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INSIDE AND OUTSIDE COLLATERAL:
IMPLICATIONS FOR FINANCIAL MARKET INSTABILITY

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ABSTRACT

This paper studies the fragility of interbank markets. Due to moral hazard and asymmetric information problems, collateral along with borrowing constraints are needed to provide appropriate incentives for banks. A key element of the analysis is the distinction between collateral based upon assets created outside of the banking system (e.g. government debt) and assets created through the lending process (e.g. cash flows). While active interbank markets help reallocate deposits across heterogeneous banks, because of incentive problems these flows are constrained and the markets are fragile, i.e. there are multiple equilibria. This paper explores the mechanism for the multiplicity, emphasizing its dependence upon inside and outside collateral. It also relates a 'crisis' in the model to recent financial events in the US economy.

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Inside and Outside Collateral: Implications for Financial Market Instability

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November 17, 2014

Abstract

This paper studies the fragility of interbank markets. Due to moral hazard and asymmetric information problems, collateral along with borrowing constraints are needed to provide appropriate incentives for banks. A key element of the analysis is the distinction between collateral based upon assets created outside of the banking system (e.g. government debt) and assets created through the lending process (e.g. cash flows). While active interbank markets help reallocate deposits across heterogeneous banks, because of incentive problems these flows are constrained and the markets are fragile, i.e. there are multiple equilibria. This paper explores the mechanism for the multiplicity, emphasizing its dependence upon inside and outside collateral. It also relates a ‘crisis’ in the model to recent financial events in the US economy.

1 Introduction

This paper studies the fragility of interbank markets. In those markets, banks with relatively profitable opportunities borrow funds from other banks. These exchanges include all market-financed transfers of assets between banks and may take a variety of forms, ranging from the overnight funds market to bank equity.¹ Despite the gains from trade, these markets, we argue, are fragile: i.e. they are susceptible to changes in beliefs of traders. One of the contributions of this paper is to understand the source of this fragility.

This is relevant insofar as models of fragility provide a framework for understanding the power of expectations in driving outcomes in financial markets. Importantly, the recent instability of

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¹For example, Acharya and Schnabl (2010) study the collapse of the asset backed commercial paper market during the crisis.

financial markets cannot be traced to the bank runs of Diamond and Dybvig (1983) but rather are related to flows between financial entities.²

In our setting, there are two frictions in the interbank market: (i) the profitability of borrowing banks is hidden information and (ii) banks are able to abscond with borrowed funds. This interaction of adverse selection and moral hazard limits the ability of the interbank market to reallocate funds and provides a basis for multiple equilibria.³ In particular, there may be an active interbank market in which relatively high productivity banks borrow from less productive banks. Lenders believe that borrowers are high quality and quantity constraints on the size of loans are relatively lax. Funds are thereby channeled to productive projects, yielding high output and, across production sites, a relatively low standard deviation of productivity.

Yet, for the same parameter values there may exist a “crisis” equilibrium in which lenders are pessimistic about the quality of borrowers. As a consequence, quantity constraints are tight. The flow of funds is reduced. Relatively low productivity projects are funded since the sorting of banks is less efficient. Output is low and the standard deviation of profitability across active production sites is much larger.

The paper provides a crisp characterization of the conditions for this type of multiplicity. Key elements in the analysis are the volume and the value of assets in the economy, which banks can use as collateral in interbank transactions. Those assets can either be outside assets, such as the pledging of public debt in a repo arrangement, or inside assets, such as the pledging of cash flows in the asset backed commercial paper market.⁴ As we shall see, the use of inside assets creates a type of “collateral cascade” in that cash flows from one loan can be pledged in another.

The analysis highlights the importance of these two sources of collateral in the fragility of the interbank market. In particular, consistent with evidence, the interbank loans backed by inside assets are much less resilient than those backed by outside assets.

The predictions of the model relate to both financial flows and real activity. Section 2 discusses the response of markets during the crisis. A key theme is the excess volatility of the Asset Backed Commercial Paper (ABCP) relative to repo markets, particularly those secured by treasuries. The model creates comparable variations in the use of inside and outside collateral. The implications of the model for output and capital reallocation also relate to the evidence presented in Foster, Grim, and Haltiwanger (2013) on factor reallocation. They argue that, in contrast to previous periods of low economic activity, the Great Recession did not correspond to a period of increased reallocation of factors of production.

²See, for example, Krishnamurthy, Nagel, and Orlov (2014), Gorton and Metrick (2012), Covitz, Liang, and Suarez (2009), Copeland, Martin, and Walker (2011), and Fleming, Hrungr, and Keane (2010) for arguments along these lines.

³The paper builds on the multiplicity discussed in Boissay (2011).

⁴While the terminology is reminiscent of the distinction between private and public liquidity in Holmstrom and Tirole (1998), our focus is on different forms of collateral as pledgeable assets rather than liquidity *per se*. In our model, as in theirs, a complementarity emerges between inside and outside collateral.

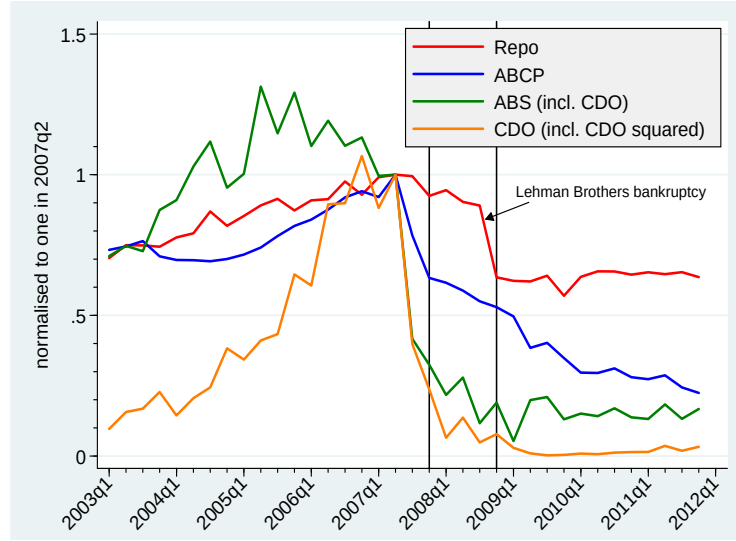
2 Facts

This section presents some pertinent facts related to the flow of funds and rates in the interbank market. These observations are used initially as motivation. Later, the discussion returns to them to evaluate our model.

Fact # 1:

During the 2007-8 crisis, wholesale funding markets secured by privately issued collateral collapsed, whereas those secured by treasuries or government guaranteed assets remained stable.

Figure 1: Volume of Secured Loans, as a ratio to Households' Deposits



Note: Volumes of repos, ABCP, and ABS, divided by households deposits; the ratio is normalised to 1 in 2007q2. Repo and Fed Funds volumes: outstanding amounts borrowed by the financial sector (excluding foreign banks offices); *Source:* US Flows of Funds (Table L.207, series FL892150005.Q). ABCP: outstanding amounts; *Source:* Federal Reserve Bank of St Louis (FRED) database (series DTBSPCKAM). ABS and CDO: quarterly issuances; *Source:* Asset Backed Alert and Dealogic. Household Deposits: total currency and deposits (including money market fund shares) by households and nonprofit organizations; *Source:* US Flows of Funds (Table L.100, series FL154000025.Q). Vertical lines: NBER business cycle troughs and peaks.

Fact #1 has been documented in a number of papers. For example, Copeland, Martin, and Walker (2011) show that the segments of the US repo market, such as tri-party repos and the bilateral treasury repos, which represent more than two thirds of the US repo market, remained strikingly stable during the crisis. By contrast, Gorton and Metrick (2012) show that the bilateral private asset repo markets, such as corporate debt or ABS repos, did exhibit instability. Those markets were small with respect to the tri-party repo market, though. Overall, the reduction in repo transactions was rather limited. Krishnamurthy, Nagel, and Orlov (2014) show that it was dwarfed by the reduction in Asset Backed Commercial Paper (ABCP) transactions. Covitz, Liang, and Suarez (2009) provide evidence that the collapse of the ABCP market resembled a run, as it was pervasive and affected all ABCP market segments, irrespective of the fundamental

quality of the underlying (private) assets. Altogether, this literature seems to indicate that one key determinant of the stability of secured wholesale funding had to do with the type of the assets used as collateral, and more specifically with whether those assets were publicly or privately issued.

