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THE POWER OF FORWARD GUIDANCE REVISITED

Alisdair McKay  
Emi Nakamura  
Jón Steinsson

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Alisdair McKay, Emi Nakamura, and Jón Steinsson  
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### **ABSTRACT**

In recent years, central banks have increasingly turned to "forward guidance" as a central tool of monetary policy, especially as interest rates around the world have hit the zero lower bound. Standard monetary models imply that far future forward guidance is extremely powerful: promises about far future interest rates have huge effects on current economic outcomes, and these effects grow with the horizon of the forward guidance. We show that the power of forward guidance is highly sensitive to the assumption of complete markets. If agents face uninsurable income risk and borrowing constraints, a precautionary savings effect tempers their responses to far future promises about interest rates. As a consequence, the ability of central banks to combat recessions using small changes in interest rates far in the future, is greatly reduced relative to the complete markets benchmark. We show that the effects of precautionary savings motives can be captured by a simplified version of our model that generates discounting in the representative agent's Euler equation. This discounted Euler equation can be easily included in standard business cycle models.

Alisdair McKay  
Boston University  
Dept. of Economics  
270 Bay State Road  
Boston, MA 02215  
amckay@bu.edu

Emi Nakamura  
Columbia Business School  
3022 Broadway, Uris Hall 820  
New York, NY 10027  
and NBER  
enakamura@columbia.edu

Jón Steinsson  
Department of Economics  
Columbia University  
1026 International Affairs Building  
420 West 118th Street  
New York, NY 10027  
and NBER  
jsteinsson@columbia.edu

# 1 Introduction

Forward guidance has become an increasingly important tool of monetary policy in recent years. Gurkaynak, Sack, and Swanson (2005) show that much of the surprise news about monetary policy at the time of FOMC announcements arises from signals about the central bank’s intentions about future monetary policy. In many cases, changes in the current Federal Funds rate are fully expected, and all of the *news* about monetary policy has to do with how the central bank is expected to set interest rates in the future.<sup>1</sup>

Promises about future interest rates have been shown to have a powerful effect on the economy in standard monetary models. Eggertsson and Woodford (2003) show that a shock to the natural rate of interest that causes the economy to hit the zero lower bound on nominal interest rates induces a powerful deflationary spiral and a crippling recession. However, the recession can be entirely abated if the central bank commits from the outset to holding interest rates at the zero lower bound for a few additional quarters beyond what is justified by contemporaneous economic conditions.

Recent work argues that the magnitude of the effects of forward guidance in New Keynesian models stretches the limits of credibility. Carlstrom, Fuerst, and Paustian (2012) show that a promise by the central bank to peg interest rates below the natural rate of interest for roughly two years generates explosive dynamics for inflation and output in a workhorse New Keynesian model (the Smets and Wouters (2007) model).<sup>2</sup> Del Negro, Giannoni, and Patterson (2013) refer to this phenomenon as the *forward guidance puzzle*. Along the same lines, consider an experiment whereby the central bank promises a 1 percentage point lower real interest rate *for a single quarter* at some point in the future. We show that in the plain vanilla New Keynesian model, this promise has an eighteen times greater impact on inflation when the promise pertains to interest rates 5 years in the future than when it pertains to the current interest rate.

It may seem unintuitive that an interest rate cut far in the future has a greater effect than a near-term one. To see why this arises in standard models, consider the response of consumption to a decrease in the real interest rate for a single quarter 5 years in the future. The consumption Euler equation dictates that consumption will rise immediately to a higher level and stay constant at that

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<sup>1</sup>Campbell et al. (2012) reinforce these results using a longer sample period spanning the Great Recession.

<sup>2</sup>Carlstrom, Fuerst, and Paustian (2012) show that the responses of output and inflation experience a sign reversal when the interest rate peg is extended from eight quarters to nine quarters in the Smets and Wouters (2007) model. We interpret this as the model “blowing up” at this point, i.e., the monetary stimulus becomes so large that the response of output and inflation is infinite.

higher level for 5 years before returning to its normal level.<sup>3</sup> The cumulative response of consumption to the shock is therefore quite large and gets larger the further in the future the interest rate shocks occurs. It is the cumulative response of consumption (with some discounting) that determines the response of current inflation in the basic New Keynesian model. So, the further in the future is the interest rate that the monetary authority announces it will change, the larger is the current response of inflation. At the zero lower bound, this large effect on inflation will lower real rates and thus create a powerful feedback loop on output.

But is it a realistic prediction of the standard model that agents increase their consumption by the same amount in response to an interest rate change 5 years in the future as they do to a change in the current interest rate? An individual who raises his consumption 5 years in advance of an anticipated interest rate cut will need to run down his savings for 5 years. For many agents, this is not feasible since it would entail hitting a borrowing constraint. More generally, as an agent's assets fall, the marginal value of wealth increases for precautionary reasons, raising the motive to save. This precautionary savings effect counteracts the intertemporal substitution motive to dissave in anticipation of the low interest rate. As the low interest rate is further in the future, the change in assets needed to take full advantage of intertemporal substitution grows and the precautionary savings effect therefore grows stronger, tempering the effects of forward guidance.

To investigate the quantitative magnitude of these effects, we consider a general equilibrium model in which agents face uninsurable, idiosyncratic income risk and borrowing constraints. In this model, the effect of forward guidance about future interest rates on current output falls the further out in the future the interest rate change is. For forward guidance about the interest rate 5 years in the future, the effect on output and inflation is roughly 40% as large as in the standard model. For forward guidance about the interest rate 10 years in the future, the effect on current output is essentially zero.

Our results indicate that forward guidance is a much less effective policy tool at the zero lower bound in a model with a realistic degree of precautionary savings than it is in standard macro models.

We consider a shock that lowers the natural rate of interest enough that the zero lower bound binds

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<sup>3</sup>The response is a step function because consumption growth only deviates from normal when the real interest rate deviates from normal and this only occurs in the single quarter that the forward guidance pertains to. Another way to see this is that the forward guidance does not change the relative price of consumption for any two dates before the date of the interest rate change. All these dates must therefore have the same level of consumption. The end-point of consumption is pinned down at the old steady state by the fact that monetary shocks have no effect on real outcomes in the long run.

for 5 years and the initial fall in output is -4% in the absence of forward guidance. If we assume markets are complete (and precautionary savings thus absent), a policy of maintaining interest rates at zero for a little more than three quarters beyond what a strict inflation targeting central bank would do completely eliminates the fall in output. In contrast, in our incomplete markets model with idiosyncratic risk and borrowing constraints, the effect of this amount of forward guidance is substantially smaller and a significant recession remains.

Returning to the intuition for the forward guidance puzzle, a key reason why the puzzle occurs is that the sensitivity of current output to far future interest rates is the same as the sensitivity to current interest rates. The important property of our incomplete markets model that reduces the effects of far future forward guidance is that this is no longer the case. We show that the responsiveness of output in the incomplete markets model can be approximated by a consumption Euler equation with “discounting.”<sup>4</sup> Furthermore, we show that this “discounted Euler equation” can be micro-founded with a simplified version of our incomplete markets model. This formulation has the advantage of being highly tractable, and easy to incorporate into the DSGE models used for policy analysis at central banks around the world.

We use the “discounted Euler equation model” to revisit the question of how severe are the effects of shocks that lead the zero lower bound (ZLB) to bind. We show that a patience shock of the type often assumed to cause the ZLB to bind that generates a large depression (-14% output and -10% inflation effect) in the plain vanilla New Keynesian model yields a much smaller recession in our model (-3% output and -2% inflation). It is well known that shocks like these can lead the economy into a “deflationary death spiral” (i.e., log output and inflation go to negative infinity) if they are persistent enough. In the plain vanilla New Keynesian model, the deflationary death spiral occurs even for shocks that are expected to last only roughly 3 years on average. In our discounted Euler equation model, however, the deflationary death spiral occurs only for shocks that are considerably more persistent.

An alternative simple adjustment to the standard model would be to assume that a fraction of agents are hand-to-mouth. This also reduces the sensitivity of aggregate output to interest rates. However, it reduces the sensitivity of output to interest rates by the same amount at all horizons—in this respect, it is equivalent to reducing the intertemporal elasticity of substitution. Adding hand-

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<sup>4</sup>Interestingly, such a formulation of the consumption Euler equation has actually been used in policy calculations by the Central Bank of Norway, to combat the forward guidance puzzle. We thank Oistein Roisland for pointing this out to us.

to-mouth agents does not alter the prediction that an interest rate change 5 years in the future has an eighteen times larger effect on current output as an equally large contemporaneous interest rate increase. Our incomplete markets model and the discounted Euler equation model reduce the effect of far future interest rates relative interest rate changes closer to the present.

Our work builds on recent papers that incorporate market incompleteness and idiosyncratic uncertainty into New Keynesian models starting with Guerrieri and Lorenzoni (2011) and Oh and Reis (2012). The papers closest in spirit to ours are McKay and Reis (2014) who investigate the power of automatic stabilizers at the zero lower bound and Gornemann, Kuester, and Nakajima (2014) who investigate the distributional implications of monetary policy shocks. Several other recent papers suggest “solutions” to the forward guidance puzzle. Del Negro, Giannoni, and Patterson (2013) argue that the experiment that gives rise to the puzzle is, itself, unreasonable. They argue that it is unreasonable to assume that the central bank really can engender substantial changes in long-term interest rates, which are at the core of why the forward guidance puzzle arises. Carlstrom, Fuerst, and Paustian (2012) and Kiley (2014) show that the magnitude of the forward guidance puzzle is substantially reduced in a sticky information (as opposed to a sticky price) model. Caballero and Farhi (2014) argue that forward guidance is less effective if the reason why the zero lower bound binds is a shortage of safe assets in the economy—a safety trap—as opposed to a deleveraging or patience shock.

The paper proceeds as follows. Section 2 explains why forward guidance is so powerful in standard New Keynesian models. Section 3 presents our incomplete markets model featuring uninsurable idiosyncratic income risk and borrowing constraints. Section 4 describes our results about the reduced power of forward guidance in our incomplete markets model relative to the standard complete markets models. Section 5 show how the behavior of the incomplete markets model can be approximated by a model with discounting in the consumption Euler equation. Section 6 concludes.

## 2 Why Is Forward Guidance So Powerful?

It is useful to start with an explanation for why forward guidance is so powerful in standard monetary models. Consider the basic New Keynesian model as developed, e.g., in Woodford (2003) and Gali (2008). The implications of private sector behavior for output and inflation in this model can be

described up to a linear approximation by an intertemporal “IS” equation of the form

$$x_t = E_t x_{t+1} - \sigma(i_t - E_t \pi_{t+1} - r_t^n), \quad (1)$$

and a Phillips curve of the form

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t. \quad (2)$$

Here,  $x_t$  denotes the output gap—i.e., the percentage difference between actual output and the natural rate of output that would prevail if prices are fully flexible— $\pi_t$  denotes inflation,  $i_t$  denotes the nominal, short-term, risk-free interest rate,  $r_t^n$  denotes the natural real rate of interest—i.e., the real interest rate that would prevail if prices were fully flexible— $\sigma$  denotes the intertemporal elasticity of substitution,  $\beta$  denotes the subjective discount factor of households, and  $\kappa$  is the slope of the Phillips curve which is determined by the degree of nominal and real rigidities in the economy. All variables are denoted as percentage deviations from their steady state values.

Suppose for simplicity that the monetary policy of the central bank is given by an exogenous rule for the real interest rate where the real interest rate tracks the natural real rate with some error:  $r_t = i_t - E_t \pi_{t+1} = r_t^n + \epsilon_{t,t-j}$ , where  $\epsilon_{t,t-j}$  denotes the shock to the short term real rate in period  $t$  that becomes known in period  $t-j$ .<sup>5</sup> Absent any monetary shocks, the real interest rate will perfectly track the natural real rate and both the output gap and inflation will be zero. Suppose we start in such a state, but then the monetary authority announces that the real interest rate will be lower by 1% for a single quarter 5 years in the future, but maintained at the natural real rate of interest in all other quarters (i.e.,  $\epsilon_{t+20,t} = -0.01$ ).

