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STRUCTURAL CHANGE WITH LONG-RUN INCOME AND PRICE EFFECTS

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ABSTRACT

We present a new multi-sector growth model that accommodates long-run demand and supply drivers of structural change. The model generates nonhomothetic Engel curves at all levels of development and is consistent with the decline in agriculture, the hump-shaped evolution of manufacturing and the rise of services over time. The economy converges to a constant aggregate growth rate that depends on sectoral income elasticities, capital intensities and rates of technological progress. We estimate the demand system derived from the model using historical data on sectoral employment shares from twenty-ve countries and household survey data from the US. Our estimated model parsimoniously accounts for the broad patterns of sectoral reallocation observed among rich, miracle and developing economies in the post-war period. We find that income effects play a major role in generating structural change.

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A data appendix is available at:
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1 Introduction

Economies undergo large scale sectoral reallocations of employment and capital as they develop, in a process commonly known as structural change (Kuznets, 1973; Maddison, 1980; Herrendorf et al., 2014; Vries et al., 2014). These reallocations lead to a gradual fall in the relative size of the agricultural sector and a corresponding rise in manufacturing. As income continues to grow, services eventually emerge as the largest sector in the economy. Leading theories of structural change attempt to understand these sweeping transformations through mechanisms involving either supply or demand. Supply-side theories focus on differences across sectors in the rates of technological growth and capital intensities, which create trends in the composition of consumption through price (substitution) effects (Baumol, 1967; Ngai and Pissarides, 2007; Acemoglu and Guerrieri, 2008). Demand-side theories, in contrast, emphasize the role of heterogeneity in income elasticities of demand across sectors (nonhomotheticity in preferences) in driving the observed reallocations accompanying income growth (Kongsamut et al., 2001).

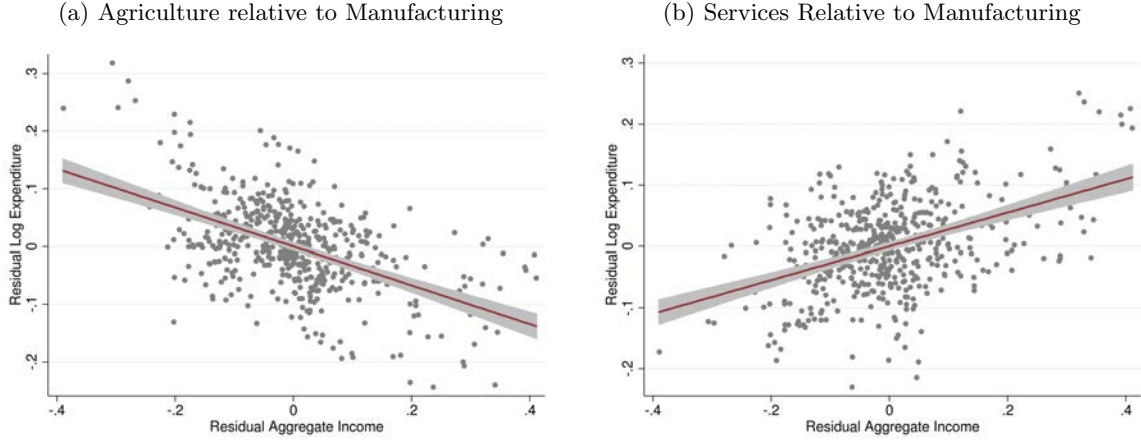
The shapes of sectoral Engel curves play a crucial role in determining the contribution of supply and demand channels to structural change.¹ If the differences in the slopes of Engel curves are large and persistent, demand channels can readily explain reallocation of resources toward sectors with higher income elasticities. For instance, steep upward Engel curves for services, flat Engel curves for manufacturing, and steep downward Engel curves for agricultural products can give rise to sizable shifts of employment from agriculture toward services. However, demand-side theories have generally relied on specific classes of nonhomothetic preferences, e.g., generalized Stone-Geary preferences, that imply Engel curves that level off quickly as income grows. Because of this rapid flattening-out of the slopes of Engel curves across sectors, these specifications limit the explanatory power of the demand channel in the long-run.

The empirical evidence suggests that Engel curves remain constant at different income levels and do not level off rapidly as income grows. At the micro level, Engel curves have been shown to be well approximated by log-linear functions with constant slopes.² We complement

¹We define Engel curves as the relationship between sectoral consumption shares and aggregate real consumption holding prices constant.

²For instance, Aguiar and Bils (2015) use the U.S. Consumer Expenditure Survey (CEX) to estimate Engel curves for 20 different consumption categories. Their estimates for the income elasticities are different from unity and vary significantly across consumption categories. Young (2012) employs the Demographic and Health Survey (DHS) to infer the elasticity of real consumption of 26 goods and services with respect to income for 29 sub-Saharan and 27 other developing countries. He estimates the elasticity of consumption for the different categories with respect to the education of the household head and then uses the estimates of the return to education from Mincerian regressions to back out the income elasticity of consumption. Young also uses a log-linear Engel curve formulation and finds that the slopes of Engel curves greatly differ across consumption categories but appear stable over time. Olken (2010) discusses Young’s exercise using Indonesia survey data and finds similar results for a small sample of three goods and services he studies. Young (2013) also makes use of log-linear Engel curves to infer consumption inequality.

Figure 1: Partial Correlations of Sectoral Expenditure and Aggregate Consumption



Notes: Data for OECD countries, 1970-2005. Each point corresponds to a country-year observation after partialling-out sectoral prices and country fixed effects. The red line depicts the OLS fit, the shaded regions, the 95% confidence interval.

this evidence and show that income elasticities stay quite similar across different income brackets using the Consumption Expenditure survey (CEX). At the macro level, log-linear Engel curves also provide reasonable approximations to aggregate consumption variables. Figure (1) plots the relationship between the residual (log) expenditure share in agriculture (Figure 1a) and services (Figure 1b) relative to manufacturing on the y-axis and residual (log) income on the x-axis after controlling for relative prices. The depicted log-linear fit shows that a constant slope captures a considerable part of the variation in the data.³

Motivated by this evidence, we develop a multi-sector model of structural transformation that accommodates for nonhomotheticity in the form of log-linear Engel curves, as well as trends in relative prices. The model builds on the standard framework used in recent empirical work on structural transformation (e.g., [Herrendorf et al., 2013](#)). Our key departure from the standard framework is the introduction of a class of utility functions that generates heterogeneous, nonhomothetic sectoral demands for all levels of income, including when income grows toward infinity. These preferences, which we will refer to as nonhomothetic Constant Elasticity of Substitution (CES) preferences, have been studied by [Gorman \(1965\)](#), [Hanoch \(1975\)](#), [Sato \(1975\)](#), and [Blackorby and Russell \(1981\)](#) in the context of static, partial-equilibrium models. We show how to embed these preferences into a general equilibrium model of economic growth.

³The partial R^2 of the regressions shown in Figure 1 are 27% and 20%, respectively. In fact, if we split the sample into observations below and above the median income in the sample and estimate the relative Engel curves separately, we cannot reject the hypothesis of identical slopes of the Engel curves. See Table G.1 in the online appendix. If we reported separately the Engel curves for agriculture, manufacturing and services, we would find a negative, zero and positive slope, respectively.

Nonhomothetic CES preferences present a number of advantages. In addition to their tractability, they allow for income elasticity parameters to be independent of the elasticity of substitution between sectors, a feature that is unique to these preferences. Since our framework does not impose functional relationships between income and substitution elasticities, it lends itself to the task of decomposing the contributions of the demand and supply channels to structural change. In addition, our framework can accommodate an arbitrary number of sectors with heterogeneous and independent income elasticities. As a result, they generate Engel curves for different sectors that match the evidence discussed above: the logarithm of relative demand for the output of each sector has an approximately linear relationship with the logarithm of income. More specifically, this relationship is characterized by a sector-specific income elasticity parameter. We take advantage of these features to study a standard three-sector setting with agriculture, manufacturing and services. We then extend our analysis to a richer sectoral disaggregation (ten sectors) to explore reallocation patterns within manufacturing and services.

Our theory of structural change yields a number of theoretical and empirical results. First, the equilibrium in our model asymptotically converges to a path of constant real consumption growth. The asymptotic growth rate of real consumption depends on parameters characterizing both the supply and demand channels; it is a function of the sectoral income elasticities as well as sectoral growth rates of TFP and sectoral factor intensities. In this respect, our model differs from standard models using Stone-Geary preferences in which long-run growth is pinned down solely by the growth rate of TFP, and generalizes the findings of [Ngai and Pissarides \(2007\)](#) and [Acemoglu and Guerrieri \(2008\)](#). Second, our theory can produce similar evolutions for nominal and real sectoral measures of economic activity, which is a robust feature of the data.⁴ This is a consequence of the role of income elasticities in generating sectoral reallocation patterns. Third, our framework can generate hump-shaped patterns for the evolution of manufacturing consumption shares, which is a well-documented feature in the data ([Buera and Kaboski, 2012a](#)).

To evaluate the model empirically, we use structural equations derived from our theory to estimate the elasticities that characterize our utility function. We use historical cross-country sectoral data and household expenditure data that vary in their geographies and periods covered, and in their measures of economic activity used to capture structural change. A major finding is that the estimates of the elasticity of substitution and the relative slopes of the Engel curves across sectors are robust to the sample of countries, time periods and economic measures of sectoral activity. This demonstrates that the patterns presented in [Figure 1](#) not only characterize the Engel curves in the OECD but also apply more broadly to countries at other stages of development. We take this ability to parsimoniously account for

⁴[Herrendorf et al., 2014](#) show that supply-side driven structural transformation cannot account for the similar evolution of nominal and real sectoral measures of activity.

structural transformation in a variety of contexts as evidence in favor of our model.⁵

Finally, we use our model to study the drivers of structural transformation. Both relative prices and income effects turn out to be significant contributors. However, in contrast to previous studies (e.g., [Dennis and Iscan, 2009](#), [Boppart, 2014](#)), we find that income effects are more important than sectoral substitution driven by relative price trends. A potential reason for this discrepancy is that in our framework income effects are not hard-wired to have only transitory effects on the structural transformation or to be correlated with price effects. Once we do not impose these constraints on income effects, our estimates are consistent with a predominant role of income effects in accounting for the structural transformation during the postwar period in a large sample of countries at different stages of development.

Our paper relates to a large literature that aims to quantify the role of nonhomotheticities on growth and development (see, among others, [Matsuyama \(1992\)](#), [Echevarria, 1997](#), [Gollin et al., 2002](#), [Duarte and Restuccia, 2010](#), [Alvarez-Cuadrado and Poschke, 2011](#)).⁶ [Buera and Kaboski \(2009\)](#) and [Dennis and Iscan \(2009\)](#) have noted the limits of the generalized Stone-Geary utility function to match long time series or cross-sections of countries with different income levels.

The paper that is the closest to ours is [Boppart \(2014\)](#). Boppart studies the evolution of consumption of goods relative to services by introducing a sub-class of price-independent-generalized-linear (PIGL) preferences that also yield income effects in the long-run. There are several important differences between the PIGL preferences and nonhomothetic CES preferences. First, just like explicitly separable preferences such as Stone-Geary, PIGL preferences also presuppose specific parametric correlations for the evolution of income and price elasticities over time ([Gorman, 1965](#)). In contrast, nonhomothetic CES preferences do not build in any connection between price and income effects. Second, PIGL preferences can only accommodate two sectors with distinct income elasticities. In contrast, our framework allows for an arbitrary number of sectors.⁷ These differences between the two models are further reflected in their empirical implications. Whereas we find a larger contribution for demand nonhomotheticity in accounting for structural change, Boppart concludes that supply and

⁵A key parameter singled out in the literature is the price elasticity of substitution between consumption of different goods and services. Our baseline estimate of the elasticity of substitution is around 0.7. We find a very similar estimate using household level data from the Consumer Expenditure Survey (CEX), for which we can directly control for sectoral demand shocks and use an IV strategy. We also find that, compared to previous estimates based on Stone-Geary preferences ([Herrendorf et al., 2013](#)), the estimate of the elasticity of substitution is more robust to using either value added or expenditure measures.

⁶An alternative formulation that can reconcile demand being asymptotically nonhomothetic with balanced growth path is given by hierarchical preferences (e.g., [Foellmi and Zweimüller, 2006, 2008](#) and [Foellmi et al., 2014](#)). In recent work, [Świącki \(2014\)](#) estimates a demand system that features non-vanishing income effects in combination with subsistence levels à la Stone-Geary. However, his demand system also imposes a parametric relation between income and price effects.

⁷One can extend PIGL preferences to more than two sectors by nesting other functions, e.g., CES aggregators, as composites within the two-good utility function ([Boppart, 2014](#)). However, the resulting utility function does not allow for heterogeneity in income elasticity among the goods within each nested composite.

demand make roughly similar contributions.⁸

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 contains the estimation and model evaluation for a panel of 25 countries for the period 1947-2005. Section 4 analyzes household expenditure data and aggregate macroeconomic time series for the United States. Section 5 concludes. Appendix A presents some general properties of nonhomothetic CES. All proofs are in Appendix B.

2 Model

In this section, we develop the model that guides our empirical investigation of structural transformation in Sections 3 and 4, and characterize its asymptotic properties. The model closely follows workhorse models of structural transformation (e.g., Buera and Kaboski, 2009; Herrendorf et al., 2013, 2014). We replace the standard aggregators of sectoral consumption goods with a nonhomothetic CES aggregator. This single departure from the standard workhorse model delivers the main theoretical results of the paper and the demand system later used in the estimation. On the production side, the model combines two distinct potential drivers of sectoral reallocation previously highlighted in the literature: heterogeneous rates of technological growth (Ngai and Pissarides, 2007) and heterogeneous capital-intensity across sectors (Acemoglu and Guerrieri, 2008). We show that our empirical framework can account for both of these supply-side channels through the price effect.

2.1 Preferences and the Household Problem

A representative household has the following intertemporal preferences over goods and services produced in I different sectors

$$\sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\theta} - 1}{1-\theta} \right), \quad (1)$$

where $\beta \in (0, 1)$ is the discount factor, and θ is the reciprocal of the elasticity of intertemporal substitution. Aggregate consumption, C_t , combines sectoral goods, $\{C_{it}\}_{i=1}^I$, according to the implicitly defined function

$$\sum_{i=1}^I \Omega_i^{\frac{1}{\sigma}} C_t^{\frac{\epsilon_i - \sigma}{\sigma}} C_{it}^{\frac{\sigma-1}{\sigma}} = 1, \quad (2)$$

⁸In terms of the scope of the empirical exercise, while Boppart (2014) estimates his model with U.S. data and considers two sectors, the empirical evaluation of our model includes, in addition to the U.S., a wide range of other rich and developing countries and more than two sectors.

where $\sigma \in (0, 1)$ is the elasticity of substitution, and Ω_i 's are constant weights for all $i \in \mathcal{I} \equiv \{1, \dots, I\}$.⁹ Each sectoral good i is identified with a parameter $\epsilon_i \geq 1$, which is a measure of the income elasticity of demand for that good. Equation (2) introduces a nonhomothetic generalization of the standard Constant Elasticity of Substitution (CES) aggregator, which corresponds to the special case where $\epsilon_i = 1$ for all sectors. Intuitively, as aggregate consumption C_t increases, the weight given to the consumption of good i varies at a rate controlled by parameter ϵ_i . As a result, the household's demand for sectoral good i features a constant elasticity in terms of the aggregate consumption C_t , which is in turn determined by household income.

A number of unique features of the nonhomothetic CES aggregator makes it a natural choice for our model. In particular, consider the static expenditure minimization problem with sectoral prices $\{p_i\}_{i=1}^I$ and aggregate consumption C_t defined as in equation (2). The resulting demand function has the following properties.