Figure 1 illustrates Fact #1. It reports the volumes of loans on several wholesale funding markets (repos, ABCP, and Asset Backed Securities —ABS) as a ratio to households’ total deposits, from 2003q1 until 2011q4.⁵ To ease the comparison, the ratio is normalized to one in 2007q2, one quarter before the start of the crisis. In the run-up to the crisis, the three markets essentially evolved together, and their size increased. When in Summer 2007 the crisis hit, the volume of transactions fell by 60% in the ABS market, by 40% in the ABCP market, and by a bit less than 10% in the repo market, which confirms the resilience of the latter documented in earlier work. In effect, the repo market remained stable until the fourth quarter of 2008, when Lehman Brothers went bankrupt.⁶ Interestingly, Figure 1 also points to some heterogeneity within the private collateral market segment. For example, the ABCP market was more resilient than the ABS market and, within the ABS market, the CDO segment completely dried up. This latter observation suggests that whether or not the collateralized assets were publicly or privately issued is probably not enough to explain the various degrees of resilience of secured loan markets.

Fact # 2:

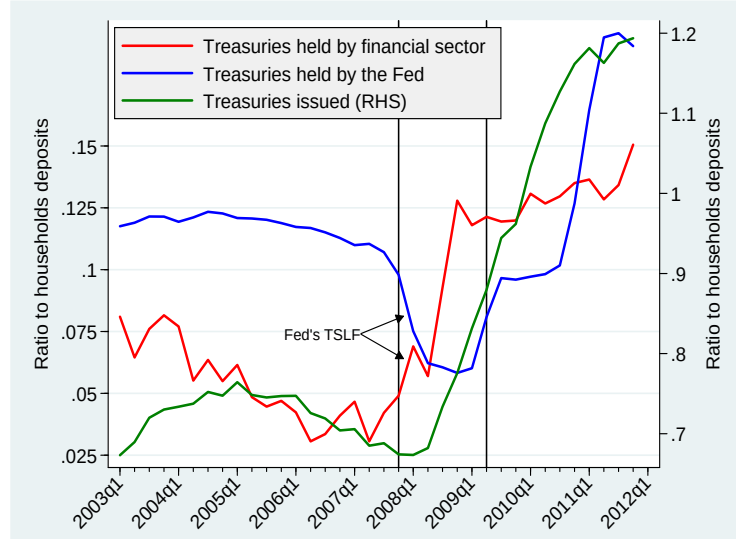
The US financial sector gradually reduced its holding of US treasury securities in the run-up to the crisis, and significantly increased it during the crisis.

Figure 2 reports the evolution of the outstanding amount of US treasuries issued by the US federal government (on the right scale) and held by the US financial sector and the Federal Reserve System (on the left scale), as a fraction of households’ deposits. The figure shows that the crisis arose at a time when both the supply of US treasury securities and the US financial intermediaries’ holding of those assets were low by historical standards: in 2007q2, notably, only 2.5% of the funds deposited were invested into treasuries. As the repo market was becoming

⁵ABSs are securities whose income payments is derived from and “backed” by a specified pool of assets. The pool of assets is typically a group of small and illiquid assets such as mortgage, credit card loans etc, which cannot be sold individually. Collateralized Debt Obligations (CDO) are structured financial products that pools together cash flow-generating assets and repackage those assets into discrete tranches that can be sold to investors. CDOs can be viewed as complex –because more structured– types of ABSs. The maturity of those claims is usually longer (above eighteen months) than that of repos and ABCPs, which does not exceed a quarter (overnight for the repos and less than six weeks for ABCPs). In the latter cases, there is therefore little difference between the outstanding volume of loans and the volume of loan issuances. So, to compare the evolution of those markets during the crisis, we report the quarterly issuances of ABS together with the outstanding volumes (end-of-quarter) of repos and ABCPs.

⁶In its Primary Dealer statistics, the Federal Reserve Bank of New York reports a \$1.5 trillion fall in outstanding repos in that quarter. This suggests that most of the collapse in the repo market in 2008q4 was imputable to primary dealers, and especially to Lehman’s bankruptcy. Copeland et al. (2012) point that, in anticipation of Lehman Brothers upcoming demise, hedge funds and other Lehman Brothers clients moved their business to other broker-dealers, thus depriving Lehman from repo-able assets. The Flows of Fund statistics for the whole broker-dealer sector indicate that only about half of the \$1.5 trillion slack was picked up by other broker-dealers, hence the sudden fall in repo activities in 2008q4.

Figure 2: US Treasuries, as a ratio of Households' Bank Deposits



Note: Total outstanding amount of US treasuries securities and Total outstanding amount of US treasuries securities held by the US financial sector and by the monetary authorities, divided by households' total deposits; *Source:* US Flows of Funds Accounts (Table L.209, series FL313161505.Q, FL713061100.Q, and FL763061100.Q + FL753061103.Q + FL743061103.Q + FL633061105.Q + FL653061105.Q + FL663061105.Q, and Table L.100, series FL154000025.Q). Vertical lines: NBER business cycle troughs and peaks.

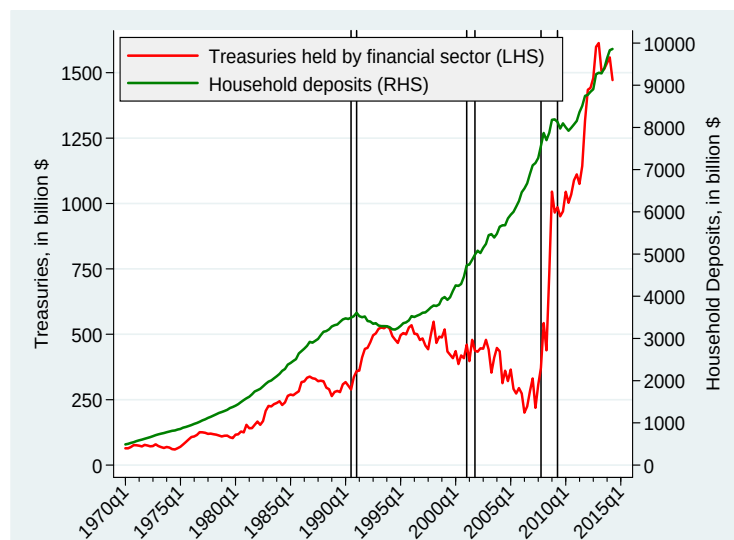
illiquid in the last quarter of 2007 (see Figure 1), and following Bear Stearns' near-bankruptcy in the first quarter of 2008, the Federal Reserve Bank launched the Term Securities Lending Facility program (TSLF). This program offered primary dealers the possibility to swap less liquid collateral for more liquid Treasury collateral held by the federal reserve bank system.⁷ Following the TSLF, financial intermediaries significantly and rapidly increased their holdings of treasuries (red line), which more than tripled within one year, while treasury holdings by the federal reserve system concomitantly decreased (blue line). The overall supply of treasuries by the US government increased too, but later, with a three quarter lag. Fleming, Hrungr, and Keane (2010) provide empirical evidence that this provision of treasury collateral through the TSLF mitigated a more general shortage of collateral on the repo market, which may explain—at least in part— why this market did not collapse as much as other secured loan markets at that time (see Figure 1).

Figure 3 shows the evolution of the levels, rather than ratios, of treasuries held by the financial sector and household deposits, from a historical perspective. As is clear from the figure, treasuries and deposits both increased from 1970 until 1995. After 1995, deposits continued to grow, but

⁷The Fed also put in place other new liquidity facilities during the early stage of the crisis, like the Term Auction Facility (TAF) or the Primary Dealer Credit Facility (PDCF). One important difference with the TAF is that the TSLF was available to Primary Dealers only (whereas the TAF was intended for depository institutions) and addressed conditions in the secured funding markets. One important difference with the PDCF is that the TSLF was an auction facility (whereas the PDCF was a standing facility) and therefore less subject to stigma effects. See Fleming, Hrungr, and Keane (2010) for a detailed discussion.

the level of treasuries actually fell steadily, until the end of 2006, just before the start of the crisis. At that time, banks deposits were at a historical high, and the level of treasuries held by the financial sector was at a historical low.

Figure 3: US Treasuries held by the financial sector and household deposits



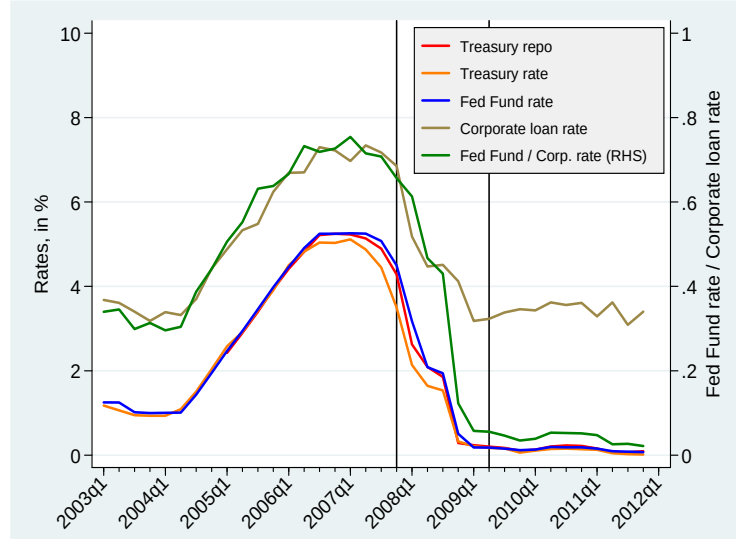
Note: Total outstanding amount of US treasuries securities held by the US financial sector: *Source:* US Flows of Funds Accounts (Table L.209, series FL763061100.Q + FL753061103.Q + FL743061103.Q + FL633061105.Q + FL653061105.Q + FL663061105.Q). Household Deposits: total currency and deposits (including money market fund shares) by households and nonprofit organizations; *Source:* US Flows of Funds (Table L.100, series FL154000025.Q). Vertical lines: NBER business cycle troughs and peaks.

