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ABSTRACT

Governments are present-biased toward spending. Fiscal rules are deficit limits that trade off commitment to not overspend and flexibility to react to shocks. We compare centralized rules — chosen jointly by all countries — to decentralized rules. If governments' present bias is small, centralized rules are tighter than decentralized rules: individual countries do not internalize the redistributive effect of interest rates. However, if the bias is large, centralized rules are slacker: countries do not internalize the disciplining effect of interest rates. Surplus limits and money burning enhance welfare, and inefficiencies arise if some countries adopt stricter rules than imposed centrally.

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Governments often impose fiscal rules on themselves to constrain their spending and borrowing. Fiscal rules can be *decentralized* — chosen independently by each country — or *centralized* — chosen jointly by a group of countries.

In 2013, 97 countries had fiscal rules in place, a dramatic increase from 1990 when only 7 countries had them. Of these 97 countries, 49 countries were subject to national rules, 48 to supranational rules, and 14 to both types of rules.[1] For example, Germany was constrained not only by the guidelines of the European Union's Stability and Growth Pact (SGP), but also by its own constitutionally mandated "debt brake" which imposed a tighter limit on the government's structural deficit than the SGP.[2]

This paper studies the optimal design of centralized fiscal rules and how they compare to decentralized fiscal rules. Are centralized rules tighter or more lax than decentralized rules? How does this depend on governments' deficit bias? What happens if some countries — like Germany in the case of the European Union — can adopt fiscal constraints to supplement those imposed centrally?

Our theory of fiscal rules is motivated by a fundamental tradeoff between commitment and flexibility: on the one hand, rules provide valuable commitment as they can limit distorted incentives in policymaking that result in a spending bias and excessive deficits; on the other hand, there is a cost of reduced flexibility as fiscal constitutions cannot spell out policy prescriptions for every single shock or contingency, and some discretion may be optimal. Under decentralized fiscal rules, each country resolves this commitment-versus-flexibility tradeoff independently. In contrast, under a centralized fiscal rule, countries resolve this tradeoff jointly.

We consider a two-period model in which a continuum of identical governments choose deficit-financed public spending. At the beginning of the first period, each government receives an idiosyncratic shock to the social value of spending in this period. Governments are benevolent ex ante, prior to the real-

[1] See IMF Fiscal Rules Data Set, 2013 and Budina et al. (2012). The treaties that encompass the supranational rules correspond to the European Union's Stability and Growth Pact, the West African Economic and Monetary Union, the Central African Economic and Monetary Community, and the Eastern Caribbean Currency Union.

[2] See Truger and Will (2012). Other countries with both national and supranational rules in 2013 were Austria, Bulgaria, Croatia, Denmark, Estonia, Finland, Lithuania, Luxembourg, Poland, Slovak Republic, Spain, Sweden, and the United Kingdom.

ization of the shock, but present-biased ex post, when it is time to choose spending. This preference structure results naturally from the aggregation of heterogeneous, time-consistent citizens' preferences (Jackson and Yariv, 2014a,b), or as a consequence of turnover in political economy models (e.g., Aguiar and Amador, 2011).[3] We assume that the shock to the value of spending is a government's private information, or *type*, capturing the fact that not all contingencies are contractible or observable. The combination of a present bias and private information implies that governments face a tradeoff between commitment and flexibility. We define a fiscal rule in this context as a fully enforceable deficit limit, imposed prior to the realization of the shock.

Our environment is the same as that considered in Amador, Werning and Angeletos (2006) and Halac and Yared (2014). These papers characterize optimal decentralized fiscal rules, which are chosen independently by each government taking global interest rates as given. We depart by studying centralized fiscal rules, which are chosen by a central authority representing all governments, taking into account the impact that fiscal rules have on global interest rates. Centralized rules internalize the fact that lowering flexibility affects countries not only directly by limiting their borrowing and spending, but also indirectly by reducing interest rates.[4]

An optimal decentralized fiscal rule is a deficit limit such that, on average, the distortion above the limit is zero. Specifically, consider a government that, ex post, would like to borrow more than allowed by the imposed limit. If the government experienced a relatively low shock to the value of spending, it will be overborrowing compared to its ex-ante optimum, as the government is present-biased ex post. On the other hand, if the government experienced a relatively

[3]See also Alesina and Perotti (1994), Alesina and Tabellini (1990), Battaglini and Coate (2008), Caballero and Yared (2010), Lizzeri (1999), Persson and Svensson (1989), and Tornell and Lane (1999). Our formulation of governments' preferences corresponds to the quasi-hyperbolic consumption model; see Laibson (1997).

[4]In our model, a government's debt exerts an externality on other governments solely through the interest rate. Centralized rules may differ from decentralized rules for reasons different from those studied here if higher debt by some governments entails other externalities, such as a higher risk of crisis and contagion, inflation, or future fiscal transfers. Our main results regarding the role of the interest rate, however, would be unchanged by the introduction of these effects. Beetsma and Uhlig (1999) and Chari and Kehoe (2007) study settings in which the existence of a common monetary policy generates an externality.

high shock, it will be underborrowing, as the government is constrained by the deficit limit. For a fixed interest rate, an optimal deficit limit equalizes the marginal benefit of providing more flexibility to underborrowing types to the marginal cost of providing more discretion to overborrowing types.

Our results contrast these decentralized rules with centralized rules. We first show that if governments' present bias is small, the optimal centralized fiscal rule is tighter than the decentralized one, and hence interest rates are lower under centralization. Intuitively, governments choosing rules independently do not internalize the fact that by allowing themselves more flexibility, they increase interest rates, thus redistributing resources away from governments that borrow more toward governments that borrow less. Committing ex ante to tighter constraints is socially beneficial: the cost of reducing flexibility for underborrowing countries is mitigated by the drop in the interest rate, which benefits more indebted countries whose marginal value of spending is higher. We note that this *redistributive effect* of the interest rate is present even when governments are not present-biased; in fact, this effect is most powerful when the bias is small.

Our main result, on the other hand, shows that if governments' present bias is large, the optimal centralized fiscal rule is slacker than the decentralized one, and hence interest rates are higher under centralization. This result arises because interest rates also have a natural *disciplining effect*. Governments choosing rules independently do not internalize the fact that by reducing their own discretion, they lower interest rates, thus increasing governments' desire to borrow and worsening fiscal discipline for all. Committing ex ante to more flexibility is socially beneficial: the cost of increasing discretion for overborrowing countries is mitigated by the rising interest rate, which induces everyone to borrow less. Paradoxically, in some cases, the externality is large enough that all governments can be made ex ante better off by abandoning their decentralized fiscal rules and allowing themselves full flexibility. Unlike the redistributive effect of the interest rate, the disciplining effect relies on governments being present-biased; in fact, this effect is dominant when the bias is large.

We explore different mechanisms that can enhance welfare when governments' present bias is large and thus the disciplining effect of the interest rate dominates the redistributive effect. We show that supplementing maximum

deficit limits with maximum *surplus* limits can be socially beneficial in this case. Maximum surplus limits are never used in an optimal decentralized fiscal rule, as these limits force low government types which are overborrowing to borrow even more. However, surplus limits also serve to increase interest rates, and through this channel they can improve overall fiscal discipline. Additionally, we show that welfare can be further increased by introducing *money burning*: a centralized fiscal rule where governments whose surplus exceeds a certain level must incur losses can improve upon simply using deficit and surplus limits, even when these losses are a pure resource cost.

Our final set of results is motivated by the observation that, in practice, some countries can adopt fiscal constraints to supplement those imposed centrally. Clearly, if governments' present bias is small, this possibility is irrelevant: governments are subject to (fully enforceable) centralized rules that are tighter than they would individually prefer. However, if governments' present bias is large, an inefficiency emerges: governments that have the ability to adopt more stringent constraints choose to do so. Tighter national rules in some countries depress global interest rates, thus reducing fiscal discipline in other countries.[5] Moreover, we show that the optimal response of the central authority is to tighten fiscal restrictions for all governments. Therefore, if a subset of countries can adopt rules on top of those imposed centrally, all countries face lower interest rates and less flexibility as a consequence.

This paper is related to several literatures. First, the paper fits into the mechanism design literature that studies the tradeoff between commitment and flexibility in self-control settings, including Amador, Werning and Angeletos (2006), Athey, Atkeson and Kehoe (2005), and Halac and Yared (2014).[6,7] Un-

[5] Fernández-Villaverde, Garicano and Santos (2013) argue that the drop in interest rates that followed European integration led to the abandonment of reforms and institutional deterioration in the peripheral European countries. Germany, on the other hand, had stricter fiscal policies and did experience a reform process.

[6] These papers solve for the optimal mechanism, whereas for most of our analysis we restrict attention to rules that take the form of deficit limits (exploring variations in Section 4). Deficit limits can be shown to correspond to the optimal decentralized mechanism under weak conditions. Characterizing the optimal centralized mechanism, however, is difficult because the problem is not convex.

[7] See also Ambrus and Egorov (2012) and Sleet (2004), as well as Bernheim, Ray and Yeltekin (2015) which considers the self-enforcement of commitment contracts. More generally, the paper relates to the literature on delegation in principal-agent settings, including

like this literature, we endogenize the effective price of the temptation good — which in our environment corresponds to the interest rate — and we show how this price can serve as a natural disciplining device, affecting the optimal mechanism for a group of agents. Our analysis and results can be applied to different self-control problems; see Section 6 for a discussion. Second, the paper is related to an extensive literature on the political economy of fiscal policy.[8] Most closely related is Azzimonti, Battaglini and Coate (2015), which considers the quantitative welfare implications of a balanced budget rule when the government is present-biased. In contrast to this work, we study the design of fiscal rules in a global economy in which individual rules affect global interest rates. In this regard, our paper is related to the literature on fiscal policy coordination across countries, including Chari and Kehoe (1990) and Persson and Tabellini (1995).[9] Whereas these papers emphasize the inefficiencies that arise when unconstrained governments freely choose policies, our interest is in the design of rules that can curb these inefficiencies. Finally, more broadly, our paper contributes to the literature on hyperbolic discounting and the benefits of commitment devices.[10]

1 Model

1.1 Setup

We study a simple model of fiscal policy in which a continuum of governments each make a spending and borrowing decision. Our setup is the same as that analyzed in Amador, Werning and Angeletos (2006), with the exception that we allow for multiple governments and an endogenous interest rate.

There are two periods and a unit mass of ex-ante identical governments.[11] At

Alonso and Matouschek (2008), Amador and Bagwell (2013), Ambrus and Egorov (2013), and Holmström (1977, 1984).

[8] In addition to the work previously cited, see Acemoglu, Golosov and Tsyvinski (2008), Azzimonti (2011), Krusell and Rios-Rull (1999), Song, Storesletten and Zilibotti (2012), and Yared (2010).

[9] See also the discussion in fn. 4.

[10] See, for example, Barro (1999), Bisin, Lizzeri and Yariv (2015), Krusell, Kruscu and Smith, Jr. (2010), Krusell and Smith, Jr. (2003), Laibson (1997), Lizzeri and Yariv (2014), and Phelps and Pollak (1968).

[11] We purposely abstract away from heterogeneity in order to study differences between

the beginning of the first period, each government observes a shock to its economy, $\theta > 0$, which is the government's private information or *type*. θ is drawn from a bounded set $\Theta \equiv \left[\underline{\theta}, \overline{\theta}\right]$ with a continuously differentiable distribution function $F(\theta)$, normalized so that $\mathbb{E}[\theta] = 1$.

Following the realization of θ, each government chooses first-period public spending g and second-period public spending x subject to a budget constraint:

$$g + \frac{x}{R} = \tau + \frac{\tau}{R}, \tag{1}$$

where τ is the revenue of the government in each period and R is the endogenously determined gross interest rate.

The government's welfare *prior* to the realization of its type θ is

$$\mathbb{E}\left[\theta U(g) + \delta U(x)\right], \tag{2}$$

where $\delta \in (0, 1]$ is the discount factor and $U(\cdot)$ is increasing, strictly concave, and continuously differentiable. The government's welfare *after* the realization of its type θ, when choosing spending g and x, is

$$\theta U(g) + \beta\delta U(x), \tag{3}$$

where $\beta \leq 1$.

Because the world consists of a continuum of governments which can only borrow and lend from one another, total spending in the aggregate must equal the value of total resources available. Let $g(\theta, R)$ be the level of first-period spending chosen by a government of type θ when the interest rate is R. Note that since governments are ex-ante identical, the distribution of realized types across governments is the same as the distribution of types for each government. Thus, given that the density function is $f(\theta)$ and each government has resources

centralized and decentralized fiscal rules that are not due to countries having different characteristics. Introducing heterogeneity would generate additional differences between centralized and decentralized rules as well as differences in decentralized rules across countries.

τ in each period, the global resource constraint in the first period is

$$\int_{\underline{\theta}}^{\overline{\theta}} g(\theta, R) f(\theta) d\theta = \tau. \tag{4}$$

The interest rate R must adjust so that governments' spending decisions satisfy (4). Equations (1) and (4) imply that the global resource constraint is satisfied in the second period. That is, letting $x(\theta, R)$ be the level of second-period spending chosen by a government of type θ when the interest rate is R, the second-period resource constraint, $\int_{\underline{\theta}}^{\overline{\theta}} x(\theta, R) f(\theta) d\theta = \tau$, holds.

We note that our setting does not allow for cross-subsidization across types. Specifically, the net present value of public spending cannot be different for a lower type relative to a higher type,[12] and hence fiscal transfers across countries are ruled out. Also, to simplify the exposition and without loss of generality, we have abstracted away from borrowing and lending of the household sector.[13]

1.2 Fiscal Rules

There are two frictions in our setting. First, if $\beta < 1$, a government's objective (3) following the realization of its type does not coincide with its objective (2) prior to this realization. In particular, the government is present-biased: its welfare after θ is realized overweighs the importance of current spending compared to its welfare before θ is realized. As mentioned in the Introduction, this structure arises naturally when the government's preferences aggregate heterogeneous citizens' preferences, even if the latter are time consistent (see Jackson and Yariv, 2014a,b). This formulation can also be motivated by political turnover; for instance, preferences such as these emerge in settings with political uncertainty where policymakers place a higher value on public spending when they hold power and can make spending decisions (see Aguiar and Amador, 2011).

The second friction in our setting is that the realization of θ — which affects

[12] This is in contrast to other models such as Atkeson and Lucas (1992) and Thomas and Worrall (1990).

