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Equilibrium Technology Diffusion, Trade, and Growth
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ABSTRACT

We study how opening to trade affects economic growth in a model where heterogeneous firms can adopt new technologies already in use by other firms in their home country. We characterize the growth rate using a summary statistic of the profit distribution—the mean-min ratio. Opening to trade increases the profit spread through increased export opportunities and foreign competition, induces more rapid technology adoption, and generates faster growth. Faster growth comes with costs: labor is reallocated away from production and fewer varieties are produced domestically. Quantitatively, these forces balance to produce large consumption-equivalent welfare gains from trade—especially along the transition path.

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1. Introduction

A large body of evidence documents trade-induced productivity effects at the firm level (see, e.g., Pavcnik (2002) and Holmes and Schmitz (2010)). Why does opening to trade lead to productivity gains at the firm level? What are the consequences of these within-firm productivity gains for aggregate economic growth and welfare?

This paper contributes new answers to these questions. We develop a model where heterogeneous firms choose either to produce with their existing technology or adopt a better technology already in use by other firms in their home country. These choices determine the productivity distribution from which firms can acquire new technologies and, hence, the equilibrium rate of technological diffusion and economic growth. We provide a closed form characterization of the economy showing how the reallocation effects of a trade liberalization (i.e., low productivity firms contract, high-productivity exporting firms expand) change firms' incentives to adopt a better technology and lead to faster within-firm productivity gains. Because these choices lead to more adoption and technology diffusion, the aggregate consequence is faster economic growth.

The starting point of our analysis is a standard heterogeneous firm model in differentiated product markets as in Melitz (2003). Firms are monopolistic competitors who differ in their productivity/technology and have the opportunity to export after paying a fixed cost. There is free entry from a large measure of potentially active firms and firms exit at an exogenous rate. Our model of technology adoption and diffusion builds on Perla and Tonetti (2014), where firms choose to either upgrade their technology or continue to produce with their existing technology in order to maximize expected discounted profits for the infinite horizon. If a firm decides to upgrade its technology, it pays a fixed cost in return for a random productivity draw from the equilibrium distribution of firms within the domestic economy. We interpret this process as technology diffusion, since firms upgrade by adopting technologies already in use by other firms. Economic growth is a result, as firms are continually able to upgrade their technology by imitating other, better firms in the economy. Thus, this is a model of how endogenous technology diffusion contributes to growth.

We study how opening to trade affects firms' technology choices and the aggregate consequences for growth and welfare. To do so we first study a simplified economy and characterize the profit and value functions of a firm, the evolution of the productivity distribution, and the growth rate of the economy on the balanced growth path equilibrium. We then study the quantitative model, which features productivity and exit shocks, and explore how changes in iceberg trade costs affect growth rates and the welfare gains from trade along the transition path.

We provide a closed form characterization of the growth rate as a simple, increasing function of a summary statistic of the profit distribution—the ratio of profits between the average and marginal adopting firm. A firm's incentive to adopt depends on two competing forces: the expected benefit of a new productivity draw and the opportunity cost of taking that draw. The expected benefit relates to the profits that the average new technology would yield. The opportunity cost of adopting a new technology is the forgone profits from producing with the current technology. Thus, the aggregate growth rate of the economy encodes the trade-off that firms face in a simple and intuitive manner.

Reductions in iceberg trade costs increase the rate of technology adoption and economic growth because they widen the ratio of profits between the average and marginal adopting firm. As trade costs decline, low productivity firms contract as competition from foreign firms reduce their profits; high productivity firms expand and export, increasing their profits. For low productivity firms, this process reduces both the opportunity cost and increases the benefit of a new technology. This leads to more frequent technology adoption at the firm level. Because the frequency at which firms upgrade their technology is intimately tied to aggregate growth, the growth rate is higher in more open economies.

The underlying mechanism in our model is distinct from the standard “market size” effect, i.e., opening to trade increases the size of the market and, hence, raises the value of adoption. We show this by studying a special case of our model with no fixed exporting cost in which all firms export. In this model, growth is the same function of the spread in profits between the average and marginal adopting firm. The difference is that trade has no effect on growth. In this model, opening to trade benefits all firms by increasing firms’ profits and values by the same proportional amount. Consistent with the well understood benefits of a larger market, opening to trade increases the expected value of adopting a new technology. However, a larger market also raises the forgone profits of adoption by the exact same amount. Thus, opening to trade does not affect the *relative* benefit of adoption and, hence, there is no change in growth.

We provide a closed form characterization of the change in welfare from these growth effects. The change in welfare is a weighted sum of the increase in economic growth and the change in the initial level of consumption. The change in consumption is a sum of three components: a static gain from reallocation of resources, a change in the measure of varieties consumed, and a change in the amount of labor allocated to the production of goods. We prove that opening to trade reduces the initial level of consumption and dampens the gains from faster economic growth. Similar to the quantitative results in Atkeson and Burstein (2010), we prove that the static gains from trade are always offset by a loss in varieties produced and consumed and a reallocation of labor away from goods production towards entry and adoption activity. Varieties decrease because the expected benefits of entering rise by less than the expected cost of entry—an increase in wages increases the cost of entry for all entering firms, but only some of the more productive firms benefit from lower trade costs. This results in a decrease in the level of initial consumption.

How faster growth competes with the loss in consumption is a quantitative question. To provide a quantitative answer, we extend the analysis in two directions. First, we extended our model to include firm-specific productivity shocks to incumbents and exit. Thus, the dynamics of the firm are now driven by two forces: an exogenous stochastic component and the endogenous component that works through the adoption process. This modeling enrichment allows us to calibrate the model by having the model match moments regarding micro-level firm dynamics and, in turn, provide quantitative discipline on the results.

The second extension is in the nature of the exercise: our main quantitative exercise focuses on transition paths—not across steady states as in our theoretical analysis. The focus on transition paths is an important, quantitative detail to address the concern that the across steady-state analysis overstates the welfare gains as the benefits of higher growth are in the future and they require costly investment to

implement. The quantitative version of the model and the transition path cannot be solved analytically, so we implement numerical methods to solve the PDEs.

The baseline exercise focuses on a ten percent permanent and unanticipated reduction in trade costs. In response to the trade liberalization, the transition path of the economy is important as productivity growth, the number of varieties, and consumption take time to adjust to the change in openness to trade. An important feature of the transition path is that consumption initially overshoots its new long-run steady state value. Consistent with our theoretical results, the measure of domestically produced varieties is smaller in the long run and, thus, net-exit must occur along the path. Along the transition to the new steady state, the result is that labor is allocated away from the creation of new firms. This labor is reallocated toward adoption activities to facilitate faster growth and toward the increased production of consumption goods. Thus, consumption is elevated along a portion of the path as it converges to a new lower relative level.

From a welfare perspective, the gains from trade are large and the transition path amplifies them. The consumption-equivalent gains, inclusive of the transition path, are over eight percent. As one point of comparison, they are an order of magnitude larger than what static models would predict using the work of Arkolakis, Costinot, and Rodriguez-Clare (2012). Moreover, these gains are about one percentage point larger than the steady-state to steady-state comparison would suggest. In other words, consumption overshooting dominates any other forces which would tend to dampen the gains from trade as the economy adjusts to the trade liberalization. We show how the size of the welfare gains are related to the moments used to calibrate the model, with a particular focus on how important the firm dynamics data, and not just data on firm heterogeneity in the cross-section, are in determining the welfare gains from trade.

1.1. Related Literature

We contribute to the theoretical literature on trade and growth. The standard mechanisms creating a relationship between trade and growth typically take two forms. First, openness leads to the cross-country diffusion of new and better ideas. Second, opening to trade increases the size of the market and, hence, raises the value of new idea creation/innovation. Depending on the details of the model, these mechanisms have been shown to increase economic growth as a country opens up to trade (see, for example, the pioneering works of Rivera-Batiz and Romer (1991) and Grossman and Helpman (1993) and their extensions to heterogeneous firm environments in Baldwin and Robert-Nicoud (2008)).

Our model differs from these traditional mechanisms. First, to focus on our distinct mechanism, we deliberately shut down the cross-country diffusion of new and better ideas. In our model, firms only acquire ideas already present inside their country. Thus, our model delivers growth without any increase in the amount or quality of ideas coming from abroad as a country opens to trade. This distinction is also salient relative to recent work such as Alvarez, Buera, and Lucas (2017) and Buera and Oberfield

(2019).¹ Second, as mentioned above, when only market size effects are present, opening to trade has no effect on economic growth. The relationship between growth and trade in our model is not because a larger market increases the value of adoption; it's due to a change in the value of adoption that arises because a trade liberalization has differential effects on firms with different productivity levels.

More broadly, we relate to the literature on the relationship between competition and productivity. Arrow's (1962) "replacement effect" is a theoretical explanation for the positive effects of competition on growth. Broadly speaking, Arrow's (1962) idea is that a monopolist will prefer the status quo whereas an entrant or firm with lower market power has less to lose, and hence there is a greater incentive to innovate.

Our mechanism is very similar with the insight that competition reduces the opportunity cost of adoption, i.e., the benefits of the status quo fall with exposure to import competition. As our theoretical results make transparent, the adoption decision and aggregate growth rate depend on the comparison between the potential benefits of adoption versus the forgone profits of operating with the old technology. On its own, the erosion of profits from import competition incentivizes firms to adopt more frequently. In this sense, our model shares similarities with Holmes, Levine, and Schmitz Jr (2012) who show how competition reduces the cost of a switch-over disruption from a new technology and leads to more technology adoption.

Closely related to our work is Bloom, Romer, Terry, and Van Reenen (2013) which focuses explicitly on import competition, within-firm productivity improvements, and aggregate growth. Motivated by the evidence in Bloom, Draca, and Reenen (2016), they show import competition forces firms to innovate more than otherwise. While similar in spirit, the underlying mechanisms are different. Central to their results is the costly adjustment of factors of production within the firm. As firms face import competition, the resources within the firm that are costly to shed are redirected toward innovative activities. Furthermore, their mechanism amplifies, but is not distinct from the traditional market size effects on innovation.

Our normative results share similarities to Atkeson and Burstein (2010). In a different model of innovation, Atkeson and Burstein (2010) show how the welfare losses from the entry margin of product innovation offset the gains from within-firm process innovation. In our model, the analog of this result is the drag on welfare from a loss in domestically-produced varieties. Moreover, like in their framework, we find that selection into exporting is an important ingredient for these forces not to completely offset. Our positive results, however, are different. In Atkeson and Burstein (2010), it is the large, exporting firms that innovate more and small import-competing firms that reduce their innovation. In our model, it is the small import-competing firms that speed up their adoption of better technologies. As our model focuses on adoption and does not feature innovation, these represent different mechanisms contributing to welfare.

¹We study the case of symmetric countries, since absence international technology diffusion, balanced growth paths with asymmetric countries likely exist only for knife-edge conditions. We hope in this paper to study a particular mechanism whose illustration is aided by the analytical tractability obtained via the symmetry assumption. We conjecture that the mechanism we study would still be relevant in a model with international technology diffusion in which a balanced growth path equilibrium would robustly exist even with asymmetric countries.

Sampson (2015) studies the effects of trade on growth when there is a dynamic complementarity between the ideas of entrants and those of the incumbents: trade induces exit of the worst performing firms and this implies that entrants are able to receive better ideas; because entrants are now better, this induces more selection and so on, leading to faster economic growth. While our model has entry and exit, it is incumbents that adopt new technologies. This distinction is empirically relevant as Sampson's (2015) model—following Luttmer (2007)—is one in which most of aggregate productivity growth (and its response to trade) is from the entry margin. In contrast, our model implies that most of aggregate productivity growth comes from within-firm improvements by incumbents, as is indicated by Garcia-Macia, Hsieh, and Klenow (2019).

A second distinction relative to Sampson (2015) is our calibration and quantitative approach. There are two dimensions to our contribution. First, we generalize the Perla and Tonetti (2014) framework with the dynamics of the firm shaped by exogenous, firm-specific geometric Brownian motion (GBM) shocks to productivity and the endogenous technology adoption decision. This allows our quantitative model to match the observed dynamics of firms and, hence, provide an important element of discipline on the growth and welfare effects of the adoption mechanism. A second contribution is an evaluation of the welfare gains from trade along the transition path. As we show, transition dynamics feature consumption overshooting, and hence, the welfare effects are larger than in steady-state to steady-state comparisons. We also show how the calibrated variance of the GBM shocks affects the calibrated value of adoption costs, which has a large impact on the size of the welfare gains from trade.

We also contribute to the literature on idea flow models of economic growth in several ways. The most important is the introduction and study of competition effects which are new, additional, forces not present in the economies of Lucas and Moll (2014) and Perla and Tonetti (2014). We extend Perla and Tonetti (2014) in important directions: general equilibrium with labor and goods markets in continuous time with firm-specific geometric Brownian motion shocks to productivity and firm entry and exit, leading to an endogenous measure of varieties. These features allow our model to confront data on the dynamics of firms and shape our quantitative results. Also, our characterization of growth as a function of summary statistics of the profit distribution is a core contribution and is not present in Perla and Tonetti (2014). As our closed form characterization of the growth rate shows, without any relative change in firms' profits, opening to trade has no effect on economic growth. Thus the competition effects that we introduce, which act through a reallocation of profits due to the fixed cost of exporting, are key to delivering interesting relationships between trade, firms' technology choices, and economic growth. We also develop numerical methods needed to study the transition dynamics of the economy.

1.2. Motivating Evidence: Trade-Induced Productivity Gains

Motivating our work is the empirical evidence that import competition gives rise to within-firm productivity improvements.

Pavcnik (2002) was an important empirical study of the establishment level productivity effects from a trade liberalization using frontier measurement techniques. Pavcnik (2002) studied Chile's trade liberalization in the late 1970s and she found large within-plant productivity improvements for import-competing firms that are attributable to trade. There was no evidence that exporters had any productiv-

ity improvements attributable to trade and no evidence of trade induced productivity gains from exit. To be clear, Pavcnik (2002) observed productivity improvements from exit, but there were no *differential* gains from exit across sectors of different trade orientation (i.e., import competing vs. non-traded, etc.). In contrast, import-competing firms had differentially larger within-plant productivity improvements.

Many subsequent studies for different countries and/or data sets have found similar results. In Brazil, Muendler (2004) found import competition led to within-firm productivity gains. Several studies of India's trade liberalization find related results. Topalova and Khandelwal (2011) found large within-firm productivity gains associated with declines in output tariffs which proxy for increases in import competition. Also in India, Sivadasan (2009) finds increases in industry TFP from tariff reductions, with 55 percent of these gains associated with within-firm productivity gains.

Bloom et al. (2016) find within-firm productivity gains in Europe from Chinese import competition. Most importantly, they associate these gains with explicit measures of technical change, e.g., information technology, management practices, and other measures of innovation. Their evidence suggests that firms undertook activities to change the technology with which they operate in response to import competition. Autor, Dorn, Hanson, Pisano, and Shu (2019), however, find that U.S. firms reduced their patenting in response to a rise in Chinese import-competition. In response to tariff reductions in China, Fielor and Harrison (2019) find evidence for larger productivity increases among the initially low-productivity firms and Bombardini, Li, and Wang (2018) find evidence for increases in patenting among the initially high-productivity firms.

Despite the large body of empirical work, theory has lagged.² Two common explanations for these within-firm productivity gains fall under the category of imperfect measurement. The first explanation is that these gains may reflect changes in the mix of intermediate inputs. For the cases of Indonesia (studied in Amiti and Konings, 2007) and India (Goldberg, Khandelwal, Pavcnik, and Topalova, 2010), there is strong evidence for this mechanism. A second explanation is that they reflect changes in product mix (see, e.g., Bernard, Redding, and Schott, 2011). While these are likely contributing forces, there is evidence they are not the whole story. For example, Bloom et al. (2016) find little evidence that they are the source of the gains in their study.

Non-measurement explanations fall under the guise of "X-efficiency" gains (Leibenstein, 1966). X-efficiency gains can be difficult to understand since it is natural to ask the question: If it was possible for a firm to improve its efficiency after a change in competition, why did it not do it in the first place? One mechanism for X-efficiency gains is that competition relaxes the agency problems within the firm (see, e.g., Raith, 2003; Schmidt, 1997). In our model, increased import competition increases the profitability of technological improvement by lowering the opportunity cost of adoption relative to the returns of adoption. This competition driven increase in the pace of technology adoption leads to within-firm productivity gains that generate faster aggregate economic growth.

Admittedly, there are aspects of firm-level adjustments to trade liberalizations about which we have

²The standard heterogeneous firm framework of Melitz (2003) does not provide an explanation for these effects. Melitz (2003) deals exclusively with the reallocation of activity across firms; there is no mechanism to generate within-firm productivity growth in response to a trade liberalization. One caveat to this statement is how the overhead costs are treated in the model and, in turn, measured in the data.

little to say. In particular, the evidence on the productivity enhancing role of becoming an exporter (see, e.g., Bustos (2011) or Marin and Voigtländer (2019)). Theoretical mechanisms studying these aspects of exporters and their quantitative evaluation is far more developed with some examples being, Melitz and Costantini (2007), Atkeson and Burstein (2010), and Akcigit, Ates, and Impullitti (2018). We deliberately focuses on the adoption behavior and how it's the import exposed firms that respond. Jointly modeling adoption and innovation at the firm level, as in Benhabib, Perla, and Tonetti (2017), and studying how firm behavior responds to changes in trade and competition is an important area for future research. Further empirical research on the differential impact of changes in trade costs across firms throughout the productivity distribution will be invaluable in this endeavor.

2. Model

2.1. Countries, Time, Consumers

There are N symmetric countries. Time is continuous and evolves for the infinite horizon. Utility of the representative consumer in country i is

$$U_i(t) = \int_t^\infty e^{-\rho(\tau-t)} \log(C_i(\tau)) d\tau. \quad (1)$$

The utility function $U_i(t)$ is the present discounted value of the instantaneous utility of consuming the final good. The discount rate is $\rho > 0$ and instantaneous utility is logarithmic.³ The final consumption good is an aggregate bundle of varieties, aggregated with a constant elasticity of substitution (CES) function by a competitive final goods producer.

Consumers supply labor to firms for the production of varieties and the fixed costs of exporting, technology adoption, and entry. Labor is supplied inelastically and the total units of labor in a country are \bar{L}_i . Consumers also own a diversified portfolio of all firms operating within their country. Thus, consumer income is the sum of net profits from firms and total payments to labor. The consumer budget constraint is

$$C_i(t) \leq \frac{W_i(t)}{P_i(t)} \bar{L}_i + \bar{\Pi}_i(t) - \bar{I}_i(t) \quad (2)$$

where $W_i(t)$ is the nominal wage rate in country i , $P_i(t)$ is the standard CES price index of the aggregate consumption good, $\bar{\Pi}_i(t)$ is aggregate profits, and $\bar{I}_i(t)$ is aggregate investment in entry and technology adoption. See the Appendix for more details.

2.2. Firms

In each country there is a final good producer that supplies the aggregate consumption good competitively. The final good is produced by aggregating an endogenous measure of intermediate varieties produced by monopolistically competitive firms, both domestically and abroad. Variety producing firms are

³The model easily generalizes to CRRA power utility, as shown in the Appendix, but analytical characterizations are less sharp.

heterogeneous over productivity, Z , with cumulative distribution function $\Phi_i(Z, t)$ and probability distribution function $\phi_i(Z, t)$ describing how productivity varies across firms, within a country. Each firm alone can supply variety v . As is standard, a final good producer aggregates these individual varieties using a constant elasticity of substitution production function.

2.2.1. Final Good Producer.

Dropping the time index for expositional clarity, a representative final good producer chooses the quantity of intermediate goods to maximize profits

$$\max_{Q_{ij}(v)} \left[\sum_{j=1}^N \int_{\Omega_{ij}} Q_{ij}(v)^{(\sigma-1)/\sigma} \right]^{\sigma/(\sigma-1)} \quad (3)$$

$$\text{s.t. } \sum_{j=1}^N \int_{\Omega_{ij}} p_{ij}(v) Q_{ij}(v) = Y_i.$$

Y_i is defined to be nominal aggregate expenditure on consumption goods. The parameter $\sigma > 1$ is the elasticity of substitution across varieties. The measure Ω_{ij} defines the endogenous set of varieties consumed in country i produced in country j . Furthermore, the total measure of varieties produced in country i , Ω_i , is also determined in equilibrium, as domestic firms can enter after paying a fixed cost and exit if hit with an exogenous death shock that arrives at rate $\delta \geq 0$.

We will drop the notation carrying around the variety identifier, as it is sufficient to identify each firm with its location and productivity level, Z . Additionally, to focus on the interactions between technology adoption, trade, and growth, we assume that all countries are symmetric in that they have identical parameter values, although each intermediate producer in each country produces a unique good. Because all countries are symmetric, we focus on the results for a typical country and abstract from notation indicating the country's location.

This final good producer problem yields the familiar variety demand and price index equations:

$$Q(Z) = \left(\frac{p(Z)}{P} \right)^{-\sigma} \frac{Y}{P}, \quad (4)$$

$$P^{1-\sigma} = \Omega \left(\int_M^\infty p_d(Z)^{1-\sigma} d\Phi(Z) + (N-1) \int_{\hat{Z}}^\infty p_x(Z)^{1-\sigma} d\Phi(Z) \right). \quad (5)$$

where p_d and p_x are the prices of domestic and imported varieties, M is the minimum of support of the distribution, and \hat{Z} is an export threshold—all endogenous variables determined in equilibrium.

2.2.2. Individual Variety Producers.

Variety producing firms hire labor, ℓ , to produce quantity Q with a linear production technology: $Q = Z\ell$. Firms can sell freely in their domestic market and also have the ability to export at some cost,

controlled by parameter κ . To export, a firm must pay a fixed flow labor cost, $\kappa\bar{L}W/P$, per foreign export market. This fixed cost is paid in market real wages and is proportional to the number of consumers accessed.⁴ Exporting firms also face iceberg trade costs, $d \geq 1$, to ship goods abroad.

Firm productivity evolves according to geometric Brownian motion, which captures the high empirical persistence of firm productivity, while also allowing for changes in firms' relative productivity to occur throughout the productivity distribution. Furthermore, at each instant, any firm can pay a real fixed cost $X(t)$ to adopt a new technology. $X(t)$ represents the cost of hiring labor to upgrade to a higher-efficiency production technology (detailed below). If a firm decides to pay this cost, it receives a random draw from the distribution of producers within its own country, as in Perla and Tonetti (2014).

Given this environment, firms must make choices regarding how much to produce, how to price their product, whether to export, and whether to change their technology. These choices can be separated into problems that are static and dynamic. Below we first describe the more standard static problem of a firm and then describe the dynamic problem of the firm to derive the optimal technology adoption policy.

Firms' Static Problem. Given a firm's location, productivity level, and product demand, the firm's static decision is to choose the amount of labor to hire, the prices to set, and whether to export for each destination to maximize profits each instant. The firm's problem when operating within the domestic market is to choose a price and quantity of labor to maximize profits. Using the standard demand function for individual varieties (equation (4)), the optimal domestic real profit function is

$$\Pi_d(Z) = \frac{1}{\sigma} \left(\frac{\bar{\sigma}W}{ZP} \right)^{1-\sigma} \frac{Y}{P}, \quad \text{where } \bar{\sigma} := \frac{\sigma}{\sigma - 1}. \quad (6)$$

$\bar{\sigma}$ is the standard markup over marginal cost.

The decision to (possibly) operate in an export market is similar, but differs in that the firm faces variable iceberg trade costs and a fixed cost to sell in the foreign market. Optimal per-market real export profits are

$$\Pi_x(Z) = \max \left\{ 0, \frac{1}{\sigma} \left(\frac{\bar{\sigma}dW}{ZP} \right)^{1-\sigma} \frac{Y}{P} - \frac{\kappa\bar{L}W}{P} \right\}. \quad (7)$$

where d is an iceberg trade costs and $\kappa\bar{L}W/P$ is the fixed cost to sell in the foreign market. Given profits described in equation (7), only firms earning positive profits from exporting—those above a productivity threshold \hat{Z} —actually enter a foreign market. Total real firm profits equal the sum of domestic profits plus the sum of exporting profits across export markets,

$$\Pi(Z, t) := \Pi_d(Z, t) + (N - 1)\Pi_x(Z, t). \quad (8)$$

⁴Export costs that are proportional to the number of consumers is consistent with the customer access interpretation featured in Arkolakis (2010). As discussed in Section 5, this influences the population scale effect properties of the model, but has no other impact.

Firms' Dynamic Problem. Given the static profit functions and a perceived law of motion for the productivity distribution and adoption cost, each firm has the choice of when to acquire a new technology, Z . Define $g(t)$ as the growth rate of total expenditures.⁵ Firms choose technology adoption policies to maximize the present discounted expected value of real profits. Since firms are owned by consumers, the firm's effective discount rate, $r(t)$, equals the interest rate derived from the consumer's Euler equation plus the firm's probability of exit, δ . That is, $r(t) = (\rho + g(t) + \delta)$.

The productivity of a non-adopting firm evolves exogenously according to geometric Brownian motion (GBM):

$$dZ_t/Z_t = (\mu + v^2/2)dt + v dW_t, \quad (9)$$

where $\mu \geq 0$ is related to the drift of the productivity process, $v \geq 0$ is the volatility, and W_t is standard Brownian motion. For tractability and clarity of exposition, our theoretical analysis in Sections 5 and 6 focuses on the case in which productivity only changes via adoption (i.e., $\mu = 0, v = 0$). In Section 7 we calibrate the model with GBM to quantify the welfare gains from trade.

The recursive formulation of the firm's problem is as follows. Each instant, a firm with productivity Z chooses between continuing with its existing technology and earning flow profits of $\Pi(Z, t)$ or stopping and adopting a new technology at cost $X(t)$. In a growing economy, adoption opportunities will improve and the firm's profits will erode, decreasing the benefits of continuing to operate its existing technology until the firm enters the stopping region and it chooses to adopt a new technology.

Define the value of the firm in the continuation region as $V(Z, t)$, $M(t)$ as the time dependent productivity boundary between continuation and adoption, and $V_s(t)$ as the expected value of adoption net of costs. $M(t)$ is a reservation productivity function, such that all firms with productivity less than or equal to $M(t)$ choose to adopt and all other firms produce with their existing technology. If a firm chooses to adopt a new technology it pays a cost and immediately receives a new productivity. This new productivity is a random draw from the cross-sectional productivity distribution of firms.⁶ This distribution will be a function of the optimal policy of all firms, i.e., the firm choice of when to draw a new productivity. Recursively, the optimal policy of firms will depend on the expected evolution of this distribution. With rational expectations, the expected net value of adoption in equilibrium is

$$V_s(t) = \int_{M(t)}^{\infty} V(Z, t) d\Phi(Z, t) - X(t). \quad (10)$$

There are several interpretations of this technology adoption choice. The literal interpretation is that firms are randomly matching and learning from each other. Empirically, this technology choice can be thought of as tangible or intangible investments that manifest themselves as improvements in productivity like improved production practices, work practices, supply-chain and inventory management, etc.

⁵Since in a balanced growth path equilibrium many growth rates will be equal (e.g., the growth rate of total expenditures, consumption, and the minimum of the productivity distribution), we will abuse notation for the sake of exposition and overload the definition of a single growth rate: $g(t)$.

⁶In discrete time, Perla and Tonetti (2014) presents both draws conditional on only meeting adopters and unconditionally from the whole distribution. Qualitatively, these two environments are very similar and in the limit to continuous time they become identical. See Benhabib et al. (2017) for a discussion and a proof that the unconditional and conditional draw models generate identical equilibrium laws of motion for the productivity distribution.

that are already in use by other firms (see, for example, the discussions in Holmes and Schmitz (2010) and Syverson (2011)).