1. The elasticity of the relative demand for two different goods with respect to aggregate consumption is constant, i.e.,

$$\frac{\partial \log (C_i / C_j)}{\partial \log C} = \epsilon_i - \epsilon_j. \quad (3)$$

2. The elasticity of substitution between goods of different sectors is uniquely defined and constant¹⁰

$$\frac{\partial \log (C_i / C_j)}{\partial \log (P_j / P_i)} = \sigma. \quad (4)$$

The first property ensures that the nonhomothetic features of these preferences do not systematically vary as income grows. As discussed in the introduction and in Section 4, available data on sectoral consumption, both at the macro and micro levels, suggest stable and heterogeneous income elasticities across sectors. Therefore, we find it reasonable to specify preferences that do not result in systematically vanishing patterns of nonhomotheticity, as, for instance, would be implied by the choice of Stone-Geary preferences. Similarly, the second property

⁹We focus in the empirically relevant case $\sigma \in (0, 1)$ (gross complements). However, the preferences are also well-defined when $\sigma > 1$ (gross substitutes). In general, if $\sigma > 0$ and $\Omega_i > 0$ for all $i \in \mathcal{I}$ and if $\epsilon_i > \sigma$ when $0 < \sigma < 1$, or $\epsilon_i < \sigma$ when $\sigma > 1$, then the aggregator C_t introduced in equation (2) is globally monotonically increasing and quasi-concave, yielding a well-defined utility function over the bundle of goods $(C_{1t}, \dots, C_{It})$, see [Hanoch \(1975\)](#). The additional restriction $\epsilon_i \geq 1$ ensures strict concavity, which simplifies the analysis of the dynamics below.

¹⁰Note that for preferences defined over I goods when $I > 2$, alternative definitions for elasticity of substitution do not necessarily coincide. In particular, equation (4) defines the so-called Morishima elasticity of substitution, which in general is not symmetric. This definition may be contrasted from the Allen (or Allen-Uzawa) elasticity of substitution defined as $\frac{E \cdot \partial C_i / \partial P_j}{C_i C_j}$, where E is the corresponding value of expenditure. [Blackorby and Russell \(1981\)](#) prove that the only preferences for which the Morishima elasticities of substitution between any two goods are symmetric, constant, and identical to Allen-Uzawa elasticities have the form of equation (2), albeit with a more general dependence of weights on C .

ensures that the patterns of inter-sectoral substitution have a constant price elasticity and, thus, do not systematically vary as income grows. This property is unique to this class of nonhomothetic CES preferences.¹¹ The combination of these two properties uniquely defines the aggregator in equation (2).

To complete the characterization of the household behavior, we assume that the representative household inelastically supplies one unit of perfectly divisible labor, and starts at period 0 with an initial endowment $\mathcal{A}_0$ of assets. The household takes the sequence of wages, real interest rates, and prices of goods and services $\{w_t, r_t, \{p_{it}\}_{i=1}^I\}_{t=0}^\infty$ as given, and chooses a sequence of assets stocks $\{\mathcal{A}_t\}_{t=1}^\infty$ and aggregate consumption $\{C_t\}_{t=0}^\infty$ to maximize its utility defined in Equation (1), subject to the per-period budget constraint

$$\mathcal{A}_{t+1} + \sum_{i=1}^I p_{it} C_{it} \leq w_t + (1 + r_t) \mathcal{A}_t, \quad (5)$$

where we have normalized the price of assets to 1. The next lemma characterizes the solution to the household problem.

Lemma 1. (*Household Behavior*) *Consider a household with preferences and budget constraint as described by equations (1), (2), (5), and the No-Ponzi condition $\lim_{t \rightarrow \infty} \mathcal{A}_t \left(\prod_{t'=1}^{t-1} \frac{1}{1+r_{t'}} \right) = 0$. Given a sequence of prices $\{w_t, r_t, \{p_{it}\}_{i=1}^I\}_{t=0}^\infty$ and an initial stock of assets $\mathcal{A}_0$, the problem has a unique solution, fully characterized by the following conditions.*

1. *The intratemporal allocation of consumption goods satisfies*

$$C_{it} = \Omega_i \left(\frac{p_{it}}{P_t} \right)^{-\sigma} C_t^{\epsilon_i}, \quad (6)$$

where P_t is the aggregate price index

$$P_t \equiv \frac{E_t}{C_t} = \frac{1}{C_t} \left[\sum_{i=1}^I \Omega_i C_t^{\epsilon_i - \sigma} p_{it}^{1-\sigma} \right]^{\frac{1}{1-\sigma}}, \quad (7)$$

and $E_t \equiv \sum_{i=1}^I p_{it} C_{it}$ denotes consumption expenditure at time t .

¹¹Nonhomothetic CES preferences inherit this property because they belong to the class of implicitly additively separable preferences (Hanoch, 1975). In contrast, any preferences that are explicitly additively separable in sectoral goods imply parametric links between income and substitution elasticities (this result is known as Pigou's Law, see Snow and Warren (2015) and the references therein). Appendix A illustrates how such links appear in specific case of Stone-Geary and price-independent generalized linear (PIGL) preferences, two types of specifications recently used in studies of structural change.

2. The intertemporal allocation of real aggregate consumption satisfies the Euler equation

$$\left(\frac{C_{t+1}}{C_t}\right)^{-\theta} = \frac{1}{\beta(1+r_t)} \frac{P_{t+1}}{P_t} \frac{\bar{\epsilon}_{t+1} - \sigma}{\bar{\epsilon}_t - \sigma}, \quad (8)$$

and the transversality condition

$$\lim_{t \rightarrow \infty} \beta^t (1+r_t) \frac{\mathcal{A}_t}{E_t} C_t^{1-\theta} \frac{1-\sigma}{\bar{\epsilon}_t - \sigma} = 0, \quad (9)$$

where we have defined $\bar{\epsilon}_t \equiv \sum_{i=1}^I \omega_{it} \epsilon_i$ with ω_{it} denoting the expenditure share in sector i , $p_{it} C_{it} / E_t$.

The key insight from Lemma 1 is that the household problem can be decomposed into two sub-problems: one involving the allocation of consumption and savings over time, and one involving the allocation of consumption across sectors. First, consider the *intertemporal* consumption-savings problem. The household solves for the sequence of $\{\mathcal{A}_{t+1}, C_t\}_{t=0}^{\infty}$ that maximizes utility (1) subject to the constraint

$$\mathcal{A}_{t+1} + E\left(C_t; \{p_{it}\}_{i=1}^I\right) \leq w_t + \mathcal{A}_t (1+r_t), \quad (10)$$

where $E\left(C_t; \{p_{it}\}_{i=1}^I\right)$ is the total expenditure function for the nonhomothetic CES preferences, defined in equation (7). Because of nonhomotheticity, consumption expenditure is a nonlinear function of real aggregate consumption, and the price index reflects changes in the sectoral composition of consumption as income grows.¹² The household incorporates this relationship in its Euler equation (8), where we see a wedge between the marginal cost of real consumption and the aggregate price index. The size of this wedge, given by $(\bar{\epsilon}_t - \sigma) / (1 - \sigma)$, depends on the average income elasticities across sectors, $\bar{\epsilon}_t = \sum_{i=1}^I \omega_{it} \epsilon_i$, and varies over time. In the case of homothetic CES where $\epsilon_i \equiv 1$, this wedge disappears.

The second part of the household problem involves the *intratemporal* problem of allocating consumption across different goods. Equation (6) corresponds to the sectoral demand implied by the nonhomothetic CES aggregator. Lemma 1 establishes that given aggregate consumption, C_t , allocated to period t , sectoral demand simply follows the solution to the static allocation problem.

Note that Equation (6) restates the two main features of the nonhomothetic CES aggregator expressed in equations (3) and (4): constant and independent elasticities of income and substitution for different goods. We can rewrite this relation in terms of the logarithm of

¹²An equivalent definition for the price index in terms of total and sectoral expenditure is

$$P_t^{1-\sigma} = \sum_i (\Omega_i E_t^{\epsilon_i - 1} p_{it}^{1-\sigma})^{\frac{1-\sigma}{\epsilon_i - \sigma}} \omega_{it}^{\frac{\epsilon_i - 1}{\epsilon_i - \sigma}}. \quad (11)$$

relative real consumption and consumption expenditure shares between sectors i and j as a function of the logarithm of relative prices and aggregate consumption,

$$\log \left(\frac{C_{it}}{C_{jt}} \right) = -\sigma \log \left(\frac{p_{it}}{p_{jt}} \right) + (\epsilon_i - \epsilon_j) \log C_t + \log \left(\frac{\Omega_i}{\Omega_j} \right), \quad (12)$$

$$\log \left(\frac{\omega_{it}}{\omega_{ji}} \right) = (1 - \sigma) \log \left(\frac{p_{it}}{p_{jt}} \right) + (\epsilon_i - \epsilon_j) \log C_t + \log \left(\frac{\Omega_i}{\Omega_j} \right), \quad (13)$$

where the second equation simply states the relationship in terms of expenditure shares, which plays an important role in our theory, as we will see later. Equation (12) once again highlights the key features of the demand system implied by this nonhomothetic CES preferences. Interpreting C_{it} as the Hicksian demand for good i with aggregate consumption C_t under prices p_{it} 's, we find a perfect separation of the price and the income effects. The first term on the right hand side shows the price effects characterized by constant elasticity of substitution σ . More interestingly, the second term on the right hand side shows the change in relative sectoral demand as consumers move across indifference curves. This income effect is characterized by constant sectoral income elasticity parameters ϵ_i 's. If $\epsilon_i > \epsilon_j$, demand for good i rises relative to good j as consumers become wealthier.¹³

Equations (12) and (13) also show how our model can generate a positive correlation between relative sectoral consumption in real and expenditure terms, as it is observed in the data. As in the case with homothetic CES aggregators, the combination of the price effect and gross complementarity ($\sigma < 1$) imply that relative real sectoral consumption should negatively correlate with relative sectoral prices. To see why, note that relative real consumption is decreasing in relative prices with an elasticity of $-\sigma$, while relative expenditure is increasing with an elasticity of $1 - \sigma$. However, our demand system has an additional force, income effects, which makes both time series co-move in aggregate consumption. Thus, if income effects are sufficiently strong, both time series can be positively correlated. In Section 3.4 we show that this is the case when we estimate our demand system.

¹³The expenditure elasticity of demand for sectoral good i is given by

$$\eta_{it} \equiv \frac{\partial \log C_{it}}{\partial \log E_t} = 1 + \frac{1 - \sigma}{\bar{\epsilon}_t - \sigma} (\epsilon_i - \bar{\epsilon}_t), \quad (14)$$

which, as Engel aggregation requires, averages to 1 when sectoral weights are given by expenditure shares. If sector i has an income elasticity parameter, ϵ_i , that exceeds the economy-wide average elasticity parameter, $\bar{\epsilon}_t$, at time t , then sector i has an expenditure elasticity greater than 1 at that point in time. The expenditure elasticity of relative demand is

$$\frac{\partial \log (C_{it}/C_{jt})}{\partial \log E_t} = \frac{\epsilon_i - \epsilon_j}{\bar{\epsilon}_t - \sigma}, \quad (15)$$

which parallels equation (3) now expressed in terms of expenditure, rather than real aggregate consumption.

2.2 Production and Competitive Equilibrium

The supply side of the economy allows for two distinct sources of heterogeneity in sectoral production. Our model combines the heterogeneous sectoral productivity growth framework of [Ngai and Pissarides \(2007\)](#) with the heterogeneous sectoral factor intensity model of [Acemoglu and Guerrieri \(2008\)](#).

A representative firm in each consumption sector produces sectoral output under perfect competition. In addition, a representative firm in a perfectly competitive investment sector produces investment good, Y_{0t} , that is used in the process of capital accumulation. We assume Cobb-Douglas production functions with time-varying Hicks-neutral sector-specific productivities,

$$Y_{it} = A_{it} K_{it}^{\alpha_i} L_{it}^{1-\alpha_i}, \quad i \in \{0\} \cup \mathcal{I},$$

where K_{it} and L_{it} are capital and labor used in the production of output Y_{it} in sector i at time t (we have identified the sector producing investment good as $i = 0$) and $\alpha_i \in (0, 1)$ denotes sector-specific capital intensity. The aggregate capital stock of the economy, K_t , accumulates using investment goods and depreciates at rate δ , $Y_{0t} = K_{t+1} - (1 - \delta) K_t$.

We focus on the features of the competitive equilibrium of this economy that motivate our empirical specifications.¹⁴ Firm profit maximization and equalization of the prices of labor and capital across sectors pin down prices of sectoral consumption goods,

$$p_{it} = \frac{p_{it}}{p_{0t}} = \frac{\alpha_0^{\alpha_0} (1 - \alpha_0)^{1-\alpha_0}}{\alpha_i^{\alpha_i} (1 - \alpha_i)^{1-\alpha_i}} \left(\frac{w_t}{R_t} \right)^{\alpha_0 - \alpha_i} \frac{A_{0t}}{A_{it}}, \quad (16)$$

where, since the units of investment good and capital are the same, we normalize the price of investment good, $p_{0t} \equiv 1$. Equation (16) shows that price effects capture both supply-side drivers of sectoral reallocation: heterogeneity in productivity growth rates and heterogeneity in capital intensities.

Goods market clearing ensures that household sectoral consumption expenditure equals the value of sectoral production output, $P_{it} C_{it} = P_{it} Y_{it}$.¹⁵ Competitive goods markets and profit maximization together imply that a constant share of sectoral output is spent on the wage bill,

$$L_{it} = (1 - \alpha_i) \frac{P_{it} C_{it}}{w_t} \omega_{it}, \quad (17)$$

¹⁴Given initial stock of capital K_0 and a sequence of sectoral productivities $\left\{ \{A_{it}\}_{i=1}^I \right\}_{t \geq 0}$, a competitive equilibrium is defined as a sequence of allocations $\left\{ C_t, K_{t+1}, Y_{0t}, L_{0t}, K_{0t}, \{Y_{it}, C_{it}, K_{it}, L_{it}\}_{i=1}^I \right\}_{t \geq 0}$ and a sequence of prices $\left\{ w_t, R_t, \{p_{it}\}_{i=1}^I \right\}_{t \geq 0}$ such that (i) agents maximize the present discounted value of their utility given their budget constraint, (ii) firms maximize profits and (iii) markets clear.

¹⁵In our empirical applications, we account for sectoral trade flows.

where ω_{it} is the share of sector i in household consumption expenditure.

Equation (17), together with equations (13) and (16) summarize the main insights from the theory that we employ in our empirical strategy. First, equation (17) implies

$$\frac{L_{it}}{L_{jt}} = \frac{1 - \alpha_i}{1 - \alpha_j} \frac{\omega_{it}}{\omega_{jt}}, \quad i, j \in \mathcal{I}. \quad (18)$$

Equation (18) shows that the paths of relative sectoral employment shares follow those of relative consumption expenditure shares. Second, equation (13) characterizes the paths of relative consumption expenditure shares as a function of relative prices and aggregate real consumption. Thus, equations (13) and (18) together predict the evolution of relative employment shares across sectors. We will use these two equations extensively in our empirical exercise in the next two sections.

2.3 Constant Growth Path

Before moving into the empirical sections, we characterize the asymptotic dynamics of the economy when sectoral total factor productivities grow at heterogeneous but constant rates. In particular, let us assume that sectoral productivity growth is

$$\frac{A_{it+1}}{A_{it}} = 1 + \gamma_i, \quad i \in \{0\} \cup \mathcal{I}. \quad (19)$$

The next proposition characterizes the asymptotic dynamics of the competitive equilibrium.

Proposition 1. *Let γ^* be defined as*

$$\gamma^* = \min_{i \in \mathcal{I}} \left[(1 + \gamma_0)^{\frac{\alpha_i}{1 - \alpha_0}} (1 + \gamma_i) \right]^{\frac{1 - \sigma}{\epsilon_i - \sigma}} - 1. \quad (20)$$

Assume that γ^ satisfies the following condition*

$$(1 + \gamma_0)^{-\frac{\alpha_0}{1 - \alpha_0}} < \beta (1 + \gamma^*)^{1 - \theta} < \min \left\{ \frac{(1 + \gamma_0)^{-\frac{\alpha_0}{1 - \alpha_0}}}{\alpha_0 + (1 - \alpha_0) (1 + \gamma_0)^{-\frac{1}{1 - \alpha_0}} (1 - \delta)}, 1 \right\}. \quad (21)$$

Then, for any initial level of capital stock, K_0 , there exists a unique competitive equilibrium along which consumption asymptotically grows at rate γ^ ,¹⁶*

$$\lim_{t \rightarrow \infty} \frac{C_{t+1}}{C_t} = 1 + \gamma^*. \quad (22)$$

Along the this constant growth path, (i) the real interest rate is constant, $r^ \equiv (1 + \gamma_0)/\beta(1 +$*

¹⁶Here we follow the terminology of [Acemoglu and Guerrieri \(2008\)](#) in referring to our equilibrium path as a constant growth path. [Kongsamut et al. \(2001\)](#) refer to this concept as generalized balanced growth path.