Fact # 3:

The yield on treasury repos and three-month treasuries increased together in the run-up to the crisis. During the crisis, they both fell sharply, and the spread between corporate loan rates and those risk-free rates rose significantly.

Figure 4 shows that the treasury repo rate is essentially the same as the risk-free Federal Fund and three month treasury rates. Those rates increased in the run-up to the crisis, and then plummeted from 5.25% in 2007q2 to about 0.25% in 2009q1. Interestingly, during the crisis the returns on treasuries and treasury repos were both slightly below the Federal Fund rate, which reflects the high demand for safe collateral at that time as well as a “collateral premium” on those assets (Krishnamurthy and Vissing-Jorgensen (2012)). The figure also shows how the average corporate loan rate (brown line) and the ratio of the Federal Fund rate to this average rate (green line) evolved during those times. The latter ratio—which is the inverse of the spread on corporate loans—will play an important role in our analysis. Its sudden fall during the crisis is indicative of the impairment of the financial intermediation process at that time.

Figure 4: Nominal Interest Rates by Asset Classes



Note: Quarterly averages. Repo rates are the DTCC GCF Repo Index of the interest rates paid for the most-traded GCF Repo contracts on US treasury securities, federal agency securities, and mortgage-backed securities (MBS) issued by Fannie Mae and Freddie Mac; *Source*: DTCC. The treasury rate is the interest rate on the 3-month treasury securities (constant maturity); *Source*: Federal Reserve Bank of St Louis FRED database. The corporate loan rate is the average of commercial and industrial loan rates; *Source*: Federal Reserve Board Survey of Terms of Business Lending. Vertical lines: NBER business cycle troughs and peaks.

3 Environment

We consider an economy populated by a government, a representative household, firms, and banks. All agents live one period.⁸ We present their choice problems and constraints in turn.

3.1 Government

The government finances expenditures b by borrowing from banks at rate R^b , and repays debt by raising taxes bR^b at the end of the period. This budget constraint is built into the consumption of the representative household in (1). Public expenditures have no direct social value. As we will see though, the debt will have indirect social value through its use as collateral by banks.

3.2 Household

The representative household is endowed with savings, which take the form of s units of the single good deposited into the banks at the beginning of the period. At the end of the period the household consumes the return on deposits, sR^d , minus the taxes bR^b . For simplicity, assume

⁸The model is static as the interactions take place just once. Note though that there are two stages to these interactions.

linear utility over consumption, c :

$$u(c) = c = sR^d - bR^b. \quad (1)$$

3.3 Firms

There is a continuum of firms each endowed with one project but no funds. They borrow from banks to undertake their project. At the beginning of the period, firms are matched randomly with a single bank.⁹ The interest rate on corporate loan is denoted by R , and will be determined in equilibrium by an *ex post* bargain between the firm and the bank. (This will be described in detail later.) When successful, the project returns a per unit of investment, with $a > R$. If the project fails, there is no repayment to the bank. The probability a project succeeds is given by ε , which reflects the efficiency of the firm's bank.

3.4 Banks

There is a continuum of banks collecting deposits from the household. Banks are endowed with intermediation skills as well as with their own investment technology, and can invest in government bonds, corporate loans, or in their own projects. Banks also lend to each other through the interbank market.

There are two stages of bank investments. In the first stage, banks are identical and use their deposits s to purchase government bonds $\omega^b s$, and set the remainder, $(1 - \omega^b)s$, aside for second stage investments. In the second stage, banks realize their intermediation productivity (efficiency) ε , and become heterogenous. They can make loans either directly to firms, or indirectly through loans in the interbank market to more productive banks. An interbank market thus arises endogenously to allow banks to reallocate their funds once they have learned their productivity. In the second stage, banks may also run their own projects.

We first study the choices of the bank in the second stage. Using this, we formulate the optimization problem of the bank in the first stage.

3.4.1 Stage 2 Choices

In the second stage banks learn their productivity ε and choose to borrow or lend on the interbank market, to lend to firms or invest in their own projects. The choice between these options is made at the start of stage 2.

⁹Banks are matched with multiple firms but a firm is only matched with a single bank; this assumption is used to mimic observed banking relationships between firms and banks. To be clear, there is no heterogeneity across firm productivity *per se*. Differences across firms arise from the nature of the relationship between the firms and their bank.

A firm's project financed by bank ε yields gross unit return a with probability ε , and zero otherwise. Bank productivity ε can be interpreted as a match-specific shock between the bank and the firms it is matched with *ex ante*, thus reflecting the quality of the lending relationship. In this way, some banks are more productive intermediaries than others.¹⁰ Given that the bank earns R when the firm pays back the loan, and that a bank lends to a continuum of firms, bank ε 's *ex post* gross return on corporate loans is εR .

Banks also have the possibility to operate their own project. The latter returns a with probability γ , and is independent of the bank's ε . Clearly, a bank with $\varepsilon R < \gamma a$ always prefers to run her own project, than lend to firms. Since the loan rate R is endogenous, whether a bank makes corporate loans or invests in its own project will be determined in equilibrium. In the discussion that follows, we proceed under the assumption that banks' own technology is (weakly) inefficient, i.e. banks (weakly) prefer to either lend to corporations or other banks rather than undertake their own project (see later Assumption 2). This project nonetheless matters for the equilibrium outcome through its effects on incentives.

The ε 's are distributed over $[0, 1]$ with a continuously differentiable cumulative distribution function $G(\varepsilon)$, with $G'(\varepsilon) > 0, \forall \varepsilon \in [0, 1]$. Once the ε 's are drawn, banks allocate their available funds, $(1 - \omega^b)s$, into interbank loans or corporate loans in order to maximize their profit. Let r be the interbank loan rate. Then, the net unit profit of a bank ε on the funds borrowed at rate r and lent to firms at rate R is $\varepsilon R - r$. There is a critical bank efficiency, denoted $\bar{\varepsilon}$, with

$$\bar{\varepsilon} = \frac{r}{R}, \quad (2)$$

such that a bank with $\varepsilon = \bar{\varepsilon}$ is indifferent between lending and borrowing on the interbank market. Banks with $\varepsilon \geq \bar{\varepsilon}$ borrow and those with $\varepsilon < \bar{\varepsilon}$ lend.

Although banks with $\varepsilon > \bar{\varepsilon}$ have a strictly positive return on the leveraged funds, there is a limit, denoted ϕs and determined in equilibrium, on how much they can borrow on the interbank market. This limit takes into account the presence of asymmetric information and moral hazard. These frictions will limit the ability to reallocate deposits to the most productive banks and create a role for collateral. They will also be a source of the multiplicity of equilibria in the interbank market.

Asymmetric Information Asymmetric information arises from the assumption that the realized ε is known to the bank only. Thus, in equilibrium lenders will be able to compute the expected value of ε for borrowers but contracts cannot be conditioned on bank specific variables.

¹⁰The heterogeneity across banks generates an active interbank market. Alternatively, the heterogeneity could be at the firm level which then translates into bank heterogeneity since a firm is associated with a single bank.

Moral Hazard We assume that the cash flows generated by the banks' own projects cannot be seized by creditors. As a result, banks may run away and default on their interbank debt, while earning the net return γa . To fix ideas, we will also refer to the banks' own projects as 'private' projects. This creates a moral hazard problem, which will be important for determining an equilibrium.

Both valued collateral and borrowing restrictions are needed to deal with banks' incentive issues. As banks cannot easily divert the cash flows generated by government bonds and the corporate loans, they can pledge those assets as collateral to secure their interbank funding. Banks issue several types of securities, depending on which type of asset they pledge as collateral, government bonds or corporate loans. In effect, the pledging of those assets implies a transfer of ownership *ex ante* from the borrower to the lenders, who will return the assets at the end of the period only if the loan is paid back. One important difference built into the model is that lenders incur a fixed cost f whenever a security backed by corporate loans is defaulted upon. This captures the notion that, in contrast to government bonds, collecting the promised corporate cash flows is costly to a lender who assumes ownership of the assets post default. This fixed cost may include, for instance, the transaction costs related to the transfer of ownership of those loans, to monitoring the firms, servicing the loans, collecting the loan payments, etc. Another difference is that government bonds can be liquidated at no cost, whereas the liquidation of corporate loans is costly for the lenders. More precisely, lenders can only seize up to a fraction θ of the cash flows from corporate loans.

