover, it is now possible for some regions of the world to escape the liquidity trap if their inflation expectations are sufficiently high.

A.2.1 The Extended Model

Phillips curve. We wish to capture the idea that wages, or prices, are rigid downwards, but not upwards. We follow the literature and assume that prices and wages cannot fall faster than a certain limit pace, perhaps determined by a "social norm" and that this limit pace is faster if there is more slack in the economy:[24]

$$\pi_{H,t} \geq -\kappa_0 - \kappa_1(1-\xi_t),$$
$$\pi^*_{F,t} \geq -\kappa^*_0 - \kappa^*_1(1-\xi^*_t),$$

where $\pi_{H,t} = \dot{P}_{H,t}/P_{H,t}$ (resp. $\pi^*_{F,t} = \dot{P}^*_{F,t}/P^*_{F,t}$) denotes the *domestic* (resp. foreign) inflation rate, and where $\kappa_1 \geq 0$ and $\kappa^*_1 \geq 0$. Moreover, we assume that if there is slack in the economy, prices or wages fall as fast as they can: $\xi_t < 1$ implies that $\pi_{H,t} = -\kappa_0 - \kappa_1(1-\xi_t)$ and $\xi^*_t < 1$ implies that $\pi^*_{F,t} = -\kappa^*_0 - \kappa^*_1(1-\xi^*_t)$. We capture this requirement with the complementary slackness conditions $[\pi_{H,t}+\kappa_0+\kappa_1(1-\xi_t)](1-\xi_t) = 0$ and $[\pi^*_{F,t}+\kappa^*_0+\kappa^*_1(1-\xi^*_t)](1-\xi^*_t) = 0$.

To summarize, there are two Phillips curves, one for Home and one for Foreign. The home Phillips curve traces out an increasing curve in the $(\pi_{H,t}, \xi_t)$ space, which becomes vertical at $\xi_t = 1$. The foreign Phillips curve is similar.

Monetary policy. We extend Eqs. (2) and (3) and assume that monetary policy is conducted according to simple truncated Taylor rules, where the nominal interest rate responds to domestic inflation:

$$i_t = \max\{r^n_t + \bar{\pi} + \psi_\pi(\pi_{H,t} - \bar{\pi}) + \psi_\xi(\xi_t - 1), 0\},$$
$$i^*_t = \max\{r^{n*}_t + \bar{\pi}^* + \psi^*_\pi(\pi^*_{F,t} - \bar{\pi}^*) + \psi^*_\xi(\xi^*_t - 1), 0\}.$$

In these equations r^n_t and r^{n*}_t are the relevant natural interest rates at Home and in Foreign, which depend on whether we analyze the financial integration equilibrium or the financial autarky equilibrium. We denote by $\bar{\pi} \geq \max\{-\kappa_0, 0\}$ and $\bar{\pi}^* \geq \max\{-\kappa^*_0, 0\}$ the home and foreign inflation targets, while $\psi_\pi > 1$ and ψ_ξ denote the Taylor rule coefficients on domestic price inflation and the output gap respectively.

For simplicity, we take the limit of large Taylor rule coefficients $\psi_\pi, \psi^*_\pi \to \infty$ and $\psi_\xi, \psi^*_\xi \to \infty$. This specification of monetary policy implies that inflation in any given country is equal to its target and that there is no recession as long as the country's interest rate is positive. For example, for Home, either $\pi_{H,t} = \bar{\pi}$, $\xi_t = 1$, and $i_t = r^n_t + \bar{\pi} \geq 0$ or $\pi_{H,t} \leq -\kappa_0 \leq \bar{\pi}$, $\xi_t \leq 1$, and $i_t = 0$. The same holds for Foreign.

A.2.2 Equilibria

We assume that the world natural interest rate $r^{w,n} = -\rho + \bar{\delta}\theta/(1-\bar{d}\theta) < 0$ is low enough that $r^{w,n} < \min\{\kappa_0, \kappa^*_0\}$, which yields the existence of a *global* liquidity trap equilibrium. We show that in this case, there are *several* possible equilibrium configurations once prices are allowed to adjust. First, there can be equilibria with no liquidity traps either at Home or in Foreign. Second, there can be equilibria with a symmetric global liquidity trap both at Home and in Foreign. Third, there can be asymmetric liquidity trap equilibria with a liquidity trap only in one country. We treat each in turn.

No liquidity trap equilibrium. We solve for the no-liquidity trap case. This equilibrium is such that $\xi = 1$, $\xi^* = 1$, $\pi_H = \bar{\pi}$, $\pi^*_F = \bar{\pi}^*$, $i = r^{w,n} + \bar{\pi}$, and $i^* = r^{w,n} + \bar{\pi}^*$.

It is straightforward to show that the terms of trade $S_t = E_t P^*_{F,t}/P_{H,t}$ is constant at $S_t = 1$ which implies that $\dot{E}_t/E_t = \pi_H - \pi^*_F = \bar{\pi} - \bar{\pi}^*$. The condition for this equilibrium to exist is that $i \geq 0$ and $i^* \geq 0$, i.e. $\min\{r^{w,n} + \bar{\pi}, r^{w,n} + \bar{\pi}^*\} \geq 0$. This condition shows that *the no-liquidity trap equilibrium exists if and only if the inflation targets $\bar{\pi}$ and $\bar{\pi}^*$ in both countries are high enough.*

Note, however, that this is an existence, not a uniqueness result. In fact, as we shall see next, other equilibria exist even if inflation targets are high enough to make the no-liquidity trap equilibrium feasible.

[24] The introduction of this kind of Phillips curves borrows heavily from Eggertsson and Mehrotra (2014) and Caballero and Farhi (2017).

Symmetric global liquidity trap equilibrium. Let us now focus on the other extreme and solve for the symmetric global liquidity trap case.

Observe that in a stationary equilibrium the terms of trade $S_t = E_t P^*_{F,t}/P_{H,t}$ must be constant at $S_t = \xi/\xi^*$, so that $\dot{E}_t/E_t = \pi_H - \pi^*_F$. Uncovered Interest Parity then requires that $i = i^* + \dot{E}_t/E_t$, which combined with $i = i^* = 0$ implies that $\dot{E}_t/E_t = 0$ and hence $\pi^*_F = \pi_H = \pi^w$. That is, *in a global liquidity trap, inflation rates are equal across countries,* hence real interest rates are equalized, $r = r^* = -\pi^w$.

In Online Appendix B.1 we provide a detailed exposition of the equilibrium equations. We can represent the equilibrium as an Aggregate Demand (AD)-Aggregate Supply (AS) diagram which constitutes a system of four equations in four unknowns π_H, π^*_F, ξ, and ξ^*. The home and foreign AD curves are given by:

$$\xi = \frac{(1-\frac{\pi_H}{\rho})y\theta d}{(1-\frac{\bar{\delta}\theta}{\rho}-\frac{\pi_H}{\rho})-(1-\frac{\pi_H}{\rho})\frac{1-y}{\xi^*}\theta d^*} \quad ; \quad \xi^* = \frac{(1-\frac{\pi^*_F}{\rho})(1-y)\theta d^*}{(1-\frac{\bar{\delta}\theta}{\rho}-\frac{\pi^*_F}{\rho})-(1-\frac{\pi^*_F}{\rho})\frac{y}{\xi}\theta d}.$$

The home and foreign AS curves are given by:

$$\pi_H = -\kappa_0 - \kappa_1(1-\xi) \quad ; \quad \pi^*_F = -\kappa^*_0 - \kappa^*_1(1-\xi^*)$$

as long as $\xi < 1$ and $\xi^* < 1$, and become vertical at $\xi = 1$ and $\xi^* = 1$.

It can be verified that the home and foreign AD equations imply $\pi_H = \pi^*_F = \pi^w$. If $\kappa_0 = \kappa^*_0$, this implies that

$$\frac{1-\xi^*}{1-\xi} = \frac{\kappa_1}{\kappa^*_1},$$

so that *Home has a smaller recession than Foreign, $\xi > \xi^*$, if and only if home prices or wages are more flexible than foreign prices or wages: $\kappa_1 > \kappa^*_1$.* More (downward) price or wage flexibility reduces the size of the recession at Home relative to Foreign because it depreciates the domestic terms of trade. In a stationary equilibrium, deflation rates are equalized across countries so relatively more wage flexibility implies a relatively smaller recession.

