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ABSTRACT

This paper develops a dynamic model of trade and growth that we use to study how openness affects economic growth. In our model, heterogeneous firms choose to either produce with their existing technology or search within the domestic economy to adopt a better technology. These choices determine the productivity distribution from which firms can acquire new technologies and, hence, the equilibrium rate of technological diffusion. Opening to trade changes the relative profitability between high and low productivity firms through expanded export opportunities and foreign competition. These reallocation effects change the timing of when firms adopt new technologies and, thus, the rate of technological diffusion. This results in growth effects from openness via within-firm productivity improvements.

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1. Introduction

How and why does opening to trade affect growth? The standard answers to these questions typically take two forms. First, openness leads to the cross-country diffusion of new and better ideas. Second, opening to trade increases the size of the market and, hence, raises the value of new idea creation/innovation. Depending on the details of the model, these mechanisms have been shown to increase economic growth as a country opens up to trade (see, for example, the pioneering works of Rivera-Batiz and Romer (1991) and Grossman and Helpman (1993)).

This paper provides a new theoretical answer to these questions. We develop a dynamic model of trade and growth where heterogeneous firms choose to produce with their existing technology or search within the domestic economy to adopt a better technology. These choices determine the productivity distribution from which firms can acquire new technologies and, hence, the equilibrium rate of technological diffusion. We use this model to show how the static reallocation effects of a trade liberalization across heterogeneous firms (i.e., low productivity firms contract, high-productivity exporting firms expand) lead to dynamic growth effects by changing the timing of when firms acquire a new technology. Because firms only acquire ideas/technology within the country there is no international technology diffusion. Moreover, the reallocation of profits incentivizes faster adoption, not market size effects. Thus, we provide a new perspective on the relationship between openness and growth.

The starting point of our analysis is a standard heterogeneous firm model in differentiated product markets as in Melitz (2003). Firms are monopolistic competitors who differ in their productivity/technology, with each firm having the opportunity to export after paying a fixed cost. Our model of technology adoption and diffusion builds on Perla and Tonetti (2014), where firms choose to either upgrade their technology or continue to produce with their existing technology in order to maximize expected discounted profits for the infinite horizon. If a firm decides to upgrade its technology, it pays a fixed cost in return for a random productivity draw from the distribution of non-adopting firms in equilibrium. Economic growth is a result, as firms are continually able to upgrade their technology by imitating other, better firms in the economy. Thus, this is a model of growth driven by endogenous technology diffusion.

We use this model to study how and why opening to trade affects the growth rate of the economy. To do so we characterize the value function of a firm and the growth rate of the economy on a balanced growth path equilibrium. We then study several issues: how changes in iceberg trade costs affect growth rates, the welfare gains from openness, and the model implied dynamics of a firm (during both normal times and trade liberalizations) in comparison to the large body of evidence on firm/establishment dynamics.

Reductions in trade costs affect growth rates because the static reallocation effects associated with a trade liberalization change firms' incentives to upgrade their technology. As trade costs

decline, low productivity firms contract as competition from foreign firms reduce their profits and value; high productivity firms expand and export, increasing their profits and value. These are the standard—static—reallocation effects of a trade liberalization emphasized in the literature (see, e.g., Melitz (2003)). Dynamic effects arise from changes in the relative value of a low productivity firm to a high productivity firm. By lowering the value of continuing to operate, opening to trade induces low productivity firms to change the time they adopt a new technology. Because the frequency at which firms upgrade their technology is intimately tied to aggregate growth, the growth rate changes as the economy becomes more open.

The underlying mechanism in our model—trade induced distributional effects giving rise to dynamic effects from trade—is theoretically distinct from the “market size” mechanisms emphasized in the previous literature. We show this distinction by studying a special case of our model with no fixed cost in which all firms export. In this model, opening to trade benefits all firms by increasing firms’ profits and values by the same proportional amount. Consistent with the well understood benefits of a larger market, opening to trade increases the expected value of adopting a new technology. However, technology adoption in our model depends on a comparison between the expected value of adoption versus the value of continuing to operate with the existing technology. A larger market also raises the value of continuing to operate by the same exact amount as the expected value of adoption. Thus, opening to trade does not affect the *relative* benefit of adoption and, hence, there is no change in a firm’s timing of adoption, no change in the rate of technology diffusion, and no change in the rate of economic growth. This result mirrors the neutrality findings in Eaton and Kortum (2001) and Atkeson and Burstein (2010) where opening to trade does not change the relative benefits of innovation.

The previous paragraphs emphasized the direct effects that opening to trade has on firms’ incentives to upgrade their technology. However, there are indirect, general equilibrium effects that can lead to either increases or decreases in economic growth. Whether reductions in trade costs increase or decrease growth depends on the technology adoption cost function and how general equilibrium effects feed through it. In our baseline parameterization adoption costs are paid in units of the final consumption good and growth increases with reductions in trade cost. Growth rates decrease, however, when adoption costs are paid by hiring labor. General equilibrium effects cause the decline in growth: lower trade costs induce firms to hire more labor to export, which increases the real wage, and thus, the cost of technology adoption. This increase in the cost of adoption can dominate the direct effect of a change in a firm’s expected benefit of adoption. Thus, our results do share the ambiguity on the direction of growth effects, as found in previous theoretical analysis of trade and growth (see, e.g., the discussion in Baldwin and Robert-Nicoud (2008)).

We perform several quantitative exercises to assess the welfare consequences of trade in our model. The welfare comparison focuses on how changes in iceberg trade costs affect growth

rates. When technology adoption costs must be paid in output, going from observed trade levels to autarky decreases growth and the welfare cost from autarky is seventy-five percent larger than what the static effects imply: a decrease of 14 versus 8 percent in expected present discounted utility. As discussed above, when technology adoption costs are paid by hiring labor, growth rates increase as a country moves toward autarky. This does not, however, imply that there are no welfare gains from trade. In our calibration, the loss of the static gains from trade are larger than the gains in growth as a country closes.

The final section of the paper explores the empirical content of our model by studying the model-implied dynamics of a firm in comparison to the evidence on firm/establishment dynamics during both normal times and trade liberalizations. Studies of the dynamics of establishments (see, e.g., Baily, Hulten, and Campbell (1992), Bartelsman and Doms (2000), Foster, Haltiwanger, and Krizan (2001)) have emphasized that there is much persistence in a firm's relative position in the productivity distribution and a substantial fraction of aggregate productivity growth comes from within-firm effects. While our model is stark and our calibration does not target firm time-series moments, we find that our model is quantitatively consistent with certain aspects of the observed persistence in relative productivity and the role of within firm effects versus reallocation in accounting for aggregate productivity growth.

Our model also speaks to the evidence on trade-induced productivity effects at the industry and firm level (see, e.g., Pavcnik (2002) and Holmes and Schmitz (2010) for a review of this evidence). An important finding in this literature is that import competition results in within-firm productivity growth or gains in X-efficiency (Leibenstein (1966)).¹ Our model provides a theoretical explanation for this evidence: low productivity/import-competing firms lose market share and profits as a country opens up to trade and this changes these firms' incentives to adopt new technologies. The result is within-firm productivity gains, as these import-competing firms adopt new technologies. This observation is important because it contrasts with the mechanisms emphasized in standard heterogeneous firm models of trade. For example, aggregate productivity gains in Melitz (2003) only arise from the reallocation of activity across firms; there is no mechanism to generate within-firm productivity growth in response to a trade liberalization. Thus, while our model is simple, it has the ability to speak to a large body of empirical evidence in ways that standard models do not.

Our model also contributes to recent work on the diffusion of ideas across people/firms. Recent contributions are those of Lucas (2009), Lucas and Moll (2014), Alvarez, Buera, and Lucas (2014), and Sampson (2014). The latter two works share our focus on open economy issues. The distinguishing characteristic between our work and Alvarez, Buera, and Lucas (2014) is that they focus on cross-country technology diffusion whereas we intentionally abstract from this.

¹A complete accounting of the wealth of evidence on this subject is not possible here, but other examples are Topalova and Khandelwal (2011) for India and Bloom, Draca, and Reenen (2011) for the affect of China on Europe.

Sampson (2014) focuses on entry dynamics and idea-flows from incumbents to entrants as a source of economic growth. While we abstract from entry dynamics, we are able to study to how opening to trade affects within-firm productivity growth. In so much as the trade induced growth effects arise from both entry and within-firm improvements, our work and Sampson's (2014) work are complements.

2. Model

2.1. Countries, Time, Consumers

There are N symmetric countries. Time is continuous and evolves for the infinite horizon. Utility of the representative consumer in country i is

$$U_i(t) = \int_t^\infty e^{-\rho(\tau-t)} \frac{C_i(\tau)^{1-\gamma}}{1-\gamma} d\tau. \quad (1)$$

The utility function $U_i(t)$ is the present discounted value of the instantaneous utility of consuming the final good. The discount rate is ρ and instantaneous utility is given by a power function, with γ denoting the inverse of the intertemporal elasticity of substitution. The final consumption good is an aggregate bundle of varieties, aggregated with a constant elasticity of substitution (CES) function by a competitive final goods producer.

Consumers supply labor to firms for the production of varieties, the fixed cost of exporting, and possibly the fixed costs for technology adoption. Labor is supplied inelastically and the total units of labor in a country are \bar{L}_i . Consumers also own the firms operating within their country and, thus, their income is the sum of profits and total payments to labor.

We abstract from borrowing or lending decisions, so consumers face the following budget constraint

$$P_i(t)C_i(t) = W_i(t)\bar{L}_i + P_i(t)\bar{\Pi}_i(t), \quad (2)$$

where $W_i(t)$ is the nominal wage rate in country i , $P_i(t)$ is the standard CES price index of the aggregate consumption good, and $\bar{\Pi}_i(t)$ is real aggregate profits (net of investment costs) in consumption units. These relationships are elaborated in detail below.

2.2. Firms

In each country there is a final good producer that supplies the aggregate consumption good competitively. Each country contains a unit mass of infinitely lived, monopolistically competitive firms. These firms are heterogeneous over productivity, Z , with cumulative distribution function $\Phi_i(Z, t)$ describing how productivity varies across firms, within a country. Each firm

alone can supply variety v . As is standard, a final good producer aggregates these individual varieties using a constant elasticity of substitution production function.

Final Good Producer. Dropping the time index for expositional clarity, the final good producer chooses the quantity to purchase of each variety:

$$\begin{aligned} \max_{Q_{ij}(v)} & \left[\sum_{j=1}^N \int_{\Omega_{ij}} Q_{ij}(v)^{(\sigma-1)/\sigma} \right]^{\sigma/(\sigma-1)} \\ \text{s.t.} & \sum_{j=1}^N \int_{\Omega_{ij}} p_{ij}(v) Q_{ij}(v) = Y_i. \end{aligned} \quad (3)$$

Y_i is nominal aggregate expenditures on consumption goods. The measure Ω_{ij} defines the endogenous set of varieties consumed in country i from country j . The parameter σ controls the elasticity of substitution across varieties.

We will drop the notation carrying around the variety identifier, as it is sufficient to identify each firm with its location and productivity level, Z . Additionally, to focus on the interactions between technology adoption, trade, and growth, we assume that all countries are symmetric. Because all countries are symmetric, we focus on the results for a typical country and abstract from notation indicating the country's location.

Individual Variety Producers. Variety producing firms hire labor, ℓ , to produce quantity Q with a linear production technology: $Q = Z\ell$. Firms can sell freely in their domestic market and also have the ability to export at some cost, controlled by parameter κ . To export, a firm must pay a fixed flow cost in units of labor, $\kappa\bar{L}W$, per foreign export market. This fixed cost is paid in market wages and is proportional to the number of consumers accessed.² Exporting firms also face iceberg trade costs, $d \geq 1$, to ship goods abroad.

Furthermore, at each instant, any firm can pay a real fixed cost $X(t)$ to adopt a new technology. If a firm decides to pay this cost, it receives a random draw from the distribution of non-adopting producers within a country, as in Perla and Tonetti (2014).

Given this environment, firms must make choices regarding how much to produce, how to price their product, whether to export, and whether to change their technology. These choices can be separated into problems that are static and dynamic. Below we first describe the more standard static problem of a firm and then describe the dynamic problem of the firm to derive the optimal technology adoption policy.

²Export costs that are proportional to the number of consumers is consistent with the customer access interpretation featured in Arkolakis (2010). As discussed in Section 4.6, this influences the scale effect properties of the model, but has no other impact.

2.3. Firms Static Problem

Given a firm's location, productivity level, and product demand, the firm's static decision is to choose the amount of labor to hire, the prices to set, and whether to export for each destination to maximize profits each instant. The firm's problem when operating within the domestic market is

$$P\Pi_d(Z) := \max_{p_d(Z), \ell_d(Z)} p_d Z \ell_d - W \ell_d. \quad (4)$$

Using the standard demand functions for individual varieties that solve equation (3) we can characterize the optimal domestic profit function as

$$\Pi_d(Z) = \frac{1}{\sigma} \left(\frac{\bar{\sigma} W}{PZ} \right)^{1-\sigma} \frac{Y}{P}, \quad \text{where } \bar{\sigma} := \frac{\sigma}{\sigma - 1}. \quad (5)$$

$\bar{\sigma}$ is the standard markup over marginal cost and Y is total expenditures on final goods in a country.

The decision to (possibly) operate in an export market is similar, but differs in that the firm faces variable iceberg trade costs and a fixed cost to sell in the foreign market. The firm's problem when operating in an export market is

$$P\Pi_x(Z) := \max_{p_x(Z), \ell_x(Z)} p_x Z d^{-1} \ell_x - W \ell_x - \kappa \bar{L} W, \quad (6)$$

where d is an iceberg trade costs and $\kappa \bar{L} W$ is the fixed cost to sell in the foreign market.

Optimal per-market export profits are

$$\Pi_x(Z) = \max \left\{ 0, \frac{1}{\sigma} \left(\frac{\bar{\sigma} d W}{PZ} \right)^{1-\sigma} \frac{Y}{P} - \bar{L} \kappa \frac{W}{P} \right\}. \quad (7)$$

Given profits described in equation (7), only firms earning positive profits actually enter a foreign market. This implies that only firms with productivity level greater than or equal to \hat{Z} export, where the cutoff productivity level is

$$\hat{Z} = \bar{\sigma} d (\sigma \bar{L} \kappa)^{\frac{1}{\sigma-1}} \left(\frac{W}{P} \right) \left(\frac{W}{Y} \right)^{\frac{1}{\sigma-1}}. \quad (8)$$

Total real firm profits equal the sum of domestic profits plus the sum of exporting profits across export markets,

$$\Pi(Z, t) := \Pi_d(Z, t) + (N - 1) \Pi_x(Z, t). \quad (9)$$

2.4. Firms Dynamic Problem

Given the static profit functions and a perceived law of motion for the productivity distribution, each firm has the choice of when to acquire a new technology, Z . Define $g(t)$ as the growth rate of total expenditures.³ Since firms are owned by consumers, they choose technology adoption policies to maximize the present discounted expected value of real profits, discounting with interest rate $r(t) = \rho + \gamma g(t)$.

The recursive formulation of the firm's problem is as follows. Each instant, a firm with productivity Z chooses between continuing with its existing technology and earning flow profits of $\Pi(Z, t)$ or stopping and adopting a new technology at cost $X(t)$. In a growing economy, a firm's profits will erode and the benefits of continuing to operate its existing technology will decline until the firm enters the stopping region and it chooses to adopt a new technology.

Define the value of the firm in the continuation region as $V(Z, t)$ and $M(t)$ as the time dependent productivity boundary between continuation and adoption. $M(t)$ is a reservation productivity function, such that all firms with productivity less than or equal to $M(t)$ chose to adopt and all other firms produce with their existing technology. If a firm chooses to adopt a new technology it pays a cost and immediately receives a new productivity, i.e., a random draw from the distribution of non-adopting producers. This distribution will be a function of the optimal policy of all firms, i.e., the firm choice of when to draw a new productivity. Recursively, the optimal policy of firms will depend on the expected evolution of this distribution. Restricting focus to a Rational Expectations equilibrium implies that the net value of adoption in equilibrium is

$$V_s(t) := \int_{M(t)}^{\infty} V(Z, t) d\Phi(Z, t) - X(t). \quad (10)$$

There are several interpretations of this technology adoption choice. Mathematically, it is similar to the model of Lucas and Moll (2014) where agents randomly meet and acquire each others technology. However, in this model—since in equilibrium adopters imitate a non-adopting firm above the threshold—this meeting structure represents limited directed search towards more productive firms. Empirically, this technology choice can be thought of as tangible or intangible investments that manifest themselves as improvements in productivity like improved production practices, work practices, advertising, supply-chain and inventory management, etc. (see, for example, the discussions in Holmes and Schmitz (2010) and Syverson (2011)).

Using the connection between optimal stopping and free boundary problems, a set of partial

³Since in equilibrium many growth rates will be equal (e.g., the growth rate of total expenditures, consumption, and the minimum of the productivity distribution), we will abuse notation for the sake of exposition and overload the definition of a single growth rate: $g(t)$.

differential equations (PDEs) and boundary conditions characterize the firm's value.⁴ The PDEs and boundary values determining a firm's value are

$$r(t)V(Z, t) = \Pi(Z, t) + \frac{\partial V(Z, t)}{\partial t} \quad (11)$$

$$V(M(t), t) = \int_{M(t)}^{\infty} V(Z, t)d\Phi(Z, t) - X(t) \quad (12)$$

$$\frac{\partial V(M(t), t)}{\partial Z} = 0. \quad (13)$$

Equation (11) describes how the firm's value function evolves in the continuation region. It says that the flow value of the firm equals the flow value of profits plus the change in the value of the firm over time. Equation (12) is the value matching condition. It says that at the reservation productivity level, $M(t)$, the firm should be indifferent between continuing to operate with its existing technology and adopting a new technology. Equation (13) is the smooth-pasting condition. The smooth pasting condition can be interpreted as an intertemporal no-arbitrage condition.

A couple of comments are in order regarding the economics of this problem.

There are two forces that drive the adoption decision. First, over time the productivity distribution will improve. This eventually gives firms an incentive to adopt a new technology as the benefit of adoption grows over time. This economic force is the same as in Lucas and Moll (2014) and Perla and Tonetti (2014).

