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OPTIMAL GOVERNMENT DEBT MATURITY

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ABSTRACT

This paper develops a model of optimal government debt maturity in which the government cannot issue state-contingent bonds and cannot commit to fiscal policy. If the government can perfectly commit, it fully insulates the economy against government spending shocks by purchasing short-term assets and issuing long-term debt. These positions are quantitatively very large relative to GDP and do not need to be actively managed by the government. Our main result is that these conclusions are not robust to the introduction of lack of commitment. Under lack of commitment, large and tilted positions are very expensive to finance ex-ante since they exacerbate the problem of lack of commitment ex-post. In contrast, a flat maturity structure minimizes the cost of lack of commitment, though it also limits insurance and increases the volatility of fiscal policy distortions. We show that the optimal maturity structure is nearly flat because reducing average borrowing costs is quantitatively more important for welfare than reducing fiscal policy volatility. Thus, under lack of commitment, the government actively manages its debt positions and can approximate optimal policy by confining its debt instruments to consols.

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1 Introduction

How should government debt maturity be structured? Two seminal papers by Angeletos (2002) and Buera and Nicolini (2004) argue that the maturity of government debt can be optimally structured so as to completely hedge the economy against fiscal shocks. This research concludes that optimal debt maturity is tilted long, with the government purchasing short-term assets and selling long-term debt. These debt positions allow the market value of outstanding government liabilities to decline when spending needs and short-term interest rates increase. Moreover, quantitative exercises imply that optimal government debt positions, both short and long, are large (in absolute value) relative to GDP. Finally, these positions are constant and do not need to be actively managed since the combination of constant positions and fluctuating bond prices delivers full insurance.

In this paper, we show that these conclusions are sensitive to the assumption that the government can fully commit to fiscal policy. In practice, a government chooses taxes, spending, and debt sequentially, taking into account its outstanding debt portfolio, as well as the behavior of future governments. Thus, a government can always pursue a fiscal policy which reduces (increases) the market value of its outstanding (newly-issued) liabilities ex-post, even though it would not have preferred such a policy ex-ante. We show that once the lack of commitment by the government is taken into account, it becomes costly for the government to use the maturity structure of debt to completely hedge the economy against shocks; there is a tradeoff between the cost of funding and the benefit of hedging.¹ Our main result is that, under lack of commitment, the optimal maturity structure of government debt is quantitatively nearly flat and is actively managed by the government.

We present these findings in the dynamic fiscal policy model of Lucas and Stokey (1983). This is an economy with public spending shocks and no capital in which the government chooses linear taxes on labor and issues public debt to finance government spending. Our model features two important frictions. First, as in Angeletos (2002) and Buera and Nicolini (2004), we assume that state-contingent bonds are unavailable, and that the government can only issue real non-contingent bonds of all maturities. Second, and in contrast to Angeletos (2002) and Buera and Nicolini (2004), we assume that the government lacks commitment to policy. We focus on the Markov Perfect Competitive Equilibrium in which the government dynamically chooses its policies at every date as a function of payoff relevant variables: the fiscal shock and its outstanding debt position at various maturities.

Neither of these frictions on its own leads to any inefficiency, since debt maturity can be structured so as to address each friction separately. First, the work of Angeletos (2002) and Buera and Nicolini (2004) shows that, even in the absence of contingent bonds, an optimally structured portfolio of non-contingent bonds can perfectly insulate the government from all

¹Our framework is consistent with an environment in which the legislature sequentially chooses a primary deficit and the debt management office sequentially minimizes the cost of financing subject to future risks, which is what is done in practice (see the IMF report, 2001).

shocks to the economy. Second, the work of Lucas and Stokey (1983) shows that, even in the absence of commitment by the government, an optimally structured portfolio of contingent bonds can perfectly induce a government without commitment to pursue the ex-ante optimally chosen policy ex-post. While each of these two frictions in isolation is irrelevant for welfare, the combination of the two leads to a non-trivial tradeoff between market completeness and commitment in the government's choice of maturity.

To get a sense of the importance of each friction, it is useful to consider for simplicity environments which feature only one friction and illustrate how maturity structure can be designed to address each friction separately. Suppose for instance that the government lacks commitment, but there are no shocks, implying that the government does not need to worry about insurance. In this case, the ex-ante optimal policy under full commitment is perfectly smooth taxation. Moreover, there are many maturity structures under full commitment which can lead to this perfectly smooth outcome. However, under lack of commitment, a government today can only guarantee commitment to this smooth policy by future governments by choosing a flat maturity structure. A tilted debt position would cause a future government to deviate from the optimal smooth path. Suppose, for example, that a future government were to choose policy while entering the period with zero short-term debt and positive long-term debt. Rather than pursuing smooth taxation as it is supposed to, the government has an incentive to deviate to a non-smooth tax policy which reduces short-term consumption and increases long-term consumption. This deviation benefits the government by reducing the market value of its outstanding liabilities (by increasing short-term interest rates).² Analogously, if a future government chooses policy while entering the period with positive short-term debt and zero long-term debt, then it has an incentive to deviate to a non-smooth tax policy which increases short-term consumption and decreases long-term consumption. This deviation benefits the government by increasing the market value of its newly issued short-term liabilities (by reducing short-term interest rates). Thus, only a flat maturity structure can guarantee that taxes remain smooth, since in this case the government does not have any beneficial deviation ex-post (i.e., any deviation's marginal effect on the market value of outstanding debt equals that on the market value of newly issued debt).

Importantly, while a flat maturity structure minimizes the cost of lack of commitment, this cost would increase the larger and more tilted are the debt positions. Large and tilted positions are very costly to finance ex-ante if the government cannot commit to policy ex-post. To see why, note that based on our above discussion, the larger and more tilted the debt position, the greater a future government's benefit from pursuing a non-smooth tax policy ex-post to relax its budget constraint. Households purchasing government bonds internalize ex-ante the fact that the government will choose ex-post non-smooth tax policies relative to those under commitment.

²Our observation that long-term debt positions lead to lower fiscal discipline is consistent with other arguments in the literature on debt maturity (see Missale and Blanchard, 1994; Missale et al., 2002; Chatterjee and Eyigungor, 2012; and Broner et al., 2013).

As a consequence, ex-ante, they require a greater premium for lending to the government. For example, if households are primarily buying long-term bonds ex-ante, then they appropriately anticipate that the government lacking commitment will pursue future policies which increase future short-term interest rates, thereby diluting their claims. In this case, households require a higher ex-ante interest rate (relative to commitment) to induce them to lend long-term to the government. An analogous reasoning holds if households are primarily buying short-term bonds ex-ante. Therefore, even though the maturity of debt issuance does not affect the cost of financing under full commitment, under lack of commitment, larger and more tilted debt positions are more expensive to finance.

To understand how the maturity can be used to reduce the cost of lack of insurance, suppose for simplicity now that the government has full commitment, but there are fiscal shocks against which the government must insure. The optimal policy under commitment uses debt to smooth taxation in the presence of these shocks. If fully contingent claims were available, there would be many maturity structures which would support the optimal policy. However, if the government only has access to non-contingent claims, then there is a unique maturity structure which replicates full insurance. As has been shown in Angeletos (2002) and Buera and Nicolini (2004), such a maturity structure is tilted in a manner which guarantees that the market value of outstanding government liabilities declines when the net present value of future government primary surpluses also declines. If this occurs when short-term interest rates rise—as is the case in quantitative examples—then the optimal maturity structure requires that the government purchases short-term assets and sells long-term debt. Because interest rate movements are small quantitatively, the tilted debt positions required for hedging are large. In such an environment, constraining the government to issuing a flat debt maturity (i.e., in the form of a consol) is costly. The reason is that the market value of debt does not fluctuate enough to provide full insurance, and this induces more volatility in fiscal policy distortions than would be achieved under perfect insurance.

Thus, a flat maturity structure minimizes the cost of commitment, whereas a large and tilted maturity structure minimizes the cost of volatility. In the presence of both lack of commitment and lack of insurance, the government faces a tradeoff. If it chooses a large and tilted debt position—as it would under full commitment—it would reduce the volatility of fiscal policy distortions, but because of lack of commitment, such large and tilted positions would be very expensive to finance and would entail large average tax distortions. To explore where the government positions itself in this tradeoff, we simulate a two-shock economy with both frictions in which the government issues a one-year bond and a consol, and we characterize optimal policy.

Our main result is that, under lack of commitment, the optimal maturity structure of government debt is quantitatively nearly flat. In our benchmark simulation, the short-term bond (the one-year bond plus the annual consol payout due in one year) is 2.20% of GDP and the market value of the consol is 59.7% of GDP, with annual payouts equal to 2.21% of GDP. Thus, the optimal maturity structure is essentially flat, and optimal policy under lack of commitment

can be approximated with a consol.³

This result contrasts with the case of full commitment, in which the short-term bond is -2690% of GDP and the market value of the consol is 2760% of GDP, with annual payouts equal to 101.8% of GDP. Moreover, in contrast to the case of full commitment, we find that under lack of commitment, debt is actively managed, and both taxes and debt are volatile and respond persistently to fiscal shocks.

Our quantitative result emerges because of the combination of two forces. First, substantial hedging requires massive tilted debt positions, as has been shown in Angeletos (2002) and Buera and Nicolini (2004). Due to their size, financing these positions can be very expensive in terms of average tax distortions because of the lack of commitment by the government. Second, under empirically plausible levels of volatility of public spending, the cost of lack of insurance under a flat maturity structure is small. Therefore, the optimal policy pushes in the direction of reducing *average* tax distortions versus reducing the *volatility* of tax distortions, and the result is a nearly flat maturity structure. Thus, in the presence of lack of commitment by the government, optimal government debt policy can be approximated by active consol management.⁴

This paper is connected to several literatures. As discussed, we build on the work of Angeletos (2002) and Buera and Nicolini (2004) by introducing lack of commitment.⁵ In this regard, our work is related to that of Arellano and Ramanarayanan (2012) and Aguiar and Amador (2013), but in contrast to this work, we ignore the possibility of default and focus purely on lack of commitment to taxation and debt issuance. Our work is also complementary to that of Arellano et al. (2013), but in contrast to this work, we ignore the presence of nominal frictions and the lack of commitment to monetary policy. In this regard, our work is most applicable to economies in which the risks of default and surprise inflation are not salient, but the government is still not committed to a path of deficits and debt maturity issuance.⁶

More broadly, our paper is also tied to the literature on optimal fiscal policy which explores the role of non-contingent debt and lack of commitment. A number of papers have studied optimal policy under full commitment but non-contingent debt, such as Barro (1979) and Aiyagari

³Though this policy prescription differs from current practice in advanced economies, it has been pursued historically, most notably by the British government in the Industrial Revolution, when consols were the largest component of the British government's debt during this time period (see Mokyr (2011)). Confining debt issuance to consols is also a policy which receives some support in the popular press (e.g., Leitner and Shapiro (2013) and Yglesias (2013)).

⁴It should be mentioned that the conclusion that the welfare benefit of smoothing economic shocks is small relative to that of raising economic levels is more generally tied to the insight in Lucas (1987).

⁵Additional work explores government debt maturity while continuing to maintain the assumption of full commitment. Shin (2007) explores optimal debt maturity when there are fewer debt instruments than states. Faraglia et al. (2010) explore optimal debt maturity in environments with habits, productivity shocks, and capital. Lustig et al. (2008) explore the optimal maturity structure of government debt in an economy with nominal rigidities. Guibaud et al. (2013) explore optimal maturity structure in a preferred habitat model.

⁶Chari and Kehoe (1993a,b) and Sleet and Yeltekin (2006) also consider the lack of commitment under full insurance, though they focus on settings which allow for default. Niepelt (2008) also focuses on default risk. Alvarez et al. (2004) and Persson et al. (2006) consider problems of commitment in an environment with long-term debt where the possibility of surprise inflation arises.

et al. (2002).⁷ As in this work, we find that optimal taxes respond persistently to economic shocks, though in contrast to this work, this persistence is due to the lack of commitment by the government as opposed to the ruling out of long-term government bonds. Other work has studied optimal policy in settings with lack of commitment, but with full insurance (e.g., Krusell et al., 2006 and Debortoli and Nunes, 2013). We depart from this work by introducing long-term debt, which in a setting with full insurance implies that the lack of commitment friction no longer introduces any inefficiencies.

Our paper proceeds as follows. In Section 2, we describe the model. In Section 3, we define the equilibrium and characterize it recursively. In Section 4, we review the optimal policy in the absence of lack of commitment and lack of insurance. In Section 5, we discuss the cost of lack of commitment in a deterministic environment, and we show that this cost increases the larger and more tilted is the debt maturity. In Section 6, we discuss the cost of lack of insurance in an environment with full commitment, and we show that this cost is small for empirically plausible volatilities of public spending. In Section 7, we combine lack of commitment and lack of insurance and perform our main quantitative exercise and present our main results. Section 8 concludes and the Appendix provides all of the proofs and additional results not included in the text.

2 Model

2.1 Environment

We consider an economy identical to that of Lucas and Stokey (1983) with two modifications. First, we rule out state-contingent bonds. Second, we assume that the government cannot commit to fiscal policy. There are discrete time periods $t = \{1, \dots, \infty\}$ and a stochastic state $s_t \in S$ which follows a first-order Markov process. s_0 is given. Let $s^t = \{s_0, \dots, s_t\} \in S^t$ represent a history, and let $\pi(s^{t+k}|s^t)$ represent the probability of s^{t+k} conditional on s^t for $t+k \geq t$.

