THE OPTIMAL USE OF GOVERNMENT PURCHASES FOR MACROECONOMIC
STABILIZATION

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NBER Working Paper No. 21322
July 2015
JEL No. E32,E62,H21,H41

ABSTRACT

This paper extends Samuelson's theory of optimal government purchases by considering the contribution of government purchases to macroeconomic stabilization. We consider a matching model in which unemployment can be too high or too low. We derive a sufficient-statistics formula for optimal government purchases. Our formula is the Samuelson formula plus a correction term proportional to the government-purchases multiplier and the gap between actual and efficient unemployment rate. Optimal government purchases are above the Samuelson level when the correction term is positive-for instance, when the multiplier is positive and unemployment is inefficiently high. Our formula indicates that US government purchases, which are mildly countercyclical, are optimal under a small multiplier of 0.03. If the multiplier is larger, US government purchases are not countercyclical enough. Our formula implies significant increases in government purchases during slumps. For instance, with a multiplier of 0.5 and other statistics calibrated to the US economy, when the unemployment rate rises from the US average of 5.9% to 9%, the optimal government purchases-output ratio increases from 16.6% to 19.8%. However, the optimal ratio increases less for multipliers above 0.5 because with higher multipliers, the unemployment gap can be filled with fewer government purchases. For instance, with a multiplier of 2, the optimal ratio only increases from 16.6% to 17.6%.

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1. Introduction

In the United States, the Full Employment and Balanced Growth Act of 1978 imparts the responsibility of achieving full employment to the Board of Governors of the Federal Reserve System, through the choice of the federal funds rate, and to the government, through public employment and other public expenditure. In practice however it is the Federal Reserve that has been in charge of macroeconomic stabilization. This reliance on monetary policy reflects the consensus among policymakers and academics that monetary policy is more adapted to stabilize the economy.

But the stabilization achieved through monetary policy alone remains imperfect. Of course, at the zero lower bound on nominal interest rates, monetary policy is severely constrained, and that is what happens starting in 2009.[1] But that is not all; as Figure 1 shows, much of the increase in unemployment had occurred before reaching the zero lower bound, as nominal interest rates were falling. Even in the 1991 and 2001 recessions, when monetary policy was not subject to the zero lower bound, stabilization was only partial. Thus, the unemployment rate has fluctuated noticeably over the past thirty years despite strong responses of the Federal funds rate.

This paper explores how government purchases can be used to improve macroeconomic stabilization. To that end, we embed the standard theory of optimal government purchases, developed by Samuelson [1954], within a matching model of the macroeconomy.[2] Samuelson's theory applies to a competitive model, which is efficient. In that setting, the optimal provision of government consumption is given by a simple formula: the marginal rate of substitution between government consumption and personal consumption equals the marginal rate of transformation between government consumption and personal consumption, which is one in our model.

But a matching model is not necessarily efficient. Our model builds on the matching framework from Michaillat and Saez [2015a]. There is one matching market where households sell labor services to other households and the government. In equilibrium there is some unemployment: sellers are unable to sell all the labor services that they could have produced. The unemployment rate may not be efficient: when unemployment is inefficiently low, too many resources

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[1]Krugman [1998] and Eggertsson and Woodford [2003] describe how the effectiveness of monetary policy is restricted by the existence of a zero lower bound on nominal interest rates.

[2]In a related manner, the new dynamic public finance literature connects normative public economics to macroeconomic models, although it focuses mostly on taxation, not on government purchases.
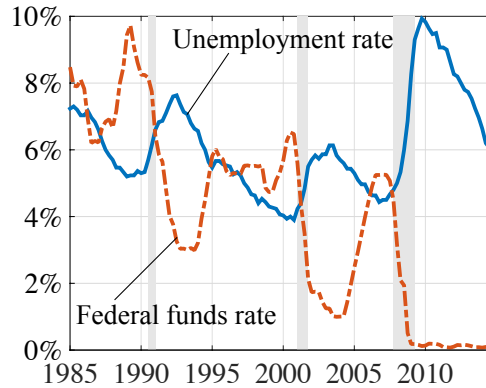
Figure 1: Unemployment and Monetary Policy in the United States, 1985–2014

Notes: The unemployment rate is the quarterly average of the seasonally adjusted monthly unemployment rate constructed by the Bureau of Labor Statistics (BLS) from the Current Population Survey (CPS). The federal funds rate is the quarterly average of the daily effective federal funds rate set by the Board of Governors of the Federal Reserve System. The shaded areas represent the recessions identified by the National Bureau of Economic Research (NBER).

are devoted to recruiting workers, and a further reduction in unemployment would reduce welfare; when unemployment is inefficiently high, too few jobseekers find a job, too much of the productive capacity of the economy is idle, and a reduction in unemployment would raise welfare. When unemployment is inefficiently high, or inefficiently low, and when government purchases influence unemployment, government purchases have an additional effect on welfare, unaccounted for in Samuelson's theory. Hence, our formula for optimal government purchases is the Samuelson formula plus a correction term that measures the effect of government purchases on welfare through their influence on unemployment.[3]

We express our formula for optimal government purchases in terms of estimable sufficient statistics [Chetty, 2009].[4] By virtue of being expressed with sufficient statistics, the formula applies to a broad range of matching models. The formula is indeed valid for any utility function, any matching function with constant returns to scale, any aggregate demand function, any mechanism for the price of services, and various ways to finance government purchases—lump-sum taxes, certain distortionary taxes, or deficit spending.

Two central sufficient statistics in our formula are the government-purchases multiplier and the

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[3]A small literature also analyzes optimal government purchases in disequilibrium models. See for instance Roberts [1982] and Drèze [1985]. Since our model of unemployment is simpler and richer than the disequilibrium model (see the discussion in Michaillat and Saez [2015a]), our analysis is more transparent and provides new insights.

[4]The new dynamic public finance literature has also recently strived to express optimal policy formulas in terms of estimable statistics [Golosov and Tsyvinski, 2015].

gap between actual and efficient unemployment rate because the correction term is proportional to the product of these two statistics. The gap between actual unemployment and efficient unemployment determines the effect of unemployment on welfare, and the government-purchases multiplier determines the effect of government purchases on unemployment. Hence, our formula connects the effect of government purchases on welfare to the estimates of government-purchase multipliers obtained by a voluminous literature.[5] This literature empirically estimates or numerically computes multipliers to describe the effects of government purchases on output and other variables, abstracting from welfare considerations. However, the multiplier plays a large role in actual policy discussions about stimulus. Stimulus advocates believe that multipliers are substantial and hence that government purchases can help fill the output gap in recessions [for example, Romer and Bernstein, 2009]. Conversely, stimulus skeptics believe that multipliers are small or negative and warn that additional government spending could be wasteful [for example, Barro and Redlick, 2011]. Our theory contributes to this debate by showing how optimal government purchases in recessions depend on both the multiplier and the social value of such purchases.

The relationship between the optimal level of government purchases and the Samuelson level is conditioned by the correction term. When the unemployment rate is efficient or the multiplier is zero, the correction term is zero and optimal government purchases follow the Samuelson formula. But when the unemployment rate is inefficient and the multiplier is nonzero, optimal government purchases systematically depart from the Samuelson formula. Optimal government purchases are above the Samuelson level when the correction term is positive and below it when the correction term is negative. When the multiplier is positive, the correction term is positive when unemployment is inefficiently high and negative when unemployment is inefficiently low.

The structure of our formula—the Samuelson formula plus a correction term capturing stabilization motives—provides results that are similar in spirit to those obtained by others in new Keynesian models. Woodford [2011] notes that away from the zero lower bound, monetary policy perfectly stabilizes the economy; hence, there is no need to use government expenditure for stabilization, and government purchases should follow the Samuelson formula. We obtain the same

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[5]In the literature estimating multipliers on US data, a few representative studies include Rotemberg and Woodford [1992], Ramey and Shapiro [1998], Blanchard and Perotti [2002], Galí, Lopez-Salido and Valles [2007], Mountford and Uhlig [2009], Hall [2009], Auerbach and Gorodnichenko [2012], Barro and Redlick [2011], Ramey [2011b], and Nakamura and Steinsson [2014]. See Ramey [2011a] for an excellent survey.

result when unemployment is efficient in our model. Werning [2012] describes the optimal use of government purchases in a liquidity trap, and decomposes the optimal level of government purchases as the Samuelson level plus a correction term arising when government purchases stimulate the economy. Galí and Monacelli [2008] and Farhi and Werning [2012] study the optimal use government purchases for stabilization in multicountry unions. They find that government purchases should be provided beyond the Samuelson level when government purchases are stabilizing.

Since our formula is expressed with statistics that can be estimated in US data, it is easy to use the formula to address several policy questions. First, we evaluate the response of US government purchases to unemployment fluctuations. We find that actual US government purchases, which are mildly counter-cyclical, would be optimal under a minuscule multiplier of 0.03. If the multiplier is larger than 0.03, US government purchases are not countercyclical enough.

Second, we determine the optimal response of government purchases to a given increase in unemployment. Our formula implies significant increases in government purchases during slumps, even for small multipliers. For instance, with a multiplier of 0.1 and other statistics calibrated to the US economy, when the unemployment rate rises from the US average of 5.9% to a high level of 9%, the optimal government purchases-output ratio increases from 16.6% to 17.8%; and with a multiplier of 0.5, the optimal ratio increases more, from 16.6% to 19.8%.

However, the optimal ratio increases less for multipliers above 0.5. For instance, with a multiplier of 2, the optimal government purchases-output ratio only increases from 16.6% to 17.6% when the unemployment rate rises from 5.9% to 9%. The intuition for the hump-shaped relation between the multiplier and the optimal increase of the government purchases-output ratio is the following. For small multipliers, the optimal amount of government purchases is determined by the crowding out of personal consumption by government consumption; a higher multiplier means less crowding out and thus higher optimal government purchases. For large multipliers, it is optimal to fill the unemployment gap and a higher multiplier means that fewer government purchases are required to fill this gap.

2. A Generic Model of Unemployment with Government Purchases

This section proposes a dynamic model of unemployment with government purchases. The model is set in continuous time. The model is generic in that we do not place much structure on the utility function, matching function, aggregate demand, price mechanism, and tax system. The components of the model that we introduce are sufficient to define a feasible allocation and describe the mathematical structure of an equilibrium, which are the only elements on which the optimal policy analysis relies. By maintaining this degree of generality, we will be able to show in Section 3 that our sufficient-statistics formula for optimal government purchases applies to a broad range of models. In Section 6, we will provide a specific model as an example.

The model builds on the matching framework from Michaillat and Saez [2015a]. The economy is composed of a measure 1 of identical households. Households are self-employed, producing and selling services on a market with matching frictions.[6] Services are purchased by the government and by other households.[7]

A household has a productive capacity normalized to 1. The productive capacity indicates the maximum amount of services that a household could deliver at any point in time. At time t, a household sells $Y(t) < 1$ units of services. An amount $C(t)$ of these services are purchased by other households and an amount $G(t)$ is purchased by the government such that $Y(t) = C(t) + G(t)$. The services are sold through long-term relationships that separate at rate $s > 0$. The idle capacity of the household at time t is $1 - Y(t)$. Since some of the capacity of the household is idle, some household members are unemployed. The rate of unemployment, defined as the share of workers who are idle, is $u(t) = 1 - Y(t)$, where $Y(t)$ is the aggregate output of services.

To purchase labor services at time t, households and government advertise $v(t)$ vacancies. New long-term relationships are formed at a rate $h(1 - Y(t), v(t))$, where h is the matching function, $1 - Y(t)$ is the aggregate idle capacity, and $v(t)$ is the aggregate number of vacancies. The matching function is continuously differentiable, is increasing in both its arguments, is concave, and has constant returns to scale. We also impose that $h(0, v) = 0$ and $h(1 - Y, 0) = 0$.

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[6]To simplify the analysis, we abstract from firms and assume that all production directly takes place within households. Michaillat and Saez [2015a] show how the model can be extended to include a labor market and a product market and firms hiring workers on the labor market and selling their production on the product market.

[7]We assume that households cannot consume their own labor services.

The market tightness x is defined by $x(t) = v(t)/(1 - Y(t))$. The market tightness is the ratio of the two arguments in the matching function: aggregate vacancies and aggregate idle capacity. Since it is an aggregate variable, households take the market tightness as given. With constant returns to scale in matching, the market tightness determines the rates at which sellers and buyers enter into new long-term trading relationships. At time t, each of the $1 - Y(t)$ units of available productive capacity is committed to a long-term relationship at rate $f(x(t)) = h(1 - Y(t), v(t))/(1 - Y(t)) = h(1, x(t))$ and each of the $v(t)$ vacancy is filled with a long-term relationship at rate $q(x(t)) = h(1 - Y(t), v(t))/v(t) = h(1/x(t), 1)$. The function f is increasing and concave. The function q is decreasing. Hence, when the market tightness is higher, it is easier to sell services but harder to buy them. Other useful properties are that $f(0) = 0$, $\lim_{x \to +\infty} q(x) = 0$, and $q(x) = f(x)/x$. We denote the elasticities of $f(x)$ and $q(x)$ with respect to x by $1 - \eta \in (0, 1)$ and $-\eta \in (-1, 0)$.

According to the matching process, output $Y(t)$ and the unemployment rate $u(t) = 1 - Y(t)$ are state variables. However, in practice, because the transitional dynamics of these variables are fast, output and unemployment rate rapidly adjust to their steady-state levels.[8] Throughout the paper, we therefore simplify the analysis by modeling output and unemployment rate as jump variables equal to their steady-state values. With this simplification, output and unemployment rate become functions of market tightness defined by

$$Y(x) = \frac{f(x)}{f(x) + s}, \quad u(x) = \frac{s}{s + f(x)}. \tag{1}$$

Appendix A derives these expressions. Appendix B shows that transitional dynamics are quantitatively unimportant. The function $Y(x)$ is in $[0, 1]$, increasing on $[0, +\infty)$, with $Y(0) = 0$. Intuitively, when the market tightness is higher, if it easier to sell services so output is higher. The elasticity of $Y(x)$ is $(1 - \eta) \cdot u(x)$.

Advertising vacancies is costly. Posting one vacancy costs $\rho > 0$ services per unit time.[9] Hence, a total of $\rho \cdot v(t)$ services are spent at time t on filling vacancies. These services represent the resources devoted by households and government to matching with appropriate providers of services. Since these resources devoted to matching do not enter households' utility function, we define two concepts of consumption. We refer to the quantities $C(t)$ and $G(t)$ purchased by

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[8] Hall [2005], Pissarides [2009], and Shimer [2012] make this point.
[9] Expressing vacancy costs directly in terms of labor services simplifies the model [Michaillat and Saez, 2015a].

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households and government as *gross personal consumption* and *gross government consumption*. Following common usage, *government consumption* designates the consumption by households of services purchased by the government. We define the gross consumptions net of consumption of matching services as *net personal consumption* $c(t) < C(t)$ and *net government consumption* $g(t) < G(t)$. As $C(t)$ and $G(t)$ are fast-moving state variables that are well approximated by their steady-state levels, net and gross consumptions are related by $C(t) = [1 + \tau(x(t))] \cdot c(t)$ and $G(t) = [1 + \tau(x(t))] \cdot g(t)$, where

$$\tau(x) = \frac{\rho \cdot s}{q(x) - \rho \cdot s}. \tag{2}$$

We also refer to the quantity $Y(t) = C(t) + G(t)$ as *gross output* and the quantity $y(t) = c(t) + g(t)$ as *net output*. Of course, $Y(t) = [1 + \tau(x(t))] \cdot y(t)$. All these expressions are derived in Appendix A.

The concepts of gross consumption and gross output correspond to the quantities measured in national accounts.[10] In our model, gross output is proportional to employment. Part of employment is used to create matches (for instance, human resource workers, procurement workers, buyers). This share of employment is part of total employment measured in national accounts, even though the services they provide are used for matching and do not enter households' utility.

Because of the matching cost, enjoying one service requires to purchase $1 + \tau$ services—one service that enters the utility function plus τ services for matching. From the buyer's perspective, it is as if it purchased 1 service at a unit price $1 + \tau$, so τ acts as a wedge on the price of services. The wedge $\tau(x)$ is positive and increasing on $[0, x^m)$, where $x^m \in (0, +\infty)$ is defined by $q(x^m) = \rho \cdot s$.[11] In addition, $\lim_{x \to x^m} \tau(x) = +\infty$. Intuitively, when the market tightness is higher, it is more difficult to match with a seller so the matching wedge is higher. The elasticity of $1 + \tau(x)$ is $\eta \cdot \tau(x)$.

It is useful to write net output as a function of market tightness:

$$y(x) = \frac{Y(x)}{1 + \tau(x)}. \tag{3}$$

This function $y(x)$ plays a central role in the analysis because it gives the amount of services that

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[10]In the US National Income and Product Accounts (NIPA), $C(t)$ is "personal consumption expenditures" and $G(t)$ "government consumption expenditures".

[11]We assume that $q(0) > \rho \cdot s$ such that $x^m > 0$ is well defined. Since $\lim_{x \to +\infty} q(x) = 0$, x^m is necessarily finite.
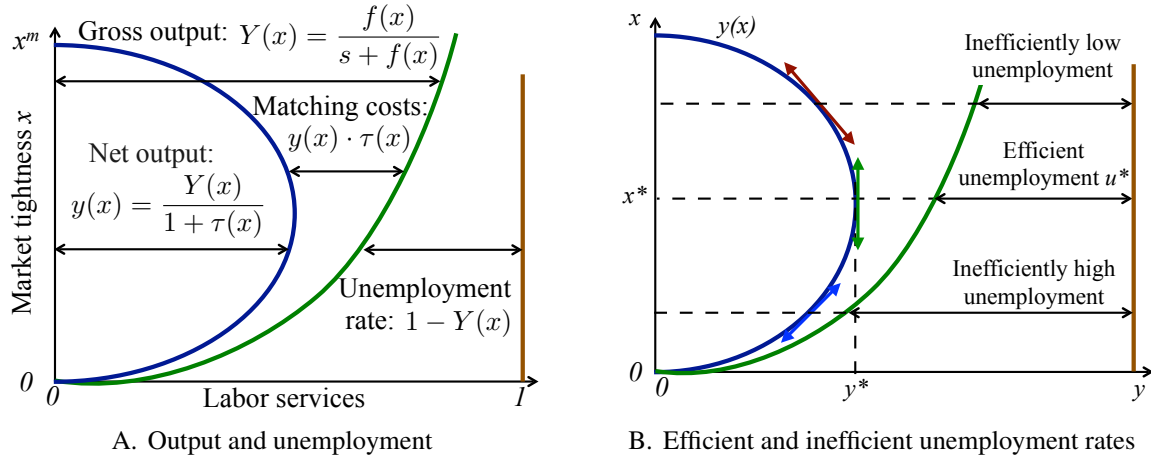
Figure 2: The Market for Labor Services

can be allocated between net personal consumption and net government consumption for a given tightness. The function $y(x)$ is defined on $[0, x^m]$, positive, with $y(0) = 0$ and $y(x^m) = 0$. The elasticity of $y(x)$ is $(1 - \eta) \cdot u(x) - \eta \cdot \tau(x)$.