To see how this forward guidance announcement affects the output gap, it is useful to solve the intertemporal IS equation forward to get

$$x_t = -\sigma \sum_{j=0}^{\infty} E_t (i_{t+j} - E_{t+j} \pi_{t+j+1} - r_{t+j}^n). \quad (3)$$

Notice, that there is no discounting in the sum on the right hand side of this equation. This implies that the output gap will rise immediately by 1% (if we assume for simplicity that  $\sigma = 1$ ) and will stay at that higher level for the next five years and then fall back to zero all at once when the low

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<sup>5</sup>Given this specification of monetary policy, the model has a unique solution for which  $\lim_{j \rightarrow \infty} E_t x_{t+j} = 0$  and inflation is bounded. We could alternatively assume that the monetary authority sets the nominal rate according to the following rule  $i_t = r_t^n + \phi \pi_t + \tilde{\epsilon}_{t,t-j}$  and  $\phi > 1$ . In this case, the model has a unique bounded solution (without the additional restriction that  $\lim_{j \rightarrow \infty} E_t x_{t+j} = 0$ ) and there exists a path for  $\tilde{\epsilon}_{t,t-j}$  that gives the same solution as the model with monetary policy given by the exogenous path for the real rate we assume. We prefer to describe the monetary policy as an exogenous rule for the real interest rate because this simplifies our exposition substantially.

interest rate period passes. The same is true of forward guidance for any other horizon.<sup>6</sup> So, the further out in the future the forward guidance is, the larger is the *cumulative* response of output.

To see how the announcement affects inflation, it is useful to solve the Phillips curve forward to get

$$\pi_t = \kappa \sum_{j=0}^{\infty} \beta^j E_t x_{t+j}. \quad (4)$$

This shows that it is the entire cumulative response of the output gap (albeit with some discounting) that determines the current response of inflation to forward guidance. The further in the future is the interest rate that the monetary authority announces it will change, the larger is the current response of inflation. While the response of inflation to a 1% change in the current real rate is  $\kappa\sigma$ , the response of inflation to a 1% change in the real rate for one quarter in the infinite future is  $\kappa\sigma/(1 - \beta)$ . If  $\beta = 0.99$ , the current response of inflation to forward guidance about a single quarter in the infinite future is 100 times larger than the response of inflation to an equally large change in the current real interest rate. Figure 1 plots the response of inflation to forward guidance about interest rates at different horizons relative to the response of inflation to an equally large change in the current real interest rate. We see that the response of inflation to forward guidance about interest rates five years in the future is roughly 18 times larger than the response of inflation to an equally sized change in the current real interest rate.

To build intuition, we have assumed that there is no endogenous feedback from changes in output and inflation back onto real interest rates. Actual monetary policies are more complicated. In normal times, forward guidance about lower real interest rates in the future may be partly undone by higher real interest rates in the intervening period. On the other hand, when monetary policy is constrained by the zero lower bound on short-term nominal interest rates, the higher inflation associated with forward guidance about future interest rates will actually lower current real interest rates and this will in turn raise current output and inflation further. In this case, the outsized effects of forward guidance we describe above will be further reinforced by subsequent endogenous interest rate movements.

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<sup>6</sup>Here it is also important that after the interest rate shock passes, the output gap will go back to zero. In other words, the monetary shock has no effect on output in the long run. This is what pins down the level of the output gap.



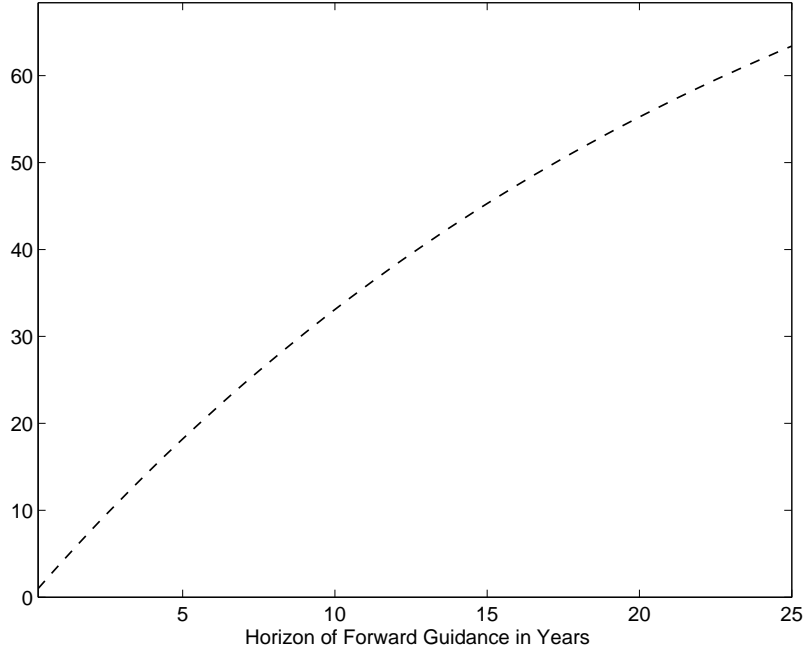


Figure 1: Response of current inflation to forward guidance about interest rates at different horizons relative to response to equally large change in current real interest rate.

### 3 An Incomplete Markets Model with Nominal Rigidities

Section 2 shows that the huge power of far future forward guidance in standard monetary models depends crucially on the prediction of the model that the current response of output to an expected change in real interest rates in the far future (say 20 years in the future) is equally large as the response of output to a change in the current real interest rate. But is this realistic? Increasing consumption today by 1% in anticipation of a 1% change in real interest rates 20 years from today would entail a large run down of assets over the 20 years until the interest rate changes. Agents that face uninsurable idiosyncratic income risk and borrowing constraints will trade off the benefits of intertemporal substitution and the costs in terms of reduced ability to smooth consumption over time of having lower buffer stock savings. To analyze this trade-off we develop a model with uninsurable idiosyncratic shocks to household productivity, borrowing constraints, and nominal rigidities.

### 3.1 The Environment

The economy is populated by a unit continuum of ex ante identical households with preferences given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{c_{h,t}^{1-\gamma}}{1-\gamma} - \frac{\ell_{h,t}^{1+\psi}}{1+\psi} \right],$$

where  $c_{h,t}$  is consumption of household  $h$  at time  $t$  and  $\ell_{h,t}$  is labor supply of household  $h$  at time  $t$ . Households are endowed with stochastic idiosyncratic productivity  $z_{h,t}$  that generates pre-tax labor income  $W_t z_{h,t} \ell_{h,t}$ , where  $W_t$  is the aggregate real wage. Each household's productivity  $z_{h,t}$  follows a Markov chain with transition probabilities  $\Pr(z_{h,t+1}|z_{h,t})$ . We assume that the initial cross-sectional distribution of idiosyncratic productivities is equal to the ergodic distribution of this Markov chain. As the the Markov chain transition matrix is constant over time, it follows that the cross-sectional distribution of productivities is constant. We use  $\Gamma^z(z)$  to denote this distribution.

In this economy, a final good is produced from intermediate inputs according to the production function

$$Y_t = \left( \int_0^1 y_{j,t}^{1/\mu} dj \right)^\mu,$$

where  $Y_t$  denotes the quantity of the final good produced at time  $t$  and  $y_{j,t}$  denotes the quantity of the intermediate good produced by firm  $j$  in period  $t$ . The intermediate goods are produced using labor as an input according to the production function

$$y_{j,t} = n_{j,t},$$

where  $n_{j,t}$  denotes the amount of labor hired by firm  $j$  in period  $t$ .

The market structure of this model economy combines elements that are familiar from the standard New Keynesian model with elements that are familiar from the standard incomplete markets model (Bewley, undated; Huggett, 1993; Aiyagari, 1994). While the final good is produced by a representative competitive firm, the intermediate goods are produced by monopolistically competitive firms. The intermediate goods firms face frictions in adjusting their prices that imply that they can only update their prices with probability  $\theta$  per period as in Calvo (1983). These firms are controlled by a risk-neutral manager who discounts future profits at rate  $\beta$ . Whatever profits are produced are paid out immediately to the households with each household receiving an equal share  $D_t$ . Households cannot trade their stakes in the firms.

Households trade a risk-free real bond with real interest  $r_t$  between periods  $t$  and  $t + 1$ . Borrowing constraints prevent these households from taking negative bond positions. There is a stock of

government debt outstanding with real face value  $B$ . The government raises tax revenue to finance interest payments on this debt. These taxes are collected by taxing households according to their labor productivity  $z_{h,t}$ . Let  $\tau_t \bar{\tau}(z_{h,t})$  be the tax paid by a household  $h$  in period  $t$ . By levying the taxes on labor productivity, which is exogenous, the tax does not distort household decisions in the same way that a lump-sum tax does not. At the same time, the dependence of the tax on  $z_{h,t}$  allows us to manipulate the cross-sectional correlation of tax payments and earnings.

We assume that the government runs a balanced budget so as to maintain a stable level of debt in each period. The government budget constraint is

$$\frac{B}{1+r_t} + \sum_z \Gamma^z(z) \tau_t \bar{\tau}(z) = B. \quad (5)$$

To illustrate our main results about the power of forward guidance, we will consider several monetary policy experiments involving somewhat different specifications of monetary policy. These are described in Section 4. The relationship between the real interest rate, the nominal interest rate  $i_t$ , and inflation  $\pi_t$  is given by the Fisher relation in the usual way

$$1+r_t = \frac{1+i_t}{1+\pi_{t+1}} \quad (6)$$

where  $\pi_{t+1} \equiv P_{t+1}/P_t - 1$  and  $P_t$  is the aggregate price level.

### 3.2 Decision Problems

The decision problem faced by the households in the economy is

$$V_t(b_{h,t}, z_{h,t}) = \max_{c_{h,t}, b_{h,t+1}, \ell_{h,t}} \left\{ \frac{c_{h,t}^{1-\gamma}}{1-\gamma} - \frac{\ell_{h,t}^{1+\psi}}{1+\psi} + \beta \sum_{z_{h,t+1}} \Pr(z_{h,t+1}|z_{h,t}) V_{t+1}(b_{h,t+1}, z_{h,t+1}) \right\}$$

subject to

$$c_{h,t} + \frac{b_{h,t+1}}{1+r_t} = b_{h,t} + W_t z_{h,t} \ell_{h,t} - \tau_t \bar{\tau}(z_{h,t}) + D_t$$

$$b_{h,t+1} \geq 0.$$

Let  $c_t(b, z)$  be the decision rule for  $c_{h,t}$ ,  $g_t(b, z)$  be the decision rule for household bond holdings  $b_{h,t+1}$ , and  $\ell_t(b, z)$  be the decision rule for  $\ell_{h,t}$ . These policy rules vary over time in response to aggregate events that affect current or future prices, taxes, or dividends.

The final goods producer's cost minimization problem implies that

$$y_{j,t} = \left( \frac{p_{j,t}}{P_t} \right)^{\mu/(1-\mu)} Y_t, \quad (7)$$

where  $p_{j,t}$  is the price changed by firm  $j$  in period  $t$  and the aggregate price index is given by

$$P_t = \left( \int_0^1 p_{j,t}^{1/(1-\mu)} dj \right)^{1-\mu}.$$

When an intermediate goods producer updates its price it solves

$$\max_{p_t^*, \{y_{j,s}, n_{j,s}\}_{s=t}^{\infty}} \sum_{s=t}^{\infty} \beta^s (1-\theta)^s \left( \frac{p_t^*}{P_s} y_{j,s} - W_s n_{j,s} \right)$$

subject to

$$y_{j,s} = \left( \frac{p_t^*}{P_s} \right)^{\mu/(1-\mu)} Y_s,$$

$$y_{j,s} = n_{j,s},$$

where  $p_t^*$  is the price set by firms who are able to update their price at date  $t$ .

