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# BOUNDING THE LABOR SUPPLY RESPONSES TO A RANDOMIZED WELFARE EXPERIMENT: A REVEALED PREFERENCE APPROACH 

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# Bounding the Labor Supply Responses to a Randomized Welfare Experiment: A Revealed 

 Preference ApproachPatrick Kline and Melissa Tartari
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#### Abstract

We study the short-term impact of Connecticut's Jobs First welfare reform experiment on women's labor supply and program participation decisions. A non-parametric optimizing model is shown to restrict the set of counterfactual choices compatible with each woman's actual choice. These revealed preference restrictions yield informative bounds on the frequency of several intensive and extensive margin responses to the experiment. We find that welfare reform induced many women to work but led some others to reduce their earnings in order to receive assistance. The bounds on this latter "opt-in" effect imply that intensive margin labor supply responses are non-trivial.


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The U.S., like other advanced economies, has an extensive system of transfer programs designed to provide social insurance and improve equity. By affecting work incentives, these programs can induce individuals to enter or exit the labor force (extensive margin responses) or to alter how much they earn conditional on working (intensive margin responses). ${ }^{1}$ The relative magnitude of these responses is an important input to the optimal design of tax and transfer schemes (Diamond, 1980; Saez, 2002; Laroque, 2005).

Much of the empirical literature concludes that adjustment to policy reforms occurs primarily along the extensive margin. ${ }^{2}$ Two sorts of evidence are often cited in support of this position. First, several studies exploiting policy variation fail to find evidence of mean impacts on hours worked among the employed (Eissa and Liebman, 1996; Meyer and Rosenbaum, 2001; Meyer, 2002). Second, in both survey and administrative data, earnings tend not to exhibit much bunching at the budget "kinks" induced by tax and transfer policies, suggesting that intensive margin elasticities are small (Heckman, 1983; Saez, 2010). Both forms of evidence are subject to qualification. In addition to being susceptible to sample selection bias, mean impacts on hours worked ignore the potentially offsetting labor supply effects of program phase-in and phase-out provisions (Bitler, Gelbach, and Hoynes, 2006). And although excess mass at kink-points is a non-parametric indicator of intensive margin responsiveness (Saez, 2010), labor supply constraints may confound the quantitative inferences drawn from bunching approaches (Chetty et al., 2011b).

This paper studies the impact of Connecticut's Jobs First (JF) welfare reform experiment on the labor supply and program participation decisions of a sample of welfare applicants and recipients. We develop a non-parametric approach to measuring intensive and extensive margin responses to policy reforms that remains valid in the presence of labor supply constraints, impact heterogeneity, and self-selection. Conceptually, detecting adjustment along a given margin in response to a policy reform requires inferring what choices a decision maker would have made if the reform had not taken place. Because choices are only observed under the policy regime to which the decision maker is exposed, the problem of distinguishing response margins is closely tied to fundamental challenges in causal inference. To address these challenges, we use revealed preference arguments to restrict the set of counterfactual choices compatible with each decision maker's actual choice. These restrictions are shown to yield informative bounds on the frequency of intensive and extensive margin responses to reform when policy regimes are randomly assigned.

The JF experiment provides an interesting venue for studying labor supply because the reform entailed a mix of positive and negative work incentives. First, it strengthened work requirements

[^0]and increased sanctions for welfare recipients who fail to seek work. Second, it changed the manner in which welfare benefits phase out by disregarding earnings up to an eligibility threshold (or "notch") above which benefits abruptly drop to zero. Bitler, Gelbach, and Hoynes (BGH, 2006) show that the JF reform induced a nuanced pattern of quantile treatment effects (QTEs) on earnings qualitatively consistent with intensive margin responsiveness. They find that JF boosted the middle quantiles of earnings while lowering the top quantiles, yielding a mean earnings effect near zero. The negative impacts on upper quantiles provide suggestive evidence of an "opt-in" response to welfare (Ashenfelter, 1983), whereby working women are induced to lower their earnings in order to qualify for transfers.

Quantifying the frequency of intensive and extensive margin responses to this reform requires additional structure, as the experiment may have shifted women between many points in the earnings distribution. For instance, JF could have induced some skilled women to work and earn above the eligibility notch while leading others to lower their earnings below the notch through an opt-in response. To narrow down the set of possible responses to the experiment, we develop a non-parametric optimizing model of labor supply and welfare participation. ${ }^{3}$ In the model, women value consumption, potentially derive disutility from welfare participation, and may face labor supply constraints. To accommodate the fact that some women with earnings above the eligibility notch draw welfare benefits, we allow women to under-report their earnings to the welfare agency with some cost.

In taking the model to the data, we allow for unrestricted heterogeneity across women in their preferences and constraints. This allows us to rationalize any distribution of earnings and program participation choices found under a given policy regime. ${ }^{4}$ However, our model places strong testable restrictions on the experimental impacts generated by the JF reform. These restrictions follow from simple revealed preference arguments. Specifically, if the utility of a woman's choice under AFDC was not lowered by the reform, she will either make the same choice under JF or select an alternative that the reform made more attractive. We use these non-parametric restrictions to develop analytic bounds on the proportion of women responding along each of nine allowable margins defined by pairings of coarse earnings and program participation categories across policy regimes.

Applying our identification results, we find evidence of substantial intensive and extensive margin responses to reform over the first seven quarters of the JF experiment. Jobs First incentivized

[^1]at least $13 \%$ of the women who would not have worked to do so and roughly $32 \%$ of women who would have worked off welfare at low earnings to take up assistance. Importantly, we find that at least $19 \%$ of women who would have worked off welfare at relatively high earnings levels were induced to reduce their earnings and opt-in to welfare, demonstrating that reform in fact led to substantial intensive margin responses. We also find that the JF work requirements induced at least $2 \%$ of the women who would have not worked while on welfare to work and under-report their earnings in order to maintain eligibility for benefits.

Our results demonstrate that simple revealed preference arguments allow researchers studying policy reforms to derive informative bounds on the size of competing response margins under very weak assumptions. These findings extend results by Heckman, Smith, and Clements (1997) who, in the context of an application to the U.S. Job Training Partnership Act, considered the identifying power of Roy (1951)-type models of optimization for the joint distribution of potential outcomes. Our approach is applicable to more general settings that do not obey strong Roy-style dependence between choices and outcomes, and can easily be adapted to other reforms which alter the value of alternatives in known directions.

We also contribute to a recent literature on partial identification of labor supply models. The bounding approach developed here is closely related to the theoretical analysis of Manski (2014) who considers the use of revealed preference arguments to set-identify tax policy counterfactuals. While Manski conducts computational experiments involving a single tax parameter, we study a reform that changes a bundle of policy features and employ a correspondingly richer model incorporating labor supply constraints and program participation and reporting decisions. Blundell, Bozio, and Laroque (2011a,b) also implement a bounds based analysis of labor supply behavior but are concerned with a statistical decomposition of fluctuations in aggregate hours worked rather than formal identification of policy counterfactuals. Their findings, which are compatible with ours, indicate that adjustments along both the intensive and extensive margins are important contributors to fluctuations in aggregate hours worked. Finally, Chetty (2012) considers bounds on labor supply elasticities in a class of semi-parametric models with optimization frictions. He too finds evidence of non-trivial intensive margin responsiveness, but relies on strong parametric assumptions.

The remainder of the paper is structured as follows. Section 1 describes the Jobs First Experiment. Section 2 describes the data from the Jobs First experiment. Section 3 provides a test for anticipatory behavior and reports experimental impacts on the earnings distribution. Section 4 describes our optimizing model. Section 5 derives the restrictions implied by revealed preference. Section 6 studies identification and estimation of the probabilities of responding to reform along various margins. Section 7 provides our main empirical results and Section 8 discusses the robustness of our results to a variety of extensions. Section 9 concludes. Technical proofs and additional results are provided in an Online Appendix.

## 1 The Jobs First Evaluation

With the passage of the Personal Responsibility and Work Opportunity Reconciliation Act (PRWORA) in 1996, all fifty states were required to replace their Aid to Families with Dependent Children (AFDC) welfare programs with Temporary Assistance to Needy Families (TANF) programs. This change involved the imposition of time limits, work requirements, and enhanced financial incentives to work. The state of Connecticut responded to PRWORA by implementing the Jobs First (JF) program. To study the effectiveness of the reform, the state contracted with the Manpower Development Research Corporation (MDRC) to conduct a randomized evaluation comparing the Jobs First TANF program to the earlier state AFDC program. Table 1 provides a detailed summary of the JF and AFDC program features.

## Changes in the Treatment of Earnings

A primary feature of the JF reform was the enhancement of financial incentives to work while on assistance. The JF program disregarded $100 \%$ of earnings up to the monthly federal poverty line (FPL) in the determination of both welfare eligibility and transfers. This zero implicit tax on earnings is to be contrasted with the relatively steep earnings penalties faced by women on welfare under AFDC. Specifically, Connecticut AFDC recipients were eligible for a fixed earnings disregard of $\$ 120$ for the twelve months following the first month of employment while on assistance and $\$ 90$ afterwards (henceforth, the unreduced and reduced fixed disregards). Women were also eligible for a proportional disregard of any additional earnings: $51 \%$ for the four months following the first month of employment while on assistance and $27 \%$ afterwards (henceforth, the unreduced and reduced proportional disregards).

Thus, a distinguishing feature of the JF reform was the dramatic reduction in the implicit tax rate on earnings faced by welfare recipients. This change was meant to incentivize work but also created an eligibility "notch" in the transfer scheme, with a windfall loss of the entire grant amount occurring if a woman earned a dollar more than the monthly federal poverty line. ${ }^{5}$ The notch created strong incentives for some women to earn less than the poverty line.

Figure 1 provides a stylized depiction of changes to the cash assistance component of welfare faced by a woman with two children who, under AFDC (panel a), has access only to the reduced fixed and proportional disregards and, under JF (panel b), has not yet hit the time limits. The Figure plots the woman's disposal income (earned income plus welfare assistance) against her earnings $E . \bar{G}$ is the base grant amount which, per Table 1, is common to JF and AFDC. Transfers under JF exhibit a large discontinuity at the federal poverty line: at earnings below the FPL

[^2]the woman receives a transfer equal to $\bar{G}$, while at earnings beyond the FPL she is ineligible for assistance. The JF transfer scheme is to be contrasted with the AFDC scheme which exhibits no discontinuities: the transfer phases out smoothly, reflecting an implicit tax rate of $73 \%$ on earnings above a $\$ 90$ disregard.

We can formalize the rules governing welfare transfers by means of the transfer function $G_{i}^{t}(E)$ which gives the monthly grant amount associated with welfare participation at earnings level $E$ under policy regime $t \in\{a, j\}$ (AFDC or JF respectively). The $i$ subscript acknowledges that the grant amount varies across women with the same earnings due to variation in the size of their Assistance Unit (AU). ${ }^{6}$ Letting 1 [.] be an indicator for the expression in brackets being true, the regime specific transfer functions can be written:

$$
\begin{align*}
G_{i}^{a}(E) & =\max \left\{\bar{G}_{i}-\mathbf{1}\left[E>\delta_{i}\right]\left(E-\delta_{i}\right) \tau_{i}, 0\right\}  \tag{1}\\
G_{i}^{j}(E) & =\mathbf{1}\left[E \leq F P L_{i}\right] \bar{G}_{i}, \tag{2}
\end{align*}
$$

where $\delta_{i} \in\{90,120\}$ and $1-\tau_{i} \in\{.27, .51\}$ are the fixed and proportional AFDC earnings disregards, and $i$ subscripts have been added to the base grant amount $\left(\bar{G}_{i}\right)$ and the federal poverty line ( $F P L_{i}$ ) to acknowledge that they vary with AU size. Although in Figure 1 the AFDC transfer is fully exhausted at an earnings level $\bar{E}$ that is strictly below the FPL, this is not always the case. A woman with access to the unreduced proportional and fixed disregards exhausts her AFDC transfer at an earnings level slightly above the FPL.

Welfare is part of a broader web of tax and transfer programs. Figure 2 depicts the woman's monthly income accounting for the Food Stamps (FS) program, payroll and Medicaid taxes, and the Earned Income Tax Credit (EITC). The FS program interacts with welfare assistance both because welfare recipients are categorically eligible for FS and because welfare transfers are treated as income in the determination of the FS transfer. The JF reform introduced a further link between cash and in-kind assistance: conditional on joint take up, earnings up to the FPL were disregarded in the determination of both the welfare and the FS transfers. This feature is clearly visible in Figure 2: under JF, the combined welfare and FS transfer depends only on whether earnings exceed the FPL, in which case assistance is terminated. Thus, JF's impact on the FS program amplifies the notch at the FPL. ${ }^{7}$

[^3]
## Work Requirements, Sanctions, and Time Limits

AFDC recipients were subject to Connecticut's pre-existing employment mandates, which specified work requirements for all parents except those caring for a child under age two. The MDRC final report describes the AFDC employment-related services as "a small-scale, largely voluntary, education-focused welfare-to-work program" (Bloom et al., 2002, p.28) with lax enforcement. JF recipients, by contrast, were required to participate in employment services targeted toward quick job placement unless they were parents caring for a child under age one. ${ }^{8}$ Additionally, the JF reform stepped up sanctions for non-compliance with work requirements. JF recipients who failed to make good faith efforts to find work while receiving assistance could be sanctioned by having their welfare grant reduced or temporarily canceled. Under AFDC, sanctions involved removing the noncompliant adult from the grant calculation rather than closing the entire case.

Finally, under AFDC, women could remain on welfare indefinitely, provided that their children were of eligible age. By contrast, under JF, women were limited to twenty one months of assistance. However, exemptions and six month extensions from the time limit were possible. Survey evidence from Bloom et al. (2002, p.76) suggests that, in practice, a majority of the cases reaching the time limit were granted an extension and, during the first year after random assignment, nearly $20 \%$ of the JF units were exempt from time limits (p.35).

## Other Changes

Under AFDC, recipients were eligible for twelve months of Transitional Child Care (TCC) subsidies if they left welfare for work, while under JF, cases were eligible for TCC indefinitely provided that their income did not exceed $75 \%$ of the state median income. Likewise, under AFDC, assistance units leaving welfare because of increased earnings were eligible for one year of Transitional Medicaid (TM), while under JF, units were eligible for two years of TM, which might again increase incentives to work. While these programs could create additional incentives to work, Bloom et al. (2002) argue that these components of the JF reform had little impact on actual access to child- or health care because of contemporaneous state level programs covering essentially the same population. ${ }^{9}$

[^4]JF also changed the treatment of income received in the form of child support (CS) transfers. Under AFDC, recipients received only the first $\$ 50$ of CS collected each month through the Bureau of Child Support Enforcement and the entire amount received was disregarded in computing the welfare transfer, corresponding to a $\$ 50 \mathrm{CS}$ disregard. Instead, under JF, recipients received a check for the full amount of any CS collected and the first $\$ 100$ was disregarded in computing the welfare transfer. These changes could induce income effects since women receiving between $\$ 50$ and $\$ 100$ of CS received an increased transfer under JF without adjusting their behavior. However, these income effects are likely negligible given that they only apply to women within this restricted range of CS payments - payments above $\$ 100$ were deducted dollar for dollar from benefits - and since the amount of additional income per month is very small.

## 2 Data and Descriptive Statistics

Our data come from the MDRC Jobs First Public Use Files. They contain a baseline survey of demographic and family composition variables merged with longitudinal administrative information on welfare participation, rounded welfare payments, family composition, and rounded earnings covered by the state unemployment insurance (UI) system.

There are a number of limitations to the Public Use Files. While welfare payments are measured monthly, UI earnings data are only available quarterly. To put them on a consistent time scale, we aggregate welfare participation to the quarterly level. Data on hours and weeks worked are not available, which prevents us from inferring hourly wages.

Another difficulty is that the administrative measure of AU size is missing for most cases. This is problematic because AU size influences the FPL and therefore the location of the JF notch. In the JF sample, we are able to infer an AU size from the grant amount in months when a women is on welfare. But in the AFDC sample, the grant amount depends on the woman's history of past employment and welfare take up, which we observe only partially. Consequently, we cannot reliably infer an AU size from grant amounts under AFDC. For this reason, when computing treatment effects by AU size, we rely on a variable collected in the baseline survey named "kidcount" that records the number of children in the household at the time of random assignment. As might be expected, the kidcount variable tends to underestimate the true AU size as women may have additional children over the seven quarters following the baseline survey. To deal with this problem we inflate the kidcount measure of AU size by one in order to avoid understating the location of the poverty line for most AUs. ${ }^{10}$ Additional details about variable construction are provided in the

[^5]Online Appendix.

## Baseline Characteristics of the Analysis Sample

Table 2 shows descriptive statistics for our analysis sample. We have 4,642 cases with complete prerandom assignment data and non-missing values of the kidcount variable. There are some mildly significant differences between the AFDC and JF groups in their baseline characteristics, however these differences are not jointly significant. We follow BGH (2006) in using propensity score reweighting to adjust for these baseline differences. ${ }^{11}$ We also examine two subgroups defined by whether they had positive earnings seven quarters prior to random assignment (the two rightmost panels in Table 2). Because pre-assignment earnings proxy for tastes and earnings ability, the JF reform likely presented these groups with different incentives, which makes them useful for exploring treatment effect heterogeneity (see BGH, 2014 for a related subgroup analysis).

## Bunching in the JF Sample

Many labor supply models predict bunching of earnings at notches (Slemrod, 2010; Kleven and Waseem, 2013). However, BGH (2006) find no evidence of such bunching at the JF eligibility notch. Here we extend their analysis by looking for bunching in the JF sample using our improved measure of AU size. Figure 3a provides a histogram of earned income rescaled relative to the FPL. Not only do we fail to detect a spike in the mass of observations located at the notch, the earnings density actually appears to be declining through this point. Moreover, this decline is relatively smooth through the notch which should bound, to its right, a dominated earnings region. Compared to women not on welfare in the quarter (Figure 3c), there is arguably an excess "mound" in the density of earnings below the notch for women on welfare throughout the quarter (Figure 3b). While it is possible to rationalize the absence of bunching with certain distributions of preferences, this evidence is also consistent with the possibility that women face significant labor supply constraints - a conjecture that has received substantial empirical support in related settings (Altonji and Paxson, 1988; Dickens and Lundberg, 1993; Chetty et al., 2011b; Beffy et al., 2014).

## Under-reporting of Earnings

A conspicuous feature of Figure 3b is that the distribution of earnings stretches well beyond the FPL, despite the fact that women with such earnings levels should be ineligible for welfare under

[^6]JF. While it is possible that some of these observations are the result of measurement problems, under-reporting behavior is also undoubtedly at play here. The MDRC final report (Bloom et al., 2002, p. 38) provides some direct evidence on this point, noting that, in the AFDC group, the fraction of women with earnings in the UI system was about ten percentage points higher than the fraction reporting earnings to the welfare agency. In the JF group, the fraction reporting earnings to the welfare system was nearly identical to the fraction with UI earnings. However, this may be an artifact of the $100 \% \mathrm{JF}$ earnings disregard which creates incentives to report an earnings amount below the poverty line rather than no earnings at all. Evidence on such partial under-reporting was found in a related context by Hotz, Mullin, and Scholz (2003), who analyzed data from a welfare reform experiment in California. ${ }^{12}$

## 3 Anticipation Effects and Intensive Margin Responses

The JF time limits may provide households with an incentive to conserve their welfare benefits for future use (Grogger and Michalopoulos, 2003; Swann, 2005). In this Section, we test for anticipatory behavior in response to time limits during the first seven quarters of the JF experiment. Finding no evidence of such responses to the JF reform, we then implement a test designed to detect intensive margin responses to the static incentives of the JF reform.

## A Test for Anticipation

The JF time limits create incentives for a risk averse woman to save months of welfare eligibility for later periods when her earnings may be lower (e.g. due to job loss). Thus, under some conditions, JF may actually make working on welfare less attractive, as this choice requires sacrificing the option value of using welfare an additional month in the future.

Following Grogger and Michalopolous (2003), we conduct a simple test for whether the JF time limits yield anticipatory effects. Our test compares the impact of reform on the welfare use of women who at baseline had a youngest child age 16-17 (for whom the time limits were irrelevant) to impacts on the welfare use of women who had younger children. As shown in Table 3, we cannot reject the null hypothesis that the average impact of JF on monthly welfare take-up is the same for both groups of women. In fact, our point estimates suggest that the response of women with younger children to reform was actually slightly greater than the response of women with children ages $16-17$, which is the opposite of what anticipatory behavior would suggest. While this finding does not prove that the women in our sample were myopic, it does suggest that anticipatory

[^7]responses to the time limits were probably small. ${ }^{13}$

## A Test for Intensive Margin Responsiveness

The JF reform provided a mix of positive and negative labor supply incentives. While the program encouraged women to work, it also potentially encouraged some women with earnings above the federal poverty line to reduce their earnings in order to receive welfare assistance. But under the null hypothesis that women are unable (or unwilling) to adjust their earnings, the program had only one effect: to encourage work. If this is true, then we should expect the distribution of earned income in the JF sample to stochastically dominate the distribution in the AFDC sample because the reform simply shifts mass from zero to positive earnings levels. ${ }^{14}$

Figure 4a provides reweighted empirical distribution functions (EDFs) of earnings in the AFDC and JF samples using quarterly earnings data for the seven quarters following random assignment - a horizon over which no case was in danger of reaching the limit. We rescale earnings relative to three times the monthly FPLs faced by the sample women: $3 F P L_{i}$ is the maximum amount that a woman can earn in a quarter while maintaining welfare eligibility throughout the quarter. By rescaling earnings relative to the FPL, we can deduce whether mass is "missing" from the portion of the distribution predicted by the JF incentive scheme - namely, at points just above the eligibility notch. Significant opt-in behavior should lead earnings levels below the FPL to be more common in the JF sample than the AFDC sample.

A reweighted Kolmogorov-Smirnov test strongly rejects the null hypothesis that the two EDFs are identical. More quarters exhibit positive earnings in the JF sample than in the AFDC sample, indicating that JF successfully incentivized many women to work. ${ }^{15}$ The earnings EDF rises more quickly in the JF sample than under AFDC, signaling excess mass at low earnings levels. Also, the EDFs cross below the notch, leading the fraction earning less than $3 F P L_{i}$ to be slightly greater for the JF sample than among the AFDC controls. A large increase in the fraction earning less than $3 F P L_{i}$ would be suggestive evidence of an opt-in response, however the impact here is small and statistically insignificant. Using a variant of the formal testing procedure of Barrett and Donald

[^8](2003) described in the Online Appendix, we fail to reject the null hypothesis that the JF earnings distribution stochastically dominates the earnings distribution in the AFDC sample. Hence, we cannot reject the null hypothesis that these impacts were generated by extensive margin responses alone.

However, these distributional effects conceal substantial heterogeneity across subgroups. Figures 4b-4c provide corresponding EDFs in two subsamples defined by their earnings in the seventh quarter prior to random assignment. These groups are of interest because pre-random assignment earnings are a strong predictor of post-random assignment earnings and therefore proxy for the relevant range of the budget set an agent would face under AFDC. Accordingly, units with positive pre-random assignment earnings should be most likely to exhibit an opt-in effect, while units with zero earning should be more likely to be pushed into the labor force by JF. The Figures confirm that the expected pattern of heterogeneity is in fact present: the positive earnings group experienced less of an impact on the fraction of quarters spent working and a significant increase in the fraction of quarters with earnings less than or equal to three times the monthly poverty line. The zero earnings group, by contrast, exhibits a large increase in the fraction of quarters working, but essentially no impact on the fraction of quarters with earnings less than or equal to three times the monthly poverty line. First order stochastic dominance is rejected at the $5 \%$ level in the positive earnings sample, indicating that intensive margin responses did in fact occur in response to the reform.

## 4 Model

Having established the presence of both intensive and extensive margin labor supply responses to the JF reform, we now seek to infer the frequency of these responses. What fraction of women were induced to lower their earnings and take up welfare in response to the JF reform? What share of women were induced to work at earnings levels above the poverty line? How many women were induced to leave welfare? The fundamental challenge to answering such questions is that we cannot observe the choice each woman would have made under the policy regime to which she was not assigned. To make progress, we require additional structure on the set of possible responses that can occur.

In this section we develop an optimizing model that formalizes the incentives provided by the JF reform and restricts the set of possible labor supply and program participation responses to the experiment. We depart from conventional structural modeling approaches (e.g., Moffitt, 1983; Keane and Moffitt, 1998; Hoynes, 1996; Swann, 2005; Keane and Wolpin, 2002, 2007, 2010; Chan, 2013) by allowing for a non-parametric specification of preferences that vary across women in a nearly unrestricted fashion. Motivated by our finding of the absence of a spike in the earnings distribution at the JF eligibility notch, we allow for the possibility that women face constraints on their labor supply decisions. We also incorporate earnings under-reporting decisions into the model, which provides an explanation for welfare participation among earnings ineligible women.

Our analysis relies on a number of simplifying assumptions. First, the model is static. In practice, women are likely to make choices taking into account both current and future payoffs. For our purposes, these motives are only of concern if they rationalize responses that do not emerge under myopic decision making. For this to be the case, alternative specific continuation values would need to differ across AFDC and JF in ways that undermine our static conclusions regarding which choices are made more or less attractive by the reform. The JF time limits are the most obvious culprit for such effects since they could make working while on welfare less attractive under JF than under AFDC. However, our adaptation of the Grogger and Michalopoulos (2003) test failed to find evidence of anticipatory behavior, leading us to believe that the dynamic incentives of the reform are in fact weak in this sample. ${ }^{16}$ Second, the model ignores the TCC, TM, and CS components of the JF reform. We explained above why these features of the reform likely had minimal effects. Introducing them would substantially complicate our analysis and add little given that we lack data on participation in these programs. Third, to simplify exposition, the model ignores the FS program, payroll and medicare taxes, and the EITC. We explain in Section 8 why extending the model to incorporate these policies has no effect on our identification arguments.

## The Decision Problem

Consider a woman with children, call her $i$, subject to a policy regime indexed by $t \in\{a, j\}$ (AFDC or JF respectively). In a given month, woman $i$ samples $K_{i} \geq 0$ job offers, composed of wage and hours offer pairs: $\Theta_{i} \equiv\left\{\left(W_{i}^{k}, H_{i}^{k}\right)\right\}_{k=1}^{K_{i}}$. The woman's offer set $\Theta_{i}$ reflects a mix of luck and the woman's labor market skills. Woman $i$ decides which (if any) of the $K_{i}$ offers to accept, whether to participate in welfare (represented by the indicator $D \in\{0,1\}$ ), and a level ( $E^{r} \geq 0$ ) of earned income to report to the welfare agency. We assume $E^{r}$ is less than or equal to her actual earnings $E=W H$ where $W$ and $H$ refer to the wage and hours at her chosen job (which are both zero when no offer is accepted). ${ }^{17}$

Woman $i$ consumes her earnings plus any welfare transfer. Specifically, her consumption is given by:

$$
C=C_{i}^{t}\left(E, D, E^{r}\right)=E+D\left(G_{i}^{t}\left(E^{r}\right)-\kappa_{i} \mathbf{1}\left[E^{r}<E\right]\right),
$$

where $\kappa_{i}>0$ is the cost of under-reporting earnings to the welfare agency. This cost captures effort

[^9]exerted in disguising earnings and the possibility of being caught under-reporting. ${ }^{18}$ The welfare grant, $G_{i}^{t}\left(E^{r}\right)$, is determined according to the regime-specific transfer functions (1)-(2) based upon reported (as opposed to actual) earnings.

Woman $i$ 's preferences are represented by the utility function:

$$
\begin{equation*}
U_{i}^{t}(H, C, D, R), \tag{3}
\end{equation*}
$$

where $R=R\left(D, E^{r}\right)=D \mathbf{1}\left[E^{r}=0\right]$ is an indicator that equals one when the woman reports zero earnings to the welfare agency. The dependence of utility on $D$ captures the potential for a "stigma" (or, conversely, a psychic benefit) to be associated with welfare participation (Moffitt, 1983), while the dependence on $R$ captures the "hassle" associated with reporting zero earnings to the welfare agency because of work requirements. Utility is indexed by the policy regime $t$ to allow for differences in the hassle associated with the work requirements under AFDC and JF.

We assume that the utility function in (3) obeys the following restrictions:
A. $1 \quad$ utility is strictly increasing in $C$;
A. $2 U_{i}^{t}(H, C, 1,1) \leq U_{i}^{t}(H, C, 1,0)$;
A. $3 \quad U_{i}^{j}(H, C, 1,1) \leq U_{i}^{a}(H, C, 1,1)$;
A. $4 \quad U_{i}^{j}(H, C, 1,0)=U_{i}^{a}(H, C, 1,0)$ for $H>0$;
A. $5 \quad U_{i}^{j}(H, C, 0,0)=U_{i}^{a}(H, C, 0,0)$;
A. $6 \quad U_{i}^{a}\left(H, C_{i}^{a}(E, 1, E), 1,0\right)<U_{i}^{a}\left(H, C_{i}^{a}(E, 0, E), 0,0\right) \forall E \in\left(F P L_{i}, \bar{E}_{i}\right]$.

Assumption A. 1 is a standard non-satiation condition. Assumptions A.2-A. 5 formalize our institutional knowledge of the JF reform, which potentially stepped up welfare hassle, but should not have affected the psychic costs or benefits associated with program participation. Specifically, A. 2 states that reporting zero earnings weakly lowers utility (due to welfare hassle). In accord with JF's increased work requirements, A. 3 restricts the utility of reporting zero earnings on welfare to be no higher under JF than AFDC. Assumption A. 4 restricts the psychic cost or benefit of welfare participation to be regime-invariant among employed workers who report positive earnings. Assumption A. 5 requires utility to be regime-invariant when off assistance. Finally, A. 6 places a lower bound on woman $i$ 's welfare stigma that ensures she does not report earnings above the federal poverty line while on assistance under AFDC. It says that at earning levels above $F P L_{i}$, the extra income associated with welfare fails to compensate her for the stigma she incurs from being on assistance. As we discuss in Section 6, assumption A. 6 simplifies our empirical analysis by allowing us to equate earning above the poverty line while on assistance with under-reporting. We show in Section 8 that this assumption is not restrictive in practice.

[^10]The above specification of utility is extremely general. Due to the non-separability of $H$ and $C$, leisure and consumption may be complements or substitutes and preferences may be non-homothetic as in classic Stone-Geary specifications of utility. Because we do not require monotonicity with respect to $H$, the woman may value working full time more than working part time or vice versa. Likewise, participation in welfare may increase or decrease utility, except at earning levels in the range ( $F P L_{i}, \bar{E}_{i}$ ], where welfare participation must lower utility by A.6. Welfare stigma creates the possibility that woman $i$ refuses assistance despite being eligible. The effect of welfare participation on utility is allowed to vary with consumption and leisure due to the non-separability of $D$. Similarly, the hassle disutility is allowed to vary with consumption and leisure due to the further non-separability of $R$. Note that we have not assumed continuity of utility with respect to $H$ or $C$, which accommodates the possibility that woman $i$ faces fixed cost of work such as a monthly commuting cost. Fixed costs discourage work at low earnings levels and create the possibility that non-working women respond to marginal changes in work incentives by earning large amounts (Cogan, 1981).

A special case of (3) monetizes hassle disutility, welfare stigma, and fixed costs of work as follows:

$$
\begin{equation*}
U_{i}\left(H, C-\phi_{i} D-\eta_{i}^{t} R-\mu_{i} \mathbf{1}[E>0]\right), \tag{4}
\end{equation*}
$$

where $\phi_{i}$ is the monetized cost of welfare stigma, $\eta_{i}^{t}$ is the hassle cost of reporting zero earnings under regime $t$, and $\mu_{i}$ is a fixed cost of work. The parameters $\left(\mu_{i}, \eta_{i}^{a}, \eta_{i}^{j}, \phi_{i}\right)$ inherit the above restrictions on preferences. Specifically, $\mu_{i} \geq 0$ by A.1, while $\eta_{i}^{j} \geq \eta_{i}^{a} \geq 0$ in accordance with A. 2 and A.3. From A. 4 and A. $6 \phi_{i}^{a}=\phi_{i}^{j}=\phi_{i}>G_{i}^{a}\left(F P L_{i}\right)$ which implies welfare cannot generate a psychic benefit. ${ }^{19}$ Finally, in accordance with A.5, the two-argument utility function in (4) is not indexed by the policy regime $t$. We refer to the second argument of (4) as the "consumption equivalent." We selectively consider this "monetized" specification below to aid in illustrating the mechanics of the model and the implications of further restricting preferences. Our main results rely on the more general specification given in (3).

Woman $i$ 's objective is to maximize her utility under policy regime $t$. Hence, she selects a labor supply, program participation, and reporting alternative: ${ }^{20}$

$$
\begin{equation*}
X_{i}^{t *} \in \underset{(W, H) \in\left\{\Theta_{i},(0,0)\right\},}{\arg \max }{\max \{0,1\}, E^{r} \in[0, E]} U_{i}^{t}\left(H, C_{i}^{t}\left(W H, D, E^{r}\right), D, R\left(D, E^{r}\right)\right) . \tag{5}
\end{equation*}
$$

We refer to $X_{i}^{t *}$ as woman $i$ 's choice under policy regime $t$. Note that her pair $\left(X_{i}^{a *}, X_{i}^{j *}\right)$ of

[^11]regime-dependent choices is governed by the vector of primitives:
$$
\theta_{i} \equiv\left(U_{i}^{j}(., ., ., .), U_{i}^{a}(., ., ., .), \kappa_{i}, \Theta_{i}, \bar{G}_{i}, \delta_{i}, \tau_{i}\right) .
$$

## Population Heterogeneity

Consider now a sample of $N$ women with children whose preferences obey assumptions A.1-A.6. These women have primitives $\left\{\theta_{i}\right\}_{i=1}^{N}$, which we treat as i.i.d. draws from a joint distribution function $\Gamma_{\theta}$ (.). We depart from much of the structural labor supply literature by leaving the distribution $\Gamma_{\theta}$ (.) unrestricted save for the support limitations implied by assumptions A.1-A.6, the assumption that $\kappa_{i}>0$, and the logical non-negativity of hours and wage offers. Substantively, this formulation implies that preferences and constraints may vary freely across women, giving rise, for instance, to arbitrary correlations between tastes and offer sets. Such dependence poses difficult endogeneity problems bypassed in much of the recent literature on non-parametric identification of structural labor supply models, which typically treats wages (and policy rules) as exogenous (Manski, 2014; Blomquist et al, 2014).

## 5 Revealed Preference Restrictions

Despite allowing for arbitrary heterogeneity across women, our model restricts how any given woman can respond to policy variation. That is, it rules out certain pairings of choices across the two policy regimes. These restrictions stem from simple revealed preference arguments. Specifically, if the utility of a woman's choice under AFDC was not lowered by the reform, she will either make the same choice under JF or select an alternative that the reform made more attractive.

A parsimonious approach to summarizing the empirical content of these restrictions leverages the fact that the JF reform improved (or worsened) the attractiveness of large collections of alternatives based on their implied earnings. This follows because the JF reform altered the mapping between earnings and grant amounts and imposed more stringent work requirements on recipients with zero earnings. In what follows, we group labor supply alternatives into three broad categories based upon the earnings they generate. We then apply revealed preference arguments to rule out possible pairings of alternatives within these broad categories across policy regimes. ${ }^{21}$ In Section 8, we discuss what can (and cannot) be learned from working with finer earnings categories.

[^12]
## A Coarsening of Earnings

Consider the following "coarsened" earnings variable $\widetilde{E}_{i}$, defined by the relation:

$$
\widetilde{E}_{i} \equiv \begin{cases}0 & \text { if } E=0  \tag{6}\\ 1 & \text { if } E \leq F P L_{i} . \\ 2 & \text { if } E>F P L_{i}\end{cases}
$$

That is, $\widetilde{E}_{i}$ indicates whether woman $i$ works, and if so, whether her earnings make her ineligible for benefits under JF. The JF reform had qualitatively different effects on the attractiveness of alternatives within each of these ranges. Specifically, the reform made earning positive amounts below the FPL $\left(\widetilde{E}_{i}=1\right)$ at least as attractive conditional on welfare participation because of JF's higher earning disregard. Conversely, the reform potentially reduced the attractiveness of not working ( $\widetilde{E}_{i}=0$ ) while on welfare because of JF's more stringent work requirements. Finally, the reform had no effect on the utility of working at earnings levels above the FPL ( $\left.\widetilde{E}_{i}=2\right)$. To understand this last point, note that women with earnings in this range are either off assistance or underreporting their earnings to the welfare agency. We show in the Online Appendix that assumptions A.1, A.2, and A. 4 imply the transfer received by a woman who optimally under-reports is $\bar{G}_{i}$ irrespective of the regime. Thus, the utility of working and under-reporting is unaffected by the regime given optimal reporting of earnings to the welfare agency.

Pairing the earning categories with the decision to participate in welfare and the under-reporting decision yields seven earnings / participation / reporting combinations, which we henceforth refer to as states. The set of possible states is given by:

$$
\mathcal{S} \equiv\{0 n, 1 n, 2 n, 0 r, 1 r, 1 u, 2 u\}
$$

The number associated with each state refers to the woman's earnings category while the letter describes her combined welfare participation and reporting decisions. Specifically, the letter $n$ denotes welfare non-participation, $r$ denotes welfare participation with truthful reporting of earnings ( $E^{r}=E$ ), and $u$ denotes welfare participation with under-reporting of earnings $\left(E^{r}<E\right)$. Note that state $0 u$ is ruled out, as it is not meaningful to "under-report" zero earnings. Likewise, state $2 r$ is not allowed by the JF eligibility rules and cannot occur under AFDC given assumption A.6.

## Allowed and Disallowed Responses

Table 4 catalogs the possible pairings of states across the two policy regimes. Pairs of states labeled "no response" entail the same behavior under the two policy regimes. We term the remaining pairs either "disallowed" or "allowed" responses. The disallowed responses entail a change in behavior that is proscribed by the model. This occurs either because the change in behavior would entail
an alternative that is dominated or because the change in behavior is incompatible with revealed preference. In Table 4, the disallowed responses are denoted with a "-" entry. The allowed responses entail a change in behavior that is permitted by the model. These responses are represented by entries that describe the three margins along which behavior may change: welfare participation (welfare take up or exit), labor supply (extensive versus intensive labor supply response), and reporting of earnings to the welfare agency (truthful reporting versus under-reporting). We next describe the logic behind which responses are allowed and which are not. The Online Appendix provides a formal proof that the restrictions in Table 4 are exhaustive.

Starting with the disallowed responses, a woman will not make a choice corresponding to state $1 u$ under JF because under-reporting is costly $\left(\kappa_{i}>0\right)$ and earnings below the poverty line are fully disregarded. For this reason, the column of Table 4 pertaining to state $1 u$ under JF is populated with "-" entries over a horizontally striped background. The remaining prohibited responses stem from revealed preference arguments. By assumptions A. 1 and A.4, the JF reform may have made alternatives corresponding to state $1 r$ more (but not less) attractive. Conversely, by assumption A.3, the reform may have made alternatives corresponding to the state $0 r$ less (but not more) attractive. Finally, the reform had no effect on the value of alternatives corresponding to the set $\mathcal{C}_{0} \equiv\{0 n, 1 n, 2 n, 1 u, 2 u\}$ by assumptions A.1, A.2, A.4, and A.5. Therefore, by revealed preference, a woman will not pair any of the states in $\mathcal{C} \succeq \equiv\{1 r\} \cup \mathcal{C}_{0}$ under AFDC with a (different) state in $\mathcal{C}_{\preceq} \equiv\{0 r\} \cup \mathcal{C}_{0}$ under JF. This reasoning justifies the "-" entries in the cells with a greyed background.

Proceeding now to responses that are allowed, consider first the extensive margin labor supply responses. A woman who, under AFDC, chooses not to work while off welfare (state 0n) must face high welfare stigma, hassle, or under-reporting costs since she is willing to forgo the full grant amount $\bar{G}_{i}$. Under JF, she may choose to work while on assistance and earn below the FPL (state $1 r$ ), as this option entails higher consumption than under AFDC. Next, a woman who, under AFDC, would participate in welfare without working (state $0 r$ ), may respond to JF in many ways. Specifically, she may be induced to: i) work while on welfare (state $1 r$ ), ii) leave welfare and earn less than the federal poverty line (state $1 n$ ), iii) earn more than the federal poverty line (state $2 n$ ), iv) remain on welfare and earn more than the federal poverty line (state $2 u$ ), or v) opt out of welfare (state $0 n$ ). The first response can result from either the reduction in implicit tax rates on earnings or the increased hassle associated with JF. Sufficiently large fixed costs of work can induce the second, third, or fourth responses. A large increase in the hassle costs may induce the fifth response, in which case no labor supply response occurs.

Consider next the allowed intensive margin labor supply responses. The pairing of states $1 n$, $1 r$, or $1 u$ under AFDC with state $1 r$ under JF could entail intensive margin responses as a woman may (or may not) adjust her earnings within region 1. A woman working on welfare under AFDC, and earning less than the FPL, will face a reduction in her implicit tax rate under JF. Like any
uncompensated increase in the wage, this change could lead to increases or decreases in the amount of work undertaken, but in either case will lead her to continue working on welfare. Likewise, a woman working off welfare under AFDC may choose to participate in JF which would offer an increase in income for the same amount of work. This may result in a reduction in earnings due to income effects. If the woman has high enough welfare stigma, she will not participate in welfare under either regime (i.e. she will pair state $1 n$ with state $1 n$ ). The pairing of either states $2 n$ or $2 u$ under AFDC with state $1 r$ under JF also corresponds to an intensive margin response: the reform induces the woman to reduce her earnings below the FPL.

Some of the above extensive and intensive margin labor supply responses can be accompanied by an adjustment in reporting behavior. Specifically, the JF reform may induce a woman to start truthfully report her earnings (pairing states $1 u$ or $2 u$ with state $1 r$ ). Conversely, the reform may induce a woman to under-report her earnings (pairing state $0 r$ with state $2 u$ ). Thus, the JF reform may have mixed effects on reporting behavior.

## Graphical Examples

Figures 5 and 6 illustrate some of the allowed responses listed above. For convenience, both figures employ the monetized form of the utility function given in (4). ${ }^{22}$

Figure 5 illustrates allowed responses that entail either an extensive margin or intensive margin labor supply adjustment. Specifically, Figure 5a illustrates an extensive margin response, corresponding to pairing state $0 r$ under AFDC with state $1 r$ under JF. As depicted, the hassle costs $\eta_{i}^{a}$ of not working under AFDC are much smaller than the corresponding costs $\eta_{i}^{j}$ under JF. The fixed cost of work $\mu_{i}$ straddles the two hassle costs. In comparison with the fixed costs of work and hassle, the cost of under-reporting $\kappa_{i}$ is depicted as being quite large. The under-reporting line is the same under AFDC and JF because under either regime a woman can secure the base grant by concealing her earnings. A woman with the configuration of costs and preferences found in Figure 5 a would not work on welfare under AFDC (point A) but would take up work and truthfully report her earnings under JF (point B). Figure 5b illustrates the traditional opt-in response considered in the literature, corresponding to pairing state $2 n$ under AFDC with state $1 r$ under JF. As depicted, the hassle costs $\eta_{i}^{a}$ of not working under AFDC are large but smaller than the corresponding costs $\eta_{i}^{j}$ under JF. The fixed cost of work $\mu_{i}$ straddles the two hassle costs. A woman with the configuration of costs and preferences found in Figure 5b would earn above the FPL off assistance under AFDC (point A) but would earn strictly below the FPL on assistance under JF (point B).

Figure 6 illustrates allowed responses that entail an adjustment in reporting behavior. As depicted, the hassle costs $\eta_{i}^{j}$ of not working under JF are larger than the corresponding costs $\eta_{i}^{a}$ under AFDC, but both are smaller than the fixed cost of work $\mu_{i}$. In comparison with the fixed costs of work and hassle, the cost of under-reporting $\kappa_{i}$ is relatively small. A woman with the

[^13]configuration of costs and preferences found in Figure 6a would work on welfare under AFDC but under-report her earnings (point A). However, under JF, she would truthfully report her earnings (point B), as the JF disregard reduces the return to under-reporting. Hence, reform may induce a reduction in under-reporting. By contrast, Figure 6 b shows a scenario where the hassle effects of JF are larger, the costs of under-reporting are smaller, and preferences over earnings are such that the disutility of work is lower. This woman would receive benefits without working (point A) under AFDC but, under JF, will choose to earn above the poverty line and under-report her earnings (point B) in order to maintain eligibility. This occurs because the JF work requirements remove point A from her budget set - such a woman has effectively been hassled off welfare into under-reporting.

## 6 Identification and Estimation of Response Probabilities

Table 4 summarizes the restrictions our model places on how a woman may respond to the JF reform. These restrictions are not directly testable because we cannot observe the same woman under two regimes at a given point in time. ${ }^{23}$ Moreover, because we allow for unrestricted heterogeneity across women, the right mix of preferences and offers can rationalize any distribution of choices under a given policy regime. However, as we show below, our theoretical restrictions do have empirical content when applied to the JF experiment. Specifically, the model places refutable inequality restrictions on the impact of the reform that can be exploited to bound the frequency of adjustment along each allowable response margin.

## The Identification Problem

Let $S_{i}^{a}$ denote the "potential" state corresponding to woman $i$ 's choice under the AFDC regime and $S_{i}^{j}$ the state corresponding to her choice under the JF regime. Our goal is to identify response probabilities of the form:

$$
\pi_{s^{a}, s^{j}} \equiv P\left(S_{i}^{j}=s^{j} \mid S_{i}^{a}=s^{a}\right),
$$

for $\left(s^{a}, s^{j}\right) \in \mathcal{S} \times \mathcal{S}$, where $P($.$) is the probability measure induced by the distribution function$ $\Gamma_{\theta}$ (.). These probabilities summarize the frequency of adjustment to the JF reform along specific labor supply and participation margins. For example, $\pi_{2 n, 1 r}$ gives the proportion of those women who would earn above the federal poverty line off assistance under AFDC that would work on welfare under JF - that is, the share of high earning women who opt into welfare.

[^14]Let $T_{i}$ denote the treatment regime to which woman $i$ is assigned and $S_{i} \equiv \mathbf{1}\left[T_{i}=j\right] S_{i}^{j}+$ $\mathbf{1}\left[T_{i}=a\right] S_{i}^{a}$ her realized state. Random assignment ensures that her potential states are independent of the policy regime to which she is assigned. Formally,

$$
\begin{equation*}
T_{i} \perp\left(S_{i}^{a}, S_{i}^{j}\right) \tag{7}
\end{equation*}
$$

where the symbol $\perp$ denotes independence. The above condition implies that, for every $s \in \mathcal{S}$ and $t \in\{a, j\}, P\left(S_{i}=s \mid T_{i}=t\right)=P\left(S_{i}^{t}=s\right) \equiv q_{s}^{t}$, which is the well-known result that experimental variation identifies the marginal distributions of potential outcomes.

Unfortunately, experimental variation is not sufficient to identify the response probabilities $\left\{\pi_{s^{a}, s^{j}}\right\}$. To see this, observe that by the law of total probability, the marginal distributions of potential states are linked by the relation:

$$
\begin{equation*}
\mathbf{q}^{j}=\boldsymbol{\Pi}^{\prime} \mathbf{q}^{a} . \tag{8}
\end{equation*}
$$

where $\mathbf{q}^{t} \equiv\left[q_{0 n}^{t}, q_{1 n}^{t}, q_{2 n}^{t}, q_{0 r}^{t}, q_{1 r}^{t}, q_{1 u}^{t}, q_{2 u}^{t}\right]^{\prime}$ for $t \in\{a, j\}$ and the $7 \times 7$ matrix $\boldsymbol{\Pi}$ is composed of unknown response probabilities. Supposing for the moment that we know the vectors $\left(\mathbf{q}^{a}, \mathbf{q}^{j}\right)$ with certainty, the system in (8) consists of 7 equations (one of which is redundant) and $7 \times 6=42$ unknown independent response probabilities. Clearly, the response probabilities are heavily underidentified. As we show next, the theory dramatically reduces the degree of under-identification present.

## Unrestricted Response Probabilities

The economic model developed in Section 4 implies that only ten out of the forty two possible response margins cataloged in Table 4 are allowed. Accordingly, only ten out of the forty two response probabilities in matrix $\boldsymbol{\Pi}$ are not restricted to equal zero. Furthermore, theory implies that $\pi_{1 u, 1 r}$ equals one because no woman pairs state $1 u$ under AFDC with any state but $1 r$ under JF. Hence, there are nine free response probabilities, which we collect into the vector: ${ }^{24}$

$$
\begin{equation*}
\boldsymbol{\pi} \equiv\left[\pi_{0 n, 1 r}, \pi_{0 r, 0 n}, \pi_{2 n, 1 r}, \pi_{0 r, 2 n}, \pi_{0 r, 1 r}, \pi_{0 r, 1 n}, \pi_{1 n, 1 r}, \pi_{0 r, 2 u}, \pi_{2 u, 1 r}\right]^{\prime} \tag{9}
\end{equation*}
$$

Even with the model restrictions, there are still nine unknowns and only seven equations which necessitates a partial identification analysis. Moreover, because we do not directly observe underreporting behavior, we cannot distinguish between states $1 u$ and $1 r$, making the vectors $\left(\mathbf{q}^{a}, \mathbf{q}^{j}\right)$ themselves under-identified. We address both of these concerns below.

[^15]
## Observable States

Our data do not allow us to measure reporting decisions other than by contrasting a woman's administrative earnings with the eligible maximum. Hence, states $1 u$ and $1 r$ are not empirically distinguishable. Accordingly, we define a function $g: \mathcal{S} \rightarrow \widetilde{\mathcal{S}}$ that reduces the latent states $\mathcal{S}$ to observable states $\widetilde{\mathcal{S}}$ that can be measured in our data. Formally,

$$
g(s) \equiv\left\{\begin{array}{ll}
s & \text { if } s \in\{0 n, 1 n, 2 n\} \\
0 p & \text { if } s=0 r \\
1 p & \text { if } s \in\{1 u, 1 r\} \\
2 p & \text { if } s=2 u
\end{array} .\right.
$$

As before, the number of each state refers to the woman's earnings category and the letter $n$ refers to welfare non-participation. The letter $p$ denotes welfare participation, which is directly observable. Note that state $2 p$ can only be occupied via under-reporting because of assumption A.6.

Let $\widetilde{S}_{i}^{t}$ denote the potential observable state of a woman whose latent potential state under policy regime $t$ is $S_{i}^{t}$, that is, $\widetilde{S}_{i}^{t} \equiv g\left(S_{i}^{t}\right)$ for $t \in\{a, j\}$. Also, define the probability of occupying state $\widetilde{s} \in \widetilde{\mathcal{S}}$ under policy regime $t$ as $p_{\widetilde{s}}^{t} \equiv P\left(\widetilde{S}_{i}^{t}=\widetilde{s}\right)=\sum_{s: \widetilde{s}=g(s)} q_{s}^{t}$. Finally, denote the vectors of observable state probabilities as $\mathbf{p}^{t} \equiv\left[p_{0 n}^{t}, p_{1 n}^{t}, p_{2 n}^{t}, p_{0 p}^{t}, p_{1 p}^{t}, p_{2 p}^{t}\right]^{\prime}$ for $t \in\{a, j\}$. We are now ready to discuss identification of the nine free response probabilities appearing in (9) based on the regime specific state distributions $\mathbf{p}^{a}$ and $\mathbf{p}^{j}$.

## Testable Implications of Revealed Preference

Integrating the unobserved states out of (8) yields a system of six equations, one of which is redundant given that state probabilities sum to one in each policy regime. The five non-redundant equations can be given an intuitive representation as:

$$
\begin{align*}
p_{0 n}^{j}-p_{0 n}^{a} & =-p_{0 n}^{a} \pi_{0 n, 1 r}+p_{0 p}^{a} \pi_{0 r, 0 n} \\
p_{1 n}^{j}-p_{1 n}^{a} & =-p_{1 n}^{a} \pi_{1 n, 1 r}+p_{0 p}^{a} \pi_{0 r, 1 n} \\
p_{2 n}^{j}-p_{2 n}^{a} & =-p_{2 n}^{a} \pi_{2 n, 1 r}+p_{0 p}^{a} \pi_{0 r, 2 n}  \tag{10}\\
p_{0 p}^{j}-p_{0 p}^{a} & =-p_{0 p}^{a}\left(\pi_{0 r, 1 n}+\pi_{0 r, 1 r}+\pi_{0 r, 2 u}+\pi_{0 r, 2 n}+\pi_{0 r, 0 n}\right) \\
p_{2 p}^{j}-p_{2 p}^{a} & =p_{0 p}^{a} \pi_{0 r, 2 u}-p_{2 p}^{a} \pi_{2 u, 1 r}
\end{align*}
$$

The left hand side of (10) catalogs the experimental impacts of the JF reform on the observable state probabilities. The right hand side rationalizes these impacts in terms of "flows" into and out of each state as allowed by the model. The identifying power of the theory derives from the fact that only a handful of response probabilities appear in each equation. Despite these restrictions, the system in (10) is under-determined.

System (10) implies sixteen inequality restrictions. ${ }^{25}$ These restrictions exhaust the predictions of our model for the distribution of observed states $\left(\mathbf{p}^{a}, \mathbf{p}^{j}\right)$. As argued above, the restrictions pertain exclusively to the impact of the JF reform on state probabilities, namely $\mathbf{p}^{j}-\mathbf{p}^{a}$, as opposed to the cross-sectional distributions of states within a regime. Violation of any of these inequalities would imply that our framework fails to allow for a response actually present in the data. To conserve space, we list the sixteen inequality restrictions in the Online Appendix. Here we report five of them that are particularly intuitive:

$$
\begin{align*}
\left(p_{0 p}^{j}-p_{0 p}^{a}\right) & \leq 0  \tag{11a}\\
\left(p_{0 p}^{j}-p_{0 p}^{a}\right)+\left(p_{0 n}^{j}-p_{0 n}^{a}\right) & \leq 0  \tag{11b}\\
\left(p_{0 p}^{j}-p_{0 p}^{a}\right)+\left(p_{2 n}^{j}-p_{2 n}^{a}\right)+\left(p_{0 n}^{j}-p_{0 n}^{a}\right)+\left(p_{1 n}^{j}-p_{1 n}^{a}\right) & \leq 0  \tag{11c}\\
\left(p_{0 p}^{j}-p_{0 p}^{a}\right)+\left(p_{0 n}^{j}-p_{0 n}^{a}\right)+\left(p_{2 p}^{j}-p_{2 p}^{a}\right)+\left(p_{1 n}^{j}-p_{1 n}^{a}\right) & \leq 0  \tag{11d}\\
\left(p_{0 p}^{j}-p_{0 p}^{a}\right)+\left(p_{2 n}^{j}-p_{2 n}^{a}\right)+\left(p_{0 n}^{j}-p_{0 n}^{a}\right)+\left(p_{2 p}^{j}-p_{2 p}^{a}\right) & \leq 0  \tag{11e}\\
\left(p_{0 p}^{j}-p_{0 p}^{a}\right)+\left(p_{2 n}^{j}-p_{2 n}^{a}\right)+\left(p_{0 n}^{j}-p_{0 n}^{a}\right)+\left(p_{2 p}^{j}-p_{2 p}^{a}\right)+\left(p_{1 n}^{j}-p_{1 n}^{a}\right) & \leq 0 \tag{11f}
\end{align*}
$$

These restrictions state that the JF reform must (weakly): lower the fraction of women on assistance and not working (11a), raise the fraction of women working (11b), raise the fraction of women who work and receive assistance (11c), raise the fraction of women with earnings in range 1 (11e), and raise the fraction of women who receive assistance and have earnings in range 1 (11f).

## Bounds on the Response Probabilities

Subject to the above restrictions holding, we can use the system in (10) to bound the nine response probabilities. The upper and lower bounds on each of the response probabilities can be represented as the solution to a pair of linear programming problems of the form:

$$
\begin{equation*}
\max _{\pi} \boldsymbol{\pi}^{\prime} \boldsymbol{\lambda} \text { subject to (10) and } \boldsymbol{\pi} \in[0,1]^{9} \tag{12}
\end{equation*}
$$

where the layout of $\boldsymbol{\pi}$ was given in (9). For example, solving the above problem for $\boldsymbol{\lambda}=[0,0,1,0,0,0,0,0,0]^{\prime}$ yields the upper bound on $\pi_{2 n, 1 r}$, while choosing $\boldsymbol{\lambda}=[0,0,-1,0,0,0,0,0,0]^{\prime}$ yields the lower bound.