There is a continuum of mass 1 of identical households that derive the following utility:

$$\mathbb{E} \sum_{t=0}^{\infty} \beta^t (u(c_t, n_t) + \theta_t(s_t) v(g_t)), \beta \in (0, 1). \quad (1)$$

c_t is consumption, n_t is labor, and g_t is government spending. $u(\cdot)$ is strictly increasing in consumption and strictly decreasing in labor, globally concave, and continuously differentiable. $v(\cdot)$ is strictly increasing, concave, and continuously differentiable. Under this representation, $\theta_t(s_t)$ is high (low) when public spending is more (less) valuable. In contrast to the model of Lucas and Stokey (1983), we have allowed g_t in this framework to be chosen by the government, as opposed to being exogenously determined. We allow for this possibility to also consider that

⁷See also Farhi (2010).

the government may not be able to commit to the ex-ante optimal level of public spending. In our exercises, we consider commitment to taxes and commitment to spending separately, so our analysis subsumes the Lucas and Stokey (1983) environment in which there is no discretion over government spending.

Household wages are normalized to 1 and are taxed at a linear tax rate τ_t . $b_t^{t+k} \geq 0$ represents government debt purchased by a representative household at t , which is a promise to repay 1 unit of consumption at $t+k > t$, and q_t^{t+k} is its price at t . At every t , the household's allocation $\{c_t, n_t, \{b_t^{t+k}\}_{k=1}^{\infty}\}$ must satisfy the household's dynamic budget constraint

$$c_t + \sum_{k=1}^{\infty} q_t^{t+k} (b_t^{t+k} - b_{t-1}^{t+k}) = (1 - \tau_t) n_t + b_{t-1}^t. \quad (2)$$

$B_t^{t+k} \geq 0$ represents debt issued by the government at t with a promise to repay 1 unit of consumption at $t+k > t$. At every t , government policies $\{\tau_t, g_t, \{B_t^{t+k}\}_{k=1}^{\infty}\}$ must satisfy the government's dynamic budget constraint

$$g_t + B_{t-1}^t = \tau_t n_t + \sum_{k=1}^{\infty} q_t^{t+k} (B_t^{t+k} - B_{t-1}^{t+k}). \quad (3)$$

The economy is closed and bonds are in zero net supply:

$$b_t^{t+k} = B_t^{t+k} \quad \forall t, k, \quad (4)$$

which combined with (2) and (3) implies that

$$c_t + g_t = n_t. \quad (5)$$

Initial debt $\{B_{-1}^{k-1}\}_{k=1}^{\infty}$ is exogenous.⁸ We assume that there exist debt limits to prevent Ponzi schemes:

$$B_t^{t+k} \in [\underline{B}, \overline{B}]. \quad (6)$$

We let \underline{B} be sufficiently low and \overline{B} be sufficiently high so that (6) does not bind in our theoretical and quantitative exercises.

A key friction in this environment is the absence of state-contingent debt, since the value of outstanding debt B_t^{t+k} is independent of the realization of the state s_{t+k} . If state-contingent bonds were available, then at any date t , the government would own a portfolio of bonds

⁸Our model implicitly allows the government to buy back the long-term bonds from the private sector. While ruling out bond buybacks is interesting, a 2012 survey by the OECD found that 85 percent of countries conduct some form of bond buyback and 32 percent of countries conduct them on a regular basis (see the OECD report by Blommestein et al., 2012). Note furthermore, that even if bond buyback is not allowed in our environment, a government can replicate the buyback of a long-term bond by purchasing an asset with a payout on the same date (see Angeletos, 2002).

$\left\{ \left\{ B_{t-1}^{t+k} | s^{t+k} \right\}_{s^{t+k} \in S^{t+k}} \right\}_{k=0}^{\infty}$, where the value of each bond payout at date $t+k$ would depend on the realization of a history of shocks $s^{t+k} \in S^{t+k}$. In our discussion, we will refer back to this complete market case.

The government is benevolent and shares the same preferences as the households in (1). We assume that the government cannot commit to policy and therefore chooses taxes, spending, and debt sequentially.

3 Markov Perfect Competitive Equilibrium

3.1 Definition of Equilibrium

We consider a Markov Perfect Competitive Equilibrium (MPCE) in which the government must optimally choose its preferred policy at every date as a function of current payoff-relevant variables. The government takes into account that its choice affects future debt and thus affects the policies of future governments. Households rationally anticipate these future policies, and their expectations are in turn reflected in current bond prices. Thus, in choosing policy today, a government anticipates that it may affect current bond prices by impacting expectations over policy in the future.

Formally, let $\mathbf{B}_t \equiv \left\{ B_t^{t+k} \right\}_{k=1}^{\infty}$ and $\mathbf{q}_t \equiv \left\{ q_t^{t+k} \right\}_{k=1}^{\infty}$. In every period t , the government enters the period and chooses a policy $\{\tau_t, g_t, \mathbf{B}_t\}$ given $\{s_t, \mathbf{B}_{t-1}\}$. Households then choose an allocation $\left\{ c_t, n_t, \left\{ b_t^{t+k} \right\}_{k=1}^{\infty} \right\}$. An MPCE consists of: a government strategy $\rho(s_t, \mathbf{B}_{t-1})$ which depends on (s_t, \mathbf{B}_{t-1}) ; a household allocation strategy $\omega((s_t, \mathbf{B}_{t-1}), \rho_t, \mathbf{q}_t)$ which depends on (s_t, \mathbf{B}_{t-1}) , the government policy $\rho_t = \rho(s_t, \mathbf{B}_{t-1})$, and bond prices \mathbf{q}_t ; and a set of bond pricing functions $\left\{ \varphi^k(s_t, \mathbf{B}_{t-1}, \rho_t) \right\}_{k=1}^{\infty}$ with $q_t^{t+k} = \varphi^k(s_t, \mathbf{B}_{t-1}, \rho_t) \forall k > 1$ which depend on (s_t, \mathbf{B}_{t-1}) and the government policy $\rho_t = \rho(s_t, \mathbf{B}_{t-1})$. In an MPCE, these objects must satisfy the following conditions $\forall t$:

1. The government strategy maximizes (1) given (s_t, \mathbf{B}_{t-1}) , $\omega(\cdot)$, $\varphi^k(\cdot) \forall k > 1$, and the government budget constraint (3),
2. The household allocation strategy maximizes (1) given (s_t, \mathbf{B}_{t-1}) , ρ_t , \mathbf{q}_t , $\rho(\cdot)$, $\varphi^k(\cdot) \forall k > 1$, and the household budget constraint (2), and
3. The set of bond pricing functions satisfy (4) given $\rho(\cdot)$ and $\omega(\cdot)$.

While we have assumed for generality that the government can freely choose taxes, spending, and debt in every period, we focus throughout our draft on the cases in which the government does not have discretion in either setting spending or in setting taxes. These special cases highlight how the right choice of government debt maturity can induce future governments to choose the commitment policy. The exact manner in which we do this is described in Section 5.

3.2 Primal Approach

Any MPCE must be a competitive equilibrium. We follow Lucas and Stokey (1983) by taking the primal approach to the characterization of competitive equilibria since this allows us to abstract away from bond prices and taxes. Let

$$\left\{ \left\{ c_t(s^t), n_t(s^t), g_t(s^t) \right\}_{s^t \in S^t} \right\}_{t=0}^{\infty} \quad (7)$$

represent a stochastic sequence, where the resource constraint (5) implies

$$c_t(s^t) + g_t(s^t) = n_t(s^t). \quad (8)$$

We can establish necessary and sufficient conditions for (7) to constitute a competitive equilibrium. The household's optimization problem implies the following intratemporal and intertemporal conditions, respectively:

$$1 - \tau_t(s^t) = -\frac{u_{n,t}(s^t)}{u_{c,t}(s^t)} \text{ and } q_t^{t+k}(s^t) = \frac{\sum_{s^{t+k} \in S^{t+k}} \beta^k \pi(s^{t+k}|s^t) u_{c,t+k}(s^{t+k})}{u_{c,t}(s^t)}. \quad (9)$$

Substitution of these conditions into the household's dynamic budget constraint implies the following condition:

$$\begin{aligned} u_{c,t}(s^t) c_t(s^t) + u_{n,t}(s^t) n_t(s^t) + \sum_{k=1}^{\infty} \sum_{s^{t+k} \in S^{t+k}} \beta^k \pi(s^{t+k}|s^t) u_{c,t+k}(s^{t+k}) B_t^{t+k}(s^t) = \\ \sum_{k=0}^{\infty} \sum_{s^{t+k} \in S^{t+k}} \beta^k \pi(s^{t+k}|s^t) u_{c,t+k}(s^{t+k}) B_{t-1}^{t+k}(s^{t-1}). \end{aligned} \quad (10)$$

Forward substitution into the above equation taking into account the absence of Ponzi schemes implies the following implementability condition:

$$\begin{aligned} \sum_{k=0}^{\infty} \sum_{s^{t+k} \in S^{t+k}} \beta^k \pi(s^{t+k}|s^t) \left(u_{c,t+k}(s^{t+k}) c_{t+k}(s^{t+k}) + u_{n,t+k}(s^{t+k}) n_{t+k}(s^{t+k}) \right) = \\ \sum_{k=0}^{\infty} \sum_{s^{t+k} \in S^{t+k}} \beta^k \pi(s^{t+k}|s^t) u_{c,t+k}(s^{t+k}) B_{t-1}^{t+k}(s^{t-1}). \end{aligned} \quad (11)$$

By this reasoning, if a stochastic sequence in (7) is generated by a competitive equilibrium, then it necessarily satisfies (8) and (11). We prove in the Appendix that the converse is also true, which leads to the below proposition that is useful for the rest of our analysis.

Proposition 1 (*competitive equilibrium*) *A stochastic sequence (7) is a competitive equilibrium if and only if it satisfies (8) $\forall s^t$ and $\exists \left\{ \left\{ B_{t-1}^{t+k}(s^{t-1}) \right\}_{k=0}^{\infty} \right\}_{s^{t-1} \in S^{t-1}}^{\infty}$ which satisfy*

(11) $\forall s^t$.

A useful corollary to this proposition concerns the relevant implementability condition in the presence of state-contingent bonds, $B_t^{t+k}|s^{t+k}$, which provide payment at $t+k$ conditional on the realization of a history s^{t+k} .

Corollary 1 *In the presence of state-contingent debt, a stochastic sequence (7) is a competitive equilibrium if and only if it satisfies (8) $\forall s^t$ and (11) for $s^t = s^0$ given initial liabilities.*

If state-contingent debt is available, then the satisfaction of (11) at s^0 guarantees the satisfaction of (11) for all other histories s^t , since state-contingent payments can be freely chosen so as to satisfy (11) at all future histories s^t .

3.3 Recursive Representation of MPCE

We can use the primal approach to represent an MPCE recursively. Recall that $\rho(s_t, \mathbf{B}_{t-1})$ is a policy which depends on (s_t, \mathbf{B}_{t-1}) , and that $\omega((s_t, \mathbf{B}_{t-1}), \rho_t, \mathbf{q}_t)$ is a household allocation strategy which depends on (s_t, \mathbf{B}_{t-1}) , government policy $\rho_t = \rho(s_t, \mathbf{B}_{t-1})$, and bond prices \mathbf{q}_t , where these bond prices depend on (s_t, \mathbf{B}_{t-1}) and government policy. As such, an MPCE in equilibrium is characterized by a stochastic sequence in (7) and a debt sequence $\left\{ \left\{ \left\{ B_t^{t+k}(s^t) \right\}_{k=1}^{\infty} \right\}_{s^t \in S^t} \right\}_{t=0}^{\infty}$, where each element depends only on s^t through (s_t, \mathbf{B}_{t-1}) , the payoff relevant variables. Given this observation, in an MPCE, one can define a function $h^k(\cdot)$

$$h^k(s_t, \mathbf{B}_t) = \beta^k \mathbb{E}[u_{c,t+k}|s_t, \mathbf{B}_t] \quad (12)$$

for $k \geq 1$, which equals the discounted expected marginal utility of consumption at $t+k$ given (s_t, \mathbf{B}_t) at t . This function is useful since, in choosing \mathbf{B}_t at date t , the government must take into account how it affects future expectations of policy which in turn affect current bond prices through expected future marginal utility of consumption.

Note furthermore that choosing $\{\tau_t, g_t, \mathbf{B}_t\}$ at date t is equivalent to choosing $\{c_t, n_t, g_t, \mathbf{B}_t\}$ from the perspective of the government, and this follows from the primal approach delineated in the previous section. Thus, we can write the government's problem recursively as

$$V(s_t, \mathbf{B}_{t-1}) = \max_{c_t, n_t, g_t, \mathbf{B}_t} \left\{ u(c_t, n_t) + \theta_t(s_t) v(g_t) + \beta \sum_{s_{t+1} \in S} \pi(s_{t+1}|s_t) V(s_{t+1}, \mathbf{B}_t) \right\} \quad (13)$$

s.t.

$$c_t + g_t = n_t, \quad (14)$$

$$u_{c,t}(c_t - B_{t-1}^t) + u_{n,t}n_t + \sum_{k=1}^{\infty} h^k(s_t, \mathbf{B}_t) (B_t^{t+k} - B_{t-1}^{t+k}) = 0, \quad (15)$$

where (15) is a recursive representation of (10). Let $f(s_t, \mathbf{B}_{t-1})$ correspond to the solution to (13) – (15) given $V(\cdot)$ and $h^k(\cdot)$. It therefore follows that the function $f(\cdot)$ necessarily implies a function $h^k(\cdot)$ which satisfies (12). An MPCE is therefore composed of functions $V(\cdot)$, $f(\cdot)$, and $h^k(\cdot)$ which are consistent with one another and satisfy (12) – (15).

4 Full Commitment and Full Insurance Benchmark

Before considering how the lack of commitment and lack of insurance interact in our framework, it is useful to first characterize optimal policy in the absence of these two frictions. To facilitate exposition, in this section as well as in Sections 5 and 6, we assume that the government has zero initial debt liabilities. In the quantitative exercise of Section 7, we relax this assumption and take into account that the government has non-zero initial liabilities.

