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QUANTITATIVE MODELS OF WEALTH INEQUALITY: A SURVEY

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ABSTRACT

In the data, wealth is very unequally distributed, even more so than labor earnings and income, and the saving rate of wealthy people is high. Many dynamic models used for quantitative policy evaluation imply that once households get rich, they dissave. As a result, these models generate too little wealth concentration in the hands of the wealthiest compared with the observed data. This raises the question of the robustness of the policy lessons that we learn from environments in which key aspects of saving behavior in the model are not consistent with those in the observed data.

Mechanisms that raise the saving rate of richer people, and thus generate more realistic saving behavior and wealth concentration in dynamic quantitative models, have been proposed. These mechanisms include heterogeneity in patience, transmission of human capital and voluntary bequests across generations, entrepreneurship or high returns to capital coupled with borrowing constraints, and high earnings risk for the top earners. More work is needed to evaluate these explanations both individually and jointly and to quantitatively assess their importance. Additionally, more work to explore alternative or complementary mechanisms is warranted.

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1 Introduction

Wealth is much more unequally distributed than labor earnings and income, and the wealthy keep saving at high rates. What do we know about the determinants of this high wealth concentration and saving behavior? The answer to this question is important for two reasons. First, there is much debate about why some people are rich and some people are poor, and dynamic quantitative models of wealth inequality can help us understand and quantify the determinants of the outcomes that we observe in the data. Second, both the redistributive and aggregate consequences of government taxes and transfer programs crucially depend on the type and strength of people's saving motives.

This paper first discusses the workhorse framework that is generally adopted to study wealth inequality, the Bewley [12] model, an incomplete markets environment, in which people save to self-insure against idiosyncratic shocks. In the basic Bewley model, precautionary savings are the key force driving wealth concentration. However, the nature of precautionary savings implies that households save to self-insure against earnings risk but that, as a result, the saving rate decreases and then turns negative when one's net worth is large enough relative to some multiple of one's labor earnings. Hence, the saving rate of the wealthy in these models is negative. In contrast, in the U.S. data, rich people keep saving at high rates, which explains the emergence and persistence of their very large estates. The basic version of the model thus fails to generate the high concentration of wealth in the hands of the richest few because it misses the fact that rich people keep saving.

Previous work has uncovered forces that, when introduced into a Bewley model, keep the saving rates of the wealthiest high and thus generate higher wealth concentration in the hands of a small fraction of households. These forces include heterogeneity in patience, transmission of human capital and voluntary bequests across generations, entrepreneurship or high returns to capital coupled with borrowing constraints, and high earnings risk for the top earners. More work is needed to evaluate these explanations both individually and jointly and to quantitatively assess their importance. Additionally, more work to explore alternative or complementary mechanisms is warranted.

The rest of this paper is organized as follows. Section 2 briefly discusses the main facts about wealth inequality. Section 3 formalizes two versions of the Bewley model (first in an infinitely-lived framework and then in an overlapping-generations framework), discusses the main intuition behind saving motives, and then highlights the quantitative implications in terms of wealth inequality and saving behavior. Section 4 studies the effects of allowing for heterogenous preferences and, in particular, for heterogeneous patience. Section 5 discusses an economy in which there are intergenerational transmission of human capital and both accidental and voluntary bequests. Section 6 analyzes the role of entrepreneurship in an environment in which an entrepreneur can have very high returns from running a firm but faces borrowing constraints. Section 7 highlights the effects of introducing large earnings risk for the top earners. Section 8 concludes by summarizing what we have learned so far and points to directions for future research.

2 Some facts about wealth inequality

Key facts about the distribution of wealth have been highlighted in a large number of studies, including Wolff [100] and [99], Kennickell [65], and Cagetti and De Nardi (in an older survey of both data and models [25]).

The most striking aspect of the wealth distribution in the United States is its degree of concentration. Over the past 30 years or so, households in the top 1% of the wealth distribution have held about one-third of the total wealth in the economy, and those in the top 5% have held more than half. At the other extreme, many households (more than 10%) have little or no assets.

While there is agreement that the share held by the richest few is very high, the extent to which the shares of the richest have changed over time (and why) is still subject of some debate (Piketty [84], Saez and Zucman [93], Bricker et al. [16], and Kopczuk [68]). Understanding why wealth inequality changes over time in a dynamic quantitative model is very important, but understanding the determinants of inequality at a point in time seems a necessary precursor to studying the evolution of inequality over time. Hence, in this survey, we focus on understanding how wealth inequality arises in steady-state dynamic quantitative models.[1]

An important related observation is that the concentration of wealth is much higher than that of earnings and income (Díaz-Giménez et al. [35] and Budria et al. [89]). For example, in 1992 the Gini indexes for labor earnings, income (inclusive of transfers), and wealth were, respectively, .63, .57, and .78 (Díaz-Giménez et al. [35]), while in 1995 they were .61, .55 and .80 (Budria et al. [89]). In addition, the correlation between these three variables is positive, but well below one. Consistent with these findings, Hendricks [58] finds that the correlation coefficient between lifetime earnings and wealth at retirement (0.61) is much less than unity.

Several studies have documented significant differences in saving behavior across various groups. (See Browning and Lusardi [18] for a review of the literature.) In particular, Dynan et al. [41] show that in the U.S., higher-lifetime-income households save a larger fraction of their income than lower-lifetime-income households. De Nardi et al. [38] show that, among the elderly, people with higher lifetime income not only reach retirement with more wealth, but also run down their net worth during the retirement period more slowly. Quadrini [85] documents that entrepreneurs, who tend to be among the richest households, exhibit higher saving rates. Buera [19] and [20] finds high saving behavior for entrepreneurs, both before and after entering entrepreneurship.

Income inequality leads to wealth inequality, but income is much less concentrated than wealth, and economic models have had difficulties in quantitatively generating the observed degree of wealth concentration from the observed income inequality. The question is what mechanisms are necessary to generate saving behavior that leads to a distribution of asset holdings consistent with the actual data.

The extent to which rich households stay rich and poor household stay poor

[1]For an interesting modeling of how consumption and income (but not wealth) inequality evolve over time, see Kruger and Perri [70]. For empirical papers studying changes of earnings processes over time, see, among others, DeBacker et al. [32], Sabelhaus and Song [92], and Dynan et al. [42].

over time, or mobility in both earnings and wealth, is an additional important element to keep in mind when thinking about cross-sectional inequality at a point in time. Hurst et al. [61] use Panel Study of Income Dynamics (PSID) data to analyze wealth mobility between 1984 and 1994 and document that most of the mobility occurs in the mid-range deciles, while the top and bottom ones show high persistence. Using the same dataset, Quadrini [85] studies the wealth mobility for entrepreneurs and non-entrepreneurs, suggesting that entrepreneurs are more upwardly mobile. Unfortunately, the PSID does not allow us to study what happens at the top percentiles. Some progress has been made by Guvenen et al. [50] by analyzing administrative data for earnings in the U.S. Still, the question of how volatile and persistent are earnings and wealth of the super-rich remains open to investigation and likely to be of crucial importance in affecting their saving behavior.

