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ABSTRACT

We present a simple model of sovereign debt crises in which a country chooses its optimal mix of short and long-term debt contracts subject to standard contracting frictions: the country cannot commit to repay its debts nor to a specific path of future debt issues, and contracts cannot be made state contingent. We show that in order to satisfy incentive compatibility the country must issue short-term debt, which exposes it to roll-over crises and inefficient repayments. We examine two policies—restructuring and reprofiling—and show that both improve ex ante welfare if structured correctly. Key to the welfare results is the country's ability to choose its debt structure so as to neutralize any negative effects resulting from redistribution of payments across creditors in times of crises.

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1 Introduction

Sovereign debt is once again at the center stage of the academic and policy debate. This attention has been largely fueled by the growth of public debt within the Eurozone, which has raised concerns regarding the ability of the current debt-restructuring framework to deal efficiently with large-scale debt crises. These concerns were largely vindicated by the experience of Greece, where the debt restructuring of 2012 is widely perceived to have come inefficiently late, after years of low growth and increasing official indebtedness.

It is against this backdrop that various reforms of the international lending framework are being discussed. The International Monetary Fund in particular is considering modifying its lending framework to allow greater flexibility in how it deals with debt crises. The centerpiece of the proposed reform is the possibility of "reprofiling" debt payments during crises: intuitively, the idea is that when a country is faced with a debt crisis, it may be helpful to extend the maturities of its debts and postpone payments until greater certainty is obtained regarding the country's prospects. If the country recovers, it can pay its debt without engaging in a full-blown restructuring; if not, restructuring is needed. This proposal has generated a lively debate. Adherents stress its ex post benefits, i.e., it will help countries deal more efficiently with debt crises when these happen. Opponents emphasize instead its ex ante costs, i.e., it will make short-term debt more similar to long- term debt, thereby raising the cost of funding, and ultimately perhaps lead to more frequent crises or prevent some countries from borrowing altogether.[1] Assessing the merits of these views requires an analytical framework to answer the following questions: What are the costs and benefits of reprofiling? When is reprofiling likely to be used relative to restructuring? What are its effects on short- and long- term interest rates? How does it affect the likelihood and cost of debt crises?

The objective of this paper is to provide a simple framework in which to analyze questions regarding sovereign debt reforms, such as the ones above. The model we develop incorporates several ingredients which we consider important for thinking about these issues. First, borrowing is subject to standard contracting frictions: debt contracts are non-contingent, they cannot be renegotiated ex post, and the country is unable to commit to a path of future debt issues or repayment. Second, the timing of defaults can matter. When the future looks particularly bleak, short-term creditors may refuse to roll-over their debt even at very high interest rates and, by insisting upon full repayment, the impact on a country's ability to generate future output can be especially severe. Lastly, and key as we will show to understanding the welfare consequences of reforms, the country's debt maturity structure is endogenous.

Our model features a small-open economy that borrows resources from the international financial market in order to finance an ex-ante profitable investment opportunity. The project takes time to mature and interruptions or early payments diminish its associated expected output. Over time, the country is subject to shocks which affect its productivity and thus its ability and perhaps even willingness to repay its debt.

[1]For an expostision of these arguments, see "Creditors likely losers from IMF Rethink" (Financial Times, 6/4/2013), "IMF sovereign debt plans in jeopardy" (Financial Times, 1/26/2014), and "Finance: In search of a better bailout" (Financial Times, 1/26/2014).

In this environment, short-term debt is costly because creditors may be unwilling to roll it over in the face of bad economic prospects, triggering debt crises. If the country could commit to a path of future debt issues, these risks could be avoided by issuing long-term debt. Absent this ability to commit though, long-term debt is also problematic. The reason is that, once issued, the country will be tempted to dilute its long-term debt by issuing more debt in the future. In this regard, as in much of the recent sovereign debt literature that incorporates debt maturity (e.g. Hatchondo et al. (2011), Arellano and Ramanarayanan (2012), and Chatterjee and Eyigungor (2012)), short-term debt in our model plays a "disciplining" role by reducing the incentives to dilute. This gives rise to a set of incentive compatible maturity structures that keep dilution in check.

Key to choosing among these possible maturity structures is how they affect both the likelihood and the cost of debt crises. Ideally, a country would use its maturity structure to reduce the total cost of debt and the incidence of crises. But the maturity structure of debt also determines how costly a debt crisis is once it happens. The reason for this is that there are potentially two ways to pay short-term creditors in the event of a crisis: by transferring resources prematurely from the economy (e.g. through taxation or disinvestment), which is inefficient in terms of future output, or by transferring future resources from long-term creditors through dilution. If the share of short-term debt is low, diluting long-term debt is a feasible option: in this case, the country eventually defaults but it reduces the cost of the crisis by avoiding inefficient payments. If the share of short-term debt is high though, dilution may not suffice: in this case, the cost of the crisis is higher since because it requires inefficient payments.

We show how, in equilibrium, the country's choice of maturity structure optimally trades off the costs and benefits of short-term debt. We find that when a country faces ex ante "favorable" circumstances in which long-term debt is endogenously relatively cheap, there is no trade-off: the country chooses the longest maturity structure that is possible among those that are incentive compatible. This maturity structure minimizes both the likelihood and the cost of debt crises. When a country faces instead ex ante "unfavorable" circumstances in which long-term debt is endogenously relatively expensive, a tension ensues: increasing the share of long-term debt reduces the costs of crises but, since long-term debt is expensive, it may also raise their likelihood. In this case, the country may opt for a shorter maturity structure than dictated by incentive compatibility considerations alone.

We then use the model to analyze the effects of potential reforms of the sovereign debt system. We assume that a country can request help from an international financial institution (e.g., the IMF) and that the latter coordinates creditors and country by choosing the appropriate remedy, be it debt restructuring, debt reprofiling, or non-intervention in the event of a debt crisis.[2] A debt restructuring is a proportional write-down of the value of the debt to a level that allows it to be rolled over at actuarially fair rates. Because restructuring is a credit event, it imposes the same penalty as default on the country. In addition to

[2] Given the large literature that already exists on the topic of solvency crises.(e.g., Cole and Kehoe 2000, Corsetti et al. 2006), we ignore self-fulfilling crises and the role for an IFI as a potential lender of last resort; this topic is well-understood.

redistributing resources, restructuring has an efficiency-enhancing effect since it prevents inefficient payments during crises and thus raises total available resources.

A debt reprofiling is considered a lighter version of restructuring. Instead of writing down the value of the debt immediately, payments due are postponed (i.e., debt maturity is lengthened). Naturally, reprofilings can only be successful if they impose a haircut or *de facto* a write-down on short-term creditors, i.e., if they are forced to roll over their debt at an interest rate that does not appropriately reflect the expected risk of default. Relative to restructurings, reprofilings have two effects. First, they redistribute resources from short-term creditors, who absorb the entire haircut, to long-term creditors. This implies that short-term debt will become more expensive, for a given maturity structure. Second, reprofilings entail a lower haircut than restructurings, and thus presumably a lower penalty of default as well.

Both restructuring and reprofiling have the potential to be welfare improving because they eliminate the inefficiency associated with early payments to short-term creditors when these are unwilling to roll-over their debt. Both reforms also affect how resources are distributed among creditors and between creditors and the country. We show that a sufficient condition for these interventions to be welfare enhancing is that they not decrease the expected total value of payments to creditors during times of crises. This is a surprising result. After all, what guarantees that redistribution away from short-term creditors doesn't result in higher contractual rates and thus a greater frequency of debt crises? More subtly, even if all creditors are paid more in times of crises, how can one be sure that larger payments to short-term debtors do not have an adverse effect on the relative probability with which long-term debtors are paid in full versus partially? A central contribution of our paper is to show that these concerns are rendered mute but only as a consequence of the country's ability to choose its debt maturity structure. In particular, we prove a general result: any negative effect arising either from the redistribution of payments between creditors (short vs. long), or from changes in the frequency with which long term debt is paid in part vs. in full, can be neutralized by appropriately changing the debt structure in an incentive-compatible fashion. This leaves only the net positive effects arising from the ability to make greater payments to creditors in times of crises, which allows the probability of a crisis to fall. Thus, our analysis allows us to conclude that the two interventions increase ex ante welfare and that, to the extent that reprofiling is a "light restructuring" (i.e., either allows a smaller haircut and/or potentially entails a smaller default penalty), it dominates outright restructuring.

An implication of our model is that both debt restructurings and debt reprofilings make it possible to reduce the incidence of debt crises. With respect to reprofilings, however, a country may not find it optimal to do so. The reason is as follows. Ex post, the country may prefer to deal with a debt crisis by diluting long-term debt instead of reprofiling since dilution allows it to increase its own consumption. From an ex ante perspective, though, this is suboptimal because all expected dilution is priced into the long-term interest rate. Understanding this, the country may be better off shortening the maturity structure of its debt, since this increases the ex post attractiveness of reprofilings by reducing the gains from dilution. In this sense, our model delivers an intriguing result: debt reprofiling might lead to a higher short-term interest rate, a shorter maturity structure of debt and a higher likelihood of debt crises in equilibrium. Even in this case, however, reprofiling must raise welfare, because the country will only choose this maturity structure if the higher likelihood of crises is more than offset by their lower cost.

Our paper complements existing work on the optimal maturity structure of debt. Most existing theories build on the notion that, in a context of incomplete markets, different maturities have different hedging as well as incentive properties. In a closed-economy model with full commitment, for instance, Angeletos (2002) and Buera and Nicolini (2004) have shown that, even if the government has access only to non-contingent bonds, it can structure an optimal portfolio of maturities to replicate the hedging properties of contingent bonds. Recently, Debortoli et al. (2014) extend this setting to the case in which the government cannot commit to a future path of debt (but it retains commitment to repayment), and show how this limits the use of maturity structure for hedging purposes.

The maturity structure of debt has also been studied in the sovereign debt literature, which stresses governments' inability to commit to repay their debts. This lack of commitment creates an additional or "disciplinary" view of short-term debt, which posits that short-term borrowing is a useful provider of incentives in such environments. While this view of short-term debt is an old idea in corporate finance (see, for instance, Diamond and Rajan 2000, 2001), different perspectives arise in the context of sovereign debt.[3] In particular, by providing creditors with the ability to be repaid early, short-term debt may induce a borrower to undertake certain actions that are desirable *ex ante* but not incentive compatible *ex post*. Our paper falls within this view.[4]

In Jeanne (2009), for instance, short-term debt provides the government with incentives to carry out pro-market reforms. In contrast, short-term debt in our setting is disciplinary in the sense that it reduces incentives to expand borrowing and dilute long-term debt, i.e., it allows the borrower commit to a certain path of debt issues. This is similar to the work of Hatchondo et al. (2011), Arellano and Ramanarayanan (2012), and Chatterjee and Eyigungor (2012), all of which study quantitative models of sovereign debt in which short-term debt is useful to control dilution. Hatchondo et al. (2011), in particular, find that the costs imposed by dilution are quantitatively important, in the sense that eliminating the government's ability to dilute existing debt (for instance, though seniority clauses) has a substantial negative effect both on the frequency of defaults and on average spreads.

Our paper contributes three important insights to our understanding of the role of short-term debt. First, it shows that even in a setting in which the only reason to issue short-term debt is "disciplinary", countries may choose to issue more short-term debt than what would be strictly required for these purposes. The reason is that, once a country needs to issue short-term debt, the incidence of crises rises and so does the cost of long-term debt. This may make long-term debt relatively expensive and thus lead the country

[3] Brunnermeier (2013) explores a related setting and shows how the borrower's inability to commit to a specific maturity structure may lead to excessive short-term debt.

[4] In Broner et al. (2008), the hedging properties of different maturities *for international investors* determine the equilibrium maturity structure of debt.

to substitute away from it in order to lower the overall cost of debt and default. Second, the model shows that the welfare consequences of reforms to existing debt-crisis management practices depend critically on how they affect the total payments that are made to creditors during crises. We show that any potentially negative redistributional effects of such reforms on short- and long-term creditors can be addressed by appropriately changing the maturity structure of debt. Lastly, the analysis shows that short-term debt may play a novel disciplining role: creating a greater incentive to deal with a crisis via an international financial institution's (IFI) reprofiling of sovereign debt rather than via the dilution of long-term debt. This may lead countries to issue more short-term debt even when it becomes more expensive.

The paper proceeds as follows. Section 2 develops the basic model. Sections 3 and 4 characterize the properties of equilibria and uses them to understand some comparative statics properties. Section 5 introduces the IFI and explores the implications of debt restructurings and reprofilings. Section 6 concludes with a discussion of robustness and extensions.

2 The Model

In this section we develop a model that permits one to analyze the basic trade-offs a country faces regarding its debt structure: the cost of borrowing through short and long-term debt, and the implications of its maturity structure for the expected frequency and severity of any debt crisis that might arise. We make several simplifying assumptions (e.g., a finite horizon and two states of nature) that allow us to obtain analytical solutions and to understand the key mechanisms underlying the main results; these insights should carry through to more complicated environments, as discussed in section 6.

Consider therefore a model in which time is discrete and there are three periods: $t = 0, 1, 2$. There are two types of agents (in the next section we will introduce a third − the IFI): a risk-neutral country that is small in world capital markets and consumes only in period 2, and perfectly competitive risk-neutral creditors. Below we specify the timing in which decisions are made and introduce the assumptions.

In period 0, the country receives an indivisible productive investment opportunity of size 1, which matures in period 2. This project can only be financed, for simplicity, by borrowing. The latter is subject to three contracting frictions: debt contracts are non-contingent, they cannot be renegotiated ex post, and the country cannot commit to a path of future debt issues.[5]

The project can be interrupted by the country before it matures. In particular, the country can disinvest a portion δ, $1 \geq \delta \geq 0$, of its project in period 1, as will be discussed in greater detail below. Thus, at the beginning of period 2, the amount of capital remaining in the project is $k = 1 - \delta$. The return to the project in period 2 is stochastic. Specifically, period 2 output y is given by

$$y(\theta; k) = \theta k, \ \theta \in \{\theta_L, \theta_H\} \tag{1}$$

where $\theta_H > \theta_L$. Henceforth we simplify the algebra by setting $\theta_L = 0$.

[5] For a broad discussion of the role of these assumptions in the sovereign debt literature, see Aguiar and Amador (2013).

Information about the project is as follows: in period 0, before accessing capital markets, the initial probability that $\theta = \theta_H$ is given by p_0. At the beginning of period 1, a signal regarding the productivity of the project is received and the updated probability of $\theta = \theta_H$ becomes $p \sim G(p)$, where $p \in [0, 1]$ and $p_0 = \int_0^1 p dG(p)$. At the beginning of period 2, $\theta \in \{\theta_L, \theta_H\}$ is realized.

Financing the project requires the country to borrow a unit of capital from the international capital market in period 0. The country can offer its preferred mix of short and long-term contracts, but all contracts must be non-contingent. Each contract thus specifies a gross rate of return R_{0t}, where t denotes the period in which repayment is due, i.e., R_{01} is the gross return due to a unit of short-term (ST) debt in period 1 and R_{02} is the equivalent for long-term (LT) debt due in period 2. The gross safe rate of return in the international capital market is given by $R > 1$ per period. This is also the rate at which the country can save.