We assume that the matching function is well behaved such that the function $y(x)$ is concave.[12] Hence, there is a unique tightness $x^* \in (0, x^m)$ that maximizes $y(x)$. The tightness x^* satisfies

$$1 = \frac{\eta}{1 - \eta} \cdot \frac{\tau(x^*)}{u(x^*)}. \tag{4}$$

The tightness x^* is the efficient tightness. We denote the efficient unemployment rate by $u^* = u(x^*)$.

Figure 2 summarizes the results that we have established. Panel A depicts how net output, gross output, and unemployment rate depend on market tightness. Panel B depicts the function $y(x)$, the efficient tightness x^*, the efficient unemployment rate u^*, and situations in which the unemployment rate is inefficiently high ($u > u^*$) and inefficiently low ($u < u^*$).

We assume that the government sets $g(t)$ as a function of the other variables at time t and parameters.[13] In that case, the dynamical system describing the equilibrium of the model only has jump variables—no state variables. We therefore assume that the equilibrium system is a source. Accordingly, the equilibrium converges immediately to its steady-state value from any initial condition.[14]

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[12]In Section 6, we use a standard Cobb-Douglas matching function and show that $y(x)$ is indeed concave.

[13]This means that $g(t)$ does not have any persistence.

[14]Without this assumption, the model would suffer from dynamic indeterminacy, making the welfare analysis im-

We summarize the presentation of the model by defining an allocation and an equilibrium. We give a static definition since we assume that the system converges immediately to its steady-state:

DEFINITION 1. *A* feasible allocation *is a net personal consumption $c \in [0,1]$, a net government consumption, $g \in [0,1]$, and net output $y \in [0,1]$, and a market tightness $x \in [0,+\infty)$ that satisfy $y = y(x)$ and $c = y - g$. The function $y(x)$ is defined by* (3).

DEFINITION 2. *An* equilibrium function *is a mapping from a net government consumption g to a feasible allocation $[c,g,y,x]$. Given that in a feasible allocation, y and c are functions of x and g, the equilibrium function can be summarized by a mapping from a net government consumption g to a market tightness $x(g)$. We assume that the function $x(g)$ is continuously differentiable.*

In the model, an equilibrium is just a value of the equilibrium function. In practice the equilibrium function $x(g)$ arises from the household's optimal consumption choice, the price mechanism, and the tax system in place to finance government purchases. The function $x(g)$ can describe efficient prices, bargained prices, or rigid prices. It can describe economies in which government purchases are financed by lump-sum taxes or taxes proportional to output or consumption, by deficit spending with Ricardian households, or by deficit spending with non-Ricardian households. As a concrete example, we will describe the function $x(g)$ in the specific model of Section 6.

3. Sufficient-Statistics Formulas for Optimal Government Purchases

The representative household derives instantaneous utility $\mathcal{U}(c,g)$ from net personal consumption c and net government consumption g. The function $\mathcal{U}$ is twice continuously differentiable, increasing in its two arguments, concave, and homothetic.[15] Since $\mathcal{U}$ is homothetic, the marginal rate of substitution between g and c is a decreasing function of $g/c = G/C$.[16]

Since the equilibrium immediately converges to its steady state, the welfare in an equilibrium is just $\mathcal{U}(c,g)$. In a feasible allocation, net personal consumption is given by $c = y(x) - g$, so

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possible.

[15]By homothetic, we mean that the utility can be written as $\mathcal{U}(c,g) = w(p(c,g)))$ where the function w is twice continuously differentiable and increasing and the function p is twice continuously differentiable, increasing in its two arguments, concave, and homogeneous of degree 1.

[16]The marginal rate of substitution between g and c is $MRS_{gc} = (\partial \mathcal{U}/\partial g)/(\partial \mathcal{U}/\partial c) = p_g(c,g)/p_c(c,g)$. Since p is homogeneous of degree 1, the derivatives p_c and p_g are homogeneous of degree 0. Hence, we can write $MRS_{gc} = p_g(1,g/c)/p_c(c/g,1)$. Thus, MRS_{gc} is a function of g/c only. Since p is concave, $p_g(1,g/c)$ is a decreasing function of g/c and $p_c(c/g,1)$ is an increasing function of g/c. Hence, MRS_{gc} is a decreasing function of g/c.

welfare can be written as $\mathscr{U}(y(x) - g, g)$. Given an equilibrium function $x(g)$, the problem of the government is to determine g to maximize welfare

$$\mathscr{W}(g) = \mathscr{U}(y(x(g)) - g, g).$$

We assume that the government's problem is a well-behaved concave problem such that the first-order conditions are necessary and sufficient to characterize the optimum.

In this section we derive several sufficient-statistics formulas giving the optimal level of government purchases. These formulas are equivalent, but they are adapted to answer different questions. We start by expressing the formula in an abstract way to describe the economic forces at play and compare the optimal level of government purchases to the level from the Samuelson formula.

Next, we express the abstract formula in terms of sufficient statistics that can be estimated in the data. This exact formula can be calibrated empirically but it is somewhat complex. Hence, we approximate it with an extremely simple formula that relates the deviation of optimal government purchases from the Samuelson level to the deviation of the actual market tightness from the efficient market tightness and only two statistics: the elasticity of substitution between government and personal consumption, and the government-purchases multiplier.

The simple formula is helpful to evaluate actual government purchases, but, because it only implicitly defines optimal government purchases, it cannot be used to determine the optimal change in government purchases after a shock altered the unemployment rate. To address this question, we rework the simple formula and derive a formula that explicitly express optimal government purchases as a function of stable sufficient statistics and the change in unemployment rate observed after the shock.

3.1. An Abstract Formula

Taking the first-order condition of the government's problem, we obtain

$$0 = \frac{d\mathscr{W}}{dg} = -\frac{\partial \mathscr{U}}{\partial c} + \frac{\partial \mathscr{U}}{\partial g} + \frac{\partial \mathscr{U}}{\partial c} \cdot \frac{dy}{dx} \cdot \frac{dx}{dg}.$$

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Reshuffling the terms in the optimality condition and dividing the condition by $\partial \mathscr{U}/\partial c$ yields the formula for optimal government purchases:

PROPOSITION 1. *Optimal government purchases satisfy*

$$1 = MRS_{gc} + \frac{dy}{dx} \cdot \frac{dx}{dg}, \tag{5}$$

where $MRS_{gc} = (\partial \mathscr{U}/\partial g)/(\partial \mathscr{U}/\partial c)$ is the marginal rate of substitution between government consumption and personal consumption.

The Samuelson formula is $1 = MRS_{gc}$; it requires that the marginal utility from personal consumption equals the marginal utility from government consumption. Our formula is just the Samuelson formula plus a correction term equal to $(dy/dx) \cdot (dx/dg)$. The structure of the formula—a standard public-economics formula plus a correction term capturing stabilization motives—is similar to the structure of the formula for optimal unemployment insurance derived by Landais, Michaillat and Saez [2010] or for optimal taxation derived by Farhi and Werning [2013].

With homothetic preferences, MRS_{gc} is decreasing in g/c. Since $G/Y = (g/c)/(g/c+1)$, MRS_{gc} is also decreasing in G/Y. Thus, the Samuelson determines a unique government purchases-output ratio. Furthermore, our formula indicates that it is desirable to increase the government purchases-output ratio above the Samuelson ratio if the correction term is positive, and to decrease the government purchases-output ratio below the Samuelson ratio if the correction term is negative. If the correction term is zero, the optimal government purchases-output ratio satisfies the Samuelson formula.

The correction term is the product of the effect of government purchases on tightness and the effect of tightness on net output. The correction term is positive if and only if more government purchases yield higher net output in equilibrium. Given the existing links between tightness, net output, and unemployment rate (Figure 2), an equivalent statement is that the correction term is positive if and only if government purchases bring the unemployment rate toward its efficient level.

There are two situations when the correction term is zero and the optimal government purchases-output ratio is given by the Samuelson formula. The first situation is when $dy/dx = 0$, which means that the unemployment rate is efficient. In that case, the marginal effect of government purchases on unemployment has no first-order effect on welfare and the principles of Samuelson's theory

Table 1: Optimal Government Purchases-Output Ratio Compared to Samuelson Ratio

Unemployment rate	Effect of net government consumption on unemployment		
	$du/dg > 0$	$du/dg = 0$	$du/dg < 0$
Inefficiently high	lower	same	higher
Efficient	same	same	same
Inefficiently low	higher	same	lower

Notes: The government purchases-output ratio in the theory of Samuelson [1954] is given by $1 = MRS_{gc}$. Compared to the Samuelson ratio, the optimal government purchases-output ratio is higher if the correction term in (5) is positive, same if the correction term is zero, and lower if the correction term is negative. By definition, the unemployment rate is inefficiently high when $dy/dx > 0$, inefficiently low when $dy/dx < 0$, and efficient when $dy/dx = 0$. Last, $du/dg = (du/dx) \cdot (dx/dg)$ where $u(x)$ is given by (1). Since $du/dx > 0$, the signs of du/dg and dx/dg are the same.

apply. The second situation is when $dx/dg = 0$, which means that government purchases have no effect on tightness and thus on the unemployment rate. In that case, the model is isomorphic to Samuelson's framework.

In all other situations, the correction term is nonzero and the optimal government purchases-output ratio departs from the Samuelson ratio. The formula implies that the optimal government purchases-output ratio is above the Samuelson ratio if and only if government purchases bring unemployment closer to its efficient level. This occurs either if the unemployment rate is inefficiently high and government purchases lower it, or if the unemployment rate is inefficiently low and government purchases raise it. Table 1 summarizes all the possibilities.

The results on government purchases typically obtained in the Keynesian regime of disequilibrium models can easily be recovered from formula (5).[17] The correction term in (5) can be written as $(dy/dx) \cdot (dx/dg) = dy/dg$. In a disequilibrium model, there are no matching costs so $y = Y$ and $g = G$ and the correction term is the standard multiplier dY/dG. In the Keynesian regime, personal consumption is fixed because it is determined by aggregate demand and the above-market-clearing price; hence, there is no crowding out of personal consumption by government consumption and $dY/dG = 1$. On the other hand when the product market clears, crowding out is one-for-one and $dY/dG = 0$. We assume that there is some value for government purchases such that $MRS_{gc} > 0$. Our formula implies that in the Keynesian regime, it is optimal to use government purchases to fill the output gap. Indeed, $MRS + dY/dG > 1$ as long as the output gap is not closed, so additional

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[17]For a typical disequilibrium model, see Barro and Grossman [1971].

government purchases always raise welfare in the Keynesian regime.

3.2. An Exact Implicit Formula

We express formula (5) with estimable sufficient statistics to facilitate its interpretation and empirical applications. The correction term in (5) is $(dy/dx) \cdot (dx/dg) = dy/dg$. The multiplier dy/dg gives the increase in net output when net government consumption increases by one unit. However, dy/dg is not directly estimable in aggregate data because the data measure gross and not net consumption. We therefore express dy/dg as a function of the multiplier dY/dG, which gives the increase in gross output when gross government consumption increases by one unit, and other estimable statistics. The multiplier dY/dG corresponds to the government-purchases multiplier that macroeconomists estimate in aggregate data. We find that (5) can be reformulated as follows:

PROPOSITION 2. *Optimal government purchases satisfy*

$$1 = MRS_{gc} + \left(1 - \frac{\eta}{1-\eta} \cdot \frac{\tau}{u}\right) \cdot \frac{dY}{dG} \cdot \left(1 - \frac{\eta}{1-\eta} \cdot \frac{\tau}{u} \cdot \frac{G}{Y} \cdot \frac{dY}{dG}\right)^{-1}, \tag{6}$$

where MRS_{gc} is the marginal rate of substitution between government and personal consumptions, dY/dG is the government-purchases multiplier, $1 - \eta$ is the tightness elasticity of the job-finding rate, τ is the matching wedge, u is the unemployment rate, and G/Y is the government purchases-output ratio.

Proof. First, note that

$$\frac{d\ln(y)}{d\ln(g)} = \frac{d\ln(y)}{d\ln(x)} \cdot \frac{d\ln(x)}{d\ln(G)} \cdot \frac{d\ln(G)}{d\ln(g)}.$$

Next, as the elasticity of $Y(x)$ is $(1-\eta) \cdot u$, we find that

$$\frac{d\ln(x)}{d\ln(G)} = \frac{1}{(1-\eta) \cdot u} \cdot \frac{d\ln(Y)}{d\ln(G)}.$$

Last, using $G = (1 + \tau(x)) \cdot g$ and as the elasticity of $1 + \tau(x)$ is $\eta \cdot \tau$, we find that

$$\frac{d\ln(G)}{d\ln(g)} = 1 + \eta \cdot \tau \cdot \frac{d\ln(x)}{d\ln(G)} \cdot \frac{d\ln(G)}{d\ln(g)}.$$

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Combining this equation with the expression for $d\ln(x)/d\ln(G)$ obtained above, we get

$$\frac{d\ln(G)}{d\ln(g)} = \left(1 - \frac{\eta}{1-\eta} \cdot \frac{\tau}{u} \cdot \frac{d\ln(Y)}{d\ln(G)}\right)^{-1}.$$

Combining all these results and as the elasticity of $y(x)$ is $(1-\eta) \cdot u - \eta \cdot \tau$, we obtain

$$\frac{dy}{dg} = \left(1 - \frac{\eta}{1-\eta} \cdot \frac{\tau}{u}\right) \cdot \frac{dY}{dG} \cdot \left(1 - \frac{\eta}{1-\eta} \cdot \frac{\tau}{u} \cdot \frac{d\ln(Y)}{d\ln(G)}\right)^{-1}.$$

Bringing all the elements together, we obtain (6). □

The correction term in formula (6) is the product of three terms. The first term is $d\ln(y)/d\ln(Y)$. It indicates how net output responds when gross output changes because of an underlying change in tightness. This term is positive when tightness is inefficiently low and zero when tightness is efficient. The second term is the government-purchases multiplier dY/dG. It indicates how gross output responds to a change in gross government purchases. The last term is $d\ln(G)/d\ln(g)$. It indicates how gross government consumption responds when net government consumption changes.

3.3. Two Approximate Implicit Formulas

Formula (6) is a bit complex, but it can be greatly simplified with a few approximations:

PROPOSITION 3. *Let x^* and u^* be the efficient market tightness and unemployment rate. Let $(G/C)^*$ be the solution to the Samuelson formula, $1 = MRS_{gc}$. Let $(G/Y)^* = (G/C)^*/(1 + (G/C)^*)$. In the vicinity of x^* and $(G/C)^*$, optimal government purchases approximately satisfy*

$$\frac{G/C - (G/C)^*}{(G/C)^*} \approx -\varepsilon \cdot \frac{dY}{dG} \cdot \frac{x - x^*}{x^*}, \tag{7}$$

where dY/dG is the government-purchases multiplier, and ε is the elasticity of substitution between government and personal consumptions. Alternatively, in the vicinity of u^* and $(G/Y)^*$, optimal government purchases approximately satisfy

$$\frac{G/Y - (G/Y)^*}{(G/Y)^*} \approx \varepsilon \cdot \frac{dY}{dG} \cdot \frac{1 - (G/Y)^*}{1-\eta} \cdot \frac{u - u^*}{u^*}, \tag{8}$$

where $1 - \eta$ is the tightness elasticity of the job-finding rate. The statistics ε, dY/dG, and $1 - \eta$ can be estimated around x^* and $(G/C)^*$.

Proof. We start from formula (6). The first approximation is a simple first-order approximation of MRS_{gc} around $(G/C)^*$. Since we assume homothetic preferences, MRS_{gc} is a function of $g/c = G/C$ only. By definition of the elasticity of substitution between g and c, $-1/\varepsilon = d\ln(MRS_{gc})/d\ln(G/C)$. The first-order approximation of $\ln(MRS_{gc})$ around $\ln((G/C)^*)$ yields

$$\ln\left(MRS_{gc}\left(\frac{G}{C}\right)\right) - \ln\left(MRS_{gc}\left(\left(\frac{G}{C}\right)^*\right)\right) = -\frac{1}{\varepsilon} \cdot \left[\ln\left(\frac{G}{C}\right) - \ln\left(\left(\frac{G}{C}\right)^*\right)\right].$$

By definition, $MRS_{gc}((G/C)^*) = 1$ so $\ln(MRS_{gc}((G/C)^*)) = 0$. Furthermore, for G/C around $(G/C)^*$, $\ln((G/C)/(G/C)^*) \approx (G/C)/(G/C)^* - 1$ and $MRS_{gc}(G/C) \approx 1$ so $\ln(MRS_{gc}(G/C)) \approx MRS_{gc}(G/C) - 1$. Combining these first-order approximations, we obtain

$$MRS_{gc} - 1 \approx -\frac{1}{\varepsilon} \cdot \frac{G/C - (G/C)^*}{(G/C)^*}.$$

Note that the elasticity of substitution, ε, is evaluated at $(G/C)^*$.

The second approximation is that

$$\frac{\tau(x)}{u(x)} = \frac{s \cdot \rho}{q(x) - s \cdot \rho} \cdot \frac{s + f(x)}{s} \approx \frac{s \cdot \rho}{q(x)} \cdot \frac{f(x)}{s} = \rho \cdot x.$$

On average in US monthly data, $s = 3.3\%$, $s \cdot \rho = 6.5\%$, $f(x) = 56\%$, and $q(x) = 94\%$, which is why we can approximate $s + f(x)$ by $f(x)$ and $q(x) - s \cdot \rho$ by $q(x)$.[18] This approximation implies that $\tau(x^*)/u(x^*) \approx \rho \cdot x^*$. We also know that by definition of efficiency, $\tau(x^*)/u(x^*) = (1 - \eta)/\eta$. Hence, $(1 - \eta)/\eta \approx \rho \cdot x^*$. Combining these approximations, we obtain

$$1 - \frac{\eta}{1 - \eta} \cdot \frac{\tau(x)}{u(x)} \approx 1 - \frac{x}{x^*}. \tag{9}$$

This term is small around x^*, so up to a first-order approximation, all the other terms in the correction term of formula (6) can be evaluated at x^*. This includes the multiplier, dY/dG.

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[18]Using US data, Appendix B obtains the average value of s, $s \cdot \rho$, $f(x)$, and $q(x)$. Appendix B also validates the approximation at any point in time between 1951 and 2014.

The third approximation is that

$$1 - \frac{\eta}{1-\eta} \cdot \frac{\tau(x)}{u(x)} \cdot \frac{G}{Y} \cdot \frac{dY}{dG} \approx 1 - \frac{x}{x^*} \cdot \frac{d\ln(Y)}{d\ln(G)} \approx 1 - \frac{d\ln(Y)}{d\ln(G)}.$$

The first step in this approximation comes from (9). The second step is possible because we evaluate this expression at x^*, as we have just discussed.

