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NATURAL EXPERIMENT POLICY EVALUATION:
A CRITIQUE

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ABSTRACT

We argue exogenous random treatment is insufficient for valid inference regarding the sign and magnitude of causal effects in dynamic environments. In such settings, treatment responses must be understood as contingent upon the typically unmodeled policy generating process. With binary assignment, this results in quantitatively significant attenuation bias. With more than two policy states, treatment responses can be biased downward, upward, or have the wrong sign. Further, it is not only generally invalid to extrapolate elasticities across policy processes, as argued by Lucas (1976), but also to extrapolate within the same policy process. We derive auxiliary assumptions beyond exogeneity for valid inference in dynamic settings. If all possible policy transitions are rare events, treatment responses approximate causal effects. However, reliance on rare events is overly-restrictive as the necessary and sufficient conditions for equality of treatment responses and causal effects is that policy variable changes have mean zero. If these conditions are not met, we show how treatment responses can nevertheless be corrected and mapped back to causal effects or extrapolated to forecast responses to future policy changes.

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1. Introduction

The goal of most empirical work in economics is to measure the sign and magnitude of causal effects. Heckman (1999) defines a causal effect as follows.

Just as the ancient Hebrews were ‘the people of the book’ economists are ‘the people of the model.’ ...Within a model, the effects on outcomes of variation in constraints facing agents are well defined. Comparative statics exercises formalize Marshall’s notion of a *ceteris paribus* change which is what economists mean by a causal effect.

Correct empirical estimation of signs of causal effects is important given that a theory can be viewed as falsified if it incorrectly predicts a sign. Angrist and Pischke (2010) herald the search for sources of independent random assignment (or valid instruments) as amounting to a “credibility revolution” in empirical estimation of causal effects. Their textbook, *Mostly Harmless Econometrics*, states, “The goal of most empirical research is to overcome selection bias, and therefore to have something to say about the causal effect of a variable.” Indeed, an extant empirical literature spanning the fields of public finance, finance, labor, macroeconomics and environmental economics seeks to identify and exploit exogenous policy treatments, with satisfaction of the exclusion restriction equated with “credibility.”

In this paper, we show that in a broad and important class of economic environments, independent random assignment is insufficient for treatment responses to equal causal effects. Specifically, in dynamic environments in which policy variables evolve over time stochastically, treatment responses do not generally equal the causal effects empiricists seek to estimate. Critically, this is shown to be true even if policy variables are *independent* random variables. Worse still, absent a description of the data generating process beyond satisfaction of the exclusion restriction, the nature of the bias is unpredictable. Treatment responses can understate, overstate, and even have signs opposite to causal effects. Moreover, the biases are shown to be quite large under plausible parameterizations. As a final problem, we show that in terms of forecasting the economy’s response to future changes in policies, it is not only invalid to extrapolate across policy generating processes, as argued by Lucas (1976), but also *within* the same policy generating process. This is a severe limit on external validity.

These findings are far from theoretical curiosities. Rather, they cast doubt on the credibility, utilization, and interpretation of the types of elasticity estimates that are used to shape government policy decisions. As just one example, in surveying empirical evidence following the Tax Reform Act of 1986, Slemrod (1990) concludes “the short-term response has in most cases been less dramatic than many economists had expected.” Similarly, Slemrod (1992) describes a consensus view of a “downward revaluation of the responsiveness to taxation of real variables.” Based on this evidence, Slemrod (1992) and Aaron (1992) call for increased focus on the distributional consequences of taxation and less concern over excess burdens. Our arguments show that, even if one sets aside issues of policy endogeneity, such normative conclusions are premature since it is impossible to infer the economic meaning of an elasticity estimate absent a full description of the policy generating process.

We demonstrate our arguments by way of a model economy designed to serve as a perfect laboratory for conducting tests-of-tests. In particular, our model economy is hard-wired with “ideal” independent policy variable transitions. In this economy, we actually know the true theory-implied causal effects. Therefore, we are able to conduct simulated natural experiments in which treatment responses are measured and test statistics computed in relation to known-to-be-correct null hypotheses.

To illustrate how statistical inference can go awry in dynamic settings despite independent assignment, suppose the policy variable of interest, say tax depreciation rates, can take on three possible values. The theory-implied causal effect of accelerated depreciation on investment is unambiguously positive. However, the treatment response to an acceleration in the rate of tax depreciation can actually be negative. For example, suppose firms think there is a high probability of a high rate of tax depreciation rate being legislated. This expectation will be capitalized into the shadow value of installed capital, and firms will invest at a high rate. If a somewhat less generous tax depreciation schedule becomes law, the shadow value of capital will fall and firms will cut investment. Conversely, consider the same modest legislated increase in the rate of tax depreciation, but suppose firms had instead expected a deceleration. Here the treatment response will actually tend to overshoot the causal effect since firms would have invested at a low rate under the prior law in expectation of bad news.

Central to our argument is notion that treatment responses can only be understood and inter-

puted in relation to the underlying policy generating process, with satisfaction of the exclusion restriction shedding little light on the economic meaning of empirical estimates. In fact, implicit in much of the literature is the view that “more change is better.” For example, Slemrod (1992) writes, “Fortunately (for the progress of our knowledge, not for policy), since 1978 the taxation of capital gains has been changed several times, providing much new evidence on the tax responsiveness of realizations.” Similarly, in his analysis of the effect of the Clean Air Act, Greenstone (2001) writes, “The Amendments introduce substantial cross-sectional and longitudinal variation in regulatory intensity at the county level.” Cummins, Hassett and Oliner (1994) examine the response of real investment to 13 changes in the U.S. corporate income tax from 1962 until 1986. Romer and Romer (2010) identify 54 exogenous tax shocks during the post-war period.

Our results indicate that a desire to find natural experiments with numerous policy shocks is generally misplaced. Although time-series variation is useful in a purely statistical sense, in that it allows one to control for fixed effects and lowers standard errors, such variation must also be understood as arising from an underlying stochastic policy process with frequent transitions and high variance. Data on business activity, say, must be understood as generated by firms making optimal decisions when confronted with the same stochastic process exploited by the econometrician. Since, as we show, frequent policy transitions often drive treatment responses further from causal effects, the net result can be a biased estimate with a tight confidence interval.

After pointing out the pitfalls associated with the analysis of treatment responses armed only with exclusion restrictions, the paper turns next to a *constructive* analysis of auxiliary assumptions needed to ensure valid inference in dynamic settings. As a first result, we show that treatment responses converge to causal effects if all possible policy transition rates tend to zero. In other words, all possible policy changes must come as a near-complete surprise and be viewed as near-permanent. However, reliance on unanticipated permanent policy transitions is troubling as an auxiliary assumption. After all, if the empiricist is on the lookout for events taking place with infinitesimal probability, she will be waiting a rather long time for evidence. In addition, it is not clear why policymakers should be interested in historic responses to changes in a policy variable if there is infinitesimal probability of that same policy variable being changed in the future. Finally, as the work cited above illustrates, it is apparent that much empirical work in the policy arena is concerned with policy variables with non-trivial transition probabilities.

Given the inherent limitations in relying upon rare events for identification, it would be useful if valid inference of causal effects could be obtained under a broader class of policy generating processes. Therefore, the paper next turns to a general statement of auxiliary assumptions required for valid inference. Conveniently, it is shown that treatment responses are equal to causal effects if and only if the expected change in the policy variable is equal to zero. That is, each policy state must be absorbing or, if a transition out of a given state is possible, the mean change must be zero. The intuition for this result is simple. Causal effects measure responses to hypothetical policy changes that are completely unanticipated and permanent. Under the stated auxiliary assumptions, policy variables are martingales, and so the shadow value of capital and labor extrapolates the current policy state into perpetuity. That is, firms act as if the current policy state will last forever even though they know that it generally will not.

As a final constructive step, we show how causal effects can be inferred from treatment responses for all possible policy generating processes, not just those meeting the auxiliary assumptions described above. We also show how treatment responses can be extrapolated across different transitions under the same policy generating process, or across different policy generating processes.

Our paper draws inspiration from Lucas (1976). Our analysis differs along a number of dimensions. Most importantly, Lucas does not analyze the relation between exogenous natural experiments and causal effects, nor does he discuss the potential for overshooting, sign reversals, or invalid test statistics in natural experiments. Further, Lucas does not derive conditions under which treatment responses correctly gauge causal effects in natural experiments. Of course, Lucas' work pre-dates the "credibility revolution." As in Lucas (1976), we show the limits of external validity of empirical evidence, here of the experimental variety. Moving beyond his critique in relation to forecasting the effects of future policy changes, we show there is no reason to think elasticities can be extrapolated *within* the same policy generating process, while Lucas discussed problems in extrapolating elasticities across policy generating processes. Constructively, we show how to extrapolate observed treatment responses between and within policy processes, as well as back to causal effects.

In related work, Abel (1983) analyzes the effect of permanent versus temporary tax policies in a perfect foresight economy. One can view our work as extending his analysis to an economy featuring uncertainty and idealized exogenous regime-shifting policies with multiple states. As we

show, allowing for the realistic possibility of more than two policy states is critical to understanding how treatment responses can have the wrong sign, overshoot causal effects, or fail as forecasters of future policy responses. More generally, we argue that the common practice of positing a dichotomy between unanticipated versus anticipated policy changes, or between temporary and permanent changes, is too coarse an approximation of reality to capture the types of biases that can emerge. Further, such coarse categorizations prevent the type of quantitative bias corrections we show to be feasible.

Auerbach (1986) and Auerbach and Hines (1988) present investment Euler equations under stochastic tax rates. Hassett and Metcalf (1999) present a real options model with a one-off investment which they use to assess whether uncertainty regarding investment tax credits encourages or discourages investment. Gourio and Miao (2008) numerically compare the effects of permanent and temporary dividend tax cuts. The model presented here draws on the q-theoretic model of Abel and Eberly (1997) which is particularly convenient given that it yields analytic solutions even as we extend it to a more complex setting with regime shifts.

Our critique of the random assignment literature is related to, but distinct from the critique made by Heckman (1997) and Keane (2010) who argue that individuals and households can be expected to endogenously undermine random assignment—especially when the randomly assigned treatment is burdensome. In our laboratory economy, firms are incapable of avoiding the respective policy treatments. Still, they adjust their behavior in a way that can lead to biased statistical inference regarding causal effects.

Related concerns have been expressed about the interpretability and economic content of estimates arising from instrumental variables (IV) in the context of a single treatment applied to a cross-section of heterogeneous agents. Heckman (1997) points out that the probability limit of IV estimators can depend on the choice of instrument. Our critique is different in that it applies when there is no instrumentation and no cross-sectional heterogeneity.

The remainder of the paper is as follows. Section 2 develops a theory of dynamic investment and labor hiring and derives causal effects. Section 3 describes our simulated laboratory for tests-of-tests. Section 4 evaluates the potential for biases in standard treatment response estimators and test statistics. Section 5 describes limits on biases and specifies auxiliary assumptions ensuring equality of treatment responses and causal effects. Section 6 derives bias corrections for environments where

the auxiliary assumptions are violated.

2. Neoclassical Theory of Investment and Labor Demand

In order to set the stage for our tests-of-tests, we must first derive causal effects of government policies based on some underlying theory. To this end, this section articulates a neoclassical q-theory of capital and labor demand for firms facing taxation and regulation. The model of Abel and Eberly (1997) is extended to incorporate three government policy variables: tax depreciation schedules; minimum wages; and environmental taxes on variable fuel inputs. The aim is to compute the causal effects (Marshallian comparative statics) of these policy variables as implied by the underlying theory. In subsequent sections we assess whether empirical estimates obtained via natural policy experiments actually recover these causal effects.

2.1. Technology

Consider a competitive firm producing an output flow each instant utilizing the Cobb-Douglas production function $\zeta n^\alpha s^{1-\alpha}$. The productivity variable is $\zeta > 0$, which can be stochastic or constant. The production input n is fuel. It is perfectly flexible and has an effective unit price $p > 0$. The government can vary p by imposing a fuel surcharge. The production input s is a stock variable which can only be adjusted at a cost. Depending on the application, we will think of s as being either physical capital or the number of workers on the firm's payroll. The variable s depreciates at rate $\delta \geq 0$. For applications in which labor is the stock variable, δ is interpreted as the rate at which workers quit. There is a flow cost $w \geq 0$ associated with each unit of the stock variable employed by the firm.

The price-taking firm sells its output at a price ρ which is stochastic. Let $y \equiv \rho\zeta$ and let z denote a standard Wiener process. Uncertainty regarding productivity and output price is captured by the fact that y evolves as a geometric Brownian motion, with

$$dy = mydt + \nu ydz. \tag{1}$$

In applications with a cross-section, each firm will be endowed with an independent Wiener process.

Operating profits are denoted π , with

$$\pi \equiv \max_n \quad yn^\alpha s^{1-\alpha} - pn - ws. \tag{2}$$

It follows that instantaneous operating profits can be expressed as:

$$\pi(s, x) = [x\kappa(p) - w]s, \quad (3)$$

with

$$\begin{aligned} \kappa(p) &\equiv (1 - \alpha)\alpha^{\alpha/(1-\alpha)}p^{-\alpha/(1-\alpha)} \\ x &\equiv y^{\frac{1}{1-\alpha}}. \end{aligned} \quad (4)$$

Note that the profitability factor κ depends only on the deep structural parameter α and the fuel price p . For simplicity, the remainder of the paper treats the government as directly determining κ , and we will speak of κ as the fuels tax policy variable, with κ understood to be decreasing in the effective fuel price p according to equation (4).

It is most simple to let x replace y as a state variable.¹ From Ito's lemma it follows that x also evolves as a geometric Brownian motion, with

$$dx = \mu x dt + \sigma x dz \quad (5)$$

where

$$\begin{aligned} \mu &\equiv \frac{m}{1 - \alpha} + \frac{1}{2} \frac{\alpha \nu^2}{(1 - \alpha)^2} \\ \sigma &\equiv \frac{\nu}{1 - \alpha}. \end{aligned} \quad (6)$$

It is worth noting that the drift μ of the state variable x is increasing in the volatility ν of the firm's output price. Ultimately, this will imply optimal instantaneous accumulation is increasing in price volatility, a standard effect in real options models.

The firm regulates its holdings of the stock variable s through its instantaneous accumulation policy a . The stock variable evolves according to:

$$ds = (a - \delta s)dt \quad (7)$$

$$s_0 > 0.$$

Following Abel and Eberly (1997), it is assumed s_0 is sufficiently large so that full depletion of the stock can be ignored. Each unit of the stock variable can be purchased and sold at a price ψ .