In period 1, after p has been realized, the payment of short-term debt becomes due. At this point, in order not to default, the country must repay its short-term creditors or roll-over its ST debt at an endogenous gross rate R_{12}. Given a competitive market for credit, rolling over ST debt is equivalent to obtaining new loans. Failure to entirely repay or roll-over its short-term debt means that the country is in default, whereupon creditors are able to insist upon repayment, to the extent possible. We assume that obtaining funds in period 1 is more costly than in period 2.[6] This can be thought of as arising from the fact that the project takes time to mature and taxation is therefore costly, especially when future prospects look bad. For example, the country may need to engage in austerity measures that reduce spending in complementary areas (e.g. cuts in social spending and infrastructure projects). In particular, we assume that by diverting δ units of capital from the project, the country or creditors are only able to obtain $F(\delta, p)$ units of output where

$$F(\delta, p) < \frac{p\alpha\theta_H\delta}{R} \text{ for } p \in (0, 1] \text{ with } F(\delta, 0) = 0,\ F_1 > 0,\ F_2 \geq 0 \tag{2}$$

and where $\alpha \in (0, 1)$, as we shall see, is a constant that denotes the maximum share of period-2 output that can be effectively transferred to creditors. Intuitively, equation (2) implies that disinvestment is inefficient in the sense that it reduces the present value of total output that can be used to pay creditors.

In order for the country to be able to borrow in equilibrium, it must be willing and able to repay its debt in some states of nature. There are thee features of our environment that provide incentives for repayment. First, as discussed previously, default in period 1 effectively allows creditors to obtain some repayment by diverting funds from the investment project. Second, we assume that upon default in period 2 creditors are effectively able to seize a proportion $\alpha \in (0, 1)$ of output $y(\theta, k)$, *but not of savings*.[7] To simplify matters, we assume that α also denotes the maximum proportion that the country is able to pay its creditors from period-2 output (e.g., there is a limit to the country's ability to raise taxes on output). Lastly, default in

[6] See Jeane (2009) who uses a similar assumption.

[7] By assuming that savings cannot be seized, we provide the country with the maximum incentives to disinvest and divert resources through savings. As we show below, however, this option is never used in equilibrium.

any period subjects the country to a default penalty in period 2 that is proportional to productivity: we denote this penalty by by $\pi\theta$ with $0 < \pi < \alpha$. This penalty can be thought of as encapsulating any future losses that a country faces after a default.[8] Lastly, note that debt cannot be renegotiated ex post: upon default, creditors obtain funds according to whether default is early (period 1) or late (period 2) and the country suffers the default penalty $\pi\theta$.[9]

Note that when $\theta = \theta_L = 0$, the country will not be able to make any repayments in period 2 and will necessarily default. Thus, borrowing is only possible if the project is sufficiently productive to repay creditors an actuarially fair return when $\theta = \theta_H$. Given that the project lasts for two periods and that the ex ante probability that $\theta = \theta_H$ is p_0, a minimum requirement is

$$p_0 \alpha \theta_H > R^2 \tag{3}$$

which we henceforth assume.

The timeline of decisions and realizations is given in Figure 1. In brief, in period 0 the country decides its optimal mix of short and long-term debt, respectively denoted by γ and $1-\gamma$, and offers contracts specifying R_{0t}, $t = 1, 2$. In period 1, information about the true state of nature is obtained via p, some funds δ may be diverted from the project, payments may be made, the country may decide to save, and new short-term credit may be obtained via contracts specifying R_{12}. Lastly, in period 2 the state of nature θ is revealed, payments are made, a fraction α of output is seized if the country is in default, and consumption and any default penalty takes place.

Before turning to the solution of the model, we specify the rules that govern creditor repayment. Suppose that in period 1 the country does not repay in full its short-term creditors. This forces the country to disinvest its project to obtain the necessary funds. If these funds suffice, the country can continue to period 2 without declaring default in period 1. If these funds do not suffice, then only ST creditors are repaid to the extent possible and then default is declared.[10] In particular, if the entire project were disinvested (leaving the country with no period-2 output), creditors would obtain, per unit of debt:[11]

$$V_1 \left(R_{01}, R_{02}; p\right) = \begin{cases} \min\left(R_{01}, \frac{F(1,p)}{\gamma}\right) \text{ for ST debt} \\ \min\left(\frac{R_{02}}{R}, \frac{F(1,p)-\gamma R_{01}}{(1-\gamma)}\right) \text{ for LT debt} \end{cases} \tag{4}$$

[8] The costs typically emphasized in the literature encompass: loss of reputation that reduces trade in goods or assets between the defaulting country and the rest of the world; economic sanctions imposed on the defaulting country by the rest of the world; costs associated to the economic content of defaults, or; costs related to domestic holdings of public debt. For a discussion of the theoretical undeprinnings and the empirical evidence behind these different costs, see Borensztein and Panizza (2009) and Sandleris (2012).

[9] In particular, creditors are unable to negotiate as a group, forgive debt, or postpone payments. This is a potential role for outside parties, which we simply call "the IMF."

[10] We have in mind that when the ST debt refuses to roll-over, the country services debt sequentially with default declared only once there is no (liquidated) output left over. At that point some ST debt will still be unpaid as will all the LT debt. Thus when default is actually declared, there are no resources available for repayment of LT debt. See the discussion in section 6 for the case in which all debt is paid pro rata from the outset.

[11] We have assumed that R_{02} is adjusted to $\frac{R_{02}}{R}$ in order to reflect that it is being repaid early.

If instead the country fails to meet its debt obligations in period 2, then default is declared and all creditors are paid pro rata obtaining, for $\theta = \theta_H$, repayment, per unit of debt, of:

$$V_2\left(R_{12}, R_{02}\right) = \begin{cases} \begin{array}{c} R_{12} \text{ for ST debt} \\ R_{02} \text{ for LT debt} \end{array} & \text{if } D_2 \leq \alpha\theta_H k \\ \begin{array}{c} \dfrac{\alpha\theta_H k}{D_2} R_{12} \text{ for ST debt} \\ \dfrac{\alpha\theta_H k}{D_2} R_{02} \text{ for LT debt} \end{array} & \text{if } D_2 > \alpha\theta_H k \end{cases} \tag{5}$$

and 0 if $\theta = \theta_L$. Here $D_2 = (1-\gamma) R_{02} + D_{12} R_{12}$ where, abusing notation slightly, D_{12} denotes the quantity of ST debt obligations incurred in period 1 at rate R_{12}.[12]

3 Feasible Strategies

In this section we describe the feasible strategy set; we solve for the equilibrium in the next section. Most of the interesting strategic choices take place in period 1 once p has been realized. At that juncture, if the country does not wish to default it will need to roll over its ST debt or, equivalently, issue new ST debt. We will show that for p sufficiently high, subject to an incentive compatibility constraint, the country will repay its debt. For p in some intermediate range, the country will be forced to default in period 2 but will do so in the most advantageous way possible – by diluting the value of long-term debt. Lastly, very low values of p will force the country to default in the most costly manner – by disinvesting the entire project in period 1.

Let us begin with period 2. If the country has already defaulted in period 1, the only action available to it is to consume any available income. If the country has not defaulted in period 1, it can now choose whether to repay its debt obligation if feasible (i.e., if $D_2 \leq \alpha\theta k + R \cdot s$ where s denotes its period 1 savings). If full repayment is not feasible, it will necessarily default.

Turning next to period 1, the country may be able to avoid default that period by meeting its ST debt obligations either by repaying this debt or by rolling it over. When is repaying its debt or issuing new debt feasible? To answer this question we start by noting that in order to roll-over/issue ST debt in period 1, the country must compensate ST creditors for the risk associated with the realization of p (the updated period-1 belief that $\theta = \theta_H$) which has now been observed. Hence debt issued in period 1 must receive $\dfrac{R}{p}$ as the gross rate of return in period 2.[13] This implies that the country will need to make repayments of $R_{01}\dfrac{R}{p}$ to each holder of ST in period 2 in order not to disinvest any portion of its project.

To properly define the feasible strategy set for the country a few definitions will be useful. Let $r = (R_{01}, R_{02})$ be the vector of promised returns made by the country in period 0. We will often refer to (γ, r) as the country's debt structure, chosen in period 0, where γ is the proportion of the debt that is short-term.

[12] The expressions in equation (5) implicitly assume that the country has no savings that it wishes to use to repay its creditors. Later we will show that this will always be the case in equilibrium.

[13] The nominal gross rate it offers, R_{12}, may differ from $\frac{R}{p}$, however, as the payment creditors receive depend on the country's strategy as will be seen below.

We can divide the strategy space into three basic categories: full repayment, dilution and late default, and disinvestment and early default. As we show below, the feasibility of each strategy depends on the realization of p.

3.1 Full repayment in period 2: $p \geq \underline{p}$

We start by defining $\underline{p}(\gamma, r)$ as the lowest realization of p such that it is *feasible* for the country to repay its debt in the high state in period 2 given its debt structure (γ, r) and given that it has not disinvested any part of its capital ($k = 1$). Thus, for $p \geq \underline{p}(\gamma, r)$, the country is able to roll over its ST debt at $t = 1$ without defaulting in the high state at $t = 2$. Formally, $\underline{p}(\gamma, r)$ solves:

$$\alpha\theta_H - (1-\gamma) R_{02} - \gamma R_{01} \frac{R}{p} = 0. \tag{6}$$

3.2 Dilution and Late Default: $p \geq \widehat{p}$

If $p < \underline{p}(\gamma, r)$, resources are insufficient to repay the debt fully in period 2. The country need not default immediately in period 1, however. It may be able to roll over its ST debt by diluting LT debt, thereby postponing the default until period 2 when it is less costly as the project has matured.

Define $\widehat{p}(\gamma; R_{01})$ as the minimum value of p that is compatible with the country being able to roll over its debt given r. In order to derive $\widehat{p}$, note that the country can repay its current ST creditors (or equivalently have the ST debt rolled over) by issuing new debt at $R_{12} = \infty$.[14] Of course, this strategy implies that the country will default in period 2 independently of the realization of θ. Upon default, though, all feasible repayments ($\alpha\theta_H$) will be used to repay ST debt and the latter will obtain a unit return of $\frac{\alpha\theta_H}{D_{12}}$. LT debt will in this case be completely diluted and obtain a return of zero (see equation (5)).

The strategy above enables the country to issue a maximum of $D_{12} = \frac{p\alpha\theta_H}{R}$ units of new (or rolled-over) debt in period 1, since it is in essence selling the discounted expected value of its entire future payable output including any funds that would have been used to repay LT creditors. The funds generated by this strategy in period 1 are used to pay ST creditors (γR_{01}) and the remaining amount of $\frac{p\alpha\theta_H}{R} - \gamma R_{01}$ is saved at the international rate R and cannot be seized by creditors. Thus, given that the ST due in period 1 must be repaid R_{01} and any ST debt rolled over requires an expected gross return of R/p, the equation $\gamma R_{01} R = p\alpha\theta_H$ yields the minimum realization of p that is compatible with this strategy for a given γ and a given R_{01}, i.e.,

$$\widehat{p}(\gamma; R_{01}) = \frac{\gamma R_{01} R}{\alpha\theta_H} \tag{7}$$

[14] The infinite rate is because there is only one productive state of nature in this economy, which significantly simplifies the analysis. In an economy with several (or a continuum of) θ, the country would face a choice of how many states it would like to completely dilute LT debt. It would then set the rate accordingly.

3.3 Early Default: $p \geq 0$

Lastly, diverting funds from the project partially or fully in period 1 is always feasible. It is worth noting, furthermore, that it is the only feasible strategy in period 1 if $p < \widehat{p}(\gamma; R_{01})$. For such realizations of p, ST debt cannot be rolled over and will these creditors will demand repayment in period 1, necessarily leading to early default. To see this, note that by diverting all funds from the project the country obtains $F(1, p)$ which, by equation (2) is strictly smaller than the appropriately discounted expected value of repayable output it would obtain were the project not disinvested and repayments were made instead in period 2: $\frac{p\alpha\theta_H}{R}$. Given the atomistic nature of creditors, the latter cannot agree to collectively rollover their debt and obtain greater repayment (by setting $R_{12} = \infty$) than they obtain via disinvestment in period 1. Each owner of ST debt would find it a dominant strategy to holdout and insist upon early repayment. Note that the same conclusion holds for any amount of disinvestment. Hence, whenever $p < \widehat{p}(\gamma; R_{01})$, the project will be fully disinvested and the country will default early. To economize on notation, hereafter we refer to $F(1, p)$ as $f(p)$.

This concludes our description of feasible strategies and we next turn to solving for the equilibrium.[15]

4 Equilibrium Strategies

In this section we describe the equilibrium strategy of the country. As we show, depending on (γ, r) and p, the optimal strategy in period 1 will take one of three forms: if $p < \widehat{p}(\gamma, r)$, the country fully disinvests in period 1 and defaults; if $\underline{p}(\gamma, r) > p \geq \widehat{p}(\gamma, r)$, the country dilutes completely the LT debt and then defaults in period 2; finally, if $p \geq \underline{p}(\gamma, r)$, the country rolls over its short term debt at rate $R_{12} = R/p$ and repays the entire debt in period 2. In period 0, the country chooses the optimal maturity structure taking these strategies into account.

We now derive these equilibrium strategies by starting in period 2 and working our way backwards.

4.1 Period 2

The equilibrium strategy in period 2 is straightforward. Given the state θ and its debt repayments due that period, D_2, if the country repays its debt (assuming feasibility, i.e., if $\alpha\theta k + Rs - D_2 \geq 0$) its payoff is:

$$W^{nd}(\theta; k, s, D_2) = \theta k + Rs - D_2 \tag{8}$$

If it does not repay its debt, either because it is infeasible or because it chooses not to, its payoff is

$$W^{d}(\theta; k, s) = (1 - \alpha)\,\theta k + Rs - \pi\theta \tag{9}$$

[15] We have omitted dominated strategies from this list, e.g., the strategy of diverting funds from the project, partially or totally, and then using the proceeds to repay ST debtors and saving the remainder at rate R. Then in period 2 either defaulting or not. As will be shown formally below, this strategy is dominated by not diverting funds from the project.

Hence, if $\alpha\theta k + Rs - D_2 \geq 0$, the country will choose to repay its debt iff $\alpha\theta k + \pi\theta \geq D_2$. That is, a country will only repay if the cost of not doing so (having a portion α of output seized and suffering the default penalty) exceeds its debt obligations, assuming that repayment is feasible.

4.2 Period 1

We next determine the strategy the country will follow in period 1 given (γ, r) and a realization of p that period. We start by noting that if the country plans to default in period 2, it will always choose to dilute its LT debt if feasible. To see this, note that dilution allows the country to consume the expected value of output next period net of the payments to ST creditors.[16] Its expected welfare from dilution at that $p \geq \widehat{p}$ is:

$$W_{dd}(p; \gamma, r) = p\theta_H - \gamma R_{01} R - p\pi\theta_H \tag{10}$$

whereas its expected welfare from rolling-over ST debt without diluting and defaulting in period 2 is at most $(1-\alpha) p\theta_H - \gamma R_{01} R - p\pi\theta_H$, i.e., without taking into account the effects of disinvestments, so that its consumption is lower by at least $p\alpha\theta_H$. Thus default with dilution dominates default without dilution.

Will the country dilute its LT debt, however, when it doesn't have to default in period 2, i.e., when $p \geq \underline{p}$? Its expected welfare from repayment is:

$$W_{pay}(p; \gamma, r) = p\theta_H - p(1-\gamma) R_{02} - \gamma R_{01} R \tag{11}$$

Thus, for repayment to be preferred to dilution and default requires:

$$\pi\theta_H \geq (1-\gamma) R_{02} \tag{12}$$

that is, the punishment from default must be sufficiently large relative to the gain from not repaying LT debt so as to render repayment incentive compatible. Alternatively, this can be expressed as saying that the amount of ST debt issued by the country must be sufficient to make repayment of LT debt incentive compatible. For a given R_{02}, this minimum amount, $\gamma_{IC}(R_{02})$, satisfies

$$\gamma_{IC}(R_{02}) = 1 - \frac{\pi\theta_H}{R_{02}} \tag{13}$$

Note that p does not influence whether repayment is incentive compatible. This is because both the punishment and all payments made are dependent on p. The value of p, however, does influence the feasibility of repayment and the country will only repay its LT debt when it is both feasible and incentive compatible to do so.

