NBER WORKING PAPER SERIES

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Working Paper 21064
http://www.nber.org/papers/w21064

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
April 2015

We are grateful to Martin Boileau, Andrea Buffa, Joao Gomes, Dirk Hackbart, Xiaoji Lin, Lars Lochstoer, Sydney Ludvigson, and Stijn Van Nieuwerburgh, as well as seminar attendants at NHH Bergen, Boston University, Federal Reserve Bank of St. Louis, NYU, University of Southern California, Duke-UNC Asset Pricing Conference, CIREQ Workshop, Midwest Macro Meeting, EUI Macro Conference, NY/Philly Workshop on Quantitative Macroeconomics, and meetings of American Finance Association, European Finance Association, Canadian Macro Study Group, and Society for Economic Dynamics for their comments and insights. Marco Casiraghi provided excellent research assistance. All remaining errors are our own responsibility. The views expressed herein are those of the authors and do not necessarily reflect the views of the National Bureau of Economic Research.

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NBER Working Paper No. 21064
April 2015
JEL No. D24,D92,G12

ABSTRACT

When the neoclassical model of investment is serious about investment, it fails to replicate elementary cross-sectional features of equity returns. Without leverage, the model produces a value discount - i.e. value firms earn lower returns than growth firms. With large enough operating leverage, a value premium emerges, but its magnitude is always smaller than the size premium's. Furthermore, when parameters are set to match key moments of the cross--sectional distribution of investment and the average book-to-market ratio, the value premium is minuscule -- about one order of magnitude smaller than found in the data. This result holds true for different specifications of the stochastic discount factor and does not depend upon the magnitude of capital adjustment costs

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1 Introduction

Starting with the path–breaking contribution by Berk, Green, and Naik (1999), finance scholars have been busy investigating whether optimizing models of firm–level investment can rationalize the empirical evidence on the cross–sectional variation of equity returns. In particular, we refer to the systematic relation between stock returns and firm characteristics such as market value (size), book–to–market, and investment rate.[1]

As a testament to the success of this research program, we now have a number of models which, under certain conditions, are consistent with the patterns we alluded to above. Our contention is that those conditions, in turn, impose restrictions on investment and firm dynamics that are easily rejected by the data. Does that matters for equity returns? Our answer is yes.

We reach this conclusion by studying the cross–sectional implications for asset returns of a model of investment – a simple variant of the neoclassical framework – which gained popularity in macroeconomics and industrial organization exactly because of its success in yielding data–conforming stochastic processes for firm–level investment and size dynamics.

Several key conclusions reached by the extant literature do not hold true in our framework. We find that, counterfactually, the value premium is always bounded above by the size premium. Furthermore, when calibrated to match key moments of the cross–sectional distribution of investment and the average book-to-market ratio, the model produces a value premium which is much smaller than found in the data. These results are robust to different assumptions on both the stochastic discount factors and the magnitude of capital adjustment costs.

Firms produce by means of a decreasing returns to scale production function and are subject to capital adjustment costs. Their productivity depends upon a common and an idiosyncratic component – mean–reverting and orthogonal to each other. Future cash-flows are discounted by means of an exogenous time–varying stochastic discount factor.

As long as assets in place – i.e. installed capital and next period's operating cash flows – are less risky than the whole firm, small stocks earn higher expected returns than large stocks. That is, the model produces a size premium.

The intuition is straightforward. Small firms tend to have low levels of capital and idiosyncratic productivity. Since the process driving the latter is mean–reverting, such firms owe a smaller fraction of their value to assets in place and are therefore riskier.

The investment rate is negatively associated with subsequent stock returns. High–

[1] See Fama and French (1992), Anderson and Garcia-Feijoo (2006), and Xing (2008) among others.

investment firms add to their capital to catch up with a recently risen idiosyncratic productivity. On the one hand, the improvement in efficiency lowers risk, as it raises the loading on assets in place. On the other hand, since investment is positively autocorrelated, the expectation of continued investment tends to increase risk. The former effect always dominates.

The plain–vanilla version of the model – one without operating leverage – delivers a value discount. This is the case because size and book–to–market ratio are counterfactually positively correlated in stationary distribution. Growth firms – i.e. firms with low book-to-market ratio – are characterized by low idiosyncratic productivity, low marginal q, and low investment rate. As a result, they command higher returns than value firms.

Introducing an operating cost, which in our formulation is constant across all firms, has two distinct effects. First, it allows the cross-sectional correlation between size and book–to–market to turn negative – a necessary condition for the value premium to arise. Second, it affects stocks' expected returns by changing the riskiness of cash flows.

With a large enough fixed operating cost, the distribution of book–to–market over the state space is consistent with a value premium. Growth firms have seen their idiosyncratic efficiency rise and are investing to reach the new efficient size. Value firms, on the contrary, are shrinking to adjust their capacity to a diminished idiosyncratic efficiency.

In common with small firms, value firms have low idiosyncratic productivity. This makes them risky. Unlike small firms, however, they have plenty of capital in place, and are expected to pay out dividends as they reduce the size of their operations to levels consistent with their current efficiency. Such payouts lower their risk. It follows that value stocks earn a lower average return than small stocks. By the same token, growth firms earn a higher return than large firms. The value premium is always smaller than the size premium.

We compute the cross-sectional distribution of returns that obtains when parameters are chosen to match the mean, standard deviation, and autocorrelation of the firm–level investment rate. The stochastic discount factor is calibrated to yield a counter–cyclical risk–free rate as well as data-conforming values for the average Sharpe ratio and for the first two moments of the risk–free rate itself.

The upshot is that the value premium is substantially smaller than in the data and is largely the result of counter–cyclical variation in the risk–free rate. By means of a comparative statics exercise, we also show that increasing capital adjustment costs has the effect of reducing the value premium further.

Our work is motivated by the large literature that attempts to rationalize the evidence on the cross-section of returns through the lens of an investment model. Indeed, we start out from the realization that most extant models are not consistent with key features of the investment process. Our main contribution is to show that carefully modeling investment behavior severely impairs the model's ability to produce cross-sectional patterns of stock returns consistent with the evidence.

In the fundamental contributions by Berk, Green, and Naik (1999) and Gomes, Kogan, and Zhang (2003), firms are collections of projects, or business lines, heterogeneous in their productivity and in their systematic risk. Necessary conditions for the value premium to arise are that firms cannot reallocate inputs across their business lines and that investment costs are sunk. Value firms are entities that, having lost their high–productivity, low–risk business lines out of bad luck, are left with low–productivity, high–risk projects. Their book–to–market ratio is high because there is no cross–project correlation between book and market values. Their risk is high because the disposition of assets provides no cash considerations in return.

In Carlson, Fisher, and Giammarino (2004), Cooper (2006), and Gala (2006), the value premium results from the assumption of investment irreversibility, in scenarios in which fixed operating cost are monotone increasing in the size of installed capital. Value firms are riskier, because they are more levered. Their book value and overhead expenses grew large, because rising productivity in the past led to substantial investment. Their market value, however, reflects the current – lower – level of efficiency and operating cash–flows.

The data does not support the assumptions that are necessary to generate a value premium in either strand of literature. The evidence against investment irreversibility and the sunk nature of investment costs is overwhelming.

In our own dataset of publicly traded firms, on average about 15 percent of firms reduce their capital stock every year. Eisfeldt and Rampini (2006) find that every year the average firm in their dataset sells physical assets for 9.16 million in 1996 dollars, or about 10% of capital expenditures. According to Eckbo and Thorburn (2008), in 2006 alone U.S. corporation announced 3,375 divestitures – sales of a portion of firm's assets to a third party – for a total value of $342 billion.[2] In return for their assets, sellers typically received cash, but sometimes also securities or a combination of both.

In our framework, we rule out any form of irreversibility or sunkness. When hit by low productivity shocks, large firms relinquish capital in exchange for cash. This is consistent

[2] Eckbo and Thorburn (2008) report that the number of transaction was relatively stable between 1980 and 2005, but grew at a fast pace until the start of the Great Recession.

with the findings of Maksimovic and Phillips (2001), who show that asset sellers have below average productivity, and those of Schoar (2002), who reports that firms dispose of plants whose productivity is declining and lower than average.

Our framework is closest to Zhang (2005), in that we assume decreasing returns to scale, mean–reverting shocks, and capital adjustment costs. Our novel results are as follows.

First, we show that in the absence of operating leverage, the model produces a value discount. The reason has nothing to do with the direct effect of leverage on cash flow risk. Rather, it depends on the association between market value and book–to–market, which is counter–factually positive in that scenario. Second, we show that the value premium is always bounded above by the size premium. Third: Differently from Zhang (2005), we calibrate the model to match key moments of the investment process. Under those parametric assumptions, the model cannot generate a value premium of the magnitude evidenced in the data. Last, but not least, we argue that the magnitude and degree of asymmetry of adjustment costs are not key determinants of the model's quantitative performance.

The role played by the option to divest in shaping equity risk is also outlined by Guthrie (2011) in an illustrative example and by Hackbarth and Johnson (2014) in the context of an elegant production-based model. Hackbarth and Johnson (2014) make strong assumptions – among which linear homogeneity of the profit function and unit root in idiosyncratic productivity – for the purpose of obtaining a sharp characterization of equity risk as it relates to Tobin's q, the only state variable in their model. The downside is that the model is not easily amenable to a quantitative analysis. In particular, the implied stochastic process for investment is inconsistent with the data, and the model cannot produce a conditional size premium.

Last, but not least, our paper is also related to a recent literature that investigates the role of investment–specific technology shocks in rationalizing the cross–section of equity returns. In particular, we think of the contributions by Papanikolaou (2011), Garlappi and Song (2013), and Kogan and Papanikolaou (2015).

The remainder of the paper is organized as follows. In Section 2 we consider a simple three–period version of our model, with the purpose of developing intuition that may help us comprehend the implications of the fully fledged infinite-horizon model introduced in Section 3. The results are illustrated in Section 4. In sections 5 and 6 we analyze the role played by capital adjustment costs and the stochastic discount factor, respectively. Section 7 concludes.

2 A Three–Period Model

In this section, we lay out a simple three–period model of investment and we explore analytically its implications for the cross–section of equity returns. The time periods are indexed by $t = 0, 1, 2$. Firms produce output by means of $y_t = e^{s_t + z_t} k_t^{\alpha}$, where $\alpha \in (0, 1)$ and $k_t \geq 0$ denotes the capital stock. We assume one–period time–to–build and geometric depreciation. Let $\delta \in (0, 1)$ be the depreciation rate. Dividends equal cash flows minus investment.

The variables s_t and z_t denote the idiosyncratic and aggregate components of productivity, respectively. Both evolve according to first–order autoregressive processes with independent, normally distributed innovations. That is,

$$
\begin{aligned}
s_{t+1} &= \rho_s s_t + \varepsilon_s, \quad \varepsilon_{s,t} \sim N(0, \sigma_s^2), \\
z_{t+1} &= \rho_z z_t + \varepsilon_z, \quad \varepsilon_{z,t} \sim N(0, \sigma_z^2),
\end{aligned}
$$

where $\rho_s, \rho_z \in (0, 1)$ and $\sigma_s, \sigma_z > 0$.

At any time t, firms evaluate cash flows accruing at $t + 1$ according to the stochastic discount factor $M_{t+1} \equiv M(z_t, z_{t+1})$. It follows that, conditional on capital k_1 and productivity levels $\{s_1, z_1\}$, the cum-dividend value of equity at $t = 1$ is

$$
V_1(k_1, s_1, z_1) \equiv \max_{k_2} \; e^{s_1+z_1} k_1^{\alpha} + k_1(1 - \delta) - k_2 + E_1[M_2[e^{s_2+z_2} k_2^{\alpha} + k_2(1 - \delta)]],
$$

where the linear operator E_s denotes the expectation taken conditional on the information known at $t = s$. As of $t = 0$, the firm's optimization problem is

$$
\max_{k_1} \; -k_1 + E_0[M_1 V_1(k_1, s_1, z_1)].
$$

2.1 Characterization

Define the expected equity return at time t as the expected value of cash–flows to equity–holders divided by the ex–dividend market value. Then, we can write the expected return on equity at time $t = 0$ as

$$
E_0[R_1] = \frac{E_0[V_1(k_1, s_1, z_1)]}{E_0[M_1 V_1(k_1, s_1, z_1)]}. \tag{1}
$$

Equity is a portfolio of two risky assets whose payoffs at time $t = 1$ and $t = 2$, respectively, are listed in the table below. We will refer to them as *current* and *continuation* asset, respectively.