Given Corollary 1, in the presence of state-contingent debt and full commitment, optimal policy solves the following program:

$$\max_{\{c_t(s^t), n_t(s^t), g_t(s^t)\}_{s^t \in S^t}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \sum_{s^t \in S^t} \beta^t \pi(s^t | s^0) (u(c_t(s^t), n_t(s^t)) + \theta_t(s_t) v(g_t(s^t))) \quad (16)$$

$$\text{s.t. (8) } \forall s^t \text{ and (11) for } s^t = s^0. \quad (17)$$

Letting μ correspond to the Lagrange multiplier on the implementability constraint (11), first order conditions yield:

$$u_{c,t}(s^t) = -u_{n,t}(s^t) + \left[\frac{\mu}{1 + \mu} \begin{pmatrix} -(u_{cc,t}(s^t) + u_{cn,t}(s^t)) c_t(s^t) \\ -(u_{nn,t}(s^t) + u_{cn,t}(s^t)) n_t(s^t) \end{pmatrix} \right] \text{ and} \quad (18)$$

$$\theta_t(s_t) v_{g,t}(s^t) = u_{c,t}(s^t) \left[(1 + \mu) + \mu \frac{u_{cc,t}(s^t) c_t(s^t) + u_{cn,t}(s^t) n_t(s^t)}{u_{c,t}(s^t)} \right]. \quad (19)$$

Proposition 2 (Lucas-Stokey benchmark) *In the presence of full commitment and full insurance, optimal policy satisfies (18) and (19) for a given μ .*

There are two important points to keep in mind from this benchmark. First, (18) and (19) imply that in the presence of full insurance, taxes and spending are independent of history and depend only on the state s_t . This follows from the presence of perfect insurance markets. As in Lucas and Stokey (1983), we assume that the program in (16) – (17) is globally concave so that first order conditions are sufficient to characterize the optimum. This implies that we can define

$$\{c^*(s_t), n^*(s_t), g^*(s_t), \tau^*(s_t)\}_{s_t \in S} \quad (20)$$

as the levels of consumption, labor, spending, and taxes which depend only on the state s_t in

the full insurance and full commitment benchmark. We will refer back to these quantities when discussing the cost of lack of commitment and lack of insurance.

A second point to notice about the solution to the program is that (18) and (19) imply the possible presence of fiscal policy distortions. For example, if the term in brackets in (18) is positive, then $u_{c,t}(s^t) > -u_{n,t}(s^t)$, which means that the marginal utility of consumption exceeds the marginal utility of leisure and taxes are positive, which is distortive. In other words, if given the opportunity, the government would prefer to reduce these distortions by increasing consumption and increasing labor. Analogously, if the term in brackets in equation (19) exceeds 1, then $\theta_t(s_t)v_{g,t}(s^t) > u_{c,t}(s^t)$, which means that the marginal utility of public spending exceeds the marginal utility of consumption. Thus, if given the opportunity, the government may choose to increase public spending while reducing consumption. The government's desire to reduce distortions is useful to bear in mind since it comes into play once we take into account the government's incentives to deviate from the commitment policy.

We now move to illustrate how lack of commitment and lack of insurance alter the equilibrium described in Proposition 2. In Sections 5 and 6, we show how, under only one friction, maturity can be structured to fully alleviate the cost of the friction. In Section 7, we combine the two frictions and present our main results.

5 Cost of Lack of Commitment

In this section, we consider the problem of the lack of commitment by the government in a deterministic environment in which the maturity structure can be designed to address this problem. We show that the optimal maturity structure in this case is flat. We then show how choosing a tilted maturity structure can entail welfare losses in this setting, and we do this using a simple three-period example with a quantitative exercise.

Before proceeding, we want to note that we consider the problem of lack of commitment to taxes and lack of commitment to spending separately. We do this both to remain in line with the model of Lucas and Stokey (1983), which does not allow any discretion over the choice of spending, and also to explore the role of maturity when there is discretion over spending. We will say that the government *lacks commitment to taxes* if the government chooses taxes and debt sequentially but it cannot alter spending which is exogenously set at $g_t(s^t) = g^*(s_t)$ defined in (20) (i.e., the state-contingent level of spending under commitment). Analogously, we will say that the government *lacks commitment to spending* if the government chooses spending and debt sequentially but it cannot alter taxes which are exogenously set at $\tau_t(s^t) = \tau^*(s_t)$ (i.e., the state-contingent level of taxes under commitment). The situation in which the government *lacks commitment to taxes and spending* corresponds to the situation in which taxes, spending, and debt are chosen sequentially. As we will see, our main results hold as long as the government lacks commitment to at least one instrument.

5.1 Flat Maturity Solves Lack of Commitment

Consider a deterministic environment in which $s_0 \neq s_1$ and $s_t = s_1 \forall t \geq 1$, with $\theta_0(s_0) = \theta^H > \theta_1(s_1) = \theta^L$. As in Section 4, suppose there are zero initial government debt liabilities for expositional simplicity. In this environment, the government at date 0 finances a one time spending increase by issuing debt into the future, and the marginal value of spending remains constant in all future dates.

To see how maturity can be used to solve the problem of lack of commitment, consider first the program under full commitment. From Proposition 2, it is clear that the optimal policy sets some policy $\{c_0^*, n_0^*, g_0^*, \tau_0^*\}$ at $t = 0$ followed by a constant policy $\{c_1^*, n_1^*, g_1^*, \tau_1^*\}$ for $t \geq 1$, and this is because the state of the economy does not change from $t = 1$ onward. The government issues some debt at date 0 to finance a spending increase, and in all future periods, it runs a constant primary surplus to spread out the repayment of that debt. Under commitment, this policy sequence can be implemented with a large number of different debt maturities issued at date 0. To understand the intuition, consider the implementability condition (11) at date 1 in this deterministic economy. It is clear that a large number of different maturity structures can satisfy the right hand side of (11) given an optimal consumption and labor sequence $\{c_1^*, n_1^*\}$ which pins down the left hand side. Clearly, one such maturity structure is flat. Specifically, let

$$\bar{B} = \tau_1^* - g_1^* \text{ for } t \geq 1, \quad (21)$$

where \bar{B} represents the primary surplus in the full commitment optimum from $t = 1$ onward. The government can thus satisfy the implementability constraint with a flat maturity with $B_0^t = \bar{B} \forall t \geq 1$.

Can the government lacking commitment choose the optimal commitment policy ex-post? The answer to this question is yes, if the government only lacks commitment to either taxes or to spending (but not both), and this can be ensured by issuing debt with a flat maturity structure at date 0. To gain an intuition to this result, suppose that a government entering the period at $t = 1$ were to perform a one-time reevaluation of policy, fully committing to its new policy sequence thereafter. If it were entering date 1 with a flat maturity structure, it would solve the following problem:

$$\max_{\{c_t, n_t, g_t\}_{t=0}^{\infty}} \sum_{t=1}^{\infty} \beta^{t-1} (u(c_t, n_t) + \theta^L v(g_t)) \quad (22)$$

s.t.

$$c_t + g_t = n_t \quad \forall t, \text{ and} \quad (23)$$

$$\sum_{t=1}^{\infty} \beta^{t-1} (u_{c,t}(c_t - \bar{B}) + u_{n,t} n_t) = 0. \quad (24)$$

Under lack of commitment to taxes, such a government faces the additional constraint that

$$g_t = g_1^*, \quad (25)$$

and under lack of commitment to spending, such a government faces the additional constraint that

$$-\frac{u_{n,t}}{u_{c,t}} = (1 - \tau_1^*). \quad (26)$$

It is clear that under both lack of commitment to taxes and lack of commitment to spending, the tradeoffs faced by the government are the same in all future periods, and if the problem is globally concave, then the government will choose a constant allocation and policy. Moreover, it can be shown that this constant allocation and policy correspond to the optimum under commitment. In other words, a flat maturity induces the government to choose the ex-ante optimum ex-post.

This insight generalizes further to a situation in which the government reoptimizes in all future periods as it would in an MPCE. This is expressed formally in the below proposition.

Proposition 3 (*optimality of flat maturity*) *In the deterministic environment with $s_0 \neq s_1$ and $s_t = s_1 \forall t \geq 1$, there exists an MPCE in which $B_0^t = B_0^{t+k} \forall t \geq 1$ and $k \geq 0$ where the allocation under the MPCE is identical to the commitment optimum if*

1. *There is lack of commitment to taxes and the program in (22) – (24) and (25) is globally concave, or*
2. *There is lack of commitment to spending and the program in (22) – (24) and (26) is globally concave.*

In the absence of commitment to both taxes and to spending, it is not possible to solve the problem of lack of commitment by using debt maturity because the government has too many tools at its discretion. While a government reevaluating policy under a flat maturity would choose a smooth policy—as it does in the example above—it may not necessarily choose the same smooth policy as it would have preferred ex-ante.⁹ In our quantitative exercises, we do consider the possibility that the government lacks commitment to both taxes and to spending, and we can show that, though a flat maturity structure does not remove the cost of lack of commitment altogether, it does minimize this cost and is therefore also optimal.¹⁰

5.2 Three-Period Example

We turn to a simple three-period example to provide further intuition for the result in Proposition 3. The advantage of this example is that it can be easily solved by hand and it highlights the

⁹See Rogers (1989) for more discussion.

¹⁰In principle, there are many possible MPCEs, and our result characterizes one such MPCE. It should be noted however that the MPCE which we characterize also corresponds to the limit of the finite period economy with end date T as $T \rightarrow \infty$.

reasons for the theoretically very high welfare costs of choosing a large and tilted maturity structure under lack of commitment. The disadvantage of this example is that it is too stylized to be able to make quantitative statements, and in the next subsection we consider such a computed example to calculate the welfare cost of lack of commitment.

Let $t = 0, 1, 2$, $\theta_0 = \theta^H$, and $\theta_1 = \theta_2 = \theta^L < \theta^H$. Suppose that the government lacks commitment to spending and that taxes and labor are exogenously fixed to some τ and n , respectively, so that the government collects a constant revenue in all dates.¹¹ Assume that the government's welfare can be represented by

$$\sum_{t=0,1,2} \beta^t ((1 - \psi) \log c_t + \psi \theta_t g_t) \quad (27)$$

for $\psi \in [0, 1]$. We consider the limiting case in which $\psi \rightarrow 1$. In this environment, the government does not have any discretion over tax policy, and any ex-post deviation by the government is driven by a desire to increase spending since the marginal benefit of additional spending always exceeds the marginal benefit of consumption.

5.2.1 The Optimality of a Flat Maturity

We now present the main result in Proposition 3 in the context of this model and discuss the intuition for it. The analog of the implementability condition at date 0 in (10) can be written as a weak inequality constraint (since it binds in the optimum):

$$\frac{c_0 - n(1 - \tau)}{c_0} + \beta \frac{B_0^1}{c_1} + \beta^2 \frac{B_0^2}{c_2} \geq 0 \quad (28)$$

which after substitution yields the analog of (11):

$$\frac{c_0 - n(1 - \tau)}{c_0} + \beta \frac{c_1 - n(1 - \tau)}{c_1} + \beta^2 \frac{c_2 - n(1 - \tau)}{c_2} \geq 0. \quad (29)$$

The optimum under commitment maximizes (27) subject to $c_t + g_t = n \forall t$ and (29), which leads to the following optimality conditions:

$$\left(\frac{\theta^H}{\theta^L} \right)^{1/2} c_0 = c_1 = c_2 = n(1 - \tau) + \bar{B} \quad (30)$$

for some \bar{B} which satisfies (29) when it binds.

In this situation, households lend to the government at $t = 0$ to finance the initial spending surge, and the government provides them with net transfer of \bar{B} at $t = 1$ and $t = 2$. Clearly,

¹¹Such a situation would prevail for example if taxes are constant and the underlying preferences satisfy those of Greenwood et al. (1988).

under full commitment, any date 0 issuance of $\{B_0^1, B_0^2\}$ satisfying

$$B_0^1 + \beta B_0^2 = \overline{B} (1 + \beta) \quad (31)$$

can implement the equilibrium, where we have taken into account that the one-period bond price at date 1 is β .

However, this is not the case under lack of commitment. In this case, one must use backward induction to solve for the behavior of the government. At $t = 2$, the government's policy is trivially determined by the implementability condition which sets $c_2 = n(1 - \tau) + B_1^2$. At $t = 1$, the government takes this future policy into account, and given outstanding debt $\{B_0^1, B_0^2\}$, it maximizes its continuation welfare subject to the resource constraint and the date 1 implementability condition, (11), which can be written as a weak inequality constraint:

$$\frac{c_1 - n(1 - \tau)}{c_1} + \beta \frac{c_2 - n(1 - \tau)}{c_2} \geq \frac{B_0^1}{c_1} + \beta \frac{B_0^2}{c_2}. \quad (32)$$

It can be shown that in this case, optimal government policy satisfies

$$\frac{c_1}{c_2} = \left(\frac{n(1 - \tau) + B_0^1}{n(1 - \tau) + B_0^2} \right)^{1/2}. \quad (33)$$

Equation (33) implies that if $B_0^1 < (>) B_0^2$, then $c_0^1 < (>) c_0^2$. As such, it is only when debt maturity is flat with $B_0^1 = B_0^2$ that this guarantees that $c_0^1 = c_0^2$, so that the solution under lack of commitment in (33) coincides with that under full commitment in (30). The government at date 0 takes this outcome into account and therefore chooses a flat maturity structure so as to guarantee that the date 1 government will follow the ex-ante optimal policy.

Lemma 1 *In the three-period example, the unique MPCE is identical to the commitment optimum and the optimal debt maturity sets $B_0^1 = B_0^2$.*

Why does a flat maturity structure induce the commitment solution? A way to see this is to consider the fact that the date 1 government—which cares only about raising spending—would like to reduce the market value of what it owes to the private sector which from the intertemporal condition can be represented by

$$B_0^1 + \beta \frac{c_1}{c_2} B_0^2. \quad (34)$$

Moreover, the government would also like to increase the market value of newly issued debt which can be represented by

$$\beta \frac{c_1}{c_2} B_1^2. \quad (35)$$

If debt maturity were tilted toward the long end, then the date 1 government would deviate from a smooth policy so as to reduce the value of what it owes. For example, suppose that $B_0^1 = 0$ and $B_0^2 = \overline{B} (1 + \beta) / \beta$. Clearly, under commitment, it would be possible to achieve

the optimum under this debt arrangement. However, under lack of commitment, (33) implies that the government deviates from the smooth ex-ante optimal policy by choosing $c_0^1 < c_0^2$. This deviation, which is achieved by issuing higher levels of debt B_1^2 relative to commitment, serves to reduce the value of what the government owes in (34), therefore freeing up resources to be utilized for additional spending at date 1.