Lastly, let us consider the strategy of diverting all funds from the project. As noted previously, if $p < \widehat{p}(\gamma, r)$, the country must default without rolling over its ST debt since atomistic short-term creditors

[16] That is, it obtains output from the sum of its saving (i.e., the funds obtained via dilution in period 1, $(\alpha p\theta_H)$) and the unseizable portion of production $(1-\alpha) p\theta_H$.

understand that the expected value of seizable output in period 2 is insufficient to repay their claims. Hence, for $p < \widehat{p}(\gamma, r)$, the country's expected welfare is given by:

$$W_{\delta}(p; \gamma, r) = -p\pi\theta_H \tag{14}$$

Will the country want to divert funds (partially or fully) for other p realizations? The only reason to do so is either to save so as to increase payments to creditors or to render them immune to being seized by creditors in period 2. The inefficiency inherent in not allowing the project to mature, however, implies that diverting funds cannot increase the resources available to repay debt. Disinvesting from the project in order to save the proceeds and render them immune to seizure, on the other hand, is dominated by dilution of LT debt. To see this, compare the payoff from dilution and default, W_{dd} in equation (10), with the expected payoff from diverting a fraction δ of the capital in the project, saving the output associated with it ($F(\delta, p)$), and shielding it from creditors: $(1-\alpha)p\theta_H(1-\delta) + RF(\delta, p) - \gamma R_{01}R - p\pi\theta_H$. A quick comparison reveals that, once the country realizes that it will default in period 2, diluting LT debt dominates disinvesting from the project; its expected payoff that is greater by $p\theta_H(1-\alpha)\delta + \alpha p\theta_H - RF(\delta, p) > 0$. The intuition for this is clear: the country can shield the expected value of seizable output by *selling* it to ST creditors in period 1; there is no need to engage in inefficient diversion of funds. From this we can conclude that funds will be diverted from the project only when unavoidable, i.e., when dilution is no longer a feasible strategy.

Summarizing the country's optimal strategy in period 1: if $p < \widehat{p}(\gamma, r)$, the country entirely disinvests its project in period 1 and defaults. If $\underline{p}(\gamma, r) > p \geq \widehat{p}(\gamma, r)$, the country dilutes completely the LT debt (by offering $R_{12} = \infty$) and then defaults in period 2. If $p \geq \underline{p}(\gamma, r)$ and the IC constraint in equation (13) is satisfied, the country rolls over its short term debt at rate $R_{12} = R/p$ and repays the entire debt in period 2. If $p \geq \underline{p}(\gamma, r)$ and the IC constraint is not satisfied, the country follows the same strategy as for $\underline{p}(\gamma, r) > p \geq \widehat{p}(\gamma, r)$ – dilution and default in period 2.

Figures 2 and 3 show the feasible and optimal strategies, respectively, for the country for all p given (γ, r), where DL is default and disinvest in period 1, DD_2 is full dilution and default in period 2, and ND is no default (full repayment). The range for which ND is optimal is shown assuming that the IC constraint is met.

4.3 Period 0

We can now determine a country's optimal choice of (γ, r). We assume that the country can set both the proportions of debt and the nominal rates of return, thus ignoring the multiplicities generated solely by expectations.[17] This is equivalent, we will show, to finding, for each γ, the lowest contract rates compatible with equilibrium behavior at that γ. The country's objective thus is to choose (γ, r) so as to maximize its

[17] That is, if creditors expect the country to default for a given range of p, this can generate self-fulfilling higher nominal rates of return on debt, thereby increasing the range of p that lead to default. Calvo (1998) provides a classic example if this type of multiplicity. More recently, Lorenzoni and Werning (2014) have explored it in a dynamic setting.

period-0 expected welfare

$$\begin{aligned} W_0(\gamma, r) &= \int_0^1 W(p; \gamma, r)\, dG(p) \\ s.t.\ W_0(\gamma, r) &\geq 0 \end{aligned} \tag{15}$$

Before deriving the equilibrium choice, it is useful to characterize the first-best outcome. Note that since creditors must, in equilibrium, obtain an expected value of R per unit of debt per period, a country's welfare would be maximized by minimizing all instances of disinvestment and by avoiding all instances of default, with or without disinvestment. Both outcomes simply lower its ex ante expected welfare without reducing the expected value of the real transfers it must make to creditors.

Suppose that the country could credibly promise not to issue any short-term debt in period 1. In our environment, this would allow it to attain the first best allocation. To see this, note that it could simply set $\gamma = 0$ and – assuming that LT debt is repaid whenever $\theta = \theta_H$ – perfect competition in the capital market would imply $R_{02} = \frac{R^2}{p_0}$. But the expected productivity of the project (see equation (3)) would indeed allow for full repayment of LT debt, validating our assumption. Thus, the country's expected ex ante welfare would in this case be given by

$$W_0^{fb} = \int_0^1 p\theta_H dG(p) - R^2 = p_0\theta_H - R^2 \tag{16}$$

since disinvestment would always be avoided and default would occur only for $\theta = \theta_L = 0$.[18]

Can the country achieve the first best without the ability to commit to no new debt issuance in period 1? Only if at $\gamma = 0$ the IC constraint is not violated. In that case, the country will not be tempted to issue new ST debt that period when $p \geq \widehat{p}$, fully diluting its LT debt. Then, conditional on there being only LT debt, the IC constraint is violated iff

$$R^2 > p_0\pi\theta_H \tag{17}$$

which we henceforth assume.[19] Thus, the first-best solution is not feasible since no creditor will be willing to make LT loans to the country if the IC constraint is violated.

Under these assumptions, we now turn to finding the country's optimal choice of γ. First, for any given γ we can find $\widehat{p}(\gamma)$, $\underline{p}(\gamma)$ and its associated r vector $(R_{01}(\gamma), R_{02}(\gamma))$. Note that $\widehat{p}(\gamma)$ solves equation (7) for $R_{01}(\gamma)$ whereas $\underline{p}(\gamma)$ solves equation (6) for $R_{01}(\gamma)$ and $R_{02}(\gamma)$. The latter, in turn, must ensure that creditors obtain the competitive rate of return, i.e.,

$$R^2 = R_{02}(\gamma) \int_{\underline{p}(\gamma)}^1 p\, dG(p) \tag{18}$$

[18]Note that the result that this is a first-best relies on the absence of effective punishment when $\theta = \theta_L$ which happens only given our assumption of $\theta_L = 0$. More generally, this would be a constrained first best.

[19]Note that if, contrary to what we assumed, $\pi > \alpha$, then not diluting LT debt would be IC.

reflecting the fact that LT debt is repaid only if $p \geq \underline{p}$, and

$$R = R_{01}(\gamma)\left[1 - G\left(\widehat{p}(\gamma)\right)\right] + \int_{0}^{\widehat{p}(\gamma)} \frac{f(p)}{\gamma} dG(p) \tag{19}$$

reflecting the fact that ST debt is repaid in full whenever $p \geq \widehat{p}$ and is repaid from complete disinvestment of the project otherwise. Thus, given a choice of γ, one can use the four equations ((7), (6), (18), and (19)) to solve for the four endogenous variables: $\widehat{p}(\gamma)$, $\underline{p}(\gamma)$, $R_{01}(\gamma)$, and $R_{02}(\gamma)$.

The country's ex ante welfare is thus given by:

$$W_0(\gamma) = \int_{\widehat{p}(\gamma)}^{1} p\theta_H dG(p) + \int_{0}^{\widehat{p}(\gamma)} Rf(p)\, dG(p) - \int_{0}^{\underline{p}(\gamma)} p\pi\theta_H dG(p) - R^2 \tag{20}$$

since it defaults whenever $p < \underline{p}$ and, in addition, it disinvests whenever $p < \widehat{p}$. It should be clear that the country's welfare is only a function of $\widehat{p}$ and $\underline{p}$; the gross rates of return r and the debt composition γ matter only because they affect these cutoff values of p. In particular, a country would want to have the lowest $\widehat{p}$ and $\underline{p}$ possible. A lower $\underline{p}$ means that the country defaults (and hence incurs the default penalty) for fewer realizations of p. A lower $\widehat{p}$ implies that default is accompanied by disinvestment of the project for a smaller range of p. Thus, finding the optimal γ requires understanding how $\widehat{p}$ and $\underline{p}$ vary with γ. We next turn to deriving these comparative statics.

First note that $\widehat{p}$ and R_{01} can be solved for independently of $\underline{p}$ and R_{02}. Using (7) and (19) we can rewrite $\widehat{p}$ as

$$\widehat{p}(\gamma) = \frac{\gamma R}{\alpha\theta_H} \cdot \frac{R - \int_{0}^{\widehat{p}(\gamma)} \left(\frac{f(p)}{\gamma}\right) dG(p)}{1 - G(\widehat{p})} \tag{21}$$

Graphing both the left-hand and right-hand sides of equation (21), it is easy to see that, for any given γ, in general there are multiple values of $\widehat{p}$ consistent with equilibrium as shown in figure 4 (since the sign of the derivative of the right-hand side of (21) with respect to $\widehat{p}$ is given by $\frac{g(\widehat{p})}{1-G(\widehat{p})}\left(\frac{p\alpha\theta_H}{R} - \frac{f(p)}{\gamma}\right)\frac{R}{\alpha\theta_H} > 0$ which is positive by (2)). As the country will choose the lowest value of $\widehat{p}(\gamma)$, and hence of $R_{01}(\gamma)$, there is no multiplicity regarding its equilibrium choice of R_{01} for a given γ.[20] We denote these equilibrium values, given γ, by $R^*_{01}(\gamma)$ and $\widehat{p}^*(\gamma)$.

Turning to the comparative static properties of $\widehat{p}^*$ and R^*_{01} with respect to γ, note that the partial derivative with respect to γ of the right-hand side of equation (21) is positive. Thus, an increase in the proportion of ST debt increases $\widehat{p}$ and hence R_{01} as shown in figure 4.[21] The intuition for this result is straightforward. An increase in γ means that there will be more instances in which ST debt cannot be rolled over by diluting the long-term debt (since $p\alpha\theta_H$ must be shared among a greater number of ST creditors)

[20] What if the curves do not intersect? This means that there are no loans that will be made in equilibrium as the interest rate that ST debt must receive to obtain the competitive rate of return is not affordable.

[21] Of course, these comparative statics have the opposite signs when the RHS intersects the 45 degree line from below – but we are only interested in the first intersection as that is the one which will be chosen by the country.

and thus there will be more instances of disinvestment, i.e., $\widehat{p}$ increases. To compensate for this R_{01} must increase, further increasing $\widehat{p}$.

Second we turn to the comparative static properties of $R_{02}(\gamma)$ and $\underline{p}(\gamma)$. Note that we can write $\underline{p}$ as:[22]

$$\underline{p}(\gamma) = \frac{\gamma R_{01}^{*}(\gamma) R}{\alpha\theta_H - (1-\gamma) R_{02}(\gamma)} \tag{22}$$

To understand the properties of this equation it is useful to graph $R_{02}\left(\underline{p}\right)$, using equation (18) to obtain R_{02} as a function of $\underline{p}$, and the locus $\underline{p}(R_{02};\gamma)$ using (22) and keeping R_{01} fixed at $R_{01}^{*}(\gamma)$. These curves are shown in figure 5. Both are upward sloping and may intersect more than once. The country will choose the intersection with the lowest value of $\underline{p}$ and hence of R_{02}. We denote these equilibrium values, given γ, by $R_{02}^{*}(\gamma)$ and $\underline{p}^{*}(\gamma)$.[23]

The comparative static properties of $R_{02}^{*}(\gamma)$ and $\underline{p}^{*}(\gamma)$ with respect to γ depend on whether $\alpha\theta_H$ is greater or smaller than R_{02}. For brevity we sometimes refer to the former case as a "favorable" economy and to the latter as an "unfavorable" economy. This dependence is easy to see with respect to the shift of the curves in figure 5. Note that the $R_{02}\left(\underline{p}\right)$ curve is not affected by changes in γ. The $\underline{p}(R_{02};\gamma)$ curve is affected by γ but the way it shifts depends on the size of $R_{02}(\gamma)$ relative to $\alpha\theta_H$. To see this, note that differentiating $\underline{p}(R_{02};\gamma)$ yields:

$$\left.\frac{d\underline{p}}{d\gamma}\right|_{\underline{p}(R_{02};\gamma)} = \frac{R_{01}^{*} R\left(\alpha\theta_H - R_{02}\right) + \gamma R\left(\alpha\theta_H - (1-\gamma) R_{02}\right)\frac{dR_{01}^{*}}{d\gamma}}{\left(\alpha\theta_H - (1-\gamma) R_{02}\right)^2} \tag{23}$$

Recalling that $\frac{dR_{01}^{*}}{d\gamma} > 0$, a sufficient condition for the expression to be positive is $R_{02} \leq \alpha\theta_H$ since $\frac{dR_{01}^{*}}{d\gamma}$ and $\alpha\theta_H - (1-\gamma) R_{02}$ are both positive. Thus, the $\underline{p}(R_{02};\gamma)$ curve will shift to the right for $R_{02} \leq \alpha\theta_H$ but, for R_{02} sufficiently large, it will shift left. As shown in figure 6, the ultimate effect on $R_{02}^{*}(\gamma)$ and $\underline{p}^{*}(\gamma)$ thus depends on whether the initial equilibrium corresponded to a favorable economy (as shown in the figure) or an unfavorable one. In the first case, which is depicted in panel A of the figure, $R_{02}^{*}(\gamma)$ and $\underline{p}^{*}(\gamma)$ will increase with γ; although in the second case the effect is ambiguous, panel B depicts the case in which $R_{02}^{*}(\gamma)$ and $\underline{p}^{*}(\gamma)$ decrease with γ.

The intuition for the result above is clear. When the economy is favorable, issuing a slightly higher proportion of ST debt raises the total cost of borrowing and leads to default when p is low. It thus means that there will be fewer states of nature in which the country can afford to pay its LT debt, leading R_{02} to increase and hence $\underline{p}$ as well. If, on the other hand, the economy is unfavorable, the effect of a small increase in the proportion of ST debt is ambiguous. A higher proportion of ST debt now increases the amount of debt that is relatively cheap, decreasing $\underline{p}$, but the increase in the ST rate of return R_{01}^{*} required by the

[22] Of course this equation is valid only if $\alpha\theta_H - (1-\gamma) R_{02}(\gamma) > 0$. If the latter didn't hold, the country would not be able issue borrow long term debt ($R_{02}(\gamma)$ would be infinite).

[23] Note that these curves are drawn without taking into account incentive compatability. This consideration, while it may render some values of γ unsustainable in equilibrium because they imply that country would never repay its long-term debt, does not invalidate the definition of $R_{02}^{*}(\gamma)$ as the smallest value of R_{02} that is consistent with γ. The reason can be clearly seen from equation (13): if $R_{02}^{*}(\gamma)$ is not incentive compatible, the same will be true for any other $R_{02}(\gamma) > R_{02}^{*}(\gamma)$.

higher γ works to increase $\underline{p}$. The final effect on $\underline{p}$ and R_{02} (which in equilibrium must move in the same direction) thus depends on the relative strength of these two effects.

