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Working Paper 21595
http://www.nber.org/papers/w21595

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
September 2015, Revised July 2019

We thank Robert Barro, Sam Kortum and Kiminori Matsuyama for comments on previous drafts. We thank the editor, four anonymous referees, Timo Boppart, Paco Buera, Berthold Herrendorf, Bart Hobijn, Chad Jones, Joe Kaboski, Sam Kortum, Pete Klenow, Dirk Krueger, Robert Z. Lawrence, Erzo Luttmer, Serguei Maliar, Alex Monge-Naranjo, Martin Rotemberg, Orie Shelef, Tomasz Swiecki, Chris Tonetti, Kei-Mu Yi, Chris Udry and participants in several seminar and conference presentations for useful comments and feedback. We thank Emily Breza and Cynthia Kinnan for their help with the Indian NSS data. We thank Ana Danieli for her outstanding assistance in the revision of an earlier draft. Comin acknowledges the generous support of the National Science Foundation and the Institute for New Economic Thinking. Mestieri acknowledges the generous support of the Agence Nationale de la Recherche (JJCC - GRATE program) while at TSE. All remaining errors are our own. The views expressed herein are those of the authors and do not necessarily reflect the views of the National Bureau of Economic Research.

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NBER Working Paper No. 21595
September 2015, Revised July 2019
JEL No. E2,O11,O4,O5

ABSTRACT

We present a new multi-sector growth model that features nonhomothetic, constant-elasticity-of-substitution preferences, and accommodates long-run demand and supply drivers of structural change for an arbitrary number of sectors. The model is consistent with the decline in agriculture, the hump-shaped evolution of manufacturing, and the rise of services over time. We estimate the demand system derived from the model using household-level data from the U.S. and India, as well as historical aggregate-level panel data for 39 countries during the postwar period. The estimated model parsimoniously accounts for the broad patterns of sectoral reallocation observed among rich, miracle and developing economies. Our estimates support the presence of strong nonhomotheticity across time, income levels, and countries. We find that income effects account for over 75% of the observed patterns of structural change.

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A data appendix is available at http://www.nber.org/data-appendix/w21595

1 Introduction

Economies undergo large scale sectoral reallocations of employment and capital as they develop, in a process commonly known as structural change (Kuznets, 1973; Maddison, 1980; Herrendorf et al., 2014; Vries et al., 2014). These reallocations lead to a gradual fall in the relative size of the agricultural sector and a corresponding rise in manufacturing. As income continues to grow, services eventually emerge as the largest sector in the economy. Leading theories of structural change attempt to understand these sweeping transformations through mechanisms involving either supply or demand. Supply-side theories focus on differences across sectors in the rates of technological growth and capital intensities, which create trends in the composition of consumption through price (substitution) effects (Baumol, 1967; Ngai and Pissarides, 2007; Acemoglu and Guerrieri, 2008). Demand-side theories, in contrast, emphasize the role of heterogeneity in income elasticities of demand across sectors (nonhomotheticity in preferences) in driving the observed reallocations accompanying income growth (Kongsamut et al., 2001).

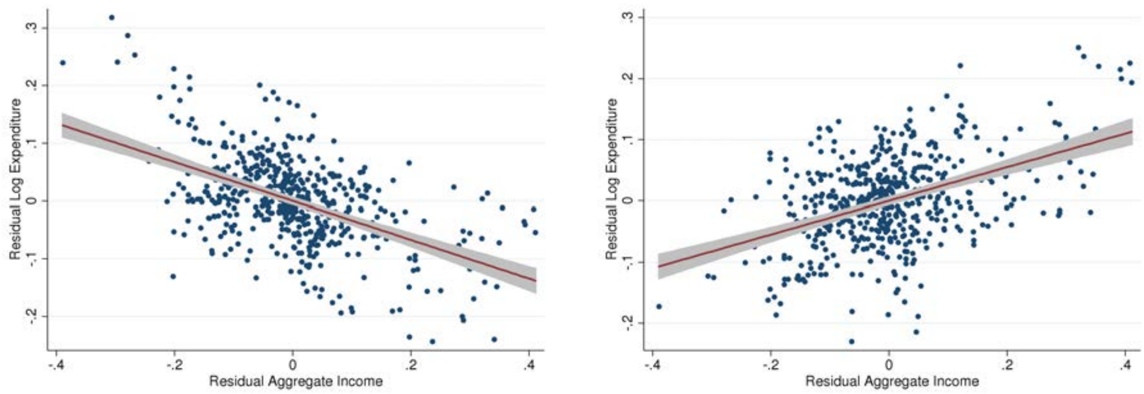
The shapes of sectoral Engel curves play a crucial role in determining the contribution of supply and demand channels to structural change. If the differences in the slopes of Engel curves are large and persistent, demand channels can readily explain the reallocation of resources toward sectors with higher income elasticities. For instance, steep upward Engel curves for services, flat Engel curves for manufacturing, and steep downward Engel curves for agricultural products can give rise to sizable shifts of employment from agriculture toward services. However, demand-side theories have generally relied on specific classes of nonhomothetic preferences, e.g., generalized Stone-Geary preferences, that imply Engel curves that level off quickly as income grows. Because of this rapid flattening-out of the slopes of Engel curves across sectors, these specifications limit the explanatory power of the demand channel in the long-run.

The empirical evidence suggests that the relationship between relative sectoral expenditure shares and income is stable, and the slopes of Engel curves do not level off rapidly as income grows. Using aggregate data from a sample of OECD countries, Figure 1 plots the residual (log) expenditure share in agriculture (Figure 1a) and services (Figure 1b) relative to manufacturing on the y-axis and residual (log) income on the x-axis after controlling for relative prices.[1] The depicted fit shows that a constant slope captures a considerable part of the variation in the data and that it does not appear that the relationship levels off as aggre-

[1] Residual Aggregate Income is constructed by taking the residuals of the following OLS regresssion: $\log Y_t^n = \alpha \log p_{at}^n + \beta \log p_{mt}^n + \gamma \log p_{st}^n + \xi^n + \nu_t^n$ where superscript n denotes country, and subscript t, time. Y_t^n, p_{at}^n, p_{mt}^n, and p_{st}^n denote aggregate income, the prices of agriculture, manufacturing, and services, respectively. ξ^n denotes a country fixed effect and ν_t^n the error term. Residual log-expenditures are constructed in an analogous manner using the log of relative sectoral expenditures as dependent variables. Table F.1 in the online appendix reports the estimates of the regression. Section 5.1.1 discusses the connection between our theory and the regression in Figure 1.

Figure 1: Partial Correlations of Sectoral Expenditure and Aggregate Consumption

(a) Agriculture relative to Manufacturing (b) Services Relative to Manufacturing



Notes: Data for OECD countries, 1970-2005. Each point corresponds to a country-year observation after partialling-out sectoral prices and country fixed effects. The red line depicts the OLS fit, the shaded regions, the 95% confidence interval.

gate consumption grows.[2] As we discuss below, we complement this aggregate-level evidence with micro-level household data from the Consumption Expenditure survey (CEX) from the US and the National Sample Survey (NSS) from India. We analyze the relationship between relative shares and expenditure in these data, and show that sectoral differences in the estimated slopes do not level off and remain stable across households with different expenditure levels.[3]

Motivated by this evidence, we develop a multi-sector model of structural change that accommodates non-vanishing nonhomotheticities. The model builds on the standard framework used in recent empirical work on structural transformation (e.g., Buera and Kaboski, 2009; Herrendorf et al., 2013). Our key departure from the standard framework is the introduction

[2]The partial R^2 of the regressions shown in Figure 1 are 27% and 20%, respectively. In fact, if we split the sample into observations below and above the median income in the sample and estimate the relative Engel curves separately, we cannot reject the hypothesis of identical slopes of the Engel curves. See Table F.1 in the online appendix. If we reported separately the Engel curves for agriculture, manufacturing and services, we would find a negative, zero and positive slope, respectively.

[3]A number of recent papers have similarly used log-linear specifications of Engel curves in analyzing micro-level expenditure data. Aguiar and Bils (2015) use the U.S. Consumer Expenditure Survey (CEX) to estimate Engel curves for 20 different consumption categories. Their estimates for the income elasticities are different from unity and vary significantly across consumption categories. Young (2012) employs the Demographic and Health Survey (DHS) to infer the elasticity of real consumption of 26 goods and services with respect to income for 29 sub-Saharan and 27 other developing countries. He estimates the elasticity of consumption for the different categories with respect to the education of the household head and then uses the estimates of the return to education from Mincerian regressions to back out the income elasticity of consumption. Young also uses a log-linear Engel curve formulation and finds that the slopes of Engel curves greatly differ across consumption categories but appear stable over time. Olken (2010) discusses Young's exercise using Indonesia survey data and finds similar results for a small sample of three goods and services. Young (2013) also makes use of log-linear Engel curves to infer consumption inequality.

of a class of utility functions that generates nonhomothetic sectoral demands for all levels of income, including when income grows toward infinity. These preferences allow for an arbitrary number of goods and feature a constant elasticity of substitution that is independent of the income elasticity parameters. Thus, our framework lends itself to the task of decomposing the contributions of the demand and supply channels to structural change. These preferences, which we will refer to as nonhomothetic Constant Elasticity of Substitution (CES) preferences, have been studied by Gorman (1965), Hanoch (1975), Sato (1975), and Blackorby and Russell (1981) in the context of static, partial-equilibrium models. Our theory embeds these preferences into a general equilibrium model of economic growth. As part of our contributions, we also derive a strategy for structurally estimating the parameters of these preferences, using both micro and aggregate data. Finally, we use the estimated model parameters to compare the contributions of income and price effects to structural change across countries.

We characterize the equilibrium paths of our growth model in the long-run and derive the dynamics of the economy along the transition path. The equilibrium in our model asymptotically converges to a path of constant real consumption growth. The asymptotic growth rate of real consumption depends on parameters characterizing both the supply and demand channels; it is a function of the sectoral income elasticities as well as sectoral growth rates of TFP and sectoral factor intensities. In this respect, our model generalizes the results of Ngai and Pissarides (2007) and Acemoglu and Guerrieri (2008) to the case featuring nonhomothetic CES demand. Our theory can produce similar evolutions for nominal and real sectoral measures of economic activity, which is a robust feature of the data.[4] This is a consequence of the role of income elasticities in generating sectoral reallocation patterns. Our framework can generate hump-shaped patterns for the evolution of manufacturing consumption shares, which is a well-documented feature in the data (Buera and Kaboski, 2012a).

In the empirical part of the paper, we first provide household-level evidence in favor of the stable effect of nonhomotheticities implied by nonhomothetic CES preferences. We estimate our demand system using household-level data from the Consumption Expenditure survey (CEX) from the US. We group household expenditures into three broad categories of products: agriculture, manufacturing, and services. The estimated income elasticity parameters are ranked such that the agriculture parameter is smaller than the manufacturing parameter, and the parameter for services is larger than that for manufacturing. We also show that the estimated income elasticity parameters are similar for households across different income brackets and time periods. Our theory also implies a log-linear linear relationship between relative sectoral consumption and the real consumption index (derived from nonhomothetic CES). We show in Figure 2 that this log-linear relationship approximately holds in our data, which suggests that the pattern of reallocation of household consumption across sectors is

[4] Herrendorf et al., 2014 show that supply-side driven structural transformation cannot account for the similar evolution of nominal and real sectoral measures of activity.

similar to the aggregate behavior discussed in Figure 1.

We then empirically evaluate the implications of our growth model for structural transformation at the macro level. We estimate the elasticities that characterize our demand function using cross-country sectoral data in a panel of 39 countries for the postwar period. The countries in our sample substantially vary in terms of their stages of development and growth experiences (e.g., Ghana, Taiwan and the US). We find that the estimated nonhomotheticity parameters are similar across different measures of sectoral activity (employment and output) and country groupings (OECD and Non-OECD countries). Crucially, we use the estimated model to demonstrate how our model matches the broad empirical patterns illustrated in Figure 1. In particular, we show that a log-linear relationship between relative sectoral shares and our theory-consistent index of real consumption captures a large share of the variation in the data, as suggested by our model (see Figure 4). We use this evidence to argue that the parsimonious specification of nonhomotheticities implied by nonhomothetic CES captures a substantial part of the sectoral reallocations experienced by countries at very different stages of development.[5]

Armed with the estimated parameters of our model, we turn to the analysis of the drivers of structural change. We use our model to decompose structural change into income and price effects. We find that income effects are the main contributors to structural transformation. They account for over 75% of the sectoral reallocation in employment predicted by the estimated model. This finding contrasts with previous studies (e.g., Dennis and Iscan, 2009, Boppart, 2014a). A potential reason for this discrepancy is that in our framework income effects are not hard-wired to have only transitory effects on the structural transformation (as in Stone-Geary preferences) or to be correlated with price effects. Without these constraints on income effects, our estimates are consistent with a predominant role of income effects in accounting for the structural transformation during the postwar period in a large sample of countries at different stages of development. We further investigate the predictive power of our model by comparing it with the two most prominent demand systems that are consistent with nonhomotheticity of preferences: the generalized Stone-Geary (Buera and Kaboski, 2009) and the price-independent generalized-linear (PIGL) preferences (Boppart, 2014a). We find that nonhomothetic CES preferences provide a better account for the patterns of structural transformation across agriculture, manufacturing and services in our cross-country sample.

Finally, we present a number of alternative approaches to the estimation of nonhomothetic CES preferences, as well as other extensions and robustness checks to our baseline empirical

[5] As we have discussed, nonhomothetic CES imposes a constant elasticity of expenditure shares on the real consumption index. The reason for the similarity between Figures 1 and 4 comes from the fact that the regression approach used in Figure 1 provides consistent estimates of the true elasticity parameters up to a scale parameter (see Section 5.1.1). We also show that the theory-consistent measure of real consumption is highly correlated with standard measures of consumption that are deflated using ideal price indices, as illustrated in Figure 6 with the Penn World Table measure of real consumption.

results. In particular, we show that a simple log-linear specification is capable of identifying the rank-ordering and the relative magnitude of income elasticity parameters across sectors. This approach allows us to extend our analysis to the National Sample Survey (NSS) data from India, where we use a fixed-effects strategy to account for (unobserved) sectoral price indices. We find that the income elasticity parameters estimated using NSS data are very similar to those estimated using US CEX data (despite the vast differences in the level of development and sectoral composition of consumption between the US and India). As another noteworthy extension, we take advantage of the fact that nonhomothetic CES can accommodate an arbitrary number of goods. We extend our empirical analysis to a richer sectoral disaggregation and document substantial heterogeneity in income elasticity within manufacturing and services.

Our paper relates to a large literature that aims to quantify the role of nonhomotheticity of demand on growth and development (see, among others, Matsuyama (1992), Echevarria, 1997, Gollin et al., 2002, Duarte and Restuccia, 2010, Alvarez-Cuadrado and Poschke, 2011).[6] Buera and Kaboski (2009) and Dennis and Iscan (2009) have noted the limits of the generalized Stone-Geary utility function to match long time series or cross-sections of countries with different income levels. More recently, Boppart (2014a) has studied the evolution of consumption of goods relative to services by introducing a sub-class of PIGL preferences that also yield non-vanishing income effects in the long-run. PIGL preferences also presuppose specific parametric correlations for the evolution of income and price elasticities over time (Gorman, 1965), and only accommodate two goods with distinct income elasticities. In contrast, our framework allows for an arbitrary number of goods.[7] The differences between the two models are further reflected in their empirical implications. Whereas we find a larger contribution for demand nonhomotheticity in accounting for structural change, Boppart concludes that supply and demand make roughly similar contributions.[8]

[6]An alternative formulation that can reconcile demand being asymptotically nonhomothetic with balanced growth path is given by hierarchical preferences (e.g., Foellmi and Zweimüller, 2006, 2008 and Foellmi et al., 2014). Swiecki (2017) estimates a demand system that features non-vanishing income effects in combination with subsistence levels à la Stone-Geary. However, this demand system also imposes a parametric relation between income and price effects. In subsequent work, Duernecker et al. (2017b) use a nested structure of nonhomothetic CES to study structural change within services. Sáenz (2017) extends our framework to time-varying capital intensities across production sectors and calibrates his model to South Korea. Matsuyama (2015, 2017) embeds nonhomothetic CES preferences in a monopolistic competition framework with international trade à la Krugman to study the patterns of structural change in a global economy and endogenizes the pattern of specialization of countries through the home market effect. Sposi (2016) and Lewis et al. (2018) incorporate nonhomothetic CES in a quantitative trade model of structural change.

[7]One can extend PIGL preferences to more than two goods by nesting other functions as composites within the two-good utility function (Boppart, 2014a), e.g., CES aggregators (this is how we proceed to estimate them in our empirical analysis). However, the resulting utility function does not allow for heterogeneity in income elasticity among the goods within each nested composite.

[8]In terms of the scope of the empirical exercise, while Boppart (2014a) estimates his model with U.S. data and considers two goods, the empirical evaluation of our model includes, in addition to the U.S., a wide range of other rich and developing countries and more than two goods. The variable elasticity implied by PIGL is

The remainder of the paper is organized as follows. Section 2 introduces the properties of the nonhomothetic CES preferences and presents the model. Section 3 presents the estimation of the model using the household level and aggregate data. Section 4 uses the model estimates to investigate the relative importance of price and income effects for the patterns of structural transformation observed in our sample. It also compares the fit of our model with those constructed based on the Stone-Geary and PIGL demand systems. Section 5 discusses a number of alternative estimation strategies and extensions of the empirical analysis (e.g., estimation to more than three sectors), as well as additional robustness checks. Section 6 presents a calibration exercise where we investigate the transitional dynamics of the model, and Section 7 concludes. Appendix A presents some general properties of nonhomothetic CES. All proofs are in Appendix B.

2 Theory

In this section, we present a class of preferences that rationalize the empirical regularities on relative sectoral consumption expenditures discussed in the Introduction. We then incorporate these preferences in a multi-sector growth model and show how we can use them to account for the patterns of structural transformation across countries. The growth model closely follows workhorse models of structural transformation (e.g., Buera and Kaboski, 2009; Herrendorf et al., 2013, 2014). The only difference with these is that we replace the standard aggregators of sectoral consumption goods with a nonhomothetic CES aggregator. This single departure from the standard workhorse model delivers the main theoretical results of the paper and the demand system later used in the estimation.

2.1 Nonhomothetic CES Preferences

Consider preferences over a bundle of goods $\boldsymbol{C} \equiv (C_1, C_2, \cdots, C_I)$ characterized by an aggregator index $C = F(\boldsymbol{C})$, implicitly defined through the constraint

$$\sum_{i=1}^{I} \left(\Omega_i C^{\epsilon_i}\right)^{\frac{1}{\sigma}} C_i^{\frac{\sigma-1}{\sigma}} = 1. \tag{1}$$

We impose the parametric restrictions that 1) $\sigma > 0$ and $\sigma \neq 1$, 2) $\Omega_i > 0$ for all $i \in \mathcal{I} \equiv \{1, \cdots, I\}$, and 3) if $\sigma < 1$ (if $\sigma > 1$), then $\epsilon_i > 0$ ($\epsilon_i < 0$) for all $i \in \mathcal{I}$.[9] Each sectoral good i

also quantitatively important in accounting for the difference in the decomposition results (see Section 4).

[9] We can show that under these parameter restrictions the aggregator C introduced in equation (1) is globally monotonically increasing and quasi-concave, yielding a well-defined utility function over the bundle of goods $\boldsymbol{C}$, see Hanoch (1975). The additional restriction $\epsilon_i \geq 1 - \sigma$ for $\sigma < 1$ ($\epsilon_i \leq 1 - \sigma$ for $\sigma > 1$) ensures strict concavity, which simplifies the analysis of the dynamics in Section 2.2.1 below. In the case of $\sigma = 1$, the only globally well-defined CES preferences are homothetic and correspond to the Cobb-Douglas preferences (Blackorby and Russell, 1981).

is identified with a parameter ϵ_i that controls the income elasticity of demand for that good. Intuitively, as the index C rises, the weight given to the consumption of good i varies at a rate controlled by parameter ϵ_i. As a result, the demand for sectoral good i features a constant elasticity in terms of the index of real consumption C.

The index of real consumption C in Equation (1) defines a specific measure of utility for a consumer with nonhomothetic CES preferences. Any alternative cardinal measure of utility U will have a distinct monotonic relationship with the index C, that is, for any such U there is some $g(\cdot)$ such that $C = g(U)$ and $g'(\cdot) > 0$ everywhere. Since consumer behavior remains invariant to such transformations, substituting $C = g(U)$ in Equation (1) leads to the same exact preferences as the one defined above.[10] In particular, choosing $C = g(U) = \xi U^{\zeta}$ for ξ and $\zeta > 0$, we can see that a constant scaling of the taste parameters to $\xi\Omega_i$ or the income elasticity parameters to $\zeta\epsilon_i$ does not affect the implied consumer behavior. Therefore, for the specification in Equation (1) to characterize distinct patterns of consumer choice for different sets of model parameters, we have to consider scaling normalizations for the income elasticity parameters $(\epsilon_1, \cdots, \epsilon_I)$ and the taste parameters $(\Omega_1, \cdots, \Omega_I)$, as we discuss below.

Hicksian Demand Consider the expenditure minimization problem with the set of prices $\boldsymbol{p} \equiv (p_1, p_2, \cdots, p_I)$ and preferences defined as in Equation (1). The nonhomothetic CES Hicksian demand function is given by

$$C_i = \Omega_i \left(\frac{p_i}{E}\right)^{-\sigma} C^{\epsilon_i}, \qquad \forall i \in \mathcal{I}, \tag{2}$$

where we have defined E as the expenditure function

$$E(C; \boldsymbol{p}) \equiv \left[\sum_{i=1}^{I} \Omega_i C^{\epsilon_i} p_i^{1-\sigma}\right]^{\frac{1}{1-\sigma}}, \tag{3}$$

that gives the cost $E = \sum_{i=1}^{I} p_i C_i$ of achieving real consumption C. Note that substituting the demand for C_i from (2) in the definition of nonhomothetic CES (1), we find that each summand in (1) corresponds to the equilibrium expenditure share. Denoting expenditure shares by $\omega_i \equiv p_i C_i / E$, Equation (1) simply implies $\sum_{i=1}^{I} \omega_i = 1$.

Two unique features of nonhomothetic CES Hicksian demand function make these preferences suitable candidates for capturing the patterns discussed in the Introduction:

1. The elasticity of the relative demand for two different goods with respect to aggregate

[10] In Section A.1 of the Online Appendix, we study an alternative index of utility that corresponds to the cost of living in a given base year.

index C is constant, i.e.,

$$\frac{\partial \log (C_i/C_j)}{\partial \log C} = \epsilon_i - \epsilon_j, \qquad \forall i, j \in \mathcal{I}. \tag{4}$$

2. The elasticity of substitution between goods of different sectors is uniquely defined and constant[11]

$$\frac{\partial \log (C_i/C_j)}{\partial \log (p_j/p_i)} = \sigma, \qquad \forall i, j \in \mathcal{I}. \tag{5}$$

The first property ensures that the nonhomothetic features of these preferences do not systematically diminish as income (and therefore utility) rises. As discussed in the Introduction, available data on sectoral consumption, both at the macro and micro levels, suggest stable and heterogeneous income elasticities across sectors. Therefore, we propose to specify preferences that do not result in systematically vanishing patterns of nonhomotheticity, as, for instance, would be implied by the choice of Stone-Geary preferences. The second property ensures that different goods have a constant elasticity of substitution and price elasticity regardless of the level of income.[12] It is because of this property that we refer to these preferences as nonhomothetic *CES*.[13]

The demand system implied by nonhomothetic CES for the *relative* consumption expenditures of goods transparently summarizes the two properties above. The Hicksian demand for any pair of expenditure shares ω_i amd ω_j, $i, j \in \mathcal{I}$, satisfies

$$\log \left(\frac{\omega_i}{\omega_j}\right) \quad = \quad (1-\sigma) \log \left(\frac{p_i}{p_j}\right) + (\epsilon_i - \epsilon_j) \log C + \log \left(\frac{\Omega_i}{\Omega_j}\right). \tag{6}$$

Equation (6) highlights one of the key features of the nonhomothetic CES demand system, which is the separation of the price and the income effects. The first term on the right hand side shows the price effects characterized by a constant elasticity of substitution σ, and the

[11]Note that for preferences defined over I goods when $I > 2$, alternative definitions for elasticity of substitution do not necessarily coincide. In particular, Equation (5) defines the so-called Morishima elasticity of substitution, which in general is not symmetric. This definition may be contrasted from the Allen (or Allen-Uzawa) elasticity of substitution defined as $\frac{E \cdot \partial C_i/\partial P_j}{C_i C_j}$, where E is the corresponding value of expenditure. Blackorby and Russell (1981) prove that the only preferences for which the Morishima elasticities of substitution between any two goods are symmetric, constant, and identical to Allen-Uzawa elasticities have the form of Equation (1), albeit with a more general dependence of weights on C.