Table 1: Payoffs

	$t = 1$	$t = 2$
Current Asset	$e^{s_1+z_1}k_1^{\alpha} + k_1(1-\delta)$	0
Continuation Asset	$-k_2$	$e^{s_2+z_2}k_2^{\alpha} + k_2(1-\delta)$

For the sake of streamlining notation, denote the $t = 1$ conditional payoffs of the two assets as

$$\Gamma_{cu,1} \equiv y_1 + k_1(1-\delta)$$
$$\Gamma_{co,1} \equiv -k_2 + E_1[M_2[y_2 + k_2(1-\delta)]],$$

where the sub-indexes *cu* and *co* are mnemonics for *current* and *continuation*, respectively. It follows that, with some abuse of notation, we can rewrite (1) as

$$E_0[R_1] = \lambda(s_0, z_0)\frac{E_0[\Gamma_{cu,1}]}{E_0[M_1\Gamma_{cu,1}]} + [1-\lambda(s_0, z_0)]\frac{E_0[\Gamma_{co,1}]}{E_0[M_1\Gamma_{co,1}]]},$$

where $\lambda(s_0, z_0)$ is the loading on the current asset – the fraction of equity value accounted for by the current asset – or

$$\lambda(s_0, z_0) = \frac{E_0[M_1\Gamma_{cu,1}]}{E_0[M_1\Gamma_{cu,1}] + E_0[M_1\Gamma_{co,1}]}.$$

Since the expected returns on current and continuation assets are independent of idiosyncratic productivity, the latter influences expected returns on equity only via its impact on the loading λ.

Idiosyncratic productivity being mean–reverting, its expected growth rate is decreasing in s_0. It follows that λ is strictly increasing in s_0. The continuation asset accounts for a larger fraction of the value of small firms. These claims are formally stated in Lemma 1. All proofs are in Appendix A.

Lemma 1 *1. Equity is a portfolio consisting of current and continuation assets, which pay off exclusively at $t = 1$ and $t = 2$, respectively;*

2. The excess return of neither asset depends on idiosyncratic productivity;

3. The current asset is itself a portfolio of the riskless asset and a risky asset with expected returns $\frac{E_0(e^{\varepsilon}_{z,1})}{E_0(M_1 e^{\varepsilon}_{z,1})}$. The loadings are both positive and are function of the risk–free rate.

4. The loading on the current asset is an increasing function of idiosyncratic productivity.

In order to determine how expected equity returns vary with s_0, we need to establish whether the current asset commands a greater or lower expected return than the continuation asset.[3] In order to answer this question, we make functional assumptions on the stochastic discount factor.

2.1.1 The Stochastic Discount Factor

For the remainder of this section, we will assume that the stochastic discount factor is given by

$$\log M_{t+1} \equiv \log \beta - \gamma \varepsilon_{z,t+1},$$

where $\gamma > 0$ disciplines aversion to risk and $\beta > 0$ is the time discount factor. This choice allows us to make the most progress in the analytical characterization, as the risk-free rate is constant. In fact, for all $t \geq 0$,

$$R_{ft} = R_f = \frac{1}{E_t[M_{t+1}]} = \frac{1}{\beta} e^{-\frac{1}{2}\gamma^2 \sigma_z^2}.$$

The maximum Sharpe ratio is also constant:

$$\frac{std(M_{t+1})}{E(M_{t+1})} = \sqrt{e^{\gamma^2 \sigma_z^2} - 1}.$$

Under these assumptions, the expected return on the risky portion of the current asset is

$$\frac{E_0(e^{\varepsilon}_{z,1})}{E_0(M_1 e^{\varepsilon}_{z,1})} = \frac{E_0(e^{\varepsilon}_{z,1})}{E_0[e^{(1-\gamma)\varepsilon_{z,1}}]} = e^{\gamma \sigma_z^2} R_f.$$

Furthermore, the expected return on the continuation asset equals

$$\begin{aligned} \frac{E_0[\Gamma_{co,1}]}{E_0[M_1 \Gamma_{co,1}]} &= e^{\frac{1}{2}\gamma^2 \sigma_z^2} \frac{E_0\left[e^{\frac{\rho_z}{1-\alpha}\varepsilon_{z,1}}\right]}{E_0\left[e^{\left(\frac{\rho_z}{1-\alpha} - \gamma\right)\varepsilon_{z,1}}\right] / R_f} \\ &= e^{\frac{\gamma \rho_z}{1-\alpha}\sigma_z^2} R_f \end{aligned}$$

With a constant risk–free rate, the only source of risk is the cash-flow volatility. As long as $\rho_z > 1 - \alpha$, the continuation asset will command a higher expected return than the current asset. This parametric condition is rather intuitive.

[3] Note that this is not equivalent to assessing the slope of the equity term structure, as the definition of assets we are using is not standard. In Section 4 we will describe in detail the model's implication for the term structure.

As of $t = 0$, the risk of the continuation asset is pinned down by the covariance between time–1 innovations to aggregate productivity ($\varepsilon_{z,1}$) and $\Gamma_{co,1}$, the time–1 conditional expectation of the asset's payoff. Such moment is greater, the higher the autocorrelation of the process ρ_z and the lower the returns to scale in production.

Returns to scale are relevant, because they shape the elasticity of the capital choice k_2 to time–1 productivity innovations. For the remainder of the section, we decide to focus on the scenario for $\rho_z > 1 - \alpha$, as it is the empirically relevant one.

In conclusion, we note that the impact of interest rate risk would depend on the sign of the covariance between interest rate and the innovation in the aggregate productivity shock. A countercyclical risk-free rate would magnify the risk of the continuation asset. Conversely, a pro-cyclical risk-free rate would lower it. This remark will be useful in interpreting the results in Section 6.

2.1.2 Size, Book-to-Market, and Investment Rate

In the simple model under consideration, the only driver of cross–sectional heterogeneity in expected equity returns is the variation in idiosyncratic productivity. It follows that the variance of the ex–dividend value of equity, or market size, is sufficient to completely characterize the distribution of expected returns.

The reason is that market size at time t=0 is pinned down by the levels of aggregate and idiosyncratic productivity, z_0 and s_0. Given the absence of capital adjustment costs, the level of installed capital k_0 is irrelevant.

As a corollary, information on indicators such as book–to–market ratio and investment rate cannot improve upon our characterization of the cross–section of returns. Because of their popularity, however, and the role they will play in the rest of the paper, we characterize the model–implied correlations between the two and expected returns.

Both book-to-market ratio and the investment rate depend on the installed capital k_0. In order to compute the cross-sectional distribution of both quantities at time $t = 0$, we need to make assumptions about the distribution of k_0.

We posit that k_0 was chosen optimally by each firm at time $t = -1$, under the assumptions that $z_{-1} = z_0 = 0$ and $s_{-1} \sim N\left(0, \frac{\sigma_s^2}{1-\rho_s^2}\right)$. In other words, we consider the scenario in which the aggregate productivity realization was equal to its unconditional mean in both $t = -1$ and $t = 0$, and that the cross-sectional distribution of idiosyncratic productivity is equal to the unconditional distribution.

Under these assumptions, the average growth rate of capital installed by firms experiencing a realization of idiosyncratic productivity s_0 is

$$E\left[\log\left(\frac{k_1}{k_0}\right)|s_0\right] = \frac{\rho_s(1-\rho_s)}{1-\alpha}s_0.$$

See Lemma 4 in Appendix A. It follows that in the cross-section the investment rate is increasing in s_0.

Finally, we want to understand how the book-to-market ratio, i.e.

$$\frac{E(k_0|s_0)}{E_0[M_1\Gamma_{cu,1}] + E_0[M_1\Gamma_{co,1}]},$$

varies with s_0 in the cross section. Above we have established that both numerator and denominator are increasing in s_0. In Lemma 2 we prove that the denominator grows faster.

Lemma 2 *Assume that the discount factor is* $M_{t+1} = \beta e^{\gamma\varepsilon_{t+1}}$ *and that* $s_{-1} \sim N\left(0, \frac{\sigma_z^2}{1-\rho_s^2}\right)$*. Along the path for the aggregate shock* $z_{-1} = z_0 = 0$*,*

1. *Size and investment rate are increasing in* s_0*;*
2. *Book-to-market is decreasing in* s_0*.*

A corollary of Lemma 2 is that cross-sectional expected returns covary positively with book-to-market and negatively with size and investment rate. However – it is worth restating it – conditional on size, both book to market and investment rate are uncorrelated with expected returns. High book-to-market and low-investment rate firms earn higher returns because on average they have low market size. We will address this issue when generalizing our model in Section 3.

2.2 Operating leverage

Now assume that at $t = 1$ firms incur a fixed operating cost $c_f > 0$. For simplicity, assume also that c_f is such that equity value is always non–negative.

Other studies, among which Carlson, Fisher, and Giammarino (2004) and Gala (2006), have assumed that the operating cost is a linear function of the capital in place. We do not follow that route, since in our framework it would be isomorphic to increasing the depreciation rate.

How does the novel assumption affect the properties stated above? Our interest in answering this question stems from the role that operating leverage will play in the quantitative analysis conducted in Section 3 and beyond.

The expected return on equity at $t = 0$ becomes

$$E_0[R_1] = \frac{E_0[V_1(k_1, s_1, z_1)] - c_f}{E_0[M_1 V_1(k_1, s_1, z_1)] - c_f/R_f}.$$

Equity is now the combination of a short position on the risk–free asset and a long position on the current and continuation assets introduced above. The position on the risk–free asset is

$$-\frac{c_f/R_f}{E_0[M_1\Gamma_{cu,1}] - \frac{c_f}{R_f} + E_0[M_1\Gamma_{co,1}]} < 0,$$

which is strictly decreasing in c_f.

Everything else equal, raising the fixed cost is equivalent to expanding the short position – i.e. increasing leverage. It follows that the expected return on equity is increasing in c_f. The impact on returns will be larger, the lower is s_0.

A greater level of s_0 is equivalent to a decline in leverage, i.e. a smaller short position on the risk–free asset and a smaller long position on the portfolio of current and continuation assets. It follows that the expected equity return still falls with s_0. This property is stated formally in Lemma 3.

Lemma 3 *For $c_f > 0$, as long as $\rho_z > 1 - \alpha$, the expected return on equity is still monotonically decreasing in the level of idiosyncratic productivity s_0.*

The fixed cost also affects the book–to–market ratio, which becomes

$$\frac{E(k_0|s_0)}{E_0[M_1\Gamma_{cu,1}] - \frac{c_f}{R_f} + E_0[M_1\Gamma_{co,1}]}.$$

The slope of the mapping between size and book-to-market increases in absolute value.

By means of a parametric example, Figure 1 illustrates the effect of introducing a fixed cost on the correlation patterns between expected returns and size, book-to-market and investment rate.[4] Recall that size (ex–dividend market value) and expected returns are pinned down by s_0. The impact of operating leverage on expected returns is strictly decreasing in size.

In general, firms characterized by either the same book–to–market or the same investment rate earn different returns, because they have different productivities. The (positive) effect of operating leverage on the conditional variance of returns is greater for lower values of s_0, which – on average – are associated with greater book–to–market and lower investment rates.

[4] We simulate 50,000 firms assuming $\alpha = 0.6$, $\delta = 0.1$, $\rho_s = 0.85$, $\sigma_s = 0.1$, $\rho_z = 0.95$, $\sigma_z = 0.1$, $\beta = 0.985$, $\gamma = 1.5$, and $c_f = 15$.

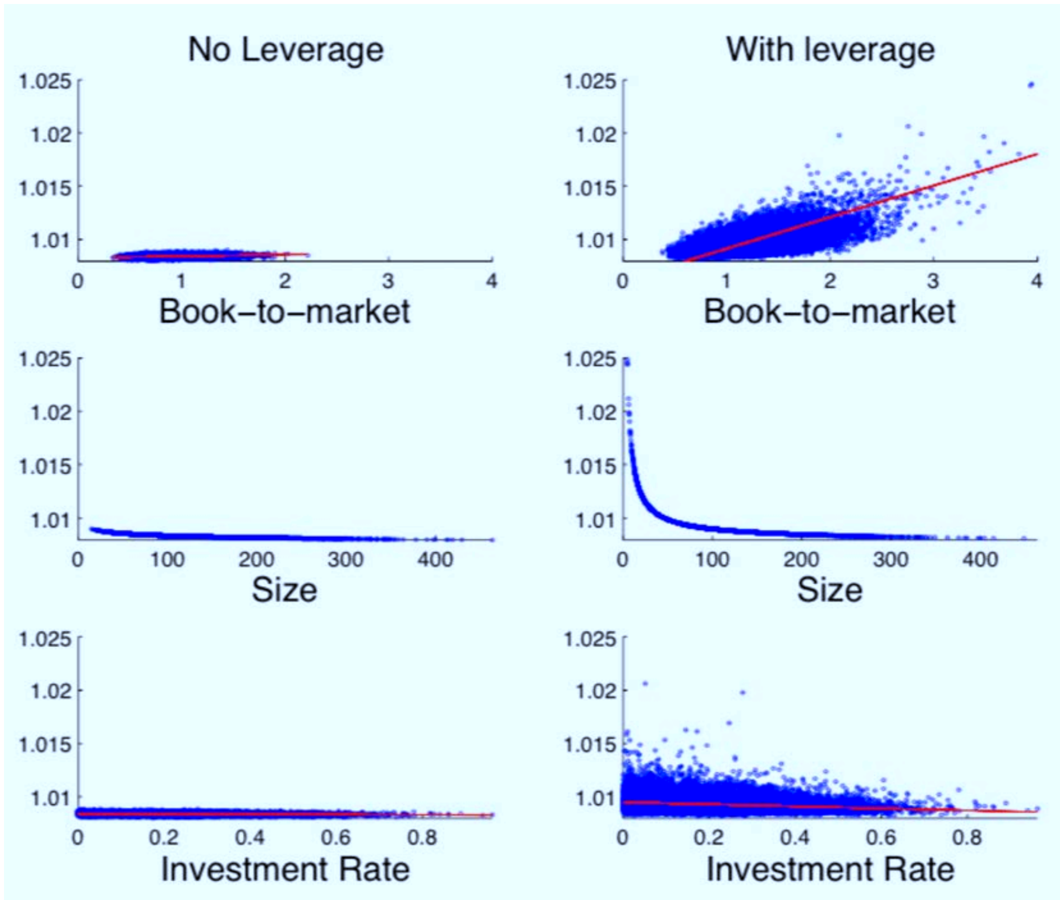

Figure 1: Cross–sectional Variation of Expected Returns.