¹Abel and Eberly (1997) treat y as the state variable.

The price ψ is treated as strictly positive in those applications where the stock variable is physical capital and is treated as zero when the stock variable is labor. For simplicity we follow Abel and Eberly, in assuming the firm faces quadratic costs to adjusting the stock variable equal to γa^2 , where $\gamma > 0$. However, as described below, our results generalize to a broader class of adjustment cost functions.

The firm pays tax at rate $\tau \geq 0$ on operating profits less depreciation deductions. Depreciation for tax purposes may differ from economic depreciation. The tax depreciation rate is $\xi\delta$, with $\xi > 1$ corresponding to accelerated depreciation.

The instantaneous cash flow (c) accruing to shareholders is equal to operating profit less taxes less accumulation costs:

$$\begin{aligned} c(s, x, a) &= (\kappa x - w)s - \tau(\kappa x - w - \xi\delta\psi)s - \psi a - \gamma a^2 \\ &= [(1 - \tau)(\kappa x - w) + \tau\xi\delta\psi]s - \psi a - \gamma a^2. \end{aligned} \quad (8)$$

All agents are risk-neutral and discount cash flows at rate $r > 0$.

2.2. Optimal Accumulation

The firm chooses its accumulation policy each instant to maximize the sum of expected capital gains and cash flows. Applying Ito's lemma, we have the following Bellman equation:

$$rV(s, x) = \max_a (a - \delta s)V_s(s, x) + \mu x V_x(s, x) + \frac{1}{2}\sigma^2 x^2 V_{xx}(s, x) + [(1 - \tau)(\kappa x - w) + \tau\xi\delta\psi]s - \psi a - \gamma a^2. \quad (9)$$

We conjecture and then verify the value function is of the form:

$$V(s, x) = sq(x) + G(x). \quad (10)$$

Notice, the conjectured value function splits firm value into two terms. The term $sq(x)$ measures the value of cash flows generated by units of the stock variable held by the firm. The term G measures the net present value generated by the optimal exercise of future growth and contraction options.

To pin down the optimal instantaneous control policy, we can isolate those terms in the Bellman equation that involve the accumulation variable a . The optimal policy solves:

$$\max_a aV_s(s, x) - \psi a - \gamma a^2. \quad (11)$$

Under the conjectured specification of the value function, $V_s(s, x) = q(x)$, and the optimal accumulation policy is:

$$a(x) = \frac{q(x) - \psi}{2\gamma}. \quad (12)$$

Evaluated at the optimal policy, the instantaneous net gain attributable to accumulation is:

$$\begin{aligned} aq - \psi a - \gamma a^2 &= (q - \psi)^2 \Gamma \\ \Gamma &\equiv \frac{1}{4\gamma}. \end{aligned} \quad (13)$$

Substituting into the Bellman equation the conjectured value function, as well as the expression for the instantaneous gain from optimal accumulation given in equation (13), we obtain:

$$\begin{aligned} rsq(x) + rG(x) &= -\delta sq(x) + \mu xsq_x(x) + \mu xG_x(x) + \frac{1}{2}\sigma^2 x^2 sq_{xx}(x) + \frac{1}{2}\sigma^2 x^2 G_{xx}(x) \\ &+ [(1 - \tau)(\kappa x - w) + \tau\xi\delta\psi]s + [q(x) - \psi]^2 \Gamma. \end{aligned} \quad (14)$$

Since the Bellman equation must hold point-wise on the state space as characterized by (s, x) pairs, the derivatives with respect to s of the left and right side of the preceding equation must match. We then have the following differential equation describing the evolution of the shadow value of the stock variable:

$$(r + \delta)q(x) = \mu xq_x(x) + \frac{1}{2}\sigma^2 x^2 q_{xx}(x) + (1 - \tau)\kappa x + \tau\xi\delta\psi - (1 - \tau)w. \quad (15)$$

From the preceding ordinary differential equation and the Feynman-Kac formula it follows that:

$$q(x_0) = E \left[\int_0^\infty e^{-(r+\delta)t} [(1 - \tau)\kappa x_t + \tau\xi\delta\psi - (1 - \tau)w] dt \mid \mathfrak{F}_0 \right]. \quad (16)$$

That is, q is simply the discounted value of the expected net marginal product, with the discount rate set to $r + \delta$ to account for depreciation of capital and worker quits.

As in Abel and Eberly (1997) we rule out bubbles causing valuations to explode as x goes to zero or infinity. We obtain the following solution to equation (15):

$$q(x) = \frac{(1 - \tau)\kappa x}{r + \delta - \mu} + \frac{\tau\xi\delta\psi - (1 - \tau)w}{r + \delta}. \quad (17)$$

Substituting the shadow value into the optimality condition (12), we obtain the following analytical expression for the optimal policy:

$$a(x; \xi, w, \kappa) = \frac{1}{2\gamma} \left[\left(\frac{(1 - \tau)\kappa}{r + \delta - \mu} \right) x + \frac{\tau\xi\delta\psi - (1 - \tau)w}{r + \delta} - \psi \right]. \quad (18)$$

A complete model solution requires computing the value of growth options (G). However, since our objective is to analyze causal effects, the policy function in equation (18) is sufficient. The growth option value is derived in the appendix.

2.3. Theory-Implied Causal Effects

We are interested in determining the causal effects (Marshallian comparative statics) as implied by the theory, which is here just the classical q-theory of investment under quadratic adjustment costs. We have the following comparative statics:

$$\begin{aligned} \frac{\partial}{\partial \xi} a(x; \xi, w, \kappa) &= \frac{1}{2\gamma} \frac{\tau \delta \psi}{r + \delta} > 0 \\ \frac{\partial}{\partial w} a(x; \xi, w, \kappa) &= -\frac{1}{2\gamma} \frac{(1 - \tau)}{r + \delta} < 0 \\ \frac{\partial}{\partial \kappa} a(x; \tau, \xi, w, \kappa) &= \frac{1}{2\gamma} \left[\frac{(1 - \tau)x}{r + \delta - \mu} \right] > 0. \end{aligned} \tag{19}$$

It is worth noting that each causal effect is linear in its respective government policy parameter since the second derivatives are equal to zero. Thus, the causal effect of a discrete change in each policy parameter is invariant to its initial point and proportional to the size of the change.

Based on the preceding comparative statics the empiricist or theorist might offer the following list of Causal Effects.

Causal Effects under the Q-Theory with Quadratic Adjustment Costs: Investment increases linearly with the depreciation rate allowed for tax purposes; Hiring decreases linearly with the minimum wage; Investment and hiring increase linearly with the profitability factor κ which is inversely related to the environmental surcharge on fuel inputs according to equation (4).

3. The Model Laboratory

Our primary objective is to assess whether standard econometric tests will correctly detect the sign and magnitude of causal effects implied by an underlying theory, here the canonical q-theory of capital and labor demand. Conveniently, we can use the model itself as an idealized laboratory for conducting tests-of-tests. After all, in contrast to real-world environments, here we actually know the true theory-implied causal effects as expressed analytically in equation (19).

In reality, endogeneity of government policy variables is a difficult hurdle to clear in correct empirical estimation of causal effects. However, we have the luxury of being able to stipulate a

stochastic environment of our own choosing. Since the problems associated with policy endogeneity are apparently well-understood, we rule it out by construction. We simply give the econometrician her “identifying assumption” of random assignment for free. In particular, we consider an economy in which the evolution of government policy variables is described by an *independent* stochastic process.

3.1. The Policy Generating Process

We assume government policy variables follow an independent N -state continuous-time Markov chain.² Government policy toward business is described by the tax depreciation schedule (ξ), minimum wage (w), and the firm profitability parameter κ (which is inversely related to the fuel per equation (4)). We assume that in all possible policy regimes there is an excess supply of workers at the minimum wage so that the sole determinant of employment is the demand for labor. The government policy vector in regime i is (ξ_i, w_i, κ_i) with $i = 1, \dots, N$, and $N \geq 2$. In each of our thought experiments, only one of the three policy variables will be treated as time-varying, since causal effect analysis entails holding fixed other policy variables.

We adopt an indexing convention that ranks policies according to their favorableness. In particular:

$$\begin{aligned} w_1 &< \dots < w_N \\ \xi_1 &> \dots > \xi_N \\ \kappa_1 &> \dots > \kappa_N. \end{aligned}$$

That is, state 1 is the *best state* and state N is the *worst state* if one ranks the states according to their implications for the firm’s present-date instantaneous after-tax cash flow.

We recall some basic properties of continuous-time Markov chains. The parameter $\lambda_{ij} \geq 0$ denotes the *transition rate* from regime i to regime j . Let

$$\Lambda_i \equiv \sum_{j \neq i} \lambda_{ij}.$$

The amount of time the policy vector remains in state i before transitioning to another state is an exponentially distributed random variable with parameter Λ_i . If $\Lambda_i > 0$, the expected life of regime

²See Ross (1996) for a detailed exposition.

i is computed as $1/\Lambda_i$. If $\Lambda_i = 0$, then state i is *absorbing*. If Λ_i is tending to ∞ , then state i is said to be *ephemeral*.³ Conditional upon a transition out of state i taking place, the probability of transitioning into state j is

$$P_{ij} \equiv \frac{\lambda_{ij}}{\sum_{j \neq i} \lambda_{ij}}.$$

3.2. Model Solution

Having described the environment, we turn next to a characterization of optimal accumulation. To begin, we recognize that equation (8), which described the instantaneous cash flow accruing to shareholders under constant government policies, must be rewritten to account for regime-shifts. Instantaneous cash flow in regime i is:

$$c^i(s, x, a) = [(1 - \tau)(\kappa_i x - w_i) + \tau \xi_i \delta \psi]s - \psi a - \gamma a^2. \quad (20)$$

The value of the firm in policy regime i is denoted V^i . Accounting for the possibility of regime changes, we replace the original Bellman equation (9) with the following system of N Bellman equations:

$$\begin{aligned} rV^1(s, x) &= \max_a (a - \delta s)V_s^1(s, x) + \mu x V_x^1(s, x) + \frac{1}{2} \sigma^2 x^2 V_{xx}^1(s, x) + \sum_{j \neq 1} \lambda_{1j} [V^j(s, x) - V^1(s, x)] \\ &\quad + [(1 - \tau)(\kappa_1 x - w_1) + \tau \xi_1 \delta \psi]s - \psi a - \gamma a^2 \\ &\dots \\ rV^N(s, x) &= \max_a (a - \delta s)V_s^N(s, x) + \mu x V_x^N(s, x) + \frac{1}{2} \sigma^2 x^2 V_{xx}^N(s, x) + \sum_{j \neq N} \lambda_{Nj} [V^j(s, x) - V^N(s, x)] \\ &\quad + [(1 - \tau)(\kappa_N x - w_N) + \tau \xi_N \delta \psi]s - \psi a - \gamma a^2. \end{aligned} \quad (21)$$

Notice, the sole change from the original program in equation (9) is the need to account for the possibility of instantaneous regime changes leading to concomitant capital gains to shareholders.

Following the same procedure as in the model with constant government policies, we conjecture and then verify value functions that are separable between the value of assets in place and growth options. We conjecture the solution to the Bellman system (21) has the following functional form:

$$V^1(s, x) = q^1(x)s + G^1(x); \dots; V^N(s, x) = q^N(x)s + G^N(x). \quad (22)$$

³Ross (1996) calls such states "instantaneous states."

To pin down the optimal control policy, we isolate terms in the Bellman equation system involving the accumulation variable. It follows that an optimal control policy solves:

$$a^i(x) \in \arg \max_a \quad aV_s^i(s, x) - \psi a - \gamma a^2; \quad i = 1, \dots, N. \quad (23)$$

Under the value function conjectured in equation (22), we have $V_s^i(s, x) = q^i(x)$ and the optimal regime-contingent accumulation policy is:

$$a^i(x) = \frac{q^i(x) - \psi}{2\gamma}; \quad i = 1, \dots, N. \quad (24)$$

Notice, equation (24) implies that investment and hiring will jump each time there is a policy transition giving rise to a jump in the shadow value.

Evaluated at the optimal policy, the instantaneous net gain attributable to accumulation is:

$$a^i(x)q^i(x) - \psi a^i(x) - \gamma[a^i(x)]^2 = [q^i(x) - \psi]^2\Gamma. \quad (25)$$

Substituting the optimized accumulation gain from equation (25) and the conjectured value functions into the original system of Bellman equations, we can rewrite the Bellman system as:

$$\begin{aligned} & \left(r + \delta + \sum_{j \neq 1} \lambda_{1j} \right) q^1(x)s + \left(r + \sum_{j \neq 1} \lambda_{1j} \right) G^1(x) \\ = & \mu x[sq_x^1(x) + G_x^1(x)] + \frac{1}{2}\sigma^2 x^2[sq_{xx}^1(x) + G_{xx}^1(x)] + \sum_{j \neq 1} \lambda_{1j}[q^j(x)s + G^j(x)] \\ & + [(1 - \tau)(\kappa_1 x - w_1) + \tau \xi_1 \delta \psi]s + [q^1(x) - \psi]^2\Gamma \\ & \dots \\ & \left(r + \delta + \sum_{j \neq N} \lambda_{Nj} \right) q^N(x)s + \left(r + \sum_{j \neq N} \lambda_{Nj} \right) G^N(x) \\ = & \mu x[sq_x^N(x) + G_x^N(x)] + \frac{1}{2}\sigma^2 x^2[sq_{xx}^N(x) + G_{xx}^N(x)] + \sum_{j \neq N} \lambda_{Nj}[q^j(x)s + G^j(x)] \\ & + [(1 - \tau)(\kappa_N x - w_N) + \tau \xi_N \delta \psi]s + [q^N(x) - \psi]^2\Gamma. \end{aligned} \quad (26)$$

Since the Bellman equations must be satisfied point-wise, the derivatives with respect to s of the preceding equations must match for each i . Thus, the following system of N equations must be

satisfied:

$$\begin{aligned}
\left(r + \delta + \sum_{j \neq 1} \lambda_{1j} \right) q^1(x) &= \mu x q_x^1(x) + \frac{1}{2} \sigma^2 x^2 q_{xx}^1(x) + \sum_{j \neq 1} \lambda_{1j} q^j(x) + (1 - \tau)(\kappa_1 x - w_1) + \tau \xi_1 \delta \psi \quad (27) \\
&\dots \\
\left(r + \delta + \sum_{j \neq N} \lambda_{Nj} \right) q^N(x) &= \mu x q_x^N(x) + \frac{1}{2} \sigma^2 x^2 q_{xx}^N(x) + \sum_{j \neq N} \lambda_{Nj} q^j(x) + (1 - \tau)(\kappa_N x - w_N) + \tau \xi_N \delta \psi.
\end{aligned}$$

Applying the Feynman-Kac formula to an arbitrary differential equation in the preceding system allows us to write:

$$\begin{aligned}
q^i(x_0) &= E \left[\int_0^\infty e^{-(r+\delta+\Lambda_i)t} \left((1 - \tau)(\kappa_i x_t - w_i) + \tau \xi_i \delta \psi + \Lambda_i \sum_{j \neq i} P_{ij} q^j(x_t) \right) dt \mid \mathfrak{F}_0 \right] \quad (28) \\
&= E \left[\int_0^\infty \left(\int_0^T e^{-(r+\delta)t} ((1 - \tau)(\kappa_i x_t - w_i) + \tau \xi_i \delta \psi) dt \right. \right. \\
&\quad \left. \left. + e^{-(r+\delta)T} \sum_{j \neq i} P_{ij} q^j(x_T) \right) (\Lambda_i e^{-\Lambda_i T}) dT \mid \mathfrak{F}_0 \right].
\end{aligned}$$

The second expression for the shadow value offered above follows directly from the first via integration by parts. It states that the shadow value is just an expectation over the current regime life, which is exponentially distributed, of the net marginal product up to the regime change plus the expectation of the new shadow value after the regime change.