3 Basic Bewley models, saving behavior, and wealth inequality

Bewley models are incomplete-market models in which households are often ex-ante identical,[2] in the sense that they face the same stochastic processes, but are ex-post heterogeneous, because they receive different sequences of realizations of the shocks. An exogenously specified earnings process is typically the source of these shocks, and its properties are usually estimated from micro-level data on earnings. Aiyagari [2] and Hansen and İmrohoroğlu [51] provide early general equilibrium versions of Bewley models.

While computing transitions is sometimes feasible, these models are usually solved for stationary equilibria. Since it is assumed that there is no aggregate uncertainty, in a stationary equilibrium there is a constant distribution of people over state variables, hence the economy is time-invariant. However, people move up and down the distribution and thus face considerable uncertainty at

[2]See Ljungqvist and Sargent [79] for an overview of Bewley models, including properties and solution methods. See Quadrini and Ríos-Rull [87] for a discussion about why we need incomplete-market models to study wealth inequality.

the individual level.

These models endogenously generate differences in asset holdings and hence wealth concentration as a result of the household's desire to save and the realization of the exogenous shocks.

3.1 A basic infinitely-lived Bewley model

Consider a Bewley model populated by a continuum of infinitely-lived agents with preferences

$$E\left\{\sum_{t=1}^{\infty}\beta^t u(c_t)\right\},$$

where $u(c_t)$ is a constant relative-risk aversion utility function.

The labor endowment of each household is given by an idiosyncratic labor productivity shock z, which assumes a finite number of possible values and follows a first order Markov process with transition matrix $\Gamma(z)$. There is only one asset, a, that people can use to self-insure against earnings risk.

A constant-returns-to-scale production technology converts aggregate capital (K) and aggregate labor (L) into aggregate output (Y).

During each period, each household chooses how much to consume (c) and save for next period by holding risk free assets (a'). The household's state variables are thus denoted by $x = (a, z)$, where a is asset holdings carried into the period and z is the labor shock endowment.

The household's recursive problem can thus be written as

$$V(x) = \max_{(c,a')}\left\{u(c) + \beta E\Big[V(a', z')|x\Big]\right\}$$

subject to

$$c + a' = (1 + r)a + zw$$

$$c \geq 0, \quad a' \geq \underline{a},$$

where r is the interest rate net of taxes and depreciation, w is the wage, and $\underline{a}$ is a net borrowing limit. See Ljungqvist and Sargent [79] for an excellent discussion of ad-hoc borrowing limits, as opposed to "natural" borrowing limits in Bewley models.

At every point in time, this model economy can be described by a probability distribution of people over assets a and earnings shocks z.

A stationary equilibrium for this economy is a set of consumption and saving rules, prices, aggregate capital and labor, and an invariant distribution of households over the state variables of the system such that: i. Given prices, the decision rules solve the household's recursive problem. ii. Aggregate capital is equal to total savings by the households in the economy, while aggregate labor is equal to total labor supplied by the households in the economy. iii. The interest rate and the wage rate equal the marginal product of capital, net of depreciation, and the marginal product of labor. iv. The constant distribution of people is induced by the law of motion of the system (determined by the exogenous earnings shocks) and by the endogenous policy functions of the households.

A version of this model is quantified by Aiyagari [2], who adopts a yearly labor earnings following a first-order autoregressive process in logs, with an autocorrelation of 0.6 and a standard deviation of the innovations of 0.2. This results in an unconditional coefficient of variation of 0.31. These figures are based on estimates from Abowd and Card [1], who use micro-level panel data to compute their estimates. These figures are also consistent with the findings of Heaton and Lucas [53], who use data from the PSID. Aiyagari also considers a process with twice the standard deviation of the innovation for earnings, which results in an unconditional coefficient of variation of 0.63; this is a much higher variability process than typically estimated in the literature. These continuous stochastic processes are discretized into Markov Chains using quadrature methods to solve the model. Quadrini and Ríos-Rull [87] summarize the implications of this model and these two earnings parameterizations, which also correspond to different levels of cross-sectional earnings inequality.

Table 1 reports values for the wealth distribution. The first line refers to data from the 1989 Survey of Consumer Finances (SCF) and shows that, in the data, wealth is highly unevenly distributed. The Gini coefficient is 0.8 and the

	% wealth in top		
Gini	1%	5%	20%
U.S. data, 1989 SCF			
.78	29	53	80
Aiyagari Baseline			
.38	3.2	12.2	41.0
Aiyagari higher variability			
.41	4.0	15.6	44.6

Table 1: A Bewley model with infinitely-lived agents. Data from the 1989 Survey of Consumer Finances (SCF) in the top line of data and corresponding simulated models in the bottom two lines of data.

wealthiest 1% of people hold 29% of net worth, while the wealthiest 5% hold 53% of total net worth. The second line of data refers to the baseline Aiyagari calibration of an infinitely-lived Bewley model, while the last line increases earnings volatility as done by Aiyagari. Comparing these lines makes it clear that this version of the model comes nowhere near to matching neither the concentration of wealth in the hands of the richest few or the main features of the wealth distribution, including the Gini coefficient. For instance, the richest 1% of people in these versions of the model hold, at most, 4% of total net worth, compared with over one-third in the data, and the Gini coefficient generated by the model is half the one in the data.

The key force driving savings in this framework is that households wish to hold a buffer stock of assets to self-insure against earnings fluctuations. The dashed line in Figure 1 reports the saving rate[3] of workers experiencing a high earnings shock during the current period (graph from Cagetti and De Nardi [24]). These workers exhibit the buffer-stock saving behavior highlighted by Carroll [28]: if their assets are low, they save because they are experiencing a high ability level as workers and want to build up their buffer-stock. If their assets are high enough, they dissave, and the richer they are, the higher their

[3]The saving rate in the graph is defined as assets in a given period minus assets in the previous period, divided by total income during the period.

rate of dissaving. In this simulation, the asset level at which the saving rate goes from positive to negative is below $1 million. Once such a buffer stock is reached, the agents don't save any more; and for this reason, the model is not capable of explaining why rich people keep saving at very high rates.