2.3 Capital Adjustment Costs

We now assume that firms adjusting their capital stock incur a cost equal to

$$g(k_t, k_{t+1}) \equiv \frac{\phi}{2}\left[\frac{k_{t+1}}{k_t} - (1-\delta)\right]^2 k_t,$$

with $\phi > 0$. Conditional on capital k_1 and productivity levels $\{s_1, z_1\}$, the value of equity at $t = 1$ is

$$V_1(k_1, s_1, z_1) \equiv \max_{k_2}\; e^{s_1+z_1}k_1^{\alpha} + k_1(1-\delta) - k_2 - g(k_1, k_2) + E_1[M_2[e^{s_2+z_2}k_2^{\alpha} + k_2(1-\delta)]].$$

As of $t = 0$, the firm's optimization problem writes as

$$\max_{k_1}\; -k_1 - g(k_0, k_1) + E_0[M_1 V_1(k_1, s_1, z_1)].$$

Capital adjustment costs introduce a novel dimension of heterogeneity, as expected returns at $t = 0$ are no longer pinned down by the level of idiosyncratic productivity s_0 and depend non-trivially on the installed capital k_0.

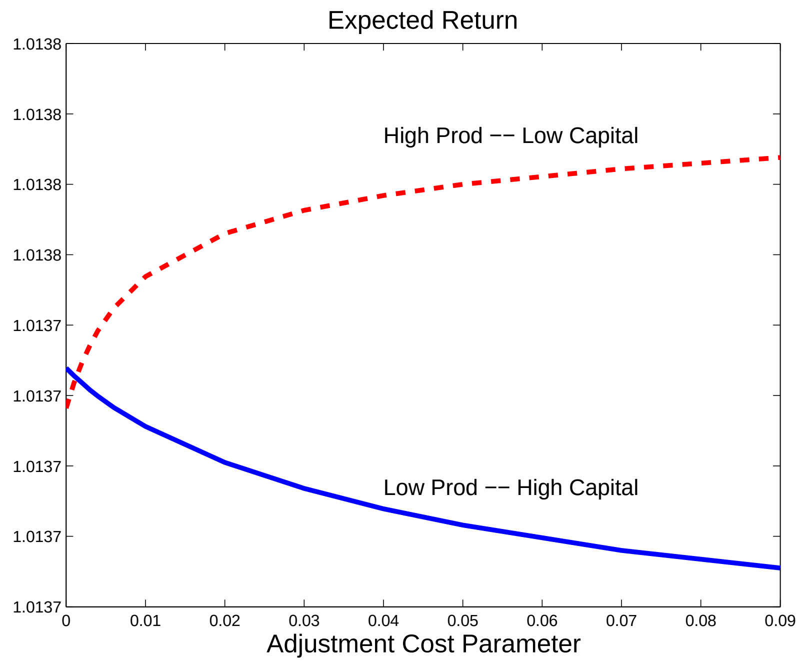

Figure 2: Comparative statics of expected returns with respect to the parameter ϕ.

The impact of the adjustment cost on returns is larger for those firms whose installed capital is farther from their static first-best level. Figure 2 shows the result of raising the value assigned to the parameter ϕ on the expected returns of two particular firms, in the case of a simple parametric example. One firm, which we will refer to as *growing*, is endowed with relatively high productivity and low capital. The other, which is *shrinking*, has low productivity and high capital. Interestingly for our purposes, the shrinking firm has a higher book-to-market ratio and a lower investment rate.

For $\phi = 0$, the model boils down to the scenario characterized above, where expected returns are pinned down by the levels of idiosyncratic productivity. Since it has lower productivity, the shrinking firm is riskier and therefore earns a higher expected return.

As the adjustment cost increases, however, the risk of the growing firm increases monotonically, while the risk of the shrinking firm declines. There is a threshold of the parameter ϕ, such that for higher values the growing firm earns a higher return. This result is interesting in that it hints that increasing the cost of adjusting the capital stock may lead to a decline in the risk spread between high- and low-book-to-market firms, eventually making it negative.

The conditional tense is warranted as we have neither assessed the generality of the result yet, nor evaluated its quantitative significance. The latter task requires a fully fledged model, such as the one introduced in Section 3.

A complete analytical characterization of the comparative statics exercise is not readily available. Yet, we can make some progress by studying the impact of varying ϕ on the three elements that shape the expected return: The return on the current asset, the return on the continuation asset, and the share of total value arising from the former, respectively. The conditional payoffs of current and continuation assets at $t = 1$ are redefined as

$$e^{s_1+z_1}k_1^{\alpha} + k_1(1-\delta)$$

and

$$-k_2 - \frac{\phi}{2}\left[\frac{k_2}{k_1} - (1-\delta)\right]^2 k_1 + E_1\left[M_2\left[e^{s_2+z_2}k_2^{\alpha} + k_2(1-\delta)\right]\right],$$

respectively.

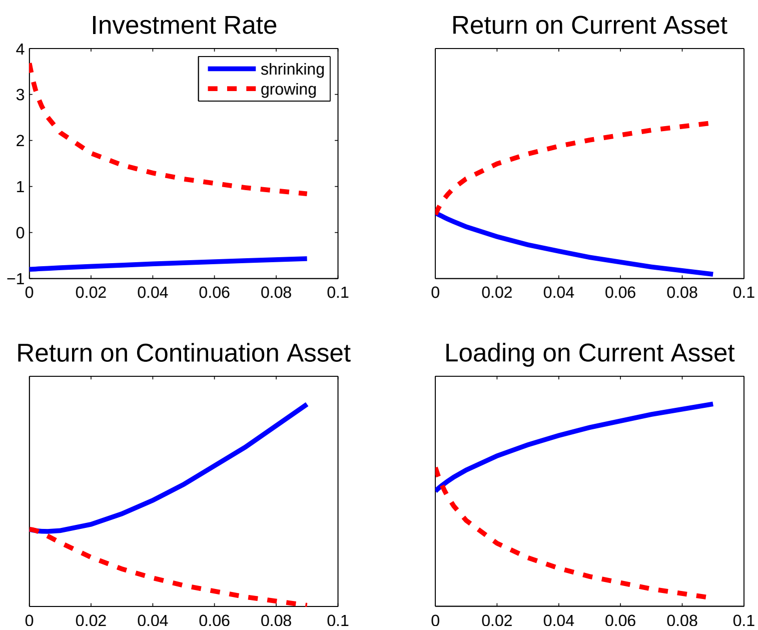

Figure 3: Comparative Statics with Respect to the Parameter ϕ.

It is easy to show that, consistent with the top–left panel of Figure 3, the investment rate of the shrinking firm increases with ϕ – i.e. the optimal choice of k_1 is monotonically increasing in the parameter value. It follows that the expected return on the current asset also declines with ϕ. This is the case, because the loading of the current asset on the risk-free asset is a strictly increasing function of k_1. Conversely, since its investment rate is strictly decreasing in the parameter value, the return on the current asset of the growing firm increases with ϕ.

For the shrinking firm, the loading on the current asset is increasing in ϕ, as cash–flows are front-loaded. A higher fraction of the firm value is accounted by the current asset. The opposite occurs for the growing firm. Cash–flows are backloaded, so that the current asset accounts for a smaller fraction of total value.

Finally, as revealed in the bottom–left panel of Figure 3, increasing ϕ means more risk for the continuation asset of shrinking firms. The intuition is that for shrinking firms, a greater k_1 means higher value at $t = 1$ contingent on a good realization of aggregate productivity and lower value contingent on a bad realization. The covariance between payoffs at $t = 1$ and the stochastic discount factor increases in absolute value.

Conversely, for the growing firm, a smaller k_1 means that the payoff of the continuation asset at $t = 1$ is larger contingent on a bad realization of aggregate productivity and smaller contingent on a good realization. The covariance with the stochastic discount factor declines in absolute value. The expected return on the continuation asset is lower.

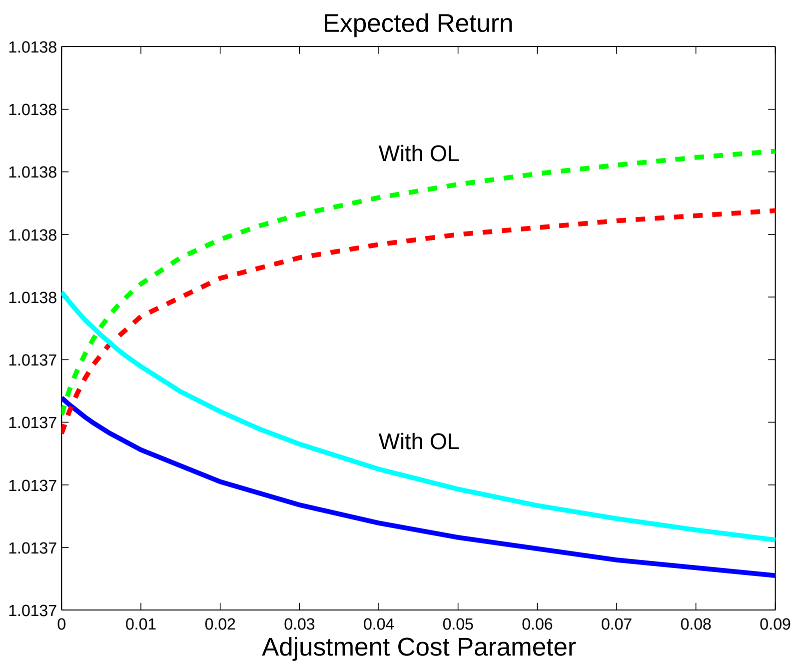

Figure 4: Effect of Operating Leverage on the Comparative Statics of Expected Returns.

We conclude this section by considering the impact of operating leverage on the comparative statics exercise just described. Refer to Figure 4. Qualitatively, nothing changes. When $c_f > 0$, it is still the case that increasing the adjustment cost parameter ϕ leads to a decline in the excess return earned by the shrinking firm with respect to the growing firm.

However, consistent with the analysis conducted above, the spread in returns for $\phi = 0$

is greater. It follows that the range of values for the parameter ϕ that produce a positive spread between shrinking and growing firm is now larger. These considerations will be relevant when we evaluate the implications for asset returns of the fully fledged model we now introduce.

3 A Fully Fledged Model

Time is discrete and is indexed by $t = 1, 2, ...$ The horizon is infinite. At every time t, a positive mass of firms produce an homogenous good by means of the production function $y_t = e^{z_t + s_t} k_t^{\alpha}$, with $\alpha \in (0, 1)$. Here $k_t \geq 0$ denotes physical capital, which depreciates at the rate $\delta \in (0, 1)$. The variables z_t and s_t are aggregate and idiosyncratic random disturbances, respectively. They are orthogonal to each other.

The common component of productivity z_t is driven by the stochastic process

$$z_{t+1} = \rho_z z_t + \sigma_z \varepsilon_{z,t+1},$$

where $\rho_z \in (0, 1)$, $\sigma_z > 0$, and $\varepsilon_{z,t} \sim N(0, 1)$ for all $t \geq 0$. The conditional distribution of z_{t+1} will be denoted as $J(z_{t+1}|z_t)$.

The dynamics of the idiosyncratic component s_t is described by

$$s_{t+1} = \rho_s s_t + \sigma_s \varepsilon_{s,t+1},$$

where $\rho_s \in (0, 1)$, $\sigma_s > 0$, and $\varepsilon_{s,t} \sim N(0, 1)$ for all $t \geq 0$. The conditional distribution of s_{t+1} will be denoted as $H(s_{t+1}|s_t)$.

Gross investment x requires firms to incur a cost $g(x, k_t)$, where

$$g(x, k_t) \equiv \chi(x)\phi_0 k_t + \phi_1 \left(\frac{x}{k_t}\right)^2 k_t, \ \ \phi_0, \ \phi_1 \geq 0,$$

and where $\chi(x) = 0$ for $x = 0$ and $\chi(x) = 1$ otherwise. The first component of $g(x, k_t)$ reflects a fixed cost, scaled by capital in place, which the firm incurs if and only if gross investment is different from zero. We also assume that each period firms incur a fixed operating cost $c_f \geq 0$. Think of that as overhead.