We conjecture a no-bubbles solution to system (27) that is linear in x , as was the case in the constant government policy model. Specifically, accounting for regime shifts, we now conjecture:

$$q^i(x) = x d_i + D_i; \quad i = 1, \dots, N. \quad (29)$$

Substituting the conjectured linear solutions into the system of equations described in (27), it follows that the unknown constants can be found as solutions to systems of linear equations. Specifically, we have:

$$\begin{aligned}
\begin{bmatrix} q^1(x) \\ \dots \\ q^N(x) \end{bmatrix} &= \underbrace{[\mathbf{T}(\tilde{R})]^{-1} \begin{bmatrix} (1 - \tau)\kappa_1 \\ \dots \\ (1 - \tau)\kappa_N \end{bmatrix}}_{\equiv \mathbf{d}} x + \underbrace{[\mathbf{T}(\hat{R})]^{-1} \begin{bmatrix} \tau \delta \psi \xi_1 - (1 - \tau)w_1 \\ \dots \\ \tau \delta \psi \xi_N - (1 - \tau)w_N \end{bmatrix}}_{\equiv \mathbf{D}}. \quad (30)
\end{aligned}$$

where $\mathbf{T}(R)$ denotes the following *augmented transition matrix*:

$$\mathbf{T}(R) \equiv \begin{bmatrix} R + \sum_{j \neq 1} \lambda_{1j} & -\lambda_{12} & \dots & -\lambda_{1N} \\ -\lambda_{21} & R + \sum_{j \neq 2} \lambda_{2j} & \dots & -\lambda_{2N} \\ \dots & \dots & \dots & \dots \\ -\lambda_{N1} & -\lambda_{N2} & \dots & R + \sum_{j \neq N} \lambda_{Nj} \end{bmatrix} \quad (31)$$

and:

$$\begin{aligned} \tilde{R} &\equiv r + \delta - \mu \\ \hat{R} &\equiv r + \delta. \end{aligned}$$

For example, if there are only two possible policy states, the preceding shadow value expressions can be written as:

$$\begin{aligned} q^i(x) = & \left(\frac{\kappa_i}{r + \delta - \mu} + \frac{\lambda_{ij}(\kappa_j - \kappa_i)}{(r + \delta - \mu)(r + \delta - \mu + \lambda_{ij} + \lambda_{ji})} \right) (1 - \tau)x \\ & + \frac{\tau \delta \psi \xi_i - (1 - \tau)w_i}{r + \delta} + \frac{\lambda_{ij}[(1 - \tau)(w_i - w_j) - \tau \delta \psi (\xi_i - \xi_j)]}{(r + \delta)(r + \delta + \lambda_{ij} + \lambda_{ji})}. \end{aligned} \quad (32)$$

Naturally, the preceding equation shows that the current-state shadow value is influenced by the probability of transitioning into the other state, a transition that would lead to a jump in q .

To complete the model solution, we must also compute the value function for growth options (G). However, since our objective is to analyze causal effects in relation to treatment responses, the policy function in equation (24) and the shadow value vector (30) are sufficient. Derivation of the growth option value is provided in the appendix.

4. Evaluating Natural Experiment Econometrics

This section considers the interpretation of empirical evidence and hypothesis tests obtained from natural policy experiments. Recall, in Section 2 we derived the causal effects implied by the underlying theory, here the neoclassical q-theory. Section 3 considered the behavior of firms embedded in an economy in which the econometrician enjoys access to exogenous policy assignments. This section considers whether the econometrician will be able to recover the causal effects implied by the theory. In addition, we will also examine the validity of using an observed treatment response, perhaps for a cross-section of firms, to forecast future treatment responses.

From equation (24) it follows that the optimal accumulation policy takes the form

$$\begin{bmatrix} a^1(x) \\ \dots \\ a^N(x) \end{bmatrix} = \frac{1}{2\gamma} \left[\begin{bmatrix} q^1(x) \\ \dots \\ q^N(x) \end{bmatrix} - \begin{bmatrix} \psi \\ \dots \\ \psi \end{bmatrix} \right] \quad (33)$$

with equation (30) providing analytical expressions for the vector of regime-contingent shadow values.

The treatment response associated with a transition from state i to j is denoted TR_{ij} . Let:

$$TR_{ij}(x) \equiv a^j(x) - a^i(x) = \frac{1}{2\gamma} [q^j(x) - q^i(x)]. \quad (34)$$

In fact, it is already apparent at this stage in the analysis that it is generally invalid to use the treatment response in one economy to forecast the treatment response in another economy, even if the policy change is the same and the economies have identical real technologies. To see this, note that equation (30) implies that the jump in the shadow value and optimal accumulation in the event of a policy change depends upon the underlying policy transition matrices, which are likely to differ across economies. This is the natural-experiment corollary of the argument made by Lucas (1976) regarding limits on extrapolation. We return to this issue in Section 6.

Consider now the estimation of causal effects. Recall, Section 2 showed that causal effects are linear in the underlying theory. Therefore, the causal effect associated with a policy change is properly computed by multiplying the relevant partial derivative in equation (19) with the respective policy variable change. Variable-by-variable, we have the following theory-implied causal effects:

$$\begin{aligned} \xi & : CE_{ij} \equiv \frac{1}{2\gamma} \left(\frac{\tau\delta\psi}{r+\delta} \right) (\xi_j - \xi_i) \\ w & : CE_{ij} \equiv -\frac{1}{2\gamma} \left(\frac{1-\tau}{r+\delta} \right) (w_j - w_i) \\ \kappa & : CE_{ij} \equiv \frac{x}{2\gamma} \left[\frac{1-\tau}{r+\delta-\mu} \right] (\kappa_j - \kappa_i). \end{aligned} \quad (35)$$

It is worth noting that the causal effect magnitudes in equation (35) can actually be computed by replacing the true transition rates in equation (30) with zeroes ($\lambda_{ij} = 0$ for all i and j). That is, causal effects can be computed by acting “as if” each policy state is absorbing. Of course, if each state were actually absorbing there would be no policy transitions to learn from.

4.1. Binary Treatments and the Problem of Attenuation Bias

We begin by considering the simplest possible setting, one featuring binary policy treatments. For example, this setting is akin to that studied by Greenstone (2001) in his analysis of the effect of environmental regulations on firm activity. Assume that each instant firms are assigned to one of two regulatory categories, Attainment or Non-Attainment, based upon measured pollutants in the jurisdiction of their respective factories. Measured pollutants evolve exogenously as a result of weather patterns and air flows. Finally, suppose that Non-Attainment status results in a constant fuel surcharge per-unit. As shown in equation (4), such fuel input surcharges would transform the profitability scalar κ into a binary random variable with the profitability factor under Non-Attainment (κ_2) falling below that under Attainment (κ_1).

Although not reported directly by Greenstone (2001), his paper is rare in the quasi-experimental literature in that one can infer the transition probabilities that are consistent with his reported sample statistics. The implied instantaneous rate of transitioning from Attainment to Non-Attainment status in his sample is $\lambda_{12} = .042$ while the implied instantaneous rate of transitioning from Non-Attainment to Attainment status is $\lambda_{21} = .42$.

Figure 1 plots the optimal investment policy functions ($a(x)$) arising from assignment to Attainment versus Non-Attainment status under the assumption that assignment to the latter category results in an increase in the fuel price sufficient to bring about a 30% reduction in κ . The figure considers two policy generating processes: the real-world policy generating process approximated by transition parameters $\lambda_{12} = .042$ and $\lambda_{21} = .42$ versus a hypothetical setting in which each assignment category is absorbing. Recall, the latter setting captures the causal effect of the contemplated regulation-induced decrease in operating profits. In particular, for each value of the stochastic demand factor x , the causal effect is computed as the wedge between investment under Attainment versus Non-Attainment status when both categories are absorbing.

As shown in Figure 1, the causal effect of environmental regulation is much larger than measured treatment responses. In particular, one can see that firm investment under the temporary designation of Non-Attainment status is far from the level of investment the firm would implement if the designation were permanent. And this is to be expected given that the expected duration of Non-Attainment status is only 2.5 years. After all, optimal investment depends upon the expected

future marginal product of capital taking into account future changes in status. In fact, in the present setting the treatment response to a transition from Non-Attainment to Attainment status can be expressed as a percentage of the causal effect as follows:

$$\begin{aligned} TR_{21}(x) &= \left(1 - \frac{\lambda_{12} + \lambda_{21}}{r + \delta - \mu + \lambda_{12} + \lambda_{21}}\right) \left(\frac{1}{2\gamma}\right) \left[\frac{(1 - \tau)(\kappa_1 - \kappa_2)}{r + \delta - \mu}\right] x \\ &= \left(1 - \frac{\lambda_{12} + \lambda_{21}}{r + \delta - \mu + \lambda_{12} + \lambda_{21}}\right) \times CE_{21}(x). \end{aligned} \quad (36)$$

The preceding equation follows directly from the formula for the shadow value of capital under binary assignment given in equation (32), with treatment responses equal to $\Delta q/2\gamma$. Using the transition parameters derived from Greenstone (2001), if one assumes a discount rate of 10%, a depreciation rate of 10% and a drift of 0%, the treatment response is less than one-third of the causal effect. A policymaker would need to take account of this large difference if she were to contemplate the imposition of permanent environmental regulations.

Moving away from the particular example of environmental regulations, the following lemma offers a simple formula for calculating treatment responses relative to causal effects for settings in which policy treatments are binary.

Lemma 1 *If there are two possible policy states, the treatment response is*

$$\begin{aligned} TR_{ij}(x) &= \underbrace{\left(1 - \frac{\lambda_{ij} + \lambda_{ji}}{r + \delta - \mu + \lambda_{ij} + \lambda_{ji}}\right)}_{\text{Attenuation}} \underbrace{\left(\frac{1}{2\gamma}\right) \left(\frac{1}{r + \delta - \mu}\right) (1 - \tau)(\kappa_j - \kappa_i)x}_{CE_{ij}} \\ &+ \underbrace{\left(1 - \frac{\lambda_{ij} + \lambda_{ji}}{r + \delta + \lambda_{ij} + \lambda_{ji}}\right)}_{\text{Attenuation}} \underbrace{\left(\frac{1}{2\gamma}\right) \left(\frac{1}{r + \delta}\right) [\delta\psi\tau(\xi_j - \xi_i) - (1 - \tau)(w_j - w_i)]}_{CE_{ij}}. \end{aligned}$$

The preceding lemma follows directly from the formula for the shadow value under binary assignment given in equation (32), with treatment responses equal to $\Delta q/2\gamma$. The lemma leads to a number of observations. First, with binary assignment, there is always a proportional attenuation bias, with severity depending on the sum of the transition rates. Second, it is apparent that if either regime is ephemeral, the treatment response will be infinitesimal and the attenuation bias is roughly 100% of the causal effect. Conversely, if both regimes are near-permanent, then treatment responses will approximate causal effects.

Third, the treatment response formula reveals that with binary assignment attenuation bias is quite severe even if one considers settings with relatively long expected policy durations. For example, suppose one considers the conservative case of expected policy durations equal to 10 years ($\lambda_{ij} = \lambda_{ji} = 0.10$). In this case, attenuation bias exceeds one-half of causal effects under standard parameterizations featuring $r + \delta < 0.20$. Similarly, the size of the attenuation bias exceeds one-half under plausible parameterizations if one considers, say, a permanent transition out of a policy regime with a five-year expected duration or an unexpected transition to a new regime with a five-year life. Although Slemrod (1990) is not clear regarding the basis for his priors regarding the expected size of responses to TRA86, these back-of-the-envelope calculations suggest that small responses to the legislation might well have been expected given the fact that firms at the time surely attached a high probability to some version of tax reform being implemented.

In fact, the problem of attenuation bias extends beyond settings with binary policy treatments. For example, consider next a setting in which all transition rates are equal. In particular, suppose that for all i and j with $i \neq j$ we have $\lambda_{ij} = \lambda > 0$. In this case, each policy regime has an expected life equal to the inverse of $(N - 1)\lambda$. And if a transition out of an arbitrary state i takes place, the conditional probability of each other regime is $1/(N - 1)$. That is, transitions are uniformly distributed over each of the remaining states. We have the following lemma.