We are now set to describe the equilibrium choice of γ. Let γ^* be the country's welfare maximizing choice, i.e., γ^* is the solution to

$$\underset{\gamma}{Max}\ W_0(\gamma) = \int_{\widehat{p}^*(\gamma)}^{1} p\theta_H dG(p) + \int_{0}^{\widehat{p}^*(\gamma)} Rf(p)\, dG(p) - \int_{0}^{\underline{p}^*(\gamma)} p\pi\theta_H dG(p) - R^2 \tag{24}$$
$$s.t. \text{ the IC constraint } (IC): \gamma \geq 1 - \frac{\pi\theta_H}{R_{02}^*(\gamma)}$$
$$\text{where } r^*(\gamma), \widehat{p}^*(\gamma), \underline{p}^*(\gamma) \text{ satisfy } (6), (7), (18), \text{ and } (19)$$

What will γ^* be? Using equation (13), let us define $\gamma_{IC}(R_{02}^*(\gamma_{IC}))$ as the *minimum* value of γ that solves

$$\gamma = 1 - \frac{\pi\theta_H}{R_{02}^*(\gamma)}$$

i.e., γ_{IC} is the minimum γ that is incentive compatible with repayment of long-term debt given that the interest rates are equilibrium ones. It follows from our previous discussion that, if $R_{02}^*(\gamma_{IC}) \leq \alpha\theta_H$ so that the economy is "favorable" when $\gamma = \gamma_{IC}$, then the country will set $\gamma^* = \gamma_{IC}$. To understand why, note that the country wishes to minimize instances of disinvestment and of default, i.e., it wishes to minimize both $\widehat{p}^*(\gamma)$ and $\underline{p}^*(\gamma)$. If $R_{02}^*(\gamma_{IC}) \leq \alpha\theta_H$, then both $\widehat{p}^*(\gamma^*)$ and $\underline{p}^*(\gamma^*)$ are increasing functions of γ, and hence the country's welfare is maximized by choosing the minimum level of γ that is incentive compatible with repayment of the LT debt.

If, however, $R_{02}^*(\gamma_{IC}) > \alpha\theta_H$ (i.e., the economy is "unfavorable"), then going up against the IC constraint may not be the welfare maximizing choice for the country since while $\frac{d\widehat{p}^*(\gamma)}{d\gamma} > 0$, now $\frac{d\underline{p}^*(\gamma)}{\partial\gamma}$ may be negative. If so, there is a welfare trade-off generated by choosing a higher γ. On the one hand, it increases the range of p associated with early default and inefficient payments because it reduces the feasibility of diluting LT debt. On the other hand, it decreases the range of p associated with late default by allowing the country to have a less costly mix of LT and ST debt. What will γ^* be in this case? Consider the set of candidates $\Gamma = \{\gamma_1, \gamma_2, ..., \gamma_n\}$ where each γ_i satisfies the first-order condition associated with the maximization problem (24). The solution with be either a $\gamma_i \in \Gamma$ or a corner solution of either γ_{IC} or 1.

Summing up, if the economy is favorable at the minimum level of ST debt compatible with incentive compatibility, the optimal maturity structure is clear. The country should issue as little ST debt as possible as the latter only increases the likelihood of both early and late default. If, on the other hand, LT debt is expensive, the country faces a trade-off. While issuing more ST debt than the minimum required by the incentive compatibility constraint may expose it to more instances of the more costly early default, it may also allow it to repay its debt over a greater range of realization of p, thereby decreasing the instances of late default. Lastly, in order for the solutions above to be the equilibrium choices, they must satisfy $W_0(\gamma^*) \geq 0$. If this participation constraint is not satisfied, the country will choose not to undertake the project.

4.4 Comparative Statics

Having fully characterized equilibrium, we can now ask how a country's optimal maturity choice is affected by changes in the environment. To simplify the exposition, we classify these changes into two groups. Changes in the *economic environment* are those related to the country's economic outlook: this category includes changes in the international interest rate R and in the distribution function over outcomes $G(p)$. Changes in the *enforcement technology* are those related to the ability of foreign creditors to extract repayment: this category includes changes in the cost of default π, the efficiency of making early payments F, and in the share of output that can be seized by creditors α.[24]

As a general rule, the optimal maturity structure depends on whether the economy is "favorable" or "unfavorable". In the former case, the maturity structure of debt is restricted by the IC constraint and thus it suffices to know how changes in the economy affect the latter. For an unfavorable economy, the choice of debt maturity may instead be the result of an optimal trade-off between the default costs associated with LT debt and the efficiency costs associated with ST debt. To the extent that both costs are affected by changes in the environment, the net effect on the choice of maturity structure is ambiguous.

Consider first an increase in the international interest rate R. From equations (18) and (19), this has a direct impact on ST and LT interest rates $R^*_{01}(\gamma)$ and $R^*_{02}(\gamma)$ and therefore increases both $\underline{p}(\gamma)$ and $\widehat{p}(\gamma)$. The increase in the probabilities of early and late default in turn feed back into $R^*_{01}(\gamma)$ and $R^*_{02}(\gamma)$, raising the spread that the country must pay on ST and LT debt. Finally, a higher $R^*_{02}(\gamma)$ tightens the IC constraint. In the favorable economy, this forces the country to shorten its maturity structure, i.e., to increase γ^*. It is worth noting that this is consistent with the evidence for emerging markets documented by Broner et al. (2008) and Arellano and Ramanarayanan (2012), who find that periods of high country spreads are accompanied by a shortening of the maturity structure of public debt.

For an unfavorable economy, the effect of an increase in R on γ is instead ambiguous. In this case, an interior equilibrium satisfies:

$$\frac{dW_0}{d\gamma} = -\pi\theta_H g(\underline{p}^*(\gamma))\frac{d\underline{p}^*(\gamma)}{d\gamma} - \left(\widehat{p}^*(\gamma)\,\theta_H - R\frac{f(p)}{\gamma}\right) g(\widehat{p}^*(\gamma))\frac{d\widehat{p}^*(\gamma)}{d\gamma} = 0. \tag{25}$$

Although it is straightforward to show that $\frac{d\widehat{p}^*(\gamma)}{d\gamma}$ increases with R, the change in $\frac{d\underline{p}^*(\gamma)}{d\gamma}$ is ambiguous.[25] Thus, without further parametric assumptions, γ^* may rises or fall in response to a higher R.

[24] This distinction between economic environment and enforcement technology is not stark, of course, and we employ these categories only to simplify the exposition. The costs of default and the efficiency of liquidation/early repayments, for instance, are clearly related to the underlying economic environment.

[25] Note that $\frac{d\underline{p}^*(\gamma)}{d\gamma}$ can be written as

$$\frac{d\underline{p}^*}{d\gamma} = \frac{\frac{\underline{p}^*}{\gamma}\left(\alpha\theta_H - R^*_{02}(\gamma)\right) + \left(\frac{\partial R^*_{01}(\gamma)}{\partial\gamma} + \frac{\partial R^*_{01}(\gamma)}{\partial\hat{p}^*}\frac{d\hat{p}^*}{d\gamma^*}\right)}{\left(\alpha\theta_H - (1-\gamma)R^*_{02}(\gamma)\right) - \underline{p}^*(1-\gamma)\frac{\partial R^*_{02}(\gamma)}{\partial\underline{p}^*}}. \tag{26}$$

The same analysis carries over to a change in the distribution function $G(p)$. In particular, one can capture the effects of deteriorating economic prospects for the country by assuming that the distribution function changes to $H(p)$, where $H(p) \geq G(p)$ for all $p \in [0,1]$. From equations (19) and (18), this shift in the distribution of p has a direct impact on ST and LT interest rates $R_{01}^{*}(\gamma)$ and $R_{02}^{*}(\gamma)$, raising the spreads paid by the country as well as the probabilities of early and late default by increasing $\underline{p}^{*}(\gamma)$ and $\widehat{p}^{*}(\gamma)$. For a favorable economy, the effect is to increase γ^{*}: once again, the rise in R_{02}^{*} tightens the incentive compatibility constraint and this forces the country to shorten its maturity structure. For the unfavorable economy, though, the probability has an ambiguous effect on γ as before. In particular, it depends on how the change affects $g(\underline{p}^{*}(\gamma))$, $g(\widehat{p}^{*}(\gamma))$, $\frac{d\underline{p}^{*}(\gamma)}{d\gamma}$, and $\frac{d\widehat{p}^{*}(\gamma)}{d\gamma}$ in equation (25) and is thus ambiguous without further parametric assumptions.

Turning next to changes in the enforcement environment of the country, consider an increase in the cost of default π. The direct effect is to relax the incentive compatibility constraint. For a favorable economy, this enables the country to lower γ, thereby reducing both $\widehat{p}^{*}(\gamma)$ and $\underline{p}^{*}(\gamma)$. Note that the welfare consequences of this change is ambiguous, though: although default becomes less likely in equilibrium, it is also more costly when it happens.[26] For an unfavorable economy, if the equilibrium is interior, an increase in π leads instead to a rise in γ. The reason is that, here, the probability of default $\underline{p}(\gamma)$ is decreasing in the share of ST debt: when default becomes more expensive relative to disinvestment, the country has an incentive to reduce its likelihood by shortening its maturity structure (see equation (25)). The welfare effects of the increase in π are clearly negative in this case as well, since it does not enlarge the country's choice set but it does increase the costs of default.[27]

Consider next the consequences of a rise in $F(\delta, p)$ – i.e., an increase in the efficiency of payments upon early default. The direct effect of this is to raise increase payments to ST creditors and thus reduce $R_{01}^{*}(\gamma)$. This in turn leads to a decrease in both $\underline{p}^{*}(\gamma)$ and $\widehat{p}^{*}(\gamma)$, which means that $R_{02}^{*}(\gamma)$ must also fall. For a favorable economy, the IC constraint is relaxed and γ^{*} falls in consequence. But for an unfavorable economy the effect is once again ambiguous. On the one hand, if early default is less costly it becomes more attractive to increase γ at the margin in order to reduce the overall probability of default associated with $\underline{p}^{*}(\gamma)$. On the other hand, these changes affect $g(\underline{p}^{*}(\gamma))$, $g(\widehat{p}^{*}(\gamma))$, $\frac{d\underline{p}^{*}(\gamma)}{d\gamma}$ and $\frac{d\widehat{p}^{*}(\gamma)}{d\gamma}$, rendering the net effect ambiguous though of course the welfare consequence is unambiguously positive.

Lastly, consider the effect of an increase in α. By raising the share of output that can be pledged to creditors, this change reduces both $\underline{p}^{*}(\gamma)$ and $\widehat{p}^{*}(\gamma)$ and, from equations (18) and (19), it also reduces $R_{01}^{*}(\gamma)$ and $R_{02}^{*}(\gamma)$. For a favorable economy, γ falls because the IC constraint is relaxed. For an unfavorable

[26] This is a standard feature of models with equilibrium default. See, e.g., Cole and Kehoe (2000), who show that a higher cost of default reduces the prevalence of self-fulfilling crises.

[27] Note that the analysis refers to local changes in π. An artifact of our model – a result of assuming $\theta_L = 0$ – is that welfare is maximized once π becomes sufficiently large. To see this, consider the limit in which π is arbitrarily close to α. From equation (3), the country in this limit is able to issue only LT debt with an interest rate $R_{02}(0) = \frac{R^2}{p_0}$. At this rate the IC constraint is satisfied, $\underline{p}(0) = \widehat{p}(0) = 0$ and, consequently, the first-best allocation is attained.

economy, the overall effect of this change depends on how the increase in α affects $\dfrac{d\widehat{p}^*(\gamma)}{d\gamma}$ relative to $\dfrac{d\underline{p}^*(\gamma)}{d\gamma}$ and hence is ambiguous though clearly welfare increasing.

To conclude, for a favorable economy,any deterioration in the economic environment or the enforcement technology leads to greater spreads, a higher frequency of early and late default and a shorter maturity structure of debt. For an unfavorable economy, the ultimate effect of any change in the environment depends on how it affects the marginal (disinvestment) cost of ST debt relative to the marginal (default) cost of LT debt. Excepting changes in π, which affect only the IC constraint, the other changes affect the behavior of both $\underline{p}^*(\gamma)$ and $\widehat{p}^*(\gamma)$ and their net effects on the maturity structure are thus ambiguous.

5 A Role for an International Financial Institution?

Our basic model concludes that a country's inability to commit to not dilute its LT debt forces it to issue some ST debt. Because debt contracts are non-contingent and renegotiation is not possible, issuing ST debt exposes the country to costly crises and inefficient repayments whenever there are signals that the economy may not perform well (i.e., whenever $p < \underline{p}(\gamma^*)$). We now ask whether there is a useful role for an international financial institution (IFI), such as the IMF, within this framework.

We focus on two types of intervention by the IFI: debt restructuring and debt reprofiling.[28] The first policy has long been in place and has been used in a variety of situations. The second is a new policy which the IMF is currently considering and which has been the subject of much recent attention.[29] Of course, the IMF does not have the power to impose or mandate debt restructurings or reprofilings in practice. But the proposed reform rests on the notion that the IMF does have leverage to "coordinate" creditors into accepting a deal, either by conditioning its resources or via its assistance to the country in distress. Thus, for simplicity we abstract from modeling the intricacies of the negotiation process by assuming that the IFI is able to overcome coordination failures.[30]

5.1 The Model with the IFI

Formally, we modify the model to include the IFI as follows. The IFI has a given tool-kit of policies. After observing p in period 1, the country decides whether to approach the IFI for help. The IFI then decides how (including whether) to intervene, given the set of policy tools at its disposal.

[28] Note that, since we ignore self-fulfilling crises, there is no room for the IFI to act as a lender of last resort. We omitted this purposely in our model as this basic role of the IFI is already well understood and including it would simply burden the exposition.

[29] See "Creditors likely losers from IMF Rethink" (Financial Times, 6/4/2013), "IMF sovereign debt plans in jeopardy" (Financial Times, 1/26/2014), , "Finance: In search of a better bailout" (Financial Times, 1/26/2014), Buchheit et al. "Guest post: The case for sovereign reprofiling the IMF way, part one" (Financial Times, 11/7/2014) and IMF (2014).

[30] This is, of course, a simplification. In reality, some countries restructure and/or reprofile their debts without any intervention by the IMF, whereas others restructure their debts in response to IMF demands. Our analysis of the welfare effects of restructurings and reprofilings, though, holds regardless of the way in which they originate.

The IFI's objective is assumed to be the maximization of the equally weighted sum of creditor and sovereign welfare. Hence, it is not concerned with redistribution per se, neither between the country and its creditors nor within the group of creditors. Because preferences are linear, this amounts to maximizing expected output net of default costs – a useful benchmark.[31] We model the IFI as having the same commitment technology as the country: it cannot commit to a policy rule in advance. Similarly, the IFI does not have a superior punishment or seizure technology than the creditor community: it cannot seize more resources nor change the default penalty. The key contribution of the IFI, then, is that it can coordinate creditors into accepting a debt restructuring or reprofiling that may not be perceived as desirable from each creditor's perspective even though it is welfare improving for creditors as a whole.

5.2 Debt Restructuring

A typical debt restructuring consists of a write-down of the value of promised future debt payments such that the probability of future default is lower. Achieving a restructuring agreement requires complicated negotiations among creditors, country, and the IFI. In this model we abstract from this procedure and instead restrict the IFI to restructurings that impose the same haircut on all creditors and collectively increase their payments in those states of nature.[32] Given that debt cannot be repaid when $\theta = \theta_L = 0$, restructuring writes down the value of the debt so that it is repayable when $\theta = \theta_H$. Debt restructuring is considered a credit event and thus we assume that it entails the same costs, $\theta\pi$, as a default.