The solution to this problem satisfies

$$\frac{p_t^*}{P_t} = \frac{\sum_{s=t}^{\infty} \beta^{s-t} (1-\theta)^{s-t} \left( \frac{P_t}{P_s} \right)^{\mu/(1-\mu)} Y_s \mu W_s}{\sum_{s=t}^{\infty} \beta^{s-t} (1-\theta)^{s-t} \left( \frac{P_t}{P_s} \right)^{1/(1-\mu)} Y_s}. \quad (8)$$

### 3.3 Equilibrium

Let  $\Gamma_t(b, z)$  be the distribution of households over idiosyncratic states at date  $t$ . This distribution evolves according to

$$\Gamma_{t+1}(\mathcal{B}, z') = \int_{\{(b,z): g_t(b,z) \in \mathcal{B}\}} \Pr(z'|z) d\Gamma_t(b, z) \quad (9)$$

for all sets  $\mathcal{B} \subset \mathbb{R}$ .

As a result of nominal rigidities, price dispersion will result in some loss of efficiency. Integrating both sides of (7) across firms and using  $y_{j,t} = n_{j,t}$  yields an aggregate production function

$$S_t Y_t = \int n_{j,t} dj \equiv N_t, \quad (10)$$

where  $N_t$  is aggregate labor demand and  $S_t \equiv \int_0^1 \left( \frac{p_{j,t}}{P_t} \right)^{\mu/(1-\mu)} dj$  reflects the efficiency loss due to price dispersion.  $S_t$  evolves according to

$$S_{t+1} = (1-\theta) S_t (1 + \pi_{t+1})^{-\mu/(1-\mu)} + \theta \left( \frac{p_{t+1}^*}{P_{t+1}} \right)^{\mu/(1-\mu)}. \quad (11)$$

Inflation can be written as a function of the relative price selected by firms that update their prices

$$1 + \pi_t = \left( \frac{1 - \theta}{1 - \theta \left( \frac{p_t^*}{P_t} \right)^{1/(1-\mu)}} \right)^{1-\mu}. \quad (12)$$

Aggregate labor supply is given by

$$L_t \equiv \int z \ell_t(b, z) d\Gamma(b, z). \quad (13)$$

and labor market clearing requires

$$L_t = N_t. \quad (14)$$

Bond market clearing requires

$$B = \int g_t(b, z) d\Gamma_t(b, z). \quad (15)$$

The aggregate dividend paid by the intermediate goods firms is

$$D_t = Y_t - W_t N_t. \quad (16)$$

Finally, integrating across the household budget constraints and using the government budget constraint and equation (16) gives

$$C_t = Y_t \quad (17)$$

as the aggregate resource constraint, where  $C_t \equiv \int c_t(b, z) d\Gamma_t(b, z)$ .

An equilibrium of this economy consists of decision rules and value functions  $\{g_t(b, z), \ell_t(b, z), V_t(b, z)\}_{t=0}^{\infty}$  that solve the household's problem, distributions  $\{\Gamma_t(b, z)\}_{t=0}^{\infty}$  that evolve according to (9). In addition, an equilibrium involves sequences  $\{C_t, L_t, N_t, Y_t, D_t, i_t, W_t, \pi_t, r_t, p_t^*/P_t, S_t, \tau_t\}_{t=0}^{\infty}$ , that satisfy the definitions of  $C_t$  and  $L_t$  and equations (5), (6), (8), (10), (11), (12), (14), (16), (17), and a monetary policy rule as described in section 4.

The main difference between this model and the model discussed in section 2 is the fact that markets are incomplete. If we modified this model to have complete markets and then linearized the equilibrium conditions, we would get the model in section 2. The introduction of incomplete markets yields a role for precautionary savings and it implies that redistribution of wealth across agents can affect the evolution of aggregate output. The fact that the present model is not linearized also implies that price dispersion affects equilibrium outcomes. Our main results regarding the reduced power of forward guidance stem from the precautionary savings motive.

Table 1: Baseline calibration

Parameter	Description	Value	Target
$\beta$	discount factor	0.986	2% annual interest rate
$\gamma$	risk aversion	2	
$1/\psi$	Frisch elasticity	1/2	Chetty (2012)
B	supply of assets	$1.4 \times$ annual GDP	aggregate liquid assets (see text)
$\mu$	markup	1.2	Christiano, Eichenbaum, and Rebelo (2011)
$\theta$	price revision rate	0.15	Christiano, Eichenbaum, and Rebelo (2011)

### 3.4 Calibration

Our model period is one quarter and our calibration is summarized in Table 1. We fix the steady state real interest rate at 2% annually and adjust the discount factor to match this.<sup>7</sup> We set the coefficient of risk aversion to 2. We set the Frisch elasticity of labor supply to 1/2, which is in line with the findings of Chetty (2012). In our baseline calibration we set the supply of government bonds,  $B$ , to match the ratio of aggregate liquid assets to GDP. We calculate liquid assets from aggregate household balance sheets reported in the Flow of Funds Accounts and take the average ratio over the period 1970 to 2013.<sup>8</sup> Our choice to calibrate the aggregate supply of assets to match liquid assets is motivated by the view that much of household net worth is illiquid and therefore not easily used for consumption smoothing and intertemporal substitution.<sup>9</sup> In a sensitivity analysis we also consider a calibration in which we match aggregate household net worth, which we also calculate from the Flow of Funds Accounts (described below).

For our choices of the desired markup of intermediate firms,  $\mu$ , and probability of maintaining a fixed price,  $\theta$ , we follow Christiano, Eichenbaum, and Rebelo (2011) and set  $\mu = 1.2$  and  $\theta = 0.15$ . The implied degree of price stickiness is on high side of values used in the business cycle literature.

<sup>7</sup>We use the term “steady state” to refer to the stationary equilibrium in which aggregate quantities and prices are constant and inflation is zero.

<sup>8</sup>We use the same definition of liquid assets as Guerrieri and Lorenzoni (2011). Flow of Funds Table B.100 Lines 10 (deposits), 17 (treasury securities), 18 (agency and GSE securities), 19 (municipal securities), 20 (corporate and foreign bonds), 24 (corporate equities), 25 (mutual fund shares).

<sup>9</sup>Kaplan and Violante (2014) present a lifecycle savings model with liquid and illiquid assets and show that the illiquidity of household net worth leads to stronger and more realistic consumption responses to transitory income fluctuations.

This tends to reduce the size of the effects we find on inflation since it makes inflation less sensitive to changes in current marginal costs. In the exercises we do where the zero lower bound on nominal interest rates binds, this also tends to reduce the size of the effects on output since the smaller effects on inflation translate into smaller effects on real interest rates (Werning, 2012).

We calibrate the idiosyncratic wage risk to the persistent component of the estimated wage process in Floden and Lindé (2001).<sup>10</sup> The estimates of Floden and Lindé are for an AR(1) with annual observations of log residual wages after the effects of age, education, and occupation have been removed. Floden and Lindé find an autoregressive coefficient of 0.9136 and an innovation variance of 0.0426. We convert these estimates to parameters of a quarterly AR(1) process for log wages by simulating the quarterly process and aggregating to annual observations. We find the parameters of the quarterly process such that estimating an AR(1) on the simulated annual data reproduces the Floden and Lindé estimates, which results in an autoregressive coefficient of 0.966 and an innovation variance of 0.017. We discretize the resulting AR(1) process for log wages to a three-point Markov chain using the Rouwenhorst (1995) method.<sup>11</sup>

Finally, to capture the fact that the bulk of tax payments are made by those with high earnings we set  $\bar{\tau}(z)$  to be positive only for the highest  $z$ . Since households are heterogeneous in the incomplete markets model, their MPCs differ widely. Households with little wealth have high MPCs, while households with a great deal of wealth have much lower MPCs. As a consequence, wealth redistribution matters for aggregate consumption dynamics. For example, a reallocation of income from high to low net worth households leads to higher consumption demand, all else equal. One way in which this shows up in our model is that the government levies taxes to finance interest payments on debt. A fall in interest rates, therefore, may lead to a redistribution of wealth due to variation across households in holdings of government debt as well as tax obligations. Our (realistic) assumption that taxes are paid mostly by the rich leads these tax effects to be relatively small.

### 3.5 Alternative Calibrations

Our baseline calibration potentially implies too little volatility in household earnings. Guvenen, Ozkan, and Song (2014) report the standard deviation of the distribution of five-year earnings growth

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<sup>10</sup>While it is common to include a transitory income shock in empirical models of wage dynamics we do not include such transitory shocks in our analysis because their impact on the quantitative results will be small as these shocks are easily smoothed by virtue of being transitory.

<sup>11</sup>Kopeccky and Suen (2010) prove that the Rouwenhorst method can match the conditional and unconditional mean and variance, and the first-order autocorrelation of any stationary AR(1) process.

rates to be 0.73.<sup>12</sup> Our model calibrated as described above implies this standard deviation is only 0.53.

We therefore consider an alternative calibration in which we raise the variance of the productivity shocks in the model so that our model matches this moment of the five-year earnings growth rate distribution. Doing so requires raising the variance of the idiosyncratic productivity innovation from 0.017 to 0.033. With more risk, the larger precautionary savings motive raises the total demand for assets by households. In this calibration we, thus, reduce the discount factor so that the model is again consistent with the total supply of assets and a 2% annual interest rate. This requires a discount factor of 0.978. We refer to this as the High Risk calibration.

We also explore the extent to which our results depend on the average level of assets in the economy. With more assets, households will generally have more self-insurance and therefore will be less concerned with running down their assets. To explore this possibility we consider an alternative calibration in which we raise the supply of government debt,  $B$ , so that the average wealth in the economy is equal to the aggregate net worth of the household sector from the Flow of Funds, including both liquid and illiquid wealth.<sup>13</sup> This yields a ratio of assets to annual GDP of 3.79. With a larger supply of assets, bond market clearing requires that households are more patient so as to increase the demand for assets at a given interest rate. In this calibration, we set the discount factor to 0.992 to be able to match a 2% annual interest rate. We refer to this as the High Asset calibration.

Finally we consider a case where we increase both the supply of assets and the extent of risk that households face. In this case we match a ratio of assets to GDP of 3.79 and the standard deviation of five-year earnings growth rates of 0.73. The discount factor needed to match a 2% annual interest rate is 0.990. We refer to this as the High Risk and Asset calibration.

### 3.6 Computation

In Section 4, we compute the perfect foresight transition paths of the economy in response to monetary policy and demand shocks. We assume that the economy begins in the steady state and returns to steady state after 250 quarters. We begin by guessing paths for all aggregate quantities and prices. We can then verify whether this guess is an equilibrium by checking that the definition of an equilibrium given above is satisfied. Part of this step involves solving and simulating the households problem at

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<sup>12</sup>This value is the average across years of the values reported in Table A8 of Guvenen, Ozkan, and Song (2014).

<sup>13</sup>Here we use the ratio of household net worth to GDP averaged over 1970 to 2013. Household net worth is taken from Table B.100 Line 42.



the guessed prices. We solve the household’s problem by iterating on the Euler equation backwards through time using the endogenous gridpoint method of Carroll (2006) to compute the policy rules for each period of the transition. We then simulate the population of households forwards through time using a non-stochastic simulation algorithm to compute the distribution of wealth at each date. We can then compute aggregate consumption, labor supply, and bond holdings using the policy rules and distribution of wealth for each date. If our guess is not an equilibrium we update it to a new guess that is closer to an equilibrium. We generate the new guess of prices and aggregate quantities by making use of an auxiliary model that approximates the aggregate behavior of the incomplete markets households and then solving for an equilibrium under this approximating model. We perform this step using a version of Newton’s method. We provide additional details of the computational methods in Appendix A.