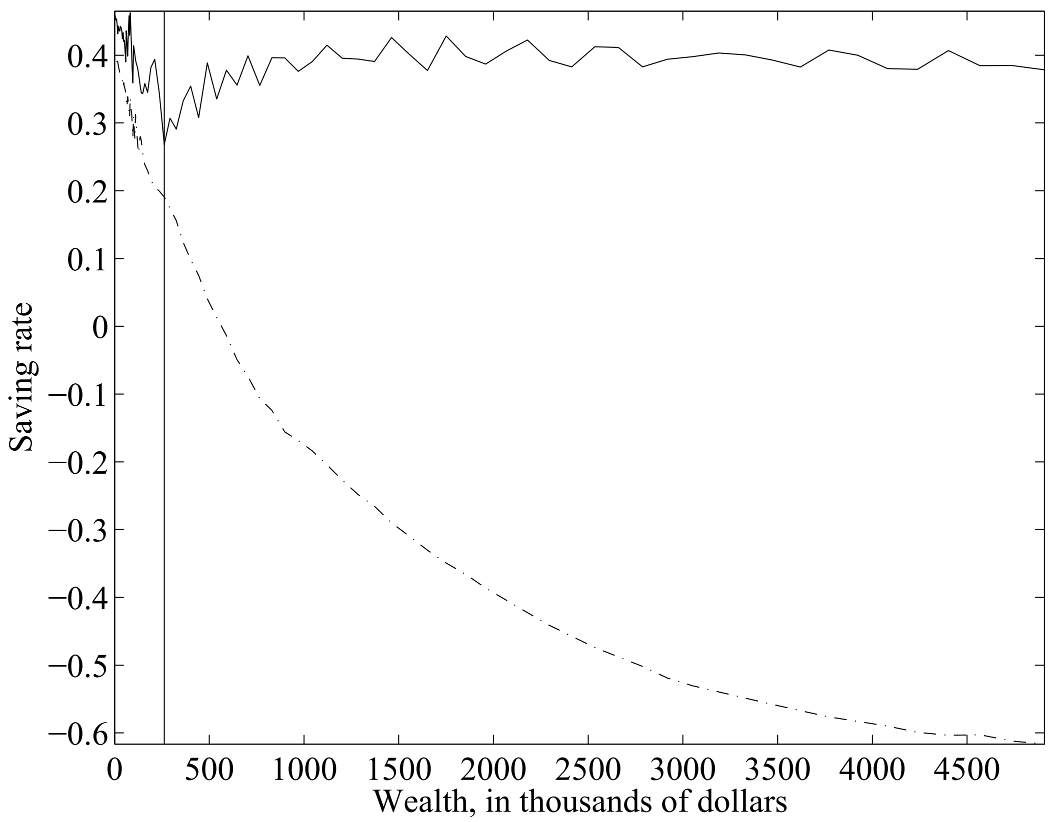

Figure 1: Saving rate for highest-ability workers. Solid line: with high entrepreneurial ability; dash-dot line: with no entrepreneurial ability; vertical line: asset level at which high-entrepreneurial-ability individuals enter entrepreneurship.

An implication of this is that what matters in generating wealth dispersion are temporary differences in earnings, not permanent ones. In addition, given that households save against idiosyncratic shocks, introducing ex-ante heterogeneity (for different skills or education levels) does not help in generating more concentration of wealth, because it does not change the nature of uncertainty that people face (see Quadrini and Ríos-Rull [87] for more details).

3.2 A basic overlapping-generations Bewley model

Huggett [59] studied wealth inequality in an overlapping-generations Bewley model, in which during each period a continuum of agents are born. These agents live at most N periods and face an age-dependent survival probability s_t of surviving up to age t, conditional on surviving up to age $t-1$. The demographic patterns are assumed to be stable, hence age t agents make up a constant fraction μ_t of the population at every point in time.

All agents value consumption as follows

$$E\left\{\sum_{t=1}^{N} \beta^t \left(\Pi_{j=1}^{t} s_t\right) u(c_t)\right\},$$

where $u(c_t)$ is the constant relative-risk aversion flow of utility from consumption, and the expected value is computed with respect to the household's earnings shocks.

The labor endowment of each household is given by a function $e(z,t)$, which depends on the agent's age t and on an idiosyncratic labor productivity shock z, which assumes a finite number of possible values and follows a first-order Markov chain with transition matrix $\Gamma(z)$.

There are no annuity markets.[4] People save to insure against earnings risk, for retirement, and in case they live a long life. People who die prematurely leave accidental bequests. Thus, compared with the previous framework, the one with infinitely-lived agents, two more saving motives are present: to smooth consumption during retirement and to self-insure against longevity risk. In principle, these additional saving motives could generate more wealth inequality and higher saving rates than the previous version of the model.

As in the infinitely-lived version of the Bewley framework, there is a constant-returns-to-scale production technology that converts aggregate capital (K) and labor (L) into output (Y).

Similarly, during each period each household chooses how much to consume

[4] This is a commonly used assumption because the annuity market is very small in practice. Eichenbaum and Peled [43] show that in the presence of moral hazard people, will choose to self-insure rather than use annuity markets, even if the rate of return on annuities is high.

(c) and save for next period by holding risk free assets (a'). At each age t, the household's state variables are (a, z), where a is asset holdings carried into the period and z is the labor shock endowment.

The household's recursive problem can be written as:

$$V(a, z, t) = \max_{(c,a')} \left\{ u(c) + \beta s_{t+1} E\Big[v(a', z', t+1) | z \Big] \right\}$$

subject to

$$c + a' = (1 + r)a + e(z, t)w + T + b_t$$

$$c \geq 0, \quad a' \geq \underline{a} \quad and \quad a' \geq 0 \quad if \quad t = N$$

where r is the interest rate net of taxes and depreciation, w is the wage net of taxes, T are accidental bequests left by all of the deceased in a period, which are assumed to be redistributed by the government to all people alive, and b_t are Social Security payments to the retirees. Modeling Social Security explicitly is important because Social Security redistributes a significant fraction of income from the young to the old and thus reduces the saving rate and changes the aggregate capital-output ratio.

At every point in time, this model economy can be described by a probability distribution of people over age t, assets a, and earnings shocks z.

A stationary equilibrium for this economy can be defined analogously to the one described for the infinitely-lived model, with the additional requirements that during each period total lump-sum transfers received by the households alive equal accidental bequests left by the deceased and the government budget constraint balances every period.