Second, general equilibrium effects are a new, additional, force that drive the adoption decision. The dependence of the firm's value function (11) on profits (which are time dependent) illustrates this feature. As an economy grows, an individual firm's profits will erode. The reason is because as other firms become better through adoption, they demand relatively more labor, and this raises wages which reduces the profits of non-adopting firms. This erosion of profits reduces the opportunity cost of continuing to operate and facilitates adoption. Our paper is about this second force—how equilibrium changes in competition and profits via trade influence adoption and growth.

Finally, we should note there is an externality in this environment. Firms are infinitesimal and do not internalize the effect their technology adoption decisions have on the evolution of the productivity distribution and, in turn, the distribution from which other firms are able to adopt.

⁴Standard references and conditions for the equivalence between optimal stopping of stochastic processes and free boundary problems are Dixit and Pindyck (1994) and Peskir and Shiryaev (2006). The deterministic stopping problem presented here is discussed on pages 110-115 of Stokey (2009).

This externality could be interpreted as a free rider problem, as firms have an incentive to wait before upgrading, and let other firms adopt first, in order to have a better chance of adopting a more productive technology.

Together with the static optimization problem described in Section 2.3, equations 11, 12, and 13 characterize optimal firm policies given equilibrium prices and a law of motion for the productivity distribution.

3. Equilibrium

In this section, we define a Balanced Growth Path (BGP) Symmetric equilibrium and then walk through the steps to characterize this equilibrium. The key steps are to determine the law of motion for the productivity distribution, impose some functional form assumptions, and normalize the economy to a stationary environment. In computing a BGP equilibrium we characterize market clearing conditions, the value functions of the firm, and the equilibrium growth rate.

3.1. Definition of a Balanced Growth Path Symmetric Equilibrium.

Definition 1. *A Balanced Growth Path Symmetric equilibrium consists of an initial distribution $\Phi(0)$ with support $[M(0), \infty)$, a sequence of distributions $\{\Phi(Z, t)\}_{t=1}^{\infty}$, firm adoption and export policies $\{M(t), \hat{Z}(t)\}_{t=0}^{\infty}$, firm price and labor policies $\{p_d(Z, t), p_x(Z, t), \ell_d(Z, t), \ell_x(Z, t)\}_{t=0}^{\infty}$, wages $\{W(t)\}_{t=0}^{\infty}$, and a growth rate $g > 0$ such that:*

- *Given aggregate prices and distributions*
 - *$M(t)$ is the optimal adoption threshold and $V(Z, t)$ is the continuation value function*
 - *$\hat{Z}(t)$ is the optimal export threshold*
 - *$p_d(Z, t)$, $p_x(Z, t)$, $\ell_d(Z, t)$, and $\ell_x(Z, t)$ are the optimal firm static policies*
- *The gross value of adoption is $\int_{M(t)}^{\infty} V(Z', t)\phi(Z', t)dZ'$*
- *Markets clear at each date t*
- *Aggregate expenditure is stationary when re-scaled: $Y(0) = Y(t)e^{-gt}$*
- *The distribution of productivities is stationary when re-scaled:*

$$\phi(Z, t) = e^{-gt}\phi(Ze^{-gt}, 0) \forall t, Z \geq M(0)e^{gt}$$

The initial distribution must have infinite right tailed support or the economy would not be able to grow indefinitely. Requiring aggregate expenditure and the productivity distributions to be

constant after re-scaling ensures that the BGP equilibrium features balanced growth. Restricting $g > 0$ ensures that the BGP equilibrium has growth.

In order to compute an equilibrium, it remains to derive the law of motion for the productivity distribution, $\frac{\partial \phi(Z,t)}{\partial t}$, the technology adoption policy $M(t)$, the value of the firm $V(Z,t)$ and technology adoption $V_s(t)$, and the growth rate, g .⁵

3.2. Law of Motion of the Productivity Distribution

This first step in computing the equilibrium takes the reservation adoption productivity function as given and derives how the productivity distribution evolves over time. Below we highlight the key elements. A complete technical derivation is provided in Appendix B.

First, the infimum of the support of the productivity distribution is $M(t)$. Recall, that when adopting, a firm receives a random productivity draw from the distribution of non-adopting producers (i.e., those in the continuation region). Therefore, firms adopting at time t only adopt from those strictly above $M(t)$. Thus, $M(t)$ is like an absorbing barrier sweeping through the distribution from below and, thus, $M(t)$ is the infimum of the support.

The Kolmogorov Forward Equation (KFE) describes the evolution of the productivity distribution for $Z > M(t)$. The KFE is simply the the flow of adopters times the density they are redistributed into.⁶ The flow of adopters is determined by the rate at which adoption threshold sweeps across the density, $M'(t)$, and the amount firms the adoption boundary collects as the adoption boundary sweeps across the density from below, $\phi(M(t), t)$. Thus, the flow of adopters is

$$S(t) := M'(t)\phi(M(t), t). \quad (14)$$

Two features of the environment determine the density the adopters are redistributed into. Since adopters only meet non-adopters above $M(t)$ and $M(t)$ is the infimum of support of $\Phi(Z, t)$, then the adopters are redistributed across the entire support of $\phi(Z, t)$. Since adopters draw directly from the productivity density, they are redistributed throughout the distribution in proportion to it. Thus, the redistribution density is $\phi(Z, t)$.

Using the expression for the flow of adopters, $S(t)$, and given that the redistribution density is $\phi(Z, t)$, this implies that the KFE for productivity distribution is

$$\frac{\partial \phi(Z, t)}{\partial t} = \underbrace{M'(t)\phi(M(t), t)}_{\text{flow of adopters}} \underbrace{\phi(Z, t)}_{\text{redistribution density}}. \quad (15)$$

⁵The firms optimal static policies are standard and details are provided in Appendix A.2.

⁶Other terms (e.g., the drift and stochastic diffusion terms) that typically show up in the KFE for other economic environments are not present here because a firm's productivity changes only when it adopts.

Given the law of motion for the productivity distribution described in equation (15), Proposition 1 below characterizes the distribution at any date t .

Proposition 1. $\phi(Z, t)$ evolves according to repeated left truncations at $M(t)$ for *any* $M(t)$ and $\Phi(0)$. A solution to the Kolmogorov Forward Equation is

$$\phi(Z, t) = \frac{\phi(Z, 0)}{1 - \Phi(M(t), 0)}. \quad (16)$$

That is, the distribution at date t is a truncation of the initial distribution at the minimum of support at time t , $M(t)$. It is important to note that Proposition 1 holds independent of the type of the initial distribution and independent of whether the economy is on a balanced growth path.

3.3. The Cost of Technology Adoption

To compute a BGP we must make a functional form assumption for the cost of technology adoption.⁷ In doing so, we want to achieve two goals. First, the parameterization should permit balanced growth by allowing costs to grow over time; otherwise the adoption cost becomes irrelevant over time. Second, the parameterization should isolate the reallocation effects on growth and remove any role that market size or scale effects play; see Section 4.4 and 4.6 for further discussion.

Given these requirements, we model the adoption cost as a convex combination of hiring domestic labor and spending final goods where the spending on final goods is proportional to average profits.

Assumption 1. *The cost of technology adoption is*

$$X(t) = \zeta \left[(1 - \eta) \frac{W(t)}{P(t)} + \eta \bar{\Pi}(t) \right]. \quad (17)$$

This cost function has two parameters: $\eta \in [0, 1]$ controls the degree to which the cost of adoption requires labor as opposed to goods and ζ affects the overall cost of upgrading technology.

To understand the implications of this functional form, consider the case when $\eta = 0$. In this case, domestic labor is required to implement technology adoption. $W(t)$ will grow on the balanced growth path at rate g and thus satisfies our first requirement of a cost function.

When $\eta = 1$, a quantity of the final good is required and that quantity is proportional to average

⁷This is a key conceptual issue with the continuous time formulation relative to the discrete time formulation in Perla and Tonetti (2014). In the discrete time formulation, firms forgo period profits when they adopt, thus the cost of adoption is lost operating profits. In the continuous time formulation, adoption happens instantaneously; thus some formal assumptions about the cost of adoption are required to make adoption costly.

profits, $\bar{\Pi}(t)$, in the economy. Tying the quantity of final goods to average profits is intuitive in so much as higher average profits imply acquiring a new technology (in expectation) is more valuable, hence the cost of acquiring this technology will adjust to reflect this.⁸ This satisfies our first requirement of a cost function, as average profits grow at rate g on the balanced growth path.

As discussed in Section 4.6, the baseline choice of scaling by average profits was constructed to isolate the role of selection. In particular, if $\kappa = 0$ and all firms are exporters, then the equilibrium will have the same growth rate for any iceberg trade costs. The reason is that the value of continuing to operate and the expected (net of the adoption cost) value of adoption all scale with profits in the same way and, thus, opening to trade does not affect the relative benefit of adoption and, hence, there is no change in the firm's adoption choice.

3.4. Market Clearing

There are two aggregate, market clearing conditions in this model. The labor market clearing condition is

$$\bar{L} = \int_{M(t)}^{\infty} \ell_d(z, t) \phi(Z, t) dz + (N - 1) \left\{ \int_{\hat{Z}(t)}^{\infty} [\ell_x(z, t) + \bar{L}\kappa] \phi(Z, t) dz \right\} + \zeta(1 - \eta)S(t), \quad (18)$$

where ℓ_d is the labor demand for domestic production, ℓ_x is variable labor demand per export market, and $\bar{L}\kappa$ is the labor used for the fixed costs of exporting. The final term $\zeta(1 - \eta)S(t)$ is the labor used in the adoption of technology. Since $\zeta(1 - \eta)$ units of labor are spent per adoption and there is a flow of $S(t)$ adopters each instant, $\zeta(1 - \eta)S(t)$ is total amount of labor used in this activity.

The final goods market clearing condition equates real final sales to consumption plus the amount paid to technology adoption costs (investment). Real final sales are defined as

$$\frac{Y(t)}{P(t)} := \frac{W(t)}{P(t)}L + \int_{M(t)}^{\infty} \Pi(Z, t) \phi(Z, t) dz, \quad (19)$$

which then must be equal to final consumption plus technology adoption costs

$$\frac{Y(t)}{P(t)} = C(t) + \zeta\eta\bar{\Pi}(t)S(t). \quad (20)$$

Like in the labor market clearing condition, the final term arises because $\zeta\eta\bar{\Pi}(t)$ final goods are spent per adoption and there is a flow of $S(t)$ adopters each instant.

⁸One way to think about this cost function empirically are switchover disruptions of the type discussed in Holmes, Levine, and Schmitz Jr (2012) and that the cost of these disruptions are proportional to average profits in the economy.

3.5. Normalizing the Environment

To compute a balanced growth path, we first transform the problem to be stationary. We do this by normalizing the economy by the reservation productivity threshold $M(t)$. Regarding notation, all normalized variables are denoted as lower case version of the relationships described above (the only exceptions are the productivity distributions). Below, we highlight the key normalized relationships.

Define the change of variables for the productivity of a firm as $z := Z/M(t)$, noting that the normalized adoption threshold equals 1. Define the growth rate of the adoption threshold as $g(t) := M'(t)/M(t)$ and of the normalized real wage as $g_w(t) := w'(t)/w(t)$. Define the following normalized, real, per-capita values: $w(t) := \frac{W(t)}{M(t)P(t)}$, $\pi(Z, t) := \frac{\Pi(Z, t)}{LM(t)w(t)}$, $x(t) := \frac{X(t)}{LM(t)w(t)}$, and $v(z, t) := \frac{V(Z, t)}{LM(t)w(t)}$.⁹ The normalized productivity distributions relative to the adoption threshold are defined as:

$$F(z, t) := \Phi(Z, t) \quad \text{and} \quad f(z, t) = M(t)\phi(zM(t), t).$$

3.5.1. Static Equilibrium Relationships

Equations (5) and (7) describe firm profits, equation (8) describes the export threshold, and equation (18) provides the market clearing condition which will determine the equilibrium wage—all in terms of equilibrium objects like aggregate output and prices. Although closed form expressions for wages, profits, and export thresholds in terms of model fundamentals can only be derived for special cases of the parameter set, in general they can be expressed as the solution to a set of simple implicit equations.¹⁰

The normalized real wage as a function of parameters and the export threshold solves

$$w = \frac{1}{\sigma} \left[\mathbb{E} [z^{\sigma-1}] + (N-1)(1-F(\hat{z}))d^{1-\sigma} \mathbb{E} [z^{\sigma-1} | z > \hat{z}] \right]^{\frac{1}{\sigma-1}}, \quad (21)$$

where the expectation is taken with respect to $F(z)$. The expression in the square brackets is a measure of normalized aggregate productivity, where the first term captures domestic producers and the second term captures exporters, accounting for the amount of exporters, the implicit productivity loss from iceberg trade costs, and the higher productivity of exporters driven by selection into exporting.

⁹Note the normalization of profits and adoption costs is relative to real, normalized wages. See Appendices C and D for a complete description of the normalization.

¹⁰See Appendix C for the complete derivation of the implicit equations that provide all of the normalized static equilibrium objects as functions of only parameters.

It is convenient to express normalized total real profits of a firm as

$$\begin{aligned}\pi(z) &= \tilde{\pi}z^{\sigma-1} + (N-1)(\tilde{\pi}d^{1-\sigma}z^{\sigma-1} - \kappa) && \text{if } z \geq \hat{z} \\ \pi(z) &= \tilde{\pi}z^{\sigma-1} && \text{otherwise}\end{aligned}\quad (22)$$

where,

$$\tilde{\pi} = \frac{1 - (N-1)(1 - F(\hat{z}))\kappa - (1 - \eta)\zeta gf(1)}{(1 - \sigma)(\bar{\sigma}w)^{\sigma-1}}. \quad (23)$$

The $\tilde{\pi}$ term is important, as it summarizes how changes to trade barriers affect profits and, in turn, technology choices.

The export productivity threshold satisfies

$$\hat{z} = d \left(\frac{\kappa}{\tilde{\pi}} \right)^{\frac{1}{\sigma-1}}. \quad (24)$$

Note that firm profits, in addition to being a function of \hat{z} and parameters are also a function of the growth rate g . To provide the full system of equations that characterize the balanced growth path equilibrium, we need to analyze the dynamic equilibrium relationships.

3.5.2. Dynamic Equilibrium Relationships

We use the definition of normalized variables and equation (23) with its explicit dependence on time to convert the continuation value function, value matching condition, and smooth pasting condition in equations (11)-(13) to a normalized system of dynamic equations:

$$(r(t) - g(t) - g_w(t))v(z, t) = \pi(z, t) - g(t)z \frac{\partial v(z, t)}{\partial z} + \frac{\partial v(z, t)}{\partial t} \quad (25)$$

$$v(1, t) = \int_1^\infty v(z, t) f(z, t) dz - x(t), \quad (26)$$

$$\frac{\partial v(1, t)}{\partial z} = 0, \quad (27)$$

Equation (25) describes how the firm's normalized value function evolves in the continuation region. Equation (26) is the normalized value matching condition and equation (27) is the normalized smooth-pasting condition. The normalized adoption cost function in terms of primitives and the key equilibrium values can be expressed as

$$x(t) = \zeta \left(\eta \left(\frac{1 - g(t)\theta(1 - \eta)\zeta}{\sigma - 1} - (N - 1)(1 - F(\hat{z}(t), t))\kappa\bar{\sigma} \right) + 1 - \eta \right). \quad (28)$$

Appendix D provides the details of their derivation.

One important point to note about the system of equations in (25)-(27) is only a function of $g(t)$ and not $M(t)$. This fact is independent of whether the economy is on a balanced growth path. Moreover, this implies that the solution is stationary as long as $g(t)$ is stationary.

To characterize the equilibrium, it only remains to solve for the firm value function and the growth rate of the economy.

3.6. The Initial Distribution

The previous subsections have characterized how the economy evolves independent of whether the economy is on a balanced growth path. To allow for a balanced growth path and maintain analytical tractability in the static firm problem, we make a key parametric assumption that the initial distribution is Pareto with tail index, θ . The shape of the productivity distribution plays an important role, affecting both the dynamic technology acquisition decision of the firm and the firm's static production and export decisions.

Assumption 2. *The initial distribution of productivity is Pareto, $\Phi(Z, 0) = 1 - \left(\frac{M(0)}{Z}\right)^\theta$, with density, $\phi(z, 0) = \theta M(0)^\theta Z^{-\theta-1}$.*

Combining Assumption 2 with our solution to the Kolmogorov Forward Equation in equation (16) is very powerful. Together, they imply that the productivity distribution always remains Pareto with shape θ . We summarize this result in Proposition 2 below.

Proposition 2. *Assumption 2 together with Proposition 1 implies*

$$\phi(Z, t) = \theta M(t)^\theta Z^{-\theta-1}. \quad (29)$$

Moreover, the normalized density is Pareto with shape θ and is stationary

$$f(z) = \theta z^{-\theta-1}.$$

Proposition 2 simplifies the derivation of static equilibrium relationships and computing for balanced growth path equilibria. On the static dimension, it allows us to compute the equilibrium relationships, for all time, as one would in a variant of Melitz (2003) (see, e.g., Chaney (2008)). On the dynamic dimension, it gives us hope of finding a balanced growth path, as this assumption has been exploited in similar models such as (Kortum (1997); Jones (2005b); Perla and Tonetti (2014)).