## 4 Results

Our main result is that the power of forward guidance is substantially muted in the incomplete markets model we present in Section 3 relative to the standard complete-markets New Keynesian model. To illustrate this, we first consider a simple policy experiment: suppose the central bank promises a 50 basis point (i.e., 2% annualized) decrease in the real interest rate for a single quarter 5 years in the future.<sup>14</sup>

Figure 2 plots the response of output to this shock in our incomplete markets model and, for comparison, in the complete markets version of this model. With complete markets, output immediately jumps up by 25bp and remains at that elevated level for 20 quarters before returning to steady state. In contrast, in the incomplete markets model the initial increase in output is only about 40% as large. Output then gradually rises as the interest rate decrease gets closer. But even in the period right before the interest rate increase, the increase in output is substantially smaller than under complete markets.

Figure 3 plots the response of inflation to this same shock. The five year output boom induced by the forward guidance about real interest rates leads to a large inflation response in the complete markets case. Since the output boom is much smaller in the incomplete markets model, the rise in inflation is also much smaller. The initial response of inflation in the incomplete markets model is

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<sup>14</sup>As in Section 2, we assume here that the monetary authority sets an exogenous path for the real interest rate.

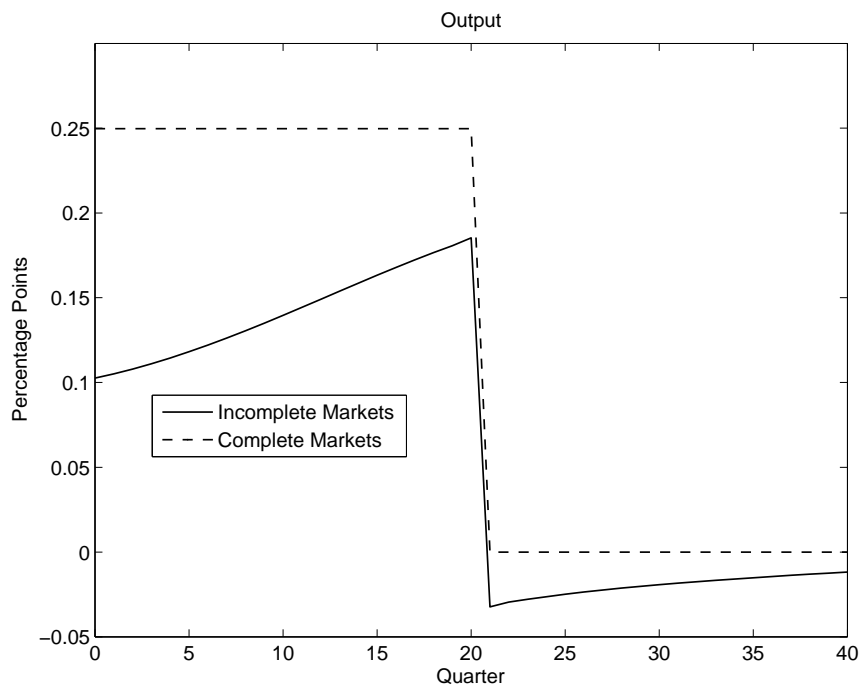


Figure 2: Response of output to 50 basis point forward guidance about the real interest rate in quarter 20 (with real interest rates in all other quarters unchanged).

again only about 40% as large as in the complete markets model.

#### 4.1 Intuition

To build intuition for why the effects of forward guidance are greatly reduced in the incomplete markets model, it is useful to start by considering the response of a single household in partial equilibrium. Figure 4 plots the partial equilibrium response of consumption and assets for a household with median productivity and wealth. For comparison, Figure 4 also plots the responses of these variables for a household with this same amount of wealth in a model with complete markets.<sup>15</sup>

As in Figure 2, the response of consumption under complete markets is to jump up immediately and remain high for 5 years. The shape of the response under complete markets is the same whether the interest rate shock is expected to occur 1 quarter or 40 quarters in the future. The reason the response has this shape is that the interest rate change alters the price of consumption before the change relative to the price of consumption after the change, but does not alter the relative price of consumption between two dates before the change or between two dates after the change.

<sup>15</sup>Specifically, the household being plotted is a household with median productivity and median wealth among households with median productivity in the incomplete markets model. The consumption and asset responses plotted are the percentage change in the evolution of these variables relative to what they would be without the shock to interest rates. We assume a realization of idiosyncratic productivity that leaves the household's productivity unchanged.

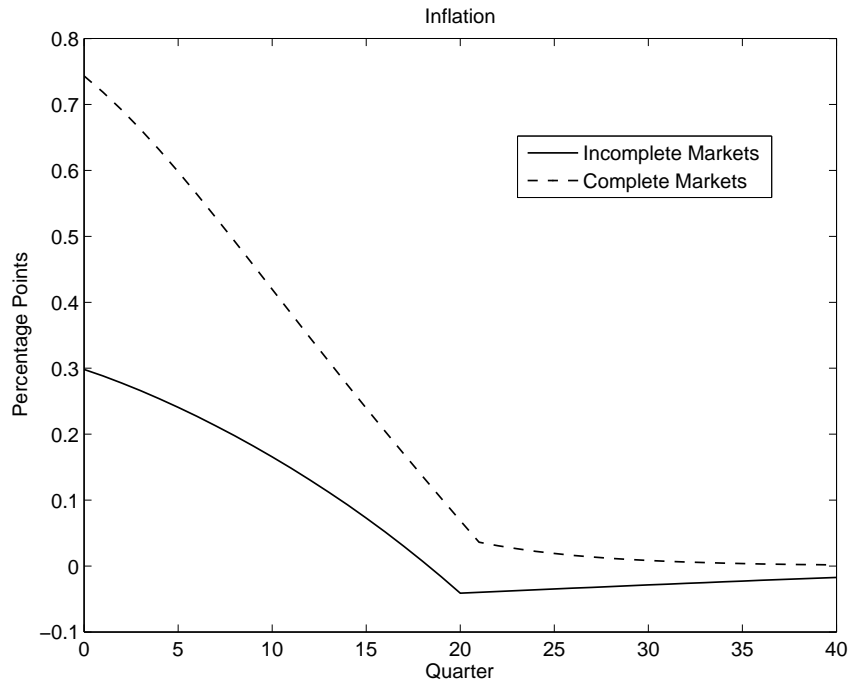


Figure 3: Response of inflation to 50 basis point forward guidance about the real interest rate in quarter 20 (with real interest rates in all other quarters unchanged).

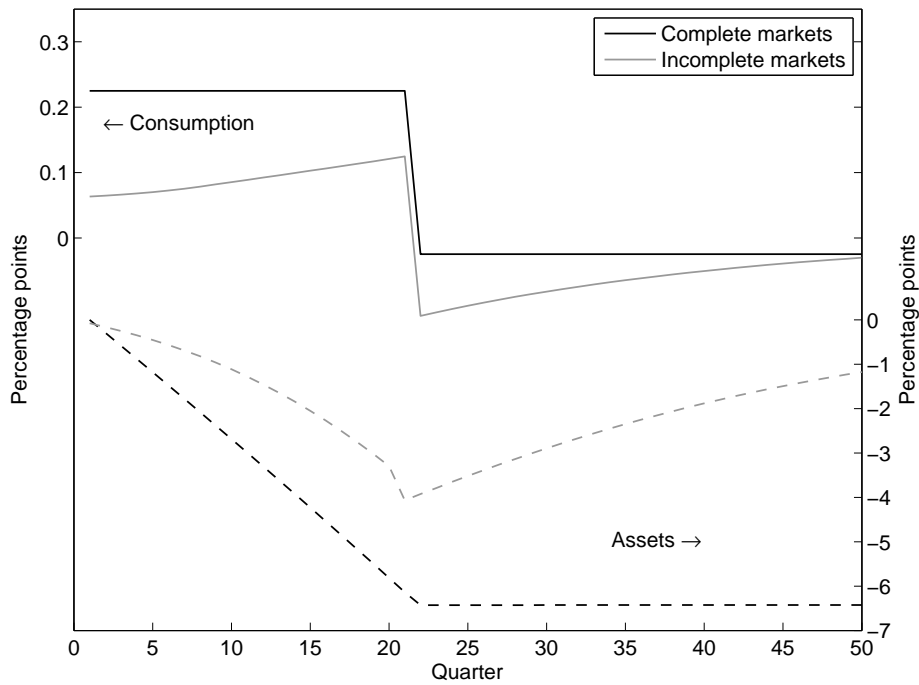


Figure 4: Partial equilibrium response of consumption and assets under complete and incomplete markets.

Under complete markets, consumption at all dates before the change must therefore be equal, and consumption at all dates after the change must also be equal. The result is a step function.

Why doesn't the incomplete markets household respond in the same way? Figure 4 shows that the complete markets response requires a substantial run-down in assets. This poses no concern for the complete markets household (since it is fully insured against all shocks). In contrast, the incomplete markets household needs to maintain a buffer stock of assets to help insulate itself from future shocks. Were the incomplete markets household to run-down its assets the way the complete markets household does, it would face permanently higher consumption volatility going forward due to its reduced buffer-stock savings. This precautionary motive leads the incomplete markets household to choose a more conservative consumption path.

In general equilibrium, households can't all run down their wealth. The increase in demand resulting from the desire to increase consumption instead results in an increase in aggregate income. This implies that if all other households responded in the way the complete markets households do, income would rise enough that a given household could do the same without having to run down its wealth. However, even if all other households did respond in this way, it is not optimal for a given household faced with incomplete markets to do the same, since it would instead want to spend a fraction of its increased income on further building up its precautionary savings. Since all households think this way, they all demand less than they would under complete markets which reduces their incomes and further reduces their demand. The result is a substantial reduction in the response of consumption (and output) to the interest rate shock. It is this trade-off between precautionary savings and intertemporal substitution that reduces the power of forward guidance in our model.

## 4.2 Dependence on Horizon of Forward Guidance

The difference between the complete and incomplete markets models grows with the horizon of the interest rate shock. Figure 5 plots the initial response of output to 50 basis point forward guidance about the real interest rate in a single quarter as the horizon of that single quarter changes from zero to 40 quarters.<sup>16</sup>

In the complete markets model, output always rises by 25 basis points, regardless of the horizon of the forward guidance. In contrast, in the incomplete markets model, the effect is only about 20

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<sup>16</sup>For example, the points at horizon 20 in Figure 5 are the first points on each line in Figure 2. And the points at horizon 10 in Figure 5 are the initial response of output in the two models if the central bank announces that it will lower the real interest rate by 50 basis points for a single quarter 10 quarters in the future.

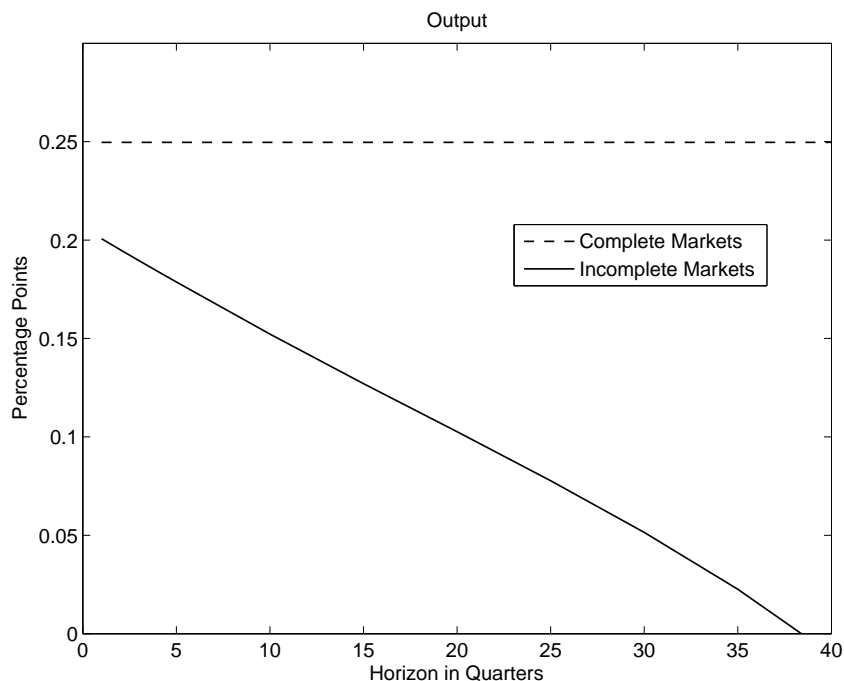


Figure 5: Initial response of output to 50 basis point forward guidance about the real interest rate for a single quarter at different horizons.

basis points for an announcement about the real interest rate next quarter and falls monotonically thereafter. It is roughly 10 basis points for an announcement about the real interest rate 5 years ahead; and essentially zero for an announcement about the real interest rate 10 years ahead.