We can also use this representation to derive bounds on linear combinations of the response

[^16]probabilities. We consider the probabilities of adjusting along four "composite" margins:
\[

$$
\begin{aligned}
\pi_{0 r, n} & \equiv \pi_{0 r, 0 n}+\pi_{0 r, 2 n}+\pi_{0 r, 1 n} \\
\pi_{p, n} & \equiv \frac{p_{0 p}^{a}}{p_{0 p}^{a}+p_{1 p}^{a}+p_{2 p}^{a}}\left(\pi_{0 r, 0 n}+\pi_{0 r, 2 n}+\pi_{0 r, 1 n}\right) \\
\pi_{n, p} & \equiv \frac{p_{0 n}^{a} \pi_{0 n, 1 r}+\pi_{1 n, 1 r} p_{1 n}^{a}+\pi_{2 n, 1 r} p_{2 n}^{a}}{p_{0 n}^{a}+p_{1 n}^{a}+p_{2 n}^{a}} \\
\pi_{0,1+} & \equiv \frac{p_{0 p}^{a}\left(\pi_{0 r, 1 r}+\pi_{0 r, 2 n}+\pi_{0 r, 2 u}+\pi_{0 r, 1 n}\right)+p_{0 n}^{a} \pi_{0 n, 1 r}}{p_{0 p}^{a}+p_{0 n}^{a}}
\end{aligned}
$$
\]

The first composite response probability gives the fraction of women who would claim benefits without working under AFDC that are induced to get off welfare under JF (denoted $\pi_{0 r, n}$ ). Upper and lower bounds for this response probability can be had by solving (12) with $\boldsymbol{\lambda}=[0,1,0,1,0,1,0,0,0]$ and $[0,-1,0,-1,0,-1,0,0,0]$ respectively. We also examine the fraction of all women who would participate in welfare under AFDC that are induced to leave welfare under JF (denoted $\pi_{p, n}$ ), the fraction of women who are induced to take up welfare under JF (denoted $\pi_{n, p}$ ), and the fraction of women who are induced by JF to work. Because no woman who would work under AFDC will choose not to work under JF (denoted $\pi_{0,1+}$ ), this last fraction is point identified by the proportional reduction in the fraction of women not working under JF relative to AFDC.

It is useful for conducting inference to obtain analytic expressions for the bounds as a function of the regime-specific marginal distributions $\left(\mathbf{p}^{a}, \mathbf{p}^{j}\right)$. We accomplished this by solving the relevant linear programming problems by hand. The resulting expressions are listed in the Online Appendix. An example is given by the bounds on the opt-in probability $\pi_{2 n, 1 r}$ which take the form:

$$
\max \left\{0, \frac{p_{2 n}^{a}-p_{2 n}^{j}}{p_{2 n}^{a}}\right\} \leq \pi_{2 n, 1 r} \leq \min \left\{\begin{array}{c}
1,  \tag{13}\\
\frac{\left(p_{2 n}^{a}-p_{2 n}^{j}\right)+\left(p_{0 p}^{a}-p_{0 p}^{j}\right),}{p_{2 n}^{a}}, \\
\frac{\left(p_{2 n}^{a}-p_{2 n}^{j}\right)+\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{0 n}^{a}-p_{0 n}^{j}\right)}{p_{2 n}^{a}}, \\
\frac{\left(p_{2 n}^{a}-p_{2 n}^{j}\right)+\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{2 p}^{a}-p_{2 p}^{j}\right)}{p_{2 n}^{a}}, \\
\frac{\left(p_{2 n}^{a}-p_{2 n}^{j}\right)+\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{1 n}^{a}-p_{1 n}^{j}\right)}{p_{2 n}^{a}}, \\
\frac{\left(p_{2 n}^{a}-p_{2 n}^{j}\right)+\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{0 n}^{a}-p_{0 n}^{j}\right)+\left(p_{2 p}^{a}-p_{2 p}^{j}\right)}{p_{2 n}^{a}}, \\
\frac{\left(p_{2 n}^{a}-p_{2 n}^{j}\right)+\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{0 n}^{a}-p_{0 n}^{j}\right)+\left(p_{1 n}^{a}-p_{1 n}^{j}\right)}{p_{2 n}^{a}}, \\
\frac{\left(p_{2 n}^{a}-p_{2 n}^{j}\right)+\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{2 p}^{a}-p_{2 p}^{j}\right)+\left(p_{1 n}^{a}-p_{1 n}^{j}\right)}{p_{2 n}^{a}}, \\
\frac{\left(p_{2 n}^{a}-p_{2 n}^{j}\right)+\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{0 n}^{a}-p_{0 n}^{j}\right)+\left(p_{2 p}^{a}-p_{2 p}^{j}\right)+\left(p_{1 n}^{a}-p_{1 n}^{j}\right)}{p_{2 n}^{a}}
\end{array}\right\} .
$$

Note that there are two possible solutions for the lower bound, one of which is zero. This is a generic feature of the lower bounds for each of the nine set-identified response probabilities. ${ }^{26}$ The

[^17]upper bound on $\pi_{2 n, 1 r}$ admits nine possible solutions. Other response probabilities can have fewer or more solutions. ${ }^{27}$

## Estimation and Inference

Consistent estimators of the upper and lower bounds of interest can be had by using sample analogs of the marginal probabilities and computing the relevant min $\{$.$\} and \max \{$.$\} expressions. Inference$ is complicated by the fact that the limit distribution of the upper and lower bounds depends upon uncertainty in which of the constraints in (12) bind - i.e. in which of the bound solutions is relevant. As discussed by Andrews and Han (2009), bootstrapping the empirical min $\{$.$\} and \max \{$.$\} of the$ sample analogues of the bound solutions will fail to capture the sampling uncertainty in the bounds, particularly when several constraints are close to binding.

We report confidence intervals for the response probabilities based upon two inference procedures described in detail in the Online Appendix. The first procedure ignores the uncertainty in which constraints bind - that is, it assumes the bound solution that appears relevant given the sample analogues binds with probability one. In such a case, results from Imbens and Manski (2002) imply a $95 \%$ confidence interval for the parameter in question can be constructed by extending the upper and lower bounds by $1.65 \widehat{\sigma}$ where $\widehat{\sigma}$ is a standard bootstrap estimate of the standard error of the sample moment used to define the relevant bound. These "naive" confidence intervals will provide valid inferences if no other constraints are close to binding.

The second approach, which is also based on a bootstrap procedure, covers the parameter with asymptotic probability greater than or equal to $95 \%$ regardless of which solutions bind. Heuristically, this procedure assumes that all bound solutions are identical, in which case sampling uncertainty in all of the solution estimates affects the composite bound. The lower limit of the resulting "conservative" confidence interval coincides with that of the naive confidence interval because sampling uncertainty only affects one of the bound solutions in the max $\{$.$\} operator. However, the$ upper limit of the conservative confidence interval generally exceeds that from the naive confidence interval, often by a substantial amount.

[^18]
## Further Structuring Preferences

As an illustration of the identifying power of further structuring preferences, we also consider the monetized form of the utility function given in (4). In the Online Appendix we show that under this restricted specification, the choice of $0 r$ under AFDC by woman $i$ reveals that her stigma cost $\phi_{i}$ is below the base grant amount $\bar{G}_{i}$. This, in turn, implies that state $1 n$ is dominated by state $1 r$ under JF. Hence, no woman pairs state $0 r$ under AFDC with state $1 n$ under JF. Accordingly, $\pi_{0 r, 1 n}=0$ which reduces the number of unknown response probabilities to eight. Imposing this restriction on system (10) reveals that the second equation uniquely identifies the response probability $\pi_{1 n, 1 r}$. Intuitively, when $\pi_{0 r, 1 n}=0$, there is a "flow" into but no "flow" out of state $1 n$. Furthermore, this version of the model implies the additional testable restriction:

$$
\begin{equation*}
p_{1 n}^{a}-p_{1 n}^{j} \geq 0 . \tag{14}
\end{equation*}
$$

Given non-rejection of this additional restriction, the derivation of the bounds, estimation, and inference can be carried out as described above with the point identified response probability $\pi_{1 n, 1 r}$ computed by plugging in its sample analogue $\frac{\hat{p}_{1 n}^{a}-\hat{p}_{1 n}^{j}}{\hat{p}_{1 n}^{a}}$.

## 7 Results

Table 5 reports the estimated probabilities of occupying the six observable earnings and welfare participation states under each policy regime in the seven quarters after random assignment. ${ }^{28}$ The sixteen testable restrictions of our baseline model, as well as the additional restriction (14) associated with the monetized form of utility, are satisfied by the point estimates. There is a small but statistically significant increase in the fraction of quarters on welfare with earnings above the quarterly poverty line indicating that, on net, JF induced more women to under-report earnings than it induced to truthfully report them.

Table 6 provides estimates of the response probabilities that rationalize the impacts in Table 5. Panel (a) of the Table reports estimates obtained under the general specification of preferences given in (3), while panel (b) reports estimates obtained under the monetized specification of preferences given in (4).

Starting with panel (a), our most important finding is that the JF reform induced a substantial opt-in response among women who would have otherwise worked off welfare at earning levels above

[^19]the poverty line. The estimated bounds imply that $\pi_{2 n, 1 r} \geq .28$. That is, at least $28 \%$ of those women with ineligible earnings under AFDC decided to work at eligible levels under JF and participate in welfare - an intensive margin labor supply response. Accounting for sampling uncertainty in the bounds extends this lower limit to $19 \%$, which is still quite substantial. The upper bounds for this parameter are not informative leading us to conclude that the opt-in probability lies in the interval $[.19,1]$ with $95 \%$ probability. We also find suggestive evidence of a second opt-in effect from non-participation, this time entailing an extensive margin labor supply response. Specifically, the sample bounds imply $\pi_{0 n, 1 r} \in[.06, .62]$. However, uncertainty in the bounds prevents us from rejecting the null that this response probability is actually zero.

We find a small but significant under-reporting response attributable to the hassle effects of JF. A conservative $95 \%$ confidence interval for $\pi_{0 r, 2 u}$ is [.02,.13]. Thus, JF induced at least one sub-population to under-report earnings. JF also had a strong effect on entry into the program by the working poor. The bootstrap confidence interval for $\pi_{1 n, 1 r}$ indicates that at least $32 \%$ of the women who would have worked off welfare under AFDC at earnings levels below the poverty line were induced to participate in JF at eligible earning levels.

The remaining response probabilities $\left(\pi_{0 r, 0 n}, \pi_{0 r, 2 n}, \pi_{0 r, 1 n}, \pi_{0 r, 1 r}, \pi_{2 u, 1 r}\right)$ each have zero lower bounds. However, we can reject the null that they are jointly zero. From (10) such a joint restriction implies $p_{0 p}^{j}-p_{0 p}^{a}=-\left(p_{2 p}^{j}-p_{2 p}^{a}\right)$, which is easily rejected by our data. Thus, at least some of these margins of adjustment are present. Among the probabilities in question, the candidate that seems most likely to be positive is $\pi_{0 r, 1 r}$ which is the extensive margin response through which welfare reform has traditionally been assumed to operate.

The last four rows of panel (a) in Table 6 report the estimated bounds, and corresponding confidence intervals, for the composite margins described in Section 6. First is the probability $\pi_{0,1+}$ that a woman responds along the extensive margin from non-work to work. A conservative $95 \%$ confidence interval for this probability is [0.13, 0.21]. Thus, JF induced a substantial fraction of women who would not have worked under AFDC to obtain employment under JF.

The confidence interval on the fraction $\pi_{n, p}$ of women induced to take up welfare by JF is relatively tight. Although JF unambiguously increased the fraction of women on welfare, our model suggests some women may also have been induced to leave welfare, breaking point identification of this margin. According to our conservative inference procedure, at least $19 \%$ (and at most $51 \%$ ) of women off welfare under AFDC were induced to claim benefits under JF. Conversely, the fraction $\pi_{p, n}$ of women induced by JF to leave welfare is estimated to be at most $17 \%$.

Finally, we cannot reject the null hypothesis that JF failed to induce any of the women who would have not worked while claiming AFDC benefits to leave welfare under JF, as the lower bound for the response probability $\pi_{0 r, n}$ is zero. We are however able to conclude that at most $24 \%$ of such women left welfare, which may limit concerns that the JF reforms pushed a large fraction of women potentially unable to work off assistance.

Consider now panel (b), which reports results when utility is assumed to be of the monetized form given in (4). Here, the response probability $\pi_{0 r, 1 n}$ is constrained to equal zero which renders $\pi_{1 n, 1 r}$ point identified. According to these estimates, the JF reform had a strong effect on entry into the program by the working poor. The bootstrap confidence interval for $\pi_{1 n, 1 r}$ indicates that between $31 \%$ and $46 \%$ of the women who would have worked off welfare under AFDC at earnings levels below the poverty line were induced to participate in JF at eligible earning levels. The estimates of the remaining response probabilities and composite margins are omitted because they are the same as in panel (a). ${ }^{29}$

## 8 Robustness and Extensions

Here we discuss potential extensions to our approach and issues which may affect the interpretation of our results. We start by exploring the implications of working with a finer coarsening of earnings. We then examine the impact of relaxing our lower bound restriction on the stigma disutility (Assumption A.6). Finally, we consider the implications of incorporating Food Stamps and taxes into the model.

## Finer Earnings Ranges

The above analysis was predicated on the coarsening of earnings dictated in (6). This coarsening scheme is "natural" in the sense that the JF reform changed the utility of all the alternatives corresponding to each of the earning ranges in (6) in the same direction. Specifically, the JF reform decreased (or left unchanged) the utility that a woman derives from any alternative corresponding to not working (range 0), increased (or left unchanged) the utility that a woman derives from any alternative corresponding to earning in range 1 , and left unchanged the utility that a woman derives from any alternative corresponding to earning in range 2 . Nevertheless, it can be of interest to consider finer coarsenings of earnings. For instance, our finding in Table 6 of a significant opt-in response could hypothetically reflect trivial earnings reductions from a dollar above the poverty line to exactly the poverty line. To assess such possibilities, consider the following finer coarsening of earnings obtained by partitioning range 2 into two sub-ranges:

$$
\widetilde{E}_{i} \equiv\left\{\begin{array}{ll}
0 & \text { if } E=0  \tag{15}\\
1 & \text { if } E \leq F P L_{i} \\
2^{\prime} & \text { if } E \in\left(F P L_{i}, 1.2 \times F P L_{i}\right] \\
2^{\prime \prime} & \text { if } E>1.2 \times F P L_{i}
\end{array} .\right.
$$

[^20]In the Online Appendix, we derive bounds on the response probabilities $\pi_{2^{\prime} n, 1 r}$ and $\pi_{2^{\prime \prime} n, 1 r}$ which correspond to the fraction of women in earnings ranges $2^{\prime}$ and $2^{\prime \prime}$ who opt-in to assistance by reducing their earnings. These bounds exploit the fact that, by revealed preference, no woman will pair state $2^{\prime} u$ under AFDC with state $2^{\prime \prime} u$ under JF. Likewise, no woman will pair state $2^{\prime} n$ under AFDC with state $2^{\prime \prime} n$ under JF.

Implementing these formulas, we find that at least $26.7 \%$ of women who would work off assistance in earnings range $2^{\prime \prime}$ under AFDC reduced their earnings below the poverty line in response to the JF reform. Accounting for sampling uncertainty yields a lower limit on a $95 \%$ confidence interval for $\pi_{2^{\prime \prime}, 1 r}$ of 0.150 . Hence, the evidence is strong that some large earnings reductions occurred in response to the JF reform. We also find that at least $30.9 \%$ of women who would work off assistance in earnings range $2^{\prime}$ under AFDC reduced their earnings below the poverty line in response to reform. The lower limit of the confidence interval for $\pi_{2^{\prime} n, 1 r}$ is 0.201 , which indicates that some opt-in responses also took place from earnings ranges closer to the poverty line. ${ }^{30}$

It is also possible to partition range 1 into two or more sub-ranges. We have experimented with such extensions but found that they fail to offer additional insights regarding the effects of the JF reform. There are good reasons for this. Recall that the theory does not constrain the sign of the labor supply responses that occur within range 1 . This theoretical indeterminacy persists if range 1 is partitioned into sub-ranges and prevents identification of the magnitude of these allowed intensive margin responses. Additionally, the possibility of under-reporting limits the utility of revealed preference arguments because states $1 u$ and $1 r$ are not empirically distinguishable. This prevents identification of the magnitude of any responses to the reform entailing adjustments in reporting behavior within range 1 .

## Stigma

Thus far, we have maintained assumption A. 6 which guarantees that women will not choose to truthfully report earnings above the FPL while on AFDC. Even without this restriction, women claiming AFDC are unlikely to earn in this range since AFDC benefit exhaustion induces a nonconvex kink in the budget set (Moffitt, 1990). Empirically, the number of observations in our sample for which this sort of behavior could be present is bounded from above by the number of quarters in the AFDC sample where women earn more than the FPL and receive a welfare transfer that is positive but no larger than $G_{i}^{a}\left(F P L_{i}\right)$. In our data, there are only 3 case-quarters (out of 14,784 ) meeting these criteria, implying that such behavior is extremely rare. ${ }^{31}$

[^21]Nevertheless, it is of pedagogical interest to consider what additional responses emerge if we do not rule out such choices a priori. In the Online Appendix we show that dropping A. 6 enables flows out of the labor force, which are absent in the model of Section 4. We show that earning constraints are essential to the emergence of these flows.

## Food Stamps, the EITC, and Payroll Taxes

In Section 4 we ignored three programs that are typically relevant for would-be welfare recipients: FS, payroll taxes, and the EITC. Here we summarize why the inclusion of these programs does not change our conclusions about the theoretically allowable effects of the JF reform.

In the Online Appendix we develop an extended model where FS participation is introduced as an additional choice, so that a woman may be off assistance, on welfare only, on FS only, or on both welfare and FS. We allow separate stigma effects for each combination of FS and welfare assistance. Under-reporting costs also vary depending on the type of assistance. Filing for EITC is assumed invariant to the policy regime, and payroll and Medicare taxes are levied on earnings under both regimes. We distinguish sixteen combinations of coarsened earnings, welfare participation, FS participation, and earnings reporting categories that a woman may occupy under either regime. Appendix Table A5 catalogs the theoretically allowed and disallowed responses: revealed preference arguments proscribe 190 out of the $16 \times 15=240$ atheoretically possible responses leaving us with 50 allowed responses. The disallowed responses imply restrictions on a corresponding $16 \times 16$ matrix of response probabilities.

An important feature of this matrix is that if we integrate out FS participation we obtain a matrix with exactly the same zero and unitary entries as the matrix $\Pi$ associated with the model of Section 4. There are two reasons for this convenient result. First, as described in Section 1, under JF earnings up to the FPL were disregarded in full for the determination of the FS grant only conditional on joint take up of welfare. Thus, JF's impact on the FS program effectively amplifies the notch at the FPL (recall Figure 2) and leaves the attractiveness of the non-welfare assistance states unaffected. Second, when deriving restrictions from the extended model we use the same coarsened earnings categories employed in conjunction with the model of Section 4. While FS can generate additional predictions about behavior within these earnings categories, it does nothing to alter predictions about pairings between them. Hence, the estimated responses presented in Table 6 can be interpreted as the responses to both the welfare and FS components of the JF reform given the tax system in place at the time of the reform.

FPL and with quarterly welfare transfers no greater than $G_{i}^{a *}\left(F P L_{i}\right)$.

## 9 Conclusion

Our analysis of the Jobs First experiment suggests that women responded to the policy incentives of welfare reform along several margins, some of which entail an intensive margin and some of which entail an extensive margin labor supply response. This finding is in accord with BGH's original interpretation of the JF experiment and with recent evidence from Blundell, Bozio, and Laroque (2011a,b) who find that secular trends in aggregate hours worked appear to be driven by both intensive and extensive margin adjustments. Our conclusions are also qualitatively consistent with recent studies relying on dynamic parametrically structured labor supply models (e.g., Blundell et al., 2012; Blundell et al., 2013).

An important question is the extent to which our finding of intensive margin responsiveness might generalize to other transfer programs that lack sharp budget notches but still involve phaseout regions that should discourage work. It seems plausible that the JF notch would yield larger disincentive effects than, say, the budget kink induced by the EITC phase-out region. However, BGH (2008) show that experimental responses to a Canadian reform inducing such a gradual benefit phaseout generated a pattern of earnings QTEs similar to that found in the JF experiment. More conclusive evidence on this question may be had via an application of the methods developed here to other policy reforms.

Though we studied a randomized experiment, our approach is easily adapted to quasi-experimental settings. Estimates of the relevant counterfactual choice probabilities can be formed using one's research design of choice (e.g., a difference in differences design), subject to the usual caveat that different designs may identify counterfactuals for different treated sub-populations. ${ }^{32}$ With the two sets of marginal choice probabilities, bounds on response probabilities can then be had by a direct application of the methods developed in this paper.

A potentially fruitful avenue for future research is to consider the application of revealed preference arguments to dynamic models. Alternatives in such models consist of sequences of possible choices, which significantly enlarges the space of potential responses that can occur. However, explicitly dynamic models also provide additional opportunities to incorporate plausible nonparametric restrictions (e.g., stationary and time-separable preferences) that may yield interesting empirical predictions. We leave the development of such methods to future research.

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Table 1: Summary of Differences Between Status Quo and Jobs First Policy Regimes

|  | Jobs First | Status Quo |
| :---: | :---: | :---: |
| Welfare: |  |  |
| Name of Program | Temporary Family Assistance (TFA) | Aid for Families with Dependent Children (AFDC) |
| Eligibility | Earnings Below Poverty Line | Earnings level at which benefits are exhausted (see disregard parameters below) |
| Earnings disregard | - Fixed Disregard: n.a. <br> - Proportional Disregard: 100\% | - Fixed Disregard: $\$ 120$ (first 12 months of work), $\$ 90$ (after 12 months) <br> - Proportional Disregard: 51\% (first 4 months of work), $27 \%$ (after month 4) |
| Time Limit | 21 months | None |
| Work requirements | Mandatory work first employment services (exempt if child <1) | Education / training (exempt if child < 2) |
| Other | - Sanctions (3 month grant reduction due to infraction: 20\% (1st), 35\% (2nd), 100\% (3rd); moderate enforcement) <br> - Asset limit \$3,000 <br> - Family cap \$50 <br> -Two years transitional Medicaid <br> -Transitional Child Care Assistance indefinitely provided as long as income is <75\% of state median <br> -Child support: \$100 disregarded; full pass-through | -Sanctions (grant reduction corresponding to removal of adult from AU; rarely enforced) <br> -Asset limit \$1,000 <br> - Family cap \$100 <br> - One year transitional Medicaid <br> - Transitional Child Care Assistance for one year as long as income is $<75 \%$ of state median <br> -Child support: \$50 disregarded; \$50 maximum pass-through |
| Food Stamps (if joint with welfare): |  |  |
| Earning Disregard | - Fixed Disregard: n.a. <br> - Proportional Disregard: $100 \%$ of earnings up to FPL | - Fixed Disregard: n.a. <br> - Proportional Disregard: 76\% of earnings up to the eligibility threshold |
| Sources: Bloom et al. (2002). |  |  |
|  <br>  <br>  <br>  <br>  <br>  <br>  <br>  <br>  <br>  <br>  <br>  <br>  <br>  <br>  earnings are disregarded as long as recipients are earning below the FPL. When JF recipients lose their welfare grant, they also lose the enhanced FS earned income disregard. |  |  |

Table 2: Mean Sample Characteristics

|  | Overall Sample |  |  |  | Zero Earnings Q7 pre-RA |  |  |  | Positive Earnings Q7 pre-RA |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Jobs First | AFDC | Difference | Difference (adjusted) | Jobs First | AFDC | Difference | Difference (adjusted) | Jobs First | AFDC | Difference | Difference (adjusted) |
| Demographic Characteristics |  |  |  |  |  |  |  |  |  |  |  |  |
| White | 0.374 | 0.360 | 0.014 | 0.001 | 0.340 | 0.331 | 0.009 | -0.001 | 0.453 | 0.421 | 0.032 | 0.003 |
| Black | 0.380 | 0.384 | -0.004 | 0.000 | 0.370 | 0.360 | 0.010 | 0.001 | 0.404 | 0.435 | -0.031 | -0.002 |
| Hispanic | 0.214 | 0.224 | -0.010 | -0.001 | 0.258 | 0.275 | -0.017 | 0.000 | 0.110 | 0.117 | -0.007 | -0.002 |
| Never married | 0.654 | 0.661 | -0.007 | 0.000 | 0.658 | 0.654 | 0.003 | 0.000 | 0.645 | 0.674 | -0.029 | 0.000 |
| Div/wid/sep/living apart | 0.332 | 0.327 | 0.005 | 0.000 | 0.327 | 0.334 | -0.007 | 0.000 | 0.345 | 0.312 | 0.032 | 0.000 |
| HS dropout | 0.350 | 0.334 | 0.017 | 0.000 | 0.390 | 0.394 | -0.004 | 0.000 | 0.257 | 0.209 | 0.048 | 0.000 |
| HS diploma/GED | 0.583 | 0.604 | -0.021 | 0.000 | 0.550 | 0.555 | -0.005 | -0.001 | 0.661 | 0.706 | -0.045 | 0.001 |
| More than HS diploma | 0.066 | 0.062 | 0.004 | 0.000 | 0.060 | 0.051 | 0.009 | 0.000 | 0.082 | 0.085 | -0.003 | -0.001 |
| More than 2 Children | 0.235 | 0.214 | 0.021 | 0.000 | 0.260 | 0.250 | 0.010 | 0.000 | 0.176 | 0.139 | 0.037 | 0.001 |
| Mother younger than 25 | 0.287 | 0.298 | -0.011 | -0.003 | 0.287 | 0.268 | 0.019 | -0.001 | 0.288 | 0.361 | -0.074 | 0.000 |
| Mother age 25-34 | 0.412 | 0.414 | -0.003 | 0.005 | 0.410 | 0.419 | -0.009 | 0.000 | 0.416 | 0.405 | 0.010 | 0.000 |
| Mother older than 34 | 0.301 | 0.287 | 0.014 | -0.002 | 0.303 | 0.313 | -0.010 | 0.001 | 0.297 | 0.233 | 0.063 | 0.000 |
| Average quarterly pretreatment values |  |  |  |  |  |  |  |  |  |  |  |  |
| Earnings | 673 | 750 | -76* | 4 | 174 | 185 | -11 | 2 | 1856 | 1935 | -79 | 11 |
|  | [1306] | [1379] | (40) | (6) | [465] | [479] | (17) | (4) | [1802] | [1828] | (99) | (21) |
| Cash welfare | 903 | 845 | 58** | 1 | 1050 | 1022 | 28 | 0 | 555 | 475 | 80** | -4 |
|  | [805] | [784] | (23) | (2) | [811] | [799] | (28) | (3) | [679] | [602] | (35) | (7) |
| Food stamps | 356 | 344 | 12 | 0 | 399 | 398 | 1 | 1 | 253 | 230 | 23 | -2 |
|  | [320] | [304] | (9) | (1) | [326] | [310] | (11) | (1) | [281] | [256] | (15) | (4) |
| Fraction of pretreatment quarters with |  |  |  |  |  |  |  |  |  |  |  |  |
| Any earnings | 0.319 | 0.347 | -0.029*** | 0.000 | 0.137 | 0.143 | -0.007 | 0.000 | 0.751 | 0.776 | -0.025* | 0.000 |
|  | [0.362] | [0.370] | (0.011) | (0.001) | [0.211] | [0.215] | (0.008) | (0.001) | [0.262] | [0.238] | (0.014) | (0.002) |
| Any welfare assistance | 0.581 | 0.551 | 0.030* | -0.001 | 0.650 | 0.636 | 0.014 | 0.000 | 0.418 | 0.373 | 0.045* | -0.002 |
|  | [0.451] | [0.450] | (0.013) | (0.001) | [0.439] | [0.439] | (0.015) | (0.001) | [0.438] | [0.416] | (0.023) | (0.004) |
| Any Food Stamp assistance | 0.613 | 0.605 | 0.008 | 0.000 | 0.670 | 0.674 | -0.004 | 0.001 | 0.480 | 0.460 | 0.020 | -0.003 |
|  | [0.437] | [0.431] | (0.012) | (0.001) | [0.427] | [0.421] | (0.015) | (0.001) | [0.433] | [0.418] | (0.023) | (0.004) |
| \# of cases | 2,318 | 2,324 |  |  | 1,630 | 1,574 |  |  | 688 | 750 |  |  |

Notes: Sample units missing baseline data on number of children (kidcount) are excluded. Adjusted differences are computed via propensity score reweighting. Numbers in brackets are standard deviations and numbers in parentheses are standard errors calculated via 1,000 block bootstrap replications (resampling at case level). ***, **, and * indicate statistical significance at the 1-percent, 5-percent, and 10-percent levels, respectively (significance indicators provided only for difference estimates).

Table 3: Fraction of Months on Welfare by Experimental Status and Age of Youngest Child

| Status and Age of Youngest Child |  |  |
| :---: | :---: | :---: |
| Age of Youngest Child at Baseline: | 16 or 17 | 15 or less |
| AFDC | 0.441 | 0.651 |
|  | $(0.038)$ | $(0.008)$ |
| JF | 0.508 | 0.740 |
|  | $(0.039)$ | $(0.007)$ |
| Difference | 0.066 | 0.089 |
|  | $(0.055)$ | $(0.010)$ |
| Difference in Differences | $\mathbf{- 0 . 0 2 2}$ |  |
|  |  | $\mathbf{( 0 . 0 5 6 )}$ |

Notes: Sample consists of 87,717 case-months: 21 months of data on each of 4,177 cases with non-missing baseline information on age of youngest child. Table gives reweighted fraction of case-months that women participated in welfare by experimental status and age of youngest child at baseline. Standard errors computed using 1,000 block bootstrap
replications (resampling at case level).

State under Jobs First

| State under AFDC | On | 1n | $2 n$ | Or | $1 r$ | 14 | $2 u$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| On | No Response | - | - | - | Extensive LS (+) <br> Take Up Welfare | - | - |
| 1 n | - | No Response | - | - | Intensive LS (+/0/-) Take Up Welfare | - | - |
| 2n | - | - | No Response | - | Intensive LS (-) Take Up Welfare (Figure 5 b) | - | - |
| Or | No LS Response Exit Welfare | Intensive LS (+) Exit Welfare | Extensive LS (+) Exit Welfare | No Response | Extensive LS (+) <br> (Figure 5 a) | - | Extensive LS (+) <br> Under-reporting <br> (Figure 6 b) |
| $1 r$ | - | - | - | - | Intensive LS (+/0/-) | - | - |
| 14 | - | - | - | - | Intensive LS (+/0/-) <br> Truthful Reporting | - | - |
| $2 u$ | - | - | - | - | Intensive LS (-) Truthful Reporting (Figure 6 a) | - | No Response |

 positive earnings at or below the FPL, and 2 for earnings strictly above the FPL), and participation status in the relevant welfare assistance program along with an earnings reporting decision ( n stands for "not on assistance", r for "on assistance and truthfully reporting earnings", and u for "on assistance and under-reporting earnings"). The cells termed "no response" entail the same behavior under the two policy regimes. The cells containing a "一" represent responses that are not allowed based on revealed preference arguments derived from the nonparametric model of Section 4 . Specifically, (a) state 1 u is unpopulated under JF ("一" in cells with a horizontally striped background fill) and (b) a woman will not leave a state at least as attractive under JF as under AFDC for a state that is no more attractive under JF than under AFDC ("-" in cells with a solid greyed-out background fill). The remaining cells represent responses that are allowed by the model. Their content summarizes the three possible sorts of responses: (a) the labor supply "LS" response (intensive versus extensive and its sign: " + " for increase, " 0 " for no change, and "-" for decrease), (b) the program participation response (take up of versus exit from welfare assistance), and (c) the reporting of earnings to the welfare agency margin (to truthfully report versus to under-report). See Online Appendix for proof.

Table 5: Probability of Earnings / Participation States

|  | Overall |  |  | Overall - Adjusted |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Jobs First | AFDC | Difference | Jobs First | AFDC | Difference |
| $\operatorname{Pr}$ (State=On) | 0.127 | 0.136 | -0.009 | 0.128 | 0.135 | -0.007 |
|  |  |  |  | (0.006) | (0.006) | (0.008) |
| $\operatorname{Pr}($ State $=1 \mathrm{n})$ | 0.076 | 0.130 | -0.055 | 0.078 | 0.126 | -0.048 |
|  |  |  |  | (0.004) | (0.005) | (0.006) |
| $\operatorname{Pr}($ State $=2 \mathrm{n})$ | 0.068 | 0.099 | -0.031 | 0.069 | 0.096 | -0.027 |
|  |  |  |  | (0.004) | (0.005) | (0.006) |
| $\operatorname{Pr}($ State $=0 \mathrm{p})$ | 0.366 | 0.440 | -0.074 | 0.359 | 0.449 | -0.090 |
|  |  |  |  | (0.008) | (0.008) | (0.012) |
| $\operatorname{Pr}($ State $=1 \mathrm{p})$ | 0.342 | 0.185 | 0.157 | 0.343 | 0.184 | 0.159 |
|  |  |  |  | (0.008) | (0.006) | (0.009) |
| $\operatorname{Pr}($ State $=2 \mathrm{p}$ ) | 0.022 | 0.009 | 0.013 | 0.023 | 0.009 | 0.014 |
|  |  |  |  | (0.002) | (0.001) | (0.002) |
| \# of quarterly observations | 16,226 | 16,268 |  | 16,226 | 16,268 |  |

Notes: Sample covers quarters 1-7 post-random assignment during which individual is either always on or always off welfare. Sample cases with kidcount missing are excluded. Number of state refers to earnings level, with 0 indicating no earnings, 1 indicating earnings below 3 times the monthly FPL, and 2 indicating earnings above 3FPL. The letter " $n$ " indicates welfare nonparticipation throughout the quarter while the letter " p " indicates welfare participation throughout the quarter. Poverty line computed under assumption AU size is one greater than amount implied by baseline kidcount variable. Adjusted probabilities are computed via propensity score reweighting. Standard errors computed using 1,000 block bootstrap replications (resampling at case level).

Table 6: Point and Set-identified Response Probabilities

| Response Type | State Occupied under |  | (a) General Specification of Preferences |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |
|  | AFDC | JF | Symbol | Estimate | Standard Error | $\begin{aligned} & 95 \% \mathrm{Cl} \\ & \text { (naive) } \end{aligned}$ | $\begin{gathered} 95 \% \mathrm{Cl} \\ \text { (conservative) } \end{gathered}$ |
| Detailed | On | $1 r$ | $\pi_{0 n, 1 \mathrm{r}}$ | $\{0.055,0.620\}$ |  | [0.000, 0.758] | [0.000, 0.879] |
|  | 1 n | $1 r$ | $\pi_{1 \mathrm{n}, 1 \mathrm{r}}$ | \{0.382, 0.987\} |  | [0.320, 1.000] | [0.320, 1.000] |
|  | 2 n | $1 r$ | $\pi_{2 n, 1 r}$ | \{0.280, 1.000\} |  | [0.193, 1.000] | [0.193, 1.000] |
|  | Or | On | $\pi_{0 r, 0 n}$ | \{0.000, 0.170\} |  | [0.000, 0.211] | [0.000, 0.247] |
|  |  | 1 n | $\pi_{0 r, 1 \mathrm{n}}$ | \{0.000, 0.170\} |  | [0.000, 0.211] | [0.000, 0.251] |
|  | " | 2 n | $\pi_{0 r, 2 n}$ | \{0.000, 0.154\} |  | [0.000, 0.171] | [0.000, 0.304] |
|  | " | 1 r | $\pi_{0 r, 1 \mathrm{r}}$ | \{0.000, 0.170\} |  | [0.000, 0.211] | [0.000, 0.251] |
|  | " | 2u | $\pi_{0 r, 2 u}$ | \{0.031, 0.051\} |  | [0.022, 0.058] | [0.022, 0.131] |
|  | 2u | $1 r$ | $\pi_{2 u, 1 \mathrm{r}}$ | \{0.000, 1.000\} |  | [0.000, 1.000] | [0.000, 1.000] |
| Composite | Not working | Working | $\pi_{0,1+}$ | 0.167 | 0.020 | [0.129, 0.206] | [0.129, 0.206] |
|  | Off welfare | On welfare | $\pi_{n, p}$ | \{0.231, 0.445\} |  | [0.187, 0.487] | [0.187, 0.510] |
|  | On welfare | Off welfare | $\pi_{p, n}$ | \{0.000, 0.119\} |  | [0.000, 0.148] | [0.000, 0.174] |
|  | On welfare, not working | Off welfare | $\pi_{0 r, n}$ | $\{0.000,0.170\}$ |  | [0.000, 0.211] | [0.000, 0.245] |
| Detailed | Or$1 n$ |  | (b) Restricted Specification of Preferences |  |  |  |  |
|  |  | 1n | $\pi_{0 r, 1 \mathrm{n}}$ | 0 |  |  |  |
|  |  | $1 r$ | $\pi_{1 \mathrm{n}, 1 \mathrm{r}}$ | 0.382 | 0.038 | [0.308, 0.456] | [0.308, 0.456] |

Notes: Number of state refers to earnings level, with 0 indicating no earnings, 1 indicating earnings below 3 times the monthly FPL, 2 indicating earnings above 3FPL, and $1+$ indicating positive earnings. The letter " $n$ " indicates welfare nonparticipation, the letter " $r$ " indicates welfare participation with truthful reporting of earnings, the letter " $u$ " indicates welfare participation with under-reporting of earnings, the letter " p " indicates welfare participation (irrespective of reporting). Composite response probabilities are linear combination of the detailed response probabilities (see Section 6 for the exact expressions). Estimates inferred from probabilities in Table 5 , see text for formulas. Numbers in braces are estimated upper and lower bounds, numbers in brackets are $95 \%$ confidence intervals. "Naive" $95 \%$ confidence interval ignores uncertainty in which moment inequalities bind. "Conservative" $95 \%$ confidence interval assumes all constraints bind. See Online Appendix for details. Panel a refers to the general specification of preferences (expression (3) in the paper). Panel $b$ refers to the restricted specification (expression (4) in the paper). Panel bomits all response probabilities whose estimates are the same as in Panel a.

Figure 1: Monthly Earnings plus Welfare Transfer under AFDC and Jobs First
a) AFDC

b) Jobs First


Notes: The figures (not drawn to scale) depict the sum of monthly earnings and welfare transfers for a woman with 2 children under AFDC (panel a) and Jobs First (panel b) policy rules as of 1997. FPL refers to federal poverty line $(\$ 1,111)$ and $\bar{G}$ is the base grant amount ( $\$ 543$ ). The illustration assumes that the woman only has access to the fixed $\$ 90$ disregard and the proportional $73 \%$ disregard under AFDC which implies that the AFDC transfer is exhausted at earnings level $\bar{E}$ corresponding to $\bar{G} / 0.73+90$ ( $\$ 835$ ). The JF welfare transfer falls to zero at earnings levels above FPL. Under JF, a woman who earns between FPL $(\$ 1,111)$ and $F P L+\bar{G}(\$ 1,654)$ can increase the sum of earnings and welfare transfers by taking up welfare assistance and working less.

Figure 2: Net Income under Status Quo and Jobs First Policies, Accounting for Food Stamps and Taxes


Notes: Figure depicts net income as a function of gross earnings for an assistance unit of size 3 in 1997 under the status-quo policy and the JF policy rules. Illustration assumes household only has access to fixed $\$ 90$ disregard under AFDC and faces $\$ 366$ in monthly rental expenses. Net income is earnings net of federal income taxes and inclusive of EITC and welfare and Food Stamps transfers (given participation in either program). Vertical lines: at the AFDC fixed earning disregard and break-even level (\$90 and \$835), at the end of the EITC's phase-in and start and end of the phase-out regions ( $\$ 762, \$ 994$ and $\$ 2,441$ ), at the minimum taxable earnings ( $\$ 1,167$ ), at the FPL ( $\$ 1,111$ ), and at $1.3 x F P L$ $(\$ 1,444)$ which is a FS eligibility threshold under AFDC. Horizontal ticks: at maximum FS and welfare grants.

Figure 3: Distribution of Quarterly Earnings Centered at 3 x Monthly Federal Poverty Line
a) Unconditional

b) On Assistance all 3 Months of the Quarter

c) Off Assistance all 3 Months of the Quarter


Notes: Restricted to the Jobs First sample in quarters 1-7 post random assignment. Assistance unit (AU) size has been inferred from monthly aid payment. AU sizes above eight have been excluded. The bins in the histograms are $\$ 100$ wide with bin 0 containing three times the monthly federal poverty line corresponding to the size and the calendar year of the quarterly observation. Vertical line indicates Jobs First eligibility threshold at three times the monthly federal poverty line.

Figure 4: CDFs of Quarterly Earnings Relative to 3 x Federal Poverty Line
a) Unconditional

b) Zero Earnings Prior to Random Assignment

c) Positive Earnings Prior to Random Assignment


Notes: Figures give reweighted CDFs of quarterly UI earnings (in quarters 1-7 post-RA) in JF and AFDC samples relative to three times the monthly federal poverty line associated with year and $A U$ size. Panel (b) refers to women with zero earnings in the $7^{\text {th }}$ quarter prior to random assignment, while panel (c) refers to women with positive earnings in that quarter. AU size determined by baseline survey variable "kidcount." To deal with increases in family size since random assignment, we use one plus the AU size directly implied by kidcount. The "p-value for equality" refers to a Kolmogorov-Smirnov test of equality of the two distributions, while "p-value for FOSD" refers to a Barrett-Donald test for first order stochastic dominance of the JF distribution over the AFDC distribution (both based on 1,000 block bootstrap replications at case level, see Online Appendix for details).

Figure 5: Extensive and Intensive Margin Responses to Reform
a) From Not Working on Assistance under AFDC to Earning in Range 1 under Jobs First


b) From Working in Range 2 off Assistance under AFDC to Earning in Range 1 under Jobs First



Notes: Panels a and b are drawn in the earnings (horizontal axis) and consumption equivalent (vertical axis) plane. The consumption equivalent equals earnings plus transfer income from welfare (if any) net of monetized hassle, stigma, work, and under-reporting costs (if any, see text for details). At each level of earnings, the bold lines correspond to consumption either off welfare or on welfare with truthful reporting of earnings to the welfare agency. The dashed lines correspond to consumption on welfare with under-reporting. Vertical lines represent the same earnings levels depicted in Figure 1: the fixed earning disregard under AFDC (\$90), the earnings level $\bar{E}$ at which welfare assistance is exhausted under AFDC, and the FPL. For clarity, the graphs assume away earnings constraints and use a fixed wage rate. Panel a depicts a scenario where the JF reform induces a woman who would participate in welfare and not work under AFDC (point A) to take up work and truthfully report her earnings under JF (point B) - an extensive margin response. Panel b depicts a scenario where the JF reform induces a woman who be off assistance and earn in range 2 (point A) to reduce her earnings to range 1 and take up assistance under JF (point B) - an intensive margin response.

Figure 6: Earnings and Participation Choices with Under-reporting
a) From Under-reporting under AFDC to Truthful Reporting under Jobs First


b) From Truthful Reporting under AFDC to Under-reporting under Jobs First



Notes: Panels $a$ and $b$ are drawn in the earnings (horizontal axis) and consumption equivalent (vertical axis) plane. The consumption equivalent equals earnings plus transfer income from welfare (if any) net of monetized hassle, stigma, work, and under-reporting costs (if any, see text for details). At each level of earnings, the bold lines correspond to consumption either off welfare or on welfare with truthful reporting of earnings to the welfare agency. At each level of earnings, the dashed lines correspond to consumption on welfare with under-reporting. Vertical lines represent the same earnings levels depicted in Figure 1: the fixed earning disregard under AFDC (\$90), the earnings level $\bar{E}$ at which welfare assistance is exhausted under AFDC, and the FPL. For clarity, the graphs assume away earnings constraints and use a fixed wage rate. Panel a depicts a scenario where the JF reform induces a woman who would participate in welfare, work, and under-report her earnings under AFDC (point A) to work and truthfully report her earnings under JF (point B) thanks to the $100 \%$ earning disregard under JF. Panel b depicts a scenario where the JF reform induces a woman who would participate in welfare without work under AFDC (point A) to work and under-report her earnings under JF (point B) to avoid the hassle cost under JF.

## Online Appendix

## Outline

- Section 1 provides additional details about the data and variable construction.
- Section 2 describes the propensity score re-weighting method used to adjust for chance imbalances in baseline characteristics.
- Section 3 explains how we construct the tests for equality and first order stochastic dominance whose p-values are reported in Figure 4 of the paper.
- Section 4 presents the baseline model described in Sections 4. We start by introducing definitions and restating the assumptions made in the paper. We then prove a few intermediate lemmas and conclude with the main propositions and their proofs which support the revealed preference restrictions summarized in Section 5 of the paper. Specifically,
- Lemma 1 establishes that no woman truthfully reports earnings above the federal poverty level while on assistance. Lemma 2 characterizes optimal reporting of earnings to the welfare agency. Corollary 1 describes the implication of optimal reporting for the dependence of preferences on the policy regime.
- Lemma 3 characterizes the relative attractiveness of each state under the two policy regimes. Lemma 4 provides the main revealed preference argument regarding pairing of states under JF and AFDC.
- Propositions 1 and 2 formally establish Table 4 in the paper. Corollary 2 establishes additional disallowed responses under the special form of the utility function introduced in Section 5 of the paper.
- Section 5, specifically Lemma 5, describes the exhaustive set of testable restrictions on state probabilities implied by revealed preference, as presented in Section 6 of the paper.
- Section 6 lists the analytical expressions for the bounds on the response probabilities and explain how they were derived. An example of such bounds is reproduced in Section 6 of the paper.
- Section 7 describes the construction of the $95 \%$ confidence intervals reported in Table 6 of the paper.
- Section 8 develops an extended model that relaxes the lower bound on stigma assumed in Section 4 the paper. This model is briefly referenced in Section 8 of the paper. Specifically,
- Propositions 3 and 4 establishes the effect of this relaxation on the response margins, as summarized in Table A4. Corollary 4 establishes additional disallowed responses under the special form of the utility function introduced in Section 4 of the paper.
- Corollary 5 shows that the relaxation of the lower bound on stigma may enable exit from the labor force in response to the JF reform only in the presence of labor market constraints.
- Section 9 develops an extended model that allows for participation in the FS program and accounts for taxes, including the EITC. This model is summarized in Section 8 of the paper. Specifically,
- Lemmas 6 and 7 characterize the combined welfare and FS transfer.
- Lemma 8 establishes that no woman truthfully reports earnings above the federal poverty level while on welfare assistance. Lemma 9 characterizes optimal reporting. Corollary 6 describes the implication of optimal reporting for the dependence of preferences on the policy regime.
- Lemma 10 provides the main revealed preference argument regarding pairing of states under JF and AFDC. Lemma 11 characterizes the relative attractiveness of each state under the two policy regimes.
- Propositions 6 and 7 establish the allowed and disallowed responses, as summarized in Table A5.
- Proposition 8 derives the response matrix. Proposition 9 and Remark 11 demonstrate that integrating out FS yields a response matrix with the same zero and unitary entries as the response matrix presented in Section 6 of the paper.
- Section 10 establishes the form of the response matrix when a finer coarsening of earnings is adopted. The results of this extension are summarized in Section 8 of the paper and the marginal distributions used for inference are reported in Table A6.
- Appendix Figures and Tables are provided at the end, along with references.


## 1 Data

## From Monthly to Quarterly Data

The public use files do not report the month of randomization. However, we were able to infer it by contrasting monthly assistance payments with an MDRC constructed variable providing quarterly assistance payments. For each case, we found that a unique month of randomization leads the aggregation of the monthly payments to match the quarterly measure to within rounding error.

## Measures of AU Size

The administrative measure of AU size is missing for most cases, which is problematic because the JF notch occurs at the FPL which varies with AU size. For the Jobs First sample we are able to infer an AU size in most months from the grant amount while the women are on welfare. However if AU size changes while off welfare we are not able to detect this change. ${ }^{1}$ Moreover, in some cases the grant amount does not match any of the base grant amounts. This can result when a woman reports some unearned income or because of sanctions. In both of these situations, we use the grant amount in other months to impute AU size. For the AFDC sample, the grant amount depends on many unobserved factors, preventing us from inferring the AU size from the administrative data.

The kidcount variable described in the text records the number of children in the household at the time of random assignment and is top-coded at three children. Appendix Table A1 gives a cross-tabulation, in the JF sample, of kidcount with our more reliable AU size measure inferred from grant amounts. The tabulation suggests the kidcount variable is a reasonably accurate measure of AU size over the first 7 quarters post-random assignment conditional on the number of children at baseline being less than three. As might be expected, the kidcount variable tends to underestimate the true AU size as women may have additional children over the 7 quarters following the baseline survey. To deal with this problem we inflate the kidcount based AU size by one in order to avoid understating the location of the poverty line for most assistance units. That is, we use the following mapping from kidcount to AU size: $0 \rightarrow 3,1 \rightarrow 3,2 \rightarrow 4,3 \rightarrow 5$, which maps each kidcount value to the modal inferred AU size in Appendix Table A1 plus one. This mapping is conservative in ensuring that earnings levels below the FPL are indeed below it.

[^23]
## 2 Propensity Score Re-weighting

We use propensity score re-weighting methods to adjust for the chance imbalances in baseline characteristics between the AFDC and JF groups. Following BGH (2006) we estimate a logit of the JF assignment dummy on: quarterly earnings in each of the 8 pre-assignment quarters, separate variables representing quarterly AFDC and quarterly food stamps payments in each of the 7 pre-assignment quarters, dummies indicating whether each of these 22 variables is nonzero, and dummies indicating whether the woman was employed at all or on welfare at all in the year preceding random assignment or in the applicant sample. We also include dummies indicating each of the following baseline demographic characteristics: being white, black, or Hispanic; being never married or separated; having a high-school diploma/GED or more than a high-school education; having more than two children; being younger than 25 or age $25-34$; and dummies indicating whether baseline information is missing for education, number of children, or marital status.

Denote the predicted values from this model by $\widehat{p}_{i}$. The propensity score weights used to adjust the moments of interest are given by:

$$
\omega_{i}=\frac{\frac{1\left[T_{i}=j\right]}{\hat{p}_{i}}}{\sum_{n=1}^{N} \frac{1\left[T_{n}=j\right]}{\hat{p}_{n}}}+\frac{\frac{1-\mathbf{1}\left[T_{i}=j\right]}{1-\hat{p}_{i}}}{\sum_{n=1}^{N} \frac{1-\mathbf{1}\left[T_{n}=j\right]}{1-\hat{p}_{n}}} .
$$

where $N$ is the number of cases. These are inverse probability weights, re-normalized to sum to one within policy group. When examining subgroups we always recompute a new set of propensity score weights and re-normalize them.

## 3 Distributional Tests

## Kolmogorov-Smirnov Test for Equality of Distributions

We use a bootstrap procedure to compute the p-values for our re-weighted Kolmogorov-Smirnov (KS) tests for equality of distribution functions across treatment groups. Let $F_{n}^{t}(e)$ be the propensity score re-weighted EDF of earnings in treatment group $t$. That is,

$$
F_{n}^{t}(e) \equiv \sum_{i} \omega_{i} \mathbf{1}\left[E_{i} \leq e, T_{i}=t\right]
$$

Define the corresponding bootstrap EDF as:

$$
F_{n}^{t *}(e) \equiv \sum_{i} \omega_{i}^{*} \mathbf{1}\left[E_{i}^{*} \leq e, T_{i}^{*}=t\right]
$$

where stars refer to resampled values (we resampled at the case level in order to preserve serial correlation in the data). The K-S test statistic is given by:

$$
\widehat{K S} \equiv \sup _{e}\left|F_{n}^{j}(e)-F_{n}^{a}(e)\right| .
$$

To obtain a critical value for this statistic, we compute the bootstrap distribution of the recentered K-S statistic:

$$
K S^{*} \equiv \sup _{e}\left|F_{n}^{j *}(e)-F_{n}^{a *}(e)-\left(F_{n}^{j}(e)-F_{n}^{a}(e)\right)\right| .
$$

Recentering is necessary to impose the correct null hypothesis on the bootstrap DGP (Giné and Zinn, 1990). We compute an estimated p-value $\widehat{\alpha}_{K S}$ for the null hypothesis that the two distributions are equal as:

$$
\widehat{\alpha}_{K S} \equiv \frac{1}{1000} \sum_{b=1}^{1000} \mathbf{1}\left[K S^{*(b)}>\widehat{K S}\right]
$$

where $b$ indexes the bootstrap replication.

## Barrett-Donald test for stochastic dominance

Our test statistic for detecting violations of the null hypothesis that the JF distribution of earnings stochastically dominates the AFDC distribution is given by:

$$
\widehat{B D} \equiv \sup _{e} F_{n}^{j}(e)-F_{n}^{a}(e) .
$$

As suggested by Barrett and Donald (2003), we bootstrap the re-centered version of this statistic given by:

$$
B D^{*} \equiv \sup _{e}\left[F_{n}^{j *}(e)-F_{n}^{a *}(e)-\left(F_{n}^{j}(e)-F_{n}^{a}(e)\right)\right] .
$$

We compute an estimated p-value $\widehat{\alpha}_{B D}$ as:

$$
\widehat{\alpha}_{B D} \equiv \frac{1}{1000} \sum_{b=1}^{1000} \mathbf{1}\left[B D^{*(b)}>\widehat{B D}\right] .
$$

## 4 Baseline Model

## Notation, Definitions, and Assumptions

Notation (Policy Regimes). Throughout, we use $a$ to refer to AFDC and $j$ to refer to JF. The policy regime is denoted by $t \in\{a, j\}$.

Definition 1 (Earnings, Reported Earnings, and Program Participation). Let $D$ be an indicator for a woman participating in welfare: $D=1$ if she is on assistance and $D=0$ otherwise. Let $E$ denote a woman's earnings. Earnings are the product of hours of work, $H$, and an hourly wage rate, $W$. Let $E^{r}$ denote the earnings a woman reports to the welfare agency and let $R$ be an indicator that takes the value 1 when a welfare recipient reports zero earnings and takes the value 0 otherwise, that is, $R=R\left(D, E^{r}\right) \equiv \mathbf{1}\left[E^{r}=0\right] D$.

Definition 2 (Earning Ranges). Earnings range 0 refers to zero earnings. Earnings range 1 refers to the interval $\left(0, F P L_{i}\right.$ ] where $F P L_{i}$ is woman $i$ 's federal poverty line. Earnings range 2 refers to the interval $\left(F P L_{i}, \infty\right)$.

Definition 3 (Welfare Transfer Functions). For any reported earnings $E^{r}$, the regime dependent transfers are

$$
G_{i}^{a}\left(E^{r}\right) \equiv \max \left\{\bar{G}_{i}-\mathbf{1}\left[E^{r}>\delta_{i}\right]\left(E-\delta_{i}\right) \tau_{i}, 0\right\},
$$

and

$$
G_{i}^{j}\left(E^{r}\right) \equiv \mathbf{1}\left[E^{r} \leq F P L_{i}\right] \bar{G}_{i} .
$$

The parameter $\delta_{i} \in\{90,120\}$ gives woman $i$ 's fixed disregards and the parameter $\tau_{i} \in\{.49, .73\}$ governs her proportional disregard. $\bar{G}_{i}$, the base grant amount, and $F P L_{i}$, the federal poverty level, vary across women due to differences in AU size. Define woman $i$ 's break-even earnings level under regime $a$ as $\bar{E}_{i} \equiv \bar{G}_{i} / \tau_{i}+\delta_{i}$, this is the level at which benefits are exhausted.

Definition 4 (Consumption Equivalent). Consider the triple ( $E, D, E^{r}$ ). Under regime $t \in$ $\{a, j\}$, woman $i$ 's consumption equivalent corresponding to $\left(E, D, E^{r}\right)$ is

$$
\begin{equation*}
C_{i}^{t}\left(E, D, E^{r}\right) \equiv E+D\left(G_{i}^{t}\left(E^{r}\right)-\kappa_{i} \mathbf{1}\left[E^{r}<E\right]\right) . \tag{1}
\end{equation*}
$$

For simplicity we refer to $C_{i}^{t}=C_{i}^{t}\left(E, D, E^{r}\right)$ as consumption. Below, when the consumption associated with a triple ( $E, D, E^{r}$ ) and calculated according to (1) does not vary across regimes we omit the superscript $t$, and we omit the subscript $i$ when it does not vary across women.