The rest of the equilibrium simplifies greatly when the Phillips curves are identical in both countries so that $\kappa^*_0 = \kappa_0$ and $\kappa^*_1 = \kappa_1$. Indeed, this requires that the recession is identical at Home and in Foreign: $\xi = \xi^* = \xi^w$, and $S = 1$. Moreover, in this case, we have the following simpler *global* AD-AS representation:

$$\xi^w = \frac{1-\frac{\pi^w}{\rho}}{1-\frac{\bar{\delta}\theta}{\rho}-\frac{\pi^w}{\rho}}\theta\bar{d}, \tag{A.1a}$$

$$\pi^w = -\kappa_0 - \kappa_1(1-\xi^w). \tag{A.1b}$$

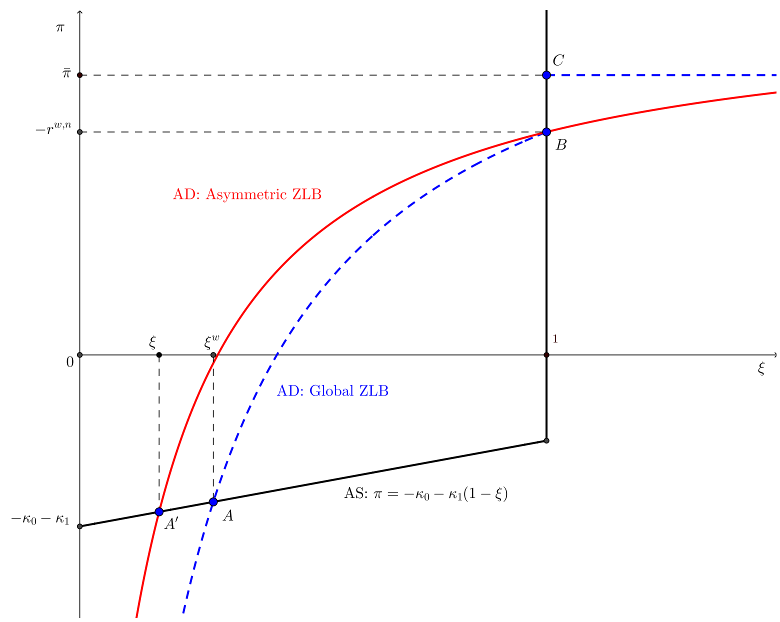
This representation makes clear that compared with the case with no inflation, *there is now a negative feedback loop between the global recession and inflation.* A larger recession reduces inflation, which in turn raises the real interest rate, causing a further recession etc. *ad infinitum.* This *feedback loop is stronger, the more flexible prices and wages are*, as captured by the slope of the Phillips curve κ_1. That is, wage flexibility plays out differently across countries and at the global level: Countries with more price flexibility bear a smaller share of the global recession than countries with less wage flexibility; but at the global level, more wage flexibility exacerbates the global recession.

The equilibrium is guaranteed to exist under some technical conditions on the Phillips curves parameters κ_0 and κ_1, which ensure that the feedback loop is not so powerful to lead to a total collapse of the economy.[25]

Figure A.1 reports the global AD-AS diagram and displays both the no liquidity trap equilibrium (if it exists) and the symmetric liquidity trap equilibrium. For simplicity the figure is drawn in the case where Philips curves and inflation targets are identical in both countries so that $\kappa^*_0 = \kappa_0$ and $\kappa^*_1 = \kappa_1$, $\bar{\pi}^* = \bar{\pi}$.

We focus on the no liquidity trap equilibrium and the symmetric liquidity trap equilibrium for now. The AS curve (black solid line) slopes upwards, then becomes vertical at $\xi = \xi^w = 1$: A smaller recession is associated with less deflation, until full employment is achieved. At the ZLB, the global AD curve (blue dashed line) also slopes upwards since an increase in inflation reduces the real interest rate, which increases output. Away from the ZLB

[25]For $\xi^w = 1^-$, the AD curve implies $\pi^w = -r^{w,n}$, while the AS curve implies $\pi^w = -\kappa_0 < -r^{w,n}$. For $\xi^w = 0$, the AD curve implies $\pi^w = \rho$, while the AS curve implies $\pi^w = -(\kappa_0 + \kappa_1)$. A sufficient condition for a unique intersection is that $\kappa_0 + \kappa_1 \leq -\rho$.



The figure reports Aggregate Supply (solid black line) and Aggregate Demand (dashed black line) in a symmetric liquidity trap equilibrium (point A) and in a no liquidity trap equilibrium (point C), when $\kappa_0^* = \kappa_0$, $\kappa_1^* = \kappa_1$, and $\bar{\pi}^* = \bar{\pi}$. The red solid line represents the home AD curve in the asymmetric equilibrium where Foreign is out of the liquidity trap (point A').

Figure A.1: Aggregate Demand and Aggregate Supply in a symmetric and asymmetric liquidity trap equilibria.

($\xi \geq 1$), the AD curve becomes horizontal at $\bar{\pi}$. We always assume that the upward sloping part of the AD curve is steeper than the non-vertical part of the AS curve and that they intersect at one point, A. The AD and AS schedules intersect at either exactly point A, or at three points, A, B, and C. Point A is the symmetric liquidity trap equilibrium: $i = 0$, $\pi^w = -\kappa_0 - \kappa_1(1-\xi^w) < \bar{\pi}$, and $\xi^w < 1$. Point C, if it exists, corresponds to the no-liquidity trap equilibrium with $i = i^* = \bar{\pi} > 0$, $\pi^w = \bar{\pi}$, and $\xi = \xi^w = 1$. Point B, if it exists, is unstable, and we ignore it.

Asymmetric liquidity trap equilibria. We now show that it is *always* possible to have an asymmetric equilibrium where one country is in a liquidity trap but not the other. These asymmetric liquidity trap equilibria are associated with different values of the real exchange rate, and are a manifestation of the same fundamental indeterminacy that we identified in the case with no inflation.

Suppose that one country is in a liquidity trap (say Home) but not the other (say Foreign). Then because the terms of trade must be constant at $S_t = \xi$, we must have $i = 0$, $i^* = i - \dot{E}_t/E_t = \pi_F^* - \pi_H > 0$, $\xi < 1$, $\xi^* = 1$, $\pi_F^* = \bar{\pi}^*$, and $\pi_H + \kappa_0 + \kappa_1(1-\xi) = 0$. In Online Appendix B.1, we provide a detailed exposition of the equilibrium equations. We find:

$$\xi = \frac{(1 - \frac{\pi_H}{\rho})y\theta d}{(1 - \frac{\bar{\delta}\theta}{\rho} - \frac{\pi_H}{\rho}) - (1 - \frac{\pi_H}{\rho})(1-y)\theta d^*}, \tag{A.2a}$$

$$\pi_H = -\kappa_0 - \kappa_1(1-\xi). \tag{A.2b}$$

The equilibrium is guaranteed to exist under the same technical conditions on Phillips curves that the ones derived above.

Comparing Eq. (A.2) and Eq. (A.1), it is easy to see that *the home recession is larger and home inflation is lower in this asymmetric liquidity trap equilibrium where only Home is in a liquidity trap, than in the symmetric equilibrium where both Home and Foreign are in a liquidity trap.* In Figure A.1, the red solid line reports the Home AD curve in the asymmetric equilibrium when Foreign is not in a liquidity trap. Point A' is the corresponding equilibrium. We can verify immediately that $\xi < \xi^w$, that is: The recession is more severe for the country that remains in the trap.

Inflation, exchange rates, and the structure of equilibria. Let us take stock and summarize the structure of equilibria when $r^{w,n} < \min\{\kappa_0, \kappa_0^*\}$. There may exist an equilibrium with no liquidity trap, which occurs if and only if $\min\{r^{w,n} + \bar{\pi}, r^{w,n} + \bar{\pi}^*\} \geq 0$. But there always exists a symmetric global liquidity trap equilibrium, as well as two asymmetric liquidity trap equilibria where only one country is in a liquidity trap. These symmetric and asymmetric liquidity trap equilibria are associated with different values of the real exchange rate and this multiplicity is a manifestation of the same fundamental indeterminacy that we identified in the case with no inflation. Indeed, it is immediate to see that terms of trade S are the most depreciated in the asymmetric liquidity trap equilibrium where Home is not in a liquidity trap but Foreign is, the most appreciated in the asymmetric liquidity trap equilibrium where Home is in a liquidity trap but Foreign is not, and intermediate between these two values in the symmetric liquidity trap equilibrium where both Home and Foreign are in a liquidity trap. The severity of the recession at Home is directly commensurate with the degree of appreciation of the terms of trade S.

A.3 Home Bias

We now relax the assumption that there is no home bias. Assume that the spending share on home goods of home agents is $y + (1-y)\beta$, and similarly that the spending share on foreign goods of foreign agents is $y^* + (1-y^*)\beta$, where $y^* = 1 - y$ and $\beta \in [0,1]$ indexes the degree of home bias. Full home bias corresponds to $\beta = 1$. The case of no home bias analyzed in the main text corresponds to $\beta = 0$.

With home bias in preferences, the good market clearing conditions become:

$$[y + (1-y)\beta]\theta W + (1-y^*)(1-\beta)\theta W^* E = \xi \bar{Y}, \tag{A.3a}$$

$$(1-y)(1-\beta)\theta W + [y^* + (1-y^*)\beta]\theta W^* E = E\xi^* \bar{Y}^*. \tag{A.3b}$$

This applies both under financial integration and under financial autarky, whether Home is in a liquidity trap ($\xi < 1$) or not ($\xi = 1$), and similarly whether Foreign is in a liquidity trap ($\xi^* < 1$) or not ($\xi^* = 1$).

For conciseness, we only consider the case where there is a global liquidity trap under financial integration. In that case, just like in the case of no home bias, there is a degree of indeterminacy indexed by the exchange rate E.

