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DISTINGUISHING CONSTRAINTS ON FINANCIAL INCLUSION AND THEIR IMPACT ON GDP, TFP, AND THE DISTRIBUTION OF INCOME

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Distinguishing Constraints on Financial Inclusion and Their Impact on GDP, TFP, and the Distribution of Income
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ABSTRACT

We develop a tractable general equilibrium model with heterogeneous agents and multiple sources of financial frictions to study how different constraints interact in equilibrium. We highlight, distinguish, and quantitatively evaluate their differential impacts, thereby uncovering the rich interactions among these constraints. The impact of financial inclusion in an economy depends not only on which constraint is alleviated, but also on the tightness of other constraints. Policy instruments should target the most binding constraint, which likely varies across countries. Moreover, there are important between financial inclusion, GDP, and the distribution of income. The transitional dynamics also differ from what happens in steady states. Policy makers should be concerned about both.

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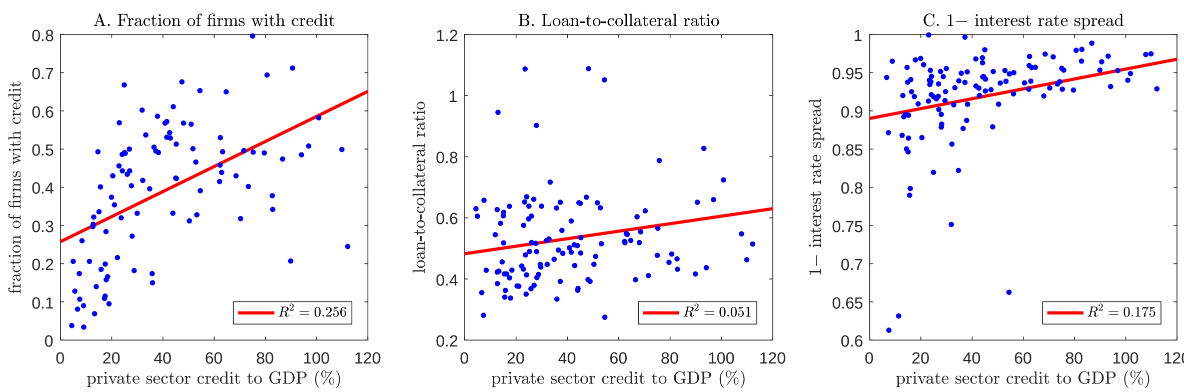
1 Introduction

The development of a financial system is multi-faceted in nature. In general, the stage of financial development and the extent of financial inclusion in a country is mainly reflected by the breadth (ability of firms to access credit), depth (the amount of collateral required for borrowing), and efficiency (ability of financial intermediaries to provide services at low cost) of its financial system. These three characteristics can be proxied by the fraction of firms with credit, the loan-to-collateral ratio, and one minus the interest rate spread. More developed financial systems are usually associated with greater breadth, depth, and efficiency. However, large heterogeneities also exist, as the correlations among the three measures are not high (see Figure 1).



Note: We obtain the data from the World Bank Enterprise Surveys and World Development Indicators. The breadth, depth, and efficiency of a country's financial system can be measured by the fraction of firms with credit, the loan-to-collateral ratio, and one minus the interest rate spread, respectively. Among the 139 countries in our sample, the correlation is 0.11 between the measures of breadth and depth, 0.09 between the measures of depth and efficiency, and 0.37 between the measures of efficiency and breadth.

Figure 1: Ratio of private credit to GDP versus various financial system indicators.

For this reason, policy instruments to foster financial inclusion are likely to vary across countries. When formulating financial inclusion strategies, policy makers should first be aware of the differences and the interactions among various policy instruments. Given their objectives and the cost of available instruments, policy makers should then choose which policies to implement. Instead of deriving optimal policies based on specific objectives and cost functions, the goal of this paper is to highlight, distinguish, and systematically examine the tradeoffs between various policy instruments as an essential first step in policy design. Reduced-form empirical techniques, however, are less likely to offer revealing answers, because different constraints have nonlinear interactions with each other and economy-wide policies inevitably bring general-equilibrium effects. These challenges lend themselves to a structural approach.

Our model has two key implications. First, policy instruments should target the most binding constraint, which likely varies across countries. For example, policy instruments that boost GDP in one country may not be as effective in other countries. Second, there are important tradeoffs among financial inclusion, GDP, and the distribution of income. For example, policies that increase GDP may lead to high income inequality. Further, short-run transitional effects may differ from outcomes in long-run steady states, which may be years away. For example, the short-run interest rate could rise and overshoot the target, resulting in high borrowing costs. Similarly, reducing the cost of intermediation could generate a moderately higher GDP and widen income inequality in the short run. But GDP would be higher and inequality would be lower in the long run. Such intertemporal tradeoffs may raise concerns for policy makers or lead to time-inconsistent behavior. We provide a roadmap for policy makers so that they know what they are likely to face in the future, thereby allowing them to build in some commitment if possible or to choose another path if not.

Our model is built on the workhorse model of occupational choice in the macro-development literature (see, e.g., Banerjee and Newman, 1993; Gine and Townsend, 2004; Buera and Shin, 2011, 2013). In the model, agents are heterogeneous in wealth and entrepreneurial productivity, and can choose between being workers or being entrepreneurs. Workers supply labor for wages in return. Entrepreneurs use capital and labor for production.

Our major deviation from standard models is the more realistic assumption that agents face multiple sources of financial constraints rather than a single one. First, agents borrow from financial intermediaries, incurring a credit entry cost. This cost captures the fixed transaction costs, documentation requirements, and other access barriers. Second, the amount of loans is constrained by a collateral constraint as in Jermann and Quadrini (2009, 2012). Third and finally, loans charge a higher interest rate than the deposit rate, reflecting an informational or intermediation cost. Existing studies have emphasized the quantitative importance of each type of financial friction. For example, Greenwood and Jovanovic (1990) and Townsend and Ueda (2006) study the credit entry cost. Buera, Kaboski and Shin (2011) and Buera and Shin (2013) focus on the collateral constraint. Greenwood, Sanchez and Wang (2010, 2013) quantify the importance of the intermediation cost. These three types of financial constraints distort the allocation of capital and entrepreneurial productivity. They naturally give rise conceptually to three policy instruments, with each one alleviating a specific type of constraint.

Real-world counterparts are easy to find. For example, the existing literature has documented that the distance to a bank branch matters for credit access, suggesting that policies to promote branch openings in rural locations with unbanked populations could help reduce the credit entry cost in our model.[1] During the global financial crisis, many countries relaxed the collateral constraint by widening the range of securities that could be accepted as collateral with the aim of boosting lending to firms and households. Finally, financial inclusion can lead to increased competition among financial institutions, accelerating investment in financial technology, thereby improving intermediation efficiency and lowering the intermediation cost.[2]

In order to assess and quantify the interactions and tradeoffs of policy instruments, we calibrate the model using data from the World Bank's Enterprise Surveys and World Development Indicators for a number of countries. We choose the model's key parameters to match the data moments reflecting key characteristics of a financial system, in particular the percentage of firms with credit, the collateral-to-loan ratio, and the interest rate spread, as well as other moments, such as the real interest rate, the gross savings as a percentage of GDP, and the firm employment distribution.

Our lead illustrative example is taken from the Philippines, although we also consider other countries below. We first examine the steady-state implications of alleviating different financial constraints on GDP and TFP by conducting counterfactual experiments. Policies that relax the collateral constraint away from its calibrated value have a larger impact on increasing GDP and TFP than policies that reduce the credit entry cost or the intermediation cost. Moreover, the model implies that the increase in GDP and TFP is attributed to different margins depending on which policy instruments is used. Specifically, when the credit entry cost is reduced, GDP and TFP initially increase mostly through the intensive margin; that is, the few entrepreneurs who already have access to finance expand their scale of production. This is because most entrepreneurs do not have sufficient wealth to pay the upfront cost and are excluded from the credit market. When the credit entry cost is further lowered, GDP and TFP increase through the extensive margin, as productive but wealth-constrained entrepreneurs who have previously been excluded from the credit market start to gain access to finance. By contrast, when the collateral constraint is relaxed, both intensive and extensive margins will contribute significantly to the increase in GDP and TFP. When the intermediation cost is lowered,

[1] Many developing countries have implemented such policies to increase credit access. For example, after nationalizing a bank in 1969, the Indian government launched an ambitious social banking program which sought to improve the access of the rural poor to formal credit and savings opportunities (see Burgess and Pande, 2005). Also see Assuncao, Mityakov and Townsend (2012), Alem and Townsend (2013), Gilje, Loutskina and Strahan (2016), Aguirregabiria, Clark and Wang (2017), Celerier and Matray (2017), and Nguyen (2017).

[2] For example, from 1985 to 1994, the Thai banking sector became a more capital-intensive industry, substituting physical capital for labor. The average cost of raising funds decreased from 14.40% in 1985 to 5.61% in 1994 for large banks (see Okuda and Mieno, 1999).

the increase in GDP and TFP is attributed only to the extensive margin. More generally, the model structure provides these decompositions in theory with formulas, which are also quantified in the data, including capturing general equilibrium effects on wages and interest rates.

In terms of the distributional implications, our model implies that reducing the credit entry cost and relaxing the collateral constraint initially increase the steady-state income GINI coefficient. However, when these two constraints are further alleviated, income inequality starts to decline. The income share of the top 10% of the income distribution, consisting mainly of wealthy and productive entrepreneurs, also displays a similar inverted U shape as the two constraints are relaxed. The model implies that the main driver for the reduction in income GINI is the middle class which experiences a consistent rise in income due to improved credit access. By contrast, reducing the intermediation cost always leads to higher income inequality, because it benefits entrepreneurs more than workers, especially the more productive and wealthier entrepreneurs who already have higher income than others. The income share of the top 10% steadily increases when the intermediation cost is lowered, whereas the middle class's income share remains unchanged. We also investigate the welfare implications of financial inclusion policies. We find that policies that relax different constraints tend to bring differential benefits to agents, depending on their wealth and productivity. Although reducing the credit entry cost benefits every agent in the economy, the welfare of some agents falls when the collateral constraint is relaxed or when the intermediation cost is reduced due to general equilibrium effects.

Next, we study the equilibrium interactions of these financial constraints. In partial equilibrium, our analytical results reveal that when multiple constraints are relaxed simultaneously, the effect is amplified through the intensive margin but dampened through the extensive margin. This implies that policies that alleviate multiple constraints could, in principle, yield a larger or smaller effect than policies targeting individual constraints, depending on whether the overall interaction effect between different constraints is positive or negative. For example, if the interaction effect through the intensive margin dominates, policy makers should develop a more balanced financial system, as different financial constraints are complements in restraining GDP. In this case, policies that alleviate the most binding financial constraint would be most effective. On the other hand, if the interaction effect through the extensive margin dominates, policy makers should focus on alleviating a single constraint while ignoring all other constraints.

Using the calibrated model, we conduct a sequence of counterfactual experiments to study the equilibrium interactions between financial constraints. We find that the interaction effect between different constraints is positive, suggesting that effective financial inclusion policies should be designed to develop a well-rounded financial system by alleviating the currently most binding constraint. As noted earlier, policies that are effective in increasing GDP in one country may not be equally effective in others. Therefore, identifying the most binding constraint is important for designing financial policies.

To demonstrate how our model can be harnessed to help identify the bottleneck constraint in a financial system, we separately calibrate the model to six representative countries. Three of them, Pakistan, Bangladesh, and Brazil, have relatively extreme financial constraints. For example, only 6.7% of firms in Pakistan have access to credit; the average collateral requirement in Bangladesh is 271.4% of the loan size; and in Brazil, the interest spread between lending rates and deposit rates is as high as 35.4%. The model implies that policies that relax the collateral constraint in Bangladesh and reduce the intermediation cost in Brazil would be more effective in increasing GDP and TFP, which is consistent with the inference directly drawn from the tight collateral constraint in Bangladesh and the high interest rate spread in Brazil. However, choosing the policy instrument entirely based on descriptive statistics could be misleading because these statistics are endogenously determined in equilibrium. For instance, the model implies that relaxing the collateral constraint rather than reducing the credit entry cost would increase the GDP of Pakistan more, even though the country's credit access ratio is low.

The other three countries analyzed are the Philippines, Kenya, and Zambia, which have relatively balanced financial constraints. It is difficult to tell which financial constraint is most binding simply by looking at the descriptive statistics for the credit access ratio, the collateral-to-loan ratio, and the interest rate spread. By calibrating the model to these countries, we are able to systematically evaluate the potential impacts of different policy instruments, and hence shed light on the real underlying bottleneck. Our model provides a structural framework to systematically study the potential implications of different financial inclusion policies, providing complementary and arguably more accurate evaluations than simple summary statistics.

Importantly, the model also allows us to evaluate financial inclusion policies during economic transitions, uncovering the tradeoffs between short-run and long-run effects. For example, the model implies that the short-run effect of policies on GDP is smaller than the long-run effect because it takes time for entrepreneurs to fully adjust their wealth accumulation and production decisions in the presence of financial frictions (see, e.g., Jermann and Quadrini, 2007; Buera and Shin, 2011). However, interest rates may overshoot in the short run, leading to high borrowing costs, the opposite of the intended effect. For Brazil, reducing the intermediation cost generates a moderately higher GDP but widens income inequality in the short run, although GDP is higher and inequality is lower in the long run. Such intertemporal tradeoffs could raise concerns for policy makers depending on their objectives and their levels of commitment.

Related Literature. Our paper is related to the literature that uses models of occupational choice and financial frictions to study the aggregate and distributional impacts of financial intermediation (see, e.g., Banerjee and Newman, 1993; Lloyd-Ellis and Bernhardt, 2000; Gine and Townsend, 2004; Cagetti and Nardi, 2006; Jeong and Townsend, 2007, 2008; Amaral and Quintin, 2010; Buera, Kaboski and Shin, 2011, 2012; Greenwood, Sanchez and Wang, 2013; Moll, 2014). In contrast to these studies, our model introduces multiple financial frictions to the occupational choice framework to highlight multiple dimensions of financial inclusion within an economy. The model also uncovers how different frictions interact with each other.

Although the financial frictions we highlight have typically been considered separately in the literature, our paper provides a unified framework for examining them both individually and jointly. In this case, our paper is particularly related to studies in which multiple financial frictions co-exist and are compared. For example, Albuquerque and Hopenhayn (2004) and Clementi and Hopenhayn (2006) find that moral hazard and limited commitment have different implications for firm dynamics. Abraham and Pavoni (2005) and Doepke and Townsend (2006) illustrate how consumption allocations differ under moral hazard with and without hidden savings versus full information. Ahlin and Townsend (2007) emphasize regional variations in obstacles to trade and financial constraints. Martin and Taddei (2013) study the implications of adverse selection on macroeconomic aggregates and contrast them with those under limited commitment. Karaivanov and Townsend (2014) estimate the financial/information regime in place for households (including those running businesses) in Thailand and find that a financial regime constrained by moral hazard fits the data best in urban areas, while a regime that is more limited by savings is more applicable for rural areas. Relatedly, Paulson, Townsend and Karaivanov (2006) argue that moral hazard best fits the data in the more urban central region of Thailand but not in the more rural northeast. Kinnan (2014) uses a different metric based on the first-order conditions characterizing optimal insurance under moral hazard, limited commitment, and hidden income to distinguish between these regimes in Thai data. Moll, Townsend and Zhorin (2014) develop a general equilibrium framework that encompasses different types of frictions, and examine their interactions. More recently, Nikolov, Schmid and Steri (2018) develop a rich structural corporate model incorporating multiple financial frictions, with the goal of identifying sources of financial constraints using firm-level data. Our paper is different from these studies in that we emphasize the rich interactions among financial constraints and the tradeoffs between different policy instruments.

Our paper is also related to the literature on misallocation (see, e.g., Hsieh and Klenow, 2009; Banerjee and Moll, 2010; Caselli and Gennaioli, 2013; Midrigan and Xu, 2014; Moll, 2014) and inequality (see, e.g., Davies, 1982; Huggett, 1996; Aghion and Bolton, 1997; Castaneda, Diaz-Gimenez and Rios-Rull, 2003; Nardi, 2004). Our contribution is to show that policy instruments that target different financial sector frictions have differential impacts on resource allocation and inequality.

The remainder of the paper is organized as follows. The next section sets out the structure of the model. Section 3 introduces the data and calibrates the model. Section 4 evaluates the aggregate and distributional implications of different policy instruments on GDP, TFP, and the income distribution. Section 5 analyzes how different financial constraints and policy instruments interact with each other. Section 6 applies the model to several countries to demonstrate its ability in identifying the most binding constraint and evaluating transitional dynamics. Finally, Section 7 concludes.

2 Model

We develop a general-equilibrium model of heterogeneous agents with multiple sources of financial frictions to study the equilibrium interactions among different financial constraints and their implications on GDP, TFP, and the distribution of income.

2.1 Agents

Heterogeneity and Demographics. Time is indexed by t. The economy is populated by a continuum of agents of measure one and there is no population growth. Agents live indefinitely and are heterogeneous in terms of wealth b_t and entrepreneurial productivity z_t. Wealth b_t evolves endogenously, determined by forward-looking savings decisions. Productivity z_t follows an exogenous stochastic process. In particular, with probability γ, agents retain their productivity in the previous period; with probability $1-\gamma$, agents draw a new productivity from a time-invariant Pareto distribution $\mu(z)$ governed by the tail parameter θ. The shocks to productivity can be interpreted as changes in market conditions that affect the profitability of individual skills (see Buera, Kaboski and Shin, 2011). In our calibration, θ is determined to match the employment share distribution of firms and γ is determined to match the gross savings as a percentage of GDP.

Preferences. Agents derive utility from consumption c_t and have preferences

$$\mathbb{E}_t \left[\sum_{s=t}^{\infty} \beta^{s-t} \frac{c_s^{1-\sigma}}{1-\sigma} \right], \tag{2.1}$$

where β is the discount factor, calibrated to match the real interest rate, and σ is the risk-aversion parameter.

Technology. In any period t, agents can make occupation choices between workers and entrepreneurs. Each worker supplies one unit of labor inelasticly and earns the equilibrium wage w_t. Each entrepreneur operates a technology that uses capital k_t and labor l_t as inputs to produce output. The profitability of an entrepreneurial business depends on the agent's productivity; specifically, the output $f(k_t, l_t, z_t)$ is given by

$$f(k_t, l_t, z_t) = z_t (k_t^{\alpha} l_t^{1-\alpha})^{1-\nu}, \tag{2.2}$$

where $1-\nu$ is the span-of-control parameter, representing the share of output accruing to variable factors. Out of this, a fraction α goes to capital and $1-\alpha$ goes to labor. Production exhibits diminishing returns to scale, with $\nu > 0$. Capital depreciates at rate δ.