Intuitively, the response of consumption is governed by two forces: an intertemporal substitution motive (as in the complete markets case) and a precautionary savings motive. For far-future interest rate changes, the run-down in assets needed to reap the benefits of intertemporal substitution are very large. This implies that eventually the benefits from intertemporal substitution are simply too small to make it worth it for households to incur the costs associated with running down their buffer-stock savings.

The results for inflation are even starker. Figure 6 plots the initial response of inflation to forward guidance about the real interest rate at different horizons. In the complete markets model, the response of inflation rises explosively with the horizon of the forward guidance. Above 20 quarters, the model “explodes”: the inflation response grows so quickly that we can no longer compute it numerically. In the incomplete markets model, in contrast, the inflation response is smaller to start out with, grows more slowly, and therefore generates very different results at long horizons.

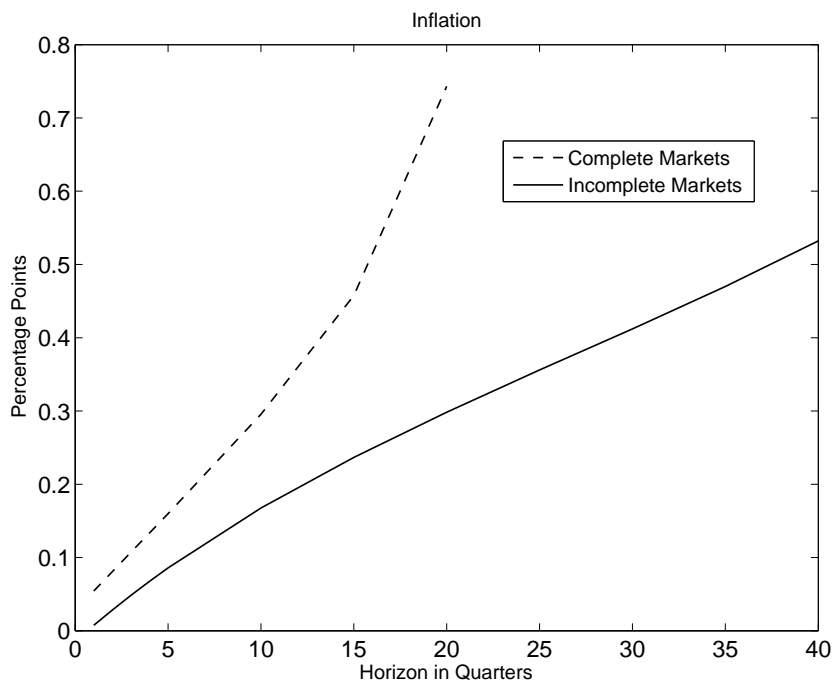


Figure 6: Initial response of inflation to 50 basis point forward guidance about the real interest rate for a single quarter at different horizons.

### 4.3 Results for Alternative Calibrations

Table 2 presents the results of the forward guidance experiment described above for our baseline incomplete markets model as well as for several alternative calibrations of our incomplete markets model. We also present the results for the the complete markets version of our model, for comparison. In each case, we present the initial response of output and inflation to 50 basis point forward guidance about the real interest rate for a single quarter 5 years in the future.

The High Risk calibration features greater uninsurable risk than our baseline calibration. We roughly double the volatility of idiosyncratic productivity shocks relative to our baseline calibration, allowing us to match recent evidence on the volatility of earnings growth from Guvenen, Ozkan, and Song (2014). This boosts the precautionary savings motive and further reduces the power of forward guidance relative to the complete markets benchmark. The response of output in this case is only about 20% of the complete markets benchmark and the response of inflation only about 32% of the complete markets benchmark.

In the High Asset calibration, we set the ratio of assets to GDP in the model to be almost three times higher than in our baseline calibration (3.79 versus 1.4). We do this to match the ratio of total net worth in the economy to GDP (as opposed to total liquid assets as in our baseline calibration).

Table 2: Power of 20 Quarter Ahead Forward Guidance

	Initial Responses of	
	Output	Inflation
Baseline	10.3	29.8
High Risk	4.8	23.8
High Asset	14.5	36.2
High Risk and Asset	11.6	33.8
Complete Markets	25.0	74.3

Initial response of output and inflation (in bases points) to forward guidance that reduces the expected real interest rate 20 quarters ahead by 50 basis point for four different calibrations of our incomplete markets model.

Increasing the quantity of available assets in the economy increases the size of the precautionary savings buffers available to households and thus reduces their reluctance to engage in intertemporal substitution. This change therefore moves the incomplete markets model closer to the complete markets benchmark. The output response rises to 58% of the complete markets benchmark, while the inflation effect rises to 49%.

Finally, we consider a High Risk and Asset calibration with both of the above-mentioned alternative parameter values. These two modifications largely offset each other. As a consequence, the results lie between the two calibrations described above and close to the baseline calibration. The response of output in this calibration is 46% of the complete markets benchmark, while the response of inflation is 45% of the complete markets benchmark.

An alternative way of calibrating the model would be to choose parameters to fit empirical estimates of the marginal propensity to consume (MPC) out of additional wealth. The average MPC in our baseline calibration is only 12%, and even in our High Risk calibration it rises only to 14%. In contrast, a substantial amount of empirical evidence suggests larger values for the MPC, with many studies estimating values close to 20%.<sup>17</sup> On this basis, one might argue for calibrations in

<sup>17</sup>Parker (1999) estimates that household spend 20% of increases in disposable income when they hit the Social Security tax cap. Johnson, Parker, and Souleles (2006) estimate that households spent 20-40% of tax rebate checks they received in 2001, and Parker et al. (2013) estimate that households spent 12-30% of tax rebate checks they received in 2008. These studies as well as most others on this topic consider anticipated changes in income. They therefore provide a lower bound for responses to unanticipated changes in income. See Jappelli and Pistaferri (2010) for an excellent recent survey of the literature on the response of consumption to changes in income.

which households are more credit constrained than even our High Risk calibration implies. Such a calibration would further amplify the effects we emphasize regarding the differences between the complete and incomplete markets models.

#### 4.4 Zero Lower Bound Analysis

In recent years, risk-free nominal interest rates around the world have hit zero. At the zero lower bound (ZLB), forward guidance has become an indispensable policy tool, since it is no longer possible to implement monetary policy via the current policy rate. Eggertsson and Woodford (2003) show how a persistent shock to the natural interest rate that causes the economy to hit the ZLB can provoke a massive recession if the central bank does not engage in unconventional monetary policy. They show, however, that the recession can be fully abated by a relatively modest amount of forward guidance about future interest rates.

Our conclusions above suggest that forward guidance may not be as powerful at the ZLB in our incomplete markets model. To investigate this question, we follow Eggertsson and Woodford (2003) in assuming that the ZLB is brought on by a temporary shock to the subjective discount factor of households in the economy that depresses the natural rate of interest below zero. In other words, we now consider a case where the discount factor can vary over time. The specific shock we consider is an increase in the discount factor that lasts for a known number of quarters and then reverts to normal.<sup>18</sup> We choose the size and persistence of the shock so that the initial output decline is 4% and the ZLB binds for 20 quarters under a naive monetary policy (described below).<sup>19</sup>

We consider two alternative monetary policies. First, we consider a policy where the central bank sets the nominal interest rate equal to a simplified Taylor rule whenever this yields an interest rate greater than zero, and, otherwise, sets the nominal rate to zero:  $i_t = \max[0, \bar{r} + \phi\pi_t]$ , where  $\phi = 1.5$  and  $\bar{r}$  is the steady state real interest rate. We refer to this policy as the “naive” policy. We also

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<sup>18</sup>Our shock differs from the shock considered in Eggertsson and Woodford (2003) in that its persistence is known implying that agents have perfect foresight about the evolution of the aggregate economy. Eggertsson and Woodford (2003) consider a shock that reverts back to normal with constant probability each period. Clearly, both formulations are approximations. Eggertsson and Woodford’s formulation abstracts from time-variation in the probability of the ZLB period ending, while our framework abstracts from uncertainty about when it will end. However, the incomplete markets model is more difficult to solve computationally without the assumption of perfect foresight for aggregate shocks. In Section 5, we consider Eggertsson and Woodford’s formulation of the shock in a more tractable approximation of the incomplete markets model.

<sup>19</sup>Hitting these targets requires slightly different calibrations of the discount factor shock in the complete versus the incomplete markets model: it corresponds to a decline in the natural rate of 16.4 basis points in the complete markets model, but only 14.8 basis points in the incomplete markets model. In each case, the duration of the shock is 33 quarters.



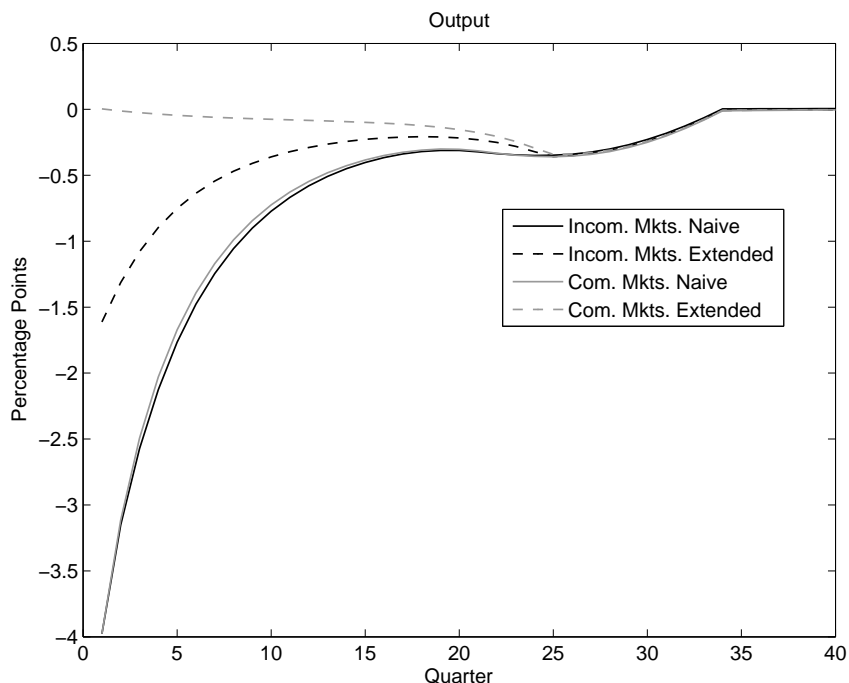


Figure 7: Response of output to the ZLB shock.

consider an “extended” policy whereby the central bank sets the nominal rate to zero for several additional quarters beyond what is implied by the naive policy and then reverts back to the policy rule. We choose the length of the additional monetary stimulus to fully eliminate the initial fall in output in the complete markets model.

