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James D. Hamilton
Ethan S. Harris
Jan Hatzius
Kenneth D. West

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ABSTRACT

We examine the behavior, determinants, and implications of the equilibrium level of the real federal funds rate, defined as the rate consistent with full employment and stable inflation in the medium term. We draw three main conclusions. First, the uncertainty around the equilibrium rate is large, and its relationship with trend GDP growth much more tenuous than widely believed. Our narrative and econometric analysis using cross-country data and going back to the 19th Century supports a wide range of plausible central estimates for the current level of the equilibrium rate, from a little over 0% to the pre-crisis consensus of 2%. Second, despite this uncertainty, we are skeptical of the “secular stagnation” view that the equilibrium rate will remain near zero for many years to come. The evidence for secular stagnation before the 2008 crisis is weak, and the disappointing post-2008 recovery is better explained by protracted but ultimately temporary headwinds from the housing supply overhang, household and bank deleveraging, and fiscal retrenchment. Once these headwinds had abated by early 2014, US growth did in fact accelerate to a pace well above potential. Third, the uncertainty around the equilibrium rate implies that a monetary policy rule with more inertia than implied by standard versions of the Taylor rule could be associated with smaller deviations of output and inflation from the Fed’s objectives. Our simulations using the Fed staff’s FRB/US model show that explicit recognition of this uncertainty results in a later but steeper normalization path for the funds rate compared with the median “dot” in the FOMC’s Summary of Economic Projections.

James D. Hamilton
Department of Economics, 0508
University of California, San Diego
9500 Gilman Drive
La Jolla, CA 92093-0508
and NBER
jhamilton@ucsd.edu

Ethan S. Harris
Bank of America
ethan.harris@baml.com

Jan Hatzius
Goldman Sachs
200 West Street
New York, NY 10282-2198
jan.hatzius@gs.com

Kenneth D. West
Department of Economics
University of Wisconsin
1180 Observatory Drive
Madison, WI 53706
and NBER
kdwest@wisc.edu

1. Introduction

What is the steady-state value of the real federal funds rate? Is there a new neutral, with a low equilibrium value for the foreseeable future?

By the beginning of 2015, a consensus seemed to be building that the answer to the second question is yes. Starting in 2012 FOMC members have been releasing their own estimates of the "longer run" nominal rate in the now somewhat infamous "dot plot." As Exhibit 1.1 shows, the longer run projection for PCE inflation has remained steady at 2.0%, but longer run projections for both the GDP and the nominal funds rate projections have dropped 25 bp. The implied equilibrium real rate has fallen from 2.0% to 1.75% and the current range among members extends from 1.25 to 2.25%. Indeed, going back to January 2012, the first FOMC projections for the longer run funds rate had a median of 4.25%, suggesting an equilibrium real rate of 2.25%. Forecasters at the CBO, OMB, Social Security Administration and other longer term official forecasts show a similar cut in the assumed equilibrium rate, typically from 2% to 1.5%.

The consensus outside official circles points to an even lower equilibrium rate. A hot topic of discussion in the past year or so is whether the U.S. has drifted into "secular stagnation," a period of chronically low equilibrium rates due to a persistent weak demand for capital, rising propensity to save and lower trend growth in the economy (see Summers (2013b,2014)). A similar view holds that there is a "new neutral" for the funds rate of close to zero in real terms (see McCulley (2003) and Clarida (2014)). The markets seem to agree. As of February 2015, the bond market was pricing in a peak nominal funds rate of less than 2½% (see Misra (2015)).

The view that the equilibrium rate is related to trend growth is long standing. For example, in Taylor's (1993) seminal paper the equilibrium rate—the real funds rate consistent with full employment and stable inflation—was assumed to be 2%. Why 2%? Because it was "close to the assumed steady state growth rate of 2.2%" which, as Taylor noted at the time, was the average growth rate from 1984:1 to 1992:3. Perhaps the best known paper to formally estimate a time-varying equilibrium rate is Laubach and Williams (2003), which makes trend growth the central determinant of the equilibrium rate.

A tight link between the equilibrium rate and growth is common in theoretical models. The Ramsey model relates the safe real rate to a representative consumer's discount factor and expected consumption growth. So, too, does the baseline New Keynesian model, whose generalization is central to much policy and academic work. Thus these familiar models tie the equilibrium rate to the trend rate of growth in consumption and thus the economy. In those models, shifts in trend growth will shift the equilibrium rate. In more elaborate models, shifts in the level of uncertainty or other model forces can also shift the equilibrium rate. Empirical estimates of the New Keynesian models such as Barsky et al. (2014) and Curdia et al. (2014) find considerable variation in the natural rate of interest.

In other words, the equilibrium rate may be time varying. Such time variation is very important for much of the discussion of current monetary policy.

In this paper, we address the question of a “new neutral” by examining the experience from a large number of countries, though focusing on the U.S. In Section 2 we describe the data and procedures that we will use to construct the ex-ante real rates used in our analysis. These go back as far as two centuries for some countries, and also include more detailed data on the more recent experience of OECD economies. We also note the strategy we often use to make empirical statements about the equilibrium rate: for the most part we will look to averages or moving averages of our measures of real rates; at no point will we estimate a structural model.

Section 3 summarizes and interprets some of the existing theoretical and empirical work and highlights the theoretical basis for anticipating a relation between the equilibrium real rate and the trend growth rate. In this and the next section, we look to moving averages as (noisy) measures of the equilibrium rate and the trend growth rate. Using both long time-series observations for the United States as well as the experience across OECD countries since 1970, we investigate the relation between safe real rates and trend output growth. We uncover some evidence that higher trend growth rates are associated with higher average real rates. However, that finding is sensitive to the particular sample of data that is used. And even for the samples with a positive relation, the correlation between growth and average rates is modest. We conclude that factors in addition to changes in the trend growth rate are central to explaining why the equilibrium real rate changes over time.

In Section 4 we provide a narrative history of determinants of the real rate in the U.S. trying to identify the main factors that may have moved the equilibrium rate over time. We conclude that changes over time in personal discount rates, financial regulation, trends in inflation, bubbles and cyclical headwinds have had important effects on the real rate observed on average over any given decade. We discuss the secular stagnation hypothesis in detail. On balance, we find it unpersuasive, arguing that it probably confuses a delayed recovery with chronically weak aggregate demand. Our analysis suggests that the current cycle could be similar to the last two, with a delayed “normalization” of both the economy and the funds rate. Our narrative approach suggests the equilibrium rate may have fallen, but probably only slightly. Presumptively lower trend growth implies an equilibrium rate below the 2% average that has recently prevailed, perhaps somewhere in the 1% to 2% range.

In Section 5 we perform some statistical analysis of the long-run U.S. data and find, consistent with our narrative history as well as with empirical results found by other researchers in postwar datasets, that we can reject the hypothesis that the real interest rate converges over time to some fixed constant. We do find a relation that appears to be stable. The U.S. real rate is cointegrated with a measure that is similar to the median of a 30-year-average of real rates around the world. When the U.S. rate is below that long-run world rate (as it is as of the beginning of 2015), we could have some confidence that the U.S. rate is going to rise, consistent with the conclusion from our narrative analysis in Section 4. The model forecasts the U.S. and world long-run real rate settling down at a value around a half a percent within about three years. However, because the world rate itself is also nonstationary with no clear tendency to revert to a fixed mean, the uncertainty associated with this forecast grows larger the farther we try to look into the future.

Indeed, the confidence interval two years ahead is wide, from 1 to 2 percentage points wide depending how far out one forecasts. This confidence interval only partially overlaps with Section 4's narrative range of 1%-2%. Both ranges include the FOMC forecast implied by the numbers in Exhibit 1.1. We do not attempt to formally reconcile our two ranges. Rather, we conclude that the U.S. real rate will rise but that it is very hard for anyone to predict what the average value might turn out to be over the next decade.

More generally, the picture that emerges from our analysis is that the determinants of the equilibrium rate are manifold and time varying. We are skeptical of analysis that puts growth of actual or potential output at the center of real interest rate determination. The link with growth is weak. Historically, that link seems to have been buried by effects from factors listed above such as regulation and bubbles. We conclude from both formal and descriptive analysis that reasonable forecasts for the equilibrium rate will come with large confidence intervals.

We close the paper in Section 6 by considering the implications of uncertainty about the equilibrium rate for the conduct of monetary policy. Orphanides and Williams (2002, 2006, 2007) have noted that if the Fed does not have a good estimate of what the equilibrium real rate should be, it may be better able to achieve its objectives by putting more inertia into its decisions than otherwise. We use simulations of the FRB/US model to gauge the relevance of this concern in the current setting. We evaluate a range of policies using an objective function that has often been applied for this kind of analysis, and consider how greater uncertainty about the equilibrium rate affects policy performance. Our results suggest that relative to the "shallow glide path" for the funds rate that has featured prominently in recent Fed communications, when there is greater uncertainty about the equilibrium rate, a policy of raising rates later but—provided the recovery does gather pace and inflation picks up—somewhat more steeply, may deliver a higher value of the objective function.

To conclude, the evidence suggests to us that the secular stagnationists are overly pessimistic. We think the long-run equilibrium U.S. real interest rate remains significantly positive, and forecasts that the real rate will remain stuck at or below zero for the next decade appear unwarranted. But we find little basis in the data for stating with confidence exactly what the value of the equilibrium real rate is going to be. In this respect our conclusion shares some common ground with the stagnationists. When the equilibrium real rate is not known, a policy of initially raising rates more slowly achieves a higher value for the objective function in our simulations compared to a policy that incorrectly assumes that the equilibrium real rate is known with certainty.

2. The real interest rate across countries and across time

Our focus is on the behavior of the real interest rate, defined as the nominal short-term policy rate minus expected inflation. The latter is of course not measured directly, and we follow the common approach in the literature of inferring expected inflation from the forecast of an autoregressive model fit to inflation. However, we differ from most previous studies in that we allow the coefficients of our inflation-forecasting relations to vary over time. We will be making use of both a very long annual data set going back up to two centuries as well as a quarterly data set available for more recent data. The countries we will be examining are listed in Exhibit 2.1. In this section we describe these data and our estimates of real interest rates.

2A. A very long-run annual data set

Our long-run analysis is based on annual data going as far back as 1800 for 17 different countries. Where available we used the discount rate set by the central bank as of the end of each year. For the Bank of England this gives us a series going all the way back to 1801, while for the U.S. we spliced together values for commercial paper rates over 1857-1913, the Federal Reserve discount rate over 1914-1953, and the average fed funds rate during the last month of the year from 1954 to present.[1] Our interest rate series for these two countries are plotted in the top row of Exhibit 2.2 and for 15 other countries in the panels of Exhibit 2.3.[2] The U.S. nominal rate shows a broad tendency to decline through World War II, rise sharply until 1980, and decline again since. The same broad trends are also seen in most other countries. However, there are also dramatic differences across countries as well, such as the sharp spike in rates in Finland and Germany following World War I.

We also assembled estimates of the overall price level for each country. For the U.S., we felt the best measure for recent data is the GDP deflator which is available since 1929. We used an estimate of consumer prices for earlier U.S. data and all other countries. The annual inflation rates are plotted in the second row of Exhibit 2.2 for the U.S. and U.K. and for 15 other countries in the panels of Exhibit 2.4. There is no clear trend in inflation for any country prior to World War I, suggesting that the downward trend in nominal rates prior to that should be interpreted as a downward trend in the real rate. Inflation rose sharply in most countries after both world wars, with hyperinflations in Germany and Finland following World War I and Japan and Italy after World War II. But the postwar spike in inflation was in every case much bigger than the rise in nominal interest rates.

How much of the variation in inflation would have been reasonable to anticipate ex ante? Barsky (1987) argued that U.S. inflation was much less predictable in the 19th century than it became later in the 20th century. Consider for example using a first-order autoregression to predict the inflation rate in country *n* for year *t*:

$$\pi_{nt} = c_n + \phi_n \pi_{n,t-1} + \varepsilon_{nt} \qquad (2.1)$$

To allow for variation over time in inflation persistence, we estimated equation (2.1) by ordinary least squares using a sample of thirty years of data ending in each year *T*. The resulting estimates of the

[1] Values of the 3 separate U.S. series are very close to each other at the dates at which they were spliced together.
[2] Our data set is largely identical to Hatzius et. al (2014) and mainly comes from the Global Financial Data Inc. database, supplemented with information from Haver Analytics. In most cases, the short-term interest rate series is a central bank discount rate (known as bank rate in UK parlance) or an overnight cash or repo rate. When more than one series is used for the same country because of changes over time in definitions and market structure, we splice the series using the discount rate as the basis.

persistence of inflation for country *n* in year *T*, $\hat{\phi}_{nT}$, are plotted as a function of *T* for the U.S. and U.K. in row 3 of Exhibit 2.2 and for other countries in the panels of Exhibit 2.5. There is indeed little persistence in realized inflation for most countries during the 19th century, implying that changes in the nominal rate should be viewed as changes in the ex-ante real rate. However, by World War I there is a fair amount of persistence in most countries, suggesting that at least some degree of postwar inflation should have been anticipated at the time. People knew there had been a war and that last year there had been significant inflation. To maintain that they nevertheless anticipated stable prices for the following year in such a setting seems an unlikely hypothesis.

In the last row of Exhibit 2.2 and the panels in Exhibit 2.6 we plot the value for the ex-ante real interest rate that is implied by the above forecasting model, that is, we plot

$$r_{nt} = i_{nt} - \hat{c}_{nt} - \hat{\phi}_{nt}\pi_{n,t} \tag{2.2}$$

where $\hat{c}_{nt}$ and $\hat{\phi}_{nt}$ are the estimated intercept and slope for a regression estimated using 30 years of data for that country ending at date *t*.[3] These suggest that ex-ante real rates were typically higher in the 19th century than they have been over the last half century. For example, a real rate above 4% was fairly often observed in the United States prior to 1900 but has been much less common since 1960. There also are strongly negative real rates for almost all countries during both world wars, as well as negative real rates over the last few years. Although one could arrive at different estimates of the ex-ante real rate using a different specification of expected inflation, the above broad conclusions are fundamentally tied to what we see in the raw interest rate and inflation data and would be unlikely to be changed under any reasonable specification of inflation expectations.

2B. Postwar quarterly data

We will also be making use of more recent, higher frequency data. For the U.S. we use the average fed funds rate over the last month of the quarter for the measure of the policy rate (available since 1954:3) and 400 times the log difference of the GDP deflator (available since 1947:1) as our series for inflation. For other countries we use the short-term interest rate (generally 3-month LIBOR or Eurocurrency rates) and the GDP deflator, as reported by the IMF World Economic Outlook and the OECD Economic Outlook database. Sample periods for which our constructed real rates are available vary across countries, as indicated in column (4) in Exhibit 2.1. For all countries but the U.S., the quarterly data end in 2013:2.

For quarterly data we replaced the forecasting equation (2.1) with a fourth-order autoregression:

$$\pi_{nt} = c_n + \phi_{n1}\pi_{n,t-1} + \phi_{n2}\pi_{n,t-2} + \phi_{n3}\pi_{n,t-3} + \phi_{n4}\pi_{n,t-4} + \varepsilon_{nt}. \tag{2.3}$$

[3] Note that the vertical scales are different for different countries.

Note that using four quarterly lags in (2.3) corresponds to the single lag in (2.1) using annual data—in each case the forecast is based on what was observed over the previous year. Because of the limited sample we begin the estimation using only 40 observations and then let the number of observations grow until we get to 80. For example, our first price-level observation for the U.S. is the value of the GDP deflator for 1947:1. Our first available estimate of expected inflation, $E_{1958:1}\pi_{US,1958:2}$, thus comes from the coefficients estimated on a sample estimated for *t* = 1948:2 to 1958:1, from which we get the 1958:1 real interest rate from $r_{US,1958:1} = i_{US,1958:1} - E_{1958:1}\pi_{US,1958:2}$. We then add one more observation (without dropping the initial data point) to infer the 1958:2 real interest rate using a sample of 41 observations and the 1958:3 real rate using 42 observations. Once we get past 1968:1, we start to drop the observation at the start of the previous sample so that each estimate from then on uses a 20-year sample.[4]

Our series for the real interest rate constructed from annual and quarterly U.S. data align quite closely (see Exhibit 2.7). We also see from Exhibit 2.8 that our quarterly series for expected inflation aligns quite well with the subsequent realized inflation, with a correlation of 0.95. Exhibit 2.9 plots the postwar U.S. series for nominal and real interest rates.

Exhibit 2.10 presents some summary statistics for the U.S. The use of rolling regressions means that one could in principle have rather different means for inflation and expected inflation; in fact the two are quite similar. Use of rolling regressions also means that expected inflation need not be less variable than actual inflation. But our series for expected inflation is indeed less variable.

2C. Real rate vs. equilibrium rate

We close with a note on terminology**.** A prominent monetary policy maker (Ferguson (2004, p2)) once complained about the multiplicity of terms for the equilibrium real interest rate:

> Economists famously cannot agree on much. In this case, we cannot even agree on the name of the benchmark concept that I have just described. The real interest rate consistent with the eventual full utilization of resources has been called the equilibrium real federal funds rate, the natural rate of interest, and the neutral real rate. I prefer the first name, the equilibrium real federal funds rate, because, by using the word 'equilibrium', it reminds us that it is a concept related to the clearing of markets.

We follow Ferguson and use "equilibrium." As well, we substitute "safe rate" or "policy rate" for "federal funds rate" when we reference data from outside the U.S. or from distant dates in the U.S. To

[4] In preliminary work, we also experimented with keeping the window size fixed at 40 quarters through the whole sample. This led to a very similar series; the correlation was 0.98 between the expected inflation series with a 40 quarter window and the equation (2.3) version that we actually used.

state the obvious, the equilibrium real federal funds rate is distinct from the equilibrium real or nominal rate of return on business capital, on equities, on long term government debt, or on short or long term consumer or corporate debt, though of course those returns are related to the equilibrium real federal funds rate.

This notion of an equilibrium real rate is a rate that is consistent, on average, with output at potential and stable inflation. Of course, over the cycle, there may be time variation in the rate that sets output at potential or inflation at target. In much of our discussion we will be looking at averages over a cycle or longer moving averages as giving us one measure of the equilibrium rate, while acknowledging that such empirical constructs are at best a noisy indicator of the theoretical construct.

3. The real rate and aggregate growth

What could account for the dramatic changes over time in real rates seen in the long term data in Exhibits 2.2 and 2.6 or the shorter recent sample in Exhibit 2.7? Much scholarly and blog discussion has tied interest rates to growth in output or potential output. This is central to the much cited paper by Laubach and Williams (2003). It is also central to discussions of secular stagnation. Gordon (2012, 2014) has argued that the trend rate of growth will be lower, which, given a presumed link between real rates and growth, suggests lower real rates. Summers (2013a) argues that in the near term, interest rates might have to be negative if output is to be at potential.

This section considers the link between the real rate and aggregate growth. In section 3A we review a standard theoretical reason for the real rate to be tied to consumption and output growth. In section 3B, we review existing evidence suggesting that, historically, the link between the real rate and consumption growth is weak. We then present new evidence of a weak link to output growth using US (section 3C)[5] and cross-country (section 3D) data. Finally, section 3E summarizes the empirical results in sections 3C and 3D.

3A. Growth and the real rate of interest in the New Keynesian model

A basic building block in macro models used in scholarly and policy work is one that links real interest rates with consumption. We do not exploit that relationship in our quantitative work. But we do think it necessary to both motivate the relationship and, in the next section 3B, explain why we did not think it productive to make such a relationship a key part of our empirical work. We do so in the context of the basic New Keynesian model, in part so that we can also briefly link the equilibrium rate that is our focus to the natural rate of New Keynesian models.

[5] Using a different approach, Leduc and Rudebusch (2014) also conclude the link in the U.S. is weak.

In New Keynesian models, the basic building block referenced in the previous paragraph is a dynamic IS equation that relates the intertemporal marginal rate of substitution in consumption to the real interest rate. We exposit this relationship in its simplest and very familiar form.

The dynamic IS equation is a formal statement of the following condition: the consumer cannot expect to be made better off by consuming one fewer unit this period, investing in the nominally safe asset, and consuming the proceeds next period. To exposit this textbook relationship, let

$i_t - \pi_{t+1}$ = ex-post real return on a nominally safe asset, (3.1)

i_t = safe nominal rate, π_{t+1} = realized inflation,

$\pi_{t+1} = \ln(P_{t+1}/P_t)$, P_t = price level;

$r_t \equiv i_t - E_t\pi_{t+1}$ = ex-ante real rate, with E_t denoting conditional expectation

C_t = consumption in period t,

$c_t = \ln(C_t)$;

β = consumer's per period discount factor = $\frac{1}{1+\delta}$,

(e.g., δ=0.04 and β=.96 if data are yearly);

$U(C_t)$ = per period utility;

σ_p^2, σ_c^2, σ_{pc} = conditional variances of inflation and of consumption growth, and conditional covariance between inflation and of consumption growth, assumed constant; for example, $\sigma_p^2 = E_t(\pi_{t+1} - E_t\pi_{t+1})^2$.

For the moment, let utility be isoelastic,

$$U(C_t) = C_t^{1-\alpha}/(1-\alpha),\ \alpha>0. \tag{3.2}$$

Then after a second order loglinearization (or conditional lognormality of inflation and consumption growth)

$$r_t \equiv i_t - E_t\pi_{t+1} \approx \delta + \alpha E_t\Delta c_{t+1} - 0.5(\sigma_p^2 + \alpha^2\sigma_c^2 + 2\alpha\sigma_{pc}). \tag{3.3}$$

Write this as

$$r_t \approx \rho + \alpha E_t\Delta c_{t+1},$$
$$\rho = \delta - 0.5(\sigma_p^2 + \alpha^2\sigma_c^2 + 2\alpha\sigma_{pc}). \tag{3.4}$$

This intertemporal condition ties the ex-ante short rate to expected consumption growth each period: Higher expected consumption growth is associated with higher real rates.

To formally tie consumption growth to output and potential output, we follow Galí (2008, ch. 3), modulo the fact that we have second order terms in our definition of ρ and he does not.

Rearrange (3.4) so that c_t is on the left. Using the definition of r_t,

$$c_t = E_t c_{t+1} - \frac{1}{\alpha}(i_t - E_t \pi_{t+1} - \rho). \tag{3.5}$$

Next, in the baseline New Keynesian model,

$$\text{consumption} = \text{output}, \tag{3.6}$$

and in all New Keynesian models, baseline or not, output can deviate from the flexible price equilibrium. Let

$$y_t^n = \text{potential output} = \text{flexible price output}, \tag{3.7}$$

$$y_t = c_t - y_t^n = \text{output gap} = \text{deviation from flexible price equilibrium.}$$

Then (3.5) can be written

$$y_t = E_t y_{t+1} - \frac{1}{\alpha}[i_t - E_t \pi_{t+1} - r_t^n], \tag{3.8}$$

$$r_t^n \equiv \rho + \alpha E_t \Delta y_{t+1}^n = \text{natural rate of interest.} \tag{3.9}$$

Equation (2) in Laubach and Williams (2003) corresponds to our equation (3.9), with a shock added on by Laubach and Williams.

The natural rate of interest has normative properties; it may be desirable for the Fed to set the expected short rate to the natural rate (see Galí (2008)). But the empirical counterpart is model dependent (see below).

If the steady state, or average, value of the output gap is zero, then in this baseline model the average value of the real interest rate (3.4) and the natural rate (3.9) are the same. But once one departs from the baseline model there may no longer be a simple connection between (1) growth of actual or potential output and (2) the real rate or the natural rate of interest. Expression (3.9) was derived assuming that consumption = output. That may be a fine simplification in some contexts but perhaps not here. The theoretical implications if consumption ≠ output are simply stated when the only departure from the baseline model is to allow two kinds of goods, one of which is imported. Then Clarida et al. (2002, p890) conclude that when, as well, $\alpha \neq 1$, the natural rate of interest is a weighted sum of the growth of potential output in (1) the home country, and (2) the rest of the world, with the weight on rest of world proportional to the share of imported goods in consumption.

$$r_t^n \equiv \rho + \omega_1 E_t \Delta y_{t+1}^n + \omega_2 E_t \Delta y_{t+1}^*, \qquad (3.10)$$

where Δy_{t+1}^* is the growth rate of potential in the rest of the world and ω_1 and ω_2 are parameters that depend on the intertemporal elasticity α and the share of imported goods in consumption.

In the U.S., an adjustment of imported goods would likely be quantitatively small. The point is that (3.9) holds only in very special circumstances. Adjustments for other departures, such as fixed capital and wage and price markup shocks, come in various forms, and are quantitatively substantial. See Barsky et al. (2014), for example.

Hence the New Keynesian model does not give a strong a priori reason for a tight short-run relation between the real rate or the natural rate on the one hand and growth of potential or actual output on the other.

3B. Mean consumption growth and the equilibrium rate

The New Keynesian model does, however, provide an a priori reason for a tight link between the real rate and consumption growth, in the form of Equation (3.4): this equation does require that utility be of the form (3.2) but is agnostic about the presence or absence of capital, imports, wage and price shocks, etc. And equation (3.4) has some intuitively appealing implications.

●Higher uncertainty about either inflation or consumption growth (as indexed by the variance terms) lowers the safe real rate. This is consistent with stories about flight to safety.

●The more one discounts the future (higher δ) the higher the safe real rate, which again makes sense—if you are very impatient, a high return is what makes you cut back on consumption today so that you can consume tomorrow.

Unfortunately, a huge literature has documented that (3.4) does not work well empirically. See Kocherlakota (1996) and Mehra and Prescott (2003) for surveys. Given our topic, the most salient failure of the model relates to its implications for the average or equilibrium level of the real rate. The second order terms are small compared to the other terms (see, for example, Table 1 in Kocherlakota (1996)). So for quantitative purposes ignore them for the moment, setting $\rho \approx \delta$. Expressing things at annual rates: average per capita consumption growth is about .02; we generally put annual discount rates at something like δ=0.04. With α=1 (log utility), that implies an average value of the safe rate of

.06—an implausibly high value. Since Weil (1989), the fact that this widely used model implies an implausibly high risk free rate is called the "risk free rate puzzle."

The huge literature referenced in the previous paragraph has examined various solutions to the puzzle. These efforts include among others varying the discount factor δ, varying risk aversion α, varying the utility function, and dropping the representative agent/complete markets paradigm. In New Keynesian models rich enough to be used quantitatively in monetary policy analysis, there usually is a representative agent, the discount factor and risk aversion are generally similar to what is above, but the utility function often incorporates what is called habit persistence.

It is our reading that habit persistence does not deliver a reasonable value for the equilibrium rate, though the evidence is a bit mixed. Habit can be modeled as internal or external. Internal persistence means utility this period depends on consumption this period relative to one's own consumption in the previous period. Internal habit is used in the influential Smets and Wouters (2003) or Christiano et al. (2005) models. External persistence means one's consumption this period is compared instead to aggregate consumption the previous period. External habit appears in papers such as de Paoli and Zabczyk (2013). In either case, let

$$X_t = \text{habit level of consumption}, \tag{3.11}$$

$$U(C_t\text{-}X_t) = (C_t\text{-}X_t)^{1-\alpha}/(1-\alpha),\ \alpha>0.$$

Then X_t varies either with one's own consumption (internal habit) or aggregate consumption (external habit).

Dennis (2009, equations (6), (7), (11) and (12)) supplies the first order analogues to (3.3) when utility is (a) of the form (3.11), or (b) when habit is multiplicative rather than additive. It follows from Dennis's expressions that neither internal nor external habit substantially affects the mean level of the safe rate when parameters are varied within the plausible range. Specifically, for additive habit, such as in (3.11) above, it follows analytically from Dennis's (11) and (12) that variation in habit has no effect on the mean safe rate. For multiplicative habit we have solved numerically for a range of plausible parameters and find habit has little effect on the mean rate. (Dennis's expressions are log linearized around a zero growth steady state. We have derived the log linearization in the presence of nonzero growth in one case (additive external habit), and the conclusion still holds.)

Campbell and Cochrane (1999) let conditional second moments vary over time. They assume that the conditional variance of what they call "surplus consumption" rises as consumption C_t approaches habit X_t. They parameterize this in a way that delivers an equilibrium real rate that is indeed plausibly low on average. The model, however, implies counterfactual relations between nominal and real rates (Canzoneri et al. (2007)).

Hence our review of existing literature leads us to conclude that it is unlikely to be productive to focus on consumption when modeling the real rate, despite the strong theoretical presumption of a link between consumption growth and the real rate. The remaining parts of this section focus on GDP growth instead.

3C. Output growth and the real rate in the U.S.

There are theoretical reasons to expect a long-run relation between the real rate and GDP growth. In a model with balanced growth, consumption will, in the long run, grow at the same rate as output and potential output. Thus the combination of the intertemporal condition (3.4) and balanced growth means that over long periods of time, the average short real rate will be higher when the growth rate of output is higher and lower when output growth is lower. Perhaps there is a clear long-run relationship between output and the real rate, despite the weak evidence of such a relationship between consumption and the real rate. In this section we use our long-run U.S. dataset to investigate the correlation, over business cycles or over 10 year averages, between GDP growth and real rates. Our focus is on the sign of the correlation between average GDP growth and average real rates. We do not attempt to rationalize or interpret magnitudes. We generally refer to "average real rate" rather than equilibrium real rate. But of course our view is that we are taking averages over a long enough period that the average rate will closely track the equilibrium rate.