Using the connection between optimal stopping and free boundary problems, a set of partial differential equations (PDEs) and boundary conditions characterize the firm's value.⁷ The PDEs and boundary values determining a firm's value are

$$r(t)V(Z, t) = \Pi(Z, t) + \left(\mu + \frac{v^2}{2} \right) Z \frac{\partial V(Z, t)}{\partial Z} + \frac{v^2}{2} Z^2 \frac{\partial^2 V(Z, t)}{\partial Z^2} + \frac{\partial V(Z, t)}{\partial t}, \quad (11)$$

$$V(M(t), t) = \int_{M(t)}^{\infty} V(Z, t) d\Phi(Z, t) - X(t), \quad (12)$$

$$\frac{\partial V(M(t), t)}{\partial Z} = 0. \quad (13)$$

Equation (11) describes how the firm's value function evolves in the continuation region. It says that the flow value of the firm is the flow value of profits (the first term) plus the expected capital gain (the last three terms). The capital gain term is the change in the value of the firm over time. It is comprised of three terms: the first two reflect the expected change in productivity which arise from both the drift and variance in the productivity process; the third term reflects a change in value of producing with a given productivity.

Equation (12) is the value matching condition. It says that at the reservation productivity level, $M(t)$, the firm should be indifferent between continuing to operate with its existing technology and adopting a new technology. That is the definition of the reservation productivity. Equation (13) is the smooth-pasting condition. The smooth pasting condition can be interpreted as an intertemporal no-arbitrage condition that ensures the recursive system of equations is equivalent to the fundamental optimal stopping problem. It ensures that, around the optimal behavior, there is not a discontinuous increase in value associated with an infinitesimal difference in the timing of adoption (or equivalently the reservation productivity).

A couple of comments are in order regarding the economics of this problem.

There are two forces that drive the adoption decision. First, over time the productivity distribution will improve. This eventually gives firms an incentive to adopt a new technology as the benefit of adoption grows over time since a firm's *relative* position in the productivity distribution will drive its adoption behavior. This economic force is the same as in Lucas and Moll (2014) and Perla and Tonetti (2014). Similarly, negative productivity shocks may also drive a firm to abandon its current technology and adopt a new one as its relative position in the distribution will have weakened and the benefits of adoption have risen.

⁷Standard references and conditions for the equivalence between optimal stopping of stochastic processes and free boundary problems are Dixit and Pindyck (1994) and Peskir and Shiryaev (2006). The deterministic stopping problem is discussed on pages 110-115 of Stokey (2009).

Second, competition and general equilibrium effects are new, additional, forces—not present in Lucas and Moll (2014) and Perla and Tonetti (2014)—which drive the adoption decision. The dependence of the firm’s value function (equation 11) on profits (which are time dependent) illustrates this feature. As the economy grows, holding fixed an individual firm’s productivity, its profits will erode. The reason is because as other firms become better through adoption, they demand relatively more labor, and this raises wages which reduces profits. This erosion of profits reduces the opportunity cost of continuing to operate and incentivizes adoption. Our paper is about this second force—how equilibrium changes in competition and profits via trade influence adoption and growth.

Finally, there is an externality in this environment. Firms are infinitesimal and do not internalize the effect their technology adoption decisions have on the evolution of the productivity distribution and, in turn, the distribution from which other firms are able to adopt. This externality could be interpreted as a free rider problem, as firms have an incentive to wait before upgrading, and let other firms adopt first, in order to have a better chance of adopting a more productive technology.

Together with the static optimization problem, equations (11), (12), and (13) characterize optimal firm policies given equilibrium prices and a law of motion for the productivity distribution.

2.3. Adoption Costs

Technology adoption is costly. In our baseline specification, this cost takes the form of labor the firm must hire; an adopting firm must hire $\zeta \bar{L}$ workers and pay them each $\frac{W(t)}{P(t)}$. The quantity of labor required to adopt a technology scales with population size to prevent scale effects and depends on the parameter $\zeta > 0$.⁸ Thus, the real cost of adoption denoted by X , is

$$X(t) := \zeta \bar{L} \frac{W(t)}{P(t)}, \quad (14)$$

The product of this quantity and the real wage determines the real cost of adoption. Note that the specification ensures that adoption cost grows in proportion to the real wage, and, thus, ensures the cost does not become increasingly small as the economy grows.

We define $S(t)$ to be the adoption rate, such that the flow of intermediate firms adopting a new technology is $\Omega(t)S(t)$, i.e., the measure of varieties times the rate of adoption.

2.4. Entry, Exit, and the Measure of Varieties

There is a large pool of non-active firms that may enter the economy by paying an entry cost to gain a draw of an initial productivity from the same distribution from which adopters draw—the cross-section of incumbent productivities. We model the cost of entry as a multiple of the adoption cost for incumbents, $X(t)/\chi$, where $0 < \chi < 1$. Hence, χ is the ratio of adoption to entry costs. The restriction that $\chi \in (0, 1)$ means that the cost to incumbents of upgrading to a better technology is lower than the cost to

⁸Since all analysis in this paper is done holding population constant, the assumption that the cost scales with population size does not affect any result in the paper.

entrants of starting to producing a new variety from scratch. Thus, the free entry condition is

$$X(t)/\chi \geq \int_{M(t)}^{\infty} V(Z, t) d\Phi(Z, t), \quad (15)$$

which will hold with equality on the BGP, as firms enter until the value of entry equals the cost.

Exit occurs because firms die at an exogenous rate $\delta \geq 0$ that is independent of firm characteristics. For tractability, our theoretical results will focus on the limiting case in which there is no firm death ($\delta = 0$) and, hence, no entry.⁹ Even in the limiting case when ($\delta = 0$), the equilibrium number of varieties (firms), Ω , is endogenous and determined by the free entry condition. This allows us to study the growth and welfare implications of lowering trade costs on the endogenous number of varieties.

We define $E(t)$ to be the entry rate, such that the flow of firms entering the market and creating a new variety equals the measure of varieties times the entry rate, $\Omega(t)E(t)$.

2.5. The Productivity Distribution

The final element of the economic environment to describe is the law of motion for the productivity distribution. We highlight the key elements below and provide a detailed derivation in Appendix B.

First, the minimum of the support of the productivity distribution is the adoption reservation productivity $M(t)$. Recall, that when adopting, a firm receives a random productivity draw from the distribution of producers. Except, perhaps, at time 0, the probability of drawing the productivity of a fellow adopting firm is infinitesimal. Therefore, firms adopting at time t will adopt a technology above $M(t)$ almost certainly. Thus, $M(t)$ is like an absorbing barrier sweeping through the distribution from below and, thus, is the minimum of the support.

The Kolmogorov Forward Equation (KFE) describes the evolution of the productivity distribution for productivities above the minimum of the support. That is, the KFE describes the inflows and outflows of firms for each point in the support of the distribution. The KFE (written in terms of the CDF) is:

$$\begin{aligned} \frac{\partial \Phi(Z, t)}{\partial t} = & \underbrace{\Phi(Z, t) \left(\underbrace{S(t) + E(t)}_{\text{adopt or enter}} \right)}_{\text{distributed below } Z} - \underbrace{S(t)}_{\text{adopt at } M(t)} - \underbrace{\delta \Phi(Z, t)}_{\text{Death}} \dots \\ & - \underbrace{\left(\mu - \frac{v^2}{2} \right) Z \frac{\partial \Phi(Z, t)}{\partial Z}}_{\text{deterministic drift}} + \underbrace{\frac{v^2}{2} Z^2 \frac{\partial^2 \Phi(Z, t)}{\partial Z^2}}_{\text{Brownian motion}}. \end{aligned} \quad (16)$$

Let's walk through each term in (16) carefully. The left-hand side describes how the CDF evaluated at Z evolves over time. The first term reflects the inflows that arise from technology adoption and entry. Since adopting and entering firms' initial productivity is drawn from the existing distribution, $\Phi(Z, t)$,

⁹In the Appendix, we solve the model with an arbitrary death rate $\delta \geq 0$ and our quantitative analysis of the welfare gains from trade in Section 7 uses the model calibrated to a positive death rate.

the total measure of firms flowing in below Z is $\Phi(Z, t) (S(t) + E(t))$. The second term reflects the loss of mass in the distribution which arise from the adopting firms. Since all adoption is done by firms at the minimum of support of the distribution, $S(t)$ is subtracted from the CDF for all $Z \geq M(t)$. The third term reflects exit, and since death occurs uniformly across all firms $\delta\Phi(Z, t)$ enters as an outflow. Finally, the last two terms reflect the expected drift due to GBM, which has a deterministic and random component.

Example: No GBM, No Death Case. To provide intuition, consider the small noise and small death rate limiting economy ($\delta = \mu = \nu = 0$). In this case the KFE simplifies to

$$\frac{\partial\Phi(Z, t)}{\partial t} = \underbrace{\underbrace{\Phi(Z, t)}_{\text{redistribution distribution}} \times \underbrace{S(t)}_{\text{adopters}}}_{\text{distributed below } Z} - \underbrace{S(t)}_{\text{adopters}} \quad (17)$$

The inflow in the fraction of firms with productivity less than Z comes from incumbents who draw a productivity below Z when adopting. This inflow equals the likelihood an adopter draws a productivity below Z time the rate of adoption. The outflow occurs only due to adoption. Since all adoption is done by firms at the minimum of support of the distribution, $S(t)$ is subtracted from the CDF for all $Z \geq M(t)$. The rate of adoption $S(t)$ is determined by

$$S(t) = M'(t)\phi(M(t), t). \quad (18)$$

In words, the rate at which the adoption threshold sweeps across the density, $M'(t)$, and the amount of firms the adoption boundary collects as it sweeps across the density from below, $\phi(M(t), t)$.

In this case, the solution to equation (16), which characterizes the productivity distribution at any date, is the probability density function

$$\phi(Z, t) = \frac{\phi(Z, 0)}{1 - \Phi(M(t), 0)}, \quad Z \geq M(t). \quad (19)$$

That is, the distribution at date t is a truncation of the initial distribution at the reservation adoption productivity at time t , $M(t)$. The solution in equation (19) is rather general. It holds independent of the type of the initial distribution and independent of whether the economy is on a balanced growth path.

3. Computing a Balanced Growth Path Equilibrium

In this section, we define and compute a Balanced Growth Path (BGP) equilibrium. Main results are then discussed in Sections 5-7. The Appendix documents the detailed steps involved in the computation of equilibrium and derivation of our main results.

3.1. Definition of a Balanced Growth Path Equilibrium.

Definition 1. A Balanced Growth Path Equilibrium consists of an initial distribution $\Phi(0)$ with support $[M(0), \infty)$, a sequence of distributions $\{\Phi(Z, t)\}_{t=1}^{\infty}$, firm adoption and export policies $\{M(t), \hat{Z}(t)\}_{t=0}^{\infty}$, firm price and labor policies $\{p_d(Z, t), p_x(Z, t), \ell_d(Z, t), \ell_x(Z, t)\}_{t=0}^{\infty}$, wages $\{W(t)\}_{t=0}^{\infty}$, adoption costs $\{X(t)\}_{t=0}^{\infty}$, measure of varieties Ω , and a growth rate $g > 0$ such that:

- Given aggregate prices, costs, and distributions
 - $M(t)$ is the optimal adoption threshold and $V(Z, t)$ is the continuation value function
 - $\hat{Z}(t)$ is the optimal export threshold
 - Ω is consistent with free entry
 - $p_d(Z, t)$, $p_x(Z, t)$, $\ell_d(Z, t)$, and $\ell_x(Z, t)$ are the optimal firm static policies
- The gross value of adoption and entry equal $\int_{M(t)}^{\infty} V(Z', t)\phi(Z', t)dZ'$
- Markets clear at each date t
- Aggregate expenditure is stationary when re-scaled: $Y(0) = Y(t)e^{-gt}$
- The distribution of productivities is stationary when re-scaled:

$$\phi(Z, t) = e^{-gt}\phi(Ze^{-gt}, 0) \forall t, Z \geq M(0)e^{gt}$$

In order to compute an equilibrium, we proceed in three steps. First, we use the law of motion for the productivity distribution (equation 16) and combine it with an assumption about the initial distribution, given a technology adoption policy $M(t)$. Second, given the law of motion, we solve for the firms' value function and adoption policy. Third, we solve for the growth rate g that ensures consistency between the first two steps.

4. Stationary Productivity Distribution.

To maintain tractability, we assume that the initial distribution $\Phi(0)$ is Pareto

$$\Phi(Z, 0) = 1 - \left(\frac{M(0)}{Z}\right)^{\theta}, \text{ with density, } \phi(Z, 0) = \theta M(0)^{\theta} Z^{-\theta-1}, \quad (20)$$

where θ is the shape parameter and $M(0)$ is the initial minimum of support. This assumption has been used in similar models such as Kortum (1997), Jones (2005), and Perla and Tonetti (2014). Furthermore, it is shown to arise from geometric random shocks as a result in the steady-state, rather than as an assumption, in Luttmer (2012) and Benhabib et al. (2017).

With this initial distribution, we obtain a simple stationary solution to the Kolmogorov Forward Equation in equation (16), where the productivity distribution always remains Pareto with shape θ . Specifi-

cally, the density at any date t is

$$\phi(Z, t) = \theta M(t)^\theta Z^{-\theta-1}, \quad Z \geq M(t). \quad (21)$$

This stationary result facilitates a solution in two ways. On the static dimension, it allows us to compute the equilibrium relationships, for all time, as one would in a variant of Melitz (2003), e.g., Chaney (2008). On the dynamic dimension, if the technology adoption policy is stationary when re-scaled, then this distribution satisfies the final requirement in Definition 1 that the distribution of productivities is stationary when re-scaled. Thus, it provides us an opportunity to find a balanced growth path.

Perla and Tonetti (2014) and Benhabib et al. (2017) provide detailed discussions on why—in the case of no GBM—an initial distribution with a Pareto tail is necessary to support long run growth via adoption and why the Pareto distribution is the only initial condition consistent with the balanced growth path law of motion for the productivity distribution. The key reason is that power laws have a scale invariance property, which means that as the economy grows geometrically, the distribution’s shape remains constant. Economically, because the tail does not get thinner over time, there always remain enough better technologies available for adoption to incentivize sustained investment in adoption in the long run.

Once stochastic shocks are added to idea flow models, the restriction on the initial productivity distribution is not as limiting as it might seem. In the case with GBM, starting from *any* initial distribution with mass arbitrarily close to the lower barrier—including initial distributions with finite support—the stationary distribution is Pareto. In the GBM case, the real content of assuming the initial distribution is Pareto with shape θ is that the shape of the stationarity distribution is the θ estimated from the data. The particular θ would arise out of a combination of initial conditions and the GBM parameters μ and ν . For related results and further discussion see Luttmer (2012) and Benhabib et al. (2017).

4.1. Static and Dynamic Equilibrium Relationships

The second step in computing the equilibrium is to characterize the firm’s value function and adoption policy, given the law of motion described above. To do so, we first to normalize the economy and make it stationary. We then describe the important static and dynamic equilibrium relationships on which the firm value function and adoption policy depend.

Normalization. We normalize the economy to be stationary. Roughly speaking, we do this by normalizing all variables by the endogenous reservation productivity threshold $M(t)$; Appendices C and D provide the complete details. Regarding notation, all normalized variables are lower case versions of the relationships described above. For example, lower case z represents $Z/M(t)$, i.e., a firm’s productivity relative to the reservation productivity threshold. The normalized productivity distribution relative to the adoption threshold is constant over time, a feature of the Pareto distribution:

$$\begin{aligned} f(z, t) &= M(t)\phi(zM(t), t) \\ f(z) &= \theta z^{-\theta-1}, \quad z \geq 1. \end{aligned} \quad (22)$$

Static Equilibrium Relationships. There are four important static equilibrium relationships that we use repeatedly throughout the rest of the paper. Specifically, the common component to firms' profits, the export productivity threshold, average profits, and the home trade share.

Normalized profits of a firm are

$$\begin{aligned} \pi(z) &= \bar{\pi}_{\min} z^{\sigma-1} + (N-1) (\bar{\pi}_{\min} d^{1-\sigma} z^{\sigma-1} - \kappa) && \text{if } z \geq \hat{z} \\ \pi(z) &= \bar{\pi}_{\min} z^{\sigma-1} && \text{otherwise.} \end{aligned} \quad (23)$$

The common component of firms' profits, defined as $\bar{\pi}_{\min}$, is important for two reasons. First, the value $\bar{\pi}_{\min}$ represents the profits of the *marginal adopting firm*. Given our normalization where z is defined relative to the reservation productivity threshold, a firm with $z = 1$ is the firm that is on the margin between adopting and not. Since, by definition, the choices of the marginal firm determine the adoption decision, how $\bar{\pi}_{\min}$ changes with trade barriers is important to understanding how the incentives to adopt change.

Second, because $\bar{\pi}_{\min}$ is common to all firms, it summarizes how changes to trade barriers affect profits of all firms on the intensive margin. That is holding fixed firms' exporter status, it determines the benefit (or loss) to all firms from opening to trade.

The export productivity threshold, \hat{z} , in equation (23) is the productivity level at which a firm is just indifferent between entering an export market or selling only domestically. This export threshold can be expressed as

$$\hat{z} = d \left(\frac{\kappa}{\bar{\pi}_{\min}} \right)^{\frac{1}{\sigma-1}}, \quad (24)$$

which depends on variable trade costs, fixed trade costs, and the common component of profits.

The profits of the marginal firm and the exporter threshold allow us to compute two summary statistics that are useful in characterizing the relationship between growth, trade, and welfare. The first is the ratio of average profits relative to minimum profits $\bar{\pi}_{\min}$

$$\bar{\pi}_{\text{rat}} := \frac{1}{\bar{\pi}_{\min}} \int_1^{\infty} \pi(z) f(z) dz, \quad (25)$$

which integrates over the normalized profit levels (equation 23) according to the density (equation 22). As we show below, this summary statistic of the profit distribution summarizes the key trade-off for the marginal firm deciding to adopt a new technology and, thus, dictates the rate of economic growth.

The second statistic is a country's home trade share. This is the amount of expenditure a country spends on domestically produced varieties.

$$\lambda_{ii} := \frac{\bar{\pi}_{\min}}{\bar{\pi}_{\min} + (N-1)\hat{z}^{-\theta}\kappa}. \quad (26)$$

This relationship is important because this value summarizes the volume of trade in the economy and,

thus, λ_{ii} is a measure of openness. The home trade share connects with the profit ratio in equation (25) to provide a connection between growth and the observed volume of trade.

Dynamic Equilibrium Relationships. On the balanced growth path, the normalized continuation value function, value matching condition, and smooth pasting condition in equations (11–13) simplify to

$$(r - g)v(z) = \pi(z) + \left(\mu + \frac{v^2}{2} - g \right) z \frac{\partial v(z)}{\partial z} + \frac{v^2}{2} z^2 \frac{\partial^2 v(z)}{\partial z^2}, \quad (27)$$

$$v(1) = \int_1^\infty v(z)f(z)dz - \zeta, \quad (28)$$

$$\frac{\partial v(1)}{\partial z} = 0. \quad (29)$$

The major advantage of the normalized system is that it reduces the value function to one of z alone, removing the dependence on time. This mirrors the normalization of the productivity distribution. Thus, computing a balanced growth path equilibrium using the normalized system of equations involves solving an ordinary, as opposed to partial, differential equation.

The final, normalized dynamic equilibrium relationship is the free entry condition

$$\frac{\zeta}{\chi} = \int_1^\infty v(z)f(z)dz, \quad (30)$$

which relates the normalized entry cost to the gross value of entry (and adoption).

4.2. Algorithm for Computing the BGP Equilibrium

Given the law of motion for the productivity distribution and the normalized static and dynamic equilibrium relationships, we now outline how to solve for the equilibrium growth rate, with details in Appendices E and G.

We first solve the ordinary differential equation describing the firm's value function in equation (27) through the method of undetermined coefficients, using profits and the exporter threshold from equations (23) and (24) and ensuring that the smooth pasting condition in equation (29) is satisfied. The value function depends on a firm's productivity z , the common profit component $\bar{\pi}_{\min}$, the export threshold \hat{z} , and the rate of economic growth g .

We then insert this value function into the value matching condition (equation 28) which, when combined with the free entry condition (equation 30), yields the growth rate g as a function of \hat{z} and $\bar{\pi}_{\min}$. Because the continuation value function of the marginal firm and the expected value of adoption both depend on the rate of economic growth, this boils down to finding a growth rate that makes the marginal firm indifferent between continuing to operate and adopting a new technology. In the small-noise no-death limiting case ($\mu = v = \delta = 0$), using the free entry condition and the value function evaluated at the adoption threshold, $\bar{\pi}_{\min}$ can be solved for analytically, yielding g and all other equilibrium objects in closed form. In the general case, the equilibrium reduces to a system of 3 nonlinear algebraic equations

in g , Ω , and $\bar{\pi}_{\min}$ that can be solved numerically.

5. Growth and Trade

For the theoretical analysis in Sections 5 and 6, we will focus on the case without GBM and with no death ($\mu = \nu = \delta = 0$). We will return to the general model with GBM and death in Section 7 when we study the quantitative effect of lower trade costs on welfare. The economic forces in the model without GBM and death are qualitatively the same as in the model with GBM and death.

5.1. Growth and Trade

Proposition 1 provides the equilibrium growth rate as a function of parameters, completing the characterization of economic growth in a model with equilibrium technology diffusion, entry and exit, and selection into exporting. For the rest of the paper, we make two substantive parameter restrictions, which are detailed in Proposition 1. First the elasticity of substitution cannot be too large relative to the Pareto tail index to ensure finite aggregate output. Second the elasticity cannot be too small to ensure positive growth; the gain in profits associated with a larger productivity after adoption must not diminish too rapidly relative to the cost of adoption.

Proposition 1 (Growth on the BGP). *If and only if $\theta > \sigma - 1 > \theta\chi > 0$, then there exists a unique Balanced Growth Path Equilibrium with finite and positive growth rate*

$$g = \frac{\rho(1-\chi)}{\chi\theta} \bar{\pi}_{rat} - \frac{\rho}{\chi\theta}, \quad (31)$$

where the ratio of average profits to minimum profits is

$$\bar{\pi}_{rat} = \frac{\left(\theta + (N-1)(\sigma-1)d^{-\theta} \left(\frac{\kappa}{\zeta} \frac{\chi}{\rho(1-\chi)} \right)^{1-\frac{\theta}{\sigma-1}} \right)}{(1+\theta-\sigma)}. \quad (32)$$

Proof. See Appendix G. □

The most interesting feature of Proposition 1 is that the growth rate is an affine function of the ratio of profits between the average and marginal firm. This profit ratio is the key summary statistic in this model and its sensitivity to trade costs will drive many of our main results. The intuition for why the growth rate is a function of the profit ratio is that the incentive to adopt depends on two competing forces: the expected benefit of a new productivity draw and the cost of taking that draw. The opportunity cost of adopting a better technology is the forgone profits from producing with the current technology. The expected benefit relates to the profits that the average new technology would yield. Proposition 1 tells us that a larger spread in the expected benefit relative to the opportunity cost increases the incentives to adopt and, thus, leads to faster economic growth.¹⁰

¹⁰This result is closely related to Hornstein, Krusell, and Violante (2011). In a McCall (1970) labor-search model they establish a relationship between the frequency of search and a summary statistic of wage dispersion—the ratio of the average wage to the minimum wage. In our model, growth is related to the frequency of firms searching to adopt new technologies and equation (31) shows how this depends on the ratio of profits between the average and marginal firm; similar to Hornstein et al. (2011).

We can go one step further and connect the profit ratio to a country's home trade share. This establishes a connection between economic growth and the volume of trade. After some substitution, a country's home trade share in terms of primitives is

$$\lambda_{ii} = \frac{1}{1 + (N - 1)d^{-\theta} \left(\frac{\kappa\chi}{\zeta\rho(1-\chi)} \right)^{1 - \frac{\theta}{\sigma-1}}}. \quad (33)$$

Comparing equation (33) and (32) reveals that the home trade share tightly relates to the profit ratio. This connection allows us summarize growth as a function of the inverse of a country's home trade share in Corollary 1.

Corollary 1 (Growth and the Volume of Trade). *On the Balanced Growth Path, the relationship between growth and the volume of trade is*

$$g = \frac{\rho(1-\chi)}{\chi\theta} \frac{\sigma-1}{\theta-\sigma+1} \lambda_{ii}^{-1} - \frac{\rho}{\theta}, \quad (34)$$

with a country's home trade share given in equation (33).

Proof. See Appendix G. □

Corollary 1 relates to the results in Arkolakis et al. (2012) which connects the *level* of real wages to a country's home trade share. An interpretation of their results in the Melitz (2003) model is that the trade induced welfare gains from reallocation are completely summarized by the change in a country's home trade share. In our model, Proposition 1 tells us that the incentives to adopt new technologies are driven by the spread in profits between the average and the marginal firm. Similar to Arkolakis et al.'s (2012) findings, Corollary 1 says that these distributional effects are summarized by the aggregate volume of trade.

How do changes in trade costs affect growth? A quick examination of equations (32) or (33) shows that a decrease in variable trade costs will increase the spread in profits between the average and marginal firm, reduce a country's home trade share, and increase the rate of economic growth. Proposition 2 summarizes these effects in the form of elasticities for a world-wide reduction in variable trade costs. Moreover, we provide a "sufficient statistic" representation of these elasticities that depends only on several parameters and observable trade statistics.

Proposition 2 (Comparative Statics: Trade, Profits, and Growth). *A decrease in variable trade costs*

1. *Decreases a country's home trade share.*

$$\varepsilon_{\lambda_{ii},d} := \frac{d \log \lambda_{ii}(d)}{d \log(d)} = \theta(1 - \lambda_{ii}) > 0. \quad (35)$$

2. Increases the spread in profits between the average and marginal firm.

$$\varepsilon_{\bar{\pi}_{rat},d} := \frac{d \log \bar{\pi}_{rat}(d)}{d \log(d)} = \frac{-(\sigma - 1)\varepsilon_{\lambda_{ii},d}}{1 + \lambda_{ii}(\theta - 1)} < 0. \quad (36)$$

3. Increases economic growth.

$$\varepsilon_{g,d} := \frac{d \log g(d)}{d \log(d)} = \left(\frac{\chi(1 + \theta - \sigma)}{(\sigma - 1)(1 - \chi)} \lambda_{ii} - 1 \right)^{-1} \varepsilon_{\lambda_{ii},d} < 0. \quad (37)$$

Proof. See Appendix G. □

The first result defines what we will call the domestic consumption elasticity $\varepsilon_{\lambda_{ii},d}$, which is the amount the home trade share declines (and volume of trade increases) as variable trade costs decline. This elasticity takes a simple form which depends on the volume of trade and the parameter θ . As is typical in the Melitz (2003) class of models, θ is the trade elasticity.

The second result connects the domestic consumption elasticity with reallocation effects. A reduction in trade costs increases the difference in profits between the average firm and the marginal firm. And the spreading of profits is tied to the domestic consumption elasticity. The basic idea is that lower trade costs induce high productivity exporting firms to expand and low productivity firms to contract as competition from foreign firms reduce their profits. Given that these distributional effects are summarized by the home trade share (Corollary 1), the change in the profit spread is proportional to the domestic consumption elasticity.

The third result in Proposition 2 shows that reductions in trade costs increase economic growth. The sensitivity of growth to changes in trade costs crucially depends on the domestic consumption elasticity. The intuition for this connection lies in the previous result. The reallocation effects from reductions in trade costs incentivizes more frequent technology adoption by both lowering the opportunity cost of adoption and increasing the expected benefit. For low productivity (non-exiting firms), the loss of market share and profits reduces the opportunity cost of adoption. Because exporting now has a larger return, this increases the expected benefit of a new technology. Together this leads firms to more frequently improve their technology. Because the frequency at which firms upgrade their technology is intimately tied to aggregate growth, growth increases as variable trade costs decrease.

The sufficient statistic representation of these elasticities isolates which parameters are useful in quantifying how growth responds to reductions in trade costs. In particular, it shows that the important factors are the: level of trade flows λ_{ii} , extent of firm heterogeneity θ , curvature on the demand for varieties σ , and size of adoption costs relative to entry costs χ . The trade share is observed. There is an array of estimates in the literature for θ and σ . The only difficult parameter to discipline is χ , but this can be determined as a residual from equation (G.29) (along with information on the discount factor) to target a particular growth rate.

This representation also illuminates the role of scale in our economy. First, notice that in both the growth rate and elasticity formulas, population size does not appear at all. This is largely by construction as the

fixed costs of exporting and adoption are scaled by population. Second, examination of equations (31) and (32) in Proposition 1 shows that the number of countries in the economy affects economic growth. What Corollary 1 and Proposition 2 show, however, is that what ultimately matters is the volume of trade, and not the number of countries or the trade costs per se. As our discussion of market size effects in Section 5.3 makes clear, growth depends on the ratio of profits between the average and marginal firm—not the level of profits. That is, controlling for the amount of trade as summarized by λ_{ii} , the number of countries does not affect the relative benefit of adoption and, hence, the incentives of firms to adopt a new technology.