The asset and wealth dynamic equations Eqs. (4) and (5) are unchanged. After simple manipulations, we can express all equilibrium variables as a function of the nominal exchange rate:

$$\xi = \frac{\theta \bar{d}^\beta(E)}{1 - \frac{\bar{\delta}^\beta \theta}{\rho}}, \tag{A.4}$$

$$\frac{NFA}{\bar{Y}} = \frac{(1 - \frac{\delta\theta}{\rho})[\frac{\theta \bar{d}^\beta(E)}{1 - \frac{\bar{\delta}^\beta \theta}{\rho}} - \frac{\theta d}{1 - \frac{\delta\theta}{\rho}}]}{g + \theta} = \frac{(1 - \frac{\delta\theta}{\rho})(\xi - \xi^{a,l})}{g + \theta},$$

$$\frac{CA}{\bar{Y}} = g\frac{NFA}{\bar{Y}},$$

where we have defined the averages modified by home bias β as

$$\bar{d}^\beta(E) = \frac{[\beta + (1-\beta)y\left(1 + \frac{\frac{\delta^*\theta}{\rho} + \frac{\theta}{g}}{1 - \frac{\delta^*\theta}{\rho}}\right)]\theta d + (1-y)(1-\beta)\left(1 + \frac{\frac{\delta^*\theta}{\rho} + \frac{\theta}{g}}{1 - \frac{\delta^*\theta}{\rho}}\right) E\theta d^*}{[\beta + (1-\beta)y\left(1 + \frac{\frac{\delta^*\theta}{\rho} + \frac{\theta}{g}}{1 - \frac{\delta^*\theta}{\rho}}\right)] + (1-y)(1-\beta)\left(1 + \frac{\frac{\delta^*\theta}{\rho} + \frac{\theta}{g}}{1 - \frac{\delta^*\theta}{\rho}}\right)},$$

$$\bar{\delta}^\beta = \frac{[\beta + (1-\beta)y\left(1 + \frac{\frac{\delta^*\theta}{\rho} + \frac{\theta}{g}}{1 - \frac{\delta^*\theta}{\rho}}\right)]\left(1 - \frac{\delta\theta}{\rho}\right) + (1-y)(1-\beta)\frac{1 + \frac{\theta}{g}}{1 - \frac{\delta^*\theta}{\rho}}\left(1 - \frac{\delta^*\theta}{\rho}\right)}{[\beta + (1-\beta)x\left(1 + \frac{\frac{\delta^*\theta}{\rho} + \frac{\theta}{g}}{1 - \frac{\delta^*\theta}{\rho}}\right)] + (1-y)(1-\beta)\left(1 + \frac{\frac{\delta^*\theta}{\rho} + \frac{\theta}{g}}{1 - \frac{\delta^*\theta}{\rho}}\right)}.$$

and where $\xi^{a,l}$ is defined exactly as in the case with no home bias, and given by the same formula. Equations (A.4) and its equivalent for the foreign country show that, as before, home output ξ is increasing in the exchange rate E while foreign output ξ^* is decreasing in E. Finally, as before, the home Net Foreign Asset Position and Current Account are increasing in the gap between the domestic recession and the home financial autarky recession $\xi^{a,l}$. *The key difference introduced by home bias β is that the home and foreign outputs ξ and ξ^* become less responsive to the* *exchange rate E*. This can be seen directly by examining (A.4) in the case of home bias ($\beta > 0$) and comparing to Eq. (17) in the case with no home bias ($\beta = 0$). This effect is seen most transparently in the limit with full home bias ($\beta \to 1$) in which case the outputs ξ and ξ^* become completely insensitive to the exchange rate E.

Assume further that both countries are in a liquidity trap under financial autarky. Then, just like in the case with no home bias, the integrated equilibrium coincides with financial autarky when $E = E^a$. For $E > E^a$, we have $\xi > \xi^a$ and $\xi^* < \xi^a$ and vice versa for $E < E^a$.[26]

A.4 Trade Elasticities

A.4.1 The model with arbitrary elasticity

We now assume away home bias and investigate instead the role of the elasticity of substitution σ between home and foreign goods. That is, we assume that the consumption aggregator takes the form:

$$c_t = \left(\gamma^{1/\sigma} c_{H,t}^{(\sigma-1)/\sigma} + (1-\gamma)^{1/\sigma} c_{F,t}^{(\sigma-1)/\sigma}\right)^{\sigma/(\sigma-1)}. \tag{A.5}$$

As before, prices are rigid with $p_{H,t} = p^*_{F,t} = 1$. It follows that Home consumer price index satisfies:

$$P_t = \left(\gamma + (1-\gamma)E_t^{1-\sigma}\right)^{1/(1-\sigma)}. \tag{A.6}$$

The main difference in the system of equilibrium equations is once again the goods market clearing conditions, which become:

$$\begin{aligned}
\frac{y}{y + E_t^{1-\sigma}(1-y)}(W_t + E_t W_t^*) &= \frac{\xi_t \bar{Y}_t}{\theta}, \\
\frac{(1-y)E_t^{1-\sigma}}{y + E_t^{1-\sigma}(1-y)}(W_t + E_t W_t^*) &= E_t \frac{\xi_t^* \bar{Y}_t^*}{\theta}.
\end{aligned}$$

This applies both under financial integration and under financial autarky, whether Home is in a liquidity trap ($\xi < 1$) or not ($\xi = 1$), and similarly whether Foreign is in a liquidity trap ($\xi^* < 1$) or not ($\xi^* = 1$). This implies that we now have

$$E_t = \hat{E}_t^{\frac{1}{\sigma}}$$

where $\hat{E}_t$ is a renormalized exchange rate given by

$$\hat{E}_t = \frac{\xi_t}{\xi_t^*}. \tag{A.7}$$

The analysis under financial autarky is identical to the case $\sigma = 1$ except for the value of the financial autarky exchange rate. Under financial integration and outside the liquidity trap, the analysis is also unchanged compared to the case $\sigma = 1$: from Eq. (A.7), $\xi = \xi^* = 1$ implies $E = 1$. Since the asset demands and asset supplies are unchanged, so is the equilibrium. This obtains as long as $r^{w,n} \geq 0$ where $r^{w,n}$ is defined in Eq. (15).

Consider now the case of a global liquidity trap ($r^{w,n} < 0$). In that case, as before, we can index the solution by the exchange rate E (or equivalently $\hat{E}$). Using Eqs. (5) and (7), we can express the world supply of assets as:

$$\begin{aligned}
V^w + D^w &= \frac{\delta y \xi + \delta^*(1-y)E\xi^*}{\rho}\bar{Y}^w + \bar{d}(E)\bar{Y}^w \\
&= \left[\frac{\hat{\delta}(E)}{\rho}\xi P^{1-\sigma} + \bar{d}(E)\right]\bar{Y}^w.
\end{aligned}$$

where $\hat{\delta}(E) = (\delta y + \delta^*(1-y)E^{1-\sigma})/P^{1-\sigma}$ is a weighted average of δ using the relative price of H and F goods as

[26] One can readily check that the range $\left[\underline{E}, \bar{E}\right]$ increases with β, so that the model with home bias admits a larger range of indeterminacy. In the limit of full home bias, any value of the exchange rate is admissible.

weights. From Eq. (4) we can write asset demand along a BGP as:

$$W^w = \xi P^{1-\sigma} \frac{\bar{Y}^w}{\theta}.$$

Putting the two together, we can solve for the Home recession ξ:

$$\xi = \frac{\theta \bar{d}(E)}{(1 - \frac{\theta \hat{\delta}(E)}{\rho}) P^{1-\sigma}}.$$

Compared to the case of $\sigma = 1$, a change in the exchange rate now has three effects on the recession at Home ξ. First, as before, it increases the supply safe 'public' assets $\bar{d}(E)$. This increases output. In addition, an increase in E changes the value of private assets, captured by $\hat{\delta}(E)$. Finally, it increases the price level P, which affects both private asset supply and private asset demand. Formally, we can write:

$$\frac{\partial \xi}{\partial E} = \frac{\theta (1-y)}{\left(1 - \frac{\theta \hat{\delta}(E)}{\rho}\right) P^{1-\sigma}} \left[d^* - \frac{\bar{d}(E)}{\left(1 - \frac{\theta \hat{\delta}(E)}{\rho}\right) P^{1-\sigma}} (1-\sigma) E^{-\sigma} \left(1 - \frac{\delta^* \theta}{\rho}\right) \right].$$

The first term in bracket corresponds to the expansion of public assets. The second term is the net effect of the expansion of private assets and the decline in asset demands. When $\sigma > 1$, this second effect is also positive: output becomes more responsive to the exchange rate as goods become more substitutable.

A.4.2 The limit $\sigma \to \infty$

Things simplify in the limit $\sigma \to \infty$, where the goods become perfect substitutes, to which we now turn. For conciseness, we only treat the case where there is a global liquidity trap under financial integration. As we take the limit $\sigma \to \infty$, we have $E = P = 1$, but there is still a degree of indeterminacy indexed by the renormalized exchange rate $\hat{E}$. Indeed, we can compute all the equilibrium variables as a function of $\hat{E}$:

$$\xi = \frac{\theta \bar{d}}{y(1 - \frac{\delta \theta}{\rho}) + \frac{1}{\hat{E}}(1-y)(1 - \frac{\delta^* \theta}{\rho})}, \tag{A.8a}$$

$$\xi^* = \frac{\theta \bar{d}}{y(1 - \frac{\delta \theta}{\rho})\hat{E} + (1-y)(1 - \frac{\delta^* \theta}{\rho})}, \tag{A.8b}$$

$$\frac{NFA}{\bar{Y}} = \frac{(1 - \frac{\delta \theta}{\rho})\xi - \theta d}{g + \theta} = \frac{(1 - \frac{\delta \theta}{\rho})(\xi - \xi^{a,l})}{g + \theta}, \tag{A.8c}$$

$$\frac{CA}{\bar{Y}} = g \frac{NFA}{\bar{Y}}, \tag{A.8d}$$

where $r^{w,n}$ and $\xi^{a,l}$ are defined exactly as in the unitary elasticity case, and are given by the same formulas. The first two equations correspond to Eq. (32). Home output ξ is increasing in the renormalized exchange rate $\hat{E}$, and foreign output is decreasing in the renormalized exchange rate $\hat{E}$. Finally, the home Net Foreign Asset Position is increasing in the gap between home output and home financial autarky output $\xi^{a,l}$ under zero home nominal interest rates. *The key difference introduced by $\sigma > 1$ over $\sigma = 1$ is that home and foreign outputs ξ and ξ^* become more responsive to the exchange rate E.* Indeed in the limit $\sigma \to \infty$, ξ and ξ^* become infinitely sensitive to the exchange rate E. In other words, larger trade elasticities magnify the stimulative effect of an exchange rate depreciation on the home recession.