2.2 Financial Markets

The only asset in the economy is capital. This is equivalent to assuming that a perfect technology exists that can freely transform capital into consumption goods. Perfectly competitive financial intermediaries receive deposits from all agents and lend to entrepreneurs. The deposit interest rate, r_t, is determined endogenously by the capital market clearing condition at time t. Following Buera and Shin (2011), we focus on within-period credit for production purposes. We do not allow borrowing for consumption smoothing across periods by imposing $b_t \geq 0$. Therefore, only entrepreneurs borrow from intermediaries.

The key distinction of our model is that we introduce and examine three sources of financial frictions in a unified framework. Each source of friction reflects one particular aspect of financial market imperfection. These three types of financial constraints naturally give rise conceptually to three policy instruments, with each one targeting a specific constraint. The three frictions are described below.

Limited Credit Access. Entrepreneurs borrow from intermediaries, which incurs an upfront fixed credit entry cost amounting to ψ units of consumption goods. The modeling of fixed entry costs for obtaining credit follows Greenwood and Jovanovic (1990) and Townsend and Ueda (2006) among others. It captures, for example, various fees associated with financial accounts, the cost of bookkeeping and exchange (transportation), etc (see, e.g., Townsend, 1983).

We assume that an agent lives in a "credit regime" if the agent pays the cost ψ and can borrow, and in a "savings regime" if the agent does not pay ψ and can therefore only save. In equilibrium, the fixed entry cost ψ is more likely to exclude poor entrepreneurs from financial markets, as it amounts to a larger fraction of their wealth. The value of ψ will be calibrated to match the fraction of firms with credit access in a country.

Collateral Constraint. The credit contract that entrepreneurs obtain is subject to a collateral requirement due to limited enforceability of debt contracts. In particular, consider an entrepreneur with wealth b_t who approaches financial intermediaries for a loan x_t at the competitive equilibrium lending interest rate r_t^l. After obtaining the loan x_t, the entrepreneur transforms the wealth-on-hand $b_t - \psi + x_t$ costlessly into capital $k_t = b_t - \psi + x_t$, which is then used as collateral to secure the loan x_t. The entrepreneur is free to default and walk away with her income and wealth. The financial intermediaries will seize the collateral, however. Following Jermann and Quadrini (2009, 2012), we assume that the liquidation value of capital is uncertain at the time of contracting. With probability $1 - \xi$ intermediaries recover the full value k_t, but with probability ξ the recovery value is zero. Thus to avoid default, the amount of loan x_t that intermediaries are willing to lend is restricted by

$$x_t \leq (1 - \xi) k_t. \tag{2.3}$$

Substituting x_t, we can derive the entrepreneur's capital constraint in terms of her wealth b_t:

$$\xi k_t \leq b_t - \psi. \tag{2.4}$$

The single parameter $\xi \in [0, 1]$ parsimoniously captures the tightness of the borrowing constraint, with $\xi = 1$ corresponding to financial autarky, where all capital must be self-financed by entrepreneurs' wealth $b_t - \psi$. The value of ξ will be calibrated to match the collateral-to-loan ratio in a country.

Intermediation Inefficiency. We assume that the lending interest rate r_t^l is higher than the deposit interest rate r_t by a margin χ, i.e., $r_t^l = r_t + \chi$. Thus for the loan x_t, entrepreneurs pay intermediation fees equal to χx_t units of consumption goods. The interest rate spread χ reflects the efficiency of financial intermediation. For example, as noted by Townsend (1983), costly intermediation could arise from the cost of enforcement and monitoring when information is imperfect. The value of χ will be calibrated to match the difference between the average deposit rate and the lending rate in a country.

2.3 Agents' Problem

We formulate the agent's problem recursively. Denote by $V_t(z_t, b_t)$ the value function of the agent of type (z_t, b_t). Denote by $W_t(z_t, b_t)$ or $E_t(z_t, b_t)$ as the agent's value if she chooses to be a worker or an entrepreneur at time t. The occupation choice is made to maximize utility,

$$V_t(z_t, b_t) = \max \left\{ W_t(z_t, b_t), E_t(z_t, b_t) \right\}. \tag{2.5}$$

The value function also generally depends on time t because we consider transitional dynamics. For expositional purposes, we denote variables and distributions in steady states by omitting the subscript t.

The worker's value is given by

$$W_t(z_t, b_t) = \max_{c_t} \frac{c_t^{1-\sigma}}{1-\sigma} + \beta \mathbb{E}_t \left[V_{t+1}(z_{t+1}, b_{t+1}) \right] \tag{2.6}$$

subject to

$$\begin{aligned} c_t + b_{t+1} &= (1 + r_t) b_t + w_t, \\ c_t, b_{t+1} &\geq 0. \end{aligned}$$

The entrepreneur's value is determined by comparing the cases with and without borrowing from financial intermediaries. Denote by $E_t^s(z_t, b_t)$ or $E_t^c(z_t, b_t)$ the entrepreneur's value when capital is financed with savings only or with credit as well. The borrowing decision is made to maximize utility,

$$E_t(z_t, b_t) = \max \left\{ E_t^s(z_t, b_t), E_t^c(z_t, b_t) \right\}. \tag{2.7}$$

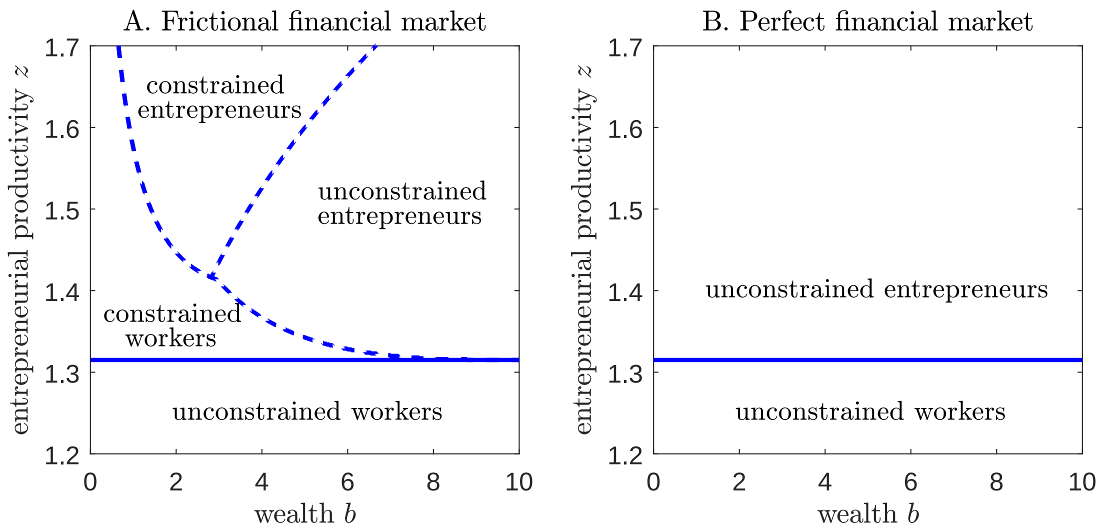
In the savings regime, the agent finances production herself and the remaining wealth is deposited in financial intermediaries for interest earnings. The value function in the savings regime is

$$
\begin{aligned}
E_t^s(z_t, b_t) = \max_{c_t, k_t, l_t} \frac{c_t^{1-\sigma}}{1-\sigma} + \beta \mathbb{E}_t \left[V_{t+1}(z_{t+1}, b_{t+1}) \right] \qquad (2.8)\\
\text{subject to}\\
c_t + b_{t+1} = z_t (k_t^\alpha l_t^{1-\alpha})^{1-\nu} + (1-\delta)k_t - w_t l_t + (1+r_t)(b_t - k_t),\\
k_t \leq b_t,\\
c_t, k_t, l_t, b_{t+1} \geq 0.
\end{aligned}
$$

In the credit regime, the agent takes out loans to finance production. Without loss of generality, we only consider the case in which the agent invests $k_t > b_t$ in the credit regime, because investing $k_t \leq b_t$ but paying the credit entry cost is obviously not optimal. Thus, we can write the value function in the credit regime as

$$
\begin{aligned}
E_t^c(z_t, b_t) = \max_{c_t, k_t, l_t} \frac{c_t^{1-\sigma}}{1-\sigma} + \beta \mathbb{E}_t \left[V_{t+1}(z_{t+1}, b_{t+1}) \right] \qquad (2.9)\\
\text{subject to}\\
c_t + b_{t+1} = z_t (k_t^\alpha l_t^{1-\alpha})^{1-\nu} + (1-\delta)k_t - w_t l_t - (1+r_t+\chi)(k_t - b_t + \psi),\\
\xi k_t \leq b_t - \psi,\\
c_t, l_t, b_{t+1} \geq 0 \text{ and } k_t > b_t.
\end{aligned}
$$

Panel A of Figure 2 illustrates the choice of occupation for agents with different wealth and productivity. Depending on whether the occupation choice is constrained by wealth, we identify four categories of agents: unconstrained workers, constrained workers, constrained entrepreneurs, and unconstrained entrepreneurs. As shown in the figure, there is a certain level of productivity (1.32) below which agents always find working for a wage better than operating a business. These agents are identified as unconstrained workers, because their productivity is so low that they never find it optimal to become entrepreneurs. Above this level of productivity, there are three regions. The left region refers to constrained workers. These agents are productive but do not have sufficient wealth to operate businesses at a profitable scale. The middle region represents constrained entrepreneurs who have sufficient wealth to operate profitable businesses, but their scale of business is constrained. Agents in the right region choose to be entrepreneurs, operating businesses at the unconstrained scale, with the marginal return on capital equal to the marginal cost. Thus, they are classified as unconstrained entrepreneurs.



Note: Panel A illustrates the occupation choice in the presence of financial frictions (i.e., $\psi = 0.05$, $\chi = 0.05$, $\xi = 0.5$). Panel B illustrates the occupation choice in a perfect financial market (i.e., $\psi = \chi = \xi = 0$) in partial equilibrium with the same interest rate and wage as Panel A. In general equilibrium, when we move from an economy with financial frictions to an economy without financial frictions, the interest rate and wage will also change, which we will analyze in Sections 4 – 6. The other parameters for plotting the figure are: $r = 0.05$, $w = 0.8$, $\delta = 0.06$, $\nu = 0.19$, $\alpha = 0.33$.

Figure 2: Occupation choice with/without financial frictions.

In the presence of financial market frictions, the decision to become an entrepreneur and the scale of business not only depends on productivity z_t but also on wealth b_t. By contrast, when the financial market is perfect (i.e., $\psi = \chi = \xi = 0$), Panel B of Figure 2 indicates that productive agents always choose to be entrepreneurs and operate their businesses at the unconstrained scale. We formally state this result in Proposition 1.

Proposition 1. *In the absence of financial frictions, there exists a threshold of entrepreneurial productivity $\underline{z}_t$, and the agent chooses to be an entrepreneur if and only if $z \geq \underline{z}_t$.*

2.4 Competitive Equilibrium

Denote by $h_t(b, z)$ the probability density function (PDF) for the joint distribution of wealth and productivity at the beginning of period t. Given an initial distribution $h_0(b, z)$, a competitive equilibrium consists of allocations $\{c_t(b, z), k_t(b, z), l_t(b, z)\}_{t=0}^{\infty}$, sequences of joint distributions of wealth and productivity $\{h_t(b, z)\}_{t=0}^{\infty}$ and prices $\{r_t, w_t\}_{t=0}^{\infty}$ such that:

(1). Agents optimally choose the underlying regime, occupation, consumption $c_t(b, z)$, capital $k_t(b, z)$, and labor $l_t(b, z)$ by solving problems (2.5)-(2.9) at all $t \geq 0$.

(2). The capital market clears at all $t \geq 0$,

$$\iint_{(b,z)\in\Phi_t^C} [k_t(b,z) - b + \psi] h_t(b,z) \mathrm{d}b\mathrm{d}z = \iint_{(b,z)\notin\Phi_t^E} b h_t(b,z) \mathrm{d}b\mathrm{d}z + \iint_{(b,z)\in\Phi_t^S} [b - k_t(b,z)] h_t(b,z) \mathrm{d}b\mathrm{d}z, \tag{2.10}$$

where Φ_t^E is the set of entrepreneurs at time t; Φ_t^S and Φ_t^C are the sets of entrepreneurs in the savings and credit regimes, respectively. We have $\Phi_t^E = \Phi_t^S \cup \Phi_t^C$.

(3). The labor market clears at all $t \geq 0$,

$$\iint_{(b,z)\in\Phi_t^E} l_t(b,z) h_t(b,z) \mathrm{d}b\mathrm{d}z = \iint_{(b,z)\notin\Phi_t^E} h_t(b,z) \mathrm{d}b\mathrm{d}z. \tag{2.11}$$

(4). $\{h_t(b,z)\}_{t=0}^{\infty}$ evolves according to the equilibrium mapping

$$h_{t+1}(b',z')\mathrm{d}b\mathrm{d}z = \gamma dz \int \mathbb{1}_{\{b^*=b'\}} h_t(b,z')\mathrm{d}b + (1-\gamma)\mu(z')dz \iint \mathbb{1}_{\{b^*=b'\}} h_t(b,z)\mathrm{d}b\mathrm{d}z, \tag{2.12}$$

where $b^* \equiv b_{t+1}(b,z)$ is the wealth at $t+1$ implied by the optimal savings decision of agents of type (b,z). $\mathbb{1}_{\{b^*=b'\}}$ is an indicator function which equals one if $b^* = b'$ and zero otherwise. The left-hand side of equation (2.12) is the probability mass of agents with (b',z') at $t+1$. The right-hand side sums the transition probability to (b',z') from any arbitrary (b,z) at t. With probability γ, the agent keeps the current productivity z' and transits to b' from (b,z') if $b^* = b'$. With probability $1-\gamma$, the agent draws a new productivity, which equals z' with probability $\mu(z')\mathrm{d}z$.

3 Data and Calibration

We use firm-level data from the World Bank Enterprise Surveys and macroeconomic indicators from the World Development Indicators to discipline the model. The Enterprise Surveys are representative firm-level surveys of a country's private sector. The surveys cover a broad range of topics including access to finance, corruption, infrastructure, crime, competition, and performance.

We calibrate the model by first matching the moments for the Philippines, which serves as our benchmark calibration. The sample contains 1,335 firms interviewed from November 2014 through May 2016. In Section 6, we calibrate the model to target moments for other countries and show that it could be used to conduct cross-country analyses.

We set the risk-aversion parameter $\sigma = 1.5$ and the span-of-control parameter $\nu = 0.19$

Table 1: Calibration and moments in data and model.

	Data	Model	Parameter
Firms with credit (%)	29.9	29.5	$\psi = 0.95$
Collateral (% of loans)	156.7	156.4	$\xi = 0.335$
Interest rate spread (%)	4.0	4.0	$\chi = 0.04$
Real interest rate (%)	6.2	6.3	$\beta = 0.88$
Top 5% employment share (%)	43.7	43.7	$\theta = 3.4$
Top 10% employment share (%)	57.6	58.6	$\gamma = 0.89$
Top 20% employment share (%)	73.2	71.5	
Top 40% employment share (%)	88.8	85.3	
Gross savings (% of GDP)	44.2	41.2	

following standard practice. The one-year depreciation rate δ is set to 0.06. The aggregate income share of capital α is set to 0.33.

The remaining six parameters ψ, ξ, χ, β, θ, and γ are calibrated by matching relevant moments in the data as shown in Table 1: the fraction of firms with access to credit; the amount of collateral as a percentage of total loans; the spread between deposit and lending rates; the real interest rate; the employment share of the top 5%, 10%, 20%, and 40% of firms; and the gross savings rate as a percentage of GDP.

The three key parameters capturing the financial frictions are calibrated to match the credit access ratio (i.e., the percentage of firms with access to credit), the collateral-to-loan ratio, and the interest rate spread. The 4% interest rate spread directly identifies the parameter χ. For firms with access to credit, the average collateral as a percentage of loans is 156.7%, indicating that for each unit of credit, the firm has to post 1.57 units of collateral, on average. This moment provides information about the tightness of the collateral constraint, governed by the parameter ξ. In the Philippines, the fraction of firms with credit access is 29.9%, which identifies the credit entry cost ψ.

Buera and Shin (2011) and Moll (2014) show that with persistent productivity, entrepreneurs can accumulate wealth on their own and overcome financial constraints. We thus identify the parameter γ by matching the gross savings as a percentage of GDP. We set the discount factor β to match the real interest rate of 6.2%. The parameter θ governs the distribution of agents' entrepreneurial productivity. A lower θ implies that the distribution is more right-skewed, and hence top entrepreneurs will employ a larger fraction of labor in the economy. We thus calibrate θ to match the employment share distribution. Specifically, we use four brackets of employment shares, corresponding to the fraction of labor employed by the top 5%, 10%, 20%, and 40% of entrepreneurs ranked in terms of their business income.

4 Distinguishing the Impact of Policy Instruments

In this section, we systematically examine the implications of different financial inclusion policy in steady states, and as companion sections, Section 5 studies the equilibrium interactions of different financial constraints, and Section 6 studies policies in different countries and their impacts on transitional dynamics.

We first present formulas to decompose changes in GDP and TFP. Next, we evaluate the aggregate implications of policy instruments in steady states. Finally, we shed light on the distributional implications of these policies on income, wealth, and welfare.

4.1 Formulas for GDP and TFP Decomposition

In our model, financial inclusion is reflected by lower values of the parameters, $\Omega \equiv (\psi, \xi, \chi)$. We distinguish the effect of financial inclusion through the extensive and intensive margins. On the one hand, relaxing financial constraints can increase GDP and TFP through the extensive margin by increasing the credit access ratio (i.e., entrepreneurs move from the savings regime to the credit regime). On the other hand, relaxing financial constraints enables entrepreneurs who already have access to credit to scale up their production, boosting GDP and TFP through the intensive margin. In particular, the effect of financial inclusion on GDP and TFP can be decomposed as follows.