Analogously, if debt maturity were tilted toward the short end, then the government would deviate from a smooth policy so as to increase the value of what it issues. For example, suppose that $B_0^1 = \bar{B}(1 + \beta)$ and $B_0^2 = 0$. As in the previous case, this debt arrangement would implement the optimum under commitment. However, rather than choosing the ex-ante optimal smooth policy, the date 1 government lacking commitment chooses policy according to (33) with $c_0^1 > c_0^2$. This deviation, which is achieved by issuing lower levels of debt B_1^2 relative to commitment, serves to increase the value of what the government issues in (34), therefore freeing up resources to be utilized for additional spending at $t = 2$.

It is only when $B_0^1 = B_0^2 = \bar{B}$ that there are no gains from deviation. In this case, it follows from (30) that $B_1^1 = B_0^2$, and therefore any deviation's marginal effect on the market value of outstanding debt is perfectly outweighed by its effect on the market value of newly issued debt. For this reason, a flat debt maturity structure induces commitment.¹²

5.2.2 What is the Welfare Cost of Lack of Commitment?

One way to investigate this question is to consider the source of the welfare losses which would ensue if the government were to choose some debt maturity structure $\{B_0^1, B_0^2\}$ satisfying (31) (so that it is optimal under commitment) but with $B_0^1 \neq B_0^2$ so that it does not coincide with the MPCE and does not induce the optimal commitment solution under lack of commitment. One can show that the further apart are B_0^1 and B_0^2 in this situation, the lower is $q_0^1 B_0^1 + q_0^2 B_0^2$, and therefore, the higher is the cost of financing for the government from a suboptimal debt maturity policy.

Lemma 2 *Suppose that the date 0 government chooses some $\{B_0^1, B_0^2\}$ satisfying (31) and that the date 1 government chooses policy given $\{B_0^1, B_0^2\}$ according to (33). The higher is $|B_0^1 - B_0^2|$, the lower is $q_0^1 B_0^1 + q_0^2 B_0^2$.*

This lemma states that, even though borrowing costs do not respond to debt maturity in the case of full commitment—that is, holding $B_0^1 + \beta B_0^2$ fixed—borrowing costs do increase under

¹²One naturally wonders how the conclusions of this example would change if $\theta_1 \neq \theta_2$. In this case, using the analogous condition to (33), it can be shown that the MPCE again coincides with the full commitment optimum under a flat maturity with $B_0^1 = B_0^2$. For example, suppose $\theta_1 > \theta_2$. The date 1 government is then not only concerned about the market value of what it owes and what it issues, but it also has a desire to borrow in order to increase spending at date 1 (at the current short-term interest rate). This desire is offset by the fact that borrowing more would raise short-term interest rates, thus deteriorating the government's financial position since $B_1^2 > B_0^2$, as is implied by $B_0^1 = B_0^2$. Note that the optimality of a flat maturity if $\theta_1 \neq \theta_2$ is not generally true theoretically in other environments, but it is approximately true quantitatively in the computed examples of this paper. Most importantly for our purpose, our result that the quantitative cost of lack of commitment is large under a highly tilted debt maturity does not depend on $\theta_1 = \theta_2$.

lack of commitment the further apart are B_0^1 and B_0^2 . Therefore, a large and highly tilted debt position can be very costly to finance.

To understand this lemma, recall that at $t = 1$, the government pursues policies which reduce the market value of outstanding debt and increase the market value of newly issued debt. More specifically, the date 1 government is interested in relaxing the implementability condition (32) by reducing the right hand side of (32) as much as possible. This is why if $B_0^1 < (>) B_0^2$, it chooses to set $c_1 < (>) c_2$. If B_0^1 and B_0^2 are very different, then there is a greater scope for a deviation from a smooth policy at $t = 1$. For example, suppose that $B_0^1 < B_0^2$ with $c_1 < c_2$ chosen according to (33). Clearly, if B_0^1 were to be reduced by some $\epsilon > 0$ and B_0^2 increased by ϵ/β so as to keep (31) satisfied, then (32) could be relaxed even further, and the date 1 government would choose a policy which further reduces the right hand side of (32).

The greater scope for deviation ex-post is very costly from an ex-ante perspective. This is because if the right hand side of (32) is lower, then the left hand side of (28) is also lower. *Therefore, by relaxing the implementability condition at date 1, the date 1 government is tightening the implementability condition at date 0, which directly reduces the ex-ante welfare at date 0.*

The simple example of the previous subsection yields some generalizable insights regarding the welfare cost of lack of commitment. A government lacking commitment at date 1 will always choose to deviate ex-post in order to relax its budget constraint. By relaxing the budget constraint, the government can reduce any ex-post distortions which were imposed ex-ante, either on the consumption-spending tradeoff or on the consumption-leisure tradeoff. The way the government relaxes the budget constraint is by pursuing policies which increase consumption in the same direction as the maturity of outstanding debt. This reduces the market value of what the government owes while increasing the market value of what the government issues. Importantly, the scope for deviation is larger if outstanding debt is highly tilted.

Households lending to the government ex-ante at date 0 anticipate this behavior from the government. Thus, they expect their consumption to be higher (relative to the case of full commitment) in future periods when currently issued debt is due. Specifically, they anticipate that the future government will take measures to reduce the market value of what they are owed or increase the cost to them of any future newly-issued savings. As such, households at date 0 require higher interest rates (relative to the case of full commitment) to induce them to lend to the government. For example, if households are primarily buying long-term bonds ex-ante, then they appropriately anticipate that the government lacking commitment will pursue future policies which increase future short-term interest rates, thereby diluting their claims. In this case, households require a higher ex-ante interest rate (relative to commitment) to induce them to lend long-term to the government. An analogous reasoning holds if households are primarily buying short-term bonds ex-ante. Therefore, the larger and more tilted are the debt positions, the costlier they are to finance under lack of commitment.

In sum, even if large and tilted positions entail no additional funding costs in the case of full commitment, in the case of lack of commitment, they significantly increase the cost of

borrowing. This tightens the date 0 budget of the government and therefore leads to larger date 0 distortions, either on the consumption-spending tradeoff or on the consumption-leisure tradeoff. For this reason, a flat debt maturity minimizes the funding costs of the government and minimizes the cost of lack of commitment.

5.3 Quantitative Cost of Lack of Commitment

In the previous subsection, we presented a deterministic example in which the optimal maturity structure is flat. We also showed that a tilted maturity structure—which would otherwise be optimal under full commitment—leads to rising funding costs the more tilted is this structure. In this subsection, we explore how large these costs are quantitatively.

To do so, we compute a three-period example following the same parametrization as in Chari et al. (1994). More specifically, we set the per period payoff of households to

$$\frac{c_t^{1-\sigma_c} - 1}{1 - \sigma_c} + \eta \frac{(1 - n_t)^{1-\sigma_l} - 1}{1 - \sigma_l} + \theta_t \log g_t, \quad (36)$$

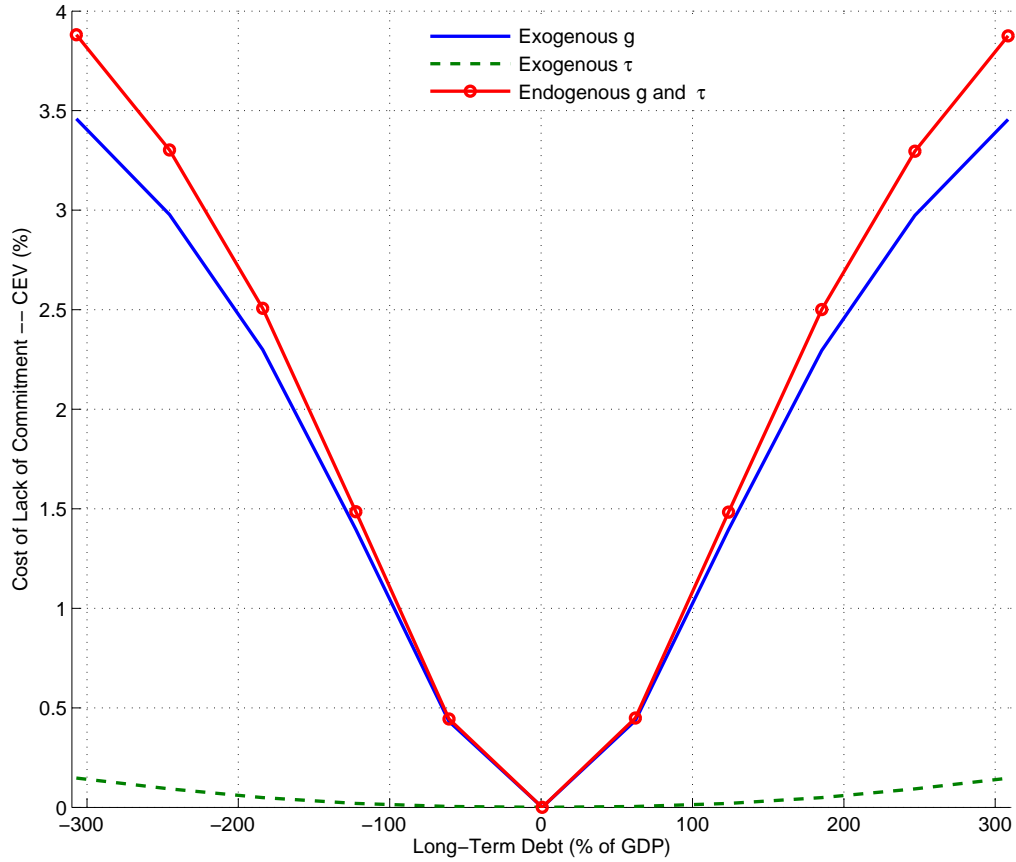
with $\sigma_c = \sigma_l = 1$. As in the three-period example of the previous subsection, we assume that $\theta_0 = \theta^H$ and $\theta_1 = \theta_2 = \theta^L$. We make the following parametric assumptions: $\beta = 0.9644$, $\eta = 3.33$, $\theta^L = 0.2195$, and $\theta^H = 0.2360$. Our choice of β implies that each period can be interpreted as representing a year. The choice of η implies that hours worked $n = 0.23$ under commitment at dates 1 and 2, and this coincides with the steady state value in Chari et al. (1994). Our choice of θ^L and θ^H implies that in the commitment benchmark, $g/y = 0.18$ at dates 1 and 2 which matches the steady state value in Chari et al. (1994). Moreover, the value of g_0 is 7% above g_1 , and 7% matches the standard deviation of g in Chari et al. (1994).

This model is analogous to that of the previous subsection. The government issues debt at date 0 in order to finance a spending increase at date 0. Under full commitment, there are many debt maturities which are consistent with the optimal policy. More specifically, any such debt maturities $\{B_0^1, B_0^2\}$ must satisfy (31) for some \bar{B} appropriate in this example. However, under lack of commitment, only a debt maturity which sets $B_0^1 = B_0^2$ is optimal.

How costly is it to choose a tilted maturity under lack of commitment? To answer this question, we consider the date 0 value of social welfare subject to the date 0 government choosing a combination of $\{B_0^1, B_0^2\}$ satisfying (31) in three different environments: (i) the government cannot commit to taxes (so spending is exogenous and fixed at the commitment optimum), (ii) the government cannot commit to spending (so taxes are exogenous and fixed at the commitment optimum), and (iii) the government cannot commit to taxes and spending (both taxes and expenditure are endogenous).

Figure 1 measures the welfare cost of a tilted debt maturity in each of these scenarios. The x-axis represents the value of B_0^2 as a fraction of GDP (with GDP measured at date 0 in the commitment optimum). Thus, the axis represents a long-term debt to output ratio. Clearly,

Figure 1: Maturity Structure and the Cost of Lack of Commitment



Notes: The figure shows the cost of lack of commitment of different maturity structures – i.e. changing the quantity of long-term debt (x-axis) while keeping constant the value of total debt issued in period 0. The welfare costs (y-axis) are expressed as percentage points of consumption equivalent variation (CEV) with respect to the commitment case, for three cases: no-commitment to taxes (solid line), no-commitment to expenditure (dashed line), and no-commitment to both taxes and expenditure (line with dots).

the higher this value, the lower is B_0^1 relative to output, and this follows from the fact that the values of $\{B_0^1, B_0^2\}$ satisfy (31). The y-axis captures the welfare cost associated with this suboptimal policy. It compares welfare under full commitment (which is constant in all maturity scenarios) to that under lack of commitment, and it represents the loss in total consumption required under full commitment to make households indifferent between a government with full commitment and a government with lack of commitment.

The figure shows that a flat maturity structure, which occurs for values of long-term debt which are low and close to zero (equal to 0.74% GDP in the simulation), has a welfare cost of zero in the cases of exogenous spending and of exogenous taxes. What the figure shows is that the further away the debt maturity structure moves from flatness, the greater the welfare cost. For example, if long-term debt exceeds 100% of GDP (implying that short-term debt is negative and also large relative to GDP), then the welfare cost of a tilted maturity structure with exogenous spending exceeds 1% of consumption. At this level of debt, the cost in a model with exogenous taxes is lower, but it is still non-negligible and equal to 0.01% of GDP.¹³ The welfare cost of lack of commitment to both taxes and spending is always higher than lack of commitment to either instrument on its own since the problem of lack of commitment gets compounded. Interestingly, this cost is also minimized at a nearly flat maturity structure, though the cost can never be exactly zero, since the government always has enough tools to deviate from the commitment optimum.