Real rate data were described in Section 2. We now describe our output data. Our U.S. GDP data runs from 1869 to the present. Balke and Gordon (1989) is the source for 1869-1929, FRED the source for 1929-present. Quarterly dates of business cycle peaks are from NBER. When we analyze annual data, quarterly turning points given by NBER were assigned to calendar years using Zarnowitz (1997, pp732-33). Zarnowitz's work precedes the 2001 and 2007 peaks so we assigned those annual dates ourselves. When, for robustness, we briefly experiment with potential output instead of GDP, the CBO is our source.

As just noted, we focus on the sign of the correlation between average GDP growth and average real rates. We find that this sign is sensitive to sample, changing sign when one or two data points are removed. We did not decide ex-ante which data points to remove. Rather, we inspected plots presented below and noted outliers whose removal might change the sign of the correlation. Ex-post, one might be able to present arguments for focusing on samples that yield a positive correlation, and thus are consistent with the positive relation suggested by theory. But one who does not come to the data with a prior of such a relation could instead conclude that there is little evidence of a positive relation.

Peak to peak results

Peak to peak results are in Exhibits 3.1-3.4. Our baseline set of data points for the peak to peak analysis are the 7 (quarterly) or 29 (annual) pairs of (GDP growth, r) averages presented in Exhibit 3.1. Here is an illustration of how we calculated peak to peak numbers. In our quarterly data, the last two peaks are 2001:1 and 2007:4. Our 2007:4 values are 2.52 for GDP growth and 0.45 for the real interest rate. Here, 2.52 is average GDP growth over the 27 quarters from 2001:2 (that is, beginning with the quarter following the previous peak) through 2007:4, with 0.45 the corresponding value for the real rate.

Let us begin with quarterly data (Exhibit 3.2, and rows (1)-(4) in Exhibit 3.4). A glance at the scatterplot Exhibit 3.2 suggests the following. First, the correlation between average GDP growth and the average real rate is negative, at -0.40 it so happens. (See line (1), column (6) of Exhibit 3.4. That exhibit reports this and other peak-to-peak correlations that we present here in the text.) Second, the negative correlation is driven by 1981:3. If we drop that observation—which, after all, reflects a cycle lasting barely more than a year (1980:2-1981:3), and is sometimes considered part of one long downturn (e.g., Mulligan (2009) and Angry Bear (2009), and our own Exhibit 4.9 below)—the correlation across the remaining six peak to peak averages is indeed positive, at +0.32 (line (2) of Exhibit 3.4)). If we continue to omit the 1981:3 peak, but substitute CBO potential output for GDP (line (3)) or ex-post interest rates for our real rate series (line (4)), the correlation falls to -0.01 or 0.17.

Of course, such sensitivity to sample or data may not surprising when there are only six or seven data points. But that sensitivity remains even when we turn to the much longer time series available with annual data, although the baseline correlation is now positive.

The averages computed from annual data in columns (5) and (6) in Exhibit 3.1 are plotted in Exhibit 3.3. A glance at the scatterplot in that exhibit reveals the positive correlation noted in the previous paragraph, at 0.23 it so happens (line (5) of Exhibit 3.4). That correlation stays positive, with a value of 0.30 (line (6) of Exhibit 3.4) if we drop 1981, the peak found anomalous in the analysis of quarterly data.

However, for annual data, one's eyes are drawn not only to 1981 but also to points such as 1918, 1920, 1944 and 1948. One can guess that the correlation may be sensitive to those points. To illustrate: Let us restore 1981, but remove the postwar 1920 and 1948 peaks, the correlation across the remaining 27 peak to peak averages is now negative, at -0.23 (line (7)). If we instead drop the three peaks that reflect the Great Depression or World War II, the correlation is again positive at 0.29 (line (8)).

The remaining rows of Exhibit 3.4 indicate that the annual data give results congruent with the quarterly data when the sample period is restricted (lines (9) and (10)) and that the annual results are not sensitive to the measure or timing aggregate output (Romer (1989) and year ahead data in lines (11) and (12)).

We defer interpretation of sensitivity until we have also looked at backward moving averages of U.S. data, and cross-country results.

Ten-year averages

We consider 40-quarter (quarterly data) or 10-year (annual data) backwards moving averages. Ten years is an arbitrary window intended to be long enough to average out transient factors and presumably will lead to reasonable alignment between average GDP growth and growth of potential output. Using annual data, we also experimented with a 20-year window, finding results similar to those about to be presented.

Numerical values of correlations are given in column (6) of Exhibit 3.5, with scatterplots presented in Exhibits 3.6 and 3.7. In Exhibit 3.6, the fourth quarter of each year is labeled with the last two digits of the year. We see in Exhibit 3.6 that for quarterly data, the correlation between the 40-quarter averages is positive, at 0.39 it so happens (line (1) in Exhibit 3.5). This is consistent with the quarterly peak-to-peak correlation of 0.32 when 1981:3 is removed (line (2) of Exhibit 3.4)). The result is robust to use of ex-post real rates (line (3)). But, as is obvious from Exhibit 3.6, if we remove the post-2007 points, which trace a path to the southwest, the correlation becomes negative, at -0.19 (line (2)). We see in Exhibit 3.7 that for annual data, the correlation between 10-year averages is negative, at -0.25 it so happens (line (4) in Exhibit 3.5). The postwar sample yields a positive correlation (line (5)). Omitting 1930-1950, so that the Depression years fall out of the sample, turns the correlation positive (line (6)). The value of 0.31 is consistent with 0.29 figure in line (8) of peak-to-peak results in Exhibit 3.4, which also removed Depression and post-World War II years.

3D. Cross-country results

Our GDP data come from the OECD. The source data were real, quarterly and seasonally adjusted. Sample coverage is dictated by our real rate series that were described in Section 2. Our real rate series for all countries had a shorter span than our GDP data. Our longest sample runs from 1971:2-2014:2.

We compute average values of GDP growth and of the real interest rate over samples of increasing size, beginning with roughly one decade (2004:1-2014:2, to be precise) and then move the start date backwards. The sample for averaging increases to approximately two (1994:1-2014:2), then three (1984:1-2014:2), and finally four (1971:2-2014:2) decades. Some countries drop out of the sample as the start of the period for averaging moves back from 2004 to 1971.

Exhibit 3.8 presents the resulting values. Exhibit 3.9 presents scatterplots of the data in Exhibit 3.9. Note that the scale of the 2004:1-2014:2 scatterplot is a little different than that of the other three scatterplots.

As suggested by the scatterplots and confirmed by the numbers presented in the "corr" row of Exhibit 3.8, the correlation between average GDP growth and average real rates is positive in all four samples, and especially so in the 20 year sample. However, the sign of the correlation is sensitive to inclusion of one or two data points. For example, in the 1984-2014 sample, if Australia is omitted, the correlation turns negative.

3E. Summary and interpretation

Both our U.S. and our international data yield a sign for the correlation between average GDP growth and the average real interest rate that is sensitive to sample, with correlations that are numerically small in almost all samples.[6] However, the theoretical presumption that there is a link between aggregate growth and real rates is very strong. One could make an argument to pay more attention to the samples that yield a positive correlation—for example, dropping 1980-81 from the set of full U.S. expansions or dropping 1930-1950 from the 10-year U.S. averages—and deduce that there is modest evidence of a modestly positive relationship between the two. For our purposes, we do not need to finely dice the results to lean either towards or against such an argument. Rather, we have two conclusions. First, if, indeed, we are headed for stagnation for supply side reasons (Gordon (2012, 2014)), any such slowdown should not be counted on to translate to a lower equilibrium rate over periods as short as a cycle or two or a decade. Second, the relation between average output growth and average real rates is so noisy that other factors play a large, indeed dominant, role in determination of average real rates. In the next section we take a narrative approach to sorting out some of these factors.

4. A narrative interpretation of historical real rates

Much of the recent discussion of the equilibrium real rate has relied on a framework similar to the simple one sketched in equation (3.5) above in which the major factor responsible for shifts in the IS curve is changes in the trend growth of the economy. Although this is a very common assumption, we found at best a weak link between trend growth and the equilibrium rate.

More generally, theoretical models suggest trend growth is not the only factor that can shift the equilibrium rate. We noted above that the literature has considered varying the discount factor, the utility function and dropping the representative agent / complete markets paradigm. In connection with the last, we note that much research assumes that the interest rate that governs consumption decisions in equation (3.5) and its generalizations for other utility functions is the risk-free real rate. However, as noted for example by Wieland (2014), in an economy with financial frictions the rate at which households and firms borrow can differ substantially from the risk-free rate. The literature on the monetary transmission mechanism suggests the equilibrium real funds rate will also be sensitive to changes in the way monetary policy is transmitted through long term rates, credit availability, the exchange rate and other asset prices. The equilibrium rate will also be sensitive to sustained changes in regulatory or fiscal policy. Finally the typical models assume that changes in the trend inflation rate have no effect on the real interest rate, an assumption that again turns out to be hard to reconcile with the observed data.

In this section we provide a narrative review of the history of the U.S. real interest rate to call attention to the important role of factors like the ones referenced in the preceding paragraph in determining changes in real rates over time. Since our focus is on the equilibrium rate we look at averages over various time periods, taking into account forces that may have shifted the equilibrium rate or caused the average to deviate from equilibrium at the time. Our ultimate goal is to understand

[6] This is consistent with the formal econometric work of Clark and Kozicki (2005,p403), who conclude that the link between trend growth and the equilibrium real rate is "quantitatively weak."

whether similar forces are at play today. We take a particularly close look at one of the most popular narrative interpretations of recent developments. This is the view that the US economy is suffering from "secular stagnation"—persistent weak demand and a near zero equilibrium rate. Our tentative conclusion from this exercise is that the equilibrium rate currently is between 1 and 2%, but there is considerable uncertainty about how quickly rates will return to equilibrium and the degree of likely overshooting at the end of the business cycle.

In this analysis we will be referring to two different measures of the real rate. The "ex-ante real rate"" is the estimate of the ex-ante real rate developed in Section 2, which proxies inflation expectations using an autoregressive model for the GDP deflator for data after 1930 or a CPI for data before 1930 that is estimated over rolling windows. The "static-expectations real rate" is the measure that people in the markets and the Fed look at most often, calculated as the nominal interest rate minus the change in the core PCE deflator over the previous 12 months. Exhibit 4.1 repeats Exhibit 2.7, with the static-expectations real rate added on. As the Exhibit shows, the two real rate series align very closely. Over the 1960 to 2014 period, the GDP-based ex-ante real rate and the PCE-based static-expectations real rate both average 2.01%.

4A. The real interest rate before World War II

Exhibit 4.2 reproduces our long history ex-ante real rate series for the United States from the lower left panel of Exhibit 2.2. The first thing that stands out in the real rate data is the notable downward shift in the real rate starting in the 1930s. U.S. real rates averaged 4.2% before World War I and only 1.3% since World War II. We found a similar drop for virtually every other country we looked at.

Three factors may account for the secular decline in real rates. First, in the earliest periods the short rate may have not been truly risk free. As Reinhart and Rogoff (2009) and others have documented, the period before World War II is laden with sovereign debt defaults. Almost all the defaults occurred when countries were in an emerging stage of development. In their data set, only Australia, New Zealand, Canada, Denmark, Thailand and the U.S. never had an external debt default. In the U.S. case, however, bouts of high inflation in the American Revolution and Civil War and the exit from the gold standard in 1933 had an effect similar to default.

Second, before the Great Depression financial markets were much less regulated. Interest rates, rather than credit and capital constraints did the work of equilibrating supply and demand.

Third, and perhaps the most important explanation in the economic history literature is low life expectancy. From 1850 to 2000 the average life expectancy for a 20 year old American male rose from 58 to 76.[7]. Shorter life expectancies in the past created two kinds of risks. First, absent a strong bequest

[7] Source: http://mappinghistory.uoregon.edu/english/US/US39-01.html

motive, a short life expectancy should mean a high time value of money. You can't take it with you. Second, shorter life expectancy increases the risk of nonpayment.[8]

Regardless of the cause of the shift, this suggests a good deal of caution in trying to extrapolate from these early years to the current economy.

History lesson #1*: The equilibrium rate is sensitive to time preference and perceptions about the riskiness of government debt.*

History lesson #2*: Judging the equilibrium rate using long historical averages can be misleading.*

4B. Financial repression (1948-1980)

Reinhart and Sbrancia (2015) define financial repression as a regulatory effort to manage sovereign debt burdens that may include "directed lending to government by captive domestic audiences (such as pension funds), explicit or implicit caps on interest rates, regulation of cross-border capital movements, and (generally) a tighter connection between government and banks." The period immediately following World War II was one of financial repression in many countries, including the U.S. If there are limited savings vehicles outside of regulated institutions and if those institutions are encouraged to lend to the government, this can lower the cost of funding government debt and the equilibrium rate. As noted by Reinhart and Rogoff (2009, p. 106),

> During the post-World War II era, many governments repressed their financial markets, with low ceilings on deposit rates and high requirements for bank reserves, among other devices, such as directed credit and minimum requirements for holding government debt in pension and commercial bank portfolios.)

Not surprisingly, real policy rates were very low for most of this period. Before the Fed Treasury Accord of 1951, interest rates were capped at 3/8% for 90 day bills, 7/8 to 1 ¼% for 12-month certificates of indebtedness and 2 ½% for Treasury bonds (Exhibit 4.3). The caps were maintained despite wild swings in inflation to as high as 25%. In the 1930s and 1940s the Fed also frequently used changes in reserve requirements as an instrument of monetary control.

The Accord gave the Fed the freedom to raise interest rates, but a variety of interest rate caps and other restrictions continued to hold down the equilibrium rate into the 1970s. When monetary policy was loose, rates fell; but when monetary policy tightened, a variety of ceilings became binding and the main restraint from monetary policy came from the quantity of credit rather than the price of credit. As Exhibit 4.4 shows, three-month T-bill rates rose above the Regulation Q deposit rate ceiling several times during this period. Indeed, many models of real activity at the time used dummy variables to capture a series of credit crunches during this period—in particular, 1966, and 1969-70. By the late

[8] Clark (2005) argued that these developments account for a decline in interest rates beginning with the industrial revolution.

1970s the constraints had become less binding and interest rate ceilings were phased out from 1980 to 1986.

History lesson #3*: The equilibrium real rate is sensitive to the degree of financial constraint imposed by regulations and by the degree to which policy relies on quantity rather than price (interest rates) to manage aggregate demand.*

4C. The inflation boom and bust (1965-1998)

The era of financial repression overlapped with the Great Inflation. Inflation was very low and stable in the early 1960s, but started to move higher in 1965. Exhibit 4.5 shows the history of headline and core PCE inflation. In 1966 the Fed tried to put on the brakes by hiking rates. This caused disintermediation out of the mortgage market and a collapse in the housing sector. The Fed then backed off, marking the beginning of a dramatic surge in inflation. From 1971 to 1977 the ex-ante real funds rate averaged just 0.3%, reflecting both persistently easy policy and a series of inflation surprises for investors.

From 1980 to 1998 the inflation upcycle was completely reversed. PCE inflation fell back to 1%. Starting with Volcker the Fed created persistently high rates. During this period the "bond vigilantes" extracted their revenge, demanding persistently high real returns. Survey measures of inflation expectations also showed a persistent upward bias. Over the period the ex-ante real funds rate averaged 4.1%. With the Fed pushing inflation lower, interest rates probably were above their long-run equilibrium level during this period.

Both inflation and real interest rates have been very low over the past two business cycles. Since 1998, year-over-year core PCE inflation has fluctuated in a narrow band of 1% to 2.4%. Consumer surveys of inflation expectations dropped to about 3% in the mid-1990s and have stayed there ever since (Exhibit 4.6). Surveys of economists, such as the Survey of Professional Forecasters have settled in right on top of the Fed's 2% PCE inflation target (also Exhibit 4.6).

History lesson #4*: Trends up or down in inflation can influence the real interest rate for prolonged periods. Real rate averages that do not take this into account are poor proxies for the equilibrium rate.*

4D. Real rates in delayed recoveries (1991-2007)

Both the 1991-2001 and 2002-2007 cycles differed significantly from past recoveries. Historically, the economy comes roaring out of a recession and the bigger the recession the faster the bounce back. Exhibit 4.7 shows a simple "spider" chart of payroll employment indexed to the trough of the last 7 business cycles.[9] Note the slow initial rebound in 1991, 2002 (and in the current cycle). This initially weak recovery prompted considerable speculation about permanent damage to growth and permanently lower rates. In 1991 Greenspan argued that heavy debt, bad loans, and lending caution by

[9] For expository purposes we have excluded the brief 1980 cycle. Also note that earlier cycles look similar to the 1970s and 1980s cycles.

banks were creating “50 mile-per-hour” headwinds for the economy. But by 1993 Greenspan was changing his tune: "The 50-miles-per-hour headwinds are probably down to 30 miles per hour.”[10] The same thing happened in the 2001-2007 cycle: fear of terrorism, corporate governance scandals, the tech overhang and fear of war in the Middle East all appeared to weigh on growth. When the Iraq War ended without a major oil shock or terrorist event, GDP growth surged at a 5.8% annual rate in the second half of 2003 and by 2005 the unemployment rate had dropped below 5%.

These delayed recoveries had a major impact on funds rate expectations. When the Fed first started hiking rates in February 1994 the market looked for the funds rate to rise about 100 bp over the next 24 months; in the event, the Fed hiked the funds rate by 300 bp in 13 months (Exhibit 4.8) The ex-ante real rate averaged 2.9% over the full business cycle, but at 4.7% at the end of the cycle as the Fed fought inflation (Exhibit 4.9). In the next cycle, when the Fed finally started to hike in June 2004, many analysts thought a normal hiking cycle was not possible.[11] When the Fed started to move, the markets were pricing in 170 bp in rate hikes over the next 24 months; in the event, the Fed hiked by 425 bp over a 24 month period. The real funds rate averaged just 0.5% over the full business cycle, but again peaked at a much higher 3.1%. The PCE-based measure yields numbers that are about two tenths higher than these averages of the ex-ante real rate.

History lesson #5: *Persistent headwinds can create a persistently low real rate, but when headwinds abate rates have tended to rise back to their historic average or higher.*

4E. Real rates, gluts, conundrums and shortages (2001-2007)

While for most of this paper we have ignored the broader global backdrop, a big story in the 2000 cycle was the unusual behavior of bond yields globally. From 2004 to 2006 the Fed hiked the funds rate by 425 bp and yet 10-year yields only rose about 40 bps. Greenspan (2005) called this the “bond conundrum,” pointing to an even bigger drop in yields outside the US, pension demand as population ages, reserve accumulation by EM central banks, and perhaps most important, a growing pool of global savings. Bernanke (2005) described this as a “glut of global savings,“ noting that after a series of crises many emerging market economies were building up massive currency reserves. He also pointed to rising savings by aging populations in Germany, Japan and other developed economies and to the attractiveness of US capital markets. Caballero (2006) and others make a related argument that there is a “safe asset shortage” caused by a rapid growth in incomes and savings in emerging markets and a shortage of safe local saving vehicles due to undeveloped capital markets.

It is not entirely clear whether the “glut”, “conundrum,” or “shortage” lowers or raises the equilibrium real funds rate. All else equal, lower US bond yields and compressed term premia stimulate the economy, forcing the Fed to hike more to achieve the same degree of financial restraint. However, not all else is equal. For example, central bank buying of US treasuries presumably put some upward

[10] http://www.nytimes.com/1993/05/27/business/mixed-outlook-from-fed.html
[11] For example McCulley (2003) argued that the equilibrium real funds rate was close to zero. He argued that “overnight money, carrying zero price risk, zero credit risk and zero liquidity risk should not yield a real after-tax return.”

pressure on the dollar, contributing to the sharp widening of the trade deficit. Indeed, as Exhibit 4.10 shows, from the peak of the previous business cycle (2000:1) to the peak of the construction boom (2005:3), housing as a share of GDP rose by 2pp and net exports as a share of GDP fell by 2 pp. On net, the saving glut may have not changed overall financial conditions, but instead made them imbalanced, contributing to both a surging trade deficit and a housing bubble. The upshot of all of this is that the glut did not prevent significant Fed rate hikes. As we noted above, the static-expectations real rate peaked at 3.3% in 2006.

> ***History lesson #6:*** *The global saving glut probably distorted overall US financial conditions, but did not have a clear impact on the equilibrium real funds rate.*

4F. Secular stagnation and the equilibrium rate (1982-?)

Our narrative approach to the history of the equilibrium rate is particularly useful in addressing a competing "narrative theory" of the last several business cycles: the idea that the economy suffers from secular stagnation. The idea goes back to the 1930s when Alvin Hansen asked whether the economy would ever be able to achieve satisfactory growth. He was concerned both about chronic deficient demand and a lower trend growth in the economy and hence a low equilibrium real rate.

The secular stagnation hypothesis.

Krugman, Dominguez, and Rogoff (1998) revived Hansen's concerns, suggesting that when the equilibrium real interest rate is negative, an economy could get stuck at suboptimal growth and deflation as a result of the zero lower bound on nominal interest rates. Summers (2013b) expressed the hypothesis this way:

> Suppose that the short-term real interest rate that was consistent with full employment had fallen to negative two or negative three percent in the middle of the last decade. Then … we may well need, in the years ahead, to think about how we manage an economy in which the zero nominal interest rate is a chronic and systemic inhibitor of economy activity, holding our economies back below their potential.

Summers (2014) suggested that secular stagnation in the U.S. goes back to the 1990s, arguing that the strong performance in the 1990s "was associated with a substantial stock market bubble." Again in 2007 the economy did "achieve satisfactory levels of capacity utilization and employment", but this was due to the housing bubble and "an unsustainable upward movement in the share of GDP devoted to residential investment." He queried "in the last 15 years: can we identify any sustained stretch during which the economy grew satisfactorily with conditions that were financially sustainable?" Finally Summers extended this argument to the rest of the industrial world, pointing to even worse performance in Japan and Europe.[12]

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[12] Summers is basically restating the "serial bubbles" view of recent business cycles popularized by Stephen Roach and many others, See for example, http://delong.typepad.com/sdj/2005/06/stephen_roach_o.html

Krugman (2013) also argued that bubbles have been necessary to achieve economic growth:

> We now know that the economic expansion of 2003-2007 was driven by a bubble. You can say the same about the latter part of the 90s expansion; and you can in fact say the same about the later years of the Reagan expansion, which was driven at that point by runaway thrift institutions and a large bubble in commercial real estate….So how can you reconcile repeated bubbles with an economy showing no sign of inflationary pressures? Summers's answer is that we may be in an economy that needs bubbles just to achieve something near full employment — that in the absence of bubbles, the economy has a negative equilibrium rate of interest. And this hasn't just been true since the 2008 financial crisis; it has arguably been true, although perhaps with increasing severity, since the 1980s.

Were near zero rates and/or asset bubbles essential to achieving full employment in the 1982, 1990 and 2000 business cycles? Is underlying demand so weak that it is impossible to create inflation pressure even with super easy policy? A close look at these cycles shows little support for either of these propositions.

Unemployment, inflation, and the real interest rate over the last 3 cycles.

The US economy has not been suffering chronic under-employment. The economy not only reached full employment in each of the last three business cycles, it actually significantly overshot full employment. This is true whether one uses typical estimates of the NAIRU from the CBO, IMF or OECD or if one takes an agnostic approach and simply use the historic average unemployment rate (5.8% in the post-war period). For example using CBO estimates, the US overshot the NAIRU rate by between 0.6 to 1.1 pp in each cycle and these periods of tight labor markets lasted between 8 and 18 quarters (Exhibit 4.11). CBO estimates of the output gap show similar results: GDP was above potential in 1988-1989, 1997-2001 and 2005-2006. Note that this success in achieving a full recovery is not an artifact of assuming low potential growth or a high NAIRU: during this period CBO estimates of potential growth rose and the estimated NAIRU fell. These extended periods where aggregate demand exceeded aggregate supply are hardly a sign of secular stagnation.

Exhibit 4.12 shows furthermore that each of the last three cycles ended with incipient inflation pressure. In the 1980s cycle, the Fed pushed inflation down to below 4%, but by 1988, it was trending up again. In the 1990s, inflation also picked up at the end of the business cycle, although core PCE inflation only briefly pierced 2%. Presumably, this was related to the unexpected surge in productivity during this period. On a 5-year basis, growth in nonfarm business productivity peaked at 3% at the end of the 1990s expansion, up from just 2% over the previous 20 years or so. Core inflation was persistently above 2% in the second half of the 2000 expansion and headline inflation was above 3% other than a brief interruption in 2006. This seems inconsistent with the idea that the Fed had trouble sustaining normal inflation.

Of course, the rise in inflation at the end of recent economic expansions has been milder than in the 1960s and 1970s. However, in our view, this is not a sign that the Fed cannot create inflation; instead, it shows that they have learned when to apply the brakes, gaining credibility along the way. The

1970s experience has taught the Fed about the risks of trying to exploit the short-run Phillips Curve and the importance of finishing the job in eradicating unwanted inflation. A good measure of their success in restoring credibility is that both survey and market measures of inflation expectations have become very stable. In Exhibit 4.13, we show the standard 10-year inflation breakeven, along with a measure from the Federal Reserve Bank of Cleveland that attempts to remove term and risk premia. The recent weak response of inflation to tight labor markets probably also reflects the unexpected productivity boom in the 1990s; increased global integration, making the US sensitive to global as well as domestic slack; the weakening of union power and low minimum wages; and host of other factors. In our view these conventional arguments for a flatter Phillips Curve are more compelling than the secular stagnation thesis.

History lesson #7: *During the period of alleged secular stagnation, the unemployment rate was below its postwar average and inflation pressures emerged at the end of each cycle.*

Over the 1982 to 2007 period as a whole the ex-ante real rate averaged 3.0% (and the static-expectations measure averaged 2.9%). This was above the 2.0% post-war average, but since the Fed was trying to lower inflation in the first half of this period, we believe the average rate was higher than its equilibrium level during this period. The 1980s cycle had the normal strong start and quick funds rate normalization. However, for both the 1990 and 2000 cycle, the economic recovery was initially weak and the funds rate was persistently low. As headwinds faded, however, eventually the funds rate surged above its long-run average. Looking at the individual cycles, the economy reached full employment with an ex-ante real rate of 3.3, 4.0 and 0.25% respectively (again see Exhibit 4.9). In each cycle, the real rate eventually peaked well above its historic average (last column of Exhibit 4.9).

History lesson #8: *During the first part of the period of alleged secular stagnation (1982-2007) the real rate averaged 3%, a percentage point higher than its post-war average.*

The role of asset bubbles in the last three recoveries

What about asset bubbles? Were they essential to achieving full employment and normal inflation? The evidence is mixed, but a close look at the three cycles offers little support for the secular stagnation thesis. As we will show, the timing of the alleged bubbles doesn't really fit the stagnation story.

1982-1990. Asset bubbles may have had some impact on the 1982-90 economic recovery, with a boom in commercial real estate and related easy lending from savings and loans. However, the economy hit full employment in 1987 and stayed there even as the tax reform in 1986 had already undercut the real estate boom and even as the stock market crashed in 1987. Thus, while nonresidential investment did surge in the early 1980s, it collapsed after tax reform in 1986. As seen in Exhibit 4.14 over the course of the recovery structures investment plunged as a share of GDP. The Savings and Loan industry followed a similar pattern. The heyday of easy S&L lending was in the early 1980s. From 1986 to 1989 the Federal Savings and Loan Corporation (FSLIC) had already closed or otherwise resolved 296 institutions. Then the Resolution Trust Corporation (RTC) took over and shuttered another 747 institutions. The boom and bust in these two sectors caused shifts in aggregate demand, but it is hard to see their role in achieving and maintaining a low unemployment rate after 1986.

> ***History Lesson #9****: The economy reached full employment in the 1980s despite high real interest rates and retrenchment in the real estate and S&L industry in the second half of the recovery.*

1990-2000. The asset bubble story is even less convincing in the 1990s recovery. The NASDAQ started to disconnect from the economy and the rest of the stock market in late 1998 and surged out of control in 1999 (Exhibit 4.15). However, before the bubble started, the unemployment rate had already dropped to 4.7% in 1997, well below both its historic average and CBO's ex post estimate of full employment. Hence the NASDAQ bubble may have contributed to the subsequent overheating at the end of the economic recovery but it is putting the cart before the horse to argue that it was necessary for achieving full employment.

> ***History Lesson #10****: The NASDAQ bubble came after the economy reached full employment and therefore was not a precondition for achieving full employment.*

2000-2008. Of the three recent business cycles, the 2000 cycle provides the best support for the argument that monetary policy is only stimulative if it creates asset bubbles. Data from Core Logic shows national home prices rising very slowly in the early 1990s, but then accelerating to double digit rates and peaking in 2005. The Case-Shiller measure of national home prices shows a slow acceleration in the early 1990s, and then an acceleration to double digit rates, peaking in 2005. Bank of America Merrill Lynch's model of the Case-Shiller data suggests home prices began to diverge from their fair value in 2001 (Exhibit 4.16). Lending standards eased during this period with a surge in exotic lending starting in the second half of 2004. Meanwhile, leverage ratios and off balance sheet asset expansion surged. [13]

Was the recovery in the economy unusually weak given the credit bubble during this period? Would the economy have reached full employment without the bubble? Getting a definitive answer on this is difficult, but at a minimum it requires looking not only at the biggest tailwind in this period—the housing bubble—but also the biggest headwinds—the sharp increase in the trade deficit and the relentless rise in energy prices. Here we compare the positives and the negatives using some simple metrics. Note that for each chart we draw a vertical line in 2005 when the unemployment rate had dropped to 5%, the CBOs estimate of NAIRU.