[13] Our model is identical to one in which households in an economy do not have access to external financial markets, and the government can borrow and lend on their behalf. The model can be extended to introduce a subset of households that can access external financial markets without affecting our analysis. Details are available from the authors upon request.

the marginal social utility of first-period spending — is privately observed by the government. One possible interpretation is that θ is not verifiable ex post by a rule-making body; therefore, even if it is observable, fiscal rules cannot explicitly depend on the value of θ. An alternative interpretation is that the exact cost of public goods is only observable to the policymaker, who may be inclined to overspend on these goods.[14]

The combination of these two frictions leads to a tradeoff between commitment and flexibility. Specifically, note that ex ante, as a function of its type θ and the interest rate R, each government would like to choose first-period spending $g^{ea}(\theta, R)$ and second-period spending $x^{ea}(\theta, R)$ satisfying

$$\theta U'\left(g^{ea}(\theta, R)\right) = \delta R U'\left(x^{ea}(\theta, R)\right) \tag{5}$$

under the budget constraint (1). However, this ex ante optimum cannot be implemented with full flexibility: if the government were given full flexibility to choose spending and borrowing, ex post it would choose $g^f(\theta, R)$ and $x^f(\theta, R)$ satisfying

$$\theta U'\left(g^f(\theta, R)\right) = \beta\delta R U'\left(x^f(\theta, R)\right), \tag{6}$$

and hence a present-biased government would overborrow relative to (5). In addition, the ex ante optimum cannot be achieved with full commitment: a spending plan cannot be made explicitly contingent on the realization of the government's type θ, and hence (5) cannot be implemented by fully committing the government to a contingent plan. Therefore, a tradeoff between commitment and flexibility arises, and the optimal mechanism is then not trivial.

We define a fiscal rule as a cutoff $\theta^* \in [\underline{\theta}, \overline{\theta}]$ such that if the government's type is $\theta > \theta^*$, its first-period and second-period spending levels are $g^f(\theta^*, R)$ and $x^f(\theta^*, R)$, whereas if the government's type is $\theta \leq \theta^*$, the spending levels are $g^f(\theta, R)$ and $x^f(\theta, R)$ (where $g^f(\cdot)$ and $x^f(\cdot)$ are given by (1) and (6)). This fiscal rule can be implemented using a maximum deficit limit, spending limit, or debt limit. Under such an implementation, all types $\theta \leq \theta^*$ can make their full-flexibility ex-post optimal choices within the limit, whereas types $\theta > \theta^*$

[14] A third possibility is that citizens have heterogeneous preferences or information on the optimal level of public spending, and only the government sees the aggregate. See Sleet (2004).

are constrained and thus choose spending at the limit. Deficit limits capture aspects of many of the fiscal rules observed in practice. Moreover, under weak conditions on the distribution function $F(\theta)$, deficit limits correspond to the optimal mechanism when the interest rate is exogenous (see Amador, Werning and Angeletos, 2006).

Our interest is in comparing the case in which the fiscal rule θ^* is decentralized — chosen independently by each government — and the case in which this rule is centralized — chosen by a central authority representing all governments. Whereas each government takes the interest rate R as given when choosing its optimal decentralized rule, the central authority takes into account the impact of θ^* on the interest rate R when choosing the optimal centralized rule.

Throughout our analysis, we assume non-increasing absolute risk aversion:

Assumption 1. *$-U''(g)/U'(g)$ is non-increasing in g.*

Let $R(\theta^*)$ denote the level of the interest rate when fiscal rule θ^* applies to all governments. The next lemma follows from Assumption 1.

Lemma 1. *$R(\theta^*)$ is strictly increasing in θ^* for all $\theta^* \in \left(\underline{\theta}, \overline{\theta}\right)$.*

Lemma 1 describes how the tightness of fiscal rules impacts the level of global interest rates. The higher is the value of the cutoff θ^*, the more flexible is the fiscal rule, so the higher is the level of borrowing and, as a result, the higher is the interest rate. This relationship between the fiscal rule and the interest rate plays a central role in our analysis of centralized versus decentralized rules.

Regarding implementation, it is worth noting that when the interest rate is endogenously determined, the mapping from θ^* to a spending or borrowing limit need not be monotonic. To see why, consider a fiscal rule θ^*, associated with a maximum allowable level of public spending $g^f(\theta^*, R(\theta^*))$. Holding the interest rate fixed, the direct effect of an increase in θ^* is to increase $g^f(\theta^*, R(\theta^*))$. But there is also an indirect effect: when θ^* increases, $R(\theta^*)$ increases, and depending on the relative strength of income and substitution effects, $g^f(\theta^*, R(\theta^*))$ can decrease. It can be shown however that if the elasticity of intertemporal substitution is sufficiently close to 1, the direct effect outweighs the indirect effect, implying that $g^f(\theta^*, R(\theta^*))$ is monotonically increasing in θ^*.

2 Decentralized Fiscal Rules

We begin by analyzing decentralized fiscal rules. Each government independently chooses a fiscal rule to maximize its expected welfare, subject to the budget constraint and taking the interest rate as given:

$$\max_{\theta^* \in [\underline{\theta}, \overline{\theta}]} \left\{ \begin{array}{l} \int_{\underline{\theta}}^{\theta^*} \left(\theta U(g^f(\theta, R)) + \delta U(x^f(\theta, R)) \right) f(\theta) d\theta \\ + \int_{\theta^*}^{\overline{\theta}} \left(\theta U(g^f(\theta^*, R)) + \delta U(x^f(\theta^*, R)) \right) f(\theta) d\theta \end{array} \right\} \tag{7}$$

$$\text{subject to (1) and (6).}$$

This program takes into account that, given a fiscal rule θ^*, all types $\theta \leq \theta^*$ exert full discretion and thus choose spending $g^f(\theta, R)$ and $x^f(\theta, R)$ (defined by (1) and (6)), whereas all types $\theta > \theta^*$ have no discretion and thus choose $g^f(\theta^*, R)$ and $x^f(\theta^*, R)$. The following assumption ensures an interior solution:

Assumption 2. $\beta \geq \underline{\theta}$.

The first-order conditions of the decentralized program yield

$$\int_{\theta_d^*}^{\overline{\theta}} \left(\theta U' \left(g^f \left(\theta_d^*, R \right) \right) - \delta R U' \left(x^f \left(\theta_d^*, R \right) \right) \right) f \left(\theta \right) d\theta = 0. \tag{8}$$

Equation (8) shows that the optimal decentralized fiscal rule sets a cutoff θ_d^* such that the average distortion above this cutoff is zero. Specifically, given the cutoff, there exists $\widehat{\theta} > \theta_d^*$ such that if the government's type is $\theta \in [\theta_d^*, \widehat{\theta})$, then

$$\theta U' \left(g^f \left(\theta_d^*, R \right) \right) < \delta R U' \left(x^f \left(\theta_d^*, R \right) \right),$$

and hence the government overborrows relative to its ex-ante optimum (defined in (5)). If instead the government's type is $\theta \in (\widehat{\theta}, \overline{\theta}]$, then

$$\theta U' \left(g^f \left(\theta_d^*, R \right) \right) > \delta R U' \left(x^f \left(\theta_d^*, R \right) \right),$$

and hence the government underborrows relative to its ex-ante optimum. The optimal decentralized rule specifies θ_d^* so that the marginal benefit of providing more flexibility to types $\theta > \widehat{\theta}$ which are underborrowing is equal to the marginal cost of providing more discretion to types $\theta < \widehat{\theta}$ which are overborrowing.

By substituting (6) into (8), we obtain the following result.

Proposition 1. *For any given interest rate R, the optimal decentralized fiscal rule specifies a cutoff θ_d^* satisfying*

$$\frac{\mathbb{E}\left[\theta|\theta \geq \theta_d^*\right]}{\theta_d^*} = \frac{1}{\beta}. \tag{9}$$

Equation (9) shows that the optimal decentralized fiscal rule is independent of the form of the utility function and the level of the interest rate. If $\beta = 1$, (9) implies $\theta_d^* = \overline{\theta}$, so the optimal decentralized rule entails full flexibility. Intuitively, in the absence of a present bias, there is no benefit to the government from constraining its borrowing and spending. At the other extreme, if $\beta = \underline{\theta}$, (9) implies $\theta_d^* = \underline{\theta}$, so the government grants itself minimal discretion. That is, only the lowest type $\underline{\theta}$ chooses its flexible optimum in this large present bias case, and all other types $\theta > \underline{\theta}$ are constrained. Finally, if $\beta \in \left(\underline{\theta}, \overline{\theta}\right)$, (9) implies that the optimal decentralized rule is bounded discretion with an interior cutoff $\theta_d^* \in (\underline{\theta}, \overline{\theta})$. This explains our motivation for Assumption 2: we take governments' present bias to be small enough that bounded discretion, as opposed to no discretion, constitutes the decentralized optimum.

Under mild restrictions on the distribution function $F(\theta)$, Proposition 1 yields that the level of discretion in the optimal decentralized fiscal rule is monotonically decreasing in the government's present bias:

Corollary 1. *If $F(\theta)$ satisfies*

$$\frac{d \log \mathbb{E}\left[\theta|\theta \geq \theta^*\right]}{d \log \theta^*} < 1 \text{ for all } \theta^* \in \left(\underline{\theta}, \overline{\theta}\right), \tag{10}$$

then θ_d^* is strictly increasing in β.

Condition (10) holds for many distributions such as exponential, log-normal,

and Gamma for a subset of their parameters.[15]

Proposition 1 characterizes the fiscal rule θ_d^* that each government chooses when taking the interest rate R as given. At the same time, note that this rule effectively determines the level of the interest rate: as described in Section 1, R must adjust so that the global resource constraint (4) is satisfied. Lemma 1 implies that if governments are present-biased (i.e. $\beta < 1$), then the interest rate that is induced by the decentralized fiscal rules θ_d^* is lower than the one that would prevail were all governments granted full flexibility.

3 Centralized Fiscal Rules

We now proceed to the main part of our analysis, which considers the optimal centralized fiscal rule. This rule is chosen by a central authority that represents all governments and takes into account the impact that rules have on the interest rate, as characterized in Lemma 1.

Subsection 3.1 describes the program that solves for the optimal centralized fiscal rule; Subsection 3.2 and Subsection 3.3 compare this rule to the optimal decentralized rule for different levels of governments' present bias; and Subsection 3.4 considers examples under log preferences.

3.1 Solving for the Optimal Centralized Fiscal Rule

An optimal centralized fiscal rule maximizes total expected welfare subject to each government's budget constraint and the global resource constraint:

$$\max_{\theta^* \in [\underline{\theta}, \bar{\theta}]} \left\{ \begin{array}{l} \displaystyle\int_{\underline{\theta}}^{\theta^*} \left(\theta U(g^f(\theta, R(\theta^*))) + \delta U(x^f(\theta, R(\theta^*))) \right) f(\theta) d\theta \\ \\ \displaystyle + \int_{\theta^*}^{\bar{\theta}} \left(\theta U(g^f(\theta^*, R(\theta^*))) + \delta U(x^f(\theta^*, R(\theta^*))) \right) f(\theta) d\theta \end{array} \right\} \tag{11}$$

$$\text{subject to (1), (4), and (6).}$$

[15] Condition (10) with a weak inequality is equivalent to Assumption A in Amador, Werning and Angeletos (2006) holding for all $\beta \in [\underline{\theta}, 1]$. Their Assumption A guarantees that a deficit limit is an optimal mechanism in a decentralized environment like this one.

This program is identical to program (7) which solves for the optimal decentralized fiscal rule, with the exception that (11) takes into account that the interest rate is a function of the cutoff θ^*. Specifically, given $g^f(\theta, R(\theta^*))$ and $x^f(\theta, R(\theta^*))$ defined by (1) and (6), the interest rate $R(\theta^*)$ is defined by the global resource constraint (4), and is characterized in Lemma 1.

The first-order conditions of the centralized program yield:

Lemma 2. *The optimal centralized fiscal rule specifies a cutoff θ_c^*, with associated interest rate $R = R(\theta_c^*)$, which whenever interior satisfies*

$$\frac{\mathbb{E}(\theta|\theta \geq \theta_c^*)}{\theta_c^*} = \frac{1}{\beta} + \frac{R'(\theta_c^*)}{(1 - F(\theta_c^*))\,\theta_c^* U'\left(g^f(\theta_c^*, R)\right)\frac{\partial g^f(\theta_c^*, R)}{\partial \theta_c^*}}(\rho + \lambda), \tag{12}$$

where

$$\rho = \frac{1}{R}\left[\begin{array}{c} \int_{\underline{\theta}}^{\theta_c^*} \delta U'\left(x^f(\theta, R)\right)\left(\tau - x^f(\theta, R)\right) f(\theta)\, d\theta \\ + \int_{\theta_c^*}^{\overline{\theta}} \delta U'\left(x^f(\theta_c^*, R)\right)\left(\tau - x^f(\theta_c^*, R)\right) f(\theta)\, d\theta \end{array}\right] \geq 0 \tag{13}$$

and

$$\lambda = \left[\begin{array}{c} \int_{\underline{\theta}}^{\theta_c^*} \left(\delta R U'\left(x^f(\theta, R)\right) - \theta U'\left(g^f(\theta, R)\right)\right)\frac{dg^f(\theta, R)}{dR} f(\theta)\, d\theta \\ + \int_{\theta_c^*}^{\overline{\theta}} \left(\delta R U'\left(x^f(\theta_c^*, R)\right) - \theta U'\left(g^f(\theta_c^*, R)\right)\right)\frac{dg^f(\theta_c^*, R)}{dR} f(\theta)\, d\theta \end{array}\right] \gtreqless 0. \tag{14}$$

Comparing Lemma 2 and Proposition 1 shows how the optimal centralized fiscal rule θ_c^* differs from the optimal decentralized fiscal rule θ_d^*. The difference is that the second term in (12) does not appear in expression (9). This term is associated with two factors, ρ and λ, which capture the effects that the interest rate has on the allocation. As we explain subsequently, ρ captures the *redistributive effect* of the interest rate, while λ is the *disciplining effect*. These effects are internalized by a centralized rule but not by a decentralized rule.