Firms discount future cash flows by means of the discount factor $M(z_t, z_{t+1})$, with

$$\log M(z_t, z_{t+1}) \equiv \log \beta + \gamma_0 z_t + \gamma_1 z_{t+1},$$

where $\beta > 0$, $\gamma_0 > 0$, and $\gamma_1 < 0$. This specification implies that the conditional risk–free rate equals

$$R_{f,t} = \frac{1}{\beta} e^{-z_t[\gamma_0 + \rho_z \gamma_1]} e^{-\frac{1}{2}\gamma_1^2 \sigma_z^2}.$$

Notice that $R_{f,t}$ is counter–cyclical if and only if $\gamma_0 > -\rho_z \gamma_1$. The price of risk is constant, as

$$\frac{std(M_{t+1})}{E_t(M_{t+1})} = \sqrt{e^{\gamma_1^2 \sigma_z^2} - 1}.$$

Abandoning the time notation for expositional convenience, we denote the firm's value function as $V(z, k, s)$, where k, z, and s, are capital in place, aggregate productivity, and idiosyncratic productivity, respectively. $V(z, k, s)$ is the fixed point of the following functional equation:

$$V(z,k,s) = \max_x e^{s+z} k^\alpha - x - g(x,k) - c_f + \int_{\Re} \int_{\Re} M(z,z') V(z',k',s') dH(s'|s) dJ(z'|z),$$
$$\text{s.t. } k' = k(1-\delta) + x.$$

Our main object of interest is the expected return on equity, defined as the ratio of expected cum-dividend value at the next date to the current ex-dividend value. Conditional on a triplet of state variables (z, k, s), it writes

$$R_e(z,k,s) = \frac{\int_{\Re} \int_{\Re} V(z',k^*,s') dH(s'|s) dJ(z'|z)}{\int_{\Re} \int_{\Re} M(z,z') V(z',k^*,s') dH(s'|s) dJ(z'|z)},$$

where k^* is the optimal choice of capital.

3.1 Calibration and simulation

For a given set of parameters, we approximate the policy function for capital by means of an algorithm based on the value function iteration method.[5] We then use such policy function, together with the stochastic processes of idiosyncratic and aggregate productivity, to generate a 20,000-quarter long time-series of the whole distribution of firms.

After ruling out the first 500 periods, we obtain our approximation of the model's ergodic distribution. Unless otherwise specified, the moments we report throughout the paper are time–series average of the cross–sectional moments.

Our calibration strategy sets our study apart from any other investigation of the cross–section of returns in production economy, as we do not target any feature of the cross–section of returns. Rather, we require our model to be consistent with the microeconomic data on investment and study the implied variation for returns.

One period is assumed to be one quarter. Consistent with most macroeconomics studies, we set $\delta = 0.030$. Following Cooley and Prescott (1995), we let $\rho_z = 0.95$ and $\sigma_z = 0.007$.

[5] See Appendix B for a summary description of our computational approach.

Table 2: Parameter Values

		I	II	III
		$\alpha = 0.6$	$\alpha = 0.6$	$\alpha = 0.3$
Description	Symbol	NO OL	OL	OL
From other studies				
Capital share	α	0.600	0.600	0.300
Depreciation rate	δ	0.030	.	.
Persist. aggregate shock	ρ_z	0.950	.	.
Variance aggregate shock	σ_z	0.007	.	.
Calibrated				
Persist. idiosync. shock	ρ_s	0.900	.	.
Variance idiosync. shock	σ_s	0.060	0.060	0.105
Fixed operating cost	c_f	0.000	0.00135	0.0070
Fixed cost of investment	ϕ_0	0.000015	.	.
Variable cost of investment	ϕ_1	0.0054	0.0054	0.009
Parameter pricing kernel	β	0.970	.	.
Parameter pricing kernel	γ_0	31.850	.	.
Parameter pricing kernel	γ_1	-33.000	.	.

We set α equal to 0.6. This is the elasticity with respect to capital that one would obtain with a more general specification of the production function where output also depended on labor, if returns to scale were 0.8 and the share of value added that accrued to capital was 0.3.[6]

The parameters of the process driving idiosyncratic productivity (ρ_s and σ_s), along with those governing the adjustment costs (ϕ_0 and ϕ_1), were chosen to match the mean and standard deviation of the investment rate, the autocorrelation of investment, and the rate of inaction, i.e. the fraction of firms with investment rate in absolute value lower than 1%. The target values are moments estimated from a large panel of public companies. The estimation procedure is detailed in Appendix C.

Clementi and Palazzo (2014) show that a simpler version of the neoclassical investment model with lognormal disturbances – one without investment adjustment costs – has the interesting properties that (i) the mean investment rate is a simple non linear function of the parameters ρ_s and σ_s and that (ii) the standard deviation of the investment rate

[6]Using a sample of U.S. public companies, Hennessy and Whited (2005) and Hennessy and Whited (2007) estimate a value for α equal to 0.551 and 0.627 respectively.

Table 3: Calibrated Targets

	Data	I	II	III
		$\alpha = 0.6$	$\alpha = 0.6$	$\alpha = 0.3$
Investment Rate		NO OL	OL	OL
Mean	0.041	0.053	0.053	0.040
Standard Deviation	0.096	0.090	0.090	0.091
Autocorrelation	0.266	0.278	0.278	0.271
Inaction Rate	0.144	0.150	0.150	0.169
Book–to–Market	0.721	0.491	0.717	0.564
Risk-Free Rate and Sharpe Ratio				
Mean (%)	0.411	0.437	0.437	0.437
Standard Deviation (%)	1.010	1.154	1.154	1.154
Sharpe Ratio	0.212	0.204	0.199	0.208
Mean Excess Return (%)	1.794	1.153	1.189	1.317
St. Dev. Excess Return (%)	8.481	5.660	5.976	6.340

is a simple non–linear function of the mean. It follows that in that framework, mean and standard deviation of the investment rate do not identify the pair $\{\rho_s, \sigma_s\}$. While these properties do not hold exact in our model, numerical results reveal that a similar restriction between the two moments exists, leaving us with a degree of freedom.

We proceed to set $\rho_s = 0.9$, a value consistent with the estimate by Imrohoroglu and Tuzel (2014), and set the remaining three parameters to minimize a weighted average of the distances between the moments and their respective targets.

The stochastic discount factor was parameterized in such a way to match the first two unconditional moments of the risk–free, as well as the mean Sharpe ratio. Because of non–linearities in the map between parameters and moments, there are two distinct sets of parameters that match the targets. One produces a counter–cyclical risk-free rate, while the other generates a pro–cyclical rate. To be consistent with the evidence,[7] we decide to go with the former.

Finally, we begin our exploration by setting $c_f = 0$. Parameter values and moments are reported in Column I of Tables 2 and 3, respectively. All returns are quarterly and the acronym "NO OL" stands for "no operating leverage."

In the last two rows of Table 3, we report the time–series average and standard deviation of the equally-weighted mean excess stock return. In spite of a data-conforming

[7] See Beaudry and Guay (1996) and Cooper and Willis (2014) among others.

Sharpe ratio, a relatively low return volatility implies that the mean excess return falls short of its data counterpart.

4 Results

We illustrate the model's implications by means of a simple methodology commonly used in the empirical asset pricing literature. In every quarter, we form portfolios of firms based on the values assumed by certain firm–level characteristics, and we compute their realized returns. Finally, we report and compare the time–series means of the returns earned by the different portfolios.

In Table 4, we list unconditional mean returns for portfolios sorted on size, i.e. the ex–dividend firm value. Stocks are classified as small if they belong to the bottom two deciles of the size distribution in the period of portfolio formation. They are classified as large if they belong to the top two deciles. Alternatively, they are included in the medium–size category. For each portfolio, we also report mean values of size, book–to–market, investment rate, capital in place, and idiosyncratic productivity.

Consistent with the empirical evidence, on average small firms earn higher returns. This is the case because, as it was the case in the simple model analyzed in Section 2, small firms have a lower idiosyncratic productivity, which in turn is associated with higher risk. Small firms also feature lower capital in place, lower investment rate and – counterfactually – lower book–to–market.

In the neoclassical model with convex adjustment costs, marginal q and investment rate, tied to each other by the Euler equation for investment, are increasing in productivity. Under constant returns to scale, Tobin's q, i.e. the reciprocal of the book-to-market ratio, is also always increasing in productivity and is therefore often used as an easy-to-compute proxy for the marginal return on capital. The latter property, however, does not generalize to the case of decreasing returns. Here, Tobin's q is decreasing in productivity in stationary distribution, and the book-to-market ratio is positively associated with size.

Refer now to Table 5, which reports statistics for portfolios sorted on the book–to–market ratio. With respect to value stocks, growth stocks feature lower productivity and lower investment rates, and command greater expected returns. The model generates a counterfactual value discount.

Figures 5 and 6 provide more evidence in support of the claim that in this model, the book-to-market criterion identifies as either growth or value, firms that in important respects are much unlike their empirical counterparts. The two figures display the dynamics

Table 4: Size Sorted Portfolios

	I	II	III
	$\alpha = 0.6$	$\alpha = 0.6$	$\alpha = 0.3$
Excess Returns (%)	NO OL	OL	OL
Small Firms	1.732	1.871	2.109
Average Size	1.595	1.624	1.730
Large Firms	1.454	1.421	1.478
Large–Small	-0.278	-0.450	-0.631
Size			
Small Firms	0.301	0.208	0.171
Average Size	0.383	0.291	0.250
Large Firms	0.502	0.411	0.360
Large–Small	0.201	0.203	0.189
Book–to–Market			
Small Firms	0.424	0.689	0.579
Average Size	0.489	0.716	0.560
Large Firms	0.556	0.746	0.562
Large–Small	0.131	0.057	-0.018
Investment Rate			
Small Firms	0.030	0.030	0.021
Average Size	0.054	0.054	0.040
Large Firms	0.069	0.069	0.059
Large–Small	0.039	0.039	0.038
Capital			
Small Firms	0.154	0.154	0.098
Average Size	0.222	0.222	0.142
Large Firms	0.322	0.322	0.207
Large–Small	0.168	0.168	0.109
Idiosyncratic Shock			
Small Firms	-0.195	-0.195	-0.336
Average Size	-0.004	-0.004	-0.001
Large Firms	0.183	0.183	0.332
Large–Small	0.379	0.379	0.668

Table 5: Book–to–Market Sorted Portfolios

	I	II	III
	$\alpha = 0.6$	$\alpha = 0.6$	$\alpha = 0.3$
Excess Returns (%)	NO OL	OL	OL
Growth Firms	1.708	1.709	1.718
Average BM	1.593	1.626	1.743
Value Firms	1.486	1.558	1.818
Value–Growth	-0.222	-0.151	0.100
Size			
Growth Firms	0.331	0.279	0.262
Average BM	0.388	0.299	0.257
Value Firms	0.453	0.320	0.250
Value–Growth	0.123	0.041	-0.012
Book–to–Market			
Growth Firms	0.399	0.619	0.477
Average BM	0.486	0.711	0.558
Value Firms	0.584	0.817	0.657
Value–Growth	0.185	0.198	0.180
Investment Rate			
Growth Firms	0.118	0.144	0.176
Average BM	0.057	0.054	0.039
Value Firms	-0.013	-0.053	-0.076
Value–Growth	-0.132	-0.197	-0.252
Capital			
Growth Firms	0.162	0.188	0.128
Average BM	0.224	0.227	0.146
Value Firms	0.303	0.273	0.164
Value–Growth	0.141	0.086	0.036
Idiosyncratic Shock			
Growth Firms	-0.130	-0.023	0.075
Average BM	0.000	0.000	0.005
Value Firms	0.110	0.019	-0.081
Value–Growth	0.240	0.042	-0.156

of productivity around portfolio formation for both types of firm, as implied by our model and by our data on public firms, respectively.[8]

According to the model, productivity of value firms rises ahead of the formation date and declines thereafter. Productivity of growth firms declines ahead of the formation date, to recover in the aftermath. The data suggests the opposite.