3.7. The Firm's Value Function

In this section, we solve for the value function of a firm analytically and discuss its properties. To solve for the value function of the firm, first note that the continuation value function in (25) on a balanced growth path simplifies to

$$(r - g)v(z) = \pi(z) - gz \frac{\partial v(z)}{\partial z}, \quad (30)$$

with the critical observation being that $\frac{\partial v(z,t)}{\partial t}$ term in equation (25) equals zero and normalized wages are constant on the balanced growth path ($g_w = 0$). Thus equation (30) is an ordinary differential equation in z and is solvable with the appropriate boundary value. Using the smooth pasting condition in equation (27) as the boundary value gives an expression for the value function of a firm with normalized productivity z . The value function of a firm that is an exporter is

$$\begin{aligned} v(z|z \geq \hat{z}) &= \frac{\tilde{\pi} (1 + (N - 1)d^{1-\sigma}) z^{\sigma-1}}{(r + g(\sigma - 2))} - \frac{(N - 1)\kappa}{r - g} - (N - 1) \left(\frac{\tilde{\pi} d^{1-\sigma} z^{1-\frac{r}{g}} \hat{z}^{\sigma+\frac{r}{g}-2}}{(r + g(\sigma - 2))} - \frac{\kappa z^{1-\frac{r}{g}} \hat{z}^{\frac{r}{g}-1}}{r - g} \right) \\ &+ \frac{\tilde{\pi} z^{1-\frac{r}{g}} g(\sigma - 1)}{(r - g)(r + g(\sigma - 2))} \end{aligned} \quad (31)$$

and the value function of a firm that is not exporting is

$$v(z|z < \hat{z}) = \frac{\tilde{\pi} z^{\sigma-1}}{(r + g(\sigma - 2))} + \frac{\tilde{\pi} z^{1-\frac{r}{g}} g(\sigma - 1)}{(r - g)(r + g(\sigma - 2))}. \quad (32)$$

These value functions are decomposable into two components: (i) profits if the firm operated its technology in perpetuity and (ii) the value of the option to adopt a new technology. To show this, we compute the discounted future profits of an exporting firm with productivity level z if it never switched technology as

$$\begin{aligned} \int_t^\infty \pi(z, t + \tau) e^{-(r-g)\tau} d\tau &= \\ \frac{\tilde{\pi} (1 + (N - 1)d^{1-\sigma}) z^{\sigma-1}}{(r + g(\sigma - 2))} - \frac{(N - 1)\kappa}{r - g} - (N - 1) \left(\frac{\tilde{\pi} d^{1-\sigma} z^{1-\frac{r}{g}} \hat{z}^{\sigma+\frac{r}{g}-2}}{(r + g(\sigma - 2))} - \frac{\kappa z^{1-\frac{r}{g}} \hat{z}^{\frac{r}{g}-1}}{r - g} \right). \end{aligned} \quad (33)$$

First, notice that the terms in (33) are exactly the same as the terms on the top line of (31). This implies that the final term in equation (31) is the option value to adopt a new technology. A similar argument can be made for the value function for a non-exporting firm, where the first

term in equation (32) is perpetuity profits and the second term is the option value. Thus the value of the firm is equal to the sum of perpetuity profits plus the value of being able to adopt a new technology.

A second point to note is how each term for perpetuity profits has an intuitive explanation. The first term is the discounted flow profits a firm earns from domestic and foreign operations (ex fixed costs). The second term nets off the discounted flow of fixed costs a firm pays to export. The final two terms adjust for the fact that an exporter eventually stops exporting—as wages and sales grow over time, the cutoff productivity level drifts upward, yet a firm’s productivity level stays the same and, thus, it stops exporting at some date in the future. Thus, the final two terms net off the fixed costs it eventually stops paying and the lost flow profits (ex fixed costs) when this firm stops exporting.

Furthermore, notice that the option value is declining in a firm’s productivity level. This occurs because the option to upgrade technology is more valuable for a firm operating with poor technology than for a firm operating with more productive technology.

Finally, note that the speed of growth is important in determining the relative contribution of the option value versus perpetuity profits to determining the total value of the firm. For example, in high growth environments, the value of producing with an existing technology forever is much lower, as competitors upgrade technology rapidly and monopolistic competition erodes a stagnant firm’s market share and profits. Thus, in high growth environments, the total value of a firm is largely determined by option value and less by perpetuity profits compared to low growth environments.

3.8. Growth

We now compute the BGP growth rate of this economy. To solve for the equilibrium growth rate, we use the value matching condition in equation (26),

$$v(1) = \int_1^{\hat{z}} v(z|z < \hat{z})f(z)dz + \int_{\hat{z}}^{\infty} v(z|z \geq \hat{z})f(z)dz - x, \quad (34)$$

which is no longer a function of t , since these normalized values are constant on a BGP. Proposition 2 determines the density $f(z)$. Using $f(z)$ and the value functions in equations (31) and (32), we can compute the gross value of adoption on the right-hand side of equation (34). We can then use these expressions in equation (34) to solve for the growth rate g as a function of x , $\tilde{\pi}$, and \hat{z} . Furthermore, given Proposition 2, the cost function can be written as a function of \hat{z} and g :

$$x = \zeta \left(\eta \left(\frac{1 - g\theta(1 - \eta)\zeta}{\sigma - 1} - (N - 1)\kappa\bar{\sigma}\hat{z}^{-\theta} \right) + 1 - \eta \right). \quad (35)$$

Similarly, $\tilde{\pi}$ reduces to a function of \hat{z} and g ,

$$\tilde{\pi} = \frac{\hat{z}d^\sigma(-\theta + \sigma - 1) (\hat{z}^\theta(\zeta g(\eta - 1)\theta + 1) + \kappa - \kappa N)}{\theta(\sigma - 1) (d^\sigma \hat{z}^{\theta+1} + d(N - 1)\hat{z}^\sigma)}. \quad (36)$$

Proposition 3 uses these relationships and collects the key results of the previous sections to provide the implicit equations that define growth on the BGP.

Proposition 3. *Given Assumptions 1 and 2, there exists a Balanced Growth Path Symmetric equilibrium with the export threshold and growth rate given by the following implicit equations in \hat{z} and g :*

$$\hat{z} = d \left(\frac{\kappa}{\tilde{\pi}} \right)^{\frac{1}{\sigma-1}} \quad (37)$$

$$g = \frac{(\sigma - 1 + (N - 1)\theta d^{1-\sigma} \hat{z}^{-\theta+\sigma-1}) \tilde{\pi} + (N - 1)(-\theta + \sigma - 1)\hat{z}^{-\theta}\kappa}{x(\gamma + \theta - 1)(\theta - \sigma + 1)} - \frac{\rho}{\gamma + \theta - 1}. \quad (38)$$

Proposition 3 characterizes what a solution to the balanced growth path is—an export threshold and growth rate that must satisfy (37) and (38). The complication is that these are implicit functions and are difficult to work with analytically outside of some special cases. Thus, to facilitate the analysis, in the next section we calibrate our model and then numerically compute several comparative statics to illustrate how and why opening to trade affects growth.

4. How and Why Openness Affects Growth

In this section we use the model to study how and why opening to trade affects the growth rate of the economy.

4.1. Calibration

Table 1 outlines the calibration of our model. We choose one set of parameters based on reference to previous literature. These are described in the top panel of Table 1. The bottom panel of Table 1 details parameters which we calibrate to target features of the data.

We set the number of countries equal to 10. The number of countries is simply a choice of the scale of the economy and has no material impact on the answer to the question regarding how growth responds to trade. Section 4.6 discusses the role of scale in our economy in more detail.

The discount rate is set to equal one percent. We set the intertemporal elasticity of substitution (IES) equal to 0.50. Given CRRA preferences, this corresponds to a γ of 2.0. Together with a growth rate of two percent, our choice of the discount rate and IES implies a real interest rate

Table 1: Calibration

Parameter	Value	Source or Target
Number of Countries, N	10	—
Discount Rate, ρ	0.01	—
IES, $1/\gamma$	0.50	—
Curvature Parameters, σ	3.0	Broda and Weinstein (2006)
Search Cost Denomination, η	1.0	—
Iceberg Trade Cost, d	4.07	23.2 percent aggregate trade share
Pareto Shape Parameter, θ	3.15	Exporters domestic shipments $4.8 \times$ non-exporters
Export Fixed Cost, κ	0.03	20 percent of establishments export
Technology Adoption Cost, ζ	7.44	2 percent growth rate

Note: The target moment for an import trade share of 23.2 comes from Simonovska and Waugh (2014); The targets regarding exporters are from Bernard, Eaton, Jensen, and Kortum (2003).

of five percent.

The parameter σ that controls the elasticity of substitution across varieties equals 3. This value is near the median estimate of the elasticity of substitution across varieties from Broda and Weinstein (2006). They built upon the methodology of Feenstra (1994) to estimate this parameter in a model that is closely related to the static trade part of our model.

The bottom panel of Table 1 reports the resulting parameters that were picked to target several moments in the data. Specifically, the iceberg cost, Pareto shape parameter, and fixed costs (export and technology) were picked to target four moments about aggregate trade, exporters, and growth: (i) US manufacturing import share is 23.2 (Simonovska and Waugh (2012)), (ii) exporters domestic shipments are 4.8 times larger than non-exporters, (iii) 20 percent of establishments export and (iv) a two percent growth rate. The second and third moments on the number of exporters are taken from the statistics reported in Bernard, Eaton, Jensen, and Kortum (2003) for the US manufacturing sector.

The fixed cost to export amounts to about forty percent of destination flow profits. The technology adoption cost is substantial, as it is 7.44 times average flow profits. Finally, the iceberg trade cost is large as well. This is not surprising given the well known fact that large frictions to trade are necessary to reconcile observed trade flows.

The Pareto shape parameter is consistent with several studies that focus on estimating these parameters in models that share the same static dimensions with our model. In particular, the value of 3.15 is close to Simonovska and Waugh's (2014) point estimate of 3.6 for the Melitz

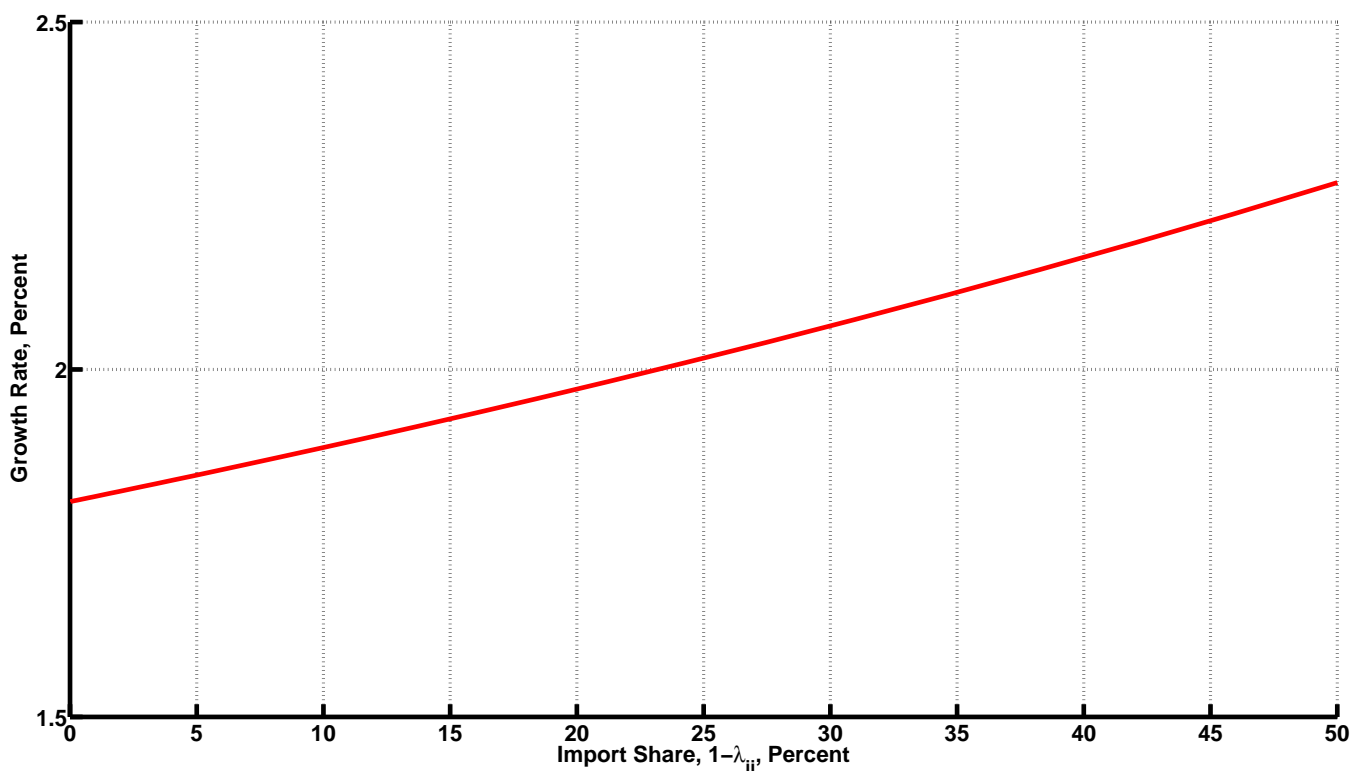


Figure 1: Growth Rate and Import Share

(2003) model. Bernard, Eaton, Jensen, and Kortum (2003) and Eaton, Kortum, and Kramarz (2011) find similar values using similar firm-level moments.

4.2. How Openness Affects Growth

To illustrate how growth depends on openness, we start from the baseline calibration and vary the iceberg trade costs to trace out how growth rates differ across economies with different degrees of openness. Figure 1 plots the results. The vertical axis reports the growth rate in percent. The horizontal axis reports the import share for a country. Recall that reductions in the trade costs increase import share, so a larger import share corresponds to lower trade costs. As an orientation device, note that when the import share equals 23 percent, the growth rate is 2 percent as calibrated.

Figure 1 shows that the growth rate is higher in equilibria in which countries are more open and trade more. For example, when trade costs are lowered such that the trade share increases from 20 percent to 50 percent, the growth rate increases from 2 percent to 2.3 percent. At the other extreme, when the economy is closed and countries do not trade with each other, the growth rate is about 1.8 percent.

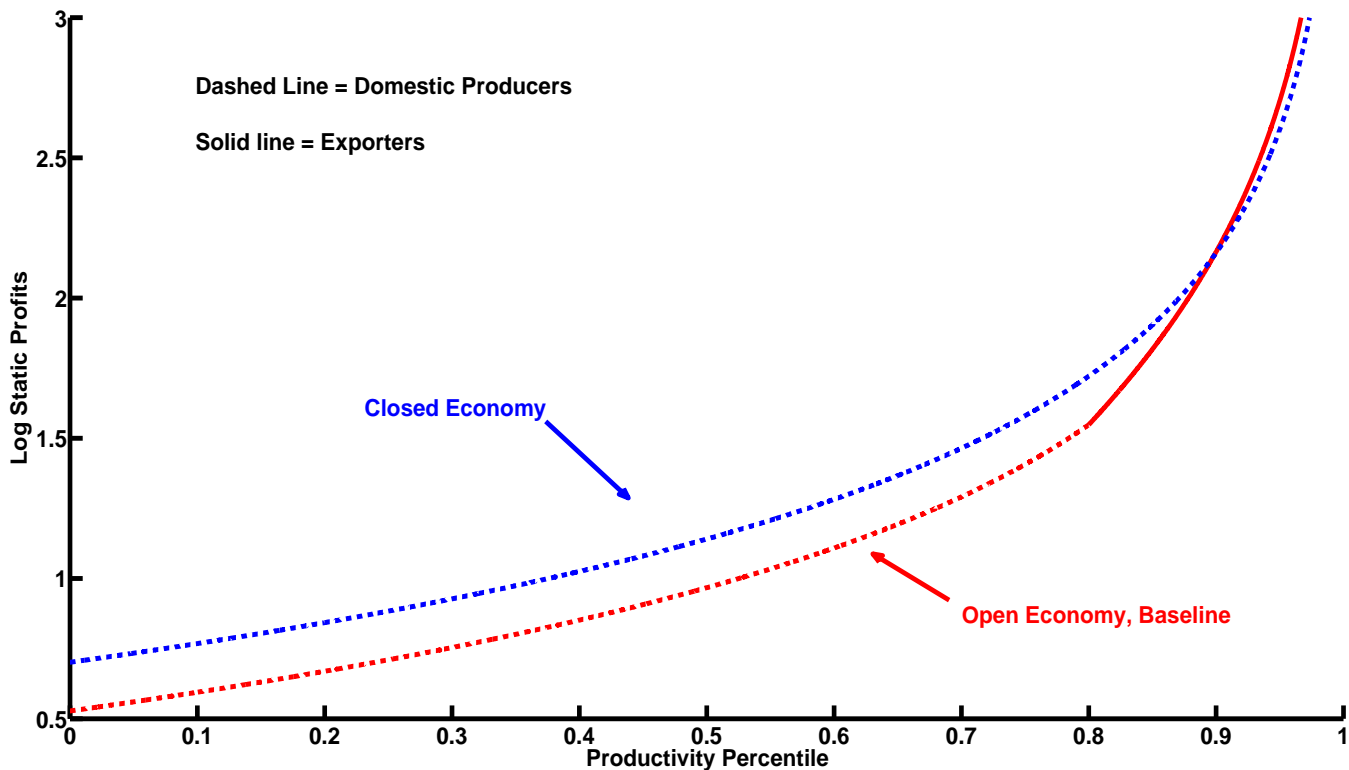


Figure 2: Static Profits and Productivity

4.3. Why Does Openness Affects Growth? Reallocation Effects.

The result in Figure 1 is interesting because these growth effects are a result of the standard—static—reallocation effects of a trade liberalization emphasized in, e.g., Melitz (2003). More specifically, opening to trade changes the relative profitability between high and low productivity firms which affects the incentives of firms to adopt a new technology.

To understand the incentives of a firm to adopt a new technology, consider the same value matching condition as in equation (34):

$$\underbrace{\frac{\tilde{\pi}g(\sigma - 1)}{(r(g) - g)(r(g) + g(\sigma - 2))} + \frac{\tilde{\pi}}{(r(g) + g(\sigma - 2))}}_{v(1)} = \underbrace{\int_1^{\infty} v(z)f(z)dz}_{v_s} - x, \quad (39)$$

where the left-hand side is the continuation value of an adopting firm, $v(1)$, and the right-hand side is the expected value of adoption, v_s , net of adoption costs. Equation (39) says that adopting firms are indifferent between continuing to produce with their existing technology or, instead, paying to receive a new productivity draw. In the language of real options, a firm exercises the option to adopt at the instant the payoff of operating with existing technology equals the payoff of exercising the option. Opening to trade changes the relative payoffs and, hence, changes the timing of when a firm exercises this option.

Opening to trade decreases the payoff to operating low productivity technologies and increases the incentive for a firm to adopt earlier. The reason the payoff to operating decreases is because lower trade barriers increase foreign competition and reduce the profits of low productivity firms who are not exporting. Mathematically, this shows up as a reduction in $\tilde{\pi}$ in equation (39) and, in turn, lowers the continuation value of a just adopting firm. Figure 2 illustrates this force by plotting the log of a firm’s static profit function versus the percentile of its productivity level. Firms at the bottom of the distribution—the same firms who are about to exercise the option to adopt—lose operating profits which lowers the value of continuing to operate. This gives firms an incentive to adopt sooner than later.

Figure 3(a) and 3(b) illustrate this by plotting the log of both the left and right-hand side of the value matching condition in equation (39) as a function of g , where the equilibrium g is determined by the intersection point. For lower trade costs, the continuation value of an adopting firm is lower at all growth rates—this is the downward shift of the $v(1)$ curve between figures 3(a) and 3(b). Holding everything else constant, the reduction in the benefits of continuing to operate with existing technology results in faster growth.