Assume further that both countries are in a liquidity trap under financial autarky. Then, just like in the case with $\sigma = 1$, the financially integrated equilibrium coincides with financial autarky when $\hat{E} = \hat{E}^a$. For $\hat{E} > \hat{E}^a$, we have $\xi > \xi^a$ and $\xi^* < \xi^a$ and vice versa for $\hat{E} < \hat{E}^a$.

A.4.3 Currency Wars in the limit $\sigma \to \infty$

Let's now consider the Nash equilibrium of the Currency Wars game of Section 3.1, where each central bank maximizes consumption $\theta W/P$ net of the cost $\mathcal{C}(a)$. The first-order conditions are:

$$\xi \frac{\frac{1}{\hat{E}}(1-y)(1-\frac{\delta^*\theta}{\rho})}{y(1-\frac{\delta\theta}{\rho})+\frac{1}{\hat{E}}(1-y)(1-\frac{\delta^*\theta}{\rho})} \frac{\theta(1+\frac{g\delta}{\rho})}{g+\theta} \frac{\mathfrak{n}}{a} = \mathcal{C}'(a), \tag{A.9a}$$

$$\xi^* \frac{y(1-\frac{\delta\theta}{\rho})}{y(1-\frac{\delta\theta}{\rho})+\frac{1}{\hat{E}}(1-y)(1-\frac{\delta^*\theta}{\rho})} \frac{\theta(1+\frac{g\delta^*}{\rho})}{g+\theta} \frac{\mathfrak{n}}{a^*} = \mathcal{C}'\left(a^*\right). \tag{A.9b}$$

The second order conditions are satisfied as long as $\mathfrak{n} < 1$. From the first order conditions, it is immediate that the Nash equilibrium satisfies $a, a^* > 1$ and we can also easily check that $\partial a/\partial d > 0$ and $\partial a^*/\partial d > 0$ so that higher public debt in either country increases the incentives to intervene to depreciate the currency.

Further, define $x = y(1-\theta\delta/\rho)$ and $x^* = (1-y)(1-\theta\delta^*/\rho)/\hat{E}$. The slope of the Home best-response is given by:

$$\left[(x+x^*)(1+\eta_c)+\mathfrak{n}(x-x^*)\right]\frac{da}{a} = \mathfrak{n}(x-x^*)\frac{da^*}{a^*},$$

while the slope of the foreign best response is given by

$$(x^*-x)\mathfrak{n}\frac{da}{a} = \left[(x+x^*)(1+\eta_c)+\mathfrak{n}(x^*-x)\right]\frac{da^*}{a^*}.$$

When $\hat{E} < \hat{E}^o \equiv ((1-y)(1-\theta\delta^*/\rho))/(y(1-\theta\delta/\rho))$, the actions are strategic substitutes, while when $\hat{E} > \hat{E}^o$, they are strategic complements. Either way, the equilibrium is asymptotically stable in the sense of Fudenberg and Tirole when

$$\left|\frac{da}{da^*}\right|_A < \left|\frac{da}{da^*}\right|_{A^*}.$$

This condition boils down to

$$\mathfrak{n} < \frac{1+\eta_c}{\sqrt{2}}\frac{\hat{E}+\hat{E}^o}{\hat{E}^o-\hat{E}} \equiv \mathcal{H}(\hat{E}).$$

It is easy to check that the function $\mathcal{H}(\hat{E})$ is increasing. Its lowest value will be reached at the lower bound $\underline{\hat{E}}$ defined such that $\xi^* = 1$:

$$\underline{\hat{E}} = \frac{\theta\bar{d}-(1-y)(1-\frac{\theta\delta^*}{\rho})}{y(1-\frac{\theta\delta}{\rho})},$$

and the condition for an asymptotically stable Nash equilibrium is always satisfied if

$$\mathfrak{n} < \mathcal{H}(\underline{\hat{E}}) = \frac{1+\eta_c}{\sqrt{2}}\frac{\theta\bar{d}}{2(1-y)(1-\frac{\theta\delta^*}{\rho})-\theta\bar{d}}.$$

A sufficient condition is

$$2(1-y)(1-\frac{\theta\delta^*}{\rho}) > \theta\bar{d} > \frac{2(1-y)(1-\frac{\theta\delta^*}{\rho})}{1+\frac{1+\eta_c}{\sqrt{2}}}.$$

We assume that this condition is satisfied.

A.5 Tariffs Wars

A.5.1 Cobb-Douglas Preferences

We first allow Home to impose an ad-valorem tariff λ on imports from Foreign, and conversely allow Foreign to impose an ad-valorem tariff λ^* on imports from Home. We consider first the case of Cobb Douglas preferences (unit elasticity of substitution). Home now faces import prices $P_{F,t} = E_t P^*_{F,t}(1+\lambda)$ while Foreign faces import prices (in foreign currency) $P^*_{H,t} = P_{H,t}(1+\lambda^*)/E_t$. As before, we normalize $P_{H,t} = P^*_{F,t} = 1$.

Home collects per unit tariff revenues $\Lambda_t = \lambda E_t P^*_{F,t} c_{F,t}$, whereas Foreign collects tariff revenues $\Lambda^*_t = \lambda^*(P_{H,t}/E_t)c^*_{H,t}$. We assume tariff proceeds are instantaneously rebated lump-sum to the consuming households. The Cobb-Douglas aggregate implies consumers in both countries spend a share γ of their accumulated wealth W plus the tariff rebates Λ on the Home good and a share $(1-\gamma)$ on the foreign good (consumers do not internalize the lump sum tariff rebate):

$$c_H = \gamma\frac{\theta W + \Lambda}{P_H},\ c_F = (1-\gamma)\frac{\theta W + \Lambda}{EP^*_F(1+\lambda)}.$$

Since Λ is the revenue raised by the tariff, we obtain

$$\Lambda = \lambda E P^*_F c_F = \lambda E P^*_F (1-\gamma)\frac{\theta W + \Lambda}{EP^*_F(1+\lambda)}.$$

Solving for Λ yields

$$\Lambda = \frac{\lambda(1-\gamma)}{1+\lambda\gamma}\theta W.$$

which implies total wealth spent is

$$\theta W + \Lambda = \frac{1+\lambda}{1+\gamma\lambda}\theta W.$$

This yields the demand functions

$$c_H = \frac{\gamma(1+\lambda)}{1+\gamma\lambda}\frac{\theta W}{P_H},\ c_F = \frac{(1-\gamma)(1+\lambda)}{1+\gamma\lambda}\frac{\theta W}{P_F(1+\lambda)}.$$

The imposition of the unilateral tariff shifts Home's expenditure share on the Home good from γ to $\tilde{\gamma} = \gamma(1+\lambda)/(1+\gamma\lambda) > \gamma$ and Home's expenditure share on Foreign goods to $1-\tilde{\gamma} = (1-\gamma)(1+\lambda)/(1+\gamma\lambda) < 1-\gamma$. In other words, the model now features home bias, with a degree of home bias β, from Section A.3, given by $\beta = \gamma\lambda/(1+\gamma\lambda)$. Similar derivations for Foreign yield:

$$c^*_H = \frac{\gamma}{1+\lambda^*(1-\gamma)}\frac{\theta W^*}{E^{-1}},\ c^*_F = \frac{(1-\gamma)(1+\lambda^*)}{1+\lambda^*(1-\gamma)}\theta W^*,$$

so that Foreign also exhibits home bias, with an expenditure share on Home goods given by $\tilde{\gamma}^* = \gamma/(1+\lambda^*(1-\gamma) < \gamma$ and an expenditure share on Foreign goods $\tilde{\gamma^*} = (1-\gamma)(1+\lambda^*)/(1+\lambda^*(1-\gamma) > 1-\gamma$. It follows that the degree of home bias in foreign is $\beta^* = (1-\gamma)\lambda^*/(1+\lambda^*(1-\gamma))$. Unlike Section A.3, the degree of home bias is asymmetric and depends on the level of the tariff set by each country.