Let $\phi^b s$ be the funds secured by the government bonds, and $\phi^c s$ those secured by corporate loans, with $\phi^b + \phi^c = \phi$. Banks raise interbank funds in several stages. Initially, they only have government bonds to pledge. So, they issue securities backed by government bonds. The incentive constraint that prevents them from borrowing funds and absconding is:

$$\gamma a (1 - \omega^b + \phi^b) \leq \omega^b R^b + \varepsilon R (1 - \omega^b + \phi^b) - \phi^b r, \quad (3)$$

which must hold for all $\varepsilon \geq \bar{\varepsilon}$. A borrower who runs away does so with all of her funds and gets $\gamma a (1 - \omega^b + \phi^b) s$. But the returns on the pledged government bonds, $\omega^b R^b s$, are lost. By contrast, a borrower ε who does not run away earns the return on government bonds plus the return on the corporate loans, $\varepsilon R (1 - \omega^b + \phi^b) s$, net of the interbank loan repayment, $\phi^b r s$. This is indicated by the right side of (3). For bank $\bar{\varepsilon}$ the net return from not running away is independent of ϕ^b , whereas the net return of running away increases with ϕ^b . It follows that, in equilibrium, (3) binds at $\varepsilon = \bar{\varepsilon}$, which yields (using (2)):

$$\phi^b = \frac{\omega^b (R^b - r) + \bar{\varepsilon} R}{\gamma a} - 1 + \omega^b. \quad (4)$$

Once ϕ^b is raised, banks lend $(1 - \omega^b + \phi^b) s$ to their firms. Bank ε can then raise additional funds

and issue a new security, by pledging the cash flows of those corporate loans, $\varepsilon R(1 - \omega^b + \phi^b)s$, as collateral. Because they depend on ε , those cash flows are subject to the asymmetric information problem, though. Their pooling and tranching can help mitigate this friction, by turning a fraction of those cash flows into information insensitive and pledgeable cash flows (see Gorton and Ordonez (2014)). The information insensitive cash flows amount to $\theta\bar{\varepsilon}(1 - \omega^b + \phi^b)s$, which can be seen as the ‘senior’ tranche of the pool; only this tranche is pledgeable. Let $\phi^{c(1)}s$ be the new funding that banks can raise against those cash flows. Again, incentive problems limit the amount that can be borrowed; the incentive constraint that prevents banks from borrowing funds and absconding is:

$$\gamma a \phi^{c(1)} - \theta \bar{\varepsilon} R(1 - \omega^b + \phi^b) \leq \phi^{c(1)}(\varepsilon R - r), \quad (5)$$

which must hold for all $\varepsilon \geq \bar{\varepsilon}$. If borrower ε runs away and defaults on this new security, then she gets $\gamma a \phi^{c(1)}s$ but loses the corporate loan cash flows that were pledged, $\theta \bar{\varepsilon} R(1 - \omega^b + \phi^b)s$. If she does not run away, then she earns the net return on the corporate loans, $\phi^{c(1)}(\varepsilon R - r)s$. Importantly, at this stage government bonds are encumbered and cannot be pledged as collateral again. Thus the collateral is not being pledged multiple times. In equilibrium, (5) must hold for all $\varepsilon \geq \bar{\varepsilon}$, and therefore binds at $\varepsilon = \bar{\varepsilon}$, which yields:

$$\phi^{c(1)} = \frac{\theta \bar{\varepsilon} R}{\gamma a} (1 - \omega^b + \phi^b). \quad (6)$$

Under Assumption 1 (see below), the term $\frac{\theta \bar{\varepsilon} R}{\gamma a}$ is smaller than one, and can be viewed as reflecting the existence of a haircut. Lenders apply a haircut on collateral to be compensated for the cost of liquidation, which they would incur if the borrower invested into her private project. Once $\phi^{c(1)}s$ is raised, banks lend $\phi^{c(1)}s$ to firms, and may then raise again additional funds $\phi^{c(2)}s$ on the interbank market by issuing another security backed by future cash flows, $\theta \bar{\varepsilon} R \phi^{c(1)}s$. In this way, the use of corporate loans as collateral gives rise to a sequence of new securities and collateral creation inside the banking sector. Accordingly, we will refer to corporate loans as ‘inside’ collateral, and to government bonds as ‘outside’ collateral.

Let $\phi^{c(n)}s$ be the funds that bank ε can raise against the $(n-1)^{\text{th}}$ corporate loans, $\theta \bar{\varepsilon} R \phi^{c(n-1)}s$. Then, the corresponding n^{th} incentive constraint is

$$\gamma a \phi^{c(n)} - \theta \bar{\varepsilon} R \phi^{c(n-1)} \leq \phi^{c(n)}(\varepsilon R - r). \quad (7)$$

In equilibrium, using (6) and the fact that (7) binds at $\varepsilon = \bar{\varepsilon}$,

$$\phi^{c(n)} = \frac{\theta \bar{\varepsilon} R}{\gamma a} \phi^{c(n-1)} = \left(\frac{\theta \bar{\varepsilon} R}{\gamma a} \right)^n (1 - \omega^b + \phi^b). \quad (8)$$

This process of loan creation is limited by the fixed cost lenders incurs from collecting once there is a default, f . The $(n-1)^{\text{th}}$ loans will be repackaged only if the associated pledgeable cash flows

cover this cost, *i.e.*:

$$\theta \bar{\varepsilon} R \phi^{c(n-1)} s \geq f. \quad (9)$$

Let N be the total number of interbank debt issuances. It is determined by the relation:

$$\left(\frac{\theta \bar{\varepsilon} R}{\gamma a} \right)^N (1 - \omega^b + \phi^b) = \frac{f}{\gamma a s}. \quad (10)$$

This representation of the interbank market, which emphasizes the stages of the lending process, can be seen as a stylized representation of how the US shadow banking industry manufactured new financial products in the run-up to the 2008 financial crisis (see Coval, Jurek, and Stafford (2009); Pozard, Adrian, Ashcraft, and Boesky (2012)). To recap, the first step is to form a diversified pool of loans. The next step is to slice the pool and to issue a security backed by the information insensitive slice of the pool. This funding then gives rise to new corporate loans, and so on. Ultimately, borrowers issue N distinct securities, and the total amount of interbank loans secured by inside collateral is given by:

$$\phi^c = \sum_{n=1}^{n=N} \phi^{c(n)} = \frac{\theta \bar{\varepsilon} R}{\gamma a} \cdot \frac{\omega^b (R^b - r) + \bar{\varepsilon} R - \frac{f}{s}}{\gamma a - \theta \bar{\varepsilon} R}. \quad (11)$$

While for simplicity the model is strictly about a single asset backed by corporate loans, notice that the interbank claims issued at each stage could potentially be rather complex and that richer models would include alternative forms of securitization, such as CDO, CDO², or CDO ^{N} . In the discussion that follows we think of N as capturing the complexity of financial markets and securities. These securities would build upon each other as in our multiple lending rounds. Of course, in equilibrium the sequencing is instantaneous, limited by the willingness of lenders to expend the securitization cost if needed. To ensure a demand for inside collateral and yet maintain a moral hazard problem, we assume:

Assumption 1. (*Liquidation cost and seizable cash flow*) (i) $f < \gamma a s$ and (ii) $\theta R \ll \gamma a$.

Point (i) means that the fixed liquidation cost is low enough, so that in equilibrium lenders can afford liquidating any corporate loan, even if the loan has a low return $\gamma a s$. Point (ii) means that the cash flow seized during a liquidation is low enough, so that borrowers still have some incentive to renege on their interbank debt.¹¹

Ultimately, the total outstanding amount of interbank loans that banks can raise is (using (4), (8) and (10)):

$$\phi = \phi^b + \phi^c = \frac{\gamma \omega^b (R^b - r) + (\gamma - \frac{\theta f}{a s}) \bar{\varepsilon} R}{\gamma (\gamma a - \theta \bar{\varepsilon} R)} - 1 + \omega^b \quad (12)$$

¹¹The determination of R is described below so that point (ii) is an assumption about model primitives.