Definition 5 (State). Consider the triple ( $E, D, E^{r}$ ). The state corresponding to ( $E, D, E^{r}$ ) is defined by the function:

$$
s\left(E, D, E^{r}\right)=\left\{\begin{array}{ll}
0 n & \text { if } E=0, D=0 \\
1 n & \text { if } E \text { in range } 1, D=0 \\
2 n & \text { if } E \text { in range } 2, D=0 \\
0 r & \text { if } E=0, D=1 \\
1 r & \text { if } E \text { in range } 1, D=1, E^{r}=E \\
1 u & \text { if } E \text { in range } 1, D=1, E^{r}<E \\
2 u & \text { if } E \text { in range } 2, D=1, E^{r}<E \\
2 r & \text { if } E \text { in range } 2, D=1, E^{r}=E
\end{array} .\right.
$$

Definition 6 (Job Offers). A woman's samples $K_{i}$ job offers, composed of wage and hours offer pairs: $\Theta_{i}=\left\{\left(W_{i}^{k}, H_{i}^{k}\right)\right\}_{k=1}^{K_{i}}$ where $K_{i}$ is an integer number (possibly zero), $\left(W_{i}^{k}, H_{i}^{k}\right) \in$ $(0, \infty) \times\left(0, \bar{H}_{i}\right]$ with $\bar{H}_{i}$ denoting the woman's total disposable time. The limiting case $K_{i}=\infty$ is treated as follows: for any $H \in\left(0, \bar{H}_{i}\right]$ a woman's samples a wage offer $W_{i}(H)$. When $K_{i}=\infty$ let $\Theta_{i}=W_{i}() \times.\left(0, \bar{H}_{i}\right]$.

Definition 7 (Alternative). An alternative is wage, hours of work, welfare participation indicator, and earning report tuple ( $W, H, D, E^{r}$ ).

Definition 8 (Sub-alternative). A sub-alternative is wage, hours of work, and welfare participation indicator tuple ( $W, H, D$ ).

Definition 9 (Alternative Compatible with a State). We say that alternative ( $W, H, D, E^{r}$ ) is compatible with state $s$ for woman $i$ if, letting $E \equiv W H, s=s\left(E, D, E^{r}\right)$.

Definition 10 (Alternative Compatible with a State and Available). We say that alternative ( $W, H, D, E^{r}$ ) is available and compatible with state $s$ for woman $i$ if $\left(W, H, D, E^{r}\right)$ is compatible with state $s$ and $(W, H) \in \Theta_{i} \cup(0,0)$.

Definition 11 (Dominated State). We say that state $s$ is dominated under regime $t$ if no available alternative compatible $s$ under regime $t$ is chosen by any woman.

Definition 12 (Utility Function). Define $U_{i}^{t}(H, C, D, R)$ as the utility woman $i$ derives from the tuple $(H, C, D, R)$ under regime $t \in\{a, j\}$. When the utility of a tuple ( $H, C, D, R$ ) is regimeinvariant we omit the superscript $t$.

Definition 13 (Relative attractiveness of a State). We say that state $s$ is:

1. no better under regime $j$ than under regime $a$ if, for any alternative ( $W, H, D, E^{r}$ ) compatible with state $s$, and letting $E \equiv W H$,

$$
U_{i}^{j}\left(H, C_{i}^{j}\left(E, D, E^{r}\right), D, R\left(D, E^{r}\right)\right) \leq U_{i}^{a}\left(H, C_{i}^{a}\left(E, D, E^{r}\right), D, R\left(D, E^{r}\right)\right) \text { for all } i .
$$

2. no worse under regime $j$ than under regime $a$ if, for any alternative $\left(W, H, D, E^{r}\right)$ compatible with state $s$, and letting $E \equiv W H$,

$$
U_{i}^{j}\left(H, C_{i}^{j}\left(E, D, E^{r}\right), D, R\left(D, E^{r}\right)\right) \geq U_{i}^{a}\left(H, C_{i}^{a}\left(E, D, E^{r}\right), D, R\left(D, E^{r}\right)\right) \text { for all } i .
$$

3. equally attractive under regime $j$ and regime $a$ if, for any alternative ( $W, H, D, E^{r}$ ) compatible with state $s$, and letting $E \equiv W H$,

$$
U_{i}^{j}\left(H, C_{i}^{j}\left(E, D, E^{r}\right), D, R\left(D, E^{r}\right)\right)=U_{i}^{a}\left(H, C_{i}^{a}\left(E, D, E^{r}\right), D, R\left(D, E^{r}\right)\right) \text { for all } i .
$$

Definition 14 (Collections of States). Define $\mathcal{S} \equiv\{0 n, 1 n, 2 n, 0 r, 1 r, 1 u, 2 u\}, \mathcal{C}_{+} \equiv\{1 r\}, \mathcal{C}_{-} \equiv$ $\{0 r\}$, and $\mathcal{C}_{0} \equiv\{0 n, 1 n, 2 n, 1 u, 2 u\}$.

Assumption 1 (Preferences). Woman $i$ 's utility functions $U_{i}^{a}(., ., .,$.$) and U_{i}^{j}(., ., .,$.$) satisfy the$ following restrictions:
A. 1 utility is strictly increasing in $C$;
A. $2 \quad U_{i}^{t}(H, C, 1,1) \leq U_{i}^{t}(H, C, 1,0)$ for all $(H, C)$ and $t \in\{a, j\}$;
A. $3 \quad U_{i}^{j}(H, C, 1,1) \leq U_{i}^{a}(H, C, 1,1)$ for all $(H, C)$;
A. $4 \quad U_{i}^{a}(H, C, 1,0)=U_{i}^{j}(H, C, 1,0)$ for all $(H, C)$ with $H>0$;
A. $5 \quad U_{i}^{a}(H, C, 0,0)=U_{i}^{j}(H, C, 0,0)$ for all $(H, C)$;
A. $6 \quad U_{i}^{a}\left(H, C_{i}^{a}(E, 1, E), 1,0\right)<U_{i}^{a}\left(H, C_{i}^{a}(E, 0, E), 0,0\right)$ for all $(H, W)$ such that $E=$ $W H \in\left(F P L_{i}, \bar{E}_{i}\right]$ whenever $\bar{E}_{i}>F P L_{i}$.

Remark 1 (Preferences: Verbalizing Assumption 1). A. 2 states that hassle does not increase utility; this "hassle disutility" can vary across alternatives. A. 3 states that regime $j$ 's hassle disutility is no smaller than regime $a$ 's; the difference in hassle disutility between two regimes may vary with the alternative. Assumption A. 4 states that the impact on utility of welfare participation does not vary with the regime whenever reported earnings are not zero. A. 5 states that the utility value of an alternative entailing no welfare recipiency is independent of the treatment. A. 6 implicitly defines a lower bound on the disutility from stigma. It says that at earning levels above $F P L_{i}$, the extra consumption due to the transfer income does not suffice to compensate the woman for the stigma disutility she incurs when being on assistance.
Remark 2 (Preferences: A Special Case). In the paper, we consider a restricted specification of the 4 -argument utility function $U_{i}^{t}(., ., .,$.$) in Assumption 1$. We do so to aid in illustrating the mechanics of the model and the implications of further restricting preferences. Specifically, we employ a 2-argument utility function $U_{i}(.,$.$) :$

$$
\begin{equation*}
U_{i}\left(H, C-\mu_{i} \mathbf{1}[E>0]-\phi_{i} D-\eta_{i}^{t} R\right), \tag{2}
\end{equation*}
$$

where $\mu_{i}$ is a fixed cost of working, $\phi_{i}$ is a stigma cost from welfare participation, and $\eta_{i}^{t}$ is a hassle cost from reporting zero earnings on assistance. The parameters $\left(\mu_{i}, \phi_{i}, \eta_{i}^{a}, \eta_{i}^{j}\right)$ are such that, for all $i, \mu_{i} \geq 0$ in accordance with A.1, the stigma cost $\phi_{i}$ is regime invariant in accordance with $\mathbf{A} .4$ in Assumption 1, $\eta_{i}^{j} \geq \eta_{i}^{a} \geq 0$ in accordance with A. 2 and A. 3 in Assumption 1, and the utility function is not indexed by regime $t$ in accordance with A. 5 in 1. A sufficient condition for A. 6 in Assumption 1 to hold, is that, $\phi_{i}>G_{i}^{a}\left(F P L_{i}\right)$ for all $i$. Furthermore, the 2-argument utility function $U_{i}(.,$.$) is strictly increasing in its second argument in accordance with A. 1$ in Assumption 1. To preview, form (2) is used below in Corollaries 2 and 4.

Remark 3 (Preferences: Another Special Case). In this Appendix, we consider a second special case of the 4 -argument utility function $U_{i}^{t}(., ., .,$.$) under Assumption 1. We do so to provide$ examples. Specifically, we let the utility that a generic woman $i$ derives under regime $t$ from alternative ( $W, H, D, E^{r}$ ) is, letting $E \equiv W H$ :

$$
\begin{align*}
U_{i}^{t}\left(H, C_{i}^{t}\left(E, D, E^{r}\right), D, R\left(D, E^{r}\right)\right)= & -\alpha_{i} H+v\left(C_{i}^{t}\left(E, D, E^{r}\right)-\mu_{i} \mathbf{1}[E>0]\right)  \tag{3}\\
& -\phi_{i} D-\eta_{i}^{t} R\left(D, E^{r}\right)
\end{align*}
$$

where $\alpha_{i}$ is the change in utility that the woman derives from one additional unit of work, $\mu_{i}$ is a fixed cost of working, $\phi_{i}$ is a stigma cost (or benefit) from welfare participation, $\eta_{i}^{t}$ is a hassle cost from reporting zero earnings on assistance, and $v($.$) is a strictly increasing function by A.1. By A.2-A. 5$ in Assumption 1, the parameters $\left(\mu_{i}, \eta_{i}^{j}, \eta_{i}^{a}, \phi_{i}\right)$ are such that $\mu_{i} \geq 0, \eta_{i}^{j} \geq \eta_{i}^{a} \geq 0$. By A. 6 in Assumption $1 \phi_{i}$ is bounded below by $\underline{\phi}_{i} \equiv \max _{E \in\left[F P L_{i}, \bar{E}_{i}\right]}\left[v\left(E-\mu_{i}+G_{i}^{a}\left(F P L_{i}\right)\right)-v\left(E-\mu_{i}\right)\right]$.

For convenience we assume that $\alpha_{i} \geq 0$, that is, leisure is a good. We consider three forms of $v$ (.): the identity function (hence $v($.$) linear), a strictly concave function (hence the marginal utility of$ consumption is strictly decreasing in consumption), a strictly convex function (hence the marginal utility of consumption is strictly increasing in consumption). When $v($.$) is linear the lower bound$ on the stigma disutility implied by A. 6 in Assumption 1 simplifies to $\underline{\phi}_{i} \equiv G_{i}^{a}\left(F P L_{i}\right)$. To preview, form (3) is used below in the proof of Propositions 2 and 4.

Assumption 2 (Under-reporting Earning Penalty). For each woman $i, \kappa_{i}>0$.
Assumption 3 (Ineligible Earning Levels). No woman may be on welfare assistance and truthfully report earnings above $F P L_{i}$ under regime $j$ or above $\bar{E}_{i}$ under regime a.

Assumption 4 (Utility Maximization). Under regime t, woman i makes choices by solving the optimization problem:

$$
\max _{(W, H) \in \Theta_{i} \cup(0,0), D \in\{0,1\}, E^{r} \in[0, W H]} U_{i}^{t}\left(H, C_{i}^{t}\left(W H, D, E^{r}\right), D, R\left(D, E^{r}\right)\right) .
$$

Assumption 5 (Breaking Indifference). Women break indifference in favor of the same alternative irrespective of the regime.

## Intermediate Lemmas

Lemma 1 (State 2r). Given Assumptions 1, 3, and 4, no woman chooses an alternative compatible with state $2 r$.

Proof. Under regime $j$ no alternative is compatible with state $2 r$ by Assumption 3. Consider now a woman with $\bar{E}_{i} \leq F P L_{i}$ under regime $a$. By Assumption 3 she may not be on assistance and truthfully report earnings above $F P L_{i}$ (range 2). Finally, consider a woman with $\bar{E}_{i}>F P L_{i}$ under regime $a$. By Assumption 3 she may not be on assistance and truthfully report earnings above $\bar{E}_{i}$. By A. 6 in Assumption 1 she will not truthfully report earnings in $\left(F P L_{i}, \bar{E}_{i}\right]$ because she can attain a higher utility level by being off assistance (Assumption 4): the extra consumption due to the transfer income does not suffice to compensate the woman for the stigma disutility she incurs when being on assistance.

Lemma 2 (Optimal Reporting). Write woman $i$ 's optimization problem (Assumption 4) as a nested maximization problem:

$$
\begin{equation*}
\max _{(W, H) \in \Theta_{i} \cup(0,0), D \in\{0,1\}}\left[\max _{E^{r} \in[0, W H]} U_{i}^{t}\left(H, C_{i}^{t}\left(W H, D, E^{r}\right), D, R\left(D, E^{r}\right)\right)\right] . \tag{4}
\end{equation*}
$$

Focus on the inner maximization problem in (4) for given sub-alternative $(W, H, D)$ with $D=1$. Let $E \equiv W H$ and $E_{i}^{r, t} \equiv E_{i}^{r, t}(W, H)$ denote woman $i$ 's utility maximizing earning report conditional on ( $W, H, 1$ ). Given Assumptions 1-5:

1. under regime $j, E_{i}^{r, t}$ entails either truthful reporting, that is, $E_{i}^{r, t}=E$, or under-reporting such that $E>E_{i}^{r, t} \in\left[0, F P L_{i}\right]$; in particular, state $1 u$ is dominated;
2. under regime a, $E_{i}^{r, t}$ entails either truthful reporting, that is, $E_{i}^{r, t}=E$, or under-reporting such that $E>E_{i}^{r, t} \in\left[0, \delta_{i}\right]$.

Proof. We prove each part of the Lemma in turn. In what follows, for convenience, we let $U_{i}^{t}$ serve as be shortcut notation for $U_{i}^{t}\left(H, C_{i}^{t}\left(E, 1, E^{r}\right), 1, R\left(1, E^{r}\right)\right)$.

1. Under regime $j$, consider three mutually exclusive pairs ( $W, H$ ) spanning the range of values for $E$ :
(a) $(W, H)$ such that $E=0$

A woman cannot over-report earnings (Assumption 4). Thus, $E_{i}^{r, j}=E$.
(b) $(W, H)$ such that $E \in\left(0, F P L_{i}\right]$

Woman $i$ 's utility while on welfare depends on reported earnings $E^{r}$ as follows (A. 4 in Assumption 1):

$$
U_{i}^{j}=\left\{\begin{array}{ll}
{[1]: U_{i}^{j}\left(H, E+\bar{G}_{i}-\kappa_{i}, 1,1\right)} & \text { if } E^{r}=0  \tag{5}\\
{[2]: U_{i}\left(H, E+\bar{G}_{i}-\kappa_{i}, 1,0\right)} & \text { if } E^{r} \in(0, E) \\
{[3]: U_{i}\left(H, E+\bar{G}_{i}, 1,0\right)} & \text { if } E^{r}=E
\end{array} .\right.
$$

By Assumption 2 and A. 1 in Assumption 1, truthful reporting yields higher utility than any under-report $E^{r} \in(0, E)$ : [3] $>[2]$ in (5). By A. 2 in Assumption 1, any underreport $E^{r} \in(0, E)$ yields at least as much utility as reporting $E^{r}=0$ : [2] $\geq[1]$ in (5). Thus, truthful reporting solves the inner maximization problem in (4) hence $E_{i}^{r, j}=E$. This shows that state $1 u$ is dominated under regime $j$ because the previous arguments holds for all $E \in\left(0, F P L_{i}\right]$ and $\left(0, F P L_{i}\right]$ corresponds to range 1 (Definition 2).
(c) $(W, H)$ such that $E>F P L_{i}$

Woman $i$ must be under-reporting (Lemma 1). Her utility while on welfare depends on reported earnings $E^{r}$ as follows (A. 4 in Assumption 1):

$$
U_{i}^{j}=\left\{\begin{array}{ll}
{[1]: U_{i}^{j}\left(H, E+\bar{G}_{i}-\kappa_{i}, 1,1\right)} & \text { if } E^{r}=0  \tag{6}\\
{[2]: U_{i}\left(H, E+\bar{G}_{i}-\kappa_{i}, 1,0\right)} & \text { if } E^{r} \in\left(0, F P L_{i}\right]
\end{array} .\right.
$$

By A. 2 in Assumption 1, any report $E^{r} \in\left(0, F P L_{i}\right.$ ] yields at least as much utility as reporting $E^{r}=0:[2] \geq[1]$ in (6). If $\mathbf{A .} 2$ in Assumption 1 holds as an equality then $[2]=[1]$ in (6) and woman $i$ is indifferent among reports in [0,FPLi]. In this case, any $E^{r} \in\left[0, F P L_{i}\right]$ solves the inner maximization problem in (4) thus $E_{i}^{r, j} \in\left[0, F P L_{i}\right]$. If A. 2 in Assumption 1 holds as a strict inequality then [2] $>$ [1] in (6) and woman $i$ is indifferent among (under-) reports in ( $\left.0, F P L_{i}\right]$ and prefers them to (under-) reporting $E^{r}=0$. In this case, any report $E^{r} \in\left(0, F P L_{i}\right]$ solves the inner maximization problem in (4) thus $E_{i}^{r, j} \in\left(0, F P L_{i}\right]$.
2. Under regime $a$, consider four mutually exclusive pairs ( $W, H$ ) spanning the range of values for $E$ :
(a) $(W, H)$ such that $E=0$

A woman cannot over-report earnings (Assumption 4). Thus, $E_{i}^{r, a}=E$.
(b) $(W, H)$ such that $E \in\left(0, \delta_{i}\right]$

Woman $i$ 's utility while on welfare depends on reported earnings as follows (A. 4 in Assumption 1):

$$
U_{i}^{a}=\left\{\begin{array}{ll}
{[1]: U_{i}^{a}\left(H, E+\bar{G}_{i}-\kappa_{i}, 1,1\right)} & \text { if } E^{r}=0  \tag{7}\\
{[2]: U_{i}\left(H, E+\bar{G}_{i}-\kappa_{i}, 1,0\right)} & \text { if } E^{r} \in(0, E) . \\
{[3]: U_{i}\left(H, E+\bar{G}_{i}, 1,0\right)} & \text { if } E^{r}=E
\end{array} .\right.
$$

By Assumption $2\left(\kappa_{i}>0\right)$, A. 1 and A. 2 in Assumption 1: $[3]>[2] \geq[1]$ in (7). Thus, truthful reporting solves the inner maximization problem in (4) hence $E_{i}^{r, a}=E$.
(c) $(W, H)$ such that $E \in\left(\delta_{i}, F P L_{i}\right]$

Woman $i$ 's utility while on welfare depends on reported earnings as follows (A. 4 in Assumption 1):

$$
U_{i}^{a}=\left\{\begin{array}{ll}
{[1]: U_{i}^{a}\left(H, E+\bar{G}_{i}-\kappa_{i}, 1,1\right)} & \text { if } E^{r}=0  \tag{8}\\
{[2]: U_{i}\left(H, E+\bar{G}_{i}-\kappa_{i}, 1,0\right)} & \text { if } E^{r} \in\left(0, \delta_{i}\right] \\
{[3]: U_{i}\left(H, E+G_{i}^{a}\left(E^{r}\right)-\kappa_{i}, 1,0\right)} & \text { if } E^{r} \in\left(\delta_{i}, E\right) \\
{[4]: U_{i}\left(H, E+G_{i}^{a}\left(E^{r}\right), 1,0\right)} & \text { if } E^{r}=E
\end{array} .\right.
$$

By Assumption 2, A. 1 and A. 2 in Assumption 1, and the fact that $\bar{G}_{i}=G_{i}^{a}(0)>$ $G_{i}^{a}\left(E^{r}\right)$ for all $E^{r}$ in $\left(\delta_{i}, F P L_{i}\right]:[1] \leq[2]$ and $[3]<[2]$ in (8). Thus, only truthful reports or under-reports in $\left[0, \delta_{i}\right]$ may solve the inner maximization problem in (4). Specifically, if A. 2 in Assumption 1 holds as an equality then $[1]=[2]$ in (8) and woman $i$ is indifferent among (under-) reports in $\left[0, \delta_{i}\right]$. In this case $E_{i}^{r, a}=E$ or $E_{i}^{r, a} \in\left[0, \delta_{i}\right]$ depending on whether $[4] \geq[2]$ or $[4] \leq[2]$. If A. 2 in Assumption 1 holds as a strict inequality then $[1]<[2]$ in (8) and woman $i$ is indifferent among (under-) reports in $\left(0, \delta_{i}\right]$. In this case $E_{i}^{r, a}=E$ or $E_{i}^{r, a} \in\left(0, \delta_{i}\right]$ depending on whether [4] $\geq[2]$ or $[4] \leq[2]$.
(d) $(W, H)$ such that $E>F P L_{i}$

Woman $i$ must be under-reporting (Lemma 1). Her utility while on welfare depends on reported earnings as follows (A. 4 in Assumption 1):

$$
U_{i}^{a}=\left\{\begin{array}{ll}
{[1]: U_{i}^{a}\left(H, E+\bar{G}_{i}-\kappa_{i}, 1,1\right)} & \text { if } E^{r}=0  \tag{9}\\
{[2]: U_{i}\left(H, E+\bar{G}_{i}-\kappa_{i}, 1,0\right)} & \text { if } E^{r} \in\left(0, \delta_{i}\right] \\
{[3]: U_{i}\left(H, E+G_{i}^{a}\left(E^{r}\right)-\kappa_{i}, 1,0\right)} & \text { if } E^{r} \in\left(\delta_{i}, F P L_{i}\right]
\end{array} .\right.
$$

By A. 1 and A. 2 in Assumption 1, and the fact that $\bar{G}_{i}=G_{i}^{a}(0)>G_{i}^{a}\left(E^{r}\right)$ for all $E^{r}$ in $\left(\delta_{i}, F P L_{i}\right]:[3]<[2]$ and $[1] \leq[2]$ in (9). Thus, only under-reports in $\left[0, \delta_{i}\right]$ may solve the inner maximization problem in (4). Specifically, if A. 2 in Assumption 1 holds as an equality then $[1]=[2]$ in (9)) and woman $i$ is indifferent among (under-) reports in $\left[0, \delta_{i}\right]$. In this case $E_{i}^{r, a} \in\left[0, \delta_{i}\right]$. If $\mathbf{A . 2}$ in Assumption 1 holds as a strict inequality then $[1]<[2]$ in (9)) and woman $i$ is indifferent among (under-) reports in ( $\left.0, \delta_{i}\right]$ and prefers them to reporting $E^{r}=0$. In this case $E_{i}^{r, a} \in\left(0, \delta_{i}\right]$.

Corollary 1 (Optimal Reporting and Policy Invariance). Given Assumptions 1-5, the utility associated with any alternative compatible with states $1 u$ and $2 u$ and entailing optimal reporting is regime invariant.

Proof. We examine each state in turn.

1. State $1 u$
(a) Consider a woman $i$ and any sub-alternative ( $W, H, 1$ ) such that, letting $E \equiv W H, E$ is in range 1 and $E_{i}^{r, j}(W, H)<E$. Thus alternative $\left(W, H, 1, E_{i}^{r, j}(W, H)\right)$ is compatible with state $1 u$ and entails optimal reporting under regime $j$. Let $C_{i}^{j} \equiv C_{i}^{j}\left(E, 1, E_{i}^{r, j}(W, H)\right)$ and $R_{i}^{j} \equiv R\left(1, E_{i}^{r, j}(W, H)\right)$. We next show that $U_{i}^{j}\left(H, C_{i}^{j}, 1, R_{i}^{j}\right)=U_{i}\left(H, C_{i}^{j}, 1, R_{i}^{j}\right)$. By Lemma $2, E_{i}^{r, j}(W, H) \in\left(0, F P L_{i}\right]$ or $E_{i}^{r, j}(W, H) \in\left[0, F P L_{i}\right]$ depending on the woman's preferences. In the first case, the utility woman $i$ enjoys is $U_{i}^{j}\left(H, E+\bar{G}_{i}-\kappa_{i}, 1,0\right)$
which equals $U_{i}\left(H, E+\bar{G}_{i}-\kappa_{i}, 1,0\right)$ by A. 4 in Assumption 1. In the second case, the utility woman $i$ enjoys is $U_{i}^{j}\left(H, E+\bar{G}_{i}-\kappa_{i}, 1,0\right)$ which also equals $U_{i}\left(H, E+\bar{G}_{i}-\kappa_{i}, 1,0\right)$ by A. 4 in Assumption 1 and because she is indifferent between (under-) reports in ( $\left.0, F P L_{i}\right]$ and reporting zero earnings, that is,
$U_{i}^{j}\left(H, E+\bar{G}_{i}-\kappa_{i}, 1,1\right)=U_{i}^{j}\left(H, E+\bar{G}_{i}-\kappa_{i}, 1,0\right)$.
(b) Consider any sub-alternative $(W, H, 1)$ such that, letting $E \equiv W H, E$ is in range 1 and $E_{i}^{r, a}(W, H)<E$. Thus alternative $\left(W, H, 1, E_{i}^{r, a}(W, H)\right)$ is compatible with state $1 u$ and entails optimal reporting under regime $a$. Let $C_{i}^{a} \equiv C_{i}^{a}\left(E, 1, E_{i}^{r, a}(W, H)\right)$ and $R_{i}^{a} \equiv$ $R\left(1, E_{i}^{r, a}(W, H)\right)$. We next show that $U_{i}^{a}\left(H, C_{i}^{a}, 1, R_{i}^{a}\right)=U_{i}\left(H, C_{i}^{a}, 1, R_{i}^{a}\right)$. By Lemma $2, E_{i}^{r, a}(W, H) \in\left(0, \delta_{i}\right]$ or $E_{i}^{r, a}(W, H) \in\left[0, \delta_{i}\right]$ depending on the woman's preferences. In the first case, the utility woman $i$ enjoys is $U_{i}^{a}\left(H, E+\bar{G}_{i}-\kappa_{i}, 1,0\right)$ which equals $U_{i}\left(H, E+\bar{G}_{i}-\kappa_{i}, 1,0\right)$ by A. 4 in Assumption 1. In the second case, the utility woman $i$ enjoys is also $U_{i}^{a}\left(H, E+\bar{G}_{i}-\kappa_{i}, 1,0\right)=U_{i}\left(H, E+\bar{G}_{i}-\kappa_{i}, 1,0\right)$ by A. 4 in Assumption 1 and because she is indifferent between (under-) reports in ( $0, \delta_{i}$ ] and reporting zero earnings, that is, $U_{i}^{a}\left(H, E+\bar{G}_{i}-\kappa_{i}, 1,1\right)=U_{i}^{a}\left(H, E+\bar{G}_{i}-\kappa_{i}, 1,0\right)$.
(c) In 1.(a) and 1.(b) we have shown that any alternative compatible with state $1 u$ and entailing optimal reporting yields regime-invariant consumption $E+\bar{G}_{i}-\kappa_{i}$ and regimeinvariant utility level $U_{i}\left(H, E+\bar{G}_{i}-\kappa_{i}, 1,0\right)$.

## 2. State $2 u$

The proof corresponding to state $2 u$ is the same as that for state $1 u$ once we consider a sub-alternative $(W, H, 1)$ such that, letting $E \equiv W H, E$ is in range 2 (Lemma 2 ).

Remark 4 (Optimal under-Reporting and Alternatives Considered). In what follows, it is without loss of generality that we only focus on alternatives entailing optimal (under-) reporting among those compatible with states $1 u$ and $2 u$. No woman would select an alternative compatible with states $1 u$ or $2 u$ not entailing optimal (under-) reporting (Assumption 4). Additionally, it is without loss of generality that we disregard alternatives compatible with state $1 u$ under regime $j$. No woman would select an alternative compatible with state $1 u$ under regime $j$ because it is dominated (Lemma 2, part 1.(b)).

Lemma 3 (Policy Impact on Attractiveness of States). Given Assumptions 1-5:

1. the states in $\mathcal{C}_{+}$are no worse under regime $j$ than under regime $a$;
2. the states in $\mathcal{C}_{-}$are no better under regime $j$ than under regime $a$;
3. the states in $\mathcal{C}_{0}$ are equally attractive under regime $j$ and regime $a$.

Proof. We prove each statement in turn.

1. The only state in $\mathcal{C}_{+}$is $1 r$. All alternatives compatible with state $1 r$ entail $E$ in range $1, D=1$, and $E^{r}=E$. Thus, the utility function associated with each of these alternatives is invariant to the treatment (A. 4 in Assumption 1). Accordingly, it suffices to show that the consumption associated with any one of these alternatives is not lower under regime $j$ than under regime $a$. Because $G_{i}^{a}(E) \leq \bar{G}_{i}$ for all $E$ in range $1, C_{i}^{j}(E, 1, E)=E+\bar{G}_{i} \geq E+G_{i}^{a}(E)=C_{i}^{a}(E, 1, E)$, which verifies the desired inequality.
2. The only state in $\mathcal{C}_{-}$is $0 r$. All the alternatives compatible with state $0 r$ entail $E=H=0$, $D=1$, and $E^{r}=0$. Thus, it suffices to show that $U_{i}^{a}\left(0, \bar{G}_{i}, 1,1\right) \geq U_{i}^{j}\left(0, \bar{G}_{i}, 1,1\right)$. This inequality holds by A. $\mathbf{3}$ in Assumption 1.
3. All alternatives compatible with states $\{0 n, 1 n, 2 n\}$ entail $D=0$. Thus, the utility associated with each of these alternatives is invariant to the policy regime (A. 5 in Assumption 1). Accordingly, it suffices to show that the consumption associated with any of these alternatives is the same under regime $j$ than under regime $a$. Because off assistance consumption is either zero, when $s_{i}=0 n$, or $E$, when $s_{i} \in\{1 n, 2 n\}$ consumption is unaffected by the regime. Finally consider states $\{1 u, 2 u\}$ entailing $0 \leq E^{r}<E$ and $D=1$. Given optimal reporting, the utility function associated with each of these alternatives is invariant to the policy regime (Corollary 1). Accordingly, it suffices to show that the consumption associated with any one of these alternatives is the same under regime $j$ and under regime $a$. If $s_{i} \in\{1 u, 2 u\}$ consumption is $E+\bar{G}_{i}-\kappa_{i}$ under both regimes (see Lemma 2). Thus consumption is also policy invariant.

Lemma 4 (Revealed Preferences). Consider any pair of states $\left(s^{a}, s^{j}\right)$ obeying: I) $s^{a} \neq s^{j}$; II) state $s^{a}$ is no worse under regime $j$ than under regime $a$; III) state $s^{j}$ is no better under regime $j$ than under regime $a$. Given Assumptions 1 and 5, no woman pairs states $s^{a}$ and $s^{j}$.

Proof. The proof is by contradiction. Consider any pair of states $\left(s^{a}, s^{j}\right)$ satisfying properties I)III). Suppose that woman $i$ chooses alternative ( $W, H, D, E^{r}$ ) under regime $a$ compatible with state $s^{a}$; and alternative $\left(H^{\prime}, W^{\prime}, D^{\prime}, E^{r \prime}\right)$ under regime $j$ compatible with state $s^{j}$. Let with $E \equiv W H$, $E^{\prime} \equiv W^{\prime} H^{\prime}, C_{i}^{t}=C_{i}^{t}\left(E, D, E^{r}\right)$ and $C_{i}^{t \prime}=C_{i}^{t}\left(E^{\prime}, D^{\prime}, E^{r \prime}\right)$ all $t \in\{a, j\}, R=R\left(D, E^{r}\right)$, and $R^{\prime}=R\left(D^{\prime}, E^{r \prime}\right)$. The woman's choice under regime $a$ reveals that

$$
U_{i}^{a}\left(H, C_{i}^{a}, D, R\right) \geq U_{i}^{a}\left(H^{\prime}, C_{i}^{a \prime}, D^{\prime}, R^{\prime}\right) .
$$

By property II)

$$
U_{i}^{j}\left(H, C_{i}^{j}, D, R\right) \geq U_{i}^{a}\left(H, C_{i}^{a}, D, R\right) .
$$

By property III)

$$
U_{i}^{a}\left(H^{\prime}, C_{i}^{a \prime}, D^{\prime}, R^{\prime}\right) \geq U_{i}^{j}\left(H^{\prime}, C_{i}^{j \prime}, D^{\prime}, R^{\prime}\right)
$$

Combining the above three inequalities we have

$$
\begin{equation*}
U_{i}^{j}\left(H, C_{i}^{j}, D, R\right) \geq U_{i}^{a}\left(H, C_{i}^{a}, D, R\right) \geq U_{i}^{a}\left(H^{\prime}, C_{i}^{a \prime}, D^{\prime}, R^{\prime}\right) \geq U_{i}^{j}\left(H^{\prime}, C_{i}^{j \prime}, D^{\prime}, R^{\prime}\right) . \tag{10}
\end{equation*}
$$

If any of the inequalities is strict, optimality of $\left(H^{\prime}, C_{i}^{j \prime}, D^{\prime}, R^{\prime}\right)$ under regime $j$ is contradicted (Assumption 4). If no inequality is strict, we have to consider $9=3^{2}$ possible situations based on the possible values of $\left(D, R, D^{\prime}, R^{\prime}\right)$. Each of these situations leads to a contradiction based on a woman breaking indifference between two alternatives in favor of the same alternative irrespective of the policy regime (Assumption 5) and Property I. Specifically, in each of the following cases expression (10) simplifies to:

1. $(D, R)=(0,0)$ and $\left(D^{\prime}, R^{\prime}\right)=(0,0)$ :

$$
U_{i}(H, C, 0,0)=U_{i}(H, C, 0,0)=U_{i}\left(H^{\prime}, C^{\prime}, 0,0\right)=U_{i}\left(H^{\prime}, C^{\prime}, 0,0\right),
$$

where we have used the fact that off assistance consumption does not vary with the regime, hence $C_{i}^{a}=C_{i}^{j}=C$ and $C_{i}^{a \prime}=C_{i}^{j \prime}=C^{\prime}$. Woman $i$ is indifferent between ( $H, C, 0,0$ ) and ( $H^{\prime}, C^{\prime}, 0,0$ ) under regime $a$ and resolves indifference in favor of ( $H, C, 0,0$ ), this contradicts resolving indifference in favor of ( $H^{\prime}, C^{\prime}, 0,0$ ) under regime $j$.
2. $(D, R)=(0,0)$ and $\left(D^{\prime}, R^{\prime}\right)=(1,0)$ :

$$
U_{i}(H, C, 0,0)=U_{i}(H, C, 0,0)=U_{i}\left(H^{\prime}, C_{i}^{a \prime}, 1,0\right)=U_{i}\left(H^{\prime}, C_{i}^{j \prime}, 1,0\right),
$$

where we have used the fact that off assistance consumption does not vary with the regime hence $C_{i}^{a}=C_{i}^{j}=C$. The last equality implies $C_{i}^{a \prime}=C_{i}^{j \prime}=C_{i}^{\prime}$ because utility is strictly increasing in consumption (Assumption 1). Woman $i$ is thus indifferent between ( $H, C, 0,0$ ) and ( $H^{\prime}, C_{i}^{\prime}, 1,0$ ) under regime $a$ and resolves indifference in favor of ( $H, C, 0,0$ ), this contradicts resolving indifference in favor of $\left(H^{\prime}, C_{i}^{\prime}, 1,0\right)$ under regime $j$.
3. $(D, R)=(0,0)$ and $\left(D^{\prime}, R^{\prime}\right)=(1,1)$ :

$$
U_{i}(H, C, 0,0)=U_{i}(H, C, 0,0)=U_{i}^{a}\left(H^{\prime}, C_{i}^{\prime}, 1,1\right)=U_{i}^{j}\left(H^{\prime}, C_{i}^{\prime}, 1,1\right),
$$

where we have used the fact that off assistance consumption does not vary with the regime, hence $C_{i}^{a}=C_{i}^{j}=C$, and that $G_{i}^{a}(0)=\bar{G}_{i}$, hence $C_{i}^{j \prime}=C_{i}^{a \prime}=C_{i}^{\prime}$. Woman $i$ is thus indifferent between $(H, C, 0,0)$ and ( $H^{\prime}, C_{i}^{\prime}, 1,1$ ) under regime $a$ and resolves indifference in favor of $(H, C, 0,0)$, this contradicts resolving indifference in favor of $\left(H^{\prime}, C_{i}^{\prime}, 1,1\right)$ under regime $j$.
4. $(D, R)=(1,1)$ and $\left(D^{\prime}, R^{\prime}\right)=(0,0)$ :

$$
U_{i}^{j}\left(H, C_{i}, 1,1\right)=U_{i}^{a}\left(H, C_{i}, 1,1\right)=U_{i}\left(H^{\prime}, C^{\prime}, 0,0\right)=U_{i}\left(H^{\prime}, C^{\prime}, 0,0\right),
$$

where we have used the fact that off assistance consumption does not vary with the regime, hence $C_{i}^{j \prime}=C_{i}^{a \prime}=C^{\prime}$, and the fact that $G_{i}^{a}(0)=\bar{G}_{i}$, hence $C_{i}^{j}=C_{i}^{a}=C_{i}$. Woman $i$ is thus indifferent between ( $H, C_{i}, 1,1$ ) and ( $H^{\prime}, C^{\prime}, 0,0$ ) under regime $a$ and resolves indifference in favor of $\left(H, C_{i}, 1,1\right)$, this contradicts resolving indifference in favor of ( $H^{\prime}, C^{\prime}, 0,0$ ) under regime $j$.
5. $(D, R)=(1,0)$ and $\left(D^{\prime}, R^{\prime}\right)=(0,0)$ :

$$
U_{i}\left(H, C_{i}^{j}, 1,0\right)=U_{i}\left(H, C_{i}^{a}, 1,0\right)=U_{i}\left(H^{\prime}, C^{\prime}, 0,0\right)=U_{i}\left(H^{\prime}, C^{\prime}, 0,0\right),
$$

where we have used the fact that off assistance consumption does not vary with the regime, hence $C_{i}^{j \prime}=C_{i}^{a \prime}=C^{\prime}$. The fist equality implies $C_{i}^{j}=C_{i}^{a}=C_{i}$ because utility is strictly increasing in consumption (Assumption 1). Woman $i$ is thus indifferent between ( $H, C_{i}, 1,0$ ) and ( $H^{\prime}, C^{\prime}, 0,0$ ) under regime $a$ and resolves indifference in favor of ( $H, C_{i}, 1,0$ ), this contradicts resolving indifference in favor of $\left(H^{\prime}, C^{\prime}, 0,0\right)$ under regime $j$.
6. $(D, R)=(1,1)$ and $\left(D^{\prime}, R^{\prime}\right)=(1,0)$ :

$$
U_{i}^{j}\left(H, C_{i}, 1,1\right)=U_{i}^{a}\left(H, C_{i}, 1,1\right)=U_{i}\left(H^{\prime}, C_{i}^{a \prime}, 1,0\right)=U_{i}\left(H^{\prime}, C_{i}^{j \prime}, 1,0\right),
$$

where we have used the fact that $G_{i}^{a}(0)=\bar{G}_{i}$ hence $C_{i}^{j}=C_{i}^{a}=C_{i}$. The last equality implies $C_{i}^{a \prime}=C_{i}^{j \prime}=C_{i}^{\prime}$ because utility is strictly increasing in consumption (Assumption 1). Woman $i$ is thus indifferent between ( $H, C_{i}, 1,1$ ) and ( $H^{\prime}, C_{i}^{\prime}, 1,0$ ) under regime $a$ and resolves indifference in favor of ( $H, C_{i}, 1,1$ ), this contradicts resolving indifference in favor of ( $H^{\prime}, C_{i}^{\prime}, 1,0$ ) under regime $j$.
7. $(D, R)=(1,0)$ and $\left(D^{\prime}, R^{\prime}\right)=(1,0)$ :

$$
U_{i}\left(H, C_{i}^{j}, 1,0\right)=U_{i}\left(H, C_{i}^{a}, 1,0\right)=U_{i}\left(H^{\prime}, C_{i}^{a \prime}, 1,0\right)=U_{i}\left(H^{\prime}, C_{i}^{j \prime}, 1,0\right)
$$

The fist equality implies $C_{i}^{j}=C_{i}^{a}=C_{i}$ and the last equality implies $C_{i}^{j \prime}=C_{i}^{a \prime}=C_{i}^{\prime}$ because utility is strictly increasing in consumption (Assumption 1). Woman $i$ is thus indifferent between $\left(H, C_{i}, 1,0\right)$ and $\left(H^{\prime}, C_{i}^{\prime}, 1,0\right)$ under regime $a$ and resolves indifference in favor of $\left(H, C_{i}, 1,0\right)$, this contradicts her resolving indifference in favor of $\left(H^{\prime}, C_{i}^{\prime}, 1,0\right)$ under regime $j$.
8. $(D, R)=(1,0)$ and $\left(D^{\prime}, R^{\prime}\right)=(1,1)$ :

$$
U_{i}\left(H, C_{i}^{j}, 1,0\right)=U_{i}\left(H, C_{i}^{a}, 1,0\right)=U_{i}^{a}\left(H^{\prime}, C_{i}^{\prime}, 1,1\right)=U_{i}^{j}\left(H^{\prime}, C_{i}^{\prime}, 1,1\right)
$$

where we have used the fact that $G_{i}^{a}(0)=\bar{G}_{i}$ hence $C_{i}^{j \prime}=C_{i}^{a \prime}=C_{i}^{\prime}$. The first equality implies $C_{i}^{j}=C_{i}^{a}=C_{i}$ because utility is strictly increasing in consumption (Assumption 1). Woman $i$ is thus indifferent between $\left(H, C_{i}, 1,0\right)$ and $\left(H^{\prime}, C_{i}^{\prime}, 1,1\right)$ under regime $a$ and resolves indifference in favor of $\left(H, C_{i}, 1,0\right)$, this contradicts resolving indifference in favor of $\left(H^{\prime}, C_{i}^{\prime}, 1,1\right)$ under regime $j$.
9. $(D, R)=(1,1)$ and $\left(D^{\prime}, R^{\prime}\right)=(1,1)$ :

$$
U_{i}^{j}\left(H, C_{i}, 1,1\right)=U_{i}^{a}\left(H, C_{i}, 1,1\right)=U_{i}^{a}\left(H^{\prime}, C_{i}^{\prime}, 1,1\right)=U_{i}^{j}\left(H^{\prime}, C_{i}^{\prime}, 1,1\right)
$$

where we have used the fact that $G_{i}^{a}(0)=\bar{G}_{i}$ hence $C_{i}^{j}=C_{i}^{a}=C_{i}$ and $C_{i}^{j \prime}=C_{i}^{a \prime}=$ $C_{i}^{\prime}$. Woman $i$ is thus indifferent between $\left(H, C_{i}, 1,1\right)$ and ( $H^{\prime}, C_{i}^{\prime}, 1,1$ ) under regime $a$ and resolves indifference in favor of $\left(H, C_{i}, 1,1\right)$, this contradicts resolving indifference in favor of $\left(H^{\prime}, C_{i}^{\prime}, 1,1\right)$ under regime $j$.

## Main Propositions

Proposition 1 (Restricted Pairings). Given Assumptions 1-5, the pairings of states corresponding to the"-" entries in Table 4 are disallowed and the pairings of states $(1 r, 1 r)$ and $(1 u, 1 r)$ must occur.

Proof. We begin with the pairings that are disallowed. State $1 u$ is dominated under regime $j$ (Lemma 2). Therefore no woman will pair state $s^{a}$ with state $s^{j}=1 u$ for any $s^{a} \in \mathcal{S}$. Next, by Lemmas 4 and 3 , no pairing of state $s^{a}$ with state $s^{j}$ can occur for all $\left(s^{a}, s^{j}\right)$ in the collection

$$
\begin{equation*}
\left\{\left(s^{a}, s^{j}\right): s^{a} \in \mathcal{C}_{0} \cup \mathcal{C}_{+}, s^{j} \in \mathcal{C}_{0} \cup \mathcal{C}_{-}, s^{a} \neq s^{j}\right\} \tag{11}
\end{equation*}
$$

Thus, it suffices to show that the properties I)-III) of Lemma 4 are met. Property I) holds trivially and properties II) and III) hold by Lemma 3. Therefore no woman will select any of the pairings in (11). We next turn to the responses that must occur. By Lemma 1, the allowable states are given by $\mathcal{S}=\{0 n, 1 n, 2 n, 0 r, 1 r, 1 u, 2 u\}$. We just argued that the pairings $\left\{\left(1 r, s^{j}\right): s^{j} \in\{0 n, 1 n, 2 n, 0 r, 1 u, 2 u\}\right\}$ are disallowed, therefore the pairing $(1 r, 1 r)$ must occur. Similarly, we just argued that the pairings $\left\{\left(1 u, s^{j}\right): s^{j} \in\{0 n, 1 n, 2 n, 0 r, 1 u, 2 u\}\right\}$ are disallowed, therefore the pairing $(1 u, 1 r)$ must occur.

Corollary 2 (Additional Restricted Pairings under Utility Specification (2)). Given Assumptions 1-5 and subject to specification (2) of the utility function, the pairing of states ( $0 r, 1 n$ ) is disallowed.

Proof. To enhance readability we employ the symbol $\left[s \succsim^{t} s^{\prime}\right]$ to signify that under regime $t$ an alternative compatible with state $s$ is weakly preferred to an alternative compatible with state $s^{\prime}$. The proof is by contradiction. Suppose there is a woman $i$ who selects an alternative compatible with state $0 r$ under regime $a$ and selects an alternative compatible with state $1 n$ under regime $j$ entailing earnings $E^{k} \equiv W^{k} H^{k}$. By Assumption 4, her choice under regime a reveals that

$$
\left[0 r \succsim^{a} 0 n\right]: U_{i}^{a}\left(0, \bar{G}_{i}-\phi_{i}-\eta_{i}^{a}\right) \geq U_{i}(0,0)
$$

which implies $\bar{G}_{i} \geq \phi_{i}+\eta_{i}^{a}$. Her choice under regime $j$ reveals that

$$
\left[1 n \succsim^{j} 1 r\right]: U_{i}\left(H^{k}, E^{k}-\mu_{i}\right) \geq U_{i}\left(H^{k}, E^{k}-\mu_{i}+\bar{G}_{i}-\phi_{i}\right)
$$

which implies $\bar{G}_{i} \leq \phi_{i}$. Thus, optimality implies $\phi_{i} \geq \bar{G}_{i} \geq \phi_{i}+\eta_{i}^{a}$. If the inequality in A. 2 of Assumption 1 holds as a strict inequality $\eta_{i}^{a}>0$ and a contradiction ensues. If the inequality in A. 2 of Assumption 1 holds as an equality $\eta_{i}^{a}=0$. Thus, $\phi_{i}=\bar{G}_{i}$ and woman $i$ must be indifferent between the alternative compatible with state $0 r$ and the alternative compatible with state $0 n$ under regime $a$ which means that $U_{i}(0,0) \geq U_{i}\left(H^{l}, E^{l}\right)$ for any offer $\left(w^{l}, H^{l}\right)$ entailing earnings $E^{l} \equiv W^{l} H^{l}$ in range 1 , including $E^{k}$. The choice of the alternative compatible with state $1 n$ under regime $j$ reveals that $U_{i}\left(H^{k}, E^{k}\right) \geq U_{i}(0,0)$. Thus, $U_{i}(0,0) \geq U_{i}\left(H^{l}, E^{l}\right) \geq U_{i}(0,0)$. If either inequality is strict a contradiction ensues. Otherwise $U_{i}(0,0)=U_{i}\left(H^{l}, E^{l}\right)=U_{i}(0,0)$ and the woman must be indifferent under regime $a$ between the alternative compatible with state $0 n$ and the alternative entailing earnings $E^{k}$ off assistance. If however she does not choose earnings $E^{k}$ off assistance under regime $a$ then she breaks indifference in the same way under $j$ (Assumption 5), which contradicts her choosing earnings $E^{k}$ off assistance under regime $j$.

Proposition 2 (Unrestricted Pairings). Given Assumptions 1-5, the pairings of states corresponding to the non "-" entries in Table 4 are allowed.

Proof. State pairings ( $1 r, 1 r$ ) and ( $1 u, 1 r$ ) must occur by Proposition 1. Table 4's remaining allowed state pairings can be conveniently organized in two collections:

$$
\begin{align*}
& \left\{\left(s^{a}, 1 r\right): s^{a} \in\{0 n, 1 n, 2 n, 2 u\}\right\},  \tag{12}\\
& \left\{\left(0 r, s_{j}\right): s^{j} \in\{0 n, 1 n, 2 n, 1 r, 2 u\}\right\} . \tag{13}
\end{align*}
$$

We start by considering the collection of pairs in (12). The common feature of the states in $\{0 n, 1 n, 2 n, 2 u\}$ is that they are equally attractive under regimes $a$ and $j$ (Lemma 3). Instead, state $1 r$ is no worse under regime $j$ than under regime $a$ (Lemma 3). In light of Proposition (1), to prove that the pairs in collection (12) are allowed it suffices to provide examples where two women occupy the same state $s^{a} \in\{0 n, 1 n, 2 n, 2 u\}$ under regime $a$, but the first woman occupies state $s^{j}=s^{a}$ under regime $j$ and the second woman occupies state $s^{j}=1 r$ under regime $j$. This also proves that no pairing in collection (12) is constrained to occur. We then turn to the collection of state pairs in (13). The common feature of the states in $\{0 n, 1 n, 2 n, 1 r, 2 u\}$ is that they are no worse under regime $j$ than under regime $a$ (Lemma 3). Instead, state $0 r$ is no better under regime $j$ than under regime $a$ (Lemma 3). To prove that the pairs in collection (13) are allowed it suffices to provide the example of a woman who occupies state $0 r$ under regime $a$ and state $s^{j} \in\{0 n, 1 n, 2 n, 1 r, 2 u\}$ under regime $j$. This also proves that no pairing in collection (13) is constrained to occur.

When providing examples we consider the specification of the utility function given in (3). Finally, we assume that woman $i$ receives either one or two job offers, that is, either $K_{i}=1$ or $K_{i}=2$. To enhance readability we employ the symbol $\left[s \succsim^{t} s^{\prime}\right]$ to signify that under regime $t$ an alternative compatible with state $s$ is weakly preferred to an alternative compatible with state $s^{\prime}$.

1. Pairings $(0 n, 1 r)$ and $(0 n, 0 n)$ are allowed, hence neither must occur.

Consider two women $i^{\prime}$ and $i^{\prime \prime}$ with preferences represented by (3) with $v(x)=x$. Let $K_{i}=1$. Assume that each woman's job offer entails earnings in range 1. That is, for $i \in\left\{i^{\prime}, i^{\prime \prime}\right\}$, $E_{i}^{k} \equiv W_{i}^{k} H_{i}^{k}$ is in range 1 . Let
(a) woman $i=i^{\prime}$ be such that $\mu_{i}=0$, and $\alpha_{i} \geq W_{i}^{k}$ and

$$
\bar{G}_{i}-\phi_{i} \leq 0
$$

(b) woman $i=i^{\prime \prime}$ be such that $\mu_{i}=0$, and $\alpha_{i} \geq W_{i}^{k}$ and

$$
H_{i}^{k}\left(\alpha_{i}-W_{i}^{k}\right)<\bar{G}_{i}-\phi_{i} \leq \min \left\{\eta_{i}^{a}, H_{i}^{k}\left(\alpha_{i}-W_{i}^{k}\right)+\kappa_{i}, H_{i}^{k}\left(\alpha_{i}-W_{i}^{k}\right)+\bar{G}_{i}-G_{i}^{a}\left(E_{i}^{k}\right)\right\}
$$

Both women chose an alternative compatible with state $0 n$ under regime $a$. We now show that woman $i^{\prime}$ chooses an alternative compatible with state $0 n$ under regime $j$ while woman $i^{\prime \prime}$ selects an alternative compatible with state $1 r$ under regime $j$. For both women, the choice of the alternative compatible with state $0 n$ under regime $a$ reveals (Assumption 4) that this alternative yields as much utility as the available alternatives compatible with states $\{0 r, 1 r, 1 u, 1 n\}$. Thus, for $i \in\left\{i^{\prime}, i^{\prime \prime}\right\}$ :

$$
\begin{align*}
& {\left[0 n \succsim^{a} 0 r\right] \quad: \quad 0 \geq \bar{G}_{i}-\phi_{i}-\eta_{i}^{a}}  \tag{14}\\
& {\left[0 n \succsim^{a} 1 r\right] \quad: \quad 0 \geq E_{i}^{k}+G_{i}^{a}\left(E_{i}^{k}\right)-\phi_{i}-\alpha_{i} H_{i}^{k}}  \tag{15}\\
& {\left[0 n \succsim^{a} 1 u\right]}  \tag{16}\\
& {\left[0 n \succsim^{a} 1 n\right]} \tag{17}
\end{align*}: 0 \geq E_{i}^{k}+\bar{G}_{i}-\phi_{i}-\kappa_{i}-\alpha_{i} H_{i}^{k}, ~ 0 \geq E_{i}^{k}-\alpha_{i} H_{i}^{k} .
$$

It is easy to verify that descriptions (1a) and (1b) are compatible with optimality under regime $a$, that is, with (14)-(17). Both women prefer state $0 n$ under regime $j$ to the available alternatives compatible with states $\{0 r, 1 n, 1 u\}$ by Proposition 1 . Woman $i=i^{\prime}$ also prefers state $0 n$ to the available alternatives compatible with state $1 r$ under regime $j$ because by description (1a) we have $\bar{G}_{i}-\phi_{i} \leq 0$ and $\alpha_{i} \geq W_{i}^{k}$ which imply (18):

$$
\begin{equation*}
\left[0 n \succsim^{j} 1 r\right]: 0 \geq E_{i}^{k}+\bar{G}_{i}-\phi_{i}-\alpha_{i} H_{i}^{k} \tag{18}
\end{equation*}
$$

By Assumption 5 she breaks an indifference situation in favor of state $0 n$. Instead, woman $i=i^{\prime \prime}$ prefers an alternative available and compatible with state $1 r$ under regime $j$ to state $0 n$ because by description (1b) we have $H_{i}^{k}\left(\alpha_{i}-W_{i}^{k}\right)<\bar{G}_{i}-\phi_{i}$ which imply (19):

$$
\begin{equation*}
\left[1 r \succsim^{j} 0 n\right]: E_{i}^{k}+\bar{G}_{i}-\phi_{i}-\alpha_{i} H_{i}^{k}>0 \tag{19}
\end{equation*}
$$

2. Pairings $(1 n, 1 r)$ and $(1 n, 1 n)$ are allowed, hence neither must occur.

Consider two women $i^{\prime}$ and $i^{\prime \prime}$ with preferences represented by (3) with $v(x)=x$. Let $K_{i}=1$. Assume that each woman's job offer entails earnings in range 1. That is, for $i \in\left\{i^{\prime}, i^{\prime \prime}\right\}$, $E_{i}^{k} \equiv W_{i}^{k} H_{i}^{k}$ is in range 1 . Let
(a) woman $i=i^{\prime}$ be such that $\mu_{i}=\eta_{i}^{a}=\eta_{i}^{j}=\alpha_{i}=0$ and

$$
\bar{G}_{i}-\phi_{i} \leq 0
$$

(b) woman $i=i^{\prime \prime}$ be such that $\mu_{i}=\eta_{i}^{a}=\eta_{i}^{j}=\alpha_{i}=0$ and

$$
0<\bar{G}_{i}-\phi_{i} \leq \min \left\{\kappa_{i}, E_{i}^{k}, \bar{G}_{i}-G_{i}^{a}\left(E_{i}^{k}\right)\right\} .
$$

Both women chose an alternative compatible with state $1 n$ under regime $a$. We now show that woman $i^{\prime}$ chooses an alternative compatible with state $1 n$ under regime $j$ while woman $i^{\prime \prime}$ selects an alternative compatible with state $1 r$ under regime $j$. For both women, the choice of the alternative compatible with state $1 n$ under regime $a$ reveals (Assumption 4) that this alternative yields as much utility as the available alternatives compatible with states $\{0 n, 0 r, 1 r, 1 u, 1 n\}$. Thus, for $i \in\left\{i^{\prime}, i^{\prime \prime}\right\}$ :

$$
\begin{align*}
& \text { [1n } \left.\succsim^{a} 0 n\right]: E_{i}^{k} \geq 0,  \tag{20}\\
& {\left[1 n \succsim^{a} 0 r\right]: E_{i}^{k} \geq \bar{G}_{i}-\phi_{i},}  \tag{21}\\
& {\left[1 n \succsim^{a} 1 n\right]: E_{i}^{k} \geq E_{i}^{l} \forall E_{i}^{l} \text {, }}  \tag{22}\\
& {\left[1 n \succsim^{a} 1 r\right]: E_{i}^{k} \geq E_{i}^{k}+G_{i}^{a}\left(E_{i}^{k}\right)-\phi_{i},}  \tag{23}\\
& {\left[1 n \succsim^{a} 1 u\right]: \quad E_{i}^{k} \geq E_{i}^{k}+\bar{G}_{i}-\phi_{i}-\kappa_{i} .} \tag{24}
\end{align*}
$$

It is easy to verify that descriptions (2a) and (2b) are compatible with optimality under regime $a$, that is, with (20)-(24). Both women prefer state $1 n$ under regime $j$ to the available alternatives compatible with states $\{0 r, 0 n, 1 u\}$, by Proposition 1. Woman $i=i^{\prime}$ also prefers state $1 n$ to the available alternatives compatible with state $1 r$ under regime $j$ because by description (2a) we have $\bar{G}_{i}-\phi_{i} \leq 0$ which implies (25):

$$
\begin{equation*}
\left[1 n \succsim^{j} 1 r\right]: E_{i}^{k} \geq E_{i}^{k}+\bar{G}_{i}-\phi_{i} . \tag{25}
\end{equation*}
$$

By Assumption 5 she breaks an indifference situation in favor of state $1 n$. Instead, woman $i=i^{\prime \prime}$ prefers earning $E_{i}^{k}$ on assistance to earning the same amount off assistance under regime $j$ because by description (2b) we have $\bar{G}_{i}-\phi_{i}>0$ which implies (26):

$$
\begin{equation*}
\left[1 r \succsim^{j} 1 n\right]: E_{i}^{k}+\bar{G}_{i}-\phi_{i}>E_{i}^{k} . \tag{26}
\end{equation*}
$$

Thus, the available alternative entailing earnings $E_{i}^{k}$ on assistance is preferred under regime $j$ to the available alternatives compatible with all states but $1 r$.