Ultimately, the response of the equilibrium growth rate depends on the change in the value of continuing to operate relative to the change in the value of adoption. How the value of adoption changes is subtle and relates to the reallocation of profits away from low productivity firms to high productivity firms.

To understand how the value of adoption changes, recall from the discussion in Section 3.7 that the net value of adoption is approximately equal to the expected perpetuity profits that a new technology can yield. Opening to trade lowers profits for low and medium productivity firms, but raises profits for high productivity firms that can export (Figure 2 illustrates this point). This implies that the value of adoption decreases by less than the decline in the value of continuing to operate with a low productivity technology. Thus, the opportunity cost of operating existing technology decreases and firms adopt sooner than later.

Between Figures 3(a) and 3(b), the expected value of adoption, v_{st} , declines, but not as much as the reduction in the continuation value of an adopting firm. Thus, the equilibrium growth rate increases from 1.8 percent to 2 percent.

4.4. Market Size Effects

The mechanism driving growth in our model is distinct from the “market size” mechanisms emphasized in the previous literature, i.e., the ability to spread the same cost of adoption across a larger market resulting in growth effects from openness. To highlight this distinction, we turn off the reallocation effects from trade and we show that the incentives to adopt operate in the *opposite* direction relative to the discussion above and that there are no growth effects from

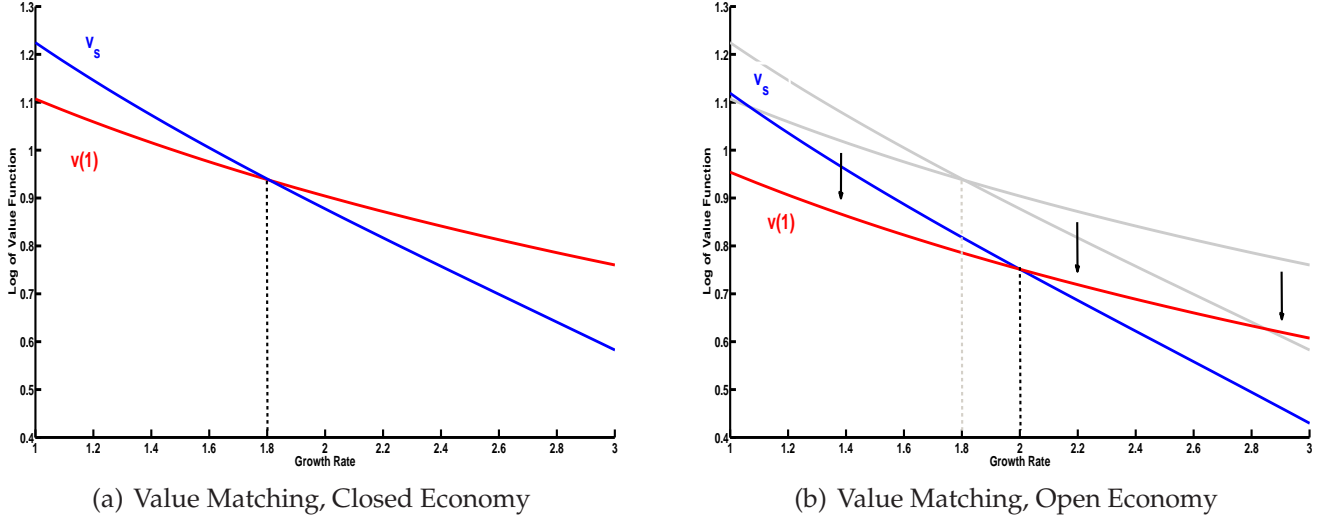


Figure 3: Value Matching in a Closed and Open Economy

opening to trade.

To focus on traditional market size forces, we set the fixed cost of exporting equal to zero, so all firms export. This trade framework is very much like a heterogeneous firm Krugman (1980) model and, for lack of a better term, we call it the “Krugman model”. In this model, real (normalized) profits are

$$\pi^k(z) = \tilde{\pi}^k z^{\sigma-1}, \quad \text{where} \quad \tilde{\pi}^k = (1 + (N - 1)d^{1-\sigma}) \frac{(\bar{\sigma}w)^{1-\sigma} y}{\sigma w}.$$

It is important to recognize that opening to trade does not reallocate profits across firms as in our baseline model. Lower trade costs result in the same proportional increase in profits for all firms. This is the sense in which we turn off reallocation effects by studying this special case.

The value matching condition for the Krugman model is

$$\tilde{\pi}^k \left(\frac{g(\sigma - 1)}{(r(g) - g)(r(g) + g(\sigma - 2))} + \frac{1}{(r(g) + g(\sigma - 2))} \right) = \quad (40)$$

$$\tilde{\pi}^k \left(\int_0^\infty \left\{ \frac{z^{\sigma-1}}{(r(g) + g(\sigma - 2))} + \left(\frac{z^{1-\frac{r(g)}{g}} g(\sigma - 1)}{(r(g) - g)(r(g) + g(\sigma - 2))} \right) \right\} f(z) dz - \tilde{\zeta} \right).$$

Like in equation (39) for the baseline model, the left-hand side is the continuation value of an adopting firm, $v(1)$. The right-hand side is expected net value of adoption, v_s .¹¹

¹¹The term $\tilde{\zeta}$ is a function of ζ and a collection of constants associated with computing average profits and then pulling out the $\tilde{\pi}^k$ specific component.

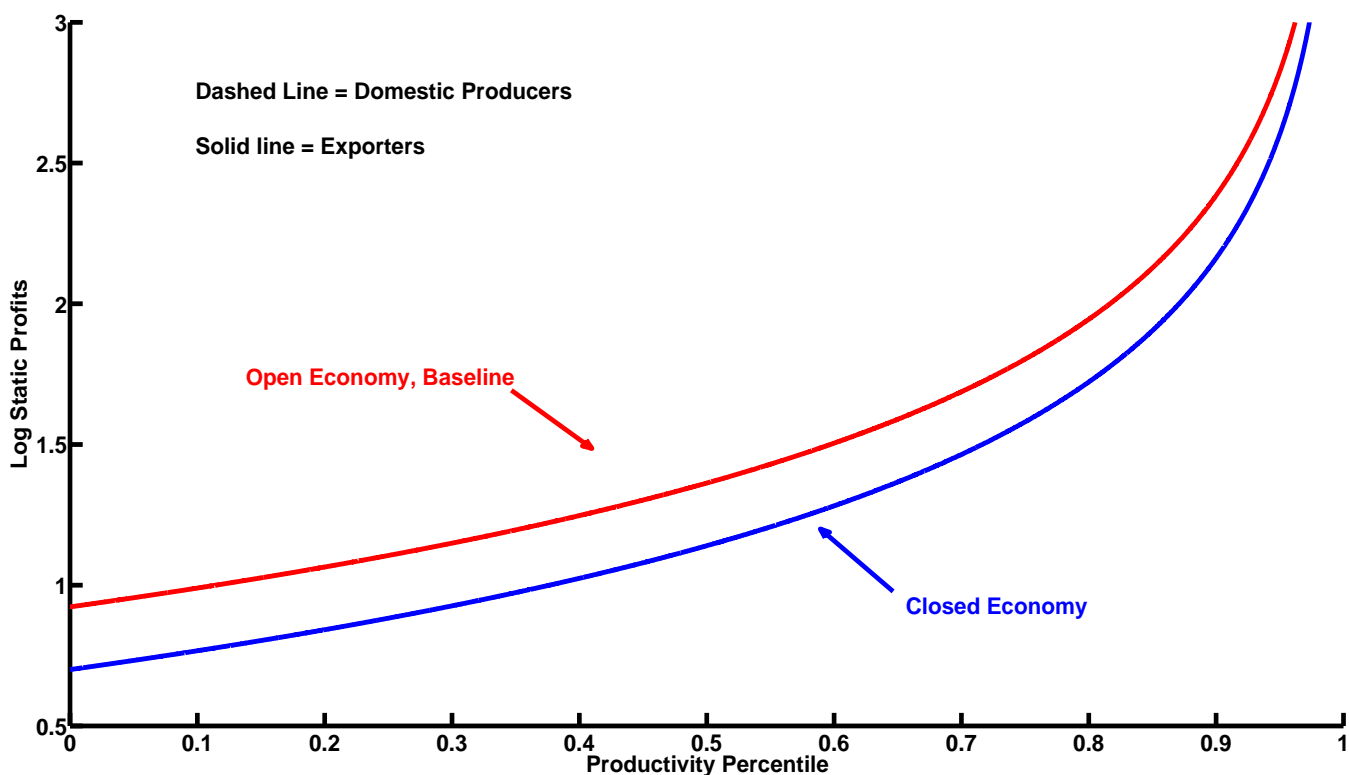


Figure 4: Krugman Model: Static Profits and Productivity

First, focus on the continuation value of an adopting firm. In contrast to our baseline model, opening to trade *increases* the payoff to operating and, thus, *decreases* the incentive of a firm to adopt earlier. The reason is a larger market increases all firms' profits—even those at the bottom of the distribution—and this shifts the continuation value up (examining how $\tilde{\pi}^k$ enters the left-hand side of equation (40) illustrates this mathematically). Figure 4 illustrates this point.¹² Profits for all firms increase by the same proportional amount and this gives firms an incentive to delay adoption.

Figure 5(a) and 5(b) illustrate this by plotting the log of each side of the value matching condition in equation (40) as a function of g . Unlike the baseline model, the continuation value of an adopting firm is higher at all growth rates—this is the upward shift of the $v(1)$ curve. Holding everything else constant, the increase in the benefits of continuing to operate with existing technology reduces firms' incentives to adopt new technologies and is a force to slow growth.

Second, focus on the net value of adoption (the right-hand side of (39)). In contrast to our baseline model, opening to trade *increases* the value of adoption. Opening to trade raises profits for all firms by the same proportional amount and this shifts the expected net value of adoption upward. This is consistent with the traditionally emphasized benefits of a larger market—

¹²We calibrated this model in the same exact way as described in Section 4.1, but dropped the moment on the number of establishments that export that disciplined the fixed cost of exporting.

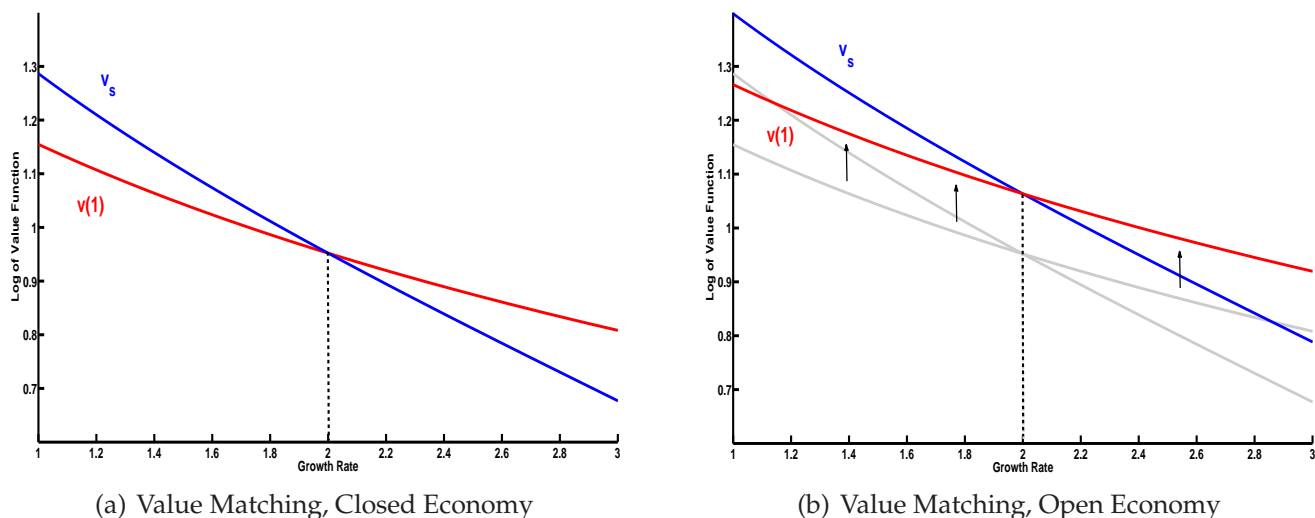


Figure 5: Krugman Model: Value Matching in a Closed and Open Economy

opening to trade increases the expected net value of adopting a new technology. As Figure 5(a) and 5(b) illustrate, the net value of adoption is higher at all growth rates. Holding everything else constant, the increase in the benefits of adoption is a force to speed growth.

What drives changes in growth, however, is the relative change between the value of continuing to operate versus adoption. These two forces—the increase in the value of operating and the increase in the value of adoption—exactly offset each other. When only market size effects operate, opening to trade has no effect on economic growth. Mathematically, equation (40) illustrates this point clearly. The profit term $\tilde{\pi}^k$ simply multiplies both sides of equation (40), so the effect of an increase in profits on the firm's continuation value is exactly canceled out by a corresponding increase in the value of adoption.

To summarize, market size mechanisms are distinct from the dynamic, reallocation effects in our baseline model. When these reallocation effects are turned off, opening to trade affects the incentives of a firm to adopt in the opposite direction by increasing both the value of adoption and the value of continuing to operate. Moreover, these effects exactly cancel out and, thus, there is no change in the relative benefit of adoption, no change in a firm's timing of adoption, and no change in the rate of economic growth.

4.5. The Adoption Cost Function

These arguments were made under the assumption that x , the adoption cost, is denominated in units of goods. Denominating the adoption cost in units of labor adds another moving part to the analysis and has the potential to change the final conclusion that growth increases with openness. A real wage increase in response to lower trade barriers would increase the value of the adoption cost. Thinking through this scenario in the context of Figure 3(b), one can see

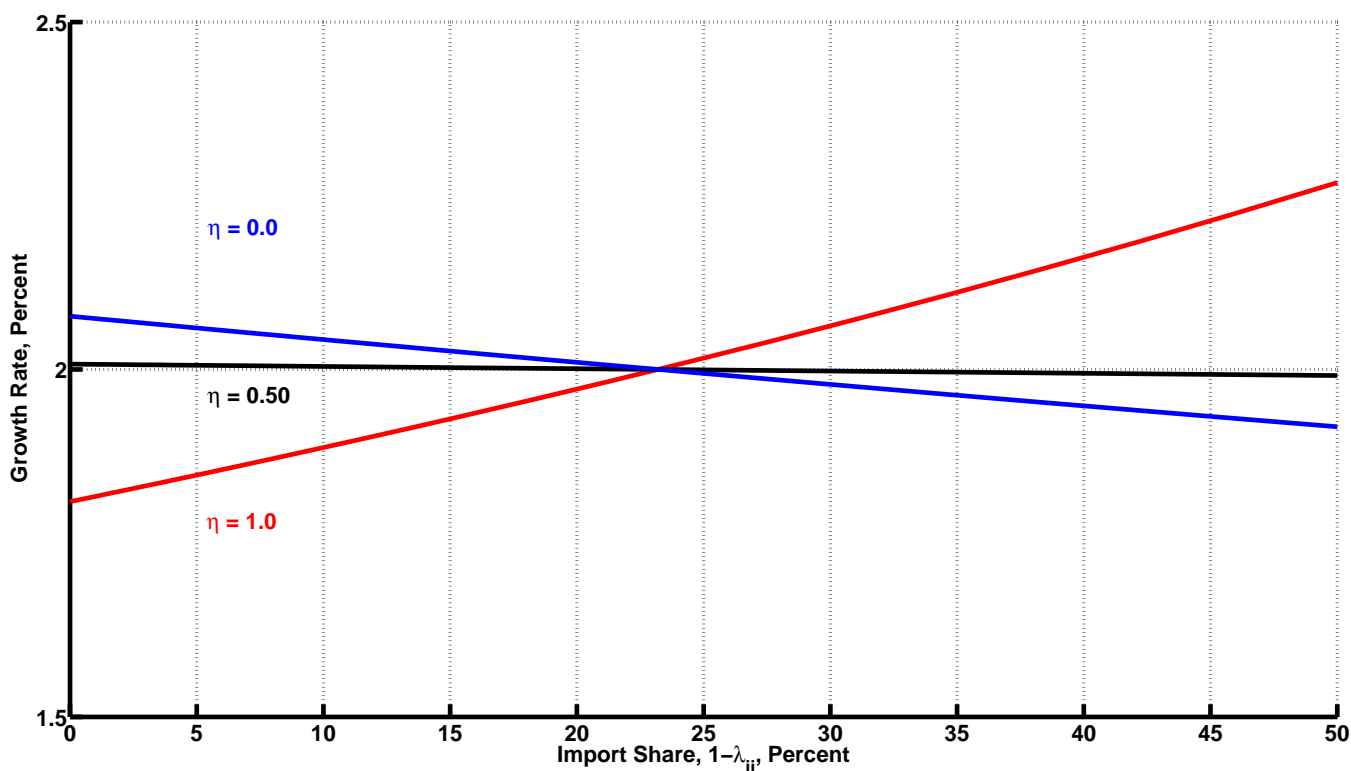


Figure 6: Adoption Costs in Labor vs. Goods

how this could potentially shift the expected net value of adoption further to the left such that growth actually declines.

Figure 6 illustrates this outcome. It repeats the exercise of reducing iceberg trade costs but for different specifications of the adoption cost (as η decreases, labor becomes a larger component of the adoption cost). Figure 6 shows that the relationship between openness and growth flattens and actually becomes negative when labor becomes a sufficiently large share of the adoption cost function. Thus, opening to trade does not always increase growth rates.

The change in the relationship between openness and growth is a result of general equilibrium effects on the wage rate. Lower trade costs lead to more demand for labor, as exporting firms produce more to sell abroad and more firms become exporters. This general equilibrium effect results in higher wages and, thus, is a force to increase the adoption cost. If labor is a sufficiently large component of the adoption cost, then this decreases the expected net value of adoption and, thus, deters adoption, resulting in slower growth. Given that the empirical evidence on the relationship between growth and trade is mixed, this result provides an insight as to why one might see different growth outcomes as countries open to trade.