First, the boosts: easy credit stimulated a boom in both construction and consumer spending. As Exhibit 4.17 shows, residential investment has historically averaged 4.7% of GDP, with a typical peak of about 6%. However, in the 2000s cycle residential investment rose from 4.9% at the end of the 2001 recession in 2001Q4 to 6.6% at the housing market peak in 2006Q1. This boom occurred despite weak

[13] Summers (2014) also argues that "fiscal policy was excessively expansive" during this period. Note, however, that official estimates of the cyclically adjusted budget deficit show fiscal policy tightening from 2004 to 2007. For example, OECD estimates show cyclically adjusted net government borrowing falling from 6.1% of potential GDP in 2004 to 4.7% in 2007. Indeed, by this metric fiscal policy tightened in the second half of each of the last three cycles with a particular big tightening in the 1992-2000 period.

demographics: the peak in first home buying is in the 30 to 39 age range, but this group shrunk about 0.9% per year in the 2000 cycle. It therefore seems quite reasonable to attribute the gain mostly to easy credit, which would imply a boost of 1.7 percentage points directly through higher homebuilding, or 0.4 percentage point at an annual rate. However, it is worth noting that at the start of the Great Recession in 2007Q4 residential construction had already fallen back to just 4.8% of GDP.

At the same time, surging home prices boosted consumer spending through both a classic wealth effect and a liquidity channel related to the surge in "mortgage equity withdrawal" (MEW) illustrated in Exhibit 4.18 and discussed in Feroli et al. (2012). To get a sense of the magnitude of these effects, we go back to the analysis in Hatzius (2006) which presented a simple model of consumer spending with separate housing wealth and MEW effects. In this analysis, the coefficient on (housing) wealth was estimated at 3.4 cents/dollar and that on "active MEW"—cash-out refinancing proceeds and home equity borrowing—at 62 cents/dollar; "passive MEW" i.e. home equity extracted in the housing turnover process was not significantly related to consumer spending. Using these estimates, the increases in the housing wealth/GDP ratio and active MEW from 2001Q4 to 2006Q1 added a total of 2.3% to the level of GDP, which implies a boost to growth of about 0.5 percentage point at an annual rate.

Second, the drags: the increase in the trade deficit and rising energy prices were important counterweights.

Regarding trade, the trade deficit increased by 2.4% of GDP from 2001Q4 to 2006Q1, subtracting 0.5 percentage point per year from growth. In our view, much of this increase was due to two forces: the direct impact of the housing and credit boom on import demand and the entry of a highly mercantilist China into the global economy post WTO accession. In our view, both need to be taken into account when evaluating how quickly the economy "should" have grown during the housing and credit bubble.

Regarding oil prices, we believe the price increase in the 2000s mostly reflected a combination of constrained supply and surging demand from emerging markets.[14] Hence, from a US perspective, much of it was an exogenous supply shock. A simple approach for estimating the size of the shock is to look at the "tax" on household incomes from energy prices rising faster than nonenergy prices. In Exhibit 4.19 we compare the growth in the overall PCE deflator to the PCE excluding energy. Based on this metric, rising energy prices imposed a tax increase of about ½ percentage point of disposable income per year on the consumer between 2001Q4 and 2006Q1. Recognizing that consumption is about 70% of GDP and assuming a marginal propensity to spend of 70%, this number suggests a GDP hit of about ¼ percentage point per year over this period. After 2006, the energy hit to GDP growth increased further as oil prices rose even faster through mid-2008.

Putting the shock variables together, we estimate that rising home construction and the housing wealth/MEW effect were adding just under 1 percentage point per year to growth from 2001Q4 to

[14] See Harris, Kasman, Shapiro and West (2009)

2006Q1. Against this, the increase in the trade deficit and the surge in energy prices were subtracting about ¾ percentage point. In other words, the negative forces probably canceled out most of the stimulative impact of the housing bubble. By the peak of the business cycle, the winds had already shifted as construction and home prices started to slide and the energy tax surged. Nonetheless, the unemployment rate fell below NAIRU, bottoming at 4.4%.

Would the economy have achieved and sustained full employment in the absence of all of these shocks, positive and negative? It is impossible to do full justice to this period in a short narrative, but if we are right that a sizable portion of the obvious bubble-induced boosts were canceled out by equally obvious drags, the fact that the unemployment rate fell below NAIRU despite a 3% real funds rate suggests that the answer may well be yes.

> ***History lesson #11****: Taking into account the offsetting headwind from the rising trade deficit and higher oil prices as well as the tailwinds from the housing bubble, it is not clear whether the economy suffered secular stagnation in the 2000s.*

4G. Outlook for the current cycle

With this historical narrative as our guide, what are the implications for the equilibrium rate today?

First, the obvious: using historical averages from some periods of history as a gauge of equilibrium today can be quite misleading. The whole period before the Fed-Treasury Accord seems of very limited value. Real rates before WW I were chronically higher, presumably reflecting higher risk premiums and discount rates. Real rates fluctuated wildly during the Depression and war years. And the period of interest rate pegging is clearly not relevant today. On a similar vein, clearly average real rates during a period when inflation is trending in one direction are a poor measure of the equilibrium rate.

Second, changes in the monetary transmission mechanism due to regulatory and developments are clearly very important to determining equilibrium. Before financial deregulation, credit crunches did most of the “dirty work” in fighting overheating in the economy. This tended to cap the upside for real interest rates, lowering the average rate for the period. Today the long period of deregulation is over and regulatory limits are growing.

Rather than dig into the deep weeds here, we would make the following observations. First, capital markets remain much less regulated than in the 1960s and 1970s. Today there is a big, active corporate debt market, global capital markets are wide open and banks play a much smaller role in the financial system. Bank capital and liquidity requirements have gone up; but restrictions on banks do not approach historic levels. Two areas may face chronically tight credit: residential mortgage lending and small business lending. But even here the constraint is tighter credit standards, not dramatic disintermediation episodes. Recall that even in the heavily regulated 1960s real rates averaged well above zero. For example, from 1960 to 1965, a period of stable 1% inflation, the real rate also averaged about 1% (recall Exhibit 4. 1) for both ex-ante and static-expectations real rates.

Third, as in the last business cycle, global forces seem to be having a big impact on the US bond market. The current negative real interest rates are a global phenomenon. Of the countries represented in Exhibit 2.1, 17 of the 20 estimated quarterly ex-ante real rates are negative as of the end of 2014, with 15 out of 17 the comparable figures for annual data. In the past 12 months US 10-year bond yields have plunged by more than 100 bp, despite the end of QE3, stable core inflation, the end of the Fed's balance sheet expansion and a looming rate hike cycle. It appears that a combination of weak global growth, falling core inflation (particularly in Europe) and expectations of further central bank balance sheet expansion is putting downward pressure on global rates. In the two years ahead, we expect the combined balance sheet of the "big four" central banks—the Fed, ECB, BOJ and BOE—to expand their balance sheets at almost double the pace of the last year.[15] As with the previous "glut" it is hard to know whether global developments are raising or lowering the equilibrium real funds rate.

Fourth, our look back at the last two economic recoveries underscores the danger of mistaking short-run headwinds for permanent weakness. Recall that one of the great dangers in formal models of the equilibrium rate is the "end point problem"—estimates of time-varying parameters tend to be skewed by the most recent data. This problem is also critical for the narrative approach: simply put, it is a lot easier to identify the equilibrium rate after the business cycle is over than in real time. In the last two tightening cycles, the Fed started slow, but eventually pushed real rates well above their historic averages.

Last, but not least, we are skeptical about the secular stagnation argument. We see two problems as it relates to the current recovery. First, it does not distinguish between a medium-term post-crisis problem and permanent stagnation. Clearly this is not a normal business cycle where a big collapse is followed by a big recovery (Exhibit 4.20). As Reinhart and Rogoff (2014) and many others note, when there is a systemic crisis both the recession and the recovery are different than in a normal business cycle. Summarizing 100 such episodes, they find that GDP typically falls by 10.3% and it typical takes 8.4 years to recover to pre-crisis levels. Their "severity index"—adding the absolute value of these numbers together—averages 19.6 for all 100 cases.

Is history repeating itself? Most of these cycles predate the modern era of automatic stabilizers and countercyclical fiscal and monetary policy. They also ignore the special status of the US as the center of capital markets. And they don't attempt to gauge the relative strength of the policy response to each crisis. Nonetheless, these historic averages are a good starting point for analyzing the current period. Indeed, as the last line of the table shows, the US has done much worse than following a normal recession, but measured in comparison to previous such cycles, the US has done quite well, with a smaller recession, a quicker recovery and a much smaller "severity index."

These systemic crises unleash extended periods of deleveraging and balance sheet repair. How long this impairs aggregate demand presumably depends on the speed of the healing process. This also suggests that the effectiveness of monetary policy should be judged by balance sheet repair as well as the speed of growth in the economy.

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[15] See Harris (2014).

Judging from a variety of metrics, easy policy seems to have accelerated the healing process:

- Banks are in better shape, with more capital, a lot less bad debt and with the ability to withstand serious stress tests.
- The housing market has worked off most of its bad loans and both price action and turnover rates are back to normal.
- There has a been a full recovery of the ratio of household net worth to income, the debt-to-income ratio has tumbled, and debt service has dropped to the lowest of its 34 year history (Exhibit 4.21).
- High-yield companies have been able to refinance and avoid defaults despite a feeble recovery.

In our view, these metrics suggest the balance sheet repair is well advanced.

A second problem with the secular stagnation argument is that it ignores the role of fiscal policy in driving aggregate demand. This economic recovery has seen major fiscal tightening, starting at the state and local level and then shifting to the federal level. Despite the weak economic recovery, the 5.5 pp improvement in deficit to GDP ratio from 2011 to 2014 was by far the fastest consolidation in modern US history (Exhibit 4.22). A number of recent studies suggest that fiscal policy is particularly potent when interest rates are stuck at the zero lower bound.[16]

Adding to the headwinds, this consolidation has been accompanied with a series of confidence shaking budget battles, including a "fiscal cliff" and repeated threats of default or shutdown. The result is a series of spikes in the "policy uncertainty index" developed by Baker, Bloom and Davis (2013) (Exhibit 4.23). It is a bit odd to have a Keynesian theory of inadequate demand such as "secular stagnation" that does not include a discussion of the role of contractionary fiscal policy in creating that shortfall.

4H. Summary: the new equilibrium

In some ways the received wisdom on the economy has come full circle: the optimistic "Great Moderation" has been replaced with its near-opposite, "Secular Stagnation." The truth seems to be somewhere in between. Some of the moderation was earned at the expense of asset bubbles. Some of the stagnation is cyclical. If our narrative is correct, the weak economic recovery of the past five years is not evidence of secular stagnation, but is evidence of severe medium-term headwinds. The real test is happening as we speak: with significant healing from the 2008-9 crisis, will the recent pick-up in growth continue, creating a full recovery in the economy? And will the economy withstand higher interest rates? Judging from the previous three business cycles (and recent growth data!), we think the answer to both questions is "yes."

[16] See Christiano, Eichenbaum, and Rebelo, (2011). One of the ironies of the secular stagnation debate is that some of its strongest advocates are also strong supporters of more simulative fiscal policy. For example, Krugman (2014) argues that the recent actions in Washington have been like someone hitting themselves with a baseball bat and now that the beating is over the economy is doing better.

Our narrative approach suggests the equilibrium rate may have fallen, but not by as much as some suggest. The last several business cycles have underscored the danger of calling a new era of lower rates in the middle of an economic recovery. We would expect the equilibrium rate to be higher than the 1% average rate during the financial repression of the 1950s and 1960s before inflation surged. On the other hand, lower trend growth in the economy may have lowered the equilibrium rate below the 2% or so average for real rates over the 1960-2007 period. Based on our narrative analysis, a reasonable range for the equilibrium rate today is between 1 and 2%. Moreover, as history has repeatedly shown, the real rates will likely peak at well above the equilibrium rate as the Fed shifts to fighting inflation late in the cycle.

5. Long-run tendencies of the real interest rate

In this section we complement the narrative analysis of the previous section with some formal econometric analysis, pursuing the reference in the previous section to the contribution of global developments to what happens in the United States. We will first present evidence of nonstationarity of the U.S. ex-ante real interest rate and then develop a bivariate vector error correction model relating U.S. rates to global factors.

A number of studies have documented instability over time in postwar measures of the real interest rate. Although Garcia and Perron (1996) and Ang and Bekaert (2002) modeled these as shifts between possibly recurrent regimes, Caporale and Grier (2000) and Bai and Perron (2003) found these were better captured as permanent breaks, with Rapach and Wohar (2005) finding statistically significant structural breaks in postwar data for each of the 13 countries they examined. Since one of the striking features in our long-run data set is the apparent higher real rate in the 19th century, here we test for stability of the mean real interest rate over our full long-run data set.

5A. Nonstationarity of the real interest rate

Consider a second-order autoregression[17] fit to annual levels of the U.S. real interest rate with a possible shift in the level beginning at some date t_0:

$$y_t = c + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \kappa\delta(t \geq t_0) + \varepsilon_t. \qquad (5.1)$$

Here $\delta(t \geq t_0)$ takes the value of one if date *t* comes after some date t_0 and is zero for earlier values. A finding that $\kappa < 0$ would mean that real interest rates tended to be lower in the second half of the sample than in the first. We estimated equation (5.1) for $y_t = r_{US,t}$ over the 1861 to 2014 sample and

[17] Using a second-order autoregression for levels allows us to include as a special case a first-order autoregression for growth rates as one of the possible specifications that we will be considering.

used White's (1980) heteroskedasticity-robust test of the null hypothesis $\kappa = 0$ for every possible break date t_0 between 1900 and 1976. The resulting $\chi^2(1)$ statistic is plotted as function of t_0 in the top panel of Exhibit 5.1, along with the 1% critical value for a $\chi^2(1)$ variable (shown in green). The hypothesis of a stable long-run interest rate would be rejected for any break date before World War I.

Of course, when one looks at a number of different possible break dates (as we have done here), the largest observed value for the test statistic no longer has a $\chi^2(1)$ distribution, but instead has an asymptotic distribution calculated by Andrews (1993, 2003), the 1% critical value for which is plotted in solid blue in Exhibit 5.1. The evidence against a constant average real rate remains quite convincing.

We also applied the approach suggested by Bai and Perron, looking for the possibility of more than one break in all the coefficients of (5.1). That approach identifies two breaks occurring at 1915 and 1921, interpreting the pre-World War I, World War I, and post-World War I episodes as three different regimes.[18] However, given the striking heteroskedasticity in these data, we feel it is more robust to use White's (1980) heteroskedasticity-robust test with a single break point, which will be the test used in the subsequent analysis.

If the real interest rate itself is nonstationary, can we identify a stable relation that can consistently describe a century and a half of U.S. and world data? This is the task we undertake next.

5B. A stable representation of long-run dynamics

Although apparently nonstationary, the real interest rate does exhibit a form of mean reversion in that episodes with real interest rates above 5% or below -5% proved to be temporary. We tried to capture this idea by fitting a first-order autoregression to 30-year rolling windows of data on each country's ex-ante real rate:

$$r_{nt} = b_n + \psi_n r_{n,t-1} + \varepsilon_{nt}. \tag{5.2}$$

In order to keep these estimates from being unduly influenced by outliers such as the extreme wartime observations, we allowed the variance of the residuals to vary over time according to an ARCH(2) process:

$$\begin{aligned} \varepsilon_{nt} &= h_{nt} v_{nt} \\ v_{nt} &\sim N(0,1) \\ h_{nt}^2 &= \kappa_n + \alpha_{n1}\varepsilon_{n,t-1}^2 + \alpha_{n2}\varepsilon_{n,t-2}^2. \end{aligned}$$

[18] The null hypothesis of a single structural break is rejected in favor of the alternative of two structural breaks with a *p* value less than 0.01 using the test described in Table 2 of Bai and Perron (1998).

We estimated the parameters by maximizing the likelihood for 30 years of observations on country *n* ending in year *t*, and calculated the long-run value for the real rate implied by those estimates:

$$\ell_{nt} = \frac{\hat{b}_{nt}}{(1-\widehat{\psi}_{nt})} \quad .$$

We then took the median value across countries as an estimate of the long-run world real rate associated with every year *t* in the sample:

$$\ell_t = \underbrace{median}_{n} \, \ell_{nt}.$$

The resulting series for the world long-run rate ℓ_t is plotted along with the actual U.S. ex-ante real rate $r_{US,t}$ in Exhibit 5.2.[19] The world long-run rate captures the broad trends noted in Section 2, such as the persistent tendency for real rates to be higher in the 19th century and a long episode of low real rates from World War II through the mid-1970s. World real rates rose through the mid-1980s and have been declining again since then.

Evidence that real rates do not show a tendency to revert to some constant value is even more dramatic when we use ℓ_t in place of $r_{US,t}$, as seen in the second panel of Exhibit 5.1.

However, it is interesting to note that the gap between the U.S. real rate and our series for the long-run real rate appears to be stationary. The third panel of Exhibit 5.1 performs the same stability test on $y_t = r_{US,t} - \ell_t$, consistent with the conclusion that the following relation is stable over time (robust standard errors in parentheses):

$$r_{US,t} - \ell_t = \underset{(0.204)}{-0.174} + \underset{(0.133)}{0.935}(r_{US,t-1} - \ell_{t-1}) - \underset{(0.103)}{0.357}(r_{US,t-2} - \ell_{t-2}) + e_t. \tag{5.3}$$

Note that with the sum of the autoregressive coefficients of only 0.58, we reject the null hypothesis of a unit root with a Dickey-Fuller robust *t* test of -4.89. In other words, although neither the U.S. nor the world rate appear to return to a single stable value over time, the two series are cointegrated and tend not to stay too far apart. Expression (5.3) implies a long-run tendency for the U.S. rate to be on average

[19] Our measure of the long-run real rate is very similar to an average value of *r* over the preceding 30 years. In fact, if (5.2) were estimated by OLS instead of with ARCH(2) residuals, and if the interest rate in year *t* was identical to the interest rate in year *t* -30, our estimate would be identical to the average rate over the 30 years ending in *t*. One can see this from the OLS identity that $\bar{r} = \hat{b} + \widehat{\psi}\bar{r}_{-1}$ and the fact that if $r_t = r_{t-30}$ then $\bar{r} = \bar{r}_{-1}$ meaning that $\hat{b}/(1-\widehat{\psi}) = \bar{r}$. If $r_t > r_{t-30}$ then ℓ_t will tend to be a little above the average rate over the 30 years ending in *t*. Also, since we estimate (5.2) allowing for ARCH errors, our estimate tends to down-weight observations that appear to be outliers in calculating the long-run real rate.

41 basis points below the world rate $(-0.174/(1-0.935+0.357)=-0.41)$ but this is poorly estimated even with a century and a half of data and cannot be distinguished statistically from zero.

An error-correction VAR for the U.S. and long-run world rate also appears to be stable:

$$\Delta r_{US,t} = \underset{(0.206)}{-0.195} + \underset{(0.104)}{0.382}\,\Delta r_{US,t-1} - \underset{(0.625)}{0.790}\,\Delta \ell_{t-1} - \underset{(0.085)}{0.403}(r_{US,t-1} - \ell_{t-1}) + e_t$$

$$\Delta \ell_t = \underset{(0.025)}{-0.020} + \underset{(0.010)}{0.026}\,\Delta r_{US,t-1} - \underset{(0.082)}{0.322}\,\Delta \ell_{t-1} + \underset{(0.009)}{0.016}(r_{US,t-1} - \ell_{t-1}) + e_t.$$

We find no evidence of a changing intercept in either of these equations, and readily accept the hypothesis that lagged levels of $r_{US,t-1}$ or ℓ_{t-1} do not enter as additional explanatory variables, supporting the conclusion that $r_{US,t}$ and ℓ_t are cointegrated with cointegrating vector (1,-1)' and error-correction term $r_{US,t}-\ell_t$. If the U.S. rate is 100 basis points above the long-run world rate, the U.S. rate would be predicted to fall by 40 basis points the following year (with a *p*-value below 10^{-5}) and the world rate to rise by 1.6 basis points, though the latter is insignificantly different from zero (p = 0.07). The intercepts in both equations are statistically insignificant. Dropping these gives the following more parsimonious representation of the error-correction VAR:

$$\Delta r_{US,t} = \underset{(0.103)}{0.379}\,\Delta r_{US,t-1} - \underset{(0.623)}{0.775}\,\Delta \ell_{t-1} - \underset{(0.085)}{0.396}(r_{US,t-1} - \ell_{t-1}) + e_{US,t} \qquad \widehat{\sigma}_{US} = 2.57 \tag{5.4}$$

$$\Delta \ell_t = -\underset{(0.082)}{0.321}\,\Delta \ell_{t-1} + \underset{(0.010)}{0.026}\,\Delta r_{US,t-1} + \underset{(0.009)}{0.017}(r_{US,t-1} - \ell_{t-1}) + e_t \qquad \widehat{\sigma}_{\ell} = 0.31. \tag{5.5}$$

Note that although we find that the U.S. and long-run world rate are cointegrated, in any given year they could differ enormously. The standard deviation of the shock in (5.4) is over 250 basis points, meaning in any given year the U.S. rate could easily be hundreds of basis points away from the long-run world rate. But what the relation says is that such deviations are likely to prove to be temporary.

5C. Implications for future real rates

It is interesting to look at what the system (5.4)-(5.5) would imply for the behavior of future interest rates. Exhibit 5.3 plots the current forecasts for the U.S. and long-run world rate that are implied by (5.4) and (5.5). The U.S. rate is expected to rise relatively quickly and the long-run world rate to fall very slightly, with both around 40 basis points within three years. Note that although the forecast levels of both rates converge to the same constant, the process for $(r_t, \ell_t)'$ implied by (5.4)-(5.5) is nonstationary. Although we expect the two to return to values close to each other as a result of the apparent cointegration between them, the particular level at which they do so becomes more uncertain the farther we look into the future, as illustrated by the continuing growth in the confidence interval for ℓ_t plotted in Exhibit 5.3. Note furthermore that the possible values for the US rate for any given year could be in a very wide interval around these points, given the standard error of the estimate in (5.4) that is over 250 basis points.

Another implication of the unit root in real rates is that the implied forecast would be different if we used a different starting point to form the forecast. The formulation implies that there is a permanent component to any shock and therefore a permanent implication of initial conditions in any forecast. Again the expanding confidence intervals as one looks farther into the future from any given initial conditions are a necessary consequence of this property.

We should also note that this is a reduced-form model that makes no use of information such as the current state of the business cycle. Our analysis in Section 4 would argue that this is a reason to think the U.S. rate will rise faster and farther than implied by the reduced-form forecasts, which again is perfectly consistent with the confidence intervals.

It's nevertheless interesting to compare these predictions for the U.S. real rate with values implied by the current term structure of Treasury Inflation Protected Securities (TIPS). By simultaneously buying a 3-year TIPS and selling a 2-year TIPS one can lock in the yield for a 1-year bond that will be purchased in two years, a rate of return known as the 2-year forward rate. Forward rates implied by the current yield curve are plotted in blue in Exhibit 5.4 along with the level for the US real rate predicted by the system (5.4)-(5.5).[20] If the U.S. real rate converges to the long-run world rate with the adjustment speed typically seen of the last century and a half, real rates will start rising at about the same rate implied by the current forward curve, though will end up at a rate somewhat below current longer-term forward rates. We nevertheless again caution that the standard errors associated with our model's forecasts are enormous.

To summarize, although it is commonly assumed in economic models that the real interest rate should eventually revert to an equilibrium or neutral value, there is little evidence for this in the data. Plausible measures of real rates were very negative in most countries for lengthy periods around the two world wars and have gone through other long periods of very high or very low values. Real rates were substantially higher in the 19th century than they have been since the middle of the 20th century. We do find some tendency for the U.S. real rate not to diverge for too long from a measure of the long-run world real rate. The fact that the latter is significantly above current U.S. real rates at present suggests a likelihood that U.S. real rates will rise over the next several years.

6. Implications for monetary policy

A key conclusion from both our narrative and statistical analysis is that the uncertainty around the equilibrium interest rate (which we denote here by r^*) is very considerable. This conclusion is very consistent with Laubach and Williams (2003). They estimate a standard error for r^* that ranges from 109bp to 258bp depending on the specification (see the bottom of Table 1 of their paper). In fact, our

[20] The forward rates implied by yields as of December 20, 2014 were calculated as in Gürkaynak, Sack, and Wright (2010) using the spreadsheet available at http://www.federalreserve.gov/pubs/feds/2008/200805/200805abs.html. Note that the forward rate cannot be reliably calculated by this method for a horizon less than 2 years into the future, which is why the plotted curve only begins in 2016.

narrative approach suggests that the uncertainty may be even greater at this point in history as policy makers try to gauge the size and persistence of a variety of economic headwinds. And predicting how r^* will change in the future is even harder, both because the linkage between r^* and its fundamental drivers such as potential GDP growth is tenuous and because these fundamental drivers themselves are difficult to forecast. In this section, we explore the implications of uncertainty about r^* for the normalization of the federal funds rate in coming years.

The study by Orphanides and Williams (2002, henceforth OW) provides a good starting point. Using a small estimated model of the US economy, they consider the policy rule

$$i_t = a_0 i_{t-1} + (1-a_0)\left(r_t^* + \pi_t\right) + a_1\left(\pi_t - \pi^*\right) + a_2\left(u_t - u^*\right) \tag{6.1}$$

where i is the nominal short-term interest rate, r^* is the real equilibrium rate, π is inflation, π^* is the central bank's inflation target, u is the unemployment rate, u^* is the structural unemployment rate, and a_0, a_1, and a_2 are parameters of the monetary policy reaction function. This model nests two extremes. If a_0 is equal to zero, it collapses to the familiar Taylor rule in which the level of short-term rates depends on the inflation and unemployment gap. But if a_0 is equal to one it collapses to a "difference rule" in which the *change* in the short-term interest rate depends on the inflation and unemployment gap. OW then derive the optimal choices of a_0, a_1, and a_2 as functions of the uncertainty around r^*.[21]

Their main conclusion is that uncertainty around r^* could be a reason for the Fed to adopt a more "inertial" policy rule, in which the current funds rate depends more on the lagged funds rate and less on the Fed's (uncertain) estimate of the equilibrium rate. In terms of equation (6.1) they show that greater uncertainty raises the optimal value of a_0, up to a value of 1 in the limiting case where the Fed knows nothing about r^*. In this limiting case, the OW analysis suggests that the greatest stability would be achieved with a difference rule, i.e. hike rates when the economy is "too hot"—i.e. inflation is too high and/or unemployment is too low—and lower rates when the economy is "too cold."

6A. Calibrating the baseline in the FRB/US model

To assess the relevance and implications of these insights for the current monetary policy outlook, we turn to FRB/US, the large-scale econometric model developed and maintained by the Federal Reserve Board staff. Since FRB/US is a rich model of the US economy that can be benchmarked to the economic and interest rate projections in the FOMC's Summary of Economic Projections (SEP), we believe it is more useful for providing insights into current US monetary policy issues than a small and more stylized model.

We use FRB/US to answer two questions. First, how does uncertainty around the true value of r^* affect the results of following a particular monetary policy rule? And second, in the current context, what does this imply for the normalization of the funds rate in terms of the timing of liftoff, the pace of rate hikes, and the peak level of the funds rate?

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[21] OW also consider uncertainty around u^* and therefore also include a potential response to the change in the unemployment rate. We set aside this issue in our formal analysis but touch on it at the end of this section.

We first generate a baseline path for *r** by assuming that the median funds rate path in the SEP is generated via a Taylor (1999) rule of the form

$$i_t = r_t^* + \pi_t + 0.5(\pi_t - \pi^*) - 2(u_t - u^*) \qquad (6.2)$$

where notation is as before. The solid line in Exhibit 6.1 denotes the baseline path for *r** that results from solving equation (6.2) given the median SEP paths for *i*, π, and *u*, setting *π**=2 and *u** equal to the structural unemployment rate in FRB/US (which converges to the SEP estimate over the longer term). The dotted lines refer to the cases in which Fed officials incorrectly perceive *r** to lie above/below its true level. In our benchmark case, we assume that the potential perception error is +/-150bp, which is well within the range of estimates in Laubach and Williams (2003). However, we also consider the implications of a smaller perception error of +/-50bp as well as a larger error of +/-250bp. We assume that the error is quite long-lasting but ultimately temporary, i.e. that Fed officials eventually converge to the true *r**.

Admittedly, there is an arbitrary aspect to the way we generated the baseline path for *r** because we assume that the median FOMC participant uses equation (6.2) to generate his or her funds rate path. A different baseline path would result if we had assumed a different reaction function, e.g. one with different weights on the inflation and unemployment gap or one that already incorporates some inertia.

Nevertheless, we believe the baseline path shown in Exhibit 6.1 is reasonable for our purposes. First, it is consistent with official Fed communications suggesting that the committee currently views *r** as depressed and only expects a slow increase back to the longer-term level of 1¾% implied by the SEP.[22] Also, it is interesting to note that the current level of *r** in Exhibit 6.1 broadly matches the estimate from the model constructed by Thomas Laubach and John Williams. Both of these authors currently hold senior positions in the Federal Reserve System, Laubach as the Director of the Division of Monetary Affairs and Secretary of the FOMC and Williams as the President of the San Francisco Fed.