[12]Nonhomothetic CES preferences inherit this property because they belong to the class of implicitly additively separable preferences (Hanoch, 1975). In contrast, any preferences that are explicitly additively separable in sectoral goods imply parametric links between income and substitution elasticities. This result is known as Pigou's Law (Snow and Warren, 2015). For a discussion of specific examples, see Appendix A.

[13]Alternatively, if we assume that consumer preferences satisfy two properties (4) and (5) for given parameter values $(\sigma, \epsilon_1, \cdots, \epsilon_I)$, the preferences will correspond to the nonhomothetic CES preferences given by Equation (1). More specifically, imposing condition (5) defines a *general* class of nonhomothetic CES preferences, defined in Equation (A.1) in the appendix. Further imposing condition (4), together with the additional restriction that we should recover homothetic CES preferences in the case of $\epsilon_i \equiv 1 - \sigma$, yields the definition in Equation (1). See Appendix A for more details.

second term on the right hand side shows the change in relative sectoral demand as consumers move across indifference curves.

Marshallian Demand Under the parametric restrictions we imposed above, the expenditure function in Equation (3) is monotonically increasing in the index C. Therefore, it implicitly defines the index C as an indirect utility function in terms of the observables, total expenditure E and the prices $\boldsymbol{p}$. Substituting this expression into the demand Equation (2) allows us to find the Marshallian demand functions for a consumer solving the utility maximization problem. Furthermore, the expenditure shares can be expressed as

$$\omega_i = \Omega_i \left(\frac{p_i}{P}\right)^{1-\sigma} \left(\frac{E}{P}\right)^{\epsilon_i-(1-\sigma)}, \qquad \forall i \in \mathcal{I}. \tag{7}$$

with the average cost index $P \equiv E/C$ satisfying

$$P = \left[\sum_i \left(\Omega_i p_i^{1-\sigma}\right)^{\chi_i} \left(\omega_i E^{1-\sigma}\right)^{1-\chi_i}\right]^{\frac{1}{1-\sigma}}, \tag{8}$$

where we have defined $\chi_i \equiv \frac{1-\sigma}{\epsilon_i}$. We retrieve the standard CES preferences for the specific case of $\epsilon_i = 1 - \sigma$ for all $i \in \mathcal{I}$. In this case, the expenditure function becomes linear in the index of real consumption C, and the average cost of real consumption corresponds to the CES price index, $P = \left[\sum_{i=1}^{I} \Omega_i p_i^{1-\sigma}\right]^{\frac{1}{1-\sigma}}$. In general, however, when ϵ_i's vary across goods, the expenditure function varies nonlinearly in the aggregator index C (due to the dependence of expenditure shares on C).

The expenditure elasticity of demand for sectoral good i is given by

$$\eta_i \equiv \frac{\partial \log C_i}{\partial \log E} = \sigma + (1-\sigma)\frac{\epsilon_i}{\bar{\epsilon}}, \tag{9}$$

where E is the consumer's total consumption expenditure, and we have defined the expenditure-weighted average of income elasticity parameters, $\bar{\epsilon} \equiv \sum_{i=1}^{I} \omega_i \epsilon_i$ with ω_i denoting the expenditure share in sector i as defined above.[14] As Engel aggregation requires, the income elasticities average to 1 when sectoral weights are given by expenditure shares, $\sum_{i=1}^{I} \omega_i \eta_i = 1$. If good i has an income elasticity parameter ϵ_i that exceeds (is less than) the consumer's average elasticity parameter $\bar{\epsilon}$, then good i is a luxury (necessity) good, in the sense that it has an expenditure elasticity greater (smaller) than 1 at that point in time. This implies that being a luxury or necessity good is not an intrinsic characteristic of a good, but rather depends on

[14] The expenditure elasticity of relative demand is $\partial \log (C_i/C_j) / \partial \log E = (1-\sigma)(\epsilon_i - \epsilon_j)/\bar{\epsilon}$. Note the difference with Equation (4) that expresses the elasticity instead in terms of the nonhomothetic CES index of real consumption.

the consumer's current composition of consumption expenditures and, ultimately, income.

As we mentioned earlier, and it is evident from Equation (9), the predictions of the model for observables remain invariant to any scaling of all income elasticity parameters ϵ_i's (and the taste parameters Ω_i's) by a constant factor. Therefore, without loss of generality, we can normalize all the income elasticity and taste parameters such that those corresponding to a specific base good b equal a given arbitrary value, e.g., $\epsilon_b = \Omega_b = 1$. With these two normalizations, we can use the expression for the demand of the base good in Equation (2) to write the real consumption index in terms of the price and expenditure of the base good, as well as the total consumption expenditure:

$$\log C = (1-\sigma)\log\left(\frac{E}{p_b}\right) + \log\omega_b. \tag{10}$$

Substituting the expression for the real consumption index (10) we find the consumption expenditure share of goods $i \in \mathcal{I}_{-b} \equiv \mathcal{I}\backslash\{b\}$ relative to the base good b satisfy

$$\log\omega_i = \log\Omega_i + (1-\sigma)\log\left(\frac{p_i}{p_b}\right) + (1-\sigma)(\epsilon_i - 1)\log\left(\frac{E}{p_b}\right) + \epsilon_i\log\omega_b. \tag{11}$$

For a given base good b, and under the normalization of the elasticity and taste parameters $\epsilon_b = \Omega_b = 1$, Equation (11) provides an expression for the consumption shares of all other goods in terms of observables. Importantly, one can easily check that the condition (11) is invariant to our choice for the cardinal measure of utility C, that is, it remains the same under any transformation $U = g(C)$. We will rely on this condition on the Marshallian demand as our main specification in the estimation of the demand system in Section 3.[15]

2.2 Multi-sector Growth with Nonhomothetic CES

We now present a growth model where we integrate the nonhomothetic CES preferences in a general-equilibrium growth model to study the effect of the demand forces documented in the Introduction on shaping the long-run patterns of structural change. On the supply side, the model combines two distinct potential drivers of sectoral reallocation previously highlighted in the literature: heterogeneous rates of technological growth (Ngai and Pissarides, 2007) and heterogeneous capital-intensity across sectors (Acemoglu and Guerrieri, 2008).

Households A unit mass of homogenous households has the following intertemporal preferences

$$\sum_{t=0}^{\infty}\beta^t\left(\frac{C_t^{1-\theta}-1}{1-\theta}\right), \tag{12}$$

[15] We discuss alternative estimation strategies in Section 5.

where $\beta \in (0,1)$ is the discount factor, θ is the parameter controlling the elasticity of intertemporal substitution,[16] and aggregate consumption, C_t, combines a bundle of I sectoral goods, $\boldsymbol{C}_t$, according to the nonhomothetic CES function defined by Equation (1). Henceforth, we focus on the empirically relevant case $\sigma \in (0,1)$, where broad categories of goods are gross complements. To complete the characterization of the household behavior, we assume that each household inelastically supplies one unit of perfectly divisible labor, and starts at period 0 with a homogeneous initial endowment $\mathcal{A}_0$ of assets.

Firms Firms in each consumption sector produce sectoral output under perfect competition. In addition, firms in a perfectly competitive investment sector produce investment good, Y_{0t}, that is used in the process of capital accumulation. We assume constant-returns-to-scale Cobb-Douglas production functions with time-varying Hicks-neutral sector-specific productivities,

$$Y_{it} \quad = \quad A_{it} K_{it}^{\alpha_i} L_{it}^{1-\alpha_i}, \qquad i \in \{0\} \cup \mathcal{I},$$

where K_{it} and L_{it} are capital and labor used in the production of output Y_{it} in sector i at time t (we have identified the sector producing investment good as $i = 0$) and $\alpha_i \in (0,1)$ denotes sector-specific capital intensity. The aggregate capital stock of the economy, K_t, accumulates using investment goods and depreciates at rate δ, $Y_{0t} = K_{t+1} - (1-\delta) K_t$.

2.2.1 Competitive Equilibrium

We focus on the features of the competitive equilibrium of this economy that motivate our empirical specifications.[17] Households take the sequence of wages, real interest rates, and prices of goods and services $\{w_t, r_t, \boldsymbol{p}_t\}_{t=0}^{\infty}$ as given, and choose a sequence of asset stocks $\{\mathcal{A}_t\}_{t=1}^{\infty}$ and aggregate consumption $\{C_t\}_{t=0}^{\infty}$ to maximize their utility defined in Equations (1) and Equations (12), subject to the per-period budget constraint

$$\mathcal{A}_{t+1} + \sum_{i=1}^{I} p_{it} C_{it} \leq w_t + (1 + r_t) \mathcal{A}_t, \tag{13}$$

where we have normalized the price of assets to 1. The next lemma characterizes the solution to the household problem.

[16] As we will explain in Section 2.2.3, the elasticity of intertemporal substitution is not constant in this model.

[17] Given an initial stock of capital K_0 and a sequence of sectoral productivities $\left\{(A_{it})_{i=0}^{I}\right\}_{t\geq 0}$, a competitive equilibrium is defined as a sequence of allocations $\left\{C_t, K_{t+1}, Y_{0t}, L_{0t}, K_{0t}, (Y_{it}, C_{it}, K_{it}, L_{it})_{i=0}^{I}\right\}_{t\geq 0}$ and a sequence of prices $\left\{w_t, R_t, (p_{it})_{i=0}^{I}\right\}_{t\geq 0}$ such that *(i)* agents maximize the present discounted value of their utility given their budget constraint, *(ii)* firms maximize profits and *(iii)* markets clear.

Lemma 1. *(Household Behavior) Consider a household with preferences as described by Equations (12) and (1) with* $\sigma \in (0,1)$ *and* $\epsilon_i \geq 1-\sigma$ *for all* $i \in \mathcal{I}$,[18] *budget constraint (13), and the No-Ponzi condition* $\lim_{t\to\infty} \mathcal{A}_t \left(\prod_{t'=1}^{t-1} \frac{1}{1+r_{t'}}\right) = 0$. *Given a sequence of prices* $\{w_t, r_t, \boldsymbol{p}_t\}_{t=0}^{\infty}$ *and an initial stock of assets* $\mathcal{A}_0$*, the problem has a unique solution, fully characterized by the following conditions.*

1. *The intratemporal allocation of consumption goods satisfies* $C_{it} = \Omega_i \left(p_{it}/E_t\right)^{-\sigma} C_t^{\epsilon_i}$ *where consumption expenditure* E_t *at time* t *satisfies* $E_t = \sum_{i=1}^{I} p_{it} C_{it} = E\left(C_t; \boldsymbol{p}_t\right)$ *for the expenditure function defined by Equation (3).*

2. *The intertemporal allocation of real aggregate consumption satisfies the Euler equation*

$$\left(\frac{C_{t+1}}{C_t}\right)^{1-\theta} = \frac{1}{\beta\left(1+r_t\right)} \frac{\bar{\epsilon}_{t+1}}{\bar{\epsilon}_t} \frac{E_{t+1}}{E_t}, \tag{14}$$

and the transversality condition

$$\lim_{t\to\infty} \beta^t \left(1+r_t\right) \frac{\mathcal{A}_t}{\bar{\epsilon}_t \, E_t / C_t^{1-\theta}} = 0. \tag{15}$$

The key insight from Lemma 1 is that the household problem can be decomposed into two sub-problems: one involving the allocation of consumption and savings over time, and one involving the allocation of consumption across sectors. The first part of the household problem involves the *intratemporal* problem of allocating consumption across different goods based on the sectoral demand implied by the nonhomothetic CES aggregator. Therefore the sequence of sectoral prices $\{\boldsymbol{p}_t\}_{t=1}^{\infty}$, consumption expenditure shares $\{\boldsymbol{\omega}_t\}_{t=1}^{\infty}$, and total consumption expenditures $\{E_t\}_{t=1}^{\infty}$ satisfy

$$\log\left(\frac{\omega_{it}}{\omega_{bt}}\right) = (1-\sigma)\log\left(\frac{p_{it}}{p_{bt}}\right) + (\epsilon_i - 1)\log C_t + \log\Omega_i, \qquad i \in \mathcal{I}_{-b} \tag{16}$$

for a base sector b, where again we have assumed $\epsilon_b = \Omega_b = 1$.

The second part is the *intertemporal* consumption-savings problem. The household solves for the sequence of $\{\mathcal{A}_{t+1}, C_t\}_{t=0}^{\infty}$ that maximizes utility (12) subject to the constraint

$$\mathcal{A}_{t+1} + E\left(C_t; \boldsymbol{p}_t\right) \leq w_t + \mathcal{A}_t\left(1+r_t\right), \tag{17}$$

where $E\left(C_t; \boldsymbol{p}_t\right)$ is the total expenditure function for the nonhomothetic CES preferences, defined in Equation (3). Because of nonhomotheticity, consumption expenditure is a nonlinear

[18] Note that we can impose the parametric constraint $\epsilon_i \geq 1-\sigma$ without loss of generality. As we discussed in Section 2.1, we have one degree of freedom in scaling all parameters ϵ_i's by a constant factor without changing the underlying preferences of households. To satisfy the constraint, it is sufficient to choose the scaling of ϵ_i parameters large enough so that $\epsilon_{min} + \sigma \geq 1$.

function of real aggregate consumption, reflecting changes in the sectoral composition of consumption as income grows. The household incorporates this relationship in its Euler equation (14), where we see a wedge between the marginal cost of real consumption and the average cost $P_t = E_t/C_t$. The size of this wedge, given by $\bar{\epsilon}_t/(1-\sigma)$, depends on the average income elasticities across sectors, $\bar{\epsilon}_t = \sum_{i=1}^{I} \omega_{it}\epsilon_i$, and varies over time. In the case of homothetic CES where $\epsilon_i \equiv 1-\sigma$, this wedge disappears.

Firm profit maximization and equalization of the prices of labor and capital across sectors pin down prices of sectoral consumption goods,

$$p_{it} = \frac{p_{it}}{p_{0t}} = \frac{\alpha_0^{\alpha_0}(1-\alpha_0)^{1-\alpha_0}}{\alpha_i^{\alpha_i}(1-\alpha_i)^{1-\alpha_i}} \left(\frac{w_t}{R_t}\right)^{\alpha_0-\alpha_i} \frac{A_{0t}}{A_{it}}, \tag{18}$$

where, since the units of investment good and capital are the same, we normalize the price of investment good, $p_{0t} \equiv 1$. Equation (18) shows that price effects capture both supply-side drivers of sectoral reallocation: heterogeneity in productivity growth rates and heterogeneity in capital intensities.

Goods market clearing ensures that household sectoral consumption expenditure equals the value of sectoral production output, $\omega_{it}E_t = P_{it}Y_{it}$.[19] Competitive goods markets and profit maximization together imply that a constant share of sectoral output is spent on the wage bill,

$$L_{it} = (1-\alpha_i)\omega_{it}\frac{E_t}{w_t}, \tag{19}$$

where ω_{it} is the share of sector i in household consumption expenditure.

The main prediction of the theory that we take to the data in the next section is the intratemporal consumption decision (Equation 16 and its empirical counterpart, 11). It provides a log-linear relationship between relative sectoral demand, relative sectoral prices, and the nonhomothetic CES index of real consumption. From the market-clearing Equation (19) note that

$$\frac{L_{it}}{L_{jt}} = \frac{1-\alpha_i}{1-\alpha_j}\,\frac{\omega_{it}}{\omega_{jt}}, \qquad i, j \in \mathcal{I}. \tag{20}$$

This implies that relative sectoral employment is proportional to relative expenditure shares. Thus, relative sectoral employment also follows the same log-linear relationship with relative prices and the index of real income. Equation (18) suggests that relative prices capture the effect of supply-side forces in the form of differential rates of productivity growth and heterogeneous capital intensities in the presence of capital deepening. Therefore, Equation (16) also offers an intuitive way to separate out the impact of demand and supply-side forces in shaping long-run patterns of structural change.

For the case in which there are three sectors, agriculture, manufacturing, and services,

[19] In our empirical applications, we account for sectoral trade flows.

Equation (16) also makes transparent how nonhomothetic CES can generate a steady decline in agricultural consumption (real and nominal), a hump-shaped pattern in manufacturing consumption and a steady increase in services. Suppose that relative prices are constant. In this case, the evolution of relative expenditure shares ω_i and real sectoral consumption C_i depend only on the evolution of aggregate real consumption C and the relative ranking of income elasticity parameters. If income elasticity parameters satisfy $\epsilon_a < \epsilon_m < \epsilon_s$, as real aggregate income C grows, the relative consumption of manufacturing to agriculture and of services to manufacturing steadily grow. Thus, the share of consumption raises monotonically for services and declines monotonically for agriculture. For manufacturing, it is clear that it asymptotically has to decline too. But, it is also easy to see that it can temporarily rise and generate an inverted U-pattern if the initial share of agricultural consumption is sufficiently high.[20]

Finally, we note that Equation (16) also shows how our model can generate a positive correlation between relative sectoral consumption in real and nominal terms, as it is observed in the data (Herrendorf et al., 2014). The combination of the price effect and gross complementarity ($\sigma < 1$) implies that relative real sectoral consumption should negatively correlate with relative sectoral prices, as is the case for homothetic aggregators whith gross complementarity.[21] However, our demand system has an additional force: income effects. The nonhomothetic effect of aggregate consumption affects both series in the same way and thus is a force that makes both time series co-move. Thus, if income effects are sufficiently strong, both time series can be positively correlated. We revisit this result in Sections 3 and 6, where we show that this is indeed the case empirically.

2.2.2 Constant Growth Path

We characterize the asymptotic dynamics of the economy when sectoral total factor productivities grow at heterogeneous but constant rates. In particular, let us assume that sectoral productivity growth is given by

$$\frac{A_{it+1}}{A_{it}} = 1 + \gamma_i, \qquad i \in \{0\} \cup \mathcal{I}. \tag{21}$$

[20]Under the assumption that relative prices remain constant, Equation (16) implies that the relative growth rate of sector i to sector j is $(\epsilon_i - \epsilon_j)g_C$, where g_C denotes the growth rate of C. Using the fact that shares add up to one, we can write the growth rate of the manufacturing sector expenditure share as $g_m = \left((\epsilon_m - \epsilon_a)\omega_a - (\epsilon_s - \epsilon_m)\omega_s\right) g_C$. Thus, the sign of g_m depends on whether $(\epsilon_m - \epsilon_a)\omega_a \lesseqgtr (\epsilon_s - \epsilon_m)\omega_s$. Since $\epsilon_a < \epsilon_m < \epsilon_s$, the sign boils down to whether $\omega_a \lesseqgtr \frac{\epsilon_s - \epsilon_m}{\epsilon_m - \epsilon_a}\omega_s$. Thus, if the initial expenditure share in agriculture is sufficiently large to satisfy the previous inequality, the evolution of manufacturing will be hump-shaped. Since ω_a decreases monotonically and ω_s increases monotonically over time, g_m changes sign at most once.

[21]To see why, note that relative real consumption is decreasing in relative prices with an elasticity of $-\sigma$, while relative nominal expenditure is increasing with an elasticity of $1 - \sigma$. Thus, with CES aggregators and gross complementarity, real and nominal variables are negatively correlated–a counterfactual prediction.

Under this assumption, the competitive equilibrium of the economy converges to a path of constant per-capita consumption growth. Along this path, nominal consumption, investment, and the stock of capital all grow at a rate dictated by the rate of growth of the investment sector γ_0. Denoting the rate of growth of real consumption by γ^*, the share of each sector i in consumption expenditure also exhibits constant growth along a constant growth path, characterized by constants

$$1 + \xi_i \equiv \lim_{t\to\infty} \frac{\omega_{it+1}}{\omega_{it}} = \frac{(1+\gamma^*)^{\epsilon_i}}{\left[(1+\gamma_0)^{\frac{\alpha_i}{1-\alpha_0}}(1+\gamma_i)\right]^{1-\sigma}}. \tag{22}$$

Given the fact that expenditures shares have to be positive and sum to 1, Equation (22) allows us to find the rate of growth of real consumption as a function of sectoral income elasticity, factor intensity, and the rates of technical growth. The next proposition presents these results that characterize the asymptotic dynamics of the competitive equilibrium.

Proposition 1. *Let γ^* be defined as*

$$\gamma^* = \min_{i\in\mathcal{I}} \left[(1+\gamma_0)^{\frac{\alpha_i}{1-\alpha_0}}(1+\gamma_i)\right]^{\frac{1-\sigma}{\epsilon_i}} - 1. \tag{23}$$

Assume that γ^* satisfies the following condition

$$(1+\gamma_0)^{-\frac{\alpha_0}{1-\alpha_0}} < \beta(1+\gamma^*)^{1-\theta} < \min\left\{\frac{(1+\gamma_0)^{-\frac{\alpha_0}{1-\alpha_0}}}{\alpha_0 + (1-\alpha_0)(1+\gamma_0)^{-\frac{1}{1-\alpha_0}}(1-\delta)}, 1\right\}. \tag{24}$$

Then, for any initial level of capital stock, K_0, there exists a unique competitive equilibrium along which consumption asymptotically grows at rate γ^*,[22]

$$\lim_{t\to\infty} \frac{C_{t+1}}{C_t} = 1 + \gamma^*. \tag{25}$$

Along the this constant growth path, (i) the real interest rate is constant, $r^* \equiv (1+\gamma_0)^{1/(1-\alpha_0)}/\beta(1+\gamma^*)^{1-\theta} - 1$, (ii) nominal expenditure, total nominal output, and the stock of capital grow at rate $(1+\gamma_0)^{\frac{1}{1-\alpha_0}}$, and (iii) only the subset of sectors $\mathcal{I}^*$ that achieve the minimum in Equation (23) employ a non-negligible fraction of workers.

Equation (23) shows how the long-run growth rate of consumption is affected by income elasticities, ϵ_i, rates of technological progress, γ_i, and sectoral capital intensities, α_i. To build intuition, consider the case in which all sectors have the same capital intensity, and

[22] Here we follow the terminology of Acemoglu and Guerrieri (2008) in referring to our equilibrium path as a constant growth path. Kongsamut et al. (2001) refer to this concept as generalized balanced growth path. As these papers, we normalize the investment sector price. See Duernecker et al. (2017a) for a discussion on the connection between this price normalization and chained-price indexing of real consumption.

preferences are homothetic. Then, since $\sigma \in (0,1)$, Equation (23) implies that the long-run growth rate of real consumption is pinned down by the sectors with the lowest technological progress, as in Ngai and Pissarides (2007). Consider now the case in which there is also heterogeneity in income elasticities. In this case, sectors with higher income elasticity and faster technological progress can co-exist in the long-run with sectors with low income elasticity and slow technological progress. The intuition is that the agents shift their consumption expenditure toward income-elastic good, as they become richer, and away from goods that are becoming cheaper due to technical progress. Finally, the role of heterogeneity in capital shares in shaping the long-run rate of consumption growth is analogous to the role of technological progress, as they both ultimately shape the evolution of prices.