3.4.2 Stage 1 Choices

Ex ante banks maximize the expected gross return on assets, denoted π , with respect to ω^b :

$$\max_{\omega^b} \pi \equiv r + \omega^b(R^b - r) + (1 - \omega^b + \phi) \int_{\bar{\varepsilon}}^1 (\varepsilon R - r) dG(\varepsilon), \quad (13)$$

given (12). The return on assets is the sum of returns on government bonds, interbank loans, and corporate loans, weighted by the share of those assets in banks' assets. The second and third terms reflect the excess return on governments bonds and corporate loans over interbank loans. This optimization problem assumes that, *ex post*, banks with $\varepsilon < \bar{\varepsilon}$ will be net lenders in the interbank market and banks with $\varepsilon \geq \bar{\varepsilon}$ will be net borrowers on the interbank market and lend to firms. Only the latter banks enjoy a excess return on corporate loans. The first order condition yields the no-arbitrage condition:

$$R^b = r, \quad (14)$$

which implies that banks are indifferent between investing into bonds and lending to other banks. Substituting this *ex ante* condition into the incentive constraints, (4), (11) and (12) yield:

$$\phi^b = \frac{\bar{\varepsilon}R}{\gamma a} - 1 + \omega^b, \quad (15)$$

$$\phi^c = \frac{\theta \bar{\varepsilon}R}{\gamma a} \cdot \frac{\bar{\varepsilon}R - \frac{f}{s}}{\gamma a - \bar{\varepsilon}\theta R}, \quad (16)$$

and

$$\phi = \frac{\bar{\varepsilon}R}{\gamma a} \cdot \frac{\gamma a - \frac{\theta f}{s}}{\gamma a - \theta \bar{\varepsilon}R} - 1 + \omega^b. \quad (17)$$

The degree of complexity of the assets, which the banking sector creates, is then (from (10)):

$$N = \frac{\ln(\bar{\varepsilon}R) + \ln s - \ln f}{\ln(\gamma a) - \ln(\theta \bar{\varepsilon}R)}. \quad (18)$$

4 Equilibrium Conditions

This section describes the set of equilibria.

4.1 Timing

The timing of the interactions between deposits, banks and firms is:

Stage 1 The sunspot is revealed. This only matters when there are multiple equilibria.

Stage 2 The household makes deposits s . The banks allocate these deposits across government bonds ($\omega^b s$) and future investment $((1 - \omega^b)s)$.

Stage 3 Firms are randomly matched with banks. Banks learn about their efficiency, ε .

Stage 4 Banks decide to make either interbank loans or corporate loans, or whether invest into private projects.

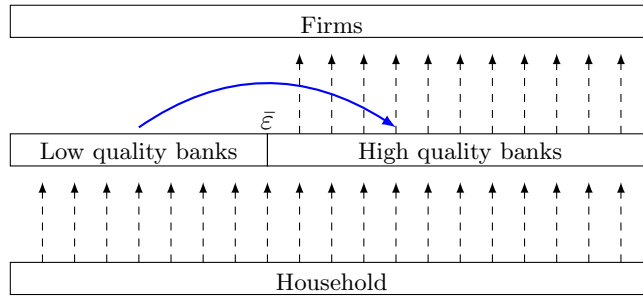
Stage 5 A fraction of the firms are funded by their bank. Banks and firms bargain over R .

4.2 Market Clearing Conditions

There are five markets: goods, government bonds, corporate loans, interbank loans, and bank deposits. We solve for the equilibria, given a stock of bonds b .

Figure 5 illustrates the reallocation of funds in equilibrium. It highlights the role of $\bar{\varepsilon}$ in determining which banks borrow and which lend. As we shall see, given $\bar{\varepsilon}$, the amount banks can borrow is determined as well. Thus the interbank market equilibrium will set $\bar{\varepsilon}$. Once this is known, the other components of the equilibria can be determined. The key to fragility comes from the multiple values of $\bar{\varepsilon}$ solving the interbank market clearing condition.

Figure 5: Interbank Reallocation and Financial Intermediation



Government Bond Market Since the government issues b bonds, and the banks demand $\omega^b s$, the market clears when:

$$\omega^b = \frac{b}{s}. \quad (19)$$

Here ω^b reflects the portfolio allocation of the bank. By our timing, this decision is made after the realization of the sunspot and hence given the rate of return on interbank loans and government debt.

Corporate Loan Market The equilibrium corporate loan rate is set as a bargain between a bank and a firm. At the time of this bargain, banks have foregone the option to operate their own project and are unable to trade further in the interbank market. Hence, they no have outside option for the loan. For bank ε , the surplus is $\varepsilon R s$. The firm obtains a surplus of $\varepsilon(a - R)s$ so

that the total surplus is $S \equiv a\varepsilon s$. The bank gets λS and the firm gets $(1 - \lambda)S$ so that:

$$R = \lambda a. \quad (20)$$

This is admittedly a simple bargaining structure as the bank has no alternative investment options. Introducing these options, such as bargaining jointly with the bank's firms, would shift the market power. As formulated the bank would then have all the bargaining power as firms have no outside option for financing. This could be offset by extending the model so that firms are matched with multiple banks, thus generating a complex bargaining problem with many banks and firms. Our approach is to parameterize the bargaining outcome by λ .

Bank Deposit Market Banks are identical *ex ante*, compete for depositors, and break even *ex post*. In equilibrium, one therefore gets

$$R^d = \pi,$$

which using (13) and (17) yields:

$$R^d = r \left(1 + \frac{\gamma - \frac{\theta f}{as}}{\gamma(\frac{\gamma}{\lambda} - \theta \bar{\varepsilon})} \int_{\bar{\varepsilon}}^1 (\varepsilon - \bar{\varepsilon}) dG(\varepsilon) \right). \quad (21)$$

Since the second term in the parentheses is positive, the deposit rate is larger than the interbank loan rate and therefore the return on government bonds. The spread between the corporate loan rate and the interbank loans rate reflects the efficiency cost of the financial intermediation process, and decreases with $\bar{\varepsilon}$. Since in equilibrium $R^b = r$, this spread also reflects the shadow value of government bonds as collateral. Despite the low return on government bonds, banks are willing to hold such assets *ex ante* because they can pledge them as collateral to relax their financing constraints *ex post*.

Interbank Market Banks with low ε are the natural suppliers of funds and those with high draws of ε demand funds in this market. On the one hand, banks with $\varepsilon \geq \bar{\varepsilon}$ demand funds ϕs ; so aggregate demand is $(1 - G(\bar{\varepsilon}))\phi s$. On the other hand, banks with $\varepsilon < \bar{\varepsilon}$ supply $s - b$, provided that $\bar{\varepsilon} > \gamma/\lambda$; in this case, those banks strictly prefer to lend than to run their own, private project, and aggregate supply is $G(\bar{\varepsilon})(s - b)$. In the special case when $\bar{\varepsilon} = \gamma/\lambda$, banks are indifferent between those two options, and so supply up to $G(\bar{\varepsilon})(s - b)$ on the interbank loan market. It follows that the market clearing condition is:

$$(1 - G(\bar{\varepsilon}))\phi \begin{cases} = G(\bar{\varepsilon})(1 - \frac{b}{s}) \text{ if } \bar{\varepsilon} > \frac{\gamma}{\lambda} \text{ (a)} \\ \in [0, G(\bar{\varepsilon})(1 - \frac{b}{s})] \text{ if } \bar{\varepsilon} = \frac{\gamma}{\lambda} \text{ (b).} \end{cases} \quad (22)$$

In (22a), banks with $\varepsilon < \bar{\varepsilon}$ lend all of their assets in the interbank market. Importantly, these banks do not fund any private projects. It is possible that the interbank market also clears with some of the suppliers investing in private loans. By arbitrage, this arises only when $\bar{\varepsilon} = \frac{\gamma}{\lambda}$, as (22b) suggests. Using (12), (19), and (20), relation (22) can be re-written as:

$$H(\bar{\varepsilon}) \equiv (1 - G(\bar{\varepsilon})) \left(1 + \phi - \frac{b}{s}\right) = (1 - G(\bar{\varepsilon})) \lambda \bar{\varepsilon} \left(\frac{1 - \frac{\theta f}{\gamma a s}}{\gamma - \lambda \theta \bar{\varepsilon}} \right) \begin{cases} = 1 - \frac{b}{s} & \text{if } \bar{\varepsilon} > \frac{\gamma}{\lambda} \quad (a) \\ \in [0, 1 - \frac{b}{s}] & \text{if } \bar{\varepsilon} = \frac{\gamma}{\lambda} \quad (b) \end{cases} \quad (23)$$

where $H(\bar{\varepsilon})$ is the fraction of savings that is ultimately channelled to firms. Banks with $\varepsilon \geq \bar{\varepsilon}$ finance those corporate loans with own funds $s - b$ and by interbank loans $\phi^b s$ and $\phi^c s$ secured by outside collateral (government bonds) and inside collateral (corporate loans), respectively.

Relation (23) is a convenient representation of the market clearing condition: its left side depends on $\bar{\varepsilon}$, while the right side is independent of $\bar{\varepsilon}$. The richness of the model will come from the non-monotonicity —the hump-shape— of $H(\bar{\varepsilon})$. As $\bar{\varepsilon}$ increases, the fraction of banks who borrow falls, but their average quality goes up. Since higher quality banks have less incentives to run their private project, the amount they can borrow increases. That is, from (12), ϕ is increasing in $\bar{\varepsilon}$. The multiplicity will come from the interaction of these extensive and intensive margins.