Second, the precise baseline path for *r** does not matter much in terms of our main goal, which is to illustrate how greater uncertainty about *r** affects the consequences of following a particular policy rule. Our formal results below show that as uncertainty is increased, more inertia in the policy rule leads to a path for output and inflation that is closer to the Fed's targets.

6B. Fed policy without inertia

Armed with our perceived paths for *r**, we can simulate the behavior of the economy using FRB/US and assuming, for starters, that Fed officials always use equation (6.2) to set the funds rate. In our main set of simulations, we use the version of FRB/US in which the private sector forms its

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[22] For example, Chairman Bernanke said in the September 2013 FOMC press conference that the equilibrium rate would likely still be depressed by the end of 2016—the point at which the committee expected to hit its mandate at that time—and that it "…looks like it will be lower for a time because of these headwinds that will be slowing aggregate demand growth."

expectations by using a backward-looking vector autoregression; later on we also explore the alternative version in which expectations are formed in in a forward-looking or model consistent fashion.

The results are shown in Exhibit 6.2. In the case where Fed officials correctly perceive *r**, both the economy and the funds rate evolve in line with the median projection in the SEP; this is true by construction. In particular, the first rate hike occurs in the middle of 2015, the pace of rate hikes peaks at 140bp per year, and the funds rate converges smoothly to its terminal rate of 3.75% without overshooting.

However, in the case where Fed officials incorrectly perceive *r** to be higher than it really is, they hike much earlier and more aggressively. The premature tightening aborts the decline in the unemployment rate, which results in a failure to return inflation back to the 2% target. In turn, the weaker economy prompts the FOMC to slow—and ultimately partly reverse—the normalization of the funds rate.

Conversely, in the case where Fed officials incorrectly perceive *r** to be lower than it really is, they hike much later. This leads to a bigger drop in unemployment to a level well below its structural rate, a slight overshooting of inflation above 2%, and a sizable overshooting of the funds rate as the FOMC reacts to the overheating of the economy. The upshot of these simulations is that perception errors around *r** introduce significant volatility into the economy's performance and the path of the funds rate.

6C. Fed policy with inertia

So what happens if Fed officials become more aware of the risk of perception errors and therefore introduce inertia into their reaction function? To gain intuition for the basic story, we first look at a simple example in which the FOMC decides to set the inertia term a_0 in equation (6.1) equal to 0.6 but keep all the other parameters of the reaction function unchanged. We later derive the value for a_0 that would optimize a particular monetary policy loss function.

Exhibit 6.3 shows the consequences of setting a_0=0.6 in terms of the funds rate and the behavior of the economy, keeping everything else the same as in Exhibit 6.2. The paths for both the economy and the funds rate when the FOMC misperceives *r** are now more similar to the baseline case in which the FOMC correctly perceives *r**. This more robust performance is the direct consequence of the fact that the policy rule now puts less weight on *r**.

The other difference compared with Exhibit 6.2 is that the normalization for the funds rate now occurs later but ultimately more steeply. In effect, Fed officials wait until there is a stronger message from the behavior of inflation and employment that rate hikes are warranted. This greater patience results in a bigger drop in the unemployment rate and more upward pressure on inflation, and ultimately a greater need to tighten policy in order to limit the overheating of the economy. To make the contrast even clearer, Exhibit 6.4 compares the two paths of the funds rate in the benchmark case where Fed officials are correct in their perception of *r**. Under the inertial policy rule, the first rate hike occurs about six months later than under the non-inertial rule, the pace of rate hikes is about one-third faster, and the funds rate overshoots its terminal value by about 40bp.

6D. Choosing the degree of inertia

Our discussion suggests that inertia has costs and benefits. On the one hand, reduced reliance on *r** leads to more robust economic outcomes. On the other hand, more inertia generally implies some degree of overshooting in the funds rate as well as the economy, which is undesirable. One way to summarize this trade-off is to calculate the degree of inertia at which the marginal benefit from greater inertia (i.e. more robust outcomes) is equal to the marginal cost of greater inertia (i.e. more overshooting).

To answer this question, we use the intertemporal monetary policy loss function

$$Loss = \sum_{t=0}^{T} \beta^t \left(\left(u_t - u^*\right)^2 + \left(\pi_t - \pi^*\right)^2 + 0.5 * \left(\Delta i_t\right)^2 \right) \quad (6.3)$$

where β is the discount factor and all other notation is as before. Equation (6.3) specifies the loss from a particular policy based on the discounted sum of future deviations from the Fed's goals for inflation and unemployment as well as quarter-to-quarter changes in the federal funds rate. In our benchmark loss function, we assume that unemployment and inflation misses are equally costly from the Fed's perspective, an assumption dubbed the "balanced approach" by Fed Chair Janet Yellen. We also assume that a 50bp quarter-to-quarter change in the funds rate is as costly as a ¼-point miss on either unemployment or inflation.[23] Finally, in evaluating the consequences of changing the degree of inertia, we assume that each of the three cases in Exhibit 6.1—i.e. that Fed officials are correct, too high, or too low in their perception of *r**—is equally likely.

Under these assumptions, Exhibit 6.5 shows that the degree of inertia that would balance the Fed's competing objectives would be equal to a_0=0.61 if the perception error is 150bp. Moreover, the specified degree of inertia rises with the size of the perception errors around *r**, from a relatively negligible 0.16 if the error is 50bp to a very substantial 0.76 if the error is 250bp.[24] We therefore conclude that the risk of perception errors around r* might lead the Fed to consider a more inertial rule and a later but steeper normalization of the funds rate relative to the current SEP baseline.

6E. Robustness

How robust is this result? We focus on two aspects of this question. First, we vary the monetary policy loss function in equation (6.3) with respect to the cost of quarter-to-quarter changes in the federal funds rate. Exhibit 6.6 shows that if we increase this cost from 0.5 to 1, as in the 2012 "optimal control" simulations by then-Vice Chair Yellen, the optimal inertia coefficient a_0 declines to 0.53. This is intuitive because a greater inherent cost of funds rate changes penalizes overshooting by more relative to our baseline assumption. Conversely, if we reduce the weight to 0—i.e. if we assume

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[23] This assumption lies in between the very low aversion to changes in the funds rate in OW and the higher aversion in Yellen (2012). We explore the implications of varying it below.

[24] A different baseline for *r** (e.g. resulting from a different assumption about the baseline policy reaction function) would generate different optimal values for a_0. However, it would not change the result that greater uncertainty around *r** increases the optimal value for a_0.

that the Fed only cares about its dual employment and inflation mandate and not about volatility in the funds rate—the optimal inertia coefficient a_0 increases to 0.75. Our assessment of these numbers is that our basic results seem fairly robust to plausible variation in the cost of funds rate changes.

Second, we switch from backward-looking VAR expectations to forward-looking model consistent expectations. Even leaving the aside the issue of uncertainty around *r**, this change in the model setup already favors a greater amount of inertia in the policy reaction function. The reason is that the private sector now anticipates future monetary policy moves, which enables the Fed to wait longer before having to step on the brakes. Beyond this, however, the basic qualitative point of our analysis is unchanged—greater uncertainty around *r** favors an even more inertial approach. As shown in Exhibit 6.7, an increase in the potential error from +/-50bp to +/-150bp and on to +/-250bp raises the optimal value of a_0 substantially.

6F. Conclusions and extensions

Summing up, our analysis confirms the findings by OW that uncertainty around *r** provides a reason the Fed might want to include more inertia in the policy reaction function, i.e. for putting greater weight on the lagged level of the funds rate relative to the uncertain estimate of *r**. This result seems to be robust to changes in the loss function and the expectations formation process. In the current context, our finding implies that if policymakers are uncertain about *r**, they may be able to keep output closer to potential by adopting a later but steeper path for normalizing the funds rate. Although such a later but steeper path tends to result in an overshooting of the funds rate, the benefit is that it limits the risk of either exiting prematurely or belatedly because of misperceptions around the true value of *r**.

It is worth noting that uncertainty around r* is only one potential rationale for a later but steeper normalization path. In fact, the work of OW suggests that the policy implications of uncertainty around the structural unemployment rate *u**—or more broadly around the correct measure of labor market utilization—are similar to those of uncertainty around the equilibrium interest rate *r**. In both cases, there is an incentive to keep interest rates low until the behavior of the economy—and particularly the behavior of inflation—sends a strong signal that tightening is appropriate.

Although it is not the focus of the present study, uncertainty about the size of the employment gap is also a very important issue at the current juncture. Some indicators of labor market utilization such as the headline unemployment rate and the job openings rate are already at or near normal levels, but others such as the broad underemployment rate U6 and the growth rate of hourly wages are still signaling substantial labor market slack. In our formal analysis, we assumed for simplicity that the gap between the headline unemployment rate and the structural unemployment rate embedded in FRB/US provides an accurate measure of how far the FOMC is from its goal of maximum employment. But this is probably not realistic, and introducing uncertainty into this assessment would likely reinforce our conclusion that a later but steeper normalization path could be associated with a higher value of the monetary authority's objective function.

Another important question is whether there are other ways, besides introducing inertia, of making monetary policy rules more robust to uncertainty around the equilibrium rate. One possibility is to place some weight on changes in financial conditions. For example, if it is true that changes in *r** partly reflect variation in the marginal product of capital, and if the equity market "sniffs out" such variation from corporate earnings results before it becomes visible in the macroeconomic data, a change in equity prices might provide an early indication of a change in *r**. If Fed officials placed some weight on changes in financial conditions when setting the funds rate, they would automatically incorporate this information and might thereby improve upon a policy rule that relies on signals from the macroeconomic data alone. Of course, since the relationship between the equity market and *r** is likely to be a noisy one, there is also the risk that an excessively large weight on equity market moves will cause Fed officials to overreact to the fleeting ups and downs of the markets. Exploring the resulting cost-benefit calculations would be a useful avenue for future research.

References

Ang, Andrew A., and Geert Bekaert, 2002, “Regime Switches in Interest Rates,” *Journal of Business and Economic Statistics* 20, 163-182.

Andrews, Donald W. K., 1993, “Parameter Instability and Structural Change with Unknown Change Point,” *Econometrica* 61(4), 821-856.

Andrews, Donald W.K., 2003, “Tests for Parameter Instability and Structural Change with Unknown Change Point: A Corrigendum,” *Econometrica* 71(1), 395-397.

Angry Bear, 2009, “Current Recession vs the 1980-82 Recession,” http://angrybearblog.com/2009/06/current-recession-vs-1980-82-recession.html

Baker, Scott R., Nicholas Bloom, and Steven J. Davis, 2013, "Measuring Economic Policy Uncertainty,", manuscript, University of Chicago.

Bai, Jushan and Pierre Perron, 1998, “Testing for and Estimation of Multiple Structural Changes,” *Econometrica* 66(1), 47-78

Bai, Jushan, and Pierre Perron, 2003, “Computation and Analysis of Multiple Structural Change Models,” *Journal of Applied Econometrics* 18, 1-22.

Balke, Nathan and Robert J. Gordon, 1989, “The Estimation of Prewar GNP: Methodology and New Evidence,” *Journal of Political Economy* 97, 38-92.

Barsky, Robert B., 1987, “The Fisher Hypothesis and the Forecastability and Persistence of Inflation,” *Journal of Monetary Economics* 19, 3-24.

Barsky, Robert, Alejandro Justiniano, and Leonardo Melosi, 2014, “The Natural Rate of Interest and Its Usefulness for Monetary Policy,” *American Economic Review: Papers & Proceedings* 104(5): 37-43.

Bernanke, Ben S., 2005, “The Global Savings Glut and the U.S. Current Account Deficit,” Sandridge Lecture (http://www.federalreserve.gov/boarddocs/speeches/2005/200503102/).

Campbell, John Y. and John Cochrane, 1999, “By Force of Habit: a Consumption-based Explanation of Aggregate Stock Market Behavior,” *Journal of Political Economy* 107 (2), 205–251.

Canzoneri, Matthew B., Robert E. Cumby, Behzad T. Diba, 2007, “Euler Equations and Money Market Interest Rates: a Challenge for Monetary Policy Models,” *Journal of Monetary Economics* 54, 1863-1881.

Caballero, Ricardo, 2006, “On the macroeconomics of asset shortages,” NBER Working Paper 12753

Caporale, Tony, and Kevin B. Grier, 2000), "Political Regime Change and the Real Interest Rate," *Journal of Money, Credit, and Banking* 32, 320-334.

Christiano, Lawrence J., Martin Eichenbaum and Charles L. Evans, 2005, "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy," *Journal of Political Economy* 113, 1-45.

Clarida, Richard, 2014, "Navigating the New Neutral", *Economic Outlook*, PIMCO, November.

Clarida, Richard, Jordi Galí and Mark Gertler, 2002, "A Simple Framework for International Monetary Policy Analysis," *Journal of Monetary Economics* 49, 879–904.

Clark, Gregory, 2005, "The Interest Rate in the Very Long Run: Institutions, Preferences, and Modern Growth," working paper, U.C. Davis.

Clark, Todd E. and Sharon Kozicki, 2005, "Estimating equilibrium real interest rates in real time," *North American Journal of Economics and Finance* 16, 395-413.

Curdia, Vasco, Andrea Ferrero, Ging Cee Ng, and Andrea Tambalotti, 2014, "Has U.S. Monetary Policy Tracked the Efficient Interest Rate?", FRB San Francisco Working Paper 2014-12.

De Paoli, Bianca and Pawel Zabczyk, 2013, "Cyclical Risk Aversion, Precautionary Saving, and Monetary Policy," *Journal of Money, Credit and Banking* 45(1), pages 1-36.

Dennis, Richard, 2009, "Consumption Habits in a New Keynesian Business Cycle Model," *Journal of Money, Credit and Banking* 41(5), 1015-1030.

Ferguson Jr., Roger W., 2004, "Equilibrium Real Interest Rate: Theory and Application", http://www.federalreserve.gov/boarddocs/speeches/2004/20041029/.

Feroli, Michael E., Ethan S. Harris, Amir Sufi and Kenneth D. West, 2012, "Housing, Monetary Policy, and the Recovery," *Proceedings of the US Monetary Policy Forum*, 2012, 3-52.

Galí, Jordi, 2008, *Monetary Policy, Inflation and the Business Cycle*, Princeton: Princeton University Press.

Garcia, Rene, and Pierre Perron, 1996, "An Analysis of the Real Interest Rate Under Regime Shifts," *Review of Economics and Statistics* 78, 111-125.

Gordon, Robert J., 2012, "Is U.S. Economic Growth Over? Faltering Innovation Confronts the Six Headwinds," NBER Working Paper No. 18315.

Gordon, Robert J., 2014, "The Demise of U.S. Economic Growth: Restatement, Rebuttal, and Reflections, " NBER Working Paper No. 19895.

Greenspan, Alan, 2005, "Testimony of Chairman Alan Greenspan," Committee on Banking, Housing and Urban Affairs, U.S. Senate

Gürkaynak, Refet S., Brian Sack, and Jonathan H. Wright, 2010, "The TIPS Yield Curve and Inflation Compensation," *American Economic Journal: Macroeconomics* 2, 70-92.Hansen, Alvin**,** 1939, "Economic Progress and Declining Population Growth." *American Economic Review* 29(1), 1-15.

Harris, Ethan, 2014, "The Great Wobble," US Economic Watch, B of A Merrill Lynch, October 13.

Harris, Ethan S., Bruce C. Kasman, Matthew D. Shapiro and Kenneth D. West, 2009, "Oil and the Macroeconomy: Lessons for Monetary Policy" *US Monetary Policy Forum.*

Hatzius, Jan, Sven Jari Stehn, and Jose Ursua, 2014, "Some Long-Term Evidence on Short-Term Rates," *Goldman Sachs US Economics Analyst*, 14/25, June 20.

Hatzius, Jan, "Housing Holds the Key to Fed Policy, 2006," Goldman Sachs Global Economics Paper No. 137, February 2, 2006.

Kocherlakota, Narayana R., 1996, "The Equity Premium: It's Still a Puzzle," *Journal of Economic Literature* 34(1), 42-71.

Krugman, Paul R., Kathryn M. Dominquez and Kenneth Rogoff, 1998, "Japan's Slump and the Return of the Liquidity Trap." *Brookings Papers on Economic Activity* 1998(2), 137-205.

Krugman, Paul R., 2013, "Secular Stagnation, Coalmines, Bubbles, and Larry Summers," New York Times, http://krugman.blogs.nytimes.com/2013/11/16/secular-stagnation-coalmines-bubbles-and-larry-summers.

Laubach, Thomas and John C. Williams, 2003, "Measuring the Natural Rate of Interest," *The Review of Economics and Statistics* 85(4): 1063-1070

Leduc, Sylvain and Glenn D. Rudebusch, 2014, "Does Slower Growth Imply Lower Interest Rates?," FRBSF Economic Letter 2014-33.

McCulley, Paul, 2003, "Needed: Central Bankers with Far Away Eyes," PIMCO, Global Central Bank Focus.

Mehra, Rajnish and Edward C. Prescott, 2003, "The Equity Premium in Retrospect," 889-938 in G. Constantinides, M. Harris and R. Stultz, (eds) *Handbook of the Economics of Finance*, vol 1B, Amsterdam: Elsevier.

Misra, Priya et. al., 2015, "A brave new World," *US Rates Weekly*, Bank of America Merrill Lynch, January 23.

Mulligan, Casey, 2009, "Worse Than 1982?", http://economix.blogs.nytimes.com/2009/06/03/worse-than-1982/?_r=0.

Orphanides, Athanasios and John C. Williams, 2002, "Robust Monetary Policy Rules with Unknown Natural Rates," *Brookings Papers on Economic Activity* 2002(2), 63-145.

Orphanides, Athanasios and John C. Williams, 2006, “Monetary Policy with Imperfect Knowledge,” *Journal of the European Economic Association* 4 (2-3), 366–375.

Orphanides, Athanasios and John C. Williams, 2007, “Robust Monetary Policy with Imperfect Knowledge,” *Journal of Monetary Economics*, 54, 1406-1435.

Rapach, David E., and Mark E. Wohar, 2005, “Regime Changes in International Real Interest Rates: Are They a Monetary Phenomenon?,” *Journal of Money, Credit and Banking* 37(5) 887-906.

Reinhart, Carmen M. and Kenneth S. Rogoff, 2009, *This Time Is Different: Eight Centuries of Financial Folly.* Princeton, N.J.: Princeton University Press.

Reinhart, Carmen M., and Kenneth S. Rogoff,. 2014,” Recovery from Financial Crises: Evidence from 100 Episodes,” *American Economic Review: Papers and Proceedings* 104(5), 50-55.

Reinhart, Carmen M., and M. Belen Sbrancia, 2015, “The Liquidation of Government Debt,” forthcoming, *Economic Policy.*

Romer, Christina D., 1989, “The Prewar Business Cycle Reconsidered: New Estimates of Gross National Product, 1869-1908,” *Journal of Political Economy* 97, 1-37.

Smets, Frank and Raf Wouters, 2003, “An Estimated Dynamic Stochastic General Equilibrium Model of the Euro Area,” *Journal of the European Economic Association* 1(5), 1123-1175.

Summers, Lawrence, 2013a, “Reflections on the ‘New Secular Stagnation Hypothesis,” 27-39 in C. Teulings and R. Baldwin (eds.), *Secular Stagnation: Facts, Causes, and Cures*, (eBook, www.voxeu.org/sites/default/files/Vox_secular_stagnation.pdf), CEPR.

Summers, Lawrence, 2013b, “Larry Summers Remarks at IMF Annual Research Conference,” https://www.facebook.com/notes/randy-fellmy/transcript-of-larry-summers-speech-at-the-imf-economic-forum-nov-8-2013/585630634864563.

Summers, Lawrence, 2014, “U.S. Economic Prospects: Secular Stagnation, Hysteresis, and the Zero Lower Bound,” *Business Economics* Vol. 49, No. 2.

Taylor, John B., 1993, “Discretion versus policy rules in practice,” Carnegie-Rochester Conference Series on Public Policy 39, 195–214.

Taylor, John B., 1999, "A Historical Analysis of Monetary Policy Rules", 319-341 in J. B. Taylor, ed., *Monetary Policy Rules*, Chicago: University of Chicago Press.

Weil, Philippe, 1989, “The Equity Premium Puzzle and the Risk-free Rate Puzzle,” *Journal of Monetary Economics* 24,401-421.

White, Halbert, 1980, “A Heteroskedasticity-Consistent Covariance Matrix Estimator and a Direct Test for Heteroskedasticity,” *Econometrica* 48, 817-838.

Wieland, Johannes, 2014, “Are Negative Supply Shocks Expansionary at the Zero Lower Bound?,” working paper, UCSD.

Zarnowitz, Victor, 1997, “Appendix”, 731-737 in Glasner, D. (ed.) *Business Cycles and Depressions: An Encyclopedia*, Garland Publishing: New York.

Exhibit 1.1. Economic projections of Federal Reserve Board members and Federal Reserve Bank presidents.

	2014	2015	2016	Longer run
GDP				
December 2014	2.35	2.80	2.75	2.15
December 2012	3.25	3.35	—	2.40
Unemployment rate				
December 2014	5.80	5.25	5.10	5.35
December 2012	7.05	6.30	—	5.60
PCE inflation				
December 2014	1.25	1.30	1.85	2.00
December 2012	1.75	1.85	—	2.00
Core PCE inflation				
December 2014	1.55	1.65	1.85	—
December 2012	1.80	1.90	—	—
Fed funds rate				
December 2014	0.13	1.13	2.50	3.75
December 2012	0.25	1.00	—	4.00

Note: Q4/Q4 percent changes, except unemployment (4Q average) and the fed funds rate (eop). Middle of the central tendency range, except fed funds rate (median).

Exhibit 2.1. Country mnemonics and start dates for real interest rate series.

	Mnemonic (1)	**Country** (2)	**Sample Start, Annual Data** (3)	**Sample Start, Quarterly Data** (4)
(1)	AUS	Australia	1893	1971:2
(2)	AUT	Austria	n.a.	1971:2
(3)	BEL	Belgium	n.a.	1971:2
(4)	CAN	Canada	1900	1971:2
(5)	CHE	Switzerland	1912	1981:2
(6)	DEU	Germany	1858	2002:2
(7)	DNK	Denmark	1864	1979:2
(8)	ESP	Spain	1874	1977:2
(9)	FIN	Finland	1946	1971:2
(10)	FRA	France	1861	1971:2
(11)	GBR	United Kingdom	1858	1971:2
(12)	IRL	Ireland	n.a.	2001:2
(13)	ITA	Italy	1893	1971:2
(14)	JPN	Japan	1900	1971:2
(15)	KOR	South Korea	n.a.	1981:2
(16)	NLD	Netherlands	1858	1971:2
(17)	NOR	Norway	1858	1971:2
(18)	PRT	Portugal	1960	1971:2
(19)	SWE	Sweden	1858	1982:2
(20)	USA	USA	1858	1958:1
(21)	NZL	New Zealand	1939	n.a.

Exhibit 2.2. U.S. and U.K. nominal interest rate, inflation rate, persistence of inflation, and ex-ante real rate, annual 1800-2014.

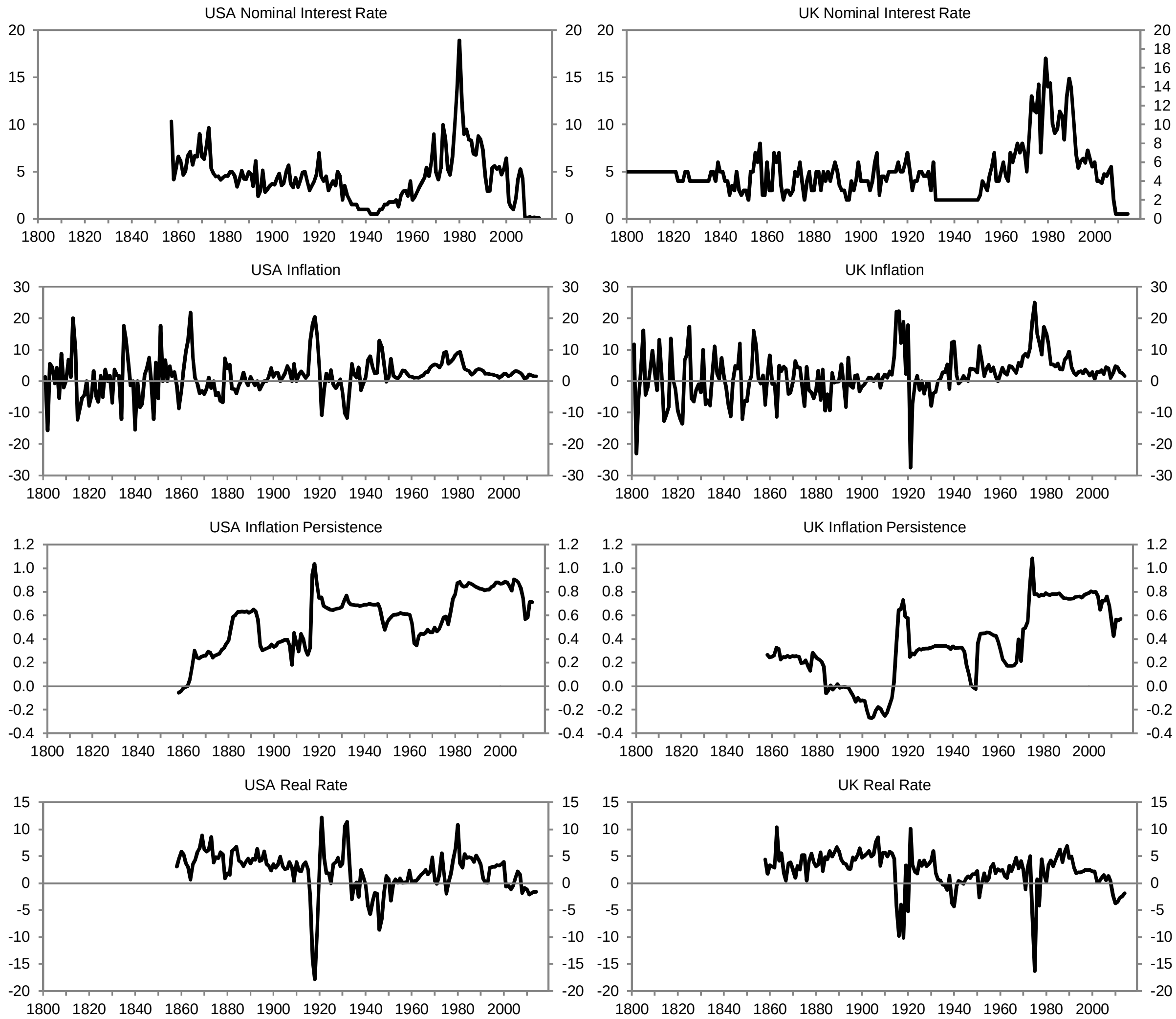


Exhibit 2.3. Nominal interest rates for 15 different countries, annual 1858-2014.

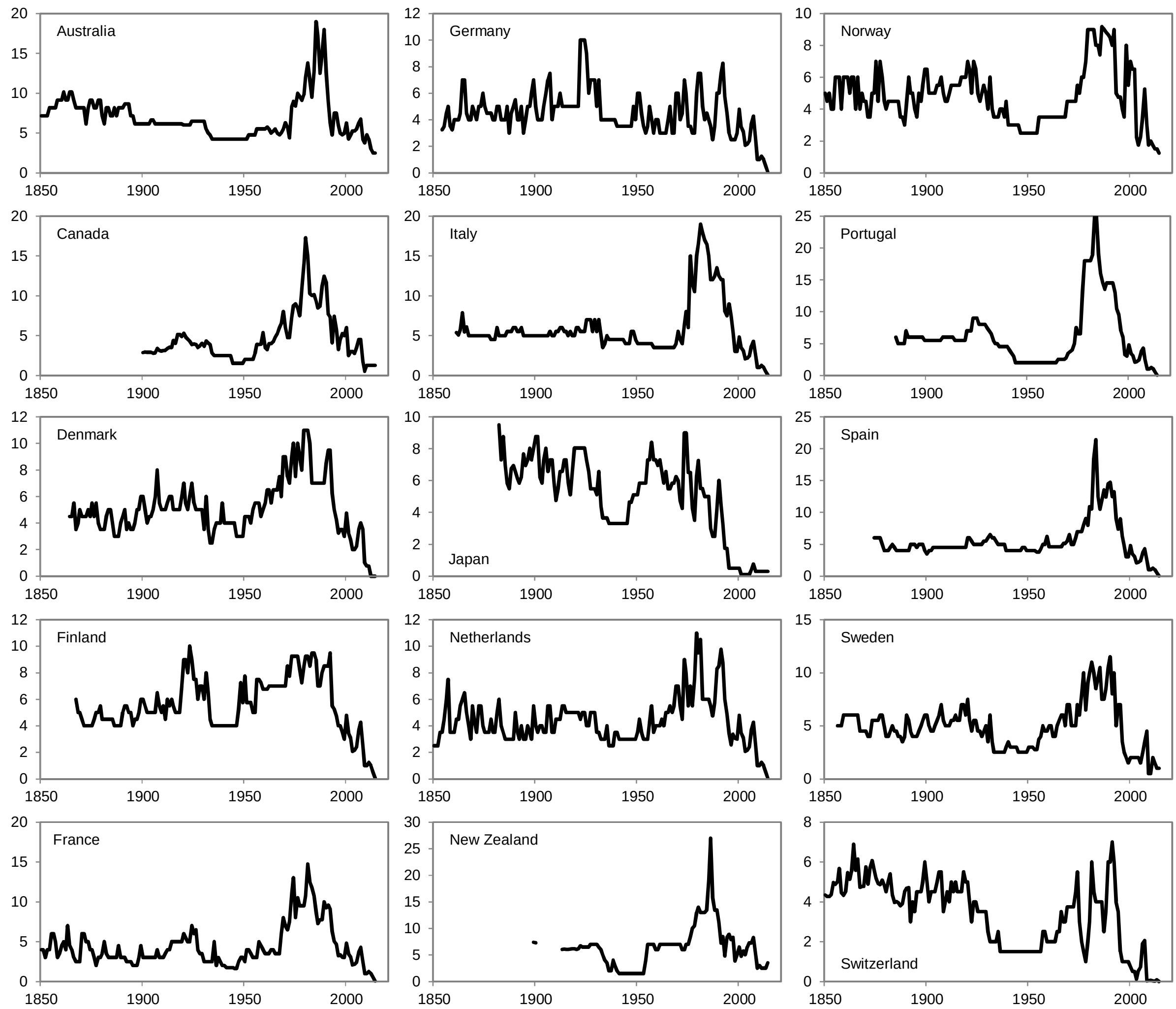


Exhibit 2.4. Inflation rate for 15 different countries, annual 1858-2014.