$\gamma^*)^{1-\theta} - 1$, (ii) nominal expenditure, total nominal output, and the stock of capital grow at rate $(1+\gamma_0)^{\frac{1}{1-\alpha_0}}$, (iii) only the subset of sectors $\mathcal{I}^*$ that achieve the minimum in equation (20) employ a non-negligible fraction of workers.¹⁷

Equation (20) shows how the long-run growth rate of consumption is affected by income elasticities, ϵ_i , rates of technological progress, γ_i , and sectoral capital intensities, α_i . To build intuition, consider the case in which all sectors have the same capital intensity, and preferences are homothetic. Then, since $\sigma \in (0, 1)$, equation (20) implies that the long-run growth rate of real consumption is pinned down by the sectors with the lowest technological progress, as in Ngai and Pissarides (2007). Consider now the case in which there is also heterogeneity in income elasticities. In this case, sectors with higher income elasticity and higher technological progress can co-exist in the long-run with sectors with low technological progress and low income elasticity. The intuition is that, as agents become richer, they want to consume more goods that have higher technological progress, as they are more income-elastic. Finally, the role of heterogeneity in capital shares also affects the long-run equilibrium through its effect on relative prices in an analogous way to technological progress.

Which sectors survive in the long run? At all points in time, all sectors produce a positive amount of goods, and its production grows over time. In relative terms, however, only the subset of sectors $\mathcal{I}^*$ satisfying equation (20) will comprise a non-negligible share of total consumption expenditure in the long-run.

3 Quantitative Exploration of a Cross-Country Panel

In this section we explore the ability of nonhomothetic CES preferences to account for the broad patterns of structural transformation observed across countries in agriculture, manufacturing and services during the postwar period. We discipline our model by using the fact that the same parameters of the utility function $\{\sigma, \epsilon_i\}_{i \in \mathcal{I}}$ for all countries. After estimating these parameters, we gauge the ability of our model to account for the very different experiences of advanced, miracle and developing economies. We conclude the section by conducting a battery of exercises that revisit critical findings in the structural change literature, and extending our analysis to more disaggregated sectoral data.

¹⁷The proof of the proposition in the appendix also constructs the equations describing the equilibrium dynamics. In addition, in the online Appendix, we provide the full characterization of the equilibrium dynamics in a continuous time rendition of the model where we introduce a more general definition of nonhomothetic CES that nests the one discussed in this section. We also present therein the specific cardinalization of the generalized nonhomothetic CES aggregator that corresponds to the definition of aggregate real consumption given in Feenstra et al. (2013).

3.1 Empirical Strategy

Our empirical strategy uses the solution of the intratemporal problem and the production decisions of firms to estimate the preference parameters of the nonhomothetic CES aggregator (2). Taking the logarithms of the sectoral demands (13) and using that the ratio of sectoral expenditures is proportional in equilibrium to the ratio of sectoral labor allocations, (18), we obtain our estimating equations

$$\log \left(\frac{L_{a,t}^c}{L_{m,t}^c} \right) = \zeta_{am}^c + (1 - \sigma) \log \left(\frac{p_{a,t}^c}{p_{m,t}^c} \right) + (\epsilon_a - \epsilon_m) \log C_t^c + \nu_{am,t}^c, \quad (23)$$

$$\log \left(\frac{L_{s,t}^c}{L_{m,t}^c} \right) = \zeta_{sm}^c + (1 - \sigma) \log \left(\frac{p_{s,t}^c}{p_{m,t}^c} \right) + (\epsilon_s - \epsilon_m) \log C_t^c + \nu_{sm,t}^c, \quad (24)$$

where a , m and s denote agriculture, manufacturing and services, respectively, and t , time. The superscript c denotes a country, and $\nu_{am,t}^c$ and $\nu_{sm,t}^c$ are the error terms. We allow for country-sector dyad fixed effects, ζ_{am}^c and ζ_{sm}^c , as there may be systematic differences in measurement across countries. These country-sector dyad fixed effects also absorb potential cross-country differences in sectoral taste parameters, Ω_i^c , and differences in factor shares in the production function, α_i . Note that there are two cross-equation restrictions. The price elasticity σ is restricted to be the same across sectors and countries. Income elasticities, ϵ_i , are also restricted to be the same across countries for a given sector s .

To construct our dependent variable, we could either use expenditure shares or employment shares. In the baseline specification, we use employment shares. This allows us to circumvent the problem that the prices that we use as regressors also enter expenditure shares, which could introduce bias.¹⁸ To account for the fact that some goods can be imported and exported, thus affecting the sectoral composition of employment, we control for the share of net sectoral exports over total production in sector i , time t and country c .¹⁹

Identification The identification strategy relies on the intra-period allocation of consumption that follows from the solution of the intratemporal allocation problem (13). That is, conditional on the observed levels of aggregate consumption C_t^c and sectoral prices p_{it}^c , we use our demand system to estimate relative consumption across sectors. Given the presence of country-sector dyad fixed effects, ζ_{am}^c , ζ_{sm}^c , the relevant variation used to identify the elasticities is the within country-sector time variation. To the extent that we have a long time series

¹⁸In practice, as we discuss below, we obtain similar estimates with either dependent variable.

¹⁹This particular specification of the trade controls follows from our theoretical model. To derive this result, note that $p_{it}^c C_{it}^c = p_{it}^c Y_{it}^c - NX_{it}^c$, where NX_{it}^c denotes the nominal value of net exports in sector i , time t and country c . Using equation (17), the amount of labor needed to produce the amount consumed in sector s needs to be adjusted by $NX_{it}^c / p_{it}^c Y_{it}^c$. We have also used more reduced-form controls, such as controlling directly for net exports or exports and imports separately, obtaining similar results.

for C_t^c and p_{it}^c , we have a super-consistent estimator of the elasticities (Hamilton, 1994).

Changes over time in aggregate and sectoral productivity contribute to the identification of the price and income elasticities. For example, sectoral productivity growth differences affect relative prices and introduce variation in the estimating equations, (23) - (24).²⁰ Sectoral and aggregate productivity shocks can also affect the level of total consumption, thereby introducing additional variation in the estimation through C_t^c .

Aggregate demand shocks, such as a rise in the propensity to spend, are captured through the aggregate consumption term $\log C_t^c$ in (23) - (24) and also contribute to the identification of our demand system.²¹ However, sectoral taste shocks that induce consumers to spend more in one sector for a given level of aggregate consumption and sectoral prices are not well-captured in this specification. Given that we have already country-sector dyad fixed effects, we cannot add an additional time fixed effect that would control for preference shocks. To the extent that these shocks are uncorrelated with other type of shocks, they enter as classical measurement error in the estimating equations (23) - (24), and our estimates are still consistent. If, on the other hand, sectoral preference shocks are correlated with other shocks (e.g., aggregate demand or productivity shocks) our estimation is going to produce biased estimates.

We deal with potential biases coming from sectoral taste shocks in two different ways. First, we estimate the elasticities separately for OECD and non-OECD countries and show that we cannot reject the null that they are statistically the same. While the estimates could in principle be biased in both cases, this would require sectoral taste shocks to be correlated with aggregate demand or productivity shocks in the same way for these two groups of countries, which we deem unlikely. Second, in Section 4.1 we use household-level data, which allows us to include sector-year fixed effects that absorb sectoral demand shocks and to use an IV strategy. Reassuringly, we find that the estimates for the elasticity of substitution and of the income elasticities are similar to our estimates from aggregate data.

3.2 Data Description

We use the GGDC 10-Sector Database for sectoral value added data (Vries et al., 2014). It provides a long-run internationally comparable dataset on sectoral measures for 10 countries in Asia, 9 in Europe, 9 in Latin America, 10 in Africa and the United States. The variables covered in the data set are annual series of production value added (nominal and real) and employment for 10 broad sectors starting in 1947. In our baseline exercise we aggregate the

²⁰Using the log of the ratio of sectoral prices in our estimation has the advantage that we can directly use nominal prices and any cross-country systematic difference in the measurement of prices is going to be captured in the fixed effect.

²¹In particular, suppose that the preference term for sector i , Ω_i , in (2) becomes stochastic rather than constant, $\tilde{\Omega}_{it} = \Omega_i + \mathbf{1}_t$ with $\mathbf{1}_t$ being white noise. Our identification strategy remains valid in this more general case, as it relies on the within period problem, which remains unaltered.

ten sectors into agriculture, manufacturing and services.²² In Section 3.6, we estimate our model for 10 sectors.

For real consumption, we use the time series on consumption per capita from the Barro-Ursua Macroeconomic Data.²³ Their data has the advantage of using the Fisher chained price index, which allows us to have a meaningful measure of real consumption.²⁴ These data do not include government services into consumption, which we exclude from our estimation. Also, the only African country covered is South Africa. Our final sample consists of 25 countries that span very different growth trajectories during the postwar period.²⁵

3.3 Estimation Results

We estimate jointly (23) and (24) imposing the cross-equation restrictions that price and income elasticities have to be the same across countries. Table 1 reports the results of estimating the system of equations for the whole sample of 25 countries and for OECD and non-OECD countries separately.

Columns (1) to (3) in Table 1 report our estimates for the entire sample of countries. Column (1) reports the estimates without using country-sector fixed effects (and thus using cross-country variation in levels to identify the parameters). Column (2) reports our estimates using country-sector fixed effects (whereby using only within country variation to identify the elasticities) and column (3) includes trade controls. Our estimates of the price elasticity of substitution across sectors range from 0.66 to 0.75 and are precisely estimated in all three specifications (standard errors are clustered at the country level). In fact, we cannot reject the null that these three estimates are statistically identical. However, we can reject the null that they are equal to one at 5%, implying that these three sectors are complements. The estimates for the difference of income elasticities yield sensible results that are very stable across the three specifications and significant at conventional levels. We find that the difference in income elasticities between agriculture and manufacturing, $\epsilon_a - \epsilon_m$, is negative and ranges from -1.09

²²The ten sectors are agriculture, mining, manufacturing, construction, public utilities, retail and wholesale trade, transport and communication, finance and business services, other market services and government services. We classify as manufacturing: mining, manufacturing and construction, while the rest are classified as services (except agriculture). Figures 5 and 6 depict the time series for each country.

²³It can be obtained at <http://scholar.harvard.edu/barro/publications/barro-ursua-macroeconomic-data>

²⁴Fisher price indices approximate to second order the ideal price index of any continuously differentiable utility, thus approximating ours (see online Appendix F for further discussion on the topic). We have also run Monte-Carlo simulations to assess the extent of the bias induced by using this approximation around the estimated parameters. We find that the error is less than 1% of the estimated parameters (Online Appendix E contains a sample code). Deaton and Muellbauer (1980) find the same result for the AIDS demand system: using the exact form of the price index or an approximation by a superlative price index makes little difference for the estimation results. In light of the small bias introduced, we prefer using the linear estimation model to a non-linear estimation of the price index, for which we would have an incidental parameters problem (see Wooldridge, 2001).

²⁵These are Denmark, France, Italy, the Netherlands, Spain, Sweden, UK, USA, West Germany, India, Indonesia, Japan, South Korea, Malaysia, Philippines, Singapore, Taiwan, Argentina, Brazil, Chile, Colombia, Mexico, Peru, Venezuela and South Africa.

Table 1: Baseline Estimates for the Cross-Country Sample

Dep. Var.:	World			OECD		Non-OECD	
Rel. Emp.	(1)	(2)	(3)	(4)	(5)	(6)	(7)
σ	0.66 (0.19)	0.75 (0.11)	0.75 (0.10)	0.69 (0.17)	0.71 (0.16)	0.69 (0.08)	0.71 (0.07)
$\varepsilon_a - \varepsilon_m$	-0.81 (0.24)	-1.09 (0.10)	-1.04 (0.10)	-0.99 (0.19)	-0.89 (0.21)	-1.16 (0.14)	-1.10 (0.11)
$\varepsilon_s - \varepsilon_m$	0.32 (0.08)	0.32 (0.10)	0.32 (0.10)	0.40 (0.19)	0.51 (0.17)	0.23 (0.09)	0.23 (0.09)
Obs.	1006	1006	1005	436	436	570	569
R^2	0.14	0.83	0.83	0.77	0.80	0.84	0.85
$c \cdot sm$ FE	N	Y	Y	Y	Y	Y	Y
Trade Controls	N	N	Y	N	Y	N	Y

Note: Standard errors clustered by country.

to -0.81, while the difference between services and manufacturing, $\varepsilon_s - \varepsilon_m$, is positive around 0.32 in all three specifications.²⁶

As we have discussed, sectoral taste shocks that are systematically correlated with other shocks can be a threat to the consistency of our estimates of elasticity parameters. To mitigate this concern, we report the estimated elasticities for OECD countries in columns (5) and (6) and non-OECD countries in columns (7) and (8) with and without trade controls, respectively. We find that the estimates of all elasticities are similar for the two sub-samples. In fact, we cannot reject the null that the estimates for the elasticities are the same for both sub-samples at conventional levels. For example, the p -value of testing the joint hypothesis of price and income elasticities being different for OECD and non-OECD countries is 0.45. We take this as reassuring evidence. In the next section, we also estimate our demand system using household data, which allows us to have more demanding specifications, finding similar results.

Table 1 also reports the R^2 of these regressions. We find that prices and aggregate consumption, along with the country-sector fixed effects, account for 83% of the variation in our panel. The fit is equally good if we split the sample between OECD and non-OECD countries (77% and 84%, respectively). The inclusion of explicit sectoral net exports controls improves the fit to 80% for OECD countries and 85% for non-OECD countries. This modest improvement suggests that the change over time of domestic demand of domestic goods is the main driver of our results. Note, however, that this does not necessarily mean that trade plays a minor role. The reason is that that our sectoral price measures are constructed from measures which include imported inputs. Thus, part of the productivity enhancing effects of trade are

²⁶Note that the scale of the difference of elasticities does not matter for the real allocation of resources. The reason is that there is one degree of freedom in the definition of real income elasticities.

reflected in our price measures.

Case Studies Figure 2 plots the actual and the predicted employment shares implied by our estimates from column (3) of Table 1 for six countries, Mexico, Colombia, Japan, Taiwan, the U.S. and Spain.²⁷ This figure confirms the good fit of the model despite the parsimony of using the same price and income elasticities for all countries. In particular, the model captures the evolution of employment shares in all sectors for countries at very different stages of development.