Our model has the prediction that changes in trade costs leads to permanent growth effects. The empirical evidence is inconclusive as to whether there are growth or level effects associated with changes in trade costs. We would also caution against using evidence from cross-country regressions (e.g., Dollar and Kraay, 2004; Frankel and Romer, 1999) to make inferences about our results for several reasons: our results imply a non-linear relationship between growth and the volume of trade, as Corollary 1 makes clear; our results are in the context of a symmetric country equilibrium; our results also abstract from important mechanisms such as cross-country technology diffusion; these results also are across BGP results not inclusive of transition paths which we find to be important in Section 7. We view our main contribution as providing a better understanding of how trade affects the relative benefits of technology adoption across firms.

5.2. Firms, Trade, and Technology Adoption

Below we focus on a firm’s adoption policy and how it relates to the aggregate environment. Proposition 3 summarizes our results.

Proposition 3 (Firms and Technology Adoption). *Given an aggregate growth rate g ,*

1. *The time $\tau(z)$ until an individual firm with productivity z adopts a new technology is*

$$\tau(z) = \frac{\log(z)}{g}. \quad (38)$$

2. *The average time until adoption is*

$$\bar{\tau} = \frac{1}{\theta g}. \quad (39)$$

3. *Over a Δ length of time, the measure of firms that adopt is*

$$S\Delta = \frac{\Delta}{\bar{\tau}}. \quad (40)$$

Proof. See Appendix G. □

The first part of the proposition focuses on an individual firm and computes the time until it changes its technology. The second and third part of the proposition aggregate. Across all firms, the average time

until adoption depends inversely on the growth rate and the Pareto shape parameter. Over an increment of time, the number of firms adopting is the flow of adopters times the length of time, which turns out to take a very simple form: the time increment multiplied by the inverse of the average time until adoption.

From a firm’s perspective, more rapid economic growth means that it optimally waits a shorter amount of time before upgrading its technology. This effect of faster growth holds for all firms and, thus, the average time across all firms is shorter. Furthermore, this result implies that over a given time increment, more firms adopt.

Connecting these observations with the aggregate growth effects of trade in Proposition 2 we have predictions about firms’ responses to a trade liberalization. Proposition 3 predicts that we should see more firms adopting new technologies in an open economy relative to a closed economy. More specifically, average, within-firm productivity growth is larger in the open economy relative to a closed economy.

These results connect with the motivating evidence discussed in Section 1.2. The predictions in Proposition 3 imply that an empirical specification which projects changes in firm level measures of technology on measures of openness should display a positive relationship. This is exactly what empirical papers using specifications of this type find in the data (see, e.g., Bloom et al., 2016; Pavcnik, 2002; Topalova and Khandelwal, 2011).

Our model makes distinct predictions about who adopts. In our model, average, within-firm productivity growth hides heterogeneity in who adopts and who does not. In particular, firms at the top of the productivity distribution continue to operate with their existing technology while firms at the bottom adopt and experience productivity growth. In contrast, in Atkeson and Burstein (2010) or Bustos (2011), it is the large, exporting firms that innovate more; small import-competing firms decrease their innovation.

5.3. Reallocation vs. Market Size Effects

Heterogeneity in firms’ incentives and actions are the essential ingredients driving the relationship between trade and growth in our model. The heterogeneous incentives induce a reallocation effect across firms that is distinct from the “market size” mechanisms emphasized in the previous literature, i.e., the ability to spread the same cost of adoption across a larger market resulting in growth effects from openness. In this section we highlight the key mechanism active in our model by removing the type of heterogeneity that links trade and growth.

To focus on traditional market size forces and abstract from the role of distributional effects, we set the fixed cost of exporting equal to zero to study an environment in which all firms export. Equilibrium objects in this environment are labeled with a superscript k , since this model resembles a heterogeneous firm version of Krugman (1980). Proposition 4 summarizes growth in this model.

Proposition 4 (Growth with No Selection into Exporting). *In the model with $\kappa = 0$ and all firms selling internationally, the growth rate is*

$$g^k = \frac{\rho(1-\chi)}{\chi\theta} \bar{\pi}_{rat}^k - \frac{\rho}{\chi\theta}, \quad (41)$$

where the ratio of average profits to minimum profits is

$$\bar{\pi}_{rat}^k = \frac{(1 + (N - 1)d^{1-\sigma})\bar{\pi}_{min}E[z^{\sigma-1}]}{(1 + (N - 1)d^{1-\sigma})\bar{\pi}_{min}} = \frac{\theta}{1 + \theta - \sigma}. \quad (42)$$

Proof. See Appendix F. □

The expression for the growth rate in equation (41) takes the same form as that in Proposition 1: Growth is an affine function of the ratio of profits between the average firm and the marginal firm. Even though the details of international trade differ, the technology adoption choice is the same and, hence, the aggregate growth rate takes the same form.

The only difference between Proposition 4 and Proposition 1 is that the profit ratio is now independent of trade costs. This implies that growth is independent of trade costs. When there is no selection and all firms export, reductions in trade costs do not induce reallocation—all firms' profits increase by the same proportion in response to lower trade costs.

The intuition for this result is the following: Technology adoption in our model depends on a comparison between the expected value of adoption versus the value of continuing to operate with the existing technology. Lower trade costs have two effects on a firm's incentive to adopt a new technology. Just as in the model with selection into exporting, lower trade costs increase the expected value of adopting a new technology. However, when all firms export, lower trade costs also increase the value of continuing to operate the old technology for the marginal firm. The profit ratio summarizes this comparison. Because all firms' profits scale up by the same proportion, lower trade costs do not affect the *relative* benefit of adoption.¹¹ Thus, changes in trade costs do not change the rate of economic growth.

In contrast to this neutrality result, with selection into exporting, the differential exposure of firms to trade opportunities affects the distribution of profits and, hence, the incentives to adopt and economic growth. To illustrate these mechanics, we decompose the profit ratio in equation (32) from Proposition 1 in our baseline model in the following way:

$$\bar{\pi}_{rat} = \underbrace{\frac{\theta}{1 + \theta - \sigma}}_{= \bar{\pi}_{rat}^k} + \frac{(N - 1)(\sigma - 1)d^{-\theta} \left(\frac{\kappa}{\zeta} \frac{\chi}{\rho(1-\chi)} \right)^{1 - \frac{\theta}{\sigma-1}}}{1 + \theta - \sigma}. \quad (43)$$

The profit ratio in our baseline economy is now written as the sum of two terms. The first term is the same as the profit ratio above in the No Selection into Exporting economy. The second term isolates the affects from trade-induced reallocation. This second term shows how reductions in either variable or fixed trade costs “spread” the distribution of profits relative to the No Selection into Exporting economy, incentivize faster adoption, and lead to increases in economic growth.

¹¹This result is similar to one in Eaton and Kortum (2001) who highlight that there are two competing forces from a larger market: a larger market makes an innovation more valuable, but also makes it more costly to achieve a new innovation. In their model, these two mechanisms cancel out leaving a result similar to Proposition 4.

6. The Welfare Gains From Trade: Theoretical Analysis

This section studies the welfare effects of lower barriers where with the focus on the case without GBM and with no death ($\mu = \nu = \delta = 0$) like in the previous section. Analytically we show how changes in trade costs affect the initial level of consumption and how consumption and growth rate effects combine to determine welfare. Section 7 studies study the quantitative effect of lower trade costs on welfare.

6.1. Consumption and Welfare

How does welfare change with trade costs? While increased trade leads to faster economic growth, we show that there are more subtle effects of opening to trade on welfare. In particular, the gains from trade are a race between the positive dynamic growth effects, the positive static reallocation effects, and the negative static effects of less varieties produced in each country, and reallocation of workers away from goods production to adoption activity.

Time zero utility and the associated initial level of consumption are

$$\bar{U} = \frac{\rho \log(c) + g}{\rho^2}, \quad (44)$$

$$c = \left(1 - \tilde{L}\right) \Omega^{\frac{1}{\sigma-1}} \lambda_{ii}^{\frac{1}{1-\sigma}} \left(\mathbb{E} [z^{\sigma-1}]\right)^{\frac{1}{\sigma-1}}. \quad (45)$$

The level of consumption depends on several factors. The $(1 - \tilde{L})$ term is the amount of labor devoted to the production of goods—as opposed to adoption and entry activity. The second term is the measure of varieties. The third term is the home trade share and the fourth term is just the $\sigma - 1$ moment of the productivity distribution. Again, recall from the discussion of Corollary 1, that the trade share is a summary statistic for the distribution of activity across producers.

With changes in trade costs, the initial level of consumption changes for several reasons: changes in the home trade share, the measure of varieties, and the share of labor engaged in goods production. Proposition 2 showed how the home trade share changes. The proposition below formalizes how the measure of varieties and the share of labor engaged in goods production changes with trade costs.

Proposition 5 (Comparative Statics: Variety and Labor Allocations). *A decrease in variable trade costs*

1. *Reduces the measure of varieties produced in each country.*

$$\varepsilon_{\Omega,d} := \frac{d \log \Omega(d)}{d \log(d)} = \left(1 - \frac{1 + \theta - \sigma}{\theta \sigma (1 - \chi)} \lambda_{ii}\right)^{-1} \varepsilon_{\lambda_{ii},d} > 0. \quad (46)$$

2. *Reduces the share of workers in goods production.*

$$\varepsilon_{\tilde{L},d} := \frac{d \log(1 - \tilde{L}(d))}{d \log(d)} = \left(\frac{\theta \sigma (1 - \chi)}{1 + \theta - \sigma} \lambda_{ii}^{-1} - 1\right)^{-1} \varepsilon_{\lambda_{ii},d} > 0. \quad (47)$$

Proof. See Appendix G. □

Lower trade costs decrease the measure varieties produced in each country. This is consistent with the Melitz (2003) logic that reductions in trade costs induce exit. Lower trade costs also result in less labor allocated to the production of goods and more labor allocated to the fixed costs of adoption and exporting. In a sense, there is more “investment” in the adoption of technology and exporting—a force towards a lower level of consumption.

Proposition 6 (Consumption Effects). *A decrease in variable trade costs decreases the initial level of consumption.*

$$\varepsilon_{c,d} = \varepsilon_{\tilde{L},d} + \frac{\varepsilon_{\Omega,d} - \varepsilon_{\lambda_{ii},d}}{\sigma - 1} > 0. \quad (48)$$

Proof. See Appendix G. □

Proposition 6 shows that there are two drags on the initial level of consumption and one offsetting gain. Reductions in trade costs reallocate labor away from the production of goods and, hence, the consumption of goods ($\varepsilon_{\tilde{L},d} > 0$). Reductions in trade costs lead to less production of domestic varieties ($\varepsilon_{\Omega,d} > 0$), which reduces consumption due to the love for variety CES final goods aggregator. Counteracting these forces are gains from an increase in the measure of foreign varieties consumed and that these foreign varieties are produced by firms that are relatively highly productive. These forces are summarized by a decrease in the home trade share ($\varepsilon_{\lambda_{ii},d} > 0$).

Which of these forces wins? The initial consumption level decreases with lower trade costs. To see this, note that from equation (46), the loss in domestic varieties is larger than the gain from importing foreign goods ($\varepsilon_{\Omega,d} > \varepsilon_{\lambda_{ii},d}$). Given that labor is always reallocated away from production as trade costs decrease, equation (48) shows that the level of initial consumption will decrease with lower trade costs.

This result deserves some discussion. First, while the initial level of consumption decreases, there is an inter-temporal gain as consumption in future periods is higher (Proposition 2).

Second, our model’s static gains from trade need not correspond with those in purely static models of trade. In Melitz (2003), the typical parameterization finds that domestic variety falls, but the gains from foreign variety more than compensate for this loss.¹² In our model, dynamics lead to additional effects on the free-entry condition. Wages increase due to increased demand for labor, which increases the cost of entry. However, the expected value of entry does not increase in equal proportion, as profits are reallocated across firms and these profits are discounted at a higher rate because of faster growth. Thus, to satisfy the free-entry condition the reduction in domestic variety is larger (relative to static models). Atkeson and Burstein (2010) have a similar quantitative finding that the gains due to increased innovation by firms are offset by losses in variety and reallocation of resources away from production for consumption.

Finally, we should note that the race between these welfare reducing and welfare enhancing effects from trade cost changes is an important normative feature of our model relative to other idea-flow models studying trade and growth. For example, in Alvarez et al. (2017), there are no resource costs associated

¹²As our phrasing suggests, this need not be the case. See the discussion on page 1713 of Melitz (2003).

with acquiring ideas. As a result, when trade facilitates faster idea acquisition and economic growth, there is no corresponding increase in costs. In contrast, in our model, faster idea acquisition comes with the cost of more labor being allocated away from production.

Using equations (44) and (45) and the results in Proposition 2, 5 and 6, the elasticity of utility with respect to a change in trade costs—the welfare gain from trade—is summarized below in Proposition 7.

Proposition 7 (Welfare Effects). *The welfare change from a decrease in variable trade costs is*

$$\varepsilon_{\bar{U},d} = \frac{\rho^2}{\bar{U}} (\rho \varepsilon_{c,d} + g \varepsilon_{g,d}). \quad (49)$$

Proof. See Appendix G. □

Comparing across BGPs, the welfare gain from trade is proportional to the change in the initial level of consumption, $\varepsilon_{c,d}$, and the change in the growth rate, $\varepsilon_{g,d}$.¹³ Proposition 2 tells us that the benefit from growth is positive and Proposition 6 shows that the effect on the level of consumption is negative. Economic logic suggests that a decrease in trade costs should increase utility. This comparative static analysis, however, ignores transition dynamics and there exist parameter values (when $\sigma - 1$ is close to its lower bound of $\theta\chi$) such that lower trade costs are associated with lower utility according to this measure (see equation G.50). Thus, understanding the welfare effects of lower trade costs requires solving for the economy along the transition path at calibrated parameter values. We perform such an analysis in Section 7.

6.2. Consumption and Welfare with No Selection into Exporting

To further isolate the race between the competing welfare reducing and welfare enhancing effects, consider the welfare effects in our model when all firms export. Proposition 8 details how trade, growth, varieties, labor allocations, and welfare are affected by trade costs in the economy in which there is no selection into exporting.

Proposition 8 (Comparative Statics with No Selection into Exporting). *In the model with $\kappa = 0$ and all firms exporting, a decrease in variable trade costs*

1. *Reduces the home trade share.*

$$\varepsilon_{\lambda_{ii},d}^k = \frac{d \log \lambda_{ii}^k(d)}{d \log(d)} = (\sigma - 1)(1 - \lambda_{ii}^k) > 0, \quad (50)$$

where,

$$\lambda_{ii}^k = (1 + (N - 1)d^{1-\sigma})^{-1}. \quad (51)$$

¹³Note that since $d > 0$, the sign of the elasticity and the derivative of utility with respect to trade are equal if and only if $\bar{U} > 0$. With log utility the sign of \bar{U} depends on initial conditions on the mean of the productivity distribution and the population size. Most importantly, the sign of the derivative of utility with respect to trade costs is independent of these initial conditions and the sign of utility.

2. Does not change the growth rate, the measure of domestic varieties, or the share of workers in goods production.

$$\varepsilon_{g,d}^k = 0, \quad \varepsilon_{\Omega,d}^k = 0, \quad \varepsilon_{\bar{L},d}^k = 0. \quad (52)$$

3. Increases the initial level of consumption.

$$\varepsilon_{c,d}^k = -\frac{\varepsilon_{\lambda_{ii},d}^k}{\sigma - 1} < 0, \quad (53)$$

4. Increases welfare.

$$\varepsilon_{U,d}^k = \frac{\rho^2}{\bar{U}} \rho \varepsilon_{c,d}^k < 0. \quad (54)$$

Proof. See Appendix F. □

Lower trade costs lower the home trade share, although now moderated by $(\sigma - 1)$ as opposed to θ . In addition to growth being independent of trade costs, the share of workers allocated to goods production does not change with trade costs either. Thus, while this model does not have dynamic gains from trade, it does not experience some of the losses associated with dynamics, such as labor reallocation.

The measure of varieties does not change either. The reason is that when all firms export, all firms' profits scale up by the same amount as the real wage and hence the entry cost. As trade costs decline, all profits increase by the same amount and the expected value of entry increases by this amount, but the cost of entry increases in the same proportion as well.¹⁴ Thus, the measure of varieties need not change for the free-entry condition to be satisfied.

These observations help explain why growth effects are associated with a loss of variety in Proposition 5. When there is selection into exporting, in response to a decrease in trade costs, the normalized expected value of entry increases less than the cost of entry because of the reallocation of profits across firms. This implies that a reduction in varieties must occur for the free-entry condition to be satisfied. Thus, the same exact reallocation effects which lead to faster economic growth, generate the loss of varieties.

The final line of Proposition 8 computes the total gains from trade. These gains are purely static and these static level effects only operate through the component associated with the home trade share. This result is similar to the welfare gain calculations in Arkolakis et al. (2012). Unlike the finding in Arkolakis et al. (2012) that the gains from trade are equivalent in models with selection or without, a comparison of Proposition 7 to Proposition 8 shows that the essential element to delivering dynamic gains from trade is the reallocation effect introduced by selection into exporting.

¹⁴While the independence of the growth rate and trade costs is true independent of whether the cost of adoption is in labor or goods, that the elasticity of varieties is independent of trade costs relies on the cost of adoption being in labor only. This result is similar to Atkeson and Burstein's (2010) Proposition 2 that states a similar neutrality result on the measure of varieties when the research good in their model uses only labor.

7. The Welfare Gains From Trade: Quantitative Analysis

This section extends the theoretical analysis in two important quantitative directions. First, we calibrate the full model with GBM productivity shocks and firm exit. While we lose the analytical tractability of the simple model featured above, these features of the model allow us to discipline parameter values using micro-level moments of firm dynamics. Second, instead of simply looking at differences across steady states, we study the transition dynamics of the economy in response to a reduction in trade costs and compute welfare changes taking the transition into account.

Since the quantitative model cannot be solved analytically, we use numerical methods to compute the BGP for calibration and to solve for transition dynamics. The key steps are to discretize the spatial dimension (i.e., productivity) of the Hamilton-Jacobi-Bellman PDE to turn it into ODEs and then to add in all necessary equilibrium conditions, which includes the value matching integral equation, to create a differential-algebraic system of equations (DAEs). This approach of solving for the transition dynamics of heterogeneous agent models by representing the economy as a differential-algebraic system of equations and using high-performance DAE/ODE solvers is applicable to a large class of models. Online Appendix Numerical Methods describes the algorithm used to compute the equilibrium of the quantitative model with GBM and exit shocks.

7.1. Calibration

We choose parameters by using a mix of normalization and calibration to match outcomes of our model on the BGP with moments in the data.

The normalizations are described in the top panel of Table 1. The technology adoption cost is normalized to the value one. This is without loss of generality as only the relative cost of adoption to entry affects allocations. Alternative normalizations result in identical results, but with re-scaled parameter values. We set the number of countries equal to ten. Per the implications of Corollary 1 and Proposition 2 a different value of N has no impact on how growth responds to trade as long as the home-expenditure share λ_{ii} is matched to its empirical value.

Our calibration procedure uses the simulated method of moments to determine the remaining parameter values. The parameter vector we are choosing is $\{\rho, \theta, \sigma, \mu, v^2, \delta, d, \kappa, \chi\}$, in words: the discount factor, Pareto shape parameter, variety elasticity of substitution, drift and variance of the GBM process, exit shock, iceberg trade cost, exporter fixed cost, and (relative) entry cost. The bottom panel of Table 1 presents a description of these nine parameters and their calibrated values.

These parameter values are jointly chosen so that the model best fits 14 moments. Because the economy with GBM shocks does not have closed-form solutions, we compute the BGP equilibrium using numerical methods and then construct sample moments in the model to compare to sample moments in the data.

The data sources are a mix of aggregate statistics, measures from other papers, and micro data. Most moments that we target are constructed for the U.S. over the time period from 1977-2000. We choose this time period because it is the span for which we have access to the micro-level firm dynamics data.

Table 1: Calibration Results: Parameters and Values

Parameter	Value	Parameter	Value
Technology Adoption Cost, ζ	1.0	Number of Countries, N	10
Discount Factor, ρ	0.0215	Pareto Shape Parameter, θ	5.01
Variety elasticity of substitution, σ	3.17	Drift of GBM process μ	-0.019
Variance of GBM process v^2	0.048	Death rate of Firms δ	0.02
Iceberg Trade Cost, d	3.04	Export Fixed Cost, κ	0.07
Entry Cost Relative to Adoption Cost $1/\chi$	5.96		

The micro-level data that we use is from the Synthetic Longitudinal Business Database (SLBD) (U.S. Census Bureau (2011)). The SLBD is a public access database that synthesizes information on establishments' employment, payroll, industry, and birth and death year. It is synthesized from the confidential, restricted-access Longitudinal Business Database that is maintained by the U.S. Census Bureau. We construct SLBD moments using only establishments with at least 20 employees which is the cutoff value used in Hurst and Pugsley (2011). Per the observations of Hurst and Pugsley (2011), the motivation for this sample restriction is that many of these small establishments appear to have no intention to grow or innovate and, thus, their motives do not correspond with the motives of firms in our model. In Section 7.4, we illustrate how alternative parameterizations of the GBM process (and, hence, firm dynamics) shape our results.

Below we describe the specific moments that we target, how we measure them, and a brief description of the parameters about which these moments are most informative.

Real interest rate. We construct the real interest rate as the difference between the rate of return on a one year U.S. treasury constant maturity nominal bond and the realized inflation rate. We use the U.S. consumer price index, all items as our measure of realized inflation. Over the 1977–2000 time period, the average real interest rate was 2.83 percent. This value, along with the growth rate of the economy, pins down the consumers discount factor ρ through the fact that the consumer's Euler equation implies that the real interest rate equals $\rho + g$.

Aggregate total factor productivity growth. To measure U.S. productivity growth, we use the Bureau of Labor and Statistics Multifactor Productivity Database. In particular, we focus on the private non-farm business sector and measure labor-augmenting productivity growth. Over the 1977-2000 time period, average productivity growth was 0.66 percent per year. Conditional on all other parameter values, the rate of economic growth is connected with the frequency of adoption and, hence, its cost χ . In Section

Table 2: Aggregate Moments: Model and Data

Moment Description	Data	Model
U.S. real interest rate	2.83	2.83
U.S. productivity growth	0.66	0.66
U.S. Import/GDP	10.47	10.47
Share of exporting establishments	3.3	3.3
Relative size of exporting establishments	4.8	4.8

Details of moment construction are provided in the body of the paper. The real interest rate, productivity growth rate, and import/GDP ratio are in percent and averages over the 1977–2000 time period.

7.4, we discuss the relationship between g and χ in more depth.

Aggregate import share. We construct the aggregate import share as the value of imports of goods and services divided by GDP. Over the 1977–2000 time period, the average import share was 10.47 percent and quite stable. As in most models of trade, this moment is informative about the costs of trade and specifically the iceberg trade cost, d .

Share of exporting establishments. This moment is constructed from the finding of Bernard, Eaton, Jensen, and Kortum (2003), who use the 1992 U.S. Census of Manufactures to report that 20 percent of manufacturing establishments export. Because we associate each variety in our model with the output of an establishment within the entire U.S. aggregate economy, we adjust the Bernard et al. (2003) number by the relative size of the manufacturing sector. In the SLBD data, only 16.5 percent of establishments are in manufacturing and we assume, as a lower bound, that only manufactures are exported. This implies that the fraction of all establishments that export is $0.2 \times 0.165 = 3.3$ percent. This moment is particularly informative about the fixed cost of exporting, κ .

Relative size of exporting establishments. We target the finding of Bernard et al. (2003) that exporters' domestic shipments are 4.8 times larger than non-exporters' shipments. This moment is particularly informative about parameters controlling the size of the firm, and thus, θ and σ .

Size dynamics of establishments. We target the observed transition probabilities of establishments as they move across different quartiles of the size distribution. We focus on the dynamics of firms at the top of the size distribution because in the model these transition probabilities are governed by the GBM process, since they are away from the adoption threshold.

We construct these moments in the following way. For each year, we sort establishments into quartiles by

Table 3: Establishment Size Dynamics, Data and Model

Transition Matrix of Relative Size: largest, quartile 1; smallest, quartile 4									
Data					Model				
	1	2	3	4		1	2	3	4
1	50.0	27.0	13.6	9.4	1	45.5	31.6	14.2	8.6
2	16.2	28.6	29.1	26.1	2	16.9	28.8	28.2	25.9

Corr(Data, Model) = 0.98

Employment share of new establishments			
Data:	0.02	Model:	0.02

Note: In the top panel, rows represent the establishment-size quartile in period t ; columns represent the establishment-size quartile in period $t + 5$. Data Source: U.S. Census Bureau (2011).

employment size. We then compute the probabilities that establishments transition across size-quartiles at a five-year horizon, conditional on continuing to operate. We then average these transition probabilities across all years in the SLBD, i.e., from 1977–2000. The focus on the five-year horizons is to sweep out any transient fluctuations that may occur at higher frequencies due to shocks that are outside of our model. To give some perspective on the overall size of establishments, firms larger than about 40 employees fall into the top two quartiles; firms larger than about 100 employees fall into the top quartile.

The left panel of Table 3 reports the observed transition probabilities. For example, 50 percent of establishments in the top quartile remain in the top quartile after 5 years, 27 percent fall to the second quartile, and the remainder are sprinkled between the bottom two quartiles.¹⁵ These moments show that there is a large degree of persistence in firm size and that the probability of moving to a nearby quartile tends to be larger than the probability of moving to a distant quartile. We target all 8 transition probabilities.

As mentioned, these transition probabilities are very informative about parameters which control the drift and variance of the GBM process. In addition, they also place restrictions on the size of firms, and thus, inform the values of θ and σ .

Employment share of new establishments. This moment is measured as the share of employment (out of total employment) that is employed in new establishments each year, in the SLBD. Averaging across all time periods, we find this value to be two percent. On the balanced growth path, the entry rate exactly corresponds with the exit rate and, hence, this moment directly pins down the exit shock faced by firms, δ . We choose to target a moment related to entry since entry dynamics are key to the behavior of the economy on the transition path.

¹⁵These transition probabilities are quite similar when restricting the sample to only manufacturing establishments, with the only difference being a bit more persistence. Moreover, they look similar to the productivity transitions reported in Baily, Hulten, and Campbell (1992).

To summarize, 9 parameters values are chosen to target these 14 moments: three aggregate moments (real interest rate, import share, productivity growth), two exporter facts (share of establishments, relative size of exporters), and nine firm dynamics moments (eight transition probabilities and the share of employment of entrants).

7.2. Calibration Results

The bottom panel of Table 1 reports the calibrated parameter values. Many of these parameter values are easily compared and are close to alternative estimates of these values in the literature.

The Pareto shape parameter value of five is exactly in line with the trade evidence discussed in Head and Mayer (2014). More broadly, the value is a bit larger than Simonovska and Waugh's (2014) point estimate for the Melitz model who use both trade and price data. The value is a bit larger than usually found when models are matched to firm-level size moments, e.g., Bernard et al. (2003) and Eaton, Kortum, and Kramarz (2011) find point estimates of around 3.6 for the Pareto shape parameter.

We find the elasticity of substitution across varieties to be 3.14, which is right in the range of the median estimates of Broda and Weinstein (2006). For example, their estimates at the most disaggregate level fall between 3.1 and 3.7 depending upon the time period considered. The values of both θ and σ determine the size distribution of firms. In our model, the size distribution is Pareto with shape parameter $\frac{\theta}{\sigma-1}$. The empirical literature suggests that this shape parameter is just slightly above one, see, e.g. Axtell (2001) or Luttmer (2007). Together our estimate of θ and σ generates a tail index of 2.3, which is larger than the near one value found in the data.

The GBM variance parameter value is 0.048 and the value for the drift is -0.019. These values are close to those used in Arkolakis (2015) and Luttmer (2007). In particular, Arkolakis (2015) uses moments related to our calibration procedure from Dunne, Roberts, and Samuelson (1988) and infers a variance of 0.067 and a drift of -0.0194. Similarly, Luttmer (2007) infers a GBM variance of 0.048.