As the bargaining power of the banks increases, i.e. λ rises, the value of outside collateral and the amount of inside collateral created through the iterations of interbank lending go up. Essentially, with more bargaining power banks not only benefit directly from larger cash flows but also have more to pledge to generate additional loans in the interbank market. As a result, $H(\bar{\varepsilon})$ goes up. $H(\bar{\varepsilon})$ also increases with the level of seizable cash flows, θ . By contrast, $H(\bar{\varepsilon})$ decreases with f and γ . Indeed, a rise in f or γ limits the creation of inside collateral, and therefore the aggregate demand for interbank loans.

4.3 Equilibria

We discuss the types of equilibria that might arise and then give conditions for their co-existence. We will show that there are potentially multiple equilibria in this model economy due to the asymmetric information and moral hazard of banks. For this discussion, we consider restrictions on $G(\cdot)$, γ , and λ so that those two frictions are meaningful:

Assumption 2. (*Initial resource mis-allocation*) (i) $g'(\bar{\varepsilon}) \geq 0 \quad \forall \bar{\varepsilon} \in [0, 1]$, and (ii) $\frac{\gamma}{\lambda} < \bar{\varepsilon}^{max} \equiv \arg \max_{\bar{\varepsilon} \in [0, 1]} H(\bar{\varepsilon})$.

Point (i) means that there is a sufficient mass of high ε -banks in the economy. Point (ii) means that the banks' private projects are inefficient. Altogether, Assumption 2 therefore implies

that deposits are initially mis-allocated across banks, and that there is scope for re-allocation through an interbank market. Under this assumption, $H(\bar{\varepsilon})$ is hump-shaped and reaches an interior maximum at $\bar{\varepsilon} = \bar{\varepsilon}^{\max} \in (\gamma/\lambda, 1)$.¹² The hump shape reflects the ambiguous effect of $\bar{\varepsilon}$ on loan demand. There are two main opposite effects: demand decreases with $\bar{\varepsilon}$ on the extensive margin but increases with $\bar{\varepsilon}$ on the intensive margin. This latter unusual effect is due to the fact that the average quality of the borrowers goes up with $\bar{\varepsilon}$, as this works to relax banks' borrowing constraint. The negative intensive margin effect dominates when $1 - G(\bar{\varepsilon})$ is large enough, that is when $\bar{\varepsilon}$ is low; in this case, the loan demand curve bends backward.

Definition 1. *An equilibrium is a vector of returns $(\omega^b, r, R^d, R^b, R)$, a critical level of ε denoted $\bar{\varepsilon}$, and borrowing limits ϕ^b, ϕ^c , such that*

- *in stage 1 banks choose ω^b to maximize their expected return on deposits;*
- *in stage 2 banks with;*
 - *$\varepsilon \geq \bar{\varepsilon}$ borrow in the interbank market up to the limit $\phi = \phi^b + \phi^c$;*
 - *$\varepsilon < \bar{\varepsilon}$ either lend in the interbank market or undertake private loans;*
- *ϕ^b, ϕ^c are determined so that banks with $\varepsilon \geq \bar{\varepsilon}$ invest rather than abscond;*
- *beliefs about the distribution of types of banks borrowing in the interbank market are consistent with the equilibrium outcome;*
- *the corporate loan rate is the outcome of the bargaining process firms and banks, so that (20) holds;*
- *markets for interbank loans, deposits and government bonds clear.*

4.3.1 Normal Times

We first study outcomes which are solutions to (23a), with $\bar{\varepsilon} > \gamma/\lambda$. These solutions become the basis for a subset of the equilibria. The right side of (23a) is predetermined and constant. Under Assumption 2, the left side is hump shaped —the exact shape will depend on the assumed form for $G(\cdot)$. As a consequence, an equilibrium does not always exist.

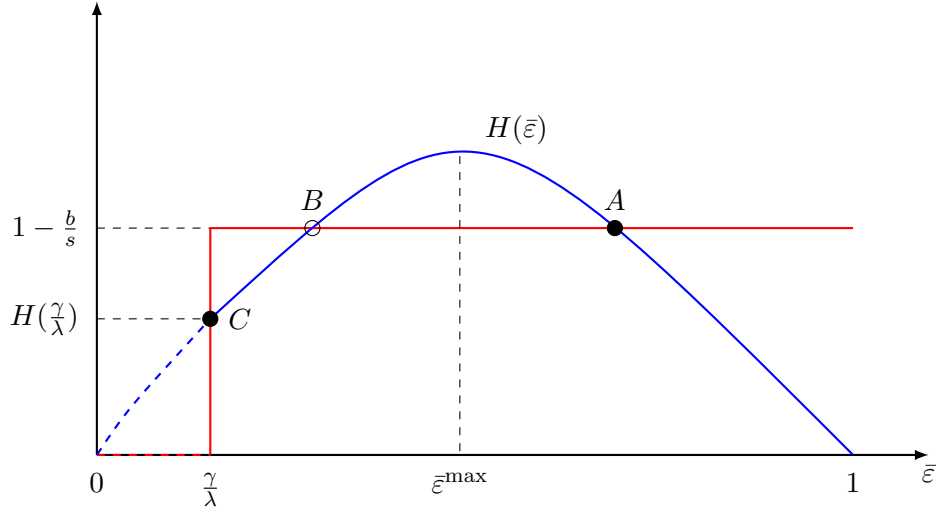
Equilibria in the interbank loan market are shown in Figure 6. Since banks always have the outside option to invest in their own project at a return γa , only the values $\bar{\varepsilon} \geq \gamma/\lambda$ are relevant. The figure illustrates one solution to (23) at point A . This equilibrium determines a value of $\bar{\varepsilon} > \gamma/\lambda$ such that banks with $\varepsilon \geq \bar{\varepsilon}$ borrow on the interbank market, and will be referred to as ‘normal times’. In this type of equilibrium, there are no private projects.

¹²For a proof of this, see Appendix 7.

From this value of $\bar{\varepsilon}$, we can construct an equilibrium as follows. Use (2) to determine the interbank rate: $r = \bar{\varepsilon}R$, where R is set in the bargain between banks and firms. From the no-arbitrage condition, (14), $R^b = r$.

Relations (15) and (16) yield ϕ^b and ϕ^c given $\bar{\varepsilon}$. Finally, the equilibrium number of ‘rounds’ in the interbank market that underlies ϕ^c is given by (18). Assumption 1 implies that ϕ^c and N are strictly positive.

Figure 6: Multiple Equilibria in the Interbank Market



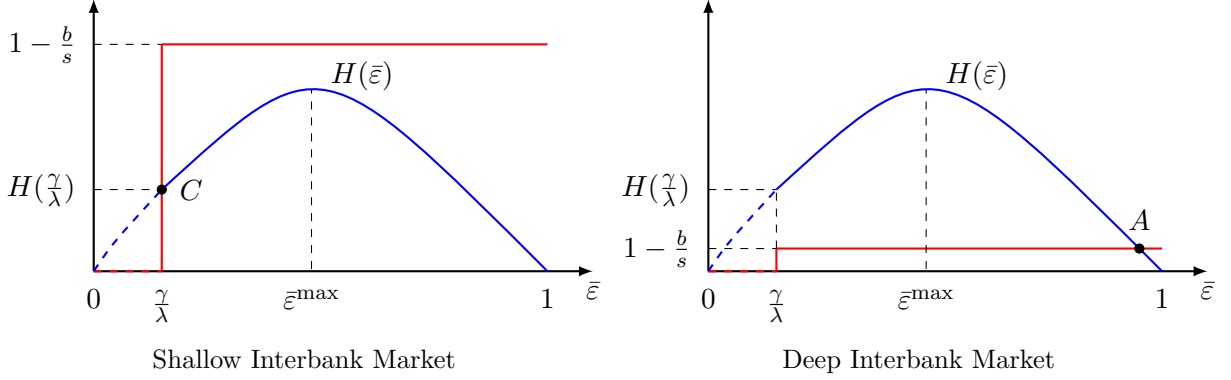
Since $H(\bar{\varepsilon})$ is hump-shaped, another interior solution (point B) can exist. Relative to A , the value of $\bar{\varepsilon}$ is lower, there are more banks borrowing but their borrowing constraint is tighter, loan quality is lower, and the intermediation process is less efficient. In the analysis that follows, we focus on the equilibrium at A as it is locally stable and thus has ‘reasonable’ comparative statics. But, as we shall see, there is another equilibrium outcome other than B that is of interest.