## 3. Pairings $(2 n, 1 r)$ and $(2 n, 2 n)$ are allowed, hence neither must occur.

Consider two women $i^{\prime}$ and $i^{\prime \prime}$ with preferences represented by (3) with $v(x)=x$. Let $K_{i}=2$. Assume that each woman's two job offers entail earnings in range 1 and in range 2 respectively. That is, for $i \in\left\{i^{\prime}, i^{\prime \prime}\right\}, E_{i}^{l} \equiv W_{i}^{l} H_{i}^{l}$ is in range 1 and $E_{i}^{k} \equiv W_{i}^{k} H_{i}^{k}$ is in range 2. Let
(a) woman $i=i^{\prime}$ be such that $\mu_{i}=\eta_{i}^{a}=\eta_{i}^{j}=\alpha_{i}=0, W_{i}^{k} \geq W_{i}^{l}$ and

$$
\bar{G}_{i}-\phi_{i} \leq 0,
$$

(b) woman $i=i^{\prime \prime}$ be such that $\mu_{i}=\eta_{i}^{a}=\eta_{i}^{j}=\alpha_{i}=0, W_{i}^{k} \geq W_{i}^{l}$ and

$$
E_{i}^{k}-E_{i}^{l}<\bar{G}_{i}-\phi_{i} \leq \min \left\{\kappa_{i}, E_{i}^{k}, \bar{G}_{i}-G_{i}^{a}\left(E_{i}^{l}\right)+E_{i}^{k}-E_{i}^{l}\right\} .
$$

Both women chose an alternative compatible with state $2 n$ under regime $a$. We now show that woman $i^{\prime}$ chooses an alternative compatible with state $2 n$ under regime $j$ while woman $i^{\prime \prime}$ selects an alternative compatible with state $1 r$ under regime $j$. For both women, the choice of the alternative compatible with state $2 n$ under regime $a$ reveals (Assumption 4) that
this alternative yields as much utility as the available alternatives compatible with states $\{0 n, 0 r, 1 n, 1 r, 1 u, 2 u\}$. Thus, for $i \in\left\{i^{\prime}, i^{\prime \prime}\right\}$ :

$$
\begin{align*}
& \text { [ } \left.2 n \succsim^{a} 0 n\right]: E_{i}^{k} \geq 0,  \tag{27}\\
& {\left[2 n \succsim^{a} 0 r\right]: \quad E_{i}^{k} \geq \bar{G}_{i}-\phi_{i},}  \tag{28}\\
& {\left[2 n \succsim^{a} 1 n\right]: \quad E_{i}^{k} \geq E_{i}^{l} \text {, }}  \tag{29}\\
& {\left[2 n \succsim^{a} 1 r\right]: \quad E_{i}^{k} \geq E_{i}^{l}+G_{i}^{a}\left(E_{i}^{l}\right)-\phi_{i},}  \tag{30}\\
& {\left[2 n \succsim^{a} 1 u\right]: E_{i}^{k} \geq E_{i}^{l}+\bar{G}_{i}-\phi_{i}-\kappa_{i},}  \tag{31}\\
& {\left[2 n \succsim^{a} 2 u\right]: \quad E_{i}^{k} \geq E_{i}^{k}+\bar{G}_{i}-\phi_{i}-\kappa_{i} .} \tag{32}
\end{align*}
$$

It is easy to verify that descriptions (3a) and (3b) are compatible with optimality under regime $a$, that is, with (27)-(32). Both women prefer state $2 n$ under regime $j$ to the available alternatives compatible with states $\{0 r, 2 u, 0 n, 1 n, 1 u\}$, by Proposition 1. Woman $i=i^{\prime}$ also prefers state $2 n$ to the available alternatives compatible with state $1 r$ under $j$ because $E_{i}^{k} \geq E_{i}^{l}$ by (29) and by description (3a) we have $\bar{G}_{i}-\phi_{i} \leq 0$ which implies (33):

$$
\begin{equation*}
\left[2 n \succsim^{j} 1 r\right]: E_{i}^{k} \geq E_{i}^{l}+\bar{G}_{i}-\phi_{i} . \tag{33}
\end{equation*}
$$

By Assumption 5 she breaks indifference in favor of state $1 n$. Instead, woman $i=i^{\prime \prime}$ prefers earning $E_{i}^{l}$ on assistance to earning $E_{i}^{k}$ off assistance under regime $j$ because by description (3b) we have $\bar{G}_{i}-\phi_{i}>E_{i}^{k}-E_{i}^{l}$ which implies (34):

$$
\begin{equation*}
\left[1 r \succsim^{j} 2 n\right]: E_{i}^{l}+\bar{G}_{i}-\phi_{i}>E_{i}^{k} . \tag{34}
\end{equation*}
$$

## 4. Pairings $(2 u, 1 r)$ and $(2 u, 2 u)$ are allowed, hence neither must occur.

Consider two women $i^{\prime}$ and $i^{\prime \prime}$ with preferences represented by (3) with $v(x)=x$. Let $K_{i}=2$. Assume that each woman's two job offers entail earnings in range 1 and in range 2 respectively. That is, for $i \in\left\{i^{\prime}, i^{\prime \prime}\right\}, E_{i}^{l} \equiv W_{i}^{l} H_{i}^{l}$ is in range 1 and $E_{i}^{k} \equiv W_{i}^{k} H_{i}^{k}$ is in range 2. Let
(a) woman $i=i^{\prime}$ be such that $\mu_{i}=\eta_{i}^{a}=\eta_{i}^{j}=\alpha_{i}=0, \phi_{i}>\underline{\phi}_{i}, W_{i}^{k} \geq W_{i}^{l}$ and

$$
\kappa_{i} \leq \min \left\{\bar{G}_{i}-\phi_{i}, E_{i}^{k}-E_{i}^{l}\right\}
$$

(b) woman $i=i^{\prime \prime}$ be such that $\mu_{i}=\eta_{i}^{a}=\eta_{i}^{j}=\alpha_{i}=0, \phi_{i}>\underline{\phi}_{i}, W_{i}^{k} \geq W_{i}^{l}$ and

$$
E_{i}^{k}-E_{i}^{l}<\kappa_{i} \leq \min \left\{\bar{G}_{i}-\phi_{i}, E_{i}^{k}-E_{i}^{l}+\bar{G}_{i}-G_{i}^{a}\left(E_{i}^{l}\right)\right\} .
$$

Both women chose an alternative compatible with state $2 u$ under regime $a$. We now show that woman $i^{\prime}$ chooses an alternative compatible with state $2 u$ under regime $j$ while woman $i^{\prime \prime}$ selects an alternative compatible with state $1 r$ under regime $j$. For both women, the choice of the alternative compatible with state $2 u$ under regime $a$ reveals (Assumption 4) that this alternative yields as much utility as the available alternatives compatible with states $\{0 n, 0 r, 1 n, 1 r, 1 u, 2 n\}$. Thus, for $i \in\left\{i^{\prime}, i^{\prime \prime}\right\}$ :

$$
\begin{align*}
& {\left[2 u \succsim^{a} 0 n\right] }:  \tag{35}\\
& {\left[2 u \succsim_{i}^{k} 0 r\right] }:  \tag{36}\\
& E_{i}^{k}+\bar{G}_{i}-\phi_{i}-\kappa_{i} \geq 0,  \tag{37}\\
& {\left[2 u \succsim^{a} 1 n\right]: } E_{i}^{k}+\bar{G}_{i}-\kappa_{i}-\kappa_{i} \geq \bar{G}_{i}-\phi_{i},  \tag{38}\\
& {\left[2 u \succsim^{a} 1 r\right] }:  \tag{39}\\
& E_{i}^{k}+\bar{G}_{i}-\phi_{i}-\kappa_{i} \geq E_{i}^{l}+G_{i}^{a}\left(E_{i}^{l}\right)-\phi_{i},  \tag{40}\\
& {\left[2 u \succsim^{a} 1 u\right]: } E_{i}^{k}+\bar{G}_{i}-\phi_{i}-\kappa_{i} \geq E_{i}^{l}+\bar{G}_{i}-\phi_{i}-\kappa_{i}, \\
& {\left[2 u \succsim^{a} 2 n\right]: } E_{i}^{k}+\bar{G}_{i}-\phi_{i}-\kappa_{i} \geq E_{i}^{k} .
\end{align*}
$$

It is easy to verify that descriptions (4a) and (4b) are compatible with optimality under regime $a$, that is, with (35)-(40). Both women prefer state $2 u$ under regime $j$ to the available alternatives compatible with states $\{0 r, 0 n, 1 n, 2 n, 1 u\}$, by Proposition 1 . Woman $i=i^{\prime}$ also prefers state $2 u$ to the available alternative compatible with state $1 r$ under regime $j$ because by description (4a) we have $\kappa_{i} \leq E_{i}^{k}-E_{i}^{l}$ which implies (41):

$$
\begin{equation*}
\left[2 u \succsim^{j} 1 r\right]: E_{i}^{k}+\bar{G}_{i}-\phi_{i}-\kappa_{i} \geq E_{i}^{l}+\bar{G}_{i}-\phi_{i} \tag{41}
\end{equation*}
$$

By Assumption 5 she breaks an indifference situation in favor of state $2 u$. Instead, woman $i=i^{\prime \prime}$ prefers earning and truthfully reporting $E_{i}^{l}$ on assistance to under-reporting earnings $E_{i}^{k}$ on assistance under regime $j$ because $\bar{G}_{i} \geq G_{i}^{a}\left(E_{i}^{l}\right)$ and by description (4b) we have $\kappa_{i}>E_{i}^{k}-E_{i}^{l}$ which implies (42):

$$
\begin{equation*}
\left[1 r \succsim^{j} 2 u\right]: E_{i}^{l}+\bar{G}_{i}-\phi_{i}>E_{i}^{k}+\bar{G}_{i}-\phi_{i}-\kappa_{i} . \tag{42}
\end{equation*}
$$

5. Pairings $\left(0 r, s^{j}\right)$ with $s^{j} \in\{0 r, 0 n, 1 n, 2 n, 1 r, 2 u\}$ are allowed.

Consider five women $\left\{i^{\prime}, i^{\prime \prime}, i^{\prime \prime \prime}, i^{I V}, i^{V}\right\}$ with preferences represented by $(3)$ with $v(x)=x$. Let $K_{i}=2$. Assume that each woman's two job offers entail earnings in range 1 and in range 2 respectively. That is, for $i \in\left\{i^{\prime}, i^{\prime \prime}, i^{\prime \prime \prime}, i^{I V}, i^{V}\right\}, E_{i}^{l} \equiv W_{i}^{l} H_{i}^{l}$ is in range 1 and $E_{i}^{k} \equiv W_{i}^{k} H_{i}^{k}$ is in range 2. Let
(a) woman $i=i^{\prime}$ be such that $\mu_{i}=0, \eta_{i}^{a}=\eta_{i}^{j}=\eta_{i}, \phi_{i}>\underline{\phi}_{i}, W_{i}^{l}=W_{i}^{k}=W_{i} \leq \alpha_{i}$, and

$$
0 \leq \eta_{i} \leq \min \left\{\bar{G}_{i}-\phi_{i}, H_{i}^{l}\left(\alpha_{i}-W_{i}\right)\right\}
$$

(b) woman $i=i^{\prime \prime}$ be such that $\mu_{i}=0, \eta_{i}^{a}=\eta_{i}^{j}=\eta_{i}, \phi_{i}>\underline{\phi}_{i}, W_{i}^{l}=W_{i}^{k}=W_{i} \leq \alpha_{i}$, and

$$
H_{i}^{l}\left(\alpha_{i}-W_{i}^{l}\right)<\eta_{i} \leq \min \left\{\bar{G}_{i}-\phi_{i}, H_{i}^{l}\left(\alpha_{i}-W_{i}\right)+\kappa_{i}, H_{i}^{l}\left(\alpha_{i}-W_{i}\right)+\bar{G}_{i}-G_{i}^{a}\left(E_{i}^{l}\right)\right\}
$$

(c) woman $i=i^{\prime \prime \prime}$ be such that $\mu_{i}=0, \eta_{i}^{a}<\eta_{i}^{j}, W_{i}^{l}=W_{i}^{k}=W_{i}<\alpha_{i}$ and

$$
\eta_{i}^{a} \leq \bar{G}_{i}-\phi_{i}<\min \left\{H_{i}^{l}\left(\alpha_{i}-W_{i}\right), \eta_{i}^{j}\right\}
$$

(d) woman $i=i^{I V}$ be such that $\eta_{i}^{a} \leq \mu_{i} \leq H_{i}^{k}\left(W_{i}^{k}-\alpha_{i}\right), \phi_{i}>\underline{\phi}_{i}, \eta_{i}^{a}<\eta_{i}^{j}, W_{i}^{k}>\alpha_{i}=W_{i}^{l}$ and

$$
H_{i}^{k}\left(W_{i}^{k}-\alpha_{i}\right)-\mu_{i}+\eta_{i}^{a} \leq \bar{G}_{i}-\phi_{i}<\min \left\{H_{i}^{k}\left(W_{i}^{k}-\alpha_{i}\right)-\mu_{i}+\eta_{i}^{j}, H_{i}^{k}\left(W_{i}^{k}-\alpha_{i}\right), \kappa_{i}\right\}
$$

(e) woman $i=i^{V}$ be such that $\eta_{i}^{a} \leq \mu_{i} \leq H_{i}^{k}\left(W_{i}^{k}-\alpha_{i}\right), \phi_{i}>\underline{\phi}_{i}, \eta_{i}^{a}<\eta_{i}^{j}, W_{i}^{k}>\alpha_{i}=W_{i}^{l}$ and

$$
H_{i}^{k}\left(W_{i}^{k}-\alpha_{i}\right)-\mu_{i}+\eta_{i}^{a} \leq \kappa_{i} \leq \min \left\{H_{i}^{k}\left(W_{i}^{k}-\alpha_{i}\right)-\mu_{i}+\eta_{i}^{j}, H_{i}^{k}\left(W_{i}^{k}-\alpha_{i}\right), \bar{G}_{i}-\phi_{i},\right\}
$$

All these women chose an alternative compatible with state $0 r$ under regime $a$. We now show that, under regime $j$, woman $i^{\prime}$ selects an alternative compatible with state $0 r$, woman $i^{\prime \prime}$ selects an alternative compatible with state $1 r$, woman $i^{\prime \prime \prime}$ selects an alternative compatible with state $0 n$, woman $i^{I V}$ selects an alternative compatible with state $2 n$, and woman $i^{V}$ selects an alternative compatible with state $2 u$. For all women, the choice of the alternative compatible with state $0 r$ under regime $a$ reveals (Assumption 4) that this alternative yields
as much utility as the available alternatives compatible with states $\{0 n, 1 n, 2 n, 1 r, 1 u, 2 u\}$. Thus, for $i \in\left\{i^{\prime}, i^{\prime \prime}, i^{\prime \prime \prime}, i^{I V}, i^{V}\right\}$ :

$$
\begin{align*}
& {\left[0 r \succsim^{a} 0 n\right]: \quad \bar{G}_{i}-\phi_{i}-\eta_{i}^{a} \geq 0,}  \tag{43}\\
& \text { [ } \left.0 r \succsim^{a} 1 n\right]: \quad \bar{G}_{i}-\phi_{i}-\eta_{i}^{a} \geq E_{i}^{l}-\mu_{i}-\alpha_{i} H_{i}^{l},  \tag{44}\\
& {\left[0 r \succsim^{a} 2 n\right]: \quad \bar{G}_{i}-\phi_{i}-\eta_{i}^{a} \geq E_{i}^{k}-\mu_{i}-\alpha_{i} H_{i}^{k},}  \tag{45}\\
& {\left[0 r \succsim^{a} 1 r\right]: \bar{G}_{i}-\phi_{i}-\eta_{i}^{a} \geq E_{i}^{l}-\mu_{i}+G_{i}^{a}\left(E_{i}^{l}\right)-\phi_{i}-\alpha_{i} H_{i}^{l} \text {, }}  \tag{46}\\
& {\left[0 r \succsim^{a} 1 u\right]: \bar{G}_{i}-\phi_{i}-\eta_{i}^{a} \geq E_{i}^{l}-\mu_{i}+\bar{G}_{i}-\phi_{i}-\kappa_{i}-\alpha_{i} H_{i}^{l} \text {, }}  \tag{47}\\
& {\left[0 r \succsim^{a} 2 u\right]: \quad \bar{G}_{i}-\phi_{i}-\eta_{i}^{a} \geq E_{i}^{k}-\mu_{i}+\bar{G}_{i}-\phi_{i}-\kappa_{i}-\alpha_{i} H_{i}^{k} .} \tag{48}
\end{align*}
$$

It is easy to verify that descriptions (5a)-(5e) are compatible with optimality under regime $a$, that is, with (43)-(48). Because $\eta_{i}^{a}=\eta_{i}^{j}$ for $i \in\left\{i^{\prime}, i^{\prime \prime}\right\}$, state $0 r$ has the same utility value under both regimes hence both women prefer state $0 r$ under regime $j$ to the available alternatives compatible with states $\{0 n, 1 n, 2 n, 1 u, 2 u\}$, by Proposition 1. Woman $i=i^{\prime}$ also prefers state $0 r$ to the available alternative compatible with state $1 r$ under regime $j$ because by description (5a) we have $\eta_{i} \leq H_{i}^{l}\left(\alpha_{i}-W_{i}\right)$ and $\mu_{i}=0$ which imply (49):

$$
\begin{equation*}
\left[0 r \succsim^{j} 1 r\right]: \bar{G}_{i}-\phi_{i}-\eta_{i} \geq E_{i}^{l}+\bar{G}_{i}-\phi_{i}-\alpha_{i} H_{i}^{l} . \tag{49}
\end{equation*}
$$

By Assumption 5 she breaks an indifference situation in favor of state $0 r$. Instead, woman $i=i^{\prime \prime}$ prefers earning and truthfully reporting $E_{i}^{l}$ on assistance to not working on assistance under regime $j$ because by description (5b) we have $\eta_{i}>H_{i}^{l}\left(\alpha_{i}-W_{i}\right)$ and $\mu_{i}=0$ which imply (50):

$$
\begin{equation*}
\left[1 r \succsim^{j} 0 r\right]: E_{i}^{l}+\bar{G}_{i}-\phi_{i}-\alpha H_{i}^{l}>\bar{G}_{i}-\phi_{i}-\eta_{i} . \tag{50}
\end{equation*}
$$

Consider now women $\left\{i^{\prime \prime \prime}, i^{I V}, i^{V}\right\}$. None selects an alternative compatible with state $1 u$ under regime $j$ by Proposition 1. Woman $i=i^{\prime \prime \prime}$ prefers not working off assistance (state $0 n$ ) to the available alternatives compatible with states $\{0 r, 1 n, 1 r, 2 n, 2 u\}$ under regime $j$ because, by description (5c), we have $\mu_{i}=0$ and, respectively, $\bar{G}_{i}-\phi_{i}<\eta_{i}^{j}$ which implies (51); $H_{i}^{l}\left(\alpha_{i}-W_{i}\right) \geq 0$ which implies (52); $H_{i}^{k}\left(\alpha_{i}-W_{i}\right) \geq 0$ which implies (53); $H_{i}^{l}\left(\alpha_{i}-W_{i}\right) \geq$ $\bar{G}_{i}-\phi_{i}$ which implies (54); and $H_{i}^{k}\left(\alpha_{i}-W_{i}\right)+\kappa_{i} \geq \bar{G}_{i}-\phi_{i}$ which implies (55):

$$
\begin{align*}
{\left[0 n \succsim^{j} 0 r\right] } & : 0>\bar{G}_{i}-\phi_{i}-\eta_{i}^{j},  \tag{51}\\
{\left[0 n \succsim^{j} 1 n\right] } & : 0 \geq E_{i}^{l}-\alpha_{i} H_{i}^{l},  \tag{52}\\
{\left[0 n \succsim^{j} 2 n\right] } & : 0 \geq E_{i}^{k}+-\alpha H_{i i}^{k},  \tag{53}\\
{\left[0 n \succsim^{j} 1 r\right] } & : 0 \geq E_{i}^{l}+\bar{G}_{i}-\phi_{i}-\alpha_{i} H_{i}^{l},  \tag{54}\\
{\left[0 n \succsim^{j} 2 u\right] } & : 0 \geq E_{i}^{k}+\bar{G}_{i}-\phi_{i}-\kappa_{i}-\alpha_{i} H_{i}^{k} . \tag{55}
\end{align*}
$$

Woman $i=i^{I V}$ prefers earning $E_{i}^{k}$ off assistance (state $2 n$ ) to the available alternatives compatible with states $\{0 n, 0 r, 1 n, 1 r, 2 u\}$ under regime $j$ because, by description ( 5 d ), we have $H_{i}^{k}\left(W_{i}^{k}-\alpha_{i}\right) \geq \mu_{i}$ which implies (56); $\bar{G}_{i}-\phi_{i}<H_{i}^{k}\left(W_{i}^{k}-\alpha_{i}\right)-\mu_{i}+\eta_{i}^{j}$ which implies (57); $W_{i}^{k}>\alpha_{i}=W_{i}^{l}$ which imply (58); $\bar{G}_{i}-\phi_{i}<H_{i}^{k}\left(W_{i}^{k}-\alpha_{i}\right)$ and $W_{i}^{k}>\alpha_{i}=W_{i}^{l}$ which imply (59); $\bar{G}_{i}-\phi_{i}<\kappa_{i}$ which implies (60):

$$
\begin{array}{ll}
{\left[2 n \succsim^{j} 0 n\right]} & : \\
{\left[2 n \succsim_{i}^{k}-\mu_{i}-\alpha_{i} H_{i}^{k} \geq 0,\right.} \\
{\left[2 n \succsim^{j} 1 n\right]} & : \\
E_{i}^{k}-\mu_{i}-\alpha_{i} H_{i}^{k}>\bar{G}_{i}-\phi_{i}-\eta_{i}^{j}, \\
{\left[2 n \succsim_{i}-\alpha_{i} H_{i}^{k} \geq E_{i}^{l}-\mu_{i}-\alpha_{i} H_{i}^{l},\right.}  \tag{60}\\
{\left[2 n \succsim^{j} 2 u\right]} & : \\
E_{i}^{k}-\mu_{i}-\alpha_{i} H_{i}^{k} \geq E_{i}^{l}-\mu_{i}+\bar{G}_{i}-\phi_{i}-\alpha_{i} H_{i}^{l}, \\
& \alpha_{i} H_{i}^{k} \geq E_{i}^{k}-\mu_{i}+\bar{G}_{i}-\phi_{i}-\kappa_{i}-\alpha_{i} H_{i}^{k} .
\end{array}
$$

Woman $i=i^{V}$ prefers under-reporting earning $E_{i}^{k}$ on assistance (state $2 u$ ) to the available alternatives compatible with states $\{0 n, 0 r, 1 n, 1 r, 2 n\}$ under regime $j$ because, by description (5e), we have $H_{i}^{k}\left(W_{i}^{k}-\alpha_{i}\right) \geq \mu_{i}$ and $\bar{G}_{i}-\phi_{i} \geq \kappa_{i}$ which imply (61); $H_{i}^{k}\left(W_{i}^{k}-\alpha_{i}\right)+\eta_{i}^{j}-\mu_{i} \geq$ $\kappa_{i}$ which implies (62); $H_{i}^{k}\left(W_{i}^{k}-\alpha_{i}\right) \geq \mu_{i}, \bar{G}_{i}-\phi_{i} \geq \kappa_{i}$ and $W_{i}^{l}=\alpha_{i}$ which imply (63); $H_{i}^{k}\left(W_{i}^{k}-\alpha_{i}\right) \geq \kappa_{i}$ and $W_{i}^{l}=\alpha_{i}$ which imply (64); $\bar{G}_{i}-\phi_{i} \geq \kappa_{i}$ which implies (65):

$$
\begin{align*}
& {\left[2 u \succsim^{j} 0 n\right] }:  \tag{61}\\
& E_{i}^{k}-\mu_{i}+\bar{G}_{i}-\phi_{i}-\kappa_{i}-\alpha_{i} H_{i}^{k} \geq 0,  \tag{62}\\
& {\left[2 u \succsim^{j} 0 r\right]: } E_{i}^{k}-\mu_{i}+\bar{G}_{i}-\phi_{i}-\kappa_{i}-\alpha_{i} H_{i}^{k}>\bar{G}_{i}-\phi_{i}-\eta_{i}^{j},  \tag{63}\\
& {\left[2 u \succsim^{j} 1 n\right]: } E_{i}^{k}-\mu_{i}+\bar{G}_{i}-\phi_{i}-\kappa_{i}-\alpha_{i} H_{i}^{k} \geq E_{i}^{l}-\mu_{i}-\alpha_{i} H_{i}^{l},  \tag{64}\\
& {\left[2 u \succsim^{j} 1 r\right]: } E_{i}^{k}-\mu_{i}+\bar{G}_{i}-\phi_{i}-\kappa_{i}-\alpha_{i} H_{i}^{k} \geq E_{i}^{l}-\mu_{i}+\bar{G}_{i}-\phi_{i}-\alpha_{i} H_{i}^{l},  \tag{65}\\
& {\left[2 u \succsim^{j} 2 n\right]: } E_{i}^{k}-\mu_{i}+\bar{G}_{i}-\phi_{i}-\kappa_{i}-\alpha_{i} H_{i}^{k} \geq E_{i}^{k}-\mu_{i}-\alpha_{i} H_{i}^{k} .
\end{align*}
$$

## 6. Pairing $(0 r, 1 n)$ is allowed.

Consider a woman $i$ with preferences represented by (3) with $v($.$) strictly concave. Let$ $K_{i}=1$. Assume that her job offer entails earnings in range 1. That is, $E_{i}^{k} \equiv W_{i}^{k} H_{i}^{k}$ is in range 1. Let
(a) woman $i$ be such that $\eta_{i}^{a}=\mu_{i}=0 \operatorname{and}^{2}$

$$
\begin{aligned}
& \max \left\{\begin{array}{c}
v\left(\bar{G}_{i}\right)-v\left(E_{i}^{k}\right)+\alpha_{i} H_{i}^{k}-\eta_{i}^{j}, \\
v\left(E_{i}^{k}+\bar{G}_{i}\right)-v\left(E_{i}^{k}\right)
\end{array}\right\} \quad<\phi_{i} \leq \min \left\{\begin{array}{c}
v\left(\bar{G}_{i}\right)-v(0), \\
v\left(\bar{G}_{i}\right)-v\left(E_{i}^{k}\right)+\alpha_{i} H_{i}^{k}
\end{array}\right\}, \\
& \max \left\{\begin{array}{c}
v\left(E_{i}^{k}+G_{i}^{a}\left(E_{i}^{k}\right)\right)-v\left(\bar{G}_{i}\right), \\
v\left(E_{i}^{k}+\bar{G}_{i}-\kappa_{i}\right)-v\left(\bar{G}_{i}\right)
\end{array}\right\} \leq \alpha_{i} H_{i}^{k} \leq v\left(E_{i}^{k}\right)-v(0) .
\end{aligned}
$$

Woman $i$ chooses an alternative compatible with state $0 r$ under $a$. We now show that, under regime $j$, she selects an alternative compatible with state $1 n$. The choice of the alternative compatible with state $0 r$ under regime $a$ reveals (Assumption 4) that this alternative yields as much utility as the available alternatives compatible with states $\{0 n, 1 n, 1 r, 1 u\}$. Thus:

$$
\begin{align*}
{\left[0 r \succsim^{a} 0 n\right] } & : v\left(\bar{G}_{i}\right)-\phi_{i} \geq v(0),  \tag{66}\\
{\left[0 r \succsim^{a} 1 n\right]: } & : v\left(\bar{G}_{i}\right)-\phi_{i} \geq v\left(E_{i}^{k}\right)-\alpha_{i} H_{i}^{k},  \tag{67}\\
{\left[0 r \succsim^{a} 1 r\right] } & : v\left(\bar{G}_{i}\right)-\phi_{i} \geq v\left(E_{i}^{k}+G_{i}^{a}\left(E_{i}^{k}\right)\right)-\phi_{i}-\alpha_{i} H_{i}^{k},  \tag{68}\\
{\left[0 r \succsim^{a} 1 u\right] } & : v\left(\bar{G}_{i}\right)-\phi_{i} \geq v\left(E_{i}^{k}+\bar{G}_{i}-\kappa_{i}\right)-\phi_{i}-\alpha_{i} H_{i}^{k} . \tag{69}
\end{align*}
$$

It is easy to verify that description (6a) is compatible with optimality under regime $a$, that is, with (66)-(69). Woman $i$ will not selected an alternative compatible with state $1 u$ under regime $j$ by Proposition 1. She prefers earning $E_{i}^{k}$ off assistance (state $1 n$ ) to the available alternatives compatible with states $\{0 n, 0 r, 1 r\}$ under $j$ because, by description (6a), we have $v\left(E_{i}^{k}\right)-v(0) \geq \alpha_{i} H_{i}^{k}$ which implies (70); $v\left(\bar{G}_{i}\right)-v\left(E_{i}^{k}\right)+\alpha H_{i}^{k}-\eta_{i}^{j}<\phi_{i}$ which implies (71); and $v\left(E_{i}^{k}+\bar{G}_{i}\right)-v\left(E_{i}^{k}\right) \leq \phi_{i}$ which implies (72):

$$
\begin{align*}
& {\left[1 n \succsim^{j} 0 n\right]: v\left(E_{i}^{k}\right)-\alpha_{i} H_{i}^{k} \geq v(0),}  \tag{70}\\
& {\left[1 n \succsim^{j} 0 r\right]:}  \tag{71}\\
& {\left[1 n \succsim^{j} 1 r\right]} \tag{72}
\end{align*}: v\left(E_{i}^{k}\right)-\alpha_{i} H_{i}^{k}>v\left(\bar{G}_{i}^{k}\right)-\phi_{i} H_{i}^{k} \geq v\left(E_{i}^{k}+\bar{G}_{i}\right)-\phi_{i}-\alpha_{i} H_{i}^{k} .
$$

[^24]7. We conclude the proof by remarking that, because pairings $\left(0 r, s^{j}\right)$ with $s^{j} \in\{0 r, 0 n, 1 n, 2 n, 1 r, 2 u\}$ are allowed, none of them must occur.

## 5 Testable Revealed Preference Restrictions

Lemma 5 (Revealed Preference Restrictions). Consider the system of equations:

$$
\begin{align*}
& p_{0 n}^{j}-p_{0 n}^{a}=-\pi_{0 n, 1 r} p_{0 n}^{a}+\pi_{0 r, 0 n} p_{0 p}^{a} \\
& p_{1 n}^{j}-p_{1 n}^{a}=-\pi_{1 n, 1 r} p_{1 n}^{a}+\pi_{0 r, 1 n} p_{0 p}^{a} \\
& p_{2 n}^{j}-p_{2 n}^{a}=-\pi_{2 n, 1 r} p_{2 n}^{a}+\pi_{0 r, 2 n} p_{0 p}^{a}  \tag{73}\\
& p_{0 p}^{j}-p_{0 p}^{a}=-\left(\pi_{0 r, 0 n}^{a}+\pi_{0 r, 2 n}+\pi_{0 r, 1 r}+\pi_{0 r, 1 n}+\pi_{0 r, 2 u}\right) p_{0 p}^{a} \\
& p_{2 p}^{j}-p_{2 p}^{a}=\pi_{0 r, 2 u} p_{0 p}^{a}-\pi_{2 u, 1 r} p_{2 p}^{a}
\end{align*}
$$

System (73) implies 16 inequality restrictions on $\mathbf{p}^{j}-\mathbf{p}^{a}$ :

$$
\begin{align*}
\left(p_{0 p}^{a}-p_{0 p}^{j}\right) & \geq 0  \tag{74}\\
\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{0 n}^{a}-p_{0 n}^{j}\right) & \geq 0  \tag{75}\\
\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{1 n}^{a}-p_{1 n}^{j}\right) & \geq 0  \tag{76}\\
\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{2 n}^{a}-p_{2 n}^{j}\right) & \geq 0  \tag{77}\\
\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{2 p}^{a}-p_{2 p}^{j}\right) & \geq 0  \tag{78}\\
\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{0 n}^{a}-p_{0 n}^{j}\right)+\left(p_{1 n}^{a}-p_{1 n}^{j}\right) & \geq 0  \tag{79}\\
\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{2 n}^{a}-p_{2 n}^{j}\right)+\left(p_{0 n}^{a}-p_{0 n}^{j}\right) & \geq 0  \tag{80}\\
\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{0 n}^{a}-p_{0 n}^{j}\right)+\left(p_{2 p}^{a}-p_{2 p}^{j}\right) & \geq 0  \tag{81}\\
\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{2 n}^{a}-p_{2 n}^{j}\right)+\left(p_{1 n}^{a}-p_{1 n}^{j}\right) & \geq 0  \tag{82}\\
\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{2 p}^{a}-p_{2 p}^{j}\right)+\left(p_{1 n}^{a}-p_{1 n}^{j}\right) & \geq 0  \tag{83}\\
\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{2 n}^{a}-p_{2 n}^{j}\right)+\left(p_{2 p}^{a}-p_{2 p}^{j}\right) & \geq 0  \tag{84}\\
\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{2 n}^{a}-p_{2 n}^{j}\right)+\left(p_{0 n}^{a}-p_{0 n}^{j}\right)+\left(p_{1 n}^{a}-p_{1 n}^{j}\right) & \geq 0  \tag{85}\\
\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{0 n}^{a}-p_{0 n}^{j}\right)+\left(p_{2 p}^{a}-p_{2 p}^{j}\right)+\left(p_{1 n}^{a}-p_{1 n}^{j}\right) & \geq 0  \tag{86}\\
\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{2 n}^{a}-p_{2 n}^{j}\right)+\left(p_{0 n}^{a}-p_{0 n}^{j}\right)+\left(p_{2 p}^{a}-p_{2 p}^{j}\right) & \geq 0  \tag{87}\\
\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{2 n}^{a}-p_{2 n}^{j}\right)+\left(p_{2 p}^{a}-p_{2 p}^{j}\right)+\left(p_{1 n}^{a}-p_{1 n}^{j}\right) & \geq 0  \tag{88}\\
\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{2 n}^{a}-p_{2 n}^{j}\right)+\left(p_{0 n}^{a}-p_{0 n}^{j}\right)+\left(p_{2 p}^{a}-p_{2 p}^{j}\right)+\left(p_{1 n}^{a}-p_{1 n}^{j}\right) & \geq 0 \tag{89}
\end{align*}
$$

Proof. Restrictions (74-89) are obtained by using the fact that, by definition, $0 \leq \pi_{s^{a}, s^{j}} \leq 1$ all $s^{a}, s^{j} \in \mathcal{S}$ and $\sum_{s^{j} \in \mathcal{S}} \pi_{s^{a}, s^{j}}=1$ all $s^{a} \in \mathcal{S}$. The response margins $\left(\pi_{0 n, 1 r}, \pi_{1 n, 1 r}, \pi_{2 n, 1 r}, \pi_{2 u, 1 r}\right)$ may each take value 0 or 1 . The response margins ( $\pi_{0 r, 0 n}, \pi_{0 r, 1 n}, \pi_{0 r, 1 r}, \pi_{0 r, 2 n}, \pi_{0 r, 2 u}$ ) may each take value 0 or 1 but if one of them takes the value 1 the others are constrained to take the value 0 . Thus, there are $2^{4}+1+5=22$ viable ordered arrangements of 9 elements each taking the boundary value 0 or 1 . Each arrangement implies restrictions on $\mathbf{p}^{j}-\mathbf{p}^{a}$ through system (73). 16 restrictions are non redundant: they are inequalities (74-89). For instance, consider the fourth equation in system (73). Letting $\pi_{0 r, 0 n}+\pi_{0 r, 2 n}+\pi_{0 r, 1 r}+\pi_{0 r, 1 n}+\pi_{0 r, 2 u}=0$, this equation implies (74). As another example, sum the first and the fourth equations in system (73) to obtain $\left(p_{0 n}^{j}-p_{0 n}^{a}\right)+\left(p_{0 p}^{j}-p_{0 p}^{a}\right)=-\pi_{0 n, 1 r} p_{0 n}^{a}-\left(\pi_{0 r, 2 n}+\pi_{0 r, 1 r}+\pi_{0 r, 1 n}+\pi_{0 r, 2 u}\right) p_{0 p}^{a}$. Letting $\pi_{0 r, 2 n}+\pi_{0 r, 1 r}+\pi_{0 r, 1 n}+\pi_{0 r, 2 u}=0$ and $\pi_{0 n, 1 r}=0$, this equation implies (75).

Remark 5 (Easy to Describe Testable Restrictions). In the paper we explicitly refer to five of the inequalities in (74-89). They are: inequality (74), inequality (75) which rewrites as $p_{0}^{a}-p_{0}^{j} \geq 0$ where $p_{0}^{t} \equiv p_{0 n}^{t}+p_{0 p}^{t}$ for $t \in\{a, j\}$; inequality (85) which rewrites as $p_{1+, p}^{a}-p_{1+, p}^{j} \leq 0$ where $p_{1+, p}^{t} \equiv p_{1 p}^{t}+p_{2 p}^{t}$ for $t \in\{a, j\} ;$ inequality (87) which rewrites as $p_{1}^{a}-p_{1}^{j} \leq 0$ where $p_{1}^{t} \equiv p_{1 n}^{t}+p_{1 p}^{t}$ for $t \in\{a, j\}$; and inequality (89) which rewrites as $p_{1 p}^{a}-p_{1 p}^{j} \leq 0$.

Corollary 3 (Additional Testable Restrictions under a Special Form of Preferences). Subject to specification (2) of the utility function, revealed preference imply a testable restriction in addition to (74-89):

$$
\begin{equation*}
p_{1 n}^{a}-p_{1 n}^{j} \geq 0 . \tag{90}
\end{equation*}
$$

Subject to (90), inequalities (76), (79), (82), (83), (85), (86), (88), and (89) are redundant.
Proof. Subject to specification (2) of the utility function, $\pi_{0 r, 1 n}=0$ by Corollary 2. System (73) simplifies accordingly. In particular, the second equation writes $p_{1 n}^{a}-p_{1 n}^{j}=\pi_{1 n, 1 r} p_{1 n}^{a}$. Letting $\pi_{1 n, 1 r}=0$ we obtain restriction (90). Redundancy of inequalities (76), (79), (82), (83), (85), (86), (88), and (89) is easily verified. For instance, inequality (76) is implied by (74) and (90).

## 6 Bounds on the Response Margins

## Derivation of Bounds

A solution to any linear programming problem has to occur at one of the vertices of the problem's constraint space (see Murty, 1983). Recall that the linear constraints are as per system (73). To obtain the set of possible solutions to the linear programming problem

$$
\max _{\pi} \boldsymbol{\pi}^{\prime} \boldsymbol{\lambda} \text { subject to (73) and } \boldsymbol{\pi} \in[0,1]^{9},
$$

we enumerated all vertices of the convex polytope defined by the intersection of the hyperplane defined by the equations in (73) with the hypercube defined by the unit constraints on the parameters. In practice, this amounted to setting all possible choices of four of the nine parameters in (73) to 0 or 1 and solving for the remaining five parameters. There were $\binom{9}{4}=126$ different possible choices of four parameters and $2^{4}=16$ different binary arrangements those parameters could take, yielding 2016 possible vertices. However we were able to use the structure of our problem to rule out the existence of solutions at certain vertices - e.g., $\pi_{2 n, 1 r}$ and $\pi_{0 r, 2 n}$ cannot both be set arbitrarily because this would lead to a violation of the third equation in (73). Such restrictions reduced the problem to solving the system at a manageable number of vertices. We then enumerated the set of minima and maxima each parameter could achieve across the relevant solutions. After eliminating dominated solutions, we arrived at the stated bounds.

## Lists of Bounds

The analytical expressions for the bounds on the response probabilities are presented below. The symbol $\left({ }^{*}\right)$ is placed next to a solution, or a term, that is redundant subject to the specification of the utility function given in (2).

## Simple Response Margins

$$
\max \left\{0, \frac{\left(p_{2 n}^{a}-p_{2 n}^{j}\right)}{p_{2 n}^{a}}\right\} \leq \pi_{2 n, 1 r} \leq \min \left\{\begin{array}{c}
1, \\
\frac{\left(p_{2 n}^{a}-p_{2 n}^{j}\right)+\left(p_{0 p}^{a}-p_{0 p}^{j}\right)}{p_{2 n}^{a},}, \\
\frac{\left(p_{2 n}^{a}-p_{2 n}^{j}\right)+\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{0 n}^{a}-p_{0 n}^{j}\right)}{p_{2 n}^{a}}, \\
\frac{\left(p_{2 n}^{a}-p_{2 n}^{j}\right)+\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{2 p}^{a}-p_{2 p}^{j}\right)}{p_{2 n}^{a}}, \\
\frac{\left(p_{2 n}^{a}-p_{2 n}^{j}\right)+\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{0 n}^{a}-p_{0 n}^{j}\right)+\left(p_{2 p}^{a}-p_{2 p}^{j}\right)}{p_{2 n}^{a}}, \\
\frac{\left(p_{2 n}^{a}-p_{2 n}^{j}\right)+\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{1 n}^{a}-p_{1 n}^{j}\right)}{p_{2 n}^{a}},(*) \\
\frac{\left(p_{2 n}^{a}-p_{2 n}^{j}\right)+\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{0 n}^{a}-p_{0 n}^{j}\right)+\left(p_{1 n}^{a}-p_{1 n}^{j}\right)}{p_{2 n}^{a}},(*) \\
\frac{\left(p_{2 n}^{a}-p_{2 n}^{j}\right)+\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{2 p}^{a}-p_{2 n}^{j}\right)+\left(p_{1 n}^{a}-p_{1 n}^{j}\right)}{p_{2 n}^{a}},(*) \\
\frac{\left(p_{2 n}^{a}-p_{2 n}^{j}\right)+\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{0 n}^{a}-p_{0 n}^{j}\right)+\left(p_{2 p}^{a}-p_{2 p}^{j}\right)+\left(p_{1 n}^{a}-p_{1 n}^{j}\right)}{p_{2 n}^{a}}
\end{array},(*),\right.
$$

$$
\begin{aligned}
& \max \left\{0, \frac{\left(p_{0 n}^{a}-p_{0 n}^{j}\right)}{p_{0 n}^{a}}\right\} \leq \pi_{0 n, 1 r} \leq \min \{ \\
& \left.\frac{\left(p_{0 n}-p_{0 n}^{j}\right)+\left(p_{0 p}-p_{0 p}^{j}\right)+\left(p_{1 n}-p_{1 n}^{j}\right)+\left(p_{2 n}-p_{2 n}^{j}\right)+\left(p_{2 p}-p_{2 p}^{j}\right)}{p_{0 n}^{a}}(*)\right) \\
& \left.\begin{array}{c}
1, \\
\frac{\left(p_{2 p}^{a}-p_{2 p}^{j}\right)+\left(p_{0 p}^{a}-p_{0 p}^{j}\right)}{p_{2 p}^{a}}, \\
\frac{\left(p_{2 p}^{a}-p_{2 p}^{j}\right)+\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{2 n}^{a}-p_{2 n}^{j}\right)}{p_{2 p}^{a}}, \\
\frac{\left(p_{2 p}^{a}-p_{2 p}^{j}\right)+\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{0 n}^{a}-p_{0 n}^{j}\right)}{p_{2 p}^{a}}, \\
\frac{\left(p_{2 p}^{a}-p_{2 p}^{j}\right)+\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{0 n}^{a}-p_{0 n}^{j}\right)+\left(p_{2 n}^{a}-p_{2 n}^{j}\right)}{p_{2 p}^{a}}, \\
\frac{\left(p_{2 p}^{a}-p_{2 p}^{j}\right)+\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{1 n}^{a}-p_{1 n}^{j}\right)}{p_{2 p}^{a}},(*) \\
\frac{\left(p_{2 p}^{a}-p_{2 p}^{j}\right)+\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{1 n}^{a}-p_{1 n}^{j}\right)+\left(p_{0 n}^{a}-p_{0 n}^{j}\right)}{p_{2 p}^{a}},(*) \\
\frac{\left(p_{2 p}^{a}-p_{2 p}^{j}\right)+\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{1 n}^{a}-p_{1 n}^{j}\right)+\left(p_{2 n}^{a}-p_{2 n}^{j}\right)}{p_{2 p}^{a}},(*) \\
\frac{\left(p_{2 p}^{a}-p_{2 p}^{j}\right)+\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{1 n}^{a}-p_{1 n}^{j}\right)+\left(p_{0 n}^{a}-p_{0 n}^{j}\right)+\left(p_{2 n}^{a}-p_{2 n}^{j}\right)}{p_{2 p}^{a}}(*)
\end{array}\right\} . \\
& \max \left\{0, \frac{\left(p_{2 p}^{a}-p_{2 p}^{j}\right)}{p_{2 p}^{a}}\right\} \leq \pi_{2 u, 1 r} \leq \min \{ \\
& \max \left\{0, \frac{\left(p_{0 p}^{a}-p_{0 p}^{j}\right)-p_{0 n}^{j}-p_{2 p}^{j}-p_{2 n}^{j}-p_{1 n}^{j}(*)}{p_{0 p}^{a}}\right\} \leq \pi_{0 r, 1 r} \leq \min \{ \\
& \frac{\left(p_{0 p}^{a}-p_{0 p}^{j}\right)}{p_{0 p}^{a}}, \\
& \frac{\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{2 p}^{a}-p_{2 p}^{j}\right)}{p_{0 p}^{a}}, \\
& \frac{\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{2 n}^{a}-p_{2 n}^{j}\right)}{p_{0 p}^{a}}, \\
& \frac{\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{0 n}^{a}-p_{0 n}^{j}\right)}{p_{0 p}^{a}}, \\
& \frac{\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{2 p}^{a}-p_{2 p}^{j}\right)+\left(p_{2 n}^{a}-p_{2 n}^{j}\right)}{p_{0 p}^{a}}, \\
& \frac{\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{2 p}^{a}-p_{2 p}^{j}\right)+\left(p_{0 n}^{a}-p_{0 n}^{j}\right)}{p_{0 p}^{a}}, \\
& \frac{\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{2 n}^{a}-p_{2 n}^{j}\right)+\left(p_{0 n}^{a}-p_{0 n}^{j}\right)}{p_{0 p}^{a}}, \\
& \frac{\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{2 n}^{a}-p_{2 n}^{j}\right)+\left(p_{0 n}^{a}-p_{0 n}^{j}\right)+\left(p_{2 p}^{a}-p_{2 p}^{j}\right)}{p_{0 p}^{a}}, \\
& \frac{\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{1 n}^{a}-p_{1 n}^{j}\right)}{p_{0 p}^{a}},(*) \\
& \frac{\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{1 n}^{a}-p_{1 n}^{j}\right)+\left(p_{2 p}^{a}-p_{2 p}^{j}\right)}{p_{0 p}^{a}},(*) \\
& \frac{\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{1 n}^{a}-p_{1 n}^{j}\right)+\left(p_{2 n}^{a}-p_{2 n}^{j}\right)}{p_{0 p}^{a}},(*) \\
& \frac{\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{1 n}^{a}-p_{1 n}^{j}\right)+\left(p_{0 n}^{a}-p_{0 n}^{j}\right)}{p_{0 p}^{a}},(*) \\
& \frac{\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{1 n}^{a}-p_{1 n}^{j}\right)+\left(p_{2 p}^{a}-p_{2 p}^{j}\right)+\left(p_{0 n}^{a}-p_{0 n}^{j}\right)}{p_{0 p}^{a}},(*) \\
& \frac{\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{1 n}^{a}-p_{1 n}^{j}\right)+\left(p_{2 p}^{a}-p_{2 p}^{j}\right)+\left(p_{2 n}^{a}-p_{2 n}^{j}\right)}{p_{0 p}^{a}},(*) \\
& \left.\frac{\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{1 n}^{a}-p_{1 n}^{j}\right)+\left(p_{2 p}^{a}-p_{2 p}^{j}\right)+\left(p_{2 n}^{a}-p_{2 n}^{j}\right)+\left(p_{0 n}^{a}-p_{0 n}^{j}\right)}{p_{0 p}^{a}}(*)\right)
\end{aligned}
$$

$$
\begin{aligned}
& \left\{\begin{array}{c}
\frac{p_{1 n}^{j}}{p_{0 p}^{a}}, \\
\frac{\left(p_{0 p}^{a}-p_{0 p}^{j}\right)}{p_{0 p}^{a}}, \\
\frac{\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{0 n}^{a}-p_{0 n}^{j}\right)}{p_{0 p}^{a}}, \\
\frac{\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{2 n}^{a}-p_{2 n}^{j}\right)}{p_{0 p}^{a}}, \\
\frac{\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{2 p}^{a}-p_{2 p}^{j}\right)}{p_{0 p}^{a}}, \\
\frac{\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{0 n}^{a}-p_{0 n}^{j}\right)+\left(p_{2 n}^{a}-p_{2 n}^{j}\right)}{p_{0 p}^{a}}, \\
\frac{\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{0 n}^{a}-p_{0 n}^{j}\right)+\left(p_{2 p}^{a}-p_{2 p}^{j}\right)}{p_{0 p}^{a}}, \\
\frac{\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{2 n}^{a}-p_{2 n}^{j}\right)+\left(p_{2 p}^{a}-p_{2 p}^{j}\right)}{p_{0 p}^{a}}, \\
\frac{\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{0 n}^{a}-p_{0 n}^{j}\right)+\left(p_{2 n}^{a}-p_{2 n}^{j}\right)+\left(p_{2 p}^{a}-p_{2 p}^{j}\right)}{p_{0 p}^{a}}
\end{array}\right\} . \\
& \left\{\begin{array}{c}
1, \\
\frac{\left(p_{1 n}^{a}-p_{1 n}^{j}\right)+\left(p_{0 p}^{a}-p_{0 p}^{j}\right)}{p_{1 n}^{a}}, \\
\frac{\left(p_{1 n}^{a}-p_{1 n}^{j}\right)+\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{0 n}^{a}-p_{0 n}^{j}\right)}{p_{1 n}^{a}}, \\
\frac{\left(p_{1 n}^{a}-p_{1 n}^{j}\right)+\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{2 p}^{a}-p_{2 p}^{j}\right)}{p_{1 n}^{a}}, \\
\frac{\left(p_{1 n}^{a}-p_{1 n}^{j}\right)+\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{2 n}^{a}-p_{2 n}^{j}\right)}{p_{1 n}^{a}}, \\
\frac{\left(p_{1 n}^{a}-p_{1 n}^{j}\right)+\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{0 n}^{a}-p_{0 n}^{j}\right)+\left(p_{2 p}^{a}-p_{2 p}^{j}\right)}{p_{1 n}^{a}}, \\
\frac{\left(p_{1 n}^{a}-p_{1 n}^{j}\right)+\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{0 n}^{a}-p_{0 n}^{j}\right)+\left(p_{2 n}^{a}-p_{2 n}^{j}\right)}{p_{1 n}^{a}}, \\
\frac{\left(p_{1 n}^{a}-p_{1 n}^{j}\right)+\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{2 p}^{a}-p_{2 p}^{j}\right)+\left(p_{2 n}^{a}-p_{2 n}^{j}\right)}{\left.p_{1 n}^{a}-p_{1 n}^{j}\right)+\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{0 n}^{a}-p_{0 n}^{j}\right)+\left(p_{2 p}^{a}-p_{2 p}^{j}\right)+\left(p_{2 n}^{a}-p_{2 n}^{j}\right)} \\
p_{1 n}^{a}
\end{array},\right. \\
& \max \left\{0, \frac{\left(p_{1 n}^{a}-p_{1 n}^{j}\right)}{p_{1 n}^{a}}\right\} \leq \pi_{1 n, 1 r} \leq \min \{
\end{aligned}
$$

## Composite Response Margins

$$
\begin{aligned}
& \pi_{0 r, n} \geq \max \left\{0,-\frac{\left(p_{0 n}^{a}-p_{0 n}^{j}\right)+\left(p_{2 n}^{a}-p_{2 n}^{j}\right)+\left(p_{1 n}^{a}-p_{1 n}^{j}\right)(*)}{p_{0 p}^{a}}\right\}, \\
& \pi_{0 r, n} \leq \min \left\{\frac{p_{0 n}^{j}+p_{2 n}^{j}+p_{1 n}^{j}(*)}{p_{0 p}^{a}}, \frac{\left(p_{0 p}^{a}-p_{0 p}^{j}\right)}{p_{0 p}^{a}}, \frac{\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{2 p}^{a}-p_{2 p}^{j}\right)}{p_{0 p}^{a}}\right\} . \\
& \pi_{p, n} \geq \max \left\{0,-\frac{\left(p_{0 n}^{a}-p_{0 n}^{j}\right)+\left(p_{2 n}^{a}-p_{2 n}^{j}\right)+\left(p_{1 n}^{a}-p_{1 n}^{j}\right)(*)}{p_{0 p}^{a}+p_{1 p}^{a}+p_{2 p}^{a}}\right\}, \\
& \pi_{p, n} \leq \min \left\{\frac{p_{0 n}^{j}+p_{2 n}^{j}+p_{1 n}^{j}(*)}{p_{0 p}^{a}+p_{1 p}^{a}+p_{2 p}^{a}}, \frac{\left(p_{0 p}^{a}-p_{0 p}^{j}\right)}{p_{0 p}^{a}+p_{1 p}^{a}+p_{2 p}^{a}}, \frac{\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{2 p}^{a}-p_{2 p}^{j}\right)}{p_{0 p}^{a}+p_{1 p}^{a}+p_{2 p}^{a}}\right\} . \\
& \pi_{n, p} \geq \max \left\{0, \frac{\left(p_{0 n}^{a}-p_{0 n}^{j}\right)+\left(p_{1 n}^{a}-p_{1 n}^{j}\right)+\left(p_{2 n}^{a}-p_{2 n}^{j}\right)}{p_{0 n}^{a}+p_{1 n}^{a}+p_{2 n}^{a}}\right\}, \\
& \pi_{n, p} \leq \min \left\{\begin{array}{c}
1, \\
\frac{\left(p_{0 n}^{a}-p_{0 n}^{j}\right)+\left(p_{1 n}^{a}-p_{1 n}^{j}\right)+\left(p_{2 n}^{a}-p_{2 n}^{j}\right)+\left(p_{0 p}^{a}-p_{0 p}^{j}\right)}{p_{0 n}^{a}+p_{1 n}^{a}+p_{2 n}^{a}}, \\
\frac{\left(p_{0 n}^{a}-p_{0 n}^{j}\right)+\left(p_{1 n}^{a}-p_{1 n}^{j}\right)+\left(p_{2 n}^{a}-p_{2 n}^{j}\right)+\left(p_{0 p}^{a}-p_{0 p}^{j}\right)+\left(p_{2 p}^{a}-p_{2 p}^{j}\right)}{p_{0 n}^{a}+p_{1 n}^{a}+p_{2 n}^{a}}
\end{array}\right\} . \\
& \pi_{0,1+}=\frac{\left(p_{0 n}^{a}-p_{0 n}^{j}\right)+\left(p_{0 p}^{a}-p_{0 p}^{j}\right)}{p_{0 p}^{a}+p_{0 n}^{a}} .
\end{aligned}
$$

## 7 Inference on Bounds

We begin with a description of the upper limit of our confidence interval. For each response probability $\pi$ we have a set of possible upper bound solutions $\left\{u b_{1}, u b_{2}, \ldots, u b_{K}\right\}$. We know that:

$$
\begin{aligned}
\pi & \leq \bar{\pi} \equiv \min \{\underline{u b}, 1\} \\
\underline{u b} & \equiv \min \left\{u b_{1}, u b_{2}, \ldots, u b_{K}\right\} .
\end{aligned}
$$

A consistent estimate of the least upper bound $\underline{u b}$ can be had by plugging in consistent sample moments $\widehat{u b}_{k} \xrightarrow{p} u b_{k}$ and using $\underline{\widehat{u b}} \equiv \min \left\{\widehat{u b}_{1}, \widehat{u b}_{2}, \ldots, \widehat{u b}_{K}\right\}$ as an estimate of $\underline{u b}$. This estimator is consistent by continuity of probability limits. We can then form a corresponding consistent estimator $\widehat{\bar{\pi}} \equiv \min \{\underline{\widehat{u b}}, 1\}$ of $\bar{\pi}$.