Under financial autarky, along a BGP, $\xi^a\bar{Y} = \theta W = \theta(\delta\xi^a\bar{Y}/(r^a+\rho) + d\bar{Y})$ where the second equality obtains from $W = V + D$. it follows that the autarky output and interest rate are as in Proposition 1. It follows that the only effect of the tariffs is to force an adjustment of the autarky exchange rate E^a (Lerner symmetry). Substituting $W = \xi^a\bar{Y}/\theta$ and $W^* = \xi^{a,*}\bar{Y}^*/\theta$ into the goods market clearing condition Eq. (36), one obtains Eq. (37):

$$E^a = \frac{\xi^a}{\xi^{a,*}}\frac{1+\lambda^*(1-y)}{1+\lambda y}.$$

A.5.2 Tariff Wars in the limit $\sigma \to \infty$

We now consider the case where the elasticity of substitution is not unitary and preferences are given by Eq. (A.5). The Home demand bloc satisfies (recall that $p_H = p_F^* = 1$):

$$c_H = \gamma C \left(\frac{1}{P}\right)^{-\sigma},$$
$$c_F = (1-\gamma) C \left(\frac{E(1+\lambda)}{P}\right)^{-\sigma},$$
$$PC = \theta W + \Lambda.$$

The revenues generated by the tariff satisfy

$$\Lambda = \lambda E c_F = \frac{\lambda}{1+\lambda}(1-\gamma)(\theta W + \Lambda)\left(\frac{E(1+\lambda)}{P}\right)^{1-\sigma},$$

from which we can recover Λ:

$$\Lambda = \frac{\lambda(1-\gamma)(E(1+\lambda))^{1-\sigma}}{(1+\lambda)\gamma + (1-\gamma)(E(1+\lambda))^{1-\sigma}}\theta W.$$

Substituting, into the demand bloc, we can recover the Home demands for the Home and Foreign goods:

$$c_H = \gamma\theta\frac{W}{\tilde{P}}\left(\frac{1}{\tilde{P}}\right)^{-\sigma}$$
$$c_F = (1-\gamma)(1+\lambda)^{-\sigma}\theta\frac{W}{\tilde{P}}\left(\frac{E}{\tilde{P}}\right)^{-\sigma},$$

where $\tilde{P} = (\gamma + (1-\gamma)(1+\lambda)^{-\sigma}E^{1-\sigma})^{1/(1-\sigma)}$. The imposition of the unilateral tariff in Home is equivalent to a shift in Home preference weight on Foreign goods from $1-\gamma$ to $(1-\gamma)/(1+\lambda)^{\sigma} < (1-\gamma)$.

Similar derivations for Foreign yield:

$$c_H^* = \gamma(1+\lambda^*)^{-\sigma}\theta\frac{W^*}{\tilde{P}^*}\left(\frac{1/E}{\tilde{P}^*}\right)^{-\sigma}$$
$$c_F^* = (1-\gamma)\theta\frac{W^*}{\tilde{P}^*}\left(\frac{1}{\tilde{P}^*}\right)^{-\sigma},$$

where $\tilde{P}^* = (\gamma(1+\lambda^*)^{-\sigma}E^{\sigma-1} + (1-\gamma))^{1/(1-\sigma)}$. The imposition of the unilateral tariff in Foreign is equivalent to a shift in Foreign preference weigh on Home goods from γ to $\gamma/(1+\lambda^*)^{\sigma} < \gamma$.

Goods market clearing requires

$$\gamma\theta\left[W\frac{1}{\gamma + (1-\gamma)(1+\lambda)^{-\sigma}E^{1-\sigma}} + EW^*\frac{1}{\gamma + (1-\gamma)(1+\lambda^*)^{\sigma}E^{1-\sigma}}\right] = \xi\bar{Y} \tag{A.10a}$$

$$(1-\gamma)\theta\left[W\frac{1}{\gamma(1+\lambda)^{\sigma}E^{\sigma-1} + (1-\gamma)} + EW^*\frac{1}{\gamma(1+\lambda^*)^{-\sigma}E^{\sigma-1} + (1-\gamma)}\right] = E\xi^*\bar{Y}^* \tag{A.10b}$$

Because tariff revenues are rebated lump sum to households, all remaining equilibrium equations are unchanged: wealth accumulation, asset pricing and government debt dynamics.

Under financial autarky, the autarky interest rate and output r^a and ξ^a are determined in asset markets and independent from the tariffs, as in Section A.5.1. It follows that Lerner symmetry still obtains: the exchange rate adjusts to offset the effect of the tariffs. Substituting $W = \xi^a\bar{Y}/\theta$ and $W^* = \xi^{a,*}\bar{Y}^*/\theta$ into the goods market clearing condition, the autarky exchange rate is defined implicitly by

$$E^{\sigma} = \frac{\xi^a}{\xi^{a,*}}\left(\frac{1+\lambda^*}{1+\lambda}\right)^{\sigma}\left(\frac{E\tilde{P}^*}{\tilde{P}}\right)^{1-\sigma}.$$

Under financial integration, we can express the wealth to output ratio from the wealth accumulation equation, using the asset pricing and government debt dynamics equation we obtain, both outside and at the ZLB:

$$(g+\theta-r^w)\frac{W}{\bar{Y}} = \left(1-\frac{\delta}{r^w+\rho}(r^w-g)\right)\xi + (g-r^w)d \tag{A.11a}$$

$$(g+\theta-r^w)\frac{W^*}{\bar{Y}^*} = \left(1-\frac{\delta^*}{r^w+\rho}(r^w-g)\right)\xi^* + (g-r^w)d^* \tag{A.11b}$$

Substituting into the good market clearing equations, we obtain:

$$\theta\left[\gamma\frac{\left(1-\frac{\delta}{r^w+\rho}(r^w-g)\right)\xi+(g-r^w)d}{\gamma+(1-\gamma)(1+\lambda)^{-\sigma}E^{1-\sigma}} + E(1-\gamma)\frac{\left(1-\frac{\delta^*}{r^w+\rho}(r^w-g)\right)\xi^*+(g-r^w)d^*}{\gamma+(1-\gamma)(1+\lambda^*)^{\sigma}E^{1-\sigma}}\right] = (g+\theta-r^w)\xi$$

$$\theta\left[\gamma\frac{\left(1-\frac{\delta}{r^w+\rho}(r^w-g)\right)\xi+(g-r^w)d}{\gamma(1+\lambda)^{\sigma}E^{\sigma-1}+(1-\gamma)} + E(1-\gamma)\frac{\left(1-\frac{\delta^*}{r^w+\rho}(r^w-g)\right)\xi^*+(g-r^w)d^*}{\gamma(1+\lambda^*)^{-\sigma}E^{\sigma-1}+(1-\gamma)}\right] = (g+\theta-r^w)E\xi^*$$

Outside of the ZLB, $\xi=\xi^*=1$, and the system of two equations can be solved for the global real rate r^w and the exchange rate E. While output is unaffected by tariffs, the Lerner symmetry breaks down, as in the case $\sigma=1$: a change in tariff forces an adjustment of the exchange rate. But the change in exchange rate also affects the global supply of assets and requires an adjustment in the global real rate r^w.

At the ZLB, $r^w=0$ and this system of equations can be solved for domestic and foreign output ξ and ξ^* for a given admissible exchange rate.

The solution takes the following form:

$$\xi = \frac{\theta}{1-\tilde{\delta}}\left(\frac{\gamma}{\kappa\kappa^*}\frac{\kappa^*+\alpha^*(1+\lambda)^{-\sigma}E^{1-\sigma}}{\tilde{P}^{1-\sigma}}d+\frac{1-\gamma}{\kappa\kappa^*}\frac{\kappa^*(1+\lambda^*)^{-\sigma}E^{\sigma-1}+\alpha^*}{\tilde{P}^{*1-\sigma}}Ed^*\right) \tag{A.12a}$$

$$E\xi^* = \frac{\theta}{1-\tilde{\delta}}\left(\frac{\gamma}{\kappa\kappa^*}\frac{\alpha+\kappa(1+\lambda)^{-\sigma}E^{1-\sigma}}{\tilde{P}^{1-\sigma}}d+\frac{1-\gamma}{\kappa\kappa^*}\frac{\alpha(1+\lambda^*)^{-\sigma}E^{\sigma-1}+\kappa}{\tilde{P}^{*1-\sigma}}Ed^*\right), \tag{A.12b}$$

where

$$\begin{aligned}
\alpha &= \frac{\gamma(1+\lambda)^{-\sigma}E^{1-\sigma}}{\tilde{P}^{1-\sigma}}\left(\frac{\theta}{g}+\frac{\theta\delta}{\rho}\right)\\
\alpha^* &= \frac{(1-\gamma)(1+\lambda^*)^{-\sigma}E^{\sigma-1}}{\tilde{P}^{*1-\sigma}}\left(\frac{\theta}{g}+\frac{\theta\delta^*}{\rho}\right)\\
\kappa &= \left(1+\frac{\theta}{g}\right)\frac{(1-\gamma)(1+\lambda)^{-\sigma}E^{1-\sigma}}{\tilde{P}^{1-\sigma}}+\frac{\gamma}{\tilde{P}^{1-\sigma}}\left(1-\frac{\theta\delta}{\rho}\right)\\
\kappa^* &= \left(1+\frac{\theta}{g}\right)\frac{\gamma(1+\lambda^*)^{-\sigma}E^{\sigma-1}}{\tilde{P}^{*1-\sigma}}+\frac{1-\gamma}{\tilde{P}^{*1-\sigma}}\left(1-\frac{\theta\delta^*}{\rho}\right)\\
\tilde{\delta} &= \frac{\alpha\alpha^*}{\kappa\kappa^*}
\end{aligned}$$

In the limit where the elasticity of substitution tends to infinity, Home and Foreign goods become perfect substitute and there is no possibility to manipulate the terms of trade. In that limit, $E=P=P^*=1$ and $\lambda=\lambda^*=0$. We consider the following change of variable $\hat{E}=E^{\sigma}$ and $\hat{\lambda}=\exp(\lambda\sigma)$. We can interpret $\hat{\lambda}$ and $\hat{\lambda}^*$ as infinitesimal attempts at manipulating tariffs in the perfectly substitutable limit.