Figure 7 shows that forward guidance is substantially less powerful at the ZLB in the incomplete markets model than in the standard New Keynesian model. The bottom two lines show the path of output under the naive monetary policy for the complete and incomplete markets cases; while the top two lines show the effects of the extended monetary policy in these two cases. While the extended monetary policy fully eliminates the recession in the complete markets case, a substantial recession remains in the incomplete markets model. Figure 8 shows the implications for inflation: the extended policy is much more successful in preventing deflation in the complete markets model versus the incomplete markets model. While the initial deflation is only about 30 basis points in the complete markets case, it is more than 100 basis points in the incomplete markets case. The fact that inflation is lower in the incomplete markets case implies that real interest rates are higher (since nominal rates are stuck at zero). This contributes to the larger fall in output.

Figure 9 plots the implications of the naive and extended monetary policies for the nominal interest rate. Under the naive policy the ZLB binds for 20 quarters and then rises gradually to its

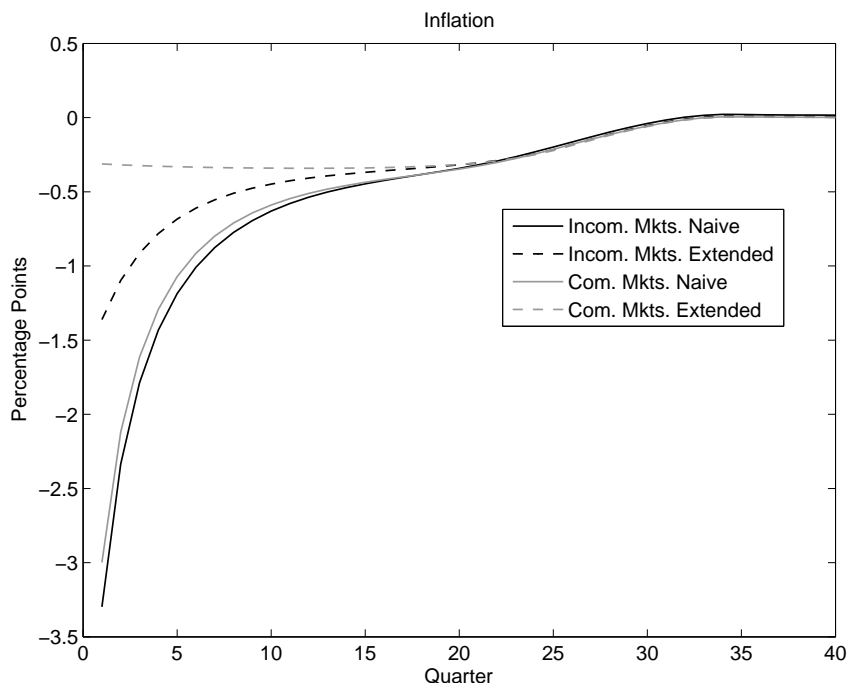


Figure 8: Response of inflation to the ZLB shock.

steady state value of 50 basis points. Under the extended policy, the nominal interest rate remains at zero for 23 quarters (an additional 3 quarters), and interest rates are somewhat lower in quarter 24 than the naive policy implies (this partial stimulus in the 24th quarter is what is needed to exactly eliminate the initial fall in output due to the shock). The difference between the dashed and solid lines, thus, indicates the amount of additional stimulus provided by the extended policy.

## 5 Discounted Euler Equation Model

We argued in Section 2 that the power of forward guidance in standard monetary models results from the fact that current output responds just as strongly to a change in the expected real interest rate 20 years in the future as it does to an equally large change in the current real interest rate (see equation (3)). Our analysis in Sections 3 and 4 indicates that this is no longer the case in a model where agents face uninsurable idiosyncratic income risk and borrowing constraints. In such a model, precautionary savings forces imply that output reacts much less to changes in far future expected real interest rates than current real interest rates (Figure 5).

However, our full model with uninsurable idiosyncratic income risk and borrowing constraints is more difficult to analyze than the linearized DSGE models commonly used in business cycle anal-

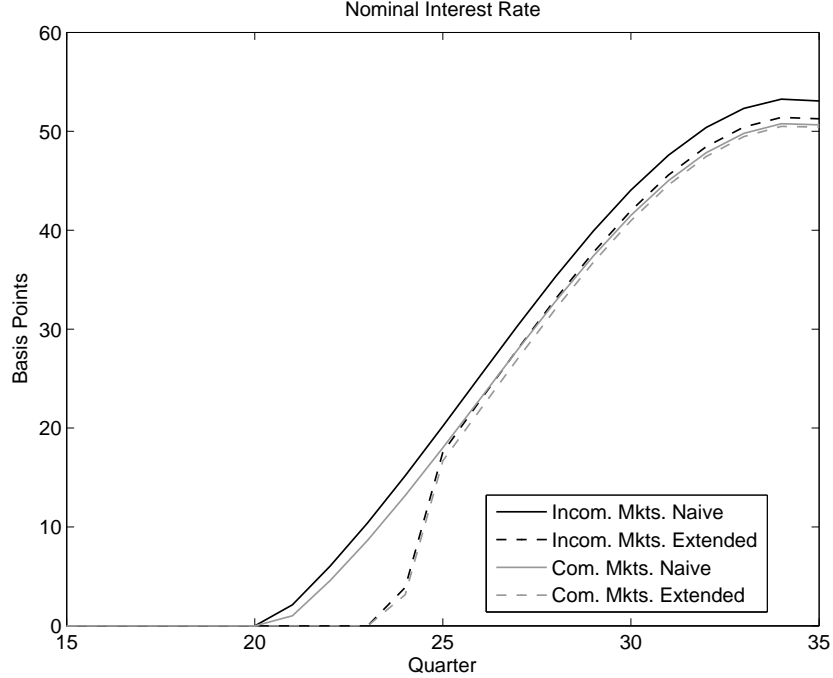


Figure 9: Response of nominal interest rate to the ZLB shock.

ysis. It is therefore useful to develop a modification to the standard linearized consumption Euler equation that better approximates a model with uninsurable idiosyncratic income risk and borrowing constraints. Figures 2 and 5 suggest that what is needed is to “discount” the effect of future interest rates on current consumption.

We therefore consider the following simple modification to the standard linearized consumption Euler equation:

$$x_t = -\zeta\sigma E_t \sum_{j=0}^{\infty} \alpha^j (i_{t+j} - E_{t+j}\pi_{t+j+1} - r_{t+j}^n), \quad (18)$$

where  $\alpha < 1$  causes future interest rates to be discounted exponentially, and  $\zeta < 1$  is a factor that reduces the the response of output to all interest rates. Equation (18) can be first differenced to yield

$$x_t = \alpha E_t x_{t+1} - \zeta\sigma (i_t - E_t \pi_{t+1} - r_t^n). \quad (19)$$

We refer to this equation as the discounted Euler equation. The key difference between the discounted Euler equation and the standard Euler equation is the factor  $\alpha < 1$  multiplying expected future consumption. This term implies that far future interest rate changes have much smaller effects on current consumption than near term interest rate changes.

In Appendix B, we show how the discounted Euler equation can be micro-founded with a simplified version of our incomplete markets model that is analytically tractable. In this simplified

version, idiosyncratic productivity is i.i.d. and takes one of two values each period, which we refer to as employed ( $z = 1$ ) and unemployed ( $z = 0$ ). The only income of the unemployed is a fixed unemployment benefit. Furthermore, we assume that there are no assets available ( $B = 0$ ).<sup>20</sup> In this simplified model, all unemployed households are borrowing constrained and the behavior of aggregate consumption is determined by the Euler equation of the employed households. Linearizing that Euler equation yields our discounted Euler equation—equation (19). The term  $\alpha$  reflects the fact that with some probability the currently employed households will be unemployed next period and next period’s expected marginal utility is therefore partly determined by the constant marginal utility in the unemployed state. As consumption in the unemployed state is not forward looking, taking an expectation over the employed and unemployed states serves to limit the forward-looking nature of consumption in the employed state. The term  $\zeta$  reflects the fact that some of aggregate consumption in each period is determined by that period’s constrained unemployed agents who do not respond to interest rates.<sup>21</sup>

Figure 10 shows that with  $\alpha = 0.97$  and  $\zeta = 0.75$ , the discounted Euler equation provides a good approximation to the response of output to a real interest rate shock 20 quarters in the future. The approximation is nearly perfect up until the time that the interest rate changes. What the discounted Euler equation misses is the fall in consumption after the interest rate shock. This fall is due to redistribution of wealth in the incomplete markets model (from households with high MPCs to households with low MPCs), which the discounted Euler equation does not capture.

To illustrate the importance of allowing for discounting in the Euler equation, we revisit the question of monetary policy at the zero lower bound using the standard linearized New Keynesian model analyzed in Section 2 with the standard Euler equation replaced by the discounted Euler equation. An advantage of this approach to incorporating the precautionary savings effects we emphasize is that we are able to deviate from our previous setting of perfectly anticipated shocks. In particular, we assume that the ZLB binds due to a shock that lowers the natural rate below zero

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<sup>20</sup>Krusell, Mukoyama, and Smith (2011) and Ravn and Sterk (2013) have also used this approach to formulate tractable incomplete markets models.

<sup>21</sup>Pierrigallini (2006) and Nistico (2012) provide an alternative micro-foundation for discounting in the Euler equation based on mortality shocks as in Blanchard’s (1985) perpetual-youth model. In their formulation, discounting only arises from aggregation over different generations, and to generate a quantitatively important deviation from the standard Euler equation, these authors assume counter-factually high death rates. Our approach rationalizes why long-lived consumers can have short planning horizons. Moreover, in Pierrigallini and Nistico’s formulation, the discounting in the Euler equation is larger the larger is the amount of financial wealth in the economy and disappears when financial wealth is in zero-net supply. In contrast, in our full model, agents discount the future more when they have little liquid financial wealth to buffer shocks to income.

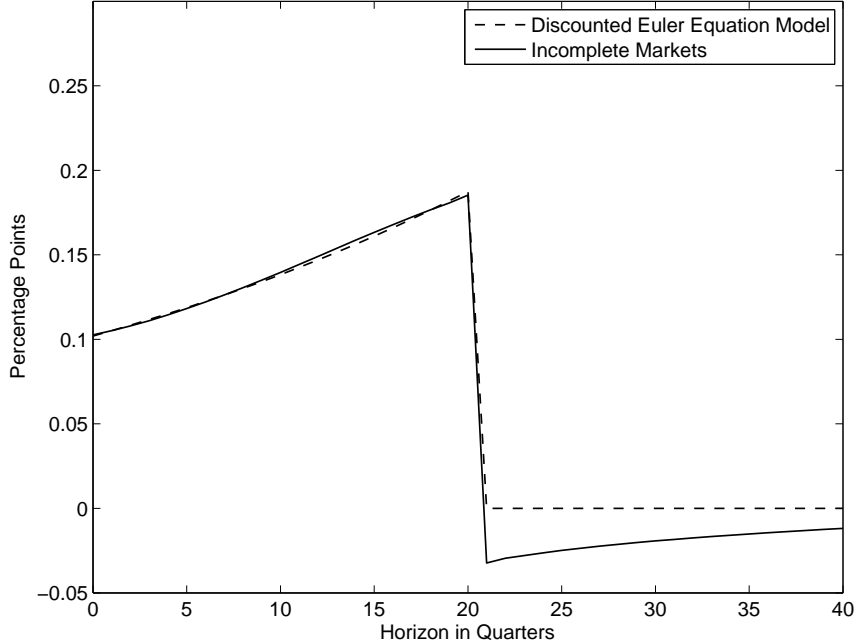


Figure 10: Response of output to a 50 bp change in the expected real interest rate in quarter 20 in the incomplete markets model and in the discounted Euler equation model.

and persists at the same negative value with probability  $\lambda$  each quarter. With probability  $1 - \lambda$ , it reverts back to normal. For simplicity, we assume that once the natural rate reverts back to normal, the zero lower bound on nominal interest rates never binds again in the future. This is the same type of shock as Eggertsson and Woodford (2003) consider.