These implications are specific to our baseline model with an active extensive margin of exporting. In the model where all firms export, the denomination of the adoption cost function plays no role in affecting growth. For example, consider the extreme case in which adoption

costs are paid exclusively in labor. In this model, the real wage is always proportional to average profits. Thus when the adoption costs are paid in units of labor, one can show that the expected net value of adopting a new technology is proportional to average profits as well. Thus, the expected net value of adopting a new technology increases by the same amount as the continuation value of an adopting firm resulting in no growth effects from openness.

4.6. Scale Effects

Unlike traditional endogenous growth models, scale (i.e. population size or number of countries) does not play a prominent role in our model. The section on market size effects foreshadowed this result already; expanding the size of the market via trade did not affect firms' incentives to adopt new technologies and, hence, did not affect economic growth. Because scale effects are an intimate part of traditional endogenous growth models, this section focuses on the role of scale via population size or the number of countries in our model.

Our model has neither strong or weak scale effects as categorized by Jones (2005a).¹³ The way to see this is to return to the special case of our model where all firms export (i.e., the fixed cost of exporting is set such that $\kappa = 0$). In this case, we can solve for the growth rate in closed form and see that population size does not show up in the expression. This result is summarized below.

Proposition 4 (No Scale Effects if $\kappa = 0$). *If $\kappa = 0$, the growth rate is independent of N and \bar{L} :*

$$g = \frac{(\sigma - 1)(\zeta g(\eta - 1)\theta + 1)}{\zeta\theta(\gamma + \theta - 1)(\eta(\zeta g(\eta - 1)\theta - \sigma + 2) + \sigma - 1)} - \frac{\rho}{\gamma + \theta - 1}. \quad (41)$$

Furthermore, special cases of the adoption cost function allow for the growth rate to be solved in closed form:

$$\text{if } \eta = 0, \quad g = \frac{1 - \zeta\theta\rho}{\gamma\zeta\theta + \zeta\theta^2}; \quad \text{if } \eta = 1, \quad g = \frac{\sigma - 1 - \zeta\theta\rho}{\zeta\theta(\gamma + \theta - 1)}. \quad (42)$$

The intuition behind this result is that an increase in population size (or population growth rate) does not change the relative benefit of adoption and, hence, the incentives of firms to adopt a new technology. A larger population has two offsetting effects: it increases the expected value of adoption because profits are now larger, but it also increases the value of continuing to operate with the existing technology. Given that our adoption cost function depends on the scale of the market as well, the relative net-benefit of adoption does not change with population size and thus there is no change in a firm's timing of adoption, no change in the rate of technology

¹³Jones (2005a) describes strong scale effects being characterized by the growth rate of output per worker increasing with the number of workers. Weak scale effects are characterized by the growth rate of output per worker depending on the growth rate of workers in the economy (not the level as in the case of strong scale effects).

diffusion, and no change in the rate of economic growth.

When there is selection into exporting (i.e., $\kappa > 0$), numerically we do not find scale effects either. This is due the specification that the fixed costs of exporting scales with population size. This modeling choice implies that an increase in population size has no affect on the mass of exporters or the volume of trade, and hence no additional affect on the incentives for firms to adopt technologies.

The dependence on the number of countries is a more subtle issue. In the special case of $\kappa = 0$ described above, mathematically it is clear that the number of countries does not affect the growth rate. The logic is exactly the same as a change in population, the benefits and costs of adoption exactly cancel such that an economy with more countries has the same growth rate as an economy with less countries.

In our baseline model with selection into exporting, examination of equation (38) suggests that the number of countries, N , will affect the growth rate. However, we find numerically that conditional on a given volume of trade, the growth rate is invariant to the number of countries. For example, the growth rate of an economy with ten countries and an import to GDP ratio of twenty percent is exactly the same as the growth rate of an economy with twenty countries and an import to GDP ratio of twenty percent. The issue here is that the volume of trade is sensitive to the number of countries.¹⁴ Any effect that the number of countries has on the growth rate is operative because it changes the volume of trade. Once this is controlled for, the number of countries plays no role in shaping the growth rate.

Relative to recent models of idea flows, our model shares some similarities and differences in the role that scale plays. Sampson (2014) finds a similar result in that scale plays no role in driving economic growth and the growth effects from openness. However, these results differ substantially from Alvarez, Buera, and Lucas (2014) where scale appears to play a prominent role, with the number of ideas a country has access to depending upon the number of countries. The result is that the growth rate on a balanced growth path with symmetric countries is linear in the number of countries in the economy. One critical difference in our work is the abstraction from cross-country idea diffusion.

To summarize, our model does not have strong or weak scale effects of the form described by Jones (2005a) and the number of countries plays no role either once their affect on the volume of trade is controlled for. We should remark that the absence of scale effects is surprising to even us. The lessons of previous endogenous growth models are intertwined with lessons about the role of scale. For example, the ability of openness to increase scale and, hence, growth is a key lesson of these models. However, as this and the previous section makes clear, growth effects

¹⁴See, e.g., the examination of country size in Alvarez and Lucas (2007) for an insightful discussion on the role of size in similar trade models.

Table 2: The Welfare Cost of Autarky

	$\eta = 1$ (cost in goods)	$\eta = 0$ (cost in labor)
Welfare	14.3	7.3
Growth Component	6.3	-2.4
Level Component	8.4	9.8
Consumption	9.2	10.3
Growth rate (Change)	-0.20	0.07

Note: Reports the percent change in variables between autarky and the calibrated economy. Growth rate reports the change in percentage points.

from openness can arise through mechanisms that are scale independent.

4.7. Welfare

Using the representative consumer's utility function, we can analyze the welfare implications of these growth patterns. Although growth rates can increase or decrease in response to reduced trade costs, there exist welfare gains from trade even if growth rates decline.

The first column of Table 2 presents the welfare costs of autarky for an economy in which the adoption cost is in units of goods ($\eta = 1$). In the open economy, welfare is fourteen percent higher than under autarky. Moreover, the change in welfare can be decomposed into two components, static and dynamic. The static component captures the time zero change in the level of consumption while the dynamic component accounts for the change in the growth rate. The static gain of eight percent results from the standard increase in varieties exported and increase in quantities produced for a given variety, as in Chaney (2008). The dynamic gains of six percent come from the increased growth rate under openness and are added to the static gains to generate total welfare improvements of 14.3 percent.

The second column of Table 2 document the welfare gains from openness when the cost of adoption is in labor ($\eta = 0$). Even though growth rates increase in autarky from 2 percent to 2.07 percent, there are still welfare gains of 7.3 percent from openness. Initial consumption is ten percent higher in the open economy due to increased production and exports, which more than offsets the drag of a lower growth rate to increase overall welfare.

5. Micro-Level Implications: Theory and Data

Behind growth and how growth responds to trade are the dynamic decisions made by individual firms. These decisions have implications for the the persistence of productivity at the firm level and the sources of aggregate growth (i.e., within versus between effects) both during normal times and trade liberalization episodes. This section explores the implications of our model and how these implications mesh with data on establishment dynamics.

5.1. Establishment Dynamics

Studies of the dynamics of establishments (see, e.g., Baily, Hulten, and Campbell (1992), Bartelsman and Doms (2000), Foster, Haltiwanger, and Krizan (2001)) have established several facts about productivity dynamics and aggregate productivity growth decompositions. Specifically, these studies have emphasized (i) there is much persistence in a firms relative position in the productivity distribution and (ii) a substantial fraction of aggregate productivity growth comes from within firm effects. Below, we discuss these facts and the implications of our model with respect to them.

Productivity Dynamics. Baily, Hulten, and Campbell (1992) represented the productivity dynamics of establishments by reporting a transition matrix showing the probability that an establishment in a productivity quintile moves to another quintile between two time periods. Thus the values in the transition matrix provide information about how persistent a firms relative position is, the likelihood firms experience large jumps in relative position, etc.

The left-hand column of Table 3 reproduces the transition matrix from Baily, Hulten, and Campbell (1992), Table 3. The data is from establishments in the The Longitudinal Research Database which contains information on manufacturing establishments in the US. This particular transition matrix is for the period from 1972-1977 and is adjusted to only reflect the mobility of incumbent firms (i.e., we adjusted the data to net out exiting establishments, about which our model is silent). Rows of the matrix report the productivity quintile in period t and columns report the productivity quintile in the subsequent period ($t + 5$); so row 1, column 1 says that 64.8 percent of firms in the first quintile of productivity remained in the first quintile of productivity five years later.

One key feature of the data is the persistence in productivity. The way to see this is to note the relatively large values on the diagonal of the matrix. Across quintiles, the most likely occurrence is the that an establishment remains in that productivity quintile. This persistence also has a particular ordering: the diagonal elements are the largest for the top and bottom quintile and more moderate in the intermediate quintiles. These features of the data have been emphasized in Baily, Hulten, and Campbell (1992), Bartelsman and Doms (2000), Foster, Haltiwanger, and Krizan (2001).

Table 3: Establishment Productivity Dynamics

Transition Matrix of Relative Productivity: highest productivity, quintile 1; lowest, quintile 5											
Data						Model					
	1	2	3	4	5		1	2	3	4	5
1	64.8	15.9	7.6	5.9	5.9	1	72.9	27.1	0	0	0
2	33.0	34.7	17.0	7.0	8.3	2	0	45.9	54.1	0	0
3	13.5	24.0	21.5	24.2	16.7	3	0	0	18.8	73.0	8.2
4	16.5	21.4	21.1	19.5	21.5	4	7.1	7.1	7.1	7.1	71.8
5	15.9	18.6	11.2	17.8	36.5	5	20.0	20.0	20.0	20.0	20.0

Correlation of Data and Model =0.56***

Note: Data Source: Baily, Hulten, and Campbell (1992). Rows represent the productivity quintile in period t ; columns represent productivity quintile in period $t + 5$; highest productivity, quintile 1; lowest, quintile 5. Three stars indicate statistical significance at the 1 percent level.

Our model is consistent with the persistence seen in the data. The right-hand column of Table 3 reports the transition matrix computed from a discretized version of our calibrated model. Quantitatively consistent with the data, the model is able to replicate the persistence in the first and second quintile. Qualitatively, it also replicates the relatively low persistence in intermediate quintiles that is seen in the data. While the match between the model and data is far from exact—the model was not designed to qualitatively or quantitatively target these dynamics—the model does capture these features of the data emphasized previously in the literature.

Another important feature of the data is the substantial mobility from the bottom quintile to all other productivity quintiles. For example, in the data, 16 percent of establishments go from the bottom to the top quintile. This observation is very important as this mobility speaks to the mechanism behind growth in our model. Adopting firms—at the bottom of the distribution—receive a new productivity draw from the equilibrium productivity distribution, thus the model predicts substantial mobility from the bottom of the distribution to all quintiles.

Quantitatively, the overall magnitude in mobility from the bottom is similar to the prediction of the model. In the model, because a firm receives a new productivity draw from the equilibrium productivity distribution, the probability that a firm enters into any quintile is exactly 20 percent.¹⁵ In the data, the median probability that a firm exits the bottom quintile is 17 percent.

¹⁵One might be puzzled, for example, that the model predicts that 7 percent of firms in the fourth quintile move up to the first quintile, but the model predicts that only firms at the very bottom of the distribution can move up. This is a feature of time aggregation; these firms started in the fourth quintile and over the 5 year period drifted down to the bottom of the distribution and then moved up.

As a summary measure, the bottom of Table 3 reports the correlation between the empirical transition matrix and the model implied transition matrix. The correlation coefficient is 0.55 and it is statistically significant at the one percent level. Given the parsimony of our model and that we did not target these dynamics, we view this result as a success.

The model is not perfect. And it should not be given the starkness of the mechanism and what ideas the mechanism was designed to capture. The key failure is the inability of the model to generate the incremental moves up seen in the data. For example, in the data 33 percent of firms in the second quintile move up to the first quintile. In the model, all firms wait until they arrive at the bottom of the distribution to adopt, so there is no scope for these incremental moves. Heterogeneous adoption costs, adding exogenous productivity shocks, and/or the modeling of innovation are avenues that may improve the model's predictions for establishment dynamics.

Aggregate Growth: Within vs. Between Effects. A second feature of data on establishment dynamics is that decompositions of aggregate growth find that within-firm productivity growth accounts for a meaningful fraction of total growth. Following Foster, Haltiwanger, and Krizan (2001), one can decompose an industry's aggregate TFP growth into several components: (i) a within component, (ii) a between component, (iii) a cross component (iv) and net entry component. The within component measures average firm level TFP growth of continuing establishments weighted by the firms previous period share of gross output. The later terms reflect how resources are reallocated across firms; how they covary with changes in TFP; and any effects from entry. Using data from the Census of Manufactures over a ten year window, Foster, Haltiwanger, and Krizan (2001) find that 48 percent of aggregate TFP growth comes from the within component. Earlier work in Baily, Hulten, and Campbell (1992) finds 37 percent comes from the within component.

In our model, we find that the within component contributes 33 percent to aggregate TFP growth. This is a bit below, but quantitatively consistent with what is found in the data. Moreover, given that there are alternative mechanisms that could generate within-firm productivity improvements, we should expect that our model would understate the role that it plays. Viewed through the lens of this decomposition, the remaining contributions of growth in our model come from the between and cross-component. Obviously, because we abstract from entry dynamics we are unable to speak to that aspect of the data.

Related to our model, Sampson (2014) focuses on entry dynamics and how idea-flows from incumbents to entrants are a source of economic growth, in the spirit of Luttmer (2007, 2012). He shows that if entrants receive a productivity draw from a scaled version of the equilibrium productivity distribution, then this is sufficient to advance the productivity distribution as firms enter and exit, resulting in endogenous growth. The empirical implication of Sampson's (2014) model is that most of aggregate productivity growth is from the net entry component. Thus, his

work provides a complementary mechanism that, in principal, helps account for the positive effect of entry on growth.

5.2. Trade-Induced Productivity Effects

There is much evidence on trade-induced productivity effects consistent with the implications of our model. A key theme in the empirical evidence is that import competition gives rise to aggregate productivity growth with much of it attributed to within firm productivity improvements. The emphasis on (i) import competition and (ii) within firm productivity effects is very much in line with the mechanism and results of our model.

Pavcnik (2002) was an important empirical study of the establishment level productivity effects from a trade liberalization using modern methods of measurement. Pavcnik (2002) studied Chile's trade liberalization in the late 1970s and she found large, within-plant productivity improvements for import-competing firms that are attributable to trade. There was no evidence that exporters had any productivity improvements attributable to trade and no evidence of trade induced productivity gains from exit.¹⁶

Many subsequent studies for different countries and/or data sets have found similar results. In Brazil, Muendler (2004) found import competition lead to within firm productivity gains. Several studies of India's trade liberalization find related results. Topalova and Khandelwal (2011) found large within-firm productivity gains associated with declines in output tariffs which proxy for increases in import competition. Also in India, Sivadasan (2009) finds increases in industry TFP from tariff reductions, with 55 percent of these gains associated with within-firm productivity gains. Bloom, Draca, and Reenen (2011) finds within-firm productivity gains in Europe from Chinese import competition. In preliminary work, Steinwender (2014) finds heterogeneous responses with the least productive firms increasing their productivity the most in response to increases in import competition.

Our model is consistent with and provides a theoretical explanation for this evidence. In our model, firms losing market share and profits from import competition become more likely to upgrade their technology. The mechanism leads to what appear to be trade-induced productivity gains at the establishment level. That our model is consistent with within-firm productivity gains from import competition contrasts with standard/static models of trade. For example, Melitz (2003) deals exclusively with the reallocation of activity across firms; there is no mechanism to generate within-firm productivity growth in response to a trade liberalization. Thus, while our model is simple, it has ability to speak to a large body of empirical evidence for which

¹⁶To be clear, Pavcnik (2002) observed productivity improvements from exit, but there were no differential gains from exit across sectors of different trade orientation (i.e., import competing vs. non-traded, etc.).

there have been few theoretical explanations.¹⁷

There are aspects of evidence on firm-level adjustments to trade liberalization that our model cannot speak to. In particular, the evidence on the productivity enhancing role of increased export opportunities (see, e.g., Trefler (2004) or Bustos (2011)). In our model, exporters do not change their technology as trade costs are reduced. This may seem puzzling because a firm can choose to acquire a new technology whenever it wants and exporters are now facing a larger market in which a better technology would be even more profitable. Exporting firms, however, do not respond immediately because technology adoption is costly and risky. Exporters are already operating relatively high productivity technologies, so the chances they will adopt an even better technology is low, making it not worthwhile to pay the adoption cost. Extending the model with an incremental innovation decision is an avenue for future research that may help the model on this dimension (see, e.g., Benhabib, Perla, and Tonetti (2014)).

6. Conclusion

This paper contributes a dynamic model of growth and international trade, driven by technology diffusion. Firms choose to upgrade their productivity through technology adoption to remain competitive and profitable, with the incentives to upgrade dependent on the shape of the endogenously determined productivity distribution. Highly productive firms benefit from a decline in trade costs, as they are the exporters who can take advantage of increased sales abroad. Low productivity firms only sell domestically and are hurt by the increased competition from foreign firms. Under most calibrations, in equilibrium this leads lower productivity firms to upgrade their technology more frequently, which increases aggregate growth. The increased pace of technology adoption has aggregate benefits beyond those to the individual firm, since other upgrading firms may adopt its improved technology. However, aggregate growth rates do not always increase in response to reduced trade costs, since the growth response to increased openness depends on the cost of technological improvement and the strength of general equilibrium wage effects. Nonetheless, while the gains and losses from reduced trade barriers are not distributed evenly across firms, the representative consumer who owns all firms benefits from openness.

¹⁷Alternative explanations (via measurement) for these productivity gains are changes in the mix of intermediate inputs (Amiti and Konings (2007) and Goldberg, Khandelwal, Pavcnik, and Topalova (2010)), and product mix rationalization (Bernard, Redding, and Schott (2011)). Also, Holmes, Levine, and Schmitz Jr (2012), Impullitti and Licandro (2013), Bloom, Romer, Terry, and Van Reenen (2014) are models which generate changes in the technology of firms via innovation in response to increases in competition.

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Appendix

A. Environment and Optimization Problems

To demonstrate the extensibility of the model, in this appendix we derive conditions for a version of the model in which the consumer can elastically supply labor and there exists a government that taxes and subsidizes. It is trivial to turn these features off, by setting taxes and subsidies to zero and labor supply equal to labor endowment.