Lemma 2 *If all transition rates are equal to λ , with N policy states the treatment response is*

$$\begin{aligned}
TR_{ij}(x) = & \left(1 - \underbrace{\frac{N\lambda}{r + \delta - \mu + N\lambda}}_{\text{Attenuation}} \right) \underbrace{\left(\frac{1}{2\gamma} \right) \left(\frac{1}{r + \delta - \mu} \right) (1 - \tau)(\kappa_j - \kappa_i)x}_{CE_{ij}} \\
& + \left(1 - \underbrace{\frac{N\lambda}{r + \delta + N\lambda}}_{\text{Attenuation}} \right) \underbrace{\left(\frac{1}{2\gamma} \right) \left(\frac{1}{r + \delta} \right) [\delta\psi\tau(\xi_j - \xi_i) - (1 - \tau)(w_j - w_i)]}_{CE_{ij}}.
\end{aligned}$$

As shown in the appendix, the preceding lemma follows from direct calculation of the shadow value using equation (30) and the fact that $\Delta a = \Delta q/2\gamma$. A number of observations emerge. First, an obvious implication of Lemma 2 is that as $N\lambda$ tends to infinity, and expected policy durations tend to zero, so too do treatment responses. Conversely, as $N\lambda$ tends to zero, and expected policy durations tend to infinity, the treatment responses approximate causal effects. Second,

it is noteworthy that in the setting considered with uniform random assignment, the treatment response associated with a transition from regime i to j only depends upon the magnitude of policy variables in these two regimes. This may seem surprising given that the shadow value capitalizes the possibility of entering all other regimes. However, in the special case of identical transition rates, the capitalized value of transitioning into each of the remaining regimes enters symmetrically into both q^i and q^j so that $q^j - q^i$ is invariant to the magnitude of policy parameters in the other regimes. As discussed below, this case is atypical in that it will generally be the case that TR_{ij} depends on the probabilities and magnitudes of counterfactual policy variables.

4.2. Overshooting and Incorrect Signing of Causal Effects

The settings described in the previous subsection, featuring binary assignment or equality of all transition rates, are comforting inasmuch as the empiricist can claim treatment responses measured in such environments are “conservative” estimates of causal effects. Of course, it is not clear why one would want conservative estimates. After all, conservative estimates of elasticities lead to downward bias in the estimated deadweight loss arising from government interventions. Further, magnitudes, as distinct from signs, are often used as a basis for falsifying underlying theories. Therefore, downward biases open up the possibility of false-falsifications. Finally, and perhaps most importantly, it is apparent that the settings described in the previous setting are atypical. It is seldom the case that policy variables are binary, and it is seldom the case that policy transitions are uniformly distributed.

These arguments notwithstanding, one might hope that the settings considered in the preceding subsection might still be instructive about the nature of the wedge between treatment responses and causal effects. After all, conventional wisdom holds that agents will respond less aggressively to transient government policies, so attenuation bias may be expected to be a general feature of natural policy experiments. To explore these issues, consider an economy with three possible tax depreciation rates: $\xi_1 = 1.5$; $\xi_2 = 1$ and $\xi_3 = 0.5$. Consider then treatment responses and test statistics under the following transition rate configurations.

$$\begin{bmatrix} \cdot & \lambda_{12} & \lambda_{13} \\ \lambda_{21} & \cdot & \lambda_{23} \\ \lambda_{31} & \lambda_{32} & \cdot \end{bmatrix} = \begin{bmatrix} \cdot & .1 & .1 \\ .1 & \cdot & .1 \\ .1 & .1 & \cdot \end{bmatrix}_A ; \begin{bmatrix} \cdot & .02 & .02 \\ .02 & \cdot & .2 \\ .02 & .02 & \cdot \end{bmatrix}_B \begin{bmatrix} \cdot & .1 & .1 \\ .1 & \cdot & .1 \\ .19 & .01 & \cdot \end{bmatrix}_C ; \begin{bmatrix} \cdot & .01 & .01 \\ .01 & \cdot & .01 \\ .24 & .01 & \cdot \end{bmatrix}_D$$

Table 1 and Figure 2 describe the results from 1000 simulated experiments using these policy transition matrices. In each experiment there are 1000 firms with i.i.d. treatment response parameters. The treatment response of an arbitrary firm m is:

$$\begin{aligned} TR_{ij}^m &= \Omega_m(q_j - q_i) \\ \Omega_m &\equiv \frac{1}{2\gamma_m} \end{aligned} \tag{37}$$

with each random variable Ω_m drawn from a truncated normal distribution with mean $\mu_\Omega = 2.6$ and standard deviation $\sigma_\Omega = 1.2$. The expected causal effect in each experiment is 26 ($= 10\mu_\Omega$), with the change in q equal to 10 on a causal effect basis (unanticipated and permanent change). Each panel in Figure 2 is performed in relation to the different policy transition matrices described immediately above. The null hypothesis for each t-statistic is the true expected causal effect of 26. A valid estimator will cause the test statistic to have t-distribution with mean zero and variance 1. Effectively, we have a matched pairs sample since we measure each firm's investment the instant before and the instant after the policy change.

Consider first the Matrix A [Panel A] results. Here the policy generating process features equal transition rates for all regimes and so there is an attenuation bias of equal magnitude for all possible policy transitions. Importantly, attenuation bias shifts the t-statistic downward so that the null hypothesis, which is known to be true, is rejected too often.

An empiricist might view attenuation as a relatively benign form of bias, since the treatment responses still have the right sign, with attenuation bias implying that the treatment responses may be viewed as conservative estimates. However, Panel B of Figure 2 shows that it is actually possible for treatment responses to exceed causal effects, contrary to the conventional notion that transience diminishes responsiveness. The figure depicts firm responses to positive news in the form of a transition from state 2 to state 1. The overshooting effect can be understood as follows. In Economy B there is an asymmetry in that in the event of occupying the intermediate tax depreciation state (2), a transition to the slowest tax depreciation state (3) is more likely than a transition to the fast tax depreciation state (1). In this case, firms choose a low rate of investment in state 2, factoring in the likelihood of higher future taxes. Therefore, in the event of a transition to state 1, investment increases dramatically. This example illustrates that a "policy surprise" is not necessarily conducive to correct causal inference. After all, in the present setting, it is the actually

the surprising nature of the transition from state 2 to 1 that is responsible for overshooting. This overshooting leads to a tendency to over-reject the null hypothesis, which is known to be true.

The two parts of Panel C illustrate the potential for asymmetric responses. In particular, one sees that the treatment response associated with a transition from state 3 to 2 differs from the response associated with a transition from state 2 to 1. And this holds despite the fact that the true causal effects for these transitions are equal by construction. It follows then that it would be invalid to directly extrapolate treatment responses across these two transitions. That is, the historic response to one transition is not directly informative about how the economy will respond to a future transition of equal size, despite the fact that causal effects under the stated theory are, in fact, linear. This can be viewed as a more severe limit on extrapolation than argued by Lucas (1976), as here we see it is invalid to extrapolate across transitions within the same policy generating process.

Still, the empiricist may still take comfort from the fact that all examples furnished up to this point have the feature that treatment responses have the correct sign. However, Panel D shows that random assignment itself is insufficient to guarantee that treatment responses will correctly measure the *sign* of causal effects. In particular, in Economy D investment actually decreases when the tax depreciation rate transitions from low (state 3) to medium (state 2). The intuition for this effect is as follows. In Economy D, the low tax depreciation rate regime has a fairly low expected life of four years. Further, conditional upon a transition out of that regime, there is only a 4% chance of a transition to the medium rate regime. In such an environment, firms will invest at a high rate even in the worst regime. The firms will be disappointed then and find it optimal to reduce investment if there is a transition from state 3 to 2 rather than from 3 to 1. The importance of this example, and the countless others one can construct featuring sign reversals, is that even with ideal random assignment empirical estimation is susceptible to the problem of false-falsification of underlying theories.

In order for treatment responses to overshoot causal effects, or have the wrong sign, it must be the case that the number of policy states is sufficiently high. After all, it follows from Lemma 1 that if the policy variable is binary, treatment responses have the correct sign and are attenuated relative to causal effects. However, as shown in the appendix, if the policy variable can take on three or more values, one cannot rule out the possibility of an incorrect sign or overshooting absent

auxiliary assumptions placing restrictions on the policy generating process. We have the following lemma.

Lemma 3 *If there are at least three policy states, there exists a continuum of transition rates such that a proper subset of treatment responses are opposite in sign to their respective causal effects. Furthermore, there exists a continuum of policy transition intensities such that a proper subset of treatment responses are greater than their respective causal effects.*

It is instructive to consider analytically a policy process generating a sign reversal. To this end, suppose the only policy variable varying over time is the minimum wage, which can take on three possible values, $w_h > w_m > w_l$. For simplicity, assume the only positive transition rates are λ_{hl} and λ_{hm} . That is, the medium and low minimum wage regimes are absorbing. Equation (30) implies that under this policy generating process the shadow value of workers in the medium and high minimum wage states are:

$$\begin{aligned} q^m(x) &= \frac{(1-\tau)\kappa x}{\tilde{R}} + \frac{\tau\xi\delta\psi - (1-\tau)w_m}{\hat{R}} \\ q^h(x) &= \frac{(1-\tau)\kappa x}{\tilde{R}} + \frac{\tau\xi\delta\psi}{\hat{R}} - \frac{1-\tau}{\hat{R}(\hat{R} + \lambda_{hl} + \lambda_{hm})}(\lambda_{hl}w_l + \lambda_{hm}w_m + \hat{R}w_h). \end{aligned} \quad (38)$$

Firms will actually find it optimal to reduce hiring in the event of a reduction in the minimum wage from w_h to w_m , implying a treatment response opposite in sign from the causal effect, provided that λ_{hl} is sufficiently high. In particular:

$$\lambda_{hl} > \frac{\hat{R}(w_h - w_m)}{w_m - w_l} \Rightarrow q^h(x) > q^m(x) \Rightarrow TR_{hm} < 0. \quad (39)$$

Intuitively, in the present example, firms will reduce hiring in response to a small reduction in the minimum wage if they had attached a sufficiently high probability to a larger cut in the minimum wage. It is interesting to note that the critical value of λ_{hl} such that a sign reversal occurs is actually independent of λ_{hm} . That is, a sign reversal can occur regardless of the *relative* likelihood of transitioning to w_m and w_l . Similarly, the sign reversal can occur if the transition from w_h to w_m is expected (high λ_{hm}) or unexpected (low λ_{hm}). Thus, it is apparent that a loose rule of thumb such as looking for “surprises/non-surprises” is inadequate. It is also worth noting that the critical value of λ_{hl} such that sign reversals occur (equation (39)) is increasing in the value of

the counterfactual wage w_l . This implies that firms will cut employment in response to mid-size reduction in the minimum wage if they attach a high probability to a large reduction. But they will also cut employment in response to a mid-size reduction in the minimum wage if they attach a low probability to a very large reduction. Finally, it is worth recalling that Lemma 2 showed that with uniform policy assignment the treatment response associated with an arbitrary transition from regime i to j is independent of the value of policy variables in all counterfactual states $k \notin \{i, j\}$. However, the present example shows this is not a general property. Rather, understanding the potential for bias requires explicit description of the probability and nature of all states, including unrealized counterfactual states.

Let us next consider analytically a policy generating process such that the treatment response overstates the causal effect. Again, assume the only policy variable that varies over time is the minimum wage, which takes on three possible values, $w_h > w_m > w_l$. For simplicity, assume the high and low minimum wage regimes are absorbing, so that the only positive transition rates are λ_{ml} and λ_{mh} . We are interested in conditions such that the employment response to a cut in the minimum wage from w_m to w_l exceeds the causal effect, or:

$$TR_{ml} = \frac{1}{2\gamma} [q^l(x) - q^m(x)] > \left(\frac{1}{2\gamma} \right) \frac{(1 - \tau)(w_m - w_l)}{\hat{R}} \equiv CE_{ml}. \quad (40)$$

Using equation (30) to compute the shadow values in the preceding equation, one finds that the treatment response exceeds the causal effect if the conditional expectation of the change in the minimum wage is positive. That is, in the present example:

$$\left(\frac{\lambda_{mh}}{\lambda_{ml} + \lambda_{mh}} \right) (w_h - w_m) + \left(\frac{\lambda_{ml}}{\lambda_{ml} + \lambda_{mh}} \right) (w_l - w_m) > 0 \Rightarrow TR_{ml} > CE_{ml}. \quad (41)$$

Intuitively, under the stated inequality, a reduction in the minimum wage has a disproportionate effect on employment given that the shadow value of workers in the medium state was disproportionately influenced by the high wage scenario. It is also worth noting that the preceding inequality is unaffected if one multiplies λ_{ml} and λ_{mh} by some scalar $k > 0$. Thus, overshooting can occur even if a transition is unexpected (k small) or expected (k large).

Thus far, this subsection has considered treatment responses associated with one-step jumps in policy variables. One might hope that treatment responses become reliable estimators of causal effects provided that policy changes are sufficiently large. As shown in the appendix, size of treatment does not imply immunity from bias.

Lemma 4 *If there are at least four policy states, there exist policy transition rates such that the treatment response associated with a transition from the worst to best state (best to worst state) is negative (positive).*

As shown in the next subsection, it would be an overstatement to claim that “anything can happen” in the realm of natural policy experiments. However, all of the examples provided in this section illustrate starkly that a wide range of potential biases can arise in such settings, despite orthogonality of policy assignment. In particular, absent specification of restrictions on the policy generating process, one cannot guarantee that treatment responses have the right sign and that treatment responses do not overstate causal effects. It is apparent then that auxiliary assumptions are necessary.

Before proceeding, it is worth noting that the types of biases demonstrated in this section are not reliant on a quadratic adjustment cost function. Rather, the same biases would also occur if one posited that adjustment costs took the form of some other convex function of a . To see this, note that the source of bias is the wedge between the shadow value of the stock variable under transitory policies versus its shadow value under permanent policy assignment, while the shadow value is itself invariant to the adjustment cost function here.

5. Bias Mitigation and Auxiliary Assumptions

The previous section showed that a range of biases can emerge in natural policy experiments even with exogenous assignment: attenuation bias, overshooting, and sign reversals. We begin this section by describing some limits on these biases. We then move on to a statement of necessary and sufficient conditions for treatment responses to equal causal effects.

5.1. Limits on Bias in Natural Experiments

This subsection considers some limitations on the nature and severity of bias that can arise in natural policy experiments. Consider first the problem of attenuation bias. One natural question to ask is whether there exists any policy generating process such that all treatment responses are equal to zero. The answer is no, as shown in the following lemma.

Lemma 5 *There is no set of policy transition rates such that all treatment responses are equal to zero.*

Of course, the preceding lemma provides only a modicum of comfort in relation to the problem of attenuation bias. After all, the lemma still allows for the possibility of severe downward bias. And further, when it comes to attenuation bias, there is no guarantee that biases cancel out when averaged across the various possible transitions. To see this, recall that Lemma 1 showed that both possible treatment responses are biased downward in settings with binary assignment. Similarly, Lemma 2 showed that all possible treatment responses are biased downward under uniform policy assignment.

Consider next the problem of treatment responses overstating causal effects. Here the set of damage controls is a bit more extensive. For example, the following lemma shows that regardless of the assumed transition rates, overshooting cannot possibly occur in the case of worst-to-best state transitions.