Welfare $\mathcal{U}$ equals real consumption. Outside or at the ZLB, Home welfare is given by:

$$\begin{aligned}
\mathcal{U} &= \frac{\theta W + \Lambda}{P} \\
&= \frac{\theta W}{P}\left(\frac{P}{\tilde{P}}\right)^{1-\sigma} \\
&= \theta \frac{\left(1 - \frac{\delta}{r^w+\rho}(r^w - g)\right)\xi + (g - r^w)d}{(g+\theta-r^w)P} \frac{\gamma + (1-\gamma)E^{1-\sigma}}{\gamma + (1-\gamma)(1+\lambda)^{-\sigma}E^{1-\sigma}}\bar{Y}
\end{aligned}$$

In the limit of perfect substitution, this simplifies to

$$\mathcal{U} = \theta \frac{\left(1 - \frac{\delta}{r^w+\rho}(r^w - g)\right)\xi + (g - r^w)d}{(g+\theta-r^w)}\bar{Y}$$

Outside the ZLB, $\xi = 1$, $E = 1$, and the global real rate is given by the usual formula $r^w = \theta\bar{\delta}/(1-\theta\bar{d})$. It follows that welfare is independent from the tariff. If there is even an infinitesimal cost of administering the tariff, the optimal tariff is $\hat{\lambda} = \hat{\lambda}^* = 0$.

Consider now the ZLB, where $r^w = 0$. Inspecting the expression for $\mathcal{U}$, it is immediate that *maximizing welfare coincides with maximizing output* ξ. Taking the limit $\sigma \to \infty$, the system Eq. (A.12) simplifies to:

$$\begin{aligned}
\alpha &= \frac{\gamma E^{-\sigma}/\hat{\lambda}}{\gamma + (1-\gamma)E^{-\sigma}/\hat{\lambda}}\left(\frac{\theta}{g} + \frac{\theta\delta}{\rho}\right) \\
\alpha^* &= \frac{(1-\gamma)E^{\sigma}/\hat{\lambda}^*}{\gamma E^{\sigma}/\hat{\lambda}^* + (1-\gamma)}\left(\frac{\theta}{g} + \frac{\theta\delta^*}{\rho}\right) \\
\kappa &= \frac{1}{\gamma + (1-\gamma)E^{-\sigma}/\hat{\lambda}}\left[\left(1+\frac{\theta}{g}\right)(1-\gamma)E^{-\sigma}/\hat{\lambda} + \gamma\left(1 - \frac{\theta\delta}{\rho}\right)\right] \\
\kappa^* &= \frac{1}{\gamma E^{\sigma}/\hat{\lambda}^* + (1-\gamma)}\left[\left(1+\frac{\theta}{g}\right)\gamma E^{\sigma}/\hat{\lambda}^* + (1-\gamma)\left(1 - \frac{\theta\delta^*}{\rho}\right)\right]
\end{aligned}$$

and Eq. (A.12) collapses to Eq. (41).

Lastly, to evaluate $\partial_{\hat{\lambda}}\xi$, observe that

$$\begin{aligned}
\partial_{\hat{\lambda}}\alpha^* = \partial_{\hat{\lambda}}\kappa^* &= 0 \\
\partial_{\hat{\lambda}}\alpha &< 0 \\
\partial_{\hat{\lambda}}\kappa &< 0 \\
\partial_{\hat{\lambda}}\frac{\kappa^*\hat{\lambda}\hat{E} + \alpha^*}{\gamma\hat{\lambda}\hat{E} + (1-\gamma)} &> 0 \\
\partial_{\hat{\lambda}}\kappa(1-\tilde{\delta}) &< 0,
\end{aligned}$$

from which it follows that $\partial_{\hat{\lambda}}\xi > 0$. To evaluate the effect of the Home tariff on Foreign output, observe that we can manipulate Eq. (A.11) and the condition $\theta W^w = \xi\bar{Y} + E\xi^*\bar{Y}^*$ (global expenditures equals global output) to obtain

$$E = \frac{\gamma}{1-\gamma}\frac{1 - \frac{\theta\delta}{\rho}}{1 - \frac{\theta\delta^*}{\rho}}\frac{\xi - \xi^a}{\xi^{a*} - \xi^*}.$$

With perfect substitution, this simplifies to

$$(1-\gamma)\left(1 - \frac{\theta\delta^*}{\rho}\right)(\xi^{a*} - \xi^*) = \gamma\left(1 - \frac{\theta\delta}{\rho}\right)(\xi - \xi^a),$$

so that ξ^* must decrease if ξ increases.

Given $\hat{\lambda}^*$, Home's best response is to set the highest possible tariff, which increases Home output at the expense of Foreign. As each country attempts to increase its tariffs, the equilibrium converges to autarky, $\xi \to \xi^a$, $\xi^* = \xi^{a,*}$.

A.6 Within Country Heterogeneity: Borrowers and Savers

In this extension, we present a version of our model incorporating within country heterogeneity between borrowers and savers. For simplicity, we abstract away from public debt by setting $D = D^* = 0$.

We add a mass of borrowing constrained impatient borrowers (B) agents. The rest of the agents are savers (S) and are modeled as before. Borrowers consume as much as possible when they are born, and the rest when they die. They only get an endowment when they die, and they can only pledge a part of it. They must therefore borrow in order to consume when born. They then roll over their debt until they die, at which point they use their income to repay their debt and consume the remainder. In a small interval dt, a part $\eta\xi_t\bar{Y}_t dt$ of total income accrues to dying borrowers in the form of labor income. Because of the borrowing constraint, borrowers born in the interval dt can only consume $\chi\bar{Y}_t dt$, where we imagine that χ is small compared to η.[27] We assume that the new trees accrue to savers.

Note that there is now a distinction between financial wealth and human wealth for borrowers. Indeed a borrower receives income when he dies. This future income is a form a human wealth and is not part of his financial wealth. When the borrower dies, this human wealth allows him to repay the debt that he has incurred to consume when he was born and rolled over until his death (his financial wealth), and to consume the residual.

The evolution equations for the financial wealth of borrowers and savers are given by:[28,29]

$$\begin{aligned}
gW^B &= -\theta W^B - \chi\bar{Y} + r^w W^B, \\
gW^S &= -\theta W^S + (1-\delta-\eta)\xi\bar{Y} + r^w W^S + (\rho+g)V, \\
gW^{B*} &= -\theta W^{B*} - \chi^*\bar{Y}^* + r^w W^{B*}, \\
gW^{S*} &= -\theta W^{S*} + (1-\delta-\eta)\xi^*\bar{Y}^* + r^w W^{S*} + (\rho+g)V^*.
\end{aligned}$$

We continue to denote by $W = W^B + W^S$ total home wealth and by $W^* = W^{B*} + W^{S*}$ total foreign wealth and obtain the evolution equations for total wealth by aggregating the evolution equations for wealth by borrowers and savers in both countries:

$$\begin{aligned}
gW &= -\theta W + (1-\delta-\eta)\xi\bar{Y} - \chi\bar{Y} + r^w W + (\rho+g)V, \\
gW^* &= -\theta W^* + (1-\delta^*-\eta^*)\xi^*\bar{Y}^* - \chi^*\bar{Y} + r^w W + (\rho+g)V^*.
\end{aligned}$$

The good market clearing conditions are now given by:[30]

$$\begin{aligned}
\xi\bar{Y} &= y(\theta(W+EW^*) + \eta\xi\bar{Y} + E\eta^*\xi^*\bar{Y}^* + \chi\bar{Y} + E\chi^*\bar{Y}^*) \\
E\xi^*\bar{Y}^* &= (1-y)(\theta(W+EW^*) + \eta\xi\bar{Y} + E\eta^*\xi^*\bar{Y}^* + \chi\bar{Y} + E\chi^*\bar{Y}^*).
\end{aligned}$$

[27] Note that, as in Guerrieri and Lorenzoni (2011) and Eggertsson and Krugman (2012), the borrowing limit $\chi\bar{Y}$ does not depend on whether the economy is in recession. This assumption is crucial to generate a liquidity trap, as it implies that the debt issued by borrowers does not scale with output, so asset demand declines faster than asset supply in the recession. While the assumption that the credit constraint is invariant to the recession is perhaps extreme, all that is needed for our result to go through is that the borrowing limit does not scale one for one with output.

[28] Let us for example explain in details the wealth evolution equation for borrowers at Home. The wealth of borrowers is negative $W^B < 0$, it represents their debt. In an interval dt, the wealth of borrowers W^B changes because of because of dying borrowers repaying their debt ($-\theta W^B dt$), because of newborn borrowers taking on new debt ($-\chi\bar{Y}dt$), and because of the accumulation of interest ($r^w W^B$). In a steady state, the wealth of borrowers W^B must also change by $gW^B dt$. This gives the wealth evolution equation for borrowers.

[29] Note that the wealth of borrowers does not take into account the income of borrowers when they die, because it is not part of their financial wealth. But of course, the income of borrowers influences their consumption when they die. It therefore appears in the goods market clearing conditions. This explains the terms $\eta\xi\bar{Y}$ and $E\eta^*\xi^*\bar{Y}^*$ in the market clearing conditions at Home and in Foreign.