A.1. Consumer Preferences and Labor Supply

All countries are symmetric. In each country there exists a representative consumer of measure \bar{L} . The period utility of the consumer is given by a Cobb-Douglas function that combines final goods consumption (C) and leisure, which is total time endowment (\bar{L}) minus labor supplied (\hat{L}). The coefficient of relative risk aversion is γ . Period utility is¹⁸

$$U(C, \hat{L}) = \frac{\nu}{(1-\gamma)(1+\nu)} C^{1-\gamma} (\bar{L} - \hat{L})^{(1-\gamma)/\nu} \quad (\text{A.1})$$

Welfare is the present discounted value of consumer utility, discounted at rate ρ . The consumer purchases the final consumption goods and invests in technology adoption with a real cost of $X(t)$ per upgrading firm, where $S(t)$ is the mass of firms upgrading.¹⁹ Consumers earn wages for labor supplied and profits from their ownership of the firms. There is a government who can subsidize technology adoption (ς_s) and can tax labor income (τ_w) and firm profits (τ_π). Each period taxes are rebated to the consumer. Thus, welfare is

$$\begin{aligned} \bar{U}(\tilde{t}) &= \int_{\tilde{t}}^{\infty} U(C(t), \hat{L}(t)) e^{-\rho(t-\tilde{t})} dt \\ \text{s.t. } P(t)C(t) + P(t)(1 - \varsigma_s)SX(t) &= (1 - \tau_w)W(t)\hat{L}(t) + (1 - \tau_\pi) [P(t)\bar{\Pi}_d(t) + (N - 1)P(t)\bar{\Pi}_x(t)] \end{aligned} \quad (\text{A.2})$$

The first-order conditions for endogenous labor supply give

$$\frac{-\frac{\partial U(C, \hat{L})}{\partial \hat{L}}}{\frac{\partial U(C, \hat{L})}{\partial C}} = \frac{(1 - \tau_w)W}{P} \quad (\text{A.3})$$

Solving for labor supply,

$$\hat{L} = \bar{L} - \frac{CP}{(1-\tau_w)\nu W} \quad (\text{A.4})$$

¹⁸See King, Plosser, and Rebelo (1988) for a discussion of these preferences.

The frisch elasticity is

$$\bar{\nu}(C, \hat{L}) := \frac{\frac{\partial U(C, \hat{L})}{\partial \hat{L}}}{\hat{L} \left(\frac{\partial^2 U(C, \hat{L})}{\partial \hat{L}^2} - \frac{\left(\frac{\partial^2 U(C, \hat{L})}{\partial C \partial \hat{L}} \right)^2}{\frac{\partial^2 U(C, \hat{L})}{\partial C^2}} \right)} \quad (\text{A.5})$$

Which, using equations A.1 and A.4, is

$$= \frac{\nu\gamma}{1 - \gamma - \gamma\nu} \frac{\bar{L} - \hat{L}}{\hat{L}} = \frac{\gamma}{(1 - \tau_w)(1 - \gamma - \gamma\nu)} \frac{CP}{\hat{L}W} \quad (\text{A.6})$$

Note that the limit as $\nu \rightarrow \infty$ is the inelastic labor supply case, where $\hat{L} = \bar{L}$, $U(C, \bar{L}) = \frac{C^{1-\gamma}}{1-\gamma}$, and $\bar{\nu} = 0$.

A.2. The Static Firm Problem

Intermediate Goods Demand. There is a unit mass of infinitely lived, monopolistically competitive firms and a competitive final goods sector. The final goods sector takes prices as given and aggregates intermediate goods with a CES production function, with σ the elasticity of substitution between all available products. The standard solutions follow from maximizing the following final goods production problem,

$$\max_{Q_d, Q_x} \left[\int_M Q_d(Z)^{(\sigma-1)/\sigma} d\Phi(Z) + (N-1) \int_{\hat{Z}} Q_x(Z)^{(\sigma-1)/\sigma} d\Phi(Z) \right]^{\sigma/(\sigma-1)} \quad (\text{A.7})$$

$$\text{s.t. } \int_M p_d(Z) Q_d(Z) d\Phi(Z) + (N-1) \int_{\hat{Z}} p_x(Z) Q_x(Z) d\Phi(Z) = Y \quad (\text{A.8})$$

Defining a price index P , the demand for each intermediate product is,

$$Q_d(Z) = \left(\frac{p_d(Z)}{P} \right)^{-\sigma} \frac{Y}{P}, \quad Q_x(Z) = \left(\frac{p_x(Z)}{P} \right)^{-\sigma} \frac{Y}{P} \quad (\text{A.9})$$

$$P^{1-\sigma} = \int_M p_d(Z)^{1-\sigma} d\Phi(Z) + (N-1) \int_{\hat{Z}} p_x(Z)^{1-\sigma} d\Phi(Z) \quad (\text{A.10})$$

Static Profits. Given a tax rate on profits, τ_π , a monopolist operating domestically chooses static prices and labor demand to maximize profits, subject to the demand function given in equation

¹⁹Firms are maximizing real profits at an interest rate determined by the consumer. Hence, the investment choice of the consumer and firm is aligned, and consumers will lend money for upgrades to their existing firms. As consumers own a perfectly diversified portfolio of domestic firms, they are only diluting their own equity with this financing method, which becomes neutral in equilibrium.

A.9,

$$(1 - \tau_\pi)P\Pi_d(Z) := \max_{p,\ell} \{ (1 - \tau_\pi) (pZ\ell - W\ell) \} \text{ s.t. equation A.9} \quad (\text{A.11})$$

Where $\Pi_d(Z)$ is the pre-tax real profits from domestic production.

Firms face a fixed cost of exporting. To export, a firm must hire labor in the foreign country to gain access to foreign consumers. This fixed cost is paid in market wages, is proportional to the number of consumers accessed, and may be partially rebated by a fixed export cost subsidy (ς_x) paid by the government. Additionally, exports are subject to a variable iceberg trade cost, $d \geq 1$, so that firm profits from exporting to a single country (i.e., export profits per market) are

$$(1 - \tau_\pi)P\Pi_x(Z) := \max_{p,\ell} \{ (1 - \tau_\pi) (p\frac{Z}{d}\ell - W\ell - \bar{L}(\kappa - \varsigma_x)W) \} \text{ s.t. equation A.9} \quad (\text{A.12})$$

Optimal firm policies consist of $p_d(Z)$, $p_x(Z)$, $\ell_d(Z)$, and $\ell_x(Z)$ and determine $\Pi_d(Z)$ and $\Pi_x(Z)$. As is standard, it is optimal for firms to charge a constant markup over marginal cost, $\bar{\sigma}$.

$$p_d(Z) = \bar{\sigma} \frac{W}{Z} \quad (\text{A.13})$$

$$p_x(Z) = \bar{\sigma} d \frac{W}{Z} \quad (\text{A.14})$$

$$\ell_d(Z) = \frac{Q_d(Z)}{Z}, \quad \ell_x(Z) = d \frac{Q_x(Z)}{Z} \quad (\text{A.15})$$

To derive firm profits, take equation A.11 and divide by $(1 - \tau_\pi)P$ to get $\Pi_d(Z) = \frac{p_d(Z)}{P} Q_d(Z) - \frac{Q_d(Z)}{Z} \frac{W}{P}$. Substitute from equation A.13, as $W = \frac{Z p_d(Z)}{\bar{\sigma}}$, to yield $\Pi_d(Z) = \frac{1}{\bar{\sigma}} \frac{p_d(Z)}{P} Q_d(Z)$. Finally, use $Q_d(Z)$ from equation A.9 to show

$$\Pi_d(Z) = \frac{1}{\bar{\sigma}} \left(\frac{p_d(Z)}{P} \right)^{1-\sigma} \frac{Y}{P}. \quad (\text{A.16})$$

Using similar techniques, export profits per market are

$$\Pi_x(Z) = \max \left\{ 0, \frac{1}{\bar{\sigma}} \left(\frac{p_x(Z)}{P} \right)^{1-\sigma} \frac{Y}{P} - \bar{L}(\kappa - \varsigma_x) \frac{W}{P} \right\} \quad (\text{A.17})$$

Since there is a fixed cost to export, only firms with sufficiently high productivity will find it profitable to export. Solving equation A.17 for the productivity that earns zero profits gives the export productivity threshold. That is, a firm will export iff $Z \geq \hat{Z}$, where \hat{Z} satisfies

$$\left(\frac{p_x(\hat{Z})}{P}\right)^{1-\sigma} = \sigma \bar{L}(\kappa - \varsigma_x) \frac{W}{Y}, \quad (\text{A.18})$$

$$\hat{Z} = \bar{\sigma} d (\sigma \bar{L}(\kappa - \varsigma_x))^{\frac{1}{\sigma-1}} \left(\frac{W}{P}\right) \left(\frac{W}{Y}\right)^{\frac{1}{\sigma-1}}. \quad (\text{A.19})$$

A.3. Adoption Costs

In order to upgrade its technology a firm must buy some goods and hire some labor. The functional form chosen for adoption costs has implications for the presence of scale effects in this economy. The chosen baseline allows us to highlight the new mechanism relating trade and growth, and abstract from the standard scale effects present in previous papers.

The amount of labor that needs to be hired to upgrade a firm is proportional to the population of the economy.

The amount of goods that needs to be purchased to upgrade a firm is proportional to the expected profits of a firm. Hence, the goods required per customer increase with the scale of the economy—otherwise the relative costs of upgrading would become infinitesimal in the long-run. Define Θ as the amount of goods required to upgrade a firm. ζ controls the overall cost of technology adoption, while η controls how much of the costs are to hire labor versus buy goods.

The pre-subsidy real cost of upgrades is denoted by X .

$$PX := \bar{L}\zeta [(1 - \eta)W + \eta PM\Theta] \quad (\text{A.20})$$

For the majority of the appendix, the specification of Θ will be kept general so that alternative cost and scaling functions can be considered.²⁰ As will be discussed, the relationship between Θ and expected profits is constructed to isolate the role of selection, such that the growth rate in a Krugman-style economy with $\kappa = 0$ is independent of N , \bar{L} , and d for any η .

²⁰The factoring by M and division by P in the definition of Θ is to aid normalization to a stationary environment.

A.4. Static Equilibrium Conditions

Let aggregate profits from domestic production be $\bar{\Pi}_d$ and aggregate export profits per market be $\bar{\Pi}_x$.

$$\bar{\Pi}_d := \int_M^\infty \Pi_d(Z) d\Phi(Z) \quad (\text{A.21})$$

$$\bar{\Pi}_x := \int_{\hat{Z}}^\infty \Pi_x(Z) d\Phi(Z) \quad (\text{A.22})$$

Total labor demand is the sum of labor used for domestic production, export production, the fixed cost of exporting, and technology adoption. Equating labor supply and demand yields

$$\hat{L} = \int_M^\infty \ell_d(Z) d\Phi(Z) + (N-1) \int_{\hat{Z}}^\infty \ell_x(Z) d\Phi(Z) + (N-1)(1 - \Phi(\hat{Z}))\kappa\bar{L} + \bar{L}(1 - \eta)\zeta S \quad (\text{A.23})$$

The quantity of final goods must equal the sum of consumption and investment in technology adoption. Thus, the resource constraint is

$$\frac{Y}{P} = C + \bar{L}\eta\zeta M\Theta S \quad (\text{A.24})$$

The trade share, λ is

$$\lambda = \int_{\hat{Z}}^\infty \left(\frac{p_x(Z)}{P} \right)^{1-\sigma} d\Phi(Z). \quad (\text{A.25})$$

The government must run a balanced budget each instant. That is, income from taxes on profits and wages must equal subsidies to technology adoption times the amount of firms upgrading, plus the subsidies to fixed cost of export times the amount of exporting firms.

$$\tau_w W \hat{L} + \tau_\pi (P\bar{\Pi}_d + (N-1)P\bar{\Pi}_x) = \varsigma_s PXS + \varsigma_x (N-1)\bar{L}(1 - \Phi(\hat{Z}))W \quad (\text{A.26})$$

A.5. Dynamic Equilibrium Conditions

Firm's Problem. Define a firm's total pre-tax real profits as

$$\Pi(Z, t) := \Pi_d(Z, t) + (N-1)\Pi_x(Z, t) \quad (\text{A.27})$$

Where from equation A.17, $\Pi_x(Z, t) = 0$ for firms who do not export. Let $V(Z, t)$ be the value of a firm with productivity Z at time t . Given optimal static policies,

$$r(t)V(Z, t) = (1 - \tau_\pi)\Pi(Z, t) + \frac{\partial V(Z, t)}{\partial t} \quad (\text{A.28})$$

$$V(M(t), t) = \int_{M(t)}^{\infty} V(Z, t)d\Phi(Z, t) - (1 - \varsigma_s)X(t) \quad (\text{A.29})$$

$$\frac{\partial V(M(t), t)}{\partial Z} = 0 \quad (\text{A.30})$$

Equation A.28 is the bellman equation for a firm continuing to produce with it's existing technology. It receives instantaneous profits and the value of a firm with the same productivity changes over time. Equation (A.29) is the value matching condition, which states that the marginal adopter must be indifferent between adopting and not adopting.²¹ Equation (A.30) is the smooth-pasting condition.²²

B. Derivation of Law of Motion and Searchers

This section describes the details of deriving the law of motion for the distribution.

Before proceeding, one key issue (that may be particularly relevant at the beginning of time) is whether $M(t)$ is continuous. The law of motion for $\Phi(Z, t)$ is written for continuous and discontinuous regions. Recall that $M(t)$ is the boundary productivity between the continuation region in which a firm produces with its existing technology and the adoption region, in which a firm pays a fixed cost to receive a new productivity draw. As a tie-breaking rule, it is assumed that agents at the threshold adopt, and hence the function is right-continuous. Define $\mathbb{S}(t)$ as the *mass* of adopters at time t . At points of continuity in $M(t)$ this mass should be 0. Define $S(t)$ as the *flow* of adopters at time t , which is not defined at points of discontinuity of $M(t)$. It is important to recognize that $M(t)$ may not be continuous, particularly at “special times,” that reset the economy like time 0 or potentially in response to an unanticipated change in a parameter value.

The adoption technology is such that agents only match other agents in the producing/continuation region, as in Perla and Tonetti (2014). Therefore, agents searching at time t only adopt from those strictly above $M(t)$. Thus, when thinking of the evolution of the distribution $M(t)$ is an

²¹The upgrade cost is paid through equity financing from the representative consumer, as shown in equation A.2.

²²Using the standard relationship between free boundary and optimal stopping time problems, the firm's problem could equivalently be written as the firm choosing a time at which it would upgrade.

absorbing barrier, and the minimum of support of the productivity density is

$$\lim_{\Delta \rightarrow 0} \inf \text{support}\{\Phi(\cdot, t + \Delta)\} = M(t) \quad (\text{B.1})$$

$$\inf \text{support}\{\Phi(\cdot, t)\} = M(t), \quad \text{at points of continuity} \quad (\text{B.2})$$

Thus $\phi(Z, t + \Delta)$ equals $\phi(Z, t)$ at points of continuity in $M(t)$. Now the mass of agents adopting at time t are those below the $M(t)$ threshold

$$\mathbb{S}(t) := \Phi(M(t), t) \quad (\text{B.3})$$

Then from equation (B.2), at points of continuity, the mass of adopters is

$$\mathbb{S}(t) = \Phi(\inf \text{support}(\cdot, t), t) = 0. \quad (\text{B.4})$$

At points of continuity of $M(t)$, there exists a flow of adopters during each infinitesimal time period. The Kolmogorov Forward Equation (KFE) for $Z > M(t)$, describing the evolution of the density, is the flow of adopters (source) times the density they draw from (redistribution density). Determining the flow of adopters is the fact that the adoption threshold sweeps across the density at rate $M'(t)$ and as the adoption boundary sweeps across the density from below it collects $\phi(M(t), t)$ amount firms. The cdf that the flow of adopters is redistributed into is determined by two features of the environment. Since adopters only meet non-adopters above $M(t)$ and $M(t)$ is the infimum of support of $\Phi(Z, t)$, then the adopters are redistributed across the entire support of $\phi(Z, t)$. Since adopters draw directly from the productivity density, they are redistributed throughout the distribution in proportion to the density. Thus, the flow of adopters $S(t)$ multiplies the cdf, $\Phi(Z, t)$, to describe how the cdf evolves as in equation 15:

$$\frac{\partial \Phi(Z, t)}{\partial t} = \underbrace{\Phi(Z, t)S(t)}_{\text{Redistributed below } Z} - \underbrace{S(t)}_{\text{Removed at } M(t)}. \quad (\text{B.5})$$

To derive this, at time t it is convenient to use the normalized version of the cdf, as described in Appendix C.1. Define $z := Z/M(t)$, $g(t) := M'(t)/M(t)$, and $\Phi(Z, t) =: F(Z/M(t), t)$. Hence, for all t , the adoption threshold is stationary at $z = M(t)/M(t) = 1$.

To characterize the normalized KFE, first differentiate the cdf with respect to t , yielding

$$\frac{\partial \Phi(Z, t)}{\partial t} = \frac{\partial F(Z/M(t), t)}{\partial t} - \frac{Z}{M(t)} \frac{M'(t)}{M(t)} \frac{\partial F(Z/M(t), t)}{\partial z}. \quad (\text{B.6})$$

Differentiating the cdf with respect to Z yields

$$\frac{\partial \Phi(Z, t)}{\partial Z} = \frac{1}{M(t)} \frac{\partial F(Z/M(t), t)}{\partial z}. \quad (\text{B.7})$$

Given that $z := \frac{Z}{M(t)}$ and $g(t) := M'(t)/M(t)$, combining eq. B.5 and eq. B.6 provides the KFE in cdfs of the normalized distribution:

$$\frac{\partial F(z, t)}{\partial t} = g(t)z \frac{\partial F(z, t)}{\partial z} + (F(z, t) - 1)S(t). \quad (\text{B.8})$$

The interpretation of this KFE is that while operating firms in the distribution are not moving in absolute terms, they are moving backwards at rate $g(t)$ relative to $M(t)$. As the minimum of support is $z = M(t)/M(t) = 1$ for all t , a necessary condition is that $F(1, t) = 0$ for all t , and therefore $\frac{\partial F(1, t)}{\partial t} = 0$. Thus, evaluating eq. B.8 at $z = 1$ gives an expression for $S(t)$,²³

$$S(t) = g(t) \frac{\partial F(1, t)}{\partial z}. \quad (\text{B.9})$$

This equation can be transformed back to the unnormalized distribution with (B.7)

$$S(t) = \frac{M'(t)}{M(t)} M(t) \frac{\partial \Phi(M(t), t)}{\partial Z} = M'(t) \phi(M(t), t) \quad (\text{B.10})$$

At points of continuity, Equation (B.10) defines the flow of “exiting” firms.²⁴

Substitute this expression for S in eq. B.10 into eq. B.5 to arrive at the desired KFE:

$$\frac{\partial \phi(Z, t)}{\partial t} = \phi(Z, t) \phi(M(t), t) M'(t). \quad (\text{B.11})$$

A solution to equation (B.11) is a truncation for **any** $M(t)$ and $F(0)$

$$\phi(Z, t) = \frac{\phi(Z, 0)}{1 - \Phi(M(t), 0)}. \quad (\text{B.12})$$

²³Equivalently, the flow of adopters can be derived as the net flow of the probability current through the adoption threshold.