Huggett [59] calibrates this model economy to key features of the U.S. data and uses different versions of it to quantify how much wealth inequality can be generated using a pure life-cycle model with labor earnings shocks and uncertain life span.

The second line of Table 2 reports De Nardi's version of Huggett's model (this version of the results is reported for easier comparability with results coming later on, but Huggett's numbers are very similar). The first line of the table refers to the 1989 U.S. data. The second one refers to a version of Huggett's model economy in which there are only accidental bequests, which are redistributed equally to all people alive every year. The paper succeeds in matching the U.S. Gini coefficient for wealth, but the concentration is obtained by having too many people holding little wealth and by not concentrating enough wealth in the upper tail of the wealth distribution. The key reason of this failure is that in the data the rich (people with high permanent income) have a very high saving rate, while in the model households that have accumulated a sufficiently high buffer stock of assets and retirement saving don't keep saving until they reach huge levels of wealth. Thus, the additional saving motives in this version of the model (saving for retirement and for longevity risk) help bring the implications of the model a little closer to the data, but do not go far enough in that direction as they do not raise the saving rate of people by enough as they get richer.

Huggett also finds that relaxing the household's borrowing constraint increases the fraction of people bunched at zero or negative wealth, but does not increase much the asset holdings of the rich, and hence does not help in generating a distribution of wealth closer to the observed one. In addition, he documents the amount of wealth inequality generated by his model at different ages and shows that, starting from age 40, the model underpredicts the amount of wealth inequality by age.

3.3 Analytically tractable Bewley models

There are some closed-form, or analytically tractable, Bewley models. Caballero [21] studies consumption and precautionary savings with Constant Absolute Risk Aversion (CARA) preferences. Wang [97] shows that permanent-income behavior arises endogenously in a discrete-time Bewley model with CARA utility and AR(1) income processes in general equilibrium, in response to the endogenous movements of the interest rate. Wang [98] studies a continuous-time Bewley model with CARA utility and Uzawas discounting function in which the joint wealth-income distribution can be characterized in closed form. Benhabib and Bisin [9] characterize the dynamics of the distribution of wealth in an economy with intergenerational transmission of wealth and redistributive fiscal policy. They show that the stationary wealth distribution is a Pareto distribution and they study analytically the its dependence on capital income taxes, estate taxes, and welfare subsidies. Benhabib et al. [10] evaluate the dynamics of the distribution of wealth in an overlapping generation economy with finitely lived agents and intergenerational transmission of wealth. They show that in this framework the stationary wealth distribution is still a Pareto distribution in the right tail and that it is capital income risk, rather than labor income risk, that drives the properties of the right tail of the wealth distribution.

These works provide very valuable insights. However, in this survey, the focus is on richer quantitative models that cannot be solved analytically.

3.4 What we learn from basic quantitative Bewley models

The results in the two previous subsections show that basic Bewley models, whether featuring infinitely-lived agents or life-cycle agents with more realistic patterns of earnings and savings over the life cycle, are far from doing a good job of matching the observed distribution of wealth. In particular, as they match aggregate wealth held in the economy, they do so by generating rich people who are not nearly rich enough, middle-class people who are too rich, and poor people who are too poor, compared with the actual data.[5]

4 Heterogenous preferences

There is enough micro-level empirical evidence of preference heterogeneity (see for example, Lawrance [75] and Cagetti [23]) to suggest that preference heterogenity might be a plausible avenue to help explain the vastly different amounts of wealth held by people.

[5]Incomplete-market models can be applied to study many interesting and important questions that go beyond wealth inequality and thus the scope of this survey. See Quadrini and Victor Ríos-Rull [88], Krusell and Smith [72], Guvenen [49], and Heathcote et al. [52] for surveys on this.

Krusell and Smith [71] generalize the infinitely-lived version of the Bewley model by adding a stochastic process for each dynasty's discount factor and risk aversion. The discount factor (or the risk aversion) changes on average every generation and is meant to recover the fact that parents and children in the same dynasty may have different preferences. They show that it is possible to find a stochastic process for the dynasties' discount factor to match the variance of the cross-sectional distribution of wealth. They also find that heterogeneity in risk aversion does not affect the results much (however, Cagetti [22] shows that this result is sensitive to chosen utility parameter values). Instead, a small discrepancy between the possible realizations of the discount factors can generate a more disperse wealth distribution. However, while capturing the variance of the wealth distribution, their model and calibration fail to match the extreme degree of concentration of wealth in the hands of the richest.

Hendricks [57] studies the effects of preference heterogeneity in a life-cycle framework with only accidental bequests. As Krusell and Smith, he also finds that heterogeneity in risk aversion has very limited effects on saving and wealth inequality. Moreover, he shows that time preference heterogeneity makes for a modest contribution in accounting for high wealth concentration if the heterogeneity in discount factors is chosen to generate realistic patterns of consumption and wealth inequality as cohorts age.

Also in the spirit of preference heterogeneity, Heer [55] adopts a model in which richer and poorer people have different tastes for leaving bequests, while Laitner [73] assumes that all households save for life-cycle purposes but that only some of them care about their children. Laitner allows for perfect annuity markets, therefore all bequests are voluntary, and there is no earning risk over the life cycle, hence no precautionary savings. In addition, he matches the concentration in the upper tail of the wealth distribution by choosing the fraction of households that behave as a dynasty and the distribution of wealth within the dynasty (which is indeterminate in the model).

More in the spirit of experimenting with preferences, rather than of preference heterogeneity, Díaz, Pijoan-Mas, and Ríos-Rull [34] study the effect of habit formation and find that it actually decreases the concentration of wealth generated by this type of model. In fact, habits act similarly to increased risk aversion, and more risk aversion tends to increase the saving of everyone and to dampen wealth dispersion. Carroll [29], instead, suggests a "capitalist spirit" model, in which finitely-lived consumers have wealth in the utility function, which can be calibrated to make wealth a luxury good, thus generating nonhomothetic preferences.

In sum, previous work indicates that preference heterogeneity, and especially patience heterogenity, can generate increased wealth dispersion. It would be interesting to deepen the previous analysis by both studying richer processes for patience and allowing for richer formulations of the utility function in which, for instance, risk aversion and intertemporal substitution do not have to coincide (see Wang et al [96] for some interesting findings on this).

5 Transmission of human capital and voluntary bequests

Kotlikoff and Summers [69] argue that intergenerational transmission of wealth, as opposed to life-cycle savings, accounts for the majority of aggregate capital formation. Further studies have found that intergenerational transfers account for at least 50-60% of total wealth accumulation (Gale and Scholz [45]). Given that intergenerational transfers are so large in the aggregate, they might also play an important role in shaping wealth inequality.