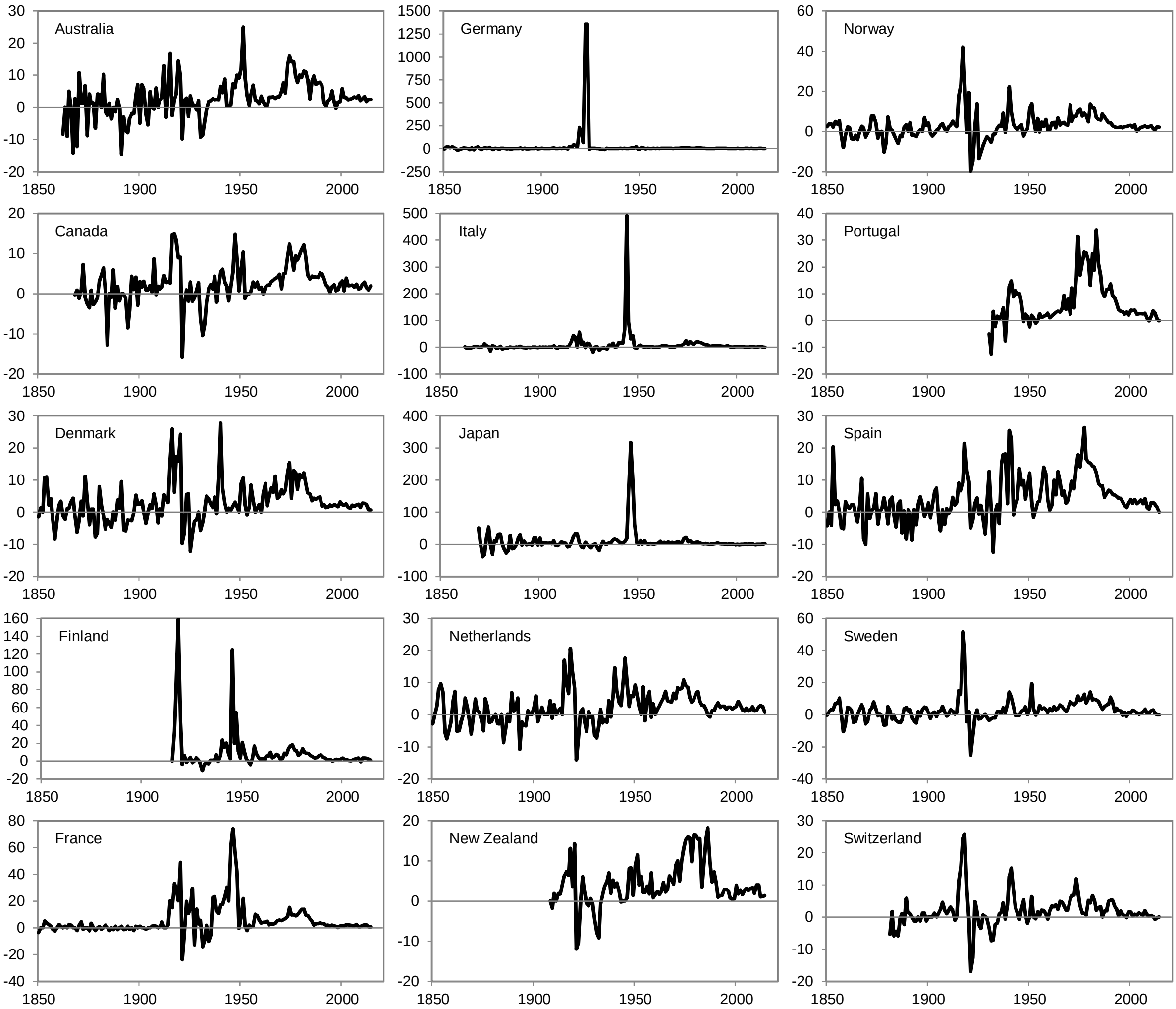


Exhibit 2.5. Inflation persistence for 15 different countries, annual 1858-2014.

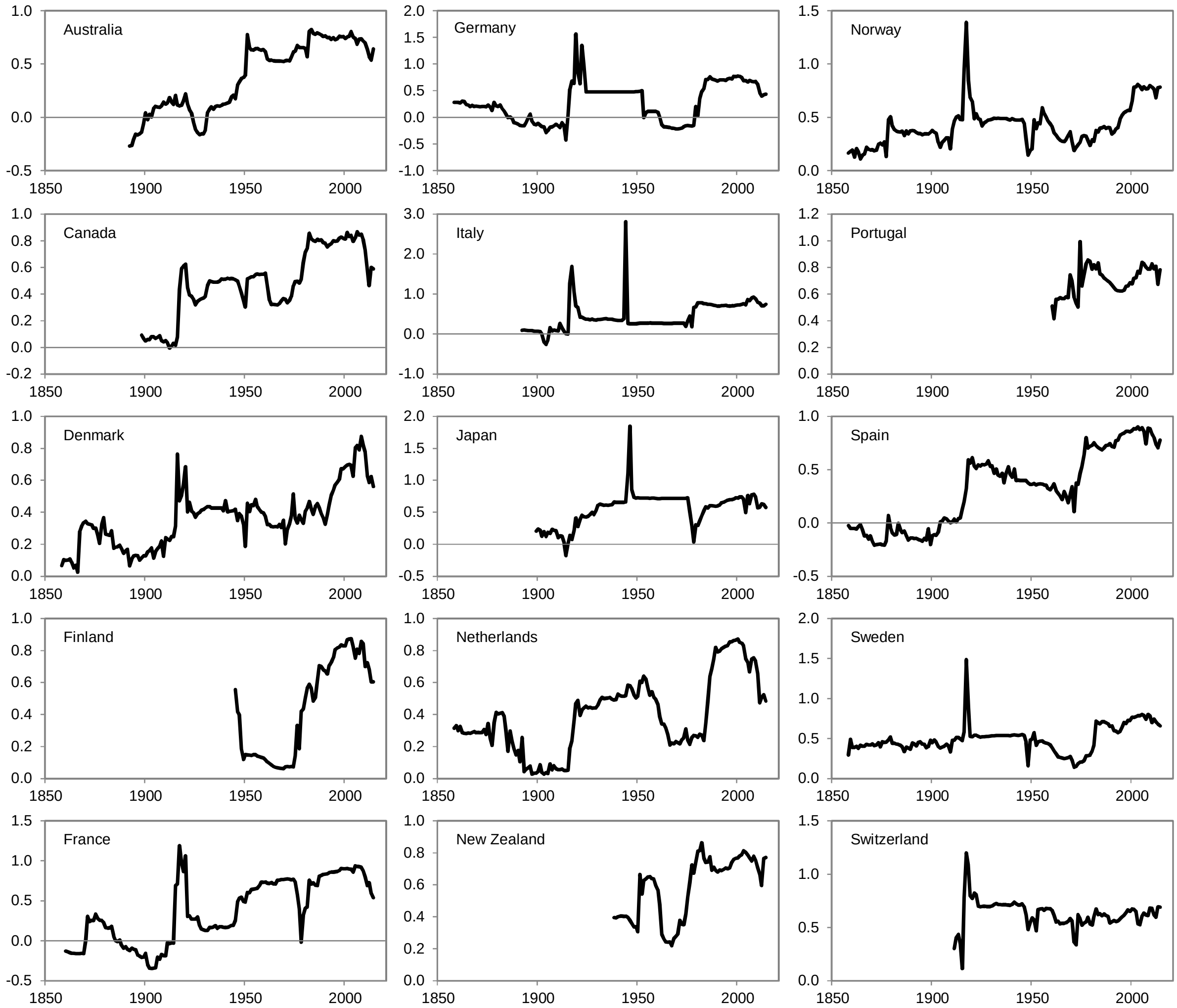


Figure 2.6. Ex-ante real interest rate for 15 different countries, annual 1858-2014.

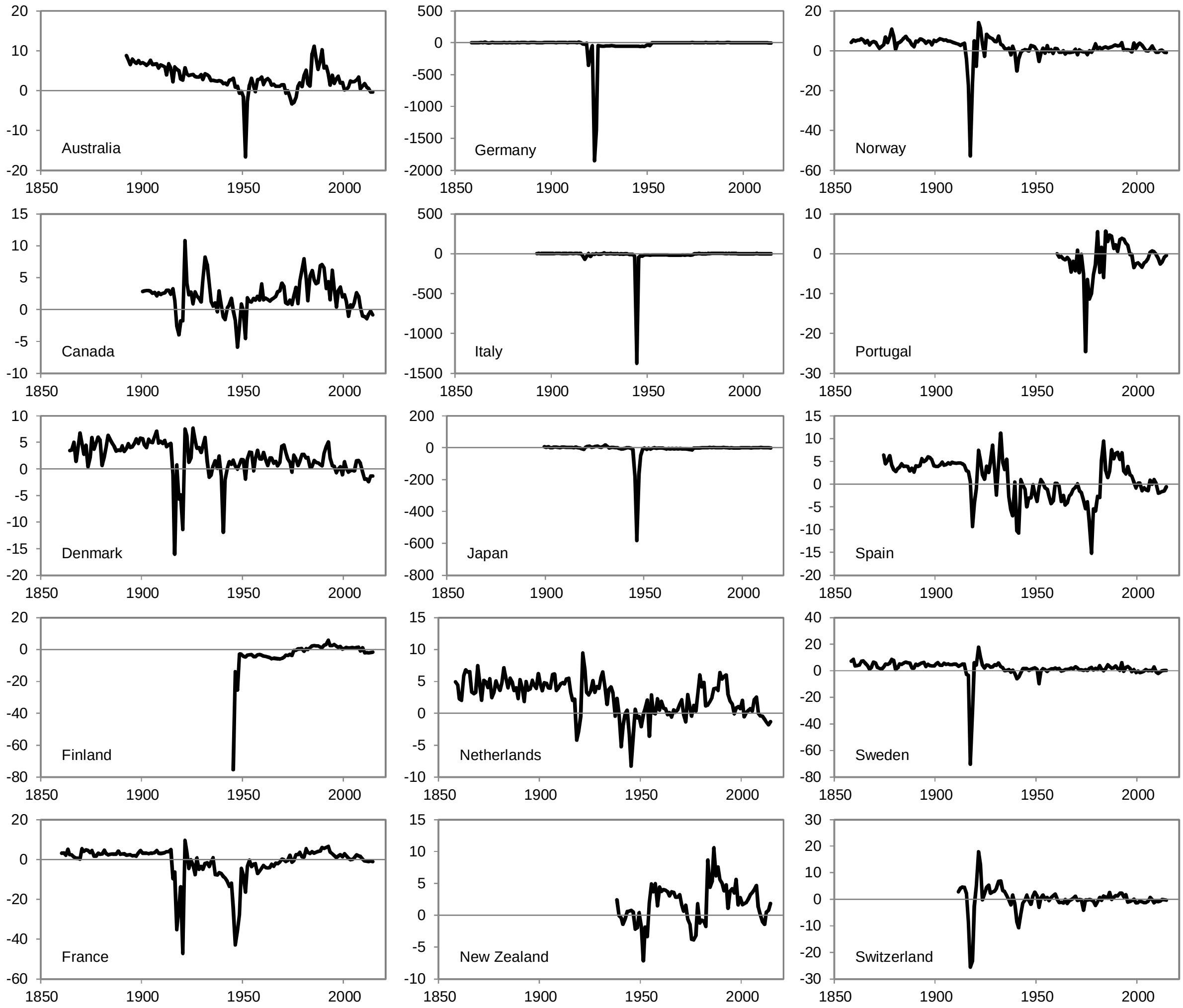


Exhibit 2.7. U.S. ex-ante real interest rate as inferred from annual and quarterly data.

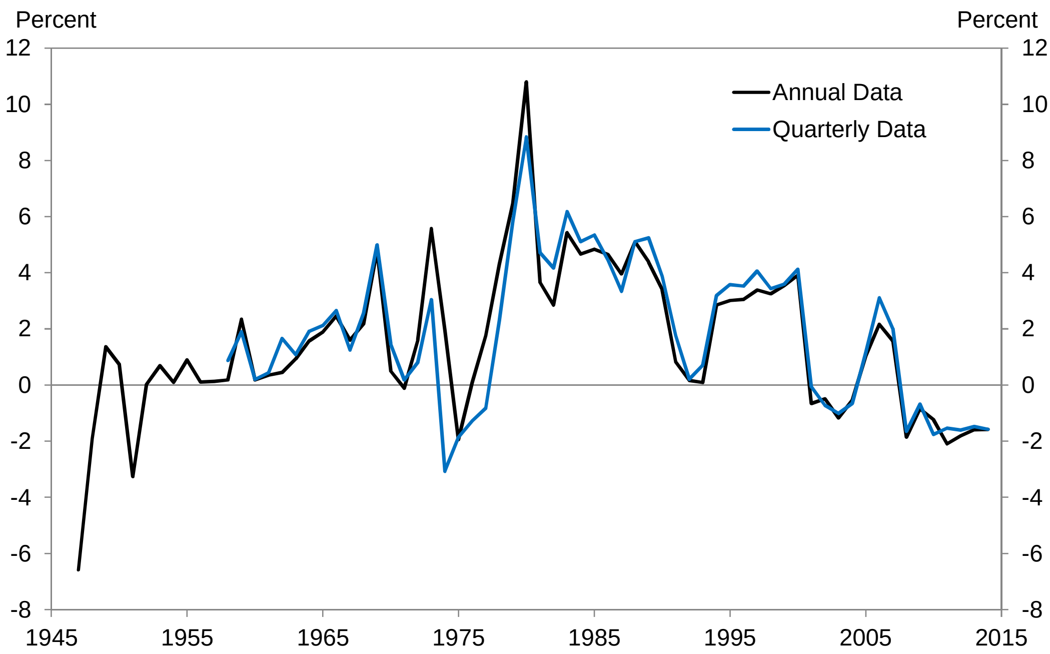


Exhibit 2.8. Expected and actual inflation for quarterly U.S. data, 1958:2 to 2014:3.

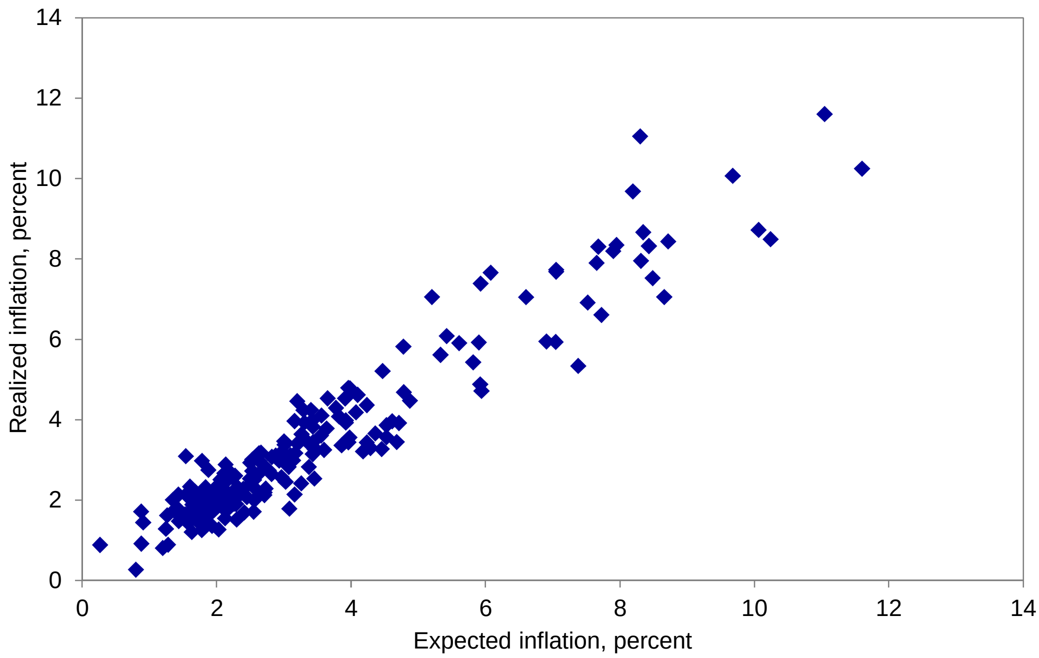
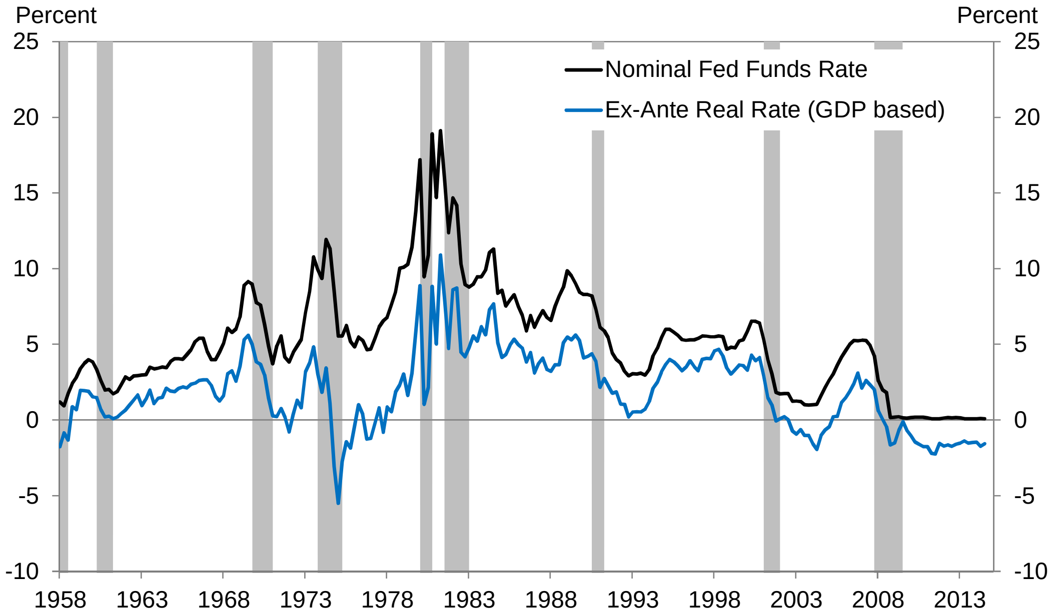


Exhibit 2.9. Nominal U.S. interest rate (average fed funds rate for last month of the quarter) and ex-ante real interest rate, 1958:1-2014:3.



Note: Gray shading indicates NBER recession.

Exhibit 2.10. Basic statistics for U.S. quarterly ex-ante real rates, 1958:2-2014:3.

	r	i	$E_t\pi_{t+1}$	π
Mean	1.95	5.27	3.32	3.30
S.D.	2.55	3.60	2.12	2.32

Notes:

1. r=ex-ante real rate=i-$E_t\pi_{t+1}$, i=nominal rate, π=inflation, $E_t\pi_{t+1}$=expected inflation.
2. Expected inflation is computed from a univariate AR(4) in inflation estimated from rolling regressions. For t=1968:2 through t=2014:3, the rolling sample size is 80 quarters, with the most distant observation dropped each time a new observation is added to the end of the sample. For t=1958:2 to t=1967:4, the sample starts at 40 quarters and then grows to 79 quarters, with no observations dropped each time a new observation is added to the end of the sample.

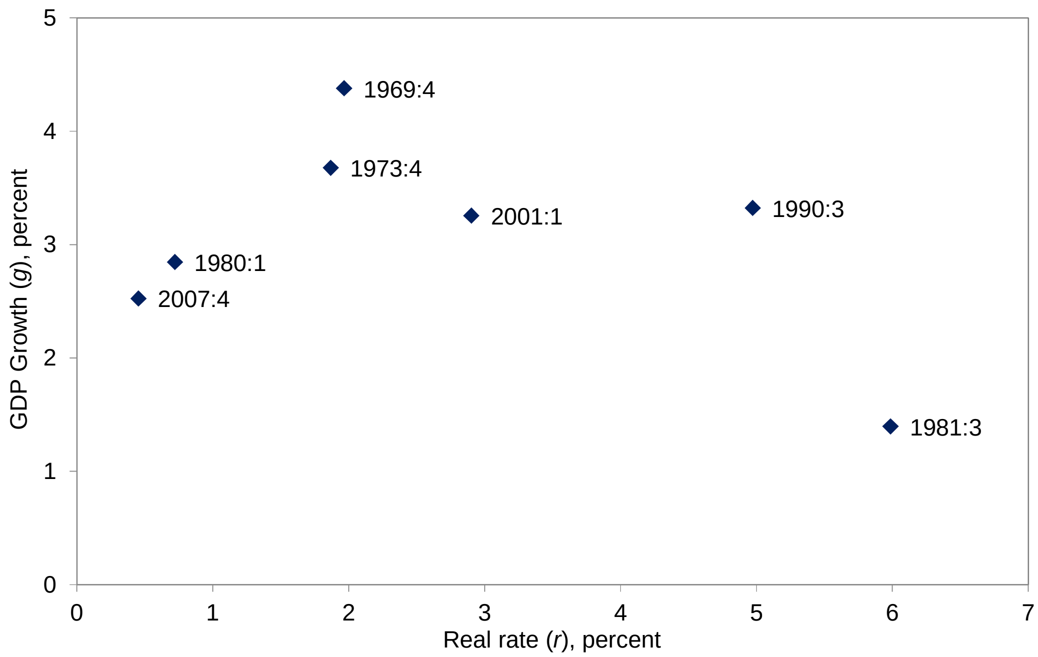
Exhibit 3.1. U.S. GDP growth and ex-ante real rate *r*, peak-to-peak averages.

	QUARTERLY			ANNUAL		
	Peak Date (1)	GDP growth (2)	*r* (3)	Peak Date (4)	GDP growth (5)	*r* (6)
(1)	2007:4	2.52	0.45	2007	2.66	0.43
(2)	2001:1	3.25	2.90	2001	3.16	2.13
(3)	1990:3	3.32	4.97	1990	3.36	4.36
(4)	1981:3	1.40	5.99	1981	1.16	7.23
(5)	1980:1	2.84	0.72	1979	2.93	2.10
(6)	1973:4	3.68	1.87	1973	3.52	1.89
(7)	1969:4	4.38	1.97	1969	4.62	1.80
(8)	1960:2	2.86	n.a.	1960	2.82	0.90
(9)	1957:3	2.43	n.a.	1957	2.63	0.30
(10)	1953:2	5.39	n.a.	1953	4.83	-0.09
(11)				1948	-2.58	-4.77
(12)				1944	9.97	-1.70
(13)				1937	0.67	2.89
(14)				1929	2.87	3.93
(15)				1926	3.56	1.79
(16)				1923	5.48	6.22
(17)				1920	-2.54	-2.35
(18)				1918	3.64	-5.60
(19)				1913	4.20	2.64
(20)				1910	1.95	2.39
(21)				1907	3.74	3.11
(22)				1903	4.51	3.68
(23)				1899	4.72	3.75
(24)				1895	2.69	4.90
(25)				1892	3.90	4.42
(26)				1890	2.33	4.07
(27)				1887	2.47	4.87
(28)				1882	5.04	3.84
(29)				1873	5.20	6.73

Notes:

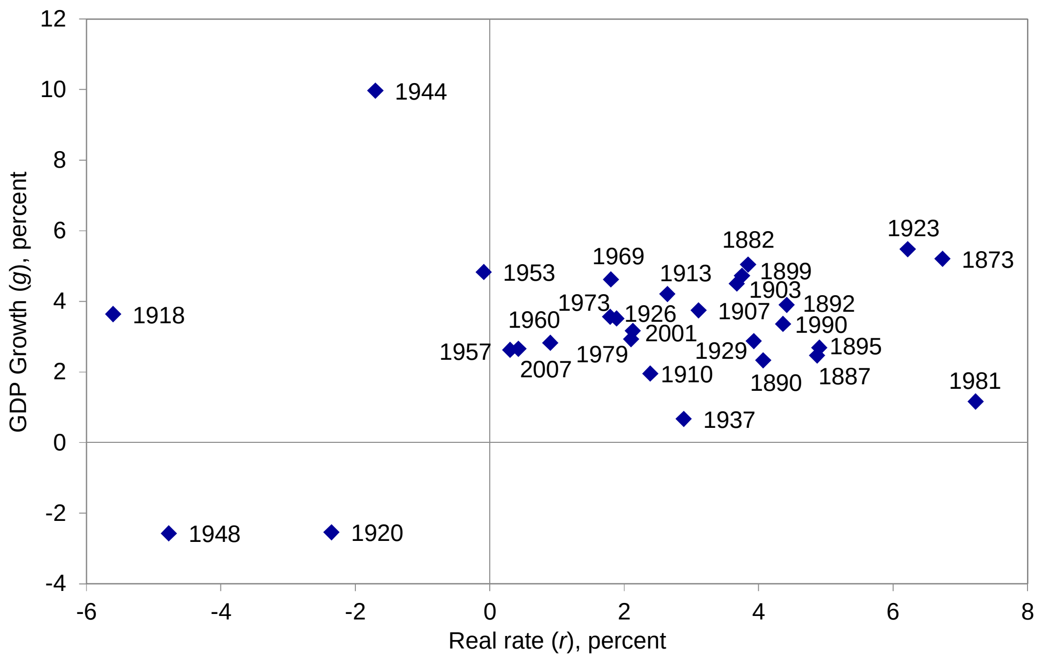
1. Each entry is a peak to peak average, expressed at annual rates. For example, the 2.52 figure in row (1), column (2), means that average GDP growth over the 27 quarters from 2001:2 through 2007:4 was 2.52%; the 0.45 figure in column (3) is the corresponding value for the ex-ante real rate *r* over this period.
2. The ex-ante real rate *r* is the short term nominal policy rate minus next quarter's (column (3)) or next year's (column (6)) expected inflation. Expected inflation is computed from an autoregression in log differences of the GDP deflator, using rolling samples. See Section 2 for details.
3. The nominal policy rate is end of period Federal funds rate from 1954 forward. See text for sources of nominal rates in earlier periods. Real GDP growth data from Balke and Gordon (1989) prior to 1930, from FRED afterwards. Business cycle peak dates from NBER and Zarnowitz (1997).

Exhibit 3.2. Peak-to-peak average real GDP growth versus average *r*, quarterly data, 1969:4-2007:4.



Note: This figure plots the quarterly data in Exhibit 3.1.

Exhibit 3.3. Peak-to-peak average real GDP growth versus average *r*, annual data, 1873-2007.



Note: This figure plots the annual data in Exhibit 3.1.

Exhibit 3.4. Correlation of U.S. GDP growth with *r*, peak-to-peak averages.

	Start (1)	End (2)	No. of Peaks (3)	Freq. (4)	Specification (5)	Correlation (6)
(1)	1969:4	2007:4	7	Q	Baseline (Exhibit 3.2)	-0.40
(2)	1969:4	2007:4	6	Q	Omit 1981:3	0.32
(3)	1969:4	2007:4	6	Q	Potential GDP growth, omit 1981:3	-0.01
(4)	1960:2	2007:4	7	Q	Ex-post real interest rate, omit 1981:3	0.17
(5)	1873	2007	29	A	Baseline (Exhibit 3.3)	0.23
(6)	1873	2007	28	A	Omit 1981	0.30
(7)	1873	2007	27	A	Omit 1920, 1948	-0.23
(8)	1873	2007	26	A	Omit 1937, 1944, 1948	0.29
(9)	1953	2007	9	A	Postwar, omit 1981	-0.04
(10)	1969	2007	6	A	Quarterly sample, omit 1981	0.18
(11)	1873	2007	29	A	Romer (1989) data used 1870-1929	0.21
(12)	1873	2007	29	A	Year ahead GDP growth	0.10

Notes:

1. In column (6), correlations are computed with a peak to peak average considered a single observation. In line (1), for example, the value of -0.40 is the sample correlation between the 7 values of GDP growth and of the real rate given in columns (2) and (3) of Exhibit 3.1.
2. In column (4), "A" denotes annual data, "Q" quarterly data.