Which sectors survive in the long-run? At all points in time, all sectors produce a positive amount of goods, and its production grows over time. In relative terms, however, only the subset of sectors $\mathcal{I}^*$ satisfying Equation (23) will comprise a non-negligible share of total consumption expenditure in the long-run. Indeed, if the initial number of sectors is finite, generically only one sector survives in the long-run.

2.2.3 Transitional Dynamics

To study the transitional dynamics of the economy, we focus on the special case where all sectors have a common capital intensity $\alpha \equiv \alpha_i$ for all i.[23] Let us normalize each of the aggregate variables by their respective rates of growth, introducing normalized consumption expenditure $\tilde{E}_t \equiv (1+\gamma_0)^{-\frac{t}{1-\alpha}} E_t$, per-capita stock of capital $\tilde{k}_t \equiv (1+\gamma_0)^{-\frac{t}{1-\alpha}} K_t$, and real per-capita consumption $\tilde{C}_t \equiv (1+\gamma^*)^{-t} C_t$. Using the assets market clearing condition, we can translate Equations (14) and (17) into equations that characterize the evolution of the normalized aggregate variables

$$\tilde{k}_{t+1} = (1+\gamma_0)^{-\frac{1}{1-\alpha}} \left[\tilde{k}_t^{\alpha} + \tilde{k}_t (1-\delta) - \tilde{E}_t\right], \tag{26}$$

$$\left(\frac{\tilde{C}_{t+1}}{\tilde{C}_t}\right)^{\theta-1} \frac{\bar{\epsilon}_{t+1}}{\bar{\epsilon}_t} \frac{\tilde{E}_{t+1}}{\tilde{E}_t} = \frac{1+\alpha\tilde{k}_t^{\alpha-1}-\delta}{1+r^*}, \tag{27}$$

[23]The online appendix characterizes the dynamics along an equilibrium path in the more general case with heterogeneous capital intensities α_i in a continuous-time rendition of the current model.

where the normalized consumption expenditure $\tilde{E}_t$ is a function of $\tilde{C}_t$ and the two functions of the growth in $\tilde{C}_t$, that is, $\tilde{C}_{t+1}/\tilde{C}_t$, as

$$\left(\frac{\tilde{E}_{t+1}}{\tilde{E}_t}\right)^{1-\sigma} = \sum_{i=1}^{I} \omega_{it} \left(\frac{\tilde{C}_{t+1}}{\tilde{C}_t}\right)^{\epsilon_i} (1+\xi_i)^t \,, \tag{28}$$

$$\frac{\bar{\epsilon}_{t+1}}{\bar{\epsilon}_t} = \left(\frac{\tilde{E}_t}{\tilde{E}_{t+1}}\right)^{1-\sigma} \sum_{i=1}^{I} \omega_{it} \left(\frac{\epsilon_i}{\bar{\epsilon}_t}\right) \left(\frac{\tilde{C}_{t+1}}{\tilde{C}_t}\right)^{\epsilon_i} (1+\xi_i)^t \,. \tag{29}$$

Starting from any initial levels of normalized per-capita consumption $\tilde{C}_0$ and stock of capital $\tilde{k}_0$, we can find that period's allocation of expenditure shares $\boldsymbol{\omega}_t$ using Equations (2) and (3), and compute the normalized per-capita consumption and stock of capital of the next period using Equations (26) and (27). Proposition (1) establishes that the equilibrium path exists, is unique, and is therefore fully characterized by the dynamic equations above.

At the aggregate level, the transitional dynamics of this economy deviates from that of the standard neoclassical growth model because the household's elasticity of intertemporal substitution (EIS) varies with income. Goods with lower income elasticity are less intertemporally substitutable. Since the relative shares of high and low income-elastic goods in the consumption expenditure of households vary over time, the effective elasticity of intertemporal substitution of households correspondingly adjusts. Typically, as income rises, low income-elastic goods constitute a smaller share of the households' expenditure and therefore the effective elasticity of intertemporal substitution rises over time. When the economy begins with a normalized stock of capital $\tilde{k}_t$ below its long-run level $\tilde{k}^*$, the interest rate along the transitional path exceeds its long-run level. With a rising elasticity of intertemporal substitution, households respond increasingly more strongly to these high interest rates. Therefore, the accumulation of capital and the fall in the interest rate both accelerate over time.[24]

In general, the transitional dynamics of the economy can generate a rich set of different patterns of structural transformation depending on relative income elasticity parameters and the rates of productivity growth of different sectors $\{\epsilon_i, \gamma_i\}_{i=1}^{I}$. In Section 3 we will estimate the demand-side parameters of the model using both micro and macro level data. We will then use these parameters to calibrate the model in Section 6 and study the implications for the evolution of sectoral shares as well as the paths of interest rate and savings.

[24]The mechanism operates with all preferences that feature nonhomotheticity (e.g., King and Rebelo (1993) discuss it in the context of a neoclassical growth model with Stone-Geary preferences). In the current model, if the rate of productivity growth in high income-elastic sectors is large enough, the share of these sectors may in fact fall over time, and the effective elasticity of intertemporal substitution of households may correspondingly fall. However, as we will see in the calibration of the model in Section 6, the empirically relevant case is one in which the share of more income-elastic goods rises as the economy grows.

3 Model Estimation

In this section, we bring our model to the data with two goals in mind. Our first goal is to show that nonhomothetic CES preferences, despite their parametric parsimony, provide a good account of the relation between the sectoral composition of the economy, relative prices and total expenditure. In this section, we focus on estimating the intraperiod problem described in the previous section between three sectors: $a$griculture, $m$anufacturing, and $s$ervices. In particular, we want to demonstrate that the estimated income elasticity parameters differ systematically across sectors (specifically, $\epsilon_a < \epsilon_m < \epsilon_s$), and that the relative sectoral demand is well-captured by our demand specification. Our second goal is to provide estimates of the structural parameters of the model (i.e., income elasticity parameters, $\{\epsilon_i\}_{i\in\mathcal{I}}$, and the elasticity of substitution, σ) that can be used to calibrate our model and study its transitional dynamics in Section 6.

We use household-level and aggregate data to estimate sectoral demands. We first analyze U.S. data on final-good household expenditure. We then estimate the model with aggregate data using a panel of 39 countries over the post-war period.[25] The sample of countries covers a wide range of growth experiences, including developing countries (e.g., Botswana and India), miracle economies (e.g., South Korea and Taiwan) and developed economies (e.g., the U.S. and Japan). Section 5 presents additional empirical results, including household-level estimates for India and finer sectoral aggregations that go beyond the three sectors studied in this section.[26]

3.1 Empirical Strategy

Our estimation strategy for both micro and macro data relies on the intratemporal allocation of consumption across different goods implied by nonhomothetic CES preferences. We consider the empirical counterpart to Equation (11), with the price and expenditure shares of manufacturing goods m as the base.[27] Let n denote the unit of observation, which can be either a household (when using micro data), or aggregate outcomes of a country (when using macro data). Equation (11) implies that the log-relative expenditure shares satisfy the

[25] The growth model developed in Section 2.2 abstracts from within-country dispersion of income and assumes all households are identical. In Section B of the online appendix, we derive approximate expressions for aggregate sectoral demand in an environment featuring within-country heterogeneity in income. In particular, we show that the equations characterizing household-level and aggregate-level allocation of expenditure are identical up to first order of approximation in the standard deviation of the logarithm of consumption expenditure, if the latter has a symmetric distribution such as the log-normal distribution (see Battistin et al., 2009, for evidence for the log-normality of the distribution of total consumption expenditure across households).

[26] We also report the estimates from U.S. household data when utility is specified over value added (rather than final good expenditure). We present alternative econometric specifications to estimate our demand system.

[27] In Section 5 we present estimation results when we use services instead of manufacturing as base goods, and show that we obtain similar results.

following moment condition

$$\log\left(\frac{\omega_{it}^{n}}{\omega_{mt}^{n}}\right)=(1-\sigma)\log\left(\frac{p_{it}^{n}}{p_{mt}^{n}}\right)+(1-\sigma)\left(\epsilon_{i}-1\right)\log\left(\frac{E_{t}^{n}}{p_{mt}^{n}}\right)+(\epsilon_{i}-1)\log\omega_{mt}^{n}+\zeta_{i}^{n}+\nu_{it}^{n} \quad (30)$$

for $i\in\mathcal{I}_{-m}=\{a,s\}$, where ω_{it}^{n} and p_{it}^{n} denote the share of consumption and the price of sector-i goods in unit n at time t, E_t^n denotes total expenditure of unit n at time t, ζ_i^n accounts for relative taste parameters, and ν_{it}^{n} denotes the error term.

Equation (30) defines a system of log-linear equations for $i\in\mathcal{I}_{-m}$ with constraints in its coefficients. First, there is the constraint that σ is the same across equations. Second, for each equation, the product of the coefficient on relative prices, $(1-\sigma)$, and expenditure share on manufacturing, (ϵ_i-1) has to be equal to the coefficient on expenditure, $(1-\sigma)(\epsilon_i-1)$. We estimate the parameters $\{\sigma,\epsilon_i,\zeta_i^n\}_{i\in\mathcal{I}_{-m}}$ of this system of equations via the generalized method of moments (GMM).[28] As we discussed in Section 2.1, these parameters fully characterize the underlying preferences, since we can freely normalize one taste parameter and one income elasticity parameter.[29]

3.2 Data Description

We use the U.S. Consumer Expenditure Survey (CEX) for studying relative sectoral demand at the household level. These data report the composition of household consumption expenditures on different final goods. For the aggregate data, we combine Groningen's 10-Sector Database with aggregate consumption measures from the ninth version of the Penn World Table (PWT). The aggregate data contains sectoral employment, value-added output, sectoral prices and consumption per capita. We briefly discuss each of these datasets in what follows. We present more details on the data sources in Section D of the online appendix.

U.S. Household Expenditure Data We use U.S. household quarterly consumption data for the period 1999-2010 from the Consumption Expenditure Survey (CEX). In the CEX, each household is interviewed about their expenditures for up to four consecutive quarters. Our data construction is based on Aguiar and Bils (2015), who in turn follow very closely Heathcote et al. (2010) and Krueger and Perri (2006). As these authors, we focus on a sample of urban households with a present household head aged between 25 and 64. We also use the same total income measure (net of taxes) and household controls as Aguiar and Bils

[28]Note that relative to Equation (11) we have subtracted $\log\omega_{mt}^{n}$ from both sides of the equation. Since we estimate ϵ_i directly and impose in the moment equation that the coefficient on $\log\omega_{mt}^{n}$ is (ϵ_i-1), we would obtain exactly the same estimates if we do not subtract $\log\omega_{mt}^{n}$ from both sides of the equation. We choose this formulation to build continuity with the estimation of aggregate data that relies on relative shares being proportional to employment shares.

[29]We also note that our estimation strategy is different from the one proposed in Hanoch (1975). Hanoch proposes an estimation based on double differences that can only idenfity $I-2$ income elasticity parameters.

(2015). These controls are demographic dummies based on age range of the household head (25-37, 38-50, 51-64), household size dummies ($\leq$2, 3-4, 5+) and dummies for the number of household earners $(1, 2+)$.

The key difference from Aguiar and Bils (2015) is that we construct our consumption categories to match expenditure in agriculture, manufacturing and services. We follow Herrendorf et al. (2013) to construct these three categories. The agricultural sector is composed by food-at-home expenditures. The main expenditure categories for the manufacturing sector are vehicles, housing equipment, other durables, clothing, shoes and personal care items. For services, these are housing, utilities, health, food away from home, television subscriptions and other entertainment fees.

We combine the CEX data with disaggregated regional quarterly price series from the BLS's urban CPI (CPI-U). Similar to Hobijn and Lagakos (2005) and Hobijn et al. (2009), we construct the price for each sector faced by a household by taking the household expenditure-weighted average of the log-price of each of the expenditure categories belonging to the sector. Since expenditure weights are household-specific, this allows us to, albeit imperfectly, account for the fact that the effective price for each sector may be different across households.

Aggregate Data Our aggregate data comes from two sources. The sectoral data comes from Groningen's 10-Sector Database (Vries et al., 2014). The 10-Sector Database provides a long-run internationally comparable dataset on sectoral measures for 10 countries in Asia, 9 in Europe, 9 in Latin America, 10 in Africa and the United States. The variables covered in the data set are annual series of production value added (nominal and real) and employment for 10 broad sectors starting in 1947. In our baseline exercise, we aggregate the ten sectors into agriculture, manufacturing and services following Herrendorf et al. (2013). In Section 5, we estimate our model for 10 sectors. Our consumption expenditure per capita data comes from the ninth version of the Penn World Tables, (Feenstra et al., 2015). Combining these two datasets gives us a final panel of 39 countries with an average number of observations of 42 years per country. As we have discussed, the countries in our sample span very different growth trajectories. For example, the ratio of the 90th to the 10th percentile of consumption per capita in year 2000 is 18.2.[30]

[30] We The countries in our sample are Argentina, Bolivia, Botswana, Brazil, Chile, Colombia, Costa Rica, Denmark Ethiopia, France, Germany, Ghana, Hong Kong, India, Indonesia, Italy, Japan, Kenya, Korea, Malawi, Malaysia, Mauritius, Mexico, Netherlands, Nigeria, Perú, Philippines, Senegal, Singapore, South Africa, Spain, Sweden, Taiwan, Tanzania, Thailand, United Kingdom, United States of America, Venezuela and Zambia. In the previous version of this paper (Comin et al., 2015), we used the Barro-Ursua dataset instead of the PWTv9.0. The main advantage of the PWTv9.0 is that it allows us to estimate our model for all countries in the 10-sector database (39) relative to Barro-Ursua, for which we only had data for 25 countries. We find very similar results using both data sets.

3.3 Household-Level Results

Empirical Strategy and Identification We use the specification in Equation (30), where in this case n denotes different households observed in the data, under the assumptions $\zeta_i^n \equiv \beta_i' X^n + \delta_{ir}$ and $\nu_{it}^n \equiv \delta_{it} + \tilde{\nu}_{it}^n$. The first assumption imposes the constraint that the cross-household heterogeneity in time-invariant taste parameters can be fully explained as a linear function of the vector X_i^n of household characteristics discussed above (age, household size, number of earners dummies) and sector-region (ir) fixed effects. The second assumption allows for a dyad of sector-time (it) fixed effects to absorb potential aggregate consumption shocks. This specification identifies income elasticities based on the within-region covariation between expenditure shares and total household expenditures, controlling for household characteristics.

To deal with potential measurement error and endogeneity issues, we use instruments for the observed measures of household expenditures and relative prices. First, we follow Aguiar and Bils (2015) and instrument household expenditures (total and on the reference good) in a given quarter with the annual household income after taxes and the income quintile of the household. The instruments capture the permanent household income and are therefore correlated with household expenditures without being affected by transitory measurement error in total expenditures.[31] Second, we instrument household relative prices with a "Hausman" relative-price instrument. Each of the prices used in the relative-price instrument is constructed in two steps. First, for each sub-component of a sector, we compute the average price across regions excluding the own region. Then, the sectoral price for a region is constructed using the average region expenditure shares in each sub-component as weights.[32] These price instruments capture the common trend in U.S. prices while alleviating endogeneity concerns due to regional shocks (and measurement error of expenditure).[33]

We present our estimation results using two alternative weighting schemes. We use the household weights provided in the CEX data to map the household sample to be representative of the entire population. Additionally, we re-weight households by their total level of expenditure to bridge the gap with the estimates with aggregate-level data.[34] Comparing the alternative weighting schemes allows us to examine the stability of the estimated parameters

[31] The measure of total household income corresponds to a separate question in the CEX and is not constructed adding household expenditures over the year. Boppart (2014a) also instrument quarterly expenditure levels by household income.

[32] Formally, we instrument relative prices $\log(p_{it}^r/p_{mt}^r)$ with $\log(p_{it}^{-r}/p_{mt}^{-r})$, where $\log p_{it}^{-r}$ for $i \in \{a, m, s\}$ constructed as follows. Suppose that for sector j we have information on the price of subcomponents $k \in \{1, \ldots, K\}$, then $\log p_{it}^{-r} = \sum_{k=1}^{K} \hat{\omega}_{kt}^r \log \hat{p}_{kt}^{-r}$ where $\hat{\omega}_{kt}^r$ denotes the average expenditure share of k in region r and $\hat{p}_{kt}^{-r}$ denotes the log of the average price in the U.S. excluding region r. We have verified that constructing the instrument using the price in the own region or the average national price delivers similar results.

[33] Using the average price in the U.S. excluding the own region addresses the concern of regional shocks, while capturing the common component of prices across regions. Using average expenditures in the region addresses the concern of mismeasurement of household expenditure shares in that region to the extent that the mismeasurement averages out in the aggregate.

[34] We thank the editor for this suggestion.

Table 1: Estimates, CEX Final Good Expenditure, $\epsilon_m = 1$.

	(1)	(2)	(3)	(4)	(5)	(6)
σ	0.26	0.28	0.28	0.20	0.31	0.33
	(0.04)	(0.04)	(0.03)	(0.05)	(0.04)	(0.05)
$\epsilon_a - 1$	-0.80	-0.83	-0.81	-0.70	-0.95	-0.97
	(0.06)	(0.07)	(0.06)	(0.07)	(0.09)	(0.10)
$\epsilon_s - 1$	0.65	0.68	0.75	0.67	0.82	0.85
	(0.07)	(0.07)	(0.06)	(0.07)	(0.09)	(0.10)
Expenditure Re-Weighted	N	Y	N	Y	N	Y
Region FE	N	N	Y	Y	Y	Y
Year × Quarter FE	N	N	N	N	Y	Y

Notes: All regressions include household controls (described in the text). Standard errors clustered at the household level shown in parenthesis. The number of observations is 60,925 in all regressions.

across income groups.

Estimation Results Table 1 reports our estimation results. Columns (1) and (2) report the estimates when we control only for household characteristics X^h but we do not include any time or region fixed effects. Column (1) corresponds to the weighting scheme that replicates the U.S. population, while column (2) corresponds to the expenditure re-weighted estimates. In both cases we find very similar estimates. The estimates show that the income elasticity parameter is lower for agriculture relative to manufacturing ($\epsilon_a - 1 = -0.80$ in column 1) and higher for services relative to manufacturing ($\epsilon_s - 1 = 0.65$ in column 1). The price elasticity estimates are less than one ($\sigma = 0.26$ in the first column) suggesting that agriculture, manufacturing and services are gross complements in household preferences.

We subsequently add region and time fixed effects in columns (3) to (6). We find very similar coefficients to those in columns (1) and (2). An important observation from Table 1 is that our estimates of relative income elasticities do not change significantly between the specifications with U.S. population weights (odd columns) and those with expenditure weights (even columns). This finding suggests that the assumption in our model of income elasticity parameters $\{\epsilon_i\}_{i\in\mathcal{I}}$ being constant across income groups, provides a good description of the data.[35]

Table 2 explores the stability of the slope of the relative demand in expenditure across different subsamples of the data. First, we split households in two groups: above and below the annual median income in the sample. Columns (1) and (2) report the estimates of specification

[35] Table E.1 in the online appendix reports the regression of our instruments on aggregate expenditure and prices, which would correspond to the "first-stage" in a 2SLS setting, and show that the coefficients have the expected sign and are significant at conventional levels.

Table 2: Sample Splits, CEX Final Good Expenditure

	< P50	> P50	Pre '05	Post '05
	(1)	(2)	(3)	(4)
σ	0.35	0.31	0.33	0.25
	(0.07)	(0.05)	(0.07)	(0.05)
$\epsilon_a - 1$	-0.89	-0.99	-0.98	-0.92
	(0.17)	(0.12)	(0.15)	(0.08)
$\epsilon_s - 1$	0.75	0.59	0.74	0.65
	(0.19)	(0.16)	(0.14)	(0.10)

Regressions estimated using CEX-replicate weights. Households controls included in all regressions (as described in the main text). All regressions include Region and Year $\times$ Quarter fixed effects. Standard errors clustered at the household level. The estimation in columns (2) and (3) is performed imposing the constraint $\epsilon_a \geq 0$ (by estimating an exponential transformation of the variable). The corresponding standard errors are computed using the delta method.

(30) when we estimate it separately for each subsample. We find that the estimated elasticities are not significantly different from each other. We also study the stability of the estimates over time and estimate our baseline regression in the pre- and post-2005 sub-samples. Columns (3) and (4) report the estimates, where we again find estimates that are close in magnitude.

A key prediction of nonhomothetic CES is that relative expenditure shares are log-linear in the real consumption index. We use the estimates from Table 1 in our demand Equation (2) to obtain a measure of the real consumption index, C_t^n.[36] Our theory (e.g., Equation 6) implies that log-relative expenditure shares are a linear function of log-relative prices and the log real consumption index. Figure 2 plots the (binned) residuals after all controls and relative price have been partialled-out from our the instrumented real consumption measure and relative expenditure shares. As implied by our model, we find that residual variation in relative shares is well approximated by a log-linear function of residual consumption, both for agriculture relative to manufacturing (2a) and services relative to manufacturing (2b).

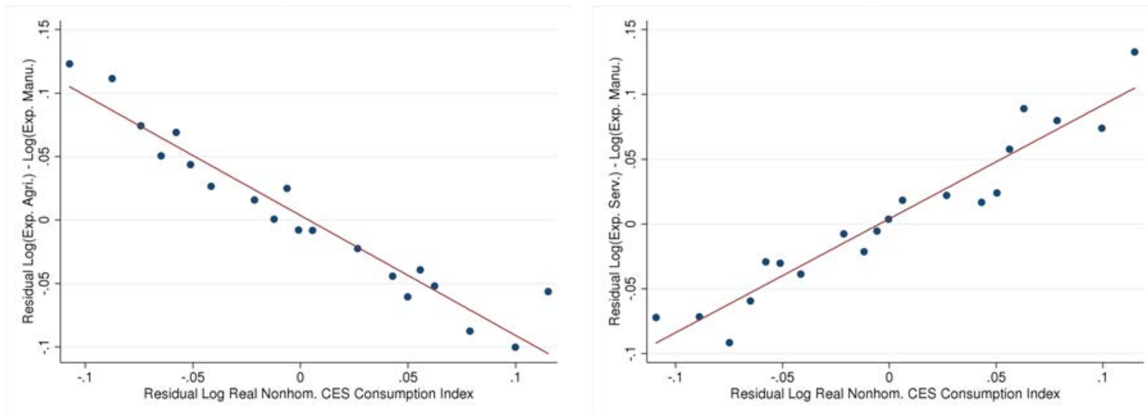
3.4 Cross-Country Aggregate-Level Results

After estimating the model with household data, we explore the ability of nonhomothetic CES preferences to account for the broad patterns of structural transformations observed across countries during the post-war period.

[36] We use the estimates in column (5). We use Equation (2) for each sector and compute our measure of C_t^n using a weighted average across the three sectors. We use the average household expenditure per sector as weights.

Figure 2: Partial Correlation of the Real Consumption Index and Relative Log Expenditure Shares, CEX

(a) Agriculture relative to Manufacturing (b) Services Relative to Manufacturing



These plots depict the (binned) residuals corresponding to the average value of 20 equal-sized bins of the data. The red line depicts the linear regression between the residualized variables.

Empirical Strategy and Identification To estimate our model with aggregate data, we employ a strategy very similar to the one we used for micro data. Since our model assumes that each country is inhabited by homogeneous households, the specification discussed in Section 3.1 applies, where n now stands for different countries observed in our data (see footnote 25 for further discussion). In our baseline exercise, we estimate our model from the patterns of structural change in employment. In particular, Equation (20) implies that relative sectoral consumption expenditures are proportional to relative sectoral employment shares, yielding[37]

$$\log\left(\frac{L_{it}^n}{L_{mt}^n}\right) = (1-\sigma)\log\left(\frac{p_{it}^n}{p_{mt}^n}\right) + (1-\sigma)(\epsilon_i - 1)\log\left(\frac{E_t^n}{p_{mt}^n}\right) + (\epsilon_i - 1)\log\omega_{mt}^n + \zeta_i^n + \nu_{it}^n. \quad (31)$$

The term ζ_i^n denotes a country-sector fixed effect. In addition, we include controls for log-sectoral exports and imports in Equation (31) to account for the fact that some goods can be traded, thus affecting the sectoral composition of employment.[38] Using employment rather

[37]In this case ζ_i^n also absorbs country-specific heterogeneity in sectoral capital intensity, α_i^n's.