Proposition 2. *Consider a financial inclusion policy that relaxes the constraints from Ω to Ω', with $\Omega' \leq \Omega$. The increase in steady-state GDP is given by*

$$\begin{aligned}\mathrm{GDP}_{\Omega'} - \mathrm{GDP}_{\Omega} = &\int_z \int_{\underline{b}(z;\Omega')}^{\underline{b}(z;\Omega)} \left[y^C(b,z;\Omega')h(b,z;\Omega') - y^S(b,z;\Omega)h(b,z;\Omega) \right] dbdz \qquad (4.1)\\ &+ \int_z \int_{\underline{b}(z;\Omega)}^{\infty} \left[y^C(b,z;\Omega')h(b,z;\Omega') - y^C(b,z;\Omega)h(b,z;\Omega) \right] dbdz\\ &+ \int_z \int_{0}^{\underline{b}(z;\Omega')} \left[y^S(b,z;\Omega')h(b,z;\Omega') - y^S(b,z;\Omega)h(b,z;\Omega) \right] dbdz.\end{aligned}$$

where $\underline{b}(z;\Omega)$ denotes the threshold of wealth above which entrepreneurs of productivity z choose to access credit in the steady state of the economy with Ω. The variables $y^S(b,z;\Omega)$ and $y^C(b,z;\Omega)$ are entrepreneurs' output net of the deadweight loss arising from financial frictions in the savings and credit regimes, respectively (see Appendix A).

The first term on the right-hand side of equation (4.1) captures the increases through the extensive margin, because when $\underline{b}(z;\Omega') < \underline{b}(z;\Omega)$, more entrepreneurs move from the savings regime to the credit regime and produce more. The second term captures the increases through the intensive margin, because when $y^C(b,z;\Omega') > y^C(b,z;\Omega)$, entrepreneurs who are already in the credit regime expand their scale of production. The third term reflects the general equilibrium effect on entrepreneurs who remain in the savings regime after the financial inclusion policy is implemented.

We define the model-implied steady-state TFP as

$$\text{TFP}_\Omega = \frac{Y_\Omega}{K_\Omega^\alpha L_\Omega^{1-\alpha}}, \tag{4.2}$$

where Y_Ω, K_Ω, and L_Ω are steady-state aggregate output, capital, and labor in the economy with Ω, defined in Appendix B.3.[3] By exploring the equivalence between growth accounting by factor and growth accounting by regime, we can decompose the economy's change in steady-state TFP along similar dimensions.[4]

Proposition 3. *Consider a financial inclusion policy that relaxes the constraints from Ω to Ω', with $\Omega' \leq \Omega$. The increase in steady-state TFP is given by*

$$\begin{aligned}\text{TFP}_{\Omega'} - \text{TFP}_\Omega =& g_{p^c}\left[s_y^c - s_y^s - \alpha(s_k^c - s_k^s) - (1-\alpha)(s_l^c - s_l^s)\right]\text{TFP}_\Omega \\ &+ \left[s_y^c g_{\overline{y}^c} - \alpha s_k^c g_{\overline{k}^c} - (1-\alpha)s_l^c g_{\overline{l}^c}\right]\text{TFP}_\Omega \\ &+ \left[s_y^s g_{\overline{y}^s} - \alpha s_k^s g_{\overline{k}^s} - (1-\alpha)s_l^s g_{\overline{l}^s}\right]\text{TFP}_\Omega,\end{aligned} \tag{4.3}$$

where s_y^s, s_y^c, s_k^s, s_k^c, s_l^s, s_l^c are the average fraction of output, capital, and labor associated with entrepreneurs in the savings regime and the credit regime. g_{p^c} is the percentage change in the fraction of entrepreneurs with credit access after the financial inclusion policy is implemented. $g_{\overline{y}^s}$, $g_{\overline{y}^c}$, $g_{\overline{k}^s}$, $g_{\overline{k}^c}$, $g_{\overline{l}^s}$, and $g_{\overline{l}^c}$ are the percentage change in the average output, capital, and labor in the savings regime and the credit regime after the financial inclusion policy is implemented.

In equation (4.3), the first term captures the increase in TFP through the extensive margin, attributed to entrepreneurs who move from the savings regime to the credit regime. The second term captures the increase in TFP through the intensive margin, attributed to entrepreneurs who are already in the credit regime before the financial inclusion policy is implemented. The third term captures the general equilibrium effect from entrepreneurs who remain in the savings regime after the financial inclusion policy is implemented.

[3] Moll (2014) shows that when entrepreneurs have constant-return-to-scale production technology, TFP defined by equation (4.2) is equal to the average individual productivity weighted by wealth.

[4] Our decomposition is in a similar spirit to the approach of Jeong and Townsend (2007), who decompose the increase in TFP into the occupational shift effect, financial deepening effect, capital heterogeneity effect, and sectoral Solow residuals.

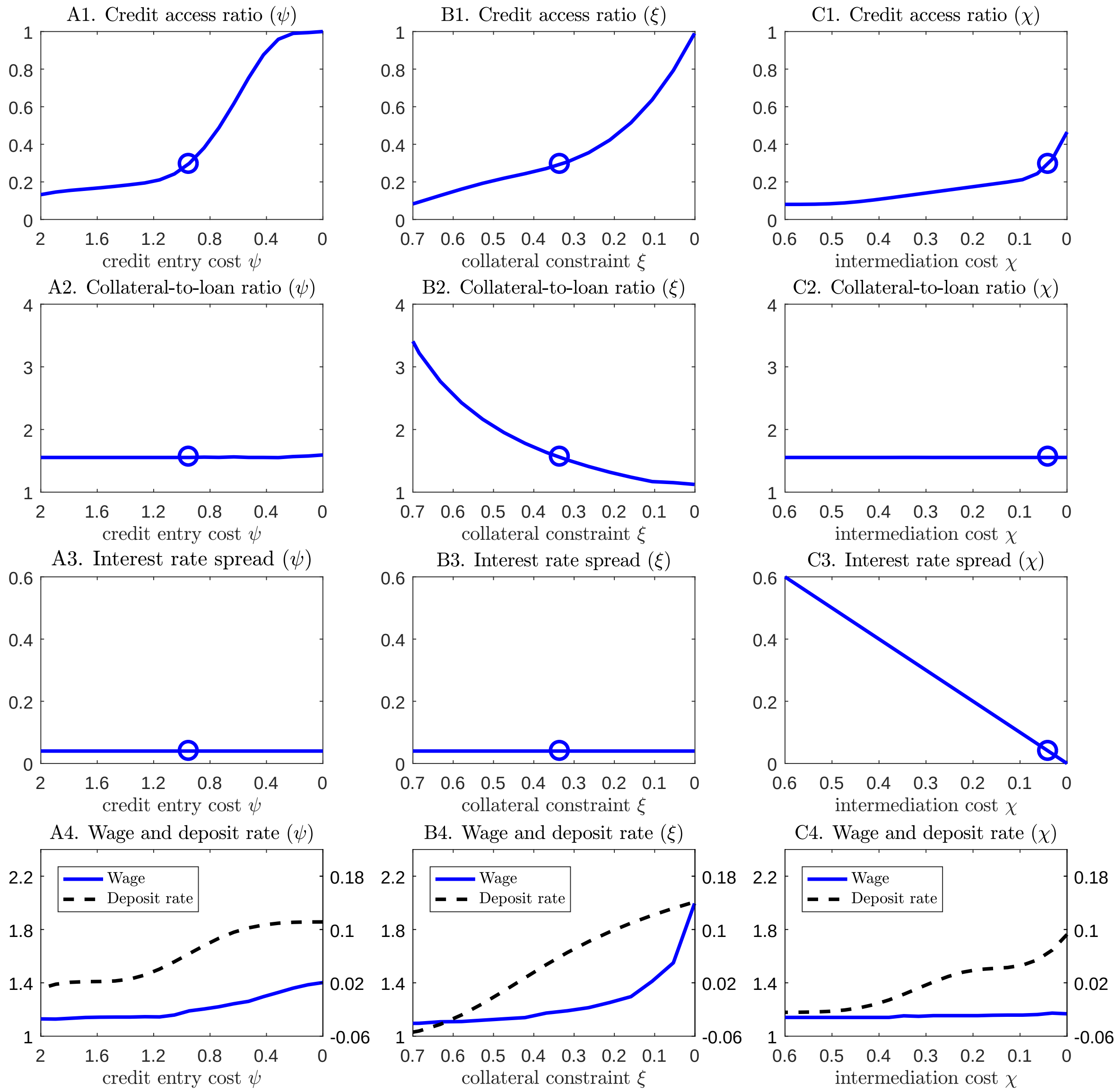
Because the extensive and intensive margins benefit different agents, by analyzing how different financial inclusion policies increase GDP and TFP through the two margins, we can better understand the implication of these policies on inequality and the distribution of income. In the rest of this section, we use these formulas to quantitatively assess the contribution of the two margins to increases in GDP and TFP, and discuss their link to inequality.

4.2 Aggregate Implications of Financial Inclusion

In this section, we analyze the steady-state implications of the three policy instruments in the Philippines. In particular, we vary the value of one of the parameters ψ, ξ, and χ, holding the other two parameters at their calibrated values. These counterfactual experiments capture what would happen in the steady state if the government uses a single policy instrument to increase financial inclusion.

Financial Characteristics and Equilibrium Prices. Figure 3 plots the equilibrium financial characteristics and prices. As shown in Panels A1, B1, and C1, reducing the credit entry cost ψ, relaxing the collateral constraint ξ, and reducing the intermediation cost χ all lead to a higher credit access ratio and more inclusion. However, as we have discussed in Section 2, these financial frictions are meant to capture different types of constraints. Particularly, the parameter ψ captures an ex-ante friction in obtaining credit access, but not the ex-post terms of loan contracts within the credit regime. Thus, a lower ψ increases the credit access ratio but has a negligible effect on the collateral-to-loan ratio and the interest rate spread (Panels A2 and A3). The other two parameters, ξ and χ, capture ex-post frictions within the credit regime. Thus, a lower ξ and a lower χ imply a lower collateral-to-loan ratio (Panel B2) and a lower interest rate spread (Panel C3), respectively. Better terms of loan contracts attract entrepreneurs to access credit by paying the credit entry cost ψ. Thus, even if ψ remains unchanged, the credit access ratio increases due to lower ξ and χ.

Panels A4, B4, and C4 show that financial inclusion along all three dimensions significantly increases the equilibrium interest rate (the black dashed line). However, the equilibrium wage (the blue solid line) takes off only for lower values of the credit entry cost ψ and the collateral constraint ξ. Moreover, reducing the intermediation cost χ has a very limited effect on the equilibrium wage. Because the interest rate and wage determine the main source of income for entrepreneurs and workers, the sharply different impacts on their equilibrium values have crucial implications for the evolution of inequality and



Note: Panels A1, B1, and C1 plot the steady-state fraction of entrepreneurs with access to credit when one of the parameters ψ, ξ, and χ is reduced while the other two parameters are held at their calibrated values. Panels A2, B2, and C2 plot the average steady-state collateral-to-loan ratio, k/x. Panels A3, B3, and C3 plot the interest rate spread χ between the deposit rate r and the lending rate r^l. In Panels A4, B4, and C4, the left y-axis refers to the steady-state wage (the blue solid line); and the right y-axis refers to the steady-state deposit rate (the black dashed line). The blue circle in each panel represents the corresponding value under our calibration in Table 1.

Figure 3: Financial characteristics and equilibrium prices.

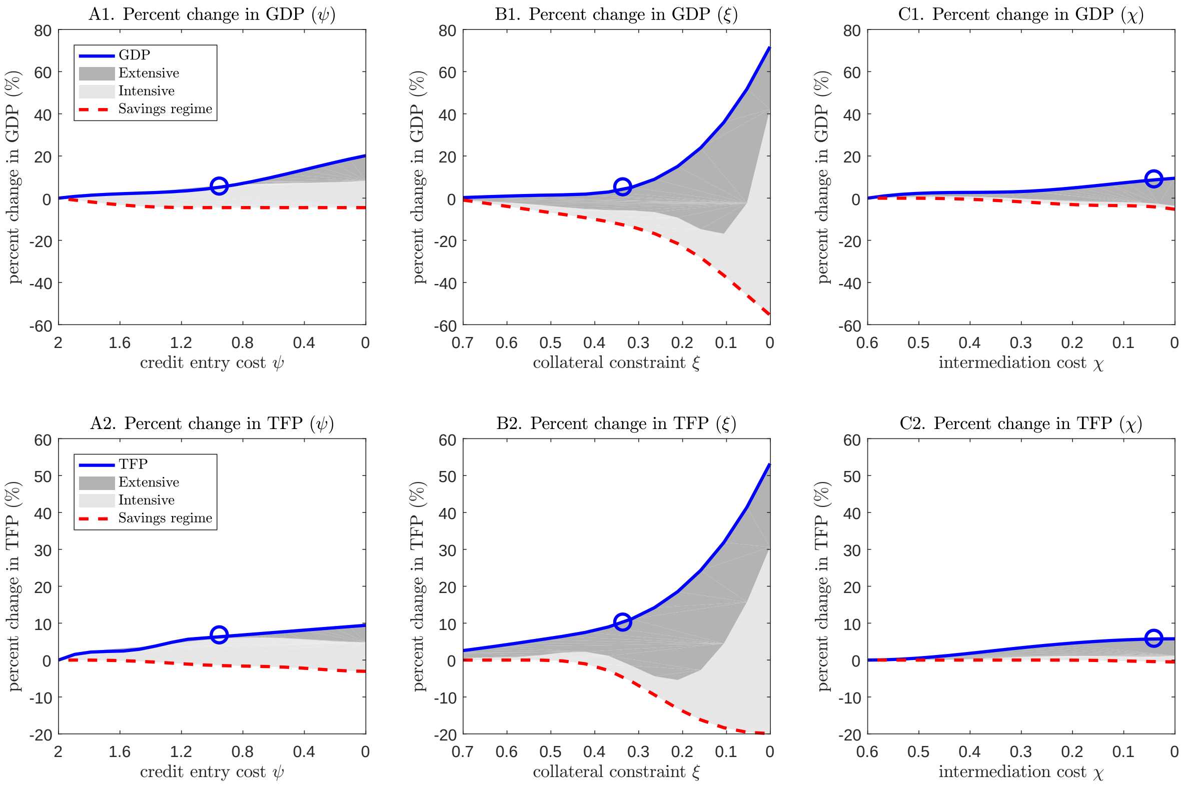
the distribution of income. We return to the discussion of these issues in Section 4.3 below.

GDP, TFP, and Their Decompositions. In Figure 4, we plot the increase in steady-state GDP (the blue solid lines in Panels A1, B1, C1) and TFP (the blue solid lines in Panels A2, B2, C2) when one of the parameters ψ, ξ, and χ is reduced while the other two parameters are held at their calibrated values.

For example, Panels A1 and A2 imply that when the credit entry cost ψ is reduced from 2 to 0, steady-state GDP and TFP increase by about 20.2% and 9.4% respectively; Panels B1 and B2 show that steady-state GDP and TFP increase by about 71.8% and 53.3% respectively when the collateral constraint ξ is lowered from 0.7 to 0; and Panels C1 and C2 show that steady-state GDP and TFP increase by about 9.5% and 5.8% respectively, when the intermediation cost χ is lowered from 0.6 to 0. These counterfactual experiments clearly illustrate that steady-state GDP and TFP respond differently to different financial constraints. In Section 6 below, we apply our model to different countries and demonstrate that it can be used to identify the most binding financial constraint in a country.

We use formulas (4.1) and (4.3) to understand the margins through which steady-state GDP and TFP increase after financial constraints are relaxed. The red dashed line shows that owing to a general equilibrium effect, the savings regime negatively contributes to the overall increase in steady-state GDP and TFP. Intuitively, as financial constraints are relaxed, resources are more efficiently allocated towards more productive entrepreneurs, increasing the demand for capital and labor, which drives up the equilibrium interest rate (the black dashed lines in Panels A4, B4, and C4 of Figure 3) and wage (the blue solid lines in Panels A4, B4, and C4 of Figure 3). Entrepreneurs who remain in the savings regime do not benefit from the relaxed financial constraints. Instead, a higher cost of production leads to them optimally choosing to produce less output. In addition, there is a composition effect. As relatively productive entrepreneurs obtain credit, the average productivity of entrepreneurs in the savings regime falls, negatively contributing to the overall increase in steady-state TFP. Quantitatively, when ψ, ξ, and χ are reduced from their leftmost values on the x-axis to zero, the savings regime contributes to about -4.5%, -55.4%, and -5.3% of the increase in steady-state GDP, and -2.7%, -20.0%, and -0.7% of the increase in steady-state TFP.

The height of the light and dark grey areas in Figure 4 represent the contribution of the intensive and extensive margins, respectively. Although both margins contribute positively to the increase in steady-state GDP and TFP when financial constraints are relaxed, our simulation suggests that the magnitude of their effects largely depends on the values of ψ, ξ, and χ. Specifically, Panel A1 of Figure 4 shows that the intensive margin contributes to a significant portion of the increase in steady-state GDP and TFP. This is because a lower credit entry cost ψ increases pledgeable collateral, allowing entrepreneurs



Note: In Panels A1, B1, and C1, the blue solid line plots the percentage increase in GDP relative to the benchmark economy associated with the leftmost parameter value on the x-axis (i.e., $\psi = 2$ in Panel A1, $\xi = 0.7$ in Panel B1, and $\chi = 0.6$ in Panel C1). For each parameter value on the x-axis, we use formula (4.1) to decompose the corresponding increase in GDP relative to the benchmark. The red dashed line plots the contribution of the savings regime (the third term in formula 4.1). The height of the light grey area represents the contribution of the intensive margin (the second term in formula 4.1). The height of the dark grey area represents the contribution of the extensive margin (the first term in formula 4.1). In Panels A2, B2, and C2, we perform the same exercise for TFP using formula (4.3). The blue circle in each panel represents the corresponding value under our calibration in Table 1.

Figure 4: Impact of financial inclusion on GDP and TFP in steady states.

in the credit regime to expand their production. By contrast, our simulation suggests that when ψ is reduced from 2 to 0.8, the extensive margin has a negligible contribution to the increase in steady-state GDP and TFP; and only when ψ is further reduced to values below 0.8 does the extensive margin start to play a role. Intuitively, this is because there is some sort of financial exclusion for high values of ψ. Unless ψ is sufficiently reduced, most entrepreneurs cannot afford the high credit entry cost. As shown in Panel A1 of Figure 3, the credit access ratio remains low when ψ is greater than 0.8, and this ratio starts to increase significantly only when ψ is further reduced.

By contrast, Panels B1 and B2 show that when the collateral constraint ξ is reduced, both intensive and extensive margins contribute significantly to the increase in steady-state GDP and TFP. Relaxing the collateral constraint allows entrepreneurs with access to credit to borrow more to expand their production, increasing steady-state GDP and TFP through the intensive margin. Moreover, the relaxed collateral constraint also encourages more entrepreneurs to pay the credit entry cost to enter the credit regime, increasing steady-state GDP and TFP through the extensive margin.