²⁴At points of discontinuity in $M(t)$, a mass $\mathbb{S}(t)$ “exit” and draw from $\lim_{\Delta \rightarrow 0} \Phi(\cdot, t + \Delta)$. The law of motion at $t+ := \lim_{\Delta \rightarrow 0} t + \Delta$ is therefore

$$\begin{aligned} \Phi(Z, t+) &= \underbrace{\Phi(Z, t)}_{\text{Was below } Z} - \underbrace{\mathbb{S}(t)}_{\text{Searched}} + \underbrace{\mathbb{S}(t)\Phi(Z, t+)}_{\text{Searched and drew } \leq Z}, \quad \text{for } Z \geq M(t+) \\ \Phi(Z, t+) - \Phi(Z, t) &= -(1 - \Phi(Z, t+))\Phi(M(t), t) \end{aligned}$$

That is the distribution at date t is a truncation of the initial distribution at $M(t)$.

C. Normalized Static Equilibrium Conditions

To aid in computing a balanced growth path equilibrium, in this section we transform the problem and derive normalized static equilibrium conditions. Define the change of variables for the productivity of a firm as $z := Z/M(t)$.

C.1. Normalizing of the Productivity Distribution

Define the normalized distribution of productivity, as the distribution of productivity relative to the adoption threshold:

$$\Phi(Z, t) =: F(Z/M(t), t) \quad (\text{C.1})$$

Differentiating to obtain the pdf yields

$$\phi(Z, t) = \frac{1}{M(t)} f(Z/M(t), t) \quad (\text{C.2})$$

$$f(z, t) = M(t) \phi(zM(t), t) \quad (\text{C.3})$$

Note that if $\phi(Z, t)$ is Pareto with minimum of support $M(t)$ and tail parameter θ , then $f(z, t)$ is independent of M , and hence t : $f(z) = \theta z^{-\theta-1}$.

If an integral of the following form exists, for some unary function $\Psi(\cdot)$, substitute for $f(z)$, then do a change of variables of $z = \frac{Z}{M}$ to obtain a useful transformation of the integral.

$$\int_M^\infty \Psi\left(\frac{Z}{M}\right) \phi(Z) dZ = \int_M^\infty \Psi\left(\frac{Z}{M}\right) f\left(\frac{Z}{M}\right) \frac{1}{M} dZ = \int_{M/M}^\infty \Psi(z) f(z) dz \quad (\text{C.4})$$

The key to this transformation is that the transformation from ϕ to f introduces a $1/M$. Thus, abusing notation by using an expectation of the normalized variable,

$$\int_M^\infty \Psi\left(\frac{Z}{M}\right) \phi(Z) dZ = \mathbb{E}[\Psi(z)] \quad (\text{C.5})$$

C.2. Normalizing the Static Equilibrium

Define the following normalized, real, per-capita values: $\hat{z} := \frac{\hat{Z}}{M}$, $L := \frac{\hat{L}}{L}$, $y := \frac{Y}{LMP}$, $c := \frac{C}{LM}$, $q_d(Z) := \frac{Q_d(Z)}{LM}$, $x := \frac{X}{LMw}$, $w := \frac{W}{MP}$ and $\pi_d(Z) := \frac{\Pi_d(Z)}{LMw}$. Note the normalization of profits and adoption costs is relative to real, normalized wages.

Combining the normalized variables with equations A.13 and A.14 provides the real prices in terms of real, normalized wages.

$$\frac{p_d(Z)}{P} = \bar{\sigma} \frac{w}{Z/M} \quad (\text{C.6})$$

$$\frac{p_x(Z)}{P} = \bar{\sigma} d \frac{w}{Z/M}. \quad (\text{C.7})$$

Substituting equations C.6 and C.7 into equation A.9 and dividing by $\bar{L}M$ yields normalized quantities,

$$q_d(Z) = \bar{\sigma}^{-\sigma} w^{-\sigma} y \left(\frac{Z}{M}\right)^{\sigma} \quad (\text{C.8})$$

$$q_x(Z) = \bar{\sigma}^{-\sigma} w^{-\sigma} d^{-\sigma} y \left(\frac{Z}{M}\right)^{\sigma}. \quad (\text{C.9})$$

Divide equation A.15 by \bar{L} , then substitute from equations A.9 and C.6. Finally, divide the top and bottom by M to obtain normalized demand for production labor

$$\ell_d(Z)/\bar{L} = \bar{\sigma}^{-\sigma} w^{-\sigma} y \left(\frac{Z}{M}\right)^{\sigma-1} \quad (\text{C.10})$$

$$\ell_x(Z)/\bar{L} = \bar{\sigma}^{-\sigma} w^{-\sigma} y d^{1-\sigma} \left(\frac{Z}{M}\right)^{\sigma-1}. \quad (\text{C.11})$$

Divide equation A.10 by $P^{1-\sigma}$ and then substitute from equation C.6 for $p_d(Z)/P$ to obtain

$$1 = \bar{\sigma}^{1-\sigma} w^{1-\sigma} \left[\int_M^{\infty} \left(\frac{Z}{M}\right)^{\sigma-1} d\Phi(Z) + (N-1) \int_{\hat{z}}^{\infty} d^{1-\sigma} \left(\frac{Z}{M}\right)^{\sigma-1} d\Phi(Z) \right] \quad (\text{C.12})$$

Simplify equation C.12 by defining \tilde{E} , a measure of effective aggregate productivity. Then use equation C.5 to give normalized real wages in terms of parameters, \hat{z} , and the productivity distribution

$$\tilde{E} := \mathbb{E} [z^{\sigma-1}] + (N-1)(1-F(\hat{z}))d^{1-\sigma} \mathbb{E} [z^{\sigma-1} | z > \hat{z}] \quad (\text{C.13})$$

$$w^{\sigma-1} = \bar{\sigma}^{1-\sigma} \tilde{E} \quad (\text{C.14})$$

$$w = \frac{1}{\bar{\sigma}} \tilde{E}^{1/(\sigma-1)} \quad (\text{C.15})$$

Note that if $d = 1$ and $\hat{z} = 1$, then $w = \frac{1}{\bar{\sigma}} (N \mathbb{E} [z^{1-\sigma}])^{1/(\sigma-1)}$. Divide equations A.16 and A.17 by $\bar{L}Mw$ and substitute with equation C.14 to obtain normalized profits,

$$\pi_d(Z) = \frac{1}{\sigma} \left(\frac{p(Z)}{P} \right)^{1-\sigma} \frac{y}{w} = \frac{1}{\sigma \bar{E}} \frac{y}{w} \left(\frac{Z}{M} \right)^{\sigma-1} \quad (\text{C.16})$$

$$\pi_x(Z) = \frac{1}{\sigma \bar{E}} \frac{y}{w} d^{1-\sigma} \left(\frac{Z}{M} \right)^{\sigma-1} - (\kappa - \varsigma_x). \quad (\text{C.17})$$

Divide equations A.21 and A.22 by $\bar{L}Mw$ and use equations C.16 and C.17 to find aggregate pre-tax profits from domestic production and from exporting to one country,

$$\bar{\pi}_d = \frac{1}{\sigma \bar{E}} \frac{y}{w} \mathbb{E} [z^{\sigma-1}] \quad (\text{C.18})$$

$$\bar{\pi}_x = \frac{1}{\sigma \bar{E}} \frac{y}{w} d^{1-\sigma} (1 - F(\hat{z})) \mathbb{E} [z^{\sigma-1} | z > \hat{z}] - (1 - F(\hat{z}))(\kappa - \varsigma_x) \quad (\text{C.19})$$

Divide equation A.23 by \bar{L} , and aggregate the total labor demand from equations C.10 and C.11 to obtain normalized aggregate labor demand

$$L = \bar{\sigma}^{-\sigma} w^{-\sigma} y (\mathbb{E} [z^{\sigma-1}] + (N-1)(1 - F(\hat{z})) d^{1-\sigma} \mathbb{E} [z^{\sigma-1} | z > \hat{z}]) + (N-1)(1 - F(\hat{z}))\kappa + (1 - \eta)\zeta S \quad (\text{C.20})$$

$$= \bar{\sigma}^{-\sigma} w^{-\sigma} y \tilde{E} + (N-1)(1 - F(\hat{z}))\kappa + (1 - \eta)\zeta S \quad (\text{C.21})$$

Define \tilde{L} as a normalized quantity of labor used outside of variable production. Multiply equation C.21 by w , and use equation C.14 to show that

$$\tilde{L} := (N-1)(1 - F(\hat{z}))\kappa + (1 - \eta)\zeta S \quad (\text{C.22})$$

$$wL = \frac{1}{\bar{\sigma}} y + \tilde{L}w \quad (\text{C.23})$$

$$L = \frac{1}{\bar{\sigma}} \frac{y}{w} + \tilde{L} \quad (\text{C.24})$$

Reorganize to find real output as a function of the productivity distribution and labor supply (net of labor used for the fixed costs of exporting and adopting technology)

$$\frac{y}{w} = \bar{\sigma} (L - \tilde{L}) \quad (\text{C.25})$$

$$y = (L - \tilde{L}) \tilde{E}^{\frac{1}{\sigma-1}} \quad (\text{C.26})$$

This equation lends interpretation to \tilde{E} as being related to the average aggregate productivity. Substituting equation C.25 into equations C.16 and C.17 to obtain a useful formulation of firm profits

$$\pi_d(Z) = \frac{L-\tilde{L}}{(\sigma-1)\tilde{E}} \left(\frac{Z}{M}\right)^{\sigma-1} \quad (\text{C.27})$$

$$\pi_x(Z) = \frac{L-\tilde{L}}{(\sigma-1)\tilde{E}} d^{1-\sigma} \left(\frac{Z}{M}\right)^{\sigma-1} - (\kappa - \varsigma_x). \quad (\text{C.28})$$

Define the common profit multiplier $\tilde{\pi}$ as

$$\tilde{\pi} = \frac{L-\tilde{L}}{(\sigma-1)\tilde{E}} = \frac{L-\tilde{L}}{(\sigma-1)(\bar{\sigma}w)^{\sigma-1}} = \frac{(\bar{\sigma}w)^{1-\sigma}y}{\sigma w} \quad (\text{C.29})$$

$$\pi_d(Z) = \tilde{\pi} \left(\frac{Z}{M}\right)^{\sigma-1} \quad (\text{C.30})$$

$$\pi_x(Z) = \tilde{\pi} d^{1-\sigma} \left(\frac{Z}{M}\right)^{\sigma-1} - \kappa + \varsigma_x \quad (\text{C.31})$$

Substitute equation C.30 and C.31 into equations C.18 and C.19 to obtain a useful formulation for aggregate profits

$$\bar{\pi}_d = \tilde{\pi} \mathbb{E} [z^{\sigma-1}] \quad (\text{C.32})$$

$$\bar{\pi}_x = (1 - F(\hat{z})) (\tilde{\pi} d^{1-\sigma} \mathbb{E} [z^{\sigma-1} | z > \hat{z}] - (\kappa - \varsigma_x)). \quad (\text{C.33})$$

Combine to calculate aggregate pre-tax total profits

$$\begin{aligned} \bar{\pi}_d + (N-1)\bar{\pi}_x &= \tilde{\pi} [\mathbb{E} [z^{\sigma-1}] + (N-1)(1 - F(\hat{z}))\tilde{\pi} d^{1-\sigma} \mathbb{E} [z^{\sigma-1} | z > \hat{z}]] \\ &\quad - (N-1)(1 - F(\hat{z}))(\kappa - \varsigma_x) \end{aligned} \quad (\text{C.34})$$

Rewriting aggregate pre-tax total profits using the definition of \tilde{E} yields

$$= \tilde{\pi} \tilde{E} - (N-1)(1 - F(\hat{z}))(\kappa - \varsigma_x). \quad (\text{C.35})$$

Note that in a closed economy, $\tilde{E} = \mathbb{E} [z^{\sigma-1}]$ and therefore aggregate profits are a markup dependent fraction of normalized output $\bar{\pi}_d = \frac{L-\tilde{L}}{\sigma-1}$. Use equation C.31 set to zero to solve for \hat{z} . This is an implicit equation as $\tilde{\pi}$ is a function of \hat{z} through \tilde{E}

$$\hat{z} = d \left(\frac{\kappa - \varsigma_x}{\tilde{\pi}} \right)^{\frac{1}{\sigma-1}}. \quad (\text{C.36})$$

Take the resource constraint in equation A.24 and divide by $M\bar{L}w$ and then equation C.25 to get an equation for normalized, per-capita consumption

$$\frac{c}{w} = \frac{y}{w} - \eta\zeta\Theta S/w = \bar{\sigma} (L - \tilde{L}) - \eta\zeta\Theta \frac{S}{w}, \quad (\text{C.37})$$

$$c = (L - \tilde{L}) \tilde{E}^{\frac{1}{\sigma-1}} - \eta\zeta\Theta S. \quad (\text{C.38})$$

Normalize the cost in equation A.20 by dividing by $\bar{L}PMw$. This is implicitly a function of \hat{z} through w

$$x = \zeta (1 - \eta + \eta\Theta/w) \quad (\text{C.39})$$

Normalize the trade share in equation A.25 by substituting from equations C.7 and C.14

$$\lambda = (1 - F(\hat{z}))d^{1-\sigma} \frac{\mathbb{E}[z^{\sigma-1} | z > \hat{z}]}{\tilde{E}} \quad (\text{C.40})$$

Take the government budget constraint in equation A.26 and divide by $PM\bar{L}w$, and use equation C.35 to obtain the normalized government budget constraint,

$$\tau_w L + \tau_\pi \left(\tilde{\pi} \tilde{E} - (N-1)(1 - F(\hat{z}))(\kappa - \varsigma_x) \right) = \varsigma_s x S + \varsigma_x (N-1)(1 - F(\hat{z})). \quad (\text{C.41})$$

To calculate the normalized labor supply and elasticity, take equation A.4, divide by \bar{L} , and rearrange the second term to obtain

$$L = 1 - \frac{1}{(1-\tau_w)\nu} \frac{c}{w} \quad (\text{C.42})$$

Use equation C.37 to get

$$L = 1 - \frac{1}{(1-\tau_w)\nu} \left(\bar{\sigma}(L - \tilde{L}) - \zeta\eta\Theta \frac{S}{w} \right) \quad (\text{C.43})$$

Solving for the labor supply L provides

$$L = \frac{1}{(1-\tau_w)\nu + \bar{\sigma}} \left[(1 - \tau_w)\nu + \bar{\sigma}\tilde{L} + \zeta\eta\Theta \frac{S}{w} \right] \quad (\text{C.44})$$

$$= \frac{(1-\tau_w)\nu + \bar{\sigma}\tilde{L} + \zeta\eta\Theta \frac{S}{w}}{(1-\tau_w)\nu + \bar{\sigma}} \quad (\text{C.45})$$

From equation A.6, the frisch elasticity is²⁵

$$\bar{\nu}(c, L) = \frac{\gamma}{(1 - \tau_w)(1 - \gamma - \gamma\nu)} \frac{c}{wL}. \quad (\text{C.46})$$

Starting from C.35, use C.29, C.22, and B.9, to derive that

$$\bar{\pi}_{\text{agg}} = \frac{L - g\theta(1 - \eta)\zeta}{\sigma - 1} - (N - 1)(1 - F(\hat{z}))(\kappa\bar{\sigma} - \varsigma_x) \quad (\text{C.47})$$

As discussed previously, in the baseline economy, the cost of goods required to upgrade technology is proportional to expected firm profits. Thus, the baseline Θ is chosen to be

$$\Theta = \left(\frac{L - g\theta(1 - \eta)\zeta}{\sigma - 1} - (N - 1)(1 - F(\hat{z}))(\kappa\bar{\sigma} - \varsigma_x) \right) w \quad (\text{C.48})$$

Note that this function is independent of d , except through the \hat{z} .

D. Normalized and Stationary Dynamic Equilibrium Conditions

This section derives normalized stationary dynamic balanced growth path equilibrium conditions.

D.1. Utility and Welfare on a BGP

Assume on the BGP that \tilde{L} is constant. Normalizing the utility function defined in equation A.1 and using the substitution $C(t) = c\bar{L}M(t)$ shows that,

$$U(c, L, t) = \frac{\nu}{(1-\gamma)(\nu+1)} \bar{L}^{\frac{(\gamma-1)(\nu+1)}{\nu}} c^{1-\gamma} (1-L)^{(1-\gamma)/\nu} M(t)^{1-\gamma}. \quad (\text{D.1})$$

The time 0 welfare for a given L and c is then

$$\bar{U}(c, L) = \frac{\nu}{(1-\gamma)(\nu+1)} \bar{L}^{\frac{(\gamma-1)(\nu+1)}{\nu}} c^{1-\gamma} (1-L)^{(1-\gamma)/\nu} \int_0^\infty M(t)^{1-\gamma} e^{-\rho t} dt. \quad (\text{D.2})$$

²⁵For any set of parameters in equation C.45, in the inelastic limit as $\nu \rightarrow \infty$, $L \rightarrow 1$. Also note that in the absence taxes, fixed costs, and adoption, $L = \frac{\nu}{\nu+\xi}$.