[38]We note also that our sectoral price measures have embedded the effect of traded intermediate inputs and that total expenditures embed the effect of trade on income. We use the "trade detail" data from the PWT to construct sectoral exports and imports. Agricultural trade flows correspond to trade in food and beverages. Manufacturing trade flows correspond to trade in industrial supplies, fuels and lubricants, capital goods, transport equipment and consumer goods. Our baseline specifications includes directly as control log-sectoral exports and imports. Alternatively, we can rely on a model with exogenous trade flows to derive less flexible estimation equations that control for trade flows and are consistent with the model. In this case, we need to assume that factor intensities are identical in the production function of the same sector across different

Table 3: Cross-Country Estimates, $\epsilon_m = 1$

	World		OECD		Non-OECD	
	(1)	(2)	(3)	(4)	(5)	(6)
σ	0.57	0.50	0.25	0.35	0.63	0.48
	[0.32,0.69]	[0.26,0.71]	[0.20,0.66]	[0.03,0.55]	[0.06,0.74]	[0.34,0.75]
$\epsilon_a - 1$	-0.98	-0.89	-0.99	-0.99	-0.91	-0.80
	[-1.13,-0.41]	[-1.14,-0.46]	[-1.00,-0.38]	[-1.00,-0.66]	[-1.15,-0.58]	[-1.14,-0.40]
$\epsilon_s - 1$	0.17	0.21	0.27	0.25	0.18	0.37
	[0.07,0.60]	[0.03,0.67]	[0.03,0.55]	[0.09,1.95]	[0.11,2.08]	[0.03,0.67]
Country × Sector FE	Y	Y	Y	Y	Y	Y
Trade Controls	N	Y	N	Y	N	Y
Observations	1626	1626	492	492	1134	1134

Notes: Bootstrapped 95% confidence intervals clustering at the country level shown in square brackets (computed through bootstrapping 50 samples with replacement). The estimation in columns (3) and (4) is performed imposing the constraint that $\epsilon_a \geq 0$ (by estimating an exponential transformation of the variable).[39]

than value-added shares in Equation (31) is our favored specification for investigating the cross-country data because it does not use the price data (an explanatory variable) to construct the dependent variables (Section 5 shows that we find similar estimates if we use value-added shares as dependent variables).

Our cross-country estimation relies on the within-country variation of employment shares, expenditure, and relative prices to identify the price and income elasticities. The identification assumption to obtain consistent estimates is that, for each country, the shocks to relative prices and income are uncorrelated with the relative demand shocks ν_{it}^n. This assumption would be violated if, for example, sectoral taste shocks (which are part of ν_{it}^n) are correlated with aggregate demand or relative price shocks. To alleviate these endogeneity concerns, we estimate our model separately for OECD and Non-OECD countries and show that the estimates do not change significantly across sub-samples. While the estimates could in principle be biased in both cases, this would require sectoral taste shocks (or any other omitted variable) to be correlated with aggregate demand or relative price shocks in the same way across sub-samples, which we deem less likely.

countries approach. We can then use the accounting identity $p_{it}^n C_{it}^n = p_{it}^n Y_{it}^n - NX_{it}^n$, where NX_{it}^n denotes the nominal value of net exports in sector i, time t and country n. It follows that the expressions for sectoral employment in sector i should be adjusted by terms involving the observed values of $NX_{it}^n / p_{it}^n Y_{it}^n$. Using this alternative model-driven controls for trade flows, we have found results very similar to what is presented here.

[39] We report bootstrapped standard errors because the weighting matrix in the second step of the GMM estimation when we allow for clustering at the country-level becomes almost singular, as we include a large number of fixed effects.

Estimation Results Table 3 reports the results obtained from estimating (31) for the full sample of 39 countries and separately for OECD and Non-OECD sub-samples. Columns (1) and (2) report the estimates for our entire sample with and without trade controls, respectively. The estimated income elasticity parameter is lower for agriculture relative to manufacturing ($\epsilon_a < 1$) and larger for services compared to manufacturing ($\epsilon_s > 1$). The price elasticity is also less than unity ($\sigma = 0.57$). Introducing trade controls hardly changes our estimates, as shown in column (2).[40]

Columns (3) and (4) report the estimated elasticities for OECD countries and columns (5) and (6) report the estimates for the Non-OECD sample. The estimates are similar for the two sub-samples. In fact, we cannot reject the null that the estimates for the income elasticity parameters are the same for both sub-samples at conventional levels. We find that our estimates of income elasticity parameters appear to be stable across countries of different levels of income. The point estimates of σ vary more across subsamples. We find values between 0.25 and 0.63. However, they always remain less than unity in all specifications. The estimates appear more stable when controlling for sectoral trade. Moreover, the estimate of any specification falls within the confidence interval of the estimates in the other sub-samples. Overall, the similarity of the estimates across sub-samples is reassuring, as we deem less likely that unobserved relative demand shocks may be correlated with relative prices and income in the same way across two such different groupings of countries.

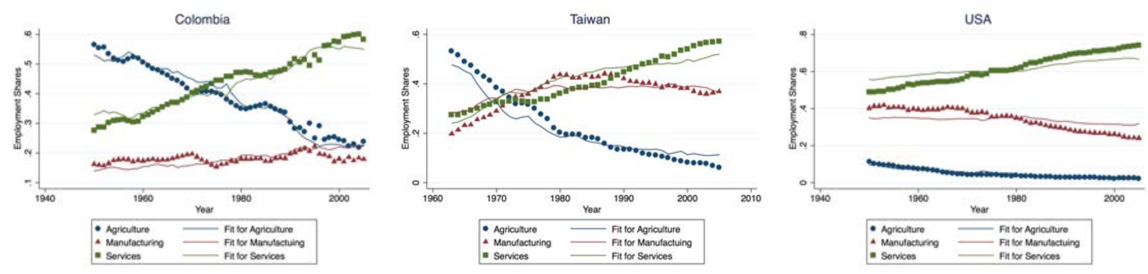
4 Accounting for Structural Change

After estimating the model, we turn to studying structural change through the lens of our estimated model. To this end, we first illustrate the overall model fit. We then quantify the contribution of changes in relative prices and income to the observed reallocations of sectoral employment.

Figure 3 plots the actual and the predicted employment shares from Column (2) of Table 3 for three countries, Colombia, Taiwan and the United States (figures F.1, F.2 and F.3 in the online Appendix show the plot for all countries in our sample). Figure 3 shows that, despite its parsimony, the fitted model is capable of capturing the patterns of reallocation across sectors for countries that experienced very different growth processes in the post-war period. Recall that we impose a common set of elasticities across all countries, $\{\sigma, \epsilon_a, \epsilon_s\}$, and only allow for constant taste differences across countries (captured as country-sector fixed

[40] In the previous version of the paper (Comin et al., 2015), we used the Barro-Ursua measures of real consumption. In that case, we only had data for 25 countries. Almost all of the differences from our current sample come from the fact that we now have more Non-OECD countries. We find similar estimates using either sample. Also, we can reject the null hypothesis that $\log(E_t)$ and $\log(E_t/p_{mt})$ have unit roots in our sample. Thus, the variables in our regression are not cointegrated. Finally, in terms of assessing fit, if we regress log-relative employment shares on the predicted log-relative employment shares from column (1) plus a constant, we find R^2 value for agriculture to manufacturing of 0.95 and 0.37 for services to manufacturing.

Figure 3: Example of Fit with common preference parameters $\{\sigma, \epsilon_a, \epsilon_s\}$.



effects, ζ_i^n, whose model counterpart would be $\log \Omega_i^n$). For Taiwan, we note that our model generates a hump-shape for the share of employment in manufacturing, albeit of a smaller magnitude than the one observed in the data. If we were to estimate country-specific elasticity parameters instead of using a common set of elasticities for all countries, we would obtain a close-to-perfect fit for almost all countries in the sample.[41]

4.1 Contribution of Relative Price and Income Effects

Next, we use our estimated model to quantify how much of the variation in employment shares is accounted for by changes in relative prices and real consumption. Table 4 reports the model fit that we obtain when we include different subsets of regressors in the estimation.[42] The first row shows the R^2 when only country-sector fixed effects are included in the regression. We find that fixed effects account for the bulk of the overall variation in the data. This is not surprising since countries differ widely in their measures of relative employment shares, and country-sector fixed effects capture the average differences across countries. Sectoral fixed effects account for 93% and 51% of the total variance in log-relative agriculture to manufacturing employment and log-services to manufacturing employment, respectively.

The second row in Table 4 reports the R^2 when we include both country fixed effects and relative prices. Effectively, this exercise corresponds to assuming a homothetic CES. We find that the increases in R^2 are relatively modest. For log-relative employment share of

[41] Figures F.4, F.5 and F.6 in the online Appendix report the fit for all countries with country-specific elasticities.

[42] Note that the objective function of the GMM estimation used in our baseline is different from OLS and thus R^2 measures are hard to interpret in a GMM context. To make sure that our R^2 measures are interpretable, we proceed as follows. We use our demand estimates from column 1 in Table 3 and construct theory-consistent measures of the nonhomothetic CES real consumption index, $\log \hat{C}_t^n$. We then use the demand structure implied by Equation (6) to re-estimate the demand system $\log\left(\frac{L_{it}^n}{L_{mt}^n}\right) = (1-\sigma)\log\left(\frac{p_{it}^n}{p_{mt}^n}\right) + (\epsilon_i - 1)\log \hat{C}_t^n + \zeta_i^n + \nu_i^n$, for $i \in \{a, m\}$, where ζ_i^n denote country-sector fixed-effects and ν_i^n is the error term. We then compare the R^2 of this specification with those found by including only a subset of the regressors. Since this approach involves linear seemingly unrelated regressions, the resulting R^2 values have the usual goodness of fit interpretation. The estimated values we obtain for the elasticity parameters are very similar (and not statistically different from) to the ones obtained in Table 3. The estimates for $\epsilon_a - 1$, $\epsilon_s - 1$ and σ are, denoting standard errors in parenthesis, $-0.77(0.32)$, $0.27(0.08)$ and $0.83(0.14)$, respectively.

Table 4: Accounting for Structural Change, R^2

	$\ln\left(\frac{\text{Agri.}}{\text{Manu.}}\right)$		$\ln\left(\frac{\text{Serv.}}{\text{Manu.}}\right)$	
Regression includes:	R^2	% Within	R^2	% Within
Only Fixed Effects	0.93	-	0.51	-
FE + Relative Prices	0.94	14%	0.52	2%
FE + Real Consumption Index	0.97	57%	0.57	12%
All Regressors	0.97	58%	0.58	14%

agriculture to manufacturing the R^2 increases from 0.93 with only fixed effects to 0.94, and for log-relative service to manufacturing it goes from 0.51 to 0.52. In the third row, we report the same exercise when only country-sector fixed effects and real consumption are included. This yields more substantial increases. For log-relative agriculture to manufacturing employment we find an R^2 of 0.97, while for log-relative services to manufacturing it goes up to 0.57. When we include all regressors, we find R^2 values of 0.97 and 0.58 for relative employment shares in agriculture to manufacturing, and services to manufacturing, respectively.[43]

Next, we focus on analyzing structural change within countries. That is, we want to quantify how much of the sectoral reallocation over time within a country can be accounted for by the model relative to the total within-country variation in the data. To this end, we compute the changes in the R^2 when we add additional regressors after we partial out country-sector fixed effects.

We start analyzing the within-country variation in log-relative agriculture to manufacturing employment. When we only include relative prices, we find that the model can account for 14% of the total within-country variation. If we only include real consumption, it accounts for 57% of the total within-country variation. Including both hardly improves the overall fit to 58% (since prices and real consumption are not independent). Thus, the maximal contribution we can attribute to relative prices of the within-country variation accounted by our model is 24% (= 14/58). Conversely, we conclude that real consumption contributed at least to 76% of the within-country variation accounted by our model. For log-relative services to manufacturing employment, we find that prices contribute to 2% of the overall within-country variation, for 12% of real consumption. Thus, out of the overall 14% of the within-country variation accounted by the model, relative prices contribute to 14%(= 2/14) of the variation and real consumption to 86%.[44] If we break down our analysis between OECD and Non-OECD countries, a similar picture emerges (see Table 14 in the appendix).[45]

[43] We also note that the overall R^2's for the World are the same whether we include trade controls or not (up to the second decimal).

[44] The relative low value of the within-R^2 partly reflects the existence of outliers, as depicted in Figure 4b.

[45] Including only sector-country fixed effects and the nonhomothetic real consumption index accounts for virtually all the within variation for both groups of countries when we look at log-relative agriculture to

Table 5: Correlation of Nominal and Real Value Added

	Data	Model
Agriculture/Manufacturing	0.96	0.94
Services/Manufacturing	0.87	0.71

Note: Model-predicted values constructed using the structural estimates from column (2) in Table 3.

Overall, Table 4 paints a picture consistent with the view that the nonhomotheticity of demand plays a dominant role in accounting for structural change in our panel of countries. If we attribute all the covariation in prices and consumption to prices in our full sample, we find that the within-country variation accounted for by real consumption is 76% for the log-relative agriculture to manufacturing equation, and 86% for log-relative services to manufacturing. Thus, we conclude that nonhomotheticities account for over 75% of the structural change in our sample.[46,47]

4.1.1 Structural Change in Real and Nominal Value Added

A salient feature of the patterns of structural transformation observed in the data on sectoral value added is that they appear regardless of whether we document them in nominal or real terms (Herrendorf et al., 2014). To investigate our model's ability to account for this fact, we combine our structural cross-country estimates from Table 3 and the sectoral demands, (16), to generate the predicted evolution of nominal and real sectoral demands.

manufacturing employment and for at least 96% when we look at log-relative services to manufacturing. In contrast, including only relative prices in the log-relative agriculture to manufacturing regression accounts for at most 11% and 25% for OECD and non-OECD countries, respectively. For log-relative services to manufacturing employment, we find that relative prices account at most for 46% and 25% for OECD and Non-OECD.

[46]In the working paper version of the paper (Comin et al., 2015), we also show that the likelihood-ratio tests of including price and consumption data to a model with only country-sector fixed effects are significant. The increase in the likelihood is much higher when we add consumption data than when we add relative price data.

[47]This conclusion differs from Boppart (2014a) who studies the evolution of services relative to the rest of the economy in the U.S. during the postwar period. He finds that the contribution of price and income effects are roughly of equal sizes. First, the differences in the results are partly due to the differences in the level of sectoral aggregation. If we confine our analysis to the U.S. and lump together agriculture and manufacturing into one sector, we find that price effects account for 26% of the variation. Second, our specification of demand is different from Boppart (2014a) because in our specification the price elasticity is constant. In contrast, Boppart's demand system implies that the price elasticity of services relative to the rest of consumption is declining as the economy grows. As noted by Buera and Kaboski (2009), since the relative expenditure and value added of services grows at a faster rate than services relative price, a declining price elasticity automatically increases the explanatory power of relative prices. We have checked that a declining variable elasticity is quantitatively important for the decomposition exercise. We have generated a synthetic panel of countries with two sectors (agriculture plus manufacturing, and services) with preferences given by nonhomothetic CES calibrated to capture the key features of our true cross-country panel. We then do two decomposition exercises with these data: one estimating a nonhomothetic CES demand, and another estimating a PIGL demand. We find that the within variation accounted for by prices is four times larger with PIGL than with nonhomothetic CES.

Table 5 reports the correlation between nominal and real shares both in our estimated model and in the data. We find that the model is able to generate correlation similar to the data. In particular, the correlation between the nominal and real relative demand of agricultural goods to manufactures is 0.94 in our model, while in the data it is 0.96. For services, the model generates a correlation of 0.71 while in the observed correlation in the data is 0.87.

The success in generating a correlation between nominal and real measures of the same magnitude as in the data is important. Note that it is an out-of-sample test of the predictions of our model, since our estimation has not targeted the evolution of sectoral shares of real or nominal value added. As discussed in Section 2, if we had used a homothetic CES framework, the correlation generated by the model would have been negative because the price elasticity of substitution is smaller than one. This implies that real and nominal variables can not have positive co-movement with homothetic CES.[48] Of course, any specification of preferences that asymptotically converges to a homothetic CES (e.g., Stone-Geary) would face a similar problem in explaining the nominal-real co-movement. This also holds for homothetic demand systems with variable elasticity of substitution (since agriculture, manufacturing and services are gross complements). In our framework, there is a second force that makes the positive co-movement possible: income effects. The nonhomothetic effect of real consumption affects in an identical way real and nominal variables (this term is $C^{\epsilon_i-\epsilon_m}$, see Equation 2). At the estimated parameter values, the implied income effects are sufficiently strong to overcome the relative price effect and make both time series co-move positively. Therefore, we argue that the ability to simultaneously account for the evolution of real and nominal sectoral shares is a key feature of our specification of nonhomotheticity.

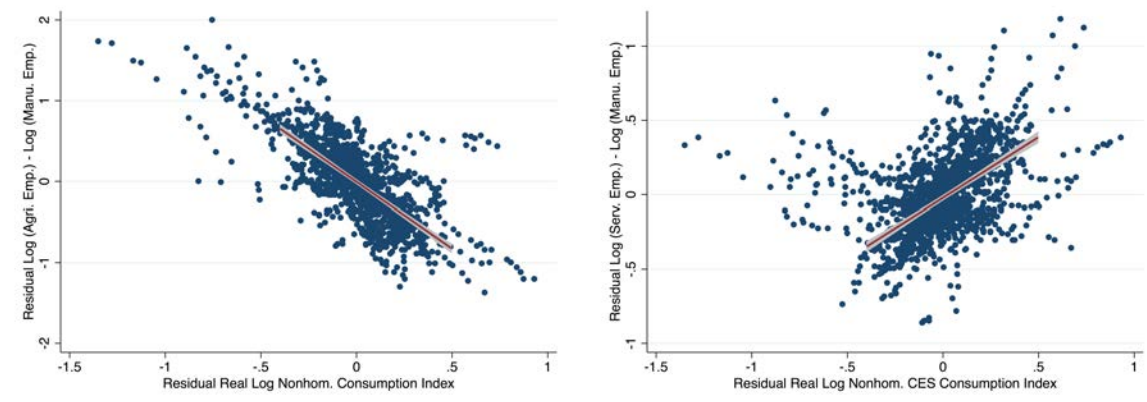
4.1.2 Partial Correlation of the Real Consumption Index and Employment Shares

As we have discussed, a key prediction of our demand system is that relative shares are well approximated by a log-linear relationship in relative prices and the nonhomothetic CES real consumption index. We depict in Figure 4 the partial correlation between employment shares and the nonhomothetic CES real consumption index (after partialling out country-sector fixed effects and relative prices). We see that for both log-relative agriculture to manufacturing and log-relative services to manufacturing employment a substantial part of the variation is well captured by a simple log-linear relationship. We take this result as additional supporting evidence for nonhomothetic CES being able to capture salient patterns in the data and nonhomotheticities playing an important role at all levels of development.

[48]To see that, note that the relative trend in nominal values $\omega_{i,t}/\omega_{j,t}$ would be proportional to $(p_{i,t}/p_{j,t})^{1-\sigma}$. For real values, $c_{i,t}/c_{j,t}$, would be proportional to $(p_{i,t}/p_{j,t})^{-\sigma}$. As $0 < \sigma < 1$, both trends would move in opposite directions.

[Figure 4](#): Partial Correlation of the Real Consumption Index and Relative Log Employment Shares Shares, Cross-Country Data

(a) Agriculture relative to Manufacturing (b) Services Relative to Manufacturing



4.2 Fit Comparison with Stone-Geary and PIGL prefrences

We compare the cross-country fit of our model to alternative specifications where we replace the nonhomothetic CES aggregator with Stone-Geary (Herrendorf et al., 2014) and PIGL preferences (augmented to three sectors as described in Boppart, 2014b). Appendix D introduces these two demand systems and the estimation procedure. Here, we highlight two similarities between these demand specifications and ours that allow us to perform this comparison. First, the number of parameters to be estimated in these two demand systems is the same as that in nonhomothetic CES. Second, as with nonhomothetic CES, there exist sets of parameters for these two demand systems such that the expenditure shares are constant for each country-sector (they correspond to Cobb-Douglas with expenditure shares equal to country-sector averages). Thus, we benchmark the fit of these three demand systems relative to using the country-sector average as a prediction for each sector. This amounts to computing the R^2 for agriculture, manufacturing and services shares after subtracting country-sector means in each sector.[49]

We find that the within-R^2 for Stone-Geary is 0.14, meaning that 14% of the residual

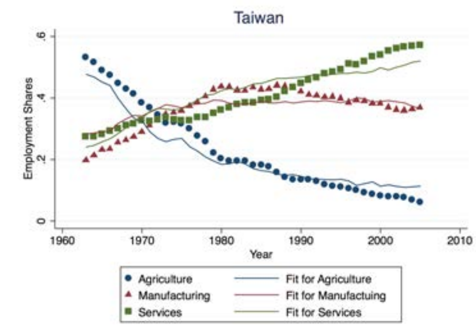
[49] The R^2 compares the sum of squared errors of the model fit to the sum of squared errors obtained by using the country-sector average as a prediction. Formally, $R^2 = 1 - \frac{1}{I}\sum_{i=1}^{I}\left(\sum_{t=1}^{N}(y_{it}^c - \hat{y}_{it}^c)^2 / \sum_{t=1}^{N}(y_{it}^c - \bar{y}_i^c)^2\right)$ where N denotes the total number of observations per sector, I, the number of sectors, y_{it}^c, observed employment shares in sector i and country c, $\hat{y}_{it}^c$, predicted employment shares, $\bar{y}_i^c$ the sample average of y_{it} for country c in sector i, and $i \in \mathcal{I} = \{a, m, s\}$. We also note that the estimates used to compute the within-R^2 for nonhomoethetic CES correspond to the structural estimates in column (1) of Table 3. Finally, note that in this exercise we are computing the R^2 on employment shares (and not relative log-shares). The reason is that the estimation of the three demand systems is based on different left-hand-side variables (e.g., Stone-Geary is not log-linear and it is estimated on shares directly). We chose to benchmark the fit of the three demand systems based on the level of employment shares as it is arguably the most basic object of interest.

Figure 5: Comparison of Demand Systems and Fit for Taiwan

(a) Overall Fit Comparison

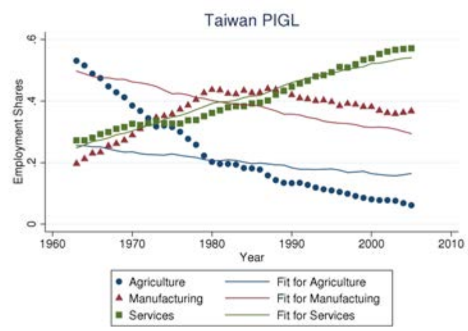
Model:	Within-R^2
Stone-Geary	0.14
PIGL	0.13
Nonhom.-CES	0.29

(b) Nonhomothetic CES Fit for Taiwan



(c) Stone-Geary Fit for Taiwan

(d) PIGL Fit for Taiwan



variation in agricultural, manufacturing and service shares after we partial out country-sector averages is accounted for by the Stone-Geary demand system. The corresponding number for nonhomothetic CES is more than two-times larger, 0.29. The intuition for the worse fit of Stone-Geary is that income effects are very low for rich countries, since for high levels of income the subsistence levels responsible for introducing the nonhomotheticity become negligible (see, also, Dennis and Iscan, 2009).[50] For the PIGL demand system we find an R^2 of 0.13, which is very similar to that of Stone-Geary. PIGL preferences track the trends in services more accurately than Stone-Geary due to the fact that they feature a non-vanishing nonhomotheticity of the service sector. However, they under-perform relative to nonhomothetic CES mostly because they assume a homothetic composite between agriculture and manufacturing, while nonhomothetic CES allows for sector-specific nonhomotheticities. Figure 5 illustrates the fit for the case of Taiwan for the three demand systems.[51]

[50] For the U.S., the value of the nonhomothetic terms $p_{it}\bar{c}_i$ relative to total expenditure is never higher (in absolute terms) than 0.1%, which suggests that nonhomotheticity are insignificant. The highest values of the nonhomotheticities in the sample are 37% for agriculture and 18% for services.