Under what p realizations would the IFI restructure debt? Since the IFI effectively maximizes expected output net of default costs, it is strictly willing to restructure whenever $p < \hat{p}(\gamma; r)$. By doing so, it prevents disinvestment and increases the resources available to redistribute between the country and its creditors. For $p \in \left[\hat{p}(\gamma; r), \underline{p}(\gamma; r)\right)$, on the other hand, debt restructurings redistribute resources without affecting efficiency and the IFI would choose not to intervene should the country ask it to.[33] Will the country be willing to call the IFI for $p < \hat{p}(\gamma; r)$? Yes, since the country will only be made better off by preventing disinvestment. It thus follows that debt payments are restructured whenever $p < \hat{p}(\gamma; r)$.

Consider thus a realization $p \leq \hat{p}(\gamma; r)$ in period 1 that is followed by an IFI restructuring. The latter consists of exchanging all outstanding present and future debt payment promises $D_2(p)$ for new promises to make payments $D^s(p)$ at $t = 2$ (where s stands for restructuring) and where

$$Rf(p) \leq pD^s(p) \leq p\alpha\theta_H < pD_2(p) = \gamma R_{01} R + p(1-\gamma) R_{02} \tag{27}$$

Note that the preceding equation encapsulates all the assumptions: creditors are being offered more, in

[31] Other commitment and preference assumptions are possible, though we think these are the most natural. Section 7 discusses how outcomes will differ under alternative assumption.

[32] Assuming the same haircut across creditors is without loss of generality. On the other hand, as discussed in Section 6, if the IFI could impose agreements that made creditors collectively worse off, restructuring may no longer improve welfare.

[33] Of course, the IMF would not intervene when there was no crisis either, $p \geq \underline{p}$, as this imposes a default cost on the country and thus decreases total welfare.

expected value, than what they would obtain if the country defaulted early ($Rf\,(p)$), and full repayment is now feasible when $\theta = \theta_H$.

Given that all creditors face the same proportional haircut (and hence repayment is *pro rata*), the unit returns to debt under restructuring are given by

$$\frac{D^s(p)}{D_2\,(p)} \cdot \frac{R}{p} \cdot R_{01} \;\; \text{and} \;\; \frac{D^s(p)}{D_2\,(p)} \cdot R_{02} \tag{28}$$

where the first term (the return on ST debt) has been adjusted to reflect both the risk that $\theta = \theta_L$ and the fact that it is being repaid in period 2 (hence, R/p).

The presence of an IFI that, at the country's bequest, restructures whenever $p < \hat{p}\,(\gamma; r)$, affects all equilibrium outcomes and welfare. In particular, perfect competition among creditors requires that gross returns reflect the possibility of this intervention. Thus, equations (18) and (19) become:

$$R^2 = R^s_{02} \left[\int_0^{\widehat{p}^s(\gamma)} \frac{D^s(p)}{D_2(p)} \cdot pdG\,(p) + \int_{\underline{p}^s(\gamma)}^{1} pdG\,(p) \right] \tag{29}$$

$$R = R^s_{01} \left[1 - G\left(\widehat{p}^s\,(\gamma)\right) + \int_0^{\widehat{p}^s(\gamma)} \frac{D^s(p)}{D_2(p)} dG\,(p) \right] \tag{30}$$

and, together with equations (7) and (22), these expressions determine equilibrium interest rates and early versus late crises probabilities for a given γ.

Note that whenever there are multiple solutions to the system of equations for a given γ, the country would pick the contract structure that yields the lowest value of $\underline{p}(\gamma)$ (and this is the $\underline{p}(\gamma)$ denoted by $\underline{p}^s(\gamma)$). To understand why, note that the country's ex ante welfare in the presence of restructuring, $W^s_0\,(\gamma)$, is given by:

$$W^s_0\,(\gamma) = \int_0^1 p\theta_H dG(p) - \int_0^{\underline{p}^s(\gamma)} p\pi\theta_H dG(p) - R^2. \tag{31}$$

and hence, for any given (feasible) γ, its welfare is maximized by minimizing $\underline{p}^s(\gamma)$ and hence the probability of default. We denote these equilibrium values by $r^s(\gamma) = (R^s_{01}(\gamma), R^s_{02}(\gamma))$, $\underline{p}^s(\gamma)$ and $\hat{p}^s\,(\gamma)$.

We next turn to the central point of the section: an analysis of the ex ante welfare implications of a restructuring policy. We will show that a sufficient condition for an IFI with only restructuring in its toolkit to increase ex ante welfare is that it increase expected payments to creditors relative to non-intervention – as we have assumed that it does. At first blush this may seem to be trivially true. If the country can increase its payments to creditors when $p < \widehat{p}$, shouldn't that decrease contractual rates and thus the probability of default? This reasoning ignores the fact that the change in rates also affect $\widehat{p}$, and thus the frequency with which one type or another of debt is repaid. In particular, when payments to ST debt increase below $\widehat{p}$, by lowering R_{01} they also decrease $\widehat{p}$. If LT debt is not being repaid between $\widehat{p}$ and $\underline{p}$, then a lower $\widehat{p}$ increases R_{02}. The final effect on the probability of default is ambiguous. Thus a deeper analysis is required.

The country's ex ante welfare in this new environment, $W_0^s(\gamma^s)$, is given by evaluating (31) at the new optimal γ, denoted γ^s. Comparing the ex ante welfare in an environment with and without restructuring (using (20) and (31)), yields:

$$W_0^s(\gamma^s) - W_0(\gamma^*) = \int_0^{\widehat{p}^*(\gamma^*)} [p\theta_H - Rf(p)]\, dG(p) + \int_{\underline{p}^s(\gamma^s)}^{\underline{p}^*(\gamma^*)} p\pi\theta_H dG(p), \tag{32}$$

where recall that γ^* denotes the optimal choice of γ in an environment without the potential of restructuring (i.e., with no IFI). This equation has a natural interpretation. The first term, which is positive, captures the benefit of restructuring that comes from avoiding inefficient payments whenever $p < \widehat{p}(\gamma^*)$. The second term captures the effect of restructuring that comes from the change in the expected likelihood of default. If debt restructuring raises the equilibrium likelihood of default, then $\underline{p}^s(\gamma^s) > \underline{p}^*(\gamma^*)$ and this term is negative; if instead restructuring lowers the equilibrium likelihood of default, then $\underline{p}^s(\gamma^s) < \underline{p}^*(\gamma^*)$ and this term is positive.

Before proving our central lemma it is useful to define *continuous sharing rules*. Sharing rules define how payments of a given size are shared as a function of p and of the proportion of debt that is ST. Since this may depend on the size of total payments to creditors $X(p)$, it too must be included in our definition below. For notational ease we refer to a sharing rule $\beta_x(\gamma, p)$ as one that shares payment $X(p)$.

Definition 1 *$\beta_x(\gamma, p)$ is a continuous sharing rule iff, for $\gamma \in [0, 1]$ and $p \in [p_1, p_2]$*

1. $\beta_x(\gamma, p) \in [0, 1]$, with $\beta_x(0, p) = 1$; $\beta_x(1, p) = 0$
2. $\beta_x(\gamma, p)$ is a continuous function of γ.

The distribution of payments between ST and LT creditors in the economy that we are studying, with or without restructuring, isn't a continuous sharing rule because the IC constraint introduces discontinuities in how resources are shared. To see this more clearly, note that we can characterize restructuring as creditors sharing total payments of size $X(p) = pD^s(p)$ for $p < \widehat{p}$, where the share obtained by LT creditors $\beta^s(\gamma, p)$ is given by:

$$\beta^s_{pD^s(p)}(\gamma, p) = \frac{(1-\gamma)R^s_{02}(\gamma)}{\gamma R^s_{01}(\gamma) \cdot \dfrac{R}{p} + (1-\gamma)R^s_{02}(\gamma)} \tag{33}$$

which, as long as the IC constraint is satisfied, is a continuous function of γ. Once the IC constraint is breached, however, R_{02} must jump to reflect the fact that LT debt is no longer repaid when $p \geq \underline{p}$, and thus β^s is not a continuous function of γ. It will be useful, for proving lemmas 1 and 2 in the appendix, to refer to a continuous version of β^s by ignoring the IC constraint and then showing that it is not violated.

We are now ready to state and prove the main lemma of this section. In what follows, we refer to an economy with a given debt structure $(\gamma_0, r(\gamma_0))$. In this economy, when the economy defaults early i.e., $p < \widehat{p}$, payments among creditors are divided according to some rule. This rule need not be the one used in the model without the IFI intervention. We follow this more general route in order to prove a more powerful result that is independent of how resources were distributed prior to the IFI intervention. For $p \geq \widehat{p}$, the rules that govern repayment are unchanged: only ST debt is repaid for $p \in [\widehat{p}, \underline{p})$ and all debt is repaid for $p \geq \underline{p}$ if the IC constraint is satisfied.[34]

Lemma 1 *Consider an economy with a debt structure $(\gamma_0, r(\gamma_0))$ that satisfies the IC constraint. Let $X(p)$ be the total payments to creditors at each $p < \widehat{p}$ and let $B_x(\gamma, p)$ be the proportion of $X(p)$ obtained by LT creditors (with $1 - B_x(\gamma, p)$ going to ST creditors) at each (γ, p) combination for $p < \widehat{p}$. Note that we do not require B to be a continuous sharing rule. Consider a reform to a new distributional regime $\beta_y(\gamma, p)$ where β_y would be a continuous sharing rule if the IC constraint was ignored. Suppose also that this regime increases total expected payments to creditors to $Y(p)$ satisfying $\int_0^{\widehat{p}} Y(p)dG(p) > \int_0^{\widehat{p}} X(p)dG(p)$. Then there exists a γ' such that $\underline{p}(\gamma'; \beta_y) < \underline{p}(\gamma_0; B_x)$ and γ' respects the IC constraint.*

Proof. We will show that one of two possible scenarios holds: (i) there exists an incentive-compatible γ' such that $\widehat{p}(\gamma'; \beta_y) = \widehat{p}(\gamma_0; B_x) = \widehat{p}_0$ which we then show implies $\underline{p}(\gamma'; \beta_y) < \underline{p}(\gamma_0; B_x)$ or, (ii) if such a γ' does not exist, then setting $\gamma = 1$ guarantees a lower $\underline{p}$.

For this proof it is useful to construct a modified *continuous sharing rule*: $\beta_{y,\widehat{p}_0}(\gamma, p)$. This rule follows similar institutional rules as $\beta_y(\gamma, p)$: that is, for $p < \widehat{p}_0$ it shares $Y(p)$ according to β_y; it pays all creditors fully above $\underline{p}(\gamma; \beta_{y,\widehat{p}_0})$ as long as $\underline{p}(\gamma; \beta_{y,\widehat{p}_0}) \geq \widehat{p}_0$, and it pays ST creditors fully and LT creditors zero for $p \in (\widehat{p}_0, \underline{p}(\gamma; \beta_{y,\widehat{p}_0}))$. It differs from β_y in two respects: i. it ignores the IC constraint and, ii. $\widehat{p}_0$ is not an equilibrium object. It is imposed and the endogenous $\widehat{p}(\gamma; \beta_{y,\widehat{p}_0})$ generated by this sharing rule is ignored. If $\underline{p}(\gamma; \beta_{y,\widehat{p}_0}) < \widehat{p}_0$, the rules followed for $p < \widehat{p}_0$ are applied; full repayment occurs in that case once $p > \widehat{p}_0$.[35]

Note that $\beta_{y,\widehat{p}_0}(\gamma, p)$ is indeed a continuous sharing rule for $p \in [0, \widehat{p}_0]$: when there is no LT debt these creditors obtain zero (so a zero share) whereas when there is no ST debt, LT creditors obtain $Y(p)$ below $\widehat{p}_0$ (i.e., a share of one). As contract rates change continuously as a function of γ once the IC constraint is ignored, so do all other endogenous variables, ensuring that the function is continuous in γ. Note furthermore that if for some γ', $\widehat{p}(\gamma'; \beta_{y,\widehat{p}_0}) = \widehat{p}_0$, this is an equilibrium $\widehat{p}$ for the economy under the true $\beta_y(\gamma', p)$ i.e., $\widehat{p}(\gamma'; \beta_{y,\widehat{p}_0}) = \widehat{p}(\gamma'; \beta_y)$, $\underline{p}(\gamma; \beta_{y,\widehat{p}_0}) = \underline{p}(\gamma; \beta_y)$, and hence $r(\gamma'; \underline{p}(\gamma'; \beta_{y,\widehat{p}_0})) = r(\gamma', \underline{p}(\gamma'; \beta_y))$ and, as we shall show, the IC constraint is met. Thus, at such a γ' we have an equilibrium for the economy governed by β_y. We are now ready to start the proof.

If there is a γ' such that both allocations have the same $\widehat{p}_0$, it follows that $\gamma' R_{01}\left(\gamma'; \beta_{y,\widehat{p}_0}\right) = \gamma_0 R_{01}(\gamma_0; B_x)$. Thus, we have

$$\gamma' \cdot \frac{R - \int_0^{\widehat{p}_0} (1 - \beta_{y,\widehat{p}_0}(\gamma', p)) \frac{Y(p)}{\gamma'} dG(p)}{1 - G(\widehat{p}_0)} = \gamma_0 \cdot \frac{R - \int_0^{\widehat{p}_0} (1 - B_x(\gamma_0, p)) \frac{X(p)}{\gamma_0} dG(p)}{1 - G(\widehat{p}_0)} \tag{34}$$

[34] Note that these are the equilibrium strategies regardless of how payments are divided among creditors below $\widehat{p}$.

[35] Note that we do not require that distributing resources in this manner be feasible (i.e., $\widehat{p}_0$ may lie below the endogenous $\widehat{p}$). This is not a concern as will become evident below.

which, after a bit of algebra, yields:

$$\gamma' = h(\gamma') = \gamma_0 + \frac{1}{R}\int_0^{\widehat{p}_0} \left[(1-\beta_{y,\widehat{p}_0}(\gamma',p))Y(p) - (1-B_x(\gamma_0,p))X(p)\right] dG(p), \tag{35}$$

where $h(\gamma')$ is a continuous function of γ' mapping $[0,1]$ to $\left[\begin{array}{c} \gamma_0 - \frac{1}{R}\int_0^{\widehat{p}_0}(1-B_x(\gamma_0,p))X(p)dG(p), \\ \gamma_0 + \frac{1}{R}\int_0^{\widehat{p}_0}\left[Y(p)-(1-B_x(\gamma_0,p))X(p)\right]dG(p) \end{array}\right]$.
Note that $R_{01}(\gamma_0; B_x(\gamma_0,p)) > 0$ guarantees that the lower limit of the range is positive (see equation (34). There are then two possibilities.

If $h(1) > 1$, we are not assured the existence of γ' for which $\gamma' R_{01}\left(\gamma'; \beta_{y,\widehat{p}_0}\right) = \gamma_0 R_{01}\left(\gamma_0; B_x\right)$. If such a γ does not exist, then setting $\gamma' = 1$ yields $R_{01}\left(1; \beta_{y,\widehat{p}_0}\right) < \gamma_0 R_{01}\left(\gamma_0; B_x\right)$. Note that this implies $R_{01}\left(1; \beta_y\right) < \gamma_0 R_{01}\left(\gamma_0; B_x\right)$ since under $\beta_{y,\widehat{p}_0}$ ST creditors are being paid less than in full below $\widehat{p}_0$ whereas under β_y they would be paid in full for $\left(\widehat{p}\left(1;\beta_y^s\right), \widehat{p}_0\right)$ and $\widehat{p}\left(1;\beta_y^s\right) < \widehat{p}_0$ (by definition of $\widehat{p}$). It then follows that the economy can obtain a lower $\underline{p}$ since $\underline{p}\left(1;\beta_y\right) = \frac{R_{01}\left(1;\beta_y\right)R}{\alpha\theta_H} < \frac{R_{01}(\gamma_0;B_x)R}{\alpha\theta_H-(1-\gamma_0)R_{02}^0(\gamma_0;B_x)} = \underline{p}\left(\gamma_0; B_x\right)$. Lastly, note that the IC constraint is trivially satisfied at $\gamma = 1$.