Lemma 6 *The treatment response associated with a transition from the worst to best state is always less than its respective causal effect.*

The preceding lemma follows from the fact that with time-varying policies, the shadow value in the best (worst) state cannot be greater than (less than) the shadow value under permanently best (worst) state government policies (formula (28)). Therefore, the change in accumulation in the event of a policy transition cannot exceed the jump that would occur in the event of a completely unanticipated and permanent shift in government policies from the worst to best state.

Intuition suggests that there might be other factors mitigating the extent to which treatment responses will overshoot causal effects. To motivate the argument, suppose there are three policy states with, say, low, medium and high minimum wages. Suppose also that the low and high minimum wage states are absorbing. In this case, the worker shadow values in those two states are equal to their shadow values under permanent assignment. Suppose now that the inequality in equation (41) is satisfied. Then in the medium wage state the conditional expectation of the wage change is positive. This implies the shadow value of workers in the medium state is less than its value under permanent assignment to that state, implying low hiring in this state. Consequently, the treatment response to a transition from medium to low minimum wages will overshoot the causal effect. However, this same argument implies the treatment response to a transition from medium to high minimum wages will actually understate its respective causal effect (in absolute

value terms). The following lemma shows this pattern is more general.

Lemma 7 *If there exists a transition from state j to a (better) state $i < j$ with treatment response exceeding its respective causal effect by $k > 0$, then the sum of the treatment responses for transitions from state i to the best state (1) and from the worst state (N) to j must fall below the sum of their respective causal effects by at least k .*

Consider finally the problem of sign reversals. Lemma 3 showed that there exist transition rates such that for some transitions the sign of the treatment response will differ from causal effects. Moreover, Lemma 4 showed sign reversals can occur even for worst to best state transitions. However, sign reversals cannot be universal. In particular, the next lemma implies that there is no set of transition rates such that each treatment response differs in sign from its respective causal effect.

Lemma 8 *There is at least one state such that the treatment response associated with a transition from it to the best state is positive, and there is at least one state such that the treatment response associated with a transition to it from the worst state is positive.*

5.2. Necessary and Sufficient Conditions for No-Bias

The examples and proofs in Section 4 demonstrate that exogenous random assignment is not a sufficient condition for equality of treatment responses and causal effects. This subsection considers the types of auxiliary assumptions needed to ensure either rough or exact equivalence between treatment responses and causal effects.

We begin with an examination of conditions that ensure a rough equivalence between treatment responses and causal effects. To motivate these conditions, it is worth returning to the method used to derive causal effects in the underlying theory. Recall, the causal effects in equation (19) were obtained by differentiating the optimal policy function with respect to policy variables that were treated as parameters. One can think of such exercises as allowing one to compare firm behavior in one parameterized economy with firm behavior in another parameterized economy. Alternatively, one can think of such exercises as measuring how firms would respond to a completely unanticipated and permanent change in a policy variable. Therefore, one would expect treatment

responses to approximate causal effects in the event that the policy generating process featured almost completely unanticipated policy changes which are near-permanent. Consistent with this intuition, we have the following lemma.

Lemma 9 *As each transition rate tends to zero, each treatment responds tends to its respective causal effect.*

The preceding lemma follows from the fact that as each transition rate tends to zero, shadow values under regime-shifting policies (equation (28)) tend to the shadow values under permanent policies (equation (16)).

In reality, exogenous policy experiments are rare. Those meeting the auxiliary assumptions of being nearly-completely unanticipated and near-permanent are even more rare. Therefore, it would be useful to derive auxiliary assumptions with broader real-world applicability. Conveniently, we have the following proposition.

Proposition 10 *Necessary and sufficient conditions for treatment responses to equal causal effects for each policy transition are that the best and worst policy states are absorbing while remaining states are either absorbing or feature policy variable changes with conditional expectation equal to zero.*

The intuition for the proposition is as follows. Causal effects can be evaluated by computing how firms will respond to completely unanticipated and permanent changes in policy variables. Therefore, treatment responses will equal causal effects if the shadow value of the state variable (in each policy regime) under time-varying government policies is equal to the respective shadow value under permanent government policies. With regime-shifts, shadow values capitalize the expected future value of policy variables at all future dates. But note that under the conditions described in Proposition 10, the expected value of the policy variable at each future date is the current value. That is, policy variables are martingales. And if policy variables are martingales, the shadow value of capital and labor effectively capitalize the current policy state as lasting into perpetuity. This same argument explains the necessity of the extreme policy states being absorbing. After all, starting at the extreme states, the policy variable can only change in one direction so the mean change cannot be zero unless no change is possible. For example, starting in the worst policy state

for depreciation deductions, say zero deduction allowed ($\xi_N = 0$), the only possible change is an acceleration of tax depreciation.

It is also worth noting at this stage that Lemma 9 and Proposition 10 actually hold for the more general class of adjustment cost functions that are convex in accumulation (a), not just the specific quadratic function we initially posited for analytical simplicity. This is because shadow values will be invariant to adjustment costs in such cases, and it is the shadow value departure from its value under permanent policy assignment that creates a wedge between treatment responses and causal effects.

Figure 3 contains the results of 1000 simulated natural experiments consisting of 1000 i.i.d. firms with heterogeneous adjustment costs. Heterogeneity in adjustment costs leads to heterogeneity in treatment effects. The results of a similar analysis was depicted in Table 1 and Figure 2 but for different policy generating processes. Figure 3 provides the distribution of simulated t-statistics against the true null hypothesis under policy generating processes meeting the conditions described in Lemma 9 and Proposition 10. Consistent with the analytical results, the tests will reject the known-true null hypothesis at the right probability level.

Many of our results can be viewed as destructive in that they show the limits, perils and pitfalls associated with causal inference via random assignment in dynamic settings. In fact, in some cases empiricists caveat their findings with a paean to vaguely-defined notions of “expectations.” In extreme case, they will look to avoid confronting policy expectations by arguing that policy changes are unexpected and expected to be permanent. And indeed, Lemma 9 shows that if the environment considered really does fit this rare event criterion, then treatment responses are a good approximation of causal effects. However, Proposition 10 shows that it is not necessary for empiricists to confine their attention to rare events. After all, the proposition shows that even in settings with frequent changes in policy variables, treatment responses capture causal effects provided the conditional mean change is zero. One expects that many policy experiments will fit this description. However, it is also important to note that many policy variables will not have this martingale property. For example, one might reasonably expect some policy variables to be mean-reverting or to have reflecting barriers at extreme values.

A natural question to ask at this point is how severe the bias will be for policy generating processes that only approximate the conditions described in Proposition 10. And further, one

might be interested in some heuristics regarding the nature of the bias. To address these questions we perform two simulated natural experiment studies. To begin, one might be interested in the quantitative importance of violations of the absorbing-state condition stipulated in Proposition 10. To address this question, we consider a setting in which the policy variable ξ has four states and transitions only to its nearest neighbors. For intermediate states, the transition rate is λ for both one-step up and one-step down moves. For the two extreme states, the transition rate for the respective nearest neighbor is 2λ . Thus, the expected duration of each regime is $1/2\lambda$. Note that all conditions in Proposition 10 have been satisfied with the exception that the extreme states are not absorbing.

Figure 4 shows the ratio of treatment response to causal effect as λ is varied. If λ is set equal to zero, i.e. the policy generating process consists only of absorbing states, treatment responses naturally coincide with causal effects. For all other values of λ , the figure shows there is attenuation bias. Moreover, the bias increases rapidly with λ , as expected regime durations shorten. For example, if λ is set to 0.2 (expected regime durations of 2.5 years), the attenuation bias ranges between 40% and 70%. The extent of the bias also depends on the policy state. In particular, transitions between a boundary state and its neighbor exhibit the largest attenuation bias. Intuitively, under the stated conditions the boundary states reflect mean policy variable changes that are not equal to zero, and this magnifies bias. However, it is apparent that the attenuation bias resulting from non-absorbing barriers spills over to the intermediate states despite the mean change in policy variables being equal to zero in those states.

Yet another natural question to ask is the nature and direction of bias if the expected change in policy variables is not zero. In order to analyze this question, consider again a setting with four states, with only transitions to nearest neighbors being possible. However, let us now assume the extreme states are absorbing while up and down transition rates for intermediate states differ so that the expected change in the policy variable is negative. That is, a slow-down in tax depreciation is expected in the intermediate states.

Figure 5 shows the ratio of treatment responses to causal effects in this simulated experiment. Note that now the direction of the bias depends on the originating state. For transitions from state 3 to state 4 (the worst state), the figure shows a substantial attenuation bias. Intuitively, when in state 3, firms expect the state to worsen and so have low investment. This behavior implies a

small observed treatment response if the anticipated negative change occurs. Conversely, the figure shows that the transition from state 2 to state 1 (the best state) is associated with a treatment response in excess of the causal effect. Intuitively, when in state 2, firms invest at a low rate in the expectation of a slow-down in tax depreciation. If a positive surprise occurs, the investment response will be large.

6. Extrapolation of Treatment Responses

Section 5 considered a number of problems that can arise with inference in dynamic environments: wedges between treatment responses and causal effects; inability to extrapolate treatment responses within the same policy generating process; and inability to extrapolate treatment responses across economies with differing policy generating processes. Regarding the first of these problems, Lemma 9 and Proposition 10 provided a degree of relief as restrictions on the policy process were described ensuring either rough (rare event transitions) or exact (mean zero changes) equality between treatment responses and causal effects. These results notwithstanding, the issues of extrapolating treatment responses within and across policy generating processes remain, as does the problem of inferring causal effects when the policy variable being analyzed does not satisfy the required auxiliary assumptions ensuring unbiasedness.

The inference and extrapolation problems confronting the econometrician can be thought of as an application of the heterogeneous treatment effects model commonly used in the fields of medicine, education and labor. To illustrate, suppose we know the policy generating process but do not know how firms will respond to policy changes since we do not know the true distribution of adjustment cost parameters. Suppose that nature provides us with “ideal” evidence in the form of an exogenous transition of the policy variable of interest, say the tax depreciation rate, from ξ_i to ξ_j . We are interested in answering three questions. First, how can the newly-acquired evidence be used to forecast how firms in the same economy will respond to future changes in the policy variable? Second, how can one extrapolate the evidence to another economy with an identical real technology but differing in the underlying policy generating process? And finally, how can one map the treatment response back to causal effects?

Treatment effects are heterogeneous since the treatment response for firm m is given by $\Omega_m(q_j - q_i)$, where $\Omega_m = 1/2\gamma_m$. One can think of $q_j - q_i$ as measuring the size of a common government

policy “treatment” expressed in units of shadow value, with heterogeneity in adjustment costs implying heterogeneous firm responses. Heterogeneous effects is a situation that commonly confronts empiricists. However, here there is an additional necessary step in performing inference since the size of the treatment ($q_j - q_i$) actually depends on expectations regarding the future path of the policy variable. Specifically, the size of treatment is determined via the policy transition matrix and equation (30).

An unbiased estimator of μ_Ω is given by the mean treatment response (\overline{TR}_{ij}) normalized by size of the treatment. Using equation (30) we have:

$$\hat{\mu}_\Omega = \frac{\overline{TR}_{ij}}{q_j - q_i} = \frac{\overline{TR}_{ij}}{D_j - D_i}. \quad (42)$$

From the preceding equation it follows that a small response to policy changes can be imputed to either high adjustment costs or small changes in the shadow value of capital.

At this point it is worth commenting on the study by Cummins, Hassett and Hubbard (1994), who use firm responses to imputed changes in the shadow value of capital to infer adjustment cost parameters and tax-response elasticities. Implicit in their shadow value imputation is the assumption that each tax code change is unanticipated and expected to be permanent. This is a troubling assumption given the frequency of tax code changes in their sample period. They argue this assumption serves to overstate the change in shadow values and thus overstate adjustment costs. However, the analysis of Section 4 shows that their imputation method may tend to overstate, understate, or have sign that is opposite to the true change in shadow values, depending on the true policy generating process. Thus, it is difficult to know the sign and direction of bias.

Armed with inference regarding adjustment costs via equation (42), it is possible to extrapolate how firms will respond to a future change in tax depreciation rates from say ξ_h to ξ_k . We have:

$$\widehat{TR}_{hk} = \hat{\mu}_\Omega(q_k - q_h) = \left[\frac{q_k - q_h}{q_j - q_i} \right] \overline{TR}_{ij} = \left[\frac{D_k - D_h}{D_j - D_i} \right] \overline{TR}_{ij}. \quad (43)$$

The preceding calculation is a straightforward application of formula (30). Given its apparent brevity, it is worth emphasizing that the last term in squared brackets in equation (43) generally depends upon all transition rates and all possible realizations of the policy variable, not just those in the set $\{h, i, j, k\}$. In other words, valid extrapolation of a given treatment response hinges upon a correct characterization of the underlying policy generating process, including counterfactual states.

As exceptional cases that prove the rule, when the auxiliary assumptions described in Lemma 9 and Proposition 10 are satisfied, the extrapolation problem is greatly simplified since we then have:

$$\frac{D_k - D_h}{D_j - D_i} = \frac{\xi_k - \xi_h}{\xi_j - \xi_i} \Rightarrow \widehat{TR}_{hk} = \left[\frac{\xi_k - \xi_h}{\xi_j - \xi_i} \right] \overline{TR}_{ij}. \quad (44)$$

Formula (44) shows that if the underlying policy generating process meets the auxiliary assumptions required for equality of treatment responses and causal effects, the extrapolation of treatment responses within a policy generating process requires a simple adjustment for the relative size of changes in the policy variable, with no need to explicitly account for expectations. Imputations along the lines of formula (44) are commonplace. However, it is apparent that making such an imputation requires making the case that auxiliary assumptions are met.