We assume that the central bank follows a “naive” monetary policy similar to the one we consider in Section 4

$$i_t = \max(0, r_t^n + \phi\pi_t), \quad (20)$$

where  $\phi > 1$ . The full model then consists of equations (2), (19), and (20). For comparability with Eggertsson and Woodford (2003), we assume that  $\beta = 0.99$ ,  $\sigma = 0.5$ , and  $\kappa = 0.02$ . To mimic the behavior of the incomplete markets model regarding the effects of future interest rates, we set  $\alpha = 0.97$  and  $\zeta = 0.75$  as in Figure 10. We also consider the standard case of  $\alpha = 1$  and  $\zeta = 1$ .

We start by solving for the level of the output gap and inflation after the shock has dissipated. Since we have assumed that the natural rate will never go negative again, it is feasible for the monetary authority to set  $i_t = r_t^n$  at all times after the shock dissipates. This implies that both the output gap and inflation will be zero at all times after the shock dissipates. Given this, it is easy to solve for the output gap and inflation while the shock persists. First, notice that all periods

while the shock persists are identical since the probability of the shock reverting to normal does not change over time. This implies that output and inflation will be constant while the shock persists. We refer to the period during which the shock persists as the short run. Next, notice that in the short run  $E_t x_{t+1} = \lambda x_t$  and  $E_t \pi_{t+1} = \lambda \pi_t$  since with probability  $1 - \lambda$  the economy will revert to normal (in which case  $x_t = \pi_t = 0$ ). Using these facts and equations (2) and (19), a few steps of algebra (presented in Appendix C) yield

$$\pi_S = \frac{\kappa}{1 - \beta p} x_S, \quad (21)$$

$$x_S = \frac{\zeta \sigma}{1 - \alpha \lambda - \frac{\zeta \sigma \lambda \kappa}{1 - \lambda \beta}} r_S^n, \quad (22)$$

where  $\pi_S$  and  $x_S$  denote inflation and the output gap in the short run, and  $r_S^n$  denotes the natural real rate of interest in the short run.

Eggertsson and Woodford (2003) present results for a shock that lowers the natural rate to  $r_S^n = -0.02$  (annualized) and reverts to normal with probability  $\lambda = 0.1$  (per quarter). They show that in the standard model, a shock of this size and persistence generates a very large recession—an output gap of -14.3%—accompanied by a large amount of deflation (-10.5%). In Table 3, we show that in the discounted Euler equation model, this same shock leads to a much more modest recession. The output gap is a mere -2.9%, and inflation falls by only 2.1%. Clearly, incorporating discounting of future interest rates radically alters the conclusions one comes to about the severity of the problem that the monetary authority faces with this type of shock.<sup>22</sup>

The strength of the deflationary forces in the standard model are due to a feedback loop that gets stronger the more persistent is the shock to the natural rate. The basic feedback loop results from the following chain of logic: The negative natural rate leads to a positive interest rate gap—a real rate that is higher than the natural rate—because the nominal rate can't fall below zero. This leads output to fall, and if the shock is persistent it leads expectations of future output to fall, which in turn leads expected inflation to fall, which causes the current real rate to rise further, and current output to fall further, etc. The more persistent is the shock, the more it affects expected inflation and the stronger this feedback loop becomes.

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<sup>22</sup>Here we have illustrated the importance of allowing for discounting in the Euler equation using a shock to expected natural real rate of interest, while elsewhere in the paper we have focused on its importance when it comes to changes in expectations about actual real rates. It is easy to see from equation (18) that discounting mutes the effect of changes in both far future natural rates and far future actual rates by an equal amount. It is only the difference between the realized rate and the natural rate at each horizon that matters for the consumption-savings decisions of households.

Table 3: How Severe a Constraint Is the ZLB?

	Output	Inflation
Standard Model ( $\alpha = 1, \zeta = 1$ )	-14.3%	-10.5%
Discounted Euler Equation Model ( $\alpha = 0.97, \zeta = 0.75$ )	-2.9%	-2.1%

Response of output and inflation when the natural rate falls to -2% (annualized) with a 10% per quarter probability of returning to normal.

It is well known that the strength of the deflationary forces associated with negative shocks to the natural real rate become infinitely strong—i.e. imply that the (log) output gap and inflation go to negative infinity—for even relatively modest levels of persistence. This can be seen by looking at the denominator of the expression for the short run output gap in equation (22). As this denominator goes to zero, the short run output gap goes to negative infinity. In the standard model (with  $\alpha = 1$  and  $\zeta = 1$ ), this occurs for a shock with an expected duration of 11 quarters.

In the discounted Euler equation model, the strength of the deflationary forces are muted and consequently the persistence of the ZLB shock needs to be greater for this “deflationary death spiral” to occur. This is depicted in Figure 11, which plots the drop in output for different levels of persistence of the ZLB shock. The solid line is the standard model, while the two broken lines are two calibrations of the discounted Euler equation model that match the baseline and high-risk calibrations of our incomplete markets model, respectively.<sup>23</sup> The deflationary death spiral occurs only for shocks that are considerably more persistent in the discounted Euler equation model than in the standard model.

## 6 Conclusion

We study the effects of forward guidance about monetary policy. We do this in a standard New Keynesian model augmented with uninsurable income risk and borrowing constraints. Our main finding is that allowing for uninsurable income risk and borrowing constraints substantially decrease

<sup>23</sup>As we discuss in the text,  $\alpha = 0.97$  and  $\zeta = 0.75$  matches the baseline calibration of the incomplete markets model well. We use the same approach to match the high-risk calibration of the incomplete markets model with the discounted Euler equation model. This yields  $\alpha = 0.94$  and  $\zeta = 0.7$ .

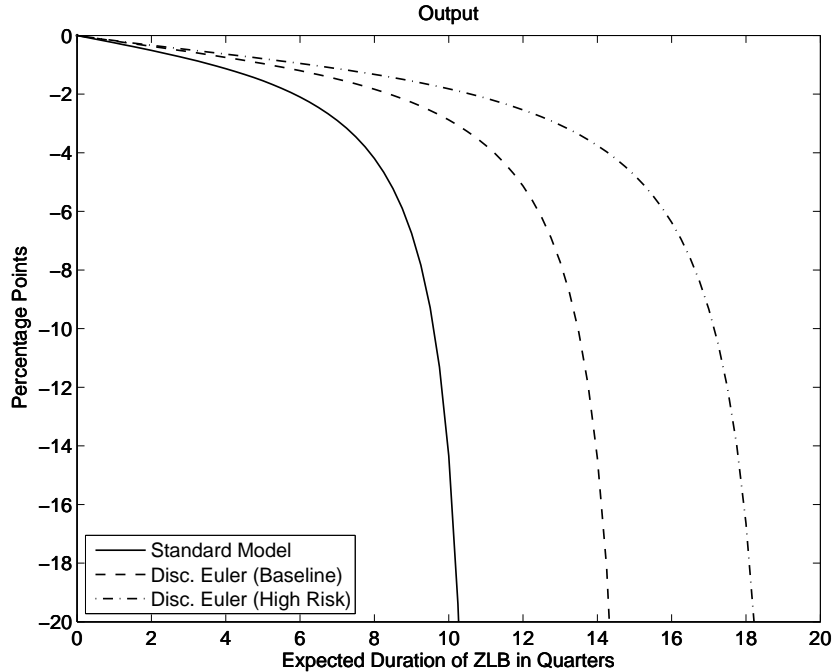


Figure 11: Response of output to shock that makes the natural real rate of interest -2% (annualized) for different levels of persistence of the shock (different values of  $\lambda$ ).

the power of forward guidance relative to the standard New Keynesian model.

While an interest rate announcement in the standard New Keynesian model has the same effect on consumption whether it occurs 1 or 40 quarters in the future, the effect declines with the horizon of the announcement in the incomplete markets model. Forward guidance 10 years ahead has essentially zero effect on output in the incomplete markets model. Intuitively, in the incomplete markets model, precautionary savings effects work to offset the standard intertemporal substitution motive. Our results have important implications for monetary policy at the zero lower bound. They imply both that persistent shocks to the natural rate of interest have smaller effects on current output and also that far future forward guidance has substantially less power to stimulate the economy.

We show that the response of consumption to interest rates at different horizons in the incomplete markets model can be approximated by a consumption Euler equation with “discounting.” Such an equation is a tractable way of incorporating the precautionary savings forces we emphasize into workhorse linearized macroeconomic models. We show that this discounted Euler equation can be micro-founded using a simple model of borrowing constraints.



## A Computational methods

Here we describe the procedure used to find an equilibrium path of the heterogeneous agent model along a perfect foresight transition for the zero-lower-bound episode considered in Section 4.4. The algorithm used to compute the results for a one-time change in the real interest rate is closely related to what we present here.

**Writing the firm's first order condition recursively.** For the numerical analysis it is convenient to rewrite equation (8) recursively. Define

$$P_t^A \equiv \sum_{s=t}^{\infty} \beta^{s-t} (1-\theta)^{s-t} \left(\frac{P_t}{P_s}\right)^{\mu/(1-\mu)} Y_s \mu W_s \quad (23)$$

$$P_t^B \equiv \sum_{s=t}^{\infty} \beta^{s-t} (1-\theta)^{s-t} \left(\frac{P_t}{P_s}\right)^{1/(1-\mu)} Y_s. \quad (24)$$

then equation (8) becomes

$$\frac{p_t^*}{P_t} = \frac{P_t^A}{P_t^B}. \quad (25)$$

Equations (23) and (24) can be written recursively

$$P_t^A = \mu W_t Y_t + (1-\theta) \beta \mathbb{E}_t (1 + \pi_{t+1})^{-\mu/(1-\mu)} P_{t+1}^A \quad (26)$$

$$P_t^B = Y_t + \beta (1-\theta) \mathbb{E}_t (1 + \pi_{t+1})^{-1/(1-\mu)} P_{t+1}^B. \quad (27)$$

**Initial guess.** We assume that the economy has returned to steady state after  $T = 250$  periods and look for equilibrium values for endogenous variables between dates  $t = 0$  to  $T$ . In this explanation of our methods we use variables without subscripts to represent sequences from 0 to  $T$ . Let  $X$  denote a path for all endogenous aggregate variables from date 0 to date  $T$ . These variables include aggregate quantities and prices

$$X \equiv \{C_t, L_t, N_t, Y_t, D_t, i_t, W_t, \pi_t, r_t, p_t^*/P_t, S_t, \tau_t, P_t^A, P_t^B\}_{t=0}^T.$$

The dimension of  $X$  is given by 14 variables for each date and 251 dates. We require an initial guess  $X^0$ . In most cases we found it sufficient to guess that the economy remains in steady state.

**Solving the household's problem.** The household's decision problem depends on  $X$  through  $r$ ,  $W$ ,  $\tau$ , and  $D$ . For a given  $X^i$  we solve the household's problem using the endogenous gridpoint method

(Carroll, 2006). We approximate the household consumption function  $c(b, z)$  with a shape-preserving cubic spline with 200 unequally-spaced knot points for each value of  $z$  with more knots placed at low asset levels where the consumption function exhibits more curvature. Given the consumption function we calculate labor supply from the household’s intratemporal optimality condition and savings from the budget constraint.

**Simulating the population of households.** We simulate the population of households in order to compute aggregate consumption and aggregate labor supply. We use a non-stochastic simulation method. We approximate the distribution of wealth with a histogram with 1000 unequally-spaced bins for each value of  $z$  again placing more bins at low asset levels. We then update the distribution of wealth according to the household savings policies and the exogenous transitions across  $z$ . When households choose levels of savings between the center of two bins, we allocate these households to the adjacent bins in a way that preserves total savings. See Young (2010) for a description of non-stochastic simulation in this manner.