Becker and Tomes [8] were the first to model the parental decision problem and to characterize the transmission of both human capital and bequests across generations. They showed that in the presence of borrowing constraints, parental transfers first come in the form of children's human capital investment; and that only after the optimal amount of human capital investment in children has been achieved, parents start giving monetary transfers, such as bequests. Bequests are thus a luxury good in this framework.

De Nardi [36] introduces two types of intergenerational links in the OLG model used by Huggett: voluntary bequests and transmission of human capital. She models the utility from bequests as providing a "warm glow" (as in Andreoni [5]). In this framework, parents and their children are linked by voluntary and accidental bequests and by the transmission of earnings ability. The households thus save to self-insure against labor earnings shocks and life-span risk, for retirement, and possibly to leave bequests to their children.

In De Nardi's model, therefore, voluntary and accidental bequests coexist and their relative size and importance are determined by the calibration. Empirically measuring the size of voluntary bequests relative to that of purely accidental ones (due to uncertainty about the life-span) is challenging. Hurd [60] estimates a very low marginal utility from leaving bequests. Altonji and Villanueva [4] also find relatively small values for the elasticity of bequests to permanent income, although they do show that this number increases with life-time resources. Most of the bequests, however, are concentrated among the top percentiles, a group that these papers ignore. Looking at a sample of wealthier retirees, Laitner and Juster [74] find that about half of the households in their sample plan to leave estates and that the amount of wealth attributable to estate building is significant, accounting for half or more of the total for those who plan to leave bequests.

Compared with Huggett, the voluntary bequest motive introduces an extra term in the value function of a retired person who faces a positive probability of death:

$$V(a,t) = \max_{c,a'} \Big\{ u(c) + s_t \beta E_t V(a', t+1) + (1 - s_t)\phi(b(a')) \Big\}, \tag{1}$$

where

$$\phi(b(a')) = \phi_1 \Big(1 + \frac{b(a')}{\phi_2} \Big)^{1-\sigma}, \tag{2}$$

where $b(a')$ are estates net of estate taxes, as a function of end of period net worth. The utility from leaving bequests hence depends on two parameters: ϕ_1, which represents the strength of the bequest motive, and ϕ_2, which measures the extent to which bequests are a luxury good because it affects the marginal utility of bequests in a nonlinear way (see De Nardi [36] for more discussion on this). These two parameters are respectively calibrated to the data on the fraction of capital due to intergenerational transfers and to match one moment of the observed distribution of bequests, which is that over 30% of singles leave estates of little or no value. This calibration implies that bequests are a luxury good.

It should be noted many papers that do not find evidence in favor of a bequest motive, such as Hurd [60] and Hendricks [56], assume that utility is homotetic in bequests ($\phi_2 = 0$), thus generating the counterfactual implication that even very poor people save to leave bequests of significant size. In contrast, De Nardi's more flexible functional form and parameterization imply a realistic distribution of estates. Her calibration is also quantitatively consistent with the elasticity of the savings of the elderly to permanent income that has been estimated from microeconomic data by Altonji and Villanueva [4].

Transfer wealth ratio	Wealth Gini	Percentage wealth in the top 1%	5%	20%	40%	60%	Percentage with negative or zero wealth
1989 U.S. data							
.60	.78	29	53	80	93	98	5.8–15.0
No intergenerational links, equal bequests to all							
.67	.67	7	27	69	90	98	17
No intergenerational links, unequal bequests to children							
.38	.68	7	27	69	91	99	17
One link: parent's bequest motive							
.55	.74	14	37	76	95	100	19
Both links: parent's bequest motive and productivity inheritance							
.60	.76	18	42	79	95	100	19

Table 2: OLG models of wealth inequality, from De Nardi [36]

Table 2 summarizes De Nardi's results. The first line of the table refers to the 1989 SCF U.S. data. The second line refers to the version of Huggett's model economy in which there are only accidental bequests, which are redistributed equally to all people alive every year, as described earlier.

The third line refers to an economy in which there are only accidental bequests, but the accidental bequests are received by the children of the deceased only once, upon their parent's death; and are thus unequally distributed and imply a realistic age of bequest recipience (rather than every period). This experiment shows that accidental bequests, even if unequally distributed, do not generate a more unequal distribution. This is because receipt of a bequest per se does not alter the saving behavior of the richest.

The comparison of lines two and three also highlights the fact that Kotlikoff and Summers' [69] measure on intergenerational transfers is sensitive to the timing of transfers because of the way that transfers are capitalized and accumulated interest is accrued to bequests. If children inherit only once, when their parent dies (rather than every year), then the fraction of wealth attributed to intergenerational transfers in the model is much lower than the one in the data.

The fourth line allows for a voluntary bequest motive and shows that voluntary bequests can explain the emergence of large estates, which are often accumulated in more than one generation and characterize the upper tail of the wealth distribution in the data. The bequest motive to save is much stronger for the richest households, who, even when very old, keep some assets to leave to their children. The rich leave more wealth to their offspring, who, in turn, tend to do the same. This behavior generates some large estates that are transmitted across generations because of the voluntary bequests

The fifth line allows for both voluntary bequests and transmission of ability and shows that a human-capital link through which children partially inherit the productivity of their parents generates an even more concentrated wealth distribution. More productive parents accumulate larger estates and leave larger bequests to their children, who, in turn, are more productive than average in the workplace.

As shown in Figure 2, the presence of a bequest motive also generates lifetime saving profiles that imply slower wealth decumulation in old age for richer people, consistent with the facts documented by De Nardi et al. [38], using micro-level data from the Health and Retirement Survey. De Nardi et al. [38] suggest that medical expenses are another important mechanism that can generate this kind of slow decumulation.

In a model with intergenerational links that abstracts from medical expenses risk, saving for precautionary purposes and saving for retirement are the primary factors for wealth accumulation at the lower tail of the distribution, while saving to leave bequests significantly affects the shape of the upper tail.