In sum, this quantitative example shows that the cost of lack of commitment increases if the tilt of debt maturity also rises. Moreover, the welfare cost can be significant if the debt to GDP ratio exceeds 100%. This observation is useful to keep in mind, since, in the presence of shocks, highly tilted debt positions are required to achieve significant hedging, as we will see in the next section.

6 Cost of Lack of Insurance

We now consider an environment in which there are fiscal shocks but the government has full commitment. The purpose of this section is to illustrate how debt maturity can be utilized to fully insulate an economy from shocks, and to highlight how doing so entails the government choosing debt positions which are highly tilted and large relative to GDP. We then move to consider how constraining the government's maturity choice to a flat one with consols affects welfare, and we show that doing so does not lead to large welfare losses quantitatively.

Recall the necessary and sufficient conditions for a competitive equilibrium in an economy with non-contingent debt as expressed in Proposition 1. These conditions are more stringent from those prevailing in an economy with state-contingent debt, which are expressed in Corollary 1. We review here the key result in Theorem 1 of Angeletos (p. 1114) which proves that *any*

¹³In this example, the reason that the welfare cost is relatively lower is that in the utility function (36), the distortion implied by (19) in the case of full commitment is zero.

allocation under state-contingent debt can be approximately implemented with non-contingent debt.

Proposition 4 (*Angeletos*) *Any allocation $\left\{ \left\{ c_t(s^t), n_t(s^t), g_t(s^t) \right\}_{s^t \in S^t} \right\}_{t=0}^{\infty}$ which can be implemented with state-contingent debt can be either implemented with non-contingent debt or be approximated arbitrarily well with non-contingent debt.*

Corollary 2 *If such an allocation is history-independent, then it can be implemented or approximated arbitrarily well with a time-invariant non-contingent debt position.*

The economic intuition for the proposition is that there are more bond instruments than shocks in our environment, and therefore, it is possible to use these instruments to achieve full insurance. In other words, fluctuations in the market value of the outstanding non-contingent debt can replicate the state-contingent payments under complete markets. Formally, given a stochastic allocation generated under complete markets, one can choose non-contingent debt positions so as to satisfy the right hand side of (11) under incomplete markets. This is possible as long as there is some fluctuation in the term structure of interest rates as a function of the shocks, even if that fluctuation is arbitrarily small. The fact that this fluctuation in interest rates can be achieved with infinitesimally small changes in policy explains why a full insurance equilibrium can be approximated arbitrarily well.

The corollary to this proposition follows from the fact that if an allocation is history-independent, and therefore only depends on the contemporaneous shock, then this also means that effective state-contingent payments and the term structure of interest rates are also history independent. In other words, the left hand side of (11) is a function of the shock, and the marginal utilities of consumption used to evaluate the right hand side of (11) are also only a function of the shocks. It thus follows that (11) can hold with a time-invariant debt position with a market value that is a function of the shock.

What the proposition and corollary imply is that, under full commitment, the optimal policy under state-contingent bonds can be implemented under non-contingent bonds with a time-invariant debt position. The combination of constant positions and fluctuating relative bond prices at different maturities delivers full insurance.

The most straightforward way to see this result is to turn to the simple three-period economy of Section 5.3. Specifically, let $t = 0, 1, 2$ and let preferences satisfy (36) with $\sigma_c = \sigma_l = 1$ and $\eta = 3.33$. Moreover, shocks satisfy $\theta_0 = \theta^H$, $\theta_1 = \theta^H$ with probability $1/2$, $\theta_1 = \theta^L < \theta^H$ with probability $1/2$, and $\theta_2 = \theta^L$. This is therefore an economy in which the government has to finance high spending at $t = 0$, and then faces a 50% probability of having to incur high spending again at $t = 1$. There is certainty at $t = 2$ that spending will be low. There are two differences between this example and that of Section 5.3: First, there is uncertainty at $t = 1$, and second, the government has full commitment. One can show in this case that the optimal

policy can replicate the commitment solution under full insurance by setting

$$B_0^1 < 0 \text{ and } B_0^2 > 0,$$

meaning that the government purchases short-term assets and sells long-term debt at $t = 0$.

Why does such a maturity structure provide full insurance? Consider the allocation at $t = 1$ and $t = 2$ under full insurance. From our discussion in Section 4, the optimal policy under full insurance assigns an allocation—that is, a level of consumption, labor, and public spending—which depends only on the current shock. As such, let $\{c^i, n^i, g^i, \tau^i\}$ correspond to the relevant policy under shock $i = H, L$. It is straightforward to argue in this case that $g^H > g^L$, since the marginal product of public spending is higher in the high state. Moreover, one can also show that $\tau^i n^i - g^i$ is lower when the fiscal shock is high, which comes from the fact that the government is more willing to run a deficit when the marginal value of public spending is high. In a complete market economy with state-contingent bonds, the government is able to offset this temporary increase in the deficit with a state-contingent payment it receives from households. The insight behind Proposition 4 is that such a state-contingent payment can be replicated with a capital gain on the government's bond portfolio in an economy without state-contingent debt.

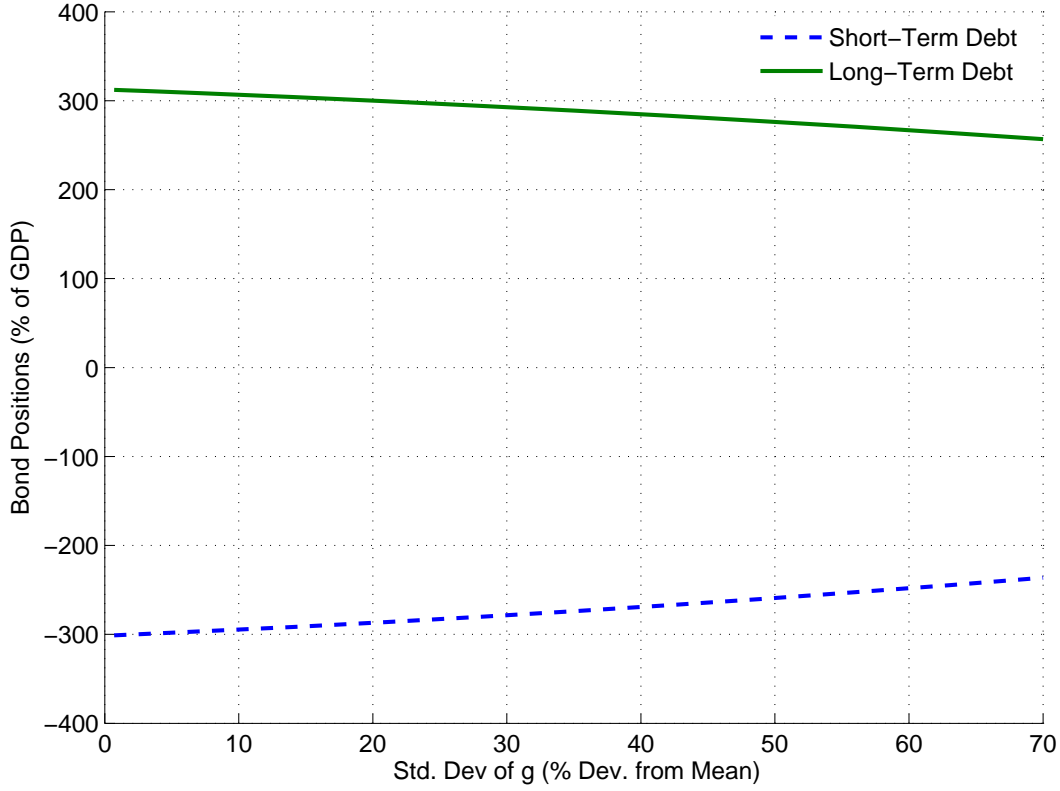
To see why, note that it can also be shown that in the optimal policy, $c^H < c^L$, since higher spending crowds out consumption whenever the fiscal shock is high. Since consumption at $t = 2$ is independent of the shock (since $\theta_2 = \theta^L$ independently of the value of θ_1), it follows that the bond price for a one-period bond at $t = 1$, which from (9) is represented by $\beta c_1/c_2$, is lower if the shock is high. Therefore, under a high fiscal shock, the deficit is larger and short-term interest rates are higher. This implies that the market value of the government's bond portfolio represented by

$$B_0^1 + \beta \frac{c_1}{c_2} B_0^2$$

declines as long as the government has issued long-term debt at date 0 ($B_0^2 > 0$). It is by this logic that the capital gain and loss on the government's bond portfolio serve to replicate state-contingent payments. The reason that the government purchases short-term assets at date 0 is to be able to buy back some of outstanding long-term debt at date 1. If the date 1 fiscal shock is high and the government needs resources, it will be able to buy this debt back at a lower price.

Figure 2 depicts the size of these debt positions in different fiscal scenarios. We initialize θ^L and θ^H so that g_1 is on average 18% of GDP, with a standard deviation of 7%, which correspond to the values used in Chari et al. (1994). We then progressively increase the standard deviation of θ around the mean, so as to increase the volatility of g_1 . The x-axis in the figure provides the percent standard deviation of government spending from its mean. The y-axis provides the size of B_0^1 and B_0^2 relative to total output n_0 under optimal policy. The figure shows that for the entire range of standard deviations from 0% to 70%, the debt positions of the government are massive. For example, for the standard deviation in public spending of 7%, which prevails in the United States sample from 1988 to 2013, the government chooses a negative short-term debt

Figure 2: Debt Positions Providing Full Insurance



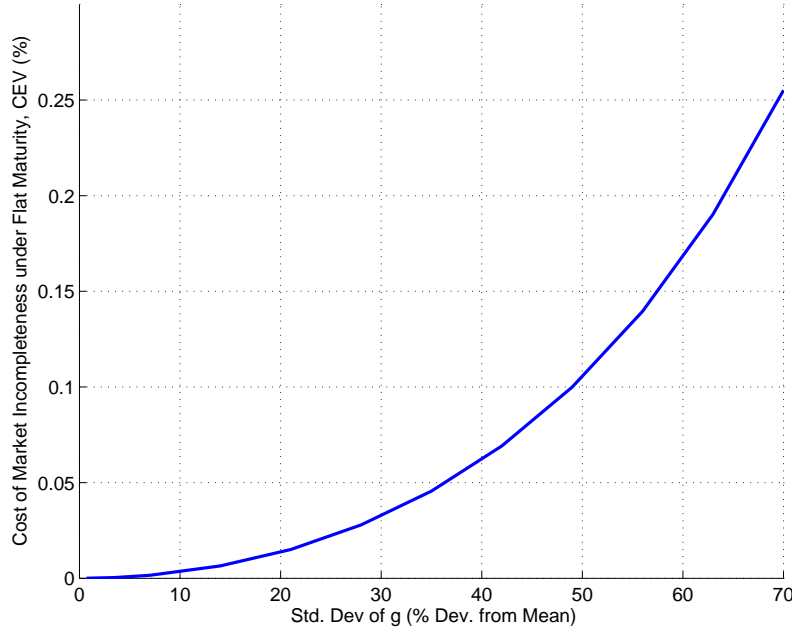
Notes: The figure shows the short-term (dashed lined) and long-term (solid line) debt positions required to provide full-insurance, as a function of the standard deviation of the underlying shock.

position of about -300% of GDP and a positive long-term debt position of a similar magnitude. These large positions, which are consistent with the results of Angeletos (2002) and Buera and Nicolini (2004), are due to the fact that the variation in short-term interest rates captured by the variation in c_1/c_2 in the efficient equilibrium is not substantial enough to be able to facilitate full hedging with smaller debt positions.¹⁴

What is the welfare benefit of choosing these massively tilted debt positions? One way to explore this question is to compare welfare under the optimal policy to the best possible equilibrium in a scenario in which the government is constrained to issuing consols, that is, a

¹⁴This observation explains why full hedging requires smaller positions as the volatility in public spending rises. Though an increase in public spending volatility implies an increase in the volatility of the deficit (which would push towards larger debt positions), it also implies an increase in the volatility of the short-term interest rate (which would push towards smaller debt positions). The figure shows that the second effect dominates in this simulation.

Figure 3: The Costs of Flat Maturity



Notes: The figure shows the welfare costs of issuing a flat maturity structure as a function of the standard deviation of the underlying shock.

scenario in which an additional constraint on the program of the government is that $B_0^1 = B_0^2$. Figure 3 depicts the welfare cost of choosing this suboptimal policy as a function of the volatility of public spending, where the volatility is calculated in different fiscal scenarios as described previously.¹⁵ The welfare cost on the y-axis represents the percent sacrifice in total consumption required to make households indifferent between a government choosing the optimal policy with heavily tilted debt positions and a government choosing the optimal policy subject to the constraint that it can only issue consols.

The figure shows that the welfare cost rises with the volatility in public spending. This is intuitive, since the entire purpose of choosing a tilted maturity structure is to insure against risk. If the risk is zero for instance, a government under commitment is indifferent between a tilted and a flat maturity structure and faces no loss in being constrained to consols. In the figure, for standard deviations of public spending below 10% (recall that the standard deviation in United States data is about 7%), the welfare cost is below 0.004% of consumption.

This exercise suggests that the welfare cost of lack of insurance is significantly smaller than the welfare cost of lack of commitment. To see this, recall that significant hedging in this

¹⁵For this exercise, the values of g_t are the same in the constrained and unconstrained equilibrium.

economy requires positive and negative debt positions, each well in excess of 100% of GDP. As we showed in Figure 1 in Section 5.3, if such positions were chosen in a deterministic economy under lack of commitment, the welfare cost relative to a flat maturity structure would be in excess of 1% of consumption. In contrast, what the current exercise shows is that if a flat debt maturity structure is chosen in a stochastic economy with full commitment, the welfare cost relative to the optimal tilted maturity structure is less than 0.004% of consumption for all reasonable volatilities of public spending.