To conduct inference on $\pi$, we seek a critical value $r$ that obeys:

$$
\begin{equation*}
P(\underline{u b} \leq \underline{\widehat{u b}}+r)=0.95, \tag{91}
\end{equation*}
$$

as such an $r$ implies:

$$
\begin{aligned}
P(\pi \leq \min \{\underline{\widehat{u b}}+r, 1\}) \geq & P(\bar{\pi} \leq \min \{\underline{\widehat{u b}}+r, 1\}) \\
= & P(\bar{\pi} \leq \min \{\underline{\widehat{u b}}+r, 1\} \mid \underline{u b} \leq \underline{\widehat{u b}}+r) 0.95 \\
& +P(\bar{\pi} \leq \min \{\underline{\widehat{u b}}+r, 1\} \mid \underline{u b}>\underline{\widehat{u b}}+r) 0.05 \\
\geq & P(\bar{\pi} \leq \min \{\underline{\widehat{u b}}+r, 1\} \mid \underline{u b} \leq \underline{\widehat{u b}}+r) 0.95 \\
= & 0.95
\end{aligned}
$$

with the first inequality binding when $\pi=\bar{\pi}$. The last line follows because $\underline{u b} \leq \underline{\widehat{u b}}+r$ implies $\min \{\underline{u b}+r, 1\} \leq \min \{\underline{\widehat{u b}}+r, 1\}$.

We can rewrite (91) as:

$$
P\left(-\min \left\{\widehat{u b}_{1}-\underline{u b}, \widehat{u b}_{2}-\underline{u b}, \ldots, \widehat{u b}_{K}-\underline{u b}\right\} \leq r\right)=0.95,
$$

or equivalently

$$
P\left(\max \left\{\underline{u b}-\widehat{u b}_{1}, \underline{u b}-\widehat{u b}_{2}, \ldots, \underline{u b}-\widehat{u b}_{K}\right\} \leq r\right)=0.95 .
$$

It is well known that the limiting distribution of $\max \left\{\underline{u b}-\widehat{u b}_{1}, \underline{u b}-\widehat{u b}_{2}, \ldots, \underline{u b}-\widehat{u b}_{K}\right\}$ depends on which and how many of the upper bound constraints bind. Several approaches to this problem have been proposed which involve conducting pre-tests for which constraints are binding (e.g. Andrews and Barwick, 2012).

We take an alternative approach to inference that is simple to implement and consistent regardless of the constraints that bind. Our approach is predicated on the observation that:

$$
\begin{equation*}
P\left(\max \left\{u b_{1}-\widehat{u b}_{1}, \ldots, u b_{K}-\widehat{u b}_{K}\right\} \leq r\right) \leq P\left(\max \left\{\underline{u b}-\widehat{u b}_{1}, \ldots, \underline{u b}-\widehat{u b}_{K}\right\} \leq r\right), \tag{92}
\end{equation*}
$$

with equality holding in the case where all of the upper bound solutions are identical. We seek an $r^{\prime}$ such that:

$$
\begin{equation*}
P\left(\max \left\{u b_{1}-\widehat{u b}_{1}, \ldots, u b_{K}-\widehat{u b}_{K}\right\} \leq r^{\prime}\right)=.95 \tag{93}
\end{equation*}
$$

From (92),

$$
P\left(\max \left\{\underline{u b}-\widehat{u b}_{1}, \ldots, \underline{u b}-\widehat{u b}_{K}\right\} \leq r^{\prime}\right) \geq .95
$$

with equality holding when all bounds are identical.
A bootstrap estimate $r^{*} \xrightarrow{p} r^{\prime}$ of the necessary critical value can be had by considering the bootstrap analog of condition (93) (see Proposition 10.7 of Kosorok, 2008). That is, by computing the 95 th percentile of:

$$
\max \left\{\widehat{u b}_{1}-\widehat{u b}_{1}^{*}, \ldots, \widehat{u b}_{K}-\widehat{u b}_{K}^{*}\right\}
$$

across bootstrap replications, where stars refer to bootstrap quantities. An upper limit $U$ of the confidence region for $\pi$ can then be formed as:

$$
U=\min \left\{\underline{\widehat{u b}}+r^{*}, 1\right\} .
$$

Note that this procedure is essentially an unstudentized version of the inference method of Chernozhukov et al. (2013) where the set of relevant upper bounds ( $\mathcal{V}_{0}$ in their notation) is taken here to be the set of all upper bounds, thus yielding conservative inference.

We turn now to the lower limit of our confidence interval. Our greatest lower bounds are all of the form:

$$
\pi \geq \underline{\pi} \equiv \max \{l b, 0\} .
$$

We have the plugin lower bound estimator $\widehat{l b} \xrightarrow{p} l b$. By the same arguments as above we want to search for an $r^{\prime \prime}$ such that

$$
P\left(l b \geq \widehat{l b}-r^{\prime \prime}\right)=0.95 .
$$

Since $\widehat{l b}$ is just a scalar sample mean, we can choose $r^{\prime \prime}=1.65 \sigma_{l b}$ where $\sigma_{l b}$ is the asymptotic standard error of $\widehat{l b}$ in order to guarantee the above condition holds asymptotically. To account for the propensity score re-weighting, we use a bootstrap standard error estimator $\widehat{\sigma}_{l b}$ of $\sigma_{l b}$ which is consistent via the usual arguments. Thus, our "conservative" $95 \%$ confidence interval for $\pi$ is:

$$
\left[\max \left\{0, \widehat{l b}-1.65 \widehat{\sigma}_{l b}\right\}, \min \left\{\underline{\widehat{u b}}+r^{*}, 1\right\}\right] .
$$

This confidence interval covers the parameter $\pi$ with asymptotic probability of at least $95 \%$.

## 8 Relaxation of Lower Bound on the Stigma Disutility

## The Issue

In the paper we restrict a woman's preferences when $F P L_{i}<\bar{E}_{i}$. Specifically, A. $\mathbf{6}$ in Assumption 1 states that for all offers $(W, H)$ such that $E \equiv W H \in\left(F P L_{i}, \bar{E}_{i}\right]$ :

$$
U_{i}^{a}\left(H, C^{a}(E, D, E), D, 0\right)<U_{i}^{a}\left(H, C_{i}^{a}(E, 0, E), 0,0\right) .
$$

A. 6 in Assumption 1 implicitly establishes a lower bound on the stigma disutility and it guarantees that woman $i$ does not report earnings above $F P L_{i}$ while on welfare under regime $a$ when $F P L_{i}<\bar{E}_{i}$. That is, state $2 r$ is dominated under regime $a$ subject to A. 6 in Assumption 1 and Assumption 3. Without A. 6 in Assumption 1, participation in welfare may decrease or increase utility (other things equal). The number of observations in our control sample corresponding to alternatives compatible with state $2 r$ is tiny. Nevertheless, it is of pedagogical interest to consider what additional responses emerge if we do not rule out such choices a priori, that is, when we do not impose A. 6 in Assumption 1.

## A Roadmap of the Results: Table A4 and Figure A1

Table A4 catalogs the allowed and disallowed responses when A. 6 in Assumption 1 is not imposed. The possible states are $\mathcal{S} \cup\{2 r\}$. Accordingly, all but the last row and last column of Table A4 appear also in Table 4. The last row of Table A4 corresponds to the responses of a woman who under regime $a$ has earnings in the range ( $\left.F P L_{i}, \bar{E}_{i}\right]$, is on assistance, and truthfully reports her earnings to the welfare agency (state $2 r$ ).

The presentation of the results is organized as follows. Proposition 3 pertains to the disallowed pairings of states in Table A4. Corollary 4 derives additional restricted pairings when the utility function is of the special form given in (2). Proposition 4 pertains to the allowed pairings of states in Table A4. Interestingly, dispensing with A. 6 in Assumption 1 enables the emergence of flows out of the labor force, which were absent in the model of Section 4 of the paper. Corollary 5 shows that labor market constraints on hours are essential to the emergence of these flows. Figure A1 illustrates this point. To ease the graphical representation, we use the special form of the utility function in (2). Figure A1 portrays a woman who receives two job offers entailing earnings $\left(E^{1}, E^{2}\right)$ that are both in range 2 and obey $\left(E^{1}, E^{2}\right) \in\left(F P L_{i}, \bar{E}_{i}\right]$. Her welfare stigma is zero. For convenience, her fixed cost of work is also zero and her cost of under-reporting is sufficiently large that under-reporting earnings to the welfare agency is always a dominated choice. Under AFDC, the woman earns $E^{1}$, is on assistance, and truthfully reports her earnings. Observe that she would make the same choice even if earning constraints were absent. Under JF, the woman does not work and is off assistance. However, if earning constraints were absent she would be better off by earning below the FPL on assistance and truthfully reporting her earnings.

## Propositions

With reference to Section 4 in this Appendix, all Lemmas and Corollaries hold but for Lemma 1 which hinges on A. 6 in Assumption 1. Proposition 1, Corollary 2, and Proposition 2 in Section 4 are superseded by the following propositions and corollary.

Proposition 3 (Restricted Pairings). Given Assumption 1 but for A.6, and Assumptions 2-5, the pairings of states corresponding to the "-" entries in Table $A_{4}$ cannot occur and the pairings of states $(1 r, 1 r)$ and $(1 u, 1 r)$ must occur.

Proof. We proved the entries in the first 7 rows and 7 columns of Table A4 in Propositions 1 and Proposition 2. State $2 r$ is not defined under regime $j$ (Assumption 3) which proves the "-" entries in Table A4 rows 1 through 7 and column 8. We are thus left to prove the disallowed pairings in row 8 and columns 1 through 7 of Table A4. No woman pairs state $2 r$ under regime $a$ with state $1 u$ under regime $j$ because $1 u$ is dominated by state $1 r$ under $j$ (Lemma 2).

Corollary 4 (Additional Restricted Pairings under Utility Specification (2)). Given Assumption 1 but for A.6, and Assumptions 2-5, and subject to specification (2) of the utility function, the pairings of states $(0 r, 1 n)$ and $(2 r, 1 n)$ are disallowed.

Proof. To enhance readability we employ the symbol $\left[s \succsim^{t} s^{\prime}\right]$ to signify that under regime $t$ an alternative compatible with state $s$ is weakly preferred to an alternative compatible with state $s^{\prime}$. The proof that the pairing of states $(0 r, 1 n)$ is disallowed is contained in Corollary 2. The proof that the pairings of state $(2 r, 1 n)$ is disallowed is by contradiction. Suppose there is a woman $i$ who selects an alternative compatible with state $2 r$ under regime $a$ entailing earnings $E^{k} \equiv W^{k} H^{k}$ and selects an alternative compatible with state $1 n$ under regime $j$ entailing earnings $E^{l} \equiv W^{l} H^{l}$. By Assumption 4, her choice under regime $a$ reveals that

$$
\left[2 r \succsim^{a} 2 n\right]: U_{i}\left(H^{k}, E^{k}-\mu_{i}+G_{i}^{a}\left(E^{k}\right)-\phi_{i}\right) \geq U_{i}\left(H^{k}, E^{k}-\mu_{i}\right),
$$

which implies $G_{i}^{a}\left(E^{k}\right) \geq \phi_{i}$. Her choice under regime $j$ reveals that

$$
\left[1 n \succsim^{j} 1 r\right]: U_{i}\left(H^{l}, E^{l}-\mu_{i}\right) \geq U_{i}\left(H^{l}, E^{l}-\mu_{i}+\bar{G}_{i}-\phi_{i}\right),
$$

which implies $\bar{G}_{i} \leq \phi_{i}$. Thus, optimality implies $\bar{G}_{i} \leq \phi_{i} \leq G_{i}^{a}\left(E^{k}\right)$ which yields a contradiction because $G_{i}^{a}(E)<\bar{G}_{i}$ for all $E \in\left(F P L_{i}, \bar{E}_{i}\right]$ including $E^{k}$.

Proposition 4 (Unrestricted Pairings). Given Assumption 1 but for A.6, and Assumptions 2-5, the non "-" entries in Table A4 correspond to pairings of states that are allowed.

Proof. The entries in the first 7 rows and 7 columns Table 4A were proven in Propositions 1 and Proposition 2. We are left to prove the allowed pairings in row 8 and columns 1 through 7 of Table A4. To prove that the pairs in collection

$$
\begin{equation*}
\left\{\left(2 r, s^{j}\right) \mid s^{j} \in\{0 n, 1 n, 2 n, 0 r, 1 r, 2 u\}\right\} \tag{94}
\end{equation*}
$$

are allowed it suffices to provide examples where six women occupy the same state $s^{a}=2 r$ under regime $a$ but occupy state $s^{j} \in\{0 n, 1 n, 2 n, 0 r, 1 r, 2 u\}$ under regime $j$. This also proves that no pairing in collection (94) is constrained to occur. When providing these examples we consider the specification of the utility function given in (3). Finally, we assume that woman $i$ receives either one or two job offers, that is, either $K_{i}=1$ or $K_{i}=2$. To enhance readability we employ the symbol $\left[s \succsim^{t} s^{\prime}\right]$ to signify that under regime $t$ an alternative compatible with state $s$ is weakly preferred to an alternative compatible with state $s^{\prime}$.

1. Pairings $(2 r, 0 n),(2 r, 0 r)$, and $(2 r, 2 u)$ are allowed.

Consider three women $i^{\prime}, i^{\prime \prime}$, and $i^{\prime \prime \prime}$ with preferences represented by (3) with $v(x)=x$. Let $K_{i}=1$ for $i \in\left\{i^{\prime}, i^{\prime \prime}, i^{\prime \prime \prime}\right\}$. Assume that all three women's job offer entails earnings in $\in\left(F P L_{i}, \bar{E}_{i}\right]$. That is, $E_{i}^{k} \equiv W_{i}^{k} H_{i}^{k} \in\left(F P L_{i}, \bar{E}_{i}\right]$ for $i \in\left\{i^{\prime}, i^{\prime \prime}, i^{\prime \prime \prime}\right\}$. Let
(a) woman $i=i^{\prime}$ be such that $\alpha_{i}=W_{i}^{k}, \mu_{i}=0$, and

$$
\eta_{i}^{a} \geq \kappa_{i} \geq \bar{G}_{i}-\phi_{i} \geq \bar{G}_{i}-G_{i}^{a}\left(E_{i}^{k}\right)
$$

(b) woman $i=i^{\prime \prime}$ be such that $\alpha_{i}=W_{i}^{k}, \mu_{i}=0$, and

$$
\kappa_{i} \geq \bar{G}_{i}-\phi_{i} \geq \eta_{i}^{j} \geq \eta_{i}^{a} \geq \bar{G}_{i}-G_{i}^{a}\left(E_{i}^{k}\right)
$$

(c) woman $i=i^{\prime \prime \prime}$ be such that $\alpha_{i}=W_{i}^{k}, \mu_{i}=0$, and

$$
\bar{G}_{i}-G_{i}^{a}\left(E_{i}^{k}\right) \leq \eta_{i}^{a} \leq \kappa_{i} \leq \min \left\{\eta_{i}^{j}, \bar{G}_{i}-\phi_{i}\right\} .
$$

All women choose to earn and truthfully report earnings in $\left(F P L_{i}, \bar{E}_{i}\right]$ on assistance under regime $a$. We now show that woman $i^{\prime}$ chooses an alternative compatible with state $0 n$ under regime $j$, woman $i^{\prime \prime}$ chooses an alternative compatible with state $0 r$ under regime $j$, and woman $i^{\prime \prime \prime}$ chooses an alternative compatible with state $2 u$ under regime $j$. For all women, the choice of the alternative compatible with state $2 r$ under regime $a$ reveals (Assumption 4) that this alternative yields as much utility as the available alternatives compatible with states $\{0 r, 0 n, 2 n, 2 u\}$. Thus, for $i \in\left\{i^{\prime}, i^{\prime \prime}, i^{\prime \prime \prime}\right\}$ :

$$
\begin{align*}
& {\left[2 r \succsim^{a} 0 r\right] }:  \tag{95}\\
& {\left[2 r \succsim_{i}^{k} 0 n\right] }:  \tag{96}\\
& {\left[G_{i}^{a}\left(E_{i}^{k}\right)-\phi_{i}-\alpha_{i} H_{i}^{k} \geq \bar{G}_{i}-\phi_{i}-\eta_{i}^{a},\right.}  \tag{97}\\
& {\left[2 r \succsim^{a} 2 n\right] }:  \tag{98}\\
& {\left[\phi_{i}^{k}+G_{i}^{a}\left(E_{i}^{k}\right)-H_{i}^{k} \geq 0,\right.} \\
& {\left[2 r \succsim_{i}\right.}2 u]
\end{align*}: \quad E_{i}^{k}+G_{i}^{a}\left(E_{i}^{k}\right)-\phi_{i}-\alpha_{i} H_{i}^{k} \geq E_{i}^{k}-\alpha_{i} H_{i}^{k}, \bar{G}_{i}-\phi_{i}-\kappa_{i}-\alpha_{i} H_{i}^{k} .
$$

It is easy to verify that descriptions (1a), (1b), and (1c) are compatible with optimality under regime $a$ for woman $i^{\prime}, i^{\prime \prime}$, and $i^{\prime \prime \prime}$ respectively, that is, with (95)-(98). No woman selects an alternative compatible with state $2 r$ under regime $j$ because it is not defined.
Woman $i^{\prime}$ prefers not working off assistance (state $0 n$ ) to the available alternatives compatible with states $\{0 r, 2 n, 2 u\}$ under regime $j$ because, by description (1a), we have $\eta_{i}^{j} \geq \bar{G}_{i}-\phi_{i}$ which implies (99); $\alpha_{i}=W_{i}^{k}$ which implies (100); and $\kappa_{i} \geq \bar{G}_{i}-\phi_{i}$ and $\alpha_{i}=W_{i}^{k}$ which imply (101):

$$
\begin{align*}
& {\left[0 n \succsim^{j} 0 r\right]: 0 \geq \bar{G}_{i}-\phi_{i}-\eta_{i}^{j} \text {, }}  \tag{99}\\
& \text { [ } \left.0 n \succsim^{j} 2 n\right]: 0 \geq E_{i}^{k}-\alpha_{i} H_{i}^{k} \text {, }  \tag{100}\\
& {\left[0 n \succsim^{j} 2 u\right]: 0 \geq E_{i}^{k}+\bar{G}_{i}-\phi_{i}-\kappa_{i}-\alpha_{i} H_{i}^{k} .} \tag{101}
\end{align*}
$$

Woman $i^{\prime \prime}$ prefers not working on assistance (state $0 r$ ) to the available alternatives compatible with states $\{0 n, 2 n, 2 u\}$ under regime $j$ because, by description (1b), we have $\bar{G}_{i}-\phi_{i} \geq \eta_{i}^{j}$ which implies (102); $\bar{G}_{i}-\phi_{i} \geq \eta_{i}^{j}$ and $\alpha_{i}=W_{i}^{k}$ which imply (103); and $\kappa_{i} \geq \eta_{i}^{j}$ which implies (104):

$$
\begin{array}{ll}
{\left[0 r \succsim^{j} 0 n\right]} & : \\
{\left[0 r \succsim^{j} 2 n\right]} & : \bar{G}_{i}-\phi_{i}-\eta_{i}^{j} \geq 0, \\
{\left[0 r \succsim^{j} 2 u\right]} & : \bar{G}_{i}-\phi_{i}-\eta_{i}^{j} \geq E_{i}^{j} \geq E_{i}^{k}+\alpha_{i} H_{i}^{k},  \tag{104}\\
& -\phi_{i}-\kappa_{i}-\alpha_{i} H_{i}^{k} .
\end{array}
$$

Woman $i^{\prime \prime \prime}$ prefers earning $E_{i}^{k}$ on assistance and under-report (state $2 u$ ) to the available alternatives compatible with states $\{0 n, 0 r, 2 n\}$ under regime $j$ because, by description (1c),
we have $\bar{G}_{i}-\phi_{i} \geq \kappa_{i}$ and $\alpha_{i}=W_{i}^{k}$ which imply (105); $\eta_{i}^{j} \geq \kappa_{i}$ and $\alpha_{i}=W_{i}^{k}$ which imply (106); and $\bar{G}_{i}-\phi_{i} \geq \kappa_{i}$ and $\alpha_{i}=W_{i}^{k}$ which imply (107):

$$
\begin{array}{ll}
{\left[2 u \succsim^{j} 0 n\right]:} & E_{i}^{k}+\bar{G}_{i}-\phi_{i}-\kappa_{i}-\alpha_{i} H_{i}^{k} \geq 0, \\
{\left[2 u \succsim^{j} 0 r\right]:} & E_{i}^{k}+\bar{G}_{i}-\phi_{i}-\kappa_{i}-\alpha_{i} H_{i}^{k} \geq \bar{G}_{i}-\phi_{i}-\eta_{i}^{j}, \\
{\left[2 u \succsim^{j} 2 n\right]:} & E_{i}^{k}+\bar{G}_{i}-\phi_{i}-\kappa_{i}-\alpha_{i} H_{i}^{k} \geq E_{i}^{k}-\alpha_{i} H_{i}^{k} . \tag{107}
\end{array}
$$

## 2. Pairing $(2 r, 1 r)$ is allowed.

Consider woman $i$ with preferences represented by (3) with $v(x)=x$. Let $K_{i}=2$. Assume that her first job offer entails earnings in $\left(F P L_{i}, \bar{E}_{i}\right]$ and her second job offer entails earnings in range 1. That is, $E_{i}^{k} \equiv W_{i}^{k} H_{i}^{k} \in\left(F P L_{i}, \bar{E}_{i}\right]$ and $E_{i}^{l} \equiv W_{i}^{l} H_{i}^{l} \in\left(0, F P L_{i}\right]$. Let
(a) woman $i$ be such that $W_{i}^{k}>\alpha_{i}=W_{i}^{l}, \mu_{i}=0$, and

$$
\max \left\{\begin{array}{c}
\bar{G}_{i}-G_{i}^{a}\left(E_{i}^{k}\right)-\eta_{i}^{a}, \\
G_{i}^{a}\left(E_{i}^{l}\right)-G_{i}^{a}\left(E_{i}^{k}\right)
\end{array}\right\} \leq H_{i}^{k}\left(W_{i}^{k}-\alpha_{i}\right) \leq \bar{G}_{i}-G_{i}^{a}\left(E_{i}^{k}\right) \leq \min \left\{\bar{G}_{i}-\phi_{i}, \kappa_{i}\right\} .
$$

Woman $i$ chooses to earn and truthfully report earnings in $\left(F P L_{i}, \bar{E}_{i}\right]$ on assistance under regime $a$. We now show that she chooses an alternative compatible with state $1 n$ under regime $j$. The choice of the alternative compatible with state $2 r$ under regime $a$ reveals (Assumption 4) that this alternative yields as much utility as the available alternatives compatible with states $\{0 r, 0 n, 1 n, 1 r, 1 u, 2 n, 2 u\}$. Thus:

$$
\begin{array}{ll}
{\left[2 r \succsim^{a} 0 r\right]} & : \\
E_{i}^{k}+G_{i}^{a}\left(E_{i}^{k}\right)-\phi_{i}-\alpha_{i} H_{i}^{k} \geq \bar{G}_{i}-\phi_{i}-\eta_{i}^{a}, \\
{\left[2 r \succsim^{a} 0 n\right]} & : \\
E_{i}^{k}+G_{i}^{a}\left(E_{i}^{k}\right)-\phi_{i}-\alpha_{i} H_{i}^{k} \geq 0, \\
{\left[2 r \succsim^{a} 1 n\right]} & : \\
{\left[2 r \succsim_{i}^{k}+G_{i}^{a}\left(E_{i}^{k}\right)-\phi_{i}-\alpha_{i} H_{i}^{k} \geq E_{i}^{l}-\alpha_{i} H_{i}^{l},\right.} \\
{[2 r]} & :  \tag{114}\\
{\left[2 r \succsim_{i}^{k}+G_{i}^{a}\left(E_{i}^{k}\right)-\phi_{i}-\alpha_{i} H_{i}^{k} \geq E_{i}^{l}+G_{i}^{a}\left(E_{i}^{l}\right)-\phi_{i}-\alpha_{i} H_{i}^{l},\right.} \\
{\left[2 r \succsim^{a} 2 n\right]} & : E_{i}^{k}+G_{i}^{a}\left(E_{i}^{k}\right)-\phi_{i}-\alpha_{i} H_{i}^{k} \geq E_{i}^{a}+\bar{G}_{i}-\phi_{i}-\kappa_{i}-\alpha_{i} H_{i}^{l}, \\
{\left[2 r \succsim^{a}\right.} & \phi_{i}-\alpha_{i} H_{i}^{k} \geq E_{i}^{k}-\alpha_{i}-H_{i}^{k}, \\
{[ } & E_{i}^{k}+G_{i}^{a}\left(E_{i}^{k}\right)-\phi_{i}-\alpha_{i} H_{i}^{k} \geq E_{i}^{k}+\bar{G}_{i}-\phi_{i}-\kappa_{i}-\alpha_{i} H_{i}^{k} .
\end{array}
$$

It is easy to verify that description (2a) is compatible with optimality under regime $a$ for woman $i$, that is, with (108)-(114). Woman $i$ does not select an alternative compatible with state $2 r$ under regime $j$ because it is not defined; she does not selects an alternative compatible with state $1 u$ under regime $j$ because it is dominated. Woman $i$ prefers earning and truthfully report $E_{i}^{l}$ on assistance (state $1 r$ ) to the available alternatives compatible with states $\{0 n, 0 r, 1 n, 2 n, 2 u\}$ under regime $j$ because, by description (2a), we have $\bar{G}_{i}-\phi_{i} \geq 0$ and $W_{i}^{l}=\alpha_{i}$ which imply (115); $\eta_{i}^{j} \geq 0$ and $W_{i}^{l}=\alpha_{i}$ which imply (116); $\bar{G}_{i}-\phi_{i} \geq 0$ which implies (117); $\bar{G}_{i}-\phi_{i} \geq H_{i}^{k}\left(W_{i}^{k}-\alpha_{i}\right)$ and $W_{i}^{l}=\alpha_{i}$ which imply (118); and $\kappa_{i} \geq H_{i}^{k}\left(W_{i}^{k}-\alpha_{i}\right)$ and $W_{i}^{l}=\alpha_{i}$ which imply (119):

$$
\begin{align*}
& {\left[1 r \succsim^{j} 0 n\right] }:  \tag{115}\\
& E_{i}^{l}+\bar{G}_{i}-\phi_{i}-\alpha_{i} H_{i}^{l} \geq 0,  \tag{116}\\
& {\left[1 r \succsim^{j} 0 r\right] }:  \tag{117}\\
& E_{i}^{l}+\bar{G}_{i}-\phi_{i}-\alpha_{i} H_{i}^{l} \geq \bar{G}_{i}-\phi_{i}-\eta_{i}^{j},  \tag{118}\\
& {\left[1 r \succsim^{j} 1 n\right]: } E_{i}^{l}+\bar{G}_{i}-\phi_{i}-\alpha_{i} H_{i}^{l} \geq E_{i}^{l}-\alpha_{i} H_{i}^{l},  \tag{119}\\
& {\left[1 r \succsim^{j} 2 n\right]: } E_{i}^{l}+\bar{G}_{i}-\phi_{i}-\alpha_{i} H_{i}^{l} \geq E_{i}^{k}-\alpha_{i} H_{i}^{k}, \\
& {\left[1 r \succsim^{j} 2 u\right]: } E_{i}^{l}+\bar{G}_{i}-\phi_{i}-\alpha_{i} H_{i}^{l} \geq E_{i}^{k}+\bar{G}_{i}-\phi_{i}-\kappa_{i}-\alpha_{i} H_{i}^{k} .
\end{align*}
$$

## 3. Pairing $(2 r, 1 n)$ is allowed.

Consider woman $i$ with preferences represented by (3) with $v(x)$ convex. Let $K_{i}=2$. Assume that her first job offer entails earnings in $\left(F P L_{i}, \bar{E}_{i}\right]$ and her second job offer entails earnings in range 1. That is, $E_{i}^{k} \equiv W_{i}^{k} H_{i}^{k} \in\left(F P L_{i}, \bar{E}_{i}\right]$ and $E_{i}^{l} \equiv W_{i}^{l} H_{i}^{l} \in\left(0, F P L_{i}\right]$. Let
(a) woman $i$ be such that $\alpha_{i}>0, \mu_{i}=0$ and $^{3}$

$$
\left.\begin{array}{rl}
\max \left\{\begin{array}{c}
v\left(E_{i}^{l}+\bar{G}_{i}\right)-v\left(E_{i}^{l}\right), \\
\left.\left[\begin{array}{c}
v\left(E_{i}^{k}+\bar{G}_{i}-\kappa_{i}\right)-v\left(E_{i}^{l}\right) \\
-\alpha_{i}\left(H_{i}^{k}-H_{i}^{l}\right)
\end{array}\right]\right\}
\end{array} \leq \phi_{i} \leq \min \left\{\begin{array}{c}
v\left(E_{i}^{k}+G_{i}^{a}\left(E_{i}^{k}\right)\right)-v\left(E_{i}^{k}\right) \\
{\left[\begin{array}{c}
v\left(E_{i}^{k}+G_{i}^{a}\left(E_{i}^{k}\right)\right)-v\left(E_{i}^{l}\right) \\
-\alpha_{i}\left(H_{i}^{k}-H_{i}^{l}\right)
\end{array}\right]}
\end{array}\right\},\right. \\
v\left(E_{i}^{k}\right)-v\left(E_{i}^{l}\right) & \leq \alpha_{i}\left(H_{i}^{k}-H_{i}^{l}\right) \leq \min \left\{\begin{array}{c}
v\left(E_{i}^{k}+G_{i}^{a}\left(E_{i}^{k}\right)\right) \\
-v\left(E_{i}^{l}+G_{i}^{a}\left(E_{i}^{l}\right)\right)
\end{array}\right], \\
{\left[\begin{array}{c}
v\left(E_{i}^{k}+G_{i}^{a}\left(E_{i}^{k}\right)\right) \\
-v\left(E_{i}^{l}+\bar{G}_{i}-\kappa_{i}\right)
\end{array}\right]}
\end{array}\right\},
$$

Woman $i$ chooses to earn and truthfully report earnings in $\left(F P L_{i}, \bar{E}_{i}\right]$ on assistance under regime $a$. We now show that she chooses an alternative compatible with state $1 n$ under regime $j$. The choice of the alternative compatible with state $2 r$ under regime $a$ reveals (Assumption 4) that this alternative yields as much utility as the available alternatives compatible with states $\{0 r, 0 n, 1 n, 1 r, 1 u, 2 n, 2 u\}$. Thus:

$$
\begin{align*}
& {\left[2 r \succsim^{a} 0 r\right]:}  \tag{120}\\
& {\left[2 r \succsim^{a} 0 n\right]: v\left(E_{i}^{k}+G_{i}^{a}\left(E_{i}^{k}\right)\right)-\phi_{i}-\alpha_{i} H_{i}^{k} \geq v\left(E_{i}^{k}+G_{i}^{a}\left(E_{i}^{k}\right)\right)-\phi_{i}-\eta_{i} H_{i}^{k},}  \tag{121}\\
& {\left[2 r \succsim^{a} 1 n\right]: v(0),}  \tag{122}\\
& {\left[2 r \succsim^{a} 1 r\right]:}  \tag{123}\\
& {\left[2 r\left(E_{i}^{k}+G_{i}^{a}\left(E_{i}^{k}\right)\right)-\phi_{i}-\alpha_{i} H_{i}^{k} \geq v\left(E_{i}^{k}+G_{i}^{a}\left(E_{i}^{k}\right)\right)-\phi_{i} H_{i}^{l},\right.}  \tag{124}\\
& {\left[2 r \succsim_{i} H_{i}^{k} \geq v\left(E_{i}^{l}+G_{i}^{a}\left(E_{i}^{l}\right)\right)-\phi_{i}-\alpha_{i} H_{i}^{l},\right.}  \tag{125}\\
& {\left[2 r \succsim^{a} 2 n\right]:}  \tag{126}\\
& \left.\left[2 r \succsim_{i}^{k}+G_{i}^{a}\left(E_{i}^{k}\right)\right)-\phi_{i}-\alpha_{i} H_{i}^{k} \geq v\left(E_{i}^{k}+E_{i}^{a}\left(E_{i}^{k}\right)\right)-\phi_{i}-\alpha_{i}\right)-\alpha_{i} H_{i}^{k} \geq v\left(E_{i}^{k}\right)-\alpha_{i} H_{i}^{l}, \\
& {\left[2 H_{i}^{k},\right.} \\
& {\left[E_{i}^{k}+G_{i}^{a}\left(E_{i}^{k}\right)\right)-\phi_{i}-\alpha_{i} H_{i}^{k} \geq v\left(E_{i}^{k}+\bar{G}_{i}-\kappa_{i}\right)-\phi_{i}-\alpha_{i} H_{i}^{k} .}
\end{align*}
$$

It is easy to verify that description (3a) is compatible with optimality under regime $a$ for woman $i$, that is, with (120)-(126). Woman $i$ does not selects an alternative compatible with state $2 r$ under regime $j$ because it is not defined; she does not selects an alternative compatible with state $1 u$ under regime $j$ because it is dominated. Woman $i$ prefers earning $E_{i}^{l}$ off assistance (state $1 n$ ) to the available alternatives compatible with states $\{0 n, 0 r, 1 r, 2 n, 2 u\}$ under regime $j$ because, by description (3a), we have $\alpha_{i} H_{i}^{l} \leq v\left(E_{i}^{l}\right)-v(0)$ which implies (127); $\alpha_{i} H_{i}^{l} \leq v\left(E_{i}^{l}\right)-v(0)$ which by convexity, and since $\eta_{i}^{j} \geq 0$, implies $\alpha_{i} H_{i}^{l} \leq v\left(E_{i}^{l}+\bar{G}_{i}\right)-$ $v\left(\bar{G}_{i}\right)+\eta_{i}^{j}$ which along with $\phi_{i} \geq v\left(E_{i}^{l}+\bar{G}_{i}\right)-v\left(E_{i}^{l}\right)$ imply (128); $\phi_{i} \geq v\left(E_{i}^{l}+\bar{G}_{i}\right)-$ $v\left(E_{i}^{l}\right)$ which implies (129); $v\left(E_{i}^{k}\right)-v\left(E_{i}^{l}\right) \leq \alpha_{i}\left(H_{i}^{k}-H_{i}^{l}\right)$ which implies (130); and $\phi_{i} \geq$

[^25]$$
v\left(E_{i}^{k}+\bar{G}_{i}-\kappa_{i}\right)-v\left(E_{i}^{l}\right)-\alpha_{i}\left(H_{i}^{k}-H_{i}^{l}\right) \text { which implies (131): }
$$
\[

$$
\begin{align*}
& {\left[1 n \succsim^{j} 0 n\right]: v\left(E_{i}^{l}\right)-\alpha_{i} H_{i}^{l} \geq v(0),}  \tag{127}\\
& {\left[1 n \succsim^{j} 0 r\right]: v\left(E_{i}^{l}-\alpha_{i} H_{i}^{l} \geq v\left(\bar{G}_{i}\right)-\phi_{i}-\eta_{i}^{j},\right.}  \tag{128}\\
& {\left[1 n \succsim^{j} 1 r\right]:}  \tag{129}\\
& {\left[1 n \succsim^{j} 2 n\right]:}  \tag{130}\\
& {\left[1 n \succsim^{j} 2 u\right]-\alpha_{i} H_{i}^{l} \geq v\left(E_{i}^{l}+\bar{G}_{i}\right)-\phi_{i}-\alpha_{i} H_{i}^{l},}  \tag{131}\\
& :
\end{align*}
$$, \alpha_{i} H_{i}^{l} \geq v\left(E_{i}^{l}\right)-\alpha_{i} H_{i}^{l} \geq v\left(E_{i}^{k}+\bar{G}_{i}-H_{i}^{k}, \phi_{i}\right)-\phi_{i} H_{i}^{k} .
\]

Corollary 5 (Additional Restricted Pairings in the absence of Labor Market Constraints). Suppose that there are no hours constraints, that is, let $\Theta_{i}=\left\{\left(W_{i}(H), H\right) \mid H \in\left(0, \bar{H}_{i}\right]\right\}$ in Assumption 4 and suppose that wages are continuous and weakly increasing in hours worked and utility is a weakly decreasing function of hours worked. Then, given Assumption 1 but for A.6, and Assumptions 2-5, no woman pairs state $2 r$ under regime a with states $\{0 n, 0 r\}$ under regime $j$.

Proof. We show that no woman pairs state $2 r$ under regime $a$ with state $0 n$ under regime $j$; the proof that no woman pairs state $2 r$ under regime a with state $0 r$ under regime $j$ is similar. The proof is by contradiction. Suppose that there is a woman $i$ who selects an alternative compatible with state $2 r$ under regime $a$, entailing earnings $E^{k} \equiv W\left(H^{k}\right) H^{k} \in\left(F P L_{i}, \bar{E}_{i}\right]$ and selects an alternative compatible with state $0 n$ under $j$. By Assumption 4, her choice under regime $a$ reveals that

$$
\begin{equation*}
\left[2 r \succsim^{a} 0 n\right]: U_{i}\left(H^{k}, E^{k}+G_{i}^{a}\left(E^{k}\right), 1,0\right) \geq U_{i}(0,0,0,0) \tag{132}
\end{equation*}
$$

Because there are no hour constraints and because program rules are such that $\bar{E}_{i}<F P L_{i}+$ $\bar{G}_{i}$, there exists a job offer $\left(W\left(H^{l}\right), H^{l}\right)$ such that $E^{l} \equiv W\left(H^{l}\right) H^{l}$ is in range 1 and $E^{l}+$ $\bar{G}_{i}=E^{k}+G_{i}^{a}\left(E^{k}\right)$. Hence, $H^{k} \geq H^{l}$ because wages are weakly increasing in hours. Thus, $U_{i}\left(H^{l}, E^{l}+\bar{G}_{i}, 1,0\right) \geq U_{i}\left(H^{k}, E^{k}+G_{i}^{a}\left(E^{k}\right), 1,0\right)$ because utility is weakly decreasing in hours worked for given $(C, D, R)$ by Assumption 1. Together with (132), this means that

$$
\begin{equation*}
U_{i}\left(H^{l}, E^{l}+\bar{G}_{i}, 1,0\right) \geq U_{i}(0,0,0,0) \tag{133}
\end{equation*}
$$

If inequality (133) holds strictly, a contradiction ensures because this shows that no alternative compatible with state $0 n$ can be optimal under regime $j$ (it is dominated by an alternative compatible with state $1 r$ ). If inequality (133) holds as an equality, woman $i$ is indifferent between earning (and truthfully reporting) $E^{k}$ and not working off assistance under regime $a$. By Assumption 5, if the woman resolved an indifference situation against not working off assistance under regime $a$, she will also resolve an indifference situation against not working off assistance under regime $j$. This contradicts her selecting not to work off assistance over earning (and truthfully reporting) $E^{l}$ on assistance under $j$.
Proposition 5. Define $\pi_{s^{a}, s^{j}} \equiv P\left(S_{i}^{j}=s^{j} \mid S_{i}^{a}=s^{a}\right)$. Given Assumption 1 but for A. $\boldsymbol{6}$, and Assumptions 2-5, the system of equations describing the impact of the JF reform on observable state probabilities is:

$$
\begin{align*}
& p_{0 n}^{j}-p_{0 n}^{a}=-\pi_{0 n, 1 r} p_{0 n}^{a}+\pi_{0 r, 0 n} p_{0 p}^{a}+\pi_{2 r, 0 n} q_{2 r}^{a} \\
& p_{1 n}^{\prime}-p_{1 n}^{a}=-\pi_{1 n, 1 r} p_{1 n}^{a}+\pi_{0 r, 1 n} p_{0 p}^{a}+\pi_{2 r, 1 n} q_{2 r}^{r} \\
& p_{2 n}^{j}-p_{2 n}^{a}=-\pi_{2 n, 1 r} p_{2 n}^{a}+\pi_{0 r, 2 n} p_{0 p}^{a}+\pi_{2 r, 2 n} q_{2 r}^{a}  \tag{134}\\
& p_{0 p}^{j}-p_{0 p}^{a}=-\left(\pi_{0 r, 0 n}+\pi_{0 r, 2 n}+\pi_{0 r, 1 r}+\pi_{0 r, 1 n}+\pi_{0 r, 2 u}\right) p_{0 p}^{a}+\pi_{2 r, 0 r} q_{2 r}^{a} \\
& p_{2 p}^{j}-p_{2 p}^{a}=\pi_{0 r, 2 u} p_{0 p}^{a}-\pi_{2 u, 1 r} p_{2 p}^{a}-\left(\pi_{2 r, 0 n}+\pi_{2 r, 1 n}+\pi_{2 r, 2 n}+\pi_{2 r, 0 r}+\pi_{2 r, 1 r}-\pi_{2 u, 1 r}\right) q_{2 r}^{a}
\end{align*}
$$

Proof. By definition $\pi_{s^{a}, s^{j}} \equiv P\left(S_{i}^{j}=s^{j} \mid S_{i}^{a}=s^{a}\right)$, Table A4, and a simple application of the law of total probability.

Remark 6. Given Assumption 1 but for A.6, and Assumptions 2-5, bounds on the response probabilities
$\pi^{\prime} \equiv\left[\pi_{0 n, 1 r}, \pi_{0 r, 0 n}, \pi_{2 n, 1 r}, \pi_{0 r, 2 n}, \pi_{0 r, 1 r}, \pi_{0 r, 1 n}, \pi_{1 n, 1 r}, \pi_{0 r, 2 u}, \pi_{2 u, 1 r}, \pi_{2 r, 0 n}, \pi_{2 r, 1 n}, \pi_{2 r, 0 n}, \pi_{2 r, 02 n}, \pi_{2 r, 0 r}, \pi_{2 r, 1 r}\right]^{\prime}$.
are implied by system (134) and $0 \leq q_{2 r}^{a} \leq \frac{3}{14,784}$. Because $\frac{3}{14,784} \approx 0$, the numerical bounds on $\boldsymbol{\pi} \equiv\left[\pi_{0 n, 1 r}, \pi_{0 r, 0 n}, \pi_{2 n, 1 r}, \pi_{0 r, 2 n}, \pi_{0 r, 1 r}, \pi_{0 r, 1 n}, \pi_{1 n, 1 r}, \pi_{0 r, 2 u}, \pi_{2 u, 1 r}\right]^{\prime}$ are indistinguishable from those obtained when A. 6 is maintained.

## 9 Extended Model with FS and Taxes

We begin with some additional notation and definitions that supersede those from Section 4 in this Appendix. All lemmas, corollaries, and propositions supersede those from Section 4 in this Appendix.

## Notation, Definitions, and Assumptions

Notation 1 (Policy Regimes). Throughout, we use $a$ to refer to the JF reform's control welfare and FS policy and $j$ to refer to JF reform's experimental welfare and FS policy. The policy regime is denoted by $t \in\{a, j\}$. The assistance program mix is denoted by $m \in\{w, f, w f\}$ where " $w$ " refers to welfare only, " $f$ " refers to FS only, and " $w f$ " refers to welfare joint with FS.

Definition 15 (Program Participation, Earnings and Reported Earnings). Let $D^{f}, D^{w}$, and $D^{w f}$ be indicators for a woman participating in, respectively, FS only, welfare only, and both FS and welfare; $D^{f}, D^{w}$, and $D^{w f}$ take values in $\{0,1\}$. These program participation alternatives are mutually exclusive: $D^{f}+D^{w}+D^{w f} \in\{0,1\}$. Let $\mathbf{D} \equiv\left(D^{w}, D^{f}, D^{w f}\right)$. Let $E$ denote a woman's earnings. Earnings are the product of hours worked, $H$, and an hourly wage rate $W$. Let $E^{r}$ denote earnings reported to the relevant assistance agency. Let $R \equiv R\left(\mathbf{D}, E^{r}\right)=\mathbf{1}\left[E^{r}=0\right]\left(D^{w}+D^{w f}\right)$ be an indicator for zero reported earnings by a welfare recipient.

Definition 16 (Transfer and Tax Functions). Throughout, we use $G^{t}(),. F^{t}($.$) , and T($.$) to$ refer to, respectively, the welfare transfer function, the FS transfer function, and the federal income tax function (inclusive of the EITC). These functions are defined as follows.

1. Welfare Transfer Functions. For any reported earning level $E^{r}$, the regime-dependent welfare transfers are

$$
\begin{align*}
G_{i}^{a}\left(E^{r}\right) & =\max \left\{\bar{G}_{i}-\mathbf{1}\left[E^{r}>\delta_{i}\right]\left(E^{r}-\delta_{i}\right) \tau_{i}, 0\right\},  \tag{136}\\
G_{i}^{j}\left(E^{r}\right) & =\mathbf{1}\left[E^{r} \leq F P L_{i}\right] \bar{G}_{i} . \tag{137}
\end{align*}
$$

The parameter $\delta_{i} \in\{90,120\}$ gives woman $i$ 's fixed disregard and the parameter $\tau_{i} \in\{.49, .73\}$ governs her proportional disregard. $\bar{G}_{i}$ and $F P L_{i}$ vary across women due to differences in AU size. Define woman $i$ 's break-even earnings level under a as $\bar{E}_{i} \equiv \bar{G}_{i} / \tau_{i}+\delta_{i}$, this is the level at which welfare benefits are exhausted.

## 2. Food Stamps (FS) Transfer Functions.

For any reported earning level $E^{r}$, the regime-dependent FS transfers are:

$$
\begin{align*}
F_{i}^{a}\left(E^{r}\right) & =F_{i}\left(E^{r}, 0\right),  \tag{138}\\
F_{i}^{j}\left(E^{r}\right) & =F_{i}\left(E^{r}, 0\right),  \tag{139}\\
F_{i}^{a, w f}\left(E^{r}\right) & =F_{i}\left(E^{r}, G_{i}^{a}\left(E^{r}\right)\right) \mathbf{1}\left[G_{i}^{a}\left(E^{r}\right)>0\right],  \tag{140}\\
F_{i}^{j, w f}\left(E^{r}\right) & =F_{i}\left(0, \bar{G}_{i}\right) \mathbf{1}\left[E^{r} \leq F P L_{i}\right], \tag{141}
\end{align*}
$$

where $F_{i}(\cdot, \cdot)$ is the standard FS formula, as described next. Let $\mathbf{1}\left[\right.$ elig $\left.g_{i}\right]$ denote the eligibility for FS. Then, for any pair of reported earnings and welfare transfer, denoted ( $E^{r}, G$ ), the FS transfer is:

$$
\begin{equation*}
F_{i}\left(E^{r}, G\right)=\max \left\{\overline{\bar{F}}_{i}-\tau_{1}^{f} \chi_{i}\left(E^{r}, G\right), 0\right\} \mathbf{1}\left[e l i g_{i}\right] \tag{142}
\end{equation*}
$$

with

$$
\begin{equation*}
\chi_{i}\left(E^{r}, G\right) \equiv \max \left\{E^{r}+G-\tau_{2}^{f} \min \left\{E^{r}, F P L_{i}\right\}-\beta_{1 i}^{f}-\beta_{2 i}^{f}\left(E^{r}, G\right), 0\right\} \tag{143}
\end{equation*}
$$

where $\overline{\bar{F}}_{i}$ is the maximum FS transfer; $\tau_{1}^{f} \chi_{i}\left(E^{r}, G\right)$ is a the net income deduction; $\tau_{2}^{f}$ is the earned income deduction rate; $\beta_{1 i}^{f}$ is the sum of the per unit standard deduction, the medical deduction, the child support deduction, and the dependent care deduction; and $\beta_{2 i}^{f}\left(E^{r}, G\right)$ is the excess shelter deduction as a function of earnings plus the welfare transfer. The variation in $\left(\beta_{1 i}^{f}, \beta_{2 i}^{f}().\right)$ across women with the same earnings and welfare transfer is due to differences in actual medical, shelter, and child care expenses. The variation in $F_{i}(.,$.$) across women$ is due to differences in AU size. To simplify notation let $\bar{F}_{i} \equiv F_{i}\left(0, \bar{G}_{i}\right)$. We remark that $\overline{\bar{F}}_{i} \equiv F_{i}(0,0)$. The eligibility indicator $\mathbf{1}\left[\right.$ elig $\left.g_{i}\right]$ reflects categorical eligibility, when FS is taken up jointly with welfare, or the FS's gross and net income tests, when FS is taken up alone:

$$
\mathbf{1}\left[e l i g_{i}\right]=\left\{\begin{array}{ll}
1 & \text { if } G>0  \tag{144}\\
\mathbf{1}\left[E^{r} \leq \tau_{3}^{f} F P L_{i}\right] \mathbf{1}\left[\chi_{i}\left(E^{r}, 0\right) \leq F P L_{i}\right] & \text { if } G=0
\end{array},\right.
$$

where $\tau_{3}^{f}$ is a multiplier factor. The parameters $\left(\tau_{1}^{f}, \tau_{2}^{f}, \tau_{3}^{f}\right)$ take values $(0.30,0.20,1.3) .{ }^{4}$

## 3. Earned Income Tax Credit (EITC) and Federal Income Tax Functions.

For any earning level $E$, earnings inclusive of one-twelfth of the total annual EITC credit, net of federal (gross) income taxes (with head of household filing status) and net of payroll and medicare taxes are given by:

$$
T_{i}(E) \equiv E-I_{i}(E)-E I T C_{i}(E)-\left(\tau^{l}+\tau^{m}\right) E .
$$

The parameters $\left(\tau^{l}, \tau^{m}\right)=(0.062,0.0145)$ give the payroll and medicare tax rates, $I_{i}(E)$ is amount of (gross) federal income taxes, and $E I T C_{i}(E)$ is the amount of the earned income tax credit. Specifically, the earned income tax function $E I T C_{i}(\cdot)$ is given by ${ }^{5}$

$$
\begin{aligned}
\operatorname{EITC}_{i}(E)= & \tau_{1 i}^{e} E 1\left[0<E \leq \bar{E}_{1 i}^{e}\right]+\tau_{1 i}^{e} \bar{E}_{1 i}^{e} 1\left[\bar{E}_{1 i}^{e}<E \leq \bar{E}_{2 i}^{e}\right]+ \\
& \left(\tau_{1 i}^{e} \bar{E}_{1 i}^{e}-\tau_{2 i}^{e}\left(E-\bar{E}_{2 i}^{e}\right)\right) 1\left[\bar{E}_{2 i}^{e}<E \leq \bar{E}_{2 i}^{e}+\frac{\tau_{1 i}^{e}}{\tau_{2 i}^{e}} \bar{E}_{1 i}^{e}\right] .
\end{aligned}
$$

The parameters $\left(\tau_{1 i}^{e}, \tau_{2 i}^{e}\right)$ give a woman $i$ 's phase-in and phase-out rates. The parameters $\left(\bar{E}_{1 i}^{e}, \bar{E}_{2 i}^{e}\right)$ give a woman $i$ 's earning thresholds defining the earnings region yielding maximum credit. Both sets of parameters vary across women due to differences in the number of children. The (gross) federal income tax function $I_{i}(\cdot)$ is given by ${ }^{6}$

$$
I_{i}(E)=\sum_{k=1}^{5} \tau_{k}^{I} \max \left\{\min \left\{Y_{i}^{I}-y_{k-1}^{I}, y_{k}^{I}-y_{k-1}^{I}\right\}, 0\right\}
$$

[^26]where $Y_{i}^{I}$ is the woman's taxable income which is given by her earnings net of the personal exemption and of the standard deduction: $Y_{i}^{I}=E-D_{1 i}^{I}-D_{2}^{I}$. The personal exemption $D_{1 i}^{I}$ varies across women due to differences in the number of children. The parameters $\left(\tau_{1}^{I}, \tau_{2}^{I}, \tau_{3}^{I}, \tau_{4}^{I}, \tau_{5}^{I}\right)$ give the marginal tax rates and the parameters $\left(y_{0}^{I}, y_{1}^{I}, y_{2}^{I}, y_{3}^{I}, y_{4}^{I}, y_{5}^{I}\right)$ give the tax brackets with $y_{0}^{I} \equiv 0$ and $y_{5}^{I} \equiv \infty$.

Definition 17 (Consumption Equivalent). Consider a tuple ( $E, \mathbf{D}, E^{r}$ ). Under regime $t$, woman $i$ 's consumption equivalent corresponding to $\left(E, \mathbf{D}, E^{r}\right)$ is

$$
\begin{align*}
C_{i}^{t}\left(E, \mathbf{D}, E^{r}\right) \equiv & T_{i}(E)+  \tag{145}\\
& \left(G_{i}^{t}\left(E^{r}\right)+F_{i}^{t, w f}\left(E^{r}\right)-\gamma_{i} 1\left[E<E^{r}\right]\right) D^{w f}+ \\
& \left(F_{i}^{t}\left(E^{r}\right)-\omega_{i} 1\left[E<E^{r}\right]\right) D^{f}+ \\
& \left(G_{i}^{t}\left(E^{r}\right)-\kappa_{i} 1\left[E<E^{r}\right]\right) D^{w} .
\end{align*}
$$

The parameters ( $\kappa_{i}, \omega_{i}, \gamma_{i}$ ) are the costs of under-reporting earnings. For simplicity, we refer to $C_{i}^{t}=$ $C_{i}^{t}\left(E, \mathbf{D}, E^{r}\right)$ as consumption. Below, when the consumption associated with a triple ( $E, \mathbf{D}, E^{r}$ ) and calculated according to (145) does not vary across regimes we omit the superscript $t$, and we omit the subscript $i$ when it does not vary across women.

Definition 18 (State). Consider a tuple ( $E, \mathbf{D}, E^{r}$ ). The "state" corresponding to ( $E, \mathbf{D}, E^{r}$ ) is defined by the function:

$$
s\left(E, \mathbf{D}, E^{r}\right)=\left\{\begin{array}{ll}
0 n n & \text { if } E=0, \mathbf{D}=\mathbf{0}, \\
1 n n & \text { if } E \text { in range } 1, \mathbf{D}=\mathbf{0}, \\
2 n n & \text { if } E \text { in range } 2, \mathbf{D}=\mathbf{0}, \\
0 n r & \text { if } E=0, D^{f}=1, \\
1 n r & \text { if } E \text { in range } 1, D^{f}=1, E^{r}=E, \\
2 n r & \text { if } E \text { in range } 2, D^{f}=1, E^{r}=E, \\
1 n u & \text { if } E \text { in range } 1, D^{f}=1, E^{r}<E, \\
2 n u & \text { if } E \text { in range } 2, D^{f}=1, E^{r}<E, \\
0 r n & \text { if } E=0, D^{w}=1, \\
1 r n & \text { if } E \text { in range } 1, D^{w}=1, E^{r}=E, \\
2 r n & \text { if } E \text { in range } 2, D^{w}=1, E^{r}=E, \\
1 u n & \text { if } E \text { in range } 1, D^{w}=1, E^{r}<E, \\
2 u n & \text { if } E \text { in range } 2, D^{w}=1, E^{r}<E, \\
0 r r & \text { if } E=0, D^{w f}=1, \\
1 r r & \text { if } E \text { in range } 1, D^{w f}=1, E^{r}=E, \\
2 r r & \text { if } E \text { in range } 2, D^{w f}=1, E^{r}=E, \\
1 u u & \text { if } E \text { in range } 1, D^{w f}=1, E^{r}<E, \\
2 u u & \text { if } E \text { in range } 2, D^{w f}=1, E^{r}<E
\end{array} .\right.
$$

Remark 7 (State: Excluded States). In Connecticut welfare and FS assistance programs are managed by the same agency. Accordingly, we do not include states $\{1 u r, 1 r u, 2 u r, 2 r u\}$ because it is not possible to make different earning reports to the same agency. Also, we do not include states $\{0 u n, 0 n u, 0 u u\}$ because it is not possible to under-report zero earnings.