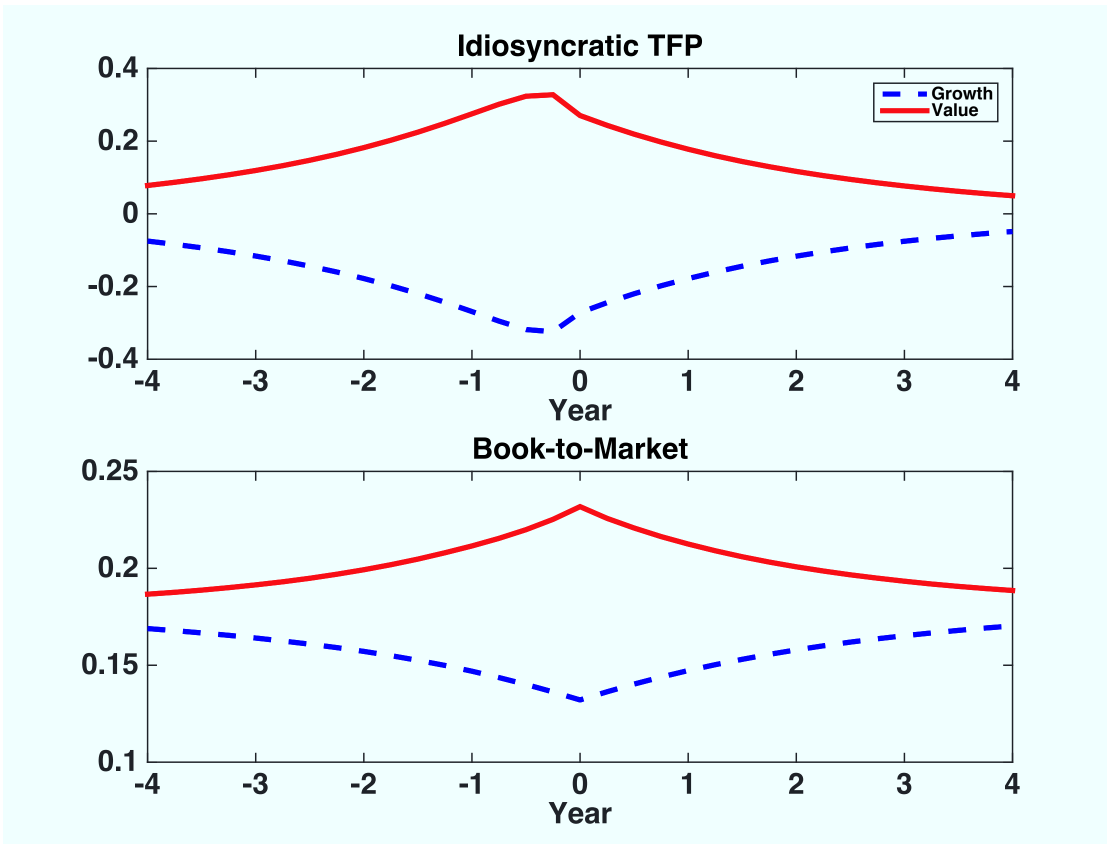

Figure 5: Dynamics Around Portfolio Formation – Model without operating leverage

4.1 Operating Leverage

Based on the insights gained in Section 2, we know that introducing operating leverage is bound to have two distinct effects. It will affect returns via its impact on cash–flow risk and it will reshape the variation of the book–to–market ratio over the state space.

We set the cost c_f in order to generate an average book-to-market ratio equal to its

[8]Figure 6 results from a production function estimation exercise described in Appendix C. To build Figure 5, we first simulate a panel of 1,500 firms for 2,500 quarters, discarding the first 50 observations. Each quarter, we identify growth and value firms, respectively, with the usual criterion. We report the time-series averages of mean firm–level TFP (top panel) and mean book–to–market (bottom panel) over a 32–quarter period centered around the date of portfolio formation.

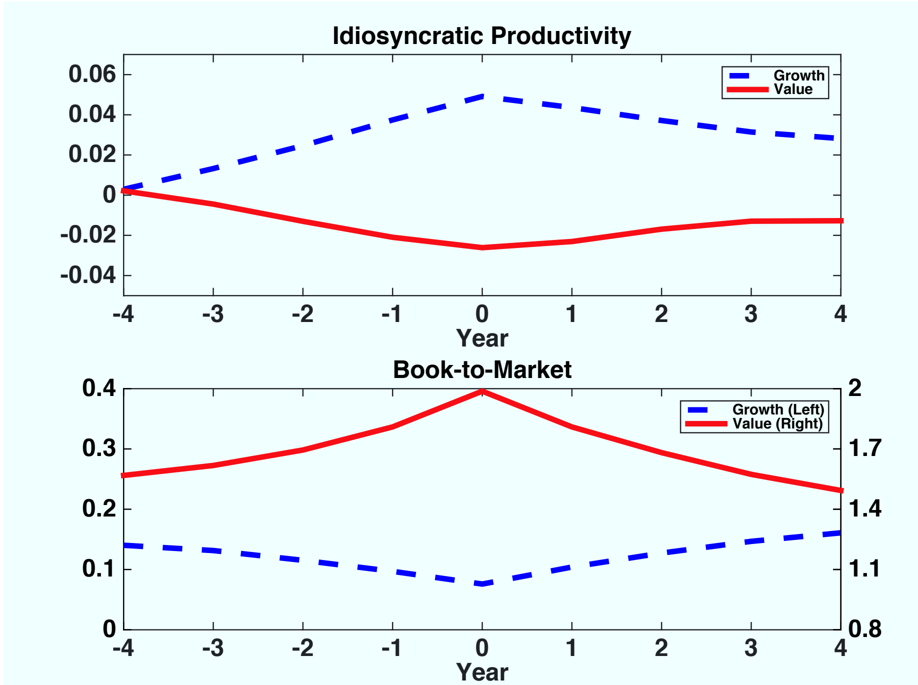

Figure 6: Dynamics Around Portfolio Formation – Data

empirical counterpart of about 0.7. Since no firm decision depends upon the value of c_f, no further change to the calibration is warranted.

The implications for portfolios sorted on size are listed in Column II of Table 4. Expected returns rise across all size categories, but grow faster for small firms. The size premium increases.

Comparing Columns I and II in Table 5, one realizes that indeed the introduction of operating leverage affects the set of firms identified as growth and value, respectively. On the one hand, value firms are now less productive, and riskier. On the other hand, growth firms are more productive, and not as risky as in the previous scenario. However, the model still produces a value discount.

In order to accommodate a greater value for c_f without generating a counterfactual average book-to-market ratio, we lower the parameter α – the elasticity of the production function – to 0.3. The operating cost is set at the highest value among those consistent with non-negativity of the firm's value function in our numerical approximation.

Figure 7 illustrates the location of small and large firms over the state space. The shaded areas consist of the pairs of capital stock and idiosyncratic productivity (k, s) associated with firms identified as either small or large in at least one period of the 19,500-quarter long simulation. Similarly, for growth and value firms in Figure 8. The

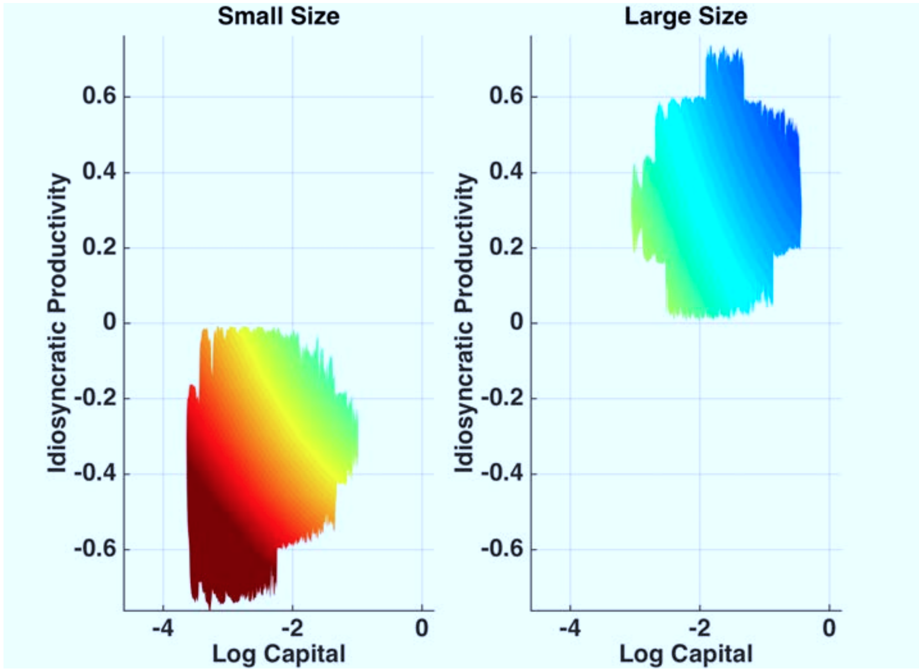

Figure 7: Location of small and large firms over the state space

color code identifies the magnitude of the time–series average of equity return returns at each location, with warm colors signaling high returns, and cold colors being associated with low returns. The key moments of size and book-to-market sorted portfolios are listed in column III of Tables 4 and 5, respectively.

The characteristics of the firms selected as either growth or value are now radically different from the case without operating leverage. On average, growth firms have higher productivity and lower capital than value firms.

The model generates a value premium, although its magnitude – 0.1% quarterly – is very limited. As a benchmark, consider that the average monthly equally weighted value premium over the period 1976:m1–2013:m12 is 0.95%, which corresponds to a quarterly return of 2.88%.[9]

Small firms earn, on average, an equally weighted excess return of around 0.6% per quarter over large firms, a value close to empirical estimates. The average monthly equally weighted size premium over the period 1976:m1–2013:m12 is 0.36%, which corresponds to a quarterly return of 1.08%.

It follows that the value premium is smaller than the size premium. This is a robust result, easily rationalized by two observations. First, the conditional dispersion of

[9]Data on equity returns are from Kenneth French's website: http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

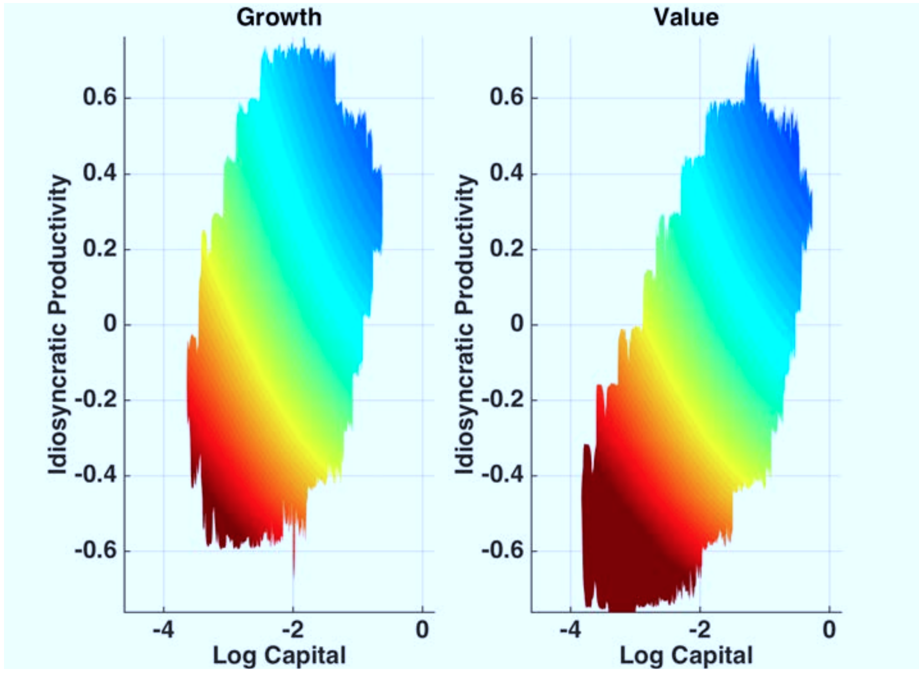

Figure 8: Location of growth and value firms over the state space

idiosyncratic productivity – the main determinant of variation in risk – is greater among firms of different size than among firms of different book-to–market. Second, and more interesting, on average value firms shed capital after portfolio formation – paying it out as dividend – regardless of the aggregate state of nature. This makes them less risky. Growth firms, on the other hand, tend to invest – drawing resources from shareholders – in all aggregate states. This feature makes them riskier.

Figure 9 helps us appreciate the dynamics of growth and value stocks in the aftermath of portfolio formation. We report the time–series averages of the portfolio means of investment rate, dividend to capital ratio, book–to–market, and idiosyncratic TFP for the 16 quarters following portfolio formation.[10]

At portfolio formation, value firms are endowed on average with capital stocks substantially greater than the efficient level dictated by their efficiency. Growth firms, on the other hand, have less capital in place than warranted by their relatively high idiosyncratic productivity. As a result, for a number of periods growth firms invest heavily, requiring new resources from their shareholders, while value firms divest and pay out cash dividends. This occurs while productivity mean–reverts for both sets of firms.

These features are in line with the available evidence. In particular, the dynamics of

[10]The data is obtained simulating a panel of 1,500 firms for 2,500 quarters and discarding the first 50 observations.