If $h(1) < 1$, there exists γ' satisfying equation (35). We want to show that $\underline{p}\left(\gamma'; \beta_y\right) = \underline{p}\left(\gamma'; \beta_{y,\widehat{p}_0}\right) < \underline{p}\left(\gamma_0; B_x\right)$. To see this, note first that $\underline{p}\left(\gamma'; \beta_y\right)$ is the minimum value of $\underline{p}$ satisfying:

$$\gamma' R_{01}(\gamma'; \beta_y)R + (1-\gamma')\frac{R^2 - R\int_0^{\widehat{p}_0}\beta_y(\gamma',p)\dfrac{Y(p)}{1-\gamma'}dG(p)}{\int_{\underline{p}}^1 pdG(p)}\underline{p} = \underline{p}\alpha\theta_H, \tag{36}$$

where the LHS of the expression is increasing in $\underline{p}$ and the RHS is increasing and linear in $\underline{p}$. Similarly, $\underline{p}\left(\gamma_0; B_x\right)$ is the minimum value of $\underline{p}$ satisfying

$$\gamma_0 R_{01}(\gamma_0; B_x)R + (1-\gamma_0)\frac{R^2 - R\int_0^{\widehat{p}_0}B_x(\gamma_0,p)\dfrac{X(p)}{1-\gamma_0}dG(p)}{\int_{\underline{p}}^1 pdG(p)}\underline{p} = \underline{p}\alpha\theta_H. \tag{37}$$

Recalling $\gamma' R_{01}(\gamma'; \beta_y) = \gamma_0 R_{01}(\gamma_0; B_x)$ note that, if evaluated at the same $\underline{p}$, the LHS of equation (36) is smaller than the LHS of (37) iff

$$\gamma' - \gamma_0 > \frac{1}{R}\int_0^{\widehat{p}_0}\left[B_x(\gamma_0,p)X(p) - \beta_y(\gamma',p)Y(p)\right]dG(p)$$

From the definition of γ' we have

$$\begin{aligned} \gamma' - \gamma_0 &= \frac{1}{R}\int_0^{\widehat{p}_0}(Y(p) - X(p))\,dG(p) + \frac{1}{R}\int_0^{\widehat{p}_0}\left[B(\gamma_0,p)X(p) - \beta_y(\gamma',p)Y(p)\right]dG(p) \\ &> \frac{1}{R}\int_0^{\widehat{p}_0}\left[B(\gamma_0,p)X(p) - \beta_y(\gamma',p)Y(p)\right]dG(p) \end{aligned}$$

where the inequality follows from our assumption regarding the expected value of payments under the new regime. Thus, evaluated at the same $\underline{p}$, the LHS of (36) is lower than the equivalent expression in (37), implying that $\underline{p}\left(\gamma'; \beta_y\right) < \underline{p}\left(\gamma_0; B_x\right)$. Note that this implies that the IC constraint is met as well since $\left(1-\gamma'\right) R_{02}\left(\gamma'; \beta_y\right) < \left(1-\gamma_0\right) R_{02}\left(\gamma_0; B_x\right)$.

We can now use this lemma to establish the main result of this section. ■

Proposition 1 *An environment which allows debt restructurings (with the property that they increase total expected payments to creditors below $\hat{p}$) strictly increases ex-ante welfare relative to a no-restructuring environment.*

Proof. As discussed previously, a country will always ask the IFI to intervene when $p < \hat{p}$, and the IFI will be willing to restructure under those p realizations.

Let γ^s be the equilibrium (hence optimal) γ chosen by the country in an environment in which the IFI can restructure and, as before, let γ^* be the country's optimal γ in the environment without the restructuring option. From equation (31), a sufficient condition for restructuring to be welfare enhancing is $\underline{p}^s\left(\gamma^s\right) < \underline{p}^*\left(\gamma^*\right)$ (i.e., once restructuring eliminates the production inefficiency associated with early default, the country need only worry about minimizing its overall probability of default). Since restructuring increases payments for $p < \widehat{p}$ (as well as changing the rules governing how resources are shared at these p's), we can simply apply the result of Lemma 1 directly, i.e., there exists an incentive-compatible γ' such that $\underline{p}^s\left(\gamma'\right) < \underline{p}^*\left(\gamma^*\right)$. The country's optimal γ^s may differ from γ' but, by revealed preference, it must increase its ex ante welfare. ■

We have now shown that the country will be made better off in an environment with restructuring; creditors are indifferent ex ante as contract rates adjust such that they earn the same expected rates of return. Restructuring eliminates early default and reduces the overall probability of default, making the country better off as well.

We can also ask how restructuring affects the country's optimal maturity structure. Unlike the welfare effect of restructuring, this is ambiguous. Before explaining why, it is useful to note that under the restructuring regime the optimal level of γ is the one that minimizes $\underline{p}^s\left(\gamma\right)$ subject to the incentive compatibility constraint. As shown by Proposition 1, however, restructuring results in a loosening of the IC constraint. Thus, if optimal, the country could choose a lower level of γ than previously. Furthermore, if the economy was "unfavorable" without restructuring, this may now no longer be true as the country can find a γ' such that $\left(1-\gamma'\right) R^s_{02}\left(\gamma'\right) < \left(1-\gamma^*\right) R^*_{02}\left(\gamma^*\right)$ which may now allow $R^s_{02}\left(\gamma'\right) \leq \alpha\theta_H$.

The previous considerations might suggest that γ must necessarily fall in the economy with restructuring. This is not true, however. Formally, an interior choice of γ in the absence of restructuring must satisfy equation (25), whereas the equivalent expression under restructuring is given by

$$\frac{dW^s_0}{d\gamma} = -\pi\theta_H g(\underline{p}^S\left(\gamma\right)))\frac{d\underline{p}^s\left(\gamma\right)}{d\gamma} = 0. \tag{38}$$

Consider first an economy that is favorable. Here the ambiguity arises because $\underline{p}^s(\gamma)$ may no longer be an increasing function of γ. To see this note that, as before, increasing γ has a negative effect when ST debt is relatively expensive (i.e., when the economy is favorable) as it substitutes away from the cheaper LT debt. But, by increasing $\widehat{p}$ an increase in γ also allows R_{02} to fall since, unlike before, there are now fewer states in which LT debt is not paid. Finally, R_{01} increases alongside γ since ST debt is now paid in full for fewer realizations of p. The net effect on $\underline{p}$ is ambiguous and, therefore, so is the difference between γ^s and γ^*. For an unfavorable economy, the same ambiguity exists. In this economy, γ still raises $\widehat{p}$ with the aforementioned effects but, in addition, there is now a positive effect of substituting away from relatively more expensive long-term debt. The final effect of γ on $\underline{p}$ is ambiguous and we cannot therefore determine the change in the optimal maturity structure without further assumptions on $G(p)$.

5.3 Debt Reprofiling

The IMF is currently considering the possibility of extending its lending framework to allow for the "reprofiling" of debt payments (IMF 2014a). Essentially, this is a postponement of payments during crises or, equivalently, a suspension of payments accompanied by a lengthening of debt maturity. The objective of reprofiling is to allow the country the possibility of avoiding a full default if the economy eventually recovers. We now use the model to analyze the main effects of this policy proposal.

The tool kit available to the IFI is now assumed to contain both restructuring and reprofiling. The latter consists of the ability to postpone all payments to creditors at $t = 1$ once p is revealed and the IFI is called in. For long-term creditors, this intervention has no direct effect. For short-term creditors, though, this intervention essentially implies that they must roll over their debt.

More formally, let $R_{12}^f(p)$ denote the rate at which short-term debt is rolled over in the event of a reprofiling, where f stands for reprofiled. Whenever debt is reprofiled, short term creditors obtain an expected return in period 2 of

$$p \cdot R_{01} \cdot R_{12}^f(p)$$

per unit of debt. It is immediate that, in order to fulfill its stated objective of helping the country avoid a default in the event of a crisis, reprofiling must necessarily impose a "haircut" on short-term creditors.[36] That is, in order for reprofiling to reduce the probability of default, it must roll-over ST debtors at a rate that is lower than what would be required by the market $(\frac{R}{p})$, i.e.,

$$R_{12}^f(p) < \frac{R}{p} \tag{39}$$

for $p < \underline{p}(\gamma; r)$. Without loss of generality, we set $R_{12}^f(p)$ at the maximum value that prevents a default

[36] As with restructuring, in an environment with many values of θ the choice of $R_{12}^f(p)$ would more generally imply a choice regarding the range of θ which would result in default in period 2.

(and hence makes ST creditors least unhappy) for $\theta = \theta_H$, i.e.,

$$R^f_{12}(p) = R^f_{12} = \frac{\alpha \cdot \theta_H - (1-\gamma) \cdot R_{02}}{\gamma \cdot R_{01}}. \tag{40}$$

As we later show, the welfare results only require that expected payments to creditors in times of crises be higher under reprofiling than under restructuring, i.e., that reprofiling entails a lower "haircut" than restructuring.

Since reprofiling requires a haircut on creditors (albeit only on short-term creditors) and thus constitutes a credit event, reprofiling is bound to impose a default penalty on the country. This is, at least, the argument made by the IMF in its proposal, which argues that this penalty should be significantly lower than that associated with a full-fledged restructuring or default.[37] We incorporate this by assuming that debt reprofiling entails a loss of $\theta \cdot \pi^f$ of output at $t = 2$, with $\pi^f < \pi$.

We are now ready to analyze the effect of debt reprofiling on equilibrium interest rates, debt maturity, and welfare. Note that, conditional on being approached by the country, the IFI always chooses to intervene and coordinates creditors and country to reprofile the debt when $p < \underline{p}(\gamma; r)$, because reprofiling has lower costs than either a debt restructuring or an outright default.

Will the country choose to approach the IFI knowing that reprofiling will be the outcome when $p < \underline{p}(\gamma; r)$. Note that reprofiling eliminates the country's ability to dilute long-term debt. Hence the country will only do so when its ex post welfare is higher. Formally, reprofiling takes place in equilibrium for $p \leq \widetilde{p}^f(\gamma; r)$, where $\widetilde{p}^f(\gamma; r) \in \left(\widehat{p}(\gamma; r), \underline{p}(\gamma; r)\right)$. That $\widetilde{p}^f(\gamma; r) < \underline{p}(\gamma; r)$ is immediate because the country only approaches the IFI if there is a crisis. That $\widetilde{p}^f(\gamma, r) > \widehat{p}(\gamma; r)$ follows from comparing the country's welfare under reprofiling

$$p(1-\alpha)\theta_H - p\pi^f\theta_H$$

to its welfare in the absence of IFI intervention

$$p\theta_H - \gamma R^f_{01} R - p\pi\theta_H,$$

i.e., whenever

$$p < \widetilde{p}^f(\gamma; r) = \min\left\{ \frac{\gamma R^f_{01} R}{\left[\alpha - (\pi - \pi^f)\right]\theta_H}, \underline{p}(\gamma; r) \right\} = \left\{ \kappa \cdot \hat{p}\left(\gamma; r\right), \underline{p}(\gamma; r) \right\}, \tag{41}$$

for $\kappa = \frac{\alpha}{[\alpha - (\pi - \pi^f)]\theta_H}$. Note that $\widetilde{p}^f(\gamma, r)$ is minimized when $\pi = \pi^f$, in which case it equals $\widehat{p}(\gamma, r)$. In all other cases, it is either equal to $\kappa \cdot \hat{p}\left(\gamma; r\right)$ or to $\underline{p}(\gamma; r)$.[38]

[37] Clearly, if $\pi^f = \pi$, reprofiling boils down to a form of restructuring and the analysis of the previous section applies. But there are reasons for which, in practice, it is easier to reprofile the debt than to restructure it outright. Reprofiling requires an agreement on the rate at which a fraction of the debt is rolled over, whereas restructuring requires an agreement on the haircut imposed on the entire debt. Intuitively, the latter is bound to entail longer and costlier negotiations. IMF (2014) reviews the available evidence and finds that, in practice, reprofilings do seem to be less costly than debt restructurings.

[38] In the specific case that in which payments to creditors under reprofiling equal $\alpha \cdot \theta_H$, it can be shown that $\widetilde{p}^f(\gamma; r) = \kappa \cdot \hat{p}\left(\gamma; r\right) < \underline{p}(\gamma; r)$ in equilibrium. To see this, assume that $\widetilde{p}^f(\gamma; r) = \underline{p}(\gamma; r)$, so that debt is reprofiled for all $p < \underline{p}(\gamma; r)$:

In this context, equations (18) and (19) become

$$R^2 = R_{02}^f \left[\int_0^{\widetilde{p}^f(\gamma)} p dG(p) + \int_{\underline{p}^f(\gamma)}^1 p dG(p) \right] \tag{42}$$

$$R = R_{01}^f \left[1 - G\left(\widetilde{p}^f(\gamma)\right) + \int_0^{\widetilde{p}^f(\gamma)} R_{12}^f \cdot p dG(p) \right] \tag{43}$$

Together with equations (7), (22) and (41), these expressions determine equilibrium interest rates and crisis and reprofiling probabilities for a given level of γ. We denote these by $r^f(\gamma) = \left(R_{01}^f(\gamma), R_{02}^f(\gamma)\right)$, $\underline{p}^f(\gamma)$, $\hat{p}^f(\gamma)$ and $\widetilde{p}^f(\gamma)$. Whenever there are multiple solutions for a given γ, we define an equilibrium with reprofiling as the solution that attains the maximum level of ex ante welfare for that particular γ:

$$W_0^f(\gamma) = \int_0^1 p\theta_H dG(p) - \int_0^{\widetilde{p}^f(\gamma)} p\pi^f \theta_H dG(p) - \int_{\widetilde{p}^f(\gamma)}^{\underline{p}^f(\gamma)} p\pi\theta_H dG(p) - R^2. \tag{44}$$

This expression summarizes welfare under reprofiling. Like restructuring, reprofiling eliminates the need for inefficient payments (disinvestment). Thus, the country suffers the penalty associated with reprofiling when $p \leq \widetilde{p}^f(\gamma)$, and the full penalty from default when $p \in \left(\widetilde{p}^f(\gamma), \underline{p}^f(\gamma)\right)$.

We now show that reprofiling is welfare-enhancing relative to restructuring.

Proposition 2 *An environment that allows reprofiling unambiguously raises ex ante welfare.*

Proof. As discussed previously, a country will always ask the IMF to intervene when $p < \widetilde{p}$, and the IMF will be willing to reprofile under those p realizations. Note also that reprofilings have a lower cost than either restructurings or outright defaults. From equation (44) then, a sufficient condition for reprofiling to be welfare enhancing relative to restructuring is that it allow the country to reduce the probability of a crisis in equilibrium, i.e., there should exist a γ^f satisfying $\underline{p}^f(\gamma^f) < \underline{p}^s(\gamma^s)$. Since reprofiling increases the expected value of payments for $p < \widetilde{p}$, we can directly apply the result of Lemma 2 in the appendix, which is analogous to Lemma 1 for the case of reprofiling. In particular, the Lemma shows that, as long as total expected payments to creditors under reprofiling are higher than they are under restructuring in times of crisis, there exists an incentive-compatible γ' such that $\underline{p}^f(\gamma') < \underline{p}^s(\gamma^s)$. The country's optimal γ^f may differ from γ' but, by revealed preference, reprofiling must lead to an increase in welfare nonetheless. ■

We have now shown that the country will be made better off in an environment with reprofiling; not only does reprofiling entail lower costs than either restructuring or outright default, it also makes it possible for the country to reduce the overall probability of default.