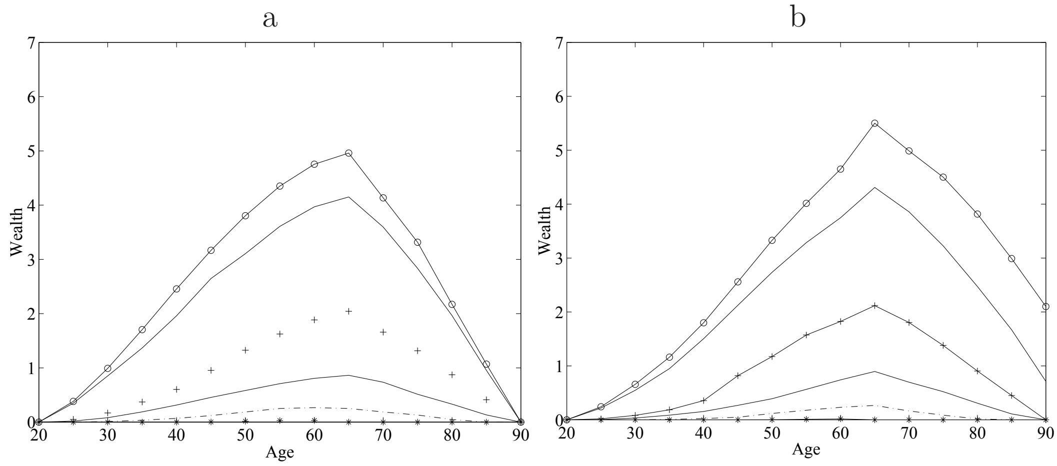

Figure 2: Wealth .1, .3, .5, .7, .9, .95 quantiles. No links, equal bequests to all, panel a, and Bequest motive, panel b .

This approach, however, takes the transmission of human capital, or productivity, as exogenous. There is a vast literature studying this channel. For instance, Ayagari et al. [3] study optimal parental investment of time and money in children, both with perfect and imperfect altruism. Brown et al. [17] develop a model in which parents and children make investments in the children's education. They show that for an identifiable set of parent-child pairs, parents will rationally under-invest in their child's education. For these parent-child pairs, additional financial aid will increase educational attainment. Their evidence thus further points to the importance of modeling this channel to better understand wealth inequality. Lee and Seshadri [77] study the importance of parental investment on the intergenerational transmission of economic status, while Lee et al. [78] attempts at identifying the causal effect of parental human capital on children's human capital. For surveys about the importance of parental background, see Heckman and Mosso [54] and Bowles et al. [15].

This approach also assumes that fertility is exogenous and that everyone has the same number of children. Scholz and Seshadri [94] examine the effects of children in a life-cycle model with endogenous fertility. They argue that children have a large effect on household's net worth and consequently are an important factor in understanding the wealth distribution. They also find that fertility and credit constraints interact in ways that significantly affect wealth accumulation.

There is also large wealth inequality within various age and demographic groups. Venti and Wise [95] and Bernheim at al. [11] show that wealth is highly dispersed at retirement even for people with similar lifetime incomes and argue that these differences cannot be explained only by events such as family status, health, and inheritances, nor by portfolio choice. Hendricks [56] focuses on the performance of a basic OLG model on cross-sectional wealth inequality at retirement age. He shows that, at retirement age, a basic version of the OLG model overstates wealth differences between earnings-rich and earnings-poor, while it understates the amount of wealth inequality conditional on similar lifetime earnings. De Nardi and Yang [39] show that the OLG model augmented with voluntary bequests and intergenerational transmission of earnings also matches the observed cross-sectional differences in wealth at retirement and their correlation with lifetime incomes quite well.

Gokhale et al. [48] aim to evaluate how much wealth inequality at retirement age arises from inheritance inequality. To do so, they construct an overlapping-generations model that allows for random death, random fertility, assortative mating, heterogeneous human capital, progressive income taxation, and Social Security. All of these elements are exogenous and calibrated to the data. The families are assumed not to care about their offspring, hence all bequests are involuntary. To solve the model, they impose that individuals are infinitely risk averse and that the rate of time preference equals the interest rate. In their framework, inheritances in the presence of Social Security play an important role in generating intra-generational wealth inequality at retirement. The intuition is that Social Security annuitizes completely the savings of poor and middle-income people but is a very small fraction of the wealth of richer people, who thus keep assets to insure against life-span risk.

Nishiyama [81] adopts an OLG model with bequests and intervivos transfers in which households in the same family line behave strategically. Like De Nardi, he concludes that the model with intergenerational transfers better explains the observed wealth distribution, although it does not fully match it.

Thus, although modeling explicitly intergenerational links helps explain the the savings of the richest, the models by De Nardi and Nishiyama are not capable of matching the wealth concentration of the richest 1% without adding complementary forces generating a high wealth concentration for the rich. De Nardi and Yang [40] merge a version of the model with intergenerational links with the high earnings risk for the top earners mechanism proposed by Castañeda et al. [30] that we discuss in Section 7 and find that these two forces together match important features of the data well. More work is warranted to evaluate the role of intergenerational links in conjunction with other complementary explanations, including preference heterogeneity and potentially out-of-pocket medical expenses after retirement.

6 Entrepreneurship

Quadrini [86] provides a nice survey on the factors affecting the decision to become an entrepreneur and the aggregate and distributional implications of entrepreneurship for savings and investment. In addition, Quadrini [85], Gentry and Hubbard [46], De Nardi et al.[37] and Buera [19] argue that entrepreneurship is a key element in understanding wealth concentration among the richest households.

Cagetti and De Nardi [24] classify as *entrepreneurs* the households who declare being self-employed, owning a privately held business (or a share of one), and having an active management role in it. According to this definition, which is consistent with the one in the model that they use, entrepreneurs constitute a small fraction of the population (about 10%), but hold a large share of total net worth (about 40%). They show that entrepreneurs make

Top %	1	5	10	20
Whole population				
percentage of total net worth held	30	54	67	81
Entrepreneurs				
percentage of households in a given percentile	63	49	39	28
percentage of net worth held in a given percentile	68	58	53	47

Table 3: From Cagetti and De Nardi [24]. Entrepreneurs and the distribution of wealth. SCF 1989.

up for a large fraction of rich people in the data. Table 3, from their paper, shows that, not only total net worth held by the richest percentiles, but also the percentage of entrepreneurs in a given wealth percentile (line two) and the percentage of wealth within that percentile that is owned by entrepreneurs (line three) are all very high. For example, among the richest 1% of people in terms of net worth, 63% are entrepreneurs and they hold 68% of the total wealth held by the wealthiest 1% of people. They also show that alternative classifications of entrepreneurship give similar results.

Cagetti and De Nardi [24] build a model of entrepreneurship with the following key elements:

1. Altruistic agents care about their children.

2. Every period, agents decide whether to run a business or work for a wage.

3. The entrepreneurial production function is given by

$$f(k) = \theta k^{\nu} + (1 - \delta)k,$$

   where k is working capital, θ is entrepreneurial ability, ν is the degree of decreasing returns to scale, and δ is depreciation. Cagetti and De Nardi [26] generalize the entrepreneurial production function to labor hiring.