[51] We report the fit for all countries for both Stone-Geary and PIGL in Online Appendix H.

5 Additional Empirical Results and Robustness Checks

In this section, we present alternative econometric specifications to estimate nonhomothetic CES in Section 5.1 (online Appendix I compares estimates of the different econometric specifications using Monte-Carlo simulations). We also discuss several extensions and robustness checks of our empirical results for both micro and macro data.

We begin by presenting two simple log-linear specifications that can be used to identify the rank-ordering and relative magnitudes of income elasticity parameters. The first one controls directly for prices, while the second specification replaces prices by fixed effects resulting from the interaction of income-level dummies with time and region dummies. The latter approach can be used to identify income elasticity parameters in the absence of price data. We apply this approach to the Indian National Sample Survey household data and to the CEX. We find very similar elasticity parameters in both samples using this specification. We also find that the estimates are consistent with our baseline specification.

We further discuss two additional strategies: *(i)* a non-linear specification that directly incorporates the average cost index and *(ii)* an iterative linear least squares approach that uses a second-order approximation of the real consumption index. We also use this second-order approximation of the real consumption index to show that using an off-the-shelf price index instead of the exact price index implied by our theory tends to over-state the growth rate of real consumption, but the bias is quantitatively small.

Next, we discuss four extensions and robustness checks for the baseline exercises presented in Section 3. First, we present estimation results that go beyond the three sectors classification. For the CEX data we separate housing from the rest of services. For the macro data, we present estimates where we disaggregate manufacturing and services into nine sub-sectors. Second, for the US CEX data, we present estimates with demand specified over value added rather than final good expenditure. We show that the same qualitative results hold for price and income elasticities. Third, we present our estimation results on the aggregate data when we use value added shares instead of employment shares as dependent variables. Finally, we show that our results are robust to using services as a base sector instead of manufacturing.

5.1 Alternative Econometric Specifications

In this subsection, we present a number of alternative strategies for estimating nonhomothetic CES demand. Let n denote the unit of observation. Consider the following empirical counterpart to Equation (16) that characterizes the intratemporal allocation of consumption across different goods $i \in \mathcal{I}_{-m}$

$$\log\left(\frac{\omega_{it}^n}{\omega_{mt}^n}\right) = (1-\sigma)\log\left(\frac{p_{it}^n}{p_{mt}^n}\right) + (\epsilon_i - \epsilon_m)\log\left(\frac{E_t^n}{P_t^n}\right) + \zeta_i^n + \nu_{it}^n, \tag{32}$$

where ω_{it}^n and p_{it}^n stand for the share of consumption expenditure and the price of sector-i goods in unit n at time t, E_t^n stands for total expenditure of the unit at time t, P_t^n stands for the price index of the unit at time t, and ζ_i^n stands for the taste parameter of the good, which potentially varies across units, and it may also include additional controls (as discussed below). The key challenge in using this specification lies in the mapping between observed data to the price index of real consumption P_t^n implied by nonhomothetic CES preferences. Below, we discuss several different strategies for dealing with this problem.[52]

5.1.1 Log-Linear Specification in Expenditures

The following observation allows us to employ a fairly simple specification to identify the rank ordering of income elasticity parameters: any valid cardinalization of the log-real consumption index $\log C$ is increasing with log-total expenditure $\log E$ after we control for prices. Building on this insight, we consider a specification where we replace the true log-real consumption index $\log C$ with log-total expenditure $\log E$ and log prices on the right hand side,

$$\log\left(\frac{\omega_{it}^n}{\omega_{mt}^n}\right) = \sum_{j\in\{a,m,s\}} \varsigma_{ij} \log p_{jt}^n + (\tilde{\epsilon}_i - \tilde{\epsilon}_m) \log E_t^n + \zeta_i^n + \nu_{it}^n \tag{33}$$

for $i \in \{a, s\}$.[53] Lemma 4 in Appendix C shows that this specification identifies $\epsilon_i - 1$ up to a scaling factor. In other words, the ratio of estimates $(\tilde{\epsilon}_s - \tilde{\epsilon}_m)/(\tilde{\epsilon}_a - \tilde{\epsilon}_m)$ is a consistent estimator of $(\epsilon_s - 1)/(\epsilon_a - 1)$. More generally, this approach identifies $I - 2$ income elasticity parameters and allows us to find the rank-ordering of income elasticity parameters with a simple log-linear specification. Tables 8 and 9 in the appendix present the results of the estimation on the household and aggregate data, respectively. In the last two rows of each table, we show that the ratios $(\tilde{\epsilon}_s - \tilde{\epsilon}_m)/(\tilde{\epsilon}_a - \tilde{\epsilon}_m)$ closely resemble the corresponding ratios $(\epsilon_s - 1)/(\epsilon_a - 1)$ found in our baseline specifications in Tables 1 and 3.[54]

5.1.2 Specification Without Price Data

In some instances, specially when dealing with household survey data, price data may not be available and it may not be possible to fully estimate Equation (32) or our baseline specification, Equation (30). This is the case for our Indian household data. Inspired by the specification in Section 5.1.1, we derive an approach that approximately retrieves the nonhomotheticity parameters of our demand system, up to a scaling factor. We estimate a model where we substitute prices with a full set of interactions between region r, time t, sector i,

[52] For other estimation strategies that also rely on structural equations see Sposi (2016), Duernecker et al. (2017b) and Lewis et al. (2018).

[53] Note that this specification corresponds to the exercise in Figure 1 discussed in the introduction.

[54] Note that Lemma 4 also implies that there exists an alternative normalization of ϵ_m such that our baseline estimation results in Tables 1 and 3 would coincide with the estimates $\tilde{\epsilon}_i - \tilde{\epsilon}_m$ in Tables 8 and 9.

and household income quintile q fixed effects. Formally, we estimate the following system of equations for $i \in \mathcal{I}_{-m}$

$$\log\left(\frac{\omega_{it}^n}{\omega_{mt}^n}\right) = (\tilde{\epsilon}_i - \tilde{\epsilon}_m)\log E_t^h + \pi_{it}^{rq} + \zeta_i^n + \nu_{it}^n, \tag{34}$$

where π_{it}^{rq} denotes the $t \times r \times q$ fixed effects. This approach allows us to capture the effect of prices in a non-parametric way through π_{it}^{rq}. It imposes the assumption that households in the same income quintile, region and time should face the same prices and choose the same consumption bundles up to the heterogeneity that we allow in household characteristics through ζ_i^n.

Evidence from Indian Household Expenditure Data We present the estimation results based on this specification for a household survey in India (and also compare it to what we would obtain using the U.S. household expenditure data). We use data from rounds 64, 66 and 68 of the India National Sample Survey (NSS), which span the years 2007 to 2012. The NSS is a representative survey of household expenditure that collects repeated cross-sections of expenditures incurred by households in goods and services. We construct total expenditure in agriculture, manufactures and services following the same classification as for the CEX data. Household total income is constructed from an earnings measure that averages (potential) different sources of income within the household from different occupations, including received benefits (net of taxes).

We construct the controls in an analogous way to the U.S., with the only difference that the requirement of a prime age household is between ages of 18 and 60. In contrast to the U.S., we do not discard rural population as it represents more than half of the Indian population (around 55%). We instead show results for the entire sample and the subsample of urban households. The composition of expenditure in India is vastly different from the CEX. The average expenditure share in food and agricultural in the sample is 52% (versus 12% in the CEX). Expenditure shares in manufactures and services in the NSS represent, on average, 27% and 21% of total expenditure (versus 27% and 61% in the CEX).

Table 6 reports our estimation results from estimating equation (34).[55] Columns (1) and (2) report our baseline estimates using the full sample for the same two weighting schemes we used for the CEX. The first makes it representative of the Indian population and the second re-weights households according to their total expenditure. We find that the relative income elasticities between agriculture and manufacturing, $\tilde{\epsilon}_a - \tilde{\epsilon}_m$, are negative, and between services

[55] We use total household annual income as instrument for household quarterly expenditure. The first stage includes all controls used in the second stage. The coefficient on household annual income is positive and significant in all first-stage regressions. We note also that for columns (1) to (4) we augment specification (34) interacting the income-quintile×time×region with a dummy that indicates whether the household is classified as rural to account for potential constant difference between rural and urban households.

Table 6: Baseline Regression for India, NSS Expenditure

			< P50	> P50	Only Urban		U.S.
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
$\tilde{\epsilon}_a - \tilde{\epsilon}_m$	-0.63	-0.55	-0.62	-0.69	-0.57	-0.52	-0.61
	(0.05)	(0.07)	(0.05)	(0.29)	(0.05)	(0.07)	(0.09)
$\tilde{\epsilon}_s - \tilde{\epsilon}_m$	0.49	0.42	0.69	0.51	0.56	0.45	0.49
	(0.07)	(0.10)	(0.53)	(0.08)	(0.08)	(0.11)	(0.09)
	Comparison to US Baseline						
$(\tilde{\epsilon}_s - \tilde{\epsilon}_m)/(\tilde{\epsilon}_a - \tilde{\epsilon}_m)$	-0.78	-0.76	-1.11	-0.74	-0.98	-0.87	-0.80
$(\epsilon_s - 1)/(\epsilon_a - 1)$, baseline estimates	-0.81	-0.82	-0.84	-0.60	-0.81	-0.81	-0.81
Expenditure Re-Weighted	N	Y	N	N	N	Y	N
Time × Region × Inc. Quintile FE	Y	Y	Y	Y	Y	Y	Y
T. × Reg. × Inc. Quint. × Rural FE	Y	Y	Y	Y	N	N	N

Notes: Standard errors clustered at the year×state×district shown in parenthesis. All regressions include household controls (discussed in the main text). Observations for the full sample are 293,007. Urban observations are 118,681. Time fixed effects are the interaction of year×month. Region fixed effects are the interactions of state×district. The ratios for the baseline estimates corresponding to columns 1, 2 and 5 to 7 are computed from columns 1 and 2 of Table 1. The ratios for columns 4 and 5 are computed from Table 2.

and manufacturing, $\tilde{\epsilon}_s - \tilde{\epsilon}_m$, are positive. Likewise, comparing the point estimates in Columns (1) and (2) we see that they again remain stable across the two weighting schemes. We further explore the stability of the parameter estimates by applying the same specification separately to the subset of households above and below the median income level. Columns (3) and (4) show that we find very similar estimates of income elasticities in the two sub-samples. We show in Columns (5) and (6) that when we restrict our attention to urban households, we also obtain very similar estimates regardless of the weighting scheme used.

Column (7) shows the coefficient we would obtain if we run the same regression for the U.S. CEX data. Despite the differences in the level of development between the US and India, we find that the US estimates, $\tilde{\epsilon}_a - \tilde{\epsilon}_m = -0.61$ and $\tilde{\epsilon}_s - \tilde{\epsilon}_m = 0.49$, are very similar in magnitude to the estimates for India. The last two rows of the table compare the ratios $(\tilde{\epsilon}_s - \tilde{\epsilon}_m)/(\tilde{\epsilon}_a - \tilde{\epsilon}_m)$ obtained in the without-price specification, Equation (34), for India and the US with the ratios $(\epsilon_s - 1)/(\epsilon_a - 1)$ from the baseline US CEX estimation. As implied by Lemma 4, we find very similar ratios. For the US, the ratio of the estimates in column (7) of Table 6 is -0.80. The corresponding number from the baseline estimates in Table 1 is -0.81. For India, the ratio in column (1) is also very close to US estimates, -0.78, and fairly stable across specifications. We take these results as evidence of nonhomothetic CES being able to capture parsimoniously with the same income elasticity parameters demand conditions from very different stages of development.

5.1.3 Alternative Exact Estimation

In this section, we present an alternative exact demand specification to our baseline regression (30). We combine Equation (32) with the expression for the price index P_t^n in terms of observables and demand parameters derived in the theory section,

$$P_t^n = \left[\sum_{i=1}^{I} \left(\Omega_i^n (p_{it}^n)^{1-\sigma} \right)^{\chi_i} \left(\omega_{it}^n (E_t^n)^{1-\sigma} \right)^{1-\chi_i} \right]^{\frac{1}{1-\sigma}}, \tag{35}$$

where $\chi_i \equiv (1-\sigma)/\epsilon_i$ to obtain the following estimating equation

$$\begin{aligned} \log\left(\frac{\omega_{it}^n}{\omega_{mt}^n}\right) &= (1-\sigma)\log\left(\frac{p_{it}^n}{p_{mt}^n}\right) + (\epsilon_i - \epsilon_m)\log\left(E_t^n\right) + \log\left(\frac{\Omega_i^n}{\Omega_m^n}\right) \\ &\quad + \frac{\epsilon_i - \epsilon_m}{1-\sigma}\log\left(\sum_{j=1}^{I} \left(\Omega_j^n (p_{jt}^n)^{1-\sigma}\right)^{\chi_j} \left(\omega_{jt}^n (E_t^n)^{1-\sigma}\right)^{1-\chi_j}\right) + \nu_{it}^n. \end{aligned} \tag{36}$$

In contrast to our baseline specification (30), this specification is non-linear in observables.[56] One potential shortcoming of this formulation is that if the demand specification of the constant taste parameters $\{\Omega_j^n\}$ contains fixed-effects, one may run into an incidental parameters problem if the time dimension in the panel is short. According to our Monte-Carlo simulations reported in online Appendix I, this method appears to be more sensitive to mismeasurement than our baseline method. Table 13 reports the estimation results of Equation (36) on our CEX household data and aggregate data. Overall, the estimates are similar to their counterparts in Tables 1 and 3 .

5.1.4 Iterative Method

We present an alternative strategy to estimate Equation (32) that relies on iterating linear regressions. We establish the following result connecting the changes in the nonhomothetic CES index of real income to the changes in standard price indices. Let Δ denote the time difference operator between $t+1$ and t. In Appendix B, we show that up to second-order approximation

$$\Delta \log C_t^n \equiv \Delta \log\left(\frac{E_t^n}{P_t^n}\right) = \frac{1-\sigma}{\overline{\mathcal{E}}_t^n}\left(\Delta \log E_t^n - \Delta \log \mathcal{P}_t^n\right), \tag{37}$$

where $\mathcal{P}_t^n$ is the chained Törnqvist price index of consumption unit n at time t, $\Delta \log \mathcal{P}_t^n \equiv \frac{1}{2}\sum_{i=1}^{I}\left(\omega_{it}^n + \omega_{it+1}^n\right)\Delta \log p_{it}^n$ and $\overline{\mathcal{E}}_t^n$ is a correspondingly chained average of income elasticity

[56] To estimate this system of equations, we proceed by normalizing $\Omega_m^n = \epsilon_m = 1$ in Equation (36). Then the system can be estimated using a non-linear version of seemingly unrelated regressions or GMM. For the household data, we use GMM with the same set of instruments as in our baseline regression

parameters ϵ_i's, $\overline{\mathcal{E}}_t^n \equiv \frac{1}{2}\sum_i \left(\omega_{it}^n + \omega_{it+1}^n\right)\epsilon_i$. Since this result applies to the change in the real consumption index, we can apply this estimation strategy to cases with consumption unit fixed effects, such as our cross-country data. This way, the initial level of the real consumption index level is controlled by the fixed effect.

The structure of the demand system in Equation (37) is log-linear in observables and parameters except for $\overline{\mathcal{E}}_t^n$. This class of demand systems falls into the "conditionally linear demand systems," and therefore we can follow the iterative strategy suggested by Blundell and Robin (1999) to estimate the model parameters. In our first step, we simply estimate (32) using the Törnqvist price index instead of the true price index,

$$\log\left(\frac{\omega_{it}^n}{\omega_{mt}^n}\right) = (1-\sigma)\log\left(\frac{p_{it}^n}{p_{mt}^n}\right) + (\tilde{\epsilon}_i - \tilde{\epsilon}_m)\log\left(\frac{E_t^n}{\mathcal{P}_t^n}\right) + \zeta_i^n + \nu_{it}^n, \tag{38}$$

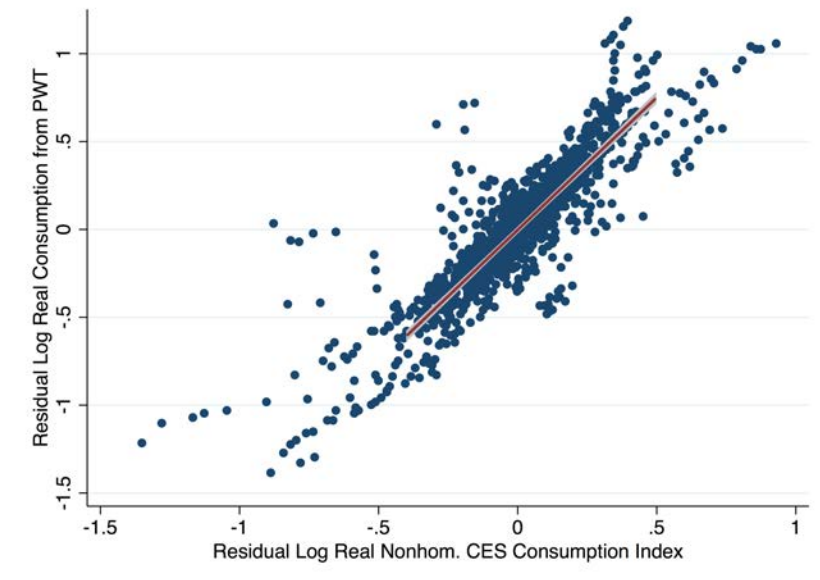
where E_t^n and $\mathcal{P}_t^n$ denote total expenditure and the corresponding specific Törnqvist price index for unit n at time t. With the estimates we obtain of σ, $\tilde{\epsilon}_a - \tilde{\epsilon}_m$ and $\tilde{\epsilon}_s - \tilde{\epsilon}_m$, we update our guess of $\log C_t^n$ according to (37). Note that this requires taking a stance on a particular normalization of $\{\epsilon_i\}$ to compute the term $\frac{1-\sigma}{\overline{\mathcal{E}}_t^n}$, e.g., $\epsilon_m = 1$. We then estimate (32), update again $\log C_t^n$ and so on, until convergence. Table F.4 in the online appendix reports our estimation results for our cross-country data. The income elasticity parameters are remarkably similiar to our baseline cross-country estimates in Table 3.[57]

Correlation between the Real Consumption Index and Real Consumption in PWT Last but not least, Equation (35) sheds light on why the relationship between relative log-value-added shares and consumption expenditure deflated by standard price indices is well approximated by a log-linear relationship as suggested in Figure 1 in the Introduction. The relationship would be exactly log-linear if the term in the denominator $\overline{\mathcal{E}}^n$ was constant. Indeed, this term is not constant by construction because, as total income grows, expenditure shares change and $\overline{\mathcal{E}_t^n}$ changes accordingly. However, $\overline{\mathcal{E}_t^n}$ is slow-moving. Consider for example the case of the United States. The ratio of nominal consumption deflated by a chained Törnqvist price in year 2000 relative to 1950 is 3.20. During the same period, we have that the ratio $\overline{\mathcal{E}_{2000}^{US}}/\overline{\mathcal{E}_{1950}^{US}} = 1.09$. implying a smaller increase of the true consumption index.[58] If we ignored the correction coming from the growth of $\overline{\mathcal{E}}$, we would expect a 3.2-fold increase in

[57] According to our Monte-Carlo simulations in online Appendix I, this method is the fastest among those that recover all demand parameters, but also the most sensitive to measurement error. We also report in Table F.5 of the online appendix the first-step estimates of this iterative procedure, Equation (38). The estimated differences in $\tilde{\epsilon}_i - \tilde{\epsilon}_m$ and $\tilde{\epsilon}_a - \tilde{\epsilon}_m$ are of the expected sign. As implied by Lemma 4, the estimates have the right magnitude, in the sense that the ratio between $(\tilde{\epsilon}_s - \tilde{\epsilon}_m)/(\tilde{\epsilon}_a - \tilde{\epsilon}_m)$ is around -0.8, which is what we find also for $(\epsilon_s - 1)/(\epsilon_m - 1)$ in our baseline estimates of Table 3.

[58] From 1950 to 2000, value added shares in agriculture, manufacturing and services went from 0.08, 0.41 and 0.51 to 0.02, 0.28 and 0.70, respectively. Normalizing $\epsilon_m = 1$ and taking the baseline estimates from the cross-country regression, this implies that $\overline{\mathcal{E}}_{1963} = 0.08 \times 0.02 + 0.41 \times 1 + 0.51 \times 1.17 = 1.01$ and $\overline{\mathcal{E}}_{2000} = 0.02 \times 0.02 + 0.28 \times 1 + 0.70 \times 1.17 = 1.10$. Note that this ratio is invariant to re-scaling ϵ_i's.

Figure 6: Real Nonhomothetic Consumption Index and Real Consumption from PWT



real consumption, while the true increase of the real consumption index is $3.2 \times \frac{1.01}{1.10} = 2.94$. This simple exercise also illustrates the broad fact that the growth rate of the real consumption index is smaller than the one implied by deflating expenditures by a chained Törnqvist index because $\overline{\mathcal{E}}_t^n$ grows as income grows. These insights hold more broadly across countries. Figure 6 plots the log-real nonhomothetic CES consumption index (constructed according to our theory) and the real consumption per capita measure reported in the Penn World Tables after partialling out country fixed effects to both measures. The figure shows that the these two measures are very well-approximated by a log-linear relationship.

5.2 Robustness Checks and Extensions

Beyond Three Sectors Jorgenson and Timmer (2011) have pointed out that in order to understand how structural transformation progresses in rich countries, it is important to zoom in the service sector, as it represents the majority of rich economies' consumption shares (see also Buera and Kaboski, 2012b). Our framework lends itself to this purpose, as it can accommodate an arbitrary number of sectors. We estimate our demand system to more than three sectors for both micro and macro data. In the CEX data, the largest broad expenditure category is housing services. We extend our analysis by separating housing from the rest of services and re-estimate our model with four sectors.[59] Table 10 in the appendix reports our findings. The price elasticity σ and the relative elasticity of agriculture to manufacturing, and

[59]We define housing as expenditure in dwellings plus utilities. We use the same set of instruments plus a price instrument for housing constructed in an analogous way to the other price instruments.

services to manufacturing (excluding housing), remain very similar to our baseline estimates in Table 1. We find that the relative income elasticity of housing to manufacturing $\epsilon_{\text{housing}} - 1$ is around 0.9, and thus somewhat larger than for the rest of services (albeit not statistically different at conventional significance levels).