4.3.2 Crisis Times

Another equilibrium, which we term ‘crisis’, can arise which is a solution to (23b). Such an equilibrium arises when $\bar{\varepsilon} = \gamma/\lambda$. In this case, there is an excess supply of loans on the interbank market, and some banks engage in private projects to soak it up.

In Figure 6, the crisis equilibrium is represented by point C . At this point, the banks with $\varepsilon \leq \gamma/\lambda$ are indifferent between lending to other banks and investing into their private project, and they are the only banks that supply interbank loans. The supply of interbank loans is infinitely elastic, as illustrated by the vertical red line. During a crisis, the borrowing constraint holds tight: productive banks, who cannot distinguish themselves from other banks, are unable to commit themselves to lending to firms, and therefore cannot raise more funding. Despite the low interbank rate, the demand for loans remains below supply, there is an excess supply of interbank

Figure 7: Unique Equilibrium Configurations



funding, and the interbank market clears by having low productivity banks invest in their own private projects. This equilibrium exists if and only if (using (23b)):

$$H\left(\frac{\gamma}{\lambda}\right) < 1 - \frac{b}{s}. \quad (24)$$

Though some private projects are financed, interbank borrowing is still possible, i.e. $\phi > 0$, because banks can pledge their assets as collateral. This is a consequence of the first part of Assumption 1. If $b = 0$ (no outside collateral) and $\theta = 0$ (no inside collateral) then in a crisis $\phi = 0$ and there is no demand for interbank loans.

Moreover, the crisis equilibrium in C co-exists with the normal times equilibrium in A . In this sense, the interbank market is susceptible to swings in confidence. It is natural to associate outcomes like A with optimism about the quality of borrowing banks, with high returns on interbank loans and relaxed borrowing restrictions. It is also natural to think of crises, like C , as the outcome of pessimism. A and C are self-fulfilling equilibria. Assume for example that banks are pessimistic: they believe that unproductive banks demand interbank funding. Since unproductive banks are prone to investing the funds into private projects, lenders require borrowers to put more ‘skin in the game’, and reduce ϕ . As every borrower demands less funding, the equilibrium interbank loan rate goes down, and the net present value of the unproductive banks’ projects increases. Hence, unproductive banks do indeed demand interbank funding.

4.3.3 Conditions for Crisis and Normal Times Equilibria

Not all economies possess multiple equilibria. Other possible outcomes are illustrated in Figure 7, which shows cases of unique equilibria. In panel (a), there are no solutions to (23a), and there is a crisis equilibrium. In panel (b), by contrast, there is a solution to (23a) but not to (23b): $H(\gamma/\lambda) > 1 - b/s$; in this case the normal times equilibrium is the only possible outcome. To describe the conditions of existence of crisis and normal times equilibria, it is useful to define two

critical levels of government debt, $\bar{b}$ and $\underline{b}$, as:

$$\bar{b} \equiv \left(1 - H\left(\frac{\gamma}{\lambda}\right)\right) s \quad (25)$$

and

$$\underline{b} \equiv (1 - H(\bar{\varepsilon}^{max})) s, \quad (26)$$

with $\underline{b} < \bar{b}$. There are three regions for the level of public debt. In the upper dominance region, with $b > \bar{b}$, banks can use so much public debt as collateral that a crisis is ruled out, irrespective of banks' beliefs about borrowers' quality in the interbank market. Then there is a lower dominance region, with $b < \underline{b}$, where instead there is a shortfall of pledgable assets in the economy. The government did not issue enough debt to avoid a crisis. Finally, there is an intermediate region, where beliefs matter and sunspot equilibrium arise; in this case, crises are self-fulfilling. Proposition 1 formalizes this characterization.

Proposition 1. (*Equilibrium*)

- When $b < \underline{b}$, there is a unique crisis equilibrium, with $\bar{\varepsilon} = \frac{\gamma}{\lambda}$ as solution to (23b);
- When $b > \bar{b}$, there is a unique normal times equilibrium, with $\bar{\varepsilon} > \bar{\varepsilon}^{max}$ as solution to (23a);
- When $b \in [\underline{b}, \bar{b}]$, there are sunspot equilibria, with either $\bar{\varepsilon} > \bar{\varepsilon}^{max}$ or $\bar{\varepsilon} = \frac{\gamma}{\lambda}$.

Proof. The results follow directly from the definition of $\bar{b}$ and $\underline{b}$ in (25) and (26). $\square$

We interpret the size the region of multiplicity as indicative of financial instability: the larger $\bar{b} - \underline{b}$, the higher the degree of financial instability. Since the thresholds $\bar{b}$ and $\underline{b}$ depend on the volume of deposits, s , the degree of financial instability also depends on s .

Proposition 2. (*Savings and Financial Stability*) *An increase in savings s widens the range of government debt levels, $\bar{b} - \underline{b}$, for which there are sunspot equilibria, and thus is conducive to interbank market instability.*

Proof. From Appendix 7, $\bar{\varepsilon}^{max}$ is independent of s . Using (23), (25) and (26), $(\bar{b} - \underline{b})/s$ is proportional to $\left(1 - \frac{\theta f}{\gamma a s}\right)$, which increases monotonically with s . $\square$

Besides influencing the range of multiplicity, the level of savings has an affect on the degree of complexity of financial markets. This reflects the presence of a fixed cost, f , so that an increase in the overall volume of funds flowing into banks can influence the number of rounds of lending that arises in equilibrium.

Proposition 3. (*Savings and Financial Complexity*) *An increase in savings s raises financial complexity, N , when $b < \frac{\theta f}{\gamma a}$.*

Proof. From relation (18) it is easy to see that N increases with both $\bar{\varepsilon}$ and s , and from relation (23), it is easy to see that $\bar{\varepsilon}$ increases with s when $b < \frac{\theta f}{\gamma a}$. Hence the result. $\square$

4.4 Comparing Outcomes: Crises versus Normal Times

The key element of a crisis is the reduction in $\bar{\varepsilon}$, as in Figure 6. The fall in $\bar{\varepsilon}$ during a crisis has a couple of important implications.

First, at a lower $\bar{\varepsilon}$, a larger fraction of banks borrow in the interbank market. This increase on the extensive margin means that lower productivity banks that would lend to higher productivity banks during normal times actually borrow during the crisis.

Countering this change on the extensive margin is a reduction in borrowing limits. With lower quality banks borrowing, these limits fall to avoid moral hazard. In addition to the reduction in size of the interbank loans, there is a composition effect of a crisis. Specifically, the share of outside collateral in total collateral rises. Essentially, the limit on the repo market is less sensitive than the borrowing backed by corporate loan repayments, because repos are secured by outside collateral, which forms a stable collateral base. Interestingly, the variability in ϕ^c reflects, in part, variations in the complexity of loans based on inside collateral, i.e. N as given in (18). The reduction in the productivity of the intermediation process is reflected in lower deposit rates.

In a crisis there is less activity in the interbank market, and some banks undertake their private projects, whose productivity is low. This reduction in the productivity of the intermediation process has not only an overall negative real effect on the *levels* of output, consumption, and welfare, but also a positive effect on the cross-sectional *dispersion* of banks' and firms' returns.

The output and productivity implications are consistent with the evidence in Foster, Grim, and Haltiwanger (2013). To the extent the Great Recession contained more of a financial collapse than other recessions, the implications of our model, which attributes recessions to the collapse of the interbank market, are relevant. A decline of economic activity is due to a reduction in the reallocation of deposits in the interbank market. This decline in activity also corresponds to a period with an increased cross-sectional dispersion in the productivity.

Proposition 4. (*Effects of a crisis*) *Relative to normal times and all other things being equal, during a crisis:*

1. *the cut-off bank productivity, $\bar{\varepsilon}$, is lower;*
2. *the interbank loan rate, r , is lower;*
3. *the deposit rate is lower;*
4. *the spread between the deposit rate and interbank loan rate (and therefore the shadow value of government bonds as collateral) increases;*

5. *the spread between the corporate loan rate and the interbank loan rate increases;*
6. *outside collateral, ϕ^b is lower;*
7. *inside collateral, ϕ^c is lower;*
8. *the ratio $\frac{\phi^b}{\phi^c}$ increases;*
9. *the complexity of interbank loans, N is reduced;*
10. *banks undertake private projects, and not all deposits (net of government bonds) are channelled to firms;*
11. *output, consumption and welfare fall;*
12. *the cross sectional dispersion of bank and firm productivity increases.*