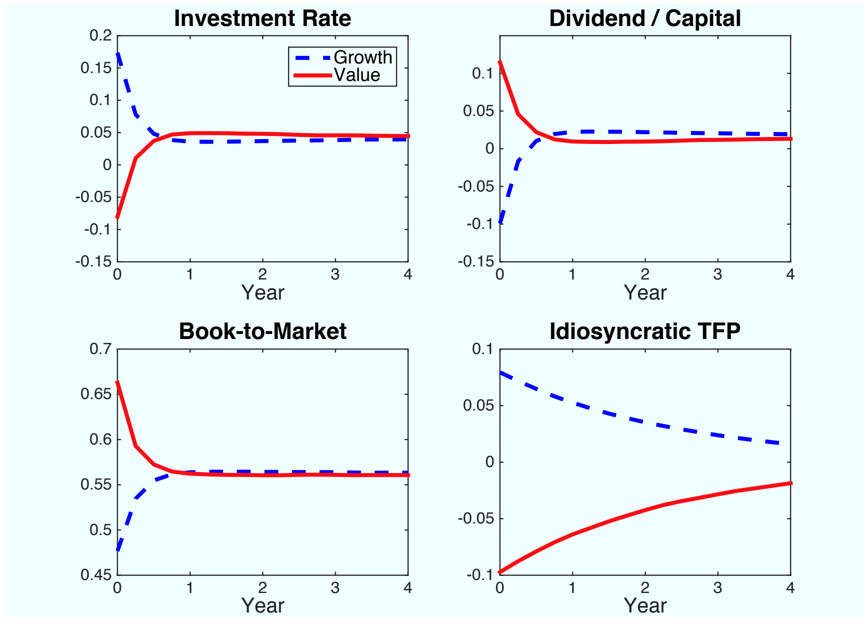

Figure 9: Firm dynamics after portfolio formation – With Operating Leverage

idiosyncratic productivity is consistent with Imrohoroglu and Tuzel (2014) as well as our own findings illustrated in Figure 6: TFP is higher for growth firms than value firms, but the gap between the two declines after portfolio formation. Value firms also have shorter cash-flow duration, as documented by Dechow, Sloan, and Soliman (2004).

At portfolio formation, value firms are deemed riskier, and command a greater equity return, because their idiosyncratic productivity is low. The divesting activity tend to reduce their risk, since the associated payouts will occur even in bad aggregate states of nature. By the same token, investors regard growth stocks as safer, and require a lower return to hold them, because their idiosyncratic productivity is high. The investment activity contributes positively to their risk, as it will require resources from shareholders in all aggregate states of nature.

The above discussion also clarifies why, consistent with the empirical findings of Xing (2008) among others, an investment strategy calling for a long position on low investment–rate stocks and a short position on high investment–rate stocks yields a positive return on average. See Table D.1.

One may argue that the model's failure to generated a sizable equity premium follows directly from its inability to generate enough cross–sectional dispersion of returns. With respect to this assertion, two considerations are in order.

To start with, the finding that in the model the size premium is always greater than the value premium is rather general. Any attempt to increase the value premium by boosting cross–sectional heterogeneity would also imply a counterfactually high size premium.

A second key observation is that the cross–sectional dispersion of equity returns – which undoubtedly falls short of empirical values – is an equilibrium object, in turn disciplined by our calibration strategy. Recall that we set $\rho_s = 0.9$, a value consistent with available estimates, and σ_s as well as the adjustment cost parameters to match cross–sectional moments of the investment rate. Operating leverage is set to match the average book-to-market ratio.

It turns out that raising ρ_s to 0.95 and re-calibrating the model does imply a larger unconditional standard deviation of idiosyncratic productivity and equity return. However, the fixed cost c_f needs to drop to ensure that equity values are non–negative in our numerical approximation. The exercise results in a even lower value premium. Conversely, the value premium rises slightly when we set $\rho_s = 0.85$. See Table D.2.

So far we have restricted our analysis to the characterization of unconditional correlations between firm-level characteristics and expected returns. With the help of Table 6, we now illustrate the model's implications for the conditional relation between expected return and size and book-to-market, respectively.

The methodology, known as double sorting, is a simple extension of the single sorting employed above. In every period, stocks are sorted in nine different portfolio, depending on size and book-to-market.

As long as market value is monotone increasing in capital and idiosyncratic productivity, conditional on size, portfolios with higher book–to–market must be characterized by greater average capital and lower productivity. It follows that risk – and expected return – increases with book–to–market. For the same reason, conditional on book-to-market, larger firms must have greater capital and idiosyncratic productivity on average. Therefore, risk – and expected return – decreases with size.

5 Capital Adjustment Costs

In this section, we explore the role of quadratic adjustment costs in shaping the cross–section of equity returns. Starting from the benchmark model – model III above – we progressively lower ϕ_1 until it reaches zero, keeping all other parameters the same.

For the sake of completeness, we report all simulated moments in Table D.3. As adjusting becomes cheaper, the mean and volatility of the investment rate increase, while the autocorrelation declines. Indeed, the autocorrelation is negative in the case without quadratic adjustment cost – a well–known result.

Table 6: Double Sorted Portfolios on Size and Book–to–Market ($\alpha = 0.3$ and OL)

	Low BM	Medium BM	High BM	H-L	Low BM	Medium BM	High BM	H-L
		Equity Returns (%)				**Size**		
Small Size	2.003	2.112	2.136	0.134	0.185	0.167	0.169	-0.016
Medium Size	1.713	1.731	1.746	0.033	0.256	0.250	0.242	-0.014
Large Size	1.494	1.462	1.504	0.010	0.355	0.368	0.343	-0.012
L–S	-0.509	-0.650	-0.632		0.170	0.202	0.174	
		Book-to-Market				**Investment Rate**		
Small Size	0.479	0.562	0.666	0.187	0.162	0.030	-0.087	-0.250
Medium Size	0.477	0.557	0.653	0.176	0.174	0.038	-0.084	-0.258
Large Size	0.476	0.560	0.647	0.171	0.200	0.055	-0.064	-0.264
L–S	-0.003	-0.002	-0.019		0.038	0.025	0.023	
		Capital				**Idiosyncratic Shock**		
Small Size	0.090	0.094	0.112	0.022	-0.224	-0.349	-0.383	-0.160
Medium Size	0.125	0.142	0.159	0.034	0.063	-0.002	-0.067	-0.130
Large Size	0.174	0.210	0.224	0.050	0.368	0.355	0.244	-0.124
L–S	0.083	0.116	0.111		0.592	0.704	0.627	
		Mass of Firms						
Small Size	0.042	0.094	0.061	0.020				
Medium Size	0.099	0.402	0.099	-0.000				
Large Size	0.048	0.103	0.052	0.005				
L–S	0.006	0.009	-0.009					

Now refer to Table 7. As we lower ϕ_1, we record an increase in the cross–sectional dispersion of all variables across book–to–market sorted portfolios, except for the idiosyncratic productivity.

In particular, the value premium increases. This finding can be rationalized with the intuition gained in Section 2. As ϕ_1 drops, the capital in place is closer to the efficient level at all times. In turn, this means that, everything else equal, value firms will pay out less dividends – this feature makes them riskier. Growth firms, on the other hand, will require less investment from shareholders – this makes them less risky.

6 The Stochastic Discount Factor

The purpose of this section is to understand to what extent our quantitative results depend upon key features of the stochastic discount factor. We start from the conditional

Table 7: Comparative Statics w.r.t. ϕ_1 – Book–to–Market Sorted Portfolios

	I	II	III	IV
	$\alpha = 0.3$; OL	$\alpha = 0.3$; OL	$\alpha = 0.3$; OL	$\alpha = 0.3$; OL
Excess Returns (%)	$\phi_1 = 0.009$	$\phi_1 = 0.006$	$\phi_1 = 0.003$	$\phi_1 = 0.000$
Growth Firms	1.708	1.712	1.701	1.701
Average BM	1.743	1.744	1.748	1.783
Value Firms	1.818	1.817	1.814	1.851
Value–Growth	0.100	0.105	0.113	0.150
Size				
Growth Firms	0.262	0.266	0.273	0.280
Average BM	0.257	0.258	0.259	0.251
Value Firms	0.250	0.254	0.259	0.257
Value–Growth	-0.012	-0.012	-0.013	-0.023
Book–to–Market				
Growth Firms	0.477	0.474	0.470	0.454
Average BM	0.558	0.559	0.560	0.561
Value Firms	0.657	0.662	0.670	0.684
Value–Growth	0.180	0.188	0.200	0.230
Investment Rate				
Growth Firms	0.176	0.193	0.223	0.302
Average BM	0.039	0.040	0.041	0.042
Value Firms	-0.076	-0.085	-0.097	-0.119
Value–Growth	-0.252	-0.278	-0.320	-0.421
Capital				
Growth Firms	0.128	0.129	0.131	0.130
Average BM	0.146	0.147	0.148	0.144
Value Firms	0.164	0.168	0.174	0.176
Value–Growth	0.036	0.039	0.043	0.046
Idiosyncratic Shock				
Growth Firms	0.075	0.072	0.071	0.0.073
Average BM	0.005	0.001	-0.004	-0.010
Value Firms	-0.081	-0.070	-0.054	-0.038
Value–Growth	-0.156	-0.142	-0.125	-0.111

variation of the risk–free rate.

Recall that our benchmark model features a countercyclical risk–free rate. In order to assess the role of this assumption, we proceed to analyze a scenario with constant risk–free rate. To that end, we set $\gamma_0 = -\rho_z \gamma_1$.

The maximum Sharpe ratio is unchanged, as it depends on γ_1 alone. The realized Sharpe ratio and the mean risk-free rate change very slightly. See Table D.4.

The same table also shows that switching to a constant risk–free rate has little impact on investment moments and on the cross-sectional mean of book–to–market. On the other hand, the effects on excess equity returns are large, due to a sizable decline in the volatility of returns.

The lower mean and volatility of realized equity returns are reflected in the returns on book–to–market sorted portfolios reported in Table 8. The value premium is only 1/4 of the benchmark value.

We have assumed throughout that the price of risk is constant. However, since Zhang (2005) emphasized the role of a time–varying price of risk in generating a sizable value premium in his framework, we are interested in assessing how our results would change if we assumed a countercyclical price of risk.

Unfortunately the functional form of our stochastic discount factor is not flexible enough to allow for such analysis. We resort instead to the pricing kernel specification of Jones and Tuzel (2013) and assume that

$$\log(M_{t+1}) = \log \beta - \frac{1}{2}\gamma_t^2 \sigma_z^2 - \gamma_t \sigma_z \varepsilon_{z,t+1},$$
$$\log \gamma_t = \gamma_0 + \gamma_1 z_t.$$

We set $\beta = 0.996$, $\gamma_0 = 3.275$, and $\gamma_1 = -15.75$ to match the first two unconditional moments of the risk–free rate, as well as the mean Sharpe ratio. Table D.4 shows that the unconditional moments of the investment rate and the average book–to–market are the same as in the case – considered above – where price of risk and risk–free rate are both constant.

A countercyclical price of risk implies higher equity returns. This is the case because – thanks to operating leverage – risk is higher in recessions.

Zeroing in on the last two columns of Table 8, one realizes that, with the exception of equity returns, the unconditional moments for book–to–market sorted portfolios are very similar to those obtained with constant price of risk. However, a countercyclical price of risk leads to a much larger value premium, of the same magnitude as in the benchmark case.

Table 8: The role of the Stochastic Discount Factor – Book–to–Market Sorted Portfolios

Excess Returns (%)	i) Countercyclical R^f ii) Constant Price of Risk	i) Constant R^f ii) Constant Price of Risk	i) Constant R^f ii) Countercyclical Price of Risk
Growth Firms	1.718	0.616	0.985
Average BM	1.743	0.622	1.017
Value Firms	1.818	0.641	1.085
Value–Growth	0.100	0.024	0.100
Size			
Growth Firms	0.262	0.233	0.243
Average BM	0.257	0.219	0.234
Value Firms	0.250	0.194	0.229
Value–Growth	-0.012	-0.039	-0.013
Book–to–Market			
Growth Firms	0.477	0.472	0.440
Average BM	0.558	0.554	0.515
Value Firms	0.657	0.653	0.604
Value–Growth	0.180	0.181	0.164
Investment Rate			
Growth Firms	0.176	0.173	0.172
Average BM	0.039	0.032	0.033
Value Firms	-0.076	-0.088	-0.083
Value–Growth	-0.252	-0.260	-0.255
Capital			
Growth Firms	0.128	0.110	0.106
Average BM	0.1146	0.121	0.120
Value Firms	0.1164	0.126	0.136
Value–Growth	0.036	0.016	0.030
Idiosyncratic Shock			
Growth Firms	0.075	0.096	0.066
Average BM	0.005	0.011	-0.001
Value Firms	-0.081	-0.122	-0.055
Value–Growth	-0.156	-0.219	-0.121

7 Conclusion

Differently from what argued in the extant literature,[11] the mechanism relating idiosyncratic productivity to risk and returns in the neoclassical model does not involve the

[11]See Zhang (2005) and Imrohoroglu and Tuzel (2014) among others.

interaction of convex adjustment costs and a countercyclical Sharpe ratio. Rather, the link between microeconomic profitability and risk is due to the simple fact that relatively unproductive firms derive most of their value from cash–flows far in the future, which are riskier. This is also why value firms – which are less productive – are less risky than growth firms.

Another difference between value and growth firms is that the former are bound to divest on average, while the latter will grow. This heterogeneity and its implication for payouts to shareholders, however, tends to reduce the value premium.

What next? It is obvious that the model needs to generate a greater variation in returns between growth and value firms. Arbitrarily increasing the volatility of idiosyncratic productivity shocks will not work, as it will produce counterfactual implications for the cross–section of investment and for the size premium. Our work also shows that tweaking with the stochastic discount factor or adjustment costs are not promising avenues.