For the macro data, we extend our estimation to the original sectors in Groningen's data: (1) agriculture, forestry and fishing, (2) mining and quarrying, (3) manufacturing, (4) public utilities, (5) construction, (6) wholesale and retail trade, hotels and restaurants, (7) transport, storage and communication, (8) finance, insurance, real state, (9) community, social and personal services.[60] Table 11 in the appendix reports the results estimating the demand system in an analogous manner to our baseline estimation (31) with additional sectors. We find that the smallest income elasticities correspond to agriculture and mining, while the highest correspond to the finance, insurance and real state category. Columns (2) and (3) show that the ranking of sectors in terms of income elasticity is very similar when we estimate OECD and Non-OECD countries separately.[61]

CEX Value-Added Demand Formulation So far, we have estimated household demand defined over households' final-good expenditure. Previous work has shown that the patterns of structural transformation are qualitatively similar whether we measure sectoral economic activity in terms of value-added or final-good expenditure shares (Herrendorf et al., 2013). We estimate our model defining household utility over the value added provided by each sector (rather than final good expenditure) and show that we obtain similar results. To do this, we follow Buera et al. (2015) and assign each consumption expenditure category to an industry of the U.S. input-output. We then compute the value added from each sector embedded in the final good expenditure of a given CEX category to express household demand over value added (see the online appendix for details). Table 12 in the appendix presents the estimates of our baseline specification where we use as dependent variable household expenditure shares measured in value added (instead of final good expenditure). We find that the estimates of ϵ_a are less than one, while the estimates of ϵ_s are above one. The estimates do not vary once we re-weight by expenditure, suggesting that estimates are stable across the income distribution. The point estimates we obtain for the price elasticity are in the 0.3 to 0.5 range. The magnitudes of all estimated elasticities appear to be overall very comparable to the expenditure formulation. The major difference is that the elasticity of agriculture ϵ_a appears to be somewhat smaller in the value added formulation (specially in the specification without

[60] We exclude government services since it is missing for a third of the observations. The data set also contains information on dwellings that are not constructed within the period, but this information is very sparse and we abstract from them. Note that in this case, the manufacturing sector is more narrowly defined than in the baseline estimation as it excludes mining and construction.

[61] In working paper version Comin et al. (2015), we exploit that the nesting properties of nonhomothetic CES are analogous to homothetic CES and we also report the estimation results from a nested CES structure where we estimate the demand for each of the sectors that belong to services or manufacturing separately.

time fixed effects, where we find values of ϵ_a between 0.02 and 0.06, for values around 0.2 for the expenditure formulation).

Macro Estimation with Value-Added Shares We investigate whether we find similar estimates to the baseline cross-country results when we use value-added output shares as dependent variable (instead of employment shares). Table F.2 in the online appendix reports the estimation results using shares of the sectoral in output value added as dependent variables in our baseline estimation, Equation (32). The estimates of the income elasticity parameters appear with the expected signs and overall similar magnitudes as in the baseline regression. The price elasticity appears to be somewhat smaller, around 0.2.

Alternative Sector as a Base Our empirical specification used manufacturing as a base sector. Here we show that our findings are robust to the use to services as reference sector in our empirical estimation.[62] Table F.3 in the online appendix shows our baseline results with services as a base sector. Columns (1) to (3) report the estimates with CEX data when we include household controls, region and time (year×quarter) fixed effects, respectively. The estimates are close to their baseline specification. In particular, the price elasticity estimates are not statistically different at conventional levels to the values found in Column (5) of Table 1. Columns (4) to (6) report the analogous exercise for the aggregate data, when estimated for the World, OECD and non-OECD countries. Again, we find similar estimates to the baseline results in Table 3.

6 Calibration Exercise

So far, we have only focused the predictions of the model regarding the intratemporal allocation of consumption expenditures. In this section, we rely on a simple calibration exercise to study the dynamic predictions of the model. As we discussed in Section 2.2.3, the qualitative properties of the transitional dynamics of the model heavily depend on the relationship between sectoral income elasticity and rates of productivity growth. In this section, we study the dynamics of the economy calibrated for the set of parameters estimated for the nonhomothetic CES preferences in Section 3. We then compare our model with simpler versions where we strip off different drivers of structural change. Relative to the Neoclassical Growth Model benchmark, we find that including any drivers of structural change in the model generates a slow-down of the convergence toward the long-run value.

[62] We choose services rather than agriculture for this exercise because services represent a more sizable fraction of expenditure, while agriculture is very small in many instances.

Table 7: Model Parameters for the Calibration Exercise

Parameter	γ_a	$\gamma_m = \gamma_0$	γ_s	σ	ϵ_a	ϵ_m	ϵ_s	α	θ	β	δ
Value	0.029	0.013	0.011	0.50	0.05	1.00	1.20	0.33	2.20	0.96	0.10

Model Calibration For the preferences, we rely on the values estimated for the sectoral income elasticity parameters ϵ_i's and the elasticity of substitution σ using the macro data in Section 3 and set $\Omega_i \equiv 1$ for all $i \in \{a, m, s\}$. We assume that capital intensity is the same across sectors and choose the standard value $\alpha = 0.33$ and a rate of depreciation of $\delta = 0.10$ for capital. For the sectoral rates of productivity growth, we assume that the rates of productivity growth in the investment sector and manufacturing are the same $\gamma_m = \gamma_0$, and calibrate them to the rate of growth of labor productivity observed in the in the postwar period in the US.[63] We then use the rates of decline in relative sectoral prices within the same period to calibrate the rates of growth of sectoral productivity for agriculture and services. Finally, we choose the value of the parameter θ such that the asymptotic value of the elasticity of intertemporal substitution matches 0.5, a reasonable number within the rangee of various estimates provided in the literature (e.g., Guvenen, 2006; Havránek, 2015). Table 7 presents the set of model parameters used for the calibration.[64]

Dynamics of Capital Accumulation First, we study how the presence of nonhomothetic CES demand changes the dynamics of the process of capital accumulation and the real interest rate. For this exercise, we compare the transitional dynamics of the calibrated model, in which both the parameters of income elasticity and the rates of productivity growth vary across sectors, with the following three different increasingly simpler models: 1) a model where the rates of productivity growth are homogeneous across sectors and the evolution of sectoral allocations is *exclusively* driven by nonhomotheticity in demand, 2) a model with homothetic preferences where income elasticity parameters are identical and, following Ngai and Pissarides (2007), the evolution of sectoral allocation are *exclusively* driven by the heterogeneity in sectoral rates of productivity growth, and 3) a standard neoclassical growth model (NGM) with homothetic CES preferences and homogeneous rates of productivity growth across sectors. We choose the parameters such that all models asymptotically converge to the same steady state as that of the calibrated model.[65]

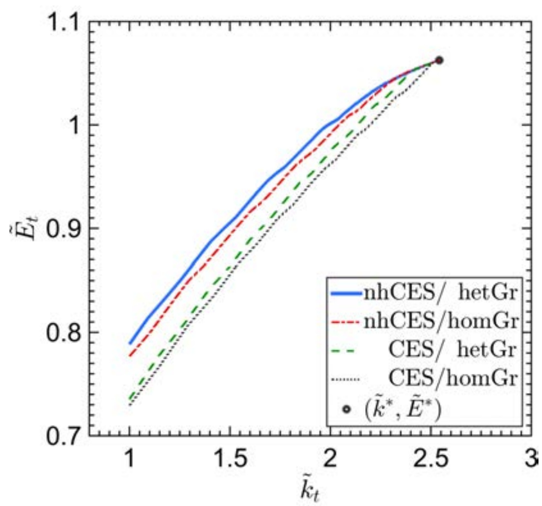
[63] Note that based on the model the rate of growth of labor productivity growth exceeds the rate of growth of multifactor productivity (TFP) γ_m by a factor $\alpha/(1-\alpha)$.

[64] Section C discusses the details of the method used for solving Equations (26) and (27) to derive the transitional dynamics of the model under these model parameters.

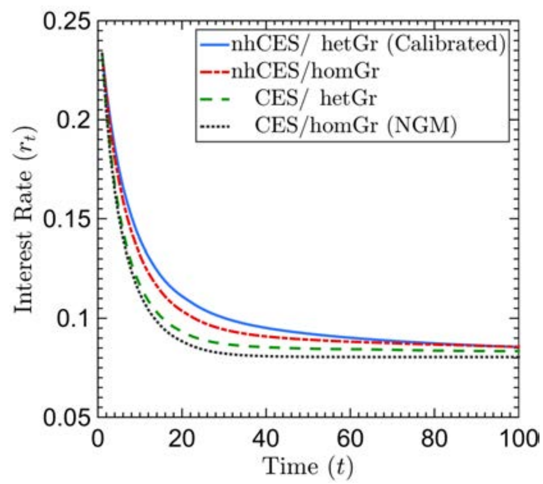
[65] Given the calibrated model parameters, the share of the service sector in consumption and employment converges to 1. Therefore, asymptotically all four models behave identical to a single-sector Neoclassical Growth Model where the instantaneous utility is defined as $C_t = C_{st}^{\epsilon_s/(1-\sigma)}$ and the productivity in the final good sector grows at rate γ_s.

Figure 7: Transitional Dynamics: Comparison with Neoclassical Growth Model (NGM)

(a) Path in $(\tilde{k}, \tilde{E})$–space



(b) Evolution of the Real Interest Rate



The evolution of the economy starting from initial per-capita stock of capital of $\tilde{k}_0 = 1 < \tilde{k}^* = 2.10$. The parameters for the Calibrated Model are given in Table 7. The nhCES/hetGr corresponds to the calibrated model with nonhomothetic CES and heterogeneous rates of sectoral productivity growth . The nhCES/homGr model corresponds to the case with nonhomothetic CES preferences and homogeneous rates of sectoral productivity growth, $\gamma_i = 0.011$ for $i \in \{a, m, s\}$. The CES/hetGr model corresponds to the case with homothetic CES preferences , $\epsilon_i = 1.20$ for $i \in \{a, m, s\}$. The CES/homGr corresponds to the case of the Neoclassical Growth Model (NGM) both the rates of productivity growth and the income elasticity parameters are homogeneous across sectors. $\tilde{k}^*$ and $\tilde{E}^*$ denote the asymptotic normalized per-capita stock fo capital and total consumption expenditure, respectively.

Beginning at an initially low level of per-capita stock of capital of $\tilde{k}_0 = 1 < \tilde{k}^* = 2.5$, Figure 7a shows the path of the economy from this initial condition toward its steady state in the space of the normalized per capita stock of capital and per-capita consumption expenditure. The figure compares these paths for all four models. All three models featuring structural change have higher values of total consumption expenditure relative to the NGM, which does not feature structural change, at all levels of per-capita stock of capital along the transitional path. As a result, we conclude based on this calibration that the presence of structural change implies a slower process of capital accumulation compared to the NGM, whether it is driven through the price or the income channel.

The slowdown in capital accumulation relative to the NGM benchmark is driven by the same two forces that shape the evolution of sectoral shares, namely, the inter-sectoral heterogeneity in the elasticities of income and the rates of productivity growth. In Section 2.2.3, we explained the mechanism behind the former force: the elasticity of intertemporal substitution gradually rises as their consumption shifts toward more income elastic goods that are also more intertemporally substitutable. The latter force is present in the benchmark theory of

Ngai and Pissarides (2007): over time, household consumption shifts toward the sectors with the slower rates of productivity growth, lowering the rate of fall in the price of consumption. If household consumption is intertemporally inelastic (in the sense that $\theta > 1$), conditional on a given level of interest rate, the slowdown in the rate of decline of prices results in faster growth of consumption expenditure.[66] As the figure shows, these two forces, as well as their potential interactions, contribute to the slowdown in the accumulation of capital in the calibrated model, although nonhomotheticity plays a larger role.

Dynamics of Interest Rate Figure 7b compares the implications of all four models above for the evolution of the real interest rate. The slower process of capital accumulation implies that the real interest rate also converges toward its steady state more slowly in all three models featuring structural change, relative to the NGM benchmark. Once again, the model that solely features nonhomotheticity grows more slowly compared to the one solely featuring heterogeneous sectoral rates of productivity growth. Nevertheless, the overall difference between the evolution of the real interest rate between the calibrated model and the corresponding NGM is relatively small: the time it takes for the real interest rate to go from 200% to 150% of its steady state level (half-life) is 9.1 years in the former relative to 4.4 years in the latter.[67]

7 Conclusion

This paper presents a tractable model of structural transformation that accommodates both long-run demand and supply drivers of structural change. Our main contributions are to introduce the nonhomothetic CES utility function to growth theory, show its empirical relevance and use its structure to decompose the overall observed structural change into the contribution of income and price effects. These preferences generate nonhomothetic Engel curves at any level of development, which are in line with the evidence that we have from both rich and developing countries. Moreover, for this class of preferences, price elasticities are independent from income elasticity parameters and they can be used for an arbitrary number of sectors. We argue that these are desirable theoretical and empirical properties.

We estimate these preferences using household-level data for the U.S. and India, and aggregate data for a panel of 39 countries during the post-war period. We argue that nonhomothetic CES preferences provide a good fit of the data despite their parsimony. Armed with the estimated price and income elasticity parameters, we then use the demand structure

[66] To better see this point, consider a constant income elasticity parameter across sectors, $\epsilon_i = \epsilon$, and log-linearize the Euler Equation (14) to find $\theta \Delta \log E_t \approx (1-\theta)(\gamma_0 - \overline{\gamma}_t) + r_t - (1-\beta)$, where $\overline{\gamma}_t \equiv \sum_i \omega_{it}\gamma_i$ is the consumption-weighted average of the sectoral rates of productivity growth (see also Equation 22 in Ngai and Pissarides, 2007). When $\sigma < 1$, over time $\overline{\gamma}_t$ falls and therefore $\Delta \log E_t$ grows if $\theta > 1$.

[67] The corresponding numbers in the model with nonhomotheticity and in the model with differential rates of productivity growth are 7.2 and 5.0, respectively.

to decompose the broad patterns of reallocation observed in our cross-country data into the contribution of nonhomotheticities and changes in relative prices. We find that over 75% of the variation is accounted for nonhomotheticities in demand.

To conclude, we believe that the proposed preferences provide a tractable departure from homothetic preferences. They can be used in other applied general equilibrium settings that currently use homothetic CES and monopolistic competition as their workhorse model, such as international trade. Also, as we discuss in Appendix A, it is possible to generalize nonhomothetic CES to generate non-constant elasticity parameters, which may be useful in some applications. Even in this case, nonhomothetic CES remains a local approximation (with constant elasticity parameters) and can be used to guide how the varying elasticities should be parametrized, e.g., by estimating nonhomothetic CES across sub-samples.

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Appendix Click Here for the Online Appendix

A General Nonhomothetic CES Preferences

In this section of the appendix, we provide an overview of the properties of the general family of nonhomothetic Constant Elasticity of Substitution (CES) preferences. We first introduce the general family and then specialize them to the case of isoelastic nonhomothetic CES functions of Section 2.1.

Prior Work Sato (1975) derived a general family of CES functions as the solution to a partial differential equation that imposes the constancy of elasticity of substitution. This family includes standard homothetic CES functions as well as two classes of separable and non-separable nonhomothetic functions. Hanoch (1975) showed that additivity of the direct or indirect utility (or production) function results in price and income effects that are non-trivially dependent on each other. He then introduced *implicit additivity* and derived a family of functions where the income elasticity of demand is not fully dependent on the elasticity of substitution. Our nonhomothetic CES functions correspond to the non-separable class of functions in the sense of Sato (1975), which also satisfy the condition of implicit additivity in the sense of Hanoch (1975). Finally, Blackorby and Russell (1981) have proved an additional property that is unique to this class of functions. In general, different generalizations of the elasticity of substitution to cases involving more than two variables, e.g., the Allen-Uzawa definition or the Morishima definition, are distinct from each other. However, for the class of nonhomothetic CES functions they become identical and elasticity of substitution can be uniquely defined similar to the case of two-variable functions.

General Definition Consider now preferences over a bundle $\boldsymbol{C}$ of I goods defined through an implicit utility function:

$$\sum_{i=1}^{I} \Omega_i^{\frac{1}{\sigma}} \left(\frac{C_i}{g_i(U)}\right)^{\frac{\sigma-1}{\sigma}} = 1, \tag{A.1}$$

where functions g_i's are differentiable in U and $\sigma \neq 1$ and $\sigma > 0$. To emphasize that this is a more general utility function, here we use U instead of C, which we reserve for the nonhomothetic CES presented in Section 2. Theorem 2 in Blackorby and Russell (1981) implies that property (5) holds if and only if the preferences can be written as equation (A.1). In this sense, the definition above corresponds to the most general class of nonhomothetic CES preferences.Standard CES preferences are a specific example of Equation (A.1) with $g_i(U) = U$ for all i's.

These preferences were first introduced, seemingly independently, by Sato (1975) and Hanoch (1975) who each characterize different properties of these functions. Here, we state and briefly prove some of the relevant results to provide a self-contained exposition of our theory in this paper.

Lemma 2. *If $\sigma > 0$ and functions $g_i(\cdot)$ are positive and monotonically increasing for all i, the function $U(\boldsymbol{C})$ defined in Equation (A.1) is monotonically increasing and quasi-concave for all $\boldsymbol{C} \gg 0$.*

Proof. Establishing monotonicity is straightforward. To establish quasi-concavity, assume to the contrary that there exists two bundles of $\boldsymbol{C}'$ and $\boldsymbol{C}''$ and their corresponding utility values U' and U'', such that $U \equiv U(\alpha \boldsymbol{C}' + (1-\alpha)\boldsymbol{C}'')$ is strictly smaller than both U' and U''. We then have for the case $\sigma \geq 1$

$$\begin{aligned}
1 &= \sum_i \Omega_i^{1/\sigma} \left(\alpha \frac{C_i'}{g_i(U)} + (1-\alpha) \frac{C_i''}{g_i(U)} \right)^{\frac{\sigma-1}{\sigma}}, \\
&> \sum_i \Omega_i^{1/\sigma} \left(\alpha \frac{C_i'}{g_i(U')} + (1-\alpha) \frac{C_i''}{g_i(U'')} \right)^{\frac{\sigma-1}{\sigma}}, \\
&\geq \alpha \sum_i \Omega_i^{1/\sigma} \left(\frac{C_i'}{g_i(U')} \right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha) \sum_i \Omega_i^{1/\sigma} \left(\frac{C_i''}{g_i(U'')} \right)^{\frac{\sigma-1}{\sigma}},
\end{aligned}$$

where in the second inequality we have used monotonicity of the g_i's and in the third we have used Jensen's inequality and the assumption that $\infty > \sigma > 1$. Since the last line equals 1 from the definition of the nonhomothetic CES functions valued at U' and U'', we arrive at a contradiction. For the case that $0 < \sigma < 1$, we can proceed analogously. In this case, the inequality signs are reversed in both lines and we also reach a contradiction. □

Demand Function Henceforth, we assume the conditions in Lemma 2 are satisfied. The next lemma characterizes the demand for general nonhomothetic CES preferences and provides the solution to the expenditure minimization problem.

Lemma 3. *Consider any bundle of goods that maximizes the utility function defined in Equation (A.1) subject to the budget constraint $\sum_i p_i C_i \leq E$. For each good i, the real consumption satisfies:*

$$C_i = \Omega_i \left(\frac{p_i}{E} \right)^{-\sigma} g_i(U)^{1-\sigma}, \tag{A.2}$$

where U satisfies

$$E = \left[\sum_{i=1}^{I} \Omega_i \left(g_i(U)\, p_i \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}. \tag{A.3}$$

and the share in consumption expenditure is given by

$$\omega_i \equiv \frac{p_i C_i}{E} = \Omega_i^{\frac{1}{\sigma}} \left(\frac{C_i}{g_i(U)} \right)^{\frac{\sigma-1}{\sigma}} = \Omega_i \left[g_i(U) \left(\frac{p_i}{E} \right) \right]^{1-\sigma}. \tag{A.4}$$

Proof. Let λ and ρ denote the Lagrange multipliers on the budget constraint and constraint (A.1), respectively:

$$\mathcal{L} = U + \rho \left(1 - \sum_i \Omega_i^{\frac{1}{\sigma}} \left(\frac{C_i}{g_i(U)} \right)^{\frac{\sigma-1}{\sigma}} \right) + \lambda \left(E - \sum_i p_i C_i \right).$$

The FOCs with respect to C_i yields:

$$\rho \frac{1-\sigma}{\sigma} \frac{\omega_i}{C_i} = \lambda p_i, \tag{A.5}$$

where we have defined $\omega_i \equiv \Omega_i^{\frac{1}{\sigma}} \left(\frac{C_i}{g_i(U)} \right)^{\frac{\sigma-1}{\sigma}}$. Equation (A.5) shows that expenditure $p_i C_i$ on good i is proportional to ω_i. Since the latter sums to one from constraint (A.1), it follows that ω_i is the expenditure share of good i, and we have: $E = \sum_{i=1}^{I} p_i C_i = \frac{1-\sigma}{\sigma} \frac{\rho}{\lambda}$. We can now substitute the definition of ω_i from Equation (A) in expression (A.5) and use the budget constraint above to find Equation (A.2), as well as Equations (A.2) and (A.4). □

Elasticities of Demand Lemma 3 implies that the equation defining the expenditure function (and implicitly the indirect utility function) for general Nonhomothetic CES preferences is given by Equation (A.2). The expenditure function is continuous in prices p_i's and U, and homogeneous of degree 1, increasing, and concave in prices. The elasticity of the expenditure function with respect to utility is

$$\eta_E^U \equiv \frac{U \partial E}{E \partial U} = \sum_i \omega_i \, \eta_{g_i}^U = \overline{\eta_{g_i}^U}, \tag{A.6}$$

which ensures that the expenditure function is increasing in utility if all g_i's are monotonically increasing. It is straightforward to also show that the elasticity of the utility function (A.1) with respect to consumption of good i is also given by

$$\eta_U^{C_i} \equiv \frac{C_i \partial U}{U \partial C_i} = \frac{\omega_i}{\overline{\eta_{g_i}^U}}, \tag{A.7}$$

where ω_i is the ratio defined in Equation (A).

Examining sectoral demand from Equation (A.2) along indifference curves, we can derive the main properties of nonhomothetic CES preferences. As expected, *on a given indifference curve*, the elasticity of substitution is constant

$$\eta_{C_i/C_j}^{p_i/p_j} \equiv \left. \frac{\partial \log \left(C_i/C_j\right)}{\partial \log \left(p_i/p_j\right)} \right|_{U=\text{const.}} = \sigma. \tag{A.8}$$

More interestingly, the elasticity of relative demand with respect to utility, in constant prices, is in different from unity:[68]

$$\eta_{C_i/C_j}^U \equiv \left. \frac{\partial \log \left(C_i/C_j\right)}{\partial \log U} \right|_{\boldsymbol{p}=\text{const.}} = (1-\sigma) \frac{\partial \log \left(g_i/g_j\right)}{\partial \log U}. \tag{A.9}$$

Since utility has a monotonic relationship with real income (and hence expenditure), it then follows that the expenditure elasticity of demand for different goods are different. More specifically, we can use (A.6) to find the expenditure elasticity of demand:

$$\eta_{C_i}^E \equiv \frac{\partial \log C_i}{\partial \log E} = \sigma + (1-\sigma) \frac{\eta_{g_i}^U}{\overline{\eta_{g_i}^U}}. \tag{A.10}$$

[68] Preferences defined by Equation (A.1) belong to the general class of preferences with *Direct Implicit Additivity*. Hanoch (1975) shows that the latter family of preferences have the nice property that is illustrated by Equations (A.8) and (A.9): the separability of the income and substitution elasticities of the Hicksian demand. This is in contrast to the stronger requirement of *Explicit Additivity* commonly assumed in nonhomothetic preferences, whereby the utility is explicitly defined as a function $U = F\left(\sum_i f_i(C_i)\right)$. In Section G.1 of the Online Appendix, we will show examples of how substitution and income elasticities of Hicksian demand are *not* separable for preferences with explicitly additivity in direct utility, e.g., generalized Stone-Geary preferences (Kongsamut et al., 2001), or indirect utility, e.g., PIGL preferences (Boppart, 2014a).