Panel C1 shows that when the intermediation cost χ is reduced, most of the increase in steady-state GDP and TFP is attributed to the extensive margin instead of the intensive margin. Intuitively, a lower intermediation cost effectively reduces the user cost of capital, which motivates entrepreneurs to invest more and expand their businesses. However, because the parameter ξ remains unchanged, the scale of business is restricted by the tight collateral constraint, limiting the effect of reducing χ. On the other hand, a lower intermediation cost induces more productive, but wealth-constrained entrepreneurs to enter the credit regime, contributing to the increase in steady-state GDP and TFP through the extensive margin.

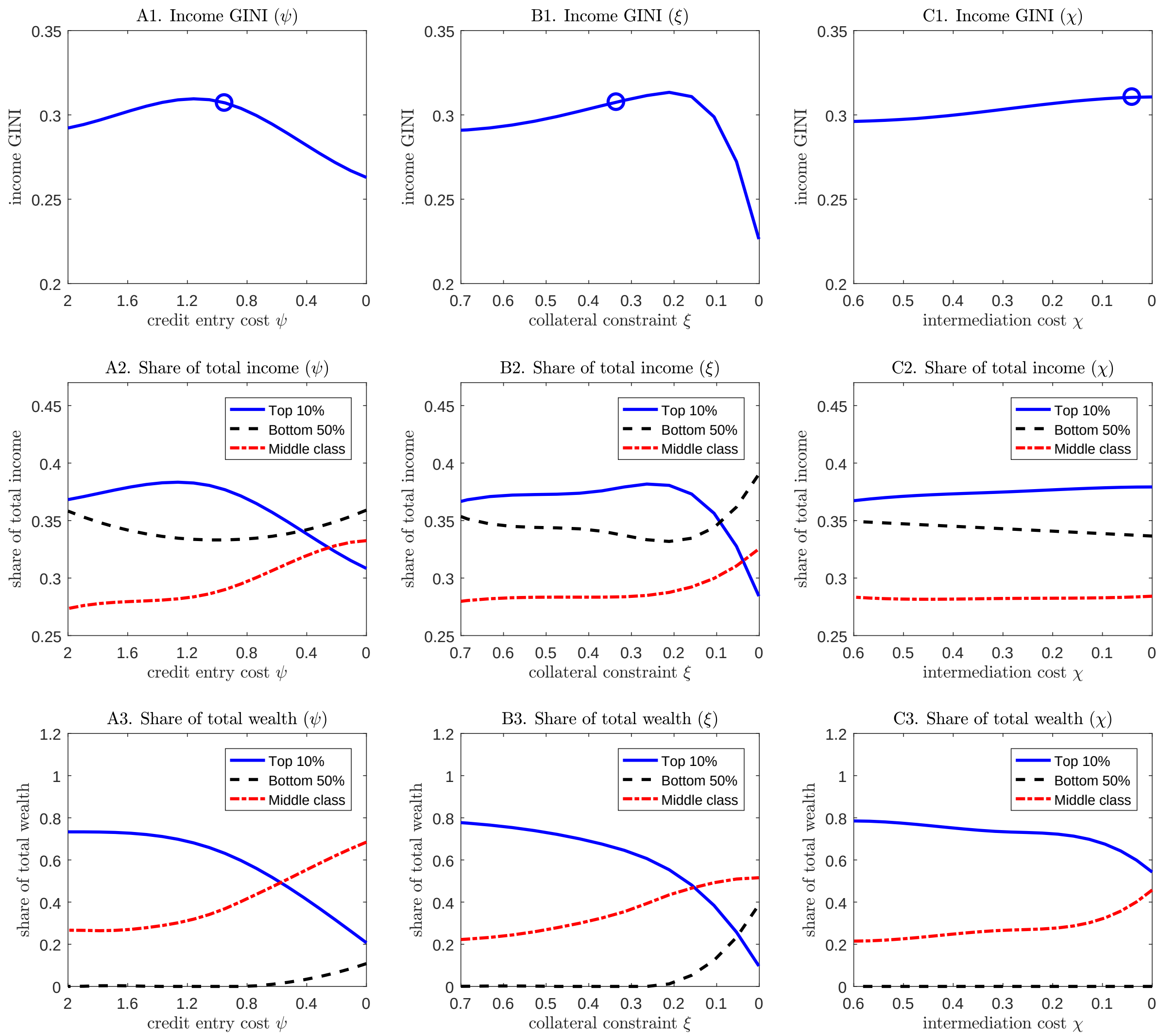
4.3 Distributional Implications of Financial Inclusion

In this section, we move from the macro to micro to shed light on the distributional implications of financial inclusion policies in steady states.

The Distribution of Income and Wealth. In Panels A1, B1, and C1 of Figure 5, we plot the steady-state income GINI when one of the parameters ψ, ξ, and χ is reduced while holding the other two parameters at their calibrated values.

Panel A1 of Figure 5 indicates that the steady-state income GINI increases before declining when the credit entry cost ψ is reduced from 2 to 0. We obtain this inverted U curve due to reasons mentioned for Figure 4. For high values of ψ, reducing the credit entry cost does not generate a significant effect through the extensive margin and GDP increases mainly through the intensive margin. However, entrepreneurs in the credit regime already earn more income than others. Thus, a policy change that further increases their income leads to higher income inequality. When ψ is further lowered, the effect through the extensive margin starts to dominate as more entrepreneurs in the savings regime enter the credit regime. The improved credit access for these entrepreneurs not only increases their income but also generates a significant upward pressure on the equilibrium wage, thereby reducing income inequality.

We can also see the rich dynamics of different income groups in Panel A2, which plots the share of total income for the top 10% and the bottom 50% of the steady-state income distribution, and for the middle class. The top 10% mainly consist of wealthy and



Note: In Panels A1, B1, and C1, we plot the steady-state income GINI when one of the parameters ψ, ξ, and χ is reduced while holding the other two parameters at their calibrated values. In Panels A2, B2, and C2, we plot the share of total income for the top 10% and the bottom 50% of the steady-state income distribution, and the middle class. In Panels A3, B3, and C3, we plot the share of total wealth for the top 10% and the bottom 50% of the steady-state wealth distribution, and the middle class. The middle class refers to people whose income/wealth is below the 10% and above the 50% threshold. The blue circle in each panel represents the corresponding value under our calibration in Table 1.

Figure 5: Impact of financial inclusion on the distribution of income and wealth.

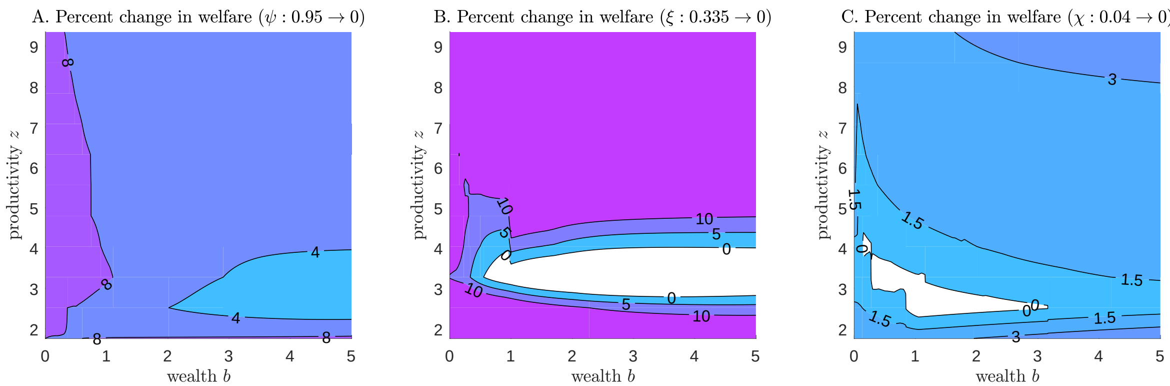
productive entrepreneurs within the credit regime, whose income share (the blue solid line) displays a shape similar to the income GINI. The bottom 50% consist of workers, whose income share (the black dashed line) displays a U shape, opposite to the top 10%. The middle class (the red dash-dotted line) mainly consists of productive, but less wealthy entrepreneurs plus some rich workers with high interest income. Clearly, the main driver for the turning point (at $\psi = 1.1$) of the steady-state income GINI curve is the rise of the middle class due to improved credit access. The dramatic expansion of the middle class's businesses results in a wage takeoff (Panel A4 of Figure 3), which increases the income share of the bottom 50% and lowers that of the top 10%.

Panel B1 shows that the steady-state income GINI also increases before declining when the collateral constraint ξ is relaxed. Similar to the turning point in the previous experiment, the turning point in the experiments of ξ also coincides with the value of ξ at which the equilibrium wage takes off (Panel B4 of Figure 3). As shown in Panel B2, the main driver for the wage takeoff is the middle class who expands production due to relaxed collateral requirements. By contrast, our simulation indicates that reducing the intermediation cost χ always leads to higher income inequality (Panel C1). Panel C2 shows that there is a moderate increase in the steady-state income share of the top 10% (the blue solid line) and a moderate decrease in that of the bottom 50% (the black dashed line). The middle class's steady-state income share is barely changed (the red dash-dotted line) and as a result, the equilibrium wage does not take off (Panel C4 of Figure 3).

In Panels A3, B3, and C3, we plot the share of total wealth for top 10% and the bottom 50% of the steady-state wealth distribution, and the middle class. The composition of each wealth group is similar to that of the corresponding income group. The steady-state wealth share of the top 10% always decreases after any of the three financial constraints is relaxed. This is because when financial constraints are relaxed, credit effectively becomes cheaper and more accessible. Thus, in steady states, wealthy and productive entrepreneurs do not need to accumulate as much wealth to operate their businesses at the unconstrained scale (Figure 2). Therefore, they reduce savings, leading to a lower wealth share of the top 10% and a higher wealth share of the rest.

Welfare Implications. To assess the steady-state welfare implications of financial inclusion policies, we conduct three counterfactual experiments for the Philippines. In the first experiment, we reduce the credit entry cost ψ from the calibrated value 0.95 to 0. In the second experiment, we assume that borrowing is frictionless by reducing the collateral constraint ξ from the calibrated value 0.335 to 0. In the third experiment, we assume perfect financial intermediation by reducing the intermediation cost χ from the calibrated value 0.04 to 0. Figure 6 presents the change in steady-state welfare (consumption-equivalent measure) for agents of different wealth and productivity.[5]

[5] Note that due to the existence of idiosyncratic productivity shocks, agents who are productive now can be unproductive in the future, and vice versa. Our value functions derived from forward-looking agents already capture the effect from such dynamic changes in individual characteristics.



Note: The numbers in Panels A, B, and C represent the percentage change in consumption-equivalent welfare in the steady state after financial inclusion policies are implemented relative to the steady state before these policies are implemented. The white area represents the region in which agents experience welfare losses after financial inclusion.

Figure 6: Impact of financial inclusion on welfare in steady states.

Panel A shows that reducing the credit entry cost ψ increases the steady-state welfare for all agents, but by different magnitudes. Agents in the region enclosed by "4" obtain relatively small welfare gains (less than 4%). These agents are mostly unconstrained entrepreneurs (see Figure 2) who are wealthy but not productive, operating businesses at small scales and do not demand much external credit. Thus reducing the credit entry cost does not benefit these entrepreneurs much as most of them are currently unconstrained. Agents in the region enclosed by "8" obtain the largest welfare gains (more than 8%). These agents are currently constrained workers, who are productive but poor. A lower credit entry cost allows them to access credit and start businesses, significantly increasing their life-time utility. The agents in the middle region obtain modest welfare gains, between 4% and 8%. These agents are currently either unconstrained workers who obtain welfare gains due to higher wages, or constrained entrepreneurs who expand their businesses due to a lower credit entry cost.

In contrast to Panel A, we find that not everyone gains when the collateral constraint ξ is relaxed or the intermediation cost χ is reduced. Panel B shows that when the collateral constraint ξ is relaxed, agents in the (white) region enclosed by "0" incur welfare losses in the new steady state. These are the entrepreneurs who are moderately productive and wealthy. Their businesses are operated at the unconstrained scale, and thus further relaxing the collateral constraint does not benefit them much. However, relaxing the collateral constraint results in a general equilibrium effect that increases the cost of production (i.e., the equilibrium interest rate and wage), reducing their profits. Our simulation implies that the biggest winners in this counterfactual experiment are the most productive but constrained entrepreneurs (the top region) and the unconstrained workers (the bottom region), whose welfare increases by more than 10%. The former group benefits from increased scale of production and the latter from the significant increase in the equilibrium wage.

Panel C shows that when the intermediation cost χ decreases, agents who are currently moderately productive but poor (in the region enclosed by "0") incur welfare losses in the new steady state. These agents are entrepreneurs who operate their businesses at small scales due to moderate productivity. They do not demand much credit and receive little benefit from a lower intermediation cost. However, they suffer from a higher production cost due to the rise in the equilibrium interest rate and wage. The agents who obtain the largest welfare gains (more than 3%) are those in the upper-right region and in the bottom-right region, enclosed by "3". The former group consists of constrained entrepreneurs who borrow the largest amount of credit. Thus, a lower intermediation cost significantly reduces their cost of production. The latter group consists of wealthy workers, who receive significantly more interest earnings from their large amounts of savings when the equilibrium interest rate goes up.

5 Equilibrium Interactions of Policy Instruments

Our previous analysis indicates that policy instruments targeting different financial constraints may have differential effects on GDP, TFP, and the distribution of income. In principle, countries may also adopt more comprehensive policies that alleviate multiple financial frictions at the same time. What would their potential effects on GDP be and through which margins? To what extent does the effect of targeting one financial constraint depend on the tightness of other financial constraints?

In this section, we explore these questions to understand how different constraints interact in equilibrium. We first examine individuals' output and credit access in partial equilibrium to analytically characterize the interaction effect. Next, we use the calibrated model to quantitatively evaluate the interaction effect. Our findings suggest that it is important for policy makers to identify and alleviate the most binding financial constraint.

5.1 Interactions in Partial Equilibrium

As we have discussed in Section 4.1, financial inclusion policies increase GDP through both the extensive margin by increasing the credit access ratio, and the intensive margin by allowing entrepreneurs with credit access to scale up their production. Below, we characterize the interaction effect through each margin of relaxing multiple constraints simultaneously. For analytical tractability, we focus on a stylized partial-equilibrium example with fixed interest rate r, wage w, and constant-returns-to-scale production function (i.e., $\nu = 0$).

Proposition 4. *(Extensive Margin) Let $\underline{b}(\Omega) \equiv \underline{b}(z;\Omega)$ be the threshold of wealth above which entrepreneurs of productivity z choose to access credit in the economy with Ω. Consider fixed interest rate r, wage w, and $\nu = 0$. We have*

i. Tightening (relaxing) each financial constraint reduces (improves) credit access:

$$\frac{\partial \underline{b}(\Omega)}{\partial \psi}, \quad \frac{\partial \underline{b}(\Omega)}{\partial \xi}, \quad \frac{\partial \underline{b}(\Omega)}{\partial \chi} \geq 0. \tag{5.1}$$

ii. $\underline{b}(\Omega)$ has submodularity in Ω, suggesting that the effect through the extensive margin of relaxing one constraint is smaller when the other constraints are more relaxed:

$$\frac{\partial^2 \underline{b}(\Omega)}{\partial \psi \partial \xi} = \frac{\partial^2 \underline{b}(\Omega)}{\partial \xi \partial \psi}, \quad \frac{\partial^2 \underline{b}(\Omega)}{\partial \xi \partial \chi} = \frac{\partial^2 \underline{b}(\Omega)}{\partial \chi \partial \xi}, \quad \frac{\partial^2 \underline{b}(\Omega)}{\partial \chi \partial \psi} = \frac{\partial^2 \underline{b}(\Omega)}{\partial \psi \partial \chi} \geq 0. \tag{5.2}$$

The first-order partial derivatives are positive, indicating that relaxing any constraint (i.e., reducing ψ, ξ, and χ) unambiguously reduces the threshold of wealth $\underline{b}(\Omega)$. Since a lower $\underline{b}(\Omega)$ means more entrepreneurs will gain access to credit, Proposition 4.i indicates that relaxing these constraints will increase the credit access ratio. The second-order cross partial derivatives are positive, indicating that the impact of relaxing any constraint on $\underline{b}(\Omega)$ would be smaller when the other two constraints are more relaxed (i.e., lower ψ, ξ, and χ). Therefore, Proposition 4 indicates that the three policy instruments dampen each other's effect through the extensive margin. For example, when the credit entry cost ψ is lower, relaxing the borrowing constraint ξ or reducing the intermediation cost χ would have a smaller effect on reducing the wealth threshold $\underline{b}(\Omega)$. Intuitively, when the credit entry cost ψ is low, most firms already have access to finance. As such, there is little room for increasing credit access further by relaxing the other two constraints.