[30] For example, the home market clearing condition can be understood as follows. The demand arising from dying savers is given by $y\theta(W - W^B + EW^* - EW^{B*})$. The demand arising from newborn borrowers is given by $y(\chi X + E\chi^*\bar{Y}^*)$. And the demand arising from dying borrowers is given by $y(\eta\xi\bar{Y} + \theta W^B + E\eta^*\xi^*\bar{Y}^* + E\theta W^{B*})$.

The asset pricing equations are unchanged:

$$
\begin{aligned}
r^w V &= -\rho V + \delta \xi \bar{Y}, \\
r^w V^* &= -\rho V^* + \delta^* \xi^* \bar{Y}^*.
\end{aligned}
$$

And we must still impose $r^w \geq 0$, $0 \leq \xi \leq 1$, $0 \leq \xi^* \leq 1$, and the complementary slackness conditions $r^w(1-\xi) = 0$ and $r^w(1-\xi^*) = 0$.

In the interest of space, we only treat the liquidity trap case. We get:

$$
\begin{aligned}
E &= \frac{\xi}{\xi^*}, \\
\xi &= \frac{\bar{\chi}(E)}{1 - \bar{\eta} - \frac{\bar{\delta}\theta}{\rho}}, \\
\frac{NFA}{\bar{Y}} &= \frac{\frac{1-\eta-\frac{\delta\theta}{\rho}}{1-\bar{\eta}-\frac{\bar{\delta}\theta}{\rho}}\bar{\chi}(E) - \chi}{g+\theta}, \\
\frac{CA}{\bar{Y}} &= g\frac{NFA}{\bar{Y}}.
\end{aligned}
$$

where for any variable z, we use the notation $\bar{z}(E) = yz(E) + (1-y)z^*(E)$.

The variable χ (χ^*) increases with home (foreign) financial development, and decreases with a home (foreign) deleveraging shock. Identifying the borrowers with the young and the savers with the middle-aged and the old, proportional decreases in the variables η and χ (η^* and χ^*) capture home (foreign) population aging.

These equations indicate that a deleveraging shock at Home (a decrease in χ) or in Foreign (a decrease in χ^*) can push the global economy into a liquidity trap. For a given exchange rate E, the larger the world deleveraging shock, the larger the recession in any given country. For a given exchange rate E and world deleveraging shock $\bar{\chi}(E)$, a larger home deleveraging shock (a lower χ) pushes the home Current Account towards a surplus.

Similarly, aging at Home (a proportional decrease in χ and η) or in Foreign (a proportional decrease in χ^* and η^*) can push the global economy in a liquidity trap. For a given exchange rate E, the larger the shock, the larger the recession in any given country. For a given exchange rate E, more aging at Home pushes the Home Current Account towards a surplus.

This analysis also shows how countries with tighter credit constraints or lower fraction of income accruing to borrowers act as if they had a larger asset demand (lower θ).

B Online Appendix: Not For Publication

B.1 Derivations for the Model with Inflation in Appendix A.2

Global liquidity trap equilibrium equations. In a global liquidity trap equilibrium, the equilibrium values of $V^w = V + SV^*$, $W^w = W + SW^*$ (expressed in terms of the home good numeraire) and π_H, π_F^*, S, ξ, and ξ^* solve the following system of equations

$$\begin{aligned}
S &= \frac{\xi}{\xi^*},\\
\theta W^w &= \xi\bar{Y} + S\xi^*\bar{Y}^*\\
-\pi_H V^w &= -\rho V^w + \delta\xi\bar{Y} + \delta^* S\xi^*\bar{Y}^*,\\
gW^w &= -\theta W^w + (1-\delta)\xi\bar{Y} + (1-\delta^*)S\xi^*\bar{Y}^* + gD^w - \pi_H W^w + (\rho+g)V^w,\\
\pi_H &= -\kappa_0 - \kappa_1(1-\xi), && \text{(B.1)}\\
\pi_F^* &= -\kappa_0^* - \kappa_1^*(1-\xi^*) && \text{(B.2)}\\
\pi_F^* &= \pi_H, && \text{(B.3)}
\end{aligned}$$

where $D^w = D + SD^*$. The first equation is the equation for the terms of trade. The second equation is the equation for total world wealth. Both result directly from combining the home and foreign goods market clearing conditions. The third equation is the asset pricing equation for world private assets. The fourth equation is the accumulation equation for world wealth, where we have used the government budget constraints to replace taxes as a function of public debt $\tau(1-\delta)\xi\bar{Y} = -gD$ and $\tau^*(1-\delta)\xi^*\bar{Y}^* = -gD^*$. The fifth and sixth equations are the home and foreign Phillips curves. The seventh equation represents the requirement derived above that the terms of trade be constant.

Asymmetric liquidity trap equilibrium equations. In an asymmetric liquidity trap equilibrium where one country (say Home) is in a liquidity trap but not the other (say Foreign), the equilibrium equations are instead given by:

$$\begin{aligned}
S &= \xi,\\
\theta W^w &= \xi\bar{Y} + S\bar{Y}^*\\
-\pi_H V^w &= -\rho V^w + \delta\xi\bar{Y} + \delta^* S\bar{Y}^*,\\
gW^w &= -\theta W^w + (1-\delta)\xi\bar{Y} + (1-\delta^*)S\bar{Y}^* + gD^w - \pi_H W^w + (\rho+g)V^w,\\
\pi_H &= -\kappa_0 - \kappa_1(1-\xi),
\end{aligned}$$

and we have $i = 0$, $i^* = \bar{\pi}^* - \pi_H = i - \dot{E}_t/E_t > 0$.

Net Foreign Assets, Current Accounts, and Metzler Diagram in quantities. In this section, we characterize Net Foreign Asset positions and Current Accounts in the model with inflation of Appendix A.2. We express these quantities in real terms in the home good numeraire. In the no liquidity trap equilibrium, these quantities are given by exactly the same formula as in the case with no inflation. In a symmetric global liquidity trap equilibrium, or in an asymmetric liquidity trap equilibrium, we have

$$\begin{aligned}
\frac{NFA}{\bar{Y}} &= \frac{W-(V+D)}{\bar{Y}} = \frac{\xi(1-\frac{\delta\theta}{r+\rho}) - \theta d}{g+\theta-r},\\
\frac{CA}{\bar{Y}} &= g\frac{NFA}{\bar{Y}},
\end{aligned}$$

where $\xi < 1$ and $r = -\pi_H$ if Home is in a liquidity trap and $\xi = 1$ and $r = -\pi_F^*$ if Home is not in a liquidity trap (but Foreign is).

The same forces that we identified in the model with no inflation are at play. For example, in a symmetric global liquidity trap equilibrium when the Phillips curves are identical across countries (so that $\kappa_0^* = \kappa_0$ and $\kappa_1^* = \kappa_1$),

$$\begin{aligned}
\frac{NFA}{\bar{Y}} &= \frac{W-(V+D)}{\bar{Y}} = \frac{(1-\frac{\delta\theta}{\rho}-\frac{\pi^w}{\rho})[\frac{\theta\bar{d}}{1-\frac{\delta\theta}{\rho}-\pi^w} - \frac{\theta d}{1-\frac{\delta\theta}{\rho}-\frac{\pi^w}{\rho}}]}{g+\theta+\pi^w}, \\
\frac{CA}{\bar{Y}} &= g\frac{NFA}{\bar{Y}}.
\end{aligned}$$

Hence to the extent that Home has a higher financial capacity than Foreign $\delta > \delta^*$, or a higher public debt ratio than Foreign $d > d^*$, then Home runs a negative Net Foreign Asset position and a Current Account deficit. We can also represent the equilibrium with a Metzler diagram in quantities augmented with a global AS curve. Indeed we have

$$\begin{aligned}
\frac{NFA}{\bar{Y}} &= \frac{\xi^w(1-\frac{\frac{\delta\theta}{\rho}}{1-\frac{\pi^w}{\rho}}) - \theta d}{g+\theta+\pi^w}, \\
S\frac{NFA^*}{\bar{Y}^*} &= \frac{\xi^w(1-\frac{\frac{\delta^*\theta}{\rho}}{1-\frac{\pi^w}{\rho}}) - \theta d^*}{g+\theta+\pi^w},
\end{aligned}$$

and we must have

$$\begin{aligned}
y\frac{NFA}{\bar{Y}} + (1-y)S\frac{NFA^*}{\bar{Y}^*} &= 0, \\
\pi^w + \kappa_0 + \kappa_1(1-\xi^w) &= 0,
\end{aligned}$$

with $S = 1$.

B.2 Recovery: Exchange Rates Movements and Interest Rates Differentials

We start with the model with permanently rigid prices, and consumption home bias of section A.3. For simplicity, we assume that there is no public debt so that $D/\bar{Y} = D^*/\bar{Y}^* = 0$.

We then assume that a Poisson shock occurs with instantaneous probability $\iota > 0$. When the Poisson shock occurs, the fraction of output δ that accrues in the form of dividends jumps instantaneously and permanently by a factor $\nu > 1$ in both countries. This alleviates the asset shortage and increases the world natural interest rate. We assume that ν is large enough that upon the realization of the Poisson shock the world natural interest rate rises above zero: $-\rho + \nu\bar{\delta}\theta > 0$. This implies that the economy may experience a liquidity trap *before* the Poisson shock, but never *after* it.