Combining the three approximations yields

$$\frac{G/C - (G/C)^*}{(G/C)^*} \approx -\varepsilon \cdot \frac{dY/dG}{1 - d\ln(Y)/d\ln(G)} \cdot \frac{x-x^*}{x^*}. \tag{10}$$

For large values of the elasticity $d\ln(Y)/d\ln(G)$, which means large values of the multiplier dY/dG combined with large values of the ratio G/Y, we could not simplify the formula further. However, in practice, the elasticity $d\ln(Y)/d\ln(G)$ is fairly small. On average in US data, $G/Y = 0.17$ and a reasonable estimate of the multiplier is $dY/dG = 0.5$. With these values, $1 - d\ln(Y)/d\ln(G) \approx 0.92$, quite close to 1. Hence, we further simplify formula (10) by approximating the term $1 - d\ln(Y)/d\ln(G)$ by 1.[19] Thus, we obtain formula (7).

To obtain formula (8), we start from (7) and use first-order approximations to express the relative deviations of u and G/Y as a function of those of x and G/C. Using $u = 1 - Y(x)$ and the elasticity of $Y(x)$, we find that for u and x near u^* and x^*,

$$\frac{u-u^*}{u^*} \approx \ln\left(\frac{u}{u^*}\right) \approx \frac{u^*-1}{u^*} \cdot (1-\eta) \cdot u^* \cdot \ln\left(\frac{x}{x^*}\right) \approx (u^*-1) \cdot (1-\eta) \cdot \frac{x-x^*}{x^*} \approx -(1-\eta) \cdot \frac{x-x^*}{x^*}.$$

We obtain the last approximation by noting that in the US, $u^* - 1 = -0.941 \approx -1$. Next, using $G/Y = (G/C)/(1+G/C)$, we find that for G/Y and G/C near $(G/Y)^*$ and $(G/C)^*$,

$$\frac{G/Y - (G/Y)^*}{(G/Y)^*} \approx \ln\left(\frac{G/Y}{(G/Y)^*}\right) \approx [1 - (G/Y)^*] \cdot \ln\left(\frac{G/C}{(G/C)^*}\right) \approx [1 - (G/Y)^*] \cdot \frac{G/C - (G/C)^*}{(G/C)^*}.$$

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[19]Section 4 discusses available estimates of dY/dG. Using US data, Appendix B obtains the average value of G/Y and validates the approximation at any point in time between 1951 and 2014.

Combining these two approximations with formula (10) yields

$$\frac{G/Y - (G/Y)^*}{(G/Y)^*} \approx \varepsilon \cdot \frac{dY/dG}{1 - d\ln(Y)/d\ln(G)} \cdot \frac{1 - (G/Y)^*}{1 - \eta} \cdot \frac{u - u^*}{u^*}. \tag{11}$$

Using again that $1 - d\ln(Y)/d\ln(G) \approx 1$, we obtain formula (8). $\qquad\square$

Formulas (7) and (8) are equivalent but they are adapted to different tasks. Formula (7) is simpler—it involves fewer statistics and fewer approximations—and it will be useful for the empirical analysis. Formula (8), on the other hand, is easier to interpret and will be useful later on.

Formulas (7) and (8) highlight the two statistics required to determine the optimal level of government purchases: the elasticity of substitution between government and personal consumptions and the government-purchases multiplier.

The elasticity of substitution plays an important role because it determines how rapidly the marginal value of government purchases relative to that of personal consumption fades with additional government purchases. The role of this elasticity has been largely neglected in previous work. If $\varepsilon = 0$, for instance $\mathcal{U}(c,g) = \min\{(c/\bar{c}), (g/\bar{g})\}$, then government purchases are useless at the margin beyond $\bar{g}/\bar{c}$, so the ratio G/C should stay at $\bar{g}/\bar{c}$. This would be a situation in which we need a number of bridges for an economy of a given size, but beyond that number, additional bridges have zero value ("bridges to nowhere"). If $\varepsilon \to +\infty$, for instance $\mathcal{U}(c,g) = c+g$, then the marginal rate of substitution is constant at 1. In that case, government purchases should be used to always fill the unemployment gap such that $x = x^*$. The intuition is that the composition of output does not matter for welfare so government consumption should be used to stabilize the economy, even if it crowds out private consumption, since the only thing that matters for welfare is total consumption. This would be a situation in which the services provided by the government substitute exactly the services that can be purchased by individuals on the market. In reality, government purchases probably have some value at the margin, without being perfect substitute for personal consumption; that is, $\varepsilon > 0$ but $\varepsilon < +\infty$. We will consider a range of values for ε.

The formulas confirm an intuition that many macroeconomists had but that had not been formalized before: optimal government purchases do depend on the government-purchases multiplier. Of course, if the multiplier is zero then government purchases should remain at the level given by the Samuelson formula. If the multiplier is positive, the government purchases-output ratio should

be above the Samuelson ratio when unemployment is inefficiently high. If multiplier is negative, the converse applies: the government purchases-output ratio should be below the Samuelson ratio when unemployment is inefficiently high.

However, fluctuations of the multiplier in response to a change in unemployment only have second-order effects. It is the departures of unemployment from its efficient level that have first-order effects on the optimal level of government purchases. To a first-order approximation, the average multiplier is sufficient to obtain the response of optimal government purchases to a shock.

3.4. An Approximate Explicit Formula

While formula (7) is useful for certain applications, we cannot use the formula to answer the following question: if the unemployment rate is 50% above its efficient level and government purchases are at the Samuelson level, what should be the increase in government purchases? This is because our formula is an implicit formula: it gives the relation that equilibrium statistics should satisfy, but it does not tell us how much government purchases should change to arrive at the optimum because the right-hand side of (7) is endogenous to G/Y. In fact, this is a typical limitation of sufficient-statistics optimal policy formulas, and a typical criticism addressed to the sufficient-statistics approach [Chetty, 2009; Golosov, Troshkin and Tsyvinski, 2011]. Here we develop an explicit sufficient-statistics formula that we can use to address this question.

Assume that the unemployment rate is initially efficient and that an unexpected permanent shock brings the unemployment rate from u^* to u_0. As government purchases G change, the unemployment rate will endogenously respond. It is this endogenous response that we need to describe to obtain our explicit formula.

PROPOSITION 4. *Let u^* be the efficient unemployment rate. Let $(G/Y)^*$ be the government purchases-output ratio given by the Samuelson formula, $1 = MRS_{gc}$. Initially, the unemployment rate is efficient ($u = u^*$) and the government purchases-output ratio is optimal ($G/Y = (G/Y)^*$). The economy is hit by an unexpected permanent shock that brings the unemployment rate to $u_0 > u^*$. Then the response of the optimal government purchases-output ratio satisfies*

$$\frac{G/Y - (G/Y)^*}{(G/Y)^*} \approx \frac{\varepsilon \cdot \frac{dY}{dG}}{\frac{1-\eta}{1-(G/Y)^*} + \varepsilon \cdot \left(\frac{dY}{dG}\right)^2 \cdot \frac{(G/Y)^*}{u^*}} \cdot \frac{u_0 - u^*}{u^*}, \tag{12}$$

where dY/dG is the government-purchases multiplier, ε the elasticity of substitution between government and personal consumptions, and $1 - \eta$ the tightness elasticity of the job-finding rate.

Proof. At u_0, government purchases are G_0 and Y_0 such that $G_0/Y_0 = (G/Y)^*$. Then, as government purchases change to G, the unemployment rate responds. We describe this response to obtain the explicit formula. Given that $u = 1 - Y$, $du/dG = -(dY/dG)$. Hence, a first-order approximation of u around G_0 yields (after subtracting u^* on both sides)

$$u - u^* \approx u_0 - u^* - \frac{dY}{dG} \cdot (G - G_0).$$

Moreover, first-order approximation when G and G/Y are around G_0 and $G_0/Y_0 = (G/Y)^*$ gives

$$\frac{G/Y - (G/Y)^*}{(G/Y)^*} \approx \ln\left(\frac{G/Y}{G_0/Y_0}\right) \approx \left(1 - \frac{d\ln(Y)}{d\ln(G)}\right) \cdot \ln\left(\frac{G}{G_0}\right) \approx \left(1 - \frac{d\ln(Y)}{d\ln(G)}\right) \cdot \frac{G - G_0}{G_0}.$$

Noting that $G_0 = (1 - u_0) \cdot (G/Y)^* \approx (G/Y)^*$ and collecting these results yields

$$\frac{u - u^*}{u^*} = \frac{u_0 - u^*}{u^*} - \frac{(G/Y)^*}{u^*} \cdot \frac{dY/dG}{1 - d\ln(Y)/d\ln(G)} \cdot \frac{G/Y - (G/Y)^*}{(G/Y)^*}.$$

We plug this expression in formula (11) and do a bit of algebra to obtain

$$\frac{G/Y - (G/Y)^*}{(G/Y)^*} \approx \frac{\varepsilon \cdot \frac{dY/dG}{1 - d\ln(Y)/d\ln(G)}}{\frac{1-\eta}{1-(G/Y)^*} + \varepsilon \cdot \left(\frac{dY/dG}{1 - d\ln(Y)/d\ln(G)}\right)^2 \cdot \frac{(G/Y)^*}{u^*}} \cdot \frac{u_0 - u^*}{u^*}. \tag{13}$$

Once more, if $d\ln(Y)/d\ln(G)$ were not small, we could not simplify the formula further. But since in practice $1 - d\ln(Y)/d\ln(G) \approx 1$, we simplify the formula to obtain (12). $\square$

Formula (12) links the relative deviation of the government purchases-output ratio with the relative deviation of the unemployment rate. The formula can be directly applied by policymakers to determine the optimal response of government purchases to a shock that leads to an increase or decrease in the unemployment rate.

Consider an increase in unemployment by 1 percentage point from the efficient unemployment rate u^*. Formula (13) gives the optimal change in the government purchases-output ratio in response to higher unemployment. We denote this optimal change, measured in percentage points,

by $\Delta(dY/dG)$. An implication from (13) is that for positive multipliers, the function Δ is positive but hump-shaped—that is, a higher multiplier does not necessarily imply a stronger increase in government purchases after an increase in unemployment. The following proposition formalizes this statement:

PROPOSITION 5. *The function Δ is negative on $(-\infty, 0]$, positive on $[0, Y/G)$, with $\Delta(0) = \Delta(Y/G) = 0$. The function Δ is decreasing on $(-\infty, (dY/dG)^{min}]$, increasing on $[(dY/dG)^{min}, (dY/dG)^{max}]$, and decreasing on $[(dY/dG)^{max}, Y/G]$. The function Δ is minimized at maximized at $(dY/dG)^{min} < 0$ and maximized at $(dY/dG)^{max} > 0$. Let Δ^m be the maximum of Δ. The minimum of Δ is $-\Delta^m$. The extremum Δ^m is given by*

$$\Delta^m = \sqrt{\varepsilon \cdot \frac{1 - (G/Y)^*}{1 - \eta} \cdot \frac{(G/Y)^*}{u^*}}.$$

Proof. The function Δ is defined by

$$\Delta(dY/dG) = \frac{\frac{dY/dG}{1 - d\ln(Y)/d\ln(G)}}{\frac{1}{\varepsilon} \cdot \frac{1 - \eta}{1 - (G/Y)^*} \cdot \frac{u^*}{(G/Y)^*} + \left(\frac{dY/dG}{1 - d\ln(Y)/d\ln(G)}\right)^2}$$

All the results follow from some routine algebra. (It is useful to make the change of variable $\theta = (dY/dG)/(1 - d\ln(Y)/d\ln(G))$.) $\qquad\square$

The maximum Δ^m gives the strongest possible response of government purchases to an increase in unemployment, for any possible multiplier. This upper bound is useful given that empirical research has not yet reached a consensus on the precise value of the multiplier. The maximum depends critically on the elasticity of substitution.

There is a simple intuition behind the apparently surprising result that the increase in government purchases is not monotonically increasing with the multiplier. Consider first a small multiplier: $dY/dG \to 0$. We can neglect the feedback effect of G on u because the multiplier is small so $u \approx u_0$. Hence, the application of formula (8) yields

$$\frac{G/Y - (G/Y)^*}{(G/Y)^*} \approx \varepsilon \cdot \frac{dY}{dG} \cdot \frac{1 - (G/Y)^*}{1 - \eta} \cdot \frac{u_0 - u^*}{u^*}.$$

From this formula, it is clear that when $dY/dG \to 0$, $G/Y - (G/Y)^*$ increases with dY/dG.[20] The intuition is that for small multipliers, the optimal amount of government purchases is determined by the crowding out of personal consumption by government consumption; a higher multiplier means less crowding out and thus higher optimal government purchases.

Consider next a very large multiplier, $d\ln(Y)/d\ln(G) \to 1$. With such a large multiplier, G/Y remains constant as G increases (formally, $d\ln(G/Y)/d\ln(G) = 1 - d\ln(Y)/d\ln(G) \to 0$). Since the marginal rate of substitution between government and personal consumptions only depends on G/Y, increasing G fills the output gap without changing the marginal rate of substitution. The optimum is to fill the output gap $Y^* - Y_0$ by increasing G while maintaining G/Y at the Samuelson level with $MRS_{gc} = 1$. A first-order approximation yields $(Y - Y_0)/Y_0 \approx \ln(Y/Y_0) \approx (d\ln(Y)/d\ln(G)) \cdot \ln(G/G_0)$. When the output gap is filled, $(Y - Y_0)/Y_0 = (Y^* - Y_0)/Y_0 \approx u_0 - u^*$. Hence, filling the output gap necessitates $\ln(G/G_0) = (u_0 - u^*)/(d\ln(Y)/d\ln(G))$. Furthermore, another first-order approximation implies that $[G/Y - (G/Y)^*]/(G/Y)^* \approx \ln((G/Y)/(G/Y)^*) = \ln((G/Y)/(G_0/Y_0)) \approx (1 - d\ln(Y)/d\ln(G)) \cdot \ln(G/G_0)$. Hence we obtain the formula

$$\frac{G/Y - (G/Y)^*}{(G/Y)^*} \approx \frac{1 - d\ln(Y)/d\ln(G)}{d\ln(Y)/d\ln(G)} \cdot (u_0 - u^*)$$

Clearly, $G/Y - (G/Y)^*$ decreases with $d\ln(Y)/d\ln(G)$ and accordingly with dY/dG.[21] The intuition is that if the multiplier is high, government purchases are a very potent policy that can bring the economy close to the efficient unemployment without distorting the allocation of output between personal and government consumption. As the multiplier rises, fewer government purchases are required to bring unemployment to its efficient level.

4.   Construction of the Sufficient Statistics for the United States

This section proposes estimates for the sufficient statistics in formulas (7) and (12). The first statistic is the government-purchases multiplier. We use the estimates reported by Ramey [2011a] in her survey of the vast literature estimating multipliers. Table 1 in Ramey [2011a] shows that in aggregate analyzes on US data, the range of estimates is 0.6–2.0 for multipliers financed by

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[20]The explicit formula (12) indeed simplifies to the same expression when $dY/dG \to 0$.

[21]The explicit formula (13) indeed simplifies to the same expression when $d\ln(Y)/d\ln(G) \to 1$.

deficit spending. If government purchases are financed by taxes but households are Ricardian, the range 0.6–2.0 would also apply.[22] If households are non-Ricardian, then the multiplier should account for the effect of current higher taxes on output. Barro and Redlick [2011] propose that the multiplier effect of taxes on output is -1.1, which implies that the relevant range of estimates is -0.7–0.9.[23] We use a multiplier of 0.5 as a baseline. Given the uncertainty of the multiplier estimates, we also consider a range of estimates centered around 0.5.

The second statistic is the elasticity of substitution between government consumption and personal consumption. A Leontief utility function has an elasticity of 0. A Cobb-Douglas utility function has an elasticity of 1. A linear utility function has an elasticity of $+\infty$. We use an elasticity of 1 as a baseline, and we also consider lower and higher values.

Using US data for the 1951–2014 period, we now estimate the remaining sufficient statistics: the ratio of government consumption to personal consumption, market tightness, unemployment, and the tightness elasticity of the job-finding rate.

4.1.   The Ratio of Government Consumption to Personal Consumption

Using employment data constructed by the BLS from the Current Employment Statistics (CES) survey, we measure G/C as the ratio of employment in the government industry to employment in the private industry.[24] Figure 3 plots G/C. The ratio G/C started at 15.5% in 1951, peaked at 24.0% in 1975, fell back to 20.0% in 1990, and averages 20.5% since 1990. The average of G/C over the 1951–2014 period is 19.9%. Using $G/Y = 1/(1+C/G)$, we find that the average of G/Y over the 1951–2014 period is 16.6%.

Under the assumption that the government determines the trend of government purchases by following the well-known Samuelson formula, the ratio $(G/C)^*$ can be measured as the low-frequency trend of G/C.[25] We produce this trend using a Hodrick-Prescott (HP) filter with smooth-

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[22]Lump-sum taxation is equivalent to deficit financing with Ricardian households [Barro, 1974].

[23]Distortionary taxation (as opposed to lump-sum taxation) can also affect output and the multiplier, a point we discuss in conclusion.

[24]Appendix C constructs an alternative measure of G/C using consumption expenditures data constructed by the Bureau of Economic Analysis (BEA) as part of the NIPA. The cyclical behavior of the two series is similar and almost undistinguishable after 1980. We measure G/C with employment data to be consistent with our measure of market tightness based on labor market data.

[25]If the trend of unemployment is efficient, it is optimal to determine the trend of government purchases with the Samuelson formula.
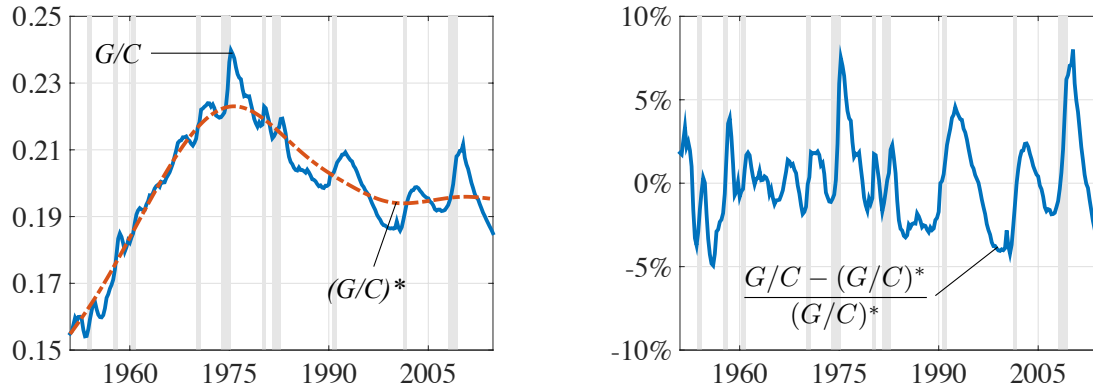
Figure 3: The Ratio of Government Consumption to Personal Consumption in the United States, 1951–2014

Notes: Government consumption G is the quarterly average of the seasonally adjusted monthly number of employees in the government industry constructed by the BLS from the CES survey. Personal consumption C is the quarterly average of the seasonally adjusted monthly number of employees in the private industry constructed by the BLS from the CES survey. The ratio $(G/C)^*$ is the low-frequency trend of G/C produced with a HP filter with smoothing parameter 10^5. The shaded areas represent the recessions identified by the NBER.

ing parameter 10^5 and the quarterly series for G/C. Figure 3 displays $(G/C)^*$. Last, Figure 3 also displays the relative deviation $(G/C - (G/C)^*)/(G/C)^*$. US government purchases are mildly countercyclical as the relative deviation increases in recessions and falls in expansions.