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in this case, LT debt is always paid in full. But then the country will choose to be at the IC constraint, because it is in the "favorable" economy and setting γ as low as possible allows it to minimize $\underline{p}(\gamma; r)$. However, a quick comparison of equations (22) and (41) reveals that $\widetilde{p}^f(\gamma; r) < \underline{p}(\gamma; r)$ when $(1-\gamma)R_{02} = \pi\theta_H$, which contradicts our original assumption. It must therefore be the case that $\widetilde{p}^f(\gamma; r) = \kappa \cdot \hat{p}(\gamma; r) < \underline{p}(\gamma; r)$ in equilibrium.

The effect of reprofiling on the maturity structure of debt, however, is ambiguous. To see this formally, note from Equation (44) that the optimal maturity structure under reprofiling must trade-off two effects: first, it seeks to minimize the probability of a default as captured by $\underline{p}^f(\gamma)$; second, conditional on defaulting, it seeks to maximize the likelihood of reprofiling, as captured by $\widetilde{p}^f(\gamma)$, thereby reducing the costs of default. Formally, whereas any interior choice of γ in the absence of reprofiling must satisfy equation (38), the equivalent expression under reprofiling is given by

$$\frac{dW_0^f}{d\gamma} = -\pi\theta_H g(\underline{p}^f(\gamma))\frac{d\underline{p}^f(\gamma)}{d\gamma} + \left(\pi - \pi^f\right)\theta_H g(\widetilde{p}^f(\gamma))\frac{d\widetilde{p}^f(\gamma)}{d\gamma} = 0. \tag{45}$$

Equation (45) shows why the effect of reprofiling on the optimal maturity of debt is ambiguous. First, and for the same reasons discussed for the economy under restructuring, $\underline{p}^f(\gamma)$ need no longer be an increasing function of γ. Second, reprofiling introduces a new additional ex ante benefit of raising γ. To see this, note from equation (41) that, at least when it is lower than $\underline{p}^f(\gamma)$, $\widetilde{p}^f(\gamma)$ is increasing in γ. Intuitively, a shorter maturity structure reduces the gains from debt dilution at $t = 1$, which provides the country with additional incentives to approach the IMF and reprofile the debt. This is beneficial from an ex ante perspective because the cost of reprofiling is lower than the cost of an outright default. Therefore, even though the possibility of reprofiling relaxes the incentive compatibility constraint, it may shorten the maturity structure of debt. In particular, the country may choose a shorter maturity structure because, even if this raises the likelihood of a crisis at the margin, it also raises ex post incentives to deal with crises by approaching the IMF and reprofiling the debt.

Our model thus confirms the view that reprofiling may raise the incidence of debt crises. Interestingly, though, this can only be the outcome if the country desires it and modifies its maturity structure accordingly. Revealed preference then guarantees that the welfare effects of this higher likelihood of crises must be more than offset by the lower cost of such crises when they can be addressed through debt reprofiling.

6 Discussion and Concluding Remarks

This paper presented a model of endogenous debt maturity and used it to analyze the effects of two alternative policies for debt-crisis resolution: restructuring and reprofiling. The main result is that these policies are guaranteed to be welfare enhancing if they raise total expected payments to creditors in times of crisis. This is not at all obvious ex ante because both policies entail redistribution among creditors and because, even in the absence of redistribution, a change in payment size affects the frequency with which creditors are paid in full versus in part and therefore impacts rates differentially on short versus long-term interest rates. We show that these effects can be dealt with by appropriately adjusting the maturity structure of debt. Of course, these results were derived in a highly stylized setting, with various simplifying assumptions. Below we discuss their robustness to different modelling assumptions and to the introduction of some additional concerns pertinent to sovereign debt.

First, how reasonable is to assume that these policy interventions raise the expected value of total payments to creditors in times of crises? In the model, this is achieved by the country sharing with creditors part of the greater potential for repayment that is generated by avoiding early default and, in the case of reprofiling, in addition not decreasing payments in those cases in which the country would have diluted long-tem debt. We think that this is a natural assumption, in the sense that it is difficult to see why a coalition of creditors would acquiesce to an agreement that did not have this property. Furthermore, although the country would prefer to minimize these payments ex post, it is nonetheless still in favor of these interventions, not only ex ante but also ex post. Lastly, note that while either short or long-term borrowing rates may increase with these policies, the *overall* cost of borrowing *in equilibrium* will fall, allaying one of the most important concerns regarding the new policy of reprofiling. If, on the contrary, the expected value of repayments fell, it is no longer possible to guarantee that these policies are welfare enhancing. If restructurings are expected to be too hard on creditors or if reprofiled debt is expected to be rolled over at a rate that is too low, this may increase the overall cost of borrowing in equilibrium, reducing the country's ex ante welfare.

Our model glossed over potential problems across different types of creditors. In the absence of collective action clauses across debt classes, there may be holdouts and reaching agreements may be difficult. These are important questions which future research should address. Our model was also silent about another potential concern related to debt reprofiling: debt overhang. It is widely perceived that an advantage of restructuring relative to reprofiling is that the former deals with debt problems "once and for all", whereas reprofiling may exacerbate debt overhang by postponing resolution. This possibility could be introduced into our model by assuming that, although reprofiling has a lower cost of default than reprofiling, it entails a loss of output due to debt overhang. Even in this case, though, the ability to reprofile debt can only be welfare reducing if it is misused. For example, if the country feared that the IFI's preferences were skewed towards creditors in such a way as to bias its coordination efforts towards reprofiling even in situations in which it would be socially optimal to restructure the debt, these fears would then lead the country not to approach the IFI and thus result in lower welfare relative to a restructuring only policy.

Our model also fails to address the role of IMF lending in debt crises. Debt restructurings are typically accompanied by IMF programs that entail some degree of lending by the institution. What would change if this role was incorporated in our framework? Assuming that the IMF breaks even on its lending, the answer depends on the institution's ability to extract payments from the country relative to private creditors, i.e. on whether the IMF faces a different α than the private-sector debt market. If the IMF and private creditors have the same capacity to extract payments from the country, then any IMF lending during crises will crowd out private lending one-for-one, and nothing in our analysis would change. If the IMF has instead a higher ability to extract payments from the country, its lending will trivially be useful: by pledging output to the IMF that it cannot pledge to private creditors, the country will be able to access additional funds during debt crises, thereby reducing the likelihood of disinvestment and default.[39]

Finally, some of the specific assumptions may seem overly restrictive although, as we now argue, they are not critical to the main results. First, we assumed that ST creditors are senior to LT creditors in the event of early default. While this assumption captures the widely-shared perception that these creditors are better able to "exit" the country in the event of a crisis, it may be overly strong. It should be clear from Lemmas 1 and 2, however, that the welfare results are independent of this specification; any alternative assumption regarding the distribution of payments in the event of early default would yield the same results. Second, the model predicts that dilution of LT debt is either zero or 100 percent (i.e., in the latter case $R_{12} = \infty$). This prediction is a consequence of the fact that we have only two realizations of output and $\theta_L = 0$. Thus, the sole choice faced by the country is whether to dilute LT debt when $\theta = \theta_H$. If there were many possible output realizations (θ) the country would face a trade-off. Whenever it issues more ST debt in period 1, it benefits at the margin by diluting LT debt payments for those realizations of θ that result in default in period 2; at the same time, though, it suffers a cost because this also increases ex ante the likelihood of default in period 2. In equilibrium, these two effects are traded-off optimally.

At a more general level, the findings in this paper are important for various debates regarding the reform of the international financial architecture. They highlight the fact that it is critical to endogenize the structure of debt in order to analyze the effects of any such reforms. In this paper, we focussed on the maturity structure of debt as the reforms have different effects on short versus long-term creditors. To examine the effects of other reforms under discussion such as the strengthening of collective action clauses, it may be important to endogenize other dimensions of the debt structure, such as the type and jurisdiction of instruments being issued.[40]

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7 Appendix

Lemma 2 *Consider an economy with a debt structure* $(\gamma_0, r(\gamma_0))$ *that satisfies the IC constraint. Let* $X(p)$ *be the total payments to creditors at each* $p < \underline{p}_0$, *and let* $B_x(\gamma, p)$ *be the proportion of* $X(p)$ *obtained by LT creditors (with* $1 - B_x(\gamma, p)$ *going to ST creditors) at each* (γ, p) *combination for* $p < \underline{p}_0$. *Consider a reform to a reprofiling regime* $\beta_y(\gamma, p)$, *where* β_y *would be a continuous sharing rule if the IC constraint was ignored.* $Y(p)$ *denotes total payments to creditors at each* $p < \underline{p}$ *under the reprofiling regime, and it is assumed to satisfy* $\int_0^{\underline{p}_0} Y(p)dG(p) \geq \int_0^{\underline{p}_0} X(p)dG(p)$. *Then there exists a* γ' *such that* $\underline{p}(\gamma'; \beta_y) < \underline{p}(\gamma_0; B_x)$ *and* γ' *respects the IC constraint.*

Proof. For this proof it is useful to construct a modified *continuous sharing rule*: $\widetilde{\beta}_y(\gamma, p)$. This rule follows the same institutional rules as $\beta_y(\gamma, p)$ except that it ignores the IC constraint and pays LT creditors as though this constraint was satisfied.

Note that $\widetilde{\beta}_y(\gamma, p)$ is indeed a continuous sharing rule for $p \in [0, \widetilde{p}]$: when there is no LT debt these creditors obtain zero (so a zero share) whereas when there is no ST debt, LT creditors obtain $Y(p)$ below $\widetilde{p}$ (i.e., a share of one). As contract rates change continuously as a function of γ once the IC constraint is ignored, so do all other endogenous variables, ensuring that the function is continuous in γ.

Given the definition of $\widetilde{p}\left(\gamma; \widetilde{\beta}_y\right)$, we first ask whether there exists a γ' such that:

$$\gamma' \cdot \frac{R - \int_0^{\widetilde{p}(\gamma'; \widetilde{\beta}_y)} \left(1 - \widetilde{p}\left(\gamma'; \widetilde{\beta}_y\right)\right) \frac{Y(p)}{\gamma'} dG(p)}{1 - G(\widetilde{p}(\gamma'; \beta_{y,\widetilde{p}}))} = \gamma_0 \frac{R - \int_0^{\widetilde{p}(\gamma'; \widetilde{\beta}_y)} (1 - B_x(\gamma_0, p)) \frac{X(p)}{\gamma_0} dG(p)}{1 - G(\widetilde{p}(\gamma'; \beta_{y,\widetilde{p}}))}, \tag{46}$$

where the RHS of the expression simply expresses $\gamma \cdot R_{01}(\gamma_0, B_x)$ in terms of $\widetilde{p}\left(\gamma'; \widetilde{\beta}_y\right)$.

After a bit of algebra, equation (46) yields

$$\gamma' = h(\gamma') = \gamma_0 + \frac{1}{R} \int_0^{\widetilde{p}(\gamma'; \widetilde{\beta}_y)} \left[(1 - \widetilde{\beta}_y(\gamma', p))Y(p) - (1 - B_x(\gamma_0, p))X(p)\right] dG(p), \tag{47}$$

where $h(\gamma')$ is a continuous function of γ' mapping $[0,1]$ to $\left[\begin{array}{c}\gamma_0, \\ \gamma_0 + \frac{1}{R}\int\limits_0^{\widetilde{p}(\gamma';\widetilde{\beta}_y)}\left[Y(p) - (1 - B_x(\gamma_0,p))X(p)\right]dG(p)\end{array}\right]$.

There are then two possibilities.

If $h(1) > 1$, we are not assured the existence of γ' satisfying equation (46). Then, we can set $\gamma' = 1$ and obtain $R_{01}\left(1;\widetilde{\beta}_y\right) < \gamma_0 R_{01}(\gamma_0; B_x)$. This implies that $\underline{p}\left(1;\widetilde{\beta}_y\right) = \widehat{p}\left(1;\widetilde{\beta}_y\right) < \widehat{p}_0 < \underline{p}_0$. Lastly, because the IC constraint is trivially satisfied at $\gamma' = 1$, we have that $\underline{p}(1;\beta_y) = \underline{p}\left(1;\widetilde{\beta}_y\right) < \underline{p}_0$, which proves the lemma for this first case.

If $h(1) < 1$, there exists γ' satisfying equation (46). We want to show that $\underline{p}(\gamma';\beta_y) = \underline{p}\left(\gamma';\widetilde{\beta}_y\right) < \underline{p}(\gamma_0; B_x)$. To see this, note first that $\underline{p}\left(\gamma';\widetilde{\beta}_y\right)$ is the minimum value of $\underline{p}$ satisfying:

$$\gamma' R_{01}(\gamma';\widetilde{\beta}_y)R + (1-\gamma')\frac{R^2 - R\int\limits_0^{\widetilde{p}(\gamma';\widetilde{\beta}_y)}\widetilde{\beta}_y(\gamma',p)\frac{Y(p)}{1-\gamma'}dG(p)}{\int\limits_{\underline{p}}^1 p\,dG(p)}\underline{p} = \underline{p}\alpha\theta_H, \tag{48}$$

where the LHS of the expression is increasing in $\underline{p}$ and the RHS is increasing and linear in $\underline{p}$. Similarly, $\underline{p}(\gamma_0; B_x)$ is the minimum value of $\underline{p}$ satisfying

$$\gamma_0 R_{01}(\gamma_0;B_x)R + (1-\gamma_0)\frac{R^2 - R\int\limits_0^{\widetilde{p}(\gamma';\widetilde{\beta}_y)}B_x(\gamma_0,p)\frac{X(p)}{1-\gamma_0}dG(p)}{\int\limits_{\underline{p}}^1 p\,dG(p)}\underline{p} = \underline{p}\alpha\theta_H. \tag{49}$$

Recalling $\gamma' R_{01}(\gamma';\widetilde{\beta}_y) = \gamma_0 R_{01}(\gamma_0;B_x)$ note that, if evaluated at the same $\underline{p}$, the LHS of equation (48) is smaller than the LHS of (49) iff

$$\gamma' - \gamma_0 > \frac{1}{R}\int\limits_0^{\widetilde{p}(\gamma';\widetilde{\beta}_y)}\left[B_x(\gamma_0,p)X(p) - \widetilde{\beta}_y(\gamma',p)Y(p)\right]dG(p)$$

From the definition of γ' we have

$$\begin{aligned}\gamma' - \gamma_0 &= \frac{1}{R}\int\limits_0^{\widetilde{p}(\gamma';\widetilde{\beta}_y)}\left(Y(p) - X(p)\right)dG(p) + \frac{1}{R}\int\limits_0^{\widetilde{p}(\gamma';\widetilde{\beta}_y)}\left[B(\gamma_0,p)X(p) - \widetilde{\beta}_y(\gamma',p)Y(p)\right]dG(p) \\ &> \frac{1}{R}\int\limits_0^{\widetilde{p}(\gamma';\widetilde{\beta}_y)}\left[B_x(\gamma_0,p)X(p) - \widetilde{\beta}_y(\gamma',p)Y(p)\right]dG(p)\end{aligned}$$

where the inequality follows from our assumption regarding the expected value of $Y(p)$ relative to $X(p)$. Thus, evaluated at the same $\underline{p}$, the LHS of (48) is lower than the equivalent expression in (49), implying that $\underline{p}\left(\gamma';\widetilde{\beta}_y\right) < \underline{p}(\gamma_0; B_x)$. Note that this implies that the IC constraint is met as well since $(1-\gamma')R_{02}(\gamma';\beta_{y,\widetilde{p}}) < (1-\gamma_0)R_{02}(\gamma_0;B_x)$, which in turn means that $\underline{p}(\gamma';\beta_y) = \underline{p}\left(\gamma';\widetilde{\beta}_y\right) < \underline{p}(\gamma_0;B_x)$.