We argue that better gains are expected from giving operating leverage another chance. This paper shows that modeling operating costs as a constant does not go far enough. Positing costs that are a linear function of installed capital won't work either, as it is almost isomorphic to increasing the depreciation rate.[12] This said, there definitely exist exogenous processes for overhead costs that are bound to improve the model's ability to generate a sizable value premium. The key questions are: 1) How do such processes compare with the empirical evidence? 2) How can they be rationalized as the outcomes of rational decision making?

The former question suggests to follow the lead of Liu, Whited, and Zhang (2009) by considering directly the empirical leverage distribution. The latter question, instead, suggests to model operating leverage as a choice, along the lines of Favilukis and Lin (2012) and Marfè (2014).

References

Anderson, C. W., and L. Garcia-Feijoo (2006): "Empirical Evidence on Capital Investment, Growth Options, and Security Returns," *Journal of Finance*, 61(1), 171–194.

Beaudry, P., and A. Guay (1996): "What Do Interest Rates Reveal About the Functioning of Realy Business Cycle Models?," *Journal of Economic Dynamics and Control*, 20, 1661–1682.

[12]It is not exactly isomorphic, because the depreciation rate enters the adjustment cost function.

Berk, J. B., R. C. Green, and V. Naik (1999): "Optimal Investment, Growth Options, and Security Returns," *Journal of Finance*, 54(5), 1553–1607.

Carlson, M., A. Fisher, and R. Giammarino (2004): "Corporate Investment and Asset Price Dynamics: Implications for the Cross–Section of Returns," *Journal of Finance*, 59(6), 2577–2603.

Clementi, G. L., and B. Palazzo (2014): "Cross–Sectional Properties of the Neoclassical Investment Model," Working paper, Boston University.

Cooley, T. F., and E. C. Prescott (1995): "Economic Growth and Business Cycles," in *Frontiers of Business Cycle Research*, ed. by T. F. Cooley, chap. Chapter 1, pp. 1–37. Princeton University Press.

Cooper, I. (2006): "Asset Pricing Implications of Nonconvex Adjustment Costs and Irreversibility of Investment," *Journal of Finance*, 61(1), 139–170.

Cooper, R., and J. Willis (2014): "Discounting: Investment Sensitivity and Aggregate Implications," Working Paper, Penn State.

Cooper, R. W., and J. C. Haltiwanger (2006): "On the Nature of Capital Adjustment Costs," *Review of Economic Studies*, 73(3), 611–633.

Dechow, P., R. Sloan, and M. Soliman (2004): "Implied Equity Duration: A New Measure of Equity Risk," *Review of Accounting Studies*, 9, 197–228.

Eckbo, E., and K. Thorburn (2008): "Corporate Restructuring: Breakups and LBOs," in *Handbook of Corporate Finance: Empirical Corporate Finance*, ed. by E. Eckbo, vol. 2, chap. 16, pp. 431–496. North-Hooland/Elsevier.

Eisfeldt, A., and A. Rampini (2006): "Capital Reallocation and Liquidity," *Journal of Monetary Economics*, 53, 369–399.

Fama, E. F., and K. R. French (1992): "The Cross-Section of Expected Stock Returns," *Journal of Finance*, 47(2), 427–465.

Favilukis, J., and X. Lin (2012): "Wage Rigidity: A Solution to Several Asset Pricing Wage Rigidity: A Solution to Several Asset Pricing Puzzles," Working Paper.

Foster, L., J. Haltiwanger, and C. Krizan (2001): "Aggregate Productivity Growth: Lessons from Microeconomic Evidence," in *New Developments in Productivity*

Analysis, ed. by C. R. Hulten, E. R. Dean, and M. J. Harper, pp. 303–363. University of Chicago Press.

Gala, V. (2006): "Irreversible Investment and the Cross-Section of Stock Returns in General Equilibrium," Working paper, Wharton.

Garlappi, L., and Z. Song (2013): "Can Investment Shocks Explain Value Premium and Momentum Profits?," Working paper, University of British Columbia.

Gomes, J. F., L. Kogan, and L. Zhang (2003): "Equilibrium Cross Section of Returns," *Journal of Political Economy*, 111(4), 693–732.

Guthrie, G. (2011): "A Note on Operating Leverage and Expected Equity Returns," *Finance Research Letters*, 8, 88–100.

Hackbarth, D., and T. Johnson (2014): "Real Options and Risk Dynamics," Working paper, Boston University.

Hennessy, C. A., and T. M. Whited (2005): "Debt Dynamics," *Journal of Finance*, 60(3), 1129–1165.

——— (2007): "How Costly is External Finance? Evidence from a Structural Estimation," *Journal of Finance*, 62(4), 1705–1745.

Imrohoroglu, A., and S. Tuzel (2014): "Firm Level Productivity, Risk, and Return," *Management Science*, 60(8), 2073–2090.

Jones, C., and S. Tuzel (2013): "Inventory Investment and the Cost of Capital," *Journal of Financial Economics*, 107(3), 557–5779.

Kogan, L., and D. Papanikolaou (2015): "Firm Characteristics and Stock Returns: The Role of Investment-Specific Shocks," *Review of Financial Studies*, 28(2), 379–402.

Liu, L. X., T. M. Whited, and L. Zhang (2009): "Investment-Based Expected Stock Returns," *Journal of Political Economy*, 117(6), 1105–1139.

Maksimovic, V., and G. Phillips (2001): "The Market for Corporate Assets: Who Engages in Mergers and asset sales and what are the efficiency gains?," *Journal of Finance*, 56, 2019–2065.

Marfè, R. (2014): "Income Insurance and the Equilibrium Term–Structure of Equity," Working Paper, Collegio Carlo Alberto.

McGrattan, E. R. (1996): "Solving the Stochastic Growth Model With a Finite Element Method," *Journal of Economic Dynamics and Control*, 20(1–3), 19–42.

Papanikolaou, D. (2011): "Investment Shocks and Asset Prices," *Journal of Political Economy*, 119(4), 639–685.

Schoar, A. (2002): "Effects of Corporate Diversification on Productivity," *Journal of Finance*, 57, 2379–2403.

Syverson, C. (2004): "Market Structure and Productivity: A Concrete Example," *Journal of Political Economy*, 112(6), 1181–1222.

Tauchen, G., and R. Hussey (1991): "Quadrature-Based Methods for Obtaining Approximate Solutions to Nonlinear Asset Pricing Models," *Econometrica*, 59(2), 371–396.

Xing, Y. (2008): "Interpreting the Value Effect Through the Q-Theory: An Empirical Investigation," *Review of Financial Studies*, 21(4), 1767–1795.

Zhang, L. (2005): "The Value Premium," *Journal of Finance*, 60(1), 67–103.

A Proofs

Proof of Lemma 1.

The current asset is itself a portfolio of two assets. One is conditionally riskless, since it pays $k_1(1-\delta)$ regardless of the state of nature. The other has a payoff $e^{s_1+z_1}k_1^\alpha$. The time–0 expected return of the latter is

$$\frac{E_0[e^{s_1+z_1}k_1^\alpha]}{E_0\left[M_1 e^{s_1+z_1}k_1^\alpha\right]} = \frac{E_0[e^{\varepsilon_{z,1}}]}{E_0[M_1 e^{\varepsilon_{z,1}}]}. \tag{2}$$

It follows that the expected return on the current asset is a weighted average of the conditional risk-free rate $R_{f,0}$ and (2), where the weight on the latter is

$$\frac{E_0\left[M_1[e^{s_1+z_1}k_1^\alpha]\right]}{E_0\left[M_1[e^{s_1+z_1}k_1^\alpha + k_1(1-\delta)]\right]} = \frac{R_{f,0}-(1-\delta)}{R_{f,0}-(1-\delta)(1-\alpha)}.$$

The weight on the short asset is

$$\frac{E_0\left[M_1[e^{s_1+z_1}k_1^\alpha + k_1(1-\delta)]\right]}{E_0\left[M_1[e^{s_1+z_1}k_1^\alpha + k_1(1-\delta)]\right] + E_0[M_1[-k_2 + E_1[M_2[y_2 + k_2(1-\delta)]]]]}.$$

The weight will be increasing in s_0 as long as the following quantity is decreasing:

$$\frac{E_0[M_1[-k_2 + E_1[M_2[y_2 + k_2(1-\delta)]]]]}{E_0\left[M_1[e^{s_1+z_1}k_1^\alpha + k_1(1-\delta)]\right]}.$$

Tedious algebra reveals that the latter can be rewritten as

$$e^{s_0\frac{\rho_s(\rho_s-1)}{1-\alpha}} \frac{\left[E[e^{\rho_s\varepsilon_s}]\right]^{1/(1-\alpha)}(1-\alpha)E_0\left[M_1\left(\frac{\alpha}{1-\frac{1-\delta}{R_{f,1}}}\right)^{\frac{\alpha}{1-\alpha}}\left[E_2[M_2e^{z_2}]\right]^{\frac{1}{1-\alpha}}\right]}{\left[E_0(M_1e^{z_1})\right]^{\frac{1}{1-\alpha}}\left[\left(\frac{\alpha}{1-\frac{1-\delta}{R_{f,0}}}\right)^{\frac{\alpha}{1-\alpha}} + \frac{1-\delta}{R_{f,0}}\left(\frac{\alpha}{1-\frac{1-\delta}{R_{f,0}}}\right)^{\frac{1}{1-\alpha}}\right]},$$

which is clearly decreasing in s_0, as $\rho_s \in (0,1)$.

■

Proof of Lemma 2.

We limit ourselves to show that the book-to-market is decreasing in s_0. Rewrite it as

$$\begin{aligned} BM =& \frac{E(k_0|s_0)}{k_1}\frac{k_1}{E_0[M_1[e^{s_1+z_1}k_1^\alpha + k_1(1-\delta)]] + E_0[M_1[-k_2 + E_1[M_2[y_2 + k_2(1-\delta)]]]]} \\ =& \frac{E(k_0|s_0)}{k_1}\widetilde{BM}. \end{aligned}$$

We have that

$$\log \widetilde{BM} = -\log\left(\frac{E_0[M_1[e^{s_1+z_1}k_1^{\alpha}+k_1(1-\delta)]]}{k_1}+\frac{E_0[M_1[-k_2+E_1[M_2[y_2+k_2(1-\delta)]]]]}{k_1}\right).$$

Since

$$\frac{E_0[M_1[e^{s_1+z_1}k_1^{\alpha}+k_1(1-\delta)]]}{k_1}=\left[\frac{\alpha}{1-\frac{1-\delta}{R_f}}\right]^{-1}+\frac{1-\delta}{R_f},$$

the first addendum in parenthesis does not depend on s_0. It follows that

$$\frac{\partial\log(\widetilde{BM})}{\partial s_0}=\frac{\rho_s(1-\rho_s)}{1-\alpha}\Omega_0,$$

where

$$\Omega_0=\frac{E_0[M_1[-k_2+E_1[M_2[y_2+k_2(1-\delta)]]]]}{E_0[M_1[e^{s_1+z_1}k_1^{\alpha}+k_1(1-\delta)]]+E_0[M_1[-k_2+E_1[M_2[y_2+k_2(1-\delta)]]]]}.$$

By Lemma 4, $s_{-1}|s_0$ is normally distributed with mean $\rho_s s_0$ and variance σ_s^2. It follows that

$$\begin{aligned}\log\left[\frac{E(k_0|s_0)}{k_1}\right]&=\log\left[E\left(e^{\frac{\rho_s s_{-1}}{1-\alpha}}|s_0\right)\right]-\frac{\rho_s s_0}{1-\alpha}\\&=\frac{\rho_s(\rho_s-1)}{1-\alpha}s_0+\frac{1}{2}\left(\frac{\rho_s\sigma_s}{1-\alpha}\right)^2.\end{aligned}$$

Finally,

$$\frac{\partial\log(BM)}{\partial s_0}=\frac{\rho_s(1-\rho_s)}{1-\alpha}\left[\Omega_0-1\right]<0.$$

■

Proof of Lemma 3.

Think of equity as being a portfolio consisting of the risk–free–asset as long as a composite of current and continuation business assets. The weight of the risk–free asset $-\frac{c_f/R^f}{E_0[M_1y_1]+E_0[M_1[-k_2+E_1[M_2y_2]]]-c_f/R^f}$ is negative and clearly increasing in s_0. That is, the short position on the risk–free asset declines with s_0. It follows that the long position on the composite of current and continuation assets also declines. Then the result follows from Lemma 1, which ensures that the return on the composite portfolio declines with s_0.