4.2. *Market Tightness and Unemployment Rate*

We measure market tightness by the ratio of vacancies to unemployment. We measure unemployment by the number of unemployed persons constructed by the BLS from the CPS. In the absence of long time series for vacancies, we construct a proxy for vacancies. We start from the help-wanted advertising index constructed by Barnichon [2010].[26] We then rescale the Barnichon index to transform it into a number of vacancies. Namely, we scale up the index such that its average value between December 2000 and December 2014 matches the average number of vacancies posted over the same period. The number of vacancies is measured by the BLS using data collected in the Job Opening and Labor Turnover Survey (JOLTS).[27]

Figure 4 plots the market tightness x. Tightness averages 0.65 between 1951 and 2014. Under

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[26]This index combines the online and print help-wanted advertising indices constructed by the Conference Board.

[27]The average value of the Barnichon index between December 2000 and December 2014 is 80.59. The average number of vacancies from JOLTS between December 2000 and December 2014 is 3.707 millions. (The JOLTS only started in December 2000.) Hence we multiply the Barnichon index by $3.707 \times 10^6/80.59 = 45,996$ to obtain a proxy for the number of vacancies since 1951.
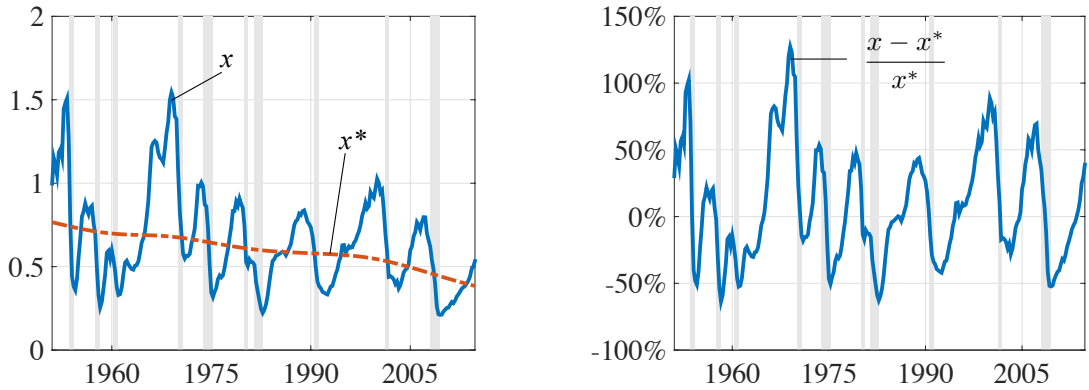
Figure 4: Market Tightness in the United States, 1951–2014

Notes: The market tightness x is the ratio of number of vacancies to unemployment level. The number of vacancies is the quarterly average of the monthly vacancy index constructed by Barnichon [2010], scaled to match the number of vacancies in JOLTS for 2001–2014. The unemployment level is the quarterly average of the seasonally adjusted monthly number of unemployed persons constructed by the BLS from the CPS. The efficient market tightness x^* is the low-frequency trend of x produced with a HP filter with smoothing parameter 10^5. The shaded areas represent the recessions identified by the NBER.

the assumption that the trend of the economy is efficient, the efficient tightness x^* can be measured as the low-frequency trend of x. We produce this trend using a HP filter with smoothing parameter 10^5 and the quarterly series for x. Figure 4 also displays x^*: it has been stable since 1951, falling only slightly over time. Last, Figure 4 displays the relative deviation $(x - x^*)/x^*$. The tightness departs from its efficient level over the business cycle: in booms the tightness is inefficiently high $(x > x^*)$, and in slumps the tightness is inefficiently low $(x < x^*)$. These departures are very large.

We measure the unemployment rate u using the unemployed rate constructed by the BLS from the CPS. Figure 5 plots the unemployment rate. The unemployment rate averages 5.9% between 1951 and 2014. Under the assumption that the trend of the economy is efficient, the efficient unemployment rate u^* can be measured as the low-frequency trend of u. We produce this trend using a HP filter with smoothing parameter 10^5 and the quarterly series for u. Figure 5 also displays u^*. We find that u^* has been stable since 1951, increasing only slightly over time. Last, Figure 5 displays the relative deviation $(u - u^*)/u^*$. The unemployment rate departs from its efficient level over the business cycle: in booms the unemployment rate is inefficiently low $(u < u^*)$, and in slumps it is inefficiently high $(u > u^*)$.
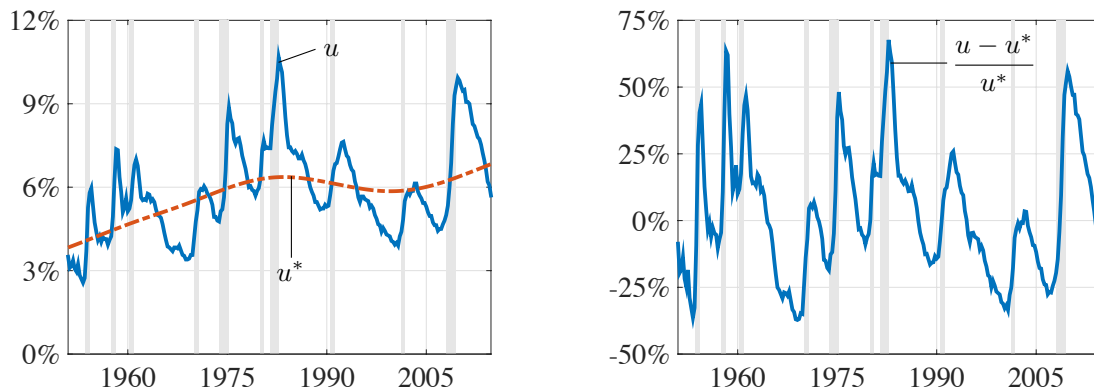
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Figure 5: Unemployment rate in the United States, 1951–2014

Notes: The unemployment rate u is the quarterly average of the seasonally adjusted monthly unemployment rate constructed by the BLS from the CPS. The efficient unemployment rate u^* is the low-frequency trend of u produced with a HP filter with smoothing parameter 10^5. The shaded areas represent the recessions identified by the NBER.

4.3. The Tightness Elasticity of the Job-Finding Rate

To estimate the tightness elasticity of the job-finding rate, $1 - \eta$, we use the measures of market tightness and unemployment rate described in Figures 4 and 5. Since the unemployment rate is given by $u = s/(f(x) + s)$, a first-order approximation of $\ln(u)$ around $\ln(x^*)$ implies

$$\ln(u) - \ln(u^*) = \frac{d \ln(u)}{d \ln(x)} \cdot (\ln(x) - \ln(x^*)) = -(1 - \eta) \cdot (1 - u^*) \cdot (\ln(x) - \ln(x^*)).$$

The scatter plot in Figure 6 indicates that the relationship between $\ln(u/u^*)$ and $\ln(x/x^*)$ is nearly linear. We therefore estimate $1 - \eta$ by running the following linear regression:

$$\ln \left(\frac{u_t}{u_t^*} \right) = \hat{\beta} \cdot \ln \left(\frac{x_t}{x_t^*} \right) + \varepsilon_t.$$

The estimate that we obtain by ordinary least squares is $\hat{\beta} = 0.54$, with standard error 0.010. As showed in Figure 5, u^* is broadly constant over time with an average value $u^* = 0.059$. Hence, we estimate that $1 - \eta = 0.54./(1 - 0.059) = 0.57$ and $\eta = 0.43$. Our estimate of η is in the range of estimates in the literature.[28]

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[28]In their survey, Petrongolo and Pissarides [2001] conclude that most estimates of η fall in the 0.4–0.7 range.
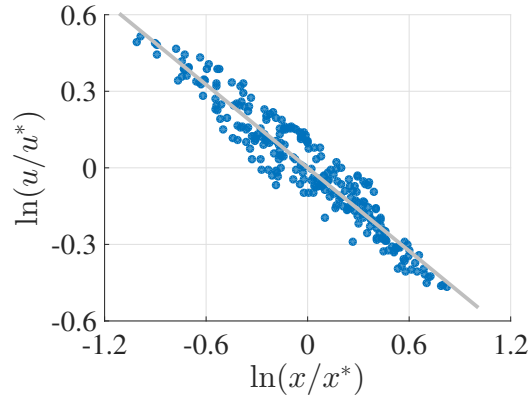
Figure 6: Estimating the Tightness Elasticity of the Job-Finding Rate in the United States

Notes: The data used for the scatter plot are quarterly US data covering the 1951–2014 period. The actual and efficient unemployment rates, *u* and *u**, are described in Figure 5. The actual and efficient market tightnesses, *x* and *x**, are described in Figure 4. The plot also displays the least-squares regression line.

5.   Policy Applications

In this section, we use our formulas and our estimates of the sufficient statistics for several policy applications. These applications focus on the United States during the 1951–2014 period. First, we show that US government purchases are not countercyclical enough for conventional estimates of the multiplier and elasticity of substitution between government and personal consumptions. Second, we show that US government purchases would only be optimal under very small values of the multiplier and the elasticity of substitution. Last, for a range of multipliers and elasticity of substitutions, we determine the optimal response of government purchases when the unemployment rate increases from its average level of 5.9% to a high level of 9%.

5.1.   *Assessment of US Government Purchases*

We determine whether US government purchases are countercyclical enough if the value of the government-purchases multiplier is given by a midrange estimate from the literature, $dY/dG = 0.5$. To evaluate US government purchases, we calculate the expression

$$\frac{G/C - (G/C)^*}{(G/C)^*} + \varepsilon \cdot \frac{dY}{dG} \cdot \frac{x - x^*}{x^*}.$$
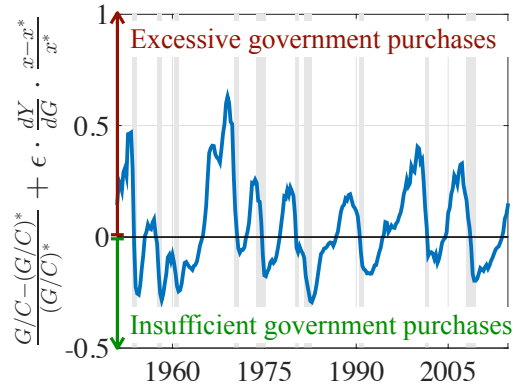
Figure 7: Assessment of US Government Purchases, 1951–2014, for $\varepsilon = 1$ and $dY/dG = 0.5$

Notes: The figure evaluates equation (7). The ratios of government consumption to personal consumption, G/C and $(G/C)^*$, are described in Figure 3. The actual and efficient market tightnesses, x and x^*, are described in Figure 4. The shaded areas represent the recessions identified by the NBER.

If this expression is zero, the level of government purchases is optimal. In that case, formula (7) holds. If this expression is positive, then government purchases are too high. Conversely, if this expression is negative, government purchases are too low.

Figure 7 plots this expression using the sufficient statistics measured in Section 4. In particular, we set the elasticity of substitution between government and personal consumption and the government-purchases multiplier to their benchmark values, $\varepsilon = 1$ and $dY/dG = 0.5$.

We find that the expression is systematically negative in slumps and positive in booms. For instance, government consumption was insufficient in the recession years of 1982, 1991, 2001, and 2009. We conclude that for a multiplier of 0.5 and an elasticity of substitution of 1, US government purchases are not countercyclical enough; that is, government purchases should be higher in slumps and lower in booms. This is not surprising because government purchases have not been actively used for stabilization in the United States.[29]

5.2. *Estimation of the Statistics Underlying US Government Purchases*

We have found that US government purchases are not countercyclical enough if the value of the government-purchases multiplier is 0.5—the best estimate from the literature. We now determine

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[29]During the Great Recession, government expenditure dramatically increased with the American Recovery and Reinvestment Act of 2009. But government purchases of goods and services did not increase; the federal government increased transfers and tax rebates, and federal government purchases increases were offset by reduced state and local government purchases. See the description of US public expenditure during the 2000–2010 period by Taylor [2011].
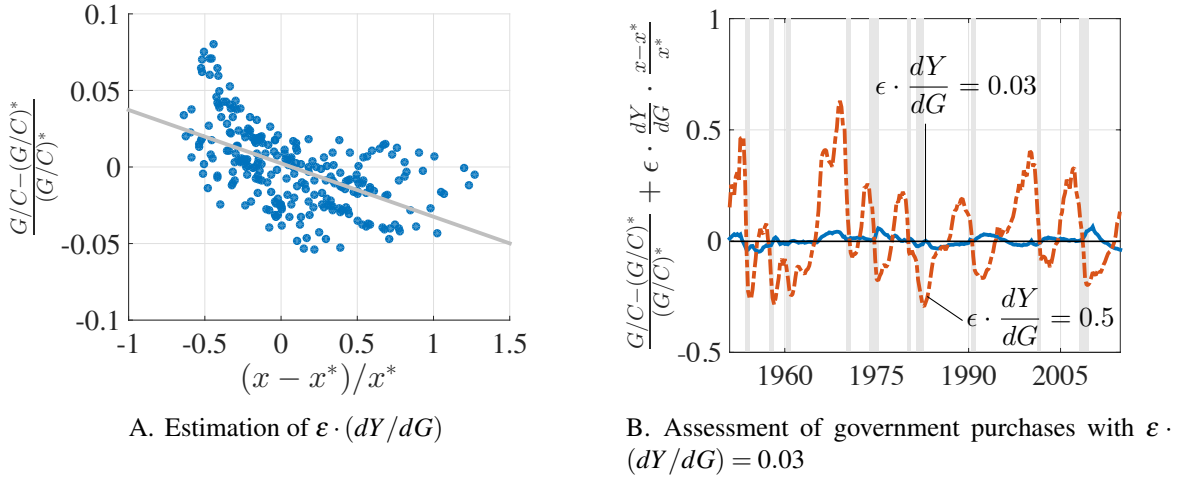
A. Estimation of $\varepsilon \cdot (dY/dG)$

B. Assessment of government purchases with $\varepsilon \cdot (dY/dG) = 0.03$

Figure 8: The Statistics Underlying US Government Purchases, 1951–2014

Notes: The data used in the figure are quarterly US data covering the 1951–2014 period. The actual and efficient market tightnesses, x and x^*, are described in Figure 4. The ratios of government consumption to personal consumption, G/C and $(G/C)^*$, are described in Figure 3. The scatter plot also displays the least-squares regression line. The shaded areas represent the recessions identified by the NBER.

the value of the government-purchases multiplier that is consistent with observed government purchases and market tightness in the United States. To do so, we exploit formula (7).

Formula (7) links the equilibrium values of optimal government purchases and market tightness. When government purchases are optimal, the observed values of government purchases and market tightness always satisfy formula (7). Therefore, we can regress $(G/C - (G/C)^*)/(G/C)^*$ on $(x - x^*)/x^*$ to estimate $\varepsilon \cdot (dY/dG)$. We run the regression

$$\frac{G_t/C_t - (G_t/C_t)^*}{(G_t/C_t)^*} = \hat{\beta} \cdot \frac{x_t - x_t^*}{x_t^*} + \varepsilon_t.$$

The estimate that we obtain by ordinary least squares is $\hat{\beta} = 0.03$, with robust standard error 0.01. The regression is illustrated in Panel A of Figure 8. The regression analysis implies that US government purchases are optimal under statistics $\varepsilon \cdot (dY/dG) = 0.03$. For an elasticity of substitution of $\varepsilon = 1$, the result implies that the US policy is optimal under a minuscule multiplier of $dY/dG = 0.03$.

Panel B of Figure 8 presents the same result under a different angle. The panel evaluates the implicit sufficient-statistics formula for optimal government purchases, given by (6), using the statistics measured in US data, an elasticity $\varepsilon = 1$, and two different multipliers: $dY/dG = 0.03$

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and $dY/dG = 0.5$. A positive value at time t means that the right-hand side of (6) is larger than the left-hand side at time t, and thus that the derivative of social welfare with respect to government consumption g is positive, which means that government consumption g is insufficient. Consistent with the previous regression result, we find that if the multiplier is $dY/dG = 0.03$, then US government purchases are nearly optimal—formula (7) nearly holds at all time. In contrast, as we saw in Figure 7, if the multiplier is $dY/dG = 0.5$, there is systematically insufficient government consumption in slumps and excessive government consumption in booms.

In sum, either the product of the government-purchases multiplier by the elasticity of substitution is tiny (i.e., $\varepsilon \cdot (dY/dG) \approx 0.03$) and US government purchases are optimal, or the product is larger and US government purchases are not countercyclical enough.

5.3. Optimal Response of Government Purchases to an Unemployment Increase

We exploit the explicit formula, given by (12) to compute the optimal response of government purchases to a given increase in unemployment. As an illustration, we consider an increase in the unemployment rate from an efficient level of 5.9% to a high level of 9%. Table 2 displays the optimal increase in government purchases for a range of estimates of the multiplier and the elasticity of substitution between government and personal consumptions.

The first observation is that even with a small multiplier of 0.2, government purchases should increase significantly when the unemployment rate increases from 5.9% to 9%. For an elasticity of substitution of 0.5, the government purchases-output ratio should increase by 1.2 percentage points from 16.6% to 17.8%; for an elasticity of substitution of 1, it should increase by 2.2 points; and for an elasticity of substitution of 2, it should increase by 3.8 points. Thus, our theory suggests that government purchases should be markedly countercyclical even for small positive multipliers.

The second observation is that the optimal increase in government purchases rises with the elasticity of substitution between government consumption and personal consumption. For instance with a multiplier of 0.5, the government purchases-output ratio should increase by 0.6 percentage points from 16.6% to 17.2% with a low elasticity of 0.1, but it should increase by 4.7 points with a high elasticity of 3. Hence, the elasticity of substitution significantly influences the optimal response of government purchases to a shock.