We infer a death shock of two percent. This is in line with a wide range of papers who have a similar shock structure. Two examples (amongst many) that cover a range inside which our value falls are: Arkolakis, Papageorgiou, and Timoshenko (2018), who calibrates this value to be 2.5 percent to match exit rates of large firms in the Colombian economy and Atkeson and Burstein (2010), who calibrate the value to be 0.55 percent to match the share of employment displaced by large firms (larger than 500 employees) in the U.S. economy.

The iceberg trade cost is large, but by no means abnormal relative to those found in the "gravity" literature that infers these costs from trade flows. For example, Waugh and Ravikumar (2016) employs a similar calibration strategy (one trade friction to match a country's import/GDP ratio) and infers the iceberg trade cost for developed countries to be around five. The large value of five (relative to our trade costs of about three) is partially a result of the lower trade elasticity θ in that paper. At the lower end of the range is Waugh (2010) who finds iceberg trade costs of around two for OECD countries—again, in part because Waugh (2010) employs a larger trade elasticity.

In terms of fit, the aggregate moments, the exporter facts, and the employment share of entrants match

the data exactly. Table 2 reports these aggregate moments in the model and the data. The right panel of Table 3 reports the model's fit for the eight firm dynamic moments. While not perfect, the model replicates the dynamics of firms well. The correlation between the model and the data is extremely high at 0.98.

The final parameter to discuss is the entry cost. Here the interpretation is that the cost of entry is about six times larger than the cost of adoption. Because this parameter value is hard to externally validate, we discuss its implications in Section 7.4 below.

7.3. The Welfare Effects of Reduction in Trade Costs

Using the model as calibrated in Table 1 as our baseline, we study how the economy is affected by a reduction in trade costs. We start our quantitative experiment from the baseline economy on a BGP and then shock the economy with an unanticipated ten percent permanent reduction in trade costs. We study how the economy transits from the baseline BGP equilibrium to the new low-trade-cost BGP equilibrium.

7.3.1. Trade, Growth, and Entry Dynamics

The reduction in trade costs at date zero causes an increase in imports relative to GDP. Figure 1 shows that imports instantly jump almost entirely to the new steady state level and then slowly converges to its final value. Part of this rise in trade simply comes from the reduction in trade costs, i.e., the cost of importing declines, and thus existing exporters expand and sell more to foreign markets. The second force that increases trade comes from the extensive margin with the entry of new exporters, i.e., the lower trade costs decrease the exporter productivity threshold, \hat{z} , leading to an expansion of exports and imports. Table 4 reports that across steady states, the ten percent reduction in trade costs causes about a 3.8 percentage point increase in imports to GDP. This is closely related, but not exactly equal, to the trade elasticity θ due to general equilibrium effects.

Unlike the rapid change in the volume of trade, the measure of domestically produced varieties takes time to adjust. Figure 2 illustrates this process, showing that the measure of domestic varieties falls gradually after the reduction of trade costs. The measure of varieties is lower in the new steady state because import competition decreases revenue for domestic non-exporting firms and because domestic exporting firms bid up the local wage as they hire more labor in order to increase their exports. Altogether, this leads to a decrease in the expected value of entry relative to the cost of entry. Thus, there are "too many" varieties produced by each country given the new lower trade costs, which reduces the incentives for firms to enter and leads to net-exit of firms. This mirrors the result in the simple analytical model that the measure of domestically produced varieties is smaller in steady states with lower trade costs (see Proposition 5).

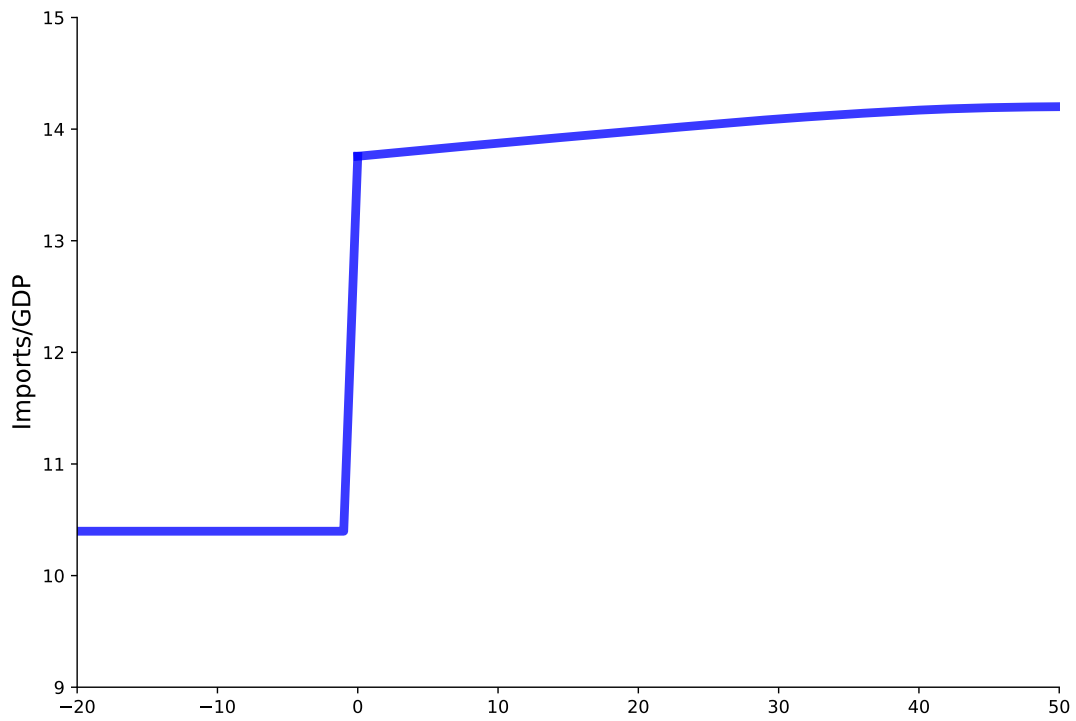


Figure 1: Imports/GDP: $1 - \lambda_{ii}(t)$

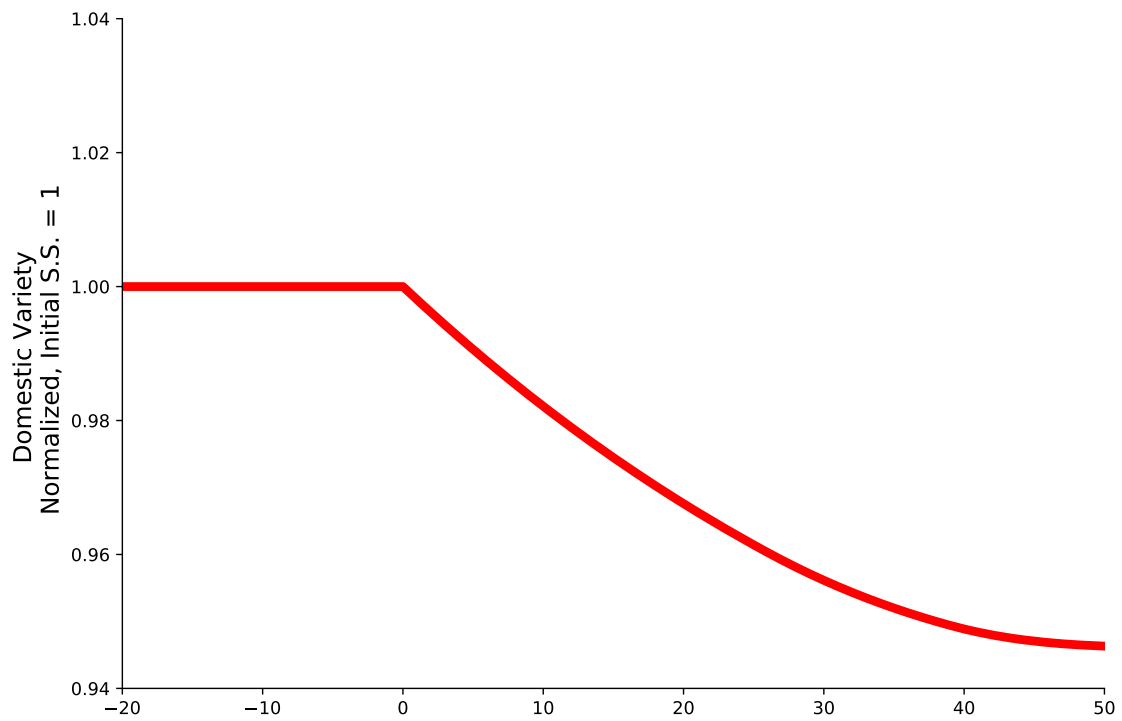


Figure 2: Domestic Variety: $\Omega(t)$

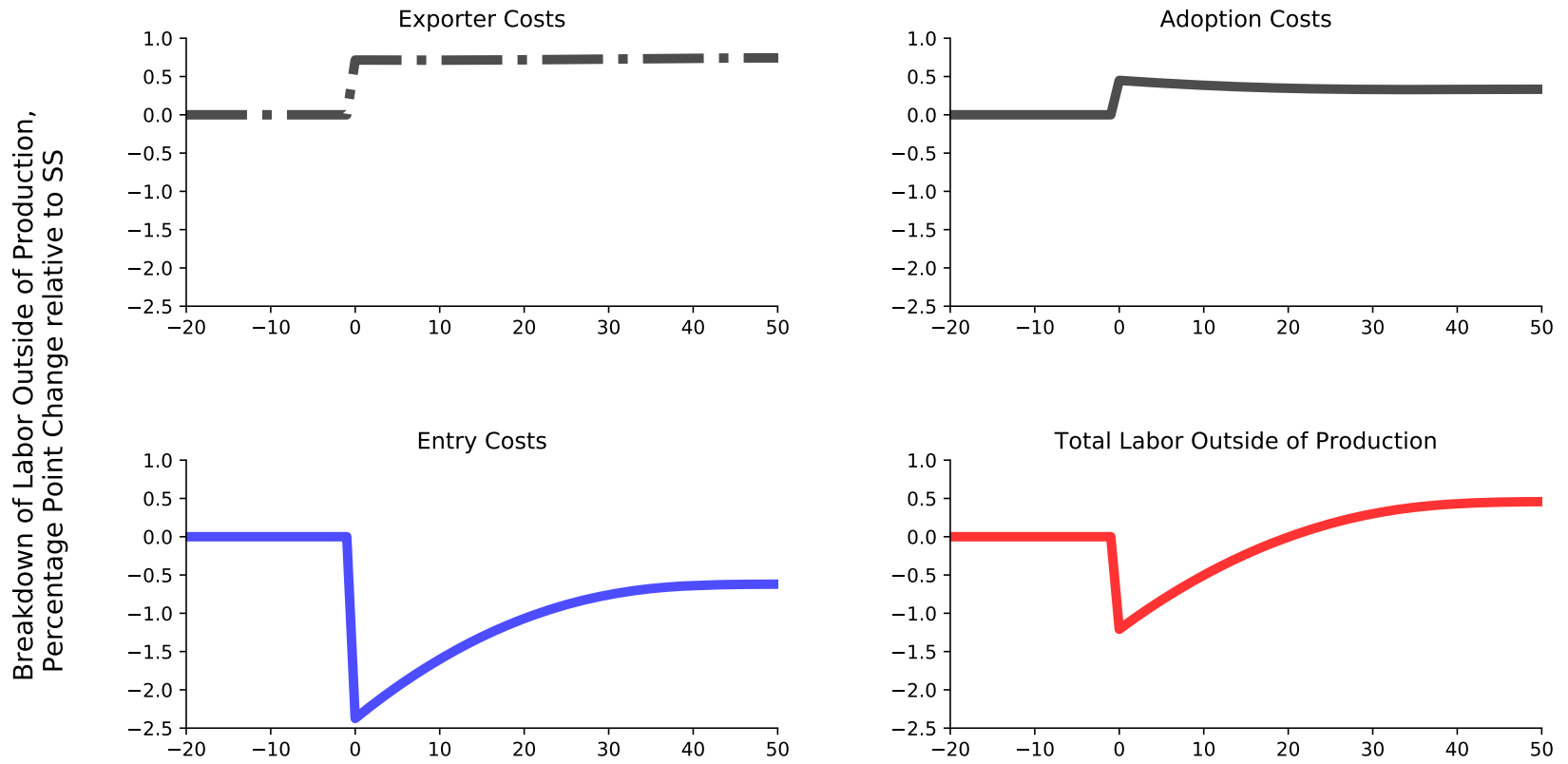


Figure 3: Reallocation Effects in Fixed Costs

Two forces are behind the gradual exit of firms. The first force is mechanical: since exit is exogenous it takes time for the measure of varieties to adjust. The second, endogenous force, relates to consumption smoothing and leads to a positive entry rate along the transition path. Thus, net-exit is less than the mechanical exit of firms would deliver.

The consumption smoothing motive is just like that in the neoclassical growth model when the capital stock is too large relative to its steady state value. In the neoclassical growth model, along the optimal transition path, investment is still positive because consumers want to smooth consumption, i.e., avoid large differences in consumption across time periods. Analogously, in our model it is feasible to enjoy initially higher consumption and have zero entry on the transition path, but the intertemporal smoothing motives dictate that the economy forgo this path to satisfy the desired consumption plan as given by the Euler equation.

The desired path of consumption across time must be supported by the allocation of labor across activities, over time. The increase in trade means more labor is allocated to the fixed costs of exporting. The decrease in entry, however, means less labor is needed for entry costs. Quantitatively, the decrease in labor allocated to entry is much larger than the increase in labor allocated to export costs, so in an accounting sense there is “excess labor” to be allocated. The two other tasks that use labor are adoption and the production of goods. Allocating all of the excess labor to the production of goods would maximize short-run consumption. In contrast, allocating all of the excess labor to adoption would increase growth rates and maximize long-run consumption. Due to the intertemporal smoothing motive, the economy does not wind up at a corner solution on the transition path, but rather an allocation of labor arises to balance out more consumption today (production) with more consumption in the future (adoption).

Figure 3 illustrates the change in the allocation of labor across activities. Note the interpretation of the y-axis is literally the change in labor allocated to that activity because the total labor endowment is normalized to one. The top left panel illustrates the 0.75 percentage point rise in labor allocated to the fixed costs of exporting, consistent with the jump in trade. The top right panel depicts the increase in adoption activities by about 0.50 percentage point. In contrast, the bottom left panel in Figure 3 shows that the amount of labor dedicated to entry falls dramatically—nearly 2.5 percentage points. The bottom right panel sums everything up: On impact, on net, the amount of labor allocated to these investment-like activities falls by about one percentage point. After the decline, the total amount of labor in these activities steadily rises to a new higher level than in the initial steady state.

These changes in the allocation of labor have two implications. First, consumption overshoots along the transition path. Figure 4 illustrates that this effect is large. As the inset in Figure 4 shows, the trade liberalization induces an initial three percent increase in consumption C_t . Along the transition path, the level of consumption $C(t)$ always lies above its previous path. Productivity grows faster than consumption, however, so that normalized consumption $c_t := C_t/M(t)$ steadily declines to its new lower level of about one and a half percent below the initial level. This effect can be seen in Figure 4 by noting that the level of consumption ends up lying below the level of productivity in period 50. Proposition 6 stated that normalized consumption is lower in BGPs with lower trade costs, but the transition dynamics analysis illustrates that consumption first overshoots before eventually converging to the new lower

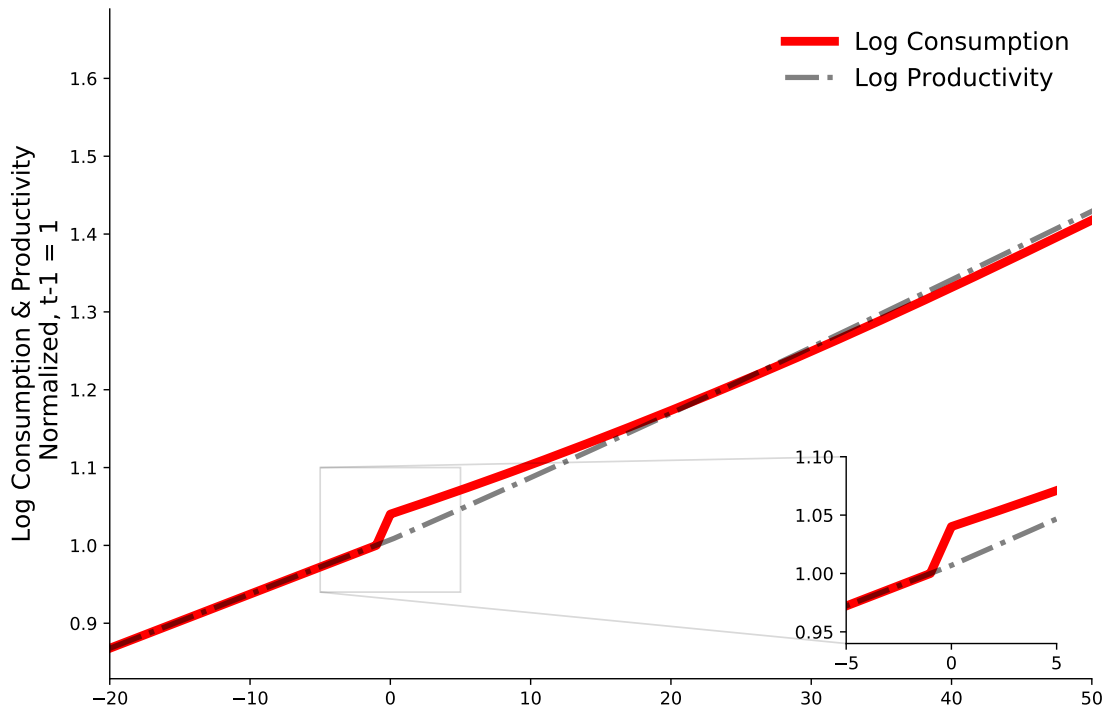


Figure 4: Log Consumption: $C(t)$

relative level.¹⁶

The second implication is that productivity growth slowly rises to its new steady state rate. Figure 5 shows that after the change in trade costs, there is an initial jump in productivity growth from the baseline of 0.66 percent to 0.75 percent. From this point on, productivity growth gradually rises at a decelerating rate as it moves towards the new BGP growth rate of 0.86 percent—0.20 percentage points higher than in the baseline. In other words, about a third of the jump is instantaneous and the remaining two-thirds of the rise in growth plays out over the next 30 years. To emphasize this point, this pattern of growth and the corresponding adoption behavior is intimately connected with the pattern of consumption. Since adoption entails a tradeoff of paying a cost today for gains that will payout in the future, the firm’s discount factor which reflects the consumer’s desired path of consumption must support investment in adoption and rising productivity growth.

7.3.2. The Welfare Gains from Trade

From a welfare perspective, the reduction in trade costs leads to several competing effects (as Proposition 7 shows for our simple economy). Productivity growth increases due to higher adoption rates. Comparing across steady states, more adoption is associated with a lower level of normalized consumption, as firms are using more labor to invest in technology upgrading. The reduction in variety along the transition path, however, more than offsets the effects from more adoption, leading to consumption

¹⁶This logic is related to the intuition for consumption overshooting in Alessandria and Choi (2014) and the amplified welfare gains from trade in Alessandria, Choi, and Ruhl (2014). An interesting element of our model is that there are different changes in “investment” activity: a rise in exporting and adoption, but a fall in entry. Quantitatively, the decline in entry is larger than the rise in other investment-like activities and, thus, consumption overshoots.

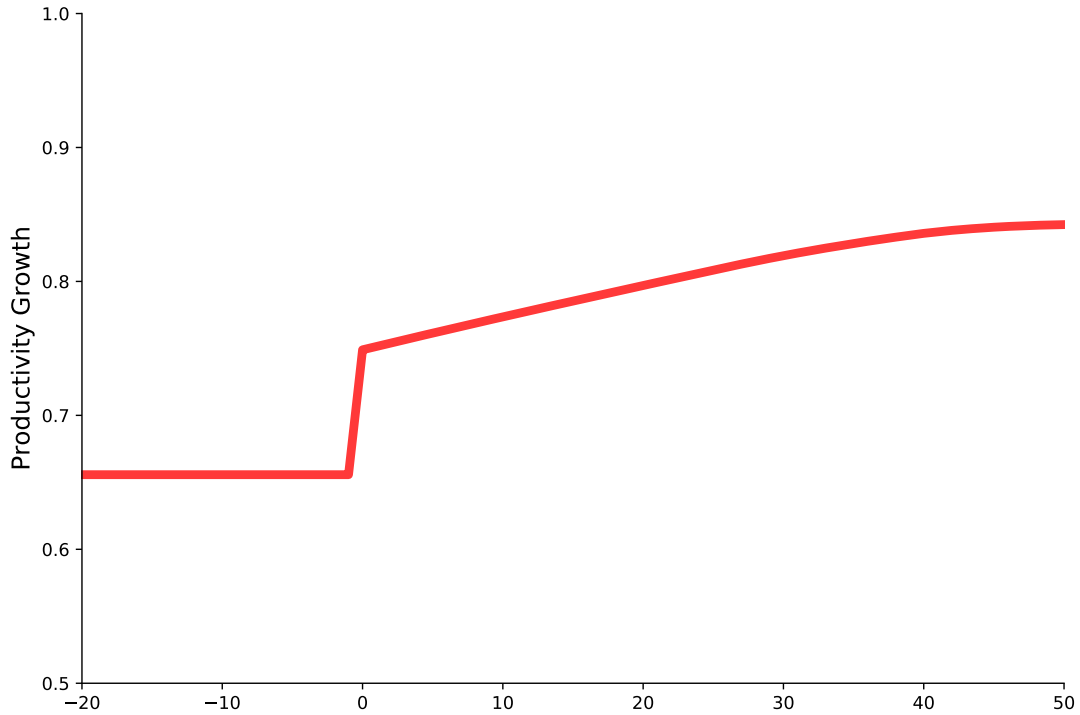


Figure 5: Productivity Growth: $g(t)$

overshooting.

To measure these effects on welfare, the final two rows of Table 4 report consumption-equivalent gains associated with the ten percent reduction in trade costs. Consumption-equivalent gains are defined as the permanent percent increase in consumption a household requires in the old regime to be indifferent between the new and old regimes.

The third row reports the welfare gains inclusive of the transition path. The gains from trade are an 8.4 percent increase in welfare. Because this is inclusive of the transition path, it includes the net-effect of two different forces: (i) the increase in growth is not immediate which tends to lower welfare due to discounting and (ii) the path of consumption overshoots which tends to increase welfare gains. The fourth row of Table 4 provides a sense of how much the transition path matters by computing the consumption-equivalent gain across steady states. Here the gains are a bit less than one percentage point lower than the gains inclusive of the transition path. In other words, the net effect of the transition path is to modestly amplify the welfare gains from trade.

The conventional logic is that the welfare gains will be smaller once the transition is included because the benefits of higher growth are in the future and they require costly investment. As we've shown, these two forces are active in our model as seen in Figure 5 and in Figure 3. However, the initial steady state has too many varieties relative to the new steady state and the decline in varieties leads to reallocation of labor away from the creation of new firms. In our model, this mechanism is strong enough to overturn any delay from the gains in future growth and generates larger gains from trade as the economy is able to enjoy higher consumption as variety adjusts.

Table 4: 10 Percent Reduction in Trade Costs: Growth, Trade, and Welfare

	Baseline BGP	New BGP
Growth	0.66	0.86
Imports/GDP	10.5	14.2
Welfare		
Consumption Equivalent Gain (Transition Path):		8.40
Consumption Equivalent Gain (SS to SS):		7.65

Note: All values are in percent. Consumption-equivalent is the permanent percent increase in consumption a household requires in the old regime to be indifferent between the new and old regimes.

These gains are larger than related static models predict. To illustrate this point, we use the formula of Arkolakis et al. (2012) (hereafter ACR) to compute the static gains from trade implied by our trade elasticity (Pareto shape parameter θ) and the model-predicted change in trade. Using their formula the welfare gains are simply $100 \times -\frac{1}{\theta} \log \left(\frac{1-0.142}{1-0.1047} \right) = 0.86\%$. This is an order of magnitude smaller than our welfare gains inclusive of the transition path. Another point of comparison is to note that the welfare gains which arise from the transition path ($8.40-7.65 = 0.75\%$) are essentially the same as what the ACR calculation implies.

Large reductions in trade flows lead to large welfare losses in our model, but welfare does not fall as much in relative terms as the ACR formula would predict. Comparing steady states of our model, a move to autarky would generate a welfare loss of about fifteen percent. In other words, starting from baseline, the loss from a move to autarky is about two times the gains from a ten percent reduction in trade costs. In contrast, the ACR formula would predict a 2.3 percent loss from autarky—more than two and a half times larger than the gains from a ten percent reduction in trade costs.

Relative to other models with dynamic gains from trade, exact quantitative comparisons are difficult to make, but several qualitative differences arise. Atkeson and Burstein (2010), for example, feature consumption overshooting (see Figure 4 in the NBER working paper version, Atkeson and Burstein (2007)), however, welfare is relatively unaffected as the magnitude of the overshooting is small. The second reason for small gains in Atkeson and Burstein (2010) is that transitions are long (approximately 200 years) and the gains from innovation accrue slowly. In contrast, our model generates a jump and then a relatively quick transition to a higher growth rate. We speculate that the rapid adjustment is intertwined with the nature of the adoption process in our model. The rapid increase in trade and variety expansion creates sudden competitive pressure on the large number of firms at the bottom of the productivity distribution to adopt and upgrade their technology.

Alessandria et al. (2014) is another point of comparison that features consumption overshooting. Simi-

larly to our quantitative experiment, they explore the welfare gains associated with a global ten percent reduction in tariffs. They find welfare gains of around six percent in their baseline model (which are in line with our results) and consumption overshooting is an important contributor to these gains. Key differences are that their model does not feature growth effects and that the exporting behavior of firms in their model is designed to generate the slow growth of exporters. This latter point is important as it amplifies the overshooting of consumption along the transition path.

Hsieh, Klenow, and Nath (2019) is a closely related paper as well. They focus on the dynamic gains from trade in a model of creative destruction along the lines of Klette and Kortum (2004) and Garcia-Macia et al. (2019). After calibrating their model, they explore the welfare gains from lower trade costs. Compared to our paper, Hsieh et al. (2019) study the gains from a somewhat larger reduction in trade costs and find that the dynamic gains from trade are around twelve percentage points—fifty percent larger than our already larger gains.

Sampson (2015) is, perhaps, the closest paper to ours, even though the mechanisms active in the two papers are quite different. Sampson (2015) studies the role of dynamic selection in generating dynamic gains from trade. The idea is that trade induces endogenous exit which makes the pool of ideas from which entering firms draw technologies better and, thus, facilitates faster economic growth. A direct comparison of our results to those of Sampson (2015) is complicated by the use of different calibration strategies and welfare metrics (we study transition paths, his model does not feature transitions). Therefore, we recalibrate our model and compute new counterfactuals to facilitate a comparison of the quantitative predictions of the two papers. Specifically, we shut down the GBM process and then recalibrate the remaining parameters of our model to match Sampson’s (2015) aggregate moments and his discount factor.¹⁷

After making these calibration adjustments, we study the experiment of Sampson (2015) by computing the equilibrium growth rates associated with the observed level of trade and autarky. By construction, in our model and in Sampson’s (2015) the steady state growth rate at observed trade levels is 1.56 percent. In our model, the autarky growth rate is 1.34 percent, while in Sampson’s (2015) it is 1.41 percent (see Table 2 of Sampson (2015)). There are also similarities in terms of welfare, as the welfare loss from autarky in our re-calibrated model is -3.23 percent, while it is -3.6 percent in Sampson (2015). Thus, when calibrating our model to mimic Sampson’s (2015) model, we find very similar steady state changes in the aggregate growth rate and in the welfare gains from trade.

Furthermore, just shutting down GBM and recalibrating our model to our aggregate moments yields similarly smaller changes in growth and welfare compared to our baseline model. The growth rate drops from 0.66 percent to 0.51 percent and the welfare loss from autarky is -4.65 percent. Thus, calibrating the model so that it generates realistic firm dynamics has a quantitatively significant effect on the response of growth and welfare to changes in trade costs. We now turn to a more detailed exploration of the relationship between firm dynamics and the welfare gains from trade.