In other words, the cost of lack of commitment is of a higher order of magnitude (at least 250 times larger in our example) than the cost of lack of insurance. To understand the intuition for this, recall that in the three-period economy, the cost of lack of commitment comes in the form of large fiscal policy distortions at date 0 required to finance large debt position. In contrast, the cost of lack of insurance comes in the form of more volatile fiscal policy distortions at dates 1 and 2. What our quantitative exercise implies is that increasing the level of distortions is more costly for welfare than increasing the volatility of distortions. In the next section, we explore what this insight implies for optimal policy in a fully dynamic economy, when both lack of commitment and lack of insurance interact.

7 Quantitative Exercise

In this section, we consider the quantitative implications of our model in an infinite horizon economy in which there is both lack of commitment and lack of insurance. Since there are as many bond maturities as dates in this environment, the state space is infinite, and this problem is very complicated to compute. To simplify the analysis, we assume that there are two shocks θ^L and $\theta^H > \theta^L$ which follow a first order Markov process, each with a symmetric persistence. Furthermore, we reduce the set of tradeable bonds in a manner analogous to the work of Woodford (2001) and Arellano et al. (2013). Namely, we consider an economy with two types of bonds: a one-period bond and a perpetuity with decaying coupons.

Let $B_{t-1}^S \gtrless 0$ denote the value of the one-period bond issued by the government at $t - 1$. Moreover, let $B_{t-1}^L \gtrless 0$ denote the value of the per period coupon associated with the perpetuity issued by the government at $t - 1$. It follows that the dynamic budget constraint of the government (3) simplifies to

$$g_t + B_{t-1}^S + B_{t-1}^L = \tau_t n_t + q_t^S B_t^S + q_t^L (B_t^L - \gamma B_{t-1}^L).$$

This budget constraint takes into account that at date t , the government makes a flow payoff to households equal to $B_{t-1}^S + B_{t-1}^L$ according to their holdings of one-period bonds and perpetuities; it issues one-period bonds B_t^S at price q_t^S ; and it exchanges non-decayed perpetuities γB_{t-1}^L for new perpetuities B_t^L at price q_t^L , where $\gamma \in (0, 1]$. The household's budget constraint (2) can be analogously modified given this set of tradeable bonds and given the market clearing condition

(4):

$$c_t + q_t^S B_t^S + q_t^L (B_t^L - \gamma B_{t-1}^L) = (1 - \tau_t) n_t + B_{t-1}^S + B_{t-1}^L.$$

In this economy, we define short-term bonds at t as representing the value of all payouts to households due at t , that is $B_{t-1}^S + B_{t-1}^L$. The level of the outstanding long-term bond can be described by its annuity value, B_{t-1}^L , or by its market value $q_t^L \gamma B_{t-1}^L$. Finally, the market value of all debt is $B_{t-1}^S + B_{t-1}^L + q_t^L \gamma B_{t-1}^L$.

There are two important points to note about this economy. First, because the number of bond instruments equals the number of shocks, the arguments of Section 6 imply that, under full commitment, the full insurance optimum can be achieved despite the absence of state-contingent bonds. As such, the cost of lack of insurance is zero in the presence of full commitment. Second, if $\gamma = 1$ so that the perpetuity does not depreciate, then in the absence of any shocks, the full commitment optimum can be implemented even under lack of commitment (to either taxes or to spending individually), and this holds because of the availability of a consol instrument and the arguments in Section 5. Therefore, the cost of lack of commitment is zero in the presence of full insurance. Given these two observations, and given that we are interested in looking at inefficiencies which arise from the *interaction* of incomplete markets and lack of commitment, we focus our quantitative exercise on the case with $\gamma = 1$, so that the long-term bond takes the form of a consol.¹⁶

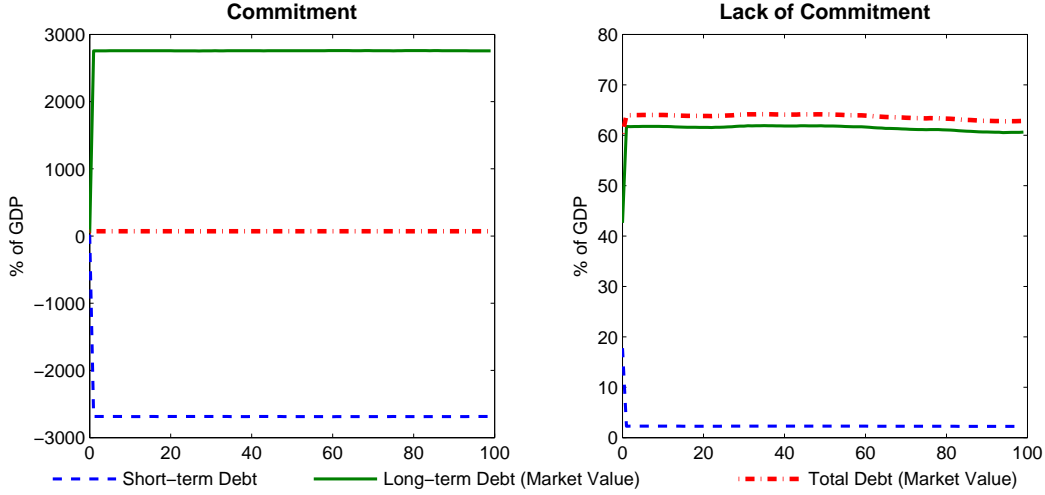
In computing the MPCE, we focus on an MPCE in which the value and policy functions are differentiable. We cannot guarantee theoretically that this MPCE is unique, but we have verified that our computational algorithm converges to the same policy starting from a large grid of many different initial guesses.¹⁷ Our benchmark simulation makes the following parametric assumptions. We let $\beta = 0.9644$ so that a period is interpreted as representing a year, with a riskless rate of 4% in a deterministic economy. We consider the same functional form and parameters value for the utility function described in Section 5.3 (i.e. $\sigma_c = \sigma_l = 1$ and $\eta = 3.33$). We let $\theta^H = 0.2360$ and $\theta^L = 0.2042$, and we choose the persistence of the Markov process so that in the full commitment optimum, the process of expenditure coincides with the simulation of Chari et al. (1994), where the average value of g/y equal to 18% of GDP, and the volatility of public spending equal to 7%, with a persistence of 0.89. We choose initial conditions $B_{-1}^S = 0.041$ and $B_{-1}^L = 0.0035$ to roughly match the US statistics for the period 1988-2013, with an average market value of total debt of 60% of GDP, out of which 28% has maturity of less than one year.¹⁸ We choose $\theta_0 = \theta^H$, and our characterization of optimal policy under lack of commitment is robust to choosing $\theta_0 = \theta^L$ instead.

¹⁶If $\gamma < 1$, then analogous arguments to those of Debortoli and Nunes (2013)—who analyze a deterministic economy with a one-period bond—imply that the government debt positions are driven towards approximately zero. Figure A-1 in the Appendix illustrates a simulation of an economy in which $\gamma = 0.5$ and shows that this is the case.

¹⁷Further details regarding our computational method are available in the Appendix Section A-3.

¹⁸This calculation ignores off-balance sheet liabilities, such as unfunded mandatory spending obligations which are significantly more long-term. Taking this additional debt into account and changing initial conditions would not change our main conclusion that the optimal debt maturity under lack of commitment is nearly flat.

Figure 4: Debt Positions with and without Commitment



Notes: The figure shows the optimal debt positions with commitment (left panel) and without commitment (right panel). For the no-commitment case we report averages across 1000 simulations.

We begin by considering an economy in which the government lacks commitment to taxes and has no discretion in setting public spending. Figure 4 displays the path of short-term debt, long-term debt, and total debt, all relative to GDP. The left panel shows the path of these quantities under full commitment. It shows that from $t \geq 1$ onward, the value of short-term debt is -2690% of GDP and the market value of long-term debt is 2760% of GDP, with an overall debt position equal to 70% of GDP. These large and highly tilted quantities are consistent with the analysis of Angeletos (2002) and Buera and Nicolini (2004) and of the three-period model in Section 6. These debt positions are not actively managed and are constant over time.¹⁹

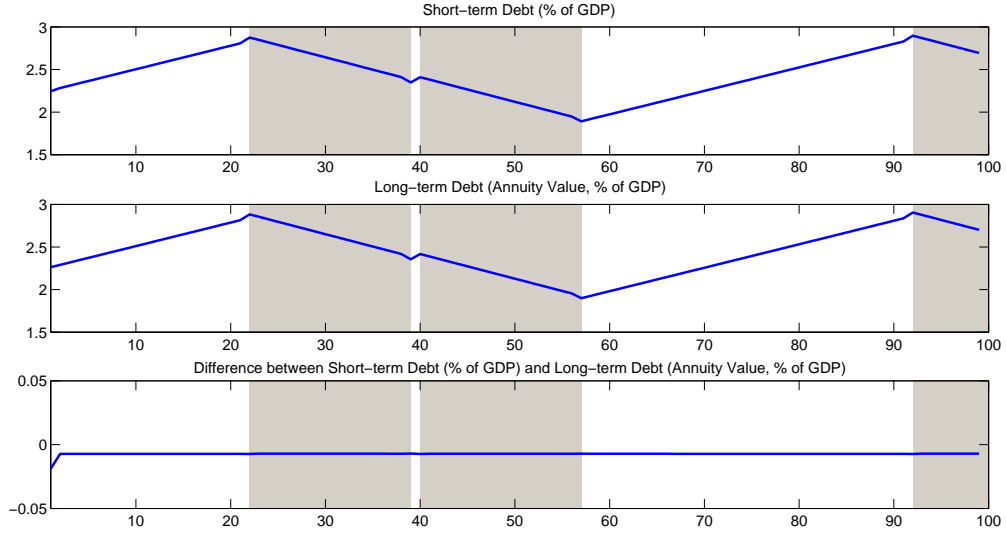
The right panel considers the economy under lack of commitment, and in this scenario debt is actively managed from $t \geq 1$ onward. Since it is actively managed, we plot the average value of debt for each time period taken from 1000 simulations. Between $t = 1$ and $t = 100$, the average value of short-term debt is 2.2% of GDP and the average market value of long-term debt is 59.7% of GDP, with an average overall debt position equal to 61.9% of GDP.²⁰ The average annuity value of the long-term debt is 2.21% of GDP, which is very close to that of the short-term debt, so the optimal maturity structure is essentially flat.

Figure 5 considers an equilibrium sequence of shocks to illustrate the active management of debt under lack of commitment. The top two panels of the figure show that the level of

¹⁹These quantitative magnitudes significantly exceed those in the i.i.d. example of Section 6 because of the persistence of the fiscal shock.

²⁰We calculate the average starting from $t = 1$ rather than $t = 0$ since the simulation suggests that debt quickly jumps towards its long-run average between $t = 0$ and $t = 1$.

Figure 5: Active Debt Management

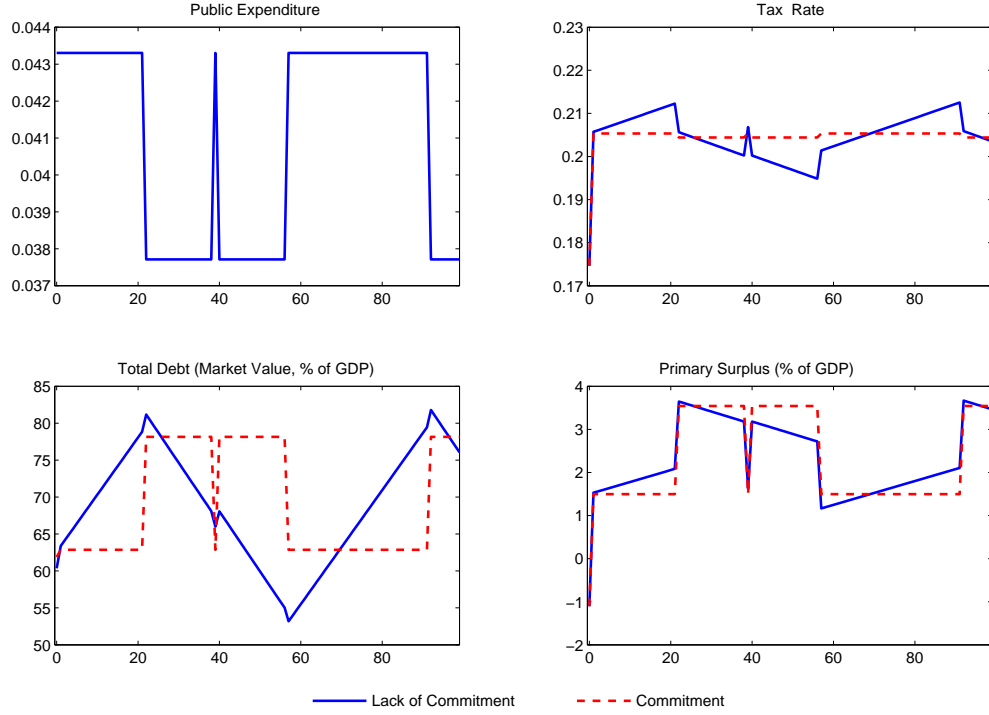


Notes: The figure shows the evolution of debt positions for a particular sequence of shocks. The shaded areas indicate periods in which the spending shock is low. The upper and middle panels plot short-term debt and the annuity value of long-term debt, respectively. The lower panel plots the one-period bond B^S , which is equal to the short-term debt minus the annuity value of the consol.

short-term debt and the annuity value of long-term debt both rise (decline) during high (low) spending shocks. This pattern occurs because the government runs larger deficits (surpluses) when spending is high (low). The bottom panel shows that the difference between the value of short-term debt and long-term debt, which is equal to the one-period bond B^S , is quantitatively very close to zero and nearly constant. Therefore, in contrast to the case of full commitment, the government actively manages its debt which primarily consists of consols. This optimal debt management amounts to adjusting the debt level while keeping a flat maturity.