Proposition 5. *(Intensive Margin) Let $y^C(\Omega) \equiv y^C(b, z; \Omega)$ be the net output of entrepreneurs of productivity z and wealth b in the credit regime (see its formula in Appendix A). Consider fixed interest rate r, wage w, and $\nu = 0$. We have*

i. Tightening (relaxing) each financial constraint reduces (raises) output:

$$\frac{\partial y^C(\Omega)}{\partial \psi}, \quad \frac{\partial y^C(\Omega)}{\partial \xi}, \quad \frac{\partial y^C(\Omega)}{\partial \chi} \leq 0. \tag{5.3}$$

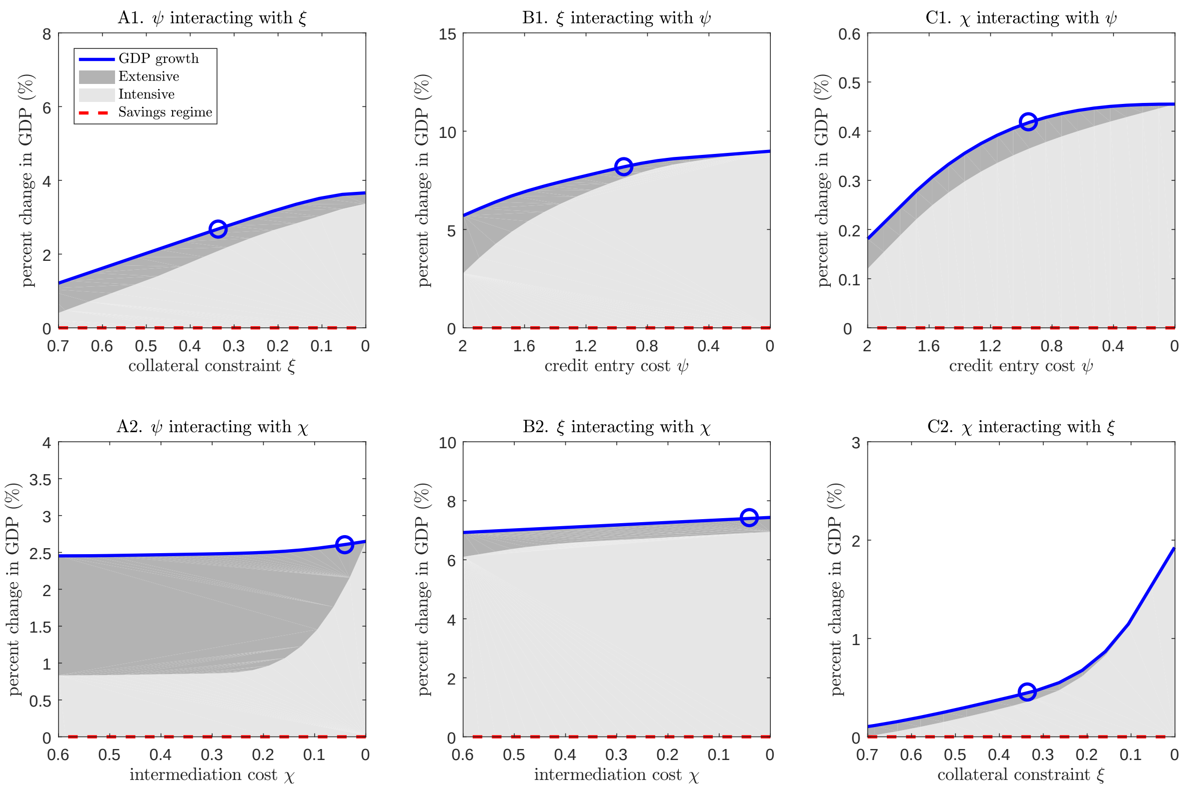
ii. $y^C(\Omega)$ has supermodularity, suggesting that the effect through the intensive margin of *relaxing one constraint is larger when the other constraints are more relaxed:*

$$\frac{\partial^2 y^C(\Omega)}{\partial\psi\partial\xi} = \frac{\partial^2 y^C(\Omega)}{\partial\xi\partial\psi}, \quad \frac{\partial^2 y^C(\Omega)}{\partial\xi\partial\chi} = \frac{\partial^2 y^C(\Omega)}{\partial\chi\partial\xi}, \quad \frac{\partial^2 y^C(\Omega)}{\partial\chi\partial\psi} = \frac{\partial^2 y^C(\Omega)}{\partial\psi\partial\chi} \geq 0. \tag{5.4}$$

The first-order partial derivatives are negative, indicating that relaxing any constraint (i.e., reducing ψ, ξ, and χ) unambiguously increases the output $y^C(\Omega)$ of entrepreneurs with credit access, increasing GDP through the intensive margin. The second-order cross partial derivatives are positive, indicating that the impact of relaxing any constraint on $y^C(\Omega)$ would be larger when the other two constraints are more relaxed (i.e., lower ψ, ξ, and χ). Therefore, Proposition 5 implies that relaxing any two constraints has complementary effects on output. For example, when the credit entry cost ψ is lower, relaxing the borrowing constraint ξ or reducing the intermediation cost χ would have a larger effect on increasing $y^C(\Omega)$. Intuitively, when the credit entry cost ψ is lower, entrepreneurs are left with more wealth after entering the credit regime, which increases pledgeable capital for borrowing. As a result, relaxing the collateral constraint ξ or reducing the intermediation cost χ would have a larger effect on output.

Decomposition of the Interaction Effect. Our analytical results reveal that different constraints are complements through the intensive margin and substitutes through the extensive margin. This implies that relaxing multiple constraints simultaneously could, in principle, bring a larger or smaller effect in increasing GDP than relaxing each constraint separately, depending on whether the overall interaction effect is positive or negative. For example, if the interaction effect through the intensive margin dominates, policy makers should develop a more balanced financial system, because different financial constraints are complements in restraining GDP. In this case, policies that alleviate the most binding financial constraint would be most effective. On the other hand, if the interaction effect through the extensive margin dominates, policy makers should alleviate a single constraint while ignoring all other constraints.

We now use our calibrated model to quantitatively study the interaction effect through these two margins. We first discuss the results in partial equilibrium by presenting a sequence of counterfactual experiments in which the interest rate, the wage, and the distribution of agents are fixed at their steady-state values of our calibrated economy. In each experiment, we measure the percentage increase in steady-state GDP after relaxing one financial constraint (i.e., the target constraint) conditional on different values of the second constraint (i.e., the moving constraint), while at the same time keeping the third constraint unchanged. Thus, our experiments essentially measure the interaction effect between the target and the moving constraints. For the three financial constraints (ψ, ξ, χ) we have, six combinations are possible as presented in Figure 7.



Note: This figure visualizes the interaction effects among financial constraints in partial equilibrium with fixed interest rate, wage, and distribution of agents. Panels A1 and A2 present the percentage increase in steady-state GDP when ψ is reduced from its calibrated value 0.95 to 0.8. Panel A1 visualizes the interaction effect between ψ and ξ by varying ξ from 0.7 to 0 while keeping χ at its calibrated value. For each value of ξ, we decompose the increase in GDP (owing to reducing ψ from 0.95 to 0.8) into three components according to formula (4.1). The red dashed line plots the contribution of the savings regime (the third term in formula 4.1). The height of the light grey area represents the contribution of the intensive margin (the second term in formula 4.1). The height of the dark grey area represents the contribution of the extensive margin (the first term in formula 4.1). Panel A2 studies the interaction effect between ψ and χ by varying χ from 0.6 to 0 while keeping ξ at its calibrated value. Panels B1 and B2 present the percentage increase in steady-state GDP when ξ is reduced from its calibrated value 0.335 to 0.29, interacting with different values of ψ and χ. Panels C1 and C2 present the percentage increase in steady-state GDP when χ is reduced from its calibrated value 0.04 to 0, interacting with different values of ψ and ξ.

Figure 7: Interaction effects among financial constraints in partial equilibrium.

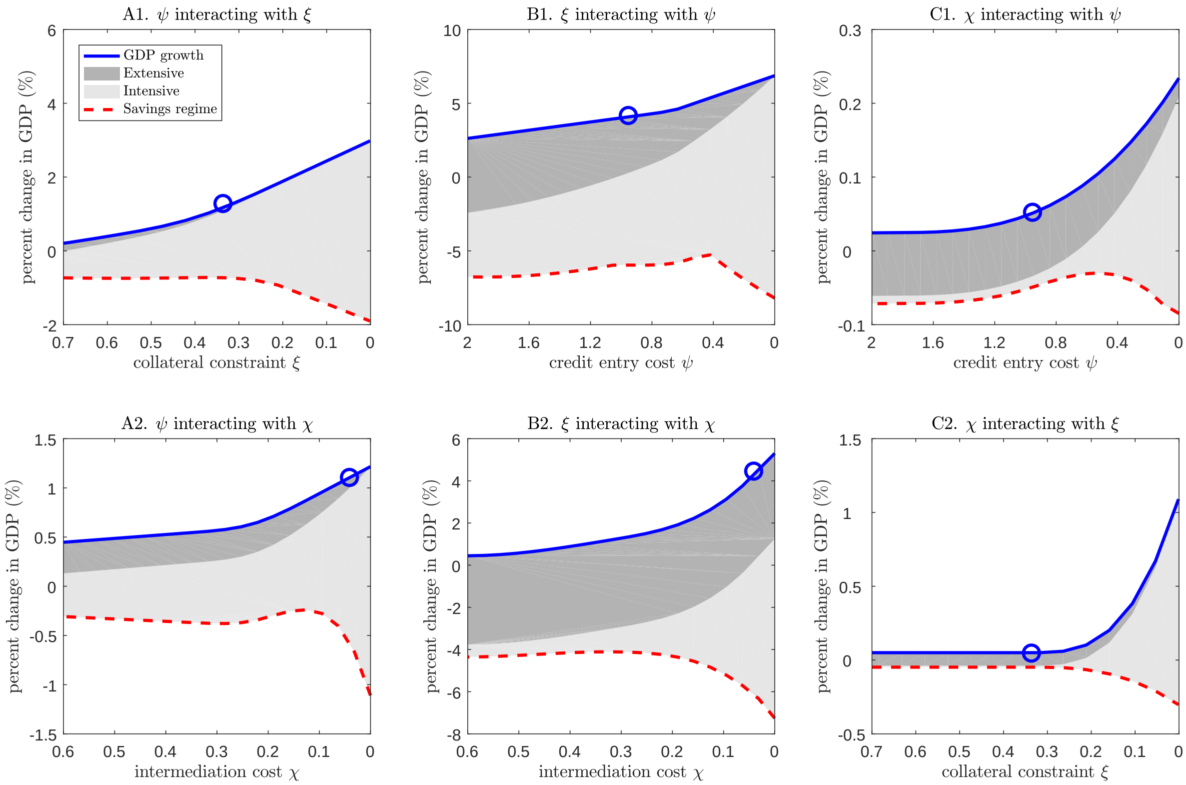
Panel A1 considers ψ as the target constraint and ξ as the moving constraint. On the y-axis, we plot the percentage increase in steady-state GDP (the blue solid line) resulting from reducing the target constraint ψ from its calibrated value 0.95 to 0.8 for different values of the moving constraint ξ (from 0.7 to 0 as shown on the x-axis). The dark-grey and light-grey areas, and the red-dashed line represent the increase in steady-state GDP attributed to the extensive margin, the intensive margin, and the savings regime respectively. Because we focus on partial equilibrium, the contribution of the savings regime to the increase in steady-state GDP is always zero. Moving from the left to the right along the x-axis, as the collateral constraint is relaxed (i.e., lowering ξ), we observe that the intensive-margin effect increases while the extensive-margin effect diminishes, consistent with the implications of Propositions 4 and 5. Interestingly, we find that the interaction effect through the intensive margin dominates that through the extensive margin. That is, when we relax the collateral constraint, the (increasing) intensive-margin effect is larger than the (decreasing) extensive-margin effect. As a result, reducing ψ increases steady-state GDP more when the collateral constraint is relaxed (see the upward-sloping blue solid line).

Similarly, in Panel A2, we hold ψ as the target constraint and consider χ as the moving constraint. Again, moving from the left to the right along the x-axis (i.e., lowering χ), the intensive-margin effect increases whereas the extensive-margin effect diminishes. The former dominates the latter, resulting in a larger increase in steady-state GDP from reducing ψ. These patterns are also observed in Panels B1 and B2, where we reduce the target constraint ξ from 0.335 to 0.29, as well as in Panels C1 and C2, where we reduce the target constraint χ from 0.04 to 0.

5.2 Interactions in General Equilibrium

In Figure 8, we conduct similar counterfactual experiments to the ones in Figure 7 to study the interaction effect in general equilibrium with the equilibrium interest rate, wage, and the distribution of wealth and productivity endogenously determined in the steady state of each counterfactual experiment.

Comparing the general-equilibrium simulation results (Figure 8) with those of the partial equilibrium (Figure 7), two differences stand out. First, the increase in steady-state GDP owing to relaxing financial constraints is smaller in general equilibrium (the blue solid line) because of the increase in the equilibrium interest rate and wage. Second, the savings regime negatively contributes to the increase in steady-state GDP in all counterfactual experiments. However, our main finding in partial equilibrium, that the interaction effect through the intensive margin dominates that through the extensive margin, remains robust. This implies that when policy makers alleviate multiple financial constraints together, the net effect on GDP is a convex combination of the effect from relaxing each financial constraint separately. In other words, effective financial inclusion policies should be designed to develop a well-rounded financial system, as reflected by a high breadth (i.e., high credit access), depth (i.e., relaxed collateral constraints), and efficiency (i.e., low intermediation costs).



Note: This figure visualizes the interaction effects among financial constraints in general equilibrium. The descriptions of each panel are similar to those in Figure 7.

Figure 8: Interaction effects among financial constraints in general equilibrium.

6 Identifying the Most Binding Constraint

The interaction effects between the three financial constraints suggest that effective financial inclusion policies should be designed to target the most binding constraint. Moreover, policies that are effective in increasing GDP in one country may not be equally effective in other countries. As an illustration, we now consider six representative countries and use the model to evaluate the effectiveness of implementing financial inclusion policies in each country.

We consider two groups of countries (see Table 2). In the first group, we intentionally choose three countries with extreme financial constraints. These countries are Pakistan, Bangladesh, and Brazil. Their financial systems exhibit sharply different characteristics. For example, fewer than 6.7% of firms in Pakistan have access to credit whereas the figures are 34.1% and 59.2% in Bangladesh and Brazil respectively. However, among those firms with credit access, those in Bangladesh have to post collateral that amounts to

Table 2: Moments in data and model in various countries.

	Countries with extreme financial constraints						Countries with balanced financial constraints					
	Pakistan		Bangladesh		Brazil		The Philippines		Kenya		Zambia	
	Data	Model	Data	Model	Data	Model	Data	Model	Data	Model	Data	Model
Firms with credit (%)	6.7	6.7	34.1	34.7	59.2	59.8	29.9	29.5	25.4	25.9	8.8	8.6
Collateral (% of loans)	153.4	154.0	271.4	271.7	95.1	100.4	156.7	156.4	120.8	120.8	236.6	236.4
Interest rate spread (%)	4.8	4.7	1.9	1.9	35.4	35.4	4.0	4.0	8.5	8.5	11.8	11.8
Real interest rate (%)	0.2	0.5	4.5	4.6	2.9	2.8	6.2	6.8	7.6	7.7	4.8	4.5
Top 5% employment share (%)	65.7	52.1	55.4	50.3	63.1	66.4	43.7	43.7	54.1	53.9	41.8	35.9
Top 10% employment share (%)	77.6	60.9	71.2	61.5	74.6	80.6	57.6	58.6	66.9	69.1	56.1	49.9
Top 20% employment share (%)	87.6	70.2	86.2	85.8	84.7	87.1	73.2	71.5	81	73.4	70.7	63.3
Top 40% employment share (%)	94.7	91.4	95.2	96.6	93.1	95.9	88.8	85.3	93.2	84.9	84.7	89.0
Gross savings (% of GDP)	21.4	20.5	21.2	20.6	23.9	28.5	44.2	41.2	15.4	14.9	35.4	29.6

as high as 271.4% of the face value of loans, whereas the figures are 153.4% and 95.1% in Pakistan and Brazil respectively. Surprisingly, although firms in Brazil tend to have access to credit and face a more relaxed collateral constraint, the loan interest rate is higher than the deposit rate by 35.4 percentage points. By calibrating our model to these countries (see Appendix Table OA.1 for parameter values), we can test the model's ability to identify the most binding constraint and quantitatively evaluate the potential gains from relaxing different constraints.

In the second group, we choose three countries with relatively balanced financial constraints: the Philippines (our benchmark country in Sections 3 – 5), Kenya, and Zambia. Kenya's financial system is relatively developed for a country with its level of income and also compared with many other Sub-Saharan African countries. Zambia is a low-income country with an underdeveloped financial system reflected in all three dimensions: a low credit access ratio, a high collateral requirement, and a high interest rate spread. Unlike countries in the first group, it is difficult to see which financial constraint is most binding from a simple eyeball test. Our calibrated model, however, allows us to identify the bottleneck problem in their financial systems.

6.1 Counterfactual Analyses in Steady States

Table 3 presents the changes in steady-state GDP, TFP and income GINI when the six countries adopt the best-possible financial intermediation technology along a single dimension (i.e., $\psi = 0$, $\xi = 0$, or $\chi = 0$).

Regarding the first group of countries, the model implies that the high intermediation cost is the key friction that hinders aggregate production in Brazil. Reducing χ to zero increases Brazil's steady-state GDP and TFP by 42.44% and 38.31%, while reducing ψ to

Table 3: Impact of financial inclusion on GDP, TFP and income GINI.

	$\psi \to 0$			$\xi \to 0$			$\chi \to 0$		
	GDP(%)	TFP(%)	GINI	GDP(%)	TFP(%)	GINI	GDP(%)	TFP(%)	GINI
Pakistan	24.96	9.71	-0.021	89.71	55.59	-0.082	0.11	0.01	0.001
Bangladesh	8.94	2.33	0.008	111.92	77.48	-0.124	0.54	0.41	0.002
Brazil	0.02	0.01	-0.000	0.00	0.00	0.000	42.44	38.31	0.056
The Philippines	9.25	4.78	-0.046	59.73	37.42	-0.083	0.07	0.05	0.002
Kenya	21.01	11.70	-0.060	13.77	7.83	-0.020	2.41	0.91	0.012
Zambia	10.22	5.34	-0.003	81.85	47.76	-0.059	1.50	1.13	0.002

zero almost has no effect on steady-state GDP and TFP due to the high intermediation cost. The model implies that Bangladesh's most binding financial friction is the tight collateral constraint. If we relax ξ to zero in Bangladesh, steady-state GDP and TFP would increase by 111.92% and 77.48% respectively, while reducing either ψ or χ to zero only increases steady-state GDP by 8.94% and 0.54%.

Regarding Pakistan, steady-state GDP and TFP would increase by 24.96% and 9.71% respectively when the credit entry cost ψ goes to zero, while keeping ξ and χ unchanged. Relaxing the collateral constraint ξ to zero boosts steady-state GDP and TFP by 89.71% and 55.9% respectively, while reducing the intermediation cost χ to zero increases steady-state GDP and TFP by only 0.11% and 0.01%. Therefore, our model suggests that the most binding financial constraint for Pakistan arises from the collateral constraint. This differs from our direct inference from the descriptive statistics in Table 2. Although only 6.7% of firms in Pakistan have access to credit, this does not necessarily mean that the credit entry cost is the most binding constraint on GDP. In fact, as we have shown in Figure 3, the credit access ratio is an endogenous variable determined by all three constraints, not simply the credit entry cost. Thus, the low credit access ratio could be largely attributed to the tight collateral constraint in Pakistan rather than the high credit entry cost.[6]

We conduct similar counterfactual experiments for the second group of countries. The last three rows of Table 3 indicate that the collateral requirement is the most binding constraint in the Philippines and Zambia whereas the credit entry cost is the most binding constraint in Kenya. In terms of the income inequality, our simulation suggests that reducing the intermediation cost always increases income GINI and relaxing the collateral constraint always reduces income GINI. Reducing the credit entry cost lowers income GINI in all six countries except for Bangladesh.