Definition 19 (Job Offers). As in Definition 6.
Definition 20 (Alternative). An alternative is a wage, hours of work, program participation indicators, and earning report tuple ( $W, H, \mathbf{D}, E^{r}$ ).

Definition 21 (Sub-alternative). A sub-alternative is a wage, hours of work, and program participation indicators tuple ( $W, H, \mathbf{D}$ ).

Definition 22 (Alternative Compatible with a State). We say that an alternative ( $W, H, \mathbf{D}, E^{r}$ ) is compatible with state $s$ for woman $i$, if letting $E \equiv W H, s=s\left(E, \mathbf{D}, E^{r}\right)$.
Definition 23 (Alternative Compatible with a State and Available). We say that an alternative $\left(W, H, \mathbf{D}, E^{r}\right)$ is compatible with state $s$ and available for woman $i$ if $\left(W, H, \mathbf{D}, E^{r}\right)$ is compatible with state $s$ and $(W, H) \in \Theta_{i} \cup(0,0)$.

Definition 24 (Utility Function). Define $U_{i}^{t}(H, C, \mathbf{D}, R)$ as the utility woman $i$ derives from the tuple $(H, C, \mathbf{D}, R)$ under regime $t \in\{a, j\}$. Below, when the utility of a tuple $(H, C, \mathbf{D}, R)$ does not vary across policy regimes we omit the superscript $t$.

Definition 25 (Attractiveness of States). We say that a state $s$ is:

1. no better under regime $j$ than under regime $a$ if, for any alternative ( $W, H, \mathbf{D}, E^{r}$ ) compatible with state $s$, and letting $E \equiv W H$,

$$
U_{i}^{j}\left(H, C_{i}^{j}\left(E, \mathbf{D}, E^{r}\right), \mathbf{D}, R\left(\mathbf{D}, E^{r}\right)\right) \leq U_{i}^{a}\left(H, C_{i}^{a}\left(E, \mathbf{D}, E^{r}\right), \mathbf{D}, R\left(\mathbf{D}, E^{r}\right)\right) \text { all } i .
$$

2. no worse under regime $j$ than under regime $a$ if, for any alternative ( $W, H, \mathbf{D}, E^{r}$ ) compatible with state $s$, and letting $E \equiv W H$,

$$
U_{i}^{j}\left(H, C_{i}^{j}\left(E, \mathbf{D}, E^{r}\right), \mathbf{D}, R\left(\mathbf{D}, E^{r}\right)\right) \geq U_{i}^{a}\left(H, C_{i}^{a}\left(E, \mathbf{D}, E^{r}\right), \mathbf{D}, R\left(\mathbf{D}, E^{r}\right)\right) \text { all } i .
$$

3. We say that a state $s$ is equally attractive under regimes $j$ and $a$ if, for any alternative $\left(W, H, \mathbf{D}, E^{r}\right)$ compatible with state $s$, and letting $E \equiv W H$,

$$
U_{i}^{j}\left(H, C_{i}^{j}\left(E, \mathbf{D}, E^{r}\right), \mathbf{D}, R\left(\mathbf{D}, E^{r}\right)\right)=U_{i}^{a}\left(H, C_{i}^{a}\left(E, \mathbf{D}, E^{r}\right), \mathbf{D}, R\left(\mathbf{D}, E^{r}\right)\right) \text { all } i .
$$

Definition 26 (Collections of States). Define

$$
\begin{aligned}
\mathcal{S} & \equiv\{0 n n, 1 n n, 2 n n, 0 n r, 1 n r, 2 n r, 1 n u, 2 n u, 0 r r, 1 r r, 0 r n, 1 r n, 1 u n, 2 u n, 1 u u, 2 u u\}, \\
\mathcal{C}_{0} & \equiv\{0 n n, 1 n n, 2 n n, 0 n r, 1 n r, 2 n r, 1 n u, 2 n u, 1 u u, 2 u u, 1 u n, 2 u n\}, \\
\mathcal{C}_{+} & \equiv\{1 r r, 1 r n\}, \\
\mathcal{C}_{-} & \equiv\{0 r r, 0 r n\}
\end{aligned}
$$

Definition 27 (Welfare Participation State). Let $\mathcal{S}_{w} \equiv\{0 n, 1 n, 2 n, 0 r, 1 r, 1 u, 2 u\}$. $\mathcal{S}_{w}$ is the list of latent states that spell out welfare participation only. The states in $\mathcal{S}_{w}$ relate to the states in $\mathcal{S}$ as follows:

$$
s_{w}=h(s)=\left\{\begin{array}{ll}
0 n & \text { if } s \in\{0 n n, 0 n r\} \\
1 n & \text { if } s \in\{1 n n, 1 n r, 1 n u\} \\
2 n & \text { if } s \in\{2 n n, 2 n r, 2 n u\} \\
0 r & \text { if } s \in\{0 r n, 0 r r\} \\
1 r & \text { if } s \in\{1 r n, 1 r r\} \\
1 u & \text { if } s \in\{1 u n, 1 u u\} \\
2 u & \text { if } s \in\{2 u n, 2 u u\}
\end{array},\right.
$$

where the number of each state $s_{w}$ refers to the woman's earnings range, the letter " $n$ " refers to welfare non-participation, the letter " $r$ " refers to welfare participation with truthful reporting of earnings, and the letter " $u$ " refers to welfare participation with under-reporting of earnings.

Definition 28 (Primitives). Let woman $i$ be described by

$$
\theta_{i} \equiv\left(U_{i}^{a}(., ., ., .), U_{i}^{j}(., ., ., .), \kappa_{i}, \omega_{i}, \gamma_{i}, \Theta_{i}, G_{i}^{a}(.), F_{i}(., .), T_{i}(.)\right) .
$$

Consider a sample of $N$ women with children. The sample women have primitives $\left\{\theta_{i}\right\}_{i=1}^{N}$, which are i.i.d. draws from a joint distribution function $\Gamma_{\theta}$ (.).

Definition 29 (Response Probabilities). Let $S_{i}^{t}$ denote woman $i$ 's potential state under regime $t \in\{a, j\}$. Define the proportion of women occupying state $s \in \mathcal{S}$ under regime $t$ as $q_{s}^{t} \equiv P\left(S_{i}^{t}=s\right)$ where $P($.$) is a probability measure induced by the distribution function \Gamma_{\theta}($.$) . Let \pi_{s^{a}, s^{j}}$ denote the proportion of women occupying state $s^{j}$ under regime $j$ among those who occupy state $s^{a}$ under regime $a$, that is, $\pi_{s^{a}, s^{j}} \equiv P\left(S_{i}^{j}=s^{j} \mid S_{i}^{a}=s^{a}\right)$ where $P($.$) is also a probability measure induced$ by the distribution function $\Gamma_{\theta}($.$) .$

Definition 30 (Integrated Response Probabilities). Let $S_{w, i}^{t}$ denote the welfare-only potential state of a woman $i$ whose potential state under regime $t$ is $S_{i}^{t}$; that is, $S_{w, i}^{t}=h\left(S_{i}^{t}\right)$. Define the proportion of women occupying state $s_{w} \in \mathcal{S}_{w}$ under regime $t$ as $p_{s_{w}}^{t} \equiv P\left(S_{w, i}^{t}=s_{w}\right)=$ $\sum_{s \in \mathcal{S}: s_{w}=h(s)} q_{s}^{t}$ where $P($.$) is a probability measure induced by the distribution function \Gamma_{\theta}$ (.). With some abuse of notation (see Definition 29), let $\pi_{s_{w}^{a}, s_{w}^{j}}$ denote the proportion of women who occupy state $s_{w}^{j}$ under regime $j$ among those who occupy state $s_{w}^{a}$ under regime $a$; that is, $\pi_{s_{w}^{a}, s_{w}^{j}} \equiv$ $P\left(S_{w, i}^{j}=s_{w}^{j} \mid S_{w, i}^{a}=s_{w}^{a}\right)$ where $P($.$) is also a probability measure induced by the distribution$ function $\Gamma_{\theta}($.$) .$

Assumption 6 (Preferences). Woman $i$ 's utility functions $U_{i}^{a}(\cdot, \cdot, \cdot, \cdot)$ and $U_{i}^{j}(\cdot, \cdot, \cdot, \cdot)$ satisfy the restrictions:
A. 1 utility is strictly increasing in $C$;
A. $2 U_{i}^{t}(H, C, \mathbf{D}, 1) \leq U_{i}^{t}(H, C, \mathbf{D}, 0)$ for all $(H, C, \mathbf{D})$ such that $D^{w}+D^{w f}=1$ and all $t \in\{a, j\}$; and $U_{i}^{t}(H, C, \mathbf{D}, 1)=U_{i}^{t}(H, C, \mathbf{D}, 0)$ for all $(H, C, \mathbf{D})$ such that $D^{f}=1$ and all $t \in\{a, j\}$;
A. $3 \quad U_{i}^{j}(H, C, \mathbf{D}, 1) \leq U_{i}^{a}(H, C, \mathbf{D}, 1)$ for all $(H, C, \mathbf{D})$ such that $D^{w}+D^{w f}=1$;
A. $4 \quad U_{i}^{a}(H, C, \mathbf{D}, 0)=U_{i}^{j}(H, C, \mathbf{D}, 0)$ for all $(H, C, \mathbf{D}, 0)$ such that $D^{w}+D^{f}+D^{w f}=1$ and $H>0$;
A. $5 \quad U_{i}^{a}(H, C, \mathbf{D}, 0)=U_{i}^{j}(H, C, \mathbf{D}, 0)$ for all $(H, C, \mathbf{D})$ such that $D^{w}+D^{f}+D^{w f}=0$ and all $t \in\{a, j\}$;
A. $6 \quad U_{i}^{a}\left(H, C_{i}^{a}(E, \mathbf{D}, E), \mathbf{D}, 0\right)<U_{i}^{a}\left(H, C_{i}^{a}(E, \mathbf{0}, E), \mathbf{0}, 0\right)$ for all $(H, W)$ such that $E \equiv$ $W H \in\left(F P L_{i}, \bar{E}_{i}\right]$ and $D^{w}+D^{w f}=1$ whenever $\bar{E}_{i}>F P L_{i}$.

Remark 8 (Preferences: Verbalizing Assumption 6). A. 2 reporting zero earnings to the welfare agency yields a hassle disutility, while reporting zero earnings to the FS agency yields no hassle disutility. A. 3 states that regime $j$ 's welfare hassle disutility is no smaller than regime $a$ 's welfare hassle disutility. A. 4 states that the utility value of alternatives entailing FS-only participation is independent of the regime. It also states that utility value of alternatives entailing welfare-only participation, or FS and welfare participation, is independent of the regime whenever reported
earnings are not zero. A. 5 states that the utility value of alternatives entailing no participation in assistance programs is independent of the regime. A. 6 states that under regime $a$ the stigma disutility associated with welfare assistance (alone or in combination with FS) is bounded from below. That is, under $a$ and at earnings levels above $F P L_{i}$, the extra consumption due to the transfer income does not suffice to compensate the woman for the welfare stigma disutility she incurs when on welfare assistance, irrespective of program mix.

Assumption 7 (Under-reporting Earning Penalties). For each woman $i,\left(\kappa_{i}, \omega_{i}, \gamma_{i}\right)>0$.
Assumption 8 (Welfare-Ineligible Earning Levels). No woman may be on welfare assistance and truthfully report earnings above $F P L_{i}$ under regime $j$ or above $\bar{E}_{i}$ under regime a.

Assumption 9 (Utility Maximization). Under regime $t$ woman $i$ makes choices by solving the optimization problem

$$
\max _{(W, H) \in \Theta_{i} \cup(0,0), \mathbf{D} \in\{0,1\}^{3}, D^{f}+D^{w}+D^{w f} \leq 1, E^{r} \in[0, W H]} U_{i}^{t}\left(H, C_{i}^{t}\left(W H, \mathbf{D}, E^{r}\right), \mathbf{D}, R\left(\mathbf{D}, E^{r}\right)\right)
$$

Assumption 10 (Population Heterogeneity). The distribution $\Gamma_{\theta}$ (.) is unrestricted save for the constraints implied by Assumptions 6-9 and the definition of wage offers (Definition 19).

Assumption 11 (Breaking Indifference). Women break indifference in favor of the same alternative irrespective of the regime.

Assumption 12 (Filing Taxes). A woman files (does not file) for federal income taxes and the EITC irrespective of the regime.

## Intermediate Lemmas

Lemma 6 (Combined Transfer). Under both regimes $j$ and $a$, for every $E^{r}$ such that $G_{i}^{t}\left(E^{r}\right)>$ 0 , the combined welfare plus FS transfer is no smaller than the sole welfare transfer or the sole FS transfer.

Proof. The proof that the combined welfare plus FS transfer is no smaller than the sole welfare transfer is trivial: the FS program has no feed-backs on the welfare program (Definition 16, expressions (136)-(137)) and the FS transfer cannot be negative (Definition 16, expression (142)). The proof that the combined welfare plus FS transfer is no smaller than the sole FS transfer is less obvious because the FS transfer is decreasing in the welfare grant which is counted as income (Definition 16, expressions (142)-(143)). Nevertheless, the FS formula in (142) shows that a $\$ 1$ increase in the welfare grant $(G)$ leads to a less than $\$ 1$ decrease in the FS transfer because $\tau_{1}^{f}<1$ and welfare assistance yields categorical FS eligibility (expression (144)). Thus, a woman whose earnings report makes her eligible for welfare can enjoy a higher transfer income by taking up both welfare and FS as opposed to taking up only FS.

Lemma 7 (Combined Transfer as a Function of Reported Earnings). Under regime a, the combined welfare plus FS transfer is weakly decreasing in reported earnings.

Proof. For any reported earning $E^{r}$, the combined transfer accruing to woman $i$ is a function $B($. defined by $B\left(E^{r}\right) \equiv G_{i}^{a}\left(E^{r}\right)+F_{i}\left(E^{r}, G_{i}^{a}\left(E^{r}\right)\right)$. Observe: 1) given $E^{r}$, the function $G+F_{i}\left(E^{r}, G\right)$ is weakly increasing in $G$ because a $\$ 1$ increase in the welfare grant $(G)$ leads to a less than $\$ 1$ decrease in the FS transfer due to $\tau_{1}^{f}<1$ (expression 142); 2) $G_{i}^{a}\left(E^{r}\right)$ is weakly decreasing in
$E^{r}$ (expression (136)); 3) given $G, F_{i}\left(E^{r}, G\right)$ is weakly decreasing in $E^{r}$ (expressions (142)-(143)). Together these facts imply that $B($.$) is a weakly decreasing function. { }^{7}$

Lemma 8 (States 2rr and 2rn). Given Assumptions 6, 8, and 9, no woman selects an allocation compatible with states $2 r r$ and $2 r n$.

Proof. Under regime $j$ no alternative is compatible with states $2 r n$ and $2 r r$ by Assumption 8. Consider now a woman with $\bar{E}_{i} \leq F P L_{i}$ under regime $a$. By Assumption 8 she may not be on assistance and truthfully report earnings above $F P L_{i}$ (range 2). Finally, consider a woman with $\bar{E}_{i}>F P L_{i}$ under regime $a$. By Assumption 8 she may not be on assistance and truthfully report earnings above $\bar{E}_{i}$. By A. 6 in Assumption 6 she will not truthfully report earnings in $\left(F P L_{i}, \bar{E}_{i}\right]$ because she can attain a higher utility level by being off welfare assistance (Assumption 9): the extra consumption due to the transfer income does not suffice to compensate the woman for the stigma disutility she incurs when being on welfare assistance.

Lemma 9 (Optimal Reporting). Write woman $i$ 's optimization problem (Assumption 9) as a nested maximization problem:

$$
\begin{equation*}
\max _{(W, H) \in \Theta_{i} \cup(0,0), \mathbf{D} \in\{0,1\}^{3}, D^{f}+D^{w}+D^{w f} \leq 1}\left[\max _{E^{r} \in[0, W H]} U_{i}^{t}\left(H, C_{i}^{t}\left(W H, \mathbf{D}, E^{r}\right), \mathbf{D}, R\left(\mathbf{D}, E^{r}\right)\right)\right] \tag{146}
\end{equation*}
$$

Focus on the inner maximization problem in (146) for given sub-alternative $(W, H, \boldsymbol{D})$ with $D^{m}=1$ for $m \in\{f, w, w f\}$. Let $E \equiv W H$ and $E_{i}^{r, t, m}=E_{i}^{r, t, m}(W, H)$ denote woman $i$ 's utility maximizing earning report conditional on sub-alternative $(W, H, \boldsymbol{D})$ with $D^{m}=1$. Given Assumptions 6-12, optimal reporting while on assistance is as follows:

## 1. Welfare Only

(a) $E_{i}^{r, j, w}$ entails either truthful reporting, that $i s, E_{i}^{r, j, w}=E$, or under-reporting such that $E>E_{i}^{r, j, w} \in\left[0, F P L_{i}\right]$; in particular, state 1 un is dominated;
(b) $E_{i}^{r, a, w}$ entails either truthful reporting, that is, $E_{i}^{r, a, w}=E$, or under-reporting such that $E>E_{i}^{r, a, w} \in\left[0, \delta_{i}\right] ;$

## 2. FS Only

For any $t \in\{a, j\}, E_{i}^{r, t, f}$ entails either truthful reporting, that is, $E_{i}^{r, t, f}=E$, or underreporting such that $E>E_{i}^{r, t, f} \in\left[0, \min \left\{\underline{E}_{i}^{f}, \bar{E}_{i}^{f}\right\}\right]$ where $\underline{E}_{i}^{f}$ is the highest level of reported earnings such that the FS transfer is unreduced and $\bar{E}_{i}^{f}$ is the highest level of reported earnings such the FS's eligibility tests are satisfied;

## 3. Welfare and FS

(a) $E_{i}^{r, j, w f}$ entails either truthful reporting, that is, $E_{i}^{r, j, w f}=E$, or under-reporting such that $E>E_{i}^{r, j, w f} \in\left[0, F P L_{i}\right]$; in particular, state $1 u u$ is dominated;

[^27](b) $E_{i}^{r, a, w f}$ entails either truthful reporting, that is, $E_{i}^{r, a, w f}=E$, or under-reporting such that $E>E_{i}^{r, a, w f}=1$ cent, or under-reporting such that $E>E_{i}^{r, a, w f} \in\left[0, \underline{E}_{i}^{w f}\right]$ where $\underline{E}_{i}^{w f}$ is the largest level of reported earnings in $\left[0, \delta_{i}\right]$ such that the corresponding FS transfer is $\bar{F}_{i}$, that is $\chi_{i}\left(\underline{E}_{i}^{w f}, \bar{G}_{i}\right)=0$ (Definition 143), or, if no such earning level exists in $\left[0, \delta_{i}\right], \underline{E}_{i}^{w f}=0$.

Proof. We prove each part of the Lemma in turn.

## 1. Welfare Only

The proofs of statements a.) and b.) mimic the proof of Lemma 2 with the appropriate adjustments in notation, namely, with $\mathbf{D}=(1,0,0)$ in place of $D=1, E_{i}^{r, t, w}$ in place of $E_{i}^{r, t}$ for $t \in\{a, j\}$ and the references to A. 2 in Assumption 6 in place of the references to A. 2 in Assumption 1.

## 2. FS Only

The stand-alone FS program rules are invariant to the regime (Definition 16). The utility associated with any alternative compatible with stand-alone FS assistance is also regime invariant (A. 4 in Assumption 6). Thus, the reported earning level that solves the inner maximization problem in (146) is the same for all $t \in\{a, j\}$ and is that which maximizes consumption. To find such level we make three preliminary observations. First, we observe that the threshold level $\underline{E}_{i}^{f}$ is strictly positive for all $i$. To see this consider a woman $i$ who enjoys no deductions other than the standard deduction, namely, $\beta_{1 i}^{f}=\$ 134$ and $\beta_{2 i}^{f}(0,0)=0$. Then, $\chi_{i}\left(E^{r}, 0\right)=E^{r}\left(1-\tau_{2}^{f}\right)-\$ 134$ (expression (143)), hence any report $E^{r} \leq \$ 134 /\left(1-\tau_{2}^{f}\right)=\$ 167.5$ yields her a FS transfer in the (maximal) amount $\overline{\bar{F}}_{i}$. A woman with deductions other than the standard deduction enjoys an even higher threshold level $\underline{E}_{i}^{f}$. Second, we observe that the threshold level $\bar{E}_{i}^{f}$ is also strictly positive for all i. To see this observe that $\bar{E}_{i}^{f}$ is the smallest level of earnings that engenders ineligibility, formally, $\bar{E}_{i}^{f}=\min \left\{\tau_{3}^{f} F P L_{i}, E^{\prime}\right\}$ where $E^{\prime}$ is such that $\chi_{i}\left(E^{\prime}, 0\right)=F P L_{i}$. Third, we observe that the threshold level $\underline{E}_{i}^{f}$ for a woman with very high deductions may be higher than $\bar{E}_{i}^{f}$. Given $E$, any report $E^{r} \in\left[0, \min \left\{\underline{E}_{i}^{f}, \bar{E}_{i}^{f}\right\}\right]$ yields the same (maximal) transfer $\overline{\bar{F}}_{i}$ hence woman $i$ enjoys consumption in the amount $T_{i}(E)+\overline{\bar{F}}_{i}-\omega_{i} \mathbf{1}\left[E<E^{r}\right]$. A report $E^{r}>\min \left\{\underline{E}_{i}^{f}, \bar{E}_{i}^{f}\right\}$ yields transfer $F_{i}\left(E^{r}, 0\right)$ hence woman $i$ enjoys consumption in the amount $T_{i}(E)+F_{i}\left(E^{r}, 0\right)-\omega_{i} \mathbf{1}\left[E<E^{r}\right]$. Because $F_{i}\left(E^{r}, 0\right)<\overline{\bar{F}}_{i}$ for all $E^{r}>\min \left\{\underline{E}_{i}^{f}, \bar{E}_{i}^{f}\right\}$ and $\omega_{i}>0$ (Assumption 7), and depending on the magnitude of woman $i$ 's under-reporting $\operatorname{cost}\left(\omega_{i}\right)$ and earning level $E$, either reporting $E_{i}^{r, t, f} \in\left[0, \min \left\{\underline{E}_{i}^{f}, \bar{E}_{i}^{f}\right\}\right]$ or truthful reporting, i.e. $E_{i}^{r, t, f}=E$, maximizes consumption hence solves the inner maximization problem in (146) for all $t \in\{a, j\}$.

## 3. Welfare and FS

The proof of statement a.) mimics the proof of Lemma 2.I.) with the appropriate adjustments in notation, namely, with $\mathbf{D}=(0,0,1)$ in place of $D=1, E_{i}^{r, j, w f}$ in place of $E_{i}^{r, j}$, the references A. 2 and A. 4 in Assumption 6 in place of the references to A. 2 and A. 4 in Assumption 1, and the expressions for consumption equal to $T_{i}(E)+\bar{G}_{i}+\bar{F}_{i}-\gamma_{i}$ in place of $T_{i}(E)+\bar{G}_{i}-\kappa_{i}$
(under-reporting) and $T_{i}(E)+\bar{G}_{i}+\bar{F}_{i}$ in place of $E+\bar{G}_{i}$ (truthful reporting). In particular, state $1 u u$ is dominated in the extended model because under regime $j$ earnings up to $F P L_{i}$ are fully disregarded in the determination of the combined welfare plus FS transfer (Definition 16, expression 141). Next we prove statement b.).
Consider first a woman $i$ who derives no disutility from hassle under regime $a$ (A. 2 in Assumption 6 holds as an equality). Thus, the utility associated with any alternative compatible with welfare plus FS assistance is regime invariant hence the reported earning level that solves the inner maximization problem in (146) is that which maximizes consumption. Reporting $E^{r} \in\left[0, \underline{E}_{i}^{w f}\right]$ yields woman $i$ the maximal combined transfer $\bar{G}_{i}+\bar{F}_{i}$ with implied consumption $T_{i}(E)+\bar{G}_{i}+\bar{F}_{i}-\gamma_{i} \mathbf{1}\left[E<E^{r}\right]$. A report $E^{r}>\underline{E}_{i}^{w f}$ yields a lower transfer (Lemma 7) with implied consumption $T_{i}(E)+G_{i}^{a}\left(E^{r}\right)+F_{i}\left(E^{r}, G_{i}^{a}\left(E^{r}\right)\right)-\gamma_{i} \mathbf{1}\left[E<E^{r}\right]$. Thus, depending on the magnitude of woman $i$ 's under-reporting cost $\left(\gamma_{i}\right)$ and earning level $E$, either report$\operatorname{ing} E_{i}^{r, a, w f} \in\left[0, \underline{E}_{i}^{w f}\right]$ or truthful reporting, i.e. $E_{i}^{r, a, w f}=E$, solves the inner maximization problem in (146).
Next, consider a woman $i$ who derives some disutility from hassle under regime $a$ (A. $\mathbf{2}$ in Assumption 6 holds as a strict inequality) and such that $\underline{E}_{i}^{w f} \in\left(0, \delta_{i}\right]$. Reporting $E^{r} \in$ $\left(0, \underline{E}_{i}^{w f}\right]$ yields her the maximal combined transfer $\bar{G}_{i}+\bar{F}_{i}$ while higher reports yield a lower transfer (Lemma 7). Depending on the magnitude of woman $i$ 's under-reporting cost and earning level $E$, either reporting $E_{i}^{r, a, w f} \in\left(0, \underline{E}_{i}^{w f}\right]$ or truthful reporting, i.e. $E_{i}^{r, a, w f}=E$, solves the inner maximization problem in equation (146). To show this we next consider $\mathbf{D}=$ $(0,0,1)$ and five mutually exclusive pairs ( $W, H$ ) spanning the range of value for $E \equiv W H$. For convenience, we let $U_{i}^{t}$ serve as shortcut notation for $U_{i}^{t}\left(H, C_{i}^{t}\left(E, \boldsymbol{D}, E^{r}\right), \boldsymbol{D}, R\left(\boldsymbol{D}, E^{r}\right)\right)$. Let $(W, H)$ be:
(a) such that $E=0$.

Woman $i$ 's cannot over-report her earnings (Assumption 146). Thus, $E_{i}^{r, a, w f}=E$.
(b) $(W, H)$ such that $E \in\left(0, \underline{E}_{i}^{w f}\right]$.

Woman $i$ 's utility while on welfare and FS depends on reported earnings as follows (A. 4 in Assumption 6):

$$
U_{i}^{a}=\left\{\begin{array}{ll}
{[1]: U_{i}^{a}\left(H, T_{i}(E)+\bar{G}_{i}+\bar{F}_{i}-\gamma_{i}, \mathbf{D}, 1\right)} & \text { if } E^{r}=0 \\
{[2]: U_{i}\left(H, T_{i}(E)+\bar{G}_{i}+\bar{F}_{i}-\gamma_{i}, \mathbf{D}, 0\right)} & \text { if } E^{r} \in(0, E) \\
{[3]: U_{i}\left(H, T_{i}(E)+\bar{G}_{i}+\bar{F}_{i}, \mathbf{D}, 0\right)} & \text { if } E^{r}=E
\end{array} .\right.
$$

By the characterization of woman $i$ 's preferences we have [1] $<[2]$. By $\gamma_{i}>0$ (Assumption 7) we have [2] < [3]. Thus, truthful reporting solves the inner maximization problem (146), that is, $E_{i}^{r, a, w f}=E$.
(c) $(W, H)$ such that $E \in\left(\underline{E}_{i}^{w f}, \delta_{i}\right]$.

Woman $i$ 's utility while on welfare and FS depends on reported earnings as follows (A. 4 in Assumption 6):

$$
U_{i}^{a}=\left\{\begin{array}{ll}
{[1]: U_{i}^{a}\left(H, T_{i}(E)+\bar{G}_{i}+\bar{F}_{i}-\gamma_{i}, \mathbf{D}, 1\right)} & \text { if } E^{r}=0 \\
{[2]: U_{i}\left(H, T_{i}(E)+\bar{G}_{i}+\bar{F}_{i}-\gamma_{i}, \mathbf{D}, 0\right)} & \text { if } E^{r} \in\left(0, \underline{E}_{i}^{w f}\right] \\
{[3]: U_{i}\left(H, T_{i}(E)+\bar{G}_{i}+F_{i}\left(E^{r}, \bar{G}_{i}\right)-\gamma_{i}, \mathbf{D}, 0\right)} & \text { if } E^{r} \in\left(\underline{E}_{i}^{w f}, E\right) \\
{[4]: U_{i}\left(H, T_{i}(E)+\bar{G}_{i}+F_{i}\left(E^{r}, \bar{G}_{i}\right), \mathbf{D}, 0\right)} & \text { if } E^{r}=E
\end{array} .\right.
$$

By the characterization of woman $i$ 's preferences we have $[1]<[2]$. Because the FS transfer $F_{i}\left(E^{r}, \bar{G}_{i}\right)$ is strictly decreasing in $E^{r}, F_{i}\left(E^{r}, \bar{G}_{i}\right)<\bar{F}_{i}$, hence $[3]<[2]$. Thus,
depending on woman $i$ 's utility function, under-reporting cost $\left(\gamma_{i}\right)$, and earnings $E$, the inner maximization problem in (146) is solved by $E_{i}^{r, a, w f} \in\left(0, \underline{E}_{i}^{w f}\right]$ (when [4] $\leq[2]$ ) or by truthful reporting, $E_{i}^{r, a, w f}=E$ (when $[4] \geq[2]$ ).
(d) $(W, H)$ such that $E \in\left(\delta_{i}, F P L_{i}\right]$.

Woman $i$ 's utility while on welfare and FS depends on reported earnings as follows (A. 6 in Assumption 6):

$$
U_{i}^{a}=\left\{\begin{array}{ll}
{[1]: U_{i}^{a}\left(H, T_{i}(E)+\bar{G}_{i}+\bar{F}_{i}-\gamma_{i}, \mathbf{D}, 1\right)} & \text { if } E^{r}=0 \\
{[2]: U_{i}\left(H, T_{i}(E)+\bar{G}_{i}+\bar{F}_{i}-\gamma_{i}, \mathbf{D}, 0\right)} & \text { if } E^{r} \in\left(0, \underline{E_{i}^{w f}}\right] \\
{[3]: U_{i}\left(H, T_{i}(E)+\bar{G}_{i}+F_{i}\left(E^{r}, \bar{G}_{i}\right)-\gamma_{i}, \mathbf{D}, 0\right)} & \text { if } E^{r} \in\left(\underline{E}_{i}^{w f}, \delta_{i}\right] \\
{[4]: U_{i}\left(H, T_{i}(E)+G_{i}^{a}\left(E^{r}\right)+F_{i}\left(E^{r}, G_{i}^{a}\left(E^{r}\right)\right)-\gamma_{i}, \mathbf{D}, 0\right)} & \text { if } E^{r} \in\left(\delta_{i}, E\right) \\
{[5]: U_{i}\left(H, T_{i}(E)+G_{i}^{a}\left(E^{r}\right)+F_{i}\left(E^{r}, G_{i}^{a}\left(E^{r}\right)\right), \mathbf{D}, 0\right)} & \text { if } E^{r}=E
\end{array} .\right.
$$

By the characterization of woman $i$ 's preferences we have [1] $<[2]$. The combined transfer $G_{i}^{a}\left(E^{r}\right)+F_{i}\left(E^{r}, G_{i}^{a}\left(E^{r}\right)\right)$ is strictly decreasing in $E^{r}$ which implies that [4] < $[3]<[2]$ (Lemma 7). Thus, depending on woman $i$ 's utility function, under-reporting cost ( $\gamma_{i}$ ), and earnings $E$, the inner maximization problem in (146) is solved by $E_{i}^{r, a, w f} \in$ $\left(0, \underline{E}_{i}^{w f}\right]$ (when $\left.[5] \leq[2]\right)$ or by truthful reporting, $E_{i}^{r, a, w f}=E$ (when $\left.[5] \geq[2]\right)$.
(e) $(W, H)$ such that $E>F P L_{i}$.

Woman $i$ must be under-reporting. Her utility while on welfare and FS depends on reported earnings as follows (A. 6 in Assumption 6):

$$
U_{i}^{a}=\left\{\begin{array}{ll}
{[1]: U_{i}^{a}\left(H, T_{i}(E)+\bar{G}_{i}+\bar{F}_{i}-\gamma_{i}, \mathbf{D}, 1\right)} & \text { if } E^{r}=0 \\
{[2]: U_{i}\left(H, T_{i}(E)+\bar{G}_{i}+\bar{F}_{i}-\gamma_{i}, \mathbf{D}, 0\right)} & \text { if } E^{r} \in\left(0, E_{i}^{w f}\right] \\
{[3]: U_{i}\left(H, T_{i}(E)+\bar{G}_{i}+F_{i}\left(E^{r}, \bar{G}_{i}\right)-\gamma_{i}, \mathbf{D}, 0\right)} & \text { if } E^{r} \in\left(E_{i}^{w f}, \delta_{i}\right] \\
{[4]: U_{i}\left(H, T_{i}(E)+G_{i}^{a}\left(E^{r}\right)+F_{i}\left(E^{r}, G_{i}^{a}\left(E^{r}\right)\right)-\gamma_{i}, \mathbf{D}, 0\right)} & \text { if } E^{r} \in\left(\delta_{i}, E\right)
\end{array} .\right.
$$

By the characterization of woman $i$ 's preferences we have [1] $<[2]$. The combined transfer $G_{i}^{a}\left(E^{r}\right)+F_{i}\left(E^{r}, G_{i}^{a}\left(E^{r}\right)\right)$ is strictly decreasing in $E^{r}$ which implies that $[4]<$ $[3]<[2]$ (Lemma 7). Thus, the inner maximization problem in (146) is solved by $E_{i}^{r, a, w f} \in\left(0, \underline{E}_{i}^{w f}\right]$.

Finally, consider a woman $i$ who derives some disutility from hassle under regime $a$ (A.2 in Assumption 6 holds as a strict inequality) and such that $\underline{E}_{i}^{w f}=0$. Depending on women $i$ 's utility function (in particular her hassle disutility), under-reporting cost, and earnings $E$, the inner maximization problem in (146) is solved by $E_{i}^{r, a, w f}=1$ cent or by truthful reporting, $E_{i}^{r, a, w f}=E$. Too show this we next consider $\mathbf{D}=(0,0,1)$ and four mutually exclusive pairs $(W, H)$ spanning the range of value for $E \equiv W H$. Again, for convenience, we let $U_{i}^{t}$ serve as shortcut notation for $U_{i}^{t}\left(H, C_{i}^{t}\left(E, \boldsymbol{D}, E^{r}\right), \boldsymbol{D}, R\left(\boldsymbol{D}, E^{r}\right)\right)$. Let $(W, H)$ be:
(a) $(W, H)$ such that $E=0$.

Woman $i$ 's cannot over-report her earnings (Assumption 146). Thus, $E_{i}^{r, a, w f}=E$.
(b) $(W, H)$ such that $E \in\left(0, \delta_{i}\right]$.

Woman $i$ 's utility while on welfare and FS depends on reported earnings as follows (A. 4 in Assumption 6):

$$
U_{i}^{a}=\left\{\begin{array}{ll}
{[1]: U_{i}^{a}\left(H, T_{i}(E)+\bar{G}_{i}+\bar{F}_{i}-\gamma_{i}, \mathbf{D}, 1\right)} & \text { if } E^{r}=0 \\
{[2]: U_{i}\left(H, T_{i}(E)+\bar{G}_{i}+F_{i}\left(E^{r}, \bar{G}_{i}\right)-\gamma_{i}, \mathbf{D}, 0\right)} & \text { if } E^{r} \in(0, E) \\
{[3]: U_{i}\left(H, T_{i}(E)+\bar{G}_{i}+F_{i}\left(E^{r}, \bar{G}_{i}\right), \mathbf{D}, 0\right)} & \text { if } E^{r}=E
\end{array} .\right.
$$

The FS transfer $F_{i}\left(E^{r}, \bar{G}_{i}\right)$ is strictly decreasing in $E^{r}$ which implies that $F_{i}\left(E^{r}, \bar{G}_{i}\right) \leq$ $\bar{F}_{i}$ for all $E^{r} \in(0, E)$ and among these reports that which yields the highest utility is $E^{r}=1$ (the smallest possible denomination). Due to rounding of the FS transfer, $F_{i}\left(1, \bar{G}_{i}\right)=\bar{F}_{i} \equiv F_{i}\left(0, \bar{G}_{i}\right)$ hence $[1]<[2]$. Thus, whether the inner maximization problem (146) has solution $E_{i}^{r, a, w f}=1$ cent or $E_{i}^{r, a, w f}=E$ depends on whether $\bar{F}_{i}-\gamma_{i} \lessgtr$ $F_{i}\left(E, \bar{G}_{i}\right)$.
(c) $(W, H)$ such that $E \in\left(\delta_{i}, F P L_{i}\right]$.

Woman $i$ 's utility while on welfare and FS depends on reported earnings as follows (A. 4 in Assumption 6):

$$
U_{i}^{a}=\left\{\begin{array}{ll}
{[1]: U_{i}^{a}\left(H, T_{i}(E)+\bar{G}_{i}+\bar{F}_{i}-\gamma_{i}, \mathbf{D}, 1\right)} & \text { if } E^{r}=0 \\
{[2]: U_{i}\left(H, T_{i}(E)+\bar{G}_{i}+F_{i}\left(E^{r}, \bar{G}_{i}\right)-\gamma_{i}, \mathbf{D}, 0\right)} & \text { if } E^{r} \in\left(0, \delta_{i}\right] \\
{[3]: U_{i}\left(H, T_{i}(E)+G_{i}^{a}\left(E^{r}\right)+F_{i}\left(E^{r}, G_{i}^{a}\left(E^{r}\right)\right)-\gamma_{i}, \mathbf{D}, 0\right)} & \text { if } E^{r} \in\left(\delta_{i}, E\right) \\
{[4]: U_{i}\left(H, T_{i}(E)+\bar{G}_{i}+F_{i}\left(E^{r}, \bar{G}_{i}\right), \mathbf{D}, 0\right)} & \text { if } E^{r}=E
\end{array} .\right.
$$

The combined transfer $G_{i}^{a}\left(E^{r}\right)+F_{i}\left(E^{r}, G_{i}^{a}\left(E^{r}\right)\right)$ is strictly decreasing in $E^{r}$ which implies that $[3]<[2]$ (Lemma 7). The FS transfer $F_{i}\left(E^{r}, G_{i}^{a}\left(E^{r}\right)\right)$ is also strictly decreasing in $E^{r}$ which implies that among reports in $\left(0, \delta_{i}\right]$ that which yields the highest utility is $E^{r}=1$ cent (the smallest possible denomination). Due to rounding of the FS transfer, $F_{i}\left(1, \bar{G}_{i}\right)=\bar{F}_{i} \equiv F_{i}\left(0, \bar{G}_{i}\right)$ hence [1] $<[2]$. Thus, whether the inner maximization problem (146) has solution $E_{i}^{r, a, w f}=1$ cent or $E_{i}^{r, a, w f}=E$ depends on whether $\bar{F}_{i}-\gamma_{i} \lessgtr F_{i}\left(E, \bar{G}_{i}\right)$.
(d) $(W, H)$ such that $E>F P L_{i}$.

Woman $i$ must be under-reporting. Her utility while on welfare and FS depends on reported earnings as follows (A. 4 in Assumption 6):

$$
U_{i}^{a}=\left\{\begin{array}{ll}
{[1]: U_{i}^{a}\left(H, T_{i}(E)+\bar{G}_{i}+\bar{F}_{i}-\gamma_{i}, \mathbf{D}, 1\right)} & \text { if } E^{r}=0 \\
{[2]: U_{i}\left(H, T_{i}(E)+\bar{G}_{i}+F_{i}\left(E^{r}, \bar{G}_{i}\right)-\gamma_{i}, \mathbf{D}, 0\right)} & \text { if } E^{r} \in\left(0, \delta_{i}\right] \\
{[3]: U_{i}\left(H, T_{i}(E)+G_{i}^{a}\left(E^{r}\right)+F_{i}\left(E^{r}, G_{i}^{a}\left(E^{r}\right)\right)-\gamma_{i}, \mathbf{D}, 0\right)} & \text { if } E^{r} \in\left(\delta_{i}, F P L_{i}\right)
\end{array} .\right.
$$

The combined transfer $G_{i}^{a}\left(E^{r}\right)+F_{i}\left(E^{r}, G_{i}^{a}\left(E^{r}\right)\right)$ is strictly decreasing in $E^{r}$ which implies that $[3]<[2]$ (Lemma 7). The FS transfer $F_{i}\left(E^{r}, \bar{G}_{i}\right)$ is also strictly decreasing in $E^{r}$ which implies that among reports in $\left(0, \delta_{i}\right]$ that which yields the highest utility is $E^{r}=1$ cent (the smallest possible denomination). Due to rounding of the FS transfer, $F_{i}\left(1, \bar{G}_{i}\right)=\bar{F}_{i} \equiv F_{i}\left(0, \bar{G}_{i}\right)$ hence $[1]<[2]$. Thus, the inner maximization problem (146) has solution $E_{i}^{r, a, w f}=1$ cent.

Corollary 6 (Optimal Reporting and Policy Invariance). Given Assumptions 6-12, the utility function associated with any alternative compatible with states $\{1 u n, 2 u n, 1 u u, 2 u u\}$ and entailing optimal reporting is regime invariant.

Proof. We examine each state in turn.

1. State 1un
(a) Consider a woman $i$ and any sub-alternative $(W, H, \boldsymbol{D})$ such that letting $E \equiv W H, E$ is in range $1, D^{w}=1$, and $E_{i}^{r, j, w}(W, H)<E$. Thus alternative $\left(W, H, \boldsymbol{D}, E_{i}^{r, j, w}(W, H)\right)$ is compatible with state 1 un and entails optimal reporting under regime $j$. Let $C_{i}^{j} \equiv$
$C_{i}^{j}\left(E,(1,0,0), E_{i}^{r, j, w}(W, H)\right)$ and $R_{i}^{j} \equiv R\left((1,0,0), E_{i}^{r, j, w}(W, H)\right)$. We next show that $U_{i}^{j}\left(H, C_{i}^{j},(1,0,0), R_{i}^{j}\right)=U_{i}\left(H, C_{i}^{j},(1,0,0), R_{i}^{j}\right)$. By Lemma $9, E_{i}^{r, j, w}(W, H) \in$ $\left(0, F P L_{i}\right]$ or $E_{i}^{r, j, w}(W, H) \in\left[0, F P L_{i}\right]$ depending on the woman's preferences. In the first case, the utility woman $i$ enjoys is $U_{i}^{j}\left(H, T_{i}(E)+\bar{G}_{i}-\kappa_{i},(1,0,0), 0\right)$ which equals $U_{i}\left(H, T_{i}(E)+\bar{G}_{i}-\kappa_{i},(1,0,0), 0\right)$ by A. 4 in Assumption 6. In the second case, the utility woman $i$ enjoys is $U_{i}^{j}\left(H, T_{i}(E)+\bar{G}_{i}-\kappa_{i},(1,0,0), 0\right)$ which also equals $U_{i}\left(H, T_{i}(E)+\bar{G}_{i}-\kappa_{i},(1,0,0), 0\right)$ by A. 4 in Assumption 6 and because she is indifferent between (under-) reports in ( $\left.0, F P L_{i}\right]$ and reporting zero earnings, that is, $U_{i}^{j}\left(H, T_{i}(E)+\bar{G}_{i}-\kappa_{i},(1,0,0), 1\right)=U_{i}^{j}\left(H, T_{i}(E)+\bar{G}_{i}-\kappa_{i},(1,0,0), 0\right)$.
(b) Consider a woman $i$ and any sub-alternative $(W, H, \boldsymbol{D})$ such that letting $E \equiv W H, E$ is in range $1, D^{w}=1$, and $E_{i}^{r, a, w}(W, H)<E$. Thus alternative $\left(W, H, \boldsymbol{D}, E_{i}^{r, a, w}(W, H)\right)$ is compatible with state $1 u n$ and entails optimal reporting under regime $a$. Let $C_{i}^{a} \equiv$ $C_{i}^{a}\left(E,(1,0,0), E_{i}^{r, a, w}(W, H)\right)$ and $R_{i}^{a} \equiv R\left((1,0,0), E_{i}^{r, a, w}(W, H)\right)$. We next show that $U_{i}^{a}\left(H, C_{i}^{a},(1,0,0), R_{i}^{a}\right)=U_{i}\left(H, C_{i}^{a},(1,0,0), R_{i}^{a}\right)$. By Lemma $9, E_{i}^{r, a, w}(W, H) \in\left(0, \delta_{i}\right]$ or $E_{i}^{r, a, w}(W, H) \in\left[0, \delta_{i}\right]$ depending on the woman's preferences. In the first case, the utility woman $i$ enjoys is $U_{i}^{a}\left(H, T_{i}(E)+\bar{G}_{i}-\kappa_{i},(1,0,0), 0\right)$ which equals
$U_{i}\left(H, T_{i}(E)+\bar{G}_{i}-\kappa_{i},(1,0,0), 0\right)$ by A. 4 in Assumption 6. In the second case, the utility woman $i$ enjoys is also
$U_{i}^{a}\left(H, T_{i}(E)+\bar{G}_{i}-\kappa_{i},(1,0,0), 0\right)=U_{i}\left(H, T_{i}(E)+\bar{G}_{i}-\kappa_{i},(1,0,0), 0\right)$ by A. 4 in Assumption 6 and because she is indifferent between (under-) reports in ( $0, \delta_{i}$ ] and reporting zero earnings, that is, $U_{i}^{a}\left(H, T_{i}(E)+\bar{G}_{i}-\kappa_{i},(1,0,0), 1\right)=U_{i}^{a}\left(H, T_{i}(E)+\bar{G}_{i}-\kappa_{i},(1,0,0), 0\right)$.
(c) In 1.(a) and 1.(b) we have shown that any alternative compatible with state $1 u n$ and entailing optimal reporting yields regime-invariant consumption $T_{i}(E)+\bar{G}_{i}-\kappa_{i}$ and regime-invariant utility level $U_{i}\left(H, T_{i}(E)+\bar{G}_{i}-\kappa_{i},(1,0,0), 0\right)$.

## 2. State $2 u n$.

The proof that the utility associated with any alternative compatible with state $2 u n$ and entailing optimal reporting is regime invariant is the same as that for state 1 un once we let the pair $(H, W)$ be such that $E \equiv W H$ is in range 2 (Lemma 9$)$.

## 3. State $1 u u$

(a) Consider a woman $i$ and any sub-alternative $(W, H, \boldsymbol{D})$ such that letting $E \equiv W H, E$ is in range $1, D^{w f}=1$, and $E_{i}^{r, j, w f}(W, H)<E$. Thus alternative $\left(W, H, \boldsymbol{D}, E_{i}^{r, j, w f}(W, H)\right)$ is compatible with state $1 u u$ and entails optimal reporting under regime $j$. Let $C_{i}^{j} \equiv$ $C_{i}^{j}\left(E,(0,0,1), E_{i}^{r, j, w f}(W, H)\right)$ and $R_{i}^{j} \equiv R\left((0,0,1), E_{i}^{r, j, w f}(W, H)\right)$. We next show that $U_{i}^{j}\left(H, C_{i}^{j},(0,0,1), R_{i}^{j}\right)=U_{i}\left(H, C_{i}^{j},(0,0,1), R_{i}^{j}\right)$. By Lemma $9, E_{i}^{r, j, w f}(W, H) \in$ $\left(0, F P L_{i}\right]$ or $E_{i}^{r, j, w f}(W, H) \in\left[0, F P L_{i}\right]$ depending on the woman's preferences. In the first case, the utility woman $i$ enjoys is $U_{i}^{j}\left(H, T_{i}(E)+\bar{G}_{i}+\bar{F}_{i}-\gamma_{i},(0,0,1), 0\right)$ which equals $U_{i}\left(H, T_{i}(E)+\bar{G}_{i}+\bar{F}_{i}-\gamma_{i},(0,0,1), 0\right)$ by A. 4 in Assumption 6. In the second case, the utility woman $i$ enjoys is $U_{i}^{j}\left(H, T_{i}(E)+\bar{G}_{i}+\bar{F}_{i}-\gamma_{i},(0,0,1), 0\right)$ which also equals $U_{i}\left(H, T_{i}(E)+\bar{G}_{i}+\bar{F}_{i}-\gamma_{i},(0,0,1), 0\right)$ by A. 4 in Assumption 6 and because she is indifferent between (under-) reports in ( $0, F P L_{i}$ ] and reporting zero earnings, that is, $U_{i}^{j}\left(H, T_{i}(E)+\bar{G}_{i}+\bar{F}_{i}-\gamma_{i},(0,0,1), 1\right)=U_{i}^{j}\left(H, T_{i}(E)+\bar{G}_{i}+\bar{F}_{i}-\gamma_{i},(0,0,1), 0\right)$.
(b) Consider a woman $i$ and any sub-alternative $(W, H, \boldsymbol{D})$ such that letting $E \equiv W H, E$ is in range $1, D^{w f}=1$, and $E_{i}^{r, a, w f}(W, H)<E$. Thus alternative $\left(W, H, \boldsymbol{D}, E_{i}^{r, a, w f}(W, H)\right)$ is compatible with state $1 u u$ and entails optimal reporting under regime $a$. Let $C_{i}^{a} \equiv$ $C_{i}^{a}\left(E,(0,0,1), E_{i}^{r, a, w f}(W, H)\right)$ and $R_{i}^{a} \equiv R\left((0,0,1), E_{i}^{r, a, w f}(W, H)\right)$. We next show that $U_{i}^{a}\left(H, C_{i}^{a},(0,0,1), R_{i}^{a}\right)=U_{i}\left(H, C_{i}^{a},(0,0,1), R_{i}^{a}\right)$. By Lemma $9, E_{i}^{r, a, w f}(W, H) \in$ $\left(0, \underline{E}_{i}^{w f}\right]$ or $E_{i}^{r, a, w f}(W, H) \in\left[0, \underline{E}_{i}^{w f}\right]$ or $E_{i}^{r, a, w f}(W, H)=1$ cent depending on the woman's preferences and $\underline{E}_{i}^{w f}$. In the first case, the utility woman $i$ enjoys is $U_{i}^{a}\left(H, T_{i}(E)+\bar{G}_{i}+\bar{F}_{i}-\gamma_{i},(0,0,1), 0\right)$ which equals $U_{i}\left(H, T_{i}(E)+\bar{G}_{i}+\bar{F}_{i}-\gamma_{i},(0,0,1), 0\right)$ by A. 6 in Assumption 6 (policy invariance). In the second case, the utility woman $i$ enjoys is also
$U_{i}^{a}\left(H, T_{i}(E)+\bar{G}_{i}+\bar{F}_{i}-\gamma_{i},(0,0,1), 0\right)=U_{i}\left(H, T_{i}(E)+\bar{G}_{i}+\bar{F}_{i}-\gamma_{i},(0,0,1), 0\right)$ by A. 4 in Assumption 6 and because she is indifferent between (under-) reports in ( $0, \underline{E}_{i}^{w f}$ ] and reporting zero earnings, that is,
$U_{i}^{a}\left(H, T_{i}(E)+\bar{G}_{i}+\bar{F}_{i}-\gamma_{i},(0,0,1), 1\right)=U_{i}^{a}\left(H, T_{i}(E)+\bar{G}_{i}+\bar{F}_{i}-\gamma_{i},(0,0,1), 0\right)$. In the third case, the utility woman $i$ enjoys is also $U_{i}\left(H, T_{i}(E)+\bar{G}_{i}+\bar{F}_{i}-\gamma_{i},(0,0,1), 0\right)$ because of the rounding of the FS transfer.
(c) In 3.(a) and 3.(b) we have shown that any alternative compatible with state $1 u u$ and entailing optimal reporting yields regime-invariant consumption $T_{i}(E)+\bar{G}_{i}+\bar{F}_{i}-\gamma_{i}$ and regime-invariant utility level $U_{i}\left(H, T_{i}(E)+\bar{G}_{i}+\bar{F}_{i}-\gamma_{i},(0,0,1), 0\right)$.

## 4. State $2 u u$

The proof that the utility associated with any alternative compatible with state $2 u u$ and entailing optimal reporting is regime-invariant is the same as that for state $1 u u$ once we let the pair $(H, W)$ be such that $E \equiv W H$ is in range 2 (Lemma 9$)$.

Remark 9 (Optimal under-Reporting and Alternatives Considered). In what follows, when considering alternatives compatible with states $\{1 u n, 1 u u, 2 u n, 2 u u\}$, it is without loss of generality that we only focus on alternatives entailing optimal (under-) reporting. No woman would select an alternative compatible with states $\{1 u n, 1 u u, 2 u n, 2 u u\}$ not entailing optimal (under-) reporting (Assumption 9). Additionally, it is without loss of generality that we disregard alternatives compatible with states $\{1 u n, 1 u u\}$ under regime $j$. No woman would select an alternative compatible with states $\{1 u n, 1 u u\}$ under regime $j$ because they are dominated (Lemma 9, parts I and III).

Lemma 10 (Revealed Preferences). Consider any pair of states ( $s^{a}, s^{j}$ ) obeying: I) $s^{a} \neq s^{j}$; II) state $s^{a}$ is no worse under regime $j$ than under regime $a$; III) state $s^{j}$ is no better under regime $j$ than under regime $a$. Then, if Assumptions 1 and 5 hold, no woman will pair states $s^{a}$ and $s^{j}$.