Figure 6 presents the path of policy under this sequence of shocks. Whereas taxes are nearly constant under full commitment—which is consistent with the complete market results of Chari et al. (1994)—they are volatile and respond persistently to shocks under lack of commitment. More specifically, during periods of high (low) expenditure, taxes jump up (down) and continue to increase (decrease) the longer the fiscal shock persists. Periods of high (low) expenditure are periods with lower (higher) primary surpluses in the case of full commitment and lack of commitment, but in contrast to the case of full commitment, under lack of commitment the surplus responds persistently to shocks. This persistence is reflected in the total market value of debt, which contrasts with the transitory response of the market value of debt in the case of

Figure 6: Fiscal Policy without Commitment



Notes: The figure shows the evolution of tax rates, primary surpluses and total debt for a particular sequence of shocks.

full commitment.²¹

In sum, our quantitative exercise shows that the optimal policy under lack of commitment involves the government actively managing a consol and with fiscal policy persistently responding to economic shocks. This characterization is very different to the case of full commitment in which policy is history-independent and debt positions are constant, quantitatively large, and very tilted.

Our quantitative result emerges because of the combination of two forces. First, substantial hedging requires massive tilted debt positions, as has been shown in Angeletos (2002) and Buera and Nicolini (2004). Due to their size, financing these positions can be very expensive in terms of average tax distortions because of the lack of commitment by the government. Second, under empirically plausible levels of volatility of public spending, the cost of lack of insurance under a

²¹Shin (2007) considers a model under full commitment and shows that if there are N possible states of the shock but at any moment only $N_1 < N$ can be reached, then N_1 bonds of different maturities can provide full insurance. Such a model would require active management of debt positions. Our model under lack of commitment also captures the active management of debt. This result, however, is not achieved by limiting the maturities available; instead it follows from the tradeoff between hedging and the cost of borrowing.

flat maturity structure is small. Therefore, the optimal policy pushes in the direction of reducing *average* tax distortions versus reducing the *volatility* of tax distortions, and the result is a nearly flat maturity structure.

We now turn to examining the robustness of our result. We have only focused on the case in which government spending is exogenous and the government lacks commitment to taxes. Figure 7, which is analogous to Figure 4, extends our results to the case of limited commitment when the government alternatively only lacks commitment to spending (middle panel) or when the government lacks commitment to both spending and taxes (bottom panel). In all cases, we find that the government under limited commitment holds similar debt positions to the case in which it only lacks commitment to taxes, and in all of these cases, the optimal maturity structure is nearly flat.

Figure 8 considers the robustness of our results to different parametric environments, where for brevity we focus on the case of lack of commitment to taxes. We plot the average value of short-term debt and the annuity value of long-term debt in the case of full commitment in the left panel and the case of lack of commitment in the right panel.²² In the top panel, starting from our benchmark environment, we alter the volatility of public spending, and we find that the optimal maturity structure under lack of commitment is nearly flat for all standard deviations of public spending below 30%. We additionally find that the debt positions decrease in size as volatility increases, and this occurs because the volatility of the marginal utility of consumption increases, which facilitates hedging through the consol with a smaller position.²³

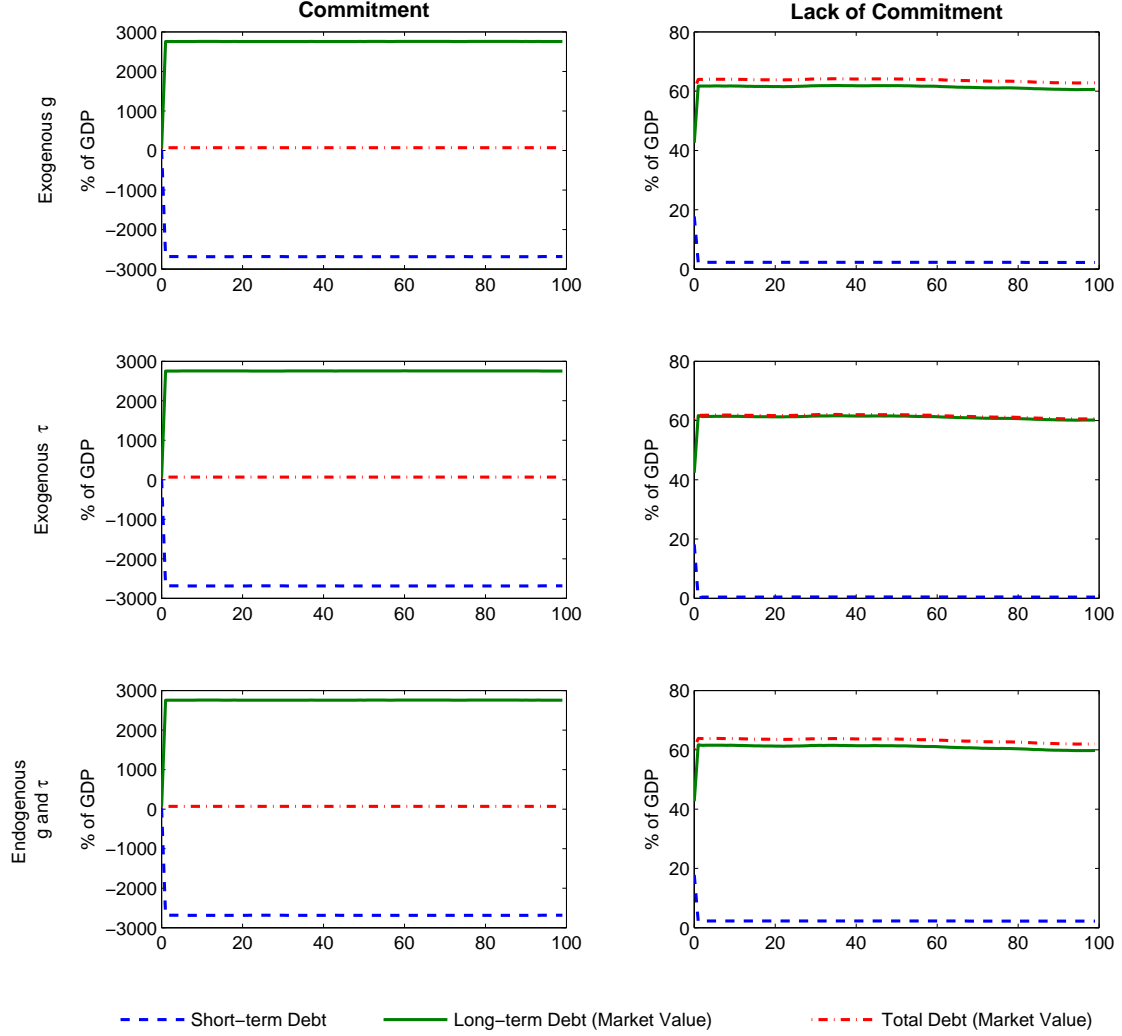
The middle panel considers the consequences of altering the coefficient of relative risk aversion σ_c . In the case of full commitment, lower values of σ_c generate larger and more tilted debt positions. A lower value of σ_c reduces the volatility in the marginal utility of consumption and therefore makes it more difficult to achieve significant hedging with smaller positions. In the case of lack of commitment, a similar force emerges since both the tilt and size of debt positions rise. Note however that, quantitatively, the maturity structure remains nearly flat as σ_c declines. The reason for this is that even though more tilted positions are useful for hedging, more tilted positions also exacerbate the problem of lack of commitment, so that the best way to deal with this problem is to still choose a nearly flat maturity structure.

The final exercise in the bottom panel considers the equilibrium under different values of σ_l , which relates to the curvature of the utility function with respect to leisure. We find that for all values of σ_l below 2, the optimal debt maturity under lack of commitment is essentially flat. The effect of higher value of σ_l is two-fold. On the one hand, higher values of σ_l imply that it is socially costly to have volatility in labor supply, and consequently, oscillations in consumption play a greater role in absorbing public spending shocks. This force increases the volatility in the marginal utility of consumption and implies that smaller debt positions are required to generate

²²In all cases, we consider the average value of these quantities relative to GDP between $t = 1$ and $t = 100$.

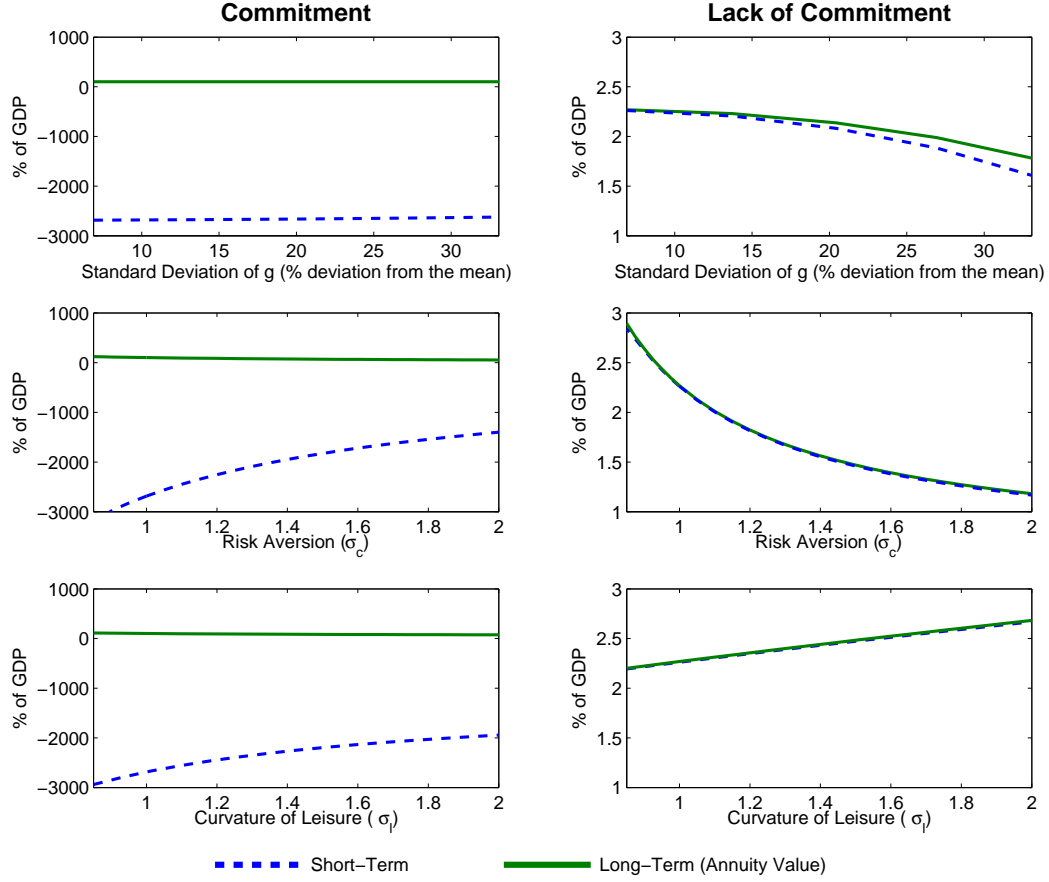
²³A similar interpretation explains the reduction in the tilt of the debt positions under full commitment as the volatility rises. See the discussion in Section 6.

Figure 7: Debt Positions in Alternative Models



Notes: The figure shows the optimal debt positions with commitment (left column) and without commitment (right column). The first panel refers to the baseline model with exogenous public expenditure (lack of commitment to taxes), the middle panel corresponds to a model with exogenous tax rates (lack of commitment to expenditure), while in the bottom panel both taxes and expenditure are endogenous (lack of commitment to both taxes and expenditure). For the no-commitment case we report averages across 1000 simulations.

Figure 8: Debt Positions under Alternative Parametrizations



Notes: The figure shows the optimal debt positions with commitment (left column) and without commitment (right column) under alternative values for the standard deviation of public expenditure (first row), relative risk aversion (second row) and the curvature of the utility function with respect to leisure (third row). For the no-commitment case we report averages across 1000 simulations.

hedging. On the other hand, higher values of σ_l also imply that it is more beneficial to engage in hedging as a way of smoothing out labor market distortions. This force implies larger debt positions since the value of hedging increases. In the case of full commitment, we find that, quantitatively, the first force dominates since debt positions become less tilted as σ_l increases. In the case of lack of commitment, we find that the second force dominates since the consol position become larger as σ_l increases, which facilitates hedging. It continues to be the case throughout, however, that the debt maturity is nearly flat under lack of commitment.

In sum, these numerical exercises confirm that our intuitions from the three-period economy are robust. Completing the market—which is done under full commitment—requires very large positions relative to the size of the economy. This fact, which is also present in Angeletos (2002) and Buera and Nicolini (2004), is due to the observation that interest rates are not sufficiently volatile so as to allow full hedging with small positions. The required enormous positions, however, exacerbate the problem of lack of commitment, which means that such positions are extremely expensive to maintain. More generally, the cost of lack of commitment significantly outweighs the cost of volatility, and for this reason, optimal policy involves a nearly flat maturity structure.²⁴

8 Conclusion

The current literature on optimal government debt maturity concludes that the government should fully insulate itself from economic shocks. This full insulation is accomplished by choosing a maturity heavily tilted towards the long end, with a constant short-term asset position and long-term debt position, both positions extremely large relative to GDP. In this paper, we show that these conclusions strongly rely on the assumption of full commitment by the government. Once lack of commitment is taken into account, then full insulation from economic shocks becomes impossible; the government faces a tradeoff between the benefit of hedging and the cost of funding. Borrowing long-term provides the government with a hedging benefit since the value of outstanding government liabilities declines when short-term interest rates rise. However, borrowing long-term lowers fiscal discipline for future governments unable to commit to policy, which leads to higher future short-term interest rates. We show through a series of exercises that the optimal debt maturity structure under lack of commitment is nearly flat, with the government actively managing its debt in response to economic shocks. Thus, optimal policy can be approximately achieved by confining government debt instruments to consols.