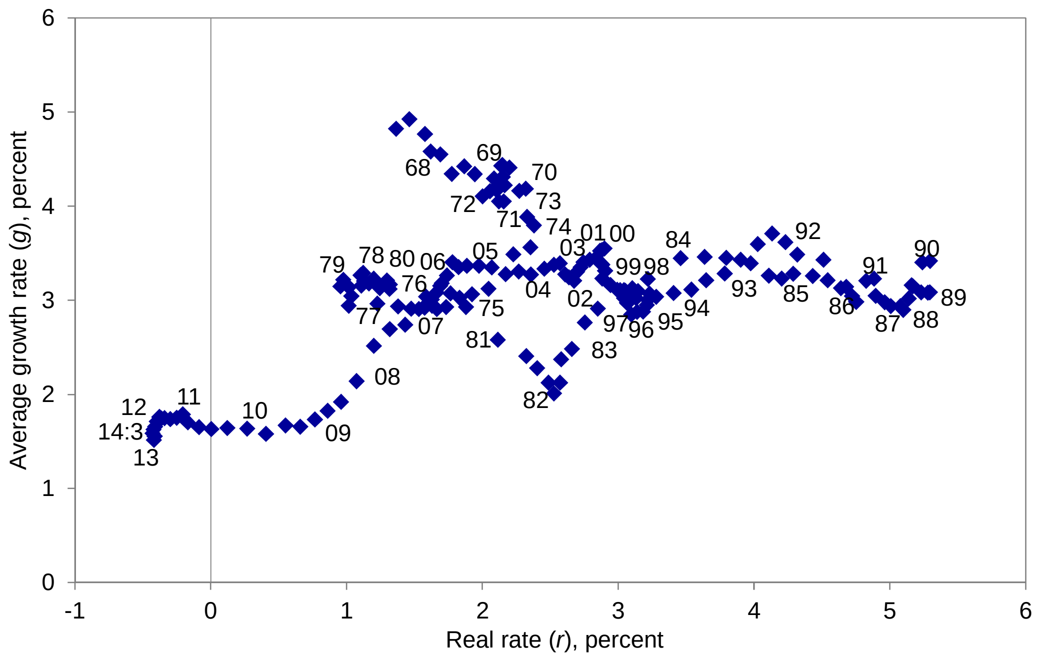
Exhibit 3.5. Correlation of U.S. GDP growth with *r,* overlapping 10-year averages.

	Start (1)	End (2)	No. of Peaks (3)	Freq. (4)	Specification (5)	Correlation (6)
(1)	1968:1	2014:3	187	Q	40Q backward averages (Exhibit 3.6)	0.39
(2)	1968:1	2007:4	160	Q	Omit 2008:1-2014:3	-0.19
(3)	1968:1	2014:2	186	Q	40 Q backward avgerages, ex-post r	0.27
(4)	1879	2014	136	A	10 year backward averages (Exhibit 3.7)	-0.25
(5)	1955	2014	60	A	Post-World War II	0.18
(6)	1889	2014	114	A	10 yr. backward avg, omit 1930-1950	0.31

Notes:

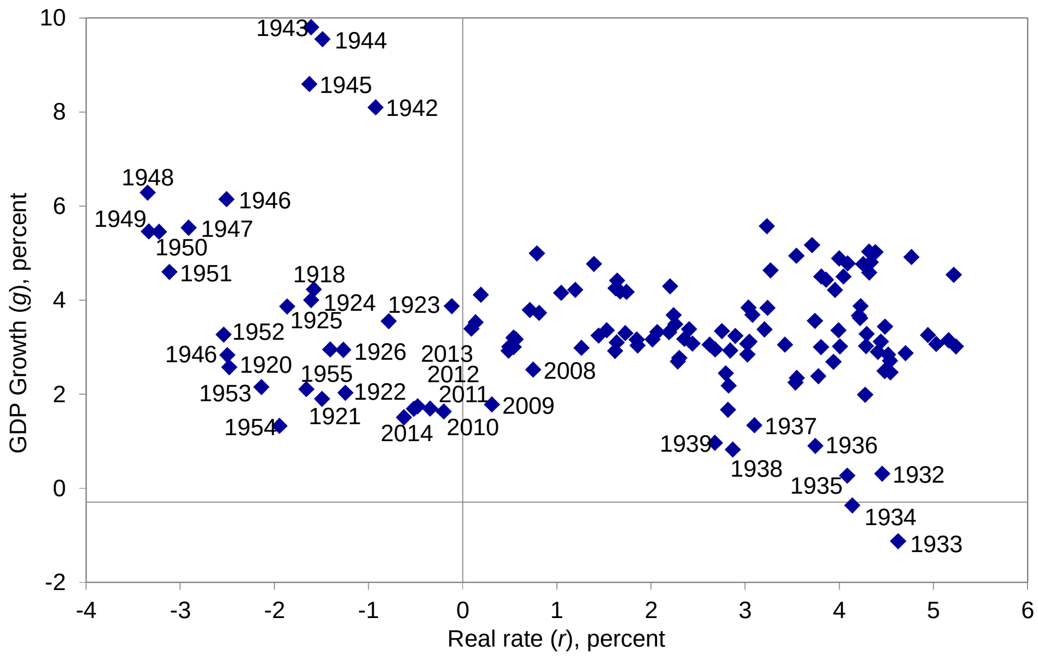
1. The units of observation are overlapping 10 year or 40 quarter averages. In row (1), for example, the first of the 187 observations is average GDP growth and average *r* over the 40 quarters from 1958:2-1968:1; the last is average GDP growth and average *r* over the 40 quarters from 2004:4-2014:3.
2. For annual data, 2014 data only use data from the first three quarters of 2014.

Exhibit 3.6. GDP growth versus *r*: 40-quarter backward moving averages, 1968:1-2014:3.



Note: The two digit label identifies the observation corresponding to the fourth quarter of that year. For example, "08" labels the 2008:4 observation: over the 40 quarter period 1999:1-2008:4, the average value of *r* and GDP growth were average *r*=1.08, average GDP growth=2.14. See notes to Exhibit 3.5.

Exhibit 3.7. GDP growth versus *r*: 10-year backward moving averages, 1879-2014.



Note: See notes to Exhibit 3.5.

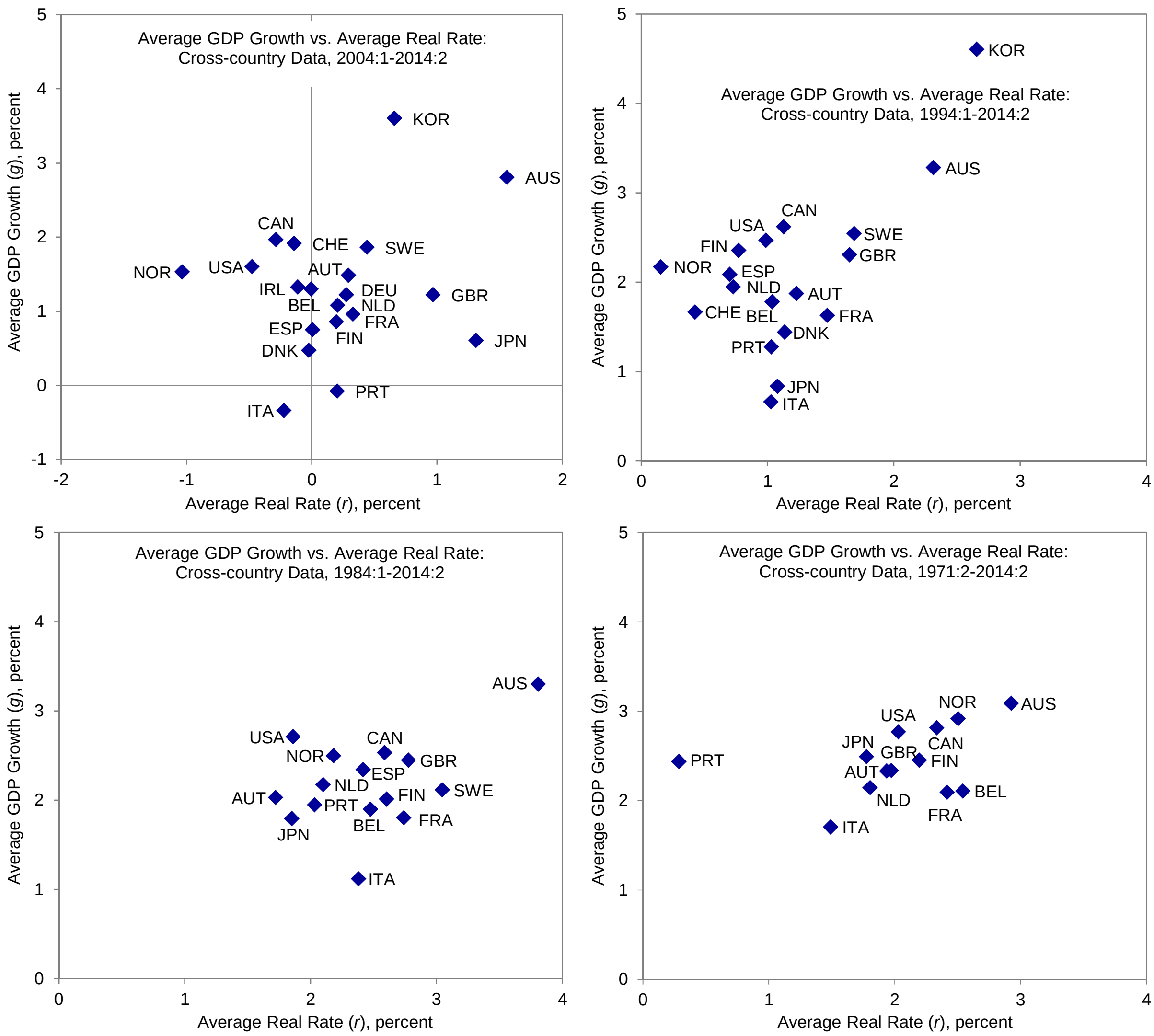
Exhibit 3.8. Cross-country GDP growth and *r*, multiyear averages.

	(1)	2004:1-2014:2		1994:1-2014:2		1984:1-2014:2		1971:2-2014:2	
		(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	Country	**GDP**	***r***	**GDP**	***r***	**GDP**	***r***	**GDP**	***r***
(1)	AUS	2.81	1.56	3.28	2.32	3.30	3.81	3.09	2.93
(2)	AUT	1.49	0.29	1.87	1.23	2.03	1.72	2.33	1.94
(3)	BEL	1.30	-0.01	1.78	1.04	1.90	2.48	2.11	2.54
(4)	CAN	1.97	-0.29	2.62	1.13	2.53	2.59	2.82	2.34
(5)	CHE	1.91	-0.14	1.67	0.43				
(6)	DEU	1.22	0.97						
(7)	DNK	0.47	-0.02	1.44	1.14				
(8)	ESP	0.75	0.01	2.09	0.70	2.34	2.42		
(9)	FIN	0.86	0.20	2.36	0.77	2.01	2.60	2.45	2.20
(10)	FRA	0.96	0.33	1.63	1.47	1.80	2.74	2.09	2.42
(11)	GBR	1.22	0.28	2.31	1.65	2.45	2.78	2.34	1.97
(12)	IRL	1.33	-0.11						
(13)	ITA	-0.34	-0.22	0.66	1.03	1.12	2.38	1.71	1.49
(14)	JPN	0.61	1.31	0.83	1.08	1.79	1.85	2.49	1.78
(15)	KOR	3.60	0.66	4.60	2.66				
(16)	NLD	1.08	0.21	1.95	0.73	2.18	2.10	2.14	1.81
(17)	NOR	1.53	-1.03	2.17	0.16	2.50	2.18	2.92	2.50
(18)	PRT	-0.08	0.20	1.28	1.03	1.95	2.03	2.44	0.29
(19)	SWE	1.86	0.44	2.54	1.69	2.12	3.05		
(20)	USA	1.60	-0.48	2.47	0.99	2.71	1.86	2.77	2.03
Corr		0.23		0.63		0.42		0.36	

Notes:

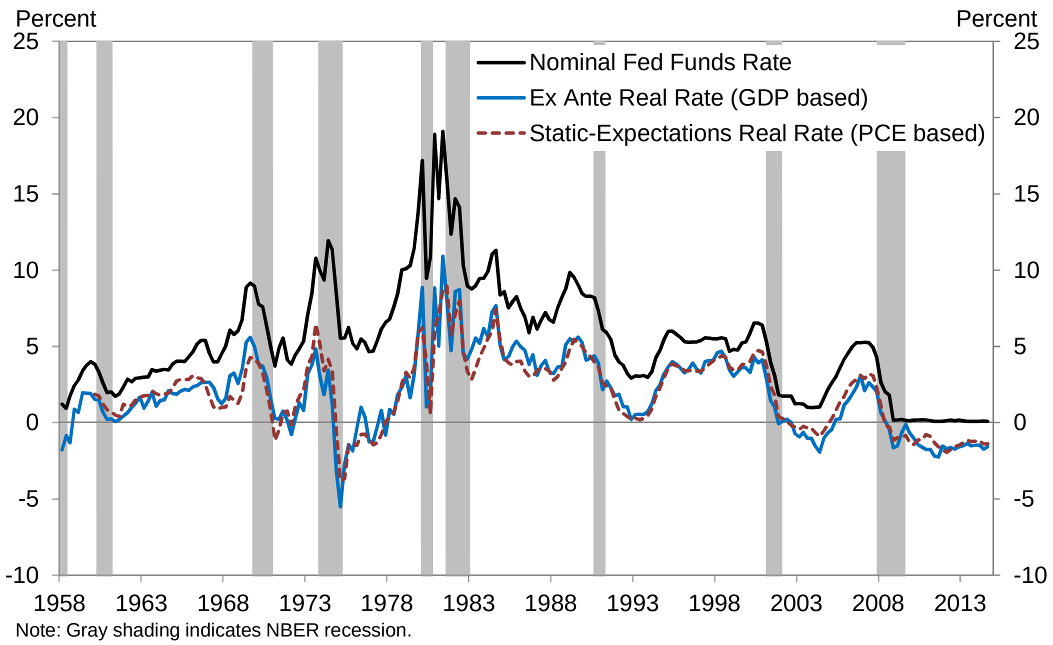
1. Each entry is an average of quarterly data over the indicated period. For example, the 2.81 figure in column (2), row (1), means that average GDP growth in Australia over the 42 quarters from 2004:1 through 2014:2 was 2.81%; the 1.56 figure in column (3) is the corresponding value for the ex-ante real rate *r* over this period.
2. The ex-ante real rate *r* is a nominal policy rate minus next quarter's expected inflation. Expected inflation is computed from an autoregression in inflation, using rolling samples. See text for details.
3. Real, quarterly GDP data is from the OECD. See text for sources of nominal policy rates and inflation.

Exhibit 3.9. Cross-country relations between GDP growth and *r* over selected samples.



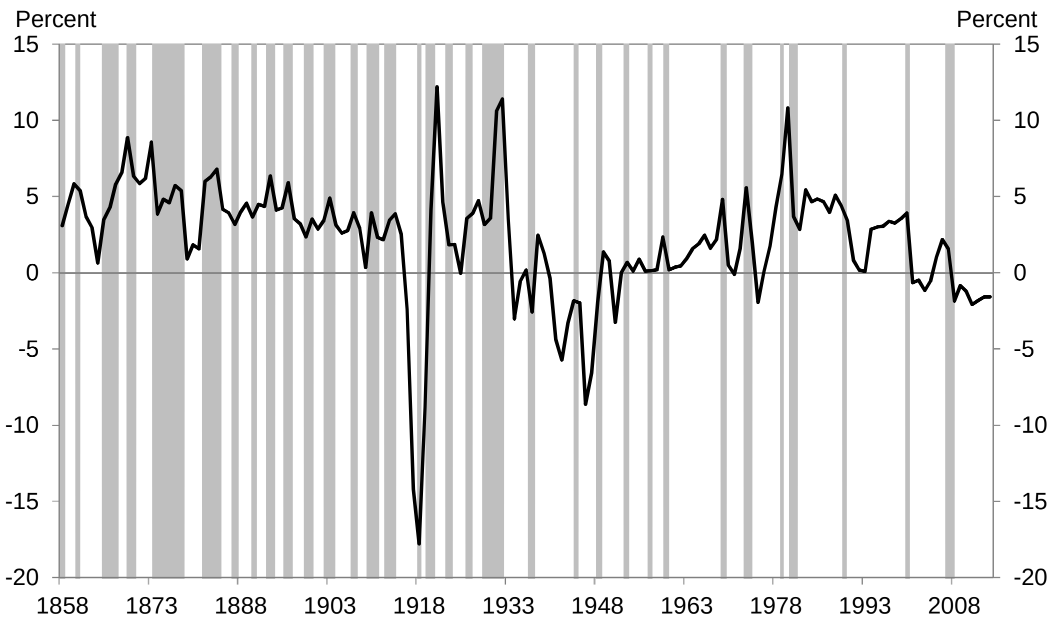
Note: This exhibit plots the data listed in Exhibit 3.8. See notes to Exhibit 3.8. Mnemonics for country names are given in Exhibit 2.1.

Exhibit 4.1. Nominal and real interest rates, 1958-2014.



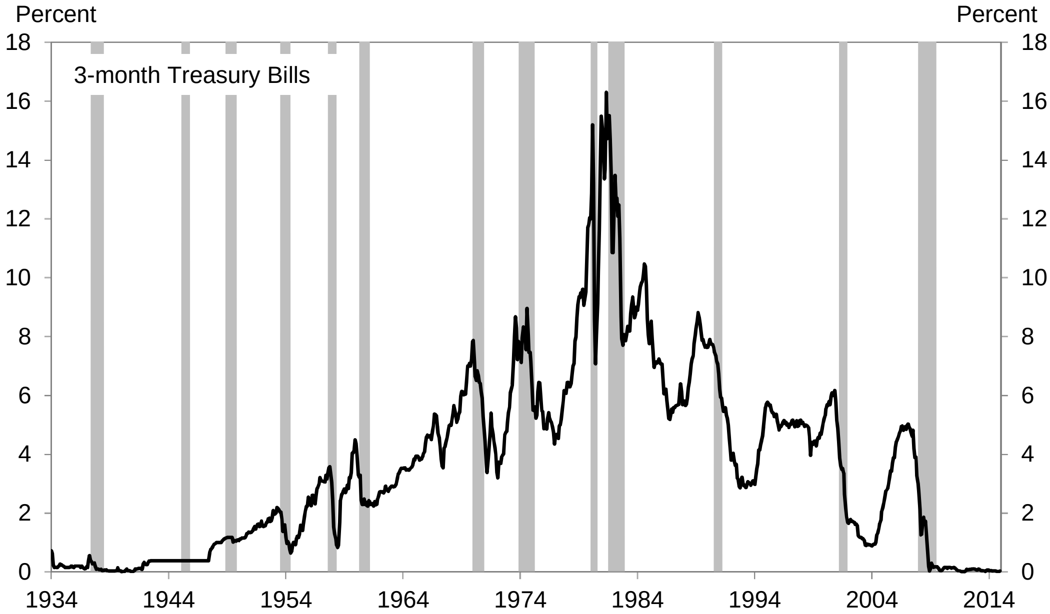
Note: Gray shading indicates NBER recession.

Exhibit 4.2. U.S. ex-ante real rate, 1858-2013.



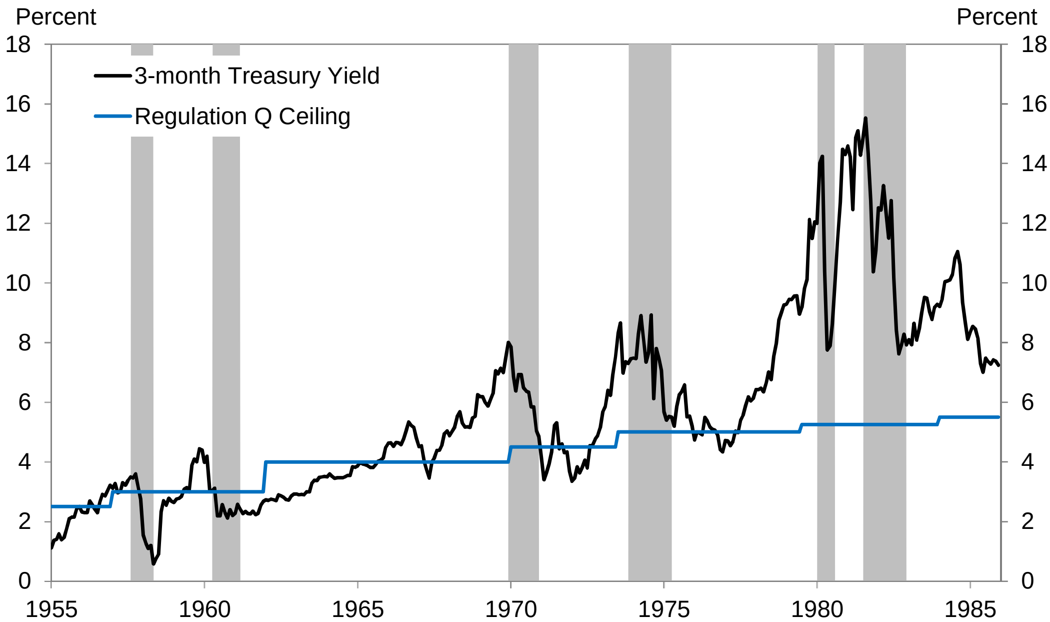
Note: Gray shading indicates NBER recession.

Exhibit 4.3. Nominal 3-month Tbill rate, 1934-2014.



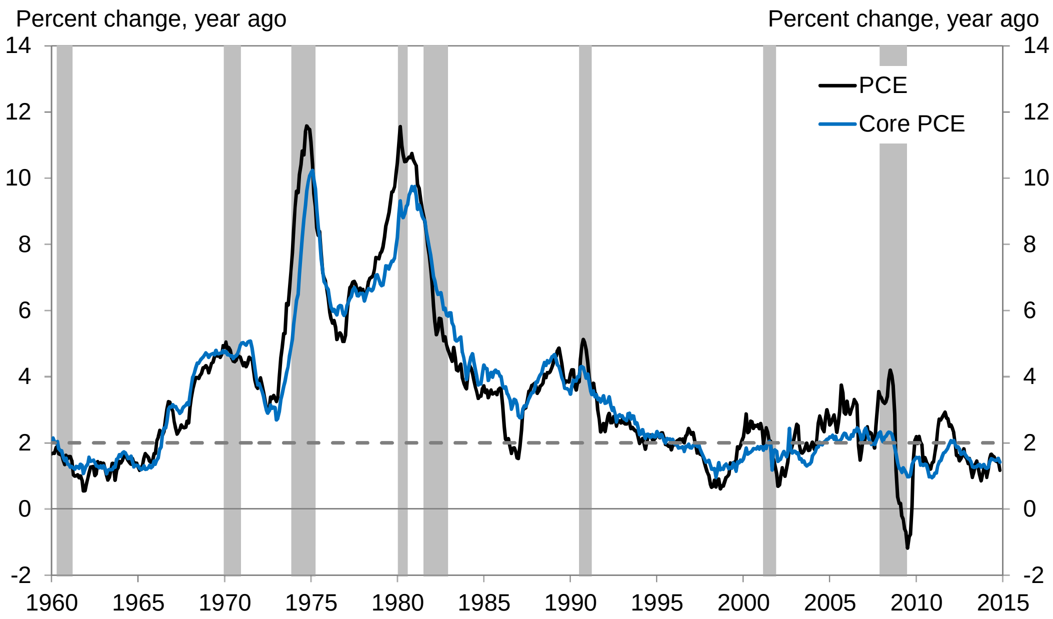
Note: Gray shading indicates NBER recession.

Exhibit 4.4. Nominal Tbill yield and regulation Q, 1955-1987.



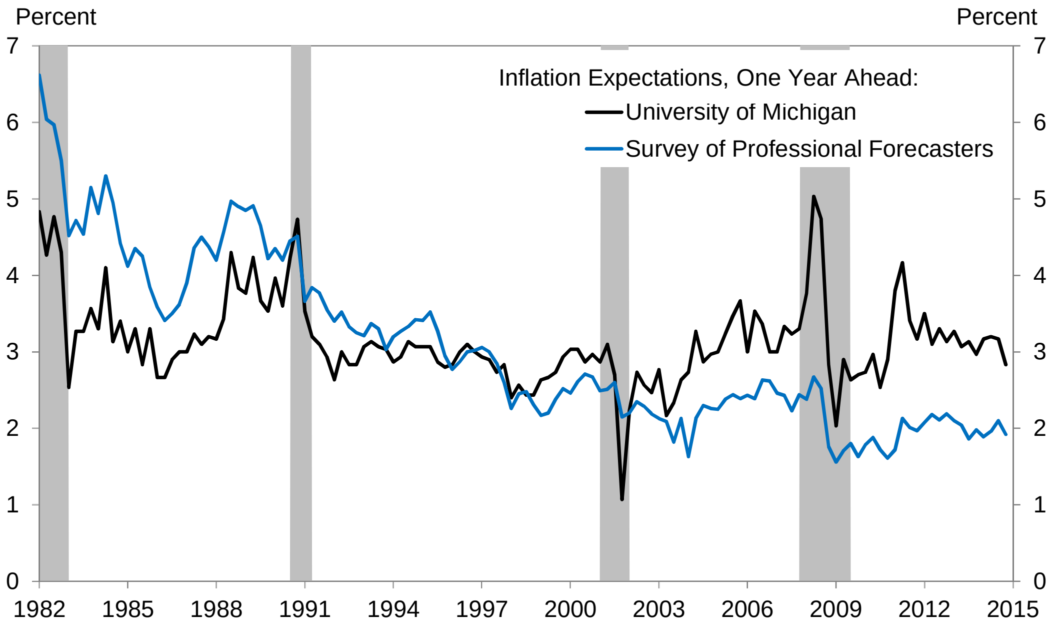
Note: Gray shading indicates NBER recession.

Exhibit 4.5. Inflation measures, 1960-2014.



Note: Gray shading indicates NBER recession.

Exhibit 4.6. Measures of expected inflation, 1982-2014.



Note: Gray shading indicates NBER recession.

Exhibit 4.7. Growth in employment following postwar business cycle troughs.

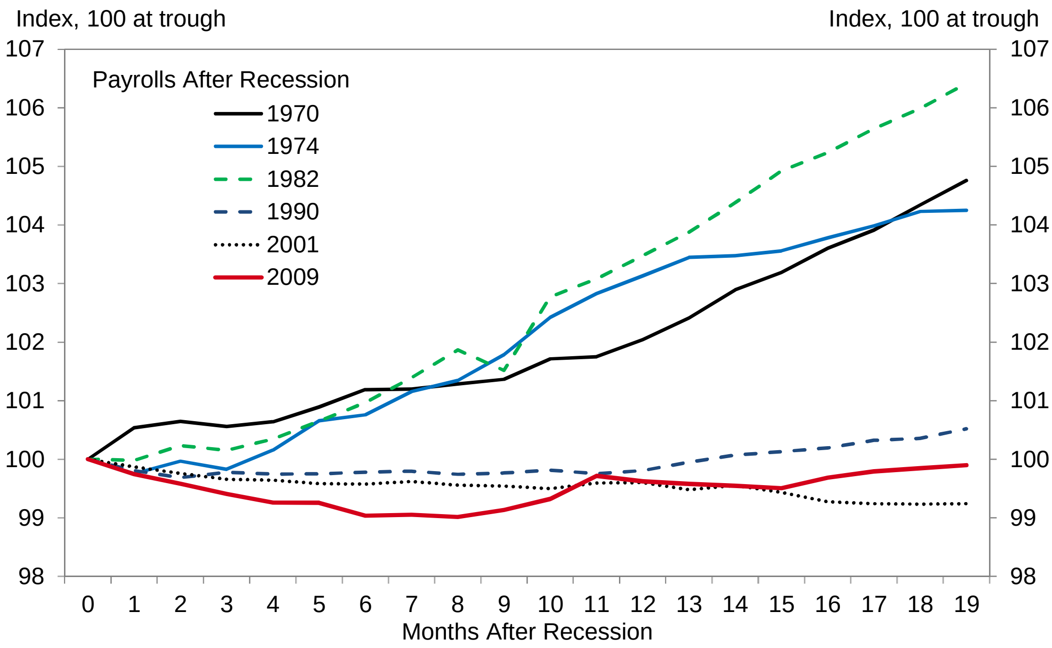


Exhibit 4.8. Fed funds rate and inflation, 1980-2014.