Convexity of the Expenditure Function in Utility First, we express the second derivative of the expenditure function in terms of elasticities,

$$\frac{\partial^2 E}{\partial U^2} = \frac{E}{U^2}\eta_E^U \left(\eta_E^U + \eta_{\eta_E^U}^U - 1\right), \tag{A.11}$$

where $\eta_{\eta_E^U}^U$ is the second order elasticity of expenditure with respect to utility. We can compute this second order elasticity as follows:[69]

$$\begin{aligned}
\eta_{\eta_E^U}^U &= U\frac{\partial}{\partial U}\log\sum_i \eta_{g_i}(U)\left(g_i(U)\,p_i\right)^{1-\sigma} - (1-\sigma)\frac{\partial \log E}{\partial \log U}, \\
&= \frac{\sum_i \eta_{\eta_{g_i}}\cdot\eta_{g_i}\left(g_i(U)\,p_i\right)^{1-\sigma} + (1-\sigma)\sum_i \eta_{g_i}^2\left(g_i(U)\,p_i\right)^{1-\sigma}}{\sum_i \eta_{g_i}\left(g_i(U)\,p_i\right)^{1-\sigma}} - (1-\sigma)\,\overline{\eta_{g_i}}, \\
&= \overline{\eta_{g_i}}\left[\frac{\overline{\eta_{\eta_{g_i}}\cdot\eta_{g_i}}}{\left(\overline{\eta_{g_i}}\right)^2} + (1-\sigma)\,Var\left(\frac{\eta_{g_i}}{\overline{\eta_{g_i}}}\right)\right],
\end{aligned} \tag{A.13}$$

where $\overline{X_i}$ and $Var\,(X_i)$ denote the expected value and variance of variable X_i across sectors with weights given by expenditure shares ω_i for prices $\boldsymbol{p}$ and utility U.

Income-Isoelastic Nonhomothetic CES Preferences We discussed that a class of preferences satisfies equation (5) if and only if we can write it as (A.1). It is easy to see that if we further impose condition (4), we find $\log g_i(U) = \epsilon_i/(1-\sigma)\log U + g(U)$ for all i. Furthermore, imposing the condition that the case of $\epsilon_i = 1-\sigma$ should correspond to homothetic CES implies that $g(U) = 0$. This gives us the definition of our basic model in Section 2, where the isoelastic functions g_i are defined as: $g_i(U) = U^{\frac{\epsilon_i}{1-\sigma}}$, where $\eta_{g_i}^U = \epsilon_i/(1-\sigma)$, and we retrieve standard CES preferences when $\epsilon_i = 1-\sigma$ for all i's. Equations (2) and (3) follow by substituting for g_i, with $C = U$, in the results of Lemma 3 above. From (A.6), the real income elasticity of the expenditure function is: $\eta_E^C \equiv \frac{C}{E}\frac{\partial E}{\partial C} = \frac{\bar{\epsilon}}{1-\sigma}$, where $\bar{\epsilon} = \sum_i \omega_i\epsilon_i$. Therefore, a sufficient condition for the function $E\,(C;\boldsymbol{p})$ to be a one-to-one mapping for all positive prices is that all sectors have an income elasticity larger than the elasticity of substitution $\epsilon_i > 0$ if $\sigma < 1$ (and $\epsilon_i < 0$ if $\sigma > 1$). This directly follows from Lemma 2. Combining Equations(A.11) and (A.13), we find

$$\frac{\partial^2 E}{\partial C^2} = \frac{E}{C^2}\frac{\bar{\epsilon}}{1-\sigma}\left(\frac{\bar{\epsilon}}{1-\sigma} - 1\right) + \frac{Var(\epsilon)}{\bar{\epsilon}}. \tag{A.14}$$

Therefore, a sufficient condition for the expenditure function to be convex in C for all prices is that $\epsilon_{min} \geq 1-\sigma$.

[69] To make sense of (A.13), consider the choice of $g_i(U) \equiv g(U)^{\varepsilon_i}$ for some monotonically increasing function $g(\cdot)$ (which corresponds to the aggregator introduced in Section A of the online appendix). We have that $\eta_{g_i} = \eta_g\,\varepsilon_i$ and $\eta_{\eta_{g_i}} = \eta_{\eta_g}$, implying:

$$\eta_{\eta_E^U}^U = \eta_g\overline{\varepsilon_i}\left[\eta_{\eta_g} + (1-\sigma)\,Var\left(\frac{\varepsilon_i}{\overline{\varepsilon_i}}\right)\right]. \tag{A.12}$$

B Proofs of Propositions and Lemmas

Proof of Lemma 1. First, we show that the household problem has a unique solution that is characterized by an Euler equation along with a standard transversality condition. Let $E_t = w_t + (1 + r_t)\,\mathcal{A}_t - \mathcal{A}_{t+1}$ be the consumption expenditure when the representative household has current stock of assets $\mathcal{A}_t$ and chooses an allocation $\mathcal{A}_{t+1}$ of assets for the next period. We can decompose the problem into two independent parts. The intratemporal problem involves allocating the expenditure E_t across I goods so as to maximize the aggregator C_t defined by Equation (1). The solution is given by Equations (2) and (3).

Let $\tilde{C}_t(E) \equiv \max C_t$ subject to the constraint $E = \sum_{i=1}^{I} p_{it} C_{it}$. The intertemporal problem then involves finding the sequence of assets $\{\mathcal{A}_{t+1}\}_{t=0}^{\infty}$ such that

$$\max \sum_{t=0}^{I} \beta^t \frac{\tilde{C}_t \left(w_t + (1 + r_t)\,\mathcal{A}_t - \mathcal{A}_{t+1}\right)^{1-\theta} - 1}{1 - \theta}. \tag{B.1}$$

From Section A, we know that when $\epsilon_i \geq 1 - \sigma$ for all i, the expenditure function is monotonically increasing and strictly convex for all prices. Therefore, its inverse, the indirect aggregate consumption function $\tilde{C}(E; \boldsymbol{p}_t)$ exists and is monotonically increasing and strictly concave for all prices. Standard results from discrete dynamic programming (e.g., see Acemoglu, 2008, Chapter 6) then imply that the Euler equation

$$C_t^{-\theta} \frac{\partial \tilde{C}_t}{\partial E_t} = \beta\,(1 + r_t)\, C_{t+1}^{-\theta} \frac{\partial \tilde{C}_{t+1}}{\partial E_{t+1}},$$

and the transversality condition

$$\lim_{t \to \infty} \beta^t \,(1 + r_t)\, \mathcal{A}_t C_t^{-\theta} \frac{\partial \tilde{C}_t}{\partial E_t} = 0, \tag{B.2}$$

provide necessary and sufficient condition for a sequence $\{\mathcal{A}_{t+1}\}_{t=0}^{\infty}$ to characterize the solution.

Using the results of Section A, we can simplify the Euler equation above to

$$C_t^{-\theta} \frac{C_t}{E_t} \frac{1 - \sigma}{\bar{\epsilon}_t} = \beta\,(1 + r_t)\, C_{t+1}^{-\theta} \frac{C_{t+1}}{E_{t+1}} \frac{1 - \sigma}{\bar{\epsilon}_{t+1}},$$

and the transversality condition to

$$\lim_{t \to \infty} \beta^t \,(1 + r_t)\, \frac{\mathcal{A}_t}{E_t} C_t^{1-\theta} \frac{1 - \sigma}{\bar{\epsilon}_t} = 0.$$

□

Proof of Proposition 1. Our proof for the proposition involves two steps. First, we use the second Welfare Theorem and consider the equivalent centralized allocation by a social planner. Due to the concavity of the aggregator C_t as a function of the bundle of goods $(C_1, \cdots, C_I)$, which is ensured by the condition $\epsilon_i \geq 1$ for all i, we can use standard arguments to establish the uniqueness of the equilibrium allocations (see Stockey et al., 1989, p. 291). Next, we construct a unique constant growth path (steady state) that satisfies the equilibrium conditions. It then follows that the equilibrium converges to the constructed Constant Growth Path (CGP).

Consider an equilibrium path along which consumption expenditure E_t, aggregate stock of capital K_t, and the capital allocated to the investment sector K_{0t} all asymptotically grow at rate $(1+\gamma_0)^{\frac{1}{1-\alpha_0}}$, and the labor employed in the investment sector asymptotically converges to $L_0^* \in (0,1)$. Henceforth, we use the tilde variables to denote normalization $A_{0t}^{-\frac{1}{1-\alpha_0}}$, for instance, $\tilde{K}_t \equiv A_{0t}^{-\frac{1}{1-\alpha_0}} K_t$. Accordingly, we can write the law of evolution of aggregate stock of capital as

$$\tilde{K}_{t+1} = \frac{1-\delta}{(1+\gamma_0)^{1/(1-\alpha_0)}} \tilde{K}_t + \frac{1}{(1+\gamma_0)^{1/(1-\alpha_0)}} \tilde{K}_{0t}^{\alpha_0} L_{0t}^{1-\alpha_0}, \tag{B.3}$$

and the interest rate and wages as

$$r_t = R_t - \delta = \alpha_0 \left(\frac{\tilde{K}_{0t}}{L_{0t}} \right)^{\alpha_0 - 1} - \delta, \tag{B.4}$$

$$\tilde{w}_t = (1-\alpha_0)\, \tilde{K}_{0t}^{\alpha_0} L_{0t}^{-\alpha_0}. \tag{B.5}$$

From the assumptions above, it follows that $\tilde{K}_{0t}/L_{0t}$ asymptotically converges to a constant, which from Equation (B.4) implies that the rate of interest also converges to a constant r^*.

We first derive an expression for the asymptotic growth of nominal consumption expenditure shares (and sectoral employment shares) of different sectors, using in equation (2),

$$\begin{aligned} 1+\xi_i \equiv \lim_{t\to\infty} \frac{\omega_{it+1}}{\omega_{it}} &= \lim_{t\to\infty} \left(\frac{E_t}{E_{t+1}} \right)^{1-\sigma} \left(\frac{p_{it+1}}{p_{it}} \right)^{1-\sigma} \left(\frac{C_{t+1}}{C_t} \right)^{\epsilon_i}, \\ &= \left(\frac{1}{1+\gamma_0} \right)^{\frac{1-\sigma}{1-\alpha_0}} \left(\frac{(1+\gamma_0)^{\frac{1-\alpha_i}{1-\alpha_0}}}{1+\gamma_i} \right)^{(1-\sigma)} (1+\gamma^*)^{\epsilon_i}, \\ &= \frac{(1+\gamma^*)^{\epsilon_i}}{\left[(1+\gamma_0)^{\frac{\alpha_i}{1-\alpha_0}} (1+\gamma_i) \right]^{1-\sigma}}, \end{aligned} \tag{B.6}$$

where in the second line we have used the definition of the constant growth path as well as the fact that from Equations (B.4) and (B.5), the relative labor-capital price grows as rate $(1+\gamma_0)^{\frac{1}{1-\alpha_0}}$ and therefore from Equation (18) we have

$$\lim_{t\to\infty} \frac{p_{it+1}}{p_{it}} = \frac{1+\gamma_0}{1+\gamma_i} (1+\gamma_0)^{\frac{\alpha_0-\alpha_i}{1-\alpha_0}}. \tag{B.7}$$

Equation (B.6) shows that the expenditure shares asymptotically grow (or diminish) monotonically. Since the shares belong to the compact $I-1$ dimensional simplex, they asymptotically converge to a time-constant set of shares.

Since shares have to add up to 1, we need to have that $\xi_i \leq 0$ for all i. Moreover, this inequality has to be satisfied with equality at least for one non-vanishing sector. Now, consider the expression defined in (23) for the growth rate of real consumption. For sectors $i \in \mathcal{I}^*$ that achieve the minimum, the growth of nominal expenditure share becomes zero, and their shares converge to constant values ω_i^*. For sectors $i \notin \mathcal{I}^*$, we find the following expression for the growth rate of nominal shares ξ_i in Equation (B.6) becomes negative. Assuming $\sigma < 1$ and $\epsilon_i > \sigma$, the expression on the right hand side becomes strictly less than 1, since we know sector i does not achieve the minimum in (23). Therefore, $\xi_i < 0$ and the nominal shares asymptotically vanish for $i \notin \mathcal{I}^*$.

Asymptotically, the expenditure-weighted average income elasticity and expenditure-weighted capital intensity in the consumption sector both converge to constants $\bar{\epsilon}^* \equiv \lim_{t\to\infty} \sum_{i=1}^{I} \epsilon_i \omega_{it} = \sum_{i\in\mathcal{I}^*} \epsilon_i \omega_i^*$ and $\bar{\alpha}^* \equiv \lim_{t\to\infty} \sum_{i=1}^{I} \alpha_i \omega_{it} = \sum_{i\in\mathcal{I}^*} \alpha_i \omega_i^*$. Henceforth, we extend our notation to use tilde to indicate variables normalized by their corresponding asymptotic rate of growth (or decline) along our proposed constant growth path. For instance, we let $\tilde{p}_{it} \equiv p_{it}(1+\gamma_0)^{-\frac{1-\alpha_i}{1-\alpha_0}t}(1+\gamma_i)^{-t}$ and $\tilde{C}_t \equiv C_t(1+\gamma^*)^{-t}$. Furthermore, we define starred notation to indicate the asymptotic value of each variable along the constant growth path, for example, we let $p_i^* \equiv \lim_{t\to\infty} \tilde{p}_{it}$ and $\tilde{C}^* \equiv \lim_{t\to\infty} \tilde{C}_t$.

We now show that a constant growth path exists and is characterized by γ^* as defined by equation (23). We also show the existence of the asymptotic values $\{\tilde{K}^*, \tilde{C}^*, \tilde{K}_0^*, L_0^*\}$. From the Euler equation (14), we have that asymptotically

$$(1+\gamma^*)^{1-\theta} = \frac{(1+\gamma_0)^{\frac{1}{1-\alpha_0}}}{\beta\,(1+r^*)}, \tag{B.8}$$

which pins down r^*, the asymptotic real interest rate in terms of γ^* given by Equation (23). Then from Equation (B.4), we find the asymptotic capital-labor ratio in the investment sector in terms of the asymptotic real interest rate

$$\kappa \equiv \frac{\tilde{K_0}^*}{L_0^*} = \left(\frac{\alpha_0}{r^*+\delta}\right)^{\frac{1}{1-\alpha_0}}. \tag{B.9}$$

This gives us the asymptotic relative labor-capital price from Equations (B.4) and (B.5) as

$$\frac{\tilde{w}^*}{R^*} = \frac{1-\alpha_0}{\alpha_0}\frac{\tilde{K}^*}{L_0^*} = \frac{1-\alpha_0}{\alpha_0}\left(\frac{\alpha_0}{r^*+\delta}\right)^{\frac{1}{1-\alpha_0}}. \tag{B.10}$$

From Equation (18), we find

$$\tilde{p}_i^* = \frac{\alpha_0^{\alpha_0}\,(1-\alpha_0)^{1-\alpha_0}}{\alpha_i^{\alpha_i}\,(1-\alpha_i)^{1-\alpha_i}}\;\left(\frac{\tilde{w}^*}{R^*}\right)^{\alpha_0-\alpha_i}\;\frac{A_{0,0}}{A_{i,0}}, \tag{B.11}$$

where $\tilde{w}^*/R^*$ is given by Equations (B.10) and (B.8) and $A_{i,0}$ denotes the initial state of technology in sector i and $A_{0,0} \equiv 1$. Given asymptotic prices

$$\tilde{E}^* = \left[\sum_{i\in\mathcal{I}^*}\left(\tilde{C}^*\right)^{\epsilon_i-\sigma}\,(\tilde{p}_i^*)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}, \tag{B.12}$$

and

$$\omega_i^* = \left(\frac{\tilde{p}_i^*}{\tilde{E}^*}\right)^{1-\sigma}\left(\tilde{C}^*\right)^{\epsilon_i-\sigma}. \tag{B.13}$$

Next, we combine the equation for accumulation of capital (B.3), the household budget constraint (17) the market clearing condition of consumption goods to establish that these exists a unique $\{\tilde{K}^*, \tilde{C}^*, \tilde{K}_0^*, L_0^*\}$ satisfying the asymptotic equilibrium conditions and $\kappa = \tilde{K}_0^*/L_0^*$ where κ is given by Equation (B.9). From market clearing, the sum of payments to labor in the consumption sector is

$\sum_{i=1}^{I}(1-\alpha_i)\omega_{it}E_{it}$, which implies $(1-\bar{\alpha}_t)\,\tilde{E}_t = \tilde{w}_t\,(1-L_{0t})$. Asymptotically, we find that

$$(1-\bar{\alpha}^*)\,\tilde{E}^* = (1-\alpha_0)\,\kappa^{\alpha_0}\,(1-L_0^*)\,. \tag{B.14}$$

Similarly, from Equation (B.3) it follows that $\left[(1+\gamma_0)^{\frac{1}{1-\alpha_0}} - (1-\delta)\right]\tilde{K}^* = \kappa^{\alpha_0}L_0^*$. Defining the expression within the square brackets at a positive constant ϑ, we use write the asymptotic employment in the investment sector in terms of the aggregate stock of capital as

$$L_0^* = \vartheta\kappa^{-\alpha_0}\tilde{K}^*. \tag{B.15}$$

Finally, using the market clearing condition in the assets market $\mathcal{A}_t = K_t$ and Equation (17), we find that $\tilde{E}_t = \tilde{w}_t + R_t\tilde{K}_t - \left(\frac{\tilde{K}_{0t}}{L_{0t}}\right)^{\alpha_0} L_{0t}$ for all t. Taking the limit, it follows that

$$\tilde{E}^* = (1-\alpha_0)\,\kappa^{\alpha_0} + \alpha_0\kappa^{\alpha_0-1}\tilde{K}^* - \kappa^{\alpha_0}L_0^*. \tag{B.16}$$

Substituting from Equation (B.15) into Equations (B.14) and (B.16) yields,

$$\bar{\alpha}^*\tilde{E}^* = \alpha_0\left(\kappa^{\alpha_0-1} - \vartheta\right)\tilde{K}^*. \tag{B.17}$$

We can show that the left hand side of this equation is a monotonically increasing function of $\tilde{C}^*$ with a given κ.[70] From condition (24), we have that $\kappa^{\alpha_0-1} - \vartheta > 0$ and therefore the right hand side is a linear increasing function of $\tilde{K}^*$. Therefore, Equation (B.17) defines $\tilde{C}^*$, and correspondingly $\tilde{E}^*$, as an increasing function of $\tilde{K}^*$. Finally, substituting this function and Equation (B.15) in Equation (B.16), we find

$$\tilde{E} + \left(\vartheta - \alpha\kappa^{\alpha_0-1}\right)\tilde{K} = (1-\alpha_0)\,\kappa^{\alpha_0}. \tag{B.18}$$

From condition (24), we know that the left hand side is a monotonically increasing function of $\tilde{K}^*$ for constant κ. This function is 0 when $\tilde{K}^*$ and limits to infinity as the latter goes to infinity. Therefore, Equation (B.18) uniquely pins down $\tilde{K}^*$ as a function of κ, which in turn is given by Equation (B.9). Condition (24) also ensures that the transversality condition (15) is satisfied. Finally, we verify that $L_0^* \in (0,1)$. Combining equations (B.15), (B.14) and (B.16) we obtain that

$$L_0^* = \frac{\bar{\alpha}}{\left[\frac{1-\bar{\alpha}}{1-\alpha_0}\left(\alpha_0\kappa^{\alpha_0-1}\vartheta^{-1} - 1\right) + 1\right]} \tag{B.19}$$

Assuming that the term in square brackets is positive, we have that $L_0^* \in (0,1)$ if and only if $\vartheta < \kappa^{\alpha_0-1}$, which in terms of fundamental parameters requires that $\beta(1+\gamma^*)^{1-\theta} < \frac{(1+\gamma_0)^{-\frac{\alpha_0}{1-\alpha_0}}}{\alpha_0 + (1-\alpha_0)(1+\gamma_0)^{-\frac{1}{1-\alpha_0}}(1-\delta)}$ which is the condition stated in (24). Also, it is readily verified that as long as $\vartheta < \kappa^{\alpha_0-1}$, L_0^* cannot be negative.

Therefore, we constructed a unique costant growth path that asymptotically satisfies the equilibrium conditions whenever the parameters of the economy satisfy Equation (24). Together with the

[70] We have that $\frac{\partial\left(\bar{\alpha}^*\tilde{E}^*\right)}{\partial\tilde{C}^*} = \frac{\bar{\alpha}^*\tilde{E}^*}{\tilde{C}^*}\frac{\bar{\epsilon}}{1-\sigma}\left[1 + (1-\sigma)\,\rho_{\epsilon_i,\alpha_i}\right]$ where $\rho_{\epsilon_i,\alpha_i}$ is the correlation coefficient between ϵ_i and α_i under a distribution implied by expenditure shares (see online Appendix for details of the derivation). Therefore, the derivative is always positive and the function is a monotonic of $\tilde{C}^*$.

uniqueness of the competitive equilibrium, this completes the proof.

□

Derivations for the Results in Section 2.2.3 We first characterize the dynamics of the state variable, the normalized per-capita stock of capital $\tilde{k}_t \equiv \tilde{K}_t/L$. Substituting in $K_{t+1} = A_{0t}K_{0t}^{\alpha}L_{0t}^{1-\alpha} + K_t(1-\delta)$ and noting the equality of per-capita stock of capital across sectors, we find

$$(1+\gamma_o)^{\frac{1}{1-\alpha}}\,\tilde{k}_{t+1} = \tilde{k}_t^{\alpha} l_{0,t} + \tilde{k}_t\,(1-\delta)\,,$$

where $l_{0,t} \equiv L_{0,t}/L$ is the share of labor employed in the investment sector. We can show that this share is given by $l_{0,t} = 1 - \tilde{E}_t/\tilde{k}_t^{\alpha}$ (see the online appendix), therefore establishing Equation (26).

For the evolution of per-capita consumption, we need to write C_{t+1}/C_t in terms of variables known at time t. Rewriting the Euler Equation (14) as $(C_{t+1}/C_t)^{1-\theta}\,\beta\,(1+r_t) = (E_{t+1}/E_t)\,\bar{\epsilon}_{t+1}/\bar{\epsilon}_t$, first note that the interest rate is given from Equation (B.4) as $r_t = \alpha\tilde{k}_t^{\alpha-1} - \delta$. Substituting for the normalized variables, we find

$$\left(\frac{\tilde{C}_{t+1}}{\tilde{C}_t}\right)^{1-\theta}\frac{(1+\gamma^*)^{1-\theta}}{(1+\gamma_0)^{\frac{1}{1-\alpha}}}\beta\,(1+r_t) = \left(\frac{\tilde{E}_{t+1}}{\tilde{E}_t}\right)\frac{\bar{\epsilon}_{t+1}}{\bar{\epsilon}_t}.$$

Using the expression for the asymptotic rate of interest r^* from (B.8) then gives us Equation (27).

Next, we can can write the growth in per-capita consumption expenditure as

$$\begin{aligned}\left(\frac{E_{t+1}}{E_t}\right)^{1-\sigma} &= \sum_{i=1}^{I}\Omega_i\left(\frac{p_{it}}{E_t}\right)^{1-\sigma}C_t^{\epsilon_i}\left(\frac{p_{it+1}}{p_{it}}\right)^{1-\sigma}\left(\frac{C_{t+1}}{C_t}\right)^{\epsilon_i},\\ &= \sum_{i=1}^{I}\omega_{it}\left(\frac{C_{t+1}}{C_t}\right)^{\epsilon_i}\left(\frac{1+\gamma_0}{1+\gamma_i}\right)^{(1-\sigma)t},\end{aligned}$$

where we have used Equation (3), Equation (18), and the expression for expenditure shares $\omega_{it} = \Omega_i(p_{it}/E_t)^{1-\sigma}C_t^{\epsilon_i}$ under the assumption of $\alpha_i \equiv \alpha$. Substituting for the normalized variables $\tilde{E}_t$ and $\tilde{C}_t$ in the expression above gives Equation (28).