The steady state of the post-Poisson shock economy is uniquely determined, and so are its dynamics from any initial position.[31] By backward induction, this means that the exchange rate during the liquidity trap phase is also pinned down, conditional on the exchange rate E_τ that occurs at the time τ of the realization of the Poisson shock. This removes the indeterminacy in the nominal exchange rate à la Kareken and Wallace (1981) that we found in our baseline model.[32]

But another form of indeterminacy appears which we can index by the exchange rate E_τ. This is because, in our model, agents are risk neutral, so that international portfolios are indeterminate.[33] Yet, a given portfolio allocation will determine relative wealths immediately after the Poisson shock. In the presence of home bias in consumption, this pins down relative demands for Home and Foreign goods and therefore the nominal exchange rate E_τ. Conversely, for a given value E_τ, one can construct international portfolios that are consistent with this value of the exchange

[31]One can verify that the dynamics of the economy are saddle-path stable.

[32]This is because we have assumed that the economy is not a in a global liquidity trap after the Poisson shock. If we assume instead that the economy is in a liquidity trap after the Poisson shock (so that the recovery is only a partial recovery which doesn't lift the economy out of the ZLB), then even without home bias, the exchange rate indeterminacy à la Kareken and Wallace (1981) is reinstated. Indeed, in this case, the exchange rate E_τ after the Poisson shock is indeterminate. The exchange rate E before the Poisson shock, which depends on its value E_τ after the Poisson shock, inherits this indeterminacy. In the interest of space, we do not develop this model formally.

[33]This other form of indeterminacy hinges on our assumption that some (here all) agents are risk neutral. If all agents were somewhat risk averse, then portfolios would be pinned down and this other form of indeterminacy would disappear.

rate at the time of the Poisson shock. We summarize by writing domestic and foreign wealth and asset values at the time of the shock as $W_\tau = w_\tau \bar{Y}/\theta$, $W_\tau^* = w_\tau^* \bar{Y}^*/\theta$, $V_\tau = v_\tau \bar{Y}/\theta$, and $V_\tau^* = v_\tau^* \bar{Y}^*/\theta$ where it is understood that the coefficients w_τ,w_τ^*, v_τ and v_τ^* are functions of the exchange rate E_τ at the time of the shock.

We focus on the stochastic steady state before the Poisson shock. Because of the jump in the exchange rate at the time of the Poisson shock, Home and Foreign typically experience different real interest rates. This is in contrast to our baseline model where real interest rates are always equalized across countries. To see this most clearly, note that financial integration imposes that in the stochastic steady state prior to the Poisson shock, we have the following UIP equation:

$$r = r^* + \iota(\frac{E_\tau}{E} - 1). \tag{B.4}$$

This implies that the home interest rate $r < r^*$ if the home currency is expected to appreciate after the Poisson shock $E_\tau/E < 1$.[34]

The asset pricing equations include news terms accounting for capital gains and losses triggered by the realization of the Poisson shock:

$$rV = -\rho V + \delta \xi \bar{Y} + \iota \left(V_\tau - V\right),$$

$$r^* V^* = -\rho V^* + \delta^* \xi^* \bar{Y}^* + \iota \left(V_\tau^* - V^*\right).$$

For example, a higher value of home assets V_τ after the Poisson shock increases the value of home assets V in the stochastic steady state before the Poisson shock.

The wealth accumulation equations include new terms accounting for the risk and return of each country's portfolio:

$$gW = -\theta W + (1-\delta)\,\xi \bar{Y} + rW + \iota\left(W - W_\tau\right) + \left(g+\rho\right)V,$$
$$gW^* = -\theta W^* + (1-\delta)\,\xi^* \bar{Y}^* + r^* W^* + \iota\left(W^* - W_\tau^*\right) + \left(g+\rho\right)V^*.$$

For example, a lower value of home wealth W_τ after the Poisson shock means that home agents have a riskier portfolio, and therefore collect higher returns as long as the Poisson shock does not materialize. This in turn increases home wealth W in the stochastic steady state before the Poisson shock.

The goods market clearing equations (A.3a) and (A.3b) are unchanged, and we must still impose $r \geq 0$, $r^* \geq 0$, $0 \leq \xi \leq 1$, $0 \leq \xi^* \leq 1$, and the complementary slackness conditions $r(1-\xi) = 0$ and $r^*(1-\xi^*) = 0$.

The jump in the exchange rate at the time of the Poisson shock opens the door to the possibility that Home and Foreign may not experience a liquidity trap simultaneously prior to the shock. Real interest rates can differ across countries, resulting in the possibility of more strongly asymmetric liquidity trap equilibria than those we have encountered so far, where one country has zero nominal interest rates, zero real interest rates and a recession, while the other country has positive nominal interest rates, positive real interest rates, and no recession.

Going back to the UIP equation (B.4), we see that for Home to be the only country in a liquidity trap, we need $r = 0$, $\xi < 1$, $r^* > 0$, $\xi^* = 1$ and $\iota\left(E_\tau/E - 1\right) = -r^* < 0$. This requires that the home exchange rate appreciate at the time of the shock, $E_\tau < E$. We focus on this configuration from here onwards.

Home output ξ is then given by

$$\xi = \frac{\beta \frac{g-\iota}{g-\iota+\theta} \frac{\iota v_\tau - \frac{(\rho+\iota)\iota}{g-\iota}(w_\tau - v_\tau)}{\rho+\iota} + y^*(1-\beta)E}{[1 - \frac{\beta \frac{g-\iota}{g-\iota+\theta}}{\beta \frac{g-\iota}{g-\iota+\theta} + y^*(1-\beta)} \frac{\delta\theta}{\rho+\iota}][\beta \frac{g-\iota}{g-\iota+\theta} + y^*\left(1-\beta\right)]}. \tag{B.5}$$

This equation shows that everything else equal, as long as there is home bias $\beta > 0$, a higher value v_τ/θ of the home asset after the Poisson shock, and a lower value of the home Net Foreign Asset position after the Poisson shock $(w_\tau^* - v_\tau^*)/\theta$, contribute to a lower home output. Both increase the value of home wealth before the Poisson shock

[34] We can also have equilibria with different values of $E = E_\tau$, with similar implications in terms of relative outputs and "currency wars" as in the main text—lower values of $E = E_\tau$ are associated with higher values of ξ and lower values of ξ^*. Interestingly, But here, this logic can be more extreme in that we can also have equilibria with asymmetric liquidity traps where there is a liquidity trap in one country but not in the other. For example, Home can be in a liquidity trap with $r = 0$ and $\xi < 1$ while Foreign is not: $r^* > 0$ and $\xi^* = 1$. In this case, going back to the UIP equation, the exchange rate appreciates when the Poisson shock occurs $E > E_\tau$.

and, because of home bias, of the demand for home goods. The reason is that a higher value of v_τ/θ increases the value of new trees, and that a lower value of $(w_\tau^* - v_\tau^*)/\theta$ indicates that home agents take more risk, and are hence rewarded by a higher return before the Poisson shock.

The foreign interest rate r^* and the exchange rate are then given by the following system of nonlinear equations

$$0 = r^* + \iota(\frac{E_\tau}{E} - 1),$$

$$1 - (\frac{\xi}{E} - 1)y\frac{1-\beta}{\beta} = \frac{\theta + \frac{g-\iota-r^*}{\rho+\iota+r^*}(\delta^*\theta + \iota v_\tau^*) - \iota\,(w_\tau^* - v_\tau^*)}{g - \iota + \theta - r^*},$$

where we use the equation above to express ξ as a function of E. This is an equilibrium as long as $\xi < 1$ and $r^* \geq 0$.

We can also compute

$$\frac{NFA}{\bar{Y}} = \frac{\xi(1 - \frac{\delta\theta}{\rho+\iota}) - \frac{\iota v_\tau}{\rho+\iota} - \iota\frac{\theta}{g-\iota}(w_\tau - v_\tau)}{g - \iota + \theta} = \frac{1 - \frac{\delta\theta}{\rho+\iota}}{g-\iota+\theta}(\xi - \hat{\xi}^{a,l}),$$

$$\frac{CA}{\bar{Y}} = g\frac{NFA}{\bar{Y}}.$$

where $\hat{\xi}^{a,l}$ is home output in the equilibrium where Home is in financial autarky before the Poisson shock, but not after the Poisson shock (and where the equilibrium coincides with that under consideration after the Poisson shock).[35]

Financial integration before the Poisson shock increases output $\xi \geq \hat{\xi}^{a,l}$ if and only if the exchange rate is more depreciated $E \geq \hat{E}^{a,l}$.[36]

[35] By financial autarky we mean that Net Foreign Asset positions at Home and in Foreign are equal to 0. We allow countries to trade actuarially fair insurance contracts on the realization of the Poisson shock. These contracts have zero ex-ante value for both home and foreign agents.

[36] The analysis simplifies drastically in the limit of full home bias ($\beta \to 1x$). In this case, we have $\xi^a = \xi^{a,l}$, $w_\tau = v_\tau = w_\tau^* = v_\tau^* = 1$. This implies that ξ and r^* are given by their financial autarky values $\xi = \xi^{a,l}$, $r^* = r^{*a,n} = \delta^*\theta - \rho$, and Net Foreign Asset Positions and Current Accounts are zero $\frac{NFA}{\bar{Y}} = \frac{CA}{\bar{Y}} = 0$. This is an equilibrium if and only if $r^{*a,n} \geq 0$ and $r^{a,n} \leq 0$ (which is equivalent to $\xi^{a,l} \leq 1$).