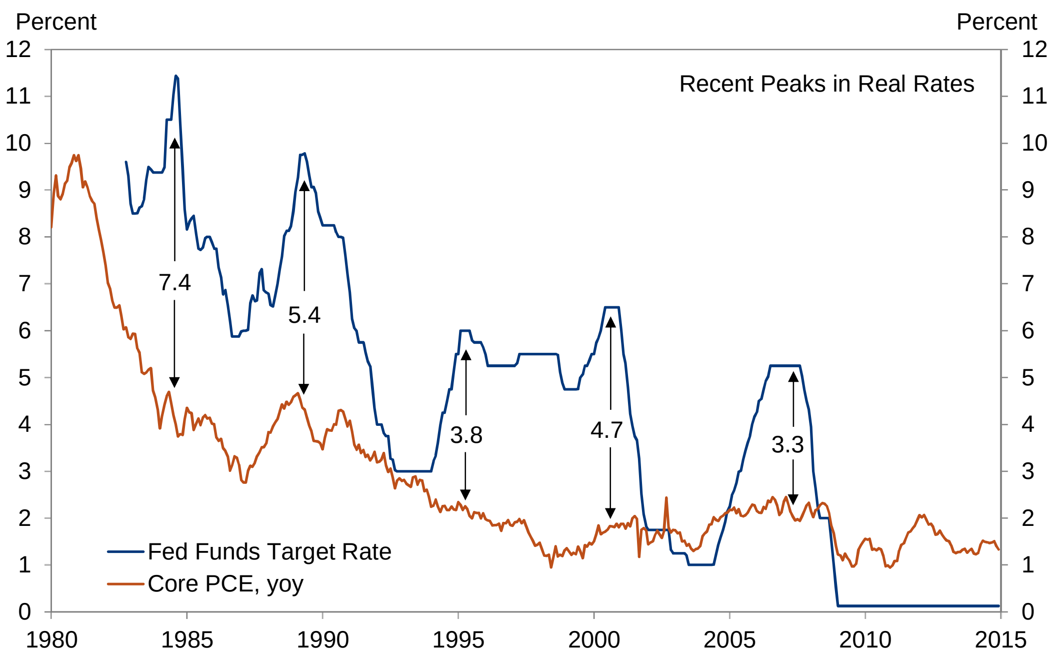
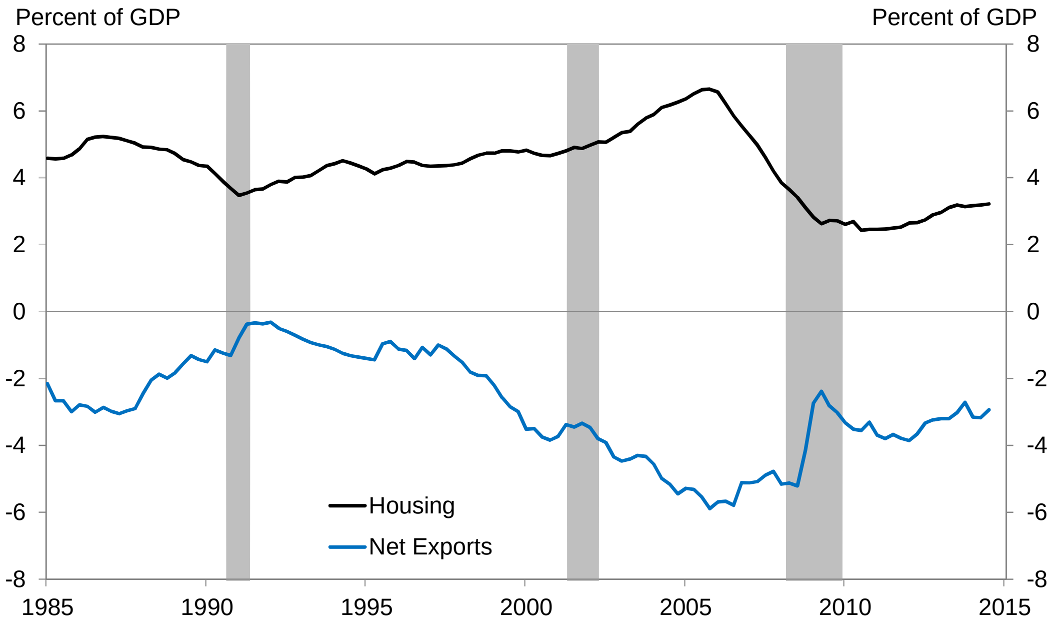


Exhibit 4.9. Nominal rates, inflation rates, and ex-ante real rates over postwar business expansions.

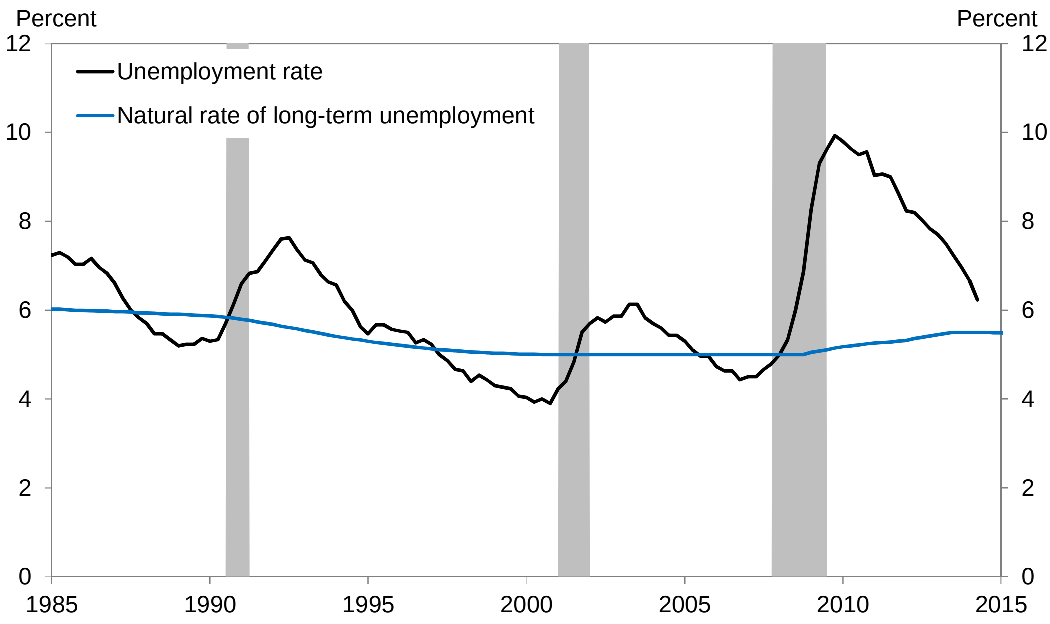
Cycle	Funds rate average over full cycle: Nominal	Inflation	Real	At Full Employment	Peak Real Funds Rate
3Q 1960 - 4Q 1969	4.26	2.48	1.97	1.44	5.59
1Q 1970 - 4Q 1973	6.23	5.11	1.87	0.18	4.84
1Q 1974 - 3Q 1981	9.39	7.64	1.74	0.55	10.91
4Q 1981 - 3Q 1990	8.80	3.51	4.97	3.34	8.73
4Q 1990 - 1Q 2001	5.04	2.04	2.90	4.01	4.68
2Q 2001 - 4Q 2007	2.92	2.43	0.45	0.25	3.10

Exhibit 4.10. Housing and net exports as a percent of GDP.



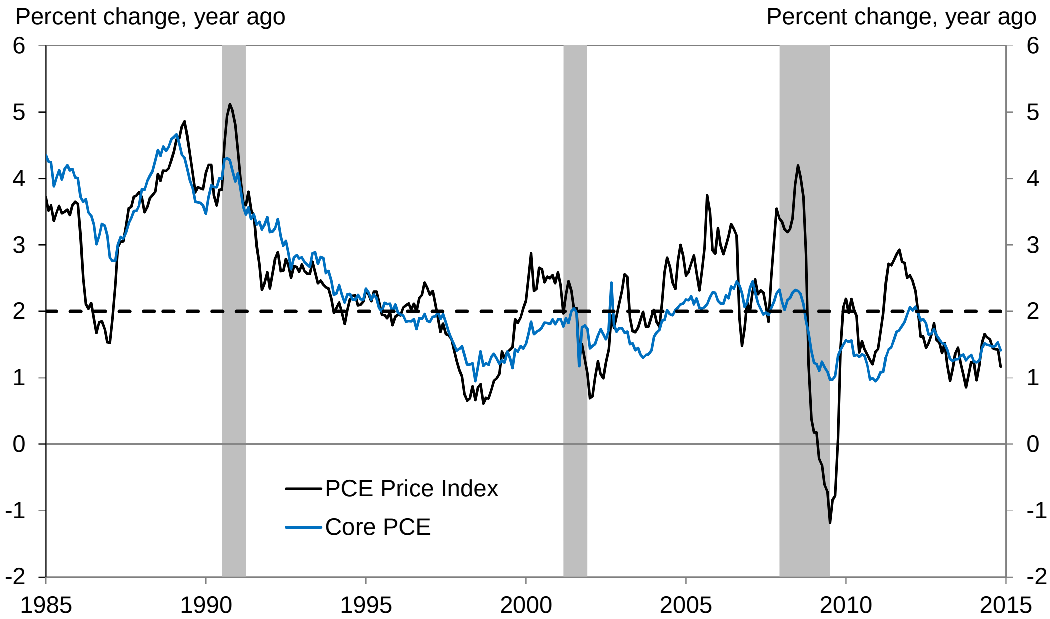
Note: Gray shading indicates NBER recession.

Exhibit 4.11. Unemployment rate, 1985-2014.



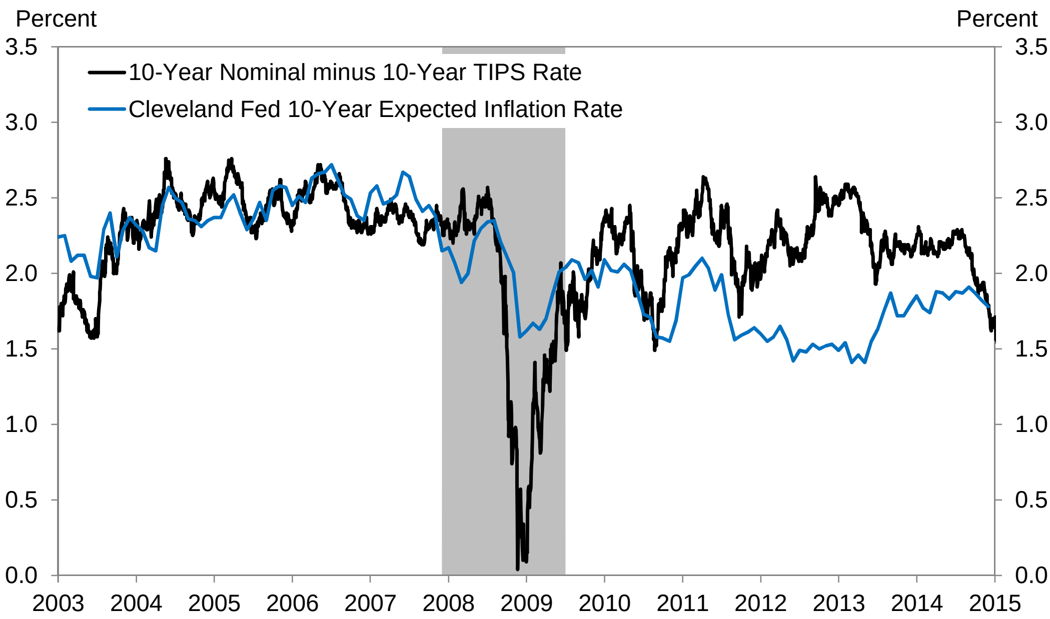
Note: Gray shading indicates NBER recession.

Exhibit 4.12. Inflation measures, 1985-2014.



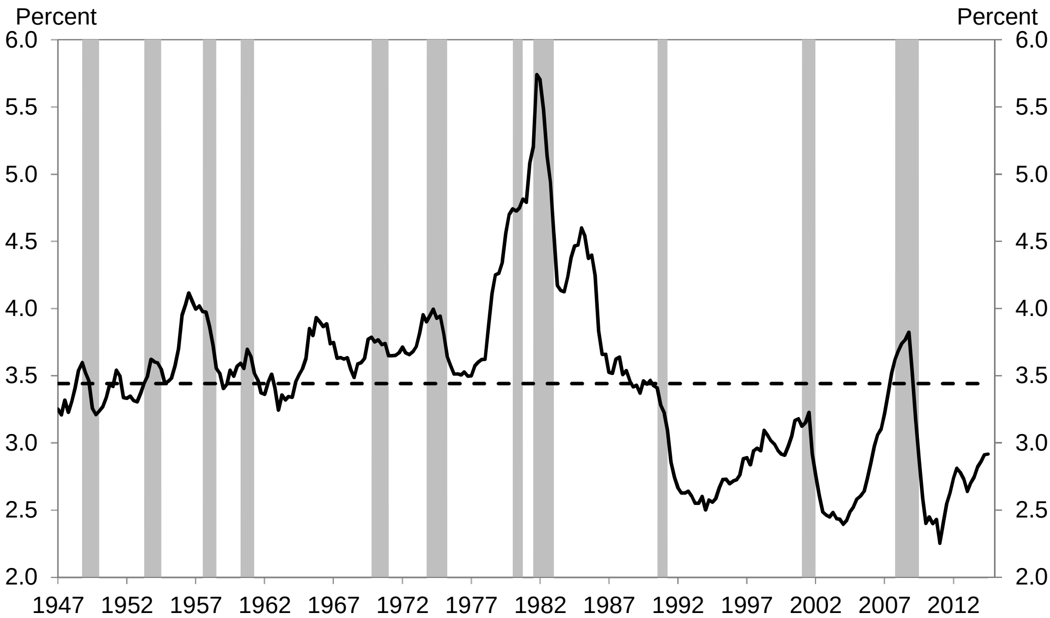
Note: Gray shading indicates NBER recession.

Exhibit 4.13. TIPS breakeven rates and expected inflation, 2003-2014.



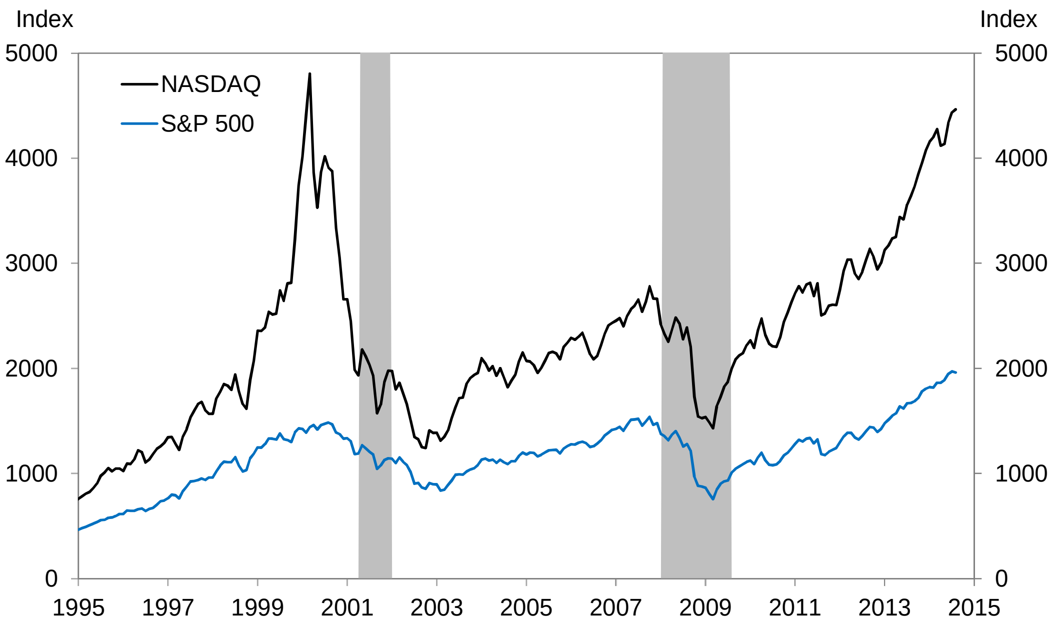
Note: Gray shading indicates NBER recession.

Exhibit 4.14. Investment in nonresidential structures as a percentage of GDP, 1947-2014.



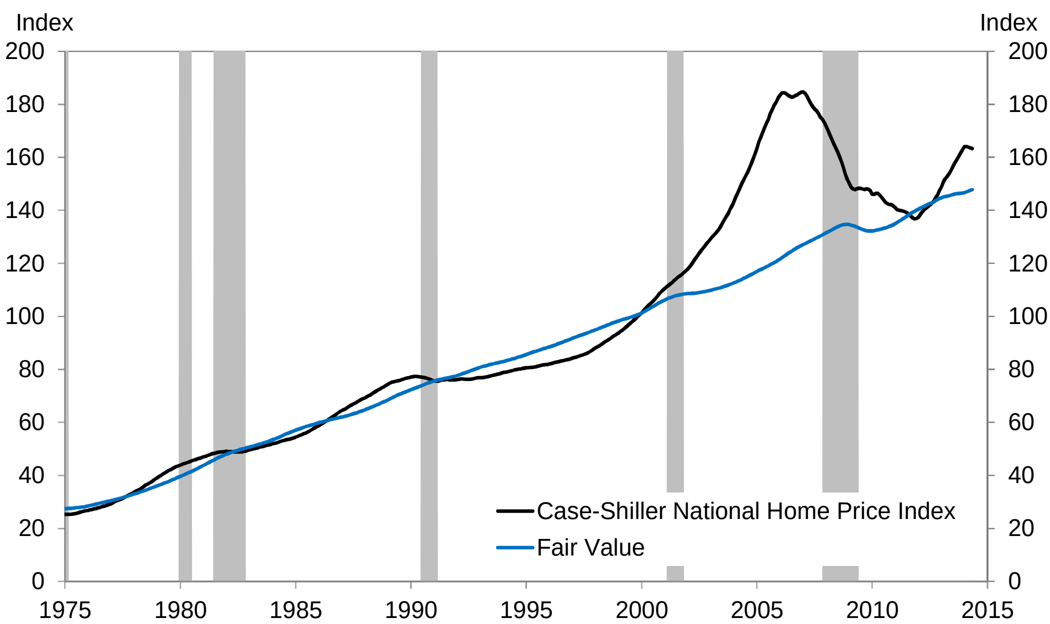
Note: Gray shading indicates NBER recession.

Exhibit 4.15. Stock prices, 1995-2014.



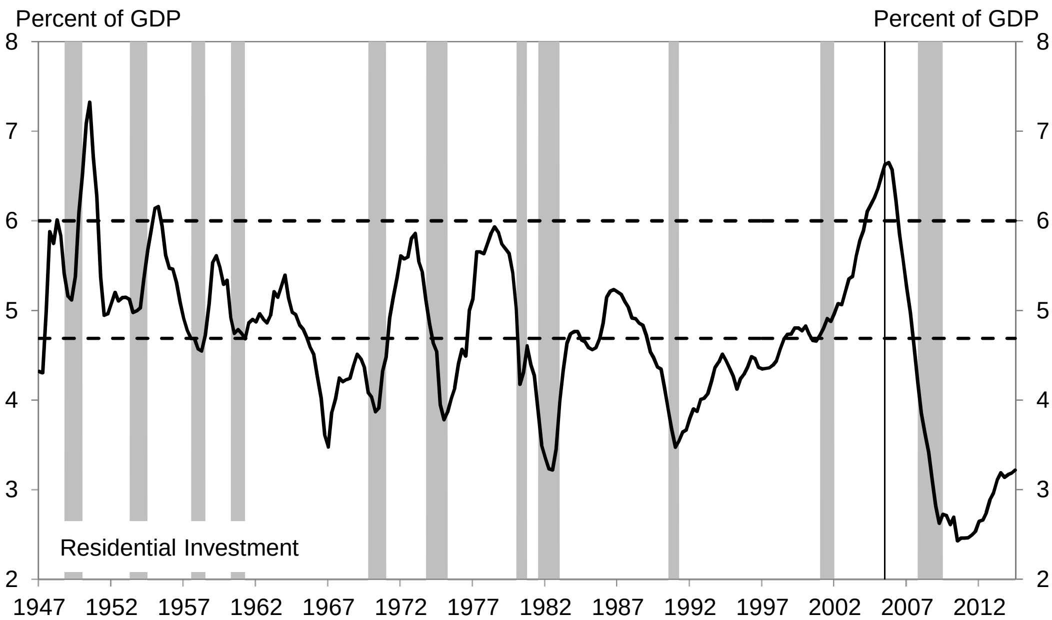
Note: Gray shading indicates NBER recession.

Exhibit 4.16. Home prices, 1975-2014.



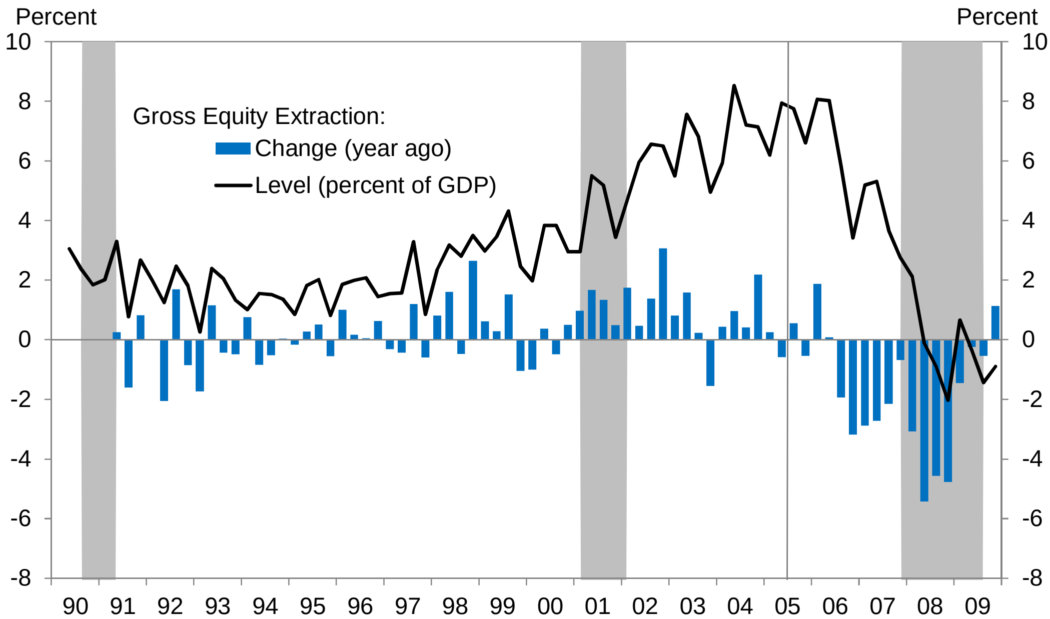
Note: Gray shading indicates NBER recession.

Exhibit 4.17. Residential fixed investment as a percent of GDP.



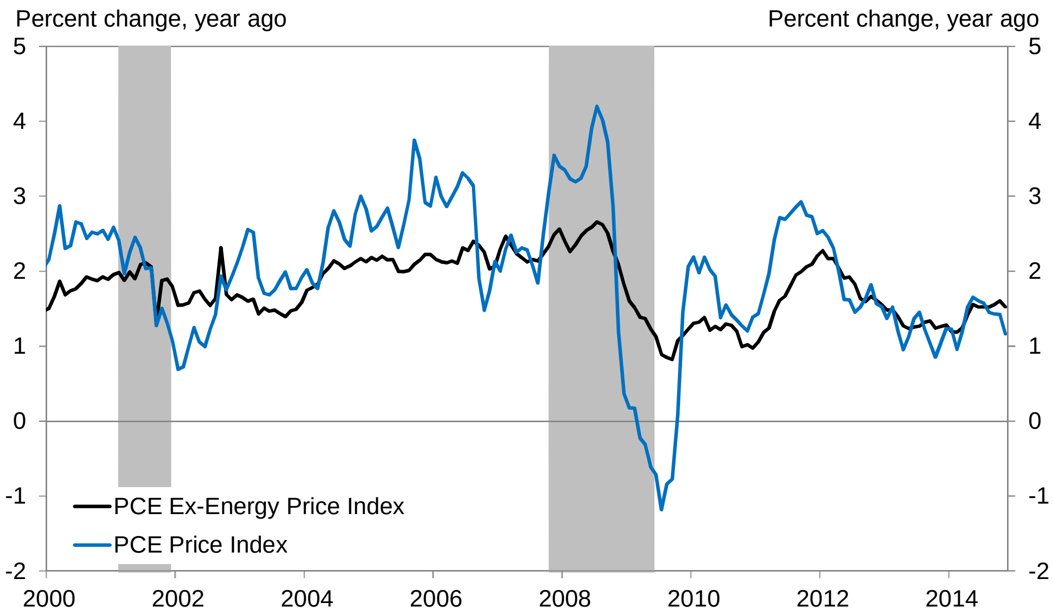
Note: Gray shading indicates NBER recession.

Exhibit 4.18. Home equity withdrawal, 1990-2009.



Note: Gray shading indicates NBER recession.

Exhibit 4.19. Inflation with and without energy prices, 2000-2014.

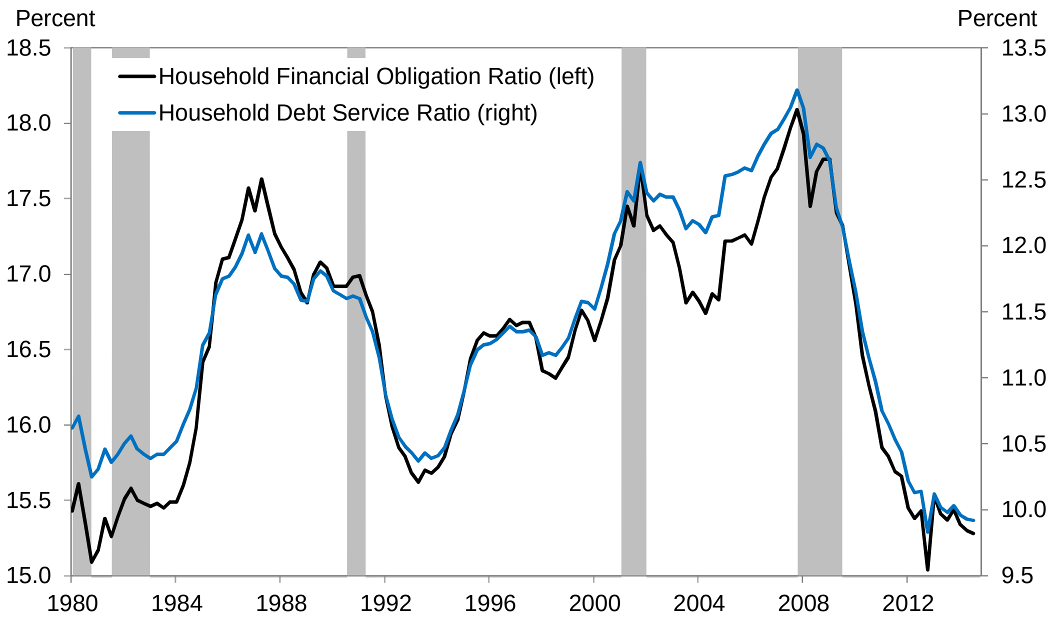


Note: Gray shading indicates NBER recession.

Exhibit 4.20. Measures of severity of historical financial crises.

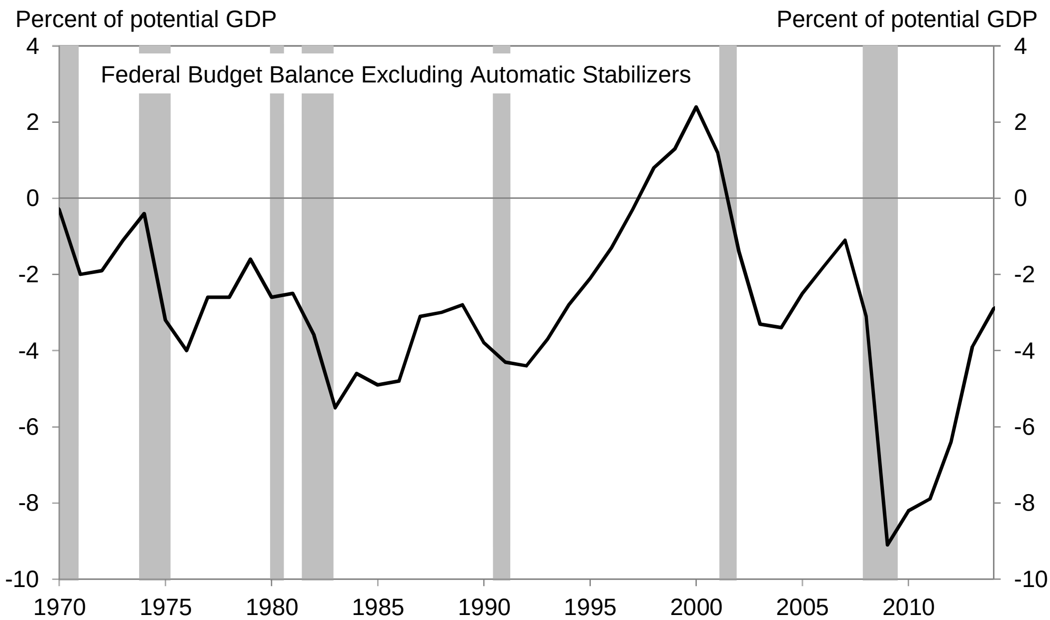
	Peak to Trough GDP (percent)	Years to Recovery	Severity Index
100 crises	-10.3	8.4	19.6
37 EM crisis	-14.2	9.9	24.2
63 DM crisis	-9.6	7.4	17.0
US 1907	-12.5	9.0	21.5
US 1929/33	-28.6	10.0	38.6
memo:			
US today	-3.1	4.0	7.1
EA today	-4.5	6.0+	10.5+
Japan today	-6.5	6.0	12.5

Exhibit 4.21. Household financial obligations and debt service as a percent of income, 1980-2014.



