# WHY DON'T THE PRICES OF STOCKS AND BONDS MOVE TOGETHER? 

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## ABSTRACT

The very low real interest rates on bonds in the 1970's were accompanied by a large drop in the value of common stocks relative to dividends and earnings. More generally, a number of authors have demonstrated that the real prices of debt and equity claims do not covary closely, and often move in opposite directions. This paper analyzes the effects of two disturbances - an increase in risk, and a slowing of productivity growth - each of which might rationalize a simultaneous drop in equity values and in real interest rates on bonds.

As long as marginal utility is a convex function of consumption, an increase in risk depresses the return on riskless bonds. When all of the wealth of the economy is traded in the stock market, equity values fall with increasing equity risk only if the intertemporal elasticity of substitution in consumption exceeds unity. This same pattern occurs in response to a fall in productivity growth. In a richer two-real-asset model, which takes account of the fact that corporate capital has rarely been more than a quarter of total wealth, it is likely that both increased risk and lower productivity growth in the corporate sector would lead to a fall in stock prices, a drop in real interest rates, and a rise in the price of the second tangible asset -the pattern seen in the 1970 's.

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Recent work has cast strong doubt on the view that observed fluctuations of stock prices can be explained by a rational expectations model in which the expected return is constant over time (see, in particular, Shiller, 1981; Leroy and Porter, 1981; and Mankiw, Romer, and Shapiro, 1985). Reluctant to drop the notion of rational valuation, many economists suspect that major movements in stock prices are the result of changes in the equilibrium discount rate. As emphasized by Shiller (1982), however, the simplest models of time-varying discount rates tend to imply that the prices of bonds, land, and other assets (relative to their real payments streams) should covary closely with stock prices, the prices of all of these assets being driven by a common underlying discount factor (perhaps the "riskless rate"). Shiller (1982) reports, to the contrary, that there is little comovement between the value of corporate equity and the prices of bonds and land, even after attempts to correct for movements in real coupons, dividends, and rents. Campbell and Shiller (1986) show that dividend price ratios are uncorrelated with subsequent real returns on bonds. In short, "the failure of land, housing, or bond prices to move with stock prices is discouraging." (Shiller, 1982, p.231).

This paper explores thoroughly the implications of two sorts of disturbances that might rationalize the puzzling behavior of the bond and stock markets. The first is an increase in the real, nondiversifiable risk associated with equity holding, which raises the required return on equity relative to the risk-free rate. The second is a decline in the rate of productivity growth. This affects equilibrium returns on the two assets symmetrically, but has an additional direct effect on equity prices and yields through a reduction in expected future, relative to current, dividends.

The models studied are the modern stochastic version of the neoclassical theory of interest associated with Irving Fisher. In their simplest form, the experiments I carry out are general equilibrium analogues of the comparative statics for an individual saver in a single risky asset studied by Sandmo (1970), Rothschild and Stiglitz (1971), and others. In addition to the general equilibrium nature of the current analysis, I go considerably beyond those papers to consider
issues of capital accumulation, the distinction between risk aversion and aversion to intertemporal substitution, and the case of more than one risky asset. The last of these turns out to be of major consequence. The model with two tangible assets, "corporate equity" and "housing", when perturbed by the two disturbances we consider, shows some promise as a potential rationalization of our macroeconomic asset pricing puzzle.

The decade of the 1970 's provides a dramatic example of movement of debt and equity values, relative to plausible estimates of their associated real earnings streams, in opposite directions. The years 1973 to 1980 stand out as a period of very low ex ante real interest rates on bonds. Agents could have consistently forecast negative returns on bills over much of this period (Blanchard and Summers, 1984; Mehra, 1983). On the other hand, the value of the S\&P 500 stock price index relative to after-tax earnings fell from 19 in 1972 to 9 in 1974, and remained below 13 for the rest of the decade. The relative prices of alternative real assets underwent a major reallignment as well. The share of corporate equity in total household wealth (from the Fed Balance Sheets for the U.S. economy) went from .27 in 1965 to .11 in 1978. The corresponding share of owner occupied housing went from .16 in 1965 to .23 in 1978.

Summers (1983) questions "whether the puzzling behavior of the interest rate and the stock market are related":

The increasing value of the earnings-price ratio coupled with the declining real interest rate substantially widened the spread between the expected return on debt and equity. The decline in real stock prices has offset the rise in bond prices associated with lower interest rates, leaving the valuation of the corporate sector relative to the income it generates unchanged. ... The spread between the relative valuation of the two types of claims on corporate income has widened substantially, from 3.1 percent in 1965 to 9.8 percent in 1978. The increase in this spread is puzzling. (Summers, 1983, p.231).

As the post-1973 period was one of high inflation by postwar U.S. standards, it is not surprising
that many authors have tried to attribute the depression of both real interest rates and equity values to some effect of inflation. Feldstein (1980) argues that the effect of extra taxes due to historic cost depreciation, FIFO inventory accounting, and taxation of nominal capital gains can account for at least some of the fall in the stock market. However, the ratio of share prices to after-tax earnings fell sharply, and Hendershott (1979), Pindyck (1984), and Modigliani and Cohn (1979) all argue that the net effect of inflation-tax interactions was to raise market values, because of the large subsidy to borrowing from the deductibility of nominal interest. Modigliani and Cohn (1979), who deserve credit for first suggesting a link between the interest rate anomaly and the stock market puzzle, despair of an explanation based on rational evaluation and suggest that inflation illusion caused investors to misvalue both stocks and bonds.

But high inflation was by no means the only distinguishing feature of the 1970's. That decade is typically thought of as one of relatively great uncertainty, and a relatively slow rate of productivity growth. Some evidence for an increase in risk is provided by the fact that stock price volatility rose markedly (Black, 1976, Pindyck, 1984; Poterba and Summers, 1985), as did the premium on "junk bonds" (Malkiel, 1979). With regard to productivity, it is noteworthy that, while real output growth per employed person in the United States averaged 2.1 percent for the period 1950 to 1973 , that growth rate averaged only 0.38 percent from 1973 to 1978. A similar pattern held for European nations (see Gordon, 1981, p.559).

Malkiel (1979) and Pindyck (1984) both attribute the large fall in equity válues to an increase in the risk premium due to increased volatility of the marginal product of capital. Neither they nor other authors have considered the possibility that increased risk could explain the depressed equity values and the low real interest rates simultaneously. Yet a drop in the riskless rate would seem to be an intuitively likely consequence of increased risk as agents substitute away from risky stocks into less risky assets. I develop rigorously the notion of such a "flight to safety" and find theoretical suppport for it. The drop in the riskless rate is a consequence not of risk aversion per se,
but of precautionary saving, an effect which involves the third derivative (rather than the second) of the utility function. Precautionary saving in response to increased risk is generally accepted by economists as a plausible and important effect, and the condition on the third derivative is weaker than that of decreasing absolute risk aversion.

Once the riskless rate is endogenized, however, a fundamental ambiguity arises in the Pindyck argument. It may well happep that, although the equity premium rises, the riskless rate falls so much that stock prices nonetheless rise. In the first model, where stocks are the only tangible asset, there is in fact a presumption in that direction. If, as is likely, agents are more averse to intertemporal substitution than agents with