Continuing with our working example, suppose instead we would like to forecast how firms in Economy B will respond to a change in tax depreciation rates from, say, ξ_h^B to ξ_k^B based upon the evidence provided by the response of firms in Economy A to a change in tax depreciation rates from ξ_i^A to ξ_j^A . Importantly, the critique of Lucas (1976) can be directly addressed. We must simply use the respective policy transition matrices for the two economies in conjunction with equation (30) to compute the respective changes in shadow values. We then have:

$$\widehat{TR}_{hk}^B = [q_k^B - q_h^B] \widehat{\mu}_\Omega = \left[\frac{q_k^B - q_h^B}{q_j^A - q_i^A} \right] \overline{TR}_{ij}^A = \left[\frac{D_k^B - D_h^B}{D_j^A - D_i^A} \right] \overline{TR}_{ij}^A. \quad (45)$$

Finally, let us confront the final problem of estimating causal effects based upon treatment responses. This analysis can be accomplished by utilizing the equation (45), since it provides a map between treatment responses generated by alternative policy generating processes. To infer implied causal effects one can think of Economy B as an economy in which policy assignment is permanent. Treating $q_k^B - q_h^B$ as the change in shadow values under permanent assignment and substituting into equation (45) we obtain:

$$\widehat{CE}_{ij} = \left[\frac{\tau \delta \psi(\xi_j - \xi_i)}{r + \delta} \right] \widehat{\mu}_\Omega = \left[\frac{\tau \delta \psi(\xi_j - \xi_i)}{r + \delta} \right] \left[\frac{\overline{TR}_{ij}}{D_j - D_i} \right]. \quad (46)$$

Applying this same algorithm to the analysis of changes in minimum wages and pollution taxes, we arrive at the following proposition.

Proposition 11 *The respective causal effects implied by responses to changes in tax depreciation*

schedules, minimum wages, and fuel taxes are

$$\begin{aligned}\widehat{CE}_{ij} &= \left[\frac{\tau\delta\psi(\xi_j - \xi_i)}{r + \delta} \right] \left[\frac{\overline{TR}_{ij}}{D_j - D_i} \right] \\ \widehat{CE}_{ij} &= - \left[\frac{(1 - \tau)(w_j - w_i)}{r + \delta} \right] \left[\frac{\overline{TR}_{ij}}{D_j - D_i} \right] \\ \widehat{CE}_{ij}(x) &= \left[\frac{(1 - \tau)(\kappa_j - \kappa_i)x}{r + \delta - \mu} \right] \left[\frac{\overline{TR}_{ij}(x)}{(d_j - d_i)x} \right]\end{aligned}$$

where equation (30) is used to obtain $D_j - D_i$ and $d_j - d_i$.

7. Concluding Remarks

This paper analyzed the effect of idealized exogenous government policy shocks on optimal firm behavior. We use the economy as a laboratory to assess natural policy experiments. It was shown that independent policy assignment is insufficient for valid inference regarding causal effects. With binary assignment, treatment responses substantially understate causal effects under plausible parameterizations. With more than two policy regimes, treatment responses can understate, overstate, or have a sign that is opposite to causal effects. We also show that there are important limits to the generalizability of historic treatment responses. In general, it is invalid to extrapolate treatment responses within, as well as across, policy generating processes. As a whole, these results cast doubt on the economic meaning of a broad array of unconditional statements about elasticities. Importantly, we derive the auxiliary assumptions required to ensure equality of treatment responses and causal effects in dynamic settings (Lemma 9 and Proposition 10). Further, we offer a general algorithm for the extrapolation of treatment responses accounting for the role of expectations (Section 6).

It follows from our analysis that an apparent falsification of a given theory based on an incorrect sign prediction may be a false-falsification. In particular, in addition to the standard stated “identifying” assumption of exogenous assignment, natural policy experiments must be understood as predicated upon auxiliary identifying assumptions about policy generating processes. Of course, this is a special example of the arguments of the philosopher W.V. Quine who pointed out that an empirical falsification only allows one to either reject the underlying theory or to reject an experimenter’s auxiliary assumptions. Unfortunately, these auxiliary assumptions have gone unrecognized, unstated and unsatisfied in much of the quasi-experimental literature.

In fact, despite the appearance of criticism, this paper simply takes the central argument of *Mostly Harmless Econometrics* a necessary step further, arguing for a need to move beyond exogeneity. In particular, Angrist and Pischke (2009) state, “The description of an ideal experiment also helps you formulate causal questions precisely. The mechanics of an ideal experiment highlight the forces you’d like to manipulate and the factors you’d like to hold constant.” In the view of many, exogenous assignment constitutes this ideal. Instead, we argue that exogenous assignment must be seen as just one ingredient in the making of an ideal policy experiment in dynamic settings.

Proofs and Derivations

Growth Option Value with Constant Government Policies

For brevity, let:

$$d \equiv \frac{(1-\tau)\kappa}{r+\delta-\mu}; D \equiv \frac{\tau\xi\delta\psi - (1-\tau)w}{r+\delta}; \Theta \equiv D - \psi$$

Since the terms in the Bellman equation scaled by s cancel each other, satisfaction of the equation demands the growth option value satisfies the following ODE:

$$rG(x) = \mu x G_x(x) + \frac{1}{2} \sigma^2 x^2 G_{xx}(x) + \underbrace{(\Gamma d^2)x^2 + (2\Theta\Gamma d)x + \Theta^2\Gamma}_{Dividend}. \quad (47)$$

It follows from the preceding equation and the Feynman-Kac formula that the growth option value G is just equal to the value of a claim to a dividend stream that is linear-quadratic in x . To value this linear-quadratic claim, conjecture a linear-quadratic value function G with unknown constants and substitute this function into the preceding differential equation. One obtains:

$$G(x) = \frac{1}{4\gamma} \left[\left(\frac{d^2}{r-2\mu-\sigma^2} \right) x^2 + \left(\frac{2\Theta d}{r-\mu} \right) x + \frac{\Theta^2}{r} \right]. \quad (48)$$

Growth Option Value with Regime Shifts

To derive the growth option value, return to the Bellman system (26) and confine attention to the remaining terms that have not been zeroed out, those terms not scaled by s . We have the following system pinning down growth option value:

$$\begin{aligned} \left(r + \sum_{j \neq 1} \lambda_{1j} \right) G^1(x) &= \mu x G_x^1(x) + \frac{1}{2} \sigma^2 x^2 G_{xx}^1(x) + \sum_{j \neq 1} \lambda_{1j} G^j(x) + [\Gamma d_1^2] x^2 + [2\Gamma d_1 \Theta_1] x + [\Gamma \Theta_1^2] \\ &\dots \\ \left(r + \sum_{j \neq N} \lambda_{Nj} \right) G^N(x) &= \mu x G_x^N(x) + \frac{1}{2} \sigma^2 x^2 G_{xx}^N(x) + \sum_{j \neq N} \lambda_{Nj} G^j(x) + [\Gamma d_N^2] x^2 + [2\Gamma d_N \Theta_N] x + [\Gamma \Theta_N^2] \\ \Theta_i &\equiv D_i - \psi. \end{aligned} \quad (49)$$

We conjecture a growth option value that is linear-quadratic in x , with regime shifts. The following lemma follows directly by substituting the conjectured linear-quadratic solution into the system of ODEs in equation (49).

Lemma: The no-bubbles solution to the differential equations

$$\begin{aligned} \left(r + \sum_{j \neq 1} \lambda_{1j} \right) J^1(x) &= \mu x J_x^1(x) + \frac{1}{2} \sigma^2 x^2 J_{xx}^1(x) + \sum_{j \neq 1} \lambda_{1j} J^j(x) + \phi_1 x^2 + \tilde{\phi}_1 x + \hat{\phi}_1 \\ &\dots \\ \left(r + \sum_{j \neq N} \lambda_{Nj} \right) J^N(x) &= \mu x J_x^N(x) + \frac{1}{2} \sigma^2 x^2 J_{xx}^N(x) + \sum_{j \neq N} \lambda_{Nj} J^j(x) + \phi_N x^2 + \tilde{\phi}_N x + \hat{\phi}_N \end{aligned} \quad (50)$$

is

$$\begin{bmatrix} J^1(x) \\ \dots \\ J^N(x) \end{bmatrix} = x^2 [\mathbf{T}(r - 2\mu - \sigma^2)]^{-1} \begin{bmatrix} \phi_1 \\ \dots \\ \phi_N \end{bmatrix} + x [\mathbf{T}(r - \mu)]^{-1} \begin{bmatrix} \tilde{\phi}_1 \\ \dots \\ \tilde{\phi}_N \end{bmatrix} + [\mathbf{T}(r)]^{-1} \begin{bmatrix} \hat{\phi}_1 \\ \dots \\ \hat{\phi}_N \end{bmatrix}. \quad (51)$$

Lemma 2

We have the following $N \times N$ augmented transition matrix:

$$\mathbf{T}(R) \equiv \begin{bmatrix} R + (N-1)\lambda & -\lambda & \dots & -\lambda \\ -\lambda & R + (N-1)\lambda & \dots & -\lambda \\ \dots & \dots & \dots & \dots \\ -\lambda & -\lambda & \dots & R + (N-1)\lambda \end{bmatrix} \Rightarrow \mathbf{T}^{-1} = \frac{1}{R(R+N\lambda)} \begin{bmatrix} R + \lambda & \lambda & \dots & \lambda \\ \lambda & R + \lambda & \dots & \lambda \\ \dots & \dots & \dots & \dots \\ \lambda & \lambda & \dots & R + \lambda \end{bmatrix} \quad (52)$$

The shadow values are:

$$\begin{aligned} \begin{bmatrix} q^1(x) \\ q^2(x) \\ \dots \\ q^N(x) \end{bmatrix} &= \frac{(1-\tau)x}{\tilde{R}(\tilde{R}+N\lambda)} \begin{bmatrix} \tilde{R} + \lambda & \lambda & \dots & \lambda \\ \lambda & \tilde{R} + \lambda & & \lambda \\ \dots & \dots & \dots & \dots \\ \lambda & \lambda & \dots & \tilde{R} + \lambda \end{bmatrix} \begin{bmatrix} \kappa_1 \\ \kappa_2 \\ \dots \\ \kappa_N \end{bmatrix} \\ &+ \frac{1}{\hat{R}(\hat{R}+N\lambda)} \begin{bmatrix} \hat{R} + \lambda & \lambda & \dots & \lambda \\ \lambda & \hat{R} + \lambda & & \lambda \\ \dots & \dots & \dots & \dots \\ \lambda & \lambda & \dots & \hat{R} + \lambda \end{bmatrix} \begin{bmatrix} \tau\delta\psi\xi_1 - (1-\tau)w_1 \\ \tau\delta\psi\xi_2 - (1-\tau)w_2 \\ \dots \\ \tau\delta\psi\xi_N - (1-\tau)w_N \end{bmatrix}. \end{aligned} \quad (53)$$

Taking differences across rows, the treatment response of the shadow value is:

$$\begin{aligned} q^j(x) - q^i(x) &= \left[1 - \frac{N\lambda}{\tilde{R} + N\lambda} \right] \frac{1}{\tilde{R}} (\kappa_j - \kappa_i) (1-\tau)x \\ &+ \left[1 - \frac{N\lambda}{\hat{R} + N\lambda} \right] \frac{1}{\hat{R}} [(\tau\delta\psi\xi_j - (1-\tau)w_j) - (\tau\delta\psi\xi_i - (1-\tau)w_i)]. \end{aligned} \quad (54)$$

And the result follows from the fact that $\Delta a = \Delta q/2\gamma$. ■

Lemma 3

To prove both parts of the lemma, suppose there are $N \geq 3$ states and assume there are three states (l, m, h) that are absorbing as a system in that once the policy process enters any one of these three states, the state never transitions to a state $i \notin \{l, m, h\}$. Transition rates amongst states outside $\{l, m, h\}$ and into $\{l, m, h\}$ can be set arbitrarily. In this case, the shadow value in any of these three states can be computed by focusing on the system confined to $\{l, m, h\}$. To fix notation, assume the states vary in the intensity of government intervention, with:

$$\begin{aligned} w_h &> w_m > w_l \\ \xi_h &< \xi_m < \xi_l \\ \kappa_h &< \kappa_m < \kappa_l. \end{aligned}$$

To prove the first part of the lemma it is sufficient to find transition rates for the three state system with sign reversals. To this end, assume the medium and low states are absorbing. We have:

$$\mathbf{T} = \begin{bmatrix} R & 0 & 0 \\ 0 & R & 0 \\ -\lambda_{hl} & -\lambda_{hm} & R + \lambda_{hl} + \lambda_{hm} \end{bmatrix} \Rightarrow \mathbf{T}^{-1} = \frac{1}{R} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \frac{\lambda_{hl}}{R + \lambda_{hl} + \lambda_{hm}} & \frac{\lambda_{hm}}{R + \lambda_{hl} + \lambda_{hm}} & \frac{R}{R + \lambda_{hl} + \lambda_{hm}} \end{bmatrix}. \quad (55)$$

This implies:

$$\begin{aligned} q^m(x) &= \left(\frac{x}{\tilde{R}}\right) (1 - \tau)\kappa_m + \left(\frac{1}{\tilde{R}}\right) [\tau\delta\psi\xi_m - (1 - \tau)w_m] \\ q^h(x) &= \frac{(1 - \tau)x}{\tilde{R}(\tilde{R} + \lambda_{hl} + \lambda_{hm})} \left[\lambda_{hl}\kappa_l + \lambda_{hm}\kappa_m + \tilde{R}\kappa_h \right] \\ &\quad + \frac{1}{\hat{R}(\hat{R} + \lambda_{hl} + \lambda_{hm})} \left[\begin{aligned} &\lambda_{hl}(\tau\delta\psi\xi_l - (1 - \tau)w_l) + \lambda_{hm}(\tau\delta\psi\xi_m - (1 - \tau)w_m) \\ &+ \hat{R}(\tau\delta\psi\xi_h - (1 - \tau)w_h) \end{aligned} \right] \end{aligned} \quad (56)$$

For each policy variable that is time-varying, we list below conditions such that $q^h(x) > q^m(x)$.

$$\begin{aligned}\kappa & : \lambda_{hl} > \frac{\tilde{R}(\kappa_m - \kappa_h)}{\kappa_l - \kappa_m} \\ \xi & : \lambda_{hl} > \frac{\hat{R}(\xi_m - \xi_h)}{\xi_l - \xi_m} \\ w & : \lambda_{hl} > \frac{\hat{R}(w_h - w_m)}{w_m - w_l}.\end{aligned}$$

To prove the second part of the lemma it is sufficient to find transition rates for the three state system with overshooting. To this end, assume that now it is the low and high states are absorbing.