¹⁷We set μ and v^2 to zero and fix σ to equal 3.17 (our calibrated value). We then choose the remaining parameter values to target our exporter moments, his aggregate growth rate of 1.56 percent, import/GDP ratio of 0.081, and set our discount factor equal to 0.04 (See Table 1 and 2 of Sampson (2015)).

7.4. The Role of Firm Dynamics and Adoption Costs

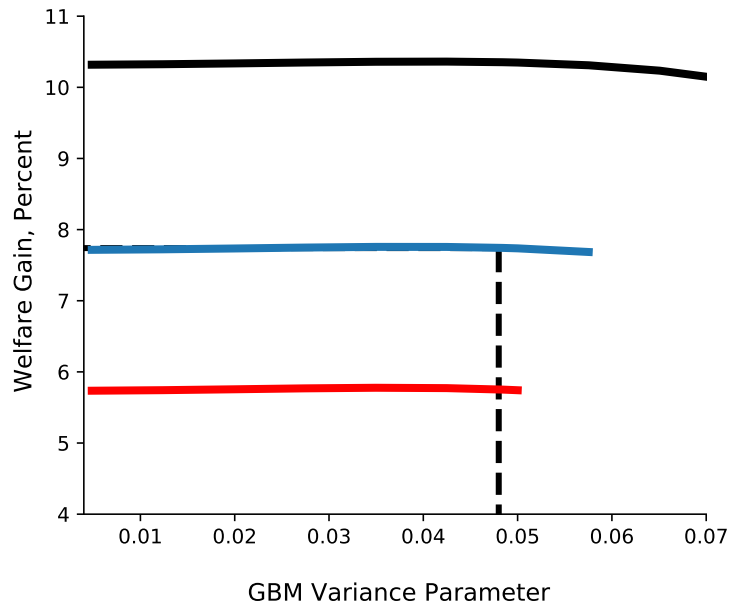
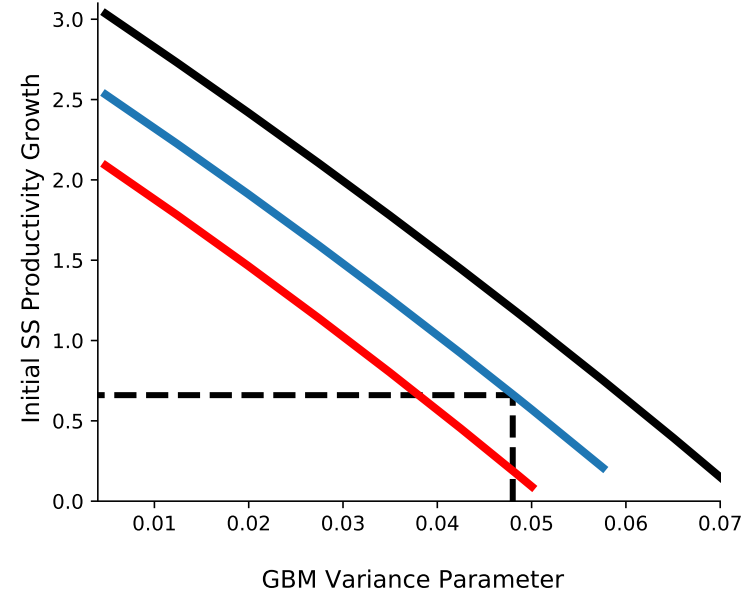
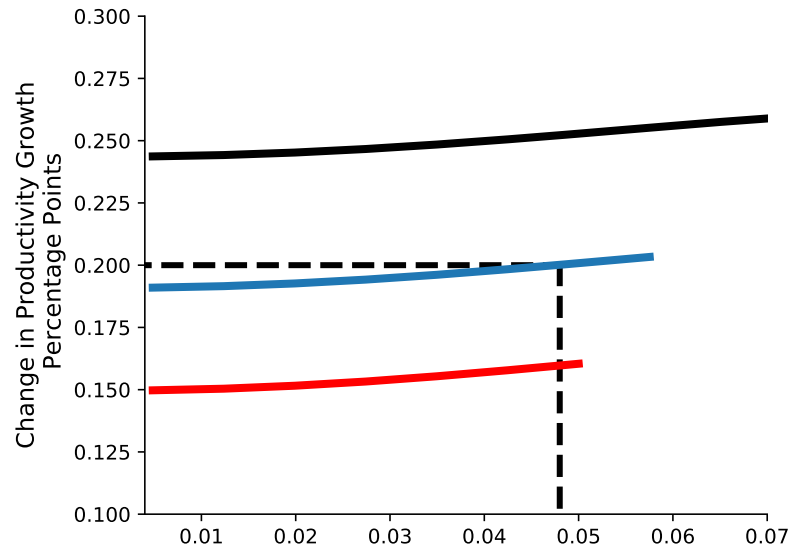
This section discusses some of the more unique parameters of our model and how their values shape the growth effects and welfare gains from trade. Relative to our theoretical results, the open question is how the firm productivity and exit shocks (and hence firm dynamics data) affect trade, consumption, and growth. Moreover, one of the parameters χ (the relative cost of adoption) is hard to interpret and discipline from firm-level data, thus it was chosen primarily to target an aggregate growth rate. Below we further analyze how these parameter values shape the key results of the paper.

Focusing on the importance of the GBM variance parameter v^2 , the left panel in Figure 6 plots the change in growth rates in response to a ten percent reduction in trade costs on the vertical axis and the variance of the GBM process on the horizontal axis. Each line is for a different value of the adoption cost parameter χ — the large χ value is ten percent larger than the baseline calibrated χ value and the small χ value is ten percent smaller than the baseline calibrated value. All other parameter values are fixed, i.e., we do not re-calibrate the model when changing these parameter values. The middle blue line is drawn for the baseline value of χ and the intersecting dashed line indicates the baseline variance. Consistent with our numerical results above, the left panel of Figure 6 reports that the aggregate growth rate changes by about 0.20 percentage points across steady states.

The first thing to observe from Figure 6 is that the percentage point change in productivity is nearly constant across different values of the variance parameter. In other words, the variance v^2 **does not** affect the response of growth to a change in trade costs.

The parameter which does influence the change in growth is the adoption cost parameter, χ . The three different lines on the left panel in Figure 6 illustrate this point. A small value of χ (top black line) corresponds to small costs of adoption. When adoption costs are small, growth is more responsive to changes in trade costs. In contrast, a large value of χ (bottom red line) corresponds to large adoption costs and a smaller response of growth to trade costs. The closed form equations available in the non-GBM version of the model deliver some insight. Equation 34 shows that the **change** in the growth rate for a given change in trade costs is larger when adoption costs are smaller.¹⁸

¹⁸In all the cases in this figure, the change in trade costs is held constant, but the change in the amount of trade is slightly different as the χ parameter shows up in the trade share. In total, the effect that χ has on the change in growth is $\left(\frac{\rho}{\chi} - \rho\right)^{\frac{\theta}{\sigma-1}}$.



- Calibrated $\chi = 0.17$
- Large $\chi = 0.187$
- Small $\chi = 0.153$
- - - Calibrated Values

Figure 6: GBM variance, Adoption Costs, and Economic Growth

Even though the elasticity of growth to trade costs is not sensitive to the value of v^2 holding adoption costs constant, the value of v^2 —and, thus, the firm dynamics data—strongly influences the calibrated value of the adoption cost. The right panel in Figure 6 illustrates this point by tracing out how the growth rate in the initial steady state varies with v^2 . For a given χ value, there is a near linear decrease in the steady state growth rate as the variance increases. Across χ values, the slope is essentially the same, but the intercept shifts, with smaller χ values leading to higher growth rates. This is intuitive—lower adoption costs lead to more adoption and faster economic growth.

The implication of these observations is that data on firm dynamics influences the inferred adoption cost and, thus, the elasticity of growth to trade costs. For example, holding fixed our target of an aggregate growth rate of 0.66 percent, if the transition matrix of relative size (Table 3) had pushed for us to find a smaller value for v^2 , then the right panel of Figure 6 shows this would have led us to calibrate a larger value for χ . Combining this observation with the left panel of Figure 6, our calibration strategy would have then led to a smaller increase in the growth rate for the same decrease in trade costs.

The lower panel of Figure 6 shows that the welfare gains from trade (comparing BGPs) are nearly constant across values of v^2 , but sensitive to the value of χ , just like the elasticity of growth to trade costs. Thus, the value of χ is crucial for determining both the change in growth and the welfare gains from trade. Even though v^2 does not much affect the welfare gains from trade when holding all other parameters constant, different values of v^2 (which are associated with different firm dynamics moments) affect the calibration of χ .

Our discussion above, which compares our gains from trade to those in Sampson (2015), strongly suggests this point as well. When the GBM process is shut down and the model is re-calibrated, the gains from trade are still larger than what a static model would imply, but they are far more modest and in line with what Sampson (2015) finds. It is in this sense that not just firm heterogeneity, but firm dynamics, matter for the welfare gains from trade in our model.

The other parameter introduced in the quantitative model is δ , which controls the rate of entry and death. Figure 7 is analogous to Figure 6, except that the horizontal axis varies δ . The top left panel shows that, holding all else equal, lower values of δ lead to less responsive changes in the growth. This is completely different from the analysis of the GBM variance presented in Figure 6.

Similarly to the GBM variance case, the δ parameter interacts with the adoption cost parameter to affect the calibrated value of χ . The right panel in Figure 6 illustrates this point by tracing out how the growth rate in the initial steady state varies with δ . For a given χ value, the steady state growth rate increases with δ ; across χ values, smaller χ values (lower adoption costs) lead to higher growth rates. Figure 6 shows that larger δ values (i.e., more entry observed in the data) would induce the calibration to infer larger χ values. But because these two parameters have opposite effects on economic growth, the change in parameter values generates offsetting effects and leaves the model's elasticity of growth to trade costs relatively unchanged.

The welfare gains from trade display a similar pattern. The bottom panel of Figure 7 shows that the welfare gains from trade increase with the value of δ , holding all else fixed. Again, however, larger values of δ generate larger calibrated values of χ , which offset to keep the welfare gains from trade largely un-

changed. Re-calibrating the model holding fixed different values for δ verifies this observation—welfare only increases slightly as δ increases.

In summary, Figure 6 shows that if the empirical transition matrix of relative size (Table 3) had pushed for us to calibrate a smaller value for v^2 , it would have resulted in a larger value for χ and, thus, a smaller response of growth to trade costs and smaller welfare gains from trade. In contrast, Figure 7 shows that the responsiveness of growth to trade costs and the welfare gains from trade would largely have been the same if the data featured more or less entry. In both contexts, an important lesson is that the adoption cost parameter, χ , has a large quantitative impact on the welfare gains from trade.

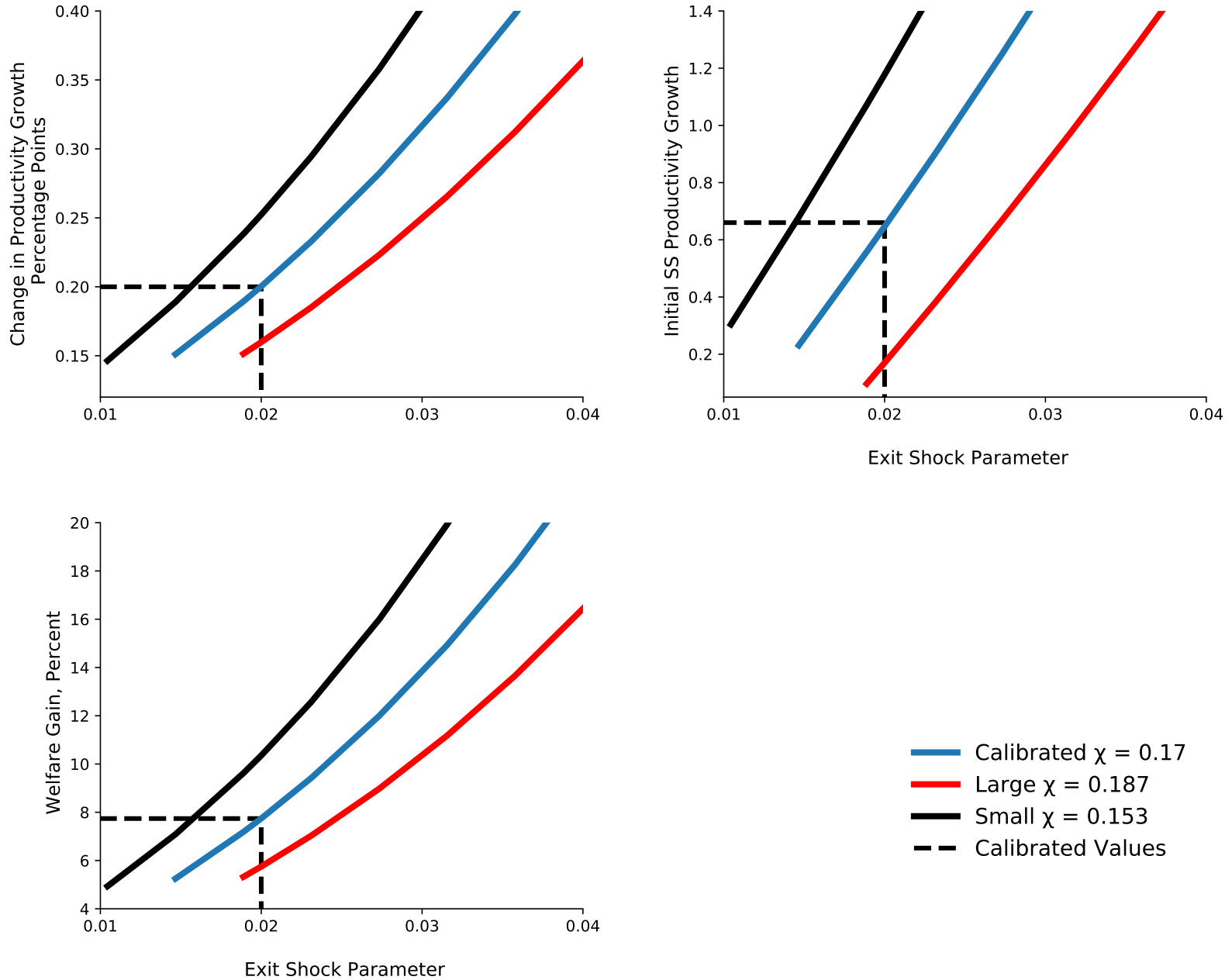


Figure 7: Exit Shock, Adoption Costs, and Economic Growth

8. Conclusion

This paper contributes a dynamic model of growth and international trade, driven by domestic technology diffusion. Firms choose to upgrade their productivity through technology adoption to remain competitive and profitable. Highly productive firms benefit from a decline in trade costs, as they are the exporters who can take advantage of increased sales abroad. Low productivity firms only sell domestically and are hurt by the increased competition from foreign firms. This paper studies a mechanism that links this increase in import competition to firm-level and aggregate growth.

We first study the BGP equilibrium of a simplified model which can be solved in closed form. The incentives to adopt, and thus the growth rate, are summarized by the ratio of profits between the average and the marginal adopting firm. In equilibrium, lower trade costs increase this profit spread, which leads lower productivity firms to upgrade their technology more frequently and increases aggregate growth. When comparing across steady states, the change in welfare from lower trade costs is a weighted sum of this increase in economic growth and a change in the initial level of consumption. The change in initial consumption is a sum of three components: a change in the home trade share, a change in the measure of domestic varieties produced, and a change in the amount of labor allocated to the production of goods. We prove that opening to trade reduces the initial level of consumption, which dampens the gains from faster economic growth.

We then study the quantitative version of the model, which adds to the simplified model productivity and exit shocks. We calibrate the model to match simulated moments to U.S. aggregate and firm-level empirical moments. Our main exercise is to study the transition dynamics of the economy in response to an unanticipated and permanent 10 percent reduction in trade costs. On the transition path imports increase rapidly while the number of domestically produced varieties falls gradually. Due to consumption smoothing desires, much less labor is allocated to entry on the transition, which frees up workers to be used in the production of existing varieties and the adoption of better technologies. This leads consumption to overshoot and productivity to grow steadily on the way to their new BGP values. Reducing trade costs by 10 percent increases welfare by 8.4 percent in consumption equivalent units. Welfare gains are larger than the steady-state to steady-state analysis would suggest due to consumption overshooting, even though the gains from higher growth are discounted further in the future. This welfare gain is an order of magnitude larger than the static ACR measurement delivers. We show how the size of the welfare gains are related to the moments used to calibrate the model, with a particular focus on how important the firm dynamics data, and not just data on firm heterogeneity in the cross-section, are in determining the welfare gains from trade.

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Appendix (For Online Publication)

A. Environment and Optimization Problems

To demonstrate the extensibility of the model, in this appendix we derive conditions for a version of the model in which there is CRRA power utility, a weakly positive probability of death δ , costs of adoption and entry can be a convex combination of labor and goods, and exogenous shocks to TFP that follow a geometric Brownian motion with drift. For expositional clarity, the body of the paper studies the special case of log utility, no exogenous productivity shocks, costs of adoption and entry in labor only, and studies the limiting economy as the death rate, and thus the BGP equilibrium entry rate, is zero.

A.1. Consumer Preferences and Labor Supply

All countries are symmetric. In each country there exists a representative consumer of measure \bar{L} . The utility of the consumer is given by a constant relative risk aversion (CRRA) function in final goods consumption (C), given an inelastic supply of labor (\bar{L}). The coefficient of relative risk aversion is $\gamma \geq 0$, and the time discount rate is ρ .

Final goods are produced through CES aggregation of an endogenous number of intermediate varieties, including those produced domestically and those imported from abroad. There is an endogenous measure $\Omega(t)$ of firms operating in each country. The flow of intermediate firms adopting a new technology is $\Omega(t)S(t)$ and the flow of firms entering the market and creating a new variety is $\Omega(t)E(t)$. The consumer purchases the final consumption good, invests in technology adoption with a real cost of $X(t)$ per upgrading intermediate firm, and invests in firm entry with a real cost of $X(t)/\chi$.¹⁹ Consumers income consists of labor earnings paid at wage $W(t)$ and profits from their ownership of the domestic firms. Aggregate profits from selling domestically are $\bar{\Pi}_d(t)$ and aggregate profits from exporting to the $N - 1$ foreign countries is $(N - 1)\bar{\Pi}_x(t)$. Thus, welfare at time \tilde{t} is

$$\begin{aligned} \bar{U}(\tilde{t}) &= \int_{\tilde{t}}^{\infty} U(C(t))e^{-\rho(t-\tilde{t})} dt \\ \text{s.t. } C(t) + \Omega(t)X(t)(S(t) + E(t)/\chi) &= \frac{W(t)}{P(t)}\bar{L} + \bar{\Pi}_d(t) + (N - 1)\bar{\Pi}_x(t). \end{aligned} \quad (\text{A.1})$$

With this, the period real profits as $\bar{\Pi}_i(t) = \bar{\Pi}_d(t) + (N - 1)\bar{\Pi}_x(t)$ and real investment as $\bar{I}_i(t) = \Omega(t)X(t)(S(t) + E(t)/\chi)$.

A.2. The Static Firm Problem

Intermediate Goods Demand. There is a measure $\Omega(t)$ of intermediate firms in each country that are monopolistically competitive, and the final goods sector is perfectly competitive. The final goods sector takes prices as given and aggregates intermediate goods with a CES production function, with $\sigma > 1$ the elasticity of substitution between all available products.

¹⁹Firms are maximizing real profits, discounting using the interest rate determined by the consumer's marginal rate of substitution plus the death rate. Hence, the investment choice of the consumer and firm is aligned, and consumers will finance upgrades to their existing firms through equity financing. As consumers own a perfectly diversified portfolio of domestic firms, they are only diluting their own equity with this financing method.

The CDF of the productivity distribution at time t is $\Phi(Z, t)$, normalized such that $\Phi(\infty, t) = 1$ for all t . Therefore, the total measure of firms with productivity below Z at time t is $\Omega(t)\Phi(Z, t)$.

Drop the t subscript for clarity. The standard solutions follow from maximizing the following final goods production problem,

$$\max_{Q_d, Q_x} \left[\Omega \int_M^\infty Q_d(Z)^{(\sigma-1)/\sigma} d\Phi(Z) + (N-1)\Omega \int_{\hat{Z}}^\infty Q_x(Z)^{(\sigma-1)/\sigma} d\Phi(Z) \right]^{\sigma/(\sigma-1)} \quad (\text{A.2})$$

$$\text{s.t. } \Omega \int_M^\infty p_d(Z)Q_d(Z)d\Phi(Z) + (N-1)\Omega \int_{\hat{Z}}^\infty p_x(Z)Q_x(Z)d\Phi(Z) = Y. \quad (\text{A.3})$$

Defining a price index P , the demand for each intermediate product is,

$$Q_d(Z) = \left(\frac{p_d(Z)}{P} \right)^{-\sigma} \frac{Y}{P}, \quad Q_x(Z) = \left(\frac{p_x(Z)}{P} \right)^{-\sigma} \frac{Y}{P} \quad (\text{A.4})$$

$$P^{1-\sigma} = \Omega \left(\int_M^\infty p_d(Z)^{1-\sigma} d\Phi(Z) + (N-1) \int_{\hat{Z}}^\infty p_x(Z)^{1-\sigma} d\Phi(Z) \right). \quad (\text{A.5})$$

Static Profits. A monopolist operating domestically chooses each instant prices and labor demand to maximize profits, subject to the demand function given in equation (A.4),

$$P\Pi_d(Z) := \max_{p_d, \ell_d} \{ (p_d Z \ell_d - W \ell_d) \} \text{ s.t. equation (A.4)}. \quad (\text{A.6})$$

Where $\Pi_d(Z)$ is the real profits from domestic production.

Firms face a fixed cost of exporting, $\kappa \geq 0$. To export, a firm must hire labor in the foreign country to gain access to foreign consumers. This fixed cost is paid in market wages, and is proportional to the number of consumers accessed. Additionally, exports are subject to a variable iceberg trade cost, $d \geq 1$, so that firm profits from exporting to a single country (i.e., export profits per market) are

$$P\Pi_x(Z) := \max_{p_x, \ell_x} \{ (p_x \frac{Z}{d} \ell_x - W \ell_x - \bar{L}\kappa W) \} \text{ s.t. equation (A.4)}. \quad (\text{A.7})$$

Optimal firm policies consist of $p_d(Z)$, $p_x(Z)$, $\ell_d(Z)$, and $\ell_x(Z)$ and determine $\Pi_d(Z)$ and $\Pi_x(Z)$. As is standard, it is optimal for firms to charge a constant markup over marginal cost, $\bar{\sigma} := (\sigma - 1)/\sigma$.

$$p_d(Z) = \bar{\sigma} \frac{W}{Z}, \quad (\text{A.8})$$

$$p_x(Z) = \bar{\sigma} d \frac{W}{Z}, \quad (\text{A.9})$$

$$\ell_d(Z) = \frac{Q_d(Z)}{Z}, \quad \ell_x(Z) = d \frac{Q_x(Z)}{Z}. \quad (\text{A.10})$$

To derive firm profits, take equation (A.6) and divide by P to get $\Pi_d(Z) = \frac{p_d(Z)}{P}Q_d(Z) - \frac{Q_d(Z)}{Z}\frac{W}{P}$. Substitute from equation (A.8), as $W = \frac{Zp_d(Z)}{\sigma}$, to yield $\Pi_d(Z) = \frac{1}{\sigma}\frac{p_d(Z)}{P}Q_d(Z)$. Finally, use $Q_d(Z)$ from equation (A.4) to show

$$\Pi_d(Z) = \frac{1}{\sigma} \left(\frac{p_d(Z)}{P} \right)^{1-\sigma} \frac{Y}{P}. \quad (\text{A.11})$$

Using similar techniques, export profits per market are

$$\Pi_x(Z) = \max \left\{ 0, \frac{1}{\sigma} \left(\frac{p_x(Z)}{P} \right)^{1-\sigma} \frac{Y}{P} - \bar{L}\kappa\frac{W}{P} \right\}. \quad (\text{A.12})$$

Since there is a fixed cost to export, only firms with sufficiently high productivity will find it profitable to export. Solving equation (A.12) for the productivity that earns zero profits gives the export productivity threshold. That is, a firm will export iff $Z \geq \hat{Z}$, where \hat{Z} satisfies

$$\left(\frac{p_x(\hat{Z})}{P} \right)^{1-\sigma} = \sigma \bar{L}\kappa\frac{W}{Y}, \quad (\text{A.13})$$

$$\hat{Z} = \bar{\sigma}d(\sigma\bar{L}\kappa)^{\frac{1}{\sigma-1}} \left(\frac{W}{P} \right) \left(\frac{W}{Y} \right)^{\frac{1}{\sigma-1}}. \quad (\text{A.14})$$

Let aggregate profits from domestic production be $\bar{\Pi}_d$ and aggregate export profits per market be $\bar{\Pi}_x$.

$$\bar{\Pi}_d := \Omega \int_M^\infty \Pi_d(Z) d\Phi(Z). \quad (\text{A.15})$$

$$\bar{\Pi}_x := \Omega \int_{\hat{Z}}^\infty \Pi_x(Z) d\Phi(Z). \quad (\text{A.16})$$

The trade share for a particular market, λ , is

$$\lambda = \Omega \int_{\hat{Z}}^\infty \left(\frac{p_x(Z)}{P} \right)^{1-\sigma} d\Phi(Z). \quad (\text{A.17})$$

A.3. Firms Dynamic Problem

Stochastic Process for Productivity Assume that operating firms have (potentially) stochastic productivity following the stochastic differential equation for geometric Brownian motion (GBM),

$$dZ_t/Z_t = (\mu + v^2/2)dt + v dW_t, \quad (\text{A.18})$$

where $\mu \geq 0$ is related to the drift of the productivity process, $v \geq 0$ is the volatility, and W_t is standard Brownian motion.

At any instant in time, a firm will exit if hit by a death shock, which follows a Poisson process with arrival rate $\delta \geq 0$. Thus, all firms have the same probability of exiting and the probability of exiting is independent of time.

The case of $\mu = v = \delta = 0$, is the baseline model studied in the body of the paper.

Firm's Problem. Define a firm's total real profits as

$$\Pi(Z, t) := \Pi_d(Z, t) + (N - 1)\Pi_x(Z, t), \quad (\text{A.19})$$

where from equation (A.12), $\Pi_x(Z, t) = 0$ for firms who do not export. Let $V(Z, t)$ be the value of a firm with productivity Z at time t . The firm's effective discount rate, $r(t)$, is the sum of the consumer's intertemporal marginal rate of substitution and the firm death rate δ .

Given the standard Bellman equation for the GBM of equation (A.18) and optimal static policies,

$$r(t)V(Z, t) = \Pi(Z, t) + \left(\mu + \frac{v^2}{2} \right) Z \frac{\partial V(Z, t)}{\partial Z} + \frac{v^2}{2} Z^2 \frac{\partial^2 V(Z, t)}{\partial Z^2} + \frac{\partial V(Z, t)}{\partial t}, \quad (\text{A.20})$$

$$V(M(t), t) = \int_{M(t)}^{\infty} V(Z, t) d\Phi(Z, t) - X(t), \quad (\text{A.21})$$

$$\frac{\partial V(M(t), t)}{\partial Z} = 0. \quad (\text{A.22})$$

Equation (A.20) is the bellman equation for a firm continuing to produce with its existing technology. It receives instantaneous profits and the value of a firm where productivity may change over time. Equation (A.21) is the value matching condition, which states that the marginal adopter must be indifferent between adopting and not adopting. $M(t)$ is the endogenous productivity threshold that defines the marginal firm. Equation (A.22) is the smooth-pasting condition.²⁰

A.4. Adoption Costs

In order to upgrade its technology a firm must buy some goods and hire some labor. These costs are in proportion to the population size, reflecting market access costs as in Arkolakis (2010) or Arkolakis (2015). Whether costs are in terms of goods or labor is a common issue in growth models, with many papers specifying goods costs and many specifying labor costs.²¹ The growth literature often uses labor costs, since new ideas cannot simply be purchased, and instead must be the result of innovators doing R&D. In the technology diffusion context, in which low productivity firms are adopting already existing technologies, an adoption cost that has some nontrivial component denominated in goods is more reasonable than in the innovation case. Since there is a paucity of empirical evidence to guide our decision in the adoption context, we model the adoption cost in a way that nests the costs being in labor exclusively, in goods exclusively, or as a mix of labor and goods. Although we solve the model for this general case, our baseline is that costs are purely labor denominated.