The third observation is that the optimal increase in government purchases does not rise mono-

Table 2: Increase in Optimal Government Purchases-Output Ratio (in Percentage Points) When the Unemployment Rate Rises from 5.9% to 9%

Elasticity of substitution between	Government-purchases multiplier				
government and personal consumptions	0.1	0.2	0.5	1	1.5
0.1	0.1	0.3	0.6	0.9	1.0
0.5	0.6	1.2	2.1	2.1	1.7
1	1.2	2.2	3.2	2.5	1.9
2	2.4	3.8	4.2	2.8	2.0
3	3.4	5.1	4.7	2.9	2.0

Notes: Initially, the unemployment rate is efficient and the government purchases-output ratio is given by the Samuelson formula, which is optimal. The efficient unemployment rate is set to $u^* = 5.9\%$ and the Samuelson ratio to $(G/Y)^* = 16.6\%$. The economy is hit by an unexpected permanent shock that brings the unemployment rate to $u_0 = 9\%$. The table displays $G/Y - (G/Y)^*$ where G/Y is the optimal government purchases-output ratio after the shock. The values of $G/Y - (G/Y)^*$ are computed using formula (12), the values indicated for the multiplier and the elasticity of substitution, and $\eta = 0.43$. For negative multipliers $dY/dG < 0$, the decrease in the optimal government purchases-output ratio is minus the value reported in the table for the positive multiplier $|dY/dG|$.

tonically with the multiplier. It is true that the optimal increase in government purchases rises for low values of the multiplier. For instance with elasticity of substitution of 1, the government purchases-output ratio should increase by 1.2 percentage points from 16.6% to 17.8% with a low multiplier of 0.1, but it should increase by 3.2 with a multiplier of 0.5. However, the increase in the optimal government purchases-output ratio diminishes for higher values of the multiplier. For instance with the same elasticity of substitution of 1, the government purchases-output ratio should only by 1.9 points with a multiplier of 1.5. The last column of Table 2 shows that, with a large multiplier of 1.5, government purchases should fill the output gap and the value of ε becomes less important.

The fourth observation is that we can directly use the table to obtain the optimal reduction in the government purchases-output ratio if the multiplier was negative. Indeed, formula (12) is an odd function of the multiplier. Hence, the optimal reduction in government purchases-output ratio for a negative multiplier $dY/dG < 0$ as the same amplitude as the optimal increase the government purchases-output ratio for the positive multiplier $-dY/dG$. For instance, with an elasticity of substitution of 1 and a multiplier of -0.5, the optimal government purchases-output ratio should decreased by 3.2 percentage points when the unemployment rate rises from 5.9% to 9%.

To illustrate the hump-shaped relation between positive multipliers and the optimal increase
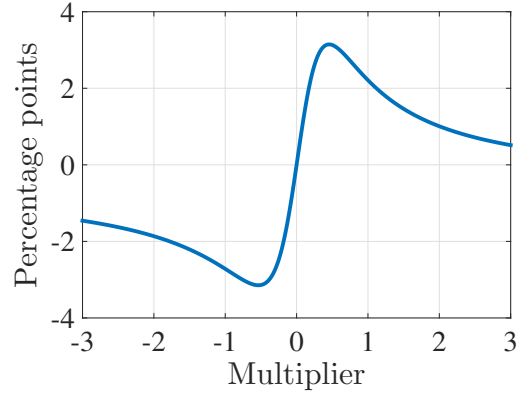
Figure 9: Change of the Optimal Government Purchases-Output Ratio When the Unemployment Rate Increases from 5.9% to 9%, for a Range of Government-Purchases Multiplier

Notes: Initially, the unemployment rate is efficient and the government purchases-output ratio is given by the Samuelson formula, which is optimal. The efficient unemployment rate is set to $u^* = 5.9\%$ and the Samuelson ratio to $(G/Y)^* = 16.6\%$. The economy is hit by an unexpected permanent shock that brings the unemployment rate to $u_0 = 9\%$. The figure displays $G/Y - (G/Y)^*$ where G/Y is the optimal government purchases-output ratio after the shock. The values of $G/Y - (G/Y)^*$ are computed using formula (13), values of the multiplier between -3 and 3, $\varepsilon = 1$, and $\eta = 0.43$.

in government purchases-output ratio, we plot the relationship in Figure 9. We consider again an increase in the unemployment rate from 5.9% to 9%. The figure displays the optimal increase in government purchases-output ratio when the multiplier ranges from -3 to 3. Since we consider a broad range of multipliers, we use formula (13), which is more precise than (12) for large multipliers. Of course, the government purchases-output ratio should not increase if the multiplier is 0. For positive multipliers, the ratio G/Y should increase. The largest increase is obtained for a multiplier of 0.5. At this multiplier, the government purchases-output ratio should increase by 3.2 percentage points. For larger multipliers, the optimal increase in the government purchases-output ratio falls: the government purchases-output ratio should only increase by 2.2 percentage points for a multiplier of 1, by 1 percentage point for a multiplier of 2, and by 0.5 percentage point for a multiplier of 3. The same pattern appears for negative multipliers, except that in that case the government purchases-output ratio should decrease after the increase in unemployment rate.

6. A Specific Model

In this section, we propose a fully specified model. We link the parameters of the model to the sufficient statistics that we introduced in Section 3. We explain in particular which parameters

determine the value of the multiplier. We calibrate and simulate the model under aggregate demand shocks.[30] We use these simulations to validate the accuracy of our approximate explicit formula.

To generate an aggregate demand, we consider a model with wealth in the utility as in Michaillat and Saez [2015b]. Aggregate demand shocks are parameterized by the marginal utility of wealth. Higher marginal utility of wealth leads households to want to save more and consume less, which depresses aggregate demand and gross output.

As in Section 4, we assume that the tightness and unemployment in the average state are efficient. We also assume that government purchases are optimal in the average state. We denote with an upper bar average state values.

6.1. Households

The representative household spends part of its labor income on labor services and saves part of it as bonds. For simplicity, we assume that government purchases are financed by a lump-sum tax.[31] The law of motion of the household's assets is

$$\dot{b}(t) = Y(x(t)) - (1 + \tau(x(t))) \cdot c(t) + r(t) \cdot b(t) - T(t). \tag{14}$$

Here, $b(t)$ are real bond holdings, $c(t)$ is net personal consumption, $(1 + \tau(x(t))) \cdot c(t)$ is gross personal consumption, $Y(x(t))$ is labor income, $r(t)$ is the real interest rate, and $T(t)$ is the lump-sum tax paid to the government.

The representative household derives utility from consuming the $c(t)$ services that it purchases, consuming the $g(t)$ services purchased by the government, and holding $b(t)$ units of real wealth. Its instantaneous utility function is separable: $\mathcal{U}(c(t), g(t)) + \mathcal{V}(b(t))$. The function $\mathcal{V}$ is differentiable, strictly increasing, and concave. Utility for wealth is a simple way to introduce an

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[30]The empirical evidence presented in Michaillat and Saez [2015a], based on the type of matching model used here, suggests that business cycles in the US are mostly caused by aggregate demand shocks.

[31]The government could finance government purchases with debt. If households are Ricardian—in the sense that they do not view government bonds as net wealth because such bonds need to be repaid with taxes later on—debt financing is economically equivalent to a lump-sum tax and budget balance, exactly as in Barro [1974]. If the government uses debt financing and households are not Ricardian, or if the government uses a distortionary tax on consumption, our formulas would remain valid, but the equilibrium function $x(g)$ would differ. This means that the expression for the multiplier that we derive below would be different.

aggregate demand in a real economy without money.[32] The function $\mathcal{U}$ is given by

$$\mathcal{U}(c,g) = \left[(1-\gamma) \cdot \left(\frac{c}{\bar{c}} \right)^{\frac{\varepsilon-1}{\varepsilon}} + \gamma \cdot \left(\frac{g}{\bar{g}} \right)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}. \tag{15}$$

The function $\mathcal{U}$ has constant returns to scale and constant elasticity of substitution $\varepsilon \geq 0$. The parameters $\bar{g}$ and $\bar{c}$ are the values of g and c in the average state. The parameter $\gamma \in (0,1)$ gives the optimal government purchases-output ratio in the average state. Indeed, the marginal rate of substitution in the average state is $\overline{MRS} = [\gamma/(1-\gamma)] \cdot (\bar{g}/\bar{c})$. Optimal government purchases satisfy $\overline{MRS} = 1$ in the average state because this state is efficient. Hence, the optimal government purchases-output ratio in the average state satisfies $\bar{g}/\bar{y} = \gamma$.

The utility function of a household at time 0 is the discounted sum of instantaneous utilities $\int_0^{+\infty} e^{-\delta \cdot t} \cdot [\mathcal{U}(c(t),g(t)) + \mathcal{V}(b(t))] \, dt$, where $\delta > 0$ is the subjective discount rate. Given initial real wealth $b(0) = 0$ and the paths for market tightness, government purchases, real interest rate, and tax $[x(t),g(t),r(t),T(t)]_{t=0}^{+\infty}$, the household chooses paths for consumption and real wealth $[c(t),b(t)]_{t=0}^{+\infty}$ to maximize its utility function subject to (14).

6.2. Market for Bonds and Real Interest Rate

Households can issue or buy riskless real bonds. Household hold bonds to smooth future consumption and because they derive utility from real wealth, which can only be stored in bonds. At time t, a household holds $b(t)$ bonds, and the rate of return on bonds is the real interest rate $r(t)$.

Bonds are traded on a perfectly competitive market. Bonds are in zero net supply because the government funds government purchases using lump-sum taxes. In equilibrium, the bond market clears and $b(t) = 0$. Hence, the aggregate real wealth in the economy is zero.

In the economy there are two goods: labor services and bonds. Hence there is only one relative price. The price of bonds relative to services is determined by the real interest rate. In a Walrasian market, the real interest rate would be determined such that supply equals demand on the market for labor services. In our matching market, things are different: we specify a price mechanisms for

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[32]See Michaillat and Saez [2015b] for more details. Another possibility would be to introduce utility for money as in Michaillat and Saez [2015a], or more generally utility for a nonproduced good. The formulation with wealth is the simplest, since it does not require the introduction of an additional good, and it captures the idea that shifts in thriftiness create aggregate demand shocks.

the real interest rate, and the market tightness will adjust such that supply equals demand on the market for labor services.

A common assumption in the matching literature is that prices are efficient—they maintain the economy at the efficient unemployment rate u^*.[33] The efficient real interest rate is

$$r^* = \delta - (1 + \tau(x^*)) \cdot \frac{\mathcal{V}'(0)}{\frac{\partial \mathcal{U}}{\partial c}(y^* - g, g)}.$$

If the interest rate always adjusts to r^*, then the economy is always efficient $x = x^*$, $y = y(x^*)$, and hence government purchases g have no effect on tightness and output and the Samuelson rule should always hold. In practice, the economy experiences business cycles with large variations in tightness. To capture this, we assume that the interest rate is not perfectly flexible. We consider a fairly general interest-rate schedule of the form:

$$r = \delta - (\delta - \bar{r}) \cdot \left(\frac{\mathcal{V}'(0)}{\overline{\mathcal{V}'(0)}} \right)^{1-\alpha} \cdot \left(\frac{\frac{\partial \mathcal{U}}{\partial c}(\bar{y} - \bar{g}, \bar{g})}{\frac{\partial \mathcal{U}}{\partial c}(y^* - g, g)} \right)^{1-\beta}. \tag{16}$$

The parameter $\bar{r}$ is the real interest rate in the average state:

$$\bar{r} = \delta - (1 + \tau(\bar{x})) \cdot \frac{\overline{\mathcal{V}'(0)}}{\frac{\partial \mathcal{U}}{\partial c}(\bar{y} - \bar{g}, \bar{g})}. \tag{17}$$

The parameter $\alpha \in [0, 1]$ measures the rigidity of the real interest rate to aggregate demand shocks. If $\alpha = 1$, the real interest rate is fully rigid: it does not respond at all to aggregate demand shocks. If $\alpha = 0$, the real interest rate is fully flexible: it responds as much to aggregate demand shocks as the efficient real interest rate, and aggregate demand shocks are fully absorbed by the real interest rate. The parameter $\beta \in [0, 1]$ measures the rigidity of the real interest rate to changes in the marginal utility of personal consumption. If $\beta = 1$, the real interest rate is fully rigid: it does not respond at all to shocks to the marginal utility of personal consumption. If $\beta = 0$, the real interest rate is fully flexible: it responds as much to shocks to the marginal utility of personal consumption as the efficient real interest rate. We will see that when $\beta = 0$, government purchases shocks are completely absorbed by the real interest rate such that the aggregate demand does not depend on

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[33] Another typical assumption is that prices are determined by bargaining. Bargained prices usually have similar properties to efficient prices [Michaillat and Saez, 2015a].

government purchases and the multiplier is zero.

The interest rate schedule can be interpreted as representing the behavior of the central bank. If $\alpha = \beta = 0$, the interest rate is always efficient: $r = r^*$. This happens when the central bank is always able to maintain the unemployment rate at its efficient level. If $\alpha = \beta = 1$, the interest rate is totally rigid: $r = \bar{r}$. This happens when the central bank is unable to affect the real interest rate. This would happen if inflation were fixed and the nominal interest rate at the zero lower bound.[34]

6.3. The Market for Labor Services

We use a Cobb-Douglas matching function $h(1-Y,v) = \omega \cdot (1-Y)^\eta \cdot v^{1-\eta}$. The parameter ω is the matching efficacy. The parameter $\eta \in (0,1)$ is the elasticity of matching with respect to available capacity. The job-finding rate is $f(x) = \omega \cdot x^{1-\eta}$ so the tightness elasticity of the job-finding rate is $1 - \eta$. The vacancy-filling rate is $q(x) = \omega \cdot x^{-\eta}$. The function $y(x)$, defined by (3), admits a unique maximum at x^*.

6.4. Equilibrium

The equilibrium is a dynamical system that we describe in Appendix D. We also show in Appendix D that the system is a source, with no state variables, so that it converges immediately to its steady state. We therefore only describe the steady state of the equilibrium. The equilibrium is composed of five variables: g, c, y, r, and x. Net government consumption g is chosen by the government. The real interest rate r is given by the schedule (16). Net output is given by $y = y(x)$, where $y(x)$ is defined by (3). Solving the household's problem, we find that c satisfies

$$\frac{\mathscr{V}'(0)}{\frac{\partial \mathscr{U}}{\partial c}(c,g)} = \frac{\delta - r}{1 + \tau(x)}. \tag{18}$$

This equation is the standard Euler equation, modified in the presence of utility of wealth, and evaluated in steady state. The only constraint on the real interest rate is $r < \delta$; hence, we can accommodate a negative real interest rate in steady state. Let $c(x,g,r)$ be the amount of net personal consumption implicitly defined by this equation. The constraint that $c + g = y$ can be written as

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[34]Michaillat and Saez [2015b] describe such a situation.

$c(x, g, r) + g = y(x)$, which determines equilibrium tightness.

6.5. Government-Purchases Multiplier

Our formulas for optimal government purchases rely on three main sufficient statistics: the elasticity of substitution between government and personal consumptions, the tightness elasticity of the job-finding rate, and the government-purchases multiplier. The first two statistics are directly linked to two parameters of the model: the parameter ε in the utility function and the parameter η in the matching function. Here we explain how the third statistic—the multiplier—relates to the parameters of the model.

PROPOSITION 6. *In the specific model the multiplier satisfies*

$$\frac{d\ln(Y)}{d\ln(G)} = \left[1 - (1-\beta) \cdot \frac{1 - G/Y}{1 - G/Y^*} \cdot \frac{z(G/(Y-G))}{z(G/(Y^*-G))}\right] \cdot \left[1 + \varepsilon \cdot \frac{\eta}{1-\eta} \cdot \frac{\tau}{u} \cdot \frac{C}{Y} \cdot z\left(\frac{G}{C}\right)\right]^{-1}$$

where the auxiliary function z is defined by

$$z(\theta) \equiv 1 + \frac{1-\gamma}{\gamma} \cdot \left(\frac{\overline{g}}{\overline{c}} \cdot \frac{1}{\theta}\right)^{\frac{\varepsilon-1}{\varepsilon}} .$$

In the average state, in which the unemployment rate is efficient and government purchases are optimal, the multiplier simplifies to $dY/dG = \beta/[\gamma + \varepsilon \cdot (1-\gamma)]$. In addition, if the elasticity of substitution between government and personal consumptions is $\varepsilon = 1$, then the multiplier in the average state simplifies to $dY/dG = \beta$.

The proof is relegated to Appendix E. The proposition explains how to calibrate the interest-rate schedule to match empirical evidence. As discussed in Section 4, the literature suggests that a good estimate of the multiplier is $dY/dG = 0.5$. By calibrating β appropriately, we will ensure that the multiplier is 0.5 in the average state.

Since τ/u is high when the unemployment rate is low, the expression for the multiplier indicates that the multiplier is high when the unemployment rate is high, in line with the empirical evidence provided by Auerbach and Gorodnichenko [2012]. The mechanism behind the fluctuations of the multiplier is similar to that described in Michaillat [2014].
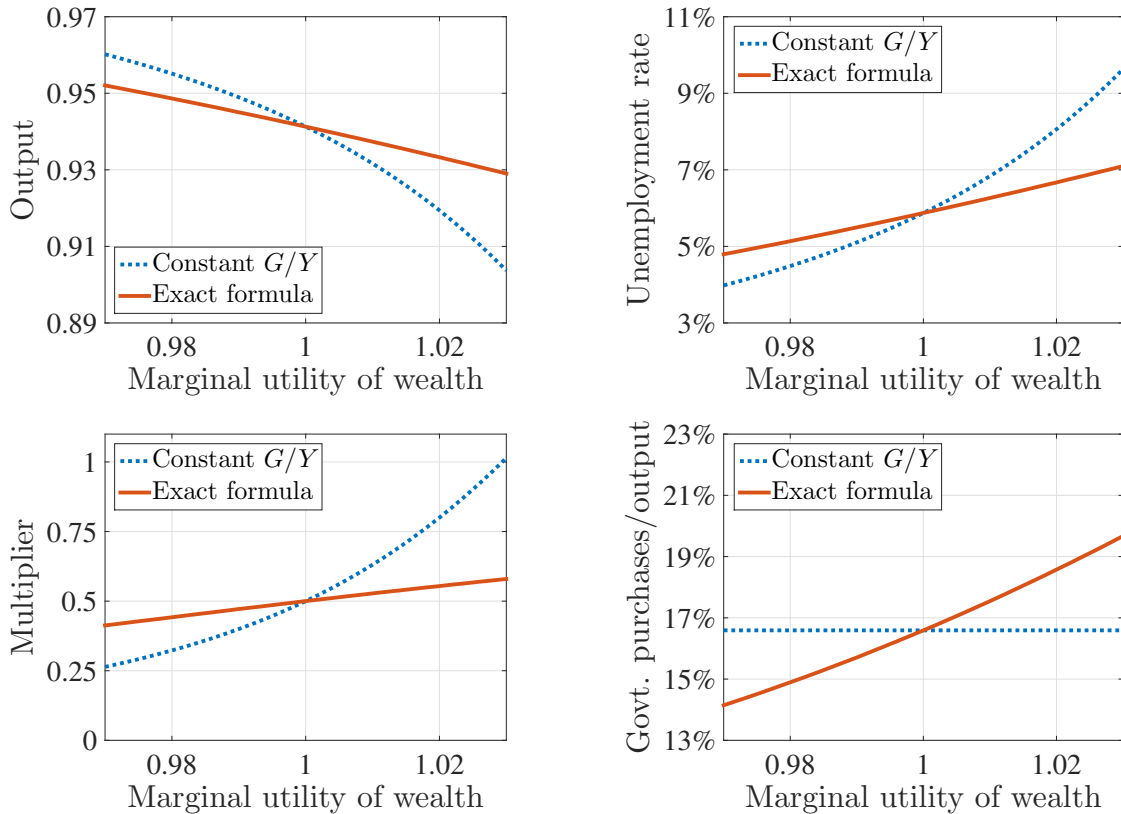
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Figure 10: Simulations of Specific Model Under Aggregate Demand Shocks

6.6. Calibration and Simulations

To describe more precisely the fluctuations of optimal government purchases over the business cycle, we calibrate and simulate our model under aggregate demand shocks. Appendix D shows that the economy jumps from one steady-state equilibrium to another in response to unexpected permanent shocks. Hence, we represent the different stages of the business cycle as a succession of steady states. We simulate business cycles generated by aggregate demand shocks by computing a collection of steady states parameterized by different values for the marginal utility of wealth, $\mathcal{V}'(0)$. In each case, we perform two simulations: one in which the government purchases-output ratio G/Y remains constant at its average value, and one in which the ratio is at its optimal level, given by formula (6).