The transition matrix is:

$$\mathbf{T} = \begin{bmatrix} R & 0 & 0 \\ -\lambda_{ml} & R + \lambda_{ml} + \lambda_{mh} & -\lambda_{mh} \\ 0 & 0 & R \end{bmatrix} \Rightarrow \mathbf{T}^{-1} = \frac{1}{\tilde{R}} \begin{bmatrix} 1 & 0 & 0 \\ \frac{\lambda_{ml}}{\tilde{R} + \lambda_{ml} + \lambda_{mh}} & \frac{R}{\tilde{R} + \lambda_{ml} + \lambda_{mh}} & \frac{\lambda_{mh}}{\tilde{R} + \lambda_{ml} + \lambda_{mh}} \\ 0 & 0 & 1 \end{bmatrix}. \quad (57)$$

We have:

$$\begin{aligned} \begin{bmatrix} q^l(x) \\ q^m(x) \\ q^h(x) \end{bmatrix} &= \frac{(1 - \tau)x}{\tilde{R}} \begin{bmatrix} 1 & 0 & 0 \\ \frac{\lambda_{ml}}{\tilde{R} + \lambda_{ml} + \lambda_{mh}} & \frac{\tilde{R}}{\tilde{R} + \lambda_{ml} + \lambda_{mh}} & \frac{\lambda_{mh}}{\tilde{R} + \lambda_{ml} + \lambda_{mh}} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \kappa_l \\ \kappa_m \\ \kappa_h \end{bmatrix} \\ &+ \frac{1}{\tilde{R}} \begin{bmatrix} 1 & 0 & 0 \\ \frac{\lambda_{ml}}{\tilde{R} + \lambda_{ml} + \lambda_{mh}} & \frac{\hat{R}}{\tilde{R} + \lambda_{ml} + \lambda_{mh}} & \frac{\lambda_{mh}}{\tilde{R} + \lambda_{ml} + \lambda_{mh}} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \tau\delta\psi\xi_l - (1 - \tau)w_l \\ \tau\delta\psi\xi_m - (1 - \tau)w_m \\ \tau\delta\psi\xi_h - (1 - \tau)w_h \end{bmatrix}. \end{aligned} \quad (58)$$

We are interested in conditions such that the treatment response exceeds the causal effect for a transition from the medium to low state:

$$q^l(x) - q^m(x) > \frac{x(1 - \tau)\kappa_l}{\tilde{R}} + \frac{\tau\delta\psi\xi_l - (1 - \tau)w_l}{\hat{R}} - \frac{x(1 - \tau)\kappa_m}{\tilde{R}} - \frac{\tau\delta\psi\xi_m - (1 - \tau)w_m}{\hat{R}}. \quad (59)$$

Listed according to the policy variable being changed, the preceding condition is met under the following conditions:

$$\begin{aligned}\kappa & : \lambda_{ml}(\kappa_l - \kappa_m) < \lambda_{mh}(\kappa_m - \kappa_h) \\ \xi & : \lambda_{ml}(\xi_l - \xi_m) < \lambda_{mh}(\xi_m - \xi_h) \\ w & : \lambda_{ml}(w_m - w_l) < \lambda_{mh}(w_h - w_m). \blacksquare\end{aligned}$$

Lemma 4

Consider a setting with at least four states. Assume the best and worst states are part of an absorbing system with two other states, in the sense that the state never exits these four states once it enters one of the four. Index these states according to their rank within this group, with 1 being the best and 4 being the worst. Consider then the following augmented transition matrix:

$$\mathbf{T} = \begin{bmatrix} R + \lambda & 0 & -\lambda & 0 \\ 0 & R & 0 & 0 \\ 0 & 0 & R & 0 \\ 0 & -\lambda & 0 & R + \lambda \end{bmatrix} \Rightarrow \mathbf{T}^{-1} = \frac{1}{\tilde{R}} \begin{bmatrix} \frac{R}{R+\lambda} & 0 & \frac{\lambda}{R+\lambda} & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & \frac{\lambda}{R+\lambda} & 0 & \frac{R}{R+\lambda} \end{bmatrix}. \quad (60)$$

We have:

$$\begin{bmatrix} q^1 \\ q^2 \\ q^3 \\ q^4 \end{bmatrix} = \frac{(1-\tau)x}{\tilde{R}} \begin{bmatrix} \frac{\tilde{R}}{\tilde{R}+\lambda} & 0 & \frac{\lambda}{\tilde{R}+\lambda} & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & \frac{\lambda}{\tilde{R}+\lambda} & 0 & \frac{\tilde{R}}{\tilde{R}+\lambda} \end{bmatrix} \begin{bmatrix} \kappa_1 \\ \kappa_2 \\ \kappa_3 \\ \kappa_4 \end{bmatrix} + \frac{1}{\hat{R}} \begin{bmatrix} \frac{\hat{R}}{\hat{R}+\lambda} & 0 & \frac{\lambda}{\hat{R}+\lambda} & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & \frac{\lambda}{\hat{R}+\lambda} & 0 & \frac{\hat{R}}{\hat{R}+\lambda} \end{bmatrix} \begin{bmatrix} \tau\delta\psi\xi_1 - (1-\tau)w_1 \\ \tau\delta\psi\xi_2 - (1-\tau)w_2 \\ \tau\delta\psi\xi_3 - (1-\tau)w_3 \\ \tau\delta\psi\xi_4 - (1-\tau)w_4 \end{bmatrix}. \quad (61)$$

In the respective experimental cases, $q^1 < q^4$ provided λ is sufficiently high to satisfy:

$$\begin{aligned} w & : \lambda > \frac{\hat{R}(w_4 - w_1)}{w_3 - w_2} \\ \kappa & : \lambda > \frac{\tilde{R}(\kappa_1 - \kappa_4)}{\kappa_2 - \kappa_3} \\ \xi & : \lambda > \frac{\hat{R}(\xi_1 - \xi_4)}{\xi_2 - \xi_3}. \blacksquare \end{aligned}$$

Lemma 5

Consider first policy experiments involving changes in κ . The treatment response is zero across all transitions only if the shadow value is constant across regimes. This holds only if there is some

constant k such that the vector $\mathbf{d} = k\mathbf{1}$. But this implies the existence of an augmented transition matrix \mathbf{T} satisfying:

$$k\mathbf{1} = \mathbf{T}^{-1}(\tilde{R}) \begin{bmatrix} (1-\tau)\kappa_1 \\ \dots \\ (1-\tau)\kappa_N \end{bmatrix} \Rightarrow \mathbf{T}(\tilde{R})\mathbf{1} = \begin{bmatrix} \tilde{R} \\ \dots \\ \tilde{R} \end{bmatrix} = \frac{1}{k} \begin{bmatrix} (1-\tau)\kappa_1 \\ \dots \\ (1-\tau)\kappa_N \end{bmatrix}. \quad (62)$$

This is a contradiction. Similarly, for policy experiments involving *ceteris paribus* changes in w or ξ , the treatment response vector will be zero if and only if there is some constant k such that vector $\mathbf{D} = k\mathbf{1}$. But this implies the existence of \mathbf{T} satisfying:

$$k\mathbf{1} = \mathbf{T}^{-1}(\hat{R}) \begin{bmatrix} \tau\delta\psi\xi_1 - (1-\tau)w_1 \\ \dots \\ \tau\delta\psi\xi_N - (1-\tau)w_N \end{bmatrix} \Rightarrow \mathbf{T}(\hat{R})\mathbf{1} = \begin{bmatrix} \hat{R} \\ \dots \\ \hat{R} \end{bmatrix} = \frac{1}{k} \begin{bmatrix} \tau\delta\psi\xi_1 - (1-\tau)w_1 \\ \dots \\ \tau\delta\psi\xi_N - (1-\tau)w_N \end{bmatrix} \quad (63)$$

a contradiction. ■

Lemma 7

Let stars denote shadow values under permanent policies. From Lemma 6 it follows:

$$q^1(x) - q^N(x) \leq q_*^1(x) - q_*^N(x). \quad (64)$$

This inequality may be rewritten as:

$$[q^1(x) - q^i(x)] + [q^j(x) - q^N(x)] + [q^i(x) - q^j(x)] \leq [q_*^1(x) - q_*^i(x)] + [q_*^j(x) - q_*^N(x)] + [q_*^i(x) - q_*^j(x)]. \quad (65)$$

Rearranging terms, it follows:

$$[q^1(x) - q^i(x)] + [q^j(x) - q^N(x)] \leq [q_*^1(x) - q_*^i(x)] + [q_*^j(x) - q_*^N(x)] - \{[q^i(x) - q^j(x)] - [q_*^i(x) - q_*^j(x)]\}. \quad (66)$$

Under the conditions stipulated in the lemma, it follows:

$$[q^1(x) - q^i(x)] + [q^j(x) - q^N(x)] \leq [q_*^1(x) - q_*^i(x)] + [q_*^j(x) - q_*^N(x)] - 2\gamma k. \quad (67)$$

The result then follows by dividing the preceding equation by 2γ . ■

Lemma 8

We claim first that q^1 must be greater than the shadow value in at least one other state. To see this, suppose to the contrary that q^1 is less than the shadow value in all other states, and let q^i denote the next lowest shadow value. Next note that since the instantaneous marginal product is highest while the regime is 1, it must be that q^1 is increasing in the expected passage time to a first transition. But as this passage time goes to zero, the shadow value goes to

$$\sum_{j \neq 1} P_{1j} q^j(x_0) \geq q^i(x_0) \Rightarrow q^1(x_0) > q^i(x_0). \quad (68)$$

We next claim that q^N must be less than the shadow value in at least one other state. To see this, suppose to the contrary that q^N is greater than the shadow value in all other states, and let q^i denote the second highest shadow value. Next note that since the instantaneous marginal product is lowest while the regime is N , it must be that q^N is decreasing in the expected passage time to a first transition. But as this passage time goes to zero, the shadow value goes to

$$\sum_{j \neq N} P_{Nj} q^j(x_0) \leq q^i(x_0) \Rightarrow q^N(x_0) < q^i(x_0). \blacksquare$$

Proposition 10

Consider an array of distinct policies 1 to N with the indexing convention being that the shadow value under permanent policies is decreasing in the index. A necessary and sufficient condition for each treatment response to equal its respective causal effect is that the difference between state-contingent shadow values is equal to shadow values under permanent policies. Thus, there must exist some k such that for all i :

$$q^i(x) = \frac{(1-\tau)\kappa_i x}{r+\delta-\mu} + \frac{\tau\delta\psi\xi_i - (1-\tau)w_i}{r+\delta} + k. \quad (69)$$

Therefore, we must identify conditions such that the following equilibrium conditions can be met under this functional form:

$$\begin{aligned} \left(r + \delta + \sum_{j \neq 1} \lambda_{1j} \right) q^1(x) &= \mu x q_x^1(x) + \frac{1}{2} \sigma^2 x^2 q_{xx}^1(x) + \sum_{j \neq 1} \lambda_{1j} q^j(x) + (1-\tau)(\kappa_1 x - w_1) + \tau\delta\psi\xi_1 \quad (70) \\ &\dots \\ \left(r + \delta + \sum_{j \neq N} \lambda_{Nj} \right) q^N(x) &= \mu x q_x^N(x) + \frac{1}{2} \sigma^2 x^2 q_{xx}^N(x) + \sum_{j \neq N} \lambda_{Nj} q^j(x) + (1-\tau)(\kappa_N x - w_N) + \tau\delta\psi\xi_N. \end{aligned}$$

Substituting in the required functional form and canceling terms we obtain:

$$\begin{aligned}
(r + \delta) k &= \sum_{j \neq 1} \lambda_{1j} \left[\frac{(1 - \tau)\kappa_j x}{r + \delta - \mu} + \frac{\tau \delta \psi \xi_j - (1 - \tau)w_j}{r + \delta} - \left(\frac{(1 - \tau)\kappa_1 x}{r + \delta - \mu} + \frac{\tau \delta \psi \xi_1 - (1 - \tau)w_1}{r + \delta} \right) \right] \\
(r + \delta) k &= \sum_{j \neq 2} \lambda_{2j} \left[\frac{(1 - \tau)\kappa_j x}{r + \delta - \mu} + \frac{\tau \delta \psi \xi_j - (1 - \tau)w_j}{r + \delta} - \left(\frac{(1 - \tau)\kappa_2 x}{r + \delta - \mu} + \frac{\tau \delta \psi \xi_2 - (1 - \tau)w_2}{r + \delta} \right) \right] \\
&\dots \\
(r + \delta) k &= \sum_{j \neq N-1} \lambda_{N-1j} \left[\frac{(1 - \tau)\kappa_j x}{r + \delta - \mu} + \frac{\tau \delta \psi \xi_j - (1 - \tau)w_j}{r + \delta} - \left(\frac{(1 - \tau)\kappa_{N-1} x}{r + \delta - \mu} + \frac{\tau \delta \psi \xi_{N-1} - (1 - \tau)w_{N-1}}{r + \delta} \right) \right] \\
(r + \delta) k &= \sum_{j \neq N} \lambda_{Nj} \left[\frac{(1 - \tau)\kappa_j x}{r + \delta - \mu} + \frac{\tau \delta \psi \xi_j - (1 - \tau)w_j}{r + \delta} - \left(\frac{(1 - \tau)\kappa_N x}{r + \delta - \mu} + \frac{\tau \delta \psi \xi_N - (1 - \tau)w_N}{r + \delta} \right) \right].
\end{aligned} \tag{71}$$

First note that any solution to the preceding system entails $k = 0$ since the right-side of the first equation is weakly negative while the right-side of the last equation is weakly-positive. It follows that any candidate solution to the system entails:

$$\begin{aligned}
\lambda_{1j} &= 0 : j = 2, \dots, N \\
\lambda_{Nj} &= 0 : j = 1, \dots, N - 1.
\end{aligned}$$

Further, it must be the case that:

$$0 = \sum_{j \neq i} \lambda_{ij} \left[\frac{(1 - \tau)\kappa_j x}{r + \delta - \mu} + \frac{\tau \xi_j \delta \psi - (1 - \tau)w_j}{r + \delta} - \left(\frac{(1 - \tau)\kappa_i x}{r + \delta - \mu} + \frac{\tau \xi_i \delta \psi - (1 - \tau)w_i}{r + \delta} \right) \right] : i = 2, \dots, N-1 \tag{72}$$

Expressed in terms of policy variable changes, the preceding equation can be written as:

$$\begin{aligned}
\sum_{j \neq i} \left(\frac{\lambda_{ij}}{\sum_{j \neq i} \lambda_{ij}} \right) (\xi_j - \xi_i) &= 0 : i = 2, \dots, N - 1 \\
\sum_{j \neq i} \left(\frac{\lambda_{ij}}{\sum_{j \neq i} \lambda_{ij}} \right) (\kappa_j - \kappa_i) &= 0 : i = 2, \dots, N - 1 \\
\sum_{j \neq i} \left(\frac{\lambda_{ij}}{\sum_{j \neq i} \lambda_{ij}} \right) (w_j - w_i) &= 0 : i = 2, \dots, N - 1. \blacksquare
\end{aligned} \tag{73}$$

References

- [1] Aaron, Henry J., 1990, Lessons for Tax Reform, in J. Slemrod Ed., *Do Taxes Matter? The Impact of the Tax Reform Act of 1986*, MIT Press: Cambridge.
- [2] Abel, Andrew, 1983, Tax Neutrality in the Presence of Adjustment Costs, *Quarterly Journal of Economics* (98), 705-712.
- [3] Abel, Andrew and Janice Eberly, 1997, An Exact Solution for the Investment and Value of a Firm Facing Uncertainty, Adjustment Costs, and Irreversibility, *Journal of Economic Dynamics and Control* (21), 831-852.
- [4] Angrist, Joshua D. and Jorn-Steffen Pischke, 2009, *Mostly Harmless Econometrics: An Empiricist's Companion*, Princeton University Press.
- [5] Angrist, Joshua D. and Jorn-Steffen Pischke, 2010, The Credibility Revolution in Economics: How Better Research Design is Taking the Con out of Econometrics, *Journal of Economic Perspectives* (24), 3-30.
- [6] Auerbach, Alan J., 1986, Tax Reform and Adjustment Costs: The Impact on Investment and Market Value, *International Economic Review* (30), 831-852.
- [7] Auerbach, Alan J. and James R. Hines, Jr., 1988, Investment Tax Incentives and Frequent Tax Reforms, *American Economic Review* (78), 211-216.
- [8] Cummins, Jason G., Kevin A. Hassett, and R. Glenn Hubbard, 1994, A Reconsideration of Investment Behavior Using Tax Reforms as Natural Experiments, *Brookings Papers on Economic Activity* (2), 1-74.
- [9] Gourio, Francois and Jianjun Miao, 2008, Transitional Dynamics of Dividend and Capital Gains Tax Cuts, *Review of Economic Dynamics* (14), 368-383.
- [10] Greenstone, Michael, 2002, The Impacts of Environmental Regulations on Industrial Activity: Evidence from the 1970 and 1977 Clean Air Act Amendments and Census of Manufacturers, *Journal of Political Economy* (110).