The amount of labor needed is parameterized by ζ , which is constant. The labor component of the

²⁰Using the standard relationship between free boundary and optimal stopping time problems, the firm's problem could equivalently be written as the firm choosing a stopping time at which it would upgrade. If $v > 0$ this stopping time is a random variable, otherwise it is deterministic.

²¹As discussed in Bollard, Klenow, and Li (2014), for examples see Hopenhayn (1992), Romer (1994), Foster, Haltiwanger, and Syverson (2008) and David (2017) for papers using output costs and Lucas (1978), Grossman and Helpman (1991), Melitz (2003), Klette and Kortum (2004), Luttmer (2007), and Acemoglu, Akcigit, Bloom, and Kerr (2018) for papers that use labor costs.

adoption cost, however, increases in equilibrium in proportion to the real wage, ensuring the cost does not become increasingly small as the economy grows. The amount of goods that needs to be purchased to adopt a technology increases with the scale of the economy—otherwise the relative costs of goods would become infinitesimal in the long-run. Θ parameterizes the amount of goods required to adopt a technology, with the cost, $M(t)\Theta$, growing as the economy grows. Essentially, ζ controls the overall cost of technology adoption, while $\eta \in [0, 1]$ controls how much of the costs are to hire labor versus buy goods. We model the mix of labor and goods as additive in order to permit a balanced growth path equilibrium.

The real cost of adopting a technology is

$$X(t) := \bar{L}\zeta \left[(1 - \eta) \frac{W(t)}{P(t)} + \eta M(t)\Theta \right]. \quad (\text{A.23})$$

A.5. Entry and Exit

There is a large pool of non-active firms that may enter the economy by paying an entry cost—equity financed by the representative consumer—to gain a draw of an initial productivity from the same distribution from which adopters draw. Since entry and adoption deliver similar gains, we model the cost of entry as a multiple of the adoption cost for incumbents, $X(t)/\chi$, where $0 < \chi < 1$. Hence, χ is the ratio of adoption to entry costs and $\chi \in (0, 1)$ reflects that incumbents have a lower cost of upgrading to a better technology than entrants have to start producing a new variety from scratch.

Thus, the free entry condition that equates the cost of entry to the value of entry is

$$X(t)/\chi = \int_{M(t)}^{\infty} V(Z, t) d\Phi(Z, t). \quad (\text{A.24})$$

If a flow $E(t)$ of firms enter, and a flow δ exit, then the differential equation for the number of firms is $\Omega'(t) = (E(t) - \delta)\Omega(t)$. Since we study a stationary equilibrium, on a BGP Ω will be constant and determined by free entry, and $E(t) = \delta$ for all t .

The costs of entry will determine the number of varieties in equilibrium, and for $\delta > 0$, there is gross entry on a balanced growth path. This model of entry and exit is very different from those in Luttmer (2007) and Sampson (2015). Here, exit is exogenous, whereas a key model mechanism studied in those papers is the endogenous selection of exit induced by fixed costs of operations. We have not modeled a fixed cost to domestic production in order to isolate our distinct mechanism. We have introduced entry and exit in our model to generate an endogenous number of varieties so that we can analyze the effect of our mechanism on welfare, taking into account changes in incumbent technology adoption behavior and changes on the extensive margin in the number of varieties produced. Given the exogenous death shock, the effect of $\delta > 0$ is only to change the firm's discount rate. For the most part, the economics are

qualitatively identical to the $\delta = 0$ case and there is no discontinuity in the limit as $\delta \rightarrow 0$.²²

A.6. Market Clearing and the Resource Constraint

Total labor demand is the sum of labor used for domestic production, export production, the fixed cost of exporting, technology adoption, and entry. Equating labor supply and demand yields

$$\bar{L} = \Omega \int_M^\infty \ell_d(Z) d\Phi(Z) + (N-1)\Omega \int_{\hat{Z}}^\infty \ell_x(Z) d\Phi(Z) + (N-1)\Omega(1 - \Phi(\hat{Z}))\kappa\bar{L} + \bar{L}(1 - \eta)\zeta\Omega(S + \delta/\chi). \quad (\text{A.25})$$

The quantity of final goods must equal the sum of consumption and investment in technology adoption. Thus, the resource constraint is

$$\frac{Y}{P} = C + \Omega\bar{L}\eta\zeta M\Theta(S + \delta/\chi). \quad (\text{A.26})$$

B. Deriving the Productivity Distribution Law of Motion and Flow of Adopters

This section describes the details of deriving the law of motion for the productivity distribution.

The Productivity Distribution Law of Motion. At points of continuity of $M(t)$, there exists a flow of adopters during each infinitesimal time period.²³ The Kolmogorov Forward Equation (KFE) for $Z > M(t)$, describes the evolution of the CDF. The KFE contains standard components accounting for the drift and Brownian motion of the exogenous GBM process detailed in equation (A.18). Furthermore, it includes the flow of adopters (source) times the density they draw from (redistribution CDF). Determining the flow of adopters is the fact that the adoption boundary $M(t)$ sweeps across the density from below at rate $M'(t)$. As adoption boundary acts as an absorbing barrier, and as it sweeps from below it collects $\phi(M(t), t)$ amount firms. The cdf that the flow of adopters is redistributed into is determined by two features of the environment. In the stationary equilibrium, $M(t)$ is the minimum of support of $\Phi(Z, t)$, so the adopters are redistributed across the entire support of $\phi(Z, t)$.²⁴ Since adopters draw directly from the productivity density, they are redistributed throughout the distribution in proportion to the density. Thus, the flow of adopters $S(t)$ multiplies the cdf, $\Phi(Z, t)$.

Since there is a constant death rate, $\delta \geq 0$, a normalized measure of $\delta\Phi(Z, t)$ exit with productivity

²²For special cases where $\delta = 0$ and the initial Ω is large relative to that which would be achieved on a BGP from a relatively small initial Ω , the free entry condition could hold as an inequality (i.e. $X(t)/\chi > \int_{M(t)}^\infty V(Z, t)d\Phi(Z)$). In that case, the lack of exit would prevent Ω from decreasing so that the free entry condition held with equality. In our baseline with $\delta = 0$, we ignore this special one-sided case, as it is economically uninteresting; it is unreasonable for the number of varieties to require major decreases in a growing economy. Also, as there is no discontinuity in the solution when taking $\delta \rightarrow 0$, we will consider our baseline economy a small δ approximation.

²³The evolution of $M(t)$, and hence the distribution itself, can only be discontinuous at time 0 or in response to unanticipated shocks. Since this paper analyzes balanced growth path equilibria, we omit derivation of the law of motion for the distribution with discontinuous $M(t)$.

²⁴Unlike in discrete time, the distinction between drawing from the unconditional distribution or the conditional distribution of non-adopting incumbents is irrelevant. The number of adopting firms is a flow, and hence measure 0, which leads to identical conditional vs. unconditional distributions.

below Z , but as new entrants of normalized flow $E(t)$ adopt a productivity through the same process as incumbents, they are added to the flow entering with measure below Z .²⁵

$$\begin{aligned} \frac{\partial \Phi(Z, t)}{\partial t} = & \underbrace{\Phi(Z, t) \left(\underbrace{S(t) + E(t)}_{\text{Adopt or Enter}} \right)}_{\text{Distributed below } Z} - \underbrace{S(t)}_{\text{Adopt at } M(t)} - \underbrace{\delta \Phi(Z, t)}_{\text{Death}} \\ & - \underbrace{\left(\mu - \frac{v^2}{2} \right) Z \frac{\partial \Phi(Z, t)}{\partial Z}}_{\text{Deterministic Drift}} + \underbrace{\frac{v^2}{2} Z^2 \frac{\partial^2 \Phi(Z, t)}{\partial Z^2}}_{\text{Brownian Motion}}. \end{aligned} \quad (\text{B.3})$$

In the baseline $\delta = \mu = v = 0$ case, this simplifies to equation (17).²⁶

Normalized Productivity Distribution. Define the change of variables $z := Z/M(t)$, $g(t) := M'(t)/M(t)$, and

$$\Phi(Z, t) =: F(Z/M(t), t). \quad (\text{B.4})$$

Differentiating,

$$\phi(Z, t) = \frac{1}{M(t)} f(Z/M(t), t). \quad (\text{B.5})$$

This normalization generates an adoption threshold that is stationary at $z = M(t)/M(t) = 1$ for all t .

Law of Motion for the Normalized Distribution. To characterize the normalized KFE, first differentiate the cdf with respect to t , yielding

$$\frac{\partial \Phi(Z, t)}{\partial t} = \frac{\partial F(Z/M(t), t)}{\partial t} - \frac{Z}{M(t)} \frac{M'(t)}{M(t)} \frac{\partial F(Z/M(t), t)}{\partial z}. \quad (\text{B.6})$$

²⁵To derive from the more common KFE written in PDFs: use the standard KFE for the pdf $\phi(Z, t)$, integrate this with respect to Z to convert into cdf $\Phi(Z, t)$, use the fundamental theorem of calculus on all terms, then the chain rule on the last term, and rearrange,

$$\frac{\partial \phi(Z, t)}{\partial t} = -\frac{\partial}{\partial Z} \left[\left(\mu + \frac{v^2}{2} \right) Z \phi(Z, t) \right] + \frac{\partial}{\partial Z^2} \left[\frac{v^2}{2} Z^2 \phi(Z, t) \right] + \dots \quad (\text{B.1})$$

$$\frac{\partial \Phi(Z, t)}{\partial t} = \left(\frac{v^2}{2} - \mu \right) Z \frac{\partial \Phi(Z, t)}{\partial Z} + \frac{v^2}{2} Z^2 \frac{\partial^2 \Phi(Z, t)}{\partial Z^2} + \dots \quad (\text{B.2})$$

²⁶While *conditional on an optimal policy* the law of motion in equation (17) and the more general equation (16) are identical to that in Luttmer (2007), this is a mechanical result of any distribution evolution with resetting of agents through direct sampling of the distribution. Economically, the forces which determine the endogenous policy are completely different, as we concentrate on the choices of incumbents rather than entrants/exit. Some of the key differences are evident in the connection to search models as discussed in Section 5.

Differentiating the cdf with respect to Z yields

$$\frac{\partial \Phi(Z, t)}{\partial Z} = \frac{1}{M(t)} \frac{\partial F(Z/M(t), t)}{\partial z}, \quad (\text{B.7})$$

$$\frac{\partial^2 \Phi(Z, t)}{\partial Z^2} = \frac{1}{M(t)^2} \frac{\partial^2 F(Z/M(t), t)}{\partial z^2}. \quad (\text{B.8})$$

Given that $z := \frac{Z}{M(t)}$ and $g(t) := M'(t)/M(t)$, combining equations (B.3), (B.6), (B.7), and (B.8) provides the KFE in cdfs of the normalized distribution:

$$\frac{\partial F(z, t)}{\partial t} = (S(t) + E(t) - \delta) F(z, t) + (g(t) - \mu + v^2/2) z \frac{\partial F(z, t)}{\partial z} + \frac{v^2}{2} z^2 \frac{\partial^2 F(z, t)}{\partial z^2} - S(t). \quad (\text{B.9})$$

The interpretation of this KFE is that while non-adopting incumbent firms are on average not moving in absolute terms, they are moving backwards at rate $g(t)$ relative to $M(t)$ (adjusted for the growth rate of the stochastic process). As the minimum of support is $z = M(t)/M(t) = 1$ for all t , a necessary condition is that $F(1, t) = 0$ for all t , and therefore $\frac{\partial F(1, t)}{\partial t} = 0$. Thus, evaluating equation (B.9) at $z = 1$ gives an expression for $S(t)$,²⁷

$$S(t) = \left(g(t) - \mu + \frac{v^2}{2} \right) \frac{\partial F(1, t)}{\partial z} + \frac{v^2}{2} \frac{\partial^2 F(1, t)}{\partial z^2}. \quad (\text{B.10})$$

This expression includes adopters caught by the boundary moving relative to their drift, as well as the flux from the GBM pushing some of them over the endogenously determined threshold. If $\mu = v = \delta = 0$, then a truncation at $M(t)$ solves equation (B.9) for any t and for any initial condition, as in Perla and Tonetti (2014).

$$\phi(Z, t) = \frac{\phi(Z, 0)}{1 - \Phi(M(t), 0)}, \quad Z \geq M(t). \quad (\text{B.11})$$

The only non-degenerate stationary $F(z)$ consistent with equation (B.11) is given by equation (B.13)—the same form as that with $v > 0$.

The Stationary Normalized Productivity Distribution. From equation (B.9), the stationary KFE is

$$0 = SF(z) + \left(g - \mu + \frac{v^2}{2} \right) z F'(z) + \frac{v^2}{2} z^2 F''(z) - S. \quad (\text{B.12})$$

subject to $F(1) = 0$ and $F(\infty) = 1$.

Moreover, for any strictly positive $v > 0$, the KFE will asymptotically generate a stationary Pareto distribution for some tail parameter from any initial condition. While many $\theta > 1$ could solve this differential equation, the particular θ tail parameter is determined by initial conditions and the evolution of $M(t)$. As in Luttmer (2007) and Gabaix (2009), the geometric random shocks leads to an endogenously determined

²⁷Equivalently, the flow of adopters can be derived as the net flow of the probability current through the adoption threshold.

power-law distribution.

$$F(z) = 1 - z^{-\theta}, \quad z \geq 1. \quad (\text{B.13})$$

From equations B.12 and B.13, evaluating at $z = 1$ for a given g and θ ,

$$S = \theta \left(g - \mu - \theta \frac{v^2}{2} \right) \quad (\text{B.14})$$

Therefore, given an equilibrium g and θ , the CDF in equation (B.13) and S from equation (B.14) characterize the stationary distribution. The relationship between g and θ is determined by the firms' decisions given S and $F(z)$. It is independent of δ on a BGP since the exit rate is constant and uniform across firms (in contrast to Luttmer (2007), where selection into exit is generated by fixed costs of operations and is not independent of firm productivity).

The Stationary Distribution with no GBM. For $v = 0$, the lack of random shocks means that the stationary distribution will not necessarily become a Pareto distribution from arbitrary initial conditions. However, if the initial distribution is Pareto, the normalized distribution will be constant. If the initial distribution is a power-law, then the stationary distribution is asymptotically Pareto.

Since a Pareto distribution is attained for any $v > 0$, we consider our baseline ($v = 0, \mu = 0$) case with an initial Pareto distribution as a small noise limit of the full model with GBM from some arbitrary initial condition. Beyond endogenously determining the tail index and changing the expected time to execute the adoption option, exogenous productivity volatility of incumbent firms has qualitatively little impact on the model. For the baseline case, from equation (B.14), the flow of adopters is $S = \theta g$.

C. Normalized Static Equilibrium Conditions

To aid in computing a balanced growth path equilibrium, in this section we transform the problem and derive normalized static equilibrium conditions.

Expectations using the Normalized Distribution. If an integral of the following form exists, for some unary function $\Psi(\cdot)$, substitute for $f(z)$ from B.5, then do a change of variables of $z = \frac{Z}{M}$ to obtain a useful transformation of the integral.

$$\int_M^\infty \Psi\left(\frac{Z}{M}\right) \phi(Z) dZ = \int_M^\infty \Psi\left(\frac{Z}{M}\right) f\left(\frac{Z}{M}\right) \frac{1}{M} dZ = \int_{M/M}^\infty \Psi(z) f(z) dz. \quad (\text{C.1})$$

The key to this transformation is that moving from ϕ to f introduces a $1/M$ term. Thus, abusing notation by using an expectation of the normalized variable,

$$\int_M^\infty \Psi\left(\frac{Z}{M}\right) \phi(Z) dZ = \mathbb{E}[\Psi(z)]. \quad (\text{C.2})$$

C.1. Normalizing the Static Equilibrium

Define the following normalized, real, per-capita values:

$$\hat{z} := \frac{\hat{Z}}{\bar{M}} \quad (\text{C.3})$$

$$y := \frac{Y}{\bar{LMP}} \quad (\text{C.4})$$

$$c := \frac{C}{\bar{LM}} \quad (\text{C.5})$$

$$q_d(Z) := \frac{Q_d(Z)}{\bar{LM}} \quad (\text{C.6})$$

$$x := \frac{X}{\bar{LM}w} \quad (\text{C.7})$$

$$w := \frac{W}{MP} \quad (\text{C.8})$$

$$\pi_d(Z) := \frac{\Pi_d(Z)}{\bar{LM}w}. \quad (\text{C.9})$$

In order to simplify algebra and notation, we normalize profits and adoption costs relative to real, normalized wages. This, for example, means the normalized cost of adoption $x = \zeta$.

Combining the normalized variables with equations (A.8) and (A.9) provides the real prices in terms of real, normalized wages.

$$\frac{p_d(Z)}{P} = \bar{\sigma} \frac{w}{Z/M}, \quad (\text{C.10})$$

$$\frac{p_x(Z)}{P} = \bar{\sigma} d \frac{w}{Z/M}. \quad (\text{C.11})$$

Substituting equations (C.10) and (C.11) into equation (A.4) and dividing by \bar{LM} yields normalized quantities,

$$q_d(Z) = \bar{\sigma}^{-\sigma} w^{-\sigma} y \left(\frac{Z}{M}\right)^{\sigma}, \quad (\text{C.12})$$

$$q_x(Z) = \bar{\sigma}^{-\sigma} w^{-\sigma} d^{-\sigma} y \left(\frac{Z}{M}\right)^{\sigma}. \quad (\text{C.13})$$

Divide equation (A.10) by \bar{L} , then substitute from equations (A.4) and (C.10). Finally, divide the top and bottom by M to obtain normalized demand for production labor

$$\ell_d(Z)/\bar{L} = \bar{\sigma}^{-\sigma} w^{-\sigma} y \left(\frac{Z}{M}\right)^{\sigma-1}, \quad (\text{C.14})$$

$$\ell_x(Z)/\bar{L} = \bar{\sigma}^{-\sigma} w^{-\sigma} y d^{1-\sigma} \left(\frac{Z}{M}\right)^{\sigma-1}. \quad (\text{C.15})$$

Divide equation (A.5) by $P^{1-\sigma}$ and then substitute from equation (C.10) for $p_d(Z)/P$ to obtain

$$1 = \Omega \bar{\sigma}^{1-\sigma} w^{1-\sigma} \left[\int_M^{\infty} \left(\frac{Z}{M}\right)^{\sigma-1} d\Phi(Z) + (N-1) \int_{\hat{Z}}^{\infty} d^{1-\sigma} \left(\frac{Z}{M}\right)^{\sigma-1} d\Phi(Z) \right]. \quad (\text{C.16})$$

Simplify equation (C.16) by defining \bar{z} , a measure of effective aggregate productivity. Then use equation (C.2) to give normalized real wages in terms of parameters, \hat{z} , and the productivity distribution

$$\bar{z} := \left[\Omega \left(\mathbb{E} [z^{\sigma-1}] + (N-1)(1-F(\hat{z}))d^{1-\sigma} \mathbb{E} [z^{\sigma-1} \mid z > \hat{z}] \right) \right]^{\frac{1}{\sigma-1}}, \quad (\text{C.17})$$

$$w^{\sigma-1} = \bar{\sigma}^{1-\sigma} \bar{z}^{\sigma-1}, \quad (\text{C.18})$$

$$w = \frac{1}{\bar{\sigma}} \bar{z}. \quad (\text{C.19})$$

Note that if $d = 1$ and $\hat{z} = 1$, then $w = \frac{1}{\bar{\sigma}} (\Omega N \mathbb{E} [z^{1-\sigma}])^{1/(\sigma-1)}$. Divide equations (A.11) and (A.12) by $\bar{L}Mw$ and substitute with equation (C.18) to obtain normalized profits,

$$\pi_d(Z) = \frac{1}{\sigma} \left(\frac{p(Z)}{P} \right)^{1-\sigma} \frac{y}{w} = \frac{1}{\bar{\sigma} \bar{z}^{\sigma-1}} \frac{y}{w} \left(\frac{Z}{M} \right)^{\sigma-1}, \quad (\text{C.20})$$

$$\pi_x(Z) = \frac{1}{\bar{\sigma} \bar{z}^{\sigma-1}} \frac{y}{w} d^{1-\sigma} \left(\frac{Z}{M} \right)^{\sigma-1} - \kappa. \quad (\text{C.21})$$

Divide equations (A.15) and (A.16) by $\bar{L}Mw$ and use equations (C.20) and (C.21) to find aggregate profits from domestic production and from exporting to one country,

$$\bar{\pi}_d = \Omega \frac{1}{\bar{\sigma} \bar{z}^{\sigma-1}} \frac{y}{w} \mathbb{E} [z^{\sigma-1}], \quad (\text{C.22})$$

$$\bar{\pi}_x = \Omega \left(\frac{1}{\bar{\sigma} \bar{z}^{\sigma-1}} \frac{y}{w} d^{1-\sigma} (1-F(\hat{z})) \mathbb{E} [z^{\sigma-1} \mid z > \hat{z}] - (1-F(\hat{z}))\kappa \right). \quad (\text{C.23})$$

Divide equation (A.25) by \bar{L} , and aggregate the total labor demand from equations (C.14) and (C.15) to obtain normalized aggregate labor demand

$$1 = \Omega \bar{\sigma}^{-\sigma} w^{-\sigma} y \left(\mathbb{E} [z^{\sigma-1}] + (N-1)(1-F(\hat{z}))d^{1-\sigma} \mathbb{E} [z^{\sigma-1} \mid z > \hat{z}] \right) + \Omega(N-1)(1-F(\hat{z}))\kappa + \Omega(1-\eta)\zeta S \quad (\text{C.24})$$

$$= \bar{\sigma}^{-\sigma} w^{-\sigma} y \bar{z}^{\sigma-1} + \Omega((N-1)(1-F(\hat{z}))\kappa + (1-\eta)\zeta(S + \delta/\chi)). \quad (\text{C.25})$$

Define \tilde{L} as a normalized quantity of labor used outside of variable production. Multiply equation (C.25) by w , and use equation (C.18) to show that

$$\tilde{L} := \Omega[(N-1)(1-F(\hat{z}))\kappa + (1-\eta)\zeta(S + \delta/\chi)], \quad (\text{C.26})$$

$$w = \frac{1}{\bar{\sigma}} y + \tilde{L}w, \quad (\text{C.27})$$

$$1 = \frac{1}{\bar{\sigma}} \frac{y}{w} + \tilde{L}. \quad (\text{C.28})$$

Reorganize to find real output as a function of the productivity distribution and labor supply (net of labor used for the fixed costs of exporting and adopting technology)

$$\frac{y}{w} = \bar{\sigma} (1 - \tilde{L}), \quad (\text{C.29})$$

$$y = (1 - \tilde{L}) \bar{z}. \quad (\text{C.30})$$

This equation lends interpretation to \bar{z} as being related to the aggregate productivity. Substituting equation (C.29) into equations (C.20) and (C.21) to obtain a useful formulation of firm profits

$$\pi_d(Z) = \frac{1-\tilde{L}}{(\sigma-1)\bar{z}^{\sigma-1}} \left(\frac{Z}{M}\right)^{\sigma-1}, \quad (\text{C.31})$$

$$\pi_x(Z) = \frac{1-\tilde{L}}{(\sigma-1)\bar{z}^{\sigma-1}} d^{1-\sigma} \left(\frac{Z}{M}\right)^{\sigma-1} - \kappa. \quad (\text{C.32})$$

Define the common profit multiplier $\bar{\pi}_{\min}$ as

$$\bar{\pi}_{\min} := \frac{1-\tilde{L}}{(\sigma-1)\bar{z}^{\sigma-1}} = \frac{1-\tilde{L}}{(\sigma-1)(\bar{\sigma}w)^{\sigma-1}} = \frac{(\bar{\sigma}w)^{1-\sigma}y}{\sigma w}, \quad (\text{C.33})$$

$$\pi_d(Z) = \bar{\pi}_{\min} \left(\frac{Z}{M}\right)^{\sigma-1}, \quad (\text{C.34})$$

$$\pi_x(Z) = \bar{\pi}_{\min} d^{1-\sigma} \left(\frac{Z}{M}\right)^{\sigma-1} - \kappa. \quad (\text{C.35})$$

Use equation (C.35) set to zero to solve for \hat{z} . This is an implicit equation as $\bar{\pi}_{\min}$ is a function of \hat{z} through \bar{z}

$$\hat{z} = d \left(\frac{\kappa}{\bar{\pi}_{\min}} \right)^{\frac{1}{\sigma-1}}. \quad (\text{C.36})$$

Substitute equations (C.34) and (C.35) into equations (C.22) and (C.23) to obtain a useful formulation for aggregate profits

$$\bar{\pi}_d = \Omega \bar{\pi}_{\min} \mathbb{E} [z^{\sigma-1}], \quad (\text{C.37})$$

$$\bar{\pi}_x = \Omega \left((1 - F(\hat{z})) (\bar{\pi}_{\min} d^{1-\sigma} \mathbb{E} [z^{\sigma-1} \mid z > \hat{z}] - \kappa) \right). \quad (\text{C.38})$$

Combine to calculate aggregate total profits

$$\begin{aligned} \bar{\pi}_d + (N-1)\bar{\pi}_x &= \Omega \bar{\pi}_{\min} \left[\mathbb{E} [z^{\sigma-1}] + (N-1)(1 - F(\hat{z})) d^{1-\sigma} \mathbb{E} [z^{\sigma-1} \mid z > \hat{z}] \right] \\ &\quad - \Omega(N-1)(1 - F(\hat{z}))\kappa. \end{aligned} \quad (\text{C.39})$$

Rewriting aggregate total profits using the definition of \bar{z} yields

$$\bar{\pi}_{\text{agg}} := \bar{\pi}_{\min} \bar{z}^{\sigma-1} - \Omega(N-1)(1 - F(\hat{z}))\kappa. \quad (\text{C.40})$$

Note that in a closed economy, $\bar{z} = (\Omega \mathbb{E} [z^{\sigma-1}])^{1/(\sigma-1)}$ and therefore aggregate profits relative to wage are a markup dependent fraction of normalized output relative to wage $\bar{\pi}_d = \frac{1-\tilde{L}}{\sigma-1}$. Take the resource constraint in equation (A.26) and divide by $M\bar{L}w$ and then use equation (C.29) to get an equation for normalized, per-capita consumption

$$\frac{c}{w} = \frac{y}{w} - \Omega \eta \zeta \Theta (S + \delta/\chi) / w = \bar{\sigma} \left(1 - \tilde{L}\right) - \Omega \eta \zeta \frac{\Theta}{w} (S + \delta/\chi), \quad (\text{C.41})$$

$$c = \left(1 - \tilde{L}\right) \bar{z} - \eta \zeta \Omega \Theta (S + \delta/\chi). \quad (\text{C.42})$$

Normalize the cost in equation (A.23) by dividing by $\bar{LPM}w$. This is implicitly a function of \hat{z} through w

$$x = \zeta (1 - \eta + \eta\Theta/w). \quad (\text{C.43})$$

Normalize the trade share in equation (A.17) by substituting from equations (C.11) and (C.18)

$$\lambda = (1 - F(\hat{z}))d^{1-\sigma} \frac{\Omega \mathbb{E} [z^{\sigma-1} \mid z > \hat{z}]}{\bar{z}^{\sigma-1}}. \quad (\text{C.44})$$

Starting from C.40, use C.33, and C.26, to derive that

$$\bar{\pi}_{agg} = \frac{1}{\sigma-1} (1 - \Omega [(1 - \eta)\zeta (S + \delta/\chi) - \sigma(N - 1)(1 - F(\hat{z}))\kappa]). \quad (\text{C.45})$$

Stationary Trade Shares and Profits. Using the stationary distribution in equation (B.13), calculate average profits from C.40,

$$\frac{\bar{\pi}_{agg}}{\Omega} = \frac{\bar{\pi}_{min}\theta}{1 + \theta - \sigma} + \frac{(\sigma - 1)(N - 1)\kappa\hat{z}^{-\theta}}{1 + \theta - \sigma}. \quad (\text{C.46})$$

Note that the minimum profits are at $z = 1$ and equal to $\bar{\pi}_{min}$ as long as $\hat{z} > 1$. Using this to define the profit spread between the average and worst firm in the economy,

$$\frac{\bar{\pi}_{agg}}{\Omega} - \bar{\pi}_{min} = \underbrace{\frac{\bar{\pi}_{min}\theta}{1 + \theta - \sigma} + \frac{(\sigma - 1)(N - 1)\kappa\hat{z}^{-\theta}}{1 + \theta - \sigma}}_{\text{ProfitSpread}} - \bar{\pi}_{min} = \frac{(\sigma - 1)\bar{\pi}_{min}}{1 + \theta - \sigma} + \frac{(\sigma - 1)(N - 1)\kappa\hat{z}^{-\theta}}{1 + \theta - \sigma}. \quad (\text{C.47})$$

Define the ratio of mean to minimum profits as $\bar{\pi}_{rat} := \frac{\bar{\pi}/\Omega}{\bar{\pi}_{min}}$. From C.47 and C.36 find that

$$\bar{\pi}_{rat} = \frac{\theta}{1 + \theta - \sigma} + (N - 1)d^{1-\sigma} \frac{(\sigma - 1)\hat{z}^{\sigma-1-\theta}}{1 + \theta - \sigma}. \quad (\text{C.48})$$

Take equations (C.44) and (C.17) to find

$$\bar{z}^{\sigma-1} = \Omega \mathbb{E} [z^{\sigma-1}] + (N - 1)\lambda \bar{z}^{\sigma-1}. \quad (\text{C.49})$$

Solving gives an expression for a function of aggregate productivity in terms of underlying productivity and trade shares. Defining the home trade share as $\lambda_{ii} := 1 - (N - 1)\lambda$,

$$\bar{z}^{\sigma-1} = \Omega \frac{\mathbb{E} [z^{\sigma-1}]}{1 - (N - 1)\lambda} = \Omega \frac{\mathbb{E} [z^{\sigma-1}]}{\lambda_{ii}}. \quad (\text{C.50})$$

From equations (C.19) and (C.50),

$$w = \frac{1}{\bar{\sigma}} \Omega^{\frac{1}{\sigma-1}} \mathbb{E} [z^{\sigma-1}]^{\frac{1}{\sigma-1}} \lambda_{ii}^{\frac{1}{1-\sigma}}. \quad (\text{C.51})$$

This relates the real normalized wage to the aggregate productivity, the home trade share, and the number of varieties. Given that $\sigma > 1$, this expression implies that the larger the share of goods purchased at home, the lower the real wage is.