Proof. The proposition compares multiple equilibria A and C in Figure 6. The effects of the reduction in $\bar{\varepsilon}$, as in Figure 6 defines a crisis. As $\bar{\varepsilon} = \frac{r}{R}$ and $R = \lambda a$, the interbank rate fall with $\bar{\varepsilon}$, and the spreads between R and r and between R^d and r increase. As more banks are involved in making loans, the cross sectional dispersion of bank and firm productivity increases. The effects of a fall in $\bar{\varepsilon}$ on outside and inside collateral come directly from (15) and (16). The complexity is given by (18) and this is increasing in $\bar{\varepsilon}$ and so falls in a crisis. The effects of a fall in the ratio of outside to inside collateral can be seen from relation $(\phi^b + 1 - b) / (\phi^c + \phi^b + 1 - b) = (\gamma - \theta\lambda\bar{\varepsilon}) / (\gamma - \theta f/(as))$: the left hand side increases with ϕ^b/ϕ^c , and the right hand side decreases with $\bar{\varepsilon}$ and hence rises in a crisis. Remembering that $H(\bar{\varepsilon})$ is the aggregate supply of corporate loans, we can write aggregate output as the sum of the outcomes of all the projects, $y = (1 - b - H(\bar{\varepsilon}))\gamma a + H(\bar{\varepsilon})\lambda a \int_{\bar{\varepsilon}}^1 \varepsilon / (1 - G(\varepsilon)) dG(\varepsilon)$, which is clearly higher in normal times than in crisis times. Aggregate consumption (or welfare) being equal to aggregate output, it too is higher in normal times. Finally, the deposit rate and comes from (21). $\square$

5 Back to the Facts

The model can be used to understand interbank market flows and interest rates. The model is structured around ‘normal’ and ‘crisis’ times. With this in mind, the pre-crisis period, roughly 2003 to mid-2007, is viewed as ‘normal’ times. The crisis begins at that point.

5.1 Pre-Crisis

From Figure 3, the level of household deposits grew steadily from 1995 to the onset of the crisis. Yet, treasuries held by the financial sector actually fell from the mid-1990s to the onset of the

crisis. As seen in Figure 2, from the 2003 to 2008, the holding of US Treasuries by the US financial sector, relative to household deposits, slowly decreased. At the same time the volumes and complex financial assets, such as repos, ABCP, or ABS-CDO, all rose (see Figure 1). These observations are consistent with the notion that the financial industry created complex ‘safe assets’ as substitutes to treasuries, as in Caballero (2009).

From the perspective of our model, this increase in the sophistication of asset markets is seen as an increase in N . Using (18), the increase in the sophistication of interbank lending could reflect the increase in deposits (s), as in Proposition 3. Or, alternatively, a reduction in the cost of liquidation borne by the lender (f), or an increase in the fraction of cash flow lenders can seize (θ). These forces increase the fraction of lending financed through inside collateral, relative to outside collateral, consistent with the pre-crisis period.

Figure 4 shows that all rates increased during this period. One way to capture these movements of interest rates pre-crisis is through an increase in the bargaining weight of banks, λ . As noted earlier, an increase in λ leads to an increase in $\bar{\varepsilon}$.¹³ The rise in the corporate loan rate, which is consistent with an increase in λ , also stimulates the creation of inside collateral in our model. The increase in the ratio of the Fed fund rate to the corporate loan rate, which comes from an increase in $\bar{\varepsilon}$, indicates that, overall, the increase in the volume of complex safe assets more than offset the reduction in the volume of outside collateral held by banks.

5.2 Crisis

Proposition 2 implies that the increase in deposits and complexity of financial markets may have had a hidden cost by increasing the likelihood of a crisis. We interpret events post-2008 as a fall in confidence of lenders in the interbank market.

The crisis is defined by a dramatic fall in cut-off productivity determining the borrowers and lenders in the interbank market. The implications of this switch from optimistic to pessimism are summarized in Proposition 4. As predicted by the model, the repo rate (r) and the ratio between the repo and the corporate loan rates ($\bar{\varepsilon}$) fell at the onset of the crisis, as seen in Figure 4.

As shown in Figure 1, repo, ABCP and ABS-CDO activity (relative to deposits) fell sharply from mid-2007 to 2009. Repo activity fell the least and stabilized in 2009, at a level lower than 2003. The ABS-CDO activity fell by over 80% from its peak in 2007 to the 2009. The ABCP volume fell more than repo but less than the ABC-CDO volumes. These relative flows are in accord with the model’s predictions, particularly items 8-9 in Proposition 4. Here we interpret the ABCP flow as indicative of $\phi^{c(1)}$ and the more dramatic reduction in ABS-CDO as a variation in the sophistication of lending, i.e. a reduction in N .

To emphasize an important point, the foundation for inside collateral is the outside collateral. When $\bar{\varepsilon}$ falls in the crisis, the value of the outside collateral is reduced as moral hazard and

¹³An increase in λ will shift $H(\bar{\varepsilon})$ up. This will increase $\bar{\varepsilon}$ at a locally stable equilibrium, such as A in Figure 6.

adverse selection problems are magnified. This reduction in ϕ^b is magnified by reduced lending in each round of interbank trades with inside collateral, $\phi^{c(n)}$, given in (8), and in the total number of these rounds. Overall, interbank activity shrinks as the result of both the reduction in the collateral value of assets and the reduction in the volume of private asset creation.

The share of Treasury securities held by US banks in the total outstanding amount of US treasuries issued increased dramatically during the crisis. In this way, repo activity was supported through government intervention. Otherwise, repo activity would have fallen even more.

During the crisis, the yield on the three-month treasuries went down below the return on repos or the Fed fund rate, which may reflect a collateral premium on Treasuries (Krishnamurthy, Nagel, and Orlov (2014); Krishnamurthy and Vissing-Jorgensen (2012)), and the shadow value of government bonds for the banking sector. In our model, this is captured by a rise in R^d/r during the crisis (item 4 in Proposition 4).

6 Conclusions

This paper highlights sudden swings in collateral value and collateral creation as a source of fragility in interbank markets. Due to the presence of moral hazard and adverse selection, loans between heterogeneous banks have two main features. First, these loans are collateralized, either by Treasuries (outside collateral) and/or cash flows (inside collateral). Second, limits emerge endogenously on the amount banks can borrow in this market.

Confidence is central to the functioning of the interbank market. When lenders are optimistic about the quality (productivity) of borrowing banks, quantity restrictions are relatively lax and interbank loan rates are high. In equilibrium, only high productivity banks borrow in the market and the reallocation of deposits across banks is relatively efficient.

But, for the same parameters, a crisis equilibrium can exist as well. During a crisis, lenders are pessimistic about bank quality. The volume of inside collateral is low, borrowing restrictions are tight, and lending rates are low. Relatively low productivity banks borrow so that the interbank market is much less efficient in the reallocation of deposits. The reduction in interbank flows is more pronounced in loans backed by inside collateral than in loans backed by outside collateral.

The disruption of financial reallocation during a crisis has real effects as well. In a crisis, banks and firms in high productivity matches are unable to attract the flow of funds they would receive in normal times. The aggregate economy is less productive: output, consumption and welfare are lower.

Though the model is purposefully simple, structured to capture the importance of expectations and the interaction of inside and outside collateral, it is capable of matching some key features of the crisis. This includes the reduction in the interbank loan rate, the fall in the flow of funds financed by inside collateral relative to outside collateral and the consequent fall in output.

7 Appendix

From the definition of $H(\bar{\varepsilon})$ in (23), one gets:

$$H'(\bar{\varepsilon}) > 0 \Leftrightarrow \underbrace{\left(\frac{\gamma}{\lambda\theta} - \bar{\varepsilon} \right) \frac{\bar{\varepsilon}g(\bar{\varepsilon})}{1 - G(\bar{\varepsilon})}}_{\chi(\bar{\varepsilon})} < \frac{\gamma}{\lambda\theta}.$$

Given Assumption 1 and relation (20), it is easy to see that $\chi(\bar{\varepsilon}) = 0$ when $\bar{\varepsilon} = 0$ and $\chi(\bar{\varepsilon}) \rightarrow +\infty$ as $\bar{\varepsilon} \nearrow 1$. It follows that $H(\bar{\varepsilon})$ reaches at least one maximum over $[0, 1]$. Moreover,

$$\chi'(\bar{\varepsilon}) > 0 \Leftrightarrow \gamma > \lambda\theta + \lambda\theta(2\bar{\varepsilon} - 1) - \frac{(\gamma - \lambda\theta\bar{\varepsilon})(g'(\bar{\varepsilon})(1 - G(\bar{\varepsilon})) + \bar{\varepsilon}g(\bar{\varepsilon})^2)}{g(\bar{\varepsilon})(1 - G(\bar{\varepsilon}))}.$$

A sufficient condition for the above inequality to hold true is $\gamma \gg \lambda\theta$, *e.g.* $\gamma > 2\lambda\theta$. Under Assumptions 1 and 2, one therefore has $\chi'(\bar{\varepsilon}) > 0 \forall \bar{\varepsilon} \in [0, 1]$, which guarantees that $H(\bar{\varepsilon})$ is hump-shaped and admits one unique maximum in $\bar{\varepsilon}^{\max} \in (0, 1)$.

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