Wealth Gini	Fraction of entrepreneurs	Percentage wealth in the top 1%	5%	20%	40%
Baseline model with entrepreneurs					
0.8	7.50%	31	60	83	94

Table 4: Cagetti and De Nardi [24] model's implications.

4. Borrowing constraints imply that

$$k = a + b(a),$$

where a is one's assets and $b(a)$ is borrowing as a function of one's assets.

In the formulation adopted in Cagetti and De Nardi [24], $b(a)$ is actually a function of all of the state variables in the economy and this outcome arises endogenously from the assumption that contracts are imperfectly enforceable and lenders take the imperfect enforceability of constracts into account when deciding how much to lend (as in Cooley et al. [31] and Kehoe and Levine [64]). Besides being more micro-funded, these kind of borrowing constraints also have the advantage of endogenously responding to economic conditions such as changing wages and interest rates (see Bassetto et al. [7] for an illustration and a discussion of this mechanism applied to the Great Recession). However, simpler kinds of borrowing constraints, such as linear functions of one's assets, make for models that are easier and faster to solve, and generate similar implications for cross-sectional wealth inequality at one point in time. For an application of the classic case in which borrowing is a linear function of one's assets in a model with wealth inequality and entrepreneurship, see Kitao [67].

In Cagetti and De Nardi [24]'s calibration, the optimal firm size is large, one's firm is very productive, and the entrepreneur is borrowing constrained. Thus, entrepreneurs, even when rich, want to keep saving to grow their firm to be able to borrow more and reap higher returns from capital. This is the mechanism that, in this framework, keeps the rich people's saving rate high and generates a high wealth concentration.

In order to compare buffer-stock saving behavior with entrepreneurial saving behavior, Figure 1 compares the the saving rates[6] for people who have the highest ability level as workers during the current period. The solid line refers to the people who get the high entrepreneurial ability level during the current period, while the dash-dot line refers to those who get the low entrepreneurial ability draw. Given the same asset level (and potential earnings as workers), the people with high entrepreneurial ability have a much higher saving rate. As we discussed, the worker with no entrepreneurial ability displays pure buffer-stock saving behavior.

The people with high entrepreneurial ability become entrepreneurs only if their wealth is above a certain level, denoted in the graph by a vertical line. The saving rate of those with high entrepreneurial ability who do not own enough assets to become entrepreneurs is higher than the one for the workers because ability is persistent, and the workers with high entrepreneurial ability save to have a chance to start a business in the future. In this region, the distance between the solid line and the dash-dot line is solely due to the higher implicit rate of return from saving that one could obtain becoming an entrepreneur in the future: all households become workers in this range and earn the same income, but the desire to become entrepreneurs generates a higher saving rate for those who have such ability.

The saving rate of those with high entrepreneurial ability and enough assets to become entrepreneurs is positive and considerably higher than that for workers. The return on the entrepreneurial activity is high, and the entrepreneur would like to increase the size of the firm by borrowing capital. However, the borrowing constraint limits the size of the firm. In order to expand the business, the entrepreneur must in part self-finance the increase in capital. The combination of higher returns from the business together with the budget constraint thus generates a very high saving rate for entrepreneurs. As the firm expands, the returns decrease. Therefore, the saving rate will also eventually decrease. (We truncate the axis of the graph for easier readability.)

[6] The saving rate in the graph is defined as assets in a given period minus assets in the previous period, divided by total income during the period.

Table 4 shows the high wealth concentration generated by this model. A few things are worth mentioning. First, the distribution of wealth is not matched by construction in the calibration procedure. Second, the model's implied returns to capital are in the range of those found by Moskowitz and Vissing-Jørgensen [80] and Kartashova [63], and hence they are not implausibly high. Third, the model generates entry probabilities as a function of one's wealth that are consistent with those estimated by Hurst and Lusardi [62] on micro-level data.

It should be noted that in Cagetti and De Nardi's parameterization, there is only one level of entrepreneurial ability; and all heterogeneity in firm size and asset holdings is due to the interaction between the borrowing constraints and the stochastic evolution of entrepreneurial and working ability, which make firms grow slowly over time. While this makes the calibration very parsimonious and matches many features of the data well, it is clear that there is a lot more heterogeneity in entrepreneurs and self-employed in the data. Kitao [67] allows for multiple entrepreneurial ability levels, but does not discuss the impact of this generalization.

Campbell and De Nardi [27] find, for instance, that aspirations about the size of the firm that one would like to run are different for men and women, and that many people who are trying to start a business also work for an employer and thus work very long hours in total. It would be interesting to generalize the model to allow, for instance, for heterogeneity in entrepreneurial total factor productivity and optimal firm size (or decreasing-returns-to-scale parameters), and to convincingly take the model to data to estimate those additional parameters. Given the data on time allocation, it would also be interesting to think more about the time allocation decision between working for an employer, starting and running one's firm, working on home production, and enjoying leisure.

Finally, Glover and Short [47] study the interplay between entrepreneurial risks and the decisions to incorporate and to go bankrupt and find that capital shocks constitute important entrepreneurial risks, which generate high welfare costs. These features are important and deserve more investigation.

Earnings level	1.0	3.0	10.0	1060
Fraction at invariant distribution	61.11%	22.25%	16.50%	0.04%

Table 5: Castañeda et al. [30] earnings' process.

7 Large earnings risk for the top earners

Castañeda et al. [30] consider a model economy with two stages of life, working time and retirement time, in which workers have a constant probability of retiring at each period, and retirees face a constant probability of dying. Each household is perfectly altruistic toward its descendants. The paper employs a number of parameters to match some features of the U.S. data, including measures of both earnings and wealth inequality.

The key feature of the model that generates a large amount of wealth holdings in the hands of the richest is the productivity shock process, whose key features are reported in Table 5. This process is thus calibrated so that the highest productivity level is more than 100 times higher than the second highest. Thus, there is a large discrepancy between the highest productivity level and all of the others. Moreover, if one is at the highest productivity level, the chance of being 100 times less productive during the next period is more than 20%.

High-earning households thus face much higher earnings risk and save at very high rates to self-insure against earnings risk and smooth consumption and thus build huge buffer stocks of assets.

This finding underscores the role of the earnings risk faced by the households in shaping saving behavior. It should be noted, however, that in this framework, earnings risk is completely independent from the size of one's wealth and business capital.