From C.44 and C.50 ,

$$\lambda = (1 - F(\hat{z})) d^{1-\sigma} \frac{\mathbb{E} [z^{\sigma-1} \mid z > \hat{z}] \lambda_{ii}}{\mathbb{E} [z^{\sigma-1}]}. \quad (\text{C.52})$$

Using the stationary distribution,

$$\lambda = \hat{z}^{-\theta} d^{1-\sigma} \frac{\hat{z}^{\sigma-1} \frac{\theta}{\theta-(\sigma-1)} \lambda_{ii}}{\frac{\theta}{\theta-(\sigma-1)}} = \hat{z}^{-\theta} d^{1-\sigma} \hat{z}^{\sigma-1} \lambda_{ii}. \quad (\text{C.53})$$

Using the definition of the home trade share,

$$\lambda_{ii} = \frac{1}{1 + (N - 1) \hat{z}^{\sigma-1-\theta} d^{1-\sigma}}. \quad (\text{C.54})$$

Furthermore, multiplying the numerator and denominator by $\bar{\pi}_{\min}$ and using equation (C.36),

$$\lambda_{ii} = \frac{\bar{\pi}_{\min}}{\bar{\pi}_{\min} + (N - 1) \hat{z}^{-\theta} \kappa}. \quad (\text{C.55})$$

Which gives an alternative expression for $\bar{\pi}_{\min}$ when $\kappa > 0$,

$$\bar{\pi}_{\min} = \frac{(N - 1) \hat{z}^{-\theta} \kappa}{1 - \lambda_{ii}}. \quad (\text{C.56})$$

D. Normalized and Stationary Dynamic Equilibrium Conditions

This section derives normalized stationary dynamic balanced growth path equilibrium conditions.

D.1. Utility and Welfare on a BGP

Using the substitution $C(t) = c\bar{L}M(t)$ shows time 0 welfare \bar{U} as a function of c and g is

$$\bar{U}(c, g) = \frac{1}{1-\gamma} \frac{(c\bar{L}M(0))^{1-\gamma}}{\rho+(\gamma-1)g}. \quad (\text{D.1})$$

With log utility

$$\bar{U}(c, g) = \frac{\rho \log(c\bar{L}M(0)) + g}{\rho^2}. \quad (\text{D.2})$$

Using the standard IES of the consumer, adjusted for stochastic death of the firm, the firms' effective discount rate on a BGP is,

$$r = \rho + \gamma g + \delta. \quad (\text{D.3})$$

With log utility

$$r = \rho + g + \delta. \quad (\text{D.4})$$

We restrict parameters such that $g(1 - \gamma) < \rho$ in equilibrium to ensure finite utility. In the log utility case, this is simply $\rho > 0$.

D.2. Normalization of the Firm's Dynamic Problem

We proceed to derive the normalized continuation value function, value matching condition, and smooth pasting condition originally specified in equations (A.20)–(A.22). Define the normalized real value of the firm relative to normalized wages as

$$v(z, t) := \frac{V(Z, t)}{LM(t)w(t)}. \quad (\text{D.5})$$

Rearranging

$$V(Z, t) = \bar{L}w(t)M(t)v(Z/M(t), t). \quad (\text{D.6})$$

First differentiate the continuation value $V(Z, t)$ with respect to t in equation (D.6) and divide by $w(t)M(t)\bar{L}$, using the chain and product rule. This gives,

$$\frac{1}{w(t)M(t)\bar{L}} \frac{\partial V(Z, t)}{\partial t} = \frac{M'(t)}{M(t)} v(z, t) - \frac{M'(t)}{M(t)} \frac{Z}{M(t)} \frac{\partial v(z, t)}{\partial z} + \frac{M(t)}{M(t)} \frac{\partial v(z, t)}{\partial t} + \frac{w'(t)}{w(t)} v(z, t). \quad (\text{D.7})$$

Defining the growth rate of $g(t) := M'(t)/M(t)$ and $g_w(t) := w'(t)/w(t)$. Substitute these into equation (D.7), cancel out $M(t)$, and group $z = Z/M(t)$ to give

$$\frac{1}{w(t)M(t)\bar{L}} \frac{\partial V(Z, t)}{\partial t} = (g(t) + g_w(t))v(z, t) - g(t)z \frac{\partial v(z, t)}{\partial z} + \frac{\partial v(z, t)}{\partial t}. \quad (\text{D.8})$$

Differentiating equation (D.6) with respect to Z yields

$$\frac{\partial V(Z, t)}{\partial Z} = \frac{\bar{L}M(t)w(t)}{M(t)} \frac{\partial v(Z/M(t), t)}{\partial z} = \bar{L}w(t) \frac{\partial v(z, t)}{\partial z}. \quad (\text{D.9})$$

Similarly,

$$\frac{\partial^2 V(Z, t)}{\partial Z^2} = \frac{\bar{L}w(t)}{M(t)} \frac{\partial^2 v(z, t)}{\partial z^2}. \quad (\text{D.10})$$

Define the normalized profits from equation (A.19) as

$$\pi(z, t) := \frac{\Pi(zM(t), t)}{w(t)M(t)\bar{L}}. \quad (\text{D.11})$$

Divide equation (A.20) by $M(t)w(t)\bar{L}$, then substitute for $\frac{\partial V(Z, t)}{\partial t}$, $\frac{\partial V(Z, t)}{\partial Z}$, and $\frac{\partial^2 V(Z, t)}{\partial Z^2}$ from eqs. (D.8), (D.9), and (D.10) in to (A.20). Finally, group the normalized profits using equation (D.11):

$$(r(t) - g(t) - g_w(t))v(z, t) = \pi(z, t) + \left(\mu + \frac{v^2}{2} - g(t) \right) z \frac{\partial v(z, t)}{\partial z} + \frac{v^2}{2} z^2 \frac{\partial^2 v(z, t)}{\partial z^2} + \frac{\partial v(z, t)}{\partial t}. \quad (\text{D.12})$$

Equation (D.12) is the normalized version of the value function of the firm in the continuation region. The stationary version of this equation is,

$$(r - g)v(z) = \pi(z) + \left(\mu + \frac{v^2}{2} - g \right) zv'(z) + \frac{v^2}{2} z^2 v''(z). \quad (\text{D.13})$$

To derive the normalized smooth pasting condition, use equation (A.22) to show that equation (D.9) evaluated at $Z = M(t)$ equals 0, delivering

$$\frac{\partial v(1, t)}{\partial z} = 0. \quad (\text{D.14})$$

To arrive at the normalized value matching condition, divide equation (A.21) by $M(t)w(t)\bar{L}$ to obtain

$$\frac{V(M(t), t)}{M(t)w(t)\bar{L}} = \int_{M(t)}^{\infty} \frac{V(Z, t)}{M(t)w(t)\bar{L}} \phi(Z, t) dZ - \frac{X(t)}{M(t)w(t)\bar{L}} \quad (\text{D.15})$$

Substituting using equation (D.5) and the definition of $x(t)$ yields

$$v(M(t)/M(t), t) = \int_M^{\infty} v(Z/M, t) \phi(Z, t) dZ - x(t). \quad (\text{D.16})$$

Finally, normalize the integral, realizing it is of the form discussed in equation (C.2), to obtain the normalized value matching condition:

$$v(1, t) = \int_1^{\infty} v(z, t) f(z, t) dz - x(t). \quad (\text{D.17})$$

D.3. Normalization of the Free Entry Condition

Normalizing the free entry condition given in equation (A.24), following similar steps that delivered the normalized value matching condition in equation (D.17), gives

$$x(t)/\chi = \int_1^{\infty} v(z, t) f(z, t) dz. \quad (\text{D.18})$$

Relating this to the value-matching condition of the adopting firm given in equation (D.17) provides a simple formulation of the stationary free entry condition that is useful in determining Ω and g :

$$v(1) = x \frac{1 - \chi}{\chi}. \quad (\text{D.19})$$

E. Solving for the Continuation Value Function

Although our baseline model does not feature exogenous productivity shocks, in this section we solve for the value function of a more general model that has GBM with $v \geq 0$. Our baseline case of $v = 0$ is nested in this formulation. The differential equation for the value function is solved using the method of undetermined coefficients. The goal of this section is to solve for the value function as a function of parameters, g , Ω , and \hat{z} (sometimes implicitly through $\bar{\pi}_{min}$).

Selection into Exporting. If $\kappa > 0$, generically some firms will choose to be exporters and some firms will only sell domestically. The value function will have a region of productivities representing the value of firms that only sell domestically and a region representing firms that also export. That is,

$$v(z) = \begin{cases} v_d(z) & \text{if } z \leq \hat{z} \\ v_x(z) & \text{if } z \geq \hat{z}. \end{cases}$$

We guess the value function is of the following form, with undetermined constants a , ν , and b ,²⁸

$$v_d(z) = a\bar{\pi}_{min} \left(z^{\sigma-1} + \frac{\sigma-1}{\nu} z^{-\nu} \right), \quad (\text{E.1})$$

$$v_x(z) = a\bar{\pi}_{min} \left((1 + (N-1)d^{1-\sigma})z^{\sigma-1} + \frac{\sigma-1}{\nu} z^{-\nu} + (N-1) \frac{1}{a(r-g)} \left(bz^{-\nu} - \frac{\kappa}{\bar{\pi}_{min}} \right) \right). \quad (\text{E.2})$$

²⁸This guess is also applying a standard transversality condition to eliminate an explosive root.

The value of a firm can be decomposed into the value of operating with its current productivity forever and the option value of adopting a better technology. The constant a is a discounting term on the value of earning the profits from producing with productivity z in perpetuity. The constant ν reflects the rate at which the option value of technology adoption goes to zero as productivity increases. The constant b is an adjustment to the perpetuity profits that reflects a firm with productivity z will eventually switch from exporting to being a domestic producer if z is constant in a growing economy.

By construction, the form of these guesses ensures that value matching and smooth pasting are satisfied, both at the adoption threshold ($z = 1$) and the exporter threshold ($z = \hat{z}$). To solve for a and ν , substitute equation (E.1) into the continuation value function in equation (D.13) using $\pi_d(z)$ from equation (C.34). This generates an ODE and, grouping terms, the method of undetermined coefficients provides a system of 2 equations in the 2 unknowns.²⁹

Solving the system gives,

$$\nu = \frac{\mu - g}{v^2} + \sqrt{\left(\frac{g - \mu}{v^2}\right)^2 + \frac{r - g}{v^2/2}}, \quad (\text{E.3})$$

$$a = \frac{1}{r - g - (\sigma - 1)(\mu - g + (\sigma - 1)v^2/2)}. \quad (\text{E.4})$$

To solve for b , plug equation (E.2) into the continuation value function in equation (D.13) using $\pi_x(z)$ from equation (C.35). This generates an ODE and, grouping terms, the method of undetermined coefficients provides a system of 3 equations in the 3 unknowns. By construction, the a and ν terms match those previously found, giving a consistent solution for b :

$$b = (1 - a(r - g)) d^{1-\sigma} \hat{z}^{\nu+\sigma-1}. \quad (\text{E.5})$$

Note, the main effect of the GBM is to modify the ν constant to reflect changes in the expected execution time of the option value of technology diffusion, and hence the exponent for discounting.

As will be useful in solving for g , evaluating at the adoption threshold yields,

$$v(1) = a\bar{\pi}_{\min} \left(1 + \frac{\sigma - 1}{\nu}\right). \quad (\text{E.6})$$

For the baseline case of $\mu = v = 0$,

$$a = \frac{1}{r + (\sigma - 2)g}, \quad (\text{E.7})$$

$$\nu = \frac{r}{g} - 1, \quad (\text{E.8})$$

$$b = \frac{\sigma - 1}{\nu + \sigma - 1} d^{1-\sigma} \hat{z}^{\nu+\sigma-1}. \quad (\text{E.9})$$

²⁹Instead of the method of undetermined coefficients, a direct solution approach would be to solve the continuation value function ODEs in the domestic sales and exporter regions, using the smooth pasting condition as the boundary value.

F. Computing the BGP Equilibrium when All Firms Export

In the case of $\kappa = 0$ all firms export (given d s.t. the economy is not in autarky), and the value function has only one region. We guess that the value function will take the following form,

$$v(z) = a\bar{\pi}_{\min} (1 + (N - 1)d^{1-\sigma}) \left(z^{\sigma-1} + \frac{\sigma-1}{\nu} z^{-\nu} \right). \quad (\text{F.1})$$

Substituting equation (F.1) into the continuation value function in equation (D.13) with profits from equation (C.35), the ν and a are identical to those from equations (E.3) and (E.4). Evaluating at the threshold,

$$v(1) = a (1 + (N - 1)d^{1-\sigma}) \bar{\pi}_{\min} \left(1 + \frac{\sigma-1}{\nu} \right). \quad (\text{F.2})$$

Solving for the Growth Rate and measure of Varieties when All Firms Export. Using the free entry condition from equation (D.19) with equation (F.2) to find,

$$\frac{x}{\bar{\pi}_{\min}} = a (1 + (N - 1)d^{1-\sigma}) \frac{\chi}{1-\chi} \frac{\sigma + \nu - 1}{\nu}. \quad (\text{F.3})$$

Substitute equations (F.1) and (F.2) into the value matching condition of equation (D.17), and divide by $a\bar{\pi}_{\min}(1 + (N - 1)d^{1-\sigma})$

$$1 + \frac{\sigma-1}{\nu} = \frac{\theta(\nu + \sigma - 1)(\theta + \nu - \sigma + 1)}{\nu(\theta + \nu)(\theta - \sigma + 1)} - \frac{x}{\bar{\pi}_{\min}a (1 + (N - 1)d^{1-\sigma})}. \quad (\text{F.4})$$

Combine equations (F.3) and (F.4), and solve for ν . For any cost function x and minimum profits $\bar{\pi}_{\min}$,

$$\nu = \frac{\chi\theta(\theta + 1 - \sigma)}{\sigma - 1 - \theta\chi}. \quad (\text{F.5})$$

The aggregate growth rate is found by equating equations (E.3) and (F.5) to find

$$g = \underbrace{\mu}_{\text{Drift}} + \underbrace{\frac{(r - \mu)((\sigma - 1)/\chi - \theta)}{\theta^2 - \theta\sigma + (\sigma - 1)/\chi}}_{\text{Deterministic}} + \frac{v^2}{2} \underbrace{\frac{\theta^2(\theta + 1 - \sigma)^2}{(\theta^2 - \theta\sigma + (\sigma - 1)/\chi)(\theta - (\sigma - 1)/\chi)}}_{\text{Stochastic}}. \quad (\text{F.6})$$

In the baseline case of $v = \mu = 0$

$$g = \frac{(\rho + \delta)(\sigma - 1 - \chi\theta)}{\theta\chi(\gamma + \theta - \sigma) - (\gamma - 1)(\sigma - 1)}, \quad (\text{F.7})$$

and,

$$\frac{x}{\bar{\pi}_{\min}} = (1 + (N - 1)d^{1-\sigma}) \frac{\chi}{1-\chi} \frac{1}{r - g}. \quad (\text{F.8})$$

The growth rate is independent of the trade costs, the population, and the number of countries. The intuition—as discussed in the body of the paper—is that the growth rate is driven by the ratio of the minimum to the mean profits, which are proportional and independent of the scale or integration of economies in the absence of any export selection. The constant death rate only enters to increase the discount rate.

Note in the baseline case of $v = \mu = \delta = 0$ and log utility

$$g = \frac{\rho(\sigma - 1 - \chi\theta)}{\theta^2\chi} \bar{\pi}_{\text{rat}}^k, \quad (\text{F.9})$$

where (from eqs. C.40 and C.17 with $\kappa = 0$ and $\hat{z} = 1$) the ratio of average profits to minimum profits is

$$\bar{\pi}_{\text{rat}}^k = \frac{\bar{\pi}_{\text{agg}}^k/\Omega}{\bar{\pi}_{\text{min}}^k} = \frac{(1 + (N - 1)d^{1-\sigma})\bar{\pi}_{\text{min}}E[z^{\sigma-1}]}{(1 + (N - 1)d^{1-\sigma})\bar{\pi}_{\text{min}}} = E[z^{\sigma-1}] = \frac{\theta}{1 + \theta - \sigma}. \quad (\text{F.10})$$

The Measure of Varieties Ω . Here we solve for the measure of varieties Ω in the baseline case of $v = \mu = \delta = 0$.

Substitute into the free entry condition equation (F.8) using the definition of $\bar{\pi}_{\text{min}}$ in terms of \bar{z} and \tilde{L} from equation (C.33), the definition of \bar{z} from equation (C.17), and the definition of \tilde{L} from equation (C.26), to obtain the implicit equation. In the baseline case where $\eta = 0$, an explicit solution is³⁰

$$\Omega = \frac{\chi((\gamma - 1)(\sigma - 1) - \theta\chi(\gamma + \theta - \sigma))}{\zeta(\theta\chi(-\gamma\delta - \sigma(\theta(\delta + \rho) + \rho) + \delta + \theta\rho + \rho) + (\gamma - 1)\delta(\sigma - 1) + \theta^2\sigma\chi^2(\delta + \rho))}. \quad (\text{F.13})$$

Note that the only place that the adoption cost, ζ , has come into the system of Ω and g is in the denominator of F.13. For this reason, the ζ parameter (along with \bar{L}) determines the scale of the economy.

It can be shown that in the Krugman model for all cases with $\eta = 0$, the number of domestic varieties is independent of trade costs d . Thus, both Ω and g are independent of d if $\eta = 0$. This implies through equations (C.26) and (B.14) that the amount of labor dedicated to technology adoption, \tilde{L} , is also independent of d if $\eta = 0$.

Through C.42, since \tilde{L} , Ω , and g are independent of d when $\eta = 0$, in response to a decrease in trade costs d , c increases only due to the $(1 + (N - 1)d^{1-\sigma})^{1/(\sigma-1)}$ term in \bar{z} .

³⁰The solution for the simplest case of $\gamma = 1$ and $\delta = 0$ is,

$$g = \frac{\rho(\sigma - 1 - \theta\chi)}{\theta\chi(1 + \theta - \sigma)} \quad (\text{F.11})$$

$$\Omega = \frac{\chi(1 + \theta - \sigma)}{\zeta\rho((1 + \theta)(\sigma - 1) - \sigma\theta\chi)} \quad (\text{F.12})$$

The key relationship can be summarized by the following elasticities.

$$\frac{d \log \bar{\pi}_{\text{rat}}^k(d)}{d \log(d)} = 0. \quad (\text{F.14})$$

Using equation (C.54) with $\hat{z} = 1$ shows

$$\frac{d \log \lambda_{ii}(d)}{d \log(d)} = (\sigma - 1) \left(1 + \frac{d^{\sigma-1}}{N-1} \right)^{-1} = (\sigma - 1)(1 - \lambda_{ii}) > 0. \quad (\text{F.15})$$

Furthermore, when $\eta = 0$,

$$\frac{d \log(1 - \tilde{L}(d))}{d \log(d)} = \frac{d \log \Omega(d)}{d \log(d)} = \frac{d \log g(d)}{d \log(d)} = 0. \quad (\text{F.16})$$

For the case with $\eta = 0$, the ratio of c for different trade costs d_1 and d_2 that both feature positive trade is

$$\frac{\bar{c}_{d_1}}{\bar{c}_{d_2}} = \frac{\bar{z}_{z_{d_1}}}{\bar{z}_{z_{d_2}}} = \left(\frac{1 + (N-1)d_1^{1-\sigma}}{1 + (N-1)d_2^{1-\sigma}} \right)^{\frac{1}{\sigma-1}}. \quad (\text{F.17})$$

Using the normalized welfare function in equation (D.1), since g is independent of d in the $\kappa = 0$ case, the ratio of welfare for different trade costs d_1 and d_2 that both feature positive trade (for $\gamma > 0$) is,

$$\frac{\bar{U}_{d_1}}{\bar{U}_{d_2}} = \left(\frac{1 + (N-1)d_1^{1-\sigma}}{1 + (N-1)d_2^{1-\sigma}} \right)^{\frac{1-\gamma}{\sigma-1}}. \quad (\text{F.18})$$

Comparing welfare between free trade ($d = 1$) and autarky (sufficiently high d s.t. there are no exporters) gives (for $\gamma \neq 1$)

$$\frac{\bar{U}_{\text{free}}}{\bar{U}_{\text{autarky}}} = N^{\frac{1-\gamma}{\sigma-1}}. \quad (\text{F.19})$$

G. Computing the BGP Equilibrium with Selection into Exporting ($\kappa > 0$)

We solve for g and Ω by reducing the equilibrium conditions to a system of two equations in these two unknowns. First, combining the free entry condition from equation (D.19) with $v(1)$ from equation (E.6) yields

$$\frac{x}{\bar{\pi}_{\text{min}}} = a \frac{\chi}{1-\chi} \frac{\sigma + \nu - 1}{\nu}. \quad (\text{G.1})$$

The second equation is found by evaluating the value matching condition of equation (D.17) by substituting in the domestic and exporter value functions in equations E.1 and E.2, the export threshold \hat{z} in equation (C.36), and the value at the adoption threshold $v(1)$ in equation (E.6) and dividing by $a\bar{\pi}_{\min}$. That is, evaluate

$$\frac{v(1)}{a\bar{\pi}_{\min}} = \frac{\int_1^{\infty} v(z, t) f(z, t) dz}{a\bar{\pi}_{\min}} - \frac{x}{a\bar{\pi}_{\min}} \quad (\text{G.2})$$

to obtain

$$1 + \frac{\sigma - 1}{\nu} = \frac{\frac{\nu(n-1)(\theta-\sigma+1)(d^{1-\sigma}(\theta+\nu)\hat{z}^{-\theta+\sigma-1} - b\theta\hat{z}^{-\theta-\nu})}{a(g-r)} + \theta(\nu(n-1)d^{1-\sigma}(\theta+\nu)\hat{z}^{-\theta+\sigma-1} + (\nu+\sigma-1)(\theta+\nu-\sigma+1))}{\nu(\theta+\nu)(\theta-\sigma+1)} - \frac{\chi}{1-\chi} \frac{\sigma+\nu-1}{\nu}. \quad (\text{G.3})$$

As detailed in Section H, use the definition of $\bar{\pi}_{\min}$ and substitute for x, ν, a, b, \hat{z} and r into equations (G.1) and (G.3) to find a system of 2 equations in Ω and g . Note that the adoption cost x does not appear in equation (G.3), which is why g is independent of the specification of the cost of adoption. Equation (G.1) does explicitly depend on x , which is why the number of varieties is a function of the adoption cost, and ultimately why welfare is also a function of x .

G.1. Case with $v = \mu = \delta = 0$

As the GBM does not qualitatively impact the solution, we concentrate our analytical theory on the simple baseline case. The cost of adoption is a function of minimum profits, parameters, and $r - g$:

$$x = \bar{\pi}_{\min} \frac{\chi}{1-\chi} \frac{1}{r-g}. \quad (\text{G.4})$$

Evaluating the general equation (G.3) with the substitutions for x, ν, a, b, \hat{z} and r that correspond to the baseline case of $v = \mu = \delta = 0$ yields a unique g that satisfies the value function and the value matching equations, given by the implicit equation,

$$g = \frac{(\sigma - 1 + (N - 1)\theta d^{1-\sigma} \hat{z}^{-\theta+\sigma-1}) \bar{\pi}_{\min} + (N - 1)(-\theta + \sigma - 1) \hat{z}^{-\theta} \kappa}{x(\gamma + \theta - 1)(\theta - \sigma + 1)} - \frac{\rho}{\gamma + \theta - 1}. \quad (\text{G.5})$$

Using $\kappa/\bar{\pi}_{\min} = \hat{z}^{\sigma-1} d^{1-\sigma}$ from equation (C.36), simplify to

$$g = \frac{(\sigma - 1) [\bar{\pi}_{\min} + (N - 1)\kappa\hat{z}^{-\theta}]}{x(\gamma + \theta - 1)(\theta - \sigma + 1)} - \frac{\rho}{\gamma + \theta - 1}. \quad (\text{G.6})$$

Using equation (C.47), realize this relates the growth rate to the difference between average and minimum profits

$$g = \left(\frac{\bar{\pi}_{\text{agg}}/\Omega - \bar{\pi}_{\min}}{x(\gamma + \theta - 1)} \right) - \frac{\rho}{\gamma + \theta - 1}. \quad (\text{G.7})$$

Substitute for x using the free entry condition in equation (G.4), use the definition of the average to minimum profit ratio ($\bar{\pi}_{\text{rat}} := \frac{\bar{\pi}/\Omega}{\bar{\pi}_{\text{min}}}$), and substitute for r using equation (D.3) to obtain

$$g = (\rho + (\gamma - 1)g) \frac{1 - \chi}{\chi(\gamma + \theta - 1)} (\bar{\pi}_{\text{rat}} - 1) - \frac{\rho}{\gamma + \theta - 1}. \quad (\text{G.8})$$

Solving for g gives an equation for g as a function exclusively of parameters and the ratio of average to minimum profits

$$g = \frac{\rho}{\chi\theta((1 - \chi)\bar{\pi}_{\text{rat}} - 1)^{-1} + 1 - \gamma}. \quad (\text{G.9})$$

Furthermore, with log utility $\gamma = 1$ and

$$g = \frac{\rho(1 - \chi)}{\chi\theta} \bar{\pi}_{\text{rat}} - \frac{\rho}{\chi\theta}. \quad (\text{G.10})$$

The relationship between growth and trade costs. First, note that the growth rate is increasing in the profit ratio, (since $\chi \in (0, 1)$):

$$\frac{dg(\bar{\pi}_{\text{rat}})}{d\bar{\pi}_{\text{rat}}} = \frac{\rho\theta\chi(1 - \chi)}{((\gamma - 1)(1 - (1 - \chi)\bar{\pi}_{\text{rat}} + \theta\chi))^2} > 0. \quad (\text{G.11})$$

To determine if whether growth is increasing in d , use the chain rule

$$\frac{dg(d)}{dd} = \frac{dg(\bar{\pi}_{\text{rat}})}{d\bar{\pi}_{\text{rat}}} \frac{d\bar{\pi}_{\text{rat}}(d)}{dd}. \quad (\text{G.12})$$

Given equations (G.11) and (G.12), a sufficient condition to conclude that $\frac{dg(d)}{dd} < 0$ is $\frac{d\bar{\pi}_{\text{rat}}(d)}{dd} < 0$. To show this differentiate C.48 w.r.t. d to find,

$$\frac{d\bar{\pi}_{\text{rat}}(d)}{dd} \propto - \left((\sigma - 1)\hat{z}(d) + d(1 + \theta - \sigma) \frac{d\hat{z}(d)}{dd} \right). \quad (\text{G.13})$$

Since $d > 0$, $\hat{z}(d) > 0$, $\sigma > 1$, and $1 + \theta - \sigma > 0$, a sufficient condition for $\frac{d\bar{\pi}_{\text{rat}}(d)}{dd} < 0$ is if $\frac{d\hat{z}(d)}{dd} > 0$. Differentiate equation (C.36) to find

$$\frac{d\hat{z}(d)}{dd} \propto (\sigma - 1) - \frac{d}{\bar{\pi}_{\text{min}}(d)} \frac{d\bar{\pi}_{\text{min}}(d)}{dd}. \quad (\text{G.14})$$

Therefore, a sufficient condition to conclude that $\frac{d\hat{z}(d)}{dd} > 0$ is

$$\frac{d \log \bar{\pi}_{\text{min}}(d)}{dd} < \frac{\sigma - 1}{d}. \quad (\text{G.15})$$

Summarizing,

$$\text{sign} \frac{dg(d)}{dd} = \text{sign} \frac{d\bar{\pi}_{\text{rat}}}{dd} = -\text{sign} \frac{d\hat{z}(d)}{dd}. \quad (\text{G.16})$$

G.2. Baseline Case with $v = \mu = \delta = \eta = 0$ and log utility.