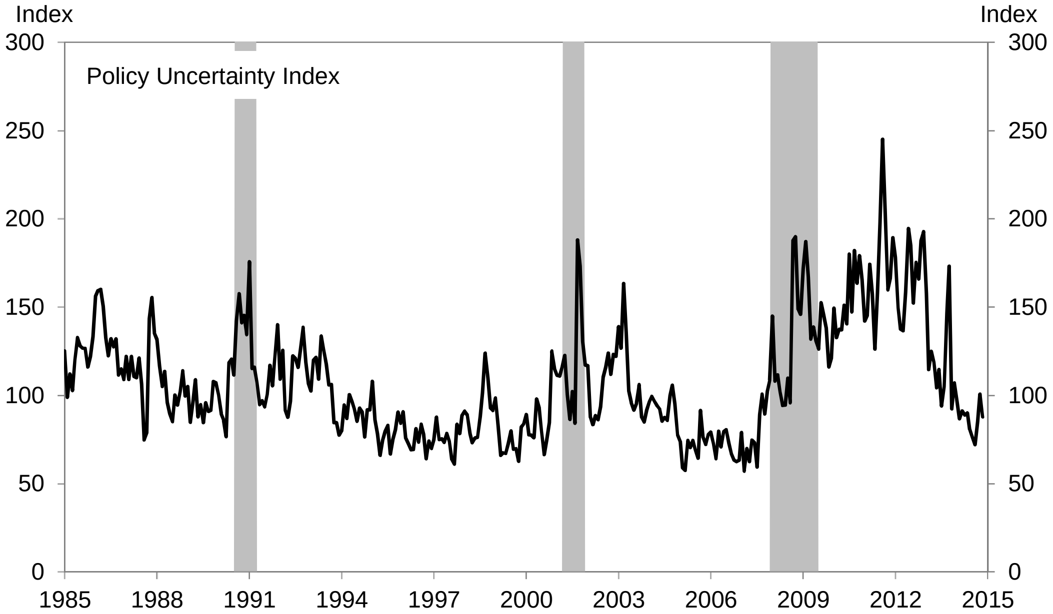
Note: Gray shading indicates NBER recession.

Exhibit 4.22. Federal surplus as percent of potential GDP, 1970-2014.



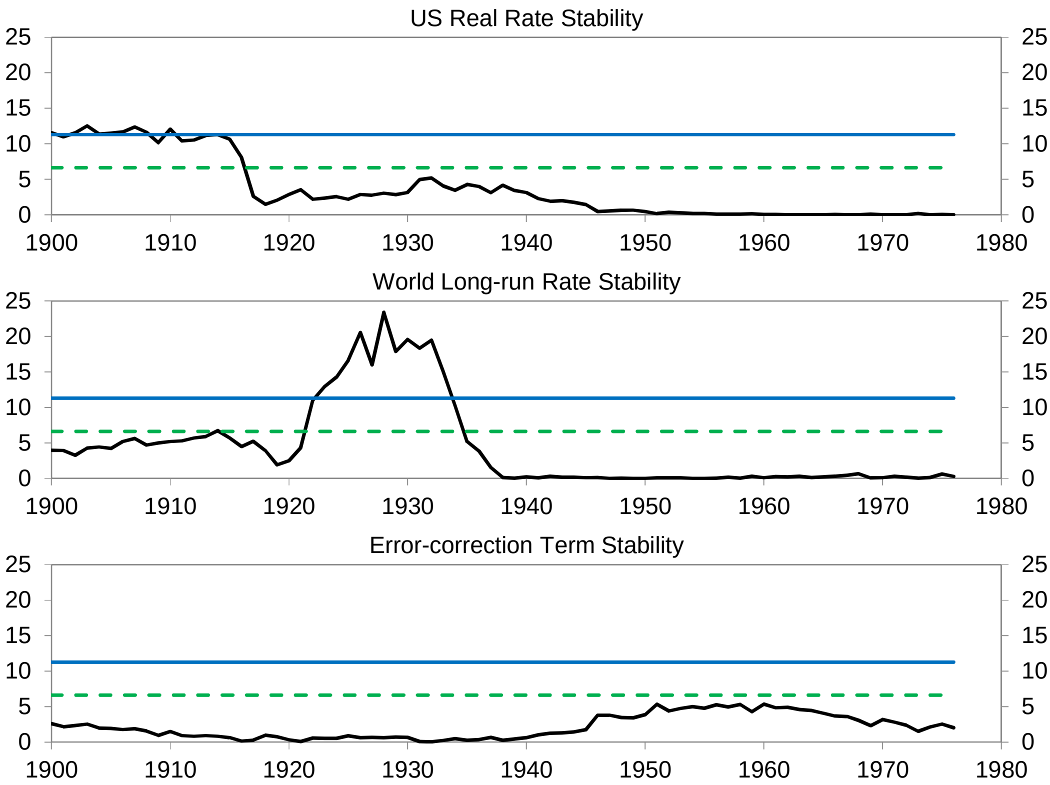
Note: Gray shading indicates NBER recession.

Exhibit 4.23. Policy uncertainty index, 1985-2014.



Note: Gray shading indicates NBER recession.

Exhibit 5.1. $\chi^2(1)$ statistic for test of null hypothesis of stability against the alternative of a break in the mean at the indicated date, 1900-1976.



Note: Dashed green: 1% critical value if test were only performed at a single date (6.63). Solid blue: 1% critical value if the maximal statistic over the range 1900-1976 were used (11.28, from Andrews (2003)). Top panel: U.S. real interest rate $r_{US,t}$; middle panel: long-run world rate ℓ_t; bottom panel: difference between U.S. real rate and long-run world rate $(r_{US,t} - \ell_t)$.

Exhibit 5.2. Long-run world real rate (ℓ_t, in blue) and U.S. ex-ante real rate ($r_{US,t}$, in black).

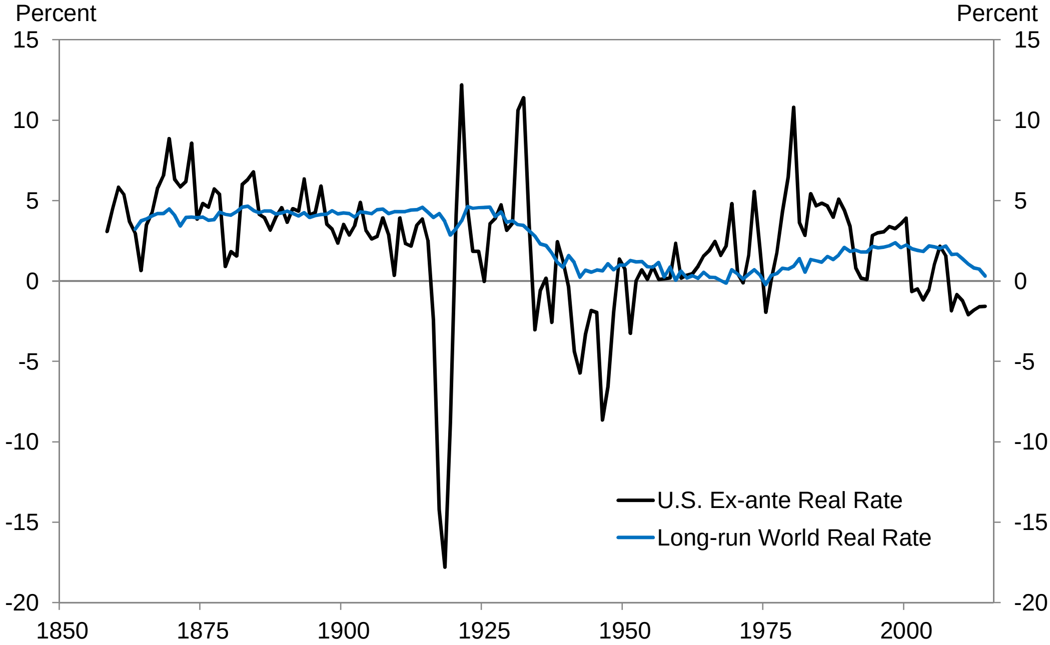


Exhibit 5.3. Forecasts for U.S. and long-run world real rates implied by (5.4) and (5.5) along with 90% confidence intervals for the latter.

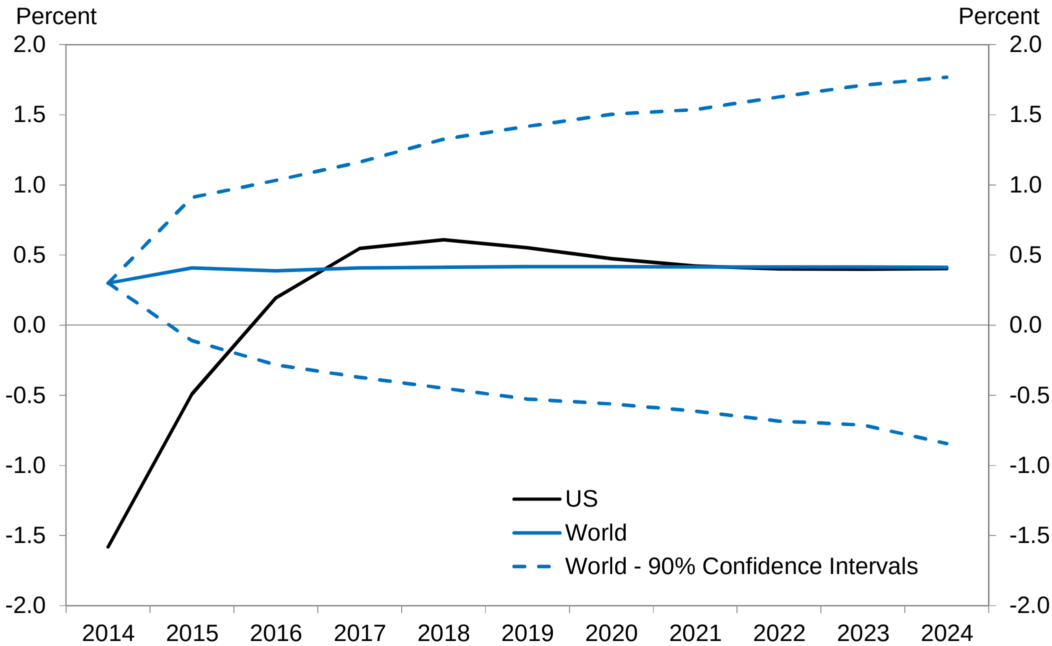


Exhibit 5.4. Predicted value (in black) for the U.S. real interest rate implied by equations (5.4)-(5.5) and forward rates (in blue) implied by the term structure of TIPS as of the end of 2014.

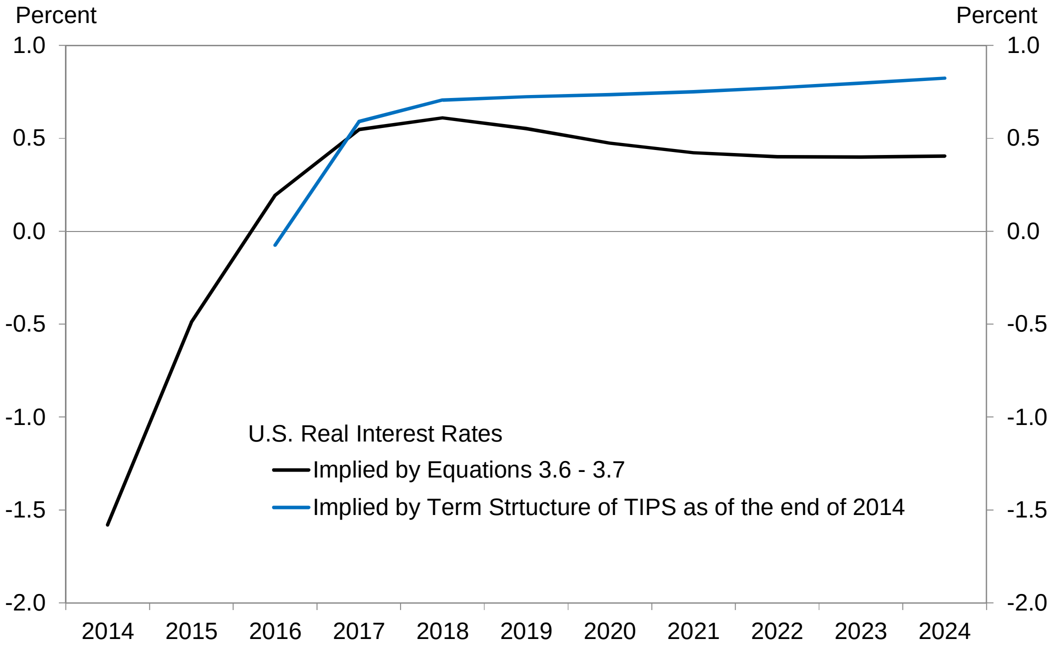


Exhibit 6.1. Baseline path around r^* and potential misperceptions.

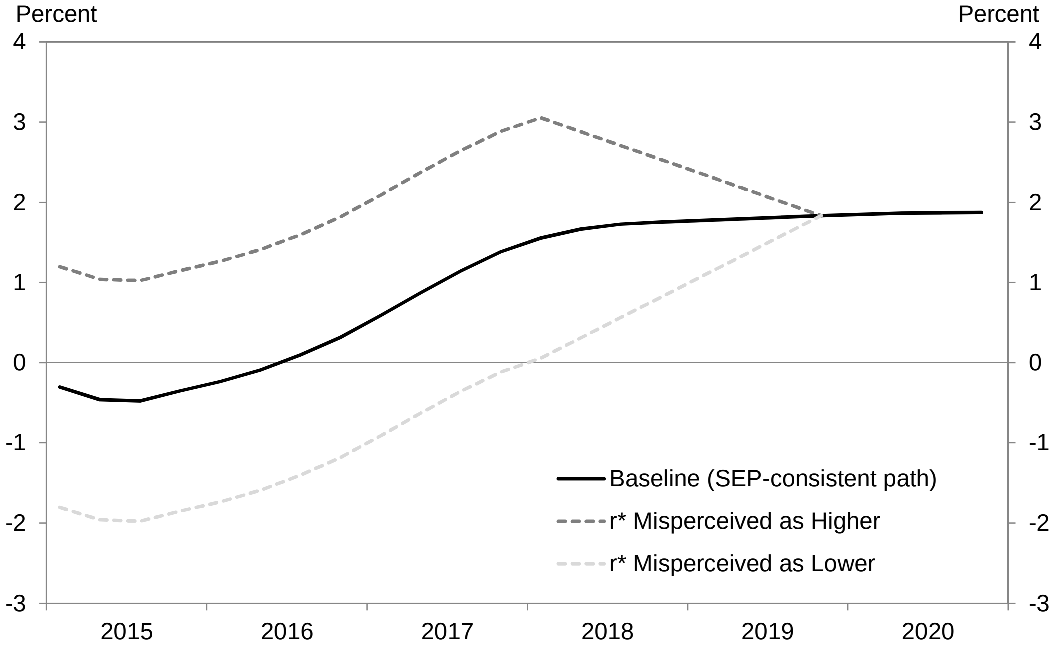


Exhibit 6.2. Behavior of the economy with perception errors but no inertia.

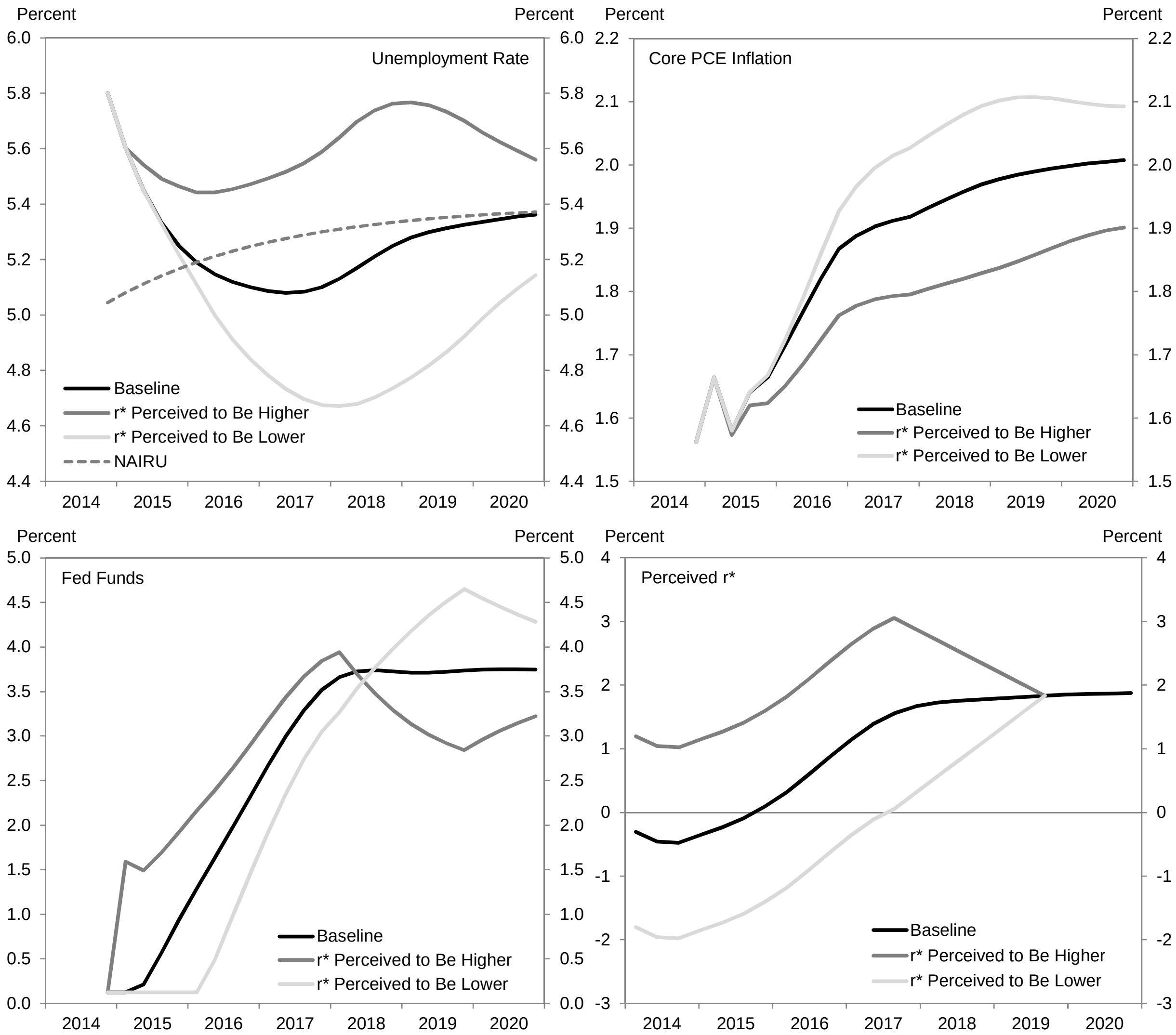


Exhibit 6.3. Behavior of the economy with perception errors and inertia.

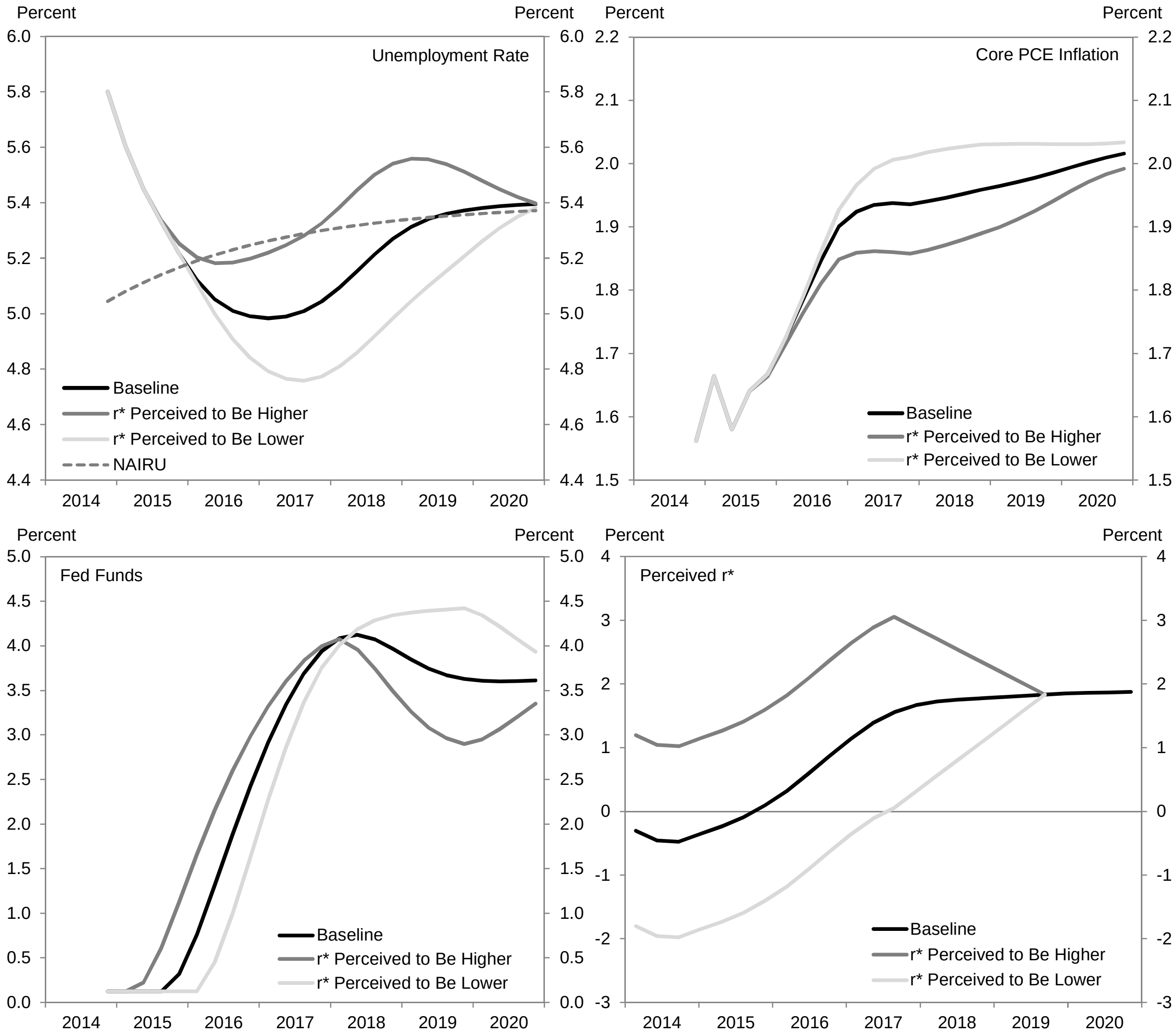


Exhibit 6.4. Funds rate paths with and without inertia.

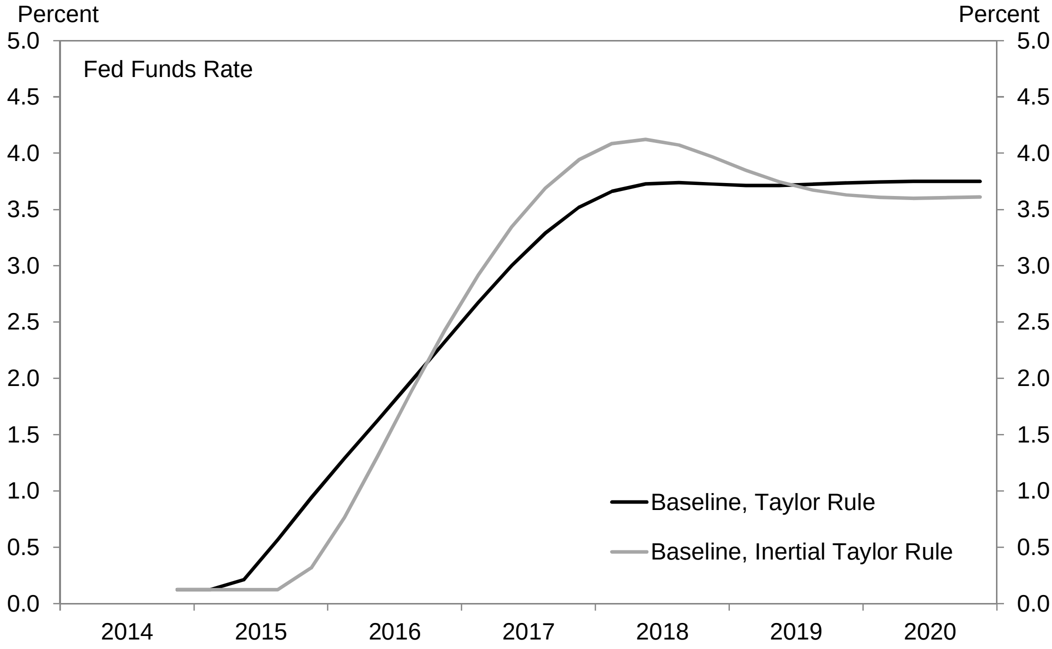
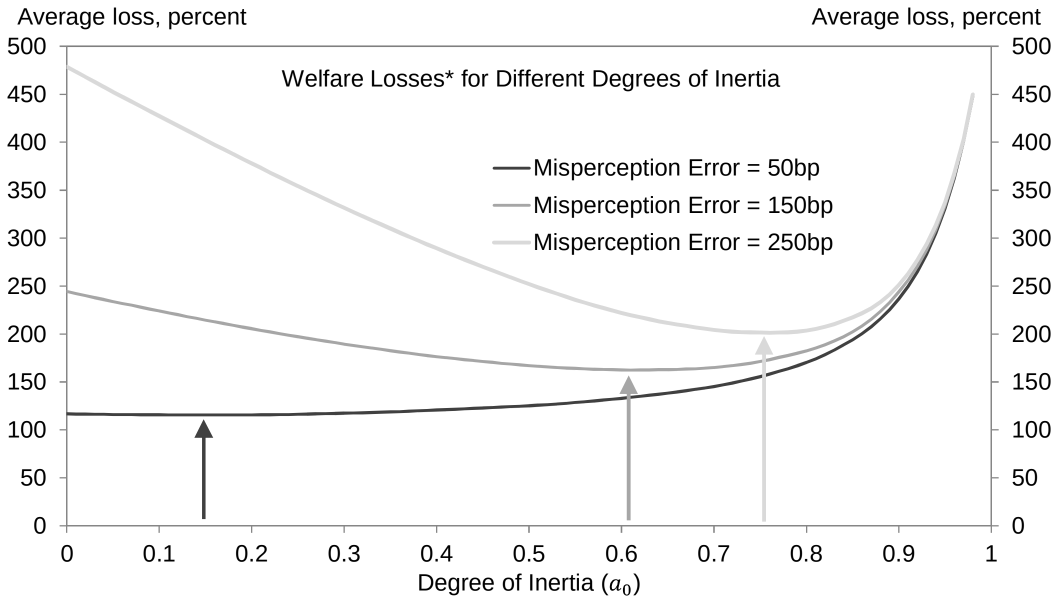


Exhibit 6.5. Optimal inertia curves for different *r** errors.



Note: Arrows indicate optimal inertia coefficients.
* Relative to no-misperception optimum.

Exhibit 6.6. Optimal inertia curves for different smoothing coefficients in loss function.

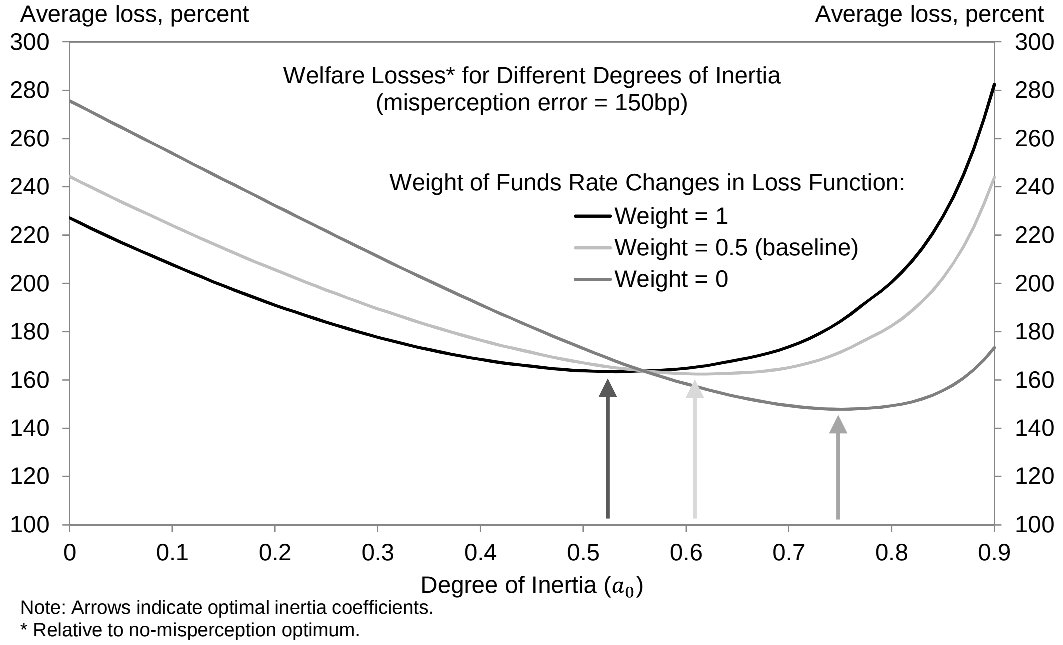


Exhibit 6.7. Optimal inertia curves for different *r** errors with model-consistent expectations.