Our analysis leaves several interesting avenues for future research. First, our framework fol-

²⁴Our conclusions should not change in the presence of additional debt instruments of different maturities. Our analysis shows that the value of the one-period bond B^S is approximately zero, and if a decaying perpetuity is introduced, then its value is also approximately zero (see the Appendix). As such, we conjecture that in a more general environment with many bond instruments, any finite maturity bond should be approximately zero under the optimal policy. This is because it is optimal for the government to constrain itself to consols, since these minimize the cost of lack of commitment.

lows Angeletos (2002) and Buera and Nicolini (2004) and therefore ignores nominal bonds and the risk of surprise inflation. Taking this issue into account is important since it incorporates a monetary authority's ability to change the value of outstanding debt in response to shocks, and it also brings forward the issues of dual commitment to monetary and fiscal policy. We believe that our work is a first step in studying this more complicated problem. Second, our framework does not incorporate investment and financing frictions which can be affected by the supply of public debt. It has been suggested that short-term government debt is useful in alleviating financial frictions (see e.g. Greenwood et al. (2014)), and an open question regards how important this friction is quantitatively relative to the lack of commitment. Finally, our analysis ignores heterogeneity and the redistributive motive for fiscal policy (see, e.g., Werning, 2007 and Bhandari et al., 2013). An interesting question for future research involves how incentives for redistribution can affect the maturity structure of public debt.

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Appendix

A-1 Proofs

Proof of Proposition 1

The necessity of these conditions is proved in the text. To prove sufficiency, let the government choose the associated level of debt $\left\{ \left\{ \left\{ B_t^{t+k}(s^t) \right\}_{k=1}^{\infty} \right\}_{s^t \in S^t} \right\}_{t=0}^{\infty}$ and a tax sequence $\left\{ \left\{ \tau_t(s^t) \right\}_{s^t \in S^t} \right\}_{t=0}^{\infty}$ which satisfies (9). Let bond prices satisfy (9). From (11), (10) is satisfied, which given (8) implies that (2) and (3) are satisfied. Therefore household optimality holds and all dynamic budget constraints are satisfied along with the market clearing, so the equilibrium is competitive. ■

Proof of Corollary 1

Let us consider an environment with state-contingent debt. Specifically, let $B_t^{t+k}|s^{t+k}(s^t)$ correspond to a state-contingent bond purchased at date t and history s^t with a payment contingent on the realization of history s^{t+k} at $t+k$. The analog in this case to condition (9) is

$$1 - \tau_t(s^t) = -\frac{u_{n,t}(s^t)}{u_{c,t}(s^t)} \text{ and } q_t^{t+k}|s^{t+k}(s^t) = \frac{\beta \pi(s^{t+k}|s^t) u_{c,t+k}(s^{t+k})}{u_{c,t}(s^t)}, \quad (\text{A-1})$$

and the analog to (11) is:

$$\begin{aligned} \sum_{k=0}^{\infty} \sum_{s^{t+k} \in S^{t+k}} \beta^k \pi(s^{t+k}|s^t) \left(u_{c,t+k}(s^{t+k}) c_{t+k}(s^{t+k}) + u_{n,t+k}(s^{t+k}) n_{t+k}(s^{t+k}) \right) = \\ \sum_{k=0}^{\infty} \sum_{s^{t+k} \in S^{t+k}} \beta^k \pi(s^{t+k}|s^t) u_{c,t+k}(s^{t+k}) B_{t-1}^{t+k}|s^{t+k}(s^{t-1}). \end{aligned} \quad (\text{A-2})$$

It is therefore necessary that (7) satisfy (8) $\forall s^t$ and (11) for $s^t = s^0$, where the last condition is identical to (A-2) for $s^t = s^0$. To prove sufficiency, let the government choose one-period state contingent debt $B_{t-1}^t|s^t(s^{t-1})$ so that the right hand side of (A-2) equals $u_{c,t}(s^t) B_{t-1}^t|s^t(s^{t-1})$ and choose $\left\{ \left\{ B_{t-1}^t|s^t(s^{t-1}) \right\}_{s^t \in S^t} \right\}_{t=0}^{\infty}$ so as to satisfy (A-2) $\forall s^t$. Let $\tau_t(s^t)$ and $q_t^{t+k}|s^{t+k}(s^t)$ satisfy (A-1). Analogous arguments then to those in the proof of Proposition 1 imply that the equilibrium is competitive. ■

Proof of Proposition 2

See text. ■

Proof of Proposition 3

We only focus here on proving part (i) since the proof of part (ii) is analogous. Suppose there is lack of commitment to taxes and the program in (22)–(24) and (25) is globally concave. Let us construct the following MPCE. Suppose government strategies are defined as follows. If

$\theta_t = \theta^H$ and $B_{t-1}^{t+k} = 0 \forall k \geq 0$, then the government chooses $\{c_0^*, n_0^*, g_0^*, \tau_0^*\}$ associated with the commitment optimum and $B_t^{t+k} = \overline{B} \forall k \geq 1$ for \overline{B} defined in (21). Because θ^H is only reached at $t = 0$ with $B_{t-1}^{t+k} = 0 \forall k \geq 0$, this is the only case to consider with $\theta_t = \theta^H$. If $\theta_t = \theta^L$ and $B_{t-1}^{t+k} = \overline{B} \forall k \geq 0$, then the government chooses $\{c_1^*, n_1^*, g_1^*, \tau_1^*\}$ associated with the date 0 commitment optimum and $B_t^{t+k} = \overline{B} \forall k \geq 1$. If instead, $\theta_t = \theta^L$ and $B_{t-1}^{t+k} \neq \overline{B} \forall k \geq 0$, then the government pursues some other strategy to be defined later.

We now check the optimality of the strategies which we have defined along the equilibrium path. Suppose that $\theta_t = \theta^H$ and $B_{t-1}^{t+k} = 0 \forall k \geq 0$. Given the future behavior of the government which sets $\{c_1^*, n_1^*, g_1^*, \tau_1^*\}$ from $t \geq 1$ onward along the equilibrium path, there does not exist a policy choice which strictly dominates setting $\{c_0^*, n_0^*, g_0^*, \tau_0^*\}$ and $B_t^{t+k} = \overline{B} \forall k \geq 1$, since the associated welfare is equivalent to that in the ex-ante optimum under full commitment. Suppose instead that $\theta_t = \theta^L$ and $B_{t-1}^{t+k} = \overline{B} \forall k \geq 0$. Given that (22) – (24) and (25) is globally concave it follows that the ex-ante optimum under full commitment, starting from $\theta_t = \theta^L$ and $B_{t-1}^{t+k} = \overline{B} \forall k \geq 0$, sets $\{c_{t+k}, n_{t+k}, g_{t+k}, \tau_{t+k}\}$ to be constant $\forall k \geq 0$. Satisfaction of (24) and (25) implies that such a constant policy necessarily coincides with the date 0 ex-ante optimal sequence $\{c_1^*, n_1^*, g_1^*, \tau_1^*\}$. It thus follows that starting from $\theta_t = \theta^L$ and $B_{t-1}^{t+k} = \overline{B} \forall k \geq 0$, there does not exist a policy choice which strictly dominates setting $\{c_1^*, n_1^*, g_1^*, \tau_1^*\}$ and $B_t^{t+k} = \overline{B} \forall k \geq 1$, given the behavior of future governments. This establishes the optimality of prescribed equilibrium strategies, and establishes that the equilibrium path of the constructed MPCE with $B_0^t = B_0^{t+k} \forall t \geq 1$ and $k \geq 1$ coincides with the date 0 commitment optimum. Note that we have established that this equilibrium behavior is optimal, independently of what strategies the government utilizes off the equilibrium path. To complete the proof, we only have to establish that such strategies exist from $\theta_t = \theta^L$ and $B_{t-1}^{t+k} \neq \overline{B} \forall k \geq 0$. It is straightforward to construct such strategies using backward induction in a T period economy as $T \rightarrow \infty$, which completes the argument. ■

Proof of Lemma 1

The date 0 government clearly cannot do better than to choose a policy which is identical to that of the commitment optimum and satisfies (30). We can show that this can be achieved with a flat maturity with $B_0^1 = B_0^2 = \overline{B}$. If this is the case, then (33) implies that c_1 and c_2 satisfy (30). Substitution of c_1 and c_2 into (29) which binds furthermore implies a value of c_0 which is also consistent with (30). Therefore, choosing $B_0^1 = B_0^2 = \overline{B}$ is optimal. Furthermore, given (33), any other choice of initial debt cannot generate the values of c_1 and c_2 satisfying (30). Therefore, $B_0^1 = B_0^2 = \overline{B}$ is the unique MPCE. ■

Proof of Lemma 2

From the government's dynamic budget constraint:

$$g_0 = \tau n + q_0^1 B_0^1 + q_0^2 B_0^2,$$

and so it is sufficient to focus on g_0 . Given $\{B_0^1, B_0^2\}$, we can use (28), (30), and (32) to write g_0 as a function of B_0^2 :

$$n - \frac{n(1-\tau)}{1 + \beta + \beta^2 - n(1-\tau) \frac{\beta}{1+\beta} \frac{\chi(B_0^2)}{\phi(B_0^2)}} \quad (\text{A-3})$$

for

$$\begin{aligned} \chi(B_0^2) &= (n(1-\tau) + B_0^1(B_0^2))^{-1/2} + \beta(n(1-\tau) + B_0^2)^{-1/2}, \\ \phi(B_0^2) &= (n(1-\tau) + B_0^1(B_0^2))^{1/2} + \beta(n(1-\tau) + B_0^2)^{1/2}, \text{ and} \\ B_0^1(B_0^2) &= (1+\beta)\bar{B} - \beta B_0^2. \end{aligned}$$

It can be shown that $\chi'(B_0^2) < (>) 0$ if $B_0^2 < (>) B_0^1$ and $\phi'(B_0^2) > (<) 0$ if $B_0^2 < (>) B_0^1$. If we fully differentiate (A-3) with respect to B_0^2 we achieve a value which is proportional to:

$$-c_0^2 \frac{\chi'(B_0^2)}{\phi(B_0^2)} + c_0^2 \frac{\chi(B_0^2)}{[\phi(B_0^2)]^2} \phi'(B_0^2) \quad (\text{A-4})$$

where c_0 is evaluated at the associated values of B_0^1 and B_0^2 . (A-4) equals zero at $B_0^2 = B_0^1$. Moreover, given the properties of $\chi'(B_0^2)$ and $\phi'(B_0^2)$, (A-4) is positive for $B_0^2 < B_0^1$ and negative for $B_0^2 > B_0^1$. This completes the proof since $|B_0^1(B_0^2) - B_0^2|$ is declining (rising) in B_0^2 for $B_0^2 < (>) B_0^1$. ■

Proof of Proposition 4 and Corollary 2

See proof of Theorem 1 in Angeletos. ■

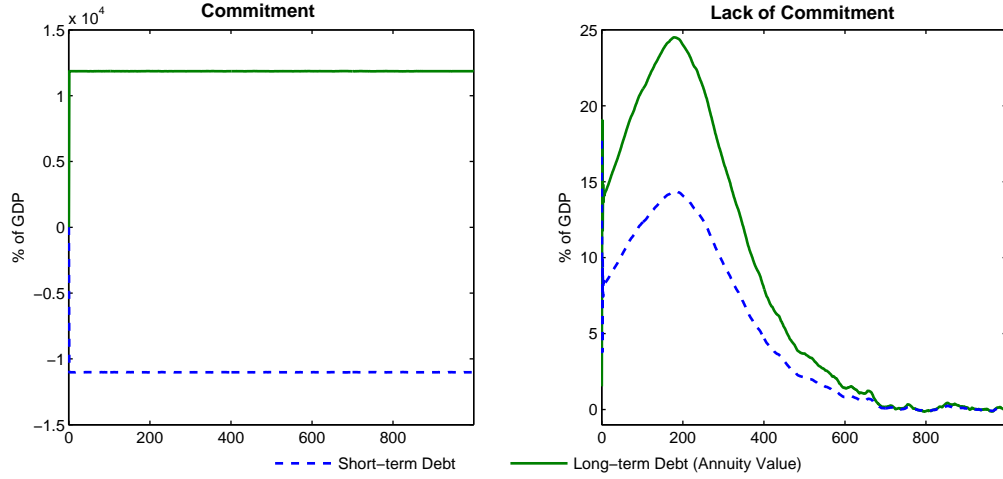
A-2 Additional Robustness Checks

Different Bond Maturities

Figure A-1 plots the optimal debt positions for a case where the government has available a one-year bond and a perpetuity with a decaying coupon at a rate $\gamma = 0.5$, so that the average maturity of the perpetuity equals 2 years. The government lacks commitment to both taxes and public expenditure. Similarly to the cases considered in the main text, the debt maturity structure under lack of commitment is much flatter than with commitment. Also, and consistently with the findings in Debortoli and Nunes (2013), debt gradually converges (on average) towards zero. This occurs because, in a deterministic environment, when debt is zero, then the maturity structure is flat and lack of commitment is no longer an issue.²⁵

²⁵Debt positions increase in expectation at first because the initial shock is θ^H .

Figure A-1: Debt Positions with Alternative Maturities



Notes: The figure shows the optimal debt positions with commitment (left column) and without commitment (right column), for a model with a one-period bond and a perpetuity with decaying payments ($\gamma = 0.5$). For the no-commitment case we report averages across 1000 simulations.

A-3 Numerical Algorithm

In the numerical algorithm, we use a collocation method on the first order conditions of the recursive problem. We solve for an MPCE in which the policy functions are differentiable and we approximate directly the set of policy functions $\{c, n, g, B^S, B^L, q^L\}$.²⁶ The solution approach finds a fixed point in the policy function space using an iteration approach. We cannot prove that this MPCE is unique, though our iterative procedure always generates the same policy functions independently of our initial guesses.

The functions are approximated on a coarse grid, where the market value of debt ranges from -700% to 700% of GDP. The results are very similar whether we use a different amplitude of the grid, and different types of functional approximation (splines, complete, or Chebyshev polynomials).

²⁶In the cases in which there is commitment to taxes or spending, we either impose the additional constraint or, equivalently, approximate a smaller set of policy functions.