The redistributive effect of the interest rate, ρ, is positive. This effect captures the fact that higher interest rates hurt first-period borrowers by reducing their spending in the second period. Countries of higher type θ borrow more in the first period and therefore benefit more from a reduction in the interest rate than countries of lower type. Moreover, because of their higher spending in the first period, higher type countries also have a higher marginal utility of spend-

ing in the second period than lower type countries. Hence, the central authority (which cares about average welfare) weighs higher type countries by more, and as a result finds it optimal to commit to a lower interest rate to redistribute resources from lower type to higher type countries.

To understand the consequences of the redistributive effect, suppose that condition (10) holds, so that the left-hand side of (12) is decreasing in θ_c^*. Then holding all else fixed, (12) shows that a higher value of ρ implies a lower value of θ_c^*.[16] That is, the redistributive effect puts downward pressure on the optimal level of discretion: by lowering flexibility, the centralized rule induces a lower interest rate, thus redistributing resources from countries that borrow less to those that borrow more. This redistribution is ex-ante beneficial for all countries.

The redistributive effect of the interest rate is present even in the absence of a self-control problem, i.e. even if governments are not present-biased and thus $\beta = 1$. In fact, this effect arises in other models that abstract from self-control issues and consider instead incomplete market economies with heterogenous agents, such as Azzimonti, de Francisco and Quadrini (2014) and Yared (2013). The redistributive channel reflects the fact that, absent perfect insurance markets, distortions such as deficit limits can improve social welfare.

Consider next the disciplining effect of the interest rate, λ. This effect captures the fact that the level of the interest rate affects the level of borrowing and spending that governments choose when given discretion. As shown in (14), λ may be positive or negative; its sign depends on how borrowing and spending change with R and how this in turn affects low versus high θ types. For intuition, suppose $dg^f(\theta, R)/dR < 0$, so that higher interest rates induce governments to borrow less. A higher interest rate in this case is beneficial for countries whose type is relatively low, as these countries overborrow relative to their ex-ante optimum. On the other hand, a higher interest rate harms countries whose type is high because these countries underborrow relative to their ex-ante optimum.

To understand the consequences of the disciplining effect, suppose again that condition (10) holds, so that left-hand side of (12) is decreasing in θ_c^*, and maintain the assumption that $dg^f(\theta, R)/dR < 0$. It can then be easily verified that

[16] Note that $\frac{R'(\theta_c^*)}{\theta_c^*(1-F(\theta_c^*))U'(g^f(\theta_c^*,R))\frac{\partial g^f(\theta_c^*,R)}{\partial \theta_c^*}} > 0$ by Lemma 1.

if θ_c^* in expression (14) were to take the value of θ_d^* given in (9), then λ would be strictly negative. Intuitively, if the cutoff is chosen at the decentralized optimum θ_d^*, then as discussed in Section 2, the average distortion above the cutoff is zero: on average, the constrained types $\theta > \theta_d^*$ are neither overborrowing nor underborrowing relative to the ex-ante optimum. This means that the disciplining effect is determined by the unconstrained types $\theta \leq \theta_d^*$, and since these types are overborrowing, a higher interest rate can improve welfare by increasing discipline. It follows that λ is negative, and by (12) this effect increases the cutoff θ_c^*. That is, a negative disciplining effect puts upward pressure on the optimal level of discretion: by increasing flexibility, the centralized rule induces a higher interest rate, thus improving fiscal discipline for overborrowing governments. This higher level of discipline is ex-ante beneficial for all countries.

The optimal centralized fiscal rule, and how it compares to the optimal decentralized fiscal rule, depends on the relative strength of the redistributive and disciplining effects of the interest rate. Subsection 3.2 shows that the redistributive effect dominates when governments' present bias is small enough. When the present bias is large enough, however, Subsection 3.3 shows that the disciplining effect is negative and dominates the redistributive effect.

3.2 Small Present Bias

We show that if governments' present bias is small, the optimal centralized fiscal rule is more stringent than the optimal decentralized fiscal rule. As a consequence, the interest rate is lower under centralization.

Proposition 2. *There exists $\overline{\beta} \in [\underline{\theta}, 1]$ such that if $\beta \geq \overline{\beta}$, then $\theta_c^* < \theta_d^*$. Therefore, the optimal centralized fiscal rule provides less flexibility than the optimal decentralized fiscal rule when governments' present bias is small enough.*

To see the idea for the proof, take $\beta = 1$, so that governments are not present-biased. The optimal decentralized fiscal rule in this case entails full flexibility, with a cutoff $\theta_d^* = \overline{\theta}$. In fact, there is no disciplining effect of the interest rate, as no government overborrows relative to the ex-ante optimum. Since the redistributive effect of the interest rate is positive, it follows that social welfare can be improved by imposing a tighter fiscal rule, $\theta_c^* < \theta_d^*$, which reduces

the interest rate. This tighter rule lowers flexibility, but it benefits all countries from an ex-ante perspective by reducing their cost of borrowing.

Proposition 2 shows that when the present bias is small, governments independently prefer lax fiscal rules. Governments do not internalize the fact that by allowing themselves more flexibility, they increase interest rates. By lowering discretion and therefore reducing R, the central authority can redistribute resources from lower type countries to higher type countries which borrow more and are harmed by high interest rates.

3.3 Large Present Bias

Consider next the case in which governments' present bias is large. We show that the optimal centralized fiscal rule is more lax than the optimal decentralized fiscal rule. As a consequence, the interest rate is higher under centralization.

Proposition 3. *There exists $\underline{\beta} \in [\underline{\theta}, 1]$ such that if $\beta \leq \underline{\beta}$, then $\theta_c^* > \theta_d^*$. Therefore, the optimal centralized fiscal rule provides more flexibility than the optimal decentralized fiscal rule when governments' present bias is large enough.*

To see the idea for the proof, take $\beta = \underline{\theta}$, so that governments' present bias is at its highest level (as defined in Assumption 2). The optimal decentralized fiscal rule in this case entails minimal discretion, with a cutoff $\theta_d^* = \underline{\theta}$. It can then be verified that setting $\theta_c^* = \underline{\theta}$ in (13) and (14) would imply $\rho = \lambda = 0$, so that the redistributive and disciplining effects of the interest rate are both zero starting from the decentralized optimum. The proof of Proposition 3 rests on showing that as θ_c^* approaches $\underline{\theta}$ from above, the disciplining effect outweighs the redistributive effect, and hence increasing flexibility is socially optimal.

More precisely, combine (13) and (14) to write the sum of the redistributive and disciplining effects of the interest rate as

$$\rho + \lambda = \int_{\underline{\theta}}^{\theta_c^*} \left[\begin{array}{c} \frac{\delta}{R} U' \left(x^f \left(\theta, R \right) \right) \left(\tau - x^f \left(\theta, R \right) \right) \\ + \left(\delta R U' \left(x^f \left(\theta, R \right) \right) - \theta U' \left(g^f \left(\theta, R \right) \right) \right) \frac{dg^f(\theta, R)}{dR} \end{array} \right] f \left(\theta \right) d\theta$$

$$+ \int_{\theta_c^*}^{\overline{\theta}} \left[\begin{array}{c} \frac{\delta}{R} U' \left(x^f \left(\theta_c^*, R \right) \right) \left(\tau - x^f \left(\theta_c^*, R \right) \right) \\ + \left(\delta R U' \left(x^f \left(\theta_c^*, R \right) \right) - \theta U' \left(g^f \left(\theta_c^*, R \right) \right) \right) \frac{dg^f(\theta_c^*, R)}{dR} \end{array} \right] f \left(\theta \right) d\theta.$$

The first integral corresponds to the redistributive and disciplining effects on government types whose spending is unconstrained; the second integral corresponds to these effects on types that are constrained by the fiscal rule. Suppose the cutoff is chosen at the decentralized optimum, $\theta_c^* = \theta_d^*$, and consider the limit as β approaches $\underline{\theta}$, so that θ_d^* and θ_c^* also approach $\underline{\theta}$. As explained in Subsection 3.1, the disciplining effect on constrained types is zero at $\theta_c^* = \theta_d^*$; moreover, as β goes to $\underline{\theta}$, the redistributive effect goes to zero because all types' primary deficits go to zero.[17] As for the unconstrained types, their mass goes to zero as β approaches $\underline{\theta}$; however, the redistributive and disciplining effects on these types differ in the limit: the redistributive effect vanishes, but the disciplining effect is strictly negative.[18] Thus, in the limit, it is possible to induce governments of type θ close to $\underline{\theta}$ to save more at little interest cost to higher government types.

Proposition 3 shows that when the present bias is large, governments independently prefer tight rules. Governments do not internalize the fact that by allowing themselves less flexibility, they reduce interest rates. By increasing discretion and therefore raising R, the central authority can provide flexibility while at the same time guaranteeing more discipline as a consequence of the higher interest rate that induces governments to borrow less. The next corollary expands on this result by showing that, under certain conditions, all countries can be made ex ante better off by jointly abandoning their decentralized fiscal rules and allowing themselves full flexibility.

Corollary 2. *Suppose $U(\cdot)$ is such that*

$$\frac{d^2\left(\theta U(g^f(\theta, R(\overline{\theta}))) + \delta U(x^f(\theta, R(\overline{\theta})))\right)}{d\theta^2} \geq 0$$

for all $\theta \in [\underline{\theta}, \overline{\theta}]$, with a strict inequality for some $\theta \in [\underline{\theta}, \overline{\theta}]$. Then there exists $\underline{\beta} \in [\underline{\theta}, 1]$ such that if $\beta \leq \underline{\beta}$, total expected welfare is strictly larger under full flexibility ($\theta_c^* = \overline{\theta}$) than under the optimal decentralized fiscal rule ($\theta_c^* = \theta_d^*$).

[17] If $\theta_c^* = \underline{\theta}$, all types' first-period spending is $g^f(\underline{\theta}, R)$, and by (4) we must have $g^f(\underline{\theta}, R) = \tau$. Consequently, at $\theta_c^* = \underline{\theta}$, each type's spending is equal to the endowment τ in each period.

[18] Note that in this limit, the unconstrained types' first-period spending is decreasing in R as their deficits are zero and hence there is no income effect of the interest rate.

To prove this result, we take $\beta = \underline{\theta}$, so that the optimal decentralized fiscal rule involves minimal discretion. Total expected welfare under this rule is equal to

$$U(\tau) + \delta U(\tau), \tag{15}$$

where we have used the fact that $\mathbb{E}[\theta] = 1$ and, under minimal discretion, each type's spending is equal to the endowment τ in each period (see fn. 17). Now suppose all governments are granted full flexibility. Expected welfare is then equal to

$$\int_{\underline{\theta}}^{\overline{\theta}} \left(\theta U\left(g^f\left(\theta, R\left(\overline{\theta}\right)\right)\right) + \delta U\left(x^f\left(\theta, R\left(\overline{\theta}\right)\right)\right)\right) f(\theta)\, d\theta. \tag{16}$$

Under full flexibility, the average value of government spending in each period is τ, the same as the value of spending for each type under minimal discretion. It follows that if realized welfare for type θ is convex in θ under full flexibility, then by Jensen's inequality (16) exceeds (15), implying that full flexibility dominates the optimal decentralized fiscal rule. The condition in Corollary 2 guarantees that welfare is indeed convex in θ under full flexibility. This condition is satisfied, for example, if $U(\cdot)$ is exponential or CRRA with elasticity of intertemporal substitution weakly less than one.

The intuition for this result is that if the present bias is large, then governments impose rules on themselves that are very stringent, and in the limit, if $\beta = \underline{\theta}$, governments do not allow themselves deficits. As a result, all countries have the same level of spending and no flexibility. If all governments jointly commit to granting themselves full flexibility, the interest rate increases, and this provides a natural disciplining device while simultaneously allowing governments flexibility to respond to shocks.

3.4 Examples

To illustrate the difference between centralized and decentralized fiscal rules, we now consider the case in which the utility function is $U(g) = \log(g)$.

Under log preferences, the elasticity of intertemporal substitution is equal to one; hence, for every cutoff θ^*, there is an implied spending limit $g^f(\theta^*, R(\theta^*))$ which is increasing in θ^*. Moreover, log preferences make the problem partic-

ularly tractable because welfare is separable with respect to the interest rate. Given $U(g) = \log(g)$, equations (1) and (6) imply

$$g^f(\theta, R(\theta^*)) = \frac{\theta}{\theta + \beta\delta}\left(\tau + \frac{\tau}{R(\theta^*)}\right), \tag{17}$$

$$x^f(\theta, R(\theta^*)) = \frac{\beta\delta}{\theta + \beta\delta}R(\theta^*)\left(\tau + \frac{\tau}{R(\theta^*)}\right), \tag{18}$$

and thus the program in (11) that solves for the optimal centralized fiscal rule becomes

$$\max_{\theta^* \in [\underline{\theta}, \overline{\theta}]} \left\{ \begin{array}{c} \int_{\underline{\theta}}^{\theta^*} \left(\theta \log\left(\frac{\theta}{\theta+\beta\delta}\right) + \delta \log\left(\frac{\beta\delta}{\theta+\beta\delta}\right)\right) f(\theta)\, d\theta \\ + \int_{\theta^*}^{\overline{\theta}} \left(\theta \log\left(\frac{\theta^*}{\theta^*+\beta\delta}\right) + \delta \log\left(\frac{\beta\delta}{\theta^*+\beta\delta}\right)\right) f(\theta)\, d\theta \\ +(1+\delta)\log\left(\tau + \frac{\tau}{R(\theta^*)}\right) + \delta \log\left(R(\theta^*)\right) \end{array} \right\} \tag{19}$$

subject to

$$\left[\int_{\underline{\theta}}^{\theta^*} \frac{\theta}{\theta + \beta\delta} f(\theta) d\theta + \int_{\theta^*}^{\overline{\theta}} \frac{\theta^*}{\theta^* + \beta\delta} f(\theta) d\theta\right]\left(\tau + \frac{\tau}{R(\theta^*)}\right) = \tau.$$

The objective (7) in the program that solves for the optimal decentralized fiscal rule, on the other hand, reduces to the first two lines of (19) when utility is log. Unlike centralized fiscal rules, decentralized fiscal rules do not internalize the effects of rules on the interest rate, and so they ignore the third line in (19).