Finally, we use the same idea to rewrite the term $\bar{\epsilon}_{t+1}$ as follows

$$\begin{aligned}\frac{\bar{\epsilon}_{t+1}}{\bar{\epsilon}_t} &= \sum_{i=1}^{I}\left(\frac{p_{it+1}}{E_{t+1}}\right)^{1-\sigma}C_{t+1}^{\epsilon_i}\epsilon_i,\\ &= \left(\frac{E_t}{E_{t+1}}\right)^{1-\sigma}\sum_{i=1}^{I}\left(\frac{p_{it}}{E_t}\right)^{1-\sigma}C_t^{\epsilon_i}\left(\frac{p_{it+1}}{p_{it}}\right)^{1-\sigma}\left(\frac{C_{t+1}}{C_t}\right)^{\epsilon_i}\left(\frac{\epsilon_i}{\bar{\epsilon}_t}\right),\\ &= \frac{\sum_{i=1}^{I}\omega_{it}\left(\frac{C_{t+1}}{C_t}\right)^{\epsilon_i}\left(\frac{1+\gamma_0}{1+\gamma_i}\right)^{(1-\sigma)t}\left(\frac{\epsilon_i}{\bar{\epsilon}_t}\right)}{\sum_{i=1}^{I}\omega_{it}\left(\frac{C_{t+1}}{C_t}\right)^{\epsilon_i}\left(\frac{1+\gamma_0}{1+\gamma_i}\right)^{(1-\sigma)t}}.\end{aligned}$$

Multiplying both the numerator and the denominator by $(1+\gamma_0)^{-\frac{t}{1-\alpha_0}}$ and substituting again for the normalized variables $\tilde{E}_t$ and $\tilde{C}_t$ gives us Equation (29).

Proof of Equation (37)**.** From the definition of the expenditure function in Equation (3), we have

$$\begin{aligned}\left(\frac{E_{t+1}}{E_t}\right)^{1-\sigma} &= \frac{\sum_i \Omega_i C_{t+1}^{\epsilon_i} P_{it+1}^{1-\sigma}}{\sum_i \Omega_i C_t^{\epsilon_i} P_{it}^{1-\sigma}} \\ &= \frac{\sum_i \Omega_i C_t^{\epsilon_i} P_{it}^{1-\sigma} \times \left(\frac{C_{t+1}}{C_t}\right)^{\epsilon_i} \left(\frac{P_{it+1}}{P_{it}}\right)^{1-\sigma}}{\sum_i \Omega_i C_{t+1}^{\epsilon_i} P_{it+1}^{1-\sigma} \times \left(\frac{C_{t+1}}{C_t}\right)^{-\epsilon_i} \left(\frac{P_{it+1}}{P_{it}}\right)^{-(1-\sigma)}} \\ &= \left(\frac{E_{t+1}}{E_t}\right)^{-(1-\sigma)} \frac{\sum_i \omega_{it} \times \left(\frac{C_{t+1}}{C_t}\right)^{\epsilon_i} \left(\frac{P_{it+1}}{P_{it}}\right)^{1-\sigma}}{\sum_i \omega_{it+1} \times \left(\frac{C_{t+1}}{C_t}\right)^{-\epsilon_i} \left(\frac{P_{it+1}}{P_{it}}\right)^{-(1-\sigma)}}.\end{aligned}$$

Assuming that $\Delta \log E_t = \log(E_{t+1}/E_t) \ll 1$ and $\Delta \log P_{it} = \log(P_{it+1}/P_{it}) \ll 1$ for all i, we can rewrite the expression above up to the second order in $\Delta \log E_t$, $\Delta \log C_t$, and $\Delta \log P_{it}$ as

$$\begin{aligned}\log \frac{E_{t+1}}{E_t} &\approx \frac{1}{2(1-\sigma)} \sum_i (\omega_{it} + \omega_{it+1}) \left((1-\sigma) \log \frac{P_{it+1}}{P_{it}} + \epsilon_i \log \frac{C_{t+1}}{C_t}\right), \\ &= \underbrace{\left[\frac{1}{2} \sum_i (\omega_{it} + \omega_{it+1}) \log \frac{P_{it+1}}{P_{it}}\right]}_{\equiv \Delta \log \mathcal{P}_{it}} + \frac{1}{1-\sigma} \underbrace{\left[\frac{1}{2} \sum_i (\omega_{it} + \omega_{it+1}) \epsilon_i\right]}_{\equiv \overline{\mathcal{E}}_t} \times \log \frac{C_{t+1}}{C_t},\end{aligned}$$

from which Equation (37) follows. □

C Discussion of the Estimation Strategy without Exact Price Index

To simplify the exposition of the derivations, we define the following notation, only to be used within this section of the Appendix: let $Y_{it}^n \equiv \log(\omega_{it}^n/\omega_{mt}^n)$, $P_{it}^n \equiv \log(p_{it}^n/p_{mt}^n)$, $X_t^n \equiv \log C_t^n$, and $Z_{it}^n \equiv \log(E_t^n/\mathcal{P}_t^n)$ for all $i \in \mathcal{I}_- = \mathcal{I} \backslash \{m\}$. We can then rewrite Equation (16) as

$$Y_{it}^n = (1-\sigma) P_{it}^n + (\epsilon_i - \epsilon_m) X_t^n + \zeta_i^n + \nu_{it}^n,\ i \in \mathcal{I}_-. \tag{C.1}$$

Henceforth, we assume i is always within set $\mathcal{I}_-$ and drop the reference to the set.

Throughout, we maintain the following assumptions.[71]

Assumption 1. *Relative prices and income are orthogonal to the errors, that is,* $\mathbb{E}\left[P_{jt}^n \nu_{it}^n\right] = \mathbb{E}\left[X_t^n \nu_{it}^n\right] = 0$ *for all* i, j*. Moreover, relative prices are not perfectly correlated with either the real income index* X_t^n *or the proxy* Z_t^n*, that is,* $|\mathbb{E}[X_t^n P_{it}^n]| < (\mathbb{E}[X_t^n] \mathbb{E}[P_{it}^n])^{1/2}$ *and* $|\mathbb{E}[Z_t^n P_{it}^n]| < (\mathbb{E}[Z_t^n] \mathbb{E}[P_{it}^n])^{1/2}$.

The different approaches discussed in Section 5.1 involve replacing the unobserved index of real consumption X_t^n by a *proxy* variable, for example, the consumption expenditure deflated by a standard

[71] In the case of household-level data, instead of assuming the orthogonality of the covariates and the error, we use instruments for both relative prices and income, which would slightly complicate the derivations that follows. However, the main insights will remain intact whether we assume the orthogonality of the covariates or the existence of instruments for them.

price index Z_t^n. For any population-level distribution of relative prices and income, without loss of generality, we can write we can write $X_t^n(Z_t^n, P_{1t}^n, \cdots, P_{It}^n)$ to

$$X_t^n = \sum_i \eta_i P_{it}^n + \gamma Z_t^n + \iota^n + u_t^n, \tag{C.2}$$

such that $\mathbb{E}\left[u_t^n\right] = \mathbb{E}\left[P_{it}^n u_t^n\right] = \mathbb{E}\left[Z_t^n u_t^n\right] = 0$ for all $i \in \mathcal{I}_{-m}$ (this corresponds to running an OLS regression if we were to observe X_t^n). It follows that we can write

$$\begin{aligned} Y_{it}^n = {} & \left(1 - \sigma + \eta_i \left(\epsilon_i - \epsilon_m\right)\right) P_{it}^n + \sum_{j \neq i} \eta_j \left(\epsilon_i - \epsilon_m\right) P_{jt}^n \\ & + \left(\epsilon_i - \epsilon_m\right) \gamma Z_t^n + \zeta_i^n + \left(\epsilon_i - \epsilon_m\right)\left(\iota^n + u_t^n\right) + \nu_{it}^n. \end{aligned} \tag{C.3}$$

The lemma below establishes that we can identify the model's elasticity parameters up to a constant factor using a system OLS estimate or a feasible GLS estimate of log relative shares on log relative prices and log real consumption expenditure, of the form[72]

$$Y_{it}^n = \sum_j \alpha_{ij} P_{jt}^n + \beta_i Z_t^n + \tilde{\zeta}_i^n + \tilde{\nu}_{it}^n. \tag{C.4}$$

Lemma 4. *Assume that the model in Equation (C.1) is well-specified, Assumption 1 holds, and that $\gamma \neq 0$, i.e., the real income index of nonhomothetic CES and the real income calculated based on standard price indices, our proxy variable, are correlated after controlling for relative prices. Let $\widehat{\beta}_i$ denote the coefficients on the real consumption expenditure based on estimating the system of Equations C.4. Then, the coefficients on the proxy variable Z_t^n satisfy plim $\widehat{\beta}_i / \widehat{\beta}_j = \left(\epsilon_i - \epsilon_m\right) / \left(\epsilon_j - \epsilon_m\right)$.*

D Comparison with Stone-Geary and PIGL prefrences

We compare the cross-country fit of our model to alternative specifications where we replace the nonhomothetic CES aggregator with Stone-Geary and PIGL preferences. A brief discussion of theses preferences and estimation is given here. We relegate a detailed discussion to Online Appendix G.

We start considering a generalized Stone-Geary formulation (Herrendorf et al., 2014). These preferences define the intra-period consumption aggregator as

$$C_t^c = \left[\Omega_a^c \left(C_{at}^c + \bar{c}_a\right)^{\frac{\sigma-1}{\sigma}} + \Omega_m^c \left(C_{mt}^c\right)^{\frac{\sigma-1}{\sigma}} + \Omega_c^c \left(C_{st}^c + \bar{c}_s\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}, \tag{D.1}$$

where C_t^c denotes aggregate consumption of country c at time t, $\Omega_i^c > 0$ are constant preference parameters that are country specific, $C_{,t}^c$ denotes consumption in sector $i = \{a, m, s\}$, $\bar{c}_a$ and $\bar{c}_s$ are constants that govern the nonhomotheticity of these preferences, and σ is a parameter that tends to the price elasticity of substitution as $C_{it}^c \gg \max\{\bar{c}_a, \bar{c}_s\}$.[73] We use the first-order conditions of the intra-

[72]This result is fairly general and can be applied to all cases where one unobserved covariate appears on the right hand side of more than one equation a system of equations, and a proxy variable exists that is correlated with the unobserved covariate and is orthogonal to the error.

[73]Since these preferences are not implicitly additive, the price and income elasticities are not independent. In Appendix G.1 we show that the elasticity of substitution between i and j is $\sigma_{ij} = \sigma \eta_i \eta_j$, where η's denote

period problem to estimate the model. As with nonhomothetic CES preferences, we estimate three parameters that are common across countries $\{\sigma, \bar{c}_a, \bar{c}_s\}$ that govern the price and income elasticities and country-specific taste parameters $\{\Omega_i^c\}_{i\in\mathcal{I},c\in C}$.

Our estimation results (reported in Table G.1 of the online appendix) imply that the three sectors are gross complements and that nonhomotheticities are significantly different from zero and of the expected sign, $\bar{c}_a < 0$ and $\bar{c}_s > 0$. To assess the goodness of fit, we compute the within-R^2 for the predicted time path of employment shares in agriculture, manufacturing and services relative to a model with only country-sector fixed effects.[74] We find that the within-R^2 of Stone-Geary is 0.14, meaning that 14% of the residual variation in sectoral shares after we partial out country-sector averages is accounted for by the Stone-Geary demand system. The corresponding number for nonhomothetic-CES is 0.29. The intuition for the worse fit of Stone-Geary is that income effects are very low for rich countries, since for high levels of income the subsistence levels responsible for introducing the nonhomotheticity $\{\bar{c}_a, \bar{c}_s\}$ are negligible.[75] Thus, only variation in prices (and trade shares) are left to account for the variation in employment shares for rich countries, and the model can miss a substantial part of structural change. This is illustrated in Figure 5 for Japan.[76]

Next, we study the cross-country fit of PIGL preferences as specified in Boppart (2014b). This preference structure features a homothetic CES aggregator between agriculture and manufacturing with price elasticity σ and a nonhomothetic aggregator between services and the agriculture-manufacturing composite. The within-period indirect utility V of a household with total expenditure E^c in country c is

$$V = \frac{1}{\varepsilon}\left(\frac{E^c}{p_{st}^c}\right)^{\varepsilon} - \frac{\Omega_s^c}{\gamma}\cdot\frac{\left(\Omega_a^c\cdot(p_{at}^c)^{1-\sigma} + \Omega_m^c\cdot(p_{mt}^c)^{1-\sigma}\right)^{\frac{\gamma}{1-\sigma}}}{(p_{st}^c)^{\gamma}} - \frac{1}{\varepsilon} + \frac{\Omega_s^c}{\gamma}, \tag{D.2}$$

with $0 \le \varepsilon \le \gamma < 1$ and $\Omega_i^c > 0$ for $i \in \{a, m, s\}$. The nonhomotheticity and price elasticity between services and the agriculture-manufacturing CES composite are governed by two parameters, ε and γ. The nonhomotheticity is not vanishing as income grows, and the price elasticity grows with income but is bounded above by 1.[77]

income elasticities.

[74]The within R^2 compares the sum of squared errors of the model fit to the sum of squared errors obtained by using the country-sector average as a prediction (in other words, assuming flat lines at the average country level in Figure 5). Formally, $R^2 = 1 - \frac{1}{I}\sum_{i=1}^{I}\left(\sum_{t=1}^{N}(y_{it}^c - \hat{y}_{it}^c)^2 / \sum_{t=1}^{N}(y_{it}^c - \bar{y}_i^c)^2\right)$ where N denotes the total number of observations per sector, I, the number of sectors, y_{it}^c, observed employment shares in sector i and country c, $\hat{y}_{it}^c$, predicted employment shares, $\bar{y}_i^c$ the sample average of y_{it} for country c in sector i, and $i \in \mathcal{I} = \{a, m, s\}$. Note also that Stone-Geary and nonhomothetic CES collapse to the same demand system in the case that we set $\sigma = 1$ and $\bar{c}_i = 0$ or $\epsilon_i = 1 - \sigma$. In this case, differences across countries and sectors would be only accounted for through Ω_i^c, which would be exactly the country-sector average levels that we use as a reference for the within R^2 (this is also true for PIGL). Finally, we also note that the estimates used to compute the within-R^2 for nonhomoethetic CES correspond to the structural estimates in column (1) of Table 3.

[75]To have a better grasp of the magnitude of the income effects, we compute the values of $\frac{p_{a,t}^c \bar{c}_a}{\sum_{i\in\{a,m,s\}} p_{it}^c C_{it}^c}$ and $\frac{p_{s,t}^c \bar{c}_s}{\sum_{i\in\{a,m,s\}} p_{it}^c C_{it}^c}$ which are the nonhomothetic part of the demand function. For the U.S., they are never higher (in absolute terms) than .1%, which suggests that nonhomtheticities are insignificant when compared to aggregate consumption. The highest values of the nonhomotheticities in the sample are 37% and 18% for services.

[76]We report the fit for all countries for both Stone-Geary and PIGL in Online Appendix H.

[77]The parameter ε governs the nonhomotheticity of preferences between services and the composite of agri-

We use the demand implied by these preferences to estimate the demand parameters. As with nonhomothetic CES and Stone-Geary, we estimate three elasticities that are common across countries $\{\varepsilon, \gamma, \sigma\}$ and we allow for country-specific constant taste parameters, $\{\Omega_i^c\}_{i\in\mathcal{I},c\in C}$. We find that, at our estimated parameter values, manufacturing and agriculture are gross complements and nonhomotheticities are significantly different from zero. In fact, the nonhomotheticity parameter that we estimate is similar in magnitude to the U.S. estimate reported in Boppart (2014a) (see table G.3). The overall fit of the PIGL demand system as measured by the within R^2 is similar in magnitude to Stone-Geary, 0.13. As illustrated in Figure 5 for the case of Japan, PIGL preferences track the trends in services more accurately than Stone-Geary due to the fact that they features a non-vanishing nonhomotheticity of the service sector. However, they under-perform relative to nonhomothetic CES mostly because they assume a homothetic composite between agriculture and manufacturing, while nonhomothetic CES allows for sector-specific nonhomotheticities.

cultural and manufacturing goods. If $\varepsilon > 0$, the expenditure elasticity is larger than one for services and less than one for agricultural and manufacturing goods (and identical for both). The price elasticity of substitution between services and the agriculture-manufacturing composite never exceeds one, it is increasing with the level of income and it asymptotes to $1-\gamma$. The baseline model in Boppart contains only two sectors. Here we follow the extension proposed in Appendix B.3.3 (Boppart, 2014b) to account for three sectors such that there can be a hump-shape in manufacturing. We have generalized the demand to allow for constant taste parameters heterogeneous across countries and not symmetric between agricultural and manufacturing goods. We have also experimented with another proposed extension such that the expenditure share in the manufacturing sector constant (Appendix B.3.2) obtaining a worse fit.

E Additional Tables and Figures

Table 8: Log-linear Specification, CEX Expenditure

	(1)	(2)	(3)	(4)	(5)	(6)
$\tilde{\epsilon}_s - \tilde{\epsilon}_m$	-0.50	-0.50	-0.50	-0.50	-0.44	-0.45
	(0.02)	(0.02)	(0.02)	(0.02)	(0.03)	(0.03)
$\tilde{\epsilon}_s - \tilde{\epsilon}_m$	0.43	0.43	0.44	0.43	0.46	0.45
	(0.02)	(0.02)	(0.02)	(0.02)	(0.03)	(0.03)
	Comparison to Baseline Results					
$(\tilde{\epsilon}_s - \tilde{\epsilon}_m)/(\tilde{\epsilon}_a - \tilde{\epsilon}_m)$	-0.86	-0.86	-0.88	-0.87	-1.03	-0.99
$(\epsilon_s - 1)/(\epsilon_a - 1)$, baseline estimates	-0.81	-0.82	-0.93	-0.96	-0.86	-0.88
Expenditure Re-Weighted	N	Y	N	Y	N	Y
Region FE	N	N	Y	Y	Y	Y
Year $\times$ Quarter	N	N	N	N	Y	Y

Notes: Standard errors clustered at the household level. All regressions include the household controls discussed in the main text. The number of observations is 60925 in all regressions. The ratios in the last row are computed from Table 1.

Table 9: Log-linear Specification, Cross-country Data

	World		OECD		Non-OECD	
	(1)	(2)	(3)	(4)	(5)	(6)
$\tilde{\epsilon}_s - \tilde{\epsilon}_m$	-0.32	-0.32	-0.65	-0.57	-0.27	-0.28
	(0.12)	(0.11)	(0.07)	(0.08)	(0.12)	(0.10)
$\tilde{\epsilon}_s - \tilde{\epsilon}_m$	0.06	0.06	0.27	0.30	0.05	0.05
	(0.03)	(0.03)	(0.08)	(0.10)	(0.02)	(0.02)
	Comparison to Baseline Results					
$(\tilde{\epsilon}_s - \tilde{\epsilon}_m)/(\tilde{\epsilon}_a - \tilde{\epsilon}_m)$	-0.20	-0.20	-0.43	-0.52	-0.18	-0.17
$(\epsilon_s - 1)/(\epsilon_a - 1)$, baseline estimates	-0.17	-0.24	-0.27	-0.25	-0.20	-0.46
Country $\times$ Sector FE	Y	Y	Y	Y	Y	Y
Trade Controls	N	Y	N	Y	N	Y
Observations	1626	1626	492	492	1134	1134

Notes: Standard errors clustered at the country level. The ratios in the last row are computed from Table 3.

Table 10: Housing as a Separate Group, CEX Expenditure, $\epsilon_m = 1$

	(1)	(2)	(3)	(4)	(5)	(6)
σ	0.17	0.19	0.20	0.17	0.19	0.23
	(0.04)	(0.04)	(0.03)	(0.04)	(0.03)	(0.03)
$\epsilon_a - 1$	-0.71	-0.74	-0.73	-0.72	-0.81	-0.85
	(0.05)	(0.06)	(0.05)	(0.05)	(0.05)	(0.06)
$\epsilon_{\text{services (excl. housing)}} - 1$	0.75	0.77	0.77	0.67	0.65	0.65
	(0.07)	(0.07)	(0.07)	(0.06)	(0.06)	(0.07)
$\epsilon_{\text{housing}} - 1$	0.94	1.00	0.96	0.89	0.78	0.82
	(0.09)	(0.10)	(0.09)	(0.09)	(0.07)	(0.08)
Expenditure Re-Weighted	N	Y	N	Y	N	Y
Region FE	N	N	Y	Y	Y	Y
Year × Quarter FE	N	N	N	N	Y	Y

Notes: Standard errors clustered at the household level shown in parentheses. The number of observations is 60925 in all regressions.

Table 11: 10-Sector Regression, $\epsilon_m = 1$

	World	OECD	Non-OECD
Price Elasticity σ	0.10	0.13	0.07
	(0.03)	(0.03)	(0.04)
Sector i Income Elasticity Parameter ϵ_i			
Agriculture	0.32	0.00	0.38
	(0.05)	(0.04)	(0.06)
Mining	0.41	0.01	0.67
	(0.06)	(0.04)	(0.05)
Public Utilities	1.59	1.32	1.61
	(0.05)	(0.03)	(0.05)
Transp., Storage, Communications	1.44	1.36	1.41
	(0.03)	(0.04)	(0.03)
Construction	1.03	0.72	1.09
	(0.02)	(0.02)	(0.02)
Community, Social and Personal Serv.	1.18	0.85	1.21
	(0.03)	(0.05)	(0.03)
Wholesale and Retail	1.62	1.59	1.58
	(0.04)	(0.05)	(0.04)
Finance, Insurance, Real State	2.17	2.36	2.04
	(0.07)	(0.11)	(0.07)
Observations	1596	492	1104

Notes: Standard errors clustered at the country level. All regressions include a sector-country fixed effect. For the OECD regression, we constain the agriculture and mining parameter to be non-negative (by estimating the exponent of ϵ_i, standard errors are adjusted using the delta method).

Table 12: Value-Added Household Estimation, CEX

	(1)	(2)	(3)	(4)	(5)	(6)
σ	0.34	0.34	0.31	0.52	0.30	0.33
	(0.02)	(0.02)	(0.02)	(0.04)	(0.03)	(0.03)
$\epsilon_a - 1$	-0.94	-0.98	-0.97	-0.99	-1.00	-0.99
	(0.08)	(0.09)	(0.08)	(0.47)	(0.11)	(0.10)
$\epsilon_s - 1$	0.64	0.62	0.66	0.90	0.88	0.94
	(0.08)	(0.09)	(0.08)	(0.58)	(0.11)	(0.11)
Household Controls	Y	Y	Y	Y	Y	Y
Region FE	N	N	Y	Y	Y	Y
Year × Quarter	N	N	N	N	Y	Y

Notes: Standard errors clustered at the household level. The number of observations is 60925 in all regressions.

Table 13: Non-Linear Demand Estimation, Household and Cross-Country, $\epsilon_m = 1$

	Household CEX		Aggregate Data	
	(1)	(2)	(3)	(4)
σ	0.09	0.12	0.65	0.62
	(0.03)	(0.03)	(0.15)	(0.13)
$\epsilon_a - 1$	-0.52	-0.54	-0.50	-0.52
	(0.03)	(0.03)	(0.22)	(0.15)
$\epsilon_s - 1$	0.59	0.60	0.24	0.24
	(0.04)	(0.04)	(0.15)	(0.14)
Household Controls	Y	Y	-	-
Expenditure Re-Weighted	N	Y	-	-
Country × Sector FE	-	-	Y	Y
Trade Controls	-	-	N	Y

Notes: Standard errors clustered at household level in (1) and (2) and at the country level in (3) and (4).

Table 14: Accounting for Structural Change, R^2

	$\ln\left(\frac{\text{Agriculture}}{\text{Manufacturing}}\right)$			$\ln\left(\frac{\text{Services}}{\text{Manufacturing}}\right)$		
Regression includes:	World	OECD	Non-OECD	World	OECD	Non-OECD
Only Fixed Effects	0.93	0.78	0.92	0.51	0.30	0.51
FE + Relative Prices	0.94	0.80	0.93	0.52	0.43	0.52
FE + Nh. CES Real Consumption	0.97	0.96	0.96	0.57	0.57	0.55
All Regressors	0.97	0.96	0.96	0.58	0.58	0.55