We calibrate the model to US data for 1951–2014. The calibration ensures that the simulations match the evidence from Section 4 on the sufficient statistics at the heart of our formulas. Thus,

we set the parameter ε in the utility function to 1 and the parameter η in the matching function to 0.43.[35] Since we target a multiplier of 0.5 in the average state, we set the parameter β in the interest-rate schedule to 0.5.[36] The rest of the calibration is standard and relegated to Appendix F.

Since there is considerable uncertainty about the values of the elasticity of substitution and multiplier, we perform additional simulations targeting $\varepsilon = 0.5$, $\varepsilon = 2$, $\overline{dY/dG} = 0.2$, and $\overline{dY/dG} = 1$. These simulations are presented in Appendix G.

Figure 10 displays the simulations of the calibrated model under aggregate demand shocks. Each steady state is indexed by a marginal utility of wealth $\mathscr{V}'(0) \in [0.97, 1.03]$. Because of the rigidity of the real interest rate, the steady states with low $\mathscr{V}'(0)$ represent booms: they have a relatively low interest rate and therefore low unemployment. Conversely, the steady states with high $\mathscr{V}'(0)$ represent slumps: they have a relatively high interest rate and high unemployment. Unemployment rises from 4.0% to 9.6%, and output falls accordingly, when $\mathscr{V}'(0)$ increases from 0.97 to 1.03 and the government purchases-output ratio remains constant. On average the multiplier is 0.5, matching exactly the empirical evidence thanks to our calibration of the parameter β. The multiplier is countercyclical, increasing from 0.26 to 1.01 when the unemployment rate increases from 4.0% to 9.6%.

The model is calibrated so that the unemployment rate is efficient in the average state ($\mathscr{V}'(0) = 1$). Hence, the unemployment rate is inefficiently high in slumps ($\mathscr{V}'(0) > 1$) and inefficiently low in booms ($\mathscr{V}'(0) < 1$). Since the multiplier is positive, the government purchases-output ratio should be more generous than the Samuelson ratio in slumps and less generous in booms. Indeed, the optimal ratio is markedly countercyclical, increasing from 14.1% when $\mathscr{V}'(0) = 0.97$ to 19.7% when $\mathscr{V}'(0) = 1.03$.

Of course, the unemployment rate responds to the adjustment of government purchases from their original level of 16.6% to their optimal level. In slumps, optimal government purchases are much higher than 16.6% so the unemployment rate is below its original level: at $\mathscr{V}'(0) = 1.03$ the unemployment rate falls by 2.5 percentage points from 9.6% to 7.1%. In booms, optimal government purchases are below 16.6% so the unemployment rate is above its original level: at $\mathscr{V}'(0) = 0.97$ the unemployment rate increases by 0.8 percentage point from 4.0% to 4.8%.

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[35]This yields a Cobb-Douglas utility function: $\mathscr{U}(c,g) = (c/\overline{c})^{\gamma} \cdot (g/\overline{g})^{1-\gamma}$.

[36]Proposition 6 establishes the link between the parameter β and the value of the multiplier.
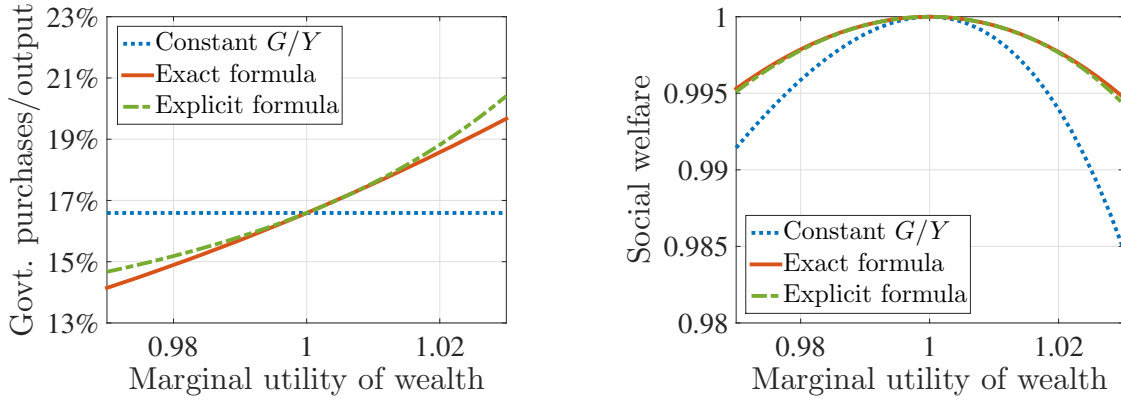
Figure 11: Accuracy of the Approximate Explicit Formula

Finally, we use simulations to evaluate the accuracy of the approximate explicit formula, given by (12). We find that despite its simplicity, the explicit formula is very accurate. Figure 11 compares the optimal government purchases obtained with the exact implicit sufficient-statistics formula, given by (6), and with the explicit formula. Around the average state ($\mathscr{V}'(0) = 1$), the government purchases-output ratio given by the two formulas are indistinguishable. The approximation is less precise further away from the average state, but it remains satisfactory: at $\mathscr{V}'(0) = 0.97$, the exact formula gives $G/Y = 14.1\%$ while the approximate explicit formula gives $G/Y = 14.7\%$; at $\mathscr{V}'(0) = 1.03$, the exact formula gives $G/Y = 19.7\%$ while the approximate explicit formula gives $G/Y = 20.4\%$. Despite these discrepancies, the social welfare values resulting from the two formulas are nearly identical.

7. Conclusion

In this paper we study the optimal use of government purchases for macroeconomic stabilization. We derive formulas for optimal government purchases expressed with estimable sufficient statistics. Our analysis provides several general insights on the conduct of government purchases to stabilize business cycles. Some of these insights confirm intuitions that macroeconomists have had for a long time. First, even in a macroeconomic model with unemployment, the Samuelson [1954] formula holds as long as the unemployment rate is efficient. Second, the government-purchases multiplier, which is one of the most commonly estimated statistic in macroeconomics, does matter for the optimal level of government purchases.

Other insights are more unexpected. First, the cutoff value of the multiplier that justifies an increase in government purchases in slumps is not 1 but 0. With any positive multiplier, it is optimal to increase government purchases above the Samuelson level when the unemployment rate is inefficiently high, even though government purchases crowd out personal consumption. In fact, with statistics calibrated to the US economy and an elasticity of substitution between government and personal consumptions of 1, the optimal government purchases-output ratio rises by significant amounts when the unemployment rate increases from its average level of 5.9% to a high level of 9%—even for small multipliers. For instance, the optimal ratio increases by 1.2, 2.2, and 3.2 percentage points for multipliers of 0.1, 0.2, and 0.5.

Second, for positive multipliers, the relation between the size of the multiplier and the increase of the optimal government purchases-output ratio following an increase in unemployment is not increasing but hump-shaped, with a peak for a multiplier of about 0.5 where the optimal ratio increases by 3.2 percentage points when the unemployment rate increases from 5.9% to 9%. The optimal ratio increases less for multipliers above 0.5 because when multipliers are large, a higher multiplier means that fewer government purchases are required to fill the unemployment gap. The optimal ratio increases less for multipliers below 0.5 because when multipliers are small, a smaller multiplier means that government purchases crowd out personal consumption more and are therefore less desirable. For example, the optimal government purchases-output ratio increases only by 1.0 percentage points for a multiplier of 2, the same as for a multiplier of 0.08.

Third, there is another statistic that has been neglected but is as important as the multiplier to determine the optimal level of government purchases: the elasticity of substitution between government and personal consumptions. The increase of the optimal government purchases-output ratio following an increase in unemployment is larger for larger elasticities of substitutions. For instance, for a multiplier of 0.5, the optimal ratio increases by 0.6, 2.1, and 4.2 percentage points for elasticities of 0.1, 0.5, and 2.

Fourth, a negative multiplier does not mean that government purchases should not respond to unemployment fluctuations; instead, it means that government purchases should be below the Samuelson level when the unemployment rate is inefficiently high and above the Samuelson level when the unemployment rate is inefficiently low. It is only for a multiplier of 0 that the government purchases-output ratio should remain constant at the Samuelson level.

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Overall, our analysis suggests that government purchases should be a key tool for macroeconomic stabilization as soon as the government-purchases multiplier and the elasticity of substitution between government and personal consumptions are positive. Even away from the zero lower bound, unemployment fluctuates a lot, which indicates that monetary policy may not be able to stabilize the economy perfectly. This could be because monetary policy takes time to become effective—for instance, if a change in the federal funds rate takes time to percolate through the economy and influence the interest rates faced by households and firms. Any time that the unemployment rate is inefficient—irrespective of monetary policy—our analysis suggests that government purchases should be adjusted, sometimes by a sizable amount. In practice, adjusting government purchases may take time and there may be a time lag to implement government purchases. To reduce time lags and make this policy effective, the government should automatize government purchases, much in the same way as unemployment insurance extensions are automatic in the United States. A possibility would be to keep a long list of useful government purchases (either services or investment projects valued by society) and go down or up the list as more or less government purchases become desirable over the business cycle.

If government purchases are financed with distortionary taxation, an important extension of the theory would be to include supply-side responses. If labor income is taxed and households choose their productive capacity, changes in government purchases will affect the tax rate and the productive capacity supplied by households. In that case, changes in government purchases affect output through its mechanical effect on aggregate demand, and through its effect on aggregate supply triggered by tax distortions. The correction term in the formula should still depend on how marginal government purchases affect tightness. However, this effect is no longer measured by the standard macro multiplier. Instead, it is measured by the macro multiplier minus a micro multiplier that measures the supply-side response to government purchases through distortionary taxation.[37] Introducing endogenous productive capacity would describe other interesting phenomena, such as the effect of government purchases on labor force participation through its influence on the job-finding rate and returns to job search.

The methodology developed in this paper could help bridge the gap between the analysis of

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[37]These results are reminiscent of those in Landais, Michaillat and Saez [2010]. They analyze optimal unemployment insurance in a matching model and find that the correction term relative to the standard Baily-Chetty formula depends on the difference between the macro and micro responses of unemployment to unemployment insurance.

optimal taxation, transfers, social insurance, and public-good provision in public economics and the analysis of stabilization policies in macroeconomics. This agenda is related to the new dynamic public finance literature that conducts optimal policy analysis in dynamic macroeconomic models [Golosov, Tsyvinski and Werning, 2006; Golosov and Tsyvinski, 2015; Golosov, Troshkin and Tsyvinski, 2011; Kocherlakota, 2010]. It is also related to the work of Farhi and Werning [2013], who propose a framework to study optimal macroprudential policies in the presence of price rigidities. In that framework they obtain the same decomposition of optimal policies into a standard public-economics policy plus a correction term capturing stabilization motives.

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Appendix A: Output and Matching Wedge

In this appendix we derive the relationships between output and tightness and matching wedge and tightnessgiven in the text.

We first derive the relationship between output and tightness. The law of motion for output is

$$\dot{Y}(t) = f(x(t)) \cdot (1 - Y(t)) - s \cdot Y(t).$$

In this law of motion, $f(x(t)) \cdot (1 - Y(t))$ is the number of long-term relationships created at t and $s \cdot Y(t)$ is the number of long-term relationships separated at t. We assume that the transitional dynamics of $Y(t)$ are fast. This implies that $Y(t)$ barely departs from its steady-state level where $\dot{Y}(t) = 0$. Imposing $\dot{Y}(t) = 0$ into the law of motion for $Y(t)$, we find that $Y(t)$ satisfies

$$0 = f(x(t)) \cdot (1 - Y(t)) - s \cdot Y(t)$$
$$Y(t) = \frac{f(x(t))}{f(x(t)) + s}.$$

We now derive the relationship between matching wedge and tightness. The household advertises vacancies to purchase services. At time t, the household adjusts its gross consumption by $\dot{C}(t)$, and it also replaces the $s \cdot C(t)$ relationships that have just separated. Making these $\dot{C}(t) + s \cdot C(t)$ new relationships requires $\left(\dot{C}(t) + s \cdot C(t)\right)/q(x(t))$ vacancies, each costing ρ services. Hence, gross and net consumption related by the following differential equation:

$$C(t) = c(t) + \frac{\rho}{q(x(t))} \cdot \left(\dot{C}(t) + s \cdot C(t)\right).$$

We assume that the transitional dynamics of $C(t)$ are fast. This implies that $C(t)$ barely departs from its steady-state level where $\dot{C}(t) = 0$. Imposing $\dot{C}(t) = 0$ into the law of motion for $C(t)$, we find that $C(t)$ satisfies

$$C(t) \cdot \left[1 - \frac{s \cdot \rho}{q(x(t))}\right] = c(t).$$

or equivalently

$$C(t) = \left[1 + \frac{s \cdot \rho}{q(x(t)) - s \cdot \rho}\right] \cdot c(t).$$

The government purchases services on the market in the same way as households do. Hence, gross and net government consumptions are also related by

$$G(t) = \left[1 + \frac{s \cdot \rho}{q(x(t)) - s \cdot \rho}\right] \cdot g(t).$$

We infer the expression for the matching wedge from these relationships between gross and net consumptions.

Appendix B: Validation of the Main Approximations

In this appendix, we use US data for the 1951-2014 period to validate the three main approximations made in the analysis.

Irrelevance of Transitional Dynamics for the Unemployment Rate

We argue here that transitional dynamics for unemployment are unimportant. We begin by constructing a time series for the job-finding rate. The monthly job-finding rate is defined by $f_t = -\ln(1 - F_t)$, where F_t is the monthly job-finding probability.[38] We construct a time series for F_t following the method developed by Shimer [2012]. We use the relationship

$$F_t = 1 - \frac{u_{t+1} - u_{t+1}^s}{u_t},$$

where u_t is the number of unemployed persons at time t and u_t^s is the number of short-term unemployed persons at time t. We measure u_t and u_t^s in the data constructed by the BLS from the CPS. The number of short-term unemployed persons is the number of unemployed persons with zero to four weeks duration, adjusted as in Shimer [2012] for the 1994–2014 period. Panel A of Figure A1 displays the monthly job-finding rate. The job-finding rate averages 56% between 1951 and 2014.

Next, we construct the separation rate following the method developed by Shimer [2012]. The separation rate s_t is implicitly defined by

$$u_{t+1} = \left(1 - e^{-f_t - s_t}\right) \cdot \frac{s_t}{f_t + s_t} \cdot h_t + e^{-f_t - s_t} \cdot u_t,$$

where h_t is the number of persons in the labor force in period t, u_t is the number of unemployed persons at time t, and f_t is the monthly job-finding rate. We measure u_t and h_t in the data constructed by the BLS from the CPS, and we use the series that we have just constructed for f_t. Panel B of Figure A1 displays the monthly separation rate. The separation rate averages 3.3% between 1951 and 2014.

Finally, we compare the fluctuations of the actual unemployment rate with those of the steady-state unemployment rate, computed using

$$u_t = \frac{s_t}{f_t + s_t} \tag{A1}$$

and our measures of the job-finding and separation rates. The two series, displayed in Panel C of Figure A1, are almost identical. Since the actual unemployment rate barely departs from its steady-state level, the transitional dynamics of the unemployment rate are unimportant.[39]

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[38]To obtain the relationship between f_t and F_t, we assume that unemployed workers find a job according to a Poisson process with arrival rate f_t.

[39]Panel C is similar to Figure 1 in Hall [2005]. Even though we use different measures of the job-finding and separation rates and a longer time period, Hall's conclusion that transitional dynamics are irrelevant remains valid.
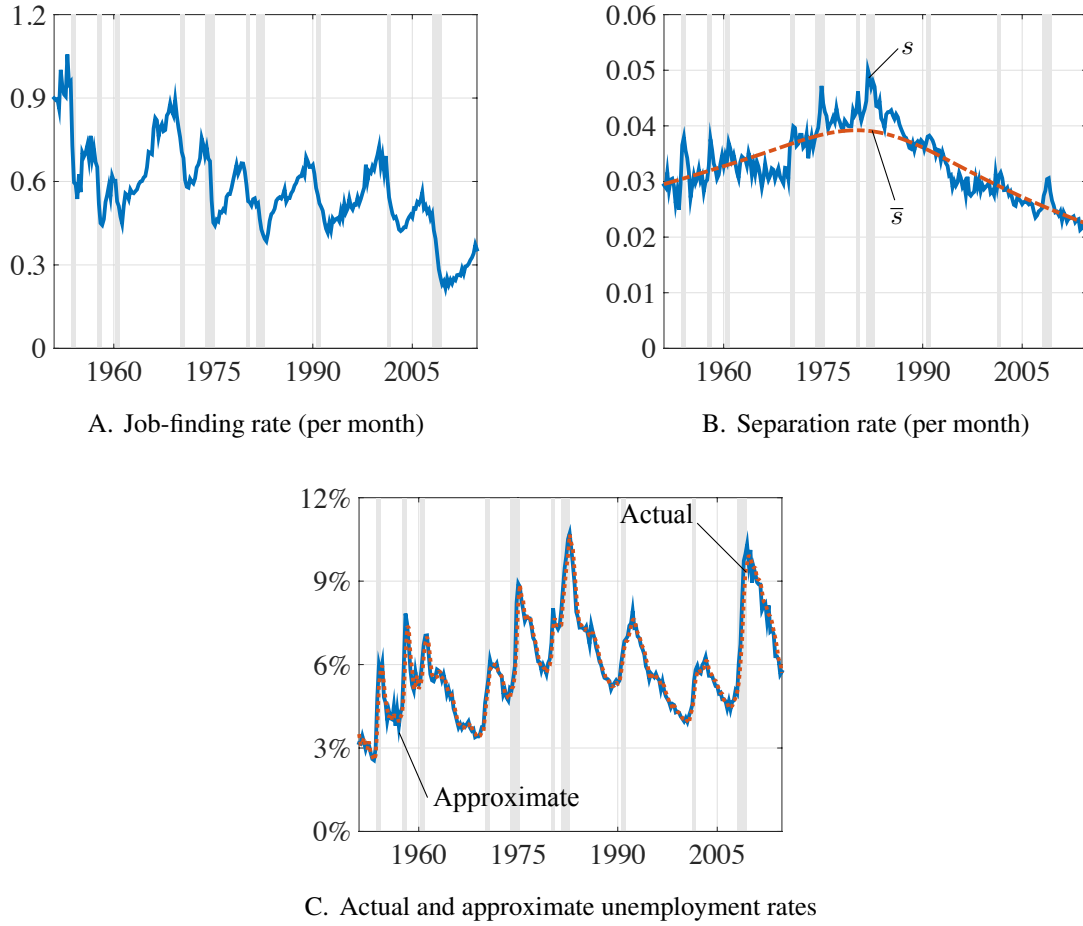
A. Job-finding rate (per month)

B. Separation rate (per month)

C. Actual and approximate unemployment rates

Figure A1: The Irrelevance of Transitional Dynamics For the Unemployment Rate

Notes: Panel A: The job-finding rate is constructed from CPS data following the methodology of Shimer [2012]. Panel B: The separation rate *s* is constructed from CPS data following the methodology of Shimer [2012]. The series $\bar{s}$ is the low-frequency trend of *s* produced with a HP filter with smoothing parameter 10^5. Panel C: The actual unemployment rate is the quarterly average of the seasonally adjusted monthly unemployment rate constructed by the BLS from the CPS. The approximate unemployment rate is computed using (A1). This approximate rate abstracts from transitional dynamics. The shaded areas represent the recessions identified by the NBER.