Proof. The proof is by contradiction. Suppose that for some pair of states $\left(s^{a}, s^{j}\right)$ satisfying properties I)-III), woman $i$ chooses a tuple ( $H, W, \mathbf{D}, E^{r}$ ) under regime $a$ obeying $s^{a}=s\left(E, \mathbf{D}, E^{r}\right)$ with $E \equiv W H$ and a tuple $\left(H^{\prime}, W^{\prime}, \mathbf{D}^{\prime}, E^{r \prime}\right)$ under regime $j$ obeying $s^{j}=s\left(E^{\prime}, \mathbf{D}^{\prime}, E^{r \prime}\right)$ with $E^{\prime} \equiv W^{\prime} H^{\prime}$. For convenience, let $C_{i}^{t}=C_{i}^{t}\left(E, \mathbf{D}, E^{r}\right)$ and $C_{i}^{t \prime}=C_{i}^{t}\left(E^{\prime}, \mathbf{D}^{\prime}, E^{r \prime}\right)$. By property II $)$, optimality of the alternative compatible with state $s^{a}$ under regime $a$, and Property III):

$$
\begin{equation*}
U_{i}^{j}\left(H, C_{i}^{j}, \mathbf{D}, R\right) \geq U_{i}^{a}\left(H, C_{i}^{a}, \mathbf{D}, R\right) \geq U_{i}^{a}\left(H^{\prime}, C_{i}^{a \prime}, \mathbf{D}^{\prime}, R^{\prime}\right) \geq U_{i}^{j}\left(H^{\prime}, C_{i}^{j \prime}, \mathbf{D}^{\prime}, R^{\prime}\right) \tag{147}
\end{equation*}
$$

As in the proof of Lemma 4, if any of the inequalities in expression (147) is strict the contradiction ensues. If no inequality is strict, we need to consider $36=7^{2}-13$ possible situations based on the possible values of $\left(\mathbf{D}, R, \mathbf{D}^{\prime}, R^{\prime}\right)$ where we subtract 13 because $R$ is functionally related to $\boldsymbol{D}$ and $R^{\prime}$ is functionally related to $\boldsymbol{D}^{\prime}$. Each of these situations leads to a contradiction based on a woman breaking indifference between two alternatives in favor of the same alternative irrespective of the policy regime (Assumption 11) or based on violation of Property I. Specifically, in each of the following cases (147) simplifies to:

1. $(\mathbf{D}, R)=(\mathbf{0}, 0)$ and $\left(\mathbf{D}^{\prime}, R^{\prime}\right)=(\mathbf{0}, 0)$ :

$$
U_{i}(H, C, \mathbf{0}, 0)=U_{i}(H, C, \mathbf{0}, 0)=U_{i}\left(H^{\prime}, C^{\prime}, \mathbf{0}, 0\right)=U_{i}\left(H^{\prime}, C^{\prime}, \mathbf{0}, 0\right)
$$

where we have used the fact that off assistance consumption is invariant to the regime, hence $C_{i}^{a}=C_{i}^{j}=C$ and $C_{i}^{a \prime}=C_{i}^{j \prime}=C^{\prime}$. Woman $i$ is thus indifferent between $(H, C, \mathbf{0}, 0)$ and $\left(H^{\prime}, C^{\prime}, \mathbf{0}, 0\right)$ under regime $a$ and resolves indifference in favor of $(H, C, \mathbf{0}, 0)$, this contradicts resolving indifference in favor of $\left(H^{\prime}, C^{\prime}, \mathbf{0}, 0\right)$ under regime $j$.
2. $(\mathbf{D}, R)=((0,1,0), 0)$ and $\left(\mathbf{D}^{\prime}, R^{\prime}\right)=(\mathbf{0}, 0)$ :

$$
U_{i}\left(H, C_{i},(0,1,0), 0\right)=U_{i}\left(H, C_{i},(0,1,0), 0\right)=U_{i}\left(H^{\prime}, C^{\prime}, \mathbf{0}, 0\right)=U_{i}\left(H^{\prime}, C^{\prime}, \mathbf{0}, 0\right)
$$

where we have used the fact that off assistance consumption is invariant to the regime, hence $C_{i}^{a \prime}=C_{i}^{j \prime}=C^{\prime}$ and that the FS-only policy is invariant to the policy regime, hence $C_{i}^{a}=C_{i}^{j}=C_{i}$. Woman $i$ is thus indifferent between $\left(H, C_{i},(0,1,0), 0\right)$ and $\left(H^{\prime}, C^{\prime}, \mathbf{0}, 0\right)$ under regime $a$ and resolves indifference in favor of $\left(H, C_{i},(0,1,0), 0\right)$, this contradicts resolving indifference in favor of $\left(H^{\prime}, C^{\prime}, \mathbf{0}, 0\right)$ under regime $j$.
3. $(\mathbf{D}, R)=((0,1,0), 0)$ and $\left(\mathbf{D}^{\prime}, R^{\prime}\right)=((0,1,0), 0)$ :

$$
U_{i}\left(H, C_{i},(0,1,0), 0\right)=U_{i}\left(H, C_{i},(0,1,0), 0\right)=U_{i}\left(H^{\prime}, C_{i}^{\prime},(0,1,0), 0\right)=U_{i}\left(H^{\prime}, C_{i}^{\prime},(0,1,0), 0\right)
$$

where we have used the fact that the FS-only policy is invariant to the regime, hence $C_{i}^{a}=$ $C_{i}^{j}=C_{i}$ and $C_{i}^{a \prime}=C_{i}^{j \prime}=C_{i}^{\prime}$. Woman $i$ is thus indifferent between $\left(H, C_{i},(0,1,0), 0\right)$ and $\left(H^{\prime}, C_{i}^{\prime},(0,1,0), 0\right)$ under regime $a$ and resolves indifference in favor of $\left(H, C_{i},(0,1,0), 0\right)$, this contradicts resolving indifference in favor of $\left(H^{\prime}, C_{i}^{\prime},(0,1,0), 0\right)$ under regime $j$.
4. $(\mathbf{D}, R)=((0,1,0), 0)$ and $\left(\mathbf{D}^{\prime}, R^{\prime}\right)=((1,0,0), 0)$ :
$U_{i}\left(H, C_{i},(0,1,0), 0\right)=U_{i}\left(H, C_{i},(0,1,0), 0\right)=U_{i}\left(H^{\prime}, C_{i}^{a \prime},(1,0,0), 0\right)=U_{i}\left(H^{\prime}, C_{i}^{j \prime},(1,0,0), 0\right)$
where we have used the fact that the FS-only policy is invariant to regime, hence $C_{i}^{a}=C_{i}^{j}=$ $C_{i}$. The last equality implies that $C_{i}^{a \prime}=C_{i}^{j \prime}=C_{i}^{\prime}$ because utility is strictly increasing in consumption (Assumption 6). Woman $i$ is thus indifferent between $\left(H, C_{i},(0,1,0), 0\right)$ and $\left(H^{\prime}, C_{i}^{\prime},(1,0,0), 0\right)$ under regime $a$ and resolves indifference in favor of $\left(H, C_{i},(0,1,0), 0\right)$, this contradicts resolving indifference in favor of $\left(H^{\prime}, C_{i}^{\prime},(1,0,0), 0\right)$ under regime $j$.
5. $(\mathbf{D}, R)=((0,1,0), 0)$ and $\left(\mathbf{D}^{\prime}, R^{\prime}\right)=((1,0,0), 1)$ :
$U_{i}\left(H, C_{i},(0,1,0), 0\right)=U_{i}\left(H, C_{i},(0,1,0), 0\right)=U_{i}^{a}\left(H^{\prime}, C_{i}^{\prime},(1,0,0), 1\right)=U_{i}^{j}\left(H^{\prime}, C_{i}^{\prime},(1,0,0), 1\right)$,
where we have used the fact that the FS-only policy is invariant to the regime, hence $C_{i}^{a}=C_{i}^{j}=C_{i}$, and the fact that $G_{i}^{a}(0)=\bar{G}_{i}$ hence $C_{i}^{j \prime}=C_{i}^{a \prime}=C_{i}^{\prime}$. Woman $i$ is thus indifferent between $\left(H, C_{i},(0,1,0), 0\right)$ and ( $\left.H^{\prime}, C_{i}^{\prime},(1,0,0), 1\right)$ under regime $a$ and resolves indifference in favor of $\left(H, C_{i},(0,1,0), 0\right)$, this contradicts resolving indifference in favor of $\left(H^{\prime}, C_{i}^{\prime},(1,0,0), 1\right)$ under regime $j$.
6. $(\mathbf{D}, R)=((0,1,0), 0)$ and $\left(\mathbf{D}^{\prime}, R^{\prime}\right)=((0,0,1), 0)$ :
$U_{i}\left(H, C_{i},(0,1,0), 0\right)=U_{i}\left(H, C_{i},(0,1,0), 0\right)=U_{i}\left(H^{\prime}, C_{i}^{a \prime},(0,0,1), 0\right)=U_{i}\left(H^{\prime}, C_{i}^{j \prime},(0,0,1), 0\right)$,
where we have used the fact that the FS-only policy is invariant to the regime, hence $C_{i}^{a}=$ $C_{i}^{j}=C_{i}$. The last equality implies that $C_{i}^{a \prime}=C_{i}^{j \prime}=C_{i}^{\prime}$ because utility is strictly increasing in consumption (Assumption 6). Woman $i$ is thus indifferent between ( $H, C_{i},(0,1,0), 0$ ) and $\left(H^{\prime}, C_{i}^{\prime},(0,0,1), 0\right)$ under regime $a$ and resolves indifference in favor of $\left(H, C_{i},(0,1,0), 0\right)$, this contradicts resolving indifference in favor of $\left(H^{\prime}, C_{i}^{\prime},(0,0,1), 0\right)$ under regime $j$.
7. $(\mathbf{D}, R)=((0,1,0), 0)$ and $\left(\mathbf{D}^{\prime}, R^{\prime}\right)=((0,0,1), 1)$ :
$U_{i}\left(H, C_{i},(0,1,0), 0\right)=U_{i}\left(H, C_{i},(0,1,0), 0\right)=U_{i}^{a}\left(H^{\prime}, C_{i}^{\prime},(0,0,1), 1\right)=U_{i}^{j}\left(H^{\prime}, C_{i}^{\prime},(0,0,1), 1\right)$,
where we have used the fact that the FS-only policy is invariant to the regime, hence $C_{i}^{a}=$ $C_{i}^{j}=C_{i}$, and the fact that $\bar{G}_{i}=G_{i}^{a}(0)$ and $\bar{F}_{i}=F_{i}^{a}(0)$ hence $C_{i}^{j \prime}=C_{i}^{a \prime}=C_{i}^{\prime}$. Woman $i$ is thus indifferent between $\left(H, C_{i},(0,1,0), 0\right)$ and $\left(H^{\prime}, C_{i}^{\prime},(0,0,1), 1\right)$ under regime $a$ and resolves indifference in favor of $\left(H, C_{i},(0,1,0), 0\right)$, this contradicts resolving indifference in favor of $\left(H^{\prime}, C_{i}^{\prime},(0,0,1), 1\right)$ under regime $j$.
8. $(\mathbf{D}, R)=((1,0,0), 0)$ and $\left(\mathbf{D}^{\prime}, R^{\prime}\right)=(\mathbf{0}, 0)$ :

$$
U_{i}\left(H, C_{i}^{j},(1,0,0), 0\right)=U_{i}\left(H, C_{i}^{a},(1,0,0), 0\right)=U_{i}\left(H^{\prime}, C_{i}^{\prime}, \mathbf{0}, 0\right)=U_{i}\left(H^{\prime}, C_{i}^{\prime}, \mathbf{0}, 0\right)
$$

where we have used the fact that off assistance consumption is invariant to the regime, hence $C_{i}^{a \prime}=C_{i}^{j \prime}=C^{\prime}$. The first equality implies that $C_{i}^{a}=C_{i}^{j}=C_{i}$ because utility is strictly increasing in consumption (Assumption 6). Woman $i$ is thus indifferent between $\left(H, C_{i},(1,0,0), 0\right)$ and ( $\left.H^{\prime}, C_{i}^{\prime}, \mathbf{0}, 0\right)$ under regime $a$ and resolves indifference in favor of $\left(H, C_{i},(1,0,0), 0\right)$, this contradicts resolving indifference in favor of $\left(H^{\prime}, C_{i}^{\prime}, \mathbf{0}, 0\right)$ under regime $j$.
9. $(\mathbf{D}, R)=((1,0,0), 1)$ and $\left(\mathbf{D}^{\prime}, R^{\prime}\right)=(\mathbf{0}, 0)$ :

$$
U_{i}^{j}\left(H, C_{i},(1,0,0), 1\right)=U_{i}^{a}\left(H, C_{i},(1,0,0), 1\right)=U_{i}\left(H^{\prime}, C_{i}^{\prime}, \mathbf{0}, 0\right)=U_{i}\left(H^{\prime}, C_{i}^{\prime}, \mathbf{0}, 0\right),
$$

where we have used the fact that off assistance consumption is invariant to the regime, hence $C_{i}^{a \prime}=C_{i}^{j \prime}=C^{\prime}$, and that $G_{i}^{a}(0)=\bar{G}_{i}$, hence $C_{i}^{j}=C_{i}^{a}=C_{i}$. Woman $i$ is thus indifferent between $\left(H, C_{i},(1,0,0), 1\right)$ and $\left(H^{\prime}, C_{i}^{\prime}, \mathbf{0}, 0\right)$ under regime $a$ and resolves indifference in favor of $\left(H, C_{i},(1,0,0), 1\right)$, this contradicts resolving indifference in favor of ( $\left.H^{\prime}, C_{i}^{\prime}, \mathbf{0}, 0\right)$ under regime $j$.
10. $(\mathbf{D}, R)=((1,0,0), 0)$ and $\left(\mathbf{D}^{\prime}, R^{\prime}\right)=((0,1,0), 0)$ :
$U_{i}\left(H, C_{i}^{j},(1,0,0), 0\right)=U_{i}\left(H, C_{i}^{a},(1,0,0), 0\right)=U_{i}\left(H^{\prime}, C_{i}^{\prime},(0,1,0), 0\right)=U_{i}\left(H^{\prime}, C_{i}^{\prime},(0,1,0), 0\right)$,
where we have used the fact that the FS-only policy is invariant to the regime, hence $C_{i}^{a \prime}=$ $C_{i}^{j \prime}=C_{i}^{\prime}$. The first equality implies that $C_{i}^{a}=C_{i}^{j}=C_{i}$ because utility is strictly increasing in consumption (Assumption 6). Woman $i$ is thus indifferent between $\left(H, C_{i},(1,0,0), 0\right)$ and $\left(H^{\prime}, C_{i}^{\prime},(0,1,0), 0\right)$ under regime $a$ and resolves indifference in favor of $\left(H, C_{i},(1,0,0), 0\right)$, this contradicts resolving indifference in favor of $\left(H^{\prime}, C_{i}^{\prime},(0,1,0), 0\right)$ under regime $j$.
11. $(\mathbf{D}, R)=((1,0,0), 1)$ and $\left(\mathbf{D}^{\prime}, R^{\prime}\right)=((0,1,0), 0)$ :
$U_{i}^{j}\left(H, C_{i},(1,0,0), 1\right)=U_{i}^{a}\left(H, C_{i},(1,0,0), 1\right)=U_{i}\left(H^{\prime}, C_{i}^{\prime},(0,1,0), 0\right)=U_{i}\left(H^{\prime}, C_{i}^{\prime},(0,1,0), 0\right)$,
where we have used the fact that the FS-only policy is invariant to the regime, hence $C_{i}^{a \prime}=C_{i}^{j \prime}=C_{i}^{\prime}$ and the fact that $G_{i}^{a}(0)=\bar{G}_{i}$, hence $C_{i}^{j}=C_{i}^{a}=C_{i}$. Woman $i$ is thus indifferent between $\left(H, C_{i},(1,0,0), 1\right)$ and $\left(H^{\prime}, C_{i}^{\prime},(0,1,0), 0\right)$ under regime $a$ and resolves indifference in favor of $\left(H, C_{i},(1,0,0), 1\right)$, this contradicts resolving indifference in favor of $\left(H^{\prime}, C_{i}^{\prime},(0,1,0), 0\right)$ under regime $j$.
12. $(\mathbf{D}, R)=((1,0,0), 0)$ and $\left(\mathbf{D}^{\prime}, R^{\prime}\right)=((1,0,0), 0)$ :
$U_{i}\left(H, C_{i}^{j},(1,0,0), 0\right)=U_{i}\left(H, C_{i}^{a},(1,0,0), 0\right)=U_{i}\left(H^{\prime}, C_{i}^{a \prime},(1,0,0), 0\right)=U_{i}\left(H^{\prime}, C_{i}^{j \prime},(1,0,0), 0\right)$.
The first equality implies that $C_{i}^{a}=C_{i}^{j}=C_{i}$ and the last equality implies $C_{i}^{a \prime}=C_{i}^{j \prime}=$ $C_{i}^{\prime}$, because utility is strictly increasing in consumption (Assumption 6). Woman $i$ is thus indifferent between $\left(H, C_{i},(1,0,0), 0\right)$ and $\left(H^{\prime}, C_{i}^{\prime},(1,0,0), 0\right)$ under regime $a$ and resolves indifference in favor of ( $H, C_{i},(1,0,0), 0$ ), this contradicts resolving indifference in favor of ( $\left.H^{\prime}, C_{i}^{\prime},(1,0,0), 0\right)$ under regime $j$.
13. $(\mathbf{D}, R)=((1,0,0), 0)$ and $\left(\mathbf{D}^{\prime}, R^{\prime}\right)=((1,0,0), 1)$ :
$U_{i}\left(H, C_{i}^{j},(1,0,0), 0\right)=U_{i}\left(H, C_{i}^{a},(1,0,0), 0\right)=U_{i}^{a}\left(H^{\prime}, C_{i}^{\prime},(1,0,0), 1\right)=U_{i}^{j}\left(H^{\prime}, C_{i}^{\prime},(1,0,0), 1\right)$,
where we have used the fact that $G_{i}^{a}(0)=\bar{G}_{i}$, hence $C_{i}^{j \prime}=C_{i}^{a \prime}=C_{i}^{\prime}$. The first equality implies that $C_{i}^{a}=C_{i}^{j}=C_{i}$ because utility is strictly increasing in consumption (Assumption $6)$. Woman $i$ is thus indifferent between $\left(H, C_{i},(1,0,0), 0\right)$ and ( $\left.H^{\prime}, C_{i}^{\prime},(1,0,0), 1\right)$ under regime $a$ and resolves indifference in favor of $\left(H, C_{i},(1,0,0), 0\right)$, this contradicts resolving indifference in favor of $\left(H^{\prime}, C_{i}^{\prime},(1,0,0), 1\right)$ under regime $j$.
14. $(\mathbf{D}, R)=((1,0,0), 1)$ and $\left(\mathbf{D}^{\prime}, R^{\prime}\right)=((1,0,0), 0)$ :
$U_{i}^{j}\left(H, C_{i},(1,0,0), 1\right)=U_{i}^{a}\left(H, C_{i},(1,0,0), 1\right)=U_{i}\left(H^{\prime}, C_{i}^{a \prime},(1,0,0), 0\right)=U_{i}\left(H^{\prime}, C_{i}^{j \prime},(1,0,0), 0\right)$,
where we have used the fact that $G_{i}^{a}(0)=\bar{G}_{i}$, hence $C_{i}^{j}=C_{i}^{a}=C_{i}$. The the last equality implies $C_{i}^{a \prime}=C_{i}^{j \prime}=C_{i}^{\prime}$ because utility is strictly increasing in consumption (Assumption 6). Woman $i$ is thus indifferent between $\left(H, C_{i},(1,0,0), 1\right)$ and ( $\left.H^{\prime}, C_{i}^{\prime},(1,0,0), 0\right)$ under regime $a$ and resolves indifference in favor of $\left(H, C_{i},(1,0,0), 1\right)$, this contradicts resolving indifference in favor of $\left(H^{\prime}, C_{i}^{\prime},(1,0,0), 0\right)$ under regime $j$.
15. $(\mathbf{D}, R)=((1,0,0), 1)$ and $\left(\mathbf{D}^{\prime}, R^{\prime}\right)=((1,0,0), 1)$ :
$U_{i}^{j}\left(H, C_{i},(1,0,0), 1\right)=U_{i}^{a}\left(H, C_{i},(1,0,0), 1\right)=U_{i}^{a}\left(H^{\prime}, C_{i}^{\prime},(1,0,0), 1\right)=U_{i}^{j}\left(H^{\prime}, C_{i}^{\prime},(1,0,0), 1\right)$,
where we have used the fact that $G_{i}^{a}(0)=\bar{G}_{i}$, hence $C_{i}^{j}=C_{i}^{a}=C_{i}$ and $C_{i}^{j \prime}=C_{i}^{a \prime}=C_{i}^{\prime}$. Woman $i$ is thus indifferent between $\left(H, C_{i},(1,0,0), 1\right)$ and ( $\left.H^{\prime}, C_{i}^{\prime},(1,0,0), 1\right)$ under regime $a$ and resolves indifference in favor of $\left(H, C_{i},(1,0,0), 1\right)$, this contradicts resolving indifference in favor of $\left(H^{\prime}, C_{i}^{\prime},(1,0,0), 1\right)$ under regime $j$.
16. $(\mathbf{D}, R)=((1,0,0), 0)$ and $\left(\mathbf{D}^{\prime}, R^{\prime}\right)=((0,0,1), 0)$ :
$U_{i}\left(H, C_{i}^{j},(1,0,0), 0\right)=U_{i}\left(H, C_{i}^{a},(1,0,0), 0\right)=U_{i}\left(H^{\prime}, C_{i}^{a \prime},(0,0,1), 0\right)=U_{i}\left(H^{\prime}, C_{i}^{j \prime},(0,0,1), 0\right)$.
The first equality implies that $C_{i}^{a}=C_{i}^{j}=C_{i}$ and the last equality implies $C_{i}^{a \prime}=C_{i}^{j \prime}=$ $C_{i}^{\prime}$, because utility is strictly increasing in consumption (Assumption 6). Woman $i$ is thus indifferent between $\left(H, C_{i},(1,0,0), 0\right)$ and ( $\left.H^{\prime}, C_{i}^{\prime},(0,0,1), 0\right)$ under regime $a$ and resolves indifference in favor of $\left(H, C_{i},(1,0,0), 0\right)$, this contradicts resolving indifference in favor of ( $\left.H^{\prime}, C_{i}^{\prime},(0,0,1), 0\right)$ under regime $j$.
17. $(\mathbf{D}, R)=((1,0,0), 0)$ and $\left(\mathbf{D}^{\prime}, R^{\prime}\right)=((0,0,1), 1)$ :
$U_{i}\left(H, C_{i}^{j},(1,0,0), 0\right)=U_{i}\left(H, C_{i}^{a},(1,0,0), 0\right)=U_{i}^{a}\left(H^{\prime}, C_{i}^{\prime},(0,0,1), 1\right)=U_{i}^{j}\left(H^{\prime}, C_{i}^{\prime},(0,0,1), 1\right)$,
where we have used the fact that $G_{i}^{a}(0)=\bar{G}_{i}$ and $F_{i}^{a}\left(0, \bar{G}_{i}\right)=\bar{F}_{i}$, hence $C_{i}^{j \prime}=C_{i}^{a \prime}=$ $C_{i}^{\prime}$. The first equality implies that $C_{i}^{a}=C_{i}^{j}=C_{i}$ because utility is strictly increasing in consumption (Assumption 6). Woman $i$ is thus indifferent between ( $\left.H, C_{i},(1,0,0), 0\right)$ and $\left(H^{\prime}, C_{i}^{\prime},(0,0,1), 1\right)$ under regime $a$ and resolves indifference in favor of $\left(H, C_{i},(1,0,0), 0\right)$, this contradicts resolving indifference in favor of $\left(H^{\prime}, C_{i}^{\prime},(0,0,1), 1\right)$ under regime $j$.
18. $(\mathbf{D}, R)=((1,0,0), 1)$ and $\left(\mathbf{D}^{\prime}, R^{\prime}\right)=((0,0,1), 0)$ :
$U_{i}^{j}\left(H, C_{i},(1,0,0), 1\right)=U_{i}^{a}\left(H, C_{i},(1,0,0), 1\right)=U_{i}\left(H^{\prime}, C_{i}^{a \prime},(0,0,1), 0\right)=U_{i}\left(H^{\prime}, C_{i}^{j \prime},(0,0,1), 0\right)$,
where we have used the fact that $G_{i}^{a}(0)=\bar{G}_{i}$ and $F_{i}^{a}\left(0, \bar{G}_{i}\right)=\bar{F}_{i}$, hence $C_{i}^{j}=C_{i}^{a}=$ $C_{i}$. The last equality implies that $C_{i}^{a \prime}=C_{i}^{j \prime}=C_{i}^{\prime}$ because utility is strictly increasing in consumption (Assumption 6). Woman $i$ is thus indifferent between ( $H, C_{i},(1,0,0), 1$ ) and $\left(H^{\prime}, C_{i}^{\prime},(0,0,1), 0\right)$ under regime $a$ and resolves indifference in favor of $\left(H, C_{i},(1,0,0), 1\right)$, this contradicts resolving indifference in favor of $\left(H^{\prime}, C_{i}^{\prime},(0,0,1), 0\right)$ under regime $j$.
19. $(\mathbf{D}, R)=((1,0,0), 1)$ and $\left(\mathbf{D}^{\prime}, R^{\prime}\right)=((0,0,1), 1)$ :
$U_{i}^{j}\left(H, C_{i},(1,0,0), 1\right)=U_{i}^{a}\left(H, C_{i},(1,0,0), 1\right)=U_{i}^{a}\left(H^{\prime}, C_{i}^{\prime},(0,0,1), 1\right)=U_{i}^{j}\left(H^{\prime}, C_{i}^{\prime},(0,0,1), 1\right)$,
where we have used the fact that $G_{i}^{a}(0)=\bar{G}_{i}$, hence $C_{i}^{j}=C_{i}^{a}=C_{i}$, and that $\bar{F}_{i}=$ $F_{i}^{a}(0)$, hence $C_{i}^{j \prime}=C_{i}^{a \prime}=C_{i}^{\prime}$. Woman $i$ is thus indifferent between $\left(H, C_{i},(1,0,0), 1\right)$ and $\left(H^{\prime}, C_{i}^{\prime},(0,0,1), 1\right)$ under regime $a$ and resolves indifference in favor of $\left(H, C_{i},(1,0,0), 1\right)$, this contradicts resolving indifference in favor of $\left(H^{\prime}, C_{i}^{\prime},(0,0,1), 1\right)$ under regime $j$.
20. $(\mathbf{D}, R)=((0,0,1), 0)$ and $\left(\mathbf{D}^{\prime}, R^{\prime}\right)=(\mathbf{0}, 0):$

$$
U_{i}\left(H, C_{i}^{j},(0,0,1), 0\right)=U_{i}\left(H, C_{i}^{a},(0,0,1), 0\right)=U_{i}\left(H^{\prime}, C^{\prime}, \mathbf{0}, 0\right)=U_{i}\left(H^{\prime}, C^{\prime}, \mathbf{0}, 0\right)
$$

where we have used the fact that off assistance consumption is invariant to the regime, hence $C_{i}^{a \prime}=C_{i}^{j \prime}=C^{\prime}$. The first equality implies that $C_{i}^{a}=C_{i}^{j}=C_{i}$ because utility is strictly increasing in consumption (Assumption 6). Woman $i$ is thus indifferent between $\left(H, C_{i},(0,0,1), 0\right)$ and ( $\left.H^{\prime}, C^{\prime}, \mathbf{0}, 0\right)$ under regime $a$ and resolves indifference in favor of ( $\left.H, C_{i},(0,0,1), 0\right)$, this contradicts resolving indifference in favor of $\left(H^{\prime}, C^{\prime}, \mathbf{0}, 0\right)$ under regime $j$.
21. $(\mathbf{D}, R)=((0,0,1), 1)$ and $\left(\mathbf{D}^{\prime}, R^{\prime}\right)=(\mathbf{0}, 0)$ :

$$
U_{i}^{j}\left(H, C_{i},(0,0,1), 1\right)=U_{i}^{a}\left(H, C_{i},(0,0,1), 1\right)=U_{i}\left(H^{\prime}, C^{\prime}, \mathbf{0}, 0\right)=U_{i}\left(H^{\prime}, C^{\prime}, \mathbf{0}, 0\right),
$$

where we have used the fact that off assistance consumption is invariant to the regime, hence $C_{i}^{a \prime}=C_{i}^{j \prime}=C^{\prime}$, and the fact that $G_{i}^{a}(0)=\bar{G}_{i}$ and $F_{i}^{a}\left(0, \bar{G}_{i}\right)=\bar{F}_{i}$, hence $C_{i}^{j}=C_{i}^{a}=C_{i}$. Woman $i$ is thus indifferent between $\left(H, C_{i},(0,0,1), 1\right)$ and ( $H^{\prime}, C^{\prime}, \mathbf{0}, 0$ ) under regime $a$ and resolves indifference in favor of $\left(H, C_{i},(0,0,1), 1\right)$, this contradicts resolving indifference in favor of ( $H^{\prime}, C^{\prime}, \mathbf{0}, 0$ ) under regime $j$.
22. $(\mathbf{D}, R)=((0,0,1), 0)$ and $\left(\mathbf{D}^{\prime}, R^{\prime}\right)=((0,1,0), 0)$ :
$U_{i}\left(H, C_{i}^{j},(0,0,1), 0\right)=U_{i}\left(H, C_{i}^{a},(0,0,1), 0\right)=U_{i}\left(H^{\prime}, C_{i}^{\prime},(0,1,0), 0\right)=U_{i}\left(H^{\prime}, C_{i}^{\prime},(0,1,0), 0\right)$,
where we have used the fact that the FS-only policy is invariant to the regime, hence $C_{i}^{a \prime}=$ $C_{i}^{j \prime}=C_{i}^{\prime}$. The first equality implies that $C_{i}^{a}=C_{i}^{j}=C_{i}$ because utility is strictly increasing in consumption (Assumption 6). Woman $i$ is thus indifferent between $\left(H, C_{i},(0,0,1), 0\right)$ and $\left(H^{\prime}, C_{i}^{\prime},(0,1,0), 0\right)$ under regime $a$ and resolves indifference in favor of $\left(H, C_{i},(0,0,1), 0\right)$, this contradicts resolving indifference in favor of $\left(H^{\prime}, C_{i}^{\prime},(0,1,0), 0\right)$ under regime $j$.
23. $(\mathbf{D}, R)=((0,0,1), 1)$ and $\left(\mathbf{D}^{\prime}, R^{\prime}\right)=((0,1,0), 0)$ :
$U_{i}^{j}\left(H, C_{i},(0,0,1), 1\right)=U_{i}^{a}\left(H, C_{i},(0,0,1), 1\right)=U_{i}\left(H^{\prime}, C_{i}^{\prime},(0,1,0), 0\right)=U_{i}\left(H^{\prime}, C_{i}^{\prime},(0,1,0), 0\right)$,
where we have used the fact that the FS-only policy is invariant to the regime, hence $C_{i}^{a \prime}=$ $C_{i}^{j \prime}=C_{i}^{\prime}$, and the fact that $G_{i}^{a}(0)=\bar{G}_{i}$ and $F_{i}^{a}\left(0, \bar{G}_{i}\right)=\bar{F}_{i}$, hence $C_{i}^{j}=C_{i}^{a}=C_{i}$. Woman $i$ is thus indifferent between $\left(H, C_{i},(0,0,1), 1\right)$ and $\left(H^{\prime}, C_{i}^{\prime},(0,1,0), 0\right)$ under regime $a$ and resolves indifference in favor of $\left(H, C_{i},(0,0,1), 1\right)$, this contradicts resolving indifference in favor of $\left(H^{\prime}, C_{i}^{\prime},(0,1,0), 0\right)$ under regime $j$.
24. $(\mathbf{D}, R)=((0,0,1), 0)$ and $\left(\mathbf{D}^{\prime}, R^{\prime}\right)=((1,0,0), 0)$ :
$U_{i}\left(H, C_{i}^{j},(0,0,1), 0\right)=U_{i}\left(H, C_{i}^{a},(0,0,1), 0\right)=U_{i}\left(H^{\prime}, C_{i}^{a \prime},(1,0,0), 0\right)=U_{i}\left(H^{\prime}, C_{i}^{j \prime},(1,0,0), 0\right)$.
The first equality implies that $C_{i}^{a}=C_{i}^{j}=C_{i}$ and the last equality implies $C_{i}^{a \prime}=C_{i}^{j \prime}=$ $C_{i}^{\prime}$, because utility is strictly increasing in consumption (Assumption 6). Woman $i$ is thus indifferent between $\left(H, C_{i},(0,0,1), 0\right)$ and ( $\left.H^{\prime}, C_{i}^{\prime},(1,0,0), 0\right)$ under regime $a$ and resolves indifference in favor of $\left(H, C_{i},(0,0,1), 0\right)$, this contradicts resolving indifference in favor of ( $\left.H^{\prime}, C_{i}^{\prime},(1,0,0), 0\right)$ under regime $j$.
25. $(\mathbf{D}, R)=((0,0,1), 0)$ and $\left(\mathbf{D}^{\prime}, R^{\prime}\right)=((1,0,0), 1)$ :
$U_{i}\left(H, C_{i}^{j},(0,0,1), 0\right)=U_{i}\left(H, C_{i}^{a},(0,0,1), 0\right)=U_{i}^{a}\left(H^{\prime}, C_{i}^{\prime},(1,0,0), 1\right)=U_{i}^{j}\left(H^{\prime}, C_{i}^{\prime},(1,0,0), 1\right)$,
where we have used the fact that $G_{i}^{a}(0)=\bar{G}_{i}$, hence $C_{i}^{j \prime}=C_{i}^{a \prime}=C_{i}^{\prime}$. The first equality implies that $C_{i}^{a}=C_{i}^{j}=C_{i}$ because utility is strictly increasing in consumption (Assumption 6 ). Woman $i$ is thus indifferent between ( $\left.H, C_{i},(0,0,1), 0\right)$ and ( $\left.H^{\prime}, C_{i}^{\prime},(1,0,0), 1\right)$ under regime $a$ and resolves indifference in favor of $\left(H, C_{i},(0,0,1), 0\right)$, this contradicts resolving indifference in favor of ( $\left.H^{\prime}, C_{i}^{\prime},(1,0,0), 1\right)$ under regime $j$.
26. $(\mathbf{D}, R)=((0,0,1), 1)$ and $\left(\mathbf{D}^{\prime}, R^{\prime}\right)=((1,0,0), 0)$ :
$U_{i}^{j}\left(H, C_{i},(0,0,1), 1\right)=U_{i}^{a}\left(H, C_{i},(0,0,1), 1\right)=U_{i}\left(H^{\prime}, C_{i}^{a \prime},(1,0,0), 0\right)=U_{i}\left(H^{\prime}, C_{i}^{j \prime},(1,0,0), 0\right)$,
where we have used the fact that $G_{i}^{a}(0)=\bar{G}_{i}$ and $F_{i}^{a}\left(0, \bar{G}_{i}\right)=\bar{F}_{i}$, hence $C_{i}^{j}=C_{i}^{a}=$ $C_{i}$. The last equality implies that $C_{i}^{a \prime}=C_{i}^{j \prime}=C_{i}^{\prime}$ because utility is strictly increasing in consumption (Assumption 6). Woman $i$ is thus indifferent between ( $H, C_{i},(0,0,1), 1$ ) and $\left(H^{\prime}, C_{i}^{\prime},(1,0,0), 0\right)$ under regime $a$ and resolves indifference in favor of $\left(H, C_{i},(0,0,1), 1\right)$, this contradicts resolving indifference in favor of $\left(H^{\prime}, C_{i}^{\prime},(1,0,0), 0\right)$ under regime $j$.
27. $(\mathbf{D}, R)=((0,0,1), 1)$ and $\left(\mathbf{D}^{\prime}, R^{\prime}\right)=((1,0,0), 1)$ :
$U_{i}^{j}\left(H, C_{i},(0,0,1), 1\right)=U_{i}^{a}\left(H, C_{i},(0,0,1), 1\right)=U_{i}^{a}\left(H^{\prime}, C_{i}^{\prime},(1,0,0), 1\right)=U_{i}^{j}\left(H^{\prime}, C_{i}^{\prime},(1,0,0), 1\right)$,
where we have used the fact that $G_{i}^{a}(0)=\bar{G}_{i}$ and $F_{i}^{a}\left(0, \bar{G}_{i}\right)=\bar{F}_{i}$, hence $C_{i}^{j}=C_{i}^{a}=$ $C_{i}$ and $C_{i}^{j \prime}=C_{i}^{a \prime}=C_{i}^{\prime}$. Woman $i$ is thus indifferent between $\left(H, C_{i},(0,0,1), 1\right)$ and $\left(H^{\prime}, C_{i}^{\prime},(1,0,0), 1\right)$ under regime $a$ and resolves indifference in favor of $\left(H, C_{i},(0,0,1), 1\right)$, this contradicts resolving indifference in favor of $\left(H^{\prime}, C_{i}^{\prime},(1,0,0), 1\right)$ under regime $j$.
28. $(\mathbf{D}, R)=((0,0,1), 0)$ and $\left(\mathbf{D}^{\prime}, R^{\prime}\right)=((0,0,1), 0)$ :
$U_{i}\left(H, C_{i}^{j},(0,0,1), 0\right)=U_{i}\left(H, C_{i}^{a},(0,0,1), 0\right)=U_{i}\left(H^{\prime}, C_{i}^{a \prime},(0,0,1), 0\right)=U_{i}\left(H^{\prime}, C_{i}^{j \prime},(0,0,1), 0\right)$.
The first equality implies that $C_{i}^{a}=C_{i}^{j}=C_{i}$ and the last equality implies $C_{i}^{a \prime}=C_{i}^{j \prime}=$ $C_{i}^{\prime}$, because utility is strictly increasing in consumption (Assumption 6). Woman $i$ is thus indifferent between $\left(H, C_{i},(0,0,1), 0\right)$ and $\left(H^{\prime}, C_{i}^{\prime},(0,0,1), 0\right)$ under regime $a$ and resolves indifference in favor of $\left(H, C_{i},(0,0,1), 0\right)$, this contradicts resolving indifference in favor of $\left(H^{\prime}, C_{i}^{\prime},(0,0,1), 0\right)$ under regime $j$.
29. $(\mathbf{D}, R)=((0,0,1), 0)$ and $\left(\mathbf{D}^{\prime}, R^{\prime}\right)=((0,0,1), 1)$ :
$U_{i}\left(H, C_{i}^{j},(0,0,1), 0\right)=U_{i}\left(H, C_{i}^{a},(0,0,1), 0\right)=U_{i}^{a}\left(H^{\prime}, C_{i}^{\prime},(0,0,1), 1\right)=U_{i}^{j}\left(H^{\prime}, C_{i}^{\prime},(0,0,1), 1\right)$,
where we have used the fact that $G_{i}^{a}(0)=\bar{G}_{i}$ and $F_{i}^{a}\left(0, \bar{G}_{i}\right)=\bar{F}_{i}$, hence $C_{i}^{j \prime}=C_{i}^{a \prime}=$ $C_{i}^{\prime}$. The first equality implies that $C_{i}^{a}=C_{i}^{j}=C_{i}$ because utility is strictly increasing in consumption (Assumption 6). Woman $i$ is thus indifferent between $\left(H, C_{i},(0,0,1), 0\right)$ and $\left(H^{\prime}, C_{i}^{\prime},(0,0,1), 1\right)$ under regime $a$ and resolves indifference in favor of $\left(H, C_{i},(0,0,1), 0\right)$, this contradicts resolving indifference in favor of $\left(H^{\prime}, C_{i}^{\prime},(0,0,1), 1\right)$ under regime $j$.
30. $(\mathbf{D}, R)=((0,0,1), 1)$ and $\left(\mathbf{D}^{\prime}, R^{\prime}\right)=((0,0,1), 0)$ :
$U_{i}^{j}\left(H, C_{i},(0,0,1), 1\right)=U_{i}^{a}\left(H, C_{i},(0,0,1), 1\right)=U_{i}\left(H^{\prime}, C_{i}^{a \prime},(0,0,1), 0\right)=U_{i}\left(H^{\prime}, C_{i}^{j \prime},(0,0,1), 0\right)$,
where we have used the fact that $G_{i}^{a}(0)=\bar{G}_{i}$ and $F_{i}^{a}\left(0, \bar{G}_{i}\right)=\bar{F}_{i}$, hence $C_{i}^{j}=C_{i}^{a}=C_{i}$. The last equality implies $C_{i}^{a \prime}=C_{i}^{j \prime}=C_{i}^{\prime}$, because utility is strictly increasing in consumption (Assumption 6 ). Woman $i$ is thus indifferent between $\left(H, C_{i},(0,0,1), 1\right)$ and $\left(H^{\prime}, C_{i}^{\prime},(0,0,1), 0\right)$ under regime $a$ and resolves indifference in favor of $\left(H, C_{i},(0,0,1), 1\right)$, this contradicts resolving indifference in favor of $\left(H^{\prime}, C_{i}^{\prime},(0,0,1), 0\right)$ under regime $j$.
31. $(\mathbf{D}, R)=((0,0,1), 1)$ and $\left(\mathbf{D}^{\prime}, R^{\prime}\right)=((0,0,1), 1)$ :
$U_{i}^{j}\left(H, C_{i},(0,0,1), 1\right)=U_{i}^{a}\left(H, C_{i},(0,0,1), 1\right)=U_{i}^{a}\left(H^{\prime}, C_{i}^{\prime},(0,0,1), 1\right)=U_{i}^{j}\left(H^{\prime}, C_{i}^{\prime},(0,0,1), 1\right)$,
where we have used the fact that $G_{i}^{a}(0)=\bar{G}_{i}$ and $F_{i}^{a}\left(0, \bar{G}_{i}\right)=\bar{F}_{i}$, hence $C_{i}^{j}=C_{i}^{a}=$ $C_{i}$ and $C_{i}^{j \prime}=C_{i}^{a \prime}=C_{i}^{\prime}$. Woman $i$ is thus indifferent between $\left(H, C_{i},(0,0,1), 1\right)$ and $\left(H^{\prime}, C_{i}^{\prime},(0,0,1), 1\right)$ under regime $a$ and resolves indifference in favor of $\left(H, C_{i},(0,0,1), 1\right)$, this contradicts resolving indifference in favor of $\left(H^{\prime}, C_{i}^{\prime},(0,0,1), 1\right)$ under regime $j$.
32. $(\mathbf{D}, R)=(\mathbf{0}, 0)$ and $\left(\mathbf{D}^{\prime}, R^{\prime}\right)=((0,1,0), 0)$ :

$$
U_{i}(H, C, \mathbf{0}, 0)=U_{i}(H, C, \mathbf{0}, 0)=U_{i}\left(H^{\prime}, C_{i}^{\prime},(0,1,0), 0\right)=U_{i}\left(H^{\prime}, C_{i}^{\prime},(0,1,0), 0\right),
$$

where we have used the fact that off assistance consumption is invariant to the regime, hence $C_{i}^{a}=C_{i}^{j}=C$ and the fact that the FS-only policy is invariant to the policy regime, hence $C_{i}^{a \prime}=C_{i}^{j \prime}=C_{i}^{\prime}$. Woman $i$ is thus indifferent between $(H, C, \mathbf{0}, 0)$ and ( $\left.H^{\prime}, C_{i}^{\prime},(0,1,0), 0\right)$ under regime $a$ and resolves indifference in favor of ( $H, C, \mathbf{0}, 0$ ), this contradicts resolving indifference in favor of $\left(H^{\prime}, C_{i}^{\prime},(0,1,0), 0\right)$ under regime $j$.
33. $(\mathbf{D}, R)=(\mathbf{0}, 0)$ and $\left(\mathbf{D}^{\prime}, R^{\prime}\right)=((1,0,0), 0)$ :

$$
U_{i}(H, C, \mathbf{0}, 0)=U_{i}(H, C, \mathbf{0}, 0)=U_{i}\left(H^{\prime}, C_{i}^{a \prime},(1,0,0), 0\right)=U_{i}\left(H^{\prime}, C_{i}^{j \prime},(1,0,0), 0\right)
$$

where we have used the fact that off assistance consumption is invariant to the regime, hence $C_{i}^{a}=C_{i}^{j}=C$. The last equality implies $C_{i}^{a \prime}=C_{i}^{j \prime}=C_{i}^{\prime}$, because utility is strictly increasing in consumption (Assumption 6). Woman $i$ is thus indifferent between ( $H, C, \mathbf{0}, 0$ ) and $\left(H^{\prime}, C_{i}^{\prime},(1,0,0), 0\right)$ under regime $a$ and resolves indifference in favor of $(H, C, \mathbf{0}, 0)$, this contradicts resolving indifference in favor of $\left(H^{\prime}, C_{i}^{\prime},(1,0,0), 0\right)$ under regime $j$.
34. $(\mathbf{D}, R)=(\mathbf{0}, 0)$ and $\left(\mathbf{D}^{\prime}, R^{\prime}\right)=((1,0,0), 1)$ :

$$
U_{i}(H, C, \mathbf{0}, 0)=U_{i}(H, C, \mathbf{0}, 0)=U_{i}^{a}\left(H^{\prime}, C_{i}^{\prime},(1,0,0), 1\right)=U_{i}^{j}\left(H^{\prime}, C_{i}^{\prime},(1,0,0), 1\right),
$$

where we have used the fact that off assistance consumption is invariant to the regime, hence $C_{i}^{a}=C_{i}^{j}=C$ and the fact that $\bar{G}_{i}=G_{i}^{a}(0)$, hence $C_{i}^{j \prime}=C_{i}^{a \prime}=C_{i}^{\prime}$. Woman $i$ is thus indifferent between $(H, C, \mathbf{0}, 0)$ and $\left(H^{\prime}, C_{i}^{\prime},(1,0,0), 1\right)$ under regime $a$ and resolves indifference in favor of $(H, C, \mathbf{0}, 0)$, this contradicts resolving indifference in favor of $\left(H^{\prime}, C_{i}^{\prime},(1,0,0), 1\right)$ under regime $j$.
35.
$(\mathbf{D}, R)=(\mathbf{0}, 0)$ and $\left(\mathbf{D}^{\prime}, R^{\prime}\right)=((0,0,1), 0)$ :

$$
U_{i}(H, C, \mathbf{0}, 0)=U_{i}(H, C, \mathbf{0}, 0)=U_{i}\left(H^{\prime}, C_{i}^{a \prime},(0,0,1), 0\right)=U_{i}\left(H^{\prime}, C_{i}^{j \prime},(0,0,1), 0\right)
$$

where we have used the fact that off assistance consumption is invariant to the regime, hence $C_{i}^{a}=C_{i}^{j}=C$. The last equality implies $C_{i}^{a \prime}=C_{i}^{j \prime}=C_{i}^{\prime}$, because utility is strictly increasing in consumption (Assumption 6). Woman $i$ is thus indifferent between ( $H, C, \mathbf{0}, 0$ ) and $\left(H^{\prime}, C_{i}^{\prime},(0,0,1), 0\right)$ under regime $a$ and resolves indifference in favor of $(H, C, \mathbf{0}, 0)$, this contradicts resolving indifference in favor of $\left(H^{\prime}, C_{i}^{\prime},(0,0,1), 0\right)$ under regime $j$.
36. $(\mathbf{D}, R)=(\mathbf{0}, 0)$ and $\left(\mathbf{D}^{\prime}, R^{\prime}\right)=((0,0,1), 1)$ :

$$
U_{i}(H, C, \mathbf{0}, 0)=U_{i}(H, C, \mathbf{0}, 0)=U_{i}^{a}\left(H^{\prime}, C_{i}^{\prime},(0,0,1), 1\right)=U_{i}^{j}\left(H^{\prime}, C_{i}^{\prime},(0,0,1), 1\right),
$$

where we have used the fact that off assistance consumption is invariant to the regime, hence $C_{i}^{a}=C_{i}^{j}=C$, and the fact that $\bar{G}_{i}=G_{i}^{a}(0)$ and $\bar{F}_{i}=F_{i}^{a}(0)$, hence $C_{i}^{j \prime}=C_{i}^{a \prime}=C_{i}^{\prime}$. Woman $i$ is thus indifferent between $(H, C, \mathbf{0}, 0)$ and ( $\left.H^{\prime}, C_{i}^{\prime},(0,0,1), 1\right)$ under regime $a$ and resolves indifference in favor of ( $H, C, \mathbf{0}, 0$ ), this contradicts resolving indifference in favor of $\left(H^{\prime}, C_{i}^{\prime},(0,0,1), 1\right)$ under regime $j$.

## Lemma 11 (Policy Impact on Attractiveness of States). Given Assumptions 6-12:

1. the states in $\mathcal{C}_{+}$are no worse under regime $j$ than under regime $a$,
2. the states in $\mathcal{C}_{-}$are no better under regime $j$ than under regime $a$,
3. the states in $\mathcal{C}_{0}$ are equally attractive under regimes $j$ and $a$.

Proof. We prove each statement in turn.

1. The states in $C_{+}$are no worse under $j$ than under regime $a$.

The only two states in $\mathcal{C}_{+}$are $1 r n$ and $1 r r$. The alternatives compatible with these states entail $E$ in range 1 , and, respectively, $\left(D^{w}, E^{r}\right)=(1, E)$ or $\left(D^{w f}, E^{r}\right)=(1, E)$. Thus, the utility function associated with each of these alternatives is invariant to the treatment (A. 4 in Assumption 6). Accordingly, it suffices to show that the consumption associated with any one of these alternatives is not lower under regime $j$ than under regime $a$, that is, $C_{i}^{j}\left(E, \mathbf{D}, E^{r}\right) \geq C_{i}^{a}\left(E, \mathbf{D}, E^{r}\right)$ for all $\left(E, \mathbf{D}, E^{r}\right)$ such that $s\left(E, \mathbf{D}, E^{r}\right) \in \mathcal{C}_{+}$. Consider first state $1 r r \in \mathcal{C}_{+}$so that $\left(E, \mathbf{D}, E^{r}\right)=(E,(0,0,1), E)$. By Lemma 7 part 1$), \bar{G}_{i}+F_{i}\left(E, \bar{G}_{i}\right) \geq$ $G_{i}^{a}(E)+F_{i}\left(E, G_{i}^{a}(E)\right)$ for all $E$ in range $1,{ }^{8}$ thus
$C_{i}^{j}(E,(0,0,1), E)=T_{i}(E)+\bar{G}_{i}+F_{i}\left(E, \bar{G}_{i}\right) \geq T_{i}(E)+G_{i}^{a}(E)+F_{i}\left(E, \bar{G}_{i}\right)=C_{i}^{a}(E,(0,0,1), E)$,
which verifies the desired inequality. Consider next state $1 r n \in \mathcal{C}_{+}$so that $\left(E, \mathbf{D}, E^{r}\right)=$ $(E,(1,0,0), E)$. Because $\bar{G}_{i} \geq G_{i}^{a}(E)$ for all $E$ in range 1,

$$
C_{i}^{j}(E,(1,0,0), E)=T_{i}(E)+\bar{G}_{i} \geq T_{i}(E)+G_{i}^{a}(E)=C_{i}^{a}(E,(1,0,0), E),
$$

which verifies the desired inequality.

## 2. The states in $C_{-}$are no better under $j$ than under regime $a$.

The only two states in $\mathcal{C}_{-}$are $0 r n$ and $0 r r$. It suffices to show that the utility associated with any alternative compatible with states $0 r n$ and $0 r r$ is at least as high under regime $a$ than under regime $j$. Consider a tuple obeying $s\left(E, \mathbf{D}, E^{r}\right) \in C_{-}$. The alternatives compatible with state $0 r n$ are such that $\left(E, \mathbf{D}, E^{r}\right)=(0,(1,0,0), 0)$ hence $C_{i}^{t}\left(E, \mathbf{D}, E^{r}\right)=\bar{G}_{i}$ all $t$ and $R\left(\mathbf{D}, E^{r}\right)=1$. The alternatives compatible with state $0 r r$ are such that $\left(E, \mathbf{D}, E^{r}\right)=$

[^28]$(0,(0,0,1), 0)$ hence $C_{i}^{t}\left(E, \mathbf{D}, E^{r}\right)=\bar{G}_{i}+\bar{F}_{i}$ all $t$ and $R\left(\mathbf{D}, E^{r}\right)=1$. Thus, it suffices to show that
$$
U_{i}^{a}\left(0, \bar{G}_{i},(1,0,0), 1\right) \geq U_{i}^{j}\left(0, \bar{G}_{i},(1,0,0), 1\right)
$$
and
$$
U_{i}^{a}\left(0, \bar{G}_{i}+\bar{F}_{i},(0,0,1), 1\right) \geq U_{i}^{j}\left(0, \bar{G}_{i}+\bar{F}_{i},(0,0,1), 1\right)
$$

Both inequalities hold by A. 3 in Assumption 6 (hassle disutility is no lower under $j$ than under $a$ ).
3. The states in $\mathcal{C}_{0}$ are equally attractive under regimes $j$ and $a$.

Write $\mathcal{C}_{0}$ as the union of two disjoint collections:

$$
\begin{align*}
& \{0 n n, 1 n n, 2 n n, 0 n r, 1 n r, 2 n r, 1 n u, 2 n u\}  \tag{148}\\
& \{1 u n, 2 u n, 1 u u, 2 u u\} \tag{149}
\end{align*}
$$

The alternatives compatible with states in collection (148) entail no assistance or FS-only assistance. Thus the utility associated with each of these alternatives is invariant to the policy regime (A. 4 and A. 5 in Assumption 6). Accordingly, it suffices to show that the consumption associated with any of these alternatives is the same under regimes $j$ and $a$. Consider first the alternatives compatible with an off-assistance state $s_{i} \in\{0 n n, 1 n n, 2 n n\}$ in collection (148). If $s_{i} \in\{0 n n\}$, consumption is zero. If $s_{i} \in\{1 n n, 2 n n\}$ consumption equals $E$. Thus, consumption is the same under either regime. Consider next the alternatives compatible with a FS-only state $s_{i} \in\{0 n r, 1 n r, 2 n r, 1 n u, 2 n u\}$ in collection (148). If $s_{i} \in$ $\{0 n r\}$, consumption equals $\overline{\bar{F}}_{i}$. If $s_{i} \in\{1 n r, 2 n r\}$, consumption equals $E+F_{i}(E, 0)$. If $s_{i} \in\{1 n u, 2 n u\}$ consumption equals $E+\overline{\bar{F}}_{i}-\omega_{i}$ by optimal reporting (Lemma 9). Thus, consumption is the same under either regime. Finally consider the alternatives compatible with states in collection (149). Given optimal reporting, the utility function associated with all the alternatives compatible with states $\{1 u n, 2 u n, 1 u u, 2 u u\}$ is invariant to the policy regime (Corollary 6). Accordingly, it suffices to show that the consumption associated with any one of these alternatives is the same under regimes $j$ and $a$. If $s_{i} \in\{1 u n, 2 u n\}$, consumption is $E+\bar{G}_{i}-\kappa_{i}$ under both regimes. If $s_{i} \in\{1 u u, 2 u u\}$, consumption is $E+\bar{G}_{i}+\bar{F}_{i}-\gamma_{i}$ under both regimes. Thus, consumption is the same under either regime.

## Main Propositions

Proposition 6 (Restricted Pairings). Given Assumptions 6-12, the pairings of states corresponding to the "-" entries in Table A5 are disallowed.

Proof. States $1 u n$ and $1 u u$ are dominated under regime $j$ (Lemma 9). Therefore no woman pairs state $s^{a}$ with state $s^{j} \in\{1 u n, 1 u u\}$ for any $s^{a} \in \mathcal{S}$. Next, by Lemmas 6 and 11, no pairing of state $s^{a}$ with state $s^{j}$ can occur for all $\left(s^{a}, s^{j}\right)$ in the collection

$$
\begin{equation*}
\left\{\left(s^{a}, s^{j}\right): s^{a} \in \mathcal{C}_{0} \cup \mathcal{C}_{+}, s^{j} \in \mathcal{C}_{0} \cup \mathcal{C}_{-}, s^{a} \neq s^{j}\right\} \tag{150}
\end{equation*}
$$

It suffices to show that the properties I)-III) of Lemma 10 are met. Property I) holds trivially and properties II) and III) hold by Lemma 11. Therefore no woman selects any of the pairings in (150).

Proposition 7 (Unrestricted Pairings). Given Assumptions 6-12, the pairings of states corresponding to the non "-" entries in Table A5 are allowed.

Remark 10 (Omitted Proof of Proposition 7). The proof of Proposition 7 would mimic the proof of Proposition 2 in that it would present examples of women who select the pairings corresponding to the non "-" entries in Table A5. We omit the proof of Proposition 7 for two reasons. First, there are 63 allowed pairings in Table A5, which makes the proof exceedingly long. Second, our interest lies in showing that the integrated response matrix of the extended model contains at least as many restrictions as the response matrix of the baseline model (Proposition 9 and Remark 11 below). The proof of Proposition 7 would only serve to confirm the additional result that the integrated response matrix of the extended model contains at most as many restrictions as the response matrix of the baseline model.