- [11] Hassett, Kevin A. and Gilbert Matcalf, 1999, Investment with Uncertain Tax Policy: Does Random Taxation Discourage Investment, *Economic Journal* (109), 372-393.
- [12] Heckman, James J., 1997, Instrumental Variables: A Study of Implicit Behavioral Assumptions Used in Making Program Evaluations, *Journal of Human Resources* (32), 441-462.
- [13] Heckman, James J., 1999, Causal Parameters and Policy Analysis in Economics: A Twentieth Century Perspective, *Quarterly Journal of Economics* (115), 45-97.
- [14] Keane, Michael, 2010, A Structural Perspective on the Experimentalist School, *Journal of Economic Perspectives* (24), 47-58.
- [15] Lucas, Robert E., Jr., 1976, Econometric Policy Evaluation: A Critique, in K. Brunner and A. Meltzer Eds., *The Phillips Curve and Labor Markets*, Amsterdam: North Holland.
- [16] Romer, Christina D. and David H. Romer, 2010, The Macroeconomic Effects of Tax Changes: Estimates Based on a New Measure of Fiscal Shocks, *American Economic Review* (100), 763-801.
- [17] Ross, Sheldon, 1996, *Stochastic Processes*, Wiley.
- [18] Slemrod, Joel, 1990, The Economic Impact of the Tax Reform Act of 1986, in J. Slemrod Ed., *Do Taxes Matter? The Impact of the Tax Reform Act of 1986*, MIT Press: Cambridge.
- [19] Slemrod, Joel, 1992, Do Taxes Matter? Lessons from the 1980's, *American Economic Review* (82), 250-256.

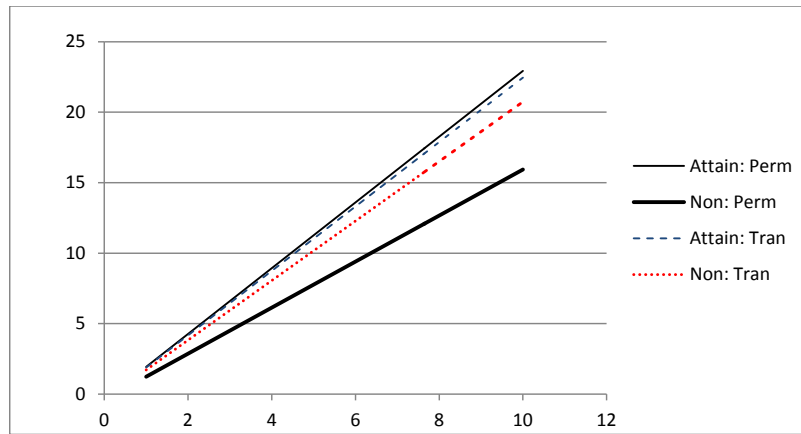


Figure 1: **Investment and Policy Transience.** This figure shows the optimal investment policy functions for Attainment and Non-Attainment status for two cases of policy generating processes. In the permanent case, both states are absorbing. In the temporary case, the transition matrix is based on estimates contained in Greenstone (2001).

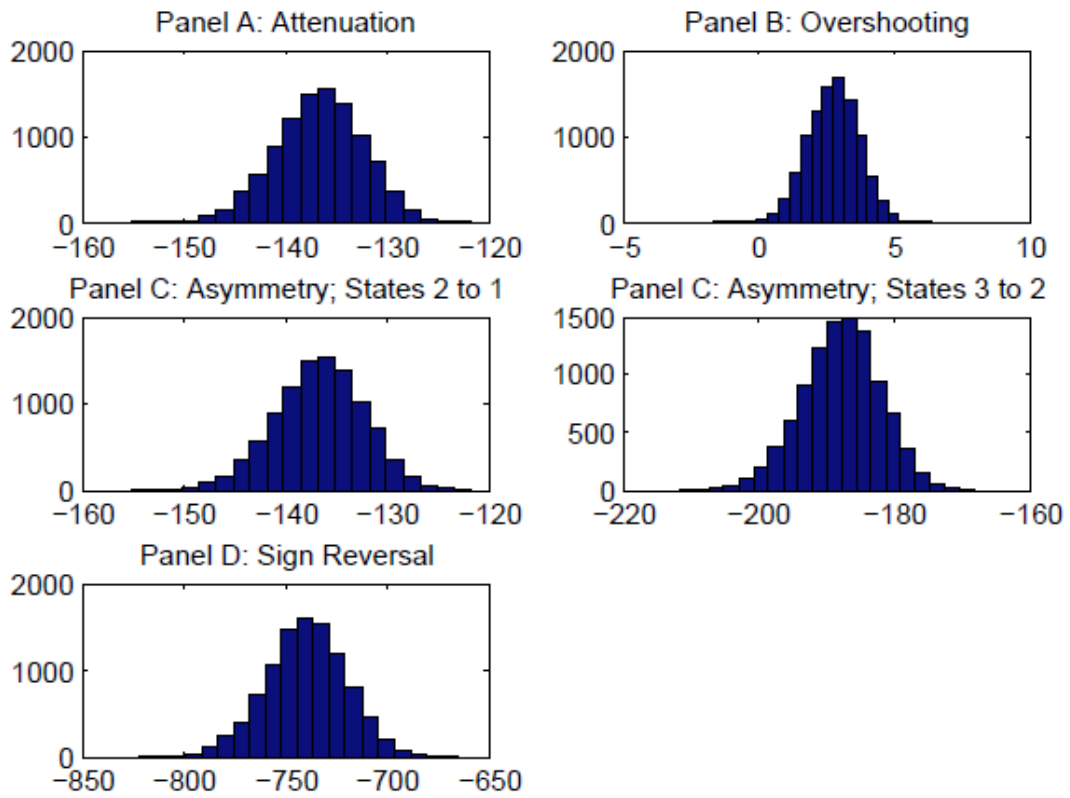


Figure 2: **Distribution of Simulated t-Statistics under Alternative Processes.** This figure shows the distribution of simulated t-statistics under alternative policy generating processes. Each t-stat is estimated under the null hypothesis of the true causal effect. In each case, the policy generating process of accelerated depreciation takes three states. Panel A shows the case of attenuation, Panel B shows the case of overshooting, Panel C shows the case of Asymmetry (the behavior of investment accumulation response between States 2 and 1, as well as between States 3 and 2), and Panel D shows the case of the sign reversal.

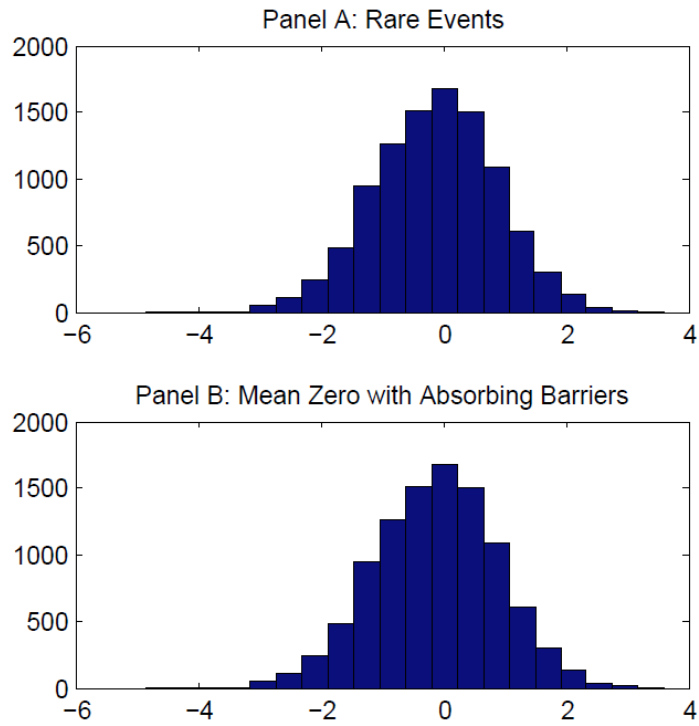


Figure 3: **Distribution of Simulated t-Statistics for Ideal Policy Processes.** This figure shows the distribution of simulated t-statistics under two ideal policy processes. Panel A shows the case of rare events, in which each state is absorbing. Panel B shows the case of a mean zero case with absorbing barriers. In both cases, each t-stat is estimated under the null hypothesis of the true causal effect and the policy generating vector of accelerated depreciation takes three states.

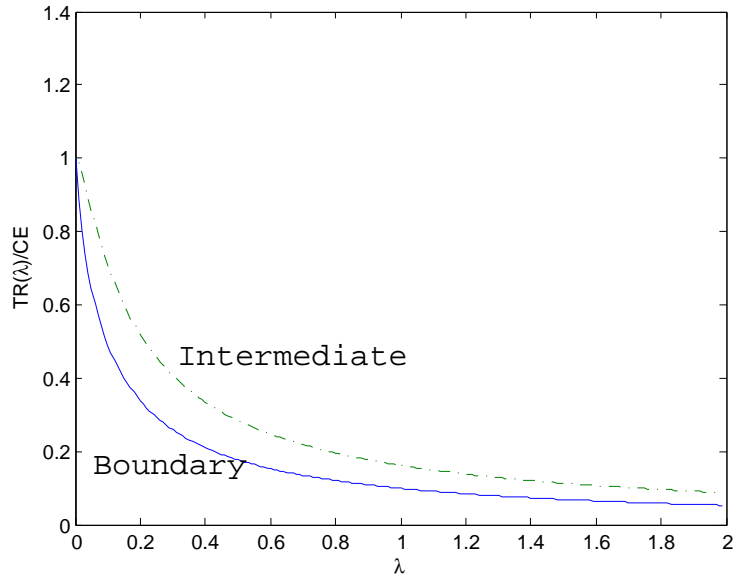


Figure 4: **Treatment Response and Causal Effect: Four-State Example.**

The figure shows the ratio of Treatment Response to Causal Effects for transitions to/from Boundary States and Nearest Neighbor (solid line). The dashed line shows the ratio of Treatment Response to Causal Effects for transitions between the two intermediate states (2,3). The transition matrix features four states with nearest neighbor transitions, mean zero changes for the intermediate states and non-absorbing boundary states.

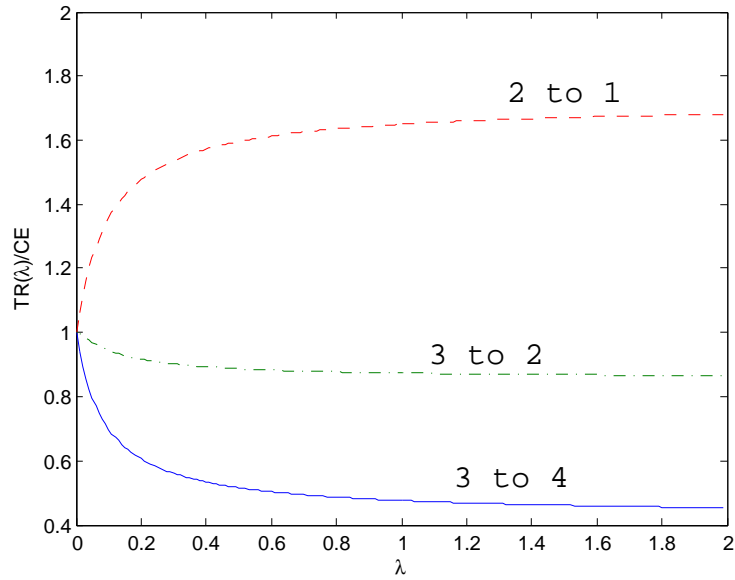


Figure 5: **Treatment Response and Causal Effect: Four-State Example with Absorbing**

The figure shows the ratio of Treatment Response to Causal Effect for transitions from: state 2 to state 1 (top line); state 3 to state 4 (bottom line); and state 3 to state 2 (middle line). The transition matrix features absorbing extreme states while other states feature an expected slow-down of tax depreciation rates.

Table 1: Distribution of Simulated Treatment Responses and t-Statistics under Alternative Processes

	Panel A	Panel B	Panel C: 2 to 1	Panel C: 3 to 2	Panel D
Mean TR	8.65	27.10	8.65	6.92	-2.64
Std TR	4.02	12.58	4.02	3.21	1.22
Median TR	7.83	24.53	7.83	6.26	-2.39
Mean T-stat	-136.69	2.74	-136.69	-187.90	-739.98
Std T-stat	4.37	0.96	4.37	5.74	19.41
Median T-stat	-136.56	2.77	-136.56	-187.73	-739.55
Prob. Rejection 95%	1.00	0.79	1.00	1.00	1.00