Approximation of $1 - [\eta/(1-\eta)] \cdot \tau/u$ by $-(x-x^*)/x^*$

In the derivation of formula (7), we approximate $1 - [\eta/(1-\eta)] \cdot \tau/u$ with $-(x-x^*)/x^*$. The term $1 - [\eta/(1-\eta)] \cdot \tau/u$ is an important part of formula (6) as it measures the effect of gross output on net output. We now validate this approximation.

We begin by measuring the matching wedge τ. To construct τ, we use the property that

$$\tau = \frac{s \cdot \rho}{q - s \cdot \rho}. \tag{A2}$$

We use the separation rate *s* described in Panel B of Figure A1. We construct the vacancy-filling rate as $q = f/x$, where *x* is the market tightness displayed in Figure 4 and *f* is the job-finding rate

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A. Vacancy-filling rate (per month)
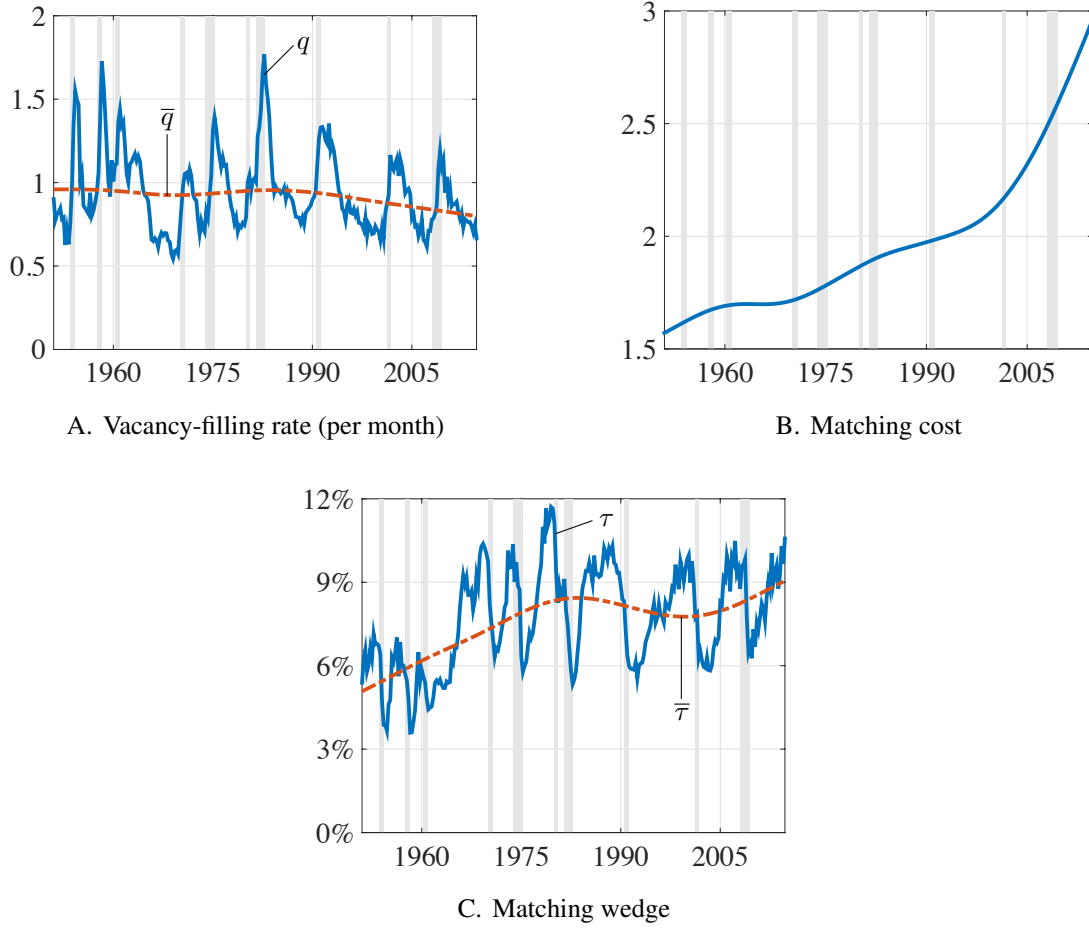
B. Matching cost

C. Matching wedge

Figure A2: Matching Cost and Matching Wedge in the United States, 1951–2014

Notes: Panel A: The vacancy-filling rate q is computed using $q = f/x$. The job-finding rate f is described in Figure A1. The market tightness x is described in Figure 4. The series $\bar{q}$ is the low-frequency trends of q produced with a HP filter with smoothing parameter 10^5. Panel B: The matching cost is computed using equation (A4). Panel C: The matching wedge τ is computed using equation (A2). The series $\bar{\tau}$ is computed using equation (A3). The shaded areas represent the recessions identified by the NBER.

described in Panel A of Figure A1. Panel A of Figure A2 displays the monthly vacancy-filling rate. The vacancy-filling rate averages 94% between 1951 and 2014.

We construct the matching cost ρ as a slow-moving variable such that on average, the market is efficient. If the market is efficient on average, then (4) implies that

$$\bar{\tau} = \frac{1 - \eta}{\eta} \cdot \bar{u}, \tag{A3}$$

where $\bar{\tau}$ is the average value of the matching wedge and $\bar{u}$ the average value of the unemployment rate. We set $\eta = 0.43$ and construct $\bar{u}$ as the low-frequency trend of u produced with a HP filter with smoothing parameter 10^5. This procedure yields $\bar{\tau}$. We also construct the average values of the vacancy-filling rate and separation rate, $\bar{q}$ and $\bar{s}$, by taking the low-frequency trends of q and s produced with a HP filter with smoothing parameter 10^5. Using the resulting series, we construct
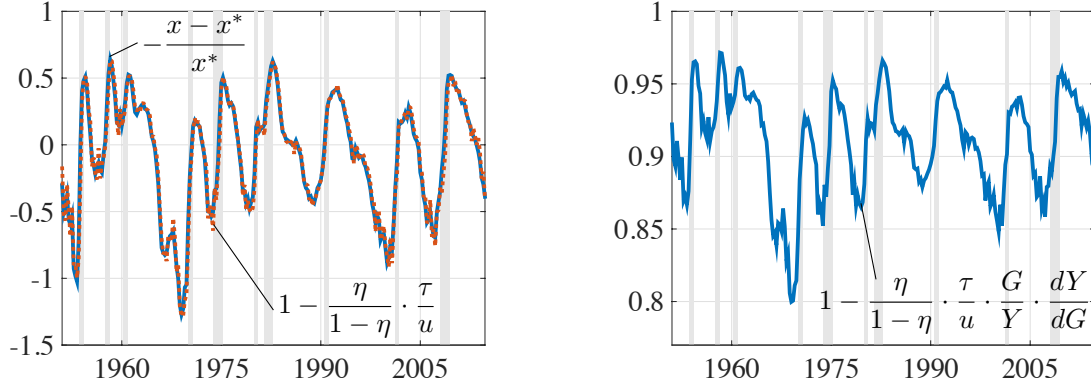
Figure A3: Approximating $1 - \frac{\eta}{1-\eta} \cdot \frac{\tau}{u}$ by $-\frac{x-x^*}{x^*}$ and $1 - \frac{\eta}{1-\eta} \cdot \frac{\tau}{u} \cdot \frac{G}{Y} \cdot \frac{dY}{dG}$ by 1

Notes: The unemployment rate u is described in Figure 5. The matching wedge τ is described in Figure A2. The term $-(x-x^*)/x^*$ is computed using the market tightness x and the efficient market tightnesses x^* described in Figure 4. The terms $1 - [\eta/(1-\eta)] \cdot \tau/u$ and $1 - [\eta/(1-\eta)] \cdot (\tau/u) \cdot (G/Y) \cdot (dY/dG)$ are computed using τ and u and setting $\eta = 0.43$. To compute the second term, we set $dY/dG = 0.5$ and use the government purchases-output ratio $G/Y = (G/C)/(1+G/C)$ from Figure 3. The shaded areas represent the recessions identified by the NBER.

ρ as the slow-moving variable

$$\rho = \frac{\overline{q}}{\overline{s}} \cdot \frac{\overline{\tau}}{1+\overline{\tau}}. \tag{A4}$$

This equation is just (A2), evaluated in the average state. Panel B of Figure A2 displays the resulting matching cost. The matching cost averages 2.0 between 1951 and 2014, and it increases significantly over time.

Using our series for s, q, and ρ, we construct a time series for τ. Panel C of Figure A2 displays the matching wedge. The matching wedge is very procyclical, with an average value of 7.8% between 1951 and 2014.

Finally, we use our series for τ, set $\eta = 0.43$, and use the unemployment rate described in Figure 5 to construct $1 - [\eta/(1-\eta)] \cdot \tau/u$. Panel A of Figure A3 displays the time series for $1 - [\eta/(1-\eta)] \cdot \tau/u$. Panel A also displays $-(x-x^*)/x^*$, constructed using the series described in Figure 4. These two series are nearly indistinguishable, which validates the approximation.

Approximation of $1 - [\eta/(1-\eta)] \cdot (\tau/u) \cdot (G/Y) \cdot (dY/dG)$ by 1

In the derivation of formula (7), we approximate $1 - [\eta/(1-\eta)] \cdot (\tau/u) \cdot (G/Y) \cdot (dY/dG)$ by 1. It is simple to assess this approximation. We set $dY/dG = 0.5$ and $\eta = 0.43$, as discussed in Section 4. We construct a series for G/Y as $(G/C)/(1+G/C)$, where G/C is the ratio of government consumption to personal consumption described in Figure 3. We use the unemployment rate u described in Figure 5 and the matching wedge τ described in Panel C of Figure A2. Panel B of Figure A3 displays the resulting time series for $1 - [\eta/(1-\eta)] \cdot (\tau/u) \cdot (G/Y) \cdot (dY/dG)$. The series is close to 1, especially in slumps: in most slumps, it is around 0.95; at all times except in the early 1970s, it is above 0.85; and on average between 1951 and 2014, it is 0.91. This means that the approximation is quite accurate.
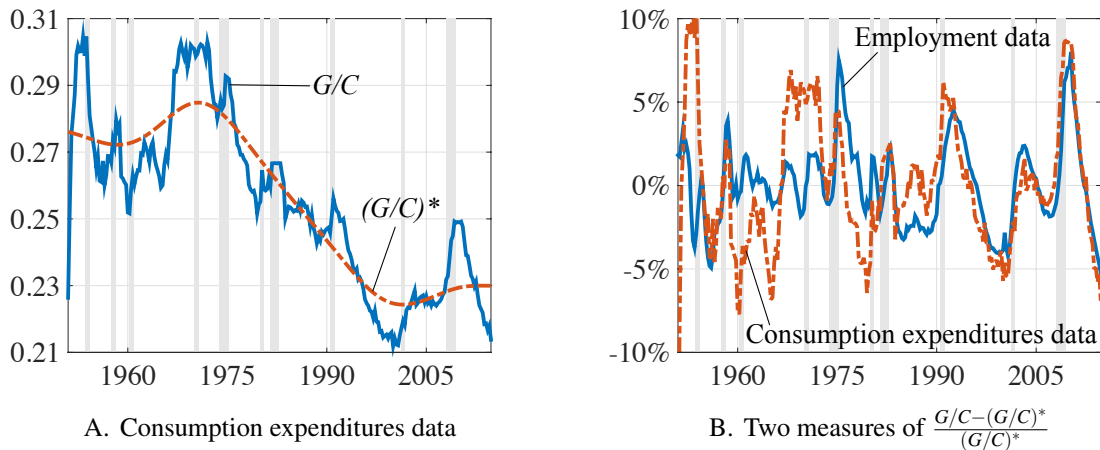
Figure A4: Government Consumption Expenditures and Government Employment in the United States, 1951–2014

Notes: Panel A: Government consumption G is seasonally adjusted quarterly government consumption expenditures in dollars constructed by the BEA as part of the NIPA. Personal consumption C is seasonally adjusted quarterly personal consumption expenditures in dollars constructed by the BEA as part of the NIPA. The ratio $(G/C)^*$ is the low-frequency trend of G/C produced with a HP filter with smoothing parameter 10^5. Panel B: The ratio $[G/C - (G/C)^*]/(G/C)^*$ measured from consumption expenditures data uses the ratios G/C and $(G/C)^*$ described in Panel A. The ratio $[G/C - (G/C)^*]/(G/C)^*$ measured from employment data uses the ratios G/C and $(G/C)^*$ described in Figure 3. The shaded areas represent the recessions identified by the NBER.

Appendix C: Government Consumption Expenditures

In this appendix, we construct an alternative measure of the ratio between government consumption and personal consumption, G/C, for the United States between 1951 and 2014. We measure G with government consumption expenditures constructed by the BEA as part of the NIPA. We measure C by the personal consumption expenditures constructed by the BEA as part of the NIPA. Figure A4 displays the resulting series.

The two measures of G/C–that based on employment data and plotted in Figure 3 and that based on consumption expenditures data and plotted in Figure A4—have fairly different levels. The ratio G/C based on consumption expenditures data is always higher. It was much higher at the beginning of the period (0.31 against 0.16 in 1953). The gap between the two measures shrank until 1990. Since then the gap has been roughly constant, the measure based on consumption expenditures data hovering between 0.21 and 0.25 and the measure based on employment data hovering between 0.19 and 0.21.

However, since 1980, the ratios $(G/C - (G/C)^*)/(G/C)^*$ obtained from government expenditures data and employment data nearly perfectly overlap. This ratio matters more than the level of G/C because it enters into most of our formulas for optimal government purchases and determines the value of $1 - MRS_{gc}$. Since changes in government expenditures compared to trend almost exactly track changes in government employment compared to trend, how we measure government purchases does not matter much for our analysis.

Before 1980, the two ratios do not overlap as well because of the Korean and Vietnam wars. The ratio based on consumption expenditures data is especially high in 1951–1953, which cor-

responds to the Korean war, and in 1967–1972, which corresponds to the Vietnam war. Since military personnel does not count as government employees in BLS data, and since wars involve important purchases of military equipment, government expenditures during wars rise whereas government employment does not change much. Accordingly, wars create a discrepancy between our two measures of $(G/C - (G/C)^*)/(G/C)^*$.

Appendix D: Equilibrium of the Specific Model

In this appendix we derive and analyze the dynamical system describing the equilibrium of the specific model of Section 6. An equilibrium consists of paths for market tightness, net personal consumption, net government consumption, net output, real wealth, and real interest rate, $[x(t),$ $c(t), g(t), y(t), b(t), r(t)]_{t=0}^{+\infty}$. The equilibrium consists of 6 variables, so it requires 6 conditions to be well defined.

The first condition is that the government chooses a fixed amount of government consumption: $g(t) = g$. The second condition is that a price mechanism determines the real interest rate: $r(t)$ is given by (16). The third condition is that the bond market is in equilibrium: $b(t) = 0$. The fourth condition is that the market for services is in equilibrium: $c(t) + g(t) = y(t)$. The fifth condition is that the matching process determines net output: $y(t) = y(x(t))$.

The sixth condition is that the representative household chooses net personal consumption to maximize utility subject to its budget constraint. To solve the household's problem, we set up the current-value Hamiltonian:

$$\mathscr{H}(t, c(t), b(t)) = \mathscr{U}(c(t), g(t)) + \mathscr{V}(b(t)) + \lambda(t) \cdot [Y(x(t)) - (1 + \tau(x(t))) \cdot c(t) + r(t) \cdot b(t) - T(t)]$$

with control variable $c(t)$, state variable $b(t)$, and current-value costate variable $\lambda(t)$. Throughout we use subscripts to denote partial derivatives. The necessary conditions for an interior solution to this maximization problem are $\partial \mathscr{H}/\partial c = 0$, $\partial \mathscr{H}/\partial b = \delta \cdot \lambda(t) - \dot{\lambda}(t)$, and the transversality condition $\lim_{t \to +\infty} e^{-\delta \cdot t} \cdot \lambda(t) \cdot b(t) = 0$. Given that $\mathscr{U}$ and $\mathscr{V}$ are concave and that $\mathscr{H}$ is the sum of $\mathscr{U}$, $\mathscr{V}$, and a linear function of (c, b), $\mathscr{H}$ is concave in (c, b) and these conditions are also sufficient. These two first-order conditions imply that

$$\frac{\partial \mathscr{U}}{\partial c}(c(t), g(t)) = \lambda(t) \cdot (1 + \tau(x(t))) \tag{A5}$$

$$\mathscr{V}'(b(t)) = (\delta - r(t)) \cdot \lambda(t) - \dot{\lambda}(t). \tag{A6}$$

Recombining these equations, we obtain the consumption Euler equation

$$(1 + \tau(x(t))) \cdot \frac{\mathscr{V}'(b(t))}{\frac{\partial \mathscr{U}}{\partial c}(c(t), g(t))} + (r(t) - \delta) = -\frac{\dot{\lambda}(t)}{\lambda(t)},$$

where $\dot{\lambda}(t)/\lambda(t)$ can be expressed as a function of $c(t)$, $g(t)$, and $x(t)$, and their time derivatives using (A5). The Euler equation represents a demand for saving in part from intertemporal consumption-smoothing considerations and in part from the utility provided by wealth. The equation implies that at the margin, the household is indifferent between spending income on consumption and holding real wealth. The equation determines the level of aggregate demand. In steady

state it defines the aggregate demand curve.