■

t=0	t=1	t=2
	p becomes known	θ realized
country chooses γ, issues short- and long-term debt specifying R_{0t}, t=1,2	country issues new debt, specifying R_{12}	
investment	disinvestment	
	debt payments	debt payments
		consumption

Figure 1

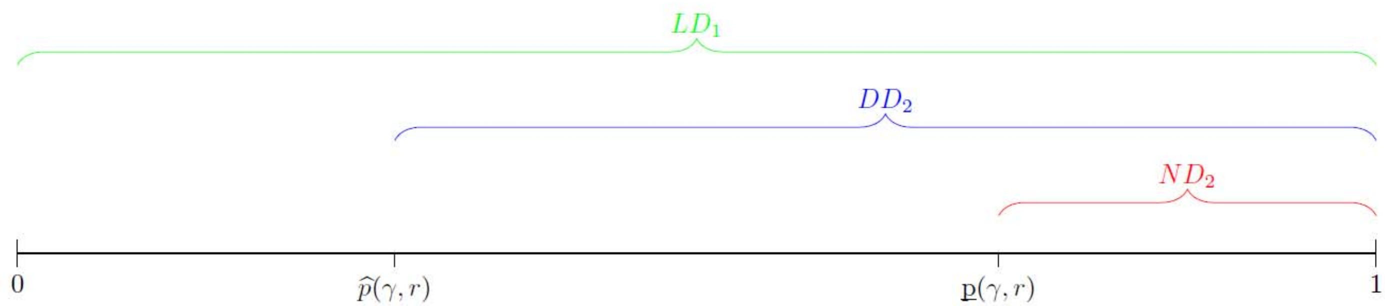

Figure 2: feasible strategies

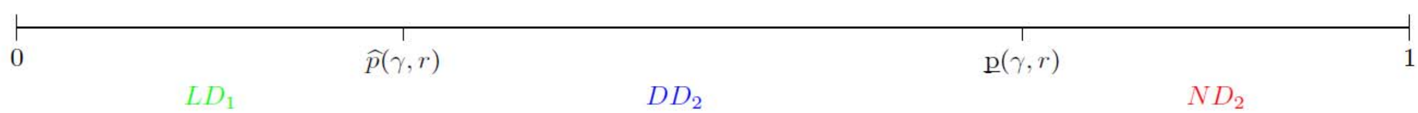

Figure 3: optimal strategies

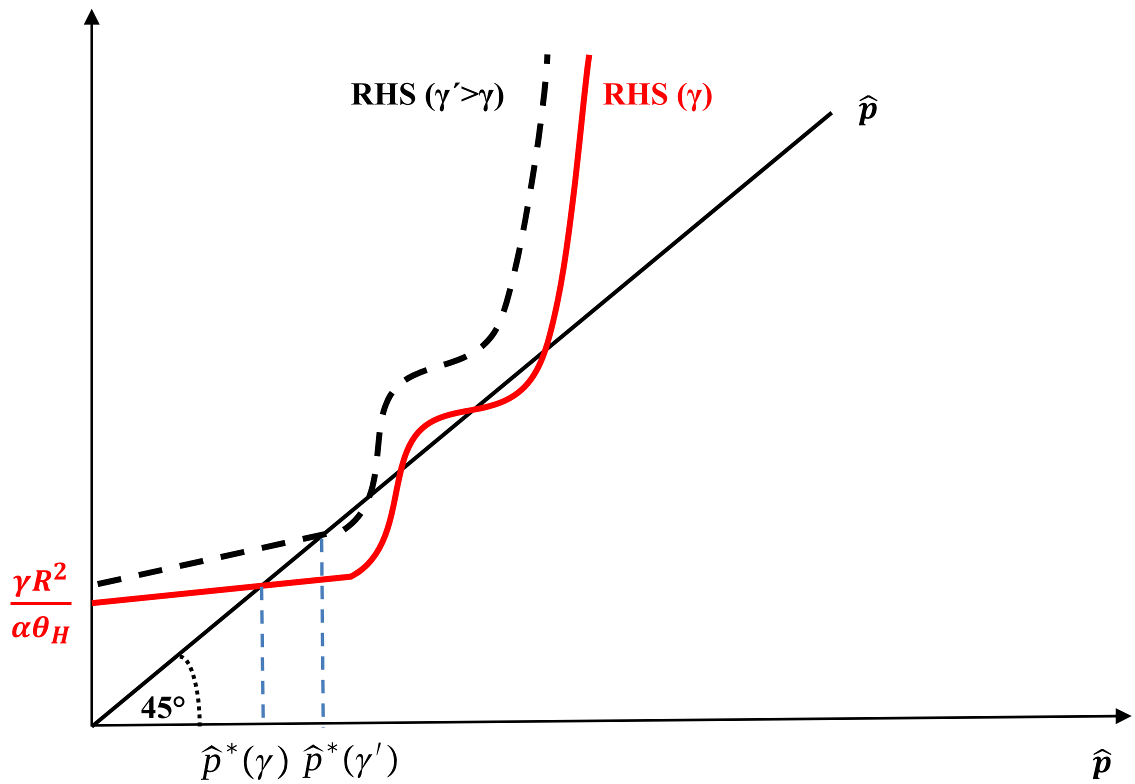

Figure 4

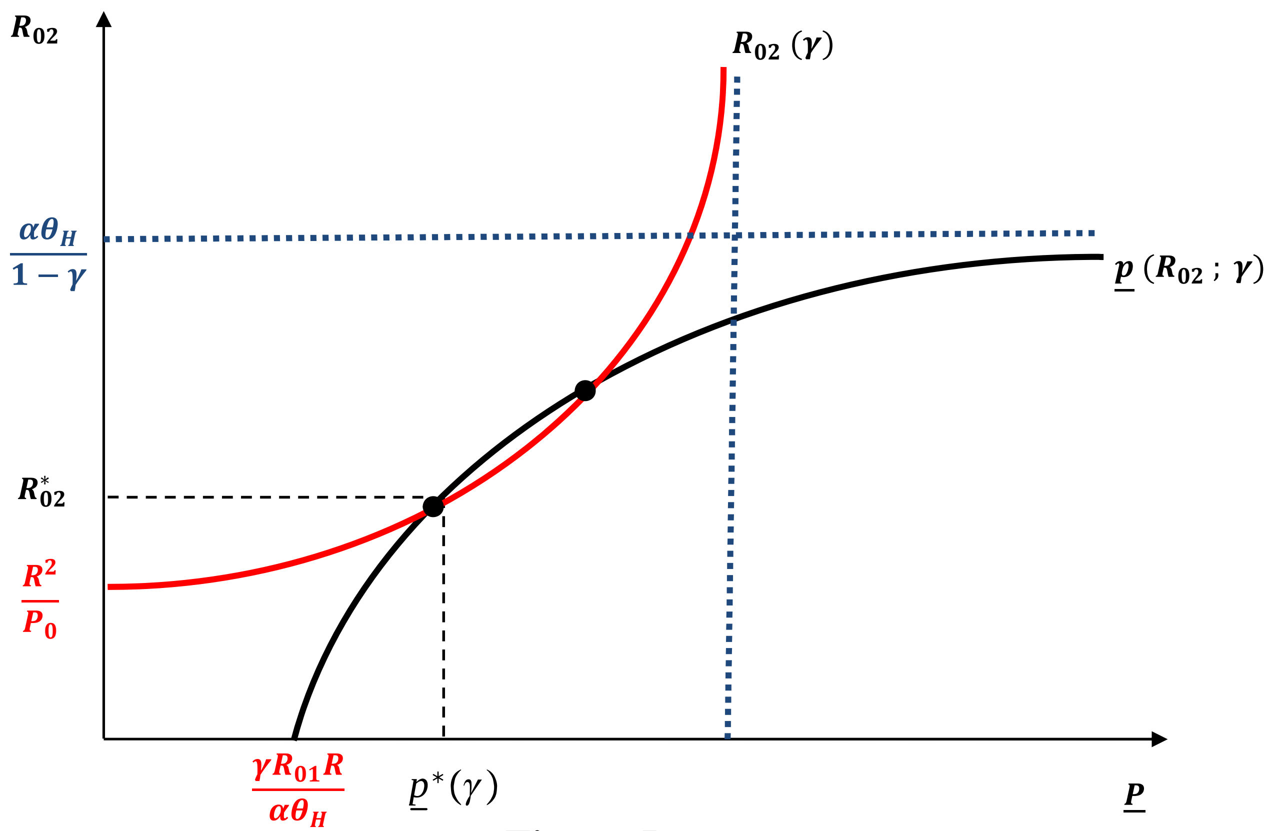

Figure 5

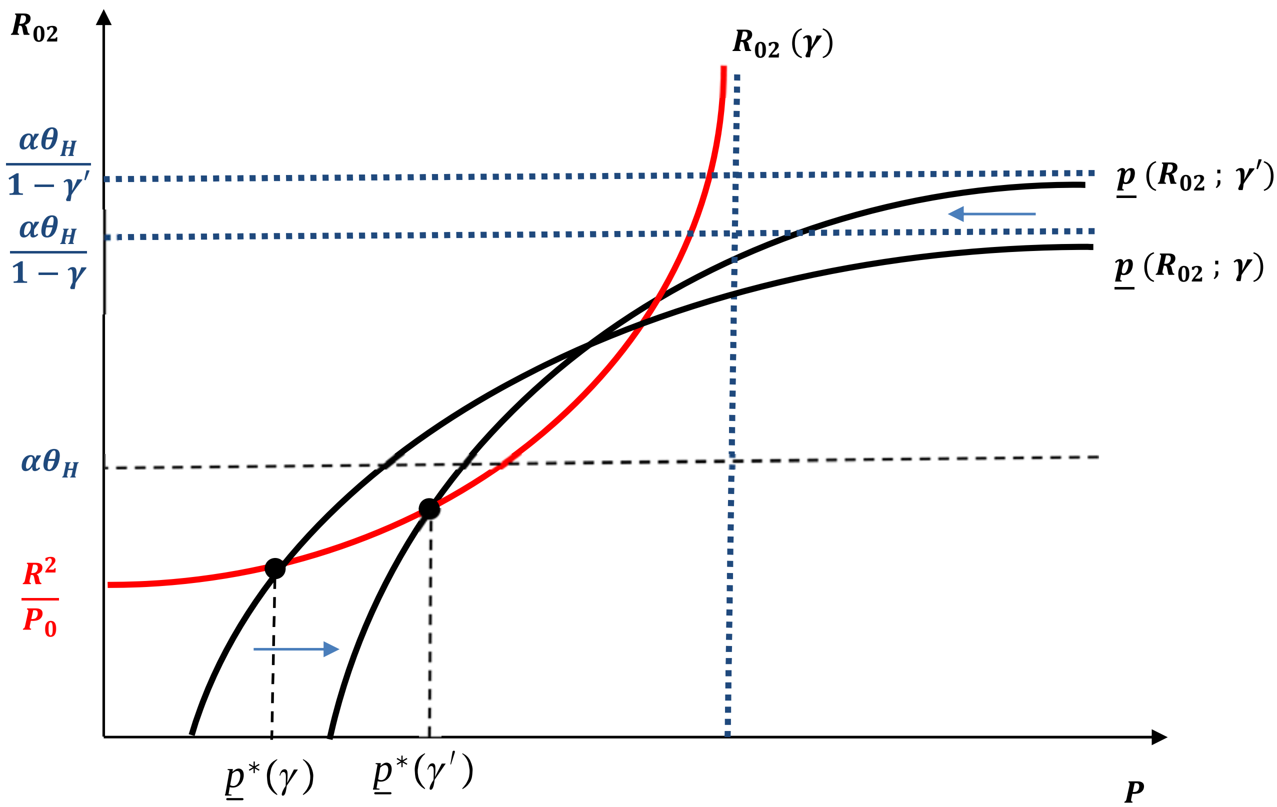

Figure 6A

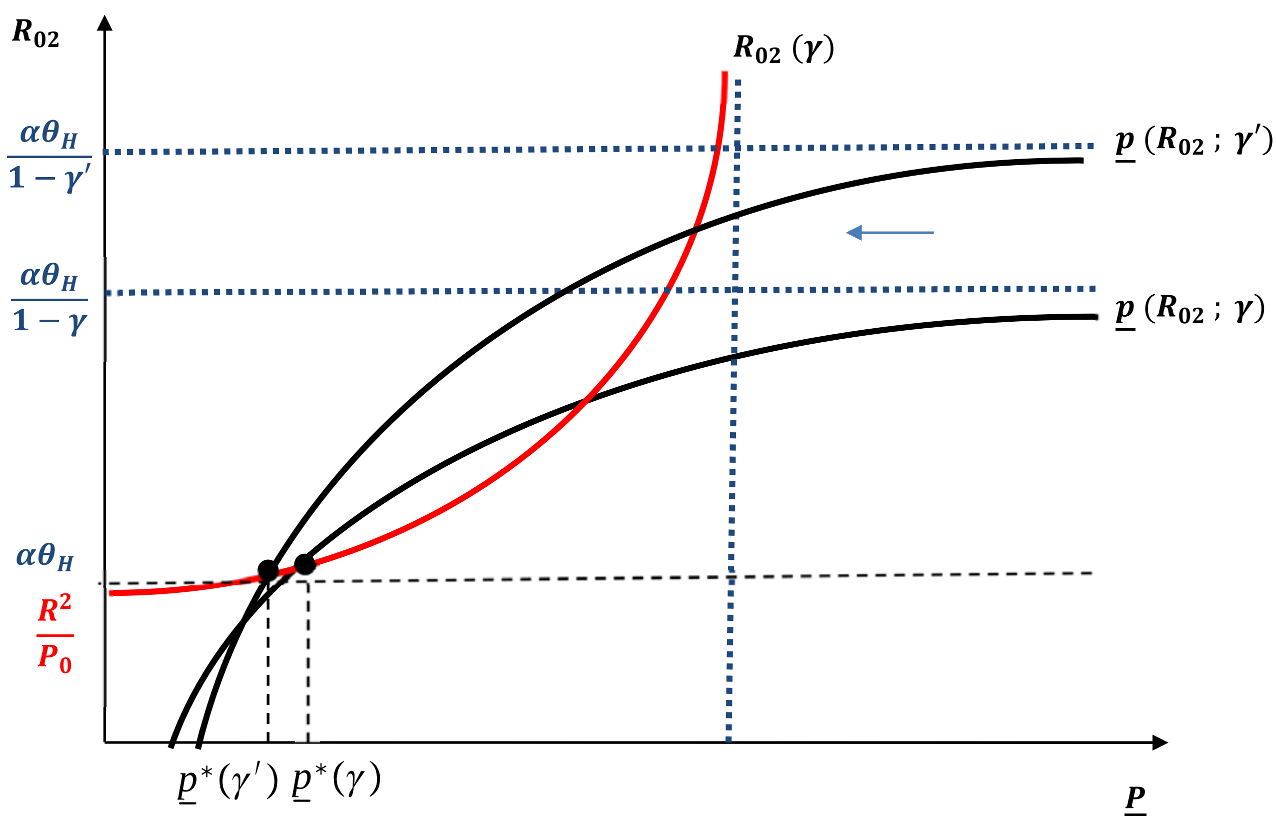

Figure 6B