Along the BGP $M(t)$ grows at rate g . Thus,

$$\bar{U}(c, L) = \frac{\nu}{(1-\gamma)(\nu+1)} \bar{L}^{\frac{(\gamma-1)(\nu+1)}{\nu}} c^{1-\gamma} (1-L)^{(1-\gamma)/\nu} M(0)^{1-\gamma} \int_0^\infty e^{(1-\gamma)gt} e^{-\rho t} dt, \quad (\text{D.3})$$

$$= \frac{\nu}{(1-\gamma)(\nu+1)} \bar{L}^{\frac{(\gamma-1)(\nu+1)}{\nu}} M(0)^{1-\gamma} c^{1-\gamma} (1-L)^{(1-\gamma)/\nu} \frac{1}{\rho + (\gamma-1)g}, \quad (\text{D.4})$$

$$\propto \frac{1}{1-\gamma} \frac{c^{1-\gamma} (1-L)^{(1-\gamma)/\nu}}{\rho + (\gamma-1)g}. \quad (\text{D.5})$$

D.2. Normalization of the Firm's Dynamic Problem

We proceed to derive the normalized continuation value function, value matching condition, and smooth pasting condition originally specified in equations (A.28)-(A.30). Define the normalized real value of the firm as

$$v(z, t) := \frac{V(Z, t)}{(1-\tau_\pi)LM(t)w(t)} \quad (\text{D.6})$$

Rearranging

$$V(Z, t) = (1 - \tau_\pi) \bar{L} w(t) M(t) v(Z/M(t), t) \quad (\text{D.7})$$

First differentiate the continuation value $V(Z, t)$ with respect to t in equation D.7 and divide by $w(t)M(t)(1 - \tau_\pi)\bar{L}$, using the chain and product rule. This gives,

$$\frac{1}{w(t)M(t)(1 - \tau_\pi)\bar{L}} \frac{\partial V(Z, t)}{\partial t} = \frac{M'(t)}{M(t)} v(z, t) - \frac{M'(t)}{M(t)} \frac{Z}{M(t)} \frac{\partial v(z, t)}{\partial z} + \frac{M(t)}{M(t)} \frac{\partial v(z, t)}{\partial t} + \frac{w'(t)}{w(t)} v(z, t) \quad (\text{D.8})$$

Defining the growth rate of $g(t) := M'(t)/M(t)$ and $g_w(t) := w'(t)/w(t)$. Substitute these into equation D.8, cancel out $M(t)$, and group $z = Z/M(t)$ to give

$$= (g(t) + g_w(t))v(z, t) - g(t)z \frac{\partial v(z, t)}{\partial z} + \frac{\partial v(z, t)}{\partial t}. \quad (\text{D.9})$$

Define the normalized profits from equation A.27 as

$$\pi(z, t) := \frac{\Pi(zM(t), t)}{w(t)M(t)\bar{L}}. \quad (\text{D.10})$$

Divide equation (A.28) by $M(t)w(t)(1 - \tau_\pi)\bar{L}$, then substitute for $\frac{\partial V(Z, t)}{\partial t}$ and $\frac{\partial V(Z, t)}{\partial Z}$ from (D.9)

and (D.13) in to (A.28). Finally, group the normalized profits using equation (D.10):

$$r(t)v(z, t) = \pi(z, t) + (g(t) + g_w(t))v(z, t) - g(t)z\frac{\partial v(z, t)}{\partial z} + \frac{\partial v(z, t)}{\partial t} \quad (\text{D.11})$$

$$(r(t) - g(t) - g_w(t))v(z, t) = \pi(z, t) - g(t)z\frac{\partial v(z, t)}{\partial z} + \frac{\partial v(z, t)}{\partial t} \quad (\text{D.12})$$

Equation (D.12) is the normalized version of the value function of the firm in the continuation region. Differentiating equation D.7 with respect to Z yields

$$\frac{\partial V(Z, t)}{\partial Z} = \frac{M(t)}{M(t)} \frac{\partial v(Z/M(t), t)}{\partial z} = \frac{\partial v(z, t)}{\partial z} \quad (\text{D.13})$$

To arrive at the normalized smooth pasting condition, use the derivative in equation D.13 to relate equation (A.30) to the normalized value function, and evaluate this expression at $M(t)$,

$$\frac{\partial v(1, t)}{\partial z} = 0. \quad (\text{D.14})$$

Equation (D.14) is the normalized version of the smooth pasting condition in (A.30). To arrive at the normalized value matching condition, divide equation A.29 by $M(t)w(t)\bar{L}(1 - \tau_\pi)$ to obtain

$$\frac{V(M(t), t)}{M(t)w(t)\bar{L}(1 - \tau_\pi)} = \int_{M(t)}^{\infty} \frac{V(Z, t)}{M(t)w(t)\bar{L}(1 - \tau_\pi)} \phi(Z, t) dZ - \frac{(1 - \varsigma_s)X(t)}{M(t)w(t)\bar{L}(1 - \tau_\pi)} \quad (\text{D.15})$$

Use the definition of $x(t)$ to rewrite equation D.6.

$$v(M(t)/M(t), t) = \int_M^{\infty} v(Z/M, t) d\Phi(Z, t) dz - \frac{1 - \varsigma_s}{1 - \tau_\pi} x(t) \quad (\text{D.16})$$

Finally, normalize the integral realizing it is of the form discussed in equation C.5

$$v(1, t) = \int_1^{\infty} v(z, t) f(z, t) dz - \frac{1 - \varsigma_s}{1 - \tau_\pi} x(t) \quad (\text{D.17})$$

D.3. Stationary Dynamic Solution for $\hat{z} > 1$

$\kappa > 0$ potentially introduces a margin of selection into the export market that creates a kink in the profit function if $\hat{z} > 1$. Heterogeneity across domestic producers and importers in how profits respond to changes in parameters is the essential channel through which trade affects growth. See Appendix D.4 for the model specified exactly at $\kappa = 0$ and $\hat{z} = 1$.

In a stationary equilibrium, the value function will be independent of t , g will be constant, and

$g_w = 0$. The value in a stationary equilibrium is

$$(r - g)v(z) = \pi(z) - gzv'(z) \quad (\text{D.18})$$

$$v(1) = \int_1^\infty v(z)dF(z) - \frac{1-\varsigma_s}{1-\tau_\pi}x \quad (\text{D.19})$$

$$v'(1) = 0 \quad (\text{D.20})$$

Define the value function in the export region of the productivity space, $z > \hat{z}$ separately as $v_x(z)$. For exposition and to simplify the algebra, split the regions into $v_d(z)$ and $v_x(z)$ and define the following constants: $K_1 := 1 + (N - 1)d^{1-\sigma}$, $K_2 := (N - 1)(\kappa - \varsigma_x)$, $K_3 := \frac{1-\varsigma_s}{1-\tau_\pi}x$. Assuming $\hat{z} > 1$, the system of equations that characterizes the stationary equilibrium is

$$(r - g)v_d(z) = \tilde{\pi}z^{\sigma-1} - gzv'_d(z) \quad (\text{D.21})$$

$$(r - g)v_x(z) = \tilde{\pi}K_1z^{\sigma-1} - K_2 - gzv'_x(z) \quad (\text{D.22})$$

$$v_d(1) = \int_1^{\hat{z}} v_d(z)dF(z) + \int_{\hat{z}}^\infty v_x(z)dF(z) - K_3 \quad (\text{D.23})$$

$$v'_d(1) = 0 \quad (\text{D.24})$$

Solving the ordinary differential equation in (D.18), using the smooth pasting condition as the boundary value in (D.20), and using the profit functions in (C.30) and (C.31) gives an expression for the value function of a firm with normalized productivity z :

$$v_d(z) = \frac{\tilde{\pi}z^{1-\frac{r}{g}}g(\sigma-1)}{(r-g)(r+g(\sigma-2))} + \frac{\tilde{\pi}z^{\sigma-1}}{(r+g(\sigma-2))},$$

$$v_x(z) = \frac{\tilde{\pi}z^{1-\frac{r}{g}}g(\sigma-1)}{(r-g)(r+g(\sigma-2))} + \frac{\tilde{\pi}(1+(N-1)d^{1-\sigma})z^{\sigma-1}}{(r+g(\sigma-2))} - \frac{(N-1)(\kappa-\varsigma_x)}{r-g}$$

$$- (N-1) \left(\frac{\tilde{\pi}d^{1-\sigma}z^{1-\frac{r}{g}}\hat{z}^{\sigma+\frac{r}{g}-2}}{(r+g(\sigma-2))} - \frac{(\kappa-\varsigma_x)z^{1-\frac{r}{g}}\hat{z}^{\frac{r}{g}-1}}{r-g} \right)$$

There is a unique g that satisfies the value function and the value matching equations, given by the implicit equation,

$$g = \frac{1-\tau_d(\sigma-1+(N-1)\theta d^{1-\sigma}\hat{z}^{-\theta+\sigma-1})\tilde{\pi}+(N-1)(-\theta+\sigma-1)\hat{z}^{-\theta}(\kappa-\varsigma_x)}{1-\varsigma_s} \frac{\rho}{x(\gamma+\theta-1)(\theta-\sigma+1)} - \frac{\rho}{\gamma+\theta-1} \quad (\text{D.25})$$

D.4. Stationary Dynamic Solution for $\hat{z} = 1$

If $\kappa = 0$, then $\hat{z} = 1$ identically. The bellman equation, value matching, and smooth pasting conditions are identical to that in Section D.3. However, when $\hat{z} = 1$, there is now only a single region for the value function. Using the same constants as in Section D.3, the system of equations is therefore

$$(r - g)v(z) = K_1 \tilde{\pi} z^{\sigma-1} - gzv'(z) \quad (\text{D.26})$$

$$v(1) = \int_1^{\infty} v(z) dF(z) - K_3 \quad (\text{D.27})$$

$$v'(1) = 0 \quad (\text{D.28})$$

Solving the system of equations in the same way as before gives the following g :

$$g = \frac{1 - \tau_d (\sigma - 1) (1 + (N - 1)d^{1-\sigma}) \tilde{\pi}}{1 - \varsigma_s x(\gamma + \theta - 1)(\theta - \sigma + 1)} - \frac{\rho}{\gamma + \theta - 1}. \quad (\text{D.29})$$

Further simplifying,

$$g = \frac{1 - \tau_d L - (1 - \eta)\theta g \zeta}{1 - \varsigma_s \theta x(\gamma + \theta - 1)} - \frac{\rho}{\gamma + \theta - 1}. \quad (\text{D.30})$$

E. Computing an Equilibrium

To compute static equilibrium objects, given a particular \hat{z} and g guess, use equations C.48 C.13, C.22, C.15, C.29, and C.39, collected below.

$$\tilde{E}(\hat{z}) = \mathbb{E} [z^{\sigma-1}] + (N - 1)(1 - F(\hat{z}))d^{1-\sigma} \mathbb{E} [z^{\sigma-1} | z > \hat{z}] \quad (\text{E.1})$$

$$\lambda(\hat{z}, \tilde{E}) = (1 - F(\hat{z}))d^{1-\sigma} \frac{\mathbb{E} [z^{\sigma-1} | z > \hat{z}]}{\tilde{E}} \quad (\text{E.2})$$

$$\Theta(\hat{z}, g, L) = \left(\frac{L - g\theta(1 - \eta)\zeta}{\sigma - 1} - (N - 1)(1 - F(\hat{z}))(\kappa\bar{\sigma} - \varsigma_x) \right) w \quad (\text{E.3})$$

$$w(\tilde{E}) = \frac{1}{\bar{\sigma}} \tilde{E}^{1/(\sigma-1)} \quad (\text{E.4})$$

$$\tilde{L}(\hat{z}, g) = (N - 1)(1 - F(\hat{z}))\kappa + (1 - \eta)\zeta g f(1) \quad (\text{E.5})$$

$$x(w) = \zeta (1 - \eta + \eta\Theta/w) \quad (\text{E.6})$$

$$\tilde{\pi}(L, \tilde{L}, \tilde{E}) = \frac{L - \tilde{L}}{(\sigma-1)\tilde{E}} \quad (\text{E.7})$$

Some Export If $\kappa > 0$ and $\hat{z} > 1$, the balanced growth path reduces to a system of equations in \hat{z} , g , and L from C.36, D.25, and C.45.

$$\hat{z}(\hat{z}, g, L; \varsigma_x, \tau_w) = d \left(\frac{\kappa - \varsigma_x}{\tilde{\pi}(\hat{z}, g)} \right)^{\frac{1}{\sigma-1}} \quad (\text{E.8})$$

$$g(\hat{z}, g, L; \varsigma_s, \tau_\pi, \varsigma_x) = \frac{1 - \tau_d (\sigma - 1 + (N - 1)\theta d^{1-\sigma} \hat{z}^{-\theta+\sigma-1}) \tilde{\pi}(\hat{z}, g) + (N - 1)(-\theta + \sigma - 1) \hat{z}^{-\theta} (\kappa - \varsigma_x)}{1 - \varsigma_s} \frac{\rho}{\gamma + \theta - 1} \frac{1}{x(\hat{z}, g)(\gamma + \theta - 1)(\theta - \sigma + 1)} \quad (\text{E.9})$$

$$L(\hat{z}, g, L; \tau_w) = \frac{(1-\tau_w)\nu+\bar{\sigma}\bar{L}+\zeta\eta\Theta\frac{gf(1)}{w}}{(1-\tau_w)\nu+\bar{\sigma}} \quad (\text{E.10})$$

All (or None) Export If $\kappa = 0$ and labor is inelastic, then from equation D.29, the balanced growth path is a solution to the implicit equation

$$\hat{z} = 1 \quad (\text{E.11})$$

$$g = \frac{1 - \tau_d (\sigma - 1) (1 + (N - 1)d^{1-\sigma}) \tilde{\pi}}{1 - \varsigma_s} \frac{1}{x(\gamma + \theta - 1)(\theta - \sigma + 1)} - \frac{\rho}{\gamma + \theta - 1} \quad (\text{E.12})$$

Welfare In either case, given an equilibrium g and \hat{z} , welfare can be calculated from equations C.38, D.5, and C.46.

$$c(L, \tilde{L}, \tilde{E}, \zeta, g) = (L - \tilde{L}) \tilde{E}^{\frac{1}{\sigma-1}} - \eta\zeta\Theta gf(1) \quad (\text{E.13})$$

$$\bar{U}(c, L, g) \propto \frac{1}{1-\gamma} \frac{c^{1-\gamma}(1-L)^{(1-\gamma)/\nu}}{\rho+(\gamma-1)g} \quad (\text{E.14})$$

$$\bar{v}(c, L) = \frac{\gamma}{(1 - \tau_w)(1 - \gamma - \gamma\nu)} \frac{c}{wL} \quad (\text{E.15})$$

F. Notation

General notation principle for normalization: move to lowercase after normalizing to the scale of the economy, from nominal to real, per-capita, and relative wages (all where appropriate). For symmetric countries, denote variables related to the trade sector with an x subscript. An overbar denotes an aggregation of the underlying variable. Drop the t subscript where possible for clarity in the static equilibrium conditions.

Notation Summary

Parameters	
Multiplicative Search Cost	$\zeta > 0$
Labor/Goods Proportion of Search Cost	$0 \leq \eta \leq 1$
Iceberg Trade Cost	$d \geq 1$
Fixed Export Cost	$\kappa \geq 0$
Consumer CRRA	$\gamma \geq 0$
Related to the (non-constant) Frisch Elasticity	$\nu > 0$
Elasticity of Substitution between Products	$\sigma > 1$
Markup	$\bar{\sigma} := \sigma/(\sigma - 1)$
Tail Parameter of the Pareto Distribution	$\theta > 1$
Consumer's Discount Factor	$\rho > 0$
Population per Country	$\bar{L} > 0$
Tax on Wages	$0 \leq \tau_w < 1$
Tax on Firm Profits	$0 \leq \tau_\pi < 1$
Search Subsidy	$0 \leq \varsigma_s < 1$
Fixed Export Cost Subsidy	$0 \leq \varsigma_x \leq \kappa$

Notation Summary

Equilibrium Variables	
Productivity	Z
CDF of the Productivity Distribution	$\Phi(Z, t)$
PDF of the Productivity Distribution	$\phi(Z, t)$
Representative Consumers Flow Utility	$U(t)$
Representative Consumers Welfare	$\bar{U}(t)$
Real Firm Value	$V(Z, t)$
Optimal Search Threshold	$M(t)$
Optimal Export Threshold	$\hat{Z}(t)$
Aggregate Nominal Expenditures on Final Goods	$Y(t)$
Aggregate Real Consumption	$C(t)$
Domestic Labor demand	$\ell_d(Z, t)$
Export Labor demand	$\ell_x(Z, t)$
Domestic Quantity	$Q_d(Z, t)$
Export Quantity	$Q_x(Z, t)$
Real Search Cost	$X(t)$
Domestic idiosyncratic prices	$p_d(Z, t)$
Export idiosyncratic prices	$p_x(Z, t)$
Nominal Wages	$W(t)$
Real Domestic Profits	$\Pi_d(Z, t)$
Real Per-market Export Profits	$\Pi_x(Z, t)$
Interest Rate	$r(t)$
Trade Share	$\lambda(t)$
Price level	$P(t)$
Labor demand/supply	$\hat{L}(t) \leq \bar{L}$
Frisch Elasticity	$\bar{\nu}(\hat{L}, C)$

Normalization Notation Summary (implicit t where appropriate)

Real, Normalized, and Per-Capita Variables	
Per-capita Labor Demand/Supply	$L := \hat{L}/\bar{L}$
Normalized Productivity	$z := Z/M$
Normalized Optimal Export Threshold	$\hat{z} := \hat{Z}/M$
Normalized CDF of the Productivity Distribution	$F(z, t) := \Phi(zM(t), t)$
Normalized PDF of the Productivity Distribution	$f(z, t) := M(t)\phi(zM(t), t)$
Expectation of the Normalized Productivity Distribution	$\mathbb{E}[\Psi(z)] := \int_1^\infty \Psi(z)f(z)dz$
Conditional Expectation of the Normalized Productivity Distribution	$\mathbb{E}[\Psi(z) z > \hat{z}] := \int_{\hat{z}}^\infty \Psi(z)\frac{f(z)}{1-F(\hat{z})}dz$
Normalized, Per-capita Real Firm Value Normalized by Real Wages/Taxes	$v(z, t) := \frac{1}{(1-\tau_\pi)LMw}V(Z, t)$
Normalized, Per-capita, Real Expenditures on Final Goods (i.e., Output)	$y := \frac{1}{LMP}Y$
Normalized, Per-capita Real consumption	$c := \frac{1}{LM}C$
Normalized, Per-capita, Pre-subsidy Real Adoption Cost Relative to Real Wages	$x := \frac{1}{LMw}X$
Normalized Real Wages	$w := \frac{1}{PM}W$
Normalized, Per-capita, Real, Pre-tax, Aggregate Domestic Profits	$\bar{\pi}_d$
Normalized, Per-capita, Real, Pre-tax, Aggregate Per-market Export Profits	$\bar{\pi}_x$