We have obtained the six equations that define the dynamical system representing the equilibrium. We now describe the transitional dynamics toward the steady state. The dynamic system is simple to study because it can be described by one single endogenous variable: the costate variable $\lambda(t)$. All the variables can be recovered from $\lambda(t)$. The law of motion for $\lambda(t)$ is given by (A6):

$$\dot{\lambda}(t) = (\delta - r) \cdot \lambda(t) - \mathcal{V}'(0) \equiv \phi(\lambda(t)).$$

Note that $r(t) = r$ is constant over time (see (16) and note that $g(t)$ is constant over time). The steady-state value of the costate variable satisfies $\phi(\lambda) = 0$ so $\lambda = \mathcal{V}'(0)/(\delta - r) > 0$. The nature of the dynamical system is given by the sign of $\phi'(\lambda)$. Since $\phi'(\lambda) = \delta - r > 0$, we infer that the system is a source. As there is no state variable, our source system jumps from one steady state to the other in response to permanent, unexpected shocks.

Appendix E: Proof of Proposition 6

Step 1. Using equation (15) and simple algebra, we write the marginal utility of consumption as

$$\frac{\partial \mathcal{U}}{\partial c}(c, g) = a \cdot d \left(\frac{G}{C} \right)^{\frac{1}{\varepsilon}}$$

where

$$a \equiv (1 - \gamma) \cdot \left(\frac{1}{\bar{c}} \right)^{\frac{\varepsilon - 1}{\varepsilon}}, \quad d(\theta) \equiv \mathcal{U}(1, \theta)$$

Additional algebra shows that the elasticity of $d(\theta)$ is $1/z(\theta)$ where

$$z(\theta) \equiv 1 + \frac{1 - \gamma}{\gamma} \cdot \left(\frac{\bar{g}}{\bar{c}} \cdot \frac{1}{\theta} \right)^{\frac{\varepsilon - 1}{\varepsilon}}$$

Step 2. Using the results from step 1 and equation (16), we rewrite the interest-rate schedule as

$$\delta - r = (\delta - \bar{r}) \cdot \left(\frac{\mathcal{V}'(0)}{\overline{\mathcal{V}'(0)}} \right)^{1 - \alpha} \cdot d \left(\frac{\bar{g}}{\bar{c}} \right)^{\frac{1 - \beta}{\varepsilon}} \cdot d \left(\frac{G}{Y^* - G} \right)^{-\frac{1 - \beta}{\varepsilon}}.$$

Using more results from step 1 and more algebra, we find that the elasticity of the interest-rate schedule is

$$\frac{d \ln(\delta - r)}{d \ln(G)} = -\frac{1 - \beta}{\varepsilon} \cdot \frac{1}{z(G/(Y^* - G))} \cdot \frac{Y^*}{Y^* - G}.$$

Note that $\delta - r$ does not depend on the market tightness.

Step 3. We implicitly define the function $C(G,x)$ as the solution of

$$d\left(\frac{G}{C}\right)^{-\frac{1}{\varepsilon}} = \frac{a}{\mathcal{V}'(0)} \cdot \frac{\delta - r}{1 + \tau(x)}.$$

The function $C(G,x)$ is the gross personal consumption that satisfies the Euler equation (18) for a gross government consumption G and market tightness x. Using the results from steps 1 and 2 and simple algebra, we obtain the two following elasticities:

$$\frac{\partial \ln(C)}{\partial \ln(x)} = -\varepsilon \cdot \eta \cdot \tau \cdot z\left(\frac{G}{C}\right)$$

$$\frac{\partial \ln(C)}{\partial \ln(G)} = 1 - (1-\beta) \cdot \frac{Y^*}{Y^* - G} \cdot \frac{z(G/C)}{z(G/(Y^* - G))}.$$

Step 4. The equilibrium condition determining market tightness is

$$Y = C(G,x) + G.$$

We differentiate this equilibrium condition with respect to G:

$$\frac{d \ln(Y)}{d \ln(G)} = \frac{C}{Y} \cdot \left(\frac{\partial \ln(C)}{\partial \ln(G)} + \frac{\partial \ln(C)}{\partial \ln(x)} \cdot \frac{d \ln(x)}{d \ln(G)}\right) + \frac{G}{Y}$$

Using the elasticity of $Y(x)$, we obtain

$$\frac{d \ln(Y)}{d \ln(G)} = (1 - \eta) \cdot u \cdot \frac{d \ln(x)}{d \ln(G)}.$$

Using these equations and the elasticities from step 3, we obtain

$$\left[1 + \frac{C}{Y} \cdot \varepsilon \cdot \frac{\eta}{1 - \eta} \cdot \frac{\tau}{u} \cdot z\left(\frac{G}{C}\right)\right] \cdot \frac{d \ln(Y)}{d \ln(G)} = 1 - (1-\beta) \cdot \frac{C}{Y} \cdot \frac{Y^*}{Y^* - G} \cdot \frac{z(G/C)}{z(G/(Y^* - G))}$$

$$\frac{d \ln(Y)}{d \ln(G)} = \frac{1 - (1-\beta) \cdot \frac{1 - G/Y}{1 - G/Y^*} \cdot \frac{z(G/(Y-G))}{z(G/(Y^*-G))}}{1 + \frac{C}{Y} \cdot \varepsilon \cdot \frac{\eta}{1-\eta} \cdot \frac{\tau}{u} \cdot z\left(\frac{G}{C}\right)}.$$

Step 5. When the unemployment rate is efficient, $Y = Y^*$ and $1 = [\eta/(1-\eta)] \cdot \tau/u$. Hence, the expression for the multiplier simplifies to

$$\frac{d \ln(Y)}{d \ln(G)} = \frac{\beta}{1 + \varepsilon \cdot \frac{C}{Y} \cdot z\left(\frac{G}{C}\right)}.$$

In the average state, $G/C = \overline{G}/\overline{C}$ so $z(G/C) = 1/\gamma$ and the multiplier further simplifies to

$$\frac{d \ln(Y)}{d \ln(G)} = \frac{\beta}{1 + \frac{\varepsilon}{\gamma} \cdot \frac{\overline{C}}{\overline{Y}}}.$$

Table A1: Parameter Values Used in the Simulations

	Description	Value	Source
	Panel A. Average values targeted in calibration		
$\overline{u}$	unemployment rate	5.9%	CPS, 1951–2014
$\overline{x}$	market tightness	0.65	Barnichon [2010], JOLTS, and CPS, 1951–2014
$\overline{G/Y}$	government purchases-output ratio	16.6%	CES, 1951–2014
$\overline{\tau}$	matching wedge	7.8%	efficiency on average (see Section 4)
$\overline{dY/dG}$	government-purchases multiplier	0.5	literature (see Section 4)
$\overline{\mathcal{V}'(0)}$	marginal utility of wealth	1	normalization
	Panel B. Calibrated parameters		
$1-\eta$	tightness elasticity of job-finding rate	0.43	Barnichon [2010], JOLTS, and CPS, 1951–2014 (see Section 4)
s	separation rate (monthly)	3.3%	CPS, 1951–2014
ω	matching efficacy	0.68	matches average values
ρ	matching cost	1.77	matches average values
ε	elasticity of substitution	1	Section 4
γ	parameter of utility function	0.166	matches $\overline{G/Y} = 16.6\%$
$\overline{c}$	parameter of utility function	0.728	matches average values
$\overline{g}$	parameter of utility function	0.145	matches average values
α	parameter of interest-rate schedule	1	normalization
β	parameter of interest-rate schedule	0.5	matches $\overline{dY/dG} = 0.5$
$\delta - \overline{r}$	parameter of interest-rate schedule	0.941	matches average values

Next, if government purchases are chosen optimally, $\gamma = \overline{G/Y}$ and the multiplier simplifies to

$$\frac{dY}{dG} = \beta \cdot \frac{1}{\gamma + \varepsilon \cdot (1 - \gamma)}.$$

Finally, if $\varepsilon = 1$, then $dY/dG = \beta$.

Appendix F: Calibration of the Specific Model

In this appendix we calibrate the specific model of Section 6 to US data for the 1951–2014 period. The calibration is summarized in Table A1.

We calibrate several parameters such that the values of key variables in the average state match the average values of these variables in the data. We target an average unemployment rate of $\overline{u} = 5.9\%$, an average market tightness of $\overline{x} = 0.65$, an average government purchases-output ratio of $\overline{G/Y} = 16.6\%$, and an average matching wedge of $\overline{\tau} = 7.8\%$. We also normalize the values of marginal utility of wealth in the average state to 1: $\overline{\mathcal{V}'(0)} = 1$. (These average values come from the times series constructed in Section 4 and Appendix B.)

We begin by calibrating the three parameters determining the value of the sufficient statistics at

the heart of our formulas for optimal government purchases. Based on the discussion in Section 4, we calibrate the model to obtain an elasticity of substitution between government and personal consumptions of 1, a tightness elasticity of the job-finding rate of 0.57, and a multiplier in the average state of 0.5. Hence we set $\varepsilon = 1$, $\eta = 0.43$, and $\beta = 0.5$.[40]

Next, we calibrate parameters related to matching. We set the separation rate to its average value over the 1951–2014 period: $s = 3.3\%$ (see Appendix B). To calibrate the matching efficacy, we exploit the relationship $\overline{u} \cdot f(\overline{x}) = s \cdot (1 - \overline{u})$, which implies $\omega = s \cdot (\overline{x})^{\eta - 1} \cdot (1 - \overline{u})/\overline{u} = 0.68$. To calibrate the vacancy-filling cost, we exploit the relationship $\overline{\tau} = \rho \cdot s / [\omega \cdot (\overline{x})^{-\eta} - \rho \cdot s]$, which implies $\rho = \omega \cdot (\overline{x})^{-\eta} \cdot \overline{\tau} / [s \cdot (1 + \overline{\tau})] = 1.77$.

Then, we calibrate the parameters of the utility function. The parameter γ determines the optimal government purchases-output ratio in the average state. As we assume that the average government purchases-output ratio is optimal in the average state, we set $\gamma = 0.166$. We set the scaling parameters $\overline{c}$ and $\overline{g}$ to $\overline{g} = (1 - \overline{u}) \cdot \overline{G/Y} \cdot = 0.145$ and $\overline{c} = (1 - \overline{G/Y}) \cdot (1 - \overline{u}) = 0.728$.

Last, we calibrate the parameters of the interest-rate schedule. For aggregate demand shocks to generate fluctuations, we need $\alpha > 0$. The value of α determines the elasticity of output to the marginal utility of wealth, $\mathcal{V}'(0)$. Since we do not know the amplitude of the fluctuations of $\mathcal{V}'(0)$, the exact value of α is irrelevant. We arbitrarily set $\alpha = 1$. Last, using (17), we set $\delta - \overline{r} = 0.941$.

Appendix G: Robustness of Simulation Results

The simulation results in the text are obtained for an elasticity of substitution between government and personal consumption of $\varepsilon = 1$ and a government-purchases multiplier in the average state of $dY/dG = 0.5$. In this appendix we repeat the simulations for alternative values of ε and dY/dG.

Figure A5 displays simulations for $\varepsilon = 0.5$ and $\varepsilon = 2$. The figure shows that when ε is lower, the optimal government purchases-output ratio responds less to fluctuations in unemployment, and consequently, fluctuations in unemployment are less attenuated. For instance when $\varepsilon = 0.5$ and the unemployment rate reaches 9.6% for $G/Y = 16.6\%$, it is optimal to increase government purchases to $G/Y = 18.9\%$, which reduces the unemployment rate to 8.0%. When $\varepsilon = 2$ and the unemployment rate reaches 9.6% for $G/Y = 16.6\%$, it is optimal to increase government purchases to $G/Y = 20.2\%$, which reduces the unemployment rate to 6.5%. The figure also shows that the explicit formula, given by (12), is more accurate for lower values of ε.

Figure A6 displays simulations for $dY/dG = 0.2$ and $dY/dG = 1$. The figure shows that a higher value of dY/dG does not imply that optimal government purchases respond more strongly to a rise in unemployment. It does imply, however, that fluctuations in unemployment are more attenuated. For instance when $dY/dG = 0.2$ and the unemployment rate reaches 9.6% for $G/Y = 16.6\%$, it is optimal to increase government purchases to $G/Y = 19.3\%$, which reduces the unemployment rate to 8.6%. When $dY/dG = 1$ and the unemployment rate reaches 9.6% for $G/Y = 16.6\%$, it is optimal to increase government purchases to $G/Y = 18.5\%$, which reduces the unemployment rate to 6.2%. The figure also shows that the explicit formula, given by (12), is more accurate for lower values of dY/dG. This is not surprising since the explicit formula is obtained by assuming that dY/dG is small enough.

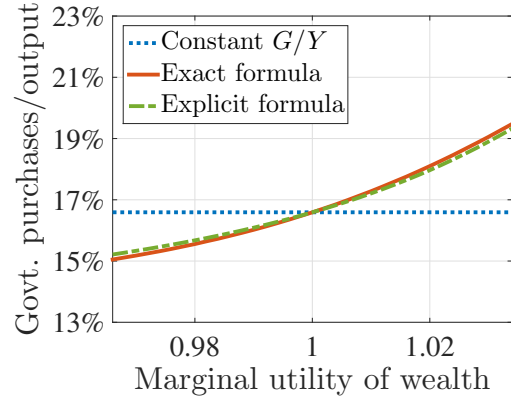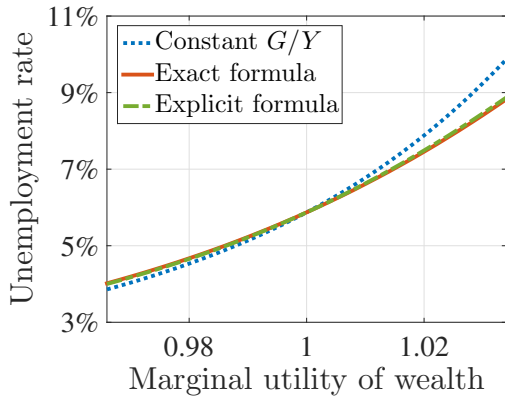[40]Proposition 6 establishes the link between β and the value of the multiplier.
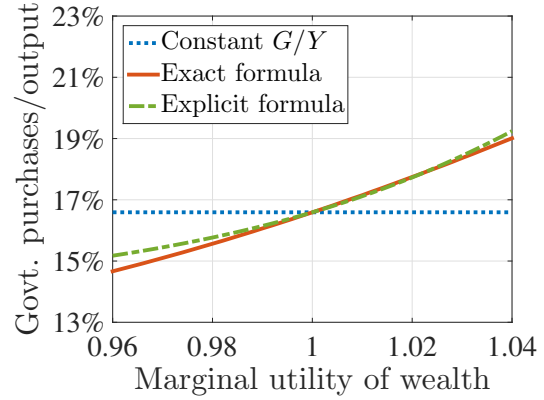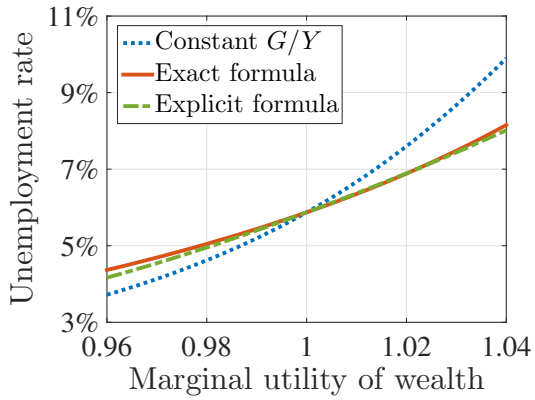
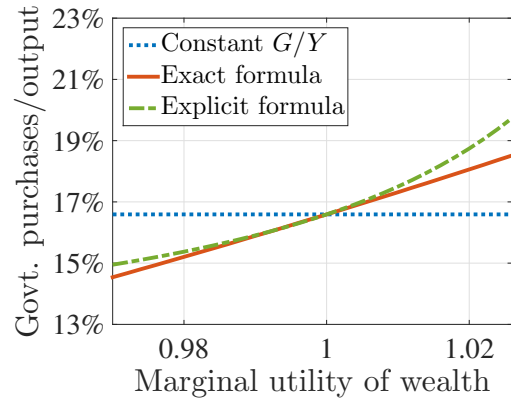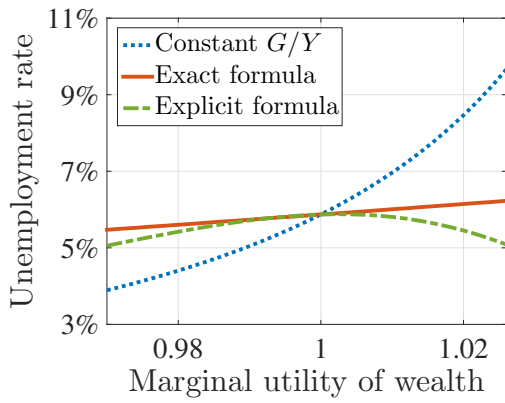A. $\varepsilon = 0.5$ and $dY/dG = 0.5$
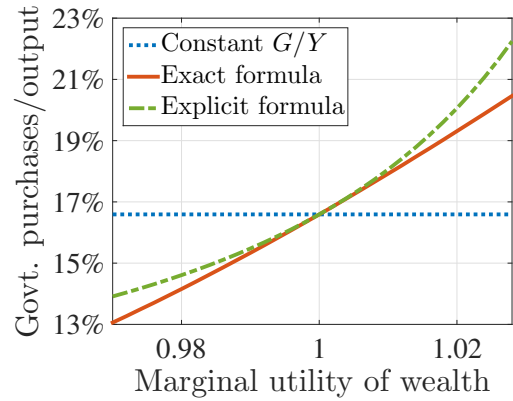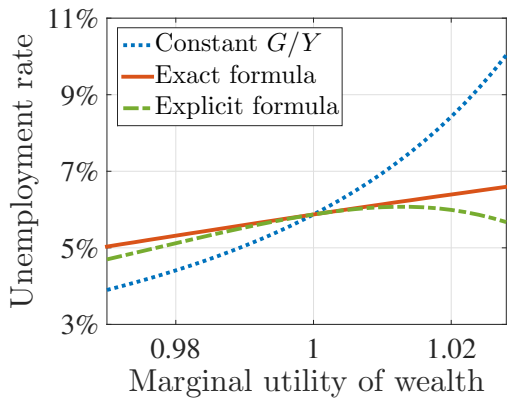
B. $\varepsilon = 2$ and $dY/dG = 0.5$

Figure A5: Simulations of the Specific Model for Various Elasticities of Substitution

A. $\varepsilon = 1$ and $dY/dG = 0.2$

B. $\varepsilon = 1$ and $dY/dG = 1$

Figure A6: Simulations of the Specific Model for Various Multipliers