In the data, DeBacker et al. [33] use a confidential panel of US income tax returns for 1987-2009 to measure business income risks. They document that, compared with labor income, business income is much riskier (even conditional staying in business), is less persistent over time, and is characterized by higher probabilities of extreme upward or downward mobility. They also show that high-income entrepreneurs are more likely to face the tail risks on both ends of the business income distribution. These features of the data are thus generally consistent with the idea that high earners are subject to larger fluctuations. However, from the standpoint of the model and its implied savings decisions, the question of how this risk is related to one's investment in capital is very important. In the data, it is not completely clear how the very risky business income of business owners found by DeBacker et al. informs the calibration of this earnings process that is assumed to be exogenous to any business investment decision.

More empirical support for this modeling assumption and calibration is provided by Parker and Vissing-Jorgensen [83], who find that incomes at the top are cyclical because of the labor component and bonuses in particular. Although for business owners the split between their wages and capital income might be somewhat flexible, these authors write "High-income households (top 1 percent) earn more than half of their noncapital gains income from wage income, and their wage income is far more exposed to aggregate fluctuations than that of lower-income households...we find even higher income exposure to aggregate fluctuations for very high-income households (top 0.01 percent) than for high-income households... ".

Interestingly, Barnett and Panousi [6] also uncover that the risks taken by business people are heteroskedastic. High-wealth agents are more likely than low-wealth agents to have big business income fluctuations (both big increases and big declines). In contrast, these "risks" do not vary along other dimensions, such as gender, level of education, and race. These findings provide support for the observation that wealth influences investment. This outcome is consistent with the existence of borrowing constraints (Cagetti and De Nardi [24]) and entrepreneurial risk aversion (a channel proposed by Panousi [82]).

From a theoretical standpoint, the "economics of superstars" by Sherwin Rosen [90] rationalizes the emergence of a small number of highly compensated individuals and a highly skewed distributions of earnings and very large rewards at the top. Gabaix and Landier [44] propose a model to rationalize increased CEO compensation. Lee [76] develops a model occupational choice for workers, entrepreneurs, and managers, that endogenously generates high managerial wages.

8 Conclusions: Lessons learned and directions for future research

Basic versions of the Bewley model in which households face earnings shocks that are assumed to be an AR(1) estimated from micro-level data sets miss key aspects of saving behavior and, in particular, the saving behavior of the rich. Previous work has shown that there are realistic mechanisms that help bring the savings implications of the model more in line with the data. These mechanisms include heterogeneity in preferences, transmission of human capital and voluntary bequests across generations, entrepreneurship or high returns to capital coupled with borrowing constraints, and high earnings risk for the top earners.

Disturbingly, even if these mechanisms give rise to similar observed wealth concentrations, they can have vastly different policy implications. For instance, modeling entrepreneurship usually implies that the adverse responses of savings and economic activity to increased taxation are significant, and especially so if taxation affects the returns to running a business (Kitao [67], Lee [76], and Cagetti and De Nardi [26]). In contrast, in a model with high earnings risk for the top earners, Kindermann and Krueger [66] conclude that the optimal marginal income tax rate is close to 90%. Thus, more work needs to be done to more conclusively determine the effects of taxation in quantitative models of wealth inequality.

The big difference in responses to taxation between these models is due to the fact that entrepreneurs' savings and investments are responsive to their implicit rate of return, net of taxes. In contrast, the very high earner facing a large probability of becoming a very low earner next period is desperate to save to smooth consumption. Hence, even if their currently high earnings become lower due to taxation, as long as potential net income tomorrow is sufficiently low compared with today's net earnings, the household will save at a high rate, even if earnings taxation is increased.

This stark contrast in policy implications stemming from different motivations to save points to the importance of understanding whether, for instance, the risk that the rich face comes from their earnings or from the capital that they invested in the firm. A combination of better data and empirical analysis and richer models that allow both for high earnings risk for the top earners and for risky entrepreneurial capital should help answering these questions.

Similarly, one might think that if households' voluntary bequest motives are an important reason why rich households keep saving, the specific bequest formulation might be quite important in determining the response to taxation. Interestingly, De Nardi and Yang [40] find that whether warm-glow bequests of the type that we have discussed in this paper depend on estates net or gross of taxes, does not generate very different responses to estate taxation reform as long as the models are calibrated to match the same facts.

Generally, more work is needed to evaluate these explanations, both individually and jointly, and to quantitatively assess their importance. Additionally, more exploration of alternative or complementary mechanisms is needed. For instance, there is evidence that a discretized AR(1) is likely a poor approximation of true income dynamics. In fact, Guvenen et al. [50] document that, in a given year, most individuals experience very small earnings shocks, and a small but non-negligible number experience very large shocks. They also find that statistical properties of earnings shocks vary significantly both over the life cycle and with the earnings level of individuals. Finally, they find important asymmetries: positive shocks to high-income individuals are quite transitory, whereas negative shocks are very persistent; the opposite is true for low-income individuals. DeBacker et al. [33] study business income and their findings are broadly consistent with those in Guvenen et al. [50], but they also find that business income is much riskier than labor income. In addition, DeBacker et al. [33] find that the persistence of business income is very similar over one-year and over five-year periods (five year results available from the authors). All of these findings indicate that modeling earnings with a richer process than an AR(1) is needed.

Importantly, it should also be noted that the properties found by Guvenen et al. [50], while very interesting, come from individual-level earnings, as opposed to household-level earnings or income. Given that many households have dual earners, it would also be interesting to contrast and compare these findings to those on household level data. For instance, Blundell at al. [14] highlight the importance of of family labor supply as an insurance mechanism to wage shocks and find strong evidence of smoothing of males and females permanent shocks to wages.

Finally, the crucial importance of the nature of idiosyncratic risk assumed in these models also raises the question of its measurement in the data. What we, as economists, measure as a shock in the data, might, to a large extent, be anticipated by the households and thus have a very different effect than shocks in these models. This might be especially the case in in administrative data sets that contain little information about the household (in contrast with survey data, which instead, might contain information on households' health, divorce, and expectations). Sabelhaus and Ackerman [91] use SCF data to derive the gap between actual and normal income from survey questions and use it as a measure of shocks. This approach stands in contrast to existing income shock measures in the literature, which are generally derived from the residuals of estimated earnings or income equations. Interestingly, the overall variance and asymmetry of shocks over the business cycle derived from this analysis are similar to those of existing residual-based estimates. Blundell et al. [13] use data on both consumption and income to draw inference on the persistence of income shocks. More work in needed to better disentangle the actual shocks that the households face and their sources.

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