For Japan and Taiwan, we see that our fitted model generates a hump-shape for employment in manufacturing that tracks the patterns observed in the data. For Taiwan, the predicted initial level of the employment share in manufacturing is 21%, it goes up to 39% and back to 35% at the end of the period. The observed levels are 20%, 43% and 37%. For Japan, the employment share in manufacturing is 26% in the initial period, it goes up to 38% in the mid 1970s and it is 30% by the end of our sample. The fitted time series starts at 26%, goes up to 35% and declines to 33% by the end of the period.²⁸

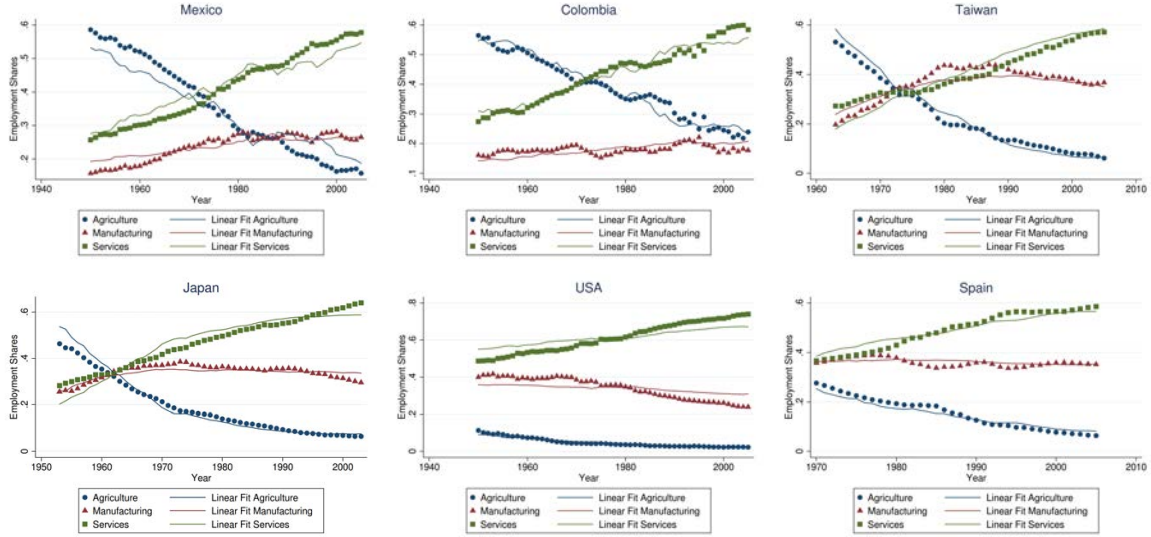
To gain better intuition on the role played by relative prices, consumption and net exports, we illustrate the case of Japan in more detail in Figure 4 in Appendix C. Panel (a) shows the overall fit of the data using the estimated parameters from the entire sample (column 3). Panels (b), (c) and (d) show the time series of relative prices, consumption and sectoral net exports. We further report the partial fit generated by each time series (and the country-sector fixed effects) at the estimated parameter values. Panel (e) shows the partial fit generated by the relative price time series. We see that, at the estimated parameter values, relative prices account for relatively little of the variation. In contrast, the evolution of aggregate consumption accounts for much of the structural transformation (see panel (f)). In particular, income effects drive the observed hump-shape in manufacturing. Intuitively, as the Japanese economy became richer in the 1950s, it reallocated labor away from agriculture to both manufacturing and services. Subsequent income growth led to the expansion of services which absorbed employment from manufacturing. Finally, panel (g) shows that changes in sectoral net exports did not play a significant role in accounting for the structural transformation in Japan.

Contribution of Relative Price and Income Effects After discussing the case of Japan, we analyze the drivers of structural transformation for our entire sample. We compute the share of the variation in employment shares that is generated by price and income effects

²⁷Figures 5 and 6 show the predicted series of the employment shares and the actual time series for all countries in our sample.

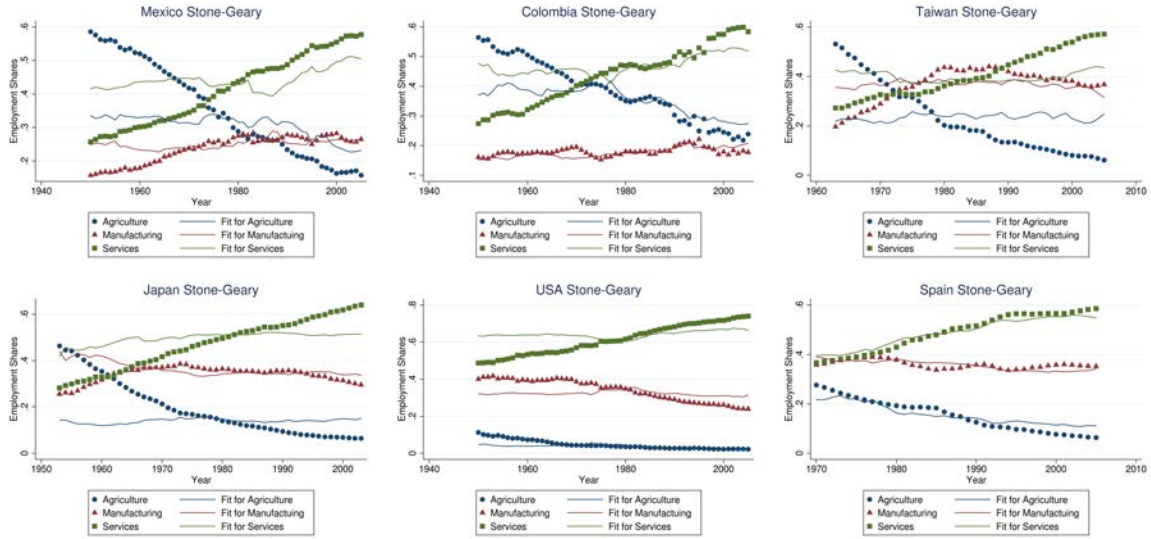
²⁸For the U.S., we see that the evolution of the employment shares in services and manufacturing are steeper than predicted by our model. This is the case also for other OECD countries. This reflects the fact that the income elasticity of services is greater for these set of countries, as column (5) in Table 1 shows. Indeed, if we plot the predicted fit using the estimates $\{\sigma, \epsilon_s - \epsilon_m, \epsilon_a - \epsilon_m\}$ for only OECD countries this problem goes entirely away, as can be seen in Figure H.4 in the online appendix.

Figure 2: Baseline fit with common preference parameters $\{\sigma, \epsilon_a - \epsilon_m, \epsilon_s - \epsilon_m\}$ for six countries



Note: Predicted fit (shown in solid lines) and actual data for six countries of the sample. Fit corresponding to the estimates for the entire sample reported in column (3) of Table 1.

Figure 3: Stone-Geary fit with common preference parameters $\{\sigma, \bar{c}_a, \bar{c}_s\}$ for six countries



Note: Predicted fit of Stone-Geary preferences (shown in solid lines) and actual data for six countries of the sample. The fit corresponds to the estimates for the entire sample. Relative to the fit of nonhomothetic CES shown in Figure 2, we see that the fit is overall worse, except perhaps for Spain for which it is comparable. For example, the Stone-Geary estimation fails to capture the hump-shaped evolution of manufacturing in Taiwan and Japan.

in our fitted model.²⁹ We find that the variation generated by prices in the median year accounts for 14% of the employment growth in agriculture, 43% in manufacturing and 17% in services. The remaining variation is accounted for by income effects. If we restrict attention to the OECD, we find a similar pattern with prices accounting for 19%, 39% and 20% of employment share growth in agriculture, manufacturing and services, respectively. Hence, we conclude that the bulk of the variation in sectoral employment growth is generated by income effects.

In the online Appendix, we study the robustness of this result to using other approaches to assess the relative contribution of the drivers of structural transformation. Table G.2 in the online appendix shows that the likelihood-ratio tests of including price and consumption data to a model with only country-sector fixed effects are significant. However, the increase in the likelihood is much higher when we add consumption data than when we add relative price data.³⁰

Heterogeneous Price Elasticity of Substitution Across Sectors So far, we have assumed that the price elasticities across sectors, σ , are identical. We can test whether this identifying restriction is a good approximation of our data or whether allowing for differential price elasticities across sectors improves the fit significantly. We run the baseline regressions (23) and (24) without imposing that the coefficient on relative prices is the same across regressions,

$$\log \left(\frac{L_{a,t}^c}{L_{m,t}^c} \right) = \zeta_{am}^c + (1 - \sigma_{am}) \log \left(\frac{p_{a,t}^c}{p_{m,t}^c} \right) + (\epsilon_a - \epsilon_m) \log C_t^c + \nu_{am,t}^c, \quad (26)$$

$$\log \left(\frac{L_{s,t}^c}{L_{m,t}^c} \right) = \zeta_{sm}^c + (1 - \sigma_{sm}) \log \left(\frac{p_{s,t}^c}{p_{m,t}^c} \right) + (\epsilon_s - \epsilon_m) \log C_t^c + \nu_{sm,t}^c, \quad (27)$$

where σ_{sm} is not restricted to be equal to σ_{am} . Table G.4 in the online appendix reports our estimates. We find that the price elasticities in the two regressions are very similar. For

²⁹To perform the decomposition, from the demand system (13), we have that the growth rate of employment in sector i relative to j is

$$\gamma_{L_i} - \gamma_{L_j} = (1 - \sigma)(\gamma_{p_i} - \gamma_{p_j}) + (\epsilon_i - \epsilon_j)\gamma_C, \quad (25)$$

where γ_{L_i} , γ_{p_i} , γ_C denote the growth rate of the employment share in sector i , the growth rate in the price of sector i and the growth rate of aggregate consumption. We compare the relative contribution of these two terms.

³⁰This conclusion differs from Boppart (2014) who studies the evolution of services relative to the rest of the economy in the U.S. during the postwar period and finds that the contribution of price and income effects are roughly of equal sizes. If we confine our analysis to the U.S. and lump together agriculture and manufacturing into one sector, we still find that price effects generate less than a third of the variation. The key difference between our analyses is in the demand systems. In our specification, the demand price elasticity is constant. In contrast, Boppart's demand system implies that the price elasticity of services relative to the rest of consumption is declining as the economy grows. As noted by Buera and Kaboski (2009), as relative services expenditure and value added grows at a faster rate than services' relative price, a declining price elasticity automatically increases the explanatory power of relative prices.

services relative to manufacturing, σ_{sm} , we find a point estimate of 0.78 with standard error of 0.18 (clustered at the country level). For agriculture relative to manufacturing, σ_{am} , we find a point estimate of 0.67 with standard error of 0.12. Thus, we cannot reject the null that the coefficients on prices are statistically different from each other. Moreover, the income elasticity estimates remain unchanged. This suggests that the CES is a good approximation for analyzing these three sectors.

Estimation with Value Added Shares Some statistical agencies impute all investment employment to manufacturing, while its service component has been increasing over time (Herrendorf et al., 2013). By measuring sectoral activity with employment shares, we are implicitly adopting this assumption. Following Herrendorf et al., we study the robustness of our findings to estimating the baseline regressions (23)- (24) using value added shares as dependent variables. Table G.5 in the online appendix reports the estimation results. The main observation is that the estimates are robust to using value added shares as dependent variable. In particular, the estimate of the price elasticity declines insignificantly to .51 (from .75 with employment shares), the income elasticity of agriculture (relative to manufacturing) is -1.17 (vs. -1.04 with employment shares) and the income elasticity of services (relative to manufacturing) is 0.1 (vs. 0.32 with employment shares).

3.4 Correlation between Real and Nominal Value Added

A salient feature of structural transformation in the data is that the sectoral time-series patterns are similar regardless of whether we document them in nominal or real terms (Herrendorf et al., 2014). To investigate our model’s ability to account for this fact, we use our estimated preference parameters $\{\sigma, \epsilon_a - \epsilon_m, \epsilon_s - \epsilon_m\}$ from column (3) in Table 1 and nominal and real sectoral demands, equations (12) and (13), to generate the predicted evolution of nominal and real sectoral demands.

Table 2 reports the correlation between nominal and real shares both in our estimated model and in the data. We find that the model is able to generate correlations similar to the data. In particular, the correlation between the nominal and real relative demand of agricultural goods to manufactures is 0.93 in our model, while in the data it is .95. For services, the model generates a correlation of .71 while in the data it is .80.

The success in matching the correlation between nominal and real measures of activity is important. First, note that it is an out-of-sample check on our model, since our analysis has not targeted the evolution of real sectoral shares (recall that the left-hand-side of our estimating equations (23)-(24) were employment shares). More significantly, simultaneously accounting for the evolution of real and nominal sectoral shares highlights the critical importance of using a nonhomothetic CES framework. If we had used an homothetic framework,

Table 2: Correlation of Nominal and Real Value Added

	Correlation	
	Data	Model
Agriculture/Manufacturing	0.95	0.93
Services/Manufacturing	0.80	0.71

Note: the correlation between real and nominal value added is generated using the estimated income and price elasticities for the entire sample reported in column (3) of Table 1.

the correlation generated by the model would have been negative because the price elasticity of substitution is smaller than one.³¹ In our framework, the estimated income effects are sufficiently strong to overcome the relative price effect.

3.5 Comparison with Stone-Geary Preferences

Given the prevalence of Stone-Geary-like preferences in quantitative models of structural transformation, we compare the fit of our model with one that replaces our nonhomothetic CES preferences with Stone-Geary preferences. To this end, we consider the aggregator

$$C_t^c = \left[\Omega_a^c (C_{a,t}^c + \bar{c}_a)^{\frac{\sigma-1}{\sigma}} + \Omega_m^c (C_{m,t}^c)^{\frac{\sigma-1}{\sigma}} + \Omega_s^c (C_{s,t}^c + \bar{c}_s)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, \quad (28)$$

where C_t^c denotes aggregate consumption of country c at time t , $C_{i,t}^c$ denotes its consumption of sector i , $\bar{c}_a$ and $\bar{c}_s$ are constants that govern the nonhomotheticity of these preferences, $\sigma > 0$ and $\Omega_i^c > 0$ are preference parameters that are country specific.³² We follow the estimation procedure described in [Herrendorf et al. \(2013\)](#).³³ As with nonhomothetic CES preferences, we estimate three parameters common across countries $\{\sigma, \bar{c}_a, \bar{c}_s\}$ that govern the price and income elasticities, and $\{\Omega_i^c\}_{i \in \mathcal{I}, c \in C}$ which are country specific parameters. As expected, we find that $\bar{c}_a < 0$ and $\bar{c}_s > 0$.

Figure 3 shows the fit of the Stone-Geary model for the same countries as in Figure 2 using the common parameters $\{\sigma, \bar{c}_a, \bar{c}_s\}$ and the country-specific preference shifters $\{\Omega_i^c\}_{i \in \mathcal{I}, c \in C}$.³⁴ We see that the overall fit is better with nonhomothetic CES preferences. For example, the fitted model is not able to reproduce the hump-shaped pattern for manufacturing of Japan and Taiwan. This is confirmed for the full sample. In Table 3, we compare the sectoral R^2

³¹To see that, note that the relative trend in nominal values $\frac{\omega_{it}}{\omega_{jt}}$ would be proportional to $\left(\frac{p_{it}}{p_{jt}}\right)^{1-\sigma}$. For real values, $\frac{C_{it}}{C_{jt}}$, would be proportional to $\left(\frac{p_{it}}{p_{jt}}\right)^{-\sigma}$. As $0 < \sigma < 1$, both trends would move in opposite directions.

³²Since these preferences are not implicitly additive, the price and income elasticities are not independent. In Appendix A.3 we show that the elasticity of substitution between i and j is $\sigma_{ij} = \sigma \eta_i \eta_j$, where η 's denote income elasticities.

³³See online Appendix D for further discussion on the estimation procedure and estimation results.

³⁴Figures G.2 and G.3 in the online appendix show the fit for all countries in our sample.

Table 3: Sectoral R^2 measures for Stone-Geary and nonhomothetic CES

	R^2	
	Stone-Geary	Nonhomothetic CES
Agriculture	0.84	0.98
Manufacturing	0.75	0.87
Services	0.74	0.90

Note: R^2 is computed for each sector as $R_i^2 = 1 - \sum_{j=1}^N (y_{ij} - \hat{y}_{ij})^2 / \sum_{j=1}^N (y_{ij} - \bar{y}_i)^2$ where N denotes the total number of observations, $\bar{y}_i$ denotes the sample average of y_i and $i \in \{a, m, s\}$.

measures generated with Stone-Geary preferences and nonhomothetic CES.³⁵ We find that in all sectors the fit improves with nonhomothetic CES. The difference in the R^2 's ranges from 12 to 16 percentage points.

The intuition for the improvement of the fit with nonhomothetic CES is that, with Stone-Geary preferences, income effects are very low for rich countries. For high levels of income, the subsistence levels responsible for introducing the nonhomotheticity $\{\sigma, \bar{c}_a, \bar{c}_s\}$ are negligible. For example, the value of $\frac{p_{it}\bar{c}_i}{P_t C_t}$ for the U.S. at our estimated parameters is less than 0.05% at any point in time. Thus, the only remaining sources of variation left to explain the variation in employment shares are prices and trade shares.³⁶

3.6 Beyond Three Sectors

Jorgenson and Timmer (2011) have pointed out that in order to understand how structural transformation progresses in rich countries, it is important to zoom in the service sector, as it represents the majority of rich economies' consumption shares (see also Buera and Kaboski, 2012b). Our framework lends itself to this purpose, as it can accommodate an arbitrary number of sectors. In this section, we use the richness of the GDDC database to extend our estimation to 10 sectors: (1) agriculture, forestry and fishing, (2) mining and quarrying, (3) manufacturing, (4) public utilities, (5) construction, (6) wholesale and retail trade, hotels and restaurants, (7) transport, storage and communication, (8) finance, insurance, real state, (9) community, social and personal services, (10) government services.³⁷

³⁵Formally, we compute $R_i^2 = 1 - \sum_{j=1}^N (y_{ij} - \hat{y}_{ij})^2 / \sum_{j=1}^N (y_{ij} - \bar{y}_i)^2$ where N denotes the total number of observations, $\bar{y}_i$ denotes the sample average of y_i and $i \in \{a, m, s\}$.