[6] On the other hand, our model suggests that reducing the credit entry cost is more effective in boosting GDP in Pakistan relative to Bangladesh. This suggests that compared to Bangladesh, the credit entry cost is indeed a more binding constraint in Pakistan.

Decomposition of the Increase in GDP and TFP. We further use formulas (4.1) and (4.3) to understand the margins through which steady-state GDP and TFP increase after relaxing various forms of financial constraints. Tables 4 and 5 present the decomposition of the increase in steady-state GDP and TFP for all six countries in general equilibrium and partial equilibrium, respectively.

The following conclusions apply to all six countries: (1) The increase in steady-state GDP and TFP caused by reducing the collateral constraint ξ to zero is mainly attributed to the intensive margin; (2) depending on country characteristics, the increase in steady-state GDP and TFP caused by reducing the credit entry cost ψ to zero could be mainly attributed to the extensive margin (Pakistan, the Philippines, and Zambia) or the intensive margin (Bangladesh and Kenya); (3) reducing the intermediation cost to zero usually has a smaller effect on steady-state GDP and TFP except for Brazil. In Brazil, the large increase in steady-state GDP and TFP from reducing the intermediation cost is mostly attributed to the intensive margin; (4) the savings regime always negatively contributes to the increase in steady-state GDP and TFP, because relaxing any financial constraint would motivate more productive entrepreneurs to move from the savings regime to the credit regime; and (5) the most binding constraints identified in partial equilibrium are the same as those in general equilibrium, although the increase in steady-state GDP and TFP is larger.

6.2 Transitional Dynamics after Relaxing Financial Constraints

We now study the transitional dynamics after implementing financial inclusion policies. Starting from the calibrated steady state of each country, we consider an unexpected shock that occurs in year 5, which permanently reduces one of the financial constraint parameters (i.e., ψ, ξ, and χ) to zero. Figures 9 and 10 plot the transitional dynamics of GDP, TFP, the income GINI coefficient, interest rate, and wage.

Across the six countries, GDP, TFP, and the income GINI coefficient converge to their new steady-state values in about 10 – 20 years. Transition is gradual because reallocation of capital is intermediated through imperfect financial markets (see, e.g., Jermann and Quadrini, 2007; Buera and Shin, 2011). For the simulated path of transitional dynamics in each country, we observe overshooting in the equilibrium interest rate, that is, the interest rate in the short run is higher than that in the long run after financial inclusion (Panels A4, B4, and C4 of Figures 9 and 10) There is no overshooting of the equilibrium wage. In all six countries, and across the three different financial inclusion policies, the equilibrium wage surges in year 5, reflecting the immediate increase in demand for workers when

Table 4: Decomposition of the increase in steady-state GDP due to financial inclusion.

		General Equilibrium			Partial Equilibrium		
		$\psi \to 0$	$\xi \to 0$	$\chi \to 0$	$\psi \to 0$	$\xi \to 0$	$\chi \to 0$
Pakistan	Extensive margin (%)	15.70	13.34	0.01	284.08	361.97	0.00
	Intensive margin (%)	15.20	128.02	0.11	41.68	833.26	0.11
	Savings regime (%)	-5.94	-51.65	-0.01	0.00	0.00	0.00
	Total (%)	24.96	89.71	0.11	325.76	1189.66	0.11
Bangladesh	Extensive margin (%)	3.67	41.34	0.12	158.76	61.53	0.18
	Intensive margin (%)	6.90	127.46	0.39	55.68	1487.82	0.54
	Savings regime (%)	-1.63	-56.87	-0.01	0.00	0.00	0.00
	Total (%)	8.94	111.92	0.50	214.45	1549.17	0.72
Brazil	Extensive margin (%)	0.01	0.00	4.25	0.32	0.00	74.44
	Intensive margin (%)	0.01	0.00	38.56	0.20	0.00	494.04
	Savings regime (%)	0.00	0.00	-0.47	0.00	0.00	0.00
	Total (%)	0.02	0.00	42.44	0.53	0.00	568.49
The Philippines	Extensive margin (%)	7.53	34.61	0.09	30.78	54.28	0.34
	Intensive margin (%)	2.76	73.81	0.01	7.18	649.66	0.06
	Savings regime (%)	-1.04	-48.69	-0.05	0.00	0.00	0.00
	Total (%)	9.25	59.73	0.05	37.96	703.78	0.40
Kenya	Extensive margin (%)	13.34	3.61	1.47	43.86	2.61	12.80
	Intensive margin (%)	10.22	18.51	0.97	38.89	54.36	1.56
	Savings regime (%)	-2.54	-8.35	-0.03	0.00	0.00	0.00
	Total (%)	21.01	13.77	2.41	82.76	56.97	14.36
Zambia	Extensive margin (%)	14.59	13.45	1.90	32.88	247.04	3.62
	Intensive margin (%)	6.49	117.45	1.26	13.13	963.18	2.17
	Savings regime (%)	-10.86	-49.05	-1.66	0.00	0.00	0.00
	Total (%)	10.22	81.85	1.50	46.01	1207.22	5.79

constraints are relaxed. Moreover, the equilibrium wage steadily increases thereafter to reach its new steady-state value due to the gradual accumulation and reallocation of capital (Panels A5, B5, and C5 of Figures 9 and 10).

GDP increases substantially in year 5 and more gradually thereafter (Panels A1, B1, and C1 of Figures 9 and 10). The positive effect of financial inclusion on GDP is larger in the long run than in the short run. Intuitively, this is because it takes time for entrepreneurs to accumulate wealth to fully reap the gains from financial inclusion policies. This intuition is closely related to the finding of Moll (2014), who shows that the negative welfare effect of financial frictions is larger in the short run than in the long run as entrepreneurs can accumulate capital to alleviate constraints.

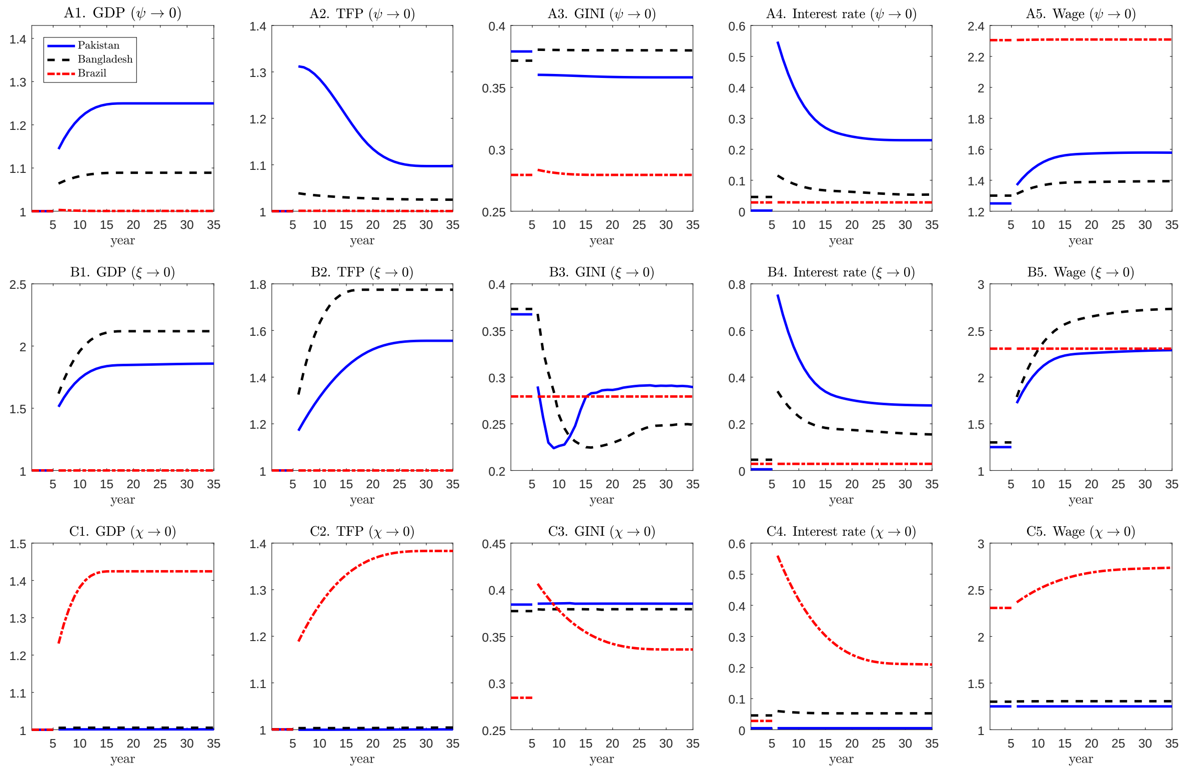
In terms of TFP, we find that reducing the credit entry cost ψ to zero generates overshooting in the short-run dynamics (Panel A2 of Figures 9 and 10) in all six countries, although the overshooting may happen with a lag in some countries (see the red dash-dotted line for Zambia in Panel A2 of Figure 10). By contrast, relaxing the collateral constraint ξ to zero does not generate an overshooting effect (Panel B2 of Figures 9 and

Table 5: Decomposition of the increase in steady-state TFP due to financial inclusion.

		General Equilibrium			Partial Equilibrium		
		$\psi \to 0$	$\xi \to 0$	$\chi \to 0$	$\psi \to 0$	$\xi \to 0$	$\chi \to 0$
Pakistan	Extensive margin (%)	21.97	14.19	0.00	21.92	12.53	0.00
	Intensive margin (%)	4.57	57.90	0.01	11.31	78.72	0.03
	Savings regime (%)	-16.83	-16.49	0.00	0.00	0.00	0.00
	Total (%)	9.71	55.59	0.01	33.24	91.25	0.03
Bangladesh	Extensive margin (%)	2.16	45.53	0.47	17.08	25.88	0.09
	Intensive margin (%)	4.43	113.54	0.16	9.26	101.41	0.46
	Savings regime (%)	-4.25	-81.59	-0.22	0.00	0.00	0.00
	Total (%)	2.33	77.48	0.41	26.34	127.29	0.55
Brazil	Extensive margin (%)	0.00	0.00	14.69	0.15	0.00	17.88
	Intensive margin (%)	0.01	0.00	24.01	0.12	0.00	49.68
	Savings regime (%)	0.00	0.00	-0.39	0.00	0.00	0.00
	Total (%)	0.01	0.00	38.31	0.27	0.00	67.56
The Philippines	Extensive margin (%)	4.65	14.78	0.05	6.15	17.35	0.10
	Intensive margin (%)	1.14	39.56	0.01	1.70	77.69	0.02
	Savings regime (%)	-1.01	-16.92	-0.01	0.00	0.00	0.00
	Total (%)	4.78	37.42	0.05	7.85	95.04	0.12
Kenya	Extensive margin (%)	8.24	2.65	0.74	10.63	3.43	3.96
	Intensive margin (%)	6.98	8.87	0.42	9.57	11.35	0.31
	Savings regime (%)	-3.52	-3.69	-0.25	0.00	0.00	0.00
	Total (%)	11.70	7.83	0.91	20.20	14.78	4.27
Zambia	Extensive margin (%)	18.08	9.33	1.14	7.12	31.62	1.41
	Intensive margin (%)	2.85	43.63	1.09	2.65	81.03	1.31
	Savings regime (%)	-15.59	-5.20	1.10	0.00	0.00	0.00
	Total (%)	5.34	47.76	1.13	9.77	112.65	2.72

10). In all six countries, TFP surges in year 5 before plateauing to reach its new steady-state value. Reducing the intermediation cost generates an overshooting effect in all six countries except for Brazil (Panel B3 of Figures 9 and 10), although the magnitude of overshooting is small due to the relatively small long-run effect on TFP in these countries. Brazil's TFP displays different dynamics because its high intermediation cost is the most binding constraint, which is not the case in other countries.

In terms of income inequality, we find that reducing the credit entry cost ψ may lead to a higher or a lower long-run GINI coefficient, depending on the country (Panel A3 of Figures 9 and 10). Relaxing the collateral constraint ξ results in a lower GINI coefficient in the long run, and overshooting occurs in the short run (Panel B3 of Figures 9 and 10). Reducing the intermediation cost χ leads to a higher GINI coefficient in the long run (Panel C3 of Figures 9 and 10). In Brazil, reducing the intermediation cost generates a moderately higher GDP but exacerbates income inequality in the short run (see the red dash-dotted line in Panel C3 of Figure 9), although GDP is higher and income inequality is lower in the long run.



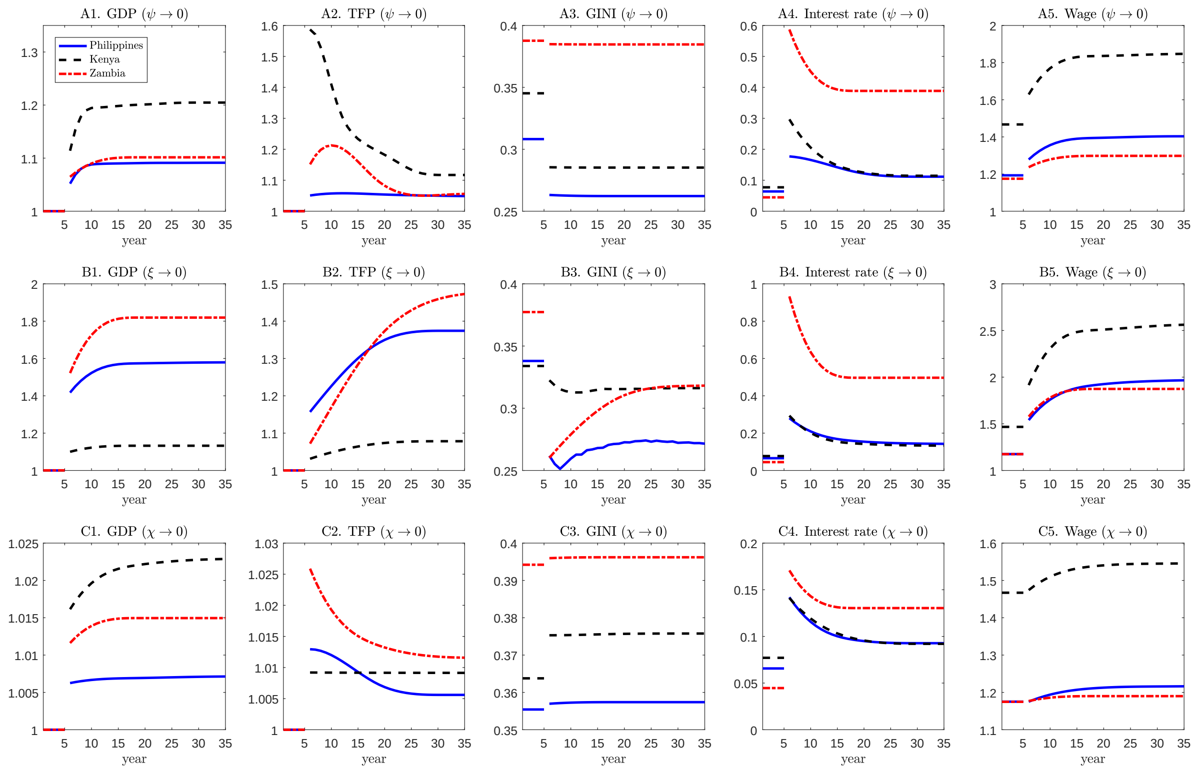
Note: In all experiments, we start with the steady state of the calibrated economy in year 1, and consider an unexpected shock in year 5, which reduces one of the financial constraint parameters (i.e., ψ, ξ, and χ) to zero permanently. Panels A1-A5, B1-B5, and C1-C5 plot the model-implied transitional dynamics of GDP, TFP, the income GINI coefficient, interest rate, and wage for reducing ψ, ξ, and χ to zero, respectively.

Figure 9: Transitional dynamics for Pakistan, Bangladesh, and Brazil.

Overall, our simulation indicates that the short-run implications of reducing different financial constraints on GDP, TFP, income inequality, and the equilibrium interest rate and wage could be very different than their long-run effects. Such intertemporal tradeoffs may raise concerns for policy makers depending on their objectives. For the same financial inclusion policy, the transitional dynamics in different countries share many common features. However, the dynamics also exhibit some degree of heterogeneity, depending on the characteristics of a country's financial system.

7 Conclusion

In this paper, we develop a tractable general-equilibrium model with heterogeneous agents to study how different financial constraints interact in equilibrium. Our model highlights three different dimensions of a financial system: breadth, depth, and efficiency.



Note: The descriptions of each experiment are similar to those in Figure 9.

Figure 10: Transitional dynamics for the Philippines, Kenya, and Zambia.

These characteristics reflect different sources of financial frictions whose tightness can be captured by three indicators, the credit access ratio, the loan-to-collateral ratio, and the interest rate spread.

Our analytical and quantitative analyses indicate that the three financial constraints should be jointly analyzed due to their rich interactions in equilibrium. The quantitative as well as qualitative implications of financial inclusion on GDP, TFP, and the distribution of income not only depend on which constraint is alleviated, but also on the extent to which other constraints are binding. Analytically, we show that relaxing these constraints increases GDP through both the extensive margin and the intensive margin. However, the interaction between different constraints tends to dampen the effect through the extensive margin and amplify the effect through the intensive margin. Quantitatively, we find that the interaction effect through the intensive margin dominates, indicating that more effective financial inclusion policies should be designed in a balanced manner. In particular, by conducting counterfactual experiments for different policy instruments, our calibrated model is able to provide suggestive guidance on identifying the most binding constraint in a financial system, as well as highlighting the short-run versus long-run tradeoffs.

Our model provides insight into the practical problems faced by policy makers. Yet, the main value of our model lies in clarifying the tradeoff among different policy instruments and in distinguishing their impacts on various macro and micro variables in both steady states and transitions. The financial frictions incorporated in our model are not meant to be exhaustive, but our intention is to suggest an approach which might be applied to more complete and realistic models.

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Online Appendix (Not for Publication)

A GDP Formula

Consider the steady-state of an economy with constraint parameters $\Omega \equiv (\psi, \xi, \chi)$. The income $I(b,z)$ for each agent of (b,z) is equal to the sum of her wage (or business profit) and interest earnings. The amount of each component depends on her occupation and credit access. In particular,

$$I(b,z;\Omega) = \tag{A.1}$$
$$\begin{cases} w + rb, & \text{workers;} \\ z(k^\alpha l^{1-\alpha})^{1-\nu} - \delta k - wl + t(b - k_t), & \text{entrepreneurs in the savings regime;} \\ z(k^\alpha l^{1-\alpha})^{1-\nu} - \delta k - wl - \psi - (r+\chi)(k - b + \psi), & \text{entrepreneurs in the credit regime,} \end{cases}$$

where k and l are short for $k(b,z;\Omega)$ and $l(b,z;\Omega)$.