Proposition 8 (Response Matrix). Let $\Pi$ denote the matrix of response probabilities $\left\{\pi_{s^{a}, s^{j}}: s^{a}, s^{j} \in \mathcal{S}\right\}$. Given Table A5, $\Pi$ is a $16 \times 16$ matrix with the following zero ( 0 ) and non-zero ( $X$ ) entries:

|  | JF's Experimental Policy: Earnings / Program Participation State |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Control | 0nn | 1 nn | 2nn | 0nr | 1 nr | 2nr | 1nu | 2nu | 0rn | 1rn | 1 un | 2un | Orr | 1 rr | 1uu | 2uu |
| 0nn | X | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | X | 0 | 0 | 0 | X | 0 | 0 |
| 1 nn | 0 | X | 0 | 0 | 0 | 0 | 0 | 0 | 0 | X | 0 | 0 | 0 | X | 0 | 0 |
| 2 nn | 0 | 0 | X | 0 | 0 | 0 | 0 | 0 | 0 | X | 0 | 0 | 0 | X | 0 | 0 |
| 0nr | 0 | 0 | 0 | X | 0 | 0 | 0 | 0 | 0 | X | 0 | 0 | 0 | X | 0 | 0 |
| 1 nr | 0 | 0 | 0 | 0 | X | 0 | 0 | 0 | 0 | X | 0 | 0 | 0 | X | 0 | 0 |
| 2 nr | 0 | 0 | 0 | 0 | 0 | X | 0 | 0 | 0 | X | 0 | 0 | 0 | X | 0 | 0 |
| 1 nu | 0 | 0 | 0 | 0 | 0 | 0 | X | 0 | 0 | X | 0 | 0 | 0 | X | 0 | 0 |
| 2nu | 0 | 0 | 0 | 0 | 0 | 0 | 0 | X | 0 | X | 0 | 0 | 0 | X | 0 | 0 |
| 0rn | X | X | X | X | X | X | X | X | X | X | 0 | X | X | X | 0 | X |
| 1 n | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | X | 0 | 0 | 0 | X | 0 | 0 |
| 1 un | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | X | 0 | 0 | 0 | X | 0 | 0 |
| 2 un | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | X | 0 | X | 0 | X | 0 | 0 |
| 0rr | X | X | X | X | X | X | X | X | X | X | 0 | X | X | X | 0 | X |
| 1 rr | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | X | 0 | 0 | 0 | X | 0 | 0 |
| 1uu | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | X | 0 | 0 | 0 | X | 0 | 0 |
| 2 uu | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | X | 0 | 0 | 0 | X | 0 | X |

Proof. By Definition 29, $\pi_{s^{a}, s^{j}} \equiv P\left(S_{i}^{j}=s^{j} \mid S_{i}^{a}=s^{a}\right)$. By Proposition 6, the pairings of states corresponding to the "-" entries in Table A5 are disallowed. Thus, $\pi_{s^{a}, s^{j}}=0$ for any pairing $\left(s^{a}, s^{j}\right)$ corresponding to a "-" entry in Table A5 because no woman occupies state $s^{a}$ under regime $a$ and state $s^{j}$ under regime $j$. By Proposition 7, the pairings of states corresponding to the non "-" entries in Table A5 are allowed. Thus, $\pi_{s^{a}, s^{j}} \neq 0$ for any pairing $\left(s^{a}, s^{j}\right)$ corresponding to a non "-" entry in Table A5 because some women may occupy state $s^{a}$ under regime $a$ and state $s^{j}$ under regime $j$.

Proposition 9 (Integrated Response Matrix). The matrix of response probabilities over the states in $\mathcal{S}, \Pi$, reduces to the following matrix $\Pi_{w}$ of response probabilities over the states in $\mathcal{S}_{w}$ :

|  | JF's Experimental Policy: Earnings / Program Participation State |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Control | $0 n$ | $1 n$ | $2 n$ | $0 r$ | $1 r$ | $1 u$ | $2 u$ |
| $0 n$ | $1-\pi_{0 n, 1 r}$ | 0 | 0 | 0 | 0 | $\pi_{0 n, 1 r}$ | 0 |
| $1 n$ | 0 | $1-\pi_{1 n, 1 r}$ | 0 | 0 | $\pi_{1 n, 1 r}$ | 0 | 0 |
| $2 n$ | 0 | 0 | $1-\pi_{2 n, 1 r}$ | $\pi_{2 n, 1 r}$ | 0 | 0 |  |
| $0 r$ | $\pi_{0 r, 0 n}$ | $\pi_{0 r, 1 n}$ | $\pi_{0 r, 2 n}$ | $1-\pi_{0 r, 0 n}-\pi_{0 r, 1 n}-\pi_{0 r, 2 n}$ | $\pi_{0 r, 1 r}$ | 0 | $\pi_{0 r, 2 u}$ |
| $1 r$ | 0 | 0 | 0 | $\pi_{0 r, 1 r}-\pi_{0 r, 2 u}$ |  |  |  |
| $1 u$ | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| $2 u$ | 0 | 0 | 0 | 0 | 1 | 0 | 0 |

where

$$
\begin{aligned}
\pi_{0 n, 1 r} & \equiv\left(\pi_{0 n n, 1 r n}+\pi_{0 n n, 1 r r}\right) \frac{q_{0 n n}^{a}}{p_{0 n}^{a}}+\left(\pi_{0 n r, 1 r n}+\pi_{0 n r, 1 r r}\right) \frac{p_{0 n}^{a}-q_{0 n n}^{a}}{p_{0 n}^{a}}, \\
\pi_{1 n, 1 r} & \equiv\left(\pi_{1 n n, 1 r n}+\pi_{1 n n, 1 r r}\right) \frac{q_{1 n n}^{a}}{p_{1 n}^{a}}+\left(\pi_{1 n r, 1 r n}+\pi_{1 n r, 1 r r}\right) \frac{q_{1 n r}^{a}}{p_{1 n}^{a}}+\left(\pi_{1 n u, 1 r n}+\pi_{1 n u, 1 r r}\right) \frac{p_{1 n}^{a}-q_{1 n n}^{a}-q_{1 n r}^{a}}{p_{1 n}^{a}}, \\
\pi_{2 n, 1 r} & \equiv\left(\pi_{2 n n, 1 r n}+\pi_{2 n n, 1 r r}\right) \frac{q_{2 n n}^{a}}{p_{2 n}^{a}}+\left(\pi_{2 n r, 1 r n}+\pi_{2 n r, 1 r r}\right) \frac{q_{2 n r}^{a}}{p_{2 n}^{a}}+\left(\pi_{2 n u, 1 r n}+\pi_{2 n u, 1 r r}\right) \frac{p_{2 n}^{a}-q_{2 n n}^{a}-q_{2 n r}^{a}}{p_{2 n}^{a}}, \\
\pi_{0 r, 0 n} & \equiv\left(\pi_{0 r n, 0 n n}+\pi_{0 r n, 0 n r}\right) \frac{q_{0 r n}^{a}}{p_{0 r}^{a}}+\left(\pi_{0 r r, 0 n n}+\pi_{0 r r, 0 n r}\right) \frac{p_{0 r}^{a}-q_{0 r n}^{a}}{p_{0 r}^{a}}, \\
\pi_{0 r, 1 n} & \equiv\left(\pi_{0 r n, 1 n n}+\pi_{0 r n, 1 n r}+\pi_{0 r n, 1 n u}\right) \frac{q_{0 r n}^{a}}{p_{0 r}^{a}}+\left(\pi_{0 r r, 1 n n}+\pi_{0 r r, 1 n r}+\pi_{0 r r, 1 n u}\right) \frac{p_{0 r}^{a}-q_{0 r n}^{a}}{p_{0 r}^{a}}, \\
\pi_{0 r, 2 n} & \equiv\left(\pi_{0 r n, 2 n n}+\pi_{0 r n, 2 n r}+\pi_{0 r n, 2 n u}\right) \frac{q_{0 r n}^{a}}{p_{0 r}^{a}}+\left(\pi_{0 r r, 2 n n}+\pi_{0 r r, 2 n r}+\pi_{0 r r, 2 n u}\right) \frac{p_{0 r}^{a}-q_{0 r n}^{a}}{p_{0 r}^{a}}, \\
\pi_{0 r, 1 r} & \equiv\left(\pi_{0 r n, 1 r n}+\pi_{0 r n, 1 r r}\right) \frac{q_{0 r n}^{a}}{p_{0 r}^{a}}+\left(\pi_{0 r r, 1 r n}+\pi_{0 r r, 1 r r}\right) \frac{p_{0 r}^{a}-q_{0 r n}^{a}}{p_{0 r}^{a}}, \\
\pi_{0 r, 2 u} & \equiv\left(\pi_{0 r n, 2 u n}+\pi_{0 r n, 2 u u}\right) \frac{q_{0 r n}^{a}}{p_{0 r}^{a}}+\left(\pi_{0 r r, 2 u n}+\pi_{0 r r, 2 u u}\right) \frac{p_{0 r}^{a}-q_{0 r n}^{a}}{p_{0 r}^{a}}, \\
\pi_{2 u, 1 r} & \equiv\left(\pi_{2 u n, 1 r n}+\pi_{2 u n, 1 r r}\right) \frac{q_{2 u n}^{a}}{p_{2 u}^{a}}+\left(\pi_{2 u u, 1 r n}+\pi_{2 u u, 1 r r}\right) \frac{p_{2 u}^{a}-q_{2 u n}^{a}}{p_{2 u}^{a}}, \\
\pi_{1 r, 1 r} & =1, \\
\pi_{1 u, 1 r} & =1 .
\end{aligned}
$$

Proof. The response probabilities over the states in $\mathcal{S}_{w}$ are of the form:

$$
\pi_{s_{w}^{a}, s_{w}^{j}} \equiv \operatorname{Pr}\left(S_{w, i}^{j}=s_{w}^{j} \mid S_{w, i}^{a}=s_{w}^{a}\right)=\sum_{s^{j} \in \mathcal{S}: s_{w}^{j}=h\left(s^{j}\right)}\left[\sum_{s^{a} \in \mathcal{S}: s_{w}^{a}=h\left(s^{a}\right)} \operatorname{Pr}\left(S_{i}^{j}=s^{j} \mid S_{i}^{a}=s^{a}\right) \frac{q_{s^{a}}^{a}}{p_{s_{w}^{a}}^{a}}\right]
$$

Remark 11 (Relationship between the Restrictions in the Baseline and in the Extended Model). The response matrix implied by the baseline model has the same zero and unitary entries as the response matrix $\Pi_{w}$ implied by the extended model.

## 10 Finer Earning Ranges

In this section we consider a finer coarsening of earnings. Specifically, we partition earnings above the federal poverty level into two sub-ranges. We begin with some definitions that supersede those in Section 4 of this Appendix. We conclude with the analytical bounds for two "opt-in" response probabilities. Proofs are omitted because they closely mimic those accompanying the baseline coarsening approach.

Definition 31 (Earning Ranges). Earnings range 0 refers to zero earnings. Earnings range 1 refers to the interval $\left(0, F P L_{i}\right]$ where $F P L_{i}$ is woman $i$ 's federal poverty line. Earnings range $2^{\prime}$ refers to the interval ( $\left.F P L_{i}, 1.2 \times F P L_{i}\right]$. Earning range $2^{\prime \prime}$ refers to the interval $\left(1.2 \times F P L_{i}, \infty\right)$.

Definition 32 (State). Consider the triple $\left(E, D, E^{r}\right)$. The state corresponding to $\left(E, D, E^{r}\right)$ is defined by the function:

$$
s\left(E, D, E^{r}\right)=\left\{\begin{array}{ll}
0 n & \text { if } E=0, D=0 \\
1 n & \text { if } E \text { in range } 1, D=0 \\
2^{\prime} n & \text { if } E \text { in range } 2^{\prime}, D=0 \\
2^{\prime \prime} n & \text { if } E \text { in range } 2^{\prime \prime}, D=0 \\
0 r & \text { if } E=0, D=1 \\
1 r & \text { if } E \text { in range } 1, D=1, E^{r}=E \\
1 u & \text { if } E \text { in range } 1, D=1, E^{r}<E \\
2^{\prime} u & \text { if } E \text { in range } 2^{\prime}, D=1, E^{r}<E \\
2^{\prime \prime} u & \text { if } E \text { in range } 2^{\prime \prime}, D=1, E^{r}<E \\
2^{\prime} r & \text { if } E \text { in range } 2^{\prime}, D=1, E^{r}=E \\
2^{\prime \prime} r & \text { if } E \text { in range } 2^{\prime \prime}, D=1, E^{r}=E
\end{array} .\right.
$$

Definition 33 (Latent and Observed States). Define $\mathcal{S}^{*} \equiv\left\{0 n, 1 n, 2^{\prime} n, 2^{\prime \prime} n, 0 r, 1 r, 1 u, 2^{\prime} u, 2^{\prime \prime} u\right\}$ and $\widetilde{S}^{*} \equiv\left\{0 n, 1 n, 2^{\prime} n, 2^{\prime \prime} n, 0 p, 1 p, 2^{\prime} p, 2^{\prime \prime} p\right\}$ where the mapping between the latent states in $\mathcal{S}^{*}$ and the observed states in $\tilde{\mathcal{S}}^{*}$ is:

$$
g(s)= \begin{cases}s & \text { if } s \in\left\{0 n, 1 n, 2^{\prime} n, 2^{\prime \prime} n\right\} \\ 0 p & \text { if } s=0 r \\ 1 p & \text { if } s \in\{1 u, 1 r\} \\ 2^{\prime} p & \text { if } s=2^{\prime} u \\ 2^{\prime \prime} p & \text { if } s=2^{\prime \prime} u\end{cases}
$$

Proposition 10 (Response Matrix). The matrix of response probabilities over the states in $\mathcal{S}^{*}$ is:

|  | JF: Earnings / Program Participation State |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $A F D C$ | $0 n$ | $1 n$ | $2^{\prime} n$ | $2^{\prime \prime} n$ | $0 r$ | $1 r$ | $1 u$ | $2^{\prime} u$ | $2^{\prime \prime} u$ |
| $0 n$ | $1-\pi_{0 n, 1 r}$ | 0 | 0 | 0 | 0 | $\pi_{0 n, 1 r}$ | 0 | 0 | 0 |
| $1 n$ | 0 | $1-\pi_{1 n, 1 r}$ | 0 | 0 | 0 | $\pi_{1 n, 1 r}$ | 0 | 0 | 0 |
| $2^{\prime} n$ | 0 | 0 | $1-\pi_{2^{\prime} n, 1 r}$ | 0 | 0 | $\pi_{2^{\prime} n, 1 r}$ | 0 | 0 | 0 |
| $2^{\prime \prime} n$ | 0 | 0 | 0 | $1-\pi_{2^{\prime \prime} n, 1 r}$ | 0 | $\pi_{2 \prime \prime \prime}{ }^{\prime \prime}{ }^{1} 1 r$ | 0 | 0 | 0 |
| $0 r$ | $\pi_{0 r, 0 n}$ | $\pi_{0 r, 1 n}$ | $\pi_{0 r, 2 n}$ |  | $\begin{gathered} 1-\pi_{0 r, 0 n} \\ -\pi_{0 r, 1 n}-\pi_{0 r, 1 r} \\ -\pi_{0 r, 2^{\prime} n}-\pi_{0 r, 2^{\prime \prime} n} \\ -\pi_{0 r, 2^{\prime} u}-\pi_{0 r, 2^{\prime \prime} u} \end{gathered}$ | $\pi_{0 r, 1 r}$ | 0 | $\pi_{0 r, 2^{\prime} u}$ | $\pi_{0 r, 2^{\prime \prime} u}$ |
| $1 r$ | 0 | 0 | 0 |  | 0 | 1 | 0 | 0 | 0 |
| $1 u$ | 0 | 0 | 0 |  | 0 | 1 | 0 | 0 | 0 |
| $2^{\prime} u$ | 0 | 0 | 0 |  | 0 | $\pi_{2^{\prime} u, 1 r}$ | 0 | $1-\pi_{2^{\prime} u, 1 r}$ | 0 |
| $2^{\prime \prime} u$ | 0 | 0 | 0 |  | 0 | $\pi_{2}{ }^{\prime \prime} u, 1 r$ | 0 | 0 | $1-\pi_{2^{\prime \prime}} u, 1 r$ |

Proof. Omitted. See proof of Propositions 1 and 2.
Corollary 7. The matrix of response probabilities in Proposition 10 implies the following system of equations describing the impact of the JF reform on observable state probabilities:

$$
\begin{align*}
& p_{0 n}^{j}-p_{0 n}^{a}=-\pi_{0 n, 1 r} p_{0 n}^{a}+\pi_{0 r, 0 n} p_{0 p}^{a} \\
& p_{1 n}^{j}-p_{1 n}^{a}=-\pi_{1 n, 1 r} p_{1 n}^{a}+\pi_{0 r, 1 n} p_{0 p}^{a} \\
& p_{2^{\prime} n}^{j}-p_{2^{\prime} n}^{a}=-\pi_{2^{\prime} n, 1 r} p_{2^{\prime} n}^{a}+\pi_{0 r, 2^{\prime} n} p_{0 p}^{a} \\
& p_{2^{\prime \prime} n}^{j}-p_{2^{\prime \prime} n}^{a}=-\pi_{2^{\prime \prime} n, 1 r}^{a} p_{2 \prime \prime n}^{a}+\pi_{0 r, 2^{\prime \prime} n} p_{0 p}^{a}  \tag{151}\\
& p_{0 p}^{j}-p_{0 p}^{a}=-\left(\pi_{0 r, 0 n}+\pi_{0 r, 2 / n}+\pi_{0 r, 2^{\prime \prime} n}+\pi_{0 r, 1 r}+\pi_{0 r, 1 n}+\pi_{0 r, 2^{\prime} u}+\pi_{0 r, 2^{\prime \prime} u}\right) p_{0 p}^{a} \\
& p_{2^{\prime} p p}^{j}-p_{2^{\prime} p}^{a}=\pi_{0 r, 2^{\prime} u} p_{0 p}^{a}-\pi_{2^{\prime} u, 1 r} p_{2^{\prime} p}^{a} \\
& p_{2^{\prime \prime} p}^{j}-p_{2^{\prime \prime} p}^{a}=\pi_{0 r, 2^{\prime \prime} u} p_{0 p}^{a}-\pi_{2^{\prime \prime} u, 1 r} p_{2^{\prime \prime} p}^{a}
\end{align*}
$$

Proof. By an application of the law of total probability given Definition 33.
Corollary 8. The analytical lower bounds of the response probabilities $\pi_{2^{\prime} n, 1 r}$ and $\pi_{2^{\prime \prime \prime} n, 1 r}$ are

$$
\begin{aligned}
\pi_{2^{\prime}, 1 r} & \geq \max \left\{0, \frac{p_{2^{\prime} n}^{a}-p_{2^{\prime} n}^{j}}{p_{2^{\prime} n}^{a}}\right\}, \\
\pi_{2^{\prime \prime} n, 1 r} & \geq \max \left\{0, \frac{p_{2^{\prime \prime} n}^{a}-p_{2^{\prime \prime} n}^{j}}{p_{2^{\prime \prime} n}^{a}}\right\} .
\end{aligned}
$$

Proof. Omitted. See Section 6 in this Appendix.

## References

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Table A1: Cross Tabulation of grant-inferred AU size and kidcount

|  | kidcount |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0 | 1 | 2 | 3 | Total |
| Inferred AU Size |  |  |  |  |  |
| 1 | 0.17 | 0.08 | 0.04 | 0.01 | 0.05 |
| 2 | 0.53 | 0.84 | 0.19 | 0.06 | 0.42 |
| 3 | 0.17 | 0.06 | 0.72 | 0.17 | 0.29 |
| 4 | 0.11 | 0.01 | 0.05 | 0.53 | 0.17 |
| 5 | 0.00 | 0.00 | 0.00 | 0.14 | 0.04 |
| 6 | 0.00 | 0.00 | 0.00 | 0.01 | 0.00 |
| 7 | 0.03 | 0.00 | 0.00 | 0.07 | 0.02 |
| 8 | 0.00 | 11,361 | 0.00 | 0.00 | 0.00 |
| \# of monthly observations | 840 |  | 8,463 | 8,043 | 28,707 |

Notes: Analysis conducted on Jobs First sample over quarters 1-7 post-random assignment. Kidcount variable, which gives the number of children reported in baseline survey, is tabulated conditional on non-missing. The AU size is inferred from (rounded) monthly grant amounts. Starting with AU size 5 , the unique correspondence between AU size and rounded grant amount obtains only for units which do not receive housing subsidies. The size inferred during months on assistance is imputed forward to months off assistance and to months that otherwise lack an inferred size.

Table A2: Mean Outcomes Post-Random Assignment

|  | Overall |  |  | Zero Earnings Q7 pre-RA |  |  | Positive Earnings Q7 pre-RA |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Jobs First | AFDC | Adjusted Difference | Jobs First | AFDC | Adjusted Difference | Jobs First | AFDC | Adjusted Difference |
| Average Earnings | $\begin{gathered} 1,191 \\ (29) \end{gathered}$ | $\begin{gathered} \hline 1,086 \\ (30) \end{gathered}$ | $\begin{aligned} & \hline 105 \\ & (36) \end{aligned}$ | $\begin{aligned} & \hline 930 \\ & \text { (32) } \end{aligned}$ | $\begin{aligned} & \hline 751 \\ & (30) \end{aligned}$ | $\begin{aligned} & \hline 179 \\ & (42) \end{aligned}$ | $\begin{gathered} 1766 \\ (65) \end{gathered}$ | $\begin{gathered} 1831 \\ (65) \end{gathered}$ | $\begin{gathered} \hline-65 \\ (84) \end{gathered}$ |
| Fraction of quarters with positive earnings | $\begin{gathered} 0.520 \\ (0.008) \end{gathered}$ | $\begin{gathered} 0.440 \\ (0.007) \end{gathered}$ | $\begin{gathered} 0.080 \\ (0.010) \end{gathered}$ | $\begin{gathered} 0.445 \\ (0.009) \end{gathered}$ | $\begin{gathered} 0.349 \\ (0.009) \end{gathered}$ | $\begin{gathered} 0.096 \\ (0.012) \end{gathered}$ | $\begin{gathered} 0.686 \\ (0.013) \end{gathered}$ | $\begin{gathered} 0.647 \\ (0.013) \end{gathered}$ | $\begin{gathered} 0.039 \\ (0.017) \end{gathered}$ |
| Fraction of quarters with earnings below 3FPL (AU size implied by kidcount+1) | $\begin{gathered} 0.906 \\ (0.007) \end{gathered}$ | $\begin{gathered} 0.897 \\ (0.007) \end{gathered}$ | $\begin{gathered} 0.009 \\ (0.009) \end{gathered}$ | $\begin{gathered} 0.938 \\ (0.008) \end{gathered}$ | $\begin{gathered} 0.940 \\ (0.008) \end{gathered}$ | $\begin{gathered} -0.002 \\ (0.010) \end{gathered}$ | $\begin{gathered} 0.837 \\ (0.011) \end{gathered}$ | $\begin{gathered} 0.803 \\ (0.011) \end{gathered}$ | $\begin{gathered} 0.034 \\ (0.014) \end{gathered}$ |
| Fraction of quarters on welfare | $\begin{gathered} 0.748 \\ (0.007) \end{gathered}$ | $\begin{gathered} 0.674 \\ (0.007) \end{gathered}$ | $\begin{gathered} 0.074 \\ (0.010) \end{gathered}$ | $\begin{gathered} 0.771 \\ (0.008) \end{gathered}$ | $\begin{gathered} 0.718 \\ (0.008) \end{gathered}$ | $\begin{gathered} 0.053 \\ (0.011) \end{gathered}$ | $\begin{gathered} 0.699 \\ (0.014) \end{gathered}$ | $\begin{gathered} 0.577 \\ (0.015) \end{gathered}$ | $\begin{gathered} 0.122 \\ (0.019) \end{gathered}$ |
| Average earnings in quarters with any month on welfare | $\begin{aligned} & 929 \\ & \text { (24) } \end{aligned}$ | $\begin{aligned} & 526 \\ & (19) \end{aligned}$ | $\begin{aligned} & 403 \\ & (28) \end{aligned}$ | $\begin{aligned} & 762 \\ & (25) \end{aligned}$ | $\begin{aligned} & 404 \\ & (18) \end{aligned}$ | $\begin{aligned} & 359 \\ & \text { (30) } \end{aligned}$ | $\begin{gathered} 1316 \\ (53) \end{gathered}$ | $\begin{aligned} & 869 \\ & (43) \end{aligned}$ | 448 <br> (64) |
| Fraction of quarters with no earnings and at least one month on welfare | $\begin{gathered} 0.363 \\ (0.007) \end{gathered}$ | $\begin{gathered} 0.437 \\ (0.007) \end{gathered}$ | $\begin{gathered} -0.074 \\ (0.010) \end{gathered}$ | $\begin{gathered} 0.426 \\ (0.009) \end{gathered}$ | $\begin{gathered} 0.508 \\ (0.009) \end{gathered}$ | $\begin{gathered} -0.082 \\ (0.012) \end{gathered}$ | $\begin{gathered} 0.227 \\ (0.011) \end{gathered}$ | $\begin{gathered} 0.272 \\ (0.012) \end{gathered}$ | $\begin{gathered} -0.045 \\ (0.016) \end{gathered}$ |
| \# of cases | 2,318 | 2,324 |  | 1,630 | 1,574 |  | 688 | 750 |  |

Notes: Sample covers quarters 1-7 post-random assignment. Sample cases with kidcount missing are excluded. Adjusted differences are computed via propensity score reweighting. Numbers in parentheses are standard errors calculated via 1,000 block bootstrap replications (resampling at case level).

Table A3: Probability of Earnings / Participation States in AFDC Sample
(Conditional on State=0p in Quarter Prior to Random Assignment)

| Quarter post-RA: | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\operatorname{Pr}($ State $=0 n)$ | 0.022 | 0.062 | 0.086 | 0.093 | 0.114 | 0.136 | 0.136 |
| $\operatorname{Pr}($ State $=1 \mathrm{n})$ | 0.021 | 0.045 | 0.058 | 0.079 | 0.084 | 0.101 | 0.112 |
| $\operatorname{Pr}($ State $=2 n)$ | 0.006 | 0.021 | 0.024 | 0.033 | 0.048 | 0.044 | 0.074 |
| $\operatorname{Pr}($ State=0p $)$ | 0.786 | 0.723 | 0.675 | 0.631 | 0.584 | 0.563 | 0.539 |
| $\operatorname{Pr}$ (State=1p) | 0.160 | 0.160 | 0.145 | 0.160 | 0.157 | 0.150 | 0.143 |
| $\operatorname{Pr}($ State=2p) | 0.002 | 0.001 | 0.004 | 0.004 | 0.004 | 0.002 | 0.005 |

Notes: Sample consists of 902 AFDC cases that were not working in the quarter prior to random assignment and were on welfare. Sample units with kidcount missing are excluded. Numbers give the reweighted fraction of sample in specified quarter after random assignment occupying each earnings / welfare paticipation state. Number of state refers to earnings level, with 0 indicating no earnings, 1 indicating earnings below 3 times the monthly FPL, and 2 indicating earnings above 3FPL. The letter $n$ indicates welfare nonparticipation throughout the quarter while the letter $p$ indicates welfare participation throughout the quarter. Poverty line computed under assumption $A U$ size is one greater than amount implied by baseline kidcount variable. Probabilities are adjusted via propensity score reweighting algorithm.

| State under Jobs First |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| State <br> under <br> AFDC | On | 1n | 2n | Or | $1 r$ | 1u | $2 u$ | $2 r$ |
| On | No Response | － | － | － | Extensive LS（＋） Take Up Welfare | － | － | － |
| 1n | － | No Response | － | － | Intensive LS（＋／0／－） Take Up Welfare | － | － | － |
| 2n | － | － | No Response | － | Intensive LS（－） <br> Take Up Welfare | － | － | － |
| Or | No LS Response Exit Welfare | Extensive LS（＋） Exit Welfare | Extensive LS（＋） Exit Welfare | No Response | Extensive LS（＋） | － | Extensive LS（＋） Under－reporting | － |
| $1 r$ | － | － | － | － | Intensive LS（＋／0／－） | － | － | － |
| 1u | － | － | － | － | Intensive LS（＋／0／－） <br> Truthful Reporting | － | － | － |
| 2u | － | － | － | － | Intensive LS（－） Truthful Reporting | － | No Response | － |
| $2 r$ | Extensive LS（－） Exit Welfare （Figure A1） | Intensive LS（－） Exit Welfare | Intensive LS（＋／0／－） Exit Welfare | Extensive LS（－） | Intensive LS（－） | － | Intensive LS（＋／0／－） Under－reporting | － |

Notes：This table catalogues the theoretically allowed response margins given the states that a woman may occupy under AFDC and Jobs First when truthful reporting of earnings above the FPL is possible under AFDC，that is，when assumption A． 8 is not maintained．A state is a pair of coarsened earnings（ 0 stands for zero earnings， 1 for positive earnings at or below the FPL，and 2 for earnings strictly above the FPL），and participation in the welfare assistance program along with an earnings reporting decision（ n stands for＂not on assistance＂，r for＂on assistance and truthfully reporting earnings＂，and u for＂on assistance and under－reporting earnings＂）．The cells termed＂no response＂entail the same behavior under the two policy regimes．The cells containing a＂－＂represent responses that are either incompatible with the policy rules or not allowed based on revealed preference arguments derived from the nonparametric model of Section 4 ．Specifically，（a） state 1 u is unpopulated under JF（＂一＂in cells with a horizontally striped background fill），（b）state $2 r$ is not defined under JF（＂一＂in cells with gridded background fill），and（c）a woman will not leave a state at least as attractive under JF a under AFDC for a state that is no more attractive under JF than under AFDC（＂一＂in cells with a solid greyed－out background fill）．The remaining cells represent responses that are allowed by the model．Their content summarizes the three possible margins of responses：（a）the labor supply＂LS＂response（intensive versus extensive and its sign：＂+ ＂for increase，＂ 0 ＂for no change，and＂- ＂for decrease），（b）the program participation response（take up of versus exit from welfare assistance），and（c）the reporting of earnings to the welfare agency margin（to truthfully report versus to under－report）．See Online Appendix for proof．

|  |  |  |  |  |  |  |  | tate under J | st |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| State under AFDC | Onn | 1nn | 2nn | Onr | 1nr | 2 nr | 1nu | 2nu | Orn | 1rn | 1 un | 2un | Orr | 1rr | 1uu | 2uu |
| Onn | No Response | - | - | - | - | - | - | - | - | Extensive LS (+) Take Up Welfare | $\underline{ }$ | - | - | $\left\lvert\, \begin{array}{c\|} \text { Extensive LI }(t) \\ \text { Take Up Weffare and } \\ \text { FS } \end{array}\right.$ | - | - |
| 1nn | - | No Response | - | - | - | - | - | - | - | Intensive LS $(+/ 0 /-$-) Take Up Welfare | - | - | - | Intensive LS $(+/ 0 /-$ - Take Up Welfare and FS | - | - |
| 2nn | - | - | No Response | - | - | - | - | - | - | Intensive LS $(-)$ Take | $\underline{ }$ | - | - | Intensive LS $(-)$ Take Up Welfare and FS | ? | - |
| Onr | - | - | - | No Response | - | - | - | - | - | $\begin{aligned} & \text { Extensive LS (+) } \\ & \text { Exit FS, Take Up } \end{aligned}$ Welfare | $\underline{\square}$ | - | - | Extensive LS (+) Take Up Welfare | - | - |
| 1nr | - | - | - | - | No Response | - | - | - | - |  | $\underline{\text { 를 }}$ | - | - | Intensive LS ( $+/ 0 /$ Take Up Welfare | - | - |
| 2nr | - | - | - | - | - | No Response | - | - | - | $\begin{gathered} \text { Intensive LS (-) } \\ \text { Exit FS, Take Up } \\ \text { Welfare } \end{gathered}$ | $\cdots$ | - | - | Intensive LS (-) Take Up Welfare | ? | - |
| 1nu | - | - | - | - | - | - | No Response | - | - | Intensive LS $(+/ / 0 /-)$ Exit FS, Take Up Welfare |  | - | - | Intensive LS (+/0/-) Exit FS, Take Up Welfare | ? | - |
| 2nu | - | - | - | - | - | - | - | No Response | - | $\begin{gathered} \text { Intensive LS (-) } \\ \text { Exit FS, Take Up } \\ \text { Welfare } \end{gathered}$ | $\cdots$ | - | - | (Intensive LS(-) | ? | - |
| Orn | No LS Response Exit Welfare | Extensive LS $(+)$ Exit Welfare | Extensive LL $(+)$ Exit Welfare | No LS Response Exit Welfare, Take Up FS | Extensive LS (+) Exit Welfare, Take Up FS | Extensive LS (t) Exit Welfare, Take Up FS | Extensive LS (+) Exit Welfare Take Up FS Under-report | $\begin{aligned} & \text { Extensive LS }(+) \\ & \text { Exit Welfare } \\ & \text { Take Up FS } \\ & \text { Under-report } \end{aligned}$ | No LS Response Exit Welfare, Take Up FS | Extensive LS(t) | V | Extensive LS (+) <br> Under-report | $\left\lvert\, \begin{gathered} \text { No Ls Response } \\ \text { Take Up Fs } \end{gathered}\right.$ | $\begin{aligned} & \text { Extensive Is (+) } \\ & \text { Take Up Fs } \end{aligned}$ | 厚 | Extensive LS (+) Take Up FS Under-report |
| 1rn | - | - | - | - | - | - | - | - | - | Intensive LS $(+/ 0 /-)$ | ? | - | - | $\begin{aligned} & \text { Intensive } \text { S S }(+0 /- \text { - } \\ & \text { Take UP FS } \end{aligned}$ | ? | - |
| 1un | - | - | - | - | - | - | - | - | - | Intensive LS $(+/ 0 /-$ ) Truthful Report |  | - | - |  | $\underline{\text { nem }}$ | - |
| 2un | - | - | - | - | - | - | - | - | - | Intensive LS (-) Truthful Report | $\underline{\square}$ | No Response | - | Intensive LS (-) Take Up FS Truthful Report | ? | - |
| Orr | No LS Response Exit Welfare | $\begin{aligned} & \text { Extensive LS }(+) \\ & \text { Exit Welfare and } \\ & \text { FS } \end{aligned}$ | Extensive LS $(+)$ <br> Exit Welfare and <br> FS | No LS Response Exit Weffare | Extensive LS (t) Exit Welfare | Extensive LS ( + ) | $\begin{array}{\|l} \text { Extensive LS (+) } \\ \text { Exit Welfare } \\ \text { Under-report FS } \end{array}$ | Extensive LS (t) Exit Welfare Under-report | $\left\lvert\, \begin{gathered} \text { No LS Response } \\ \text { Exit Fs } \end{gathered}\right.$ | $\underset{\text { Exit FS }}{\substack{\text { Extensive LS }(+)}}$ | $\cdots$ | Extensive LS (+) Exit FS Under-report | No Response | Extensive Ls (t) | " | Extensive LS (+) <br> Under-report |
| 1rr | - | - | - | - | - | - | - | - | - |  | $\underline{ }$ | - | - | Intensive LS ( $+10 /-\mathrm{H}$ | $\underline{ }$ | - |
| 1uu | - | - | - | - | - | - | - | - | - | Intensive LS ( $(+/ 0 /-)$ <br> Exit $F$. <br> Truthful report | - | - | - | Intensive LL $(+/ 0 /-$ ) Truthful Report | $\underline{\underline{\underline{2}} \text { + }}$ | - |
| 2uu | - | - | - | - | - | - | - | - | - | Intensive LS (-) Exit FS Truthful report |  | - | - | Intensive LS (-) Truthful Report | " | No Response |

 incorporated. A state isa triplet of coarsened earnings (0 stands for zero earnings, 1 for positive earnings at or below the APL , and 2 for earnings strictly above the FPLL , participation in the weffare assistance program along with an earnings reporting
decision (n stands for "not on assistance" $r$ for "on assistance and truthfully reporting earnings" and u for "on assistance and under-reporting earnings") and participation in the FS assistance program along with an earnings reporting decision (n $r$. decision ( $n$ stands for "not on assistance, $r$ for "on assistance and truthfully reporting earnings, and f for "on assistance and under-reporting earnings"), and participation in the Fs assistance program along wh an earnings reporting decision ( $n, r$,
and $u$ ). When both on welfare and FS assistance, a woman makes only one earning report to the welfare agency, hence states such as e.g. 1ru are ruled out and not included in the table The assumption of lower bounds on the stigma disutilities rules out states $\{2 r \mathrm{rn}, 2 \mathrm{rr}\}$ hence these states are not included in the table. The cells termed "no response" entail the same behavior under the two policy regimes. The cells containing a " - " represent responses that are either incompatible with the policy rules or not allowed based on revealed preference arguments derived from the extended model. Specifically, (a) states 1 uu and 1 un are unpopulated under JF (" - " in cells with a horizontally striped background fill); and (b) a woman will not leave a state at least as attractive under JF as under AFDC for a state that is no more attractive under JF than under AFDC ""-" in cells with a solid greyed-out background fill). The remaining cells represent responses that are allowed by the model. Their
content summarizes the three possible margins of responses: (a) the labor supply "LS" response (intensive versus extensive and its sign: " + " for increase " "0" for no change, and """ for decrease), (b) the program participation response (take up of content summarizes the three possible margins of responses: (a) the labor supply "LS" response (intensive versus extensive and its sign: " 4 " for increase, " 0 " for no change, and " $"$ " for decrease), (b) the program participation response (take up of versus exit from welfare assistance and/or FS assistance), and (c) the reporting of earnings to the welfare agency margin (to truthfully report versus to under-report). See Online Appendix for proof.

Table A6: Probability of Earnings / Participation States

|  | Overall |  |  | Overall - Adjusted |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Jobs First | AFDC | Difference | Jobs First | AFDC | Difference |
| Pr(State=0n) | 0.127 | 0.136 | -0.009 | 0.128 | 0.135 | -0.007 |
| $\operatorname{Pr}($ State $=1 \mathrm{n}$ ) | 0.076 | 0.130 | -0.055 | 0.078 | 0.126 | -0.048 |
| $\operatorname{Pr}($ State $=2 \mathrm{n}$ ) | 0.021 | 0.032 | -0.011 | 0.022 | 0.031 | -0.010 |
| $\operatorname{Pr}($ State $=2 \mathrm{n}$ ) | 0.047 | 0.067 | -0.020 | 0.048 | 0.065 | -0.017 |
| $\operatorname{Pr}($ State $=0 \mathrm{p})$ | 0.366 | 0.440 | -0.074 | 0.359 | 0.449 | -0.090 |
| $\operatorname{Pr}($ State $=1 \mathrm{p})$ | 0.342 | 0.185 | 0.157 | 0.343 | 0.184 | 0.159 |
| $\operatorname{Pr}($ State $=2 ' p)$ | 0.010 | 0.003 | 0.006 | 0.010 | 0.003 | 0.007 |
| $\operatorname{Pr}\left(\right.$ State $=2 \mathrm{l}{ }^{\prime \prime}$ ) | 0.012 | 0.006 | 0.006 | 0.013 | 0.006 | 0.007 |
| \# of quarterly observations | 16,226 | 16,268 |  | 16,226 | 16,268 |  |

Notes: Sample covers quarters 1-7 post-random assignment during which individual is either always on or always off welfare. Sample cases with kidcount missing are excluded. Number of state refers to earnings level, with 0 indicating no earnings, 1 indicating earnings below 3 times the monthly FPL, $2^{\prime}$ indicating earnings between $3 F P L$ and $1.2 \times 3 F P L$, and $2^{\prime \prime}$ indicating earnings above $1.2 \times \mathrm{FPL}$. The letter $n$ indicates welfare nonparticipation throughout the quarter while the letter $p$ indicates welfare participation throughout the quarter. Poverty line computed under assumption AU size is one greater than amount implied by baseline kidcount variable. Adjusted probabilities are adjusted via propensity score reweighting. Standard errors computed using 1,000 block bootstrap replications (resampling at case level).

Figure A1: Earnings and Participation Choices with Earning Constraints and no Stigma


Notes: Panels a and b are drawn in the earnings (horizontal axis) and consumption equivalent (vertical axis) plane. The consumption equivalent equals earnings plus transfer income from welfare (if any) net of monetized hassle, stigma, work, and under-reporting costs. The welfare stigma and fixed cost of work are set to zero. The cost of under-reporting is set large enough so that under-reporting is a dominated choice. Labor market constraints are imposed in the form of two earnings offers ( $E_{i}^{1}$ and $E_{i}^{2}$ ), both in range 2 (above the FPL). The wage rate is assumed fixed. Because of the labor market constraints, and the fact that a woman may always choose not to work, the only alternatives available are those identified by a solid circular symbol. Vertical lines represent the same earnings levels depicted in Figure 1 but for a situation in which the earnings level at which welfare assistance is exhausted under $\operatorname{AFDC}(\bar{E})$ is above the FPL, that is, for a woman who has access to the unreduced fixed ( $\$ 120$ ) and proportional disregards. It also displays the two earnings offers. Panel a depicts a scenario where under AFDC the woman opts to be on assistance earning $E_{i}^{1}$ and reports truthfully to the welfare agency (point A). She would make the same choice even in the absence of earnings constraints. Under JF, earning $E_{i}^{1}$ on assistance (and reporting truthfully) is no longer feasible because welfare eligibility ends at FPL. Panel b depicts a scenario where, given the earning constraints, the JF reform induces the woman to exit both welfare and the labor force (point B). However, in the absence of earning constraints, she would choose to lower her earnings below the FPL and remain on assistance as evidenced by the fact that the indifference curve through point A lies below the (dashed) JF segment in range 1 (earning levels below FPL).


[^0]:    ${ }^{1}$ Blundell and Macurdy (1999), Moffitt (2002), and Grogger and Karoly (2005) provide reviews.
    ${ }^{2}$ Heckman (1993), for instance, concludes that "elasticities are closer to 0 than 1 for hours-of-work equations (or weeks-of-work equations) estimated for those who are working. A major lesson of the past 20 years is that the strongest empirical effects of wages and nonlabor income on labor supply are to be found at the extensive margin." (emphasis in original). Likewise, many modern models of aggregate labor supply are now predicated on the notion that labor supply is "indivisible" (Hansen, 1985; Rogerson, 1988; Ljungqvist and Sargent, 2011). See Chetty et al. (2011a) for an assessment of how macro estimates of these models compare to estimates from micro data.

[^1]:    ${ }^{3}$ An alternative approach would be to invoke a statistical "rank invariance" assumption that a woman's rank in the distribution of earnings is preserved across policy regimes. Under rank invariance, QTEs can be used to identify the joint distribution of potential earnings (Heckman, Smith, and Clements, 1997) and hence to quantify extensive and intensive margin responses. However, there are many reasons to be dubious of this assumption. For example, opt-in behavior in conjunction with incentives to work may lead women to exchange ranks in the earnings distribution. BGH (2006) are also skeptical of the rank-invariance assumption. In a related analysis (BGH, 2005), they provide evidence that rank invariance is violated in the Canadian Self-Sufficiency Project experiment.
    ${ }^{4}$ This is in contrast to traditional parametric models of labor supply (e.g. Burtless and Hausman, 1978; Hoynes, 1996; Keane and Moffitt, 1998) that can be identified without policy variation. See Macurdy, Green, and Paarsch (1990) for an early critique of parametrically structured econometric models of labor supply with nonlinear budget sets.

[^2]:    ${ }^{5}$ The reform also induced a second notch specifically for applicants who faced a strict earnings test in order to establish eligibility. AFDC did not have an earnings test for applicants, but benefits for that program phased out at an amount above the JF earnings test. Hence, it became harder under JF for high earning applicants to establish eligibility.

[^3]:    ${ }^{6}$ The assistance unit consists of the woman receiving welfare plus eligible dependent children. Children are eligible if they are under age eighteen or under age nineteen and in school. Grant amounts also vary based upon the unit's assistance history.
    ${ }^{7}$ The EITC and other taxes do not directly interact with cash and in-kind assistance because income from welfare and FS is not counted in the determination of taxes and tax credits.

[^4]:    ${ }^{8}$ Regarding the AFDC work mandates, Bloom et al. (2002, p.11) state that "Connecticut, like many other states, did not strongly enforce the existing requirements for AFDC recipients to participate in employment-related activities (in fact there were waiting lists for services). Job Connection, the state's Job Opportunities and Basic Skills Training (JOBS) program, served a small proportion of the total welfare caseload in any month, and a large proportion of those who participated were in education and training activities." As to the JF work mandates, Bloom et al. (2002, p.12) state that "nearly all [non-exempted] JF participants were required to begin by looking for a job, either on their own or through Job Search Skills Training (JSST), a group activity that teaches job-seeking and job-holding skills. Education and training were generally reserved for recipients who were unable to find a job despite lengthy up-front job search activities."
    ${ }^{9}$ Regarding TCC, Bloom et al. (2002) write that "in practice, however, the difference between these two policies was minimal, because AFDC members who reached the end of their eligibility for TCC could move directly into the child care certificate program (that is, income-eligible child care) for low-income working parents." Regarding TM, they write that "the magnitude of the treatment difference related to medical assistance has diminished over time, as Connecticut has expanded the availability of health coverage to low-income children and adults who do not receive

[^5]:    welfare." In addition, they note that "the 1996 federal welfare law 'de-linked' eligibility for Medicaid from eligibility for welfare and created a new coverage category for families who are not on welfare but who meet the AFDC eligibility criteria that were in place in July 1996. These statewide expansions in health coverage for children and adults are available to both the JF group and the AFDC group." Taken together these observations suggests that the additional 12 months of TM available under JF are unlikely to have induced changes in the value of working off assistance.
    ${ }^{10}$ Appendix Table A1 tabulates the kidcount variable against the administrative measure available in the JF sample.

[^6]:    Our inflation scheme maps the kidcount measure to roughly its modal administrative value plus one. We have found that our results are robust to alternate codings including inflating the AU size by two and not inflating it at all.
    ${ }^{11}$ These techniques are described in the Appendix. After adjustment, the means of the AFDC and JF groups are very similar as evidenced by the "Adjusted Difference" column in Table 2. The baseline sample in BGH (2006) contains 4,803 cases. Relative to their analysis, we impose the additional restriction that the kidcount variable be non-missing. We also drop one AFDC case from our analysis with unrealistically high quarterly earnings that sometimes led to erratic results.

[^7]:    ${ }^{12}$ Comparing administrative earnings records from the California Unemployment Insurance system with earnings reported to welfare, they find that about a quarter of welfare cases report earning amounts to the welfare agency that are lower than the figures recorded in the state UI system. Among these cases, the average fraction of UI earnings reported varied from $64 \%$ to $84 \%$ depending on the year studied.

[^8]:    ${ }^{13}$ Grogger and Michalopolous (2003) rely on data from a randomized welfare reform where the experimental group was exposed to a twenty four month time limit (or a thirty six month limit if particularly disadvantaged). JF's more stringent twenty one month time limit might be expected to produce a larger anticipatory response than found by Grogger and Michalopoulus. It does not. One possible explanation for this discrepancy is that, as remarked above, a large fraction of JF experimental units were exempted from time limits, and a large fraction of the non-exempted units were granted six month extensions. Bloom et al. (2002, p.59) report that "written material produced by the DSS explicitly stated that extensions would be possible." Also, "staff reported that many recipients were initially skeptical that the time limit would be implemented (in fact, many staff said that they themselves were skeptical)". Based on the Interim Client Survey, it appears that "from the beginning, most recipients understood that the time limit would not necessarily result in cancellation of their welfare grant."
    ${ }^{14}$ First order stochastic dominance implies the absence of negative QTEs. Therefore the analysis of BGH (2006) already provides evidence against the extensive margin-only null hypothesis. However, focusing on particular QTEs that happen to be significant can generate a multiple testing problem. The methods used here address this problem.
    ${ }^{15}$ Appendix Table A2 provides standard errors on selected earnings impacts, which confirm the visual impression of Figures 4a-4c.

[^9]:    ${ }^{16}$ Returns to labor market experience are a second culprit. Our model posits regime-invariant earning offer functions, which implies that the attractiveness of off-welfare alternatives is assumed to be the same under AFDC and JF. If JF induces more women to work, and if returns to labor market experience are substantial, this assumption is violated. However, the magnitude of experience effects in our sample is likely to be small. For example, after studying data from a similar welfare experiment - the Canadian Self Sufficiency Project (SSP) - Card and Hyslop (2005) conclude that "work experience attributable to SSP appears to have had no detectable effect on wage opportunities." Couch (2014) uses 14 years of post-randomization earnings data from the JF reform and concludes that "the short-term intervention did not appear to have altered the long-term outcomes of participants examined in terms of employment or labor market earnings."
    ${ }^{17}$ Allowing over-reporting behavior would essentially nullify the JF work requirements. In practice, concocting a fictitious job was difficult as employment had to be verified by case workers.

[^10]:    ${ }^{18}$ See Saez (2010) for a related analysis involving a fixed "moral" cost of misreporting income to tax authorities.

[^11]:    ${ }^{19} G_{i}^{a}\left(F P L_{i}\right)$ is in the range $(0, \$ 75)$ if a woman has access to the unreduced proportional disregard under AFDC. Otherwise it is zero.
    ${ }^{20}$ This formulation acknowledges that indifferences between alternatives may arise that lead the arg max to be a set instead of a vector. We do not model how woman $i$ chooses among alternatives between which she is indifferent. We only assume that the rule she uses to choose among them is invariant to the policy regime $t$.

[^12]:    ${ }^{21}$ The structural labor supply literature often assumes labor supply choices are constrained to fall into a few data driven categories such as "part-time" and "full-time" work (e.g. Hoynes, 1996; Keane and Moffitt, 1998; Blundell et al, 2013; Manski, 2014). By contrast, we allow the choice set to vary across women in an unrestricted fashion by means of the heterogeneous offer set $\Theta_{i}$.

[^13]:    ${ }^{22}$ This depiction assumes a fixed wage rate and ignores labor market constraints, i.e. we set $K_{i}=\infty$.

[^14]:    ${ }^{23}$ Since preferences and constraints can change month to month, the panel features of our data will not aid in solving this problem without strong assumptions about how these factors evolve over time. The problem is illustrated in Online Appendix Table A3 which provides the distribution of states occupied in quarters 1 through 7 among the subsample of women assigned to AFDC who chose state $0 r$ in the quarter prior to random assignment. Even in the first quarter after random assignment, many of these women have switched states, suggesting substantial drift in preferences and constraints.

[^15]:    ${ }^{24}$ Note that the response probabilities $\pi_{0 r, 2 n}$ and $\pi_{0 r, 2 u}$ involve pairing earnings category 0 under AFDC with category 2 under JF, while the probabilities $\pi_{2 n, 1 r}$ and $\pi_{2 u, 1 r}$ involve pairing earnings category 2 under AFDC with category 1 under JF. Therefore, the model allows for "rank reversals" in earnings in response to the JF reform.

[^16]:    ${ }^{25}$ As we show in the Online Appendix, these restrictions are obtained by using the fact that $0 \leq \pi_{s^{a}, s^{j}} \leq 1$ for all $\left(s^{a}, s^{j}\right) \in \mathcal{S} \times \mathcal{S}$ and $\sum_{s^{j} \in \mathcal{S}} \pi_{s^{a}, s^{j}}=1$ for all $s^{a} \in \mathcal{S}$,

[^17]:    ${ }^{26}$ It is also an intuitive feature. Consider again the expression for the lower bound for $\pi_{2 n, 1 r}$ in (13). From (10), the

[^18]:    fraction of people occupying state $2 n$ under JF may differ from that under AFDC because of an "in-flow" from state $0 r$ (represented by $p_{0 p}^{a} \pi_{0 r, 2 n}$ ) or because of an "out-flow" to state $1 r$ (represented by $-p_{2 n}^{a} \pi_{2 n, 1 r}$ ). If $p_{2 n}^{a}-p_{2 n}^{j} \leq 0$, the in-flow from state $0 r$ must be at least as large as the out-flow to state $1 r$. But this latter quantity may be zero, in which case the lower bound on $\pi_{2 n, 1 r}$ is zero. If $p_{2 n}^{a}-p_{2 n}^{j}>0$, the in-flow from state $0 r$ can at most equal the out-flow to state $1 r$, in which case this latter quantity must be at least $p_{2 n}^{a}-p_{2 n}^{j}$. Accordingly, the lower bound on $\pi_{2 n, 1 r}$ is the $\pi$ that solves $p_{2 n}^{a}-p_{2 n}^{j}=p_{2 n}^{a} \pi$.
    ${ }^{27}$ The bounds for each parameter are functions of $\left(\mathbf{p}^{a}, \mathbf{p}^{j}\right)$, which leads to interesting patterns of dependence among them. For instance, among each pair of response probabilities $\left(\pi_{2 n, 1 r}, \pi_{0 r, 2 n}\right),\left(\pi_{0 n, 1 r}, \pi_{0 r, 0 n}\right),\left(\pi_{2 u, 1 r}, \pi_{0 r, 2 u}\right)$, and $\left(\pi_{0 r, 1 n}, \pi_{1 n, 1 r}\right)$ only one probability may have an informative lower bound.

[^19]:    ${ }^{28}$ We discard from our sample all quarters in which a woman's welfare participation status varies from month to month as it would be impossible to infer reliably whether such a women earned above the poverty line in the months when she was on welfare. This selection could confound the experimental impacts reported in Table 5 if the experiment influenced the probability of selection. However, we find that after adjusting for baseline covariates via a linear probability model, the frequency of these "mixed" quarters is roughly the same in the AFDC and JF groups: the estimated impact of JF on the probability of a quarter being mixed is 0.0063 ( $\mathrm{se}=0.0034$ ). Hence, we interpret the impacts reported in Table 5 as average treatment effects on "unmixed" quarters.

[^20]:    ${ }^{29}$ Although the expressions for the bounds differ depending on whether the utility function obeys (3) or (4), the solutions that bind in the data are the same. This is because inequality (14) holds in the sample.

[^21]:    ${ }^{30}$ Note that some of the responses involving reductions from earnings range $2^{\prime}$ to range 1 could be larger than those from earnings range $2^{\prime \prime}$ to range 1 since we don't know which earnings level in range 1 is being selected. The upper bounds on these response probabilities are uninformative.
    ${ }^{31}$ This estimate is constructed as follows: for each AFDC sample woman and quarter, we determine the welfare transfer she would receive if her earnings equaled the (AU size and quarter-specific) FPL and if she had access to the unreduced fixed and proportional disregards. We round this amount to the nearest $\$ 50$ and denote it by $G_{i}^{a *}\left(F P L_{i}\right)$. Then, we count the number of quarterly observations in the AFDC sample associated with UI earnings above the

[^22]:    ${ }^{32}$ For example, if one uses an instrumental variables design, counterfactuals are, under weak assumptions, identified only for the sub-population of "compliers" (Imbens and Rubin, 1997).

[^23]:    ${ }^{1}$ Changes in AU size are typically due to a birth or to the fact that a child becomes categorically ineligible for welfare. Under AFDC, the AU size also changes when the adult is removed from the unit due to sanctions for failure to comply with employment-related mandates. Empirically this source of time variation in AU size seems quantitatively minor. Bloom et al. (2002) report that 5 percent of AFDC group members had their benefits reduced owing to a sanction within four years after random assignment.

[^24]:    ${ }^{2}$ Concavity of $v($.$) enables the conditions imposed. For instance, the first condition requires v\left(E_{i}^{k}+\bar{G}_{i}\right)-v\left(E_{i}^{k}\right)<$ $v\left(\bar{G}_{i}\right)-v(0)$ which cannot hold unless $v($.$) is (strictly) concave.$

[^25]:    ${ }^{3}$ Convexity of $v($.$) enables the conditions imposed. For instance, the first condition requires v\left(E_{i}^{k}+G_{i}^{a}\left(E_{i}^{k}\right)\right)-$ $v\left(E_{i}^{k}\right) \geq v\left(E_{i}^{l}+\bar{G}_{i}\right)-v\left(E_{i}^{l}\right)$ which cannot hold unless $v$ is convex.

[^26]:    ${ }^{4}$ During the JF demonstration project, $\tau_{f}^{1}=0.30, \tau_{f}^{2}=0.20$ and $\tau_{f}^{3}=1.3$. The JF experimental policy effectively sets $\tau_{f}^{2}=1$ when FS is taken up jointly with welfare. This explains why we write the FS transfer as in (141), that is, as the standard transfer function evaluated at zero earnings. The eligibility formula shows that a woman with earnings above $F P L_{i}$ may be eligible for FS and the transfer formula shows that the FS transfer for which she is eligible may be positive. However, under the JF experimental policy, a woman with earnings above $F P L_{i}$ may not receive both welfare and FS because such earnings disqualify her from welfare.
    ${ }^{5}$ This function is time varying. We dispense with the time subscript for simplicity.
    ${ }^{6}$ This function is time varying. We dispense with the time subscript for simplicity.

[^27]:    ${ }^{7}$ If $B($.$) were differentiable then \frac{d B\left(E^{r}\right)}{d E r}=\frac{\partial\left(G+F\left(E^{r}, G\right)\right)}{\partial G} \frac{d G}{d E^{r}}+\frac{\partial\left(G+F\left(E^{r}, G\right)\right)}{\partial E^{r}} \frac{d E^{r}}{d E^{r}}$. To show that $\frac{d B\left(E^{r}\right)}{d E^{r}} \leq 0$ it would suffice to show that both $\frac{\partial\left(G+F\left(E^{r}, G\right)\right)}{\partial G} \frac{d G}{d E^{r}} \leq 0$ and $\frac{\partial\left(G+F\left(E^{r}, G\right)\right)}{\partial E^{r}} \leq 0$. The argument in the proof does exactly this without using calculus because neither $G($.$) nor F(.,$.$) are differentiable functions.$

[^28]:    ${ }^{8}$ There are earning levels in range 1 such that a woman is ineligible for the combined FS plus welfare assistance under JF's control policy. This comparison is meaningful only for earnings that are below the more stringent eligibility threshold; above such threshold state $1 r r$ is ruled out under JF's control policy.