³⁶An equivalent intuition provided by Dennis and Iscan (2009) is that the subsistence constants $\{\bar{c}_a, \bar{c}_s\}$ should not be stable over time to have income effects play a greater role and improve the model fit.

³⁷The data set also contains information on dwellings that are not constructed within the period, but this information is very sparse and we abstract from them. Recall also that our aggregate consumption data does not contain government services. We include them in our regressions because the important items of health and education are included in this category (along with public administration, defense and social work) and we believe it is informative to have a sense of the magnitude of the income elasticity. Excluding government services from our regressions does not change the other estimates significantly.

Table 4: 10-Sector Regression

	World	OECD	Non-OECD
Price Elasticity σ	0.82 (0.04)	0.84 (0.04)	0.82 (0.06)
Income Elasticity (relative to Manufacturing)			
Mining	-1.43 (0.17)	-1.14 (0.19)	-1.78 (0.28)
Agriculture	-0.96 (0.10)	-0.82 (0.17)	-1.15 (0.15)
Public Utilities	-0.02 (0.08)	0.12 (0.07)	-0.20 (0.14)
Transp., Storage, Communications	0.10 (0.10)	0.32 (0.11)	-0.17 (0.15)
Government Services	0.11 (0.21)	0.50 (0.20)	-0.36 (0.26)
Construction	0.18 (0.09)	0.26 (0.08)	0.12 (0.13)
Wholesale and Retail	0.37 (0.13)	0.56 (0.17)	0.12 (0.20)
Community, Social and Personal Serv.	0.44 (0.14)	0.83 (0.26)	-0.03 (0.07)
Finance, Insurance, Real State	0.94 (0.15)	1.19 (0.25)	0.64 (0.17)
R^2	0.95	0.91	0.93

Note: All sectoral elasticities computed relative to Manufacturing. Standard errors clustered at the country level. All regressions include a sector-country fixed effect. Source: GDDC 10-Sector database (Vries et al., 2014).

We estimate a demand system analogous to the one used in our baseline estimation, where we use manufacturing as a reference sector³⁸

$$\log \left(\frac{L_{i,t}^c}{L_{m,t}^c} \right) = \zeta_{im}^c + (1 - \sigma) \log \left(\frac{p_{i,t}^c}{p_{m,t}^c} \right) + (\epsilon_i - \epsilon_m) \log C_t^c + \nu_{i,t}^c, \quad (29)$$

with i denoting any of our sectors and c , a country index. Our panel estimates are reported in Table 4. The overall fit is good, with an R^2 above 0.9 in all regressions (this includes the country-sector fixed effect). Column (1) shows that we find an elasticity of substitution of 0.82 which is reasonably close to the 0.75 we found in our baseline, three-sector, estimation. We find that the smallest income elasticities correspond to mining and agriculture, while the

³⁸Note that in this case, the manufacturing sector is more narrowly defined than in the baseline estimation as it excludes mining and construction.

highest correspond to service sectors, such as finance, insurance, real state and government services. Columns (2) and (3) show that the ranking of sectors in terms of their income elasticity is very similar when we estimate OECD and non-OECD countries separately.

4 Micro and Macro Estimates for the U.S.

This section analyzes the U.S. case in more depth, for which we have more detailed data. We perform two exercises. First, we estimate our demand system using household data from the Consumption Expenditure Survey. This allows us to use an instrumental variable approach and control for sectoral preference shocks using time-sector fixed effects. Second, we estimate the parameters of the utility function using data on aggregate time series for the United States. Building on the work of [Herrendorf et al. \(2013\)](#), we specify the utility function both over final expenditure and value added and analyze the robustness of the estimates to these alternative definitions of the utility function.

4.1 Micro Estimation: Consumer Expenditure Survey

In this section we use household expenditure data to estimate our demand system. We use U.S. household quarterly consumption data for the period 1980-2006 from Consumption Expenditure Survey (CEX) data as constructed in [Heathcote et al. \(2010\)](#). We follow Heathcote et al. and focus on a sample of households with a present household head aged between 25 and 60. We also use the same consumption categories, except that we separate food from the rest of non-durables consumption.³⁹ We estimate the demand system using expenditure shares for each household on the left hand side, equation (13). To control for household fixed characteristics, we estimate the demand system using household fixed effects,

$$\log \left(\frac{\omega_{i,t}^h}{\omega_{nd,t}^h} \right) = (1 - \sigma) \log \left(\frac{p_{i,r,t}}{p_{nd,r,t}} \right) + (\epsilon_i - \epsilon_{nd}) \log C_t^h + \zeta^h + \zeta_{i,t} + \nu_{i,t}^h. \quad (30)$$

The superscript h denotes a household, and nd denotes non-durables –which we use as reference in our regressions. Prices $p_{i,r,t}$ come from the corresponding sectoral CPI-Us of the BLS.

³⁹Consumption measures are divided by the number of adult equivalents in the household. We use the categorization of [Heathcote et al. \(2010\)](#) for expenditures. The consumption categories in non-durables are: alcoholic beverages, tobacco, personal care, fuels, utilities and public services, public transportation, gasoline and motor oil, apparel, education, reading, health services and miscellaneous. Our data for services comes from entertainment expenditures. These includes among others fees for recreational lessons, TV and music related expenditures, pet-related expenditures, toys, games. Durables comprises vehicles (purchases/services derived from it and car maintenance and repair) and household equipment. Housing comprises the rents or imputed rents (if the dwelling is owned) as well as from “other dwellings” (primarily vacation homes). For each household we have a maximum of 4 observations (one per semester). The consumption data comes from the Family Characteristics and Income files except for years 1982 and 1983 for which the Detailed Expenditures files were used. See [Heathcote et al. \(2010\)](#) and [Krueger and Perri \(2006\)](#) for further discussion on the construction of the data set and its characteristics.

Table 5: Consumer Expenditure Survey, 1980-2006

	(1)	(2)	(3)	(4)
σ	0.64 (0.01)	0.64 (0.01)	0.63 (0.02)	0.63 (0.2)
Food	-0.46 (0.02)	-0.44 (0.02)	-0.49 (0.02)	-0.48 (0.02)
Housing	-0.31 (0.02)	-0.31 (0.02)	-0.27 (0.02)	-0.26 (0.02)
Services	0.57 (0.03)	0.52 (0.03)	0.62 (0.03)	0.57 (0.03)
Durables			0.94 (0.06)	0.93 (0.06)
Time FE	N	Y	N	Y
Observations	346631	346631	241470	241470

Note: Std. Errors clustered at the household level. Elasticity estimates are relative to non-durables consumption. Data from [Heathcote et al. \(2010\)](#).

They vary across regions r for each expenditure category i and time t . Aggregate consumption expenditure C_t^h is deflated using a household specific CPI, as suggested by our theory.⁴⁰ Household fixed effects are denoted by ζ^h , while time-sector fixed-effects are denoted by $\zeta_{i,t}$. Note that this time-sector fixed effect absorbs sectoral preference shocks. The error term is $\nu_{i,t}^h$.

In an analogous manner to the cross-country panel, the identification comes from within household variation in total consumption expenditure over time. Note that since the BLS CPI-U's vary regionally, we can identify the price elasticity of consumption shares even after including time-sector fixed effects. Variation in the prices is arguably exogenous to households.⁴¹

Baseline Results Table 5 reports the results of estimating (30) with and without including consumption durables, and with and without sector-year fixed effects. The estimates are very similar across all four specifications, suggesting that sector-specific demand shocks are not a significant source of bias in our demand system. The point estimate of the price elasticity is around 0.64, which is within the range of estimates we obtained in the cross-country panel

⁴⁰We use the Törnqvist price index constructed from household expenditure shares, which the superlative price index for nonhomothetic preferences ([Diewert, 1976](#)).

⁴¹One additional possible concern is the fact that even after controlling for household fixed characteristics there is an unobserved and persistent shock driving both aggregate consumption expenditure and some particular consumption category, most likely durable goods and housing. Following [Heathcote et al. \(2010\)](#), we impute the flow services obtained from housing and vehicles, which should attenuate these concerns. We also report our estimates excluding durables and show that the estimated elasticities are very similar.

(Table 1). With respect to the income elasticities, food has the lowest elasticity, followed by housing, services and we find the highest income elasticity for durables. As the purchase of durables is lumpy and most of our consumption data for durables imputes the service flow, this latter elasticity should be interpreted with caution.⁴²

IV Strategy One possible concern with the previous regression is that total consumption expenditure is an endogenous choice of the household that may be correlated with some omitted variable. To address this concern, we use the instrumental variable approach developed by Johnson et al. (2006). The instrument is based on the fact that the timing of the 2001 Federal income tax rebates for each household was a function of the last digit of the recipient’s social security number. Thus, it was effectively random.⁴³ Column (1) in Table 6 reports the OLS estimated as in our baseline estimation for the period of interest (2001-2002). Albeit some point estimates of the income elasticities differ from the baseline model, the relative ranking of the income elasticities remains the same. We also report the OLS for the households with information on tax rebates in column (2) because the households for which we can merge the tax rebate sample may not be random. We find similar estimates. Finally, column (3) reports the IV estimates. We instrument consumption expenditure with a dummy for whether or not a household received the rebate at a given point in time. The estimated price elasticity remains unaltered at .64 and is precisely estimated. The income elasticities are not precisely estimated, but remain similar to the other specifications.⁴⁴

Quartiles and Time Split Estimation A key property of the nonhomothetic CES preferences is that the income elasticity parameter ε_i is constant at different income levels. As argued in the introduction, this property is supported by evidence from prior empirical work based on mirco data (e.g., see Aguiar and Bils, 2015). Next, we complement this evidence using CEX data. We estimate the baseline regression (30) by income quartiles for the period 1980-2006. Columns (1) to (4) in Table 7 report the elasticity estimates for the first to the fourth quartiles of the CEX. We find that the value of all elasticities is quite stable at different income levels. Likewise, splitting the sample between the pre-1993 and post-1993 periods and estimating the elasticities separately also yields very similar estimates.

⁴²The service flow is measured using market rent values when available or the potential rent commanded in the market. This latter object is imputed from a regression analysis that presents a number of important challenges, such as property values being reported by only a subset of households and having missing observations for certain years (see the appendix in Heathcote et al., 2010).

⁴³Because the data requirements to construct household expenditures in Heathcote et al. (2010) are different than in Johnson et al. (2006), we can merge 60% of the tax rebate data with our baseline data. This represents around 20% of our data data for years 2001-2002. We thank Bart Hobijn for suggesting this instrument to us.

⁴⁴If we use the value of the rebate rather than an indicator as an instrument, we obtain similar results. However, as the magnitude of the rebate was not random, we prefer to use only the indicator.

Table 6: Instrumental Variables Strategy
Consumption Expenditure Survey, 2001-2002

	(1)	(2)	(3)
σ	0.62 (0.03)	0.64 (0.06)	0.64 (0.06)
Food	-0.24 (0.03)	-0.17 (0.06)	-0.15 (0.65)
Housing	-0.00 (0.03)	-0.09 (0.05)	-0.36 (0.53)
Services	0.36 (0.04)	0.42 (0.11)	0.64 (1.04)
			1st Stage
Tax Rebate Indicator			0.02 (0.006)
Time FE	Y	Y	Y
IV Sample Only	N	Y	Y
IV	N	N	Y
Observations	20921	4779	4779

Note: Std. Errors clustered at the household level. Elasticity estimates are relative to non-durables consumption. All estimates contain household and time-sector fixed effects. Data from [Heathcote et al. \(2010\)](#) and [Johnson et al. \(2006\)](#).

Table 7: Estimation by Quartiles and Sub-periods
Consumer Expenditure Survey, 1980-2006

	Quartiles				Pre 93	Post 93
	(1)	(2)	(3)	(4)	(5)	(6)
σ	0.63 (0.01)	0.76 (0.01)	0.75 (0.01)	0.67 (0.01)	0.65 (0.01)	0.70 (0.01)
Food	-0.44 (0.02)	-0.31 (0.07)	-0.42 (0.09)	-0.48 (0.02)	-0.38 (0.01)	-0.44 (0.01)
Housing	-0.20 (0.02)	-0.44 (0.06)	-0.38 (0.07)	-0.23 (0.02)	-0.24 (0.01)	-0.24 (0.01)
Services	0.47 (0.03)	0.74 (0.15)	0.78 (0.18)	0.68 (0.04)	0.72 (0.02)	0.51 (0.02)

Note: Std. Errors clustered at the household level. Elasticity estimates are relative to non-durables consumption. (1) Corresponds to the first quartile, (2), to the second, etc. All estimates contain household and time-sector fixed effects. Data from [Heathcote et al. \(2010\)](#).

4.2 Macro Estimation: Value Added and Expenditure Measures

We conclude the empirical exploration by estimating our demand system with aggregate U.S. consumption time series. We use the data from the Bureau of Economic Analysis as constructed by [Herrendorf et al. \(2013\)](#) for agriculture, manufacturing and services. Aggregate consumption data are decomposed in two different ways: final expenditure and value added. Consumption expenditure classifies sectors according to final good expenditure. Value added decomposes each dollar of final expenditure into the share of value added attributable to agriculture, manufacturing and services using U.S. input-output tables. For example, purchases of food from supermarkets is included in agriculture in the final expenditure computation, while it is broken down into food (agriculture), food processing (manufacturing) and distribution (services) when using the value added formulation. Thus, final expenditure and value added are two alternative classifications of the same underlying data.

[Herrendorf et al. \(2013\)](#) estimate a Stone-Geary utility demand system specified over expenditure and value added time series. They find that this distinction yields quantitatively very different results. Using value added measures, the elasticity of substitution across sectors is not statistically different from 0. [Buera and Kaboski \(2009\)](#) report a similar finding for the period 1870-2000. When estimating the model with final expenditure they find that the elasticity of substitution is around 0.85. [Herrendorf et al. \(2013\)](#) convincingly argue that the elasticity of substitution should be greater when using expenditure measures because they embed goods from the three sectors. However, they do not provide any justification for aggregate consumption being a Leontief aggregator of sectoral outputs. In fact, [Buera and Kaboski \(2009\)](#) consider the Leontief estimate an “implausibly low elasticity of substitution.”

We re-do the exercise of [Herrendorf et al. \(2013\)](#) using the nonhomothetic CES demand system rather than Stone-Geary, estimating

$$\log \left(\frac{\omega_{at}}{\omega_{mt}} \right) = \zeta_{am} + (1 - \sigma) \log \left(\frac{p_{at}}{p_{mt}} \right) + (\epsilon_a - \epsilon_m) \log C_t + \nu_{amt}, \quad (31)$$

$$\log \left(\frac{\omega_{st}}{\omega_{mt}} \right) = \zeta_{sm} + (1 - \sigma) \log \left(\frac{p_{st}}{p_{mt}} \right) + (\epsilon_s - \epsilon_m) \log C_t + \nu_{smt}, \quad (32)$$

where ω_{it} denotes consumption expenditure or value added in sector i at time t .⁴⁵ Our estimates are reported in Table 8. Our point estimate of the elasticity of substitution for the expenditure data is 0.88 which is very close to estimate of 0.85 reported by [Herrendorf et al. \(2013\)](#). As Herrendorf et al., we find that the elasticity of substitution is larger for expenditure data than value added. However, our estimated elasticity for value added measures is 0.57 with a standard deviation of 0.1. Thus, the preferences implied by our estimate differ significantly from the Leontief specification found using the Stone-Geary setting.

⁴⁵As we are using relative consumption shares rather than employment, there is no need to control for international trade in this regression because it is subsumed in consumption expenditure.

Table 8: Consumption Expenditure and Value Added for the U.S.