According to the income approach, the economy's GDP is equal to total national income plus depreciation

$$\text{GDP}_\Omega = \iint\limits_{(b,z)\in\Phi} I(b,z;\Omega)h(b,z;\Omega)\mathrm{d}b\mathrm{d}z + \iint\limits_{(b,z)\in\Phi^E} \delta k h(b,z;\Omega)\mathrm{d}b\mathrm{d}z. \tag{A.2}$$

We can rewrite equation (A.2) as the sum of workers' and entrepreneurs' income

$$\begin{aligned} \text{GDP}_\Omega &= \iint\limits_{(b,z)\in\Phi} I(b,z;\Omega)h(b,z;\Omega)\mathrm{d}b\mathrm{d}z + \iint\limits_{(b,z)\in\Phi^E} \delta k(b,z;\Omega)h(b,z;\Omega)\mathrm{d}b\mathrm{d}z \\ &= \iint\limits_{(b,z)\notin\Phi^E} I(b,z;\Omega)h(b,z;\Omega)\mathrm{d}b\mathrm{d}z + \iint\limits_{(b,z)\in\Phi^E} [I(b,z;\Omega) + \delta k(b,z;\Omega)]h(b,z;\Omega)\mathrm{d}b\mathrm{d}z. \end{aligned} \tag{A.3}$$

Substituting equation (A.1) into equation (A.3) yields

$$\begin{aligned} \text{GDP}_\Omega = &\iint\limits_{(b,z)\notin\Phi^E} (w + rb)h(b,z;\Omega)\mathrm{d}b\mathrm{d}z \\ &+ \iint\limits_{(b,z)\in\Phi^S} [z(k(b,z;\Omega)^\alpha l(b,z;\Omega)^{1-\alpha})^{1-\nu} - wl(b,z;\Omega) + r(b - k(b,z;\Omega))]h(b,z;\Omega)\mathrm{d}b\mathrm{d}z \\ &+ \iint\limits_{(b,z)\in\Phi^C} [z(k(b,z;\Omega)^\alpha l(b,z;\Omega)^{1-\alpha})^{1-\nu} - wl(b,z;\Omega) - \psi - (r+\chi)(k(b,z;\Omega) - b + \psi)]h(b,z;\Omega)\mathrm{d}b\mathrm{d}z. \end{aligned} \tag{A.4}$$

Taking out the terms with wage w and interest rate r, equation (A.4) is rewritten as

$$\begin{aligned}
\mathrm{GDP}_\Omega = & w\left[\iint\limits_{(b,z)\notin\Phi^E} h(b,z;\Omega)\mathrm{d}b\mathrm{d}z - \iint\limits_{(b,z)\in\Phi^E} l(b,z;\Omega)h(b,z;\Omega)\mathrm{d}b\mathrm{d}z\right] \qquad \text{(A.5)}\\
& + r\left[\iint\limits_{(b,z)\notin\Phi^E} bh(b,z;\Omega)\mathrm{d}b\mathrm{d}z + \iint\limits_{(b,z)\in\Phi^S} [b-k(b,z;\Omega)]h(b,z;\Omega)\mathrm{d}b\mathrm{d}z\right.\\
& \left. - \iint\limits_{(b,z)\in\Phi^C} [k(b,z;\Omega)-b+\psi]h(b,z;\Omega)\mathrm{d}b\mathrm{d}z\right]\\
& + \iint\limits_{(b,z)\in\Phi^S} z[k(b,z;\Omega)^\alpha l(b,z;\Omega)^{1-\alpha}]^{1-\nu}h(b,z;\Omega)\mathrm{d}b\mathrm{d}z\\
& + \iint\limits_{(b,z)\in\Phi^C} [z(k(b,z;\Omega)^\alpha l(b,z;\Omega)^{1-\alpha})^{1-\nu} - \psi - \chi(k(b,z;\Omega)-b+\psi)]h(b,z;\Omega)\mathrm{d}b\mathrm{d}z.
\end{aligned}$$

The labor and capital market clearing conditions (2.10-2.11) imply that the first two terms on the right-hand side of equation (A.5) are zero, and thus we have

$$\begin{aligned}
\mathrm{GDP}_\Omega = & \iint\limits_{(b,z)\in\Phi^S} z[k(b,z;\Omega)^\alpha l(b,z;\Omega)^{1-\alpha}]^{1-\nu}h(b,z;\Omega)\mathrm{d}b\mathrm{d}z\\
& + \iint\limits_{(b,z)\in\Phi^C} [z(k(b,z;\Omega)^\alpha l(b,z;\Omega)^{1-\alpha})^{1-\nu} - \psi - \chi(k(b,z;\Omega)-b+\psi)]h(b,z;\Omega)\mathrm{d}b\mathrm{d}z\\
= & \iint\limits_{(b,z)\in\Phi^S} y^S(b,z;\Omega)h(b,z;\Omega)\mathrm{d}b\mathrm{d}z + \iint\limits_{(b,z)\in\Phi^C} y^C(b,z;\Omega)h(b,z;\Omega)\mathrm{d}b\mathrm{d}z, \qquad \text{(A.6)}
\end{aligned}$$

where $y^S(b,z;\Omega)$ and $y^C(b,z;\Omega)$ are entrepreneurs' steady-state output net of the dead-weight loss arising from financial frictions in the savings and credit regimes:

$$y^S(b,z;\Omega) = z(k^\alpha l^{1-\alpha})^{1-\nu}, \qquad \text{(A.7)}$$

$$y^C(b,z;\Omega) = z(k^\alpha l^{1-\alpha})^{1-\nu} - \chi(k-b+\psi) - \psi. \qquad \text{(A.8)}$$

B Proofs

B.1 Proof of Proposition 1

Consider the entrepreneur's problem, where labor is a freely adjustable input. Taking the first-order condition with respect to l_t, we can derive the optimal labor demand as a function of capital k_t and productivity z_t

$$l_t(z_t, k_t) = \left[\frac{(1-\alpha)(1-\nu)z_t k_t^{\alpha(1-\nu)}}{w_t}\right]^{\frac{1}{\nu+\alpha(1-\nu)}}. \tag{B.1}$$

Because there is no uncertainty within each period, maximizing utility is equivalent to maximizing the end-of-period wealth $\tilde{b}_t$, defined by $\tilde{b}_t = c_t + b_{t+1}$. Substituting the optimal labor demand (B.1) into the entrepreneur's budget constraint yields

$$\begin{aligned}\tilde{b}_t(b_t, z_t, k_t) &= z_t(k_t^{\alpha} l_t^{1-\alpha})^{1-\nu} + (1-\delta)k_t - w_t l_t + (1+r_t)(b_t - k_t)\\ &= \left[\frac{(1-\alpha)(1-\nu)z_t}{w_t}\right]^{\frac{1}{\nu+\alpha(1-\nu)}} k_t^{\frac{\alpha(1-\nu)}{\nu+\alpha(1-\nu)}} \frac{\nu+\alpha(1-\nu)}{(1-\nu)(1-\alpha)} w_t + (1-\delta)k_t + (1+r_t)(b_t - k_t).\end{aligned} \tag{B.2}$$

In the absence of financial frictions, capital k_t is optimally chosen to maximize $\tilde{b}_t$. By taking the first-order condition with respect to k_t, we derive the optimal (unconstrained) capital demand k_t^u:

$$k_t^u(z_t) = \left[\frac{\alpha w_t}{(1-\alpha)(r_t+\delta)}\right]^{\frac{\nu+\alpha(1-\nu)}{\nu}} \left[\frac{(1-\alpha)(1-\nu)z_t}{w_t}\right]^{\frac{1}{\nu}}. \tag{B.3}$$

Substituting equation (B.3) into equation (B.2) yields the maximal end-of-period wealth $\tilde{b}_t^e$ for being an entrepreneur:

$$\tilde{b}_t^e(b_t, z_t) = (1+r_t)b_t + z_t^{\frac{1}{\nu}}\left[1-\alpha(1-\nu)\right]\left[\frac{\alpha w_t}{(1-\alpha)(r_t+\delta)}\right]^{\frac{\alpha(1-\nu)}{\nu}} \left[\frac{(1-\alpha)(1-\nu)}{w_t}\right]^{\frac{1-\nu}{\nu}}. \tag{B.4}$$

The end-of-period wealth for being a worker is $\tilde{b}_t^w(b_t) = (1+r_t)b_t + w_t$. Since $\tilde{b}_t^e(b_t, z_t)$ increases with z_t, the threshold of entrepreneurial productivity $\underline{z}_t$ at time t is given by

$\tilde{b}_t^w(b_t) = \tilde{b}_t^e(b_t, \underline{z}_t)$,

$$\underline{z}_t = \frac{(r_t+\delta)^{\alpha(1-\nu)} w_t^{1-\alpha(1-\nu)}}{[1-\alpha(1-\nu)]^{\nu}[\alpha^{\alpha}(1-\alpha)^{1-\alpha}(1-\nu)]^{1-\nu}}. \tag{B.5}$$

B.2 Proof of Proposition 2

According to equation (A.6) and the definition of $\underline{b}(z;\Omega)$, the steady-state GDP can be written as

$$\mathrm{GDP}_{\Omega} = \int_z \int_0^{\underline{b}(z;\Omega)} y^S(b,z;\Omega)h(b,z;\Omega)\mathrm{d}b\mathrm{d}z + \int_z \int_{\underline{b}(z;\Omega)}^{\infty} y^C(b,z;\Omega)h(b,z;\Omega)\mathrm{d}b\mathrm{d}z. \tag{B.6}$$

Thus,

$$\begin{aligned}\mathrm{GDP}_{\Omega'} - \mathrm{GDP}_{\Omega} = & \int_z \int_0^{\underline{b}(z;\Omega')} y^S(b,z;\Omega')h(b,z;\Omega')\mathrm{d}b\mathrm{d}z + \int_z \int_{\underline{b}(z;\Omega')}^{\infty} y^C(b,z;\Omega')h(b,z;\Omega')\mathrm{d}b\mathrm{d}z \\ & - \int_z \int_0^{\underline{b}(z;\Omega)} y^S(b,z;\Omega)h(b,z;\Omega)\mathrm{d}b\mathrm{d}z - \int_z \int_{\underline{b}(z;\Omega)}^{\infty} y^C(b,z;\Omega)h(b,z;\Omega)\mathrm{d}b\mathrm{d}z.\end{aligned} \tag{B.7}$$

When $\Omega' \leq \Omega$, we have $\underline{b}(z;\Omega') \leq \underline{b}(z;\Omega)$ for all $z \geq 0$, and thus

$$\begin{aligned}\mathrm{GDP}_{\Omega'} - \mathrm{GDP}_{\Omega} = & \int_z \int_{\underline{b}(z;\Omega')}^{\underline{b}(z;\Omega)} \left[y^C(b,z;\Omega')h(b,z;\Omega') - y^S(b,z;\Omega)h(b,z;\Omega)\right] \mathrm{d}b\mathrm{d}z \\ & + \int_z \int_{\underline{b}(z;\Omega)}^{\infty} \left[y^C(b,z;\Omega')h(b,z;\Omega') - y^C(b,z;\Omega)h(b,z;\Omega)\right] \mathrm{d}b\mathrm{d}z \\ & + \int_z \int_0^{\underline{b}(z;\Omega')} \left[y^S(b,z;\Omega')h(b,z;\Omega') - y^S(b,z;\Omega)h(b,z;\Omega)\right] \mathrm{d}b\mathrm{d}z.\end{aligned} \tag{B.8}$$

B.3 Proof of Proposition 3

The economy's steady-state aggregate output Y_{Ω}, capital K_{Ω}, and labor L_{Ω} are given by

$$Y_{\Omega} = \iint\limits_{(b,z)\in\Phi^E} [z(k(b,z;\Omega)^{\alpha} l(b,z;\Omega)^{1-\alpha}]^{1-\nu} h(b,z;\Omega)\mathrm{d}b\mathrm{d}z, \tag{B.9}$$

$$K_{\Omega} = \iint\limits_{(b,z)\in\Phi^E} k(b,z;\Omega)h(b,z;\Omega)\mathrm{d}b\mathrm{d}z, \tag{B.10}$$

$$L_{\Omega} = \iint\limits_{(b,z)\in\Phi^E} l(b,z;\Omega)h(b,z;\Omega)\mathrm{d}b\mathrm{d}z. \tag{B.11}$$

Growth Accounting by Factor. Taking the logarithm of both sides of equation (4.2) yields

$$\log(\mathrm{TFP}_\Omega) = \log(Y_\Omega) - \alpha \log(K_\Omega) - (1-\alpha)\log(L_\Omega). \tag{B.12}$$

Moving from Ω to Ω', we have a similar expression:

$$\log(\mathrm{TFP}_{\Omega'}) = \log(Y_{\Omega'}) - \alpha \log(K_{\Omega'}) - (1-\alpha)\log(L_{\Omega'}). \tag{B.13}$$

Taking the difference using equations (B.12) and (B.13) yields

$$\begin{aligned}\log(\mathrm{TFP}_{\Omega'}) - \log(\mathrm{TFP}_\Omega) = &\log(Y_{\Omega'}) - \log(Y_\Omega) - \alpha[\log(K_{\Omega'}) - \log(K_\Omega)] \\ &- (1-\alpha)[\log(L_{\Omega'}) - \log(L_\Omega)]. \end{aligned} \tag{B.14}$$

Equation (B.14) can be approximated by first difference,

$$\frac{\mathrm{TFP}_{\Omega'} - \mathrm{TFP}_\Omega}{\mathrm{TFP}_\Omega} = \frac{Y_{\Omega'} - Y_\Omega}{Y_\Omega} - \alpha\frac{K_{\Omega'} - K_\Omega}{K_\Omega} - (1-\alpha)\frac{L_{\Omega'} - L_\Omega}{L_\Omega}. \tag{B.15}$$

Let g_x denote the percentage change in the value of variable x when the economy moves from Ω to Ω'. Thus equation (B.15) can be written as

$$g_{TFP} = g_Y - \alpha g_K - (1-\alpha) g_L. \tag{B.16}$$

Since the economy consists of two regimes, aggregate capital and labor are equal to the sum of capital and labor employed by entrepreneurs living in the two regimes separately. Denote by $\bar{k}^s_\Omega$ and $\bar{l}^s_\Omega$ the average steady-state capital and labor employed by entrepreneurs in the savings regime of the economy Ω:

$$\bar{k}^s_\Omega \iint_{(b,z)\in\Phi^S} h(b,z;\Omega)\mathrm{d}b\mathrm{d}z = \iint_{(b,z)\in\Phi^S} k(b,z;\Omega)h(b,z;\Omega)\mathrm{d}b\mathrm{d}z, \tag{B.17}$$

$$\bar{l}^s_\Omega \iint_{(b,z)\in\Phi^S} h(b,z;\Omega)\mathrm{d}b\mathrm{d}z = \iint_{(b,z)\in\Phi^S} l(b,z;\Omega)h(b,z;\Omega)\mathrm{d}b\mathrm{d}z. \tag{B.18}$$

Denote by $\bar{k}^c_\Omega$ and $\bar{l}^c_\Omega$ the average steady-state capital and labor employed by entrepreneurs in the credit regime of the economy Ω:

$$\bar{k}^{c}_{\Omega} \iint\limits_{(b,z)\in\Phi^{C}} h(b,z;\Omega)\mathrm{d}b\mathrm{d}z = \iint\limits_{(b,z)\in\Phi^{C}} k(b,z;\Omega)h(b,z;\Omega)\mathrm{d}b\mathrm{d}z, \tag{B.19}$$

$$\bar{l}^{c}_{\Omega} \iint\limits_{(b,z)\in\Phi^{C}} h(b,z;\Omega)\mathrm{d}b\mathrm{d}z = \iint\limits_{(b,z)\in\Phi^{C}} l(b,z;\Omega)h(b,z;\Omega)\mathrm{d}b\mathrm{d}z. \tag{B.20}$$

Denote by E_{Ω} the percentage of agents who choose to be entrepreneurs, and by p^{c}_{Ω} the percentage of entrepreneurs living in the credit regime in the steady state of the economy Ω:

$$E_{\Omega} = \iint\limits_{(b,z)\in\Phi^{E}} h(b,z;\Omega)\mathrm{d}b\mathrm{d}z, \tag{B.21}$$

$$p^{c}_{\Omega}E_{\Omega} = \iint\limits_{(b,z)\in\Phi^{C}} h(b,z;\Omega)\mathrm{d}b\mathrm{d}z. \tag{B.22}$$

Thus the economy's steady-state aggregate capital and labor defined in equations (B.10-B.11) can be written as

$$K_{\Omega} = E_{\Omega}(1-p^{c}_{\Omega})\bar{k}^{s}_{\Omega} + E_{\Omega}p^{c}_{\Omega}\bar{k}^{c}_{\Omega}, \tag{B.23}$$

$$L_{\Omega} = E_{\Omega}(1-p^{c}_{\Omega})\bar{l}^{s}_{\Omega} + E_{\Omega}p^{c}_{\Omega}\bar{l}^{c}_{\Omega}. \tag{B.24}$$

According to equations (B.23-B.24), the growth rates of aggregate capital and labor, g_K and g_L, can be decomposed into

$$\begin{aligned} g_K &= g_E + (s^{c}_{K} - s^{s}_{K})g_{p^c} + s^{s}_{k}g_{\bar{k}^s} + s^{c}_{k}g_{\bar{k}^c}, \\ g_L &= g_E + (s^{c}_{L} - s^{s}_{L})g_{p^c} + s^{s}_{l}g_{\bar{l}^s} + s^{c}_{l}g_{\bar{l}^c}, \end{aligned} \tag{B.25}$$

where

$$
\begin{aligned}
s_k^c =& \frac{E_\Omega p_\Omega^c \bar{k}_\Omega^c}{2K_\Omega} + \frac{E_{\Omega'} p_{\Omega'}^c \bar{k}_{\Omega'}^c}{2K_{\Omega'}}, && \text{(B.26)}\\
s_k^s =& \frac{E_\Omega (1-p_\Omega^c) \bar{k}_\Omega^s}{2K_\Omega} + \frac{E_{\Omega'} (1-p_{\Omega'}^c) \bar{k}_{\Omega'}^s}{2K_{\Omega'}}, && \text{(B.27)}\\
s_l^c =& \frac{E_\Omega p_\Omega^c \bar{l}_\Omega^c}{2L_\Omega} + \frac{E_{\Omega'} p_{\Omega'}^c \bar{l}_{\Omega'}^c}{2L_{\Omega'}}, && \text{(B.28)}\\
s_l^s =& \frac{E_\Omega (1-p_\Omega^c) \bar{l}_\Omega^s}{2L_\Omega} + \frac{E_{\Omega'} (1-p_{\Omega'}^c) \bar{l}_{\Omega'}^s}{2L_{\Omega'}}. && \text{(B.29)}
\end{aligned}
$$