**Checking the equilibrium conditions.** An equilibrium value of  $X$  must satisfy equations (5), (6), (10), (11), (12), (14), (16), (17), (25), (26), and (27) and the monetary policy rule  $i_t = \max[0, \bar{r} + \phi * \pi_t + \epsilon_t]$ , where  $\epsilon_t$  is the exogenous deviation from the Taylor rule that takes a negative value under our “extended” policy. Call these 12 equations the “analytical” equilibrium conditions. The remaining two equilibrium conditions that pin down  $X$  are that  $C$  and  $L$  are consistent with household optimization and the dynamics of the distribution of wealth given the prices. Call these the “computational” equilibrium conditions.

To check whether a given  $X$  represents an equilibrium of the model is straightforward. We can easily verify whether the analytical equilibrium conditions hold at  $X$ . In addition, we can solve the household problem and simulate the population of households to verify that aggregated choices for consumption and labor supply of the heterogeneous households match with the values of  $C$  and  $L$  that appear in  $X$ .

**Updating  $X^i$**  The difficult part of the solution method arises when  $X^i$  is not an equilibrium. In this case we need to find a new guess  $X^{i+1}$  that moves us towards an equilibrium. To do this, we construct an auxiliary model by replacing the computational equilibrium conditions with additional analytical equilibrium conditions that approximate the behavior of the population of heterogeneous

households but are easier to analyze. Specifically we use the equations

$$C_t^{-\gamma} = \eta_t^1 \beta (1 + r_t) C_{t+1}^{-\gamma} \quad (28)$$

$$C_t^{-\gamma} w_t = \eta_t^2 L_t^\psi. \quad (29)$$

where  $\eta^1$  and  $\eta^2$  are treated as parameters of the auxiliary model. For a given  $X^i$ , we have computed  $C$  and  $L$  from the computational equilibrium conditions. We then calibrate  $\eta^1$  and  $\eta^2$  from (28) and (29). We then solve for a new value of  $X$  from the 12 analytical equilibrium conditions and (28) and (29). This is a problem of solving for 14 unknowns at each date from 14 non-linear equations at each date for a total of 3514 unknowns and 3514 non-linear equations. We solve this system using the method described by Juillard (1996) for computing perfect foresight transition paths for non-linear models. This method is a variant of Newton's method that exploits the sparsity of the Jacobian matrix. Call this solution  $X^{i'}$ . We then form  $X^{i+1}$  by updating partially from  $X^i$  towards  $X^{i'}$ . We iterate until  $X^i$  satisfies the equilibrium conditions within a tolerance of  $5 \times 10^{-6}$ .

## B A Simple Model of Borrowing Constraints

The discounted Euler equation (19) can be micro-founded with a special case of the model we present in Section 3 in which:

1. The idiosyncratic productivity shock takes just two values, which we will call employed ( $z = 1$ ) and unemployed ( $z = 0$ ).
2. Idiosyncratic productivity is i.i.d. across time:  $\Pr(z'|z) = \Pr(z')$ .
3. The supply of government debt is zero:  $B = 0$ .
4. The tax liability of the unemployed is  $-m$ . In other words, they receive an unemployment benefit.
5. Firm dividends  $D_t$  are distributed only to the employed households.

This special case is analytically tractable because there is no wealth in the economy and there is a strict borrowing constraint,  $b' \geq 0$ . As the gross supply of assets is zero, there is no possibility of saving in equilibrium and so the distribution of wealth remains degenerate at zero.

As individual assets are constant at zero, it follows that all households of a given employment status must choose the same consumption. Let  $c_{e,t}$  be the consumption of the employed and  $c_{u,t}$  be the consumption of the unemployed agents. Moreover, the absence of opportunities to borrow and save implies that consumption must be equal to income for all individuals. The unemployed will therefore consume  $m$  and the employed will consume

$$c_{e,t} = W_t \ell_t - \tau + D_t / (1 - u),$$

where  $\ell_t$  is labor supply per employed household,  $\tau$  is the tax liability of the employed,  $D_t$  is per capita dividends paid by firms, and  $u$  is the fraction of the population that is unemployed. The government's budget constraint requires  $\tau = um / (1 - u)$ .

The Euler equation for employed agents is

$$c_{e,t}^{-\gamma} \geq \beta R_t E_t \left[ (1 - u) c_{e,t+1}^{-\gamma} + um^{-\gamma} \right] \quad (30)$$

and the Euler equation for an unemployed agent is

$$m^{-\gamma} \geq \beta R_t E_t \left[ (1 - u) c_{e,t+1}^{-\gamma} + um^{-\gamma} \right].$$

Notice that the right hand side of the Euler equation is independent of employment status as employment is i.i.d. across time. Therefore, if  $m < c_{e,t}$  the unemployed households must be constrained and their Euler equation will not hold with equality. We assume that  $m$  is low enough that  $m < c_{e,t}$  for all  $t$ .

Following Krusell, Mukoyama, and Smith (2011) and Ravn and Sterk (2013), we will focus on the equilibrium of this economy in which the Euler equation of the employed holds with equality in all periods. In this equilibrium, the employed households choose zero savings and are therefore up against their constraint. But the constraint doesn't bind in the sense that they would not strictly prefer to borrow more if allowed. There are other equilibria of our economy in which the Euler equation for the employed household holds with inequality (implying that the employed households would strictly prefer to borrow if allowed). We focus on the equilibrium in which the employed households' Euler equation holds with equality for the following reason. In the more realistic case in which the supply of government debt is positive ( $B > 0$ ) there is a unique equilibrium in which the employed households' Euler equation holds with equality. Consider a sequence of such economies with smaller and smaller amounts of government debt. The equilibrium we focus on is the unique

equilibrium of the limiting economy for which the supply of government bonds goes to zero.<sup>24</sup>

Aggregate consumption can be written as

$$C_t = um + (1 - u)c_{e,t}. \quad (31)$$

Using this expression and the Euler equation of the employed—equation (30) (with equality)—we can construct an Euler equation for aggregate consumption

$$(C_t - um)^{-\gamma} = \beta R_t E_t [(1 - u)(C_{t+1} - um)^{-\gamma} + u(1 - u)^{-\gamma} m^{-\gamma}]$$

Log-linearizing this equation yields

$$\hat{C}_t = -\frac{1}{\gamma} \zeta \hat{R}_t + \alpha E_t \hat{C}_{t+1}, \quad (32)$$

where

$$\alpha \equiv \frac{1}{1 + \frac{u}{1-u} \left(\frac{\bar{c}_e}{m}\right)^\gamma},$$

$$\zeta \equiv \frac{\bar{C} - um}{\bar{C}},$$

and bars denote steady state values and hats denote log deviations from steady state. Notice that  $\alpha$  is decreasing in the probability of becoming unemployed,  $u$ , and the consumption impact of unemployment,  $m/\bar{c}_e$ . Notice also that  $\zeta$  falls as more of aggregate consumption is determined by the unemployed who are constrained and therefore do not respond to interest rates. Equation (32) becomes equation (19) by setting  $1/\gamma = \sigma$ ,  $\hat{R}_t = i_t - E_t \pi_{t+1}$ , noting that the natural rate of interest  $r_t^n$  is constant in our model, and noting that  $\hat{C}_t$  is equal to the output gap as  $C_t = Y_t$  and the natural rate of output is constant in our model.

The Phillips curve is almost identical to the Phillips curve in the more general model. The only difference is that the “slope” coefficient is a different function of the underlying structural parameters. To see this, notice that the labor supply condition for the employed is

$$c_{e,t}^{-\gamma} w_t = \ell_t^\psi.$$

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<sup>24</sup>Suppose the outstanding supply of government debt is positive ( $B = \varepsilon > 0$ ). In this case, bond market clearing requires some household to hold this debt at the prevailing interest rate (which is set by the monetary authority). This household must be indifferent at the margin regarding how much they hold of the debt (i.e., their Euler equation must hold with equality). If instead the Euler equation of all households held with strict inequality, no household would want to hold the outstanding bonds. There would be excess supply of bonds and excess demand for consumption goods. Since firms must meet demand at posted prices, they would produce more, which would increase income and consumption and lead marginal utility of consumption to fall. This process would continue until the agents with the lowest marginal utility of consumption—the employed agents—were willing to hold the bonds. Thus, for any positive  $\varepsilon$  the Euler equation of the employed households will hold with equality in equilibrium and the equilibrium we focus on is the limit of these equilibria as  $\varepsilon$  goes to zero.

Using the fact that aggregate labor supply is  $L_t = (1 - u)\ell_t$  and aggregate consumption is given by equation (31), this last expression can be rewritten as

$$(1 - u)^{\gamma+\psi} (C_t - um)^{-\gamma} w_t = L_t^\psi.$$

Log-linearizing this equation yields

$$-\gamma \frac{\bar{C}}{\bar{C} - um} \hat{C}_t + \hat{w}_t = \psi \hat{L}_t.$$

The term  $\bar{C}/(\bar{C} - um)$  is not present in the standard model and it will affect the slope of the Phillips curve.

Market incompleteness does not change the relationship between inflation and marginal cost. To a first-order approximation, this relationship is

$$\pi_t = \beta E_t \pi_{t+1} + \frac{(1 - \beta(1 - \theta))\theta}{1 - \theta} \hat{M}C_t.$$

Real marginal cost is

$$MC_t = w_t = \left( \psi + \gamma \frac{\bar{Y}}{\bar{Y} - um} \right) \hat{Y}_t.$$

Combining the previous three equations and using the fact that  $\hat{C}_t = \hat{Y}_t$  yields

$$\pi_t = \beta E_t \pi_{t+1} + \frac{(1 - \beta(1 - \theta))\theta}{1 - \theta} \left( \psi + \gamma \frac{\bar{Y}}{\bar{Y} - um} \right) \hat{Y}_t.$$

As the flexible price equilibrium is constant in our model,  $\hat{Y}_t$  is the output gap. This differs from the standard New Keynesian Phillips curve due to the term  $\frac{\bar{Y}}{\bar{Y} - um}$ . Intuitively, this term captures the fact that the wealth effect on labor supply is larger because the employed are only part of the population but absorb all of the fluctuations in aggregate consumption.

## C Algebra Behind Equations (21) and (22)

Consider first the Phillips curve:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t.$$

Since the output gap and inflation are constant at  $x_S$  and  $\pi_S$ , respectively, and  $E_t \pi_{t+1} = \lambda \pi_S$  in the short run, we have that

$$\pi_S = \beta \lambda \pi_S + \kappa x_S,$$

which implies

$$\pi_S = \frac{\kappa}{1 - \beta\lambda} x_S \quad (33)$$

as long as  $x_S$  and  $\pi_S$  are finite.

Consider next the discounted Euler equation

$$x_t = \alpha E_t x_{t+1} - \zeta \sigma (i_t - E_t \pi_{t+1} - r_t^n).$$

Again, since the output gap and inflation are constant at  $x_S$  and  $\pi_S$ , respectively, and  $E_t \pi_{t+1} = \lambda \pi_S$  and  $E_t x_{t+1} = \lambda x_S$  in the short run, and, in addition, since the natural real rate is  $r_S^n$  in the short run, we have that

$$x_S = \alpha \lambda x_S + \zeta \sigma (\lambda \pi_S + r_S^n).$$

If we now use equation (33) to eliminate  $\pi_S$  from this equation we get that

$$x_S = \alpha \lambda x_S + \zeta \sigma \left( \lambda \frac{\kappa}{1 - \beta\lambda} x_S + r_S^n \right),$$

which implies

$$x_S = \frac{\zeta \sigma}{1 - \alpha \lambda - \frac{\zeta \sigma \lambda \kappa}{1 - \beta\lambda}} r_S^n$$

as long as  $x_S$  is finite.

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