	σ	$\epsilon_a - \epsilon_m$	$\epsilon_s - \epsilon_m$	Obs.	R_{am}^2	R_{sm}^2
Value Added	0.57 (0.10)	-0.63 (0.12)	0.62 (0.06)	63	0.77	0.93
Expenditure	0.88 (0.03)	-0.63 (0.03)	0.55 (0.03)	63	0.98	0.91

The intuition for this difference in the estimate of the price elasticity is as follows. Expenditure shares in services measured in value-added terms raise at a faster rate than the relative price of services. The Stone-Geary demand system imposes that the income effects become less important as aggregate consumption grows. This implies that the estimation has to load the late increase in service expenditure to increases in the relative price of services. Thus, as the relative prices of services grows at a slower rate than value added, the estimation selects the minimal price elasticity to maximize the explanatory power of relative prices. In contrast, the nonhomothetic CES does not impose declining income effects. As a result, both income and price effects help account for the secular increase in expenditure shares in services and the estimation does not need to select Leontief demand system.

Our estimates of the income elasticities are very similar using measures of sectoral activity. In fact, the point estimates are identical for $\epsilon_a - \epsilon_m$, with an elasticity of -0.63 . The estimates for $\epsilon_s - \epsilon_m$ are 0.62 for value added data and 0.55 for expenditure data and we cannot reject the null that they are statistically identical. Thus, our estimates imply that the role for nonhomotheticities is very similar regardless of whether utility is specified in terms of value added or expenditure.

Forecasting U.S. Expenditure Shares What do the estimated price and income elasticities imply for the evolution of the sectoral composition of the U.S. economy?⁴⁶ Assuming that relative prices and aggregate consumption grow at the average rate of the postwar period, we forecast expenditure shares in 2025 and 2050 using the estimated price and income elasticities. The projected evolution of expenditure shares is depicted in figure G.1 in the online appendix. In the last year of our sample (2010) expenditure shares were 6% in agriculture, 20.5% in manufacturing and 74.2% in services. The projected shares in 2025 are 3.8% in agriculture, 18.1% manufacturing and 78.1% in services. In 2050, our projected shares are 2.1%, 14.5% and 83.4% in agriculture, manufacturing and services respectively. This exercise suggests that the process of structural transformation in the U.S. may continue in the next decades, with manufacturing and agriculture still accounting for a non-negligible part of the economy.

⁴⁶We thank Paco Buera and Alex Monge-Naranjo for suggesting this exercise.

5 Conclusion

This paper presents a tractable model of structural transformation that accommodates both long-run demand and supply drivers of structural change. Our main contribution is to introduce the nonhomothetic CES utility function to growth theory and to show their empirical relevance. These preferences generate nonhomothetic Engel curves at any level of development, which are in line with the evidence that we have from both rich and developing countries. Moreover, for this class of preferences, price and income elasticities are independent and they can be used for an arbitrary number of sectors. We argue that these are desirable theoretical and empirical properties.

In contrast to generalized Stone-Geary utility functions, nonhomothetic CES utility functions do not assume that preferences become asymptotically homothetic as income grows. In contrast to PIGL preferences, recently employed in a growth model by [Boppart \(2014\)](#), nonhomothetic CES utility functions accommodate an arbitrary number of sectors with heterogeneous income elasticities. In contrast to both of these models, our demand system does not assume specific functional interrelation between price and income elasticity. This property makes our specification particularly suitable for the exercise of separating the contributions of income and prices to changes in relative sectoral demand. Relative to models with differential trends in relative prices and homothetic constant elasticity of substitution preferences, our model has the advantage that can accommodate trends in both real and nominal measures.

From a quantitative stand-point, we show that our model captures well the broad sectoral reallocation patterns of structural change, in spite of the parsimony of our approach (we only make use of three elasticity parameters). We estimate our model applied to three sectors (agriculture, manufacturing and services) using a panel of 25 countries for the postwar period. The sample covers countries with very different levels and trajectories of development. The model fit captures the evolution of these three sectors of the economy. In particular, it generates a hump-shaped evolution for the manufacturing sector in all cases where this pattern appears in the data.

To conclude, we believe that the proposed preferences provide a tractable departure from homothetic preferences. We think that they can be used in many applied general equilibrium settings that currently use homothetic CES and monopolistic competition as their workhorse model, such as international trade (see [Matsuyama, 2015](#), for a subsequent application of these preferences with monopolistic competition in international trade). These preferences can be also combined with production functions without constant shares to study skilled-biased technological change.

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A Nonhomothetic CES Preferences

In this section of the appendix, we provide an overview of the properties of the general family of nonhomothetic Constant Elasticity of Substitution (CES) preferences. We first introduce the general family of nonhomothetic CES preferences in Section A.1. We then specialize them to the case of isoelastic nonhomothetic CES functions in Section A.2 and contrast the latter with Stone-Geary and PIGL preferences in Section A.3.

These preferences have a number of distinctive features, extensively studied in an early literature on theoretical foundations of preferences and production functions. [Sato \(1975\)](#) derived a general family of CES functions as the solution to a partial differential equation that imposes the constancy of elasticity of substitution. This family includes standard homothetic CES functions as well as two classes of separable and non-separable nonhomothetic functions. [Hanoch \(1975\)](#) showed that additivity of the direct or indirect utility (or production) function results in price and income effects that are non-trivially dependent on each other. He then introduced *implicit additivity* and derived in a family of functions where the income elasticity of demand is not fully dependent on the elasticity of substitution. Our nonhomothetic CES functions correspond to the non-separable class of functions in the sense of [Sato \(1975\)](#), which also satisfy the condition of implicit additivity in the sense of [Hanoch \(1975\)](#).

Finally, [Blackorby and Russell \(1981\)](#) have proved an additional property that is unique to this class of functions. In general, different generalizations of the elasticity of substitution to cases involving more than two variables, e.g., the Allen-Uzawa definition or the Morishima definition, are distinct from each other. However, for the class of nonhomothetic CES functions they become identical and elasticity of substitution can be uniquely defined similar to the case of two-variable functions.

A.1 General Nonhomothetic CES Preferences

Consider preferences over a bundle $\mathbf{C} = \{C_1, C_2, \dots, C_I\}$ of goods defined through an implicit utility function:

$$\sum_{i=1}^I \Omega_i^{\frac{1}{\sigma}} \left(\frac{C_i}{g_i(U)} \right)^{\frac{\sigma-1}{\sigma}} = 1, \quad (\text{A.1})$$

where functions g_i 's are differentiable in U and $\sigma \neq 1$ and $\sigma > 0$.⁴⁷ Standard CES preferences are a specific example of Equation (A.1) with $g_i(U) = U$ for all i 's. These preferences were first introduced, seemingly independently, by [Sato \(1975\)](#) and [Hanoch \(1975\)](#) who each characterize different properties of these functions. Here, we state and briefly prove some of the relevant results to provide a self-contained exposition of our theory in this paper.

Lemma 2. *If $\sigma > 0$ and functions $g_i(\cdot)$ are positive and monotonically increasing for all i , the function $U(\mathbf{C})$ defined in Equation (A.1) is monotonically increasing and quasi-concave for all $\mathbf{C} \gg 0$.*

Proof. Establishing monotonicity is straightforward. To establish quasi-concavity, assume to the contrary that there exists two bundles of $\mathbf{C}'$ and $\mathbf{C}''$ and their corresponding utility values U' and U'' ,

⁴⁷For the case of $\sigma = 1$, the preferences are simply defined according to $\sum_i \Omega_i \log \left(\frac{C_i}{g_i(U)} \right) = 1$.

such that $U \equiv U(\alpha \mathbf{C}' + (1 - \alpha) \mathbf{C}'')$ is strictly smaller than both U' and U'' . We then have for the case $\sigma \geq 1$

$$\begin{aligned} 1 &= \sum_i \Omega_i^{1/\sigma} \left(\alpha \frac{C'_i}{g_i(U)} + (1 - \alpha) \frac{C''_i}{g_i(U)} \right)^{\frac{\sigma-1}{\sigma}}, \\ &> \sum_i \Omega_i^{1/\sigma} \left(\alpha \frac{C'_i}{g_i(U')} + (1 - \alpha) \frac{C''_i}{g_i(U'')} \right)^{\frac{\sigma-1}{\sigma}}, \\ &\geq \alpha \sum_i \Omega_i^{1/\sigma} \left(\frac{C'_i}{g_i(U')} \right)^{\frac{\sigma-1}{\sigma}} + (1 - \alpha) \sum_i \Omega_i^{1/\sigma} \left(\frac{C''_i}{g_i(U'')} \right)^{\frac{\sigma-1}{\sigma}}, \end{aligned}$$

where in the second inequality we have used monotonicity of the g_i 's and in the third we have used Jensen's inequality and the assumption that $\infty > \sigma > 1$.⁴⁸ Since the last line equals 1 from the definition of the nonhomothetic CES functions valued at U' and U'' , we arrive at a contradiction. For the case that $0 < \sigma < 1$, we can proceed analogously. In this case, the inequality signs are reversed in both lines and we also reach a contradiction. ■

Henceforth, we assume the conditions in Lemma 2 are satisfied. The next lemma characterizes the demand for general nonhomothetic CES preferences and provides the solution to the expenditure minimization problem.

Lemma 3. *Consider any bundle of goods that maximizes the utility function defined in Equation (A.1) subject to the budget constraint $\sum_i p_i C_i \leq E$. For each good i , the real consumption satisfies:*

$$C_i = \Omega_i \left(\frac{p_i}{E} \right)^{-\sigma} g_i(U)^{1-\sigma}, \quad (\text{A.2})$$

and the share in consumption expenditure satisfies:

$$\omega_i \equiv \frac{p_i C_i}{E} = \Omega_i^{\frac{1}{\sigma}} \left(\frac{C_i}{g_i(U)} \right)^{\frac{\sigma-1}{\sigma}} = \Omega_i \left[g_i(U) \left(\frac{p_i}{E} \right) \right]^{1-\sigma}. \quad (\text{A.3})$$

Proof. Let λ and ρ denote the Lagrange multipliers on the budget constraint and constraint (A.1), respectively:

$$\mathcal{L} = U + \rho \left(1 - \sum_i \Omega_i^{\frac{1}{\sigma}} \left(\frac{C_i}{g_i(U)} \right)^{\frac{\sigma-1}{\sigma}} \right) + \lambda \left(E - \sum_i p_i C_i \right).$$

The FOCs with respect to C_i yields:

$$\rho \frac{1 - \sigma}{\sigma} \frac{\omega_i}{C_i} = \lambda p_i, \quad (\text{A.4})$$

where we have defined

$$\omega_i \equiv \Omega_i^{\frac{1}{\sigma}} \left(\frac{C_i}{g_i(U)} \right)^{\frac{\sigma-1}{\sigma}}. \quad (\text{A.5})$$

Equation (A.4) shows that expenditure $p_i C_i$ on good i is proportional to ω_i . Since the latter sums to

⁴⁸For the case $\sigma = 1$, recall that we have defined a logarithmic function, which is also concave. For the case $\sigma = \infty$ the inequality becomes an equality as we have a linear case.

one from constraint (A.1), it follows that ω_i is the expenditure share of good i , and we have:

$$E = \sum_{i=1}^I p_i C_i = \frac{1-\sigma}{\sigma} \frac{\rho}{\lambda}.$$

We can now substitute the definition of ω_i from Equation (A.5) in expression (A.4) and use (A.6) to find (A.2) and (A.3). ■

Lemma 3 implies the following relationship, defining the expenditure (and implicitly the indirect utility function) for general Nonhomothetic CES preferences:

$$E = \left[\sum_{i=1}^I \Omega_i (g_i(U) p_i)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}. \quad (\text{A.6})$$

The expenditure function is continuous in prices p_i 's and U , and homogeneous of degree 1, increasing, and concave in prices. The elasticity of the expenditure function with respect to utility is:

$$\eta_E^U \equiv \frac{U \partial E}{E \partial U} = \sum_i \omega_i \eta_{g_i}^U = \overline{\eta_{g_i}^U}, \quad (\text{A.7})$$

which ensures that the expenditure function is increasing in utility if all g_i 's are monotonically increasing. It is straightforward to also show that the elasticity of the utility function (A.1) with respect to consumption of good i is also given by:

$$\eta_U^{C_i} \equiv \frac{C_i \partial U}{U \partial C_i} = \frac{\omega_i}{\overline{\eta_{g_i}^U}}, \quad (\text{A.8})$$

where ω_i is the ratio defined in Equation (A.5).

Examining sectoral demand from Equation (A.2) along indifference curves, shows the main properties of nonhomothetic CES preferences. As expected, the elasticity of substitution is constant:

$$\eta_{C_i/C_j}^{p_i/p_j} \equiv \frac{\partial \log (C_i/C_j)}{\partial \log (p_i/p_j)} = \sigma. \quad (\text{A.9})$$

More interestingly, the elasticity of relative demand with respect to utility is in general different from unity:

$$\eta_{C_i/C_j}^U \equiv \frac{\partial \log (C_i/C_j)}{\partial \log U} = \frac{\partial \log (g_i/g_j)}{\partial \log U}. \quad (\text{A.10})$$

Since utility has a monotonic relationship with real income (and hence expenditure), it then follows that the expenditure elasticity of demand for different goods are different. More specifically, we can use (A.7) to find the expenditure elasticity of demand:

$$\eta_{C_i}^E \equiv \frac{\partial \log C_i}{\partial \log E} = \sigma + (1-\sigma) \frac{\eta_{g_i}^U}{\overline{\eta_{g_i}^U}}. \quad (\text{A.11})$$

The intuition for the normalization in expression (A.11) is that the elasticity $\eta_E^{C_i}$ has to be invariant to all monotonic transformations of utility.

Preferences defined by Equation (A.1) belong to the general class of preferences with *Direct Implicit Additivity*. Hanoch (1975) shows that the latter family of preferences have the nice property that is illustrated by Equations (A.9) and (A.10): the separability of the income and substitution elasticities of the Hicksian demand. This is in contrast to the stronger requirement of *Explicit Additivity* commonly assumed in nonhomothetic preferences, whereby the utility is explicitly defined as a function $U = F(\sum_i f_i(C_i))$. In Section A.3 below, we will show examples of how substitution and income elasticities of Hicksian demand are *not* separable for preferences with explicitly additivity in direct utility, e.g., generalized Stone-Geary preferences (Kongsamut et al., 2001), or indirect utility, e.g., PIGL preferences (Boppart, 2014).

Finally, let us investigate the convexity of the expenditure function in terms of utility. First, we express the second derivative of the expenditure function in terms of elasticities,

$$\frac{\partial^2 E}{\partial U^2} = \frac{E}{U^2} \eta_E^U \left(\eta_E^U + \eta_{\eta_E^U}^U - 1 \right), \quad (\text{A.12})$$

where $\eta_{\eta_E^U}^U$ is the second order elasticity of expenditure with respect to utility. We can compute this second order elasticity as follows:

$$\begin{aligned} \eta_{\eta_E^U}^U &= U \frac{\partial}{\partial U} \log \sum_i \eta_{g_i}(U) (g_i(U) p_i)^{1-\sigma} - (1-\sigma) \frac{\partial \log E}{\partial \log U}, \\ &= \frac{\sum_i \eta_{\eta_{g_i}} \cdot \eta_{g_i} (g_i(U) p_i)^{1-\sigma} + (1-\sigma) \sum_i \eta_{g_i}^2 (g_i(U) p_i)^{1-\sigma}}{\sum_i \eta_{g_i} (g_i(U) p_i)^{1-\sigma}} - (1-\sigma) \overline{\eta_{g_i}}, \\ &= \overline{\eta_{g_i}} \left[\frac{\overline{\eta_{\eta_{g_i}} \cdot \eta_{g_i}}}{(\overline{\eta_{g_i}})^2} + (1-\sigma) \text{Var} \left(\frac{\eta_{g_i}}{\overline{\eta_{g_i}}} \right) \right], \end{aligned} \quad (\text{A.13})$$

where $\overline{X_i}$ and $\text{Var}(X_i)$ denote the expected value and variance of variable X_i across sectors with weights given by expenditure shares ω_i for prices $\mathbf{p}$ and utility U .