■

Lemma 4 *Let* $s_{t-1}\sim N\left(0,\frac{\sigma^2}{1-\rho^2}\right)$ *and* $s_t=\rho s_{t-1}+\varepsilon$*, with* $\varepsilon\sim N(0,\sigma^2)$*,* $\sigma>0$ *and* $\rho\in(0,1)$*. Then,* $E[s_{t-1}|s_t]=\rho s_t$*.*

Proof. For simplicity, let f denote the density of a Normal distribution with parameters $\left(0, \frac{\sigma^2}{1-\rho^2}\right)$. Let also g denote the density of a Normal distribution with parameters $\left(0, \sigma^2\right)$. It follows that

$$E[s_{t-1}|s_t] = \int \frac{s_{t-1} f(s_{t-1}) g(s_t - \rho s_{t-1}) ds_{t-1}}{\int f(s_{t-1}) g(s_t - \rho s_{t-1}) ds_{t-1}}.$$

To simplify notation further, let $\eta^2 \equiv \frac{\sigma^2}{1-\rho^2}$. Then,

$$f(s_{t-1}) g(s_t - \rho s_{t-1}) = \frac{1}{2\pi\sigma\eta} \exp\left[-\frac{1}{2}\left(\frac{s_{t-1}^2}{\eta^2} + \frac{(\rho s_{t-1} - s_t)^2}{\sigma^2}\right)\right].$$

Algebraic manipulations yield

$$\begin{aligned} f(s_{t-1}) g(s_t - \rho s_{t-1}) =& \frac{1}{2\pi\sigma\eta} \exp\left(-\frac{1}{2}\frac{(s_{t-1} - \rho s_t)^2}{\sigma^2}\right) \exp\left(-\frac{1}{2}\frac{s_t^2}{\sigma^2 + \eta^2\rho^2}\right) \\ =& \frac{1}{\sqrt{2\pi}\eta} \exp\left(-\frac{1}{2}\frac{s_t^2}{\sigma^2 + \eta^2\rho^2}\right) \times \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{1}{2}\frac{(s_{t-1} - \rho s_t)^2}{\sigma^2}\right). \end{aligned}$$

The latter expression is the product of a constant and the density of a Normal with mean ρs_t and variance σ^2. It follows that

$$E[s_{t-1}|s_t] = \int s_{t-1} \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{1}{2}\frac{(s_{t-1} - \rho s_t)^2}{\sigma^2}\right] = \rho s_t.$$

■

B Approximation of the value function

Here is a sketch of the algorithm we employed to approximate the firm's value function:

1. Start by defining grids for the state variables z, k, s. The grid for capital – denoted as Ψ_k – consists of 500 points and is constructed following the method suggested by McGrattan (1996). The stochastic processes driving the two shocks are approximated following Tauchen and Hussey (1991). We use 11 grid points for each of them.

2. For all grid elements (z, k, s), guess values for the value function $V_0(z, k, s)$.

3. For $r \geq 0$, a sequence of value functions is computed using the recursion

$$\begin{aligned} V_{r+1}(z, k, s) = \max_{k' \in \Psi_k} \; & e^{s+z} k^\alpha - x - \phi_0 k \chi - \phi_1 \left(\frac{x}{k}\right)^2 k - c_f + \\ & \sum_j \sum_i M(z, z_j) V_r(z_j, k', s_i) H(s_i|s) G(z_j|z), \\ \text{s.t. } \; & x = k' - k(1-\delta), \\ & \chi = 1 \text{ if } k' \neq k \text{ and } \chi = 0 \text{ otherwise.} \end{aligned}$$

4. Keep on iterating until $\sup \left| \frac{V_{r+1}(z,k,s) - V_r(z,k,s)}{V_r(z,k,s)} \right| < 10^{-6}$.

C Data

In this appendix, we provide details on the methodology employed in estimating the calibration targets, as well the series for idiosyncratic TFP illustrated in Figure 6. Our data is from the quarterly Compustat database for the period 1976q1–2014q3.

We start in 1976 because data on capital expenditures is not available for earlier years at the quarterly frequency. We exclude financial firms (SIC codes 6000 to 6999) and utilities (SIC codes 4900 to 4999) and we only consider firms incorporated in the United States and traded on the three major exchanges: NYSE, AMEX, and NASDAQ.

For every firm, we set the initial value of the capital stock equal to the first available entry of the series $PPEGTQ$ – i.e. the gross value of property, plant, and equipment. We recover the values at later dates using the recursion

$$K_{t+1} = K_t + PPENTQ_{t+1} - PPENTQ_t.$$

The term $PPENTQ_{t+1} - PPENTQ_t$, the difference between the net value of property, plant, and equipment in contiguous quarters, proxies for net investment.[13]

Notice that this amounts to assume that accounting depreciation is a good proxy for economic depreciation. This is questionable. It would be preferable to recover gross investment and then subtract economic depreciation, calculated in the same way as in the model simulation. We do otherwise, because we have 35% more observations for the variable $PPENTQ$ than we have for either $PPEGTQ$ or $DPACTQ$ – the change in accumulated depreciation.

C.1 The investment rate

We characterize the investment process for public firms in the United States, along the lines of what accomplished by Cooper and Haltiwanger (2006) for the universe of manufacturing establishments. The gross investment rate is set equal to

$$\frac{PPENTQ_t - PPENTQ_{t-1}}{K_{t-1}} + \delta,$$

where $\delta \equiv 0.03$ is the depreciation rate for all firms and all quarters.[14]

[13] We replace missing observations for $PPENTQ$ with a linear interpolation if the values of $PPENTQ$ in the quarter immediately before and immediately after the quarter with the missing observation are available. Imputed entries account for 0.6% of the total.

[14] For the firm-quarter observations for which $DPACTQ$ is available (around 60% of the total), we also compute firm-quarter specific depreciation rates. Their average is 0.028 and the implied investment rate statistics are very similar to the ones in Table C.1.

We rule out the firm-quarter observations associated with acquisitions larger than 5% of assets in absolute value and those yielding values for the investment rate in the top and bottom 0.5% of the distribution. Our cleaned dataset consists of 375,639 firm–quarter observations.

Table C.1 reports time–series averages of the four cross–sectional moments needed for the calibration: Mean, standard deviation and autocorrelation of the investment rate, as well as the fraction of firms with investment rate in absolute value less than 0.01 – the inaction rate.[15] In the last row is the average number of firms per quarter.

Table C.1: Investment Rate Summary statistics (1976q1–2014q3)

	I	II	III
	All Firms	**Manufacturing**	**Non Manufacturing**
Mean investment rate	0.047	0.043	0.053
Std. dev. investment rate	0.099	0.092	0.108
Inaction rate	0.136	0.142	0.129
Autocorrelation	0.264	0.269	0.251
Observations	2,423	1,345	1,078

In columns II and III, we report the statistics for the subsets of manufacturing (SIC codes 2000 to 3999) and non–manufacturing firms, respectively. All moments are quite similar across columns. We select as calibration targets those for manufacturing firms.

C.2 Idiosyncratic productivity

Our measure of productivity is known in the literature as Revenue Total Factor Productivity (TFPR).[16] Its variation reflects changes in technical efficiency, as well as shifts in input supply and product demand schedules affecting input and product prices, respectively.

We estimate a iso–elastic production function, where the inputs are labor, capital, and materials. Output is proxied by annual sales (item $SALE$), capital is the series computed above, and labor is total employment (item EMP). Materials are valued at the cost of

[15] The cross-sectional autocorrelation at time t ($t = 1, ..., T$) is the estimate of α_{1t} in the equation $x_{i,t} = \alpha_{0t} + \alpha_{1t} x_{i,t-1} + \varepsilon_{i,t}, i = 1, ..., N_t$, where $x_{i,t}$ is the investment rate of firm i at time t and N_t is the total number of firms for which we have data for both $x_{i,t}$ and $x_{i,t-1}$. The value reported in Table C.1 is the time–series average of the T autocorrelations. Pooled ordinary least squares deliver very similar results.

[16] See Foster, Haltiwanger, and Krizan (2001) and Syverson (2004) among others.

goods sold (item $COGS$), net of depreciation (item DP) and wage expenditures.[17] The data is annual, since employment is not available at the quarterly frequency. All dollar figures are recast at constant prices by means of the GDP deflator.

Firm–level productivity for firm i in year t is the residual of the regression

$$\log y_{i,t} = \beta_0 + \beta_l \log l_{i,t} + \beta_k \log k_{i,t} + \beta_m \log m_{i,t} + \beta_t D_t + \nu_i + \varepsilon_{i,t},$$

where D_t is a time dummy that controls for aggregate conditions, and ν_i is a firm fixed effect. We eliminate observations in the top and bottom 1% of the residuals' distribution to minimize the impact of outliers. It is reassuring that estimating an AR(1) process yields an annual persistence of 0.620, a value in line with the quarterly persistence of 0.90 used in the calibration.[18]

D Additional Tables

[17] Wage expenditures are total employment (item EMP) multiplied by 52 times the average weekly earnings of production and nonsupervisory employees in the private sector (BLS data code CES0500000030).

[18] Restricting the sample to manufacturing firms yields an annual persistence of 0.628.

Table D.1: Investment Sorted Portfolios

	I	II	III
	$\alpha = 0.6$	$\alpha = 0.6$	$\alpha = 0.3$
Excess Returns (%)	NO OL	OL	OL
Low IK	1.600	1.644	1.804
Average IK	1.599	1.642	1.770
High IK	1.578	1.604	1.707
High–Low	-0.022	-0.044	-0.097
Size			
Low IK	0.374	0.282	0.236
Average IK	0.384	0.293	0.251
High IK	0.407	0.314	0.274
High–Low	0.033	0.033	0.038
Book–to–Market			
Low IK	0.557	0.828	0.667
Average IK	0.494	0.725	0.570
High IK	0.457	0.655	0.502
High–Low	-0.100	-0.173	-0.166
Investment Rate			
Low IK	-0.085	-0.085	-0.097
Average IK	0.033	0.033	0.023
High IK	0.149	0.149	0.138
High–Low	0.233	0.233	0.235
Capital			
Low IK	0.248	0.248	0.158
Average IK	0.228	0.228	0.146
High IK	0.222	0.222	0.141
High–Low	-0.026	-0.026	-0.018
Idiosyncratic Shock			
Low IK	-0.063	-0.063	-0.107
Average IK	-0.017	-0.017	-0.022
High IK	0.050	0.050	0.085
High–Low	0.113	0.113	0.192

Table D.2: Comparative Static with respect to ρ_s

	Low ρ_s	Benchmark	High ρ_s
Panel A - Investment Rate (IK) and Book–to–Market			
Mean IK	0.040	0.040	0.041
Standard Deviation IK	0.093	0.091	0.091
Autocorrelation IK	0.271	0.271	0.272
Average Book-to–Market	0.654	0.564	0.434
Panel B - Cross–Section of Equity Returns			
Average Equity Return (%)	1.767	1.754	1.764
Dispersion Equity Return (%)	6.042	6.290	8.128
Panel C - Book–to–Market Sorted Portfolios (Equity Returns (%))			
Growth Firms	1.702	1.718	1.731
Medium BM	1.754	1.743	1.745
Value Firms	1.863	1.818	1.807
Panel D - Book–to–Market Sorted Portfolios (Idiosyncratic Shock)			
Growth Firms	0.107	0.075	0.057
Medium BM	0.007	0.004	-0.009
Value Firms	-0.115	-0.081	-0.023
Panel E - Parameters			
ρ_s	0.850	0.900	0.950
σ_s	0.123	0.105	0.094
ϕ_1	0.0110	0.0090	0.0075
c_f	0.0074	0.0070	0.0062
$\sigma_s/\sqrt{1-\rho_s^2}$	0.233	0.241	0.301

Table D.3: Comparative Statics w.r.t. ϕ_1 – Calibrated Targets

	I	II	III	IV
Investment Rate	$\alpha = 0.3$; OL $\phi_1 = 0.009$	$\alpha = 0.3$; OL $\phi_1 = 0.006$	$\alpha = 0.3$; OL $\phi_1 = 0.003$	$\alpha = 0.3$; OL $\phi_1 = 0.000$
Mean	0.040	0.042	0.046	0.056
Standard Deviation	0.091	0.100	0.114	0.147
Autocorrelation	0.271	0.209	0.118	-0.063
Inaction Rate	0.169	0.184	0.219	0.290
Book–to–Market	0.564	0.565	0.566	0.567
Risk-Free Rate and Sharpe Ratio				
Mean (%)	0.437	0.437	0.437	0.437
Standard Deviation (%)	1.154	1.154	1.154	1.154
Sharpe Ratio	0.208	0.207	0.207	0.204
Mean Excess Return (%)	1.317	1.316	1.316	1.302
St. Dev. Excess Return (%)	6.340	6.345	6.352	6.390

Table D.4: The role of Stochastic Discount Factor – Calibrated Targets

Investment Rate	i) Countercyclical R^f ii) Constant Price of Risk	i) Constant R^f ii) Constant Price of Risk	i) Constant R^f ii) Countercyclical Price of Risk
Mean	0.040	0.034	0.034
Standard Deviation	0.091	0.092	0.092
Autocorrelation	0.271	0.257	0.257
Inaction Rate	0.169	0.228	0.236
Book–to–Market	0.564	0.558	0.512
Risk-Free Rate and Sharpe Ratio			
Mean (%)	0.437	0.422	0.408
Standard Deviation (%)	1.154	0.014	0.020
Sharpe Ratio	0.208	0.223	0.204
Mean Excess Return (%)	1.317	0.203	0.618
St. Dev. Excess Return (%)	6.340	0.909	3.033