Substituting (B.25) into (B.16) yields

$$
g_Y = g_{TFP} + g_E + \alpha(s_k^c - s_k^s) g_{p_c} + \alpha s_k^s g_{\bar{k}^s} + \alpha s_k^c g_{\bar{k}^c} + (1-\alpha)(s_l^c - s_l^s) g_{p_c} + (1-\alpha) s_l^s g_{\bar{l}^s} + (1-\alpha) s_l^c g_{\bar{l}^c}. \tag{B.30}
$$

Growth Accounting by Regime. The economy's output Y_Ω is equal to the sum of output in each regime,

$$
Y_\Omega = E_\Omega (1-p_\Omega^c) \bar{y}_\Omega^s + E_\Omega p_\Omega^c \bar{y}_\Omega^c, \tag{B.31}
$$

where $\bar{y}_\Omega^s$ and $\bar{y}_\Omega^c$ are the average output produced by entrepreneurs in the savings and credit regimes:

$$
\begin{aligned}
\bar{y}_\Omega^s \iint_{(b,z)\in\Phi^C} h(b,z;\Omega)\mathrm{d}b\mathrm{d}z &= \iint_{(b,z)\in\Phi^S} [z(k(b,z;\Omega)^\alpha l(b,z;\Omega)^{1-\alpha}]^{1-\nu} h(b,z;\Omega)\mathrm{d}b\mathrm{d}z, && \text{(B.32)}\\
\bar{y}_\Omega^c \iint_{(b,z)\in\Phi^C} h(b,z;\Omega)\mathrm{d}b\mathrm{d}z &= \iint_{(b,z)\in\Phi^C} [z(k(b,z;\Omega)^\alpha l(b,z;\Omega)^{1-\alpha}]^{1-\nu} h(b,z;\Omega)\mathrm{d}b\mathrm{d}z. && \text{(B.33)}
\end{aligned}
$$

Thus, the growth rate of output can be expressed as

$$
g_Y = g_E + (s_y^c - s_y^s) g_{p^c} + s_y^s g_{\bar{y}^s} + s_y^c g_{\bar{y}^c}, \tag{B.34}
$$

where

$$
\begin{aligned}
s_y^c =& \frac{E_\Omega p_\Omega^c \bar{y}_\Omega^c}{2Y_\Omega} + \frac{E_{\Omega'} p_{\Omega'}^c \bar{y}_{\Omega'}^c}{2Y_{\Omega'}}, && \text{(B.35)}\\
s_y^s =& \frac{E_\Omega (1-p_\Omega^c) \bar{y}_\Omega^s}{2Y_\Omega} + \frac{E_{\Omega'} (1-p_{\Omega'}^c) \bar{y}_{\Omega'}^s}{2Y_{\Omega'}}. && \text{(B.36)}
\end{aligned}
$$

TFP Decomposition. Combining equations (B.16) and (B.34) yields

$$
\begin{aligned}
g_{TFP} = &\left[s_y^c - s_y^s - \alpha(s_k^c - s_k^s) - (1-\alpha)(s_l^c - s_l^s)\right] g_{p^c} \\
&+ s_y^c g_{\overline{y}^c} - \alpha s_k^c g_{\overline{k}^c} - (1-\alpha) s_l^c g_{\overline{l}^c} \\
&+ s_y^s g_{\overline{y}^s} - \alpha s_k^s g_{\overline{k}^s} - (1-\alpha) s_l^s g_{\overline{l}^s}.
\end{aligned} \tag{B.37}
$$

Thus,

$$
\begin{aligned}
\text{TFP}_{\Omega'} - \text{TFP}_{\Omega} = &g_{p^c}\left[s_y^c - s_y^s - \alpha(s_k^c - s_k^s) - (1-\alpha)(s_l^c - s_l^s)\right] \text{TFP}_{\Omega} \\
&+ \left[s_y^c g_{\overline{y}^c} - \alpha s_k^c g_{\overline{k}^c} - (1-\alpha) s_l^c g_{\overline{l}^c}\right] \text{TFP}_{\Omega} \\
&+ \left[s_y^s g_{\overline{y}^s} - \alpha s_k^s g_{\overline{k}^s} - (1-\alpha) s_l^s g_{\overline{l}^s}\right] \text{TFP}_{\Omega}.
\end{aligned} \tag{B.38}
$$

B.4 Proof of Proposition 4

Consider an entrepreneur of (b, z). Since labor supply is not subject to the collateral constraint, the optimal labor supply l is linear in k

$$
l(z,k) = \left[\frac{(1-\alpha)z}{w}\right]^{\frac{1}{\alpha}} k. \tag{B.39}
$$

Substituting out $l(z,k)$, we derive output as a linear function of k,

$$
f(k,z) = zk^{\alpha} l^{1-\alpha} = z\left[\frac{(1-\alpha)z}{w}\right]^{\frac{1-\alpha}{\alpha}} k. \tag{B.40}
$$

With constant returns to scale, optimal capital demand has a bang-bang solution. If the agent chooses to be an entrepreneur, then $k = b$ in the savings regime and $k = (b - \psi)/\xi$ in the credit regime. Given z and b, the entrepreneur's utility is increasing in current consumption c and the future value $V(z', b')$, which increases with the next-period wealth b' for every realization of the next-period productivity z'. Therefore, the optimal borrowing decision is determined to maximize the end-of-period wealth $\tilde{b}^e = c + b'$.

Substituting equation (B.40) into the budget constraint in problems (2.8) and (2.9), we derive the end-of-period wealth in the savings regime as

$$\begin{aligned}\tilde{b}^{e,s}(b,z) &= zk^{\alpha}l^{1-\alpha} + (1-\delta)k - wl + (1+r)(b-k) \\ &= z\left[\frac{(1-\alpha)z}{w}\right]^{\frac{1-\alpha}{\alpha}} b + (1-\delta)b - w\left[\frac{(1-\alpha)z}{w}\right]^{\frac{1}{\alpha}} b,\end{aligned} \tag{B.41}$$

and the end-of-period wealth in the credit regime as

$$\begin{aligned}\tilde{b}^{e,c}(b,z) =& z(k^{\alpha}l^{1-\alpha})^{1-\nu} + (1-\delta)k - wl - (1+r+\chi)(k-b+\psi) \\ =& z\left[\frac{(1-\alpha)z}{w}\right]^{\frac{1-\alpha}{\alpha}} k + (1-\delta)k - w\left[\frac{(1-\alpha)z}{w}\right]^{\frac{1}{\alpha}} k - (1+r+\chi)(k-b+\psi) \\ =& z\left[\frac{(1-\alpha)z}{w}\right]^{\frac{1-\alpha}{\alpha}} \frac{b-\psi}{\xi} + \frac{(1-\delta)(b-\psi)}{\xi} - w\left[\frac{(1-\alpha)z}{w}\right]^{\frac{1}{\alpha}} \frac{b-\psi}{\xi} \\ & - (1+r+\chi)\left(\frac{b-\psi}{1-\xi} - b + \psi\right)\end{aligned} \tag{B.42}$$

The threshold of wealth $\underline{b}(z;\Omega)$ is determined by $\tilde{b}^{e,s}(z,\underline{b}) = \tilde{b}^{e,c}(z,\underline{b})$:

$$\underline{b}(z;\Omega) = \frac{\psi}{1-\xi} + \frac{\psi}{1-\xi}\frac{(1+r+\chi)\xi}{z\left[\frac{(1-\alpha)z}{w}\right]^{\frac{1-\alpha}{\alpha}} - w\left[\frac{(1-\alpha)z}{w}\right]^{\frac{1}{\alpha}} - \delta - r - \chi}. \tag{B.43}$$

Taking the first-order partial derivative, we derive the effect through the extensive margin from relaxing each financial constraint separately:

$$\frac{\partial\overline{b}}{\partial\psi} = \frac{1}{1-\xi} + \frac{1}{1-\xi}\frac{(1+r+\chi)\xi}{z\left[\frac{(1-\alpha)z}{w}\right]^{\frac{1-\alpha}{\alpha}} - w\left[\frac{(1-\alpha)z}{w}\right]^{\frac{1}{\alpha}} - \delta - r - \chi} \geq 0; \tag{B.44}$$

$$\frac{\partial\overline{b}}{\partial\xi} = \frac{\psi}{(1-\xi)^2} + \frac{\psi}{(1-\xi)^2}\frac{1+r+\chi}{z\left[\frac{(1-\alpha)z}{w}\right]^{\frac{1-\alpha}{\alpha}} - w\left[\frac{(1-\alpha)z}{w}\right]^{\frac{1}{\alpha}} - \delta - r - \chi} \geq 0; \tag{B.45}$$

$$\frac{\partial\overline{b}}{\partial\chi} = \frac{\psi\xi}{1-\xi}\frac{z\left[\frac{(1-\alpha)z}{w}\right]^{\frac{1-\alpha}{\alpha}} - w\left[\frac{(1-\alpha)z}{w}\right]^{\frac{1}{\alpha}} + 1 - \delta}{\left[z\left[\frac{(1-\alpha)z}{w}\right]^{\frac{1-\alpha}{\alpha}} - w\left[\frac{(1-\alpha)z}{w}\right]^{\frac{1}{\alpha}} - \delta - r - \chi\right]^2} \geq 0. \tag{B.46}$$

Taking the second-order cross-partial derivative, we derive the interaction effect through the extensive margin from relaxing two financial constraints jointly:

$$\frac{\partial^2 \overline{b}}{\partial\psi\partial\xi} = \frac{\partial^2 \overline{b}}{\partial\xi\partial\psi} = \frac{1}{(1-\xi)^2} + \frac{1}{(1-\xi)^2}\frac{1+r_t+\chi}{z\left[\frac{(1-\alpha)z}{w}\right]^{\frac{1-\alpha}{\alpha}} - w\left[\frac{(1-\alpha)z}{w}\right]^{\frac{1}{\alpha}} - \delta - r - \chi} \geq 0; \tag{B.47}$$

$$\frac{\partial^2 \overline{b}}{\partial\xi\partial\chi} = \frac{\partial^2 \overline{b}}{\partial\chi\partial\xi} = \frac{\psi}{(1-\xi)^2}\frac{z\left[\frac{(1-\alpha)z}{w}\right]^{\frac{1-\alpha}{\alpha}} - w\left[\frac{(1-\alpha)z}{w} + 1 - \delta\right]^{\frac{1}{\alpha}}}{\left[z_t\left[\frac{(1-\alpha)z}{w}\right]^{\frac{1-\alpha}{\alpha}} - w\left[\frac{(1-\alpha)z}{w}\right]^{\frac{1}{\alpha}} - \delta - r - \chi\right]^2} \geq 0; \tag{B.48}$$

$$\frac{\partial^2 \overline{b}}{\partial\chi\partial\psi} = \frac{\partial^2 \overline{b}}{\partial\psi\partial\chi} = \frac{\xi}{1-\xi}\frac{z\left[\frac{(1-\alpha)z}{w}\right]^{\frac{1-\alpha}{\alpha}} - w\left[\frac{(1-\alpha)z}{w}\right]^{\frac{1}{\alpha}} + 1 - \delta}{\left[z\left[\frac{(1-\alpha)z}{w}\right]^{\frac{1-\alpha}{\alpha}} - w\left[\frac{(1-\alpha)z}{w}\right]^{\frac{1}{\alpha}} - \delta - r - \chi\right]^2} \geq 0. \tag{B.49}$$

B.5 Proof of Proposition 5

Substituting the optimal labor demand (B.39) and $k = (b-\psi)/\xi$ into equation (A.8) yields

$$y^C(b, z; \Omega) = \left[z\left[\frac{(1-\alpha)z}{w}\right]^{\frac{1-\alpha}{\alpha}} - \chi\right]\frac{b-\psi}{\xi} + \chi(b-\psi) - \psi. \tag{B.50}$$

Taking the first-order partial derivative, we derive the effect through the intensive margin from relaxing each financial constraint separately:

$$\frac{\partial y^C}{\partial\psi} = -\left[z\left[\frac{(1-\alpha)z}{w}\right]^{\frac{1-\alpha}{\alpha}} - \chi\right]\frac{1}{\xi} + \chi(b-\psi) - 1 \leq 0; \tag{B.51}$$

$$\frac{\partial y^C}{\partial\xi} = -\left[z\left[\frac{(1-\alpha)z}{w}\right]^{\frac{1-\alpha}{\alpha}} - \chi\right]\frac{b-\psi}{\xi^2} \leq 0; \tag{B.52}$$

$$\frac{\partial y^C}{\partial\chi} = -\frac{(1-\xi)(b-\psi)}{\xi} \leq 0. \tag{B.53}$$

Taking the second-order cross-partial derivative, we derive the interaction effect through the intensive margin from relaxing two financial constraints jointly:

$$\frac{\partial^2 y^C}{\partial\psi\partial\xi} = \frac{\partial^2 y^C}{\partial\xi\partial\psi} = \left[z\left[\frac{(1-\alpha)z}{w}\right]^{\frac{1-\alpha}{\alpha}} - \delta - \chi\right]\frac{1}{\xi^2} \geq 0; \tag{B.54}$$

Table OA.1: Calibrated parameters fox six countries.

Parameters	Pakistan	Bangladesh	Brazil	The Philippines	Kenya	Zambia
ψ	2.86	0.80	0.09	0.95	2.00	1.49
ξ	0.315	0.568	0.001	0.335	0.165	0.520
χ	0.048	0.019	0.354	0.040	0.085	0.118
β	0.79	0.89	0.97	0.88	0.89	0.66
θ	3.0	3.2	3.1	3.4	3.2	3.4
γ	0.77	0.78	0.80	0.87	0.74	0.85

$$\frac{\partial^2 y^C}{\partial \xi \partial \chi} = \frac{\partial^2 y^C}{\partial \chi \partial \xi} = \frac{b - \psi}{\xi^2} \geq 0; \tag{B.55}$$

$$\frac{\partial^2 y^C}{\partial \chi \partial \psi} = \frac{\partial^2 y^C}{\partial \psi \partial \chi} = \frac{1 - \xi}{\xi} \geq 0. \tag{B.56}$$

C Numerical Algorithm

We allow wealth b to vary from 0 to 5, and verify that further increasing wealth does not affect the simulation results. To ensure accuracy, we use 1,000 grids with equal length to discretize the support of wealth when approximating the value functions. We use 100,000 grids to discretize the support of wealth when running simulations. The values between wealth grids are approximated by linear interpolation. We allow productivity z to vary from 1 to the value corresponding to 99.95% of the cumulative distribution function. We use 50 grids with equal length to discretize the support of productivity. We use the golden-section search method to find the optimal decision rules. The advantage of the golden-section search method is that it is robust to the choice of initial values because convergence is guaranteed. The numerical algorithm is implemented with parallel computing using C++ and OpenMP.

Computing the Steady State.

(0). Start from some arbitrary distribution $h_0(b, z)$.

(1). Guess the wage in the steady state, w.

(2). Guess the interest rate in the steady state, r.

(3). Given the interest rate r and wage w, solve the agent's problem (2.5-2.9) and obtain optimal policies $c(b, z; w, r)$, $k(b, z; w, r)$, $l(b, z; w, r)$ as well as the set of agents who choose to be entrepreneurs $\Phi^E(w, r)$ and the set of entrepreneurs who choose to enter the credit regime $\Phi^C(w, r)$.

(4). Forward simulate the model by T periods using the optimal policy functions. We set $T = 100$, which is long enough to ensure that the economy reaches the steady state. Calculate the steady-state joint distribution of wealth and productivity $h_T(b,z)$ according to equation (2.12).

(5). Check the capital market clearing condition (2.10) in period T. If there is excess capital demand (supply), choose a new interest rate r that is greater (smaller) than r and return to step (3). We use bi-section search to form the new guess.

(6). Check the labor market clearing condition (2.11) in period T. If there is excess labor demand (supply), choose a new wage w that is greater (smaller) than w and return to step (2).

Computing the Transitional Dynamics.

(0). Solve the steady-state distribution before implementing the financial inclusion policy $h^{pre}(b,z)$, and the steady-state distribution after implementing the financial inclusion policy $h^{post}(b,z)$.

(1). Start from the steady-state distribution before implementing the financial inclusion policy, i.e., $h_0(b,z) = h^{pre}(b,z)$.

(2). Guess the wage path $\{w_t\}_{t=0}^{T}$, where T is set long enough to ensure that the economy can reach the steady state after implementing the financial inclusion policy.

(3). Guess the interest rate path $\{r_t\}_{t=0}^{T}$.

(4). Taking the wage path $\{w_t\}_{t=0}^{T}$ and interest rate path $\{r_t\}_{t=0}^{T}$ as given, solve the agent's problem (2.5-2.9) for $t = T, ..., 0$ using backward induction, starting from $t = T$. We obtain optimal policies $\{c_t(b,z;w,r), k_t(b,z;w,r), l_t(b,z;w,r)\}_{t=0}^{T}$ as well as the set of agents who choose to be entrepreneurs $\{\Phi_t^E(w,r)\}_{t=0}^{T}$ and the set of entrepreneurs who choose to enter the credit regime $\{\Phi_t^C(w,r)\}_{t=0}^{T}$.

(5). Forward simulate the model by T periods using the optimal policy functions, starting from $t = 0$. Calculate the joint distribution of wealth and productivity $\{h_t(b,z)\}_{t=0}^{T}$ according to equation (2.12).

(6). For each $t = 0, 1, ..., T$, holding $h_t(b,z)$ and w_t constant, find the implied interest rate $\tilde{r}_t$ that clears the capital market (2.10).

(7). Calculate $\text{diff}_r = \max\{|r_t - \tilde{r}_t|, \text{for } t = 0, 1, ..., T\}$. If $\text{diff}_r > 10^{-5}$, replace r_t with $(r_t + \tilde{r}_t)/2$ for $t = 0, 1, ..., T$ and return to step (4).

(8). For each $t = 0, 1, ..., T$, holding $h_t(b, z)$ and r_t constant, find the implied wage $\tilde{w}_t$ that clears the labor market (2.11).

(9). Calculate $\text{diff}_w = \max\{|w_t - \tilde{w}_t|, \text{for } t = 0, 1, ..., T\}$. If $\text{diff}_w > 10^{-5}$, replace w_t with $(w_t + \tilde{w}_t)/2$ for $t = 0, 1, ..., T$ and return to step (3).