To make sense of (A.13), consider the choice of $g_i(U) \equiv g(U)^{\varepsilon_i}$ for some monotonically increasing function $g(\cdot)$ (which corresponds to the aggregator introduced in Section A of the online appendix). We have that $\eta_{g_i} = \eta_g \varepsilon_i$ and $\eta_{\eta_{g_i}} = \eta_{\eta_g}$, implying:

$$\eta_{\eta_E^U}^U = \eta_g \overline{\varepsilon_i} \left[\eta_{\eta_g} + (1-\sigma) \text{Var} \left(\frac{\varepsilon_i}{\overline{\varepsilon_i}} \right) \right]. \quad (\text{A.14})$$

A.2 Isoelastic Nonhomothetic CES Preferences

Now, consider the specific case used in our basic model in Section 2, where the isoelastic functions g_i are defined as:

$$g_i(U) = U^{\frac{\varepsilon_i - \sigma}{1 - \sigma}}, \quad (\text{A.15})$$

where $\eta_{g_i}^U = (\varepsilon_i - \sigma)/(1 - \sigma)$, and we retrieve standard CES preferences when $\varepsilon_i = 1$ for all i 's. To tie our exposition more closely to the discussions in Section 2, let us for now identify utility with C , aggregate real income and define a corresponding aggregate price index $P \equiv E/C$. From Equations

(A.2) and (A.3), we find demand to be:

$$C_i = \left(\frac{p_i}{P}\right)^{-\sigma} C^{\epsilon_i}, \quad (\text{A.16})$$

$$\omega_i = \frac{p_i C_i}{PC} = \left(\frac{p_i}{P}\right)^{1-\sigma} C^{\epsilon_i-1}. \quad (\text{A.17})$$

The aggregate price index is

$$P \equiv \frac{E}{C} = \left(\sum_{i=1}^I C^{\epsilon_i-1} p_i^{1-\sigma} \right)^{\frac{1}{1-\sigma}}. \quad (\text{A.18})$$

From (A.7), the real income elasticity of the expenditure function is:

$$\eta_E^C \equiv \frac{C}{E} \frac{\partial E}{\partial C} = \frac{\bar{\epsilon} - \sigma}{1 - \sigma}, \quad (\text{A.19})$$

where $\bar{\epsilon} = \sum_i \omega_i \epsilon_i$. Therefore, a sufficient condition for the function $E(C; \{p_i\}_{i=1}^I)$ to be a one-to-one mapping for all positive prices is that all sectors have an income elasticity larger than the elasticity of substitution $\epsilon_i > \sigma$ if $\sigma < 1$ (and $\epsilon_i < \sigma$ if $\sigma > 1$). This directly follows from Lemma 2.

Combining Equations (A.12) and (A.13), we find

$$\frac{\partial^2 E}{\partial C^2} = \frac{E}{C^2} \frac{\bar{\epsilon} - \sigma}{1 - \sigma} \left(\frac{\bar{\epsilon} - \sigma}{1 - \sigma} - 1 \right) + \frac{\text{Var}(\epsilon)}{\bar{\epsilon} - \sigma}. \quad (\text{A.20})$$

Therefore, a sufficient condition for the expenditure function to be convex in C for all prices is that $\epsilon_{\min} \geq 1$.

The income elasticities of demand are given by Equations (A.10) and (A.11):

$$\eta_{C_i/C_j}^C = \epsilon_i - \epsilon_j, \quad (\text{A.21})$$

$$\eta_{C_i}^E = \sigma + (1 - \sigma) \frac{\epsilon_i - \sigma}{\bar{\epsilon} - \sigma}. \quad (\text{A.22})$$

Each good i is characterized by a parameter $\epsilon_i \in \mathbb{R}$ that is a measure of its real income elasticity.

More generally, the relationship between utility U and real aggregate consumption C in Expression (A.15) can be defined by any monotonic function G such that $U = G(C)$. In particular, let us define $G(\cdot)$ such that C corresponds to consumption expenditure at constant prices $\{q_i\}_i$ such that

$$C^{1-\sigma} = \sum_{i=1}^I \Omega_i G(C)^{\epsilon_i-1} q_i^{1-\sigma}. \quad (\text{A.23})$$

Assuming $\sigma \in (0, 1)$, if $\epsilon_i > \sigma$ for all i , function $G(\cdot)$ defined through Equation (A.23) is monotonically increasing for all positive C . Therefore, we can approximate the relationship as:

$$\begin{aligned} \log G(C) &\approx \log G(\tilde{C}) + \left. \frac{\partial \log G}{\partial \log C} \right|_{C=\tilde{C}} \cdot (\log C - \log \tilde{C}), \\ &= \frac{1 - \sigma}{\bar{\epsilon} - \sigma} \log C + \text{const.}, \end{aligned} \quad (\text{A.24})$$

where $\bar{\epsilon}$ is the average elasticity parameter at constant price q and real income $\tilde{C}$.

A.3 Comparison to Generalized Stone-Geary and PIGL Preferences

For comparison, now consider generalized Stone-Geary preferences that have been widely used in previous work on structural change (see, e.g., [Kongsamut et al., 2001](#)):

$$C = \left(\sum_{i=1}^I (C_i - \underline{C}_i)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}, \quad (\text{A.25})$$

where $\underline{C}_i$ are the usual coordinate shifters.⁴⁹ The expenditure elasticity of demand for good i is given by:

$$\eta_{C_i}^E = 1 - \frac{C_i}{C_i}, \quad (\text{A.26})$$

which is different from 1 as long as the shifter $\underline{C}_i \neq 0$. However, note that due to constancy of $\underline{C}_i$, this elasticity converges to unity at the same rate as the rate of growth of C_i . Therefore, nonhomotheticity is a short-run feature of Stone-Geary preferences: as the income grows Stone-Geary preferences asymptote to homothetic CES preferences.

Another important feature of the nonhomothetic CES preferences is the fact that elasticity of substitution σ_{ij} between all goods i and j remains constant σ and remains independent of expenditure (income) elasticities. In contrast, for Stone-Geary preferences we find:

$$\sigma_{ij} = \sigma \cdot \frac{E}{E - \sum_k p_k \underline{C}_k} \cdot \left(1 - \frac{C_i}{C_i} \right) \cdot \left(1 - \frac{C_j}{C_j} \right), \quad (\text{A.27})$$

$$\eta_{C_i}^E = \frac{E}{E - \sum_k p_k \underline{C}_k} \cdot \left(1 - \frac{C_i}{C_i} \right), \quad (\text{A.28})$$

where η_i is the nominal income elasticity of demand in sector i ([Hanoch, 1975](#)). It then follows that the elasticities of substitution between goods i and j always satisfies the following equality:

$$\sigma_{ij} = \sigma \eta_{C_i}^E \eta_{C_j}^E, \quad (\text{A.29})$$

creating a direct linkage between elasticities of substitution and expenditure for different sectors. As expected, when E goes to infinity we find that $\sigma_{ij} \rightarrow \sigma$ and $\eta_{C_i}^E \rightarrow 1$ for all sectors.

An alternative specification for nonhomothetic preferences in the structural change literature, recently used by [Boppart \(2014\)](#), is the Price Independent Generalized Linear (PIGL) preferences. The canonical definition for these preferences involves a two-good system. In general, no closed-form representation for the utility function exists, but the indirect utility/expenditure function relationship can be specified as:

$$C + \frac{\vartheta}{\varrho} \left[\left(\frac{p_1}{p_2} \right)^{\varrho} - 1 \right] = \frac{1}{\xi} \left[\left(\frac{E}{p_2} \right)^{\xi} - 1 \right], \quad (\text{A.30})$$

where $\mathbf{p} = (p_1, p_2)$ is the pair of good prices, C is the aggregator (utility) and E is expenditure,

⁴⁹In particular, standard 3-sector models of structural transformation generally assume $\underline{C}_a > 0$, $\underline{C}_s < 0$ and $\underline{C}_m = 0$.

$0 \leq \xi \leq \varrho < 1$, and $\vartheta > 0$.⁵⁰ For these preferences, the expenditure elasticity of demand for good i is constant and less than unity: $\eta_{C_1}^E = 1 - \xi < 1$. Therefore, like nonhomothetic CES preferences and unlike Stone-Geary, PIGL preferences also feature nonhomotheticity at all levels of income. In contrast to nonhomothetic CES, however, there is no generalization of PIGL preferences to more than two good demand systems that preserves the independence of income elasticities across different goods.⁵¹

Needless to say, since PIGL preferences are outside the CES family, the elasticity of substitution varies with income and prices. As [Boppart \(2014\)](#) shows, the elasticity of substitution between goods 1 and 2 are given by

$$\sigma = 1 - \varrho - (\varrho - \xi) \frac{\vartheta \left(\frac{p_1}{p_2} \right)^\varrho}{\left(\frac{E}{p_2} \right)^\xi - \vartheta \left(\frac{p_1}{p_2} \right)^\varrho}. \quad (\text{A.31})$$

As a result, when PIGL preferences are embedded in a growth model, along an equilibrium path that involves growing income the elasticity of substitution will be monotonically increasing and converges toward $1 - \varrho$. Therefore, the choice of PIGL preferences involves specific assumptions about the dynamics of substitution elasticities in a two-good model.

B Proofs of Propositions and Lemmas

Proof of Lemma 1. First, we show that the household problem has a unique solution that is characterized by an Euler equation along with a standard transversality condition. Let $E_t = w_t + (1 + r_t) \mathcal{A}_t - \mathcal{A}_{t+1}$ be the consumption expenditure when the representative household has current stock of assets $\mathcal{A}_t$ and chooses an allocation $\mathcal{A}_{t+1}$ of assets for the next period. We can decompose the problem into two independent parts. The intratemporal problem involves allocating the expenditure E_t across I goods so as to maximize the aggregator C_t defined by Equation (2). The solution is given by Equations (A.16) and (A.18).

Let $\tilde{C}_t(E) \equiv \max C_t$ subject to the constraint $E = \sum_{i=1}^I p_{it} C_{it}$. The intertemporal problem then involves finding the sequence of assets $\{\mathcal{A}_{t+1}\}_{t=0}^\infty$ such that

$$\max \sum_{t=0}^I \beta^t \frac{\tilde{C}_t(w_t + (1 + r_t) \mathcal{A}_t - \mathcal{A}_{t+1})^{1-\theta} - 1}{1 - \theta}. \quad (\text{B.1})$$

From Section A.2, we know that when $\epsilon_i \geq 1$ for all i , the expenditure function is monotonically increasing and strictly convex for all prices. Therefore, its inverse, the indirect aggregate consumption function $\tilde{C}(E; \mathbf{p}_t)$ exists and is monotonically increasing and strictly concave for all prices. Standard results from discrete dynamic programming (e.g., see [Acemoglu, 2008](#), Chapter 6) then imply that the Euler equation

$$C_t^{-\theta} \frac{\partial \tilde{C}_t}{\partial E_t} = \beta (1 + r_t) C_{t+1}^{-\theta} \frac{\partial \tilde{C}_{t+1}}{\partial E_{t+1}},$$

⁵⁰ PIGL preferences are *not* additive in the sense of [Hanoch \(1975\)](#).

⁵¹ As a reminder, from Engel aggregation we know that we can have up to $I - 1$ independent income elasticities in a demand system involving I goods. This is why we have one degree of freedom in specifying the I income elasticity parameters $\bar{\epsilon}$ in nonhomothetic CES preferences defined in Section A.2.

and the transversality condition

$$\lim_{t \rightarrow \infty} \beta^t (1 + r_t) \mathcal{A}_t C_t^{-\theta} \frac{\partial \tilde{C}_t}{\partial E_t} = 0, \quad (\text{B.2})$$

provide necessary and sufficient condition for a sequence $\{\mathcal{A}_{t+1}\}_{t=0}^{\infty}$ to characterize the solution.

Using Equation (A.19) we can simplify the Euler equation above to

$$C_t^{-\theta} \frac{C_t}{E_t} \frac{1 - \sigma}{\bar{\epsilon}_t - \sigma} = \beta (1 + r_t) C_{t+1}^{-\theta} \frac{C_{t+1}}{E_{t+1}} \frac{1 - \sigma}{\bar{\epsilon}_{t+1} - \sigma},$$

and the transversality condition to

$$\lim_{t \rightarrow \infty} \beta^t (1 + r_t) \frac{\mathcal{A}_t}{E_t} C_t^{1-\theta} \frac{1 - \sigma}{\bar{\epsilon}_t - \sigma} = 0.$$

■

Proof of Proposition 1. Our proof for the proposition involves two steps. First, we use the second Welfare Theorem and consider the equivalent centralized allocation by a social planner. Due to the concavity of the aggregator C_t as a function of the bundle of goods $(C_1, \dots, C_I)$, which is ensured by the condition $\epsilon_i \geq 1$ for all i , we can use standard arguments to establish the uniqueness of the equilibrium allocations (see [Stockey et al., 1989](#), p. 291). Next, we construct a unique constant growth path (steady state) that satisfies the equilibrium conditions. It then follows that the equilibrium converges to the constructed Constant Growth Path (CGP).

Consider an equilibrium path along which consumption expenditure E_t , aggregate stock of capital K_t , and the capital allocated to the investment sector K_{0t} all asymptotically grow at rate $(1 + \gamma_0)^{\frac{1}{1-\alpha_0}}$, and the labor employed in the investment sector asymptotically converges to $L_0^* \in (0, 1)$. Henceforth, we use the tilde variables to denote normalization $A_{0t}^{-\frac{1}{1-\alpha_0}}$, for instance, $\tilde{K}_t \equiv A_{0t}^{-\frac{1}{1-\alpha_0}} K_t$. Accordingly, we can write the law of evolution of aggregate stock of capital as

$$\tilde{K}_{t+1} = \frac{1 - \delta}{(1 + \gamma_0)^{1/(1-\alpha_0)}} \tilde{K}_t + \frac{1}{(1 + \gamma_0)^{1/(1-\alpha_0)}} \tilde{K}_{0t}^{\alpha_0} L_{0t}^{1-\alpha_0}, \quad (\text{B.3})$$

and the interest rate and wages as

$$r_t = R_t - \delta = \alpha_0 \left(\frac{\tilde{K}_{0t}}{L_{0t}} \right)^{\alpha_0 - 1} - \delta, \quad (\text{B.4})$$

$$\tilde{w}_t = (1 - \alpha_0) \tilde{K}_{0t}^{\alpha_0} L_{0t}^{1-\alpha_0}. \quad (\text{B.5})$$

From the assumptions above, it follows that $\tilde{K}_{0t}/L_{0t}$ asymptotically converges to a constant, which from Equation (B.4) implies that the rate of interest also converges to a constant r^* .

We first derive an expression for the asymptotic growth of nominal consumption expenditure shares

(and sectoral employment shares) of different sectors, using in equation (6),

$$\begin{aligned}
1 + \xi_i &\equiv \lim_{t \rightarrow \infty} \frac{\omega_{it+1}}{\omega_{it}} = \lim_{t \rightarrow \infty} \left(\frac{E_t}{E_{t+1}} \right)^{1-\sigma} \left(\frac{p_{it+1}}{p_{it}} \right)^{1-\sigma} \left(\frac{C_{t+1}}{C_t} \right)^{\epsilon_i - \sigma}, \\
&= \left(\frac{1}{1 + \gamma_0} \right)^{\frac{1-\sigma}{1-\alpha_0}} \left(\frac{(1 + \gamma_0)^{\frac{1-\alpha_i}{1-\alpha_0}}}{1 + \gamma_i} \right)^{(1-\sigma)} (1 + \gamma^*)^{\epsilon_i - \sigma}, \\
&= \frac{(1 + \gamma^*)^{\epsilon_i - \sigma}}{\left[(1 + \gamma_0)^{\frac{\alpha_i}{1-\alpha_0}} (1 + \gamma_i) \right]^{1-\sigma}}, \tag{B.6}
\end{aligned}$$

where in the second line we have used the definition of the constant growth path as well as the fact that from Equations (B.4) and (B.5), the relative labor-capital price grows as rate $(1 + \gamma_0)^{\frac{1}{1-\alpha_0}}$ and therefore from Equation (16) we have

$$\lim_{t \rightarrow \infty} \frac{p_{it+1}}{p_{it}} = \frac{1 + \gamma_0}{1 + \gamma_i} (1 + \gamma_0)^{\frac{\alpha_0 - \alpha_i}{1-\alpha_0}}. \tag{B.7}$$

Equation (B.6) shows that the expenditure shares asymptotically grow (or diminish) monotonically. Since the shares belong to the compact $I - 1$ dimensional simplex, they asymptotically converge to a time-constant set of shares.