Adding the restriction that $\eta = 0$ and $\gamma = 1$ to the adoption cost equation (C.43) and the firms' discount rate equation (D.4) delivers the key simplifications that permit solving for the BGP equilibrium in closed form:

Calculating the Growth Rate and the Measure of Varieties.

$$x = \zeta, \quad (\text{G.17})$$

$$r - g = \rho. \quad (\text{G.18})$$

Using equation (G.4) gives an expression for $\bar{\pi}_{\text{min}}$ in terms of model parameters

$$\bar{\pi}_{\text{min}} = \frac{(1 - \chi)\zeta\rho}{\chi}. \quad (\text{G.19})$$

Substitute equation (G.19) into equation (C.36) to find the export threshold in terms of parameters,

$$\hat{z} = d \left(\frac{1}{\zeta} \frac{\kappa\chi}{\rho(1-\chi)} \right)^{\frac{1}{\sigma-1}}. \quad (\text{G.20})$$

Substitute $\gamma = 1$ and equations (G.17), (G.19), and (G.20) into equation (G.6) and simplify to obtain g in closed form:

$$g = \frac{\rho(1-\chi)}{\chi\theta} \frac{\sigma-1}{(\theta-\sigma+1)} \left(1 + (N-1)d^{-\theta} \left(\frac{\kappa}{\zeta} \frac{\chi}{\rho(1-\chi)} \right)^{1-\frac{\theta}{\sigma-1}} \right) - \frac{\rho}{\theta}, \quad (\text{G.21})$$

$$= \frac{\rho(1-\chi)}{\chi\theta} \frac{\left(\theta + (N-1)(\sigma-1)d^{-\theta} \left(\frac{1}{\zeta} \frac{\kappa\chi}{\rho(1-\chi)} \right)^{1-\frac{\theta}{\sigma-1}} \right)}{(\theta-\sigma+1)} - \frac{\rho}{\chi\theta}. \quad (\text{G.22})$$

For ease of comparison to g as a function of $\bar{\pi}_{\text{rat}}$ use equation (G.8) evaluated at $\gamma = 1$ with equation (G.21) to realize

$$\bar{\pi}_{\text{rat}} - 1 = \frac{\left(1 + (N-1)d^{-\theta} \left(\frac{1}{\zeta} \frac{\kappa\chi}{\rho(1-\chi)} \right)^{1-\frac{\theta}{\sigma-1}} \right)}{\left(\frac{\theta}{\sigma-1} - 1 \right)}, \quad (\text{G.23})$$

or equation (G.10) with equation (G.22) to see

$$\bar{\pi}_{\text{rat}} = \frac{\left(\theta + (N-1)(\sigma-1)d^{-\theta} \left(\frac{1}{\zeta} \frac{\kappa\chi}{\rho(1-\chi)} \right)^{1-\frac{\theta}{\sigma-1}} \right)}{(\theta - \sigma + 1)}. \quad (\text{G.24})$$

Note, g is decreasing in d and κ (since $\theta > 0$, $\sigma > 1$, and $1 + \theta - \sigma > 0$). Thus, from equation (G.16) or direct differentiation of equation (G.21),

$$\frac{dg}{dd} < 0; \quad \frac{d\bar{\pi}_{\text{rat}}}{dd} < 0; \quad \frac{d\hat{z}}{dd} > 0; \quad \frac{dg}{d\kappa} < 0. \quad (\text{G.25})$$

See by comparing to equation (F.7) that the limit of g as $d \rightarrow \infty$ in equation (G.22) equals the autarky and all-export economy growth rates, so there is no discontinuity in the economy in this direction.

Note that the parameter ζ (previously interpreted as the scale in equation F.13) and κ only enter the growth rate multiplicatively. This is because the fixed costs of adoption, entry, and export in levels are proportional to the scale of the economy. Since the calibration strategy targets relative moments (i.e., proportion of exporters, relative size of exporters to domestic firms, growth rates, trade shares), κ is not separately identifiable from ζ without some moment that targets the level of the economy.

To find the number of varieties, maintain $\gamma = 1$, $\eta = 0$. To solve for Ω , start with the definition of $\bar{\pi}_{\text{min}}$ from equation (C.33):

$$\bar{\pi}_{\text{min}} = \frac{1-\tilde{L}}{(\sigma-1)\tilde{z}^{\sigma-1}}. \quad (\text{G.26})$$

For $\bar{\pi}_{\text{min}}$, substitute from equation (G.19). For the right hand side, substitute for \tilde{L} , \tilde{z} and S with equations (C.26), (C.17), and (B.14). Then, use g and \hat{z} from equations (G.21), (G.20), and solve for Ω in terms of model parameters:

$$\Omega = \frac{1}{\zeta} \frac{\chi(1+\theta-\sigma)}{(1-\chi)\theta\sigma\rho} \left(1 + (N-1)d^{-\theta} \left(\frac{\kappa\chi}{\zeta\rho(1-\chi)} \right)^{1-\frac{\theta}{\sigma-1}} - \frac{1+\theta-\sigma}{\theta\sigma(1-\chi)} \right)^{-1}. \quad (\text{G.27})$$

Note, from equations (C.54) and (G.20), the home trade share is,

$$\lambda_{ii} = \frac{1}{1 + (N-1)d^{-\theta} \left(\frac{\kappa\chi}{\zeta\rho(1-\chi)} \right)^{1-\frac{\theta}{\sigma-1}}}. \quad (\text{G.28})$$

Note, using eqs. G.21 and G.29, the growth rate and Ω can be written as a function of the home trade share:

$$g = \frac{\rho(1-\chi)}{\chi\theta} \frac{\sigma-1}{(\theta-\sigma+1)} \lambda_{ii}^{-1} - \frac{\rho}{\theta}. \quad (\text{G.29})$$

$$\Omega = \frac{\chi}{\zeta\rho} \left(\frac{(1-\chi)\theta\sigma}{1+\theta-\sigma} \lambda_{ii}^{-1} - 1 \right)^{-1}. \quad (\text{G.30})$$

Calculating Consumption. By the resource constraint, $c = y$ when $\eta = 0$ (equation C.41). Thus, using equation (C.30), consumption is given by

$$c = y = (1 - \tilde{L})\bar{z}. \quad (\text{G.31})$$

Equations (C.26), (B.14), (C.36), and (G.29) combine to yield the amount of labor dedicated to variable goods production in terms of the home trade share:

$$1 - \tilde{L} = (\sigma - 1) \left(\sigma - \frac{1 + \theta - \sigma}{\theta(1 - \chi)} \lambda_{ii} \right)^{-1}. \quad (\text{G.32})$$

Equation (C.50) gives

$$\bar{z} = \Omega^{\frac{1}{\sigma-1}} \lambda_{ii}^{\frac{1}{1-\sigma}} (\mathbb{E} [z^{\sigma-1}])^{\frac{1}{\sigma-1}}. \quad (\text{G.33})$$

Substituting equations (G.32), (G.33), and (G.30) into equation (G.31) yields consumption as a function of parameters and the home trade share:

$$c = \frac{(\sigma - 1)\theta\sigma(1 - \chi)}{\sigma(1 + \theta - \sigma)} \left(\frac{\chi}{\rho\zeta} \right)^{\frac{1}{\sigma-1}} \left(\frac{(1 - \chi)\theta\sigma}{1 + \theta - \sigma} - \lambda_{ii} \right)^{\frac{\sigma}{1-\sigma}} (\mathbb{E} [z^{\sigma-1}])^{\frac{1}{\sigma-1}}. \quad (\text{G.34})$$

Trade Cost Elasticities. Comparative statics are analyzed by calculating elasticities with respect to trade costs using equations (G.21), (G.27), and (G.28):

$$\frac{d \log \bar{\pi}_{\text{rat}}(d)}{d \log(d)} = -\theta \left(1 + \frac{\theta d^\theta \left(\frac{\kappa\chi}{\zeta\rho(1-\chi)} \right)^{\frac{\theta}{\sigma-1} - 1}}{(N-1)(\sigma-1)} \right)^{-1} < 0, \quad (\text{G.35})$$

$$\frac{d \log g(d)}{d \log(d)} = -\theta \left(1 + \frac{\rho d^\theta (-\theta\chi + \sigma - 1) \left(\frac{\kappa\chi}{\zeta\rho(1-\chi)} \right)^{\frac{\theta}{\sigma-1}}}{\kappa(N-1)(\sigma-1)\chi} \right)^{-1} < 0, \quad (\text{G.36})$$

$$\frac{d \log \Omega(d)}{d \log(d)} = \theta \left(1 + \frac{\rho d^\theta ((\theta + 1)(\sigma - 1) - \theta\sigma\chi) \left(\frac{\kappa\chi}{\zeta\rho(1-\chi)} \right)^{\frac{\theta}{\sigma-1}}}{\theta\kappa(N-1)\sigma\chi} \right)^{-1} > 0, \quad (\text{G.37})$$

$$\frac{d \log \lambda_{ii}(d)}{d \log(d)} = \theta \left(1 + \frac{d^\theta}{N-1} \left(\frac{\kappa\chi}{\zeta\rho(1-\chi)} \right)^{\frac{\theta}{\sigma-1} - 1} \right)^{-1} > 0. \quad (\text{G.38})$$

Let $\varepsilon_{f,x}$ be the elasticity of any $f(x)$ w.r.t. x . These elasticities can be rearranged to highlight their relationship to trade volume. To see this, first define the ratio of the home trade share to the share of

goods purchased away from home:

$$\frac{\lambda_{ii}}{1 - \lambda_{ii}} = \frac{d^\theta \left(\frac{\kappa\chi}{\zeta\rho(1-\chi)} \right)^{\frac{\theta}{\sigma-1} - 1}}{N - 1}. \quad (\text{G.39})$$

Substituting this into the result above, yields

$$\frac{d \log \lambda_{ii}(d)}{d \log(d)} = \theta \left(1 + \frac{\lambda_{ii}}{1 - \lambda_{ii}} \right)^{-1} = \theta(1 - \lambda_{ii}), \quad (\text{G.40})$$

$$\frac{d \log g(d)}{d \log(d)} = -\theta \left[1 + \left(\frac{-\theta\chi + \sigma - 1}{(\sigma - 1)(1 - \chi)} \right) \frac{\lambda_{ii}}{1 - \lambda_{ii}} \right]^{-1} \quad (\text{G.41})$$

$$= \left(\frac{\chi(1 + \theta - \sigma)}{(\sigma - 1)(1 - \chi)} \lambda_{ii} - 1 \right)^{-1} \varepsilon_{\lambda_{ii},d}, \quad (\text{G.42})$$

$$\frac{d \log \Omega_{ii}(d)}{d \log(d)} = \left(1 - \frac{1 + \theta - \sigma}{\theta\sigma(1 - \chi)} \lambda_{ii} \right)^{-1} \varepsilon_{\lambda_{ii},d}. \quad (\text{G.43})$$

The elasticity of $(1 - \tilde{L})$ w.r.t. d is

$$\frac{d \log(1 - \tilde{L}(d))}{d \log(d)} = \left(\frac{\theta\sigma(1 - \chi)}{1 + \theta - \sigma} \lambda_{ii}^{-1} - 1 \right)^{-1} \varepsilon_{\lambda_{ii},d} > 0. \quad (\text{G.44})$$

The elasticity of \bar{z} w.r.t. d is

$$\frac{d \log(\bar{z}(d))}{d \log(d)} = \frac{\varepsilon_{\Omega,d} - \varepsilon_{\lambda_{ii},d}}{\sigma - 1} > 0. \quad (\text{G.45})$$

From equation (G.31)

$$\varepsilon_{c,d} = \varepsilon_{1-\tilde{L},d} + \varepsilon_{\bar{z},d}. \quad (\text{G.46})$$

Using equations (G.34) and (G.40) yields

$$\varepsilon_{c,d} = -\frac{\sigma}{\sigma - 1} \left(1 - \frac{\theta\sigma(1 - \chi)}{(1 + \theta - \sigma)} \lambda_{ii}^{-1} \right)^{-1} \varepsilon_{\lambda_{ii},d}. \quad (\text{G.47})$$

Finally, from D.2

$$\varepsilon_{\bar{U},d} = \frac{\rho \varepsilon_{c,d} + g \varepsilon_{g,d}}{g + \rho \log(c\bar{L}M(0))} \quad (\text{G.48})$$

$$= \frac{\rho^2}{\bar{U}} (\rho \varepsilon_{c,d} + g \varepsilon_{g,d}). \quad (\text{G.49})$$

This can be further organized by substitution for $\varepsilon_{c,d}$ and $\varepsilon_{g,d}$ from equations (G.47) and (G.42) into equation (G.49).³¹

$$\varepsilon_{\bar{U},d} = -\varepsilon_{\lambda_{ii},d} \frac{\rho^3}{\bar{U}} \left(\frac{\sigma}{(\sigma-1) \left(1 - \frac{\theta\sigma(1-\chi)}{(\theta-\sigma+1)\lambda_{ii}}\right)} + \frac{(\sigma-1)(1-\chi)}{\theta\chi(\theta-\sigma+1)\lambda_{ii}} \right). \quad (\text{G.50})$$

Therefore, if the term in brackets is positive, then the elasticity of utility is of the opposite sign of the home trade share, and hence always decreasing in trade costs. The first term in the brackets is always negative and the second term is always positive. There exist parameter values such that sum is negative, such that economies with lower trade costs have lower welfare in a comparison of steady states. This occurs when $\sigma-1$ is close to its lower bound of $\theta\chi$. This unintuitive result is possible because this analysis ignores transition dynamics and because this is an inefficient economy, so economics of the second best applies.

Firm Adoption Timing. A firm adopts when normalized productivity equals 1 by definition. On the BGP, firms drift backwards towards $z = 1$ at constant rate g . Thus, the time until adoption $\tau(z)$ is given by

$$\begin{aligned} e^{-g\tau} z &= 1 \\ \tau(z) &= \frac{\log(z)}{g} \end{aligned} \quad (\text{G.51})$$

The expected time until adoption for a firm that is about to draw a new productivity, $\bar{\tau}$, is just the expected adoption time integrated over the distribution of the new z :

$$\bar{\tau} = \int_1^\infty \frac{\log(z)}{g} dF(z) = \frac{1}{g} \int_1^\infty \log(z) \theta z^{-\theta-1} = \frac{1}{\theta g} \quad (\text{G.52})$$

Since firms draw a new z from the unconditional distribution, the expected time to adoption for a newly adopting firm is the same as the average time to adoption.

H. Computing the BGP Equilibrium in General

In the general case of the $\kappa = 0$, the equilibrium g can be calculated through an explicit equation, and the Ω found separately. If $\kappa > 0$, then a system of 2 non-linear algebraic equations in g and Ω are solved. Summarizing equations for easy reference against the code:

General Substitutions. The following substitutions are used in reducing the equilibrium conditions into a simple system of equations that can be solved for g and Ω . Given g and Ω all other equilibrium

³¹For determining the direction of the change, as an elasticity is $d\bar{U}'(d)/\bar{U}(d)$, and since $d > 1$, if $\bar{U}(d) > 0$ then the sign of this elasticity calculation matches the sign of the derivative. Otherwise, the sign of the derivative is negative of the elasticity. As equation (G.49) divided by the utility in the calculation, this sign cancels, and ensures that negative utility does not affect the direction of the changes (as expected with a monotone function with the possibility of arbitrarily small initial conditions).

values are determined. We use equations (B.13), (B.14), (E.3), (E.4), (E.5), (D.3), (C.26), (C.17), (C.36), (C.19), and (C.43):

$$F(z) = 1 - z^{-\theta} \quad (\text{H.1})$$

$$S = \theta \left(g - \mu - \theta \frac{v^2}{2} \right) \quad (\text{H.2})$$

$$\nu = \frac{\mu - g}{v^2} + \sqrt{\left(\frac{g - \mu}{v^2} \right)^2 + \frac{r - g}{v^2/2}} \quad (\text{H.3})$$

$$a = \frac{1}{r - g - (\sigma - 1)(\mu - g + (\sigma - 1)v^2/2)} \quad (\text{H.4})$$

$$b = (1 - a(r - g)) d^{1-\sigma} \hat{z}^{\nu+\sigma-1} \quad (\text{H.5})$$

$$r = \rho + \gamma g + \delta \quad (\text{H.6})$$

$$\tilde{L} = \Omega [(N - 1)(1 - F(\hat{z}))\kappa + (1 - \eta)\zeta(S + \delta/\chi)] \quad (\text{H.7})$$

$$\bar{z} = [\Omega (\mathbb{E}[z^{\sigma-1}] + (N - 1)(1 - F(\hat{z}))d^{1-\sigma}\mathbb{E}[z^{\sigma-1} | z > \hat{z}])]^{1/(\sigma-1)} \quad (\text{H.8})$$

$$\hat{z} = d \left(\frac{\kappa}{\bar{\pi}_{\min}} \right)^{\frac{1}{\sigma-1}} \quad (\text{H.9})$$

$$w = \frac{1}{\sigma} \bar{z} \quad (\text{H.10})$$

$$x = \zeta (1 - \eta + \eta\Theta/w) \quad (\text{H.11})$$

(Note, since $\bar{\pi}_{\min}$ is an implicit function through the \hat{z} in \bar{z} , it is easiest to add it to the system of equations instead of substituting it out).

All Firms Export Case. For any $v \geq 0$, the growth rate is given by equation (F.6), substituting for r from D.3.

$$g = \mu + \frac{(r - \mu)((\sigma - 1)/\chi - \theta)}{\theta^2 - \theta\sigma + (\sigma - 1)/\chi} + \frac{v^2}{2} \frac{\theta^2(\theta + 1 - \sigma)^2}{(\theta^2 - \theta\sigma + (\sigma - 1)/\chi)(\theta - (\sigma - 1)/\chi)}. \quad (\text{H.12})$$

Using the equilibrium g above, Ω can be found by solving the following system of equations in Ω and $\bar{\pi}_{\min}$ from equations (F.3) and (C.33) (where Ω is implicitly in the \bar{z} and \tilde{L} terms):

$$\frac{x}{\bar{\pi}_{\min}} = a (1 + (N - 1)d^{1-\sigma}) \frac{\chi}{1 - \chi} \frac{\sigma + \nu - 1}{\nu}. \quad (\text{H.13})$$

$$\bar{\pi}_{\min} = \frac{1 - \tilde{L}}{(\sigma - 1)\bar{z}^{\sigma-1}}. \quad (\text{H.14})$$

Selection into Exporting Case. Equations (G.1), (G.3), and (C.33) provide a system of 3 equations in g , Ω , and $\bar{\pi}_{\min}$. To solve this non-linear system, substitute for $\bar{\pi}_{\min}$, ν , a , b , x , r , S , \tilde{L} , \bar{z} and \hat{z} using the

general substitutions listed above to eliminate dependence on all other endogenous variables.

$$\frac{x}{\bar{\pi}_{\min}} = a \frac{\chi}{1-\chi} \frac{\sigma + \nu - 1}{\nu}, \quad (\text{H.15})$$

$$1 + \frac{\sigma - 1}{\nu} = \frac{\nu(n-1)(\theta-\sigma+1)(d^{1-\sigma}(\theta+\nu)\bar{z}^{-\theta+\sigma-1} - b\theta\bar{z}^{-\theta-\nu})}{a(g-r)} + \frac{\theta(\nu(n-1)d^{1-\sigma}(\theta+\nu)\bar{z}^{-\theta+\sigma-1} + (\nu+\sigma-1)(\theta+\nu-\sigma+1))}{\nu(\theta+\nu)(\theta-\sigma+1)} - \frac{\chi}{1-\chi} \frac{\sigma+\nu-1}{\nu}, \quad (\text{H.16})$$

$$\bar{\pi}_{\min} = \frac{1-\tilde{L}}{(\sigma-1)\bar{z}^{\sigma-1}}. \quad (\text{H.17})$$

This system of equations holds for both the general case and the baseline case of $\nu = \mu = 0$ (if using Mathematica, make sure to substitute ν to reorganize the formulas to avoid any singularity). Alternatively, for the baseline case, together with the same definition of $\bar{\pi}_{\min}$ use equations (G.4) and G.9 as the system of equations, using the same substitutions as in the more general case.

Post Solution Calculations. In either case, given the equilibrium g and Ω , the following equilibrium values can be calculated through equations (C.40), (C.30), (D.1), (D.2), (C.54), and (C.42).

$$\bar{\pi}_{\text{agg}} = \bar{\pi}_{\min} \bar{z}^{\sigma-1} - \Omega(N-1)(1-F(\hat{z}))\kappa, \quad (\text{H.18})$$

$$y = (1 - \tilde{L}) \bar{z}, \quad (\text{H.19})$$

$$\bar{U} = \begin{cases} \frac{1}{1-\gamma} \frac{(c\bar{L}M(0))^{1-\gamma}}{\rho+(\gamma-1)g} & \gamma \neq 1 \\ \frac{\rho \log(c\bar{L}M(0))+g}{\rho^2} & \gamma = 1 \end{cases}, \quad (\text{H.20})$$

$$\lambda_{ii} = \frac{1}{1 + (N-1)\hat{z}^{\sigma-1-\theta}d^{1-\sigma}}, \quad (\text{H.21})$$

$$c = (1 - \tilde{L}) \bar{z} - \eta\zeta\Omega\Theta(S + \delta/\chi). \quad (\text{H.22})$$

Consumption Equivalents. Here we focus on the log utility case. Let superscript A represent variables associated with the autarky equilibrium and superscript T denote variables in the trade equilibrium. Time zero utility under autarky is

$$U_0^A = \frac{\rho \log(c_o^A) + g^A}{\rho^2}. \quad (\text{H.23})$$

We want to know how much extra consumption is required in the autarky equilibrium to make the agent indifferent between living in the autarky and trade equilibrium. That is, we want to find μ such that

$$U_0^A = \frac{\rho \log(\mu c_o^A) + g^A}{\rho^2} = \frac{\rho^2 \log(c_o^T) + g^T}{\rho^2}. \quad (\text{H.24})$$

The μ that solves this relationship takes a simple form:

$$\mu = \exp\left(\frac{U_0^T - U_0^A}{\rho}\right). \quad (\text{H.25})$$

I. Notation

General notation principle for normalization: move to lowercase after normalizing to the scale of the economy, from nominal to real, per-capita, and relative wages (all where appropriate). For symmetric countries, denote variables related to the trade sector with an x subscript. An overbar denotes an aggregation of the underlying variable. Drop the t subscript where possible for clarity in the static equilibrium conditions.

Notation Summary

Parameters	
Multiplicative Adoption Cost	$\zeta > 0$
Labor/Goods Proportion of Adoption Cost	$0 \leq \eta \leq 1$
Goods Adoption Cost	$\Theta \geq 0$
Iceberg Trade Cost	$d \geq 1$
Fixed Export Cost	$\kappa \geq 0$
Consumer CRRA	$\gamma \geq 0$
Elasticity of Substitution between Products	$\sigma > 1$
Markup	$\bar{\sigma} := \sigma/(\sigma - 1)$
Tail Parameter of the Pareto Distribution	$\theta > 1$
Consumer's Discount Factor	$\rho > 0$
Population per Country	$\bar{L} > 0$
Adoption to Entry Cost Ratio	$\chi < 1$
Firm Death Rate	$\delta \geq 0$
Drift of Productivity Process	$\mu \geq 0$
Standard Deviation of Productivity Process	$v \geq 0$

With the simplest baseline model, and ignoring parameters which only effect the economy scale, the minimal number of parameters to calibrate is: $d, \kappa, \sigma, \rho, \theta,$ and χ .

Notation Summary

Equilibrium Variables	
Productivity	Z
CDF of the Productivity Distribution	$\Phi(Z, t)$
PDF of the Productivity Distribution	$\phi(Z, t)$
Representative Consumers Flow Utility	$U(t)$
Representative Consumers Welfare	$\bar{U}(t)$
Real Firm Value	$V(Z, t)$
Optimal Search Threshold	$M(t)$
Optimal Export Threshold	$\hat{Z}(t)$
Aggregate Nominal Expenditures on Final Goods	$Y(t)$
Aggregate Real Consumption	$C(t)$
Domestic Labor demand	$\ell_d(Z, t)$
Export Labor demand	$\ell_x(Z, t)$
Domestic Quantity	$Q_d(Z, t)$
Export Quantity	$Q_x(Z, t)$
Real Search Cost	$X(t)$
Domestic idiosyncratic prices	$p_d(Z, t)$
Export idiosyncratic prices	$p_x(Z, t)$
Nominal Wages	$W(t)$
Real Domestic Profits	$\Pi_d(Z, t)$
Real Per-market Export Profits	$\Pi_x(Z, t)$
Firm Effective Discount Rate	$r(t)$
Trade Share	$\lambda(t)$
Price level	$P(t)$
Number of Varieties	$\Omega(t)$

Normalization Notation Summary (implicit t where appropriate)

Real, Normalized, and Per-Capita Variables

Per-capita Labor Demand/Supply	$L := \bar{L}/\bar{L}$
Normalized Productivity	$z := Z/M$
Normalized Optimal Export Threshold	$\hat{z} := \hat{Z}/M$
Normalized CDF of the Productivity Distribution	$F(z, t) := \Phi(zM(t), t)$
Normalized PDF of the Productivity Distribution	$f(z, t) := M(t)\phi(zM(t), t)$
Expectation of the Normalized Productivity Distribution	$\mathbb{E}[\Psi(z)] := \int_1^\infty \Psi(z)f(z)dz$
Conditional Expectation of the Normalized Productivity Distribution	$\mathbb{E}[\Psi(z) z > \hat{z}] := \int_{\hat{z}}^\infty \Psi(z)\frac{f(z)}{1-F(\hat{z})}dz$
Normalized, Per-capita Real Firm Value Normalized by Real Wages	$v(z, t) := \frac{1}{LMw}V(Z, t)$
Normalized, Per-capita, Real Expenditures on Final Goods (i.e., Output)	$y := \frac{1}{LMP}Y$
Normalized, Per-capita Real consumption	$c := \frac{1}{LM}C$
Normalized, Per-capita, Real Adoption Cost Relative to Real Wages	$x := \frac{1}{LMw}X$
Normalized Real Wages	$w := \frac{1}{PM}W$
Normalized, Per-capita, Real, Aggregate Domestic Profits	$\bar{\pi}_d$
Normalized, Per-capita, Real, Aggregate Per-market Export Profits	$\bar{\pi}_x$