As described in Lemma 2, the difference between the optimal centralized fiscal rule and the optimal decentralized fiscal rule can be expressed as a function of the redistributive and disciplining effects of the interest rate. Solving for these rules under log preferences yields that the sum of these two effects is

$$\rho + \lambda = \frac{1 - \delta R(\theta_c^*)}{R(\theta_c^*)(1 + R(\theta_c^*))}. \tag{20}$$

Equation (20) shows that the redistributive effect of the interest rate dominates

the disciplining effect if and only if $R(\theta_c^*) < 1/\delta$. Intuitively, the redistributive effect is stronger on the margin when interest rates are low. When the interest rate declines, all types shift spending from the second to the first period, implying that their marginal utility-weighted primary surpluses in the second period increase and, as implied by (13), ρ increases. The relevant threshold for $R(\theta_c^*)$ depends on the discount factor because a reduction in δ has a similar effect as a reduction in $R(\theta_c^*)$: all types shift spending to the present when δ declines.

Under some technical conditions (described in Proposition 4 below), we can show that the relationship between the interest rate in the optimal decentralized fiscal rule and the discount factor is monotonic in the present bias parameter β. This implies that the relationship between the disciplining and redistributive effects of the interest rate, and hence that between the optimal centralized and decentralized fiscal rules, is also monotonic in β. Define

$$H(\theta^*) = \int_{\underline{\theta}}^{\theta^*} \frac{\delta}{\theta \frac{\mathbb{E}[\theta|\theta \geq \theta^*]}{\theta^*} + \delta} f(\theta)\, d\theta + \int_{\theta^*}^{\overline{\theta}} \frac{\delta}{\mathbb{E}[\theta|\theta \geq \theta^*] + \delta} f(\theta)\, d\theta, \tag{21}$$

which is equal to the aggregate savings rate in the decentralized optimum when $\theta^* = \theta_d^*$. We obtain:

Proposition 4. *Suppose (i) preferences are log; (ii) (10) holds; (iii) θ_c^* is single-valued, and (iv) the value of $\theta^* \in (\underline{\theta}, \overline{\theta})$ for which $H(\theta^*) = \delta/(1+\delta)$ is unique. There exists a unique $\beta^* \in (\underline{\theta}, 1)$ such that $\theta_c^* = \theta_d^*$ if $\beta = \beta^*$. If $\beta < \beta^*$, the optimal centralized fiscal rule provides more flexibility than the optimal decentralized fiscal rule: $\theta_c^* > \theta_d^*$. If $\beta > \beta^*$, the opposite is true: $\theta_c^* < \theta_d^*$.*

This result extends the insights of Proposition 2 and Proposition 3. Whether the optimal centralized fiscal rule provides more or less flexibility than the optimal decentralized fiscal rule is fully pinned down by governments' present bias. Under the conditions of Proposition 4, the relationship is characterized by a threshold $\beta^* \in (\underline{\theta}, 1)$ such that the centralized rule is tighter than the decentralized rule if β is above this threshold and slacker if β is below the threshold.

The conditions in Proposition 4 guarantee that both θ_d^* and θ_c^* are continuous functions of β. By Proposition 2 and Proposition 3, it follows that there exists

a value of β for which $\theta_c^* = \theta_d^*$, so that the redistributive and disciplining effects of the interest rate perfectly outweigh each other. Now our discussion of (20) implies $R(\theta_c^*) = 1/\delta$ when $\theta_c^* = \theta_d^*$, and given the conditions in Proposition 4, we show that there exists a unique value of β for which this is the case.

Figure 1 illustrates with four examples, using different distribution functions $F(\theta)$.[19] Panels A, B, and C consider the log-normal, exponential, and uniform distributions, respectively. In these three examples, θ_d^* and θ_c^* are single-valued and increasing functions of β, so fiscal rules provide more discretion as governments' present bias declines. Moreover, $H(\theta^*)$ satisfies condition (iv) in Proposition 4, and thus all the conditions of the proposition hold. Consequently, in all these examples, there exists a threshold $\beta^* \in (\underline{\theta}, 1)$ such that the optimal centralized fiscal rule is tighter than the optimal decentralized fiscal rule if $\beta > \beta^*$ and slacker if $\beta < \beta^*$.

Panel D in Figure 1 considers a mixture of a Beta distribution and a uniform distribution for which the assumptions of Proposition 4 are violated. The figure shows that the relative tightness of centralized and decentralized fiscal rules is non-monotonic in the present bias parameter β in this case. There is an intermediate range of β such that the optimal centralized rule is tighter than the optimal decentralized rule for low values of β in this range and slacker for high values of β in this range. On the other hand, as implied by Proposition 2 and Proposition 3, the centralized rule is tighter than the decentralized rule for β sufficiently high, and the opposite is true for β sufficiently low.

4 Surplus Limits and Money Burning

Our results in Section 3 show that if governments' present bias is large enough, the optimal centralized fiscal rule provides more flexibility than the optimal decentralized fiscal rule. By increasing flexibility, the centralized rule induces some government types to spend and borrow more, which increases the interest rate and therefore leads other types to spend and borrow less. A natural question in light of this result is whether other mechanisms can also increase the interest rate to achieve the same effect, and in particular whether it can be optimal to

[19] As required by our model, we consider bounded distributions; see the figure for details.

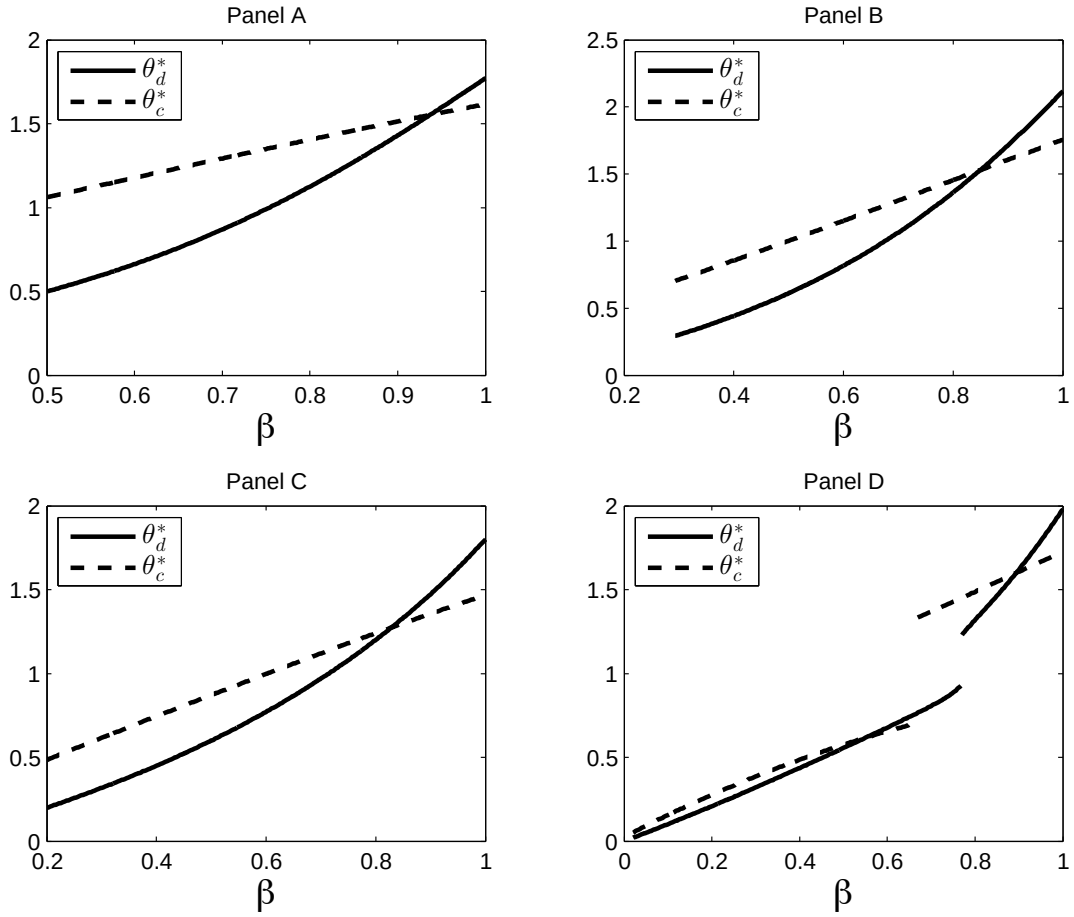

Figure 1: Optimal decentralized and centralized fiscal rules as a function of governments' present bias. Panel A uses a log-normal distribution with mean -0.596 and variance 0.75, truncated below and above the 20th and 80th percentiles. Panel B uses an exponential distribution with parameter 1.314, also truncated below and above the 20th and 80th percentiles. Panel C uses a uniform distribution on $[0.2, 1.8]$. Panel D uses $f(\theta) = f_\beta(\theta) + 2f_U(\theta)$ for $\theta \in [0.02, 1.98]$, where f_β is the pdf of a Beta distribution with parameters $(50, 50)$ and f_U is the pdf of a uniform distribution on $[0, 2]$. In all four examples, our choice of parameters ensures $\mathbb{E}[\theta] = 1$. We set $\delta = 1$. (Note that under log preferences, θ_d^* and θ_c^* are independent of the value of τ.)

force some types to spend more. Specifically, can the use of maximum surplus limits, in addition to maximum deficit limits, enhance welfare? Furthermore, can it be socially beneficial to impose losses on high-surplus countries in the form of money burning?

To address the first question, consider fiscal rules consisting of a maximum deficit limit and a maximum surplus limit. Such a rule specifies two cutoffs, $\theta^* \in [\underline{\theta}, \overline{\theta}]$ and $\theta^{**} \in [\underline{\theta}, \theta^*)$, such that: for types $\theta < \theta^{**}$, the first-period and second-period spending levels are $g^f(\theta^{**}, R)$ and $x^f(\theta^{**}, R)$; for types $\theta \in [\theta^*, \theta^{**}]$, the spending levels are $g^f(\theta, R)$ and $x^f(\theta, R)$; and for types $\theta > \theta^*$, the spending levels are $g^f(\theta^*, R)$ and $x^f(\theta^*, R)$. That is, only types $\theta \in [\theta^{**}, \theta^*]$ have full discretion; all other types are constrained by the fiscal rule and thus choose spending either at the maximum deficit limit — therefore spending less than in their flexible optimum — or at the maximum surplus limit — therefore spending more than in their flexible optimum.

It is immediate that an optimal decentralized fiscal rule always sets $\theta_d^{**} = \underline{\theta}$, so the government is not constrained by a maximum surplus limit. For an individual government that takes the interest rate as fixed, the only effect of setting a surplus limit is to force low types to borrow more. Since these types are already overborrowing relative to the ex-ante optimum in the absence of a surplus limit, a binding limit can only reduce the country's expected welfare.

Things are different, however, under a centralized fiscal rule. This rule takes into account not only the direct effect of surplus limits of increasing borrowing by low types, but also the indirect effect that operates through the interest rate.

Proposition 5. *Consider fiscal rules consisting of a maximum deficit limit and a maximum surplus limit, specified by cutoffs $\theta^* \in [\underline{\theta}, \overline{\theta}]$ and $\theta^{**} \in [\underline{\theta}, \theta^*)$ respectively. There exist $\left(U(\cdot), F(\theta), \tau, \delta\right)$ and a threshold $\underline{\beta} \in (\underline{\theta}, 1)$ such that if $\beta \leq \underline{\beta}$, the optimal centralized fiscal rule specifies a strictly higher maximum deficit limit and a strictly higher maximum surplus limit than the optimal decentralized fiscal rule: $\theta_c^* > \theta_d^*$ and $\theta_c^{**} > \theta_d^{**} = \underline{\theta}$.*

As noted above, maximum surplus limits force low types which are overborrowing to borrow even more. However, these limits also serve to increase interest rates, and through this channel they can improve fiscal discipline. Proposition 5 shows that the disciplining effect of the interest rate can more than compensate

for the distortions caused by the increased overborrowing by low types, and therefore the use of maximum surplus limits can increase social welfare.

Formally, by combining the first-order condition for θ_c^* given in (12) with the analog of that condition for θ_c^{**}, we obtain that if θ_c^* and θ_c^{**} are interior, then

$$\theta_c^{**}U'\left(g^f\left(\theta_c^{**},R\right)\right)\left(\frac{\mathbb{E}\left[\theta|\theta\leq\theta_c^{**}\right]}{\theta_c^{**}}-\frac{1}{\beta}\right)=\theta_c^{*}U'\left(g^f\left(\theta_c^{*},R\right)\right)\left(\frac{\mathbb{E}\left[\theta|\theta\geq\theta_c^{*}\right]}{\theta_c^{*}}-\frac{1}{\beta}\right).$$

The left-hand side is the cost of the marginal distortions due to overborrowing by low types. The right-hand side is the cost of the marginal distortions due to overborrowing by high types. The optimal centralized fiscal rule specifies $(\theta_c^*,\theta_c^{**})$ to equalize these costs. Thus, if governments' present bias is sufficiently large, committing to overborrowing by low types can boost welfare by increasing the interest rate and reducing overborrowing by high types.

Figure 2 considers the examples of Figure 1 but allowing for maximum surplus limits in addition to maximum deficit limits. The figure depicts the cutoff θ_c^{**} in the optimal centralized fiscal rule as a function of the present bias parameter β.[20] In Panels A, B, and C, $\theta_c^{**}>\underline{\theta}$ for a range of values of β, and thus limiting countries' surpluses enhances ex-ante welfare. The values of β for which this holds satisfy $\beta<\beta^*$, so the disciplining effect of the interest rate dominates the redistributive effect. As β increases, the redistributive effect becomes more important, so increasing interest rates becomes less beneficial and surplus limits are no longer optimal. Panel D also features $\theta_c^{**}>\underline{\theta}$ for a range of values of β, although here this range is not an interval. This occurs because the relative strength of the disciplining and redistributive effects is non-monotonic in β, as explained in our discussion of Panel D in Figure 1.[21]

We next address the question of whether introducing money burning can be socially beneficial. Consider a fiscal rule consisting of a maximum deficit limit and a maximum surplus limit, associated with cutoffs $\theta^*\in[\underline{\theta},\overline{\theta}]$ and $\theta^{**}\in[\underline{\theta},\theta^*)$, as above, but where now governments can exceed the maximum surplus limit by paying a fee, $\phi\tau>0$. Naturally, given θ^* and θ^{**}, the case in which a maximum

[20] The cutoff θ_c^* is also computed but is not displayed for expositional convenience.

[21] One may be tempted to conclude from Figure 2 that the use of maximum surplus limits always increases social welfare for low enough values of β. However, it can be shown that this is not the case; i.e. there are parametric examples for which $\theta_c^{**}=\underline{\theta}$ for all $\beta\in[\underline{\theta},1]$.

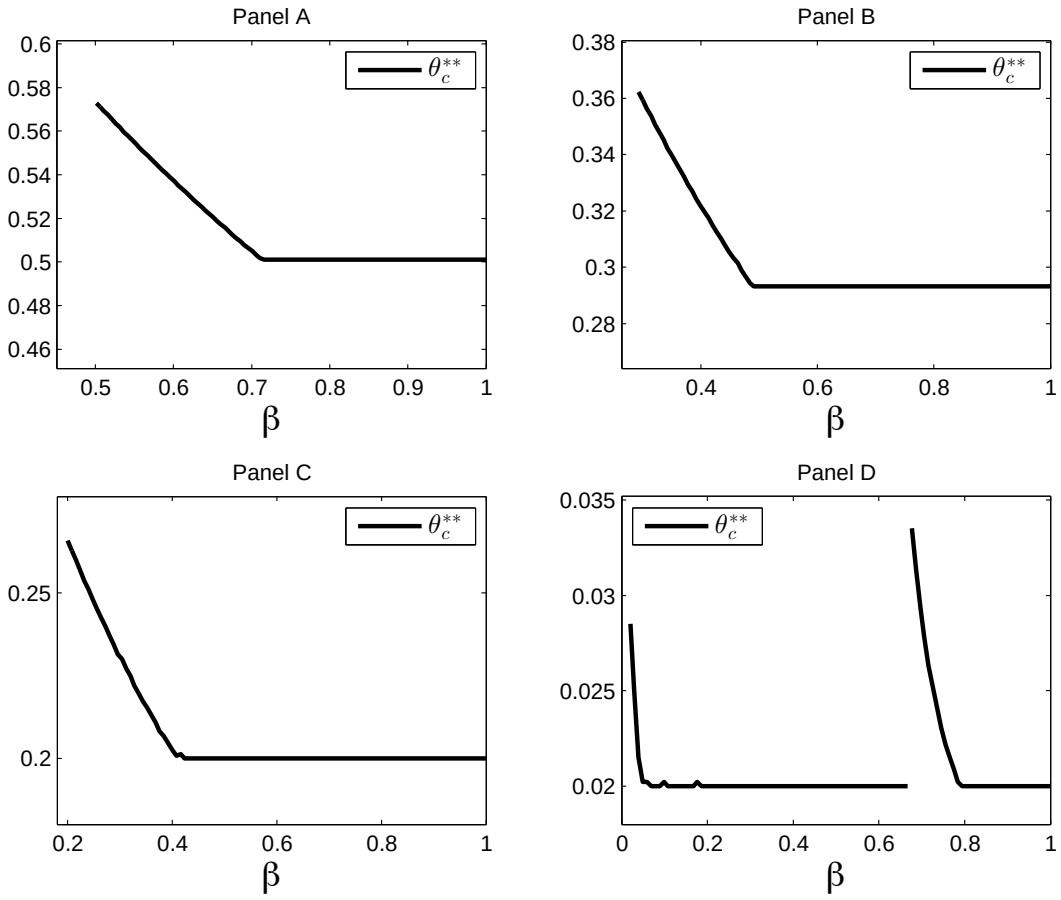

Figure 2: Maximum surplus limits in the optimal centralized fiscal rule. The parametric examples are the same as in Figure 1.

surplus limit is not imposed corresponds to one in which $\phi = 0$, whereas the case in which no type is allowed to save above the maximum surplus limit corresponds to one in which ϕ is prohibitively large. More generally, for intermediate values of ϕ, the fiscal rule is such that for some $\theta^{***} \in (\underline{\theta}, \theta^{**})$, all types $\theta \in [\underline{\theta}, \theta^{***}]$ choose to pay the fee and have spending at their flexible optimum, $g^f(\theta, R)$ and $x^f(\theta, R)$; all other types pay no fee, and their spending levels are $g^f(\theta^{**}, R)$ and $x^f(\theta^{**}, R)$ for $\theta \in (\theta^{***}, \theta^{**})$, $g^f(\theta, R)$ and $x^f(\theta, R)$ for $\theta \in [\theta^{**}, \theta^*]$, and $g^f(\theta^*, R)$ and $x^f(\theta^*, R)$ for $\theta > \theta^*$. This fiscal rule therefore gives full discretion to types $\theta \in [\underline{\theta}, \theta^{***}]$ and types $\theta \in [\theta^{**}, \theta^*]$; however, the former must pay $\phi\tau$ to obtain this flexibility. Can this rule enhance welfare, even when the fees are a pure resource cost and not transferred to other parties? The answer is yes:[22]

Proposition 6. *There exist* $\left(U(\cdot), F(\theta), \tau, \delta, \beta\right)$ *and a centralized fiscal rule featuring money burning such that total expected welfare is strictly larger under this rule than under any fiscal rule setting a maximum deficit limit and a maximum surplus limit without money burning.*

The intuition is related to that behind Proposition 5. Forcing some government types to overborrow can be socially beneficial because it allows to increase interest rates and improve overall fiscal discipline. However, a maximum surplus limit imposes a particularly large distortion on the lowest types, whose savings are forced to be far below the efficient level. By introducing fees, the fiscal rule allows these types to save more, albeit at a cost, and as a result it can induce high interest rates while limiting distortions. Proposition 6 shows that the gain in efficiency can more than compensate for the loss due to money burning.

To increase welfare relative to an environment in which only maximum deficit limits and maximum surplus limits are allowed, countries therefore can move to an institutional setting in which creditors that save beyond a certain level must incur losses which are a pure resource cost. One way to interpret this money burning is as arising from output losses that affect the entire global system, including debtor countries, where transfers from creditors to debtors guarantee that the bulk of the losses are born by creditor countries.

[22] We prove Proposition 6 by providing an example in which using a fiscal rule with money burning as described above enhances welfare under centralization, whereas *any* form of money burning is suboptimal under decentralization.

The results of this section show that there may exist mechanisms that improve upon maximum deficit limits under centralization. Solving for the optimal centralized mechanism in full generality, however, is difficult, as the problem is not convex when the interest rate is endogenous. This is in contrast to the decentralized problem, which, as shown in Amador, Werning and Angeletos (2006), can be ensured to be convex under weak conditions.

5 Interaction between Rules

Our analysis so far has considered two extreme cases: either all countries choose fiscal rules independently, or a central authority chooses a fiscal rule that applies to all countries. However, as discussed in the Introduction, reality may be in between these two extremes. Examples like that of the European Union and Germany suggest that even when a group of countries agree on a common rule, some of these countries may be able (and want) to adopt additional fiscal constraints. We investigate this possibility in this section. We study fiscal rules that consist of a maximum deficit limit, as in Section 2 and Section 3.

Consider a centralized fiscal rule θ_c^* and, to fix ideas, assume that this rule is implemented with a spending limit $g_c^* \equiv g^f(\theta_c^*, R(\theta_c^*))$. Suppose that a fraction $\psi \in (0,1)$ of governments can individually impose a different rule on themselves. Because the centralized rule is fully enforceable, governments cannot implement a cutoff $\theta^* > \theta_c^*$; that is, all countries must respect the spending limit g_c^*. However, some governments may be able to commit to a cutoff $\theta^* < \theta_c^*$, thus restricting themselves to lower spending in the first period than allowed by the central authority. Enforcing these additional fiscal constraints requires strong institutions; we are interested in the case in which only a fraction $\psi \in (0,1)$ of countries have the necessary institutional environment to set $\theta^* < \theta_c^*$.

If governments' present bias is small enough, the possibility of supplementing the centralized rule with additional fiscal constraints is irrelevant: by Proposition 2, individual governments prefer slacker constraints than those optimally imposed by the central authority. If governments' present bias is sufficiently large, on the other hand, Proposition 3 implies that governments would want to impose stricter rules on themselves than imposed centrally. In this case,

the fraction ψ of governments which have the ability to implement additional constraints would choose to do so, and they would implement their optimal decentralized fiscal rule, $\theta_d^* < \theta_c^*$. What is the impact on the world economy? How would the central authority respond?

To answer these questions, assume $U(g) = \log(g)$ as in Subsection 3.4. An analogous argument to that in Lemma 1 implies that when a fraction ψ of governments adopt tighter fiscal rules, the interest rate declines. If the centralized fiscal rule is unchanged, with a spending limit of g_c^*, then as shown by (17), the lower interest rate would lead other governments (whose rules have not changed) to spend and borrow more. That is, by imposing more discipline on themselves, the fraction ψ of governments would worsen fiscal discipline for everyone else.

In response to this, however, the central authority would optimally change the spending limit g_c^*. Given log preferences, we can solve the central authority's problem as in (19), taking into account that now a fraction ψ of governments choose a fiscal rule θ_d^* (where, recall, θ_d^* is given by equation (9)) and the interest rate is $R(\theta_c^*, \theta_d^*)$, a function of both θ_c^* and θ_d^*. We obtain:

Proposition 7. *Suppose that conditions (i)-(iv) in Proposition 4 hold and a fraction $\psi \in (0,1)$ of governments can set $\theta_d^* < \theta_c^*$. If $\beta < \beta^*$, then θ_c^* is strictly decreasing in ψ.*

When governments' present bias is large, an inefficiency arises if some governments can adopt tighter fiscal rules than those imposed centrally. These tighter rules depress global interest rates, thus reducing fiscal discipline for the rest of the governments. Moreover, the redistributive effect of the interest rate becomes more powerful as the interest rate declines (as discussed in Subsection 3.4), and hence the optimal response of the central authority is to tighten restrictions for all governments. Therefore, if a subset of countries can impose greater fiscal restrictions on themselves, all countries face lower interest rates and less flexibility as a consequence.

Figure 3 displays the optimal centralized cutoff θ_c^* and spending limit g_c^* as a function of ψ. Panels A, B, and C are analogous in their parameterization to Panels A, B, and C in Figure 1. The figure shows that as the fraction ψ of countries that can implement tighter individual rules increases, both θ_c^* and g_c^* decline, so the rule imposed by the central authority also becomes tighter.

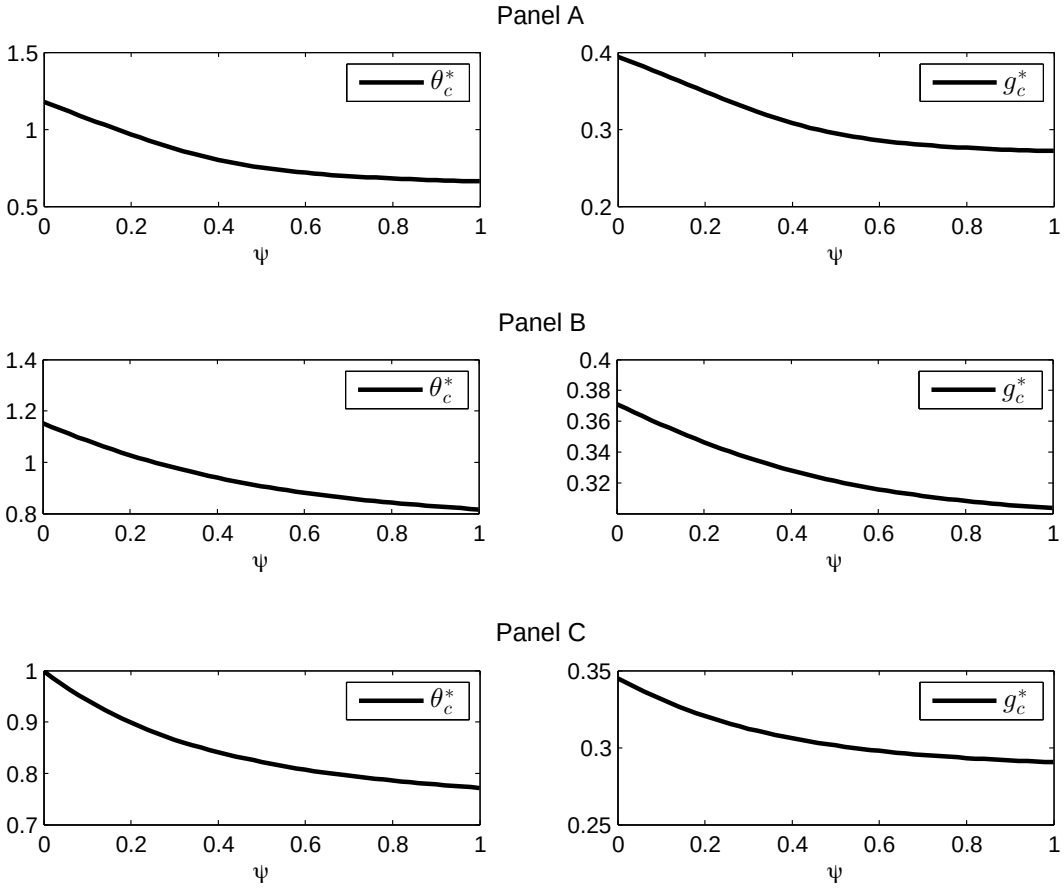

Figure 3: Optimal centralized fiscal rule and associated spending limit when a fraction ψ of governments choose their optimal decentralized fiscal rule. The parametric examples are the same as in Panels A, B, and C in Figure 1. We set $\beta = 0.6$.

6 Conclusion

This paper presented a theoretical framework to compare centralized and decentralized fiscal rules. We established that whether the optimal centralized fiscal rule is more or less constraining than the optimal decentralized fiscal rule depends on governments' present bias. In particular, if the present bias is large, a central authority optimally imposes slacker fiscal constraints than those chosen by individual governments: by increasing flexibility, the centralized rule leads to a higher interest rate, which naturally increases fiscal discipline in all countries. We showed that centrally imposed maximum surplus limits can boost welfare by increasing interest rates further and harnessing the power of their disciplining effect; moreover, introducing money burning can be socially beneficial. Finally, we studied the inefficiencies that emerge when some countries — like Germany in the case of the European Union — can adopt fiscal constraints to supplement those imposed centrally.

Although our focus has been on fiscal policy, our analysis applies more generally to any group of households, firms, or countries that face a tradeoff between commitment and flexibility. For instance, households choose forced savings plans as a means to commit to not overspend; firms impose investment rules on themselves to prevent over-expansion; and countries set environmental quotas to limit pollution. These parties face a commitment-versus-flexibility tradeoff, as they also value having discretion to respond to possible contingencies. Furthermore, in all these circumstances, the price of the temptation good — the interest rate for households, the price of investment goods for firms, and the price of polluting materials for countries — is endogenous to the rules that parties choose. Specifically, the more flexible are the rules, the higher is the price of the temptation good. As such, an ex-ante commitment to flexibility, while not necessarily privately beneficial for the parties involved, can allow to increase overall discipline and thus lead to higher social welfare.

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A Appendix

This Appendix contains all the proofs except for those of Proposition 4–Proposition 7, which are provided in the Online Appendix.

A.1 Proof of Lemma 1

Consider a fiscal rule θ^* applying to all governments. Type θ's first-period spending is $g^f\left(\theta^*, R\left(\theta^*\right)\right)$ if $\theta > \theta^*$ and $g^f\left(\theta, R\left(\theta^*\right)\right)$ if $\theta \leq \theta^*$. Substituting into the first-period global resource constraint given in (4) yields

$$\int_{\underline{\theta}}^{\theta^*} g^f\left(\theta, R\left(\theta^*\right)\right) f\left(\theta\right) d\theta + \int_{\theta^*}^{\overline{\theta}} g^f\left(\theta^*, R\left(\theta^*\right)\right) f\left(\theta\right) d\theta = \tau. \tag{22}$$

Differentiating this equation with respect to θ^*, we obtain

$$R'\left(\theta^*\right) = \frac{-\left(1 - F\left(\theta^*\right)\right) \dfrac{\partial g^f\left(\theta^*, R\right)}{\partial \theta^*}}{\displaystyle\int_{\underline{\theta}}^{\theta^*} \frac{dg^f\left(\theta, R\right)}{dR} f\left(\theta\right) d\theta + \int_{\theta^*}^{\overline{\theta}} \frac{dg^f\left(\theta^*, R\right)}{dR} f\left(\theta\right) d\theta}. \tag{23}$$

To determine the sign of $R'(\theta^*)$, note that differentiating (6) with respect to θ gives

$$\begin{aligned} \theta U''\left(g^f\left(\theta, R\left(\theta^*\right)\right)\right) \frac{dg^f\left(\theta, R\left(\theta^*\right)\right)}{d\theta} + U'\left(g^f\left(\theta, R\left(\theta^*\right)\right)\right) \\ = \beta\delta R(\theta^*) U''\left(x^f\left(\theta, R\left(\theta^*\right)\right)\right) \frac{dx^f\left(\theta, R\left(\theta^*\right)\right)}{d\theta}. \end{aligned}$$

Using (1) to substitute for $\frac{dx^f(\theta, R(\theta^*))}{d\theta}$ and (6) to substitute for $\beta\delta$ and rearranging terms, this equation yields

$$\frac{dg^f\left(\theta, R\left(\theta^*\right)\right)}{d\theta} = \frac{1}{\theta} \frac{1}{-\dfrac{U''\left(g^f\left(\theta, R\left(\theta^*\right)\right)\right)}{U'\left(g^f\left(\theta, R\left(\theta^*\right)\right)\right)} - R\left(\theta^*\right) \dfrac{U''\left(x^f\left(\theta, R\left(\theta^*\right)\right)\right)}{U'\left(x^f\left(\theta, R\left(\theta^*\right)\right)\right)}} > 0, \tag{24}$$

where the strict inequality follows from the fact that $U(\cdot)$ is strictly increasing and concave. Equation (24) implies that the numerator on the right-hand side of (23) is strictly negative for $\theta^* < \overline{\theta}$.

To sign the denominator in (23), differentiate (6) with respect to R and

follow similar steps as above to obtain

$$\frac{dg^f\left(\theta, R\left(\theta^*\right)\right)}{dR} = \frac{1}{R\left(\theta^*\right)} \frac{-1}{-\frac{U''\left(g^f\left(\theta, R\left(\theta^*\right)\right)\right)}{U'\left(g^f\left(\theta, R\left(\theta^*\right)\right)\right)} - R\left(\theta^*\right) \frac{U''\left(x^f\left(\theta, R\left(\theta^*\right)\right)\right)}{U'\left(x^f\left(\theta, R\left(\theta^*\right)\right)\right)}} + \frac{1}{R\left(\theta^*\right)} \mu\left(\theta, R\left(\theta^*\right)\right)\left(\tau - g^f\left(\theta, R\left(\theta^*\right)\right)\right), \tag{25}$$

where

$$\mu\left(\theta, R\left(\theta^*\right)\right) \equiv \frac{-R\left(\theta^*\right) \frac{U''\left(x^f\left(\theta, R\left(\theta^*\right)\right)\right)}{U'\left(x^f\left(\theta, R\left(\theta^*\right)\right)\right)}}{-\frac{U''\left(g^f\left(\theta, R\left(\theta^*\right)\right)\right)}{U'\left(g^f\left(\theta, R\left(\theta^*\right)\right)\right)} - R\left(\theta^*\right) \frac{U''\left(x^f\left(\theta, R\left(\theta^*\right)\right)\right)}{U'\left(x^f\left(\theta, R\left(\theta^*\right)\right)\right)}}.$$

The first term in (25) is strictly negative whereas the sign of the second term is ambiguous and depends on the sign of $\tau - g^f\left(\theta, R\left(\theta^*\right)\right)$. Since the denominator in (23) is equal to the integral of (25) over θ, this denominator therefore consists of a strictly negative term plus the following term:

$$\frac{1}{R\left(\theta^*\right)} \int_{\underline{\theta}}^{\theta^*} \mu\left(\theta, R\left(\theta^*\right)\right)\left(\tau - g^f\left(\theta, R\left(\theta^*\right)\right)\right) f\left(\theta\right) d\theta$$
$$+ \frac{1}{R\left(\theta^*\right)} \int_{\theta^*}^{\overline{\theta}} \mu\left(\theta^*, R\left(\theta^*\right)\right)\left(\tau - g^f\left(\theta^*, R\left(\theta^*\right)\right)\right) f\left(\theta\right) d\theta. \tag{26}$$

To determine the sign of (26), note that if $\theta'' > \theta'$, then (1) and (24) imply $g^f\left(\theta'', R\left(\theta^*\right)\right) > g^f\left(\theta', R\left(\theta^*\right)\right)$ and $x^f\left(\theta'', R\left(\theta^*\right)\right) < x^f\left(\theta', R\left(\theta^*\right)\right)$. Moreover, by Assumption 1, it follows that

$$-\frac{U''\left(g^f\left(\theta'', R\left(\theta^*\right)\right)\right)}{U'\left(g^f\left(\theta'', R\left(\theta^*\right)\right)\right)} \leq -\frac{U''\left(g^f\left(\theta', R\left(\theta^*\right)\right)\right)}{U'\left(g^f\left(\theta', R\left(\theta^*\right)\right)\right)}, \quad \text{and}$$
$$-\frac{U''\left(x^f\left(\theta'', R\left(\theta^*\right)\right)\right)}{U'\left(x^f\left(\theta'', R\left(\theta^*\right)\right)\right)} \geq -\frac{U''\left(x^f\left(\theta', R\left(\theta^*\right)\right)\right)}{U'\left(x^f\left(\theta', R\left(\theta^*\right)\right)\right)}.$$

Hence, we obtain that if $\theta'' > \theta'$, then $\tau - g^f\left(\theta'', R\left(\theta^*\right)\right) < \tau - g^f\left(\theta', R\left(\theta^*\right)\right)$ and $\mu\left(\theta'', R\left(\theta^*\right)\right) \geq \mu\left(\theta', R\left(\theta^*\right)\right)$. It follows that $\tau - g$ and μ are weakly negatively

correlated, and given (22) the expected value of $\tau - g$ is equal to zero. Therefore, the sign of (26) is weakly negative, implying that the denominator in (23) is strictly negative. Since we had established that the numerator in (23) is also strictly negative for $\theta^* \in (\underline{\theta}, \overline{\theta})$, we obtain $R'(\theta^*) > 0$ for $\theta^* \in (\underline{\theta}, \overline{\theta})$.

A.2 Proof of Proposition 1 and Corollary 1

The proof of Proposition 1 is given in the text. To prove Corollary 1, note that to establish that θ_d^* is strictly increasing in β, it is sufficient to show that the left-hand side of (9) is strictly decreasing in θ_d^*, since the right-hand side is strictly decreasing in β. The derivative of the left-hand side of (9) for a cutoff θ^* is

$$\frac{d\left(\mathbb{E}\left[\theta|\theta \geq \theta^*\right]/\theta^*\right)}{d\theta^*} = \frac{1}{\theta^*}\left(\frac{d\mathbb{E}\left[\theta|\theta \geq \theta^*\right]}{d\theta^*} - \frac{\mathbb{E}\left[\theta|\theta \geq \theta^*\right]}{\theta^*}\right). \tag{27}$$

Condition (10) implies that the right-hand side of (27) is strictly negative.

A.3 Proof of Lemma 2

For an interior solution, the first-order conditions of the program in (11) yield

$$\begin{aligned}
&\frac{\partial g^f\left(\theta_c^*, R\right)}{\partial \theta_c^*} \int_{\theta_c^*}^{\overline{\theta}} \left(\theta U'\left(g^f\left(\theta_c^*, R\right)\right) - \delta R U'\left(x^f\left(\theta_c^*, R\right)\right)\right) f\left(\theta\right) d\theta \\
&+ R'\left(\theta_c^*\right) \begin{pmatrix} \int_{\underline{\theta}}^{\theta_c^*} \delta U'\left(x^f\left(\theta, R\right)\right)\left(\tau - g^f\left(\theta, R\right)\right) f\left(\theta\right) d\theta \\ + \int_{\theta_c^*}^{\overline{\theta}} \delta U'\left(x^f\left(\theta_c^*, R\right)\right)\left(\tau - g^f\left(\theta_c^*, R\right)\right) f\left(\theta\right) d\theta \end{pmatrix} \\
&- R'\left(\theta_c^*\right) \begin{pmatrix} \int_{\underline{\theta}}^{\theta_c^*} \left(\delta R U'\left(x^f\left(\theta, R\right)\right) - \theta U'\left(g^f\left(\theta, R\right)\right)\right) \frac{dg^f(\theta,R)}{dR} f\left(\theta\right) d\theta + \\ \int_{\theta_c^*}^{\overline{\theta}} \left(\delta R U'\left(x^f\left(\theta_c^*, R\right)\right) - \theta U'\left(g^f\left(\theta_c^*, R\right)\right)\right) \frac{dg^f(\theta_c^*,R)}{dR} f\left(\theta\right) d\theta \end{pmatrix} = 0.
\end{aligned}$$

Substitution of (1) and (6) and simple algebraic manipulations yield (12)−(14).

A.4 Proof of Proposition 2

Take $\beta = 1$. By Proposition 1, the cutoff in the optimal decentralized fiscal rule is $\theta_d^* = \overline{\theta}$. Now consider the cutoff in the optimal centralized fiscal rule, θ_c^*. By Lemma 1, $R'\left(\theta^*\right) > 0$ for $\theta^* \in (\underline{\theta}, \overline{\theta})$, implying that there is no loss of generality

in maximizing (11) with respect to the interest rate. Given $\beta = 1$, first-order conditions yield

$$\frac{\partial g^f(\theta_c^*, R)}{\partial \theta_c^*} \frac{1}{R'(\theta_c^*)} \int_{\theta_c^*}^{\overline{\theta}} \left(\theta U'\left(g^f(\theta_c^*, R)\right) - \delta R U'\left(x^f(\theta_c^*, R)\right)\right) f(\theta)\, d\theta$$
$$+ \begin{pmatrix} \int_{\underline{\theta}}^{\theta_c^*} U'\left(x^f(\theta, R)\right)\left(\tau - g^f(\theta, R)\right) f(\theta)\, d\theta \\ + \int_{\theta_c^*}^{\overline{\theta}} U'\left(x^f(\theta_c^*, R)\right)\left(\tau - g^f(\theta_c^*, R)\right) f(\theta)\, d\theta \end{pmatrix} \geq 0, \tag{28}$$

which holds with equality if θ_c^* is interior. Now suppose $\theta_c^* = \overline{\theta}$. Note that using (23), we can rewrite the first term on the left-hand side of (28) as

$$-\left(\int_{\underline{\theta}}^{\theta_c^*} \frac{dg^f(\theta, R(\theta_c^*))}{dR} f(\theta)\, d\theta + \int_{\theta_c^*}^{\overline{\theta}} \frac{dg^f(\theta^*, R(\theta_c^*))}{dR} f(\theta)\, d\theta\right)$$
$$\times \mathbb{E}\left[\theta U'\left(g^f(\theta_c^*, R)\right) - \delta R U'\left(x^f(\theta_c^*, R)\right) | \theta \geq \theta_c^*\right],$$

which is equal to zero at $\theta_c^* = \overline{\theta}$ (as the expectation is equal to zero given $\beta = 1$). To sign the second term on the left-hand side of (28), note that by (1) and (24), $g^f(\theta, R)$ is strictly increasing in θ whereas $x^f(\theta, R)$ is strictly decreasing in θ. This implies that $U'\left(x^f(\theta, R)\right)$ and $\left(\tau - g^f(\theta, R)\right)$ are negatively correlated. Given (22), the expected value of $\tau - g$ is equal to zero; thus, it follows that the second term on the left-hand side of (28) is strictly negative. This implies that if $\theta_c^* = \overline{\theta}$, the left-hand side of (28) is strictly negative, a contradiction. Therefore, we must have $\theta_c^* < \overline{\theta} = \theta_d^*$.

A.5 Proof of Proposition 3 and Corollary 2

Take $\beta = \underline{\theta}$. By Proposition 1, the cutoff in the optimal decentralized fiscal rule is $\theta_d^* = \underline{\theta}$. We will show that the centralized fiscal rule specifies a cutoff $\theta_c^* > \theta_d^*$. To do this, consider a fiscal rule θ^* with associated interest rate $R = R(\theta^*)$. For $\theta \leq \theta^*$, define

$$\eta(\theta, \theta^*) \equiv 1 + \left(\frac{1}{\beta} - 1\right)\left[\frac{\gamma\left(g^f(\theta, R)\right)}{\gamma\left(g^f(\theta, R)\right) + R\gamma\left(x^f(\theta, R)\right)} + \frac{\gamma'\left(g^f(\theta, R)\right) - R^2\gamma'\left(x^f(\theta, R)\right)}{\left(\gamma\left(g^f(\theta, R)\right) + R\gamma\left(x^f(\theta, R)\right)\right)^2}\right] \tag{29}$$

where $\gamma(g) \equiv -U''(g)/U'(g)$.

The following lemma provides a sufficient condition under which $\theta_c^* > \theta_d^*$.

Lemma 3. *Take $\beta = \underline{\theta}$ and suppose that there exists $\theta^* > \underline{\theta}$ such that $\eta(\theta, \theta^*) > 0$ for all $\theta \leq \theta^*$. Then $\theta_c^* > \theta_d^*$.*

Proof. Consider fiscal rule $\theta_d^* = \underline{\theta}$. Using (4), the value of expected welfare under this rule is

$$U(\tau) + \delta U(\tau). \tag{30}$$

Consider now an alternative fiscal rule $\theta^* > \underline{\theta}$. Expected welfare in this case is

$$\begin{aligned} &\int_{\underline{\theta}}^{\theta^*} \left(\theta U\left(g^f\left(\theta, R\left(\theta^*\right)\right)\right) + \delta U\left(x^f\left(\theta, R\left(\theta^*\right)\right)\right)\right) f(\theta)\, d\theta \\ &+ \int_{\theta^*}^{\overline{\theta}} \left(\theta U\left(g^f\left(\theta^*, R\left(\theta^*\right)\right)\right) + \delta U\left(x^f\left(\theta^*, R\left(\theta^*\right)\right)\right)\right) f(\theta)\, d\theta. \end{aligned} \tag{31}$$

If the value of realized welfare for each type θ is strictly convex in θ under the fiscal rule θ^*, then it follows by Jensen's inequality that (31) is strictly greater than (30), and as a result the optimal centralized fiscal rule must specify $\theta_c^* > \underline{\theta} = \theta_d^*$. We show that welfare is indeed strictly convex in θ under a fiscal rule θ^* satisfying the condition in the lemma.

The second derivative of realized welfare for type θ with respect to θ is equal to zero if $\theta \geq \theta^*$. We are therefore left to prove that this second derivative is strictly positive for $\theta < \theta^*$. The first derivative is

$$U\left(g^f(\theta, R)\right) - \theta U'\left(g^f(\theta, R)\right)\left(\frac{1}{\beta} - 1\right)\frac{dg^f(\theta, R)}{d\theta},$$

where we have substituted in (6). The second derivative of realized welfare for type $\theta < \theta^*$ with respect to θ is therefore equal to

$$U'\left(g^f(\theta, R)\right)\frac{dg^f(\theta, R)}{d\theta}\left[1 + \left(\frac{1}{\beta} - 1\right)\left(\begin{array}{c} -\theta\dfrac{U''\left(g^f(\theta, R)\right)}{U'\left(g^f(\theta, R)\right)}\dfrac{dg^f(\theta, R)}{d\theta} \\ -1 - \theta\left(\dfrac{d^2 g^f(\theta, R)}{d\theta^2} \Big/ \dfrac{dg^f(\theta, R)}{d\theta}\right)\end{array}\right)\right]. \tag{32}$$

Note that $U'\left(g^f(\theta, R)\right) > 0$ and $\dfrac{dg^f(\theta, R)}{d\theta} > 0$ by (24). Hence, to show that

(32) is strictly positive, we must show that the term in square brackets is strictly positive. Multiplying both sides of (24) by $\theta\gamma(g^f(\theta,R))$ yields

$$-\theta\frac{U''\left(g^f\left(\theta,R\right)\right)}{U'\left(g^f\left(\theta,R\right)\right)}\frac{dg^f\left(\theta,R\right)}{d\theta}=\frac{\gamma\left(g^f\left(\theta,R\right)\right)}{\gamma\left(g^f\left(\theta,R\right)\right)+R\gamma\left(x^f\left(\theta,R\right)\right)}, \tag{33}$$

and differentiation of (24) implies

$$-\theta\left(\frac{d^2g^f\left(\theta,R\right)}{d\theta^2}\Big/\frac{dg^f\left(\theta,R\right)}{d\theta}\right)=1+\frac{\gamma'\left(g^f\left(\theta,R\right)\right)-R^2\gamma'\left(x^f\left(\theta,R\right)\right)}{\left(\gamma\left(g^f\left(\theta,R\right)\right)+R\gamma\left(x^f\left(\theta,R\right)\right)\right)^2}. \tag{34}$$

Substituting (33) and (34) in (32) shows that (32) is strictly positive if $\eta\left(\theta,\theta^*\right)>0$. Therefore, welfare is strictly convex in θ for $\theta<\theta^*$ under this condition. □

We now establish that the sufficient condition in Lemma 3 holds given $\beta=\underline{\theta}$. Take any $\theta^*\in\left[\underline{\theta},\overline{\theta}\right]$ and suppose that $\eta\left(\theta,\theta^*\right)>0$ for all $\theta\leq\theta^*$. It follows by the continuity of $\eta\left(\theta,\theta^*\right)$ that $\eta\left(\theta,\theta^*+\varepsilon\right)>0$ for all $\theta\leq\theta^*+\varepsilon$, for $\varepsilon>0$ arbitrarily small. Hence, to establish that there exists some $\theta^*>\underline{\theta}$ satisfying the condition in Lemma 3, it is sufficient to verify that $\eta\left(\underline{\theta},\underline{\theta}\right)>0$. Note that by (6), $R\left(\underline{\theta}\right)=\delta^{-1}$, and given $g^f\left(\underline{\theta},\delta^{-1}\right)=x^f\left(\underline{\theta},\delta^{-1}\right)=\tau$, it follows that

$$\begin{aligned}\gamma\left(g^f\left(\underline{\theta},\delta^{-1}\right)\right)&=\gamma\left(x^f\left(\underline{\theta},\delta^{-1}\right)\right)=\gamma\left(\tau\right),\\ \gamma'\left(g^f\left(\underline{\theta},\delta^{-1}\right)\right)&=\gamma'\left(x^f\left(\underline{\theta},\delta^{-1}\right)\right)=\gamma'\left(\tau\right).\end{aligned}$$

Thus,

$$\eta\left(\underline{\theta},\underline{\theta}\right)=1+\left(\frac{1}{\beta}-1\right)\left[\frac{1}{1+\delta^{-1}}+\frac{\gamma'\left(\tau\right)\left(1-\delta^{-2}\right)}{\left[\gamma\left(\tau\right)\left(1+\delta^{-1}\right)\right]^2}\right]>0,$$

where we have used Assumption 1 which guarantees $\gamma'\left(\tau\right)\leq 0$.

The proof of Corollary 2 follows from the arguments above and the condition in the corollary.

B Online Appendix

B.1 Proof of Proposition 4

Given the conditions in Proposition 4, the next lemma characterizes the relationship between the interest rate under the decentralized fiscal rule and the inverse rate of time preference.

Lemma 4. *Suppose conditions (i)-(iv) in Proposition 4 hold. There exists a unique value* $\beta^* \in (\underline{\theta}, 1)$ *such that* $R\left(\theta_d^*\right) = 1/\delta$ *if* $\beta = \beta^*$.

Proof. Under log preferences, $g^f\left(\theta, R(\theta^*)\right)$ and $x^f\left(\theta, R(\theta^*)\right)$ are given by equations (17) and (18). Analogous to (22), substitute $x^f\left(\theta, R(\theta^*)\right)$ into the second-period global resource constraint to obtain

$$\int_{\underline{\theta}}^{\theta^*} \frac{\beta\delta}{\theta + \beta\delta} f\left(\theta\right) d\theta + \int_{\theta^*}^{\overline{\theta}} \frac{\beta\delta}{\theta^* + \beta\delta} f\left(\theta\right) d\theta = \frac{1}{1 + R^*(\theta)}. \tag{35}$$

Under condition (10), we can define a continuous function $\theta_d^*\left(\beta\right)$ specifying the value of the decentralized cutoff θ_d^* as a function of β, as defined in equation (9). Using $\theta_d^*\left(\beta\right)$, equation (35), and the definition of $H\left(\theta^*\right)$ given in (21) yields

$$R\left(\theta_d^*\left(\beta\right)\right) = \frac{1}{H\left(\theta_d^*\left(\beta\right)\right)} - 1. \tag{36}$$

Since $\theta_d^*\left(1\right) = \overline{\theta}$, Jensen's inequality implies

$$H\left(\theta_d^*\left(1\right)\right) = \int_{\underline{\theta}}^{\overline{\theta}} \frac{\delta}{\theta + \delta} f\left(\theta\right) d\theta > \frac{\delta}{1 + \delta}.$$

Given (36), it follows that $R\left(\theta_d^*\left(1\right)\right) < 1/\delta$. Moreover, since $\theta_d^*\left(\underline{\theta}\right) = \underline{\theta}$, we obtain $H\left(\theta_d^*\left(\underline{\theta}\right)\right) = \frac{\delta}{1+\delta}$ and thus $R\left(\theta_d^*\left(\underline{\theta}\right)\right) = 1/\delta$. Finally, note that

$$H'\left(\theta_d^*\left(\underline{\theta}\right)\right) = -\int_{\theta^*}^{\overline{\theta}} \frac{\delta \frac{d\mathbb{E}[\theta|\theta \geq \theta^*]}{d\theta^*}}{\left(\mathbb{E}\left[\theta|\theta \geq \theta^*\right] + \delta\right)^2} f\left(\theta\right) d\theta < 0,$$

implying $R\left(\theta_d^*\left(\underline{\theta}+\varepsilon\right)\right) > 1/\delta$ for $\varepsilon > 0$. Therefore, given the conditions in the lemma, there exists a unique $\beta^* \in (\underline{\theta}, 1)$ satisfying $R\left(\theta_d^*\left(\beta^*\right)\right) = 1/\delta$. □

As noted above, given condition (10), we can define a continuous function $\theta_d^*(\beta)$ specifying the value of the decentralized cutoff θ_d^* given β. Analogously, if the solution to (19) is single-valued, we can define a single-valued function $\theta_c^*(\beta)$ specifying the value of the centralized cutoff θ_c^* given β. By the Theorem of the Maximum, this function is upper hemicontinuous, and since it is single-valued, it is continuous. Proposition 2 and Proposition 3 show that $\theta_c^*(1) < \theta_d^*(1)$ and $\theta_c^*(\underline{\theta}) > \theta_d^*(\underline{\theta})$. Given the continuity of $\theta_c^*(\beta)$ and $\theta_d^*(\beta)$, this implies $\theta_c^*(\beta) = \theta_d^*(\beta)$ for at least one value of $\beta \in (\underline{\theta}, 1)$. We now establish that this value of β is unique. Denote this value by $\beta^* \in (\underline{\theta}, 1)$. As shown in (9), the first-order condition that defines θ_d^* is

$$\frac{\mathbb{E}\left[\theta|\theta \geq \theta_d^*\right]}{\theta_d^*} = \frac{1}{\beta}. \tag{37}$$

Consider now the first-order condition that defines θ_c^*, which is given in (12). Substituting $(\rho + \lambda)$ in (12) with the expression in (20) yields

$$\frac{\mathbb{E}\left(\theta|\theta \geq \theta_c^*\right)}{\theta_c^*} = \frac{1}{\beta} + \frac{R'\left(\theta_c^*\right)}{\left(1 - F\left(\theta_c^*\right)\right)\theta_c^* U'\left(g^f\left(\theta_c^*, R\right)\right)\frac{\partial g^f\left(\theta_c^*, R\right)}{\partial \theta_c^*}}\left(\frac{1 - \delta R}{R\left(1+R\right)}\right). \tag{38}$$

If $\theta_d^* = \theta_c^* \in \left(\underline{\theta}, \overline{\theta}\right)$, then (37) and (38) imply

$$R\left(\theta_d^*\left(\beta\right)\right) = \frac{1}{\delta}. \tag{39}$$

By Lemma 4, the value of β for which this is true is uniquely determined. Therefore, there exists a unique $\beta^* \in (\underline{\theta}, 1)$ such that $\theta_c^*\left(\beta^*\right) = \theta_d^*\left(\beta^*\right)$, and hence $\theta_c^* > \theta_d^*$ if $\beta < \beta^*$ whereas $\theta_c^* < \theta_d^*$ if $\beta > \beta^*$.

B.2 Proof of Proposition 5

The proof of this result is given by the examples reported in Figure 2. We compute θ_c^* and θ_c^{**} by solving the following problem (recall that we consider

log preferences in these examples):

$$\max_{\theta^*,\theta^{**}} \left\{ \begin{array}{l} \int_{\underline{\theta}}^{\theta^{**}} \left(\theta \log \left(\frac{\theta^{**}}{\theta^{**}+\beta\delta} \right) + \delta \log \left(\frac{\beta\delta}{\theta^{**}+\beta\delta} \right) \right) f(\theta)\, d\theta \\ + \int_{\theta^{**}}^{\theta^*} \left(\theta \log \left(\frac{\theta}{\theta+\beta\delta} \right) + \delta \log \left(\frac{\beta\delta}{\theta+\beta\delta} \right) \right) f(\theta)\, d\theta \\ + \int_{\theta^*}^{\overline{\theta}} \left(\theta \log \left(\frac{\theta^*}{\theta^*+\beta\delta} \right) + \delta \log \left(\frac{\beta\delta}{\theta^*+\beta\delta} \right) \right) f(\theta)\, d\theta \\ + (1+\delta) \log \left(\tau + \frac{\tau}{R(\theta^*,\theta^{**})} \right) + \delta \log \left(R(\theta^*,\theta^{**}) \right) \end{array} \right\} \tag{40}$$

subject to

$$\left[\int_{\underline{\theta}}^{\theta^{**}} \frac{\theta^{**}}{\theta^{**}+\beta\delta} f(\theta) d\theta + \int_{\theta^{**}}^{\theta^*} \frac{\theta}{\theta+\beta\delta} f(\theta) d\theta + \int_{\theta^*}^{\overline{\theta}} \frac{\theta^*}{\theta^*+\beta\delta} f(\theta) d\theta \right] \left(\tau + \frac{\tau}{R(\theta^*,\theta^{**})} \right) = \tau.$$

For panels A, B, and C, Figure 2 shows that there exists $\underline{\beta} \in (\underline{\theta}, 1)$ such that $\theta_c^{**} > \underline{\theta}$ if $\beta \leq \underline{\beta}$. By the arguments in the text, it is immediate that $\theta_d^{**} = \underline{\theta}$ always holds. For all the three examples, it can also be verified that $\theta_c^* > \theta_d^*$ for $\beta \leq \underline{\beta}$.

B.3 Proof of Proposition 6

Take $U(g) = \log(g)$. Consider a fiscal rule consisting of a maximum spending limit $g^* = g^f(\theta^*, R(\theta^*))$ and a minimum spending level $g^{**} = g^f(\theta^{**}, R(\theta^{**}))$ for the first period, associated with cutoffs $\theta^* \in [\underline{\theta}, \overline{\theta}]$ and $\theta^{**} \in [\underline{\theta}, \theta^*)$. The rule also sets $\phi > 0$ such that a government must pay a fee $\phi\tau$ if it chooses spending $g < g^{**}$ in the first period. This fee is a pure resource cost.

It is immediate that if a type θ^{***} is indifferent between paying the fee $\phi\tau$ and choosing spending $g < g^{**}$, and not paying the fee and choosing spending

$g \geq g^{**}$, then it must be that $\theta^{***} < \theta^{**}$ and

$$\theta^{***}\left[\log\left(\frac{\theta^{***}}{\theta^{***}+\beta\delta}\right)-\log\left(\frac{\theta^{**}}{\theta^{**}+\beta\delta}\right)\right]+\beta\delta\left[\log\left(\frac{\beta\delta}{\theta^{***}+\beta\delta}\right)-\log\left(\frac{\beta\delta}{\theta^{**}+\beta\delta}\right)\right]$$
$$=(\theta^{***}+\beta\delta)\left[\log\left(\tau+\frac{\tau}{R(\theta^{*},\theta^{**},\theta^{***})}\right)-\log\left(\tau+\frac{\tau}{R(\theta^{*},\theta^{**},\theta^{***})}-\phi\tau\right)\right]. \quad (41)$$

Moreover, all types $\theta < \theta^{***}$ prefer to pay the fee $\phi\tau$ and choose $g < g^{**}$, whereas all types $\theta > \theta^{***}$ prefer not to pay the fee and choose $g \geq g^{**}$. The fiscal rule therefore induces money burning in equilibrium if and only if there exists such a type $\theta^{***} > \underline{\theta}$. In this case, total welfare is given by

$$\begin{aligned}
W(\theta^{*},\theta^{**},\theta^{***}) = &\int_{\underline{\theta}}^{\theta^{***}}\left(\theta\log\left(\frac{\theta}{\theta+\beta\delta}\right)+\delta\log\left(\frac{\beta\delta}{\theta+\beta\delta}\right)\right)f(\theta)\,d\theta\\
&+\int_{\theta^{***}}^{\theta^{**}}\left(\theta\log\left(\frac{\theta^{**}}{\theta^{**}+\beta\delta}\right)+\delta\log\left(\frac{\beta\delta}{\theta^{**}+\beta\delta}\right)\right)f(\theta)\,d\theta\\
&+\int_{\theta^{**}}^{\theta^{*}}\left(\theta\log\left(\frac{\theta}{\theta+\beta\delta}\right)+\delta\log\left(\frac{\beta\delta}{\theta+\beta\delta}\right)\right)f(\theta)\,d\theta\\
&+\int_{\theta^{*}}^{\overline{\theta}}\left(\theta\log\left(\frac{\theta^{*}}{\theta^{*}+\beta\delta}\right)+\delta\log\left(\frac{\beta\delta}{\theta^{*}+\beta\delta}\right)\right)f(\theta)\,d\theta\\
&+\int_{\underline{\theta}}^{\theta^{***}}(\theta+\delta)\log\left(\tau+\frac{\tau}{R(\theta^{*},\theta^{**},\theta^{***})}-\phi\tau\right)f(\theta)\,d\theta\\
&+\int_{\theta^{***}}^{\overline{\theta}}(\theta+\delta)\log\left(\tau+\frac{\tau}{R(\theta^{*},\theta^{**},\theta^{***})}\right)f(\theta)\,d\theta+\delta\log\left(R(\theta^{*},\theta^{**},\theta^{***})\right),
\end{aligned}$$

and the global resource constraint is

$$\begin{bmatrix}\phi\tau F(\theta^{***})+\int_{\underline{\theta}}^{\theta^{***}}\frac{\theta}{\theta+\beta\delta}\left(\tau+\frac{\tau}{R(\theta^{*},\theta^{**},\theta^{***})}-\phi\tau\right)f(\theta)d\theta\\ +\begin{bmatrix}\int_{\theta^{***}}^{\theta^{**}}\frac{\theta^{**}}{\theta^{**}+\beta\delta}f(\theta)d\theta+\int_{\theta^{**}}^{\theta^{*}}\frac{\theta}{\theta+\beta\delta}f(\theta)d\theta\\ +\int_{\theta^{*}}^{\overline{\theta}}\frac{\theta^{*}}{\theta^{*}+\beta\delta}f(\theta)d\theta\end{bmatrix}\left(\tau+\frac{\tau}{R(\theta^{*},\theta^{**},\theta^{***})}\right)\end{bmatrix}=\tau. \quad (42)$$

We prove the result by example. Figure 4 depicts the value of θ^{***} that

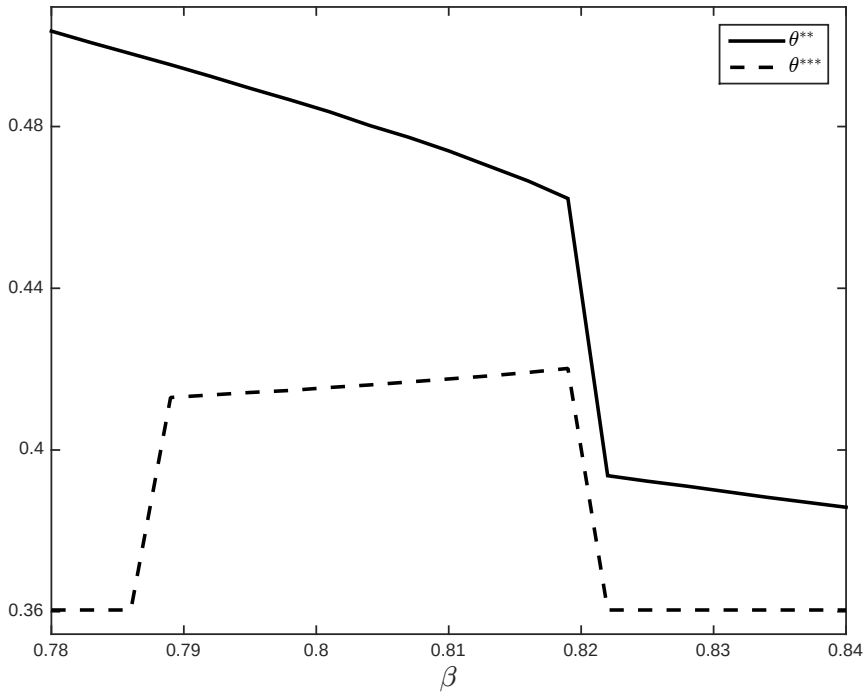

Figure 4: Money burning in the centralized fiscal rule. The figure uses $f(\theta) = 0.9999 f_L(\theta - z) + 0.0001 f_U(\theta)$, where f_L is the pdf of a log-normal distribution with mean -0.316 and variance 0.55, truncated above the 90th percentile, and with a shift z equal to the 10th percentile of the original distribution. f_U is the pdf of a uniform distribution with the same support as the shifted log-normal. The choice of parameters ensures $\mathbb{E}[\theta] = 1$. We set $\delta = 1$.

maximizes $W(\theta^*, \theta^{**}, \theta^{***})$ subject to (41) and (42) for the distributional example described in the figure.[23] The values of θ^* and θ^{**} are fixed to those that solve the problem in (40), where by assumption $\theta^{***} = \underline{\theta}$ and there is no money burning. The figure shows that given such values of θ^* and θ^{**}, the solution has $\theta^{***} > \underline{\theta}$. This implies that maximizing $W(\theta^*, \theta^{**}, \theta^{***})$ over $(\theta^*, \theta^{**}, \theta^{***})$ subject to (41) and (42) also yields $\theta^{***} > \underline{\theta}$, and hence the optimal centralized fiscal rule features money burning.

B.4 Proof of Proposition 7

Consider the program that solves for the optimal centralized fiscal rule under log preferences, given in (19). Taking now into account that a fraction ψ of governments can reduce their flexibility by setting $\theta_d^* < \theta_c^*$, the program becomes:

$$\max_{\theta^*} \left\{ \begin{array}{c} (1-\psi) \left[\begin{array}{c} \int_{\underline{\theta}}^{\theta^*} \left(\theta \log\left(\frac{\theta}{\theta+\beta\delta}\right) + \delta \log\left(\frac{\beta\delta}{\theta+\beta\delta}\right) \right) f(\theta)\, d\theta \\ + \int_{\theta^*}^{\overline{\theta}} \left(\theta \log\left(\frac{\theta^*}{\theta^*+\beta\delta}\right) + \delta \log\left(\frac{\beta\delta}{\theta^*+\beta\delta}\right) \right) f(\theta)\, d\theta \end{array} \right] \\ \psi \left[\begin{array}{c} \int_{\underline{\theta}}^{\theta_d^*} \left(\theta \log\left(\frac{\theta}{\theta+\beta\delta}\right) + \delta \log\left(\frac{\beta\delta}{\theta+\beta\delta}\right) \right) f(\theta)\, d\theta \\ + \int_{\theta_d^*}^{\overline{\theta}} \left(\theta \log\left(\frac{\theta_d^*}{\theta_d^*+\beta\delta}\right) + \delta \log\left(\frac{\beta\delta}{\theta_d^*+\beta\delta}\right) \right) f(\theta)\, d\theta \end{array} \right] \\ +(1+\delta) \log\left(\tau + \frac{\tau}{R(\theta^*, \theta_d^*)}\right) + \delta \log\left(R(\theta^*, \theta_d^*)\right) \end{array} \right\} \tag{43}$$

subject to (4) and (9).

The first-order condition is

$$(1-\psi) \int_{\theta_c^*}^{\overline{\theta}} \left(\frac{\theta}{\theta_c^*} - \frac{\theta+\delta}{\theta_c^*+\beta\delta} \right) f(\theta)\, d\theta - \left(\frac{1-\delta R}{R(1+R)} \right) \frac{dR(\theta_c^*, \theta_d^*)}{d\theta_c^*} = 0. \tag{44}$$

[23] This example satisfies Assumption A in Amador, Werning and Angeletos (2006).

Analogous steps as in the proof of Lemma 1 imply that, given $U(g) = \log(g)$,

$$R(\theta_c^*, \theta_d^*) = \frac{1}{S(\theta_c^*, \theta_d^*)} - 1, \tag{45}$$

where

$$\begin{aligned} S(\theta_c^*, \theta_d^*) &= (1-\psi)\left(\int_{\underline{\theta}}^{\theta_c^*} \frac{\beta\delta}{\theta+\beta\delta} f(\theta)\, d\theta + \int_{\theta_c^*}^{\overline{\theta}} \frac{\beta\delta}{\theta_c^*+\beta\delta} f(\theta)\, d\theta\right) \\ &\quad +\psi\left(\int_{\underline{\theta}}^{\theta_d^*} \frac{\beta\delta}{\theta+\beta\delta} f(\theta)\, d\theta + \int_{\theta_d^*}^{\overline{\theta}} \frac{\beta\delta}{\theta_d^*+\beta\delta} f(\theta)\, d\theta\right). \end{aligned} \tag{46}$$

Therefore, we obtain

$$\frac{dR(\theta_c^*, \theta_d^*)}{d\theta_c^*} = \frac{(1-\psi)\left(\int_{\theta_c^*}^{\overline{\theta}} \frac{\beta\delta}{(\theta_c^*+\beta\delta)^2} f(\theta)\, d\theta\right)}{\left[S(\theta_c^*, \theta_c^*)\right]^2}. \tag{47}$$

Substituting (45), (46), and (47) into (44) yields

$$\int_{\theta_c^*}^{\overline{\theta}} \left(\frac{\theta}{\theta_c^*} - \frac{\theta+\delta}{\theta_c^*+\beta\delta}\right) f(\theta) + \left(\frac{\delta}{S(\theta_c^*, \theta_d^*)} - \frac{1}{1-S(\theta_c^*, \theta_d^*)}\right) \int_{\theta_c^*}^{\overline{\theta}} \frac{\beta\delta}{(\theta_c^*+\beta\delta)^2} f(\theta)\, d\theta = 0. \tag{48}$$

Since θ_c^* is single valued, we can determine its comparative statics with respect to ψ by implicit differentiation of (48). Let $K(\theta_c^*, \psi)$ correspond to the left-hand side of (48); then

$$\frac{d\theta_c^*}{d\psi} = -\frac{\frac{dK(\theta_c^*, \psi)}{d\psi}}{\frac{dK(\theta_c^*, \psi)}{d\theta_c^*}}.$$

Note that the objective in (43) is concave at the optimum (since θ_c^* is uniquely determined), and thus $\frac{dK(\theta_c^*, \psi)}{d\theta_c^*} < 0$. We are therefore left to show that $\frac{dK(\theta_c^*, \psi)}{d\psi} <$

0 to complete the proof. This can be established by showing that

$$\frac{\delta}{S\left(\theta_c^*, \theta_d^*\right)} - \frac{1}{1 - S\left(\theta_c^*, \theta_d^*\right)}$$

is decreasing in ψ. Equation (46) implies that this is true as long as $\theta_c^* > \theta_d^*$. To show that $\theta_c^* > \theta_d^*$ given $\beta < \beta^*$ and $\psi \in (0, 1)$, consider $\psi = 0$. Proposition 4 implies $\theta_c^* > \theta_d^*$ in this case. As ψ increases, it is clear that θ_d^* remains the same whereas θ_c^* declines. Now suppose that $\theta_c^* = \theta_d^*$ at some point with $\psi < 1$. From (45) and (48), this would imply $\delta R\left(\theta_c^*, \theta_d^*\right) = 1$. However, since $\beta < \beta^*$, the same arguments as those used in the proof of Proposition 4 together with (45) and (48) imply $R\left(\theta_c^*, \theta_d^*\right) < 1/\delta$ for $\theta_c^* = \theta_d^*$, yielding a contradiction. The claim follows.