Since shares have to add up to 1, we need to have that $\xi_i \leq 0$ for all i . Moreover, this inequality has to be satisfied with equality at least for one non-vanishing sector. Now, consider the expression defined in (20) for the growth rate of real consumption. For sectors $i \in \mathcal{I}^*$ that achieve the minimum, the growth of nominal expenditure share becomes zero, and their shares converge to constant values ω_i^* . For sectors $i \notin \mathcal{I}^*$, we find the following expression for the growth rate of nominal shares ξ_i in Equation (B.6) becomes negative. Assuming $\sigma < 1$ and $\epsilon_i > \sigma$, the expression on the right hand side becomes strictly less than 1, since we know sector i does not achieve the minimum in (20). Therefore, $\xi_i < 0$ and the nominal shares asymptotically vanish for $i \notin \mathcal{I}^*$.

Asymptotically, the expenditure-weighted average income elasticity and expenditure-weighted capital intensity in the consumption sector both converge to constants $\bar{\epsilon}^* \equiv \lim_{t \rightarrow \infty} \sum_{i=1}^I \epsilon_i \omega_{it} = \sum_{i \in \mathcal{I}^*} \epsilon_i \omega_i^*$ and $\bar{\alpha}^* \equiv \lim_{t \rightarrow \infty} \sum_{i=1}^I \alpha_i \omega_{it} = \sum_{i \in \mathcal{I}^*} \alpha_i \omega_i^*$. Henceforth, we extend our notation to use tilde to indicate variables normalized by their corresponding asymptotic rate of growth (or decline) along our proposed constant growth path. For instance, we let $\tilde{p}_{it} \equiv p_{it}(1 + \gamma_0)^{-\frac{1-\alpha_i}{1-\alpha_0}}(1 + \gamma_i)^{-1}$ and $\tilde{C}_t \equiv C_t(1 + \gamma^*)^{-1}$. Furthermore, we define starred notation to indicate the asymptotic value of each variable along the constant growth path, for example, we let $p_i^* \equiv \lim_{t \rightarrow \infty} \tilde{p}_{it}$ and $\tilde{C}^* \equiv \lim_{t \rightarrow \infty} \tilde{C}_t$.

We now show that a constant growth path exists and is characterized by γ^* as defined by equation (20). We also show the existence of the asymptotic values $\{\tilde{K}^*, \tilde{C}^*, \tilde{K}_0^*, L_0^*\}$. From the Euler equation (8), we have that asymptotically

$$(1 + \gamma^*)^{1-\theta} = \frac{1 + \gamma_0}{\beta(1 + r^*)}, \tag{B.8}$$

which pins down r^* , the asymptotic real interest rate in terms of γ^* given by Equation (20). Then from Equation (B.4), we find the asymptotic capital-labor ratio in the investment sector in terms of

the asymptotic real interest rate

$$\kappa \equiv \frac{\tilde{K}_0^*}{L_0^*} = \left(\frac{\alpha_0}{r^* + \delta} \right)^{\frac{1}{1-\alpha_0}}. \quad (\text{B.9})$$

This gives us the asymptotic relative labor-capital price from Equations (B.4) and (B.5) as

$$\frac{\tilde{w}^*}{R^*} = \frac{1-\alpha_0}{\alpha_0} \frac{\tilde{K}^*}{L_0^*} = \frac{1-\alpha_0}{\alpha_0} \left(\frac{\alpha_0}{r^* + \delta} \right)^{\frac{1}{1-\alpha_0}}. \quad (\text{B.10})$$

From Equation (16), we find

$$\tilde{p}_i^* = \frac{\alpha_0^{\alpha_0} (1-\alpha_0)^{1-\alpha_0}}{\alpha_i^{\alpha_i} (1-\alpha_i)^{1-\alpha_i}} \left(\frac{\tilde{w}^*}{R^*} \right)^{\alpha_0-\alpha_i} \frac{A_{0,0}}{A_{i,0}}, \quad (\text{B.11})$$

where $\tilde{w}^*/R^*$ is given by Equations (B.10) and (B.8) and $A_{i,0}$ denotes the initial state of technology in sector i and $A_{0,0} \equiv 1$. Given asymptotic prices

$$\tilde{E}^* = \left[\sum_{i \in \mathcal{I}^*} \left(\tilde{C}^* \right)^{\epsilon_i - \sigma} (\tilde{p}_i^*)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}, \quad (\text{B.12})$$

and

$$\omega_i^* = \left(\frac{\tilde{p}_i^*}{\tilde{E}^*} \right)^{1-\sigma} \left(\tilde{C}^* \right)^{\epsilon_i - \sigma}. \quad (\text{B.13})$$

Next, we combine the equation for accumulation of capital (B.3), the household budget constraint (10) the market clearing condition of consumption goods to establish that there exists a unique $\{\tilde{K}^*, \tilde{C}^*, \tilde{K}_0^*, L_0^*\}$ satisfying the asymptotic equilibrium conditions and $\kappa = \tilde{K}_0^*/L_0^*$ where κ is given by Equation (B.9). From market clearing, the sum of payments to labor in the consumption sector is $\sum_{i=1}^I (1-\alpha_i) \omega_{it} E_{it}$, which implies $(1-\bar{\alpha}_t) \tilde{E}_t = \tilde{w}_t (1-L_{0t})$. Asymptotically, we find that

$$(1-\bar{\alpha}^*) \tilde{E}^* = (1-\alpha_0) \kappa^{\alpha_0} (1-L_0^*). \quad (\text{B.14})$$

Similarly, from Equation (B.3) it follows that $\left[(1+\gamma_0)^{\frac{1}{1-\alpha_0}} - (1-\delta) \right] \tilde{K}^* = \kappa^{\alpha_0} L_0^*$. Defining the expression within the square brackets at a positive constant ϑ , we use write the asymptotic employment in the investment sector in terms of the aggregate stock of capital as

$$L_0^* = \vartheta \kappa^{-\alpha_0} \tilde{K}^*. \quad (\text{B.15})$$

Finally, using the market clearing condition in the assets market $\mathcal{A}_t = K_t$ and Equation (10), we find that $\tilde{E}_t = \tilde{w}_t + R_t \tilde{K}_t - \left(\frac{\tilde{K}_{0t}}{L_{0t}} \right)^{\alpha_0} L_{0t}$ for all t . Taking the limit, it follows that

$$\tilde{E}^* = (1-\alpha_0) \kappa^{\alpha_0} + \alpha_0 \kappa^{\alpha_0-1} \tilde{K}^* - \kappa^{\alpha_0} L_0^*. \quad (\text{B.16})$$

Substituting from Equation (B.15) into Equations (B.14) and (B.16) yields,

$$\bar{\alpha}^* \tilde{E}^* = \alpha_0 (\kappa^{\alpha_0-1} - \vartheta) \tilde{K}^*. \quad (\text{B.17})$$

We can show that the left hand side of this equation is a monotonically increasing function of $\tilde{C}^*$ with a given κ .⁵² From condition (21), we have that $\kappa^{\alpha_0-1} - \vartheta > 0$ and therefore the right hand side is a linear increasing function of $\tilde{K}^*$. Therefore, Equation (B.17) defines $\tilde{C}^*$, and correspondingly $\tilde{E}^*$, as an increasing function of $\tilde{K}^*$. Finally, substituting this function and Equation (B.15) in Equation (B.16), we find

$$\tilde{E} + (\vartheta - \alpha\kappa^{\alpha_0-1})\tilde{K} = (1 - \alpha_0)\kappa^{\alpha_0}. \quad (\text{B.18})$$

From condition (21), we know that the left hand side is a monotonically increasing function of $\tilde{K}^*$ for constant κ . This function is 0 when $\tilde{K}^*$ and limits to infinity as the latter goes to infinity. Therefore, Equation (B.18) uniquely pins down $\tilde{K}^*$ as a function of κ , which in turn is given by Equation (B.9). Condition (21) also ensures that the transversality condition (9) is satisfied. Finally, we verify that $L_0^* \in (0, 1)$. Combining equations (B.15), (B.14) and (B.16) we obtain that

$$L_0^* = \frac{\bar{\alpha}}{\left[\frac{1-\bar{\alpha}}{1-\alpha_0} (\alpha_0\kappa^{\alpha_0-1}\vartheta^{-1} - 1) + 1 \right]} \quad (\text{B.19})$$

Assuming that the term in square brackets is positive, we have that $L_0^* \in (0, 1)$ if and only if $\vartheta < \kappa^{\alpha_0-1}$, which in terms of fundamental parameters requires that $\beta(1 + \gamma^*)^{1-\theta} < \frac{(1+\gamma_0)^{-\frac{\alpha_0}{1-\alpha_0}}}{\alpha_0 + (1-\alpha_0)(1+\gamma_0)^{-\frac{1}{1-\alpha_0}}(1-\delta)}$ which is the condition stated in (21). Also, it is readily verified that as long as $\vartheta < \kappa^{\alpha_0-1}$, L_0^* cannot be negative.

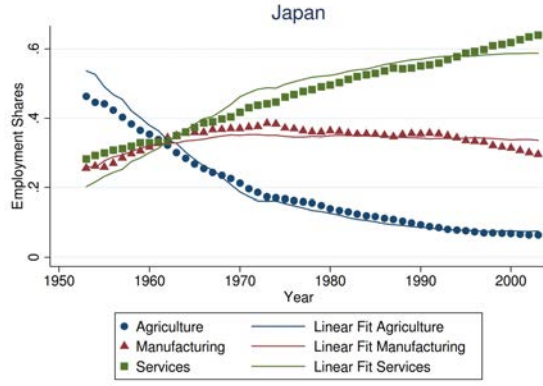
Therefore, we constructed a unique constant growth path that asymptotically satisfies the equilibrium conditions whenever the parameters of the economy satisfy Equation (21). Together with the uniqueness of the competitive equilibrium, this completes the proof.

⁵²We have that $\frac{\partial(\bar{\alpha}^*\tilde{E}^*)}{\partial\tilde{C}^*} = \frac{\bar{\alpha}^*\tilde{E}^*}{\tilde{C}^*} \frac{\bar{\varepsilon}-\sigma}{1-\sigma} [1 + (1-\sigma)\rho_{\varepsilon_i-\sigma, \alpha_i}]$ where $\rho_{\varepsilon_i-\sigma, \alpha_i}$ is the correlation coefficient between $\varepsilon_i - \sigma$ and α_i under a distribution implied by expenditure shares (see online Appendix for details of the derivation). Therefore, the derivative is always positive and the function is a monotonic of $\tilde{C}^*$.

C Additional Figures

Figure 4: Regression Fit for Japan using common world parameters $\{\sigma, \epsilon_a - \epsilon_m, \epsilon_s - \epsilon_m\}$

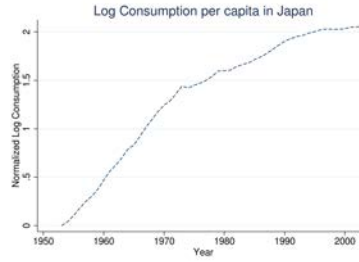
(a) Regression Fit using all regressors



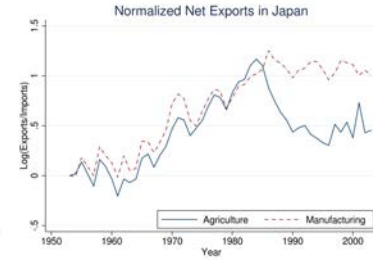
(b) Relative Prices



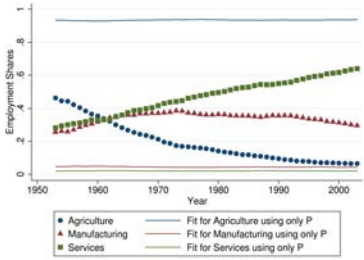
(c) Consumption



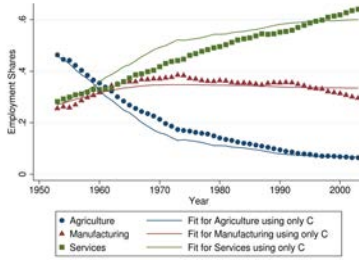
(d) Net Exports



(e) Partial fit: Prices only



(f) Partial fit: Consumption only



(g) Partial fit: Net Exports only

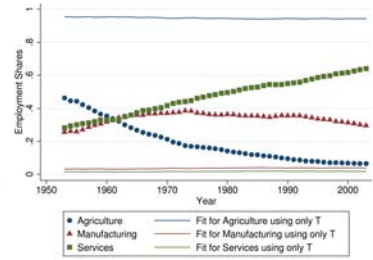


Figure 5: Baseline Country Fit

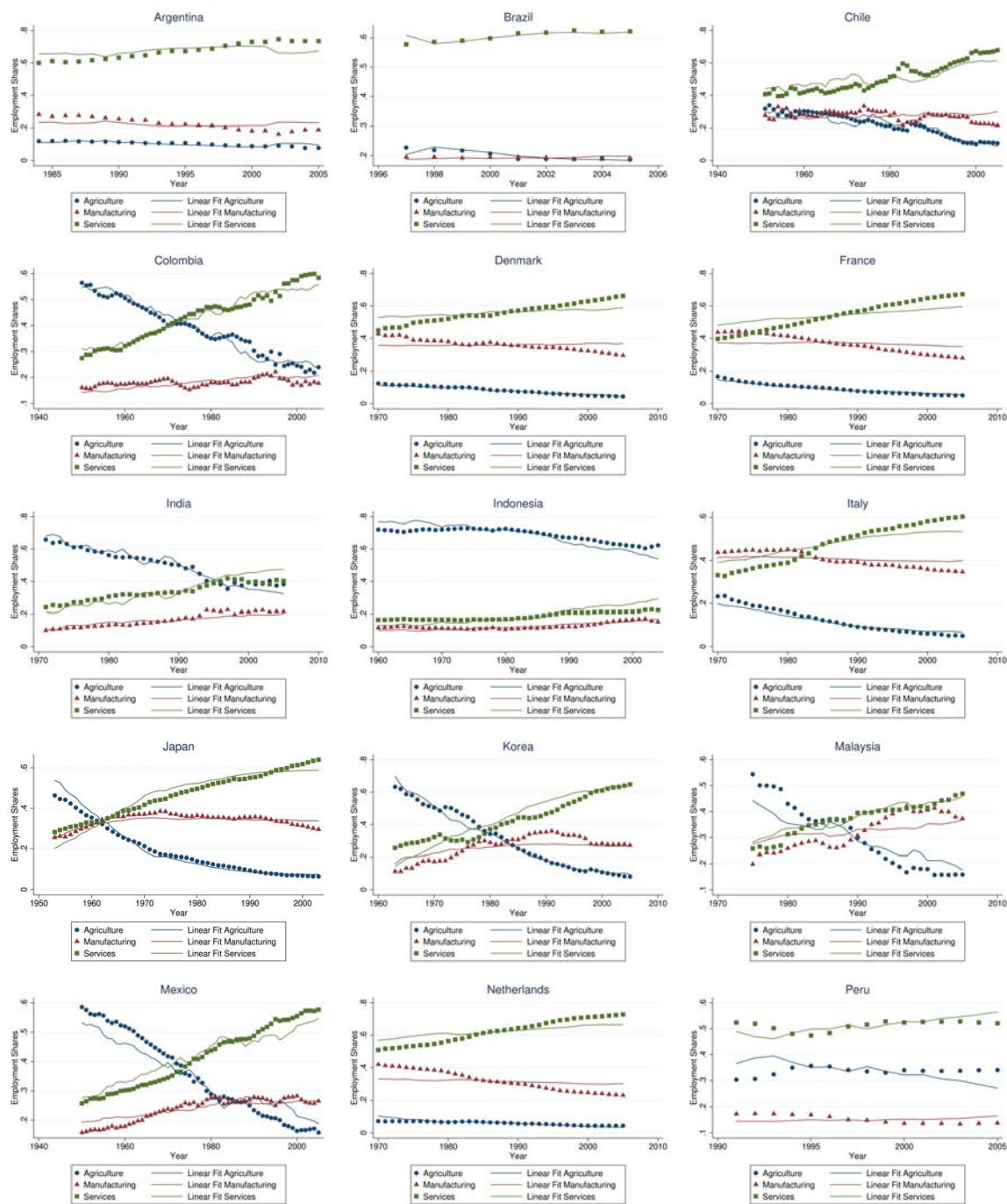


Figure 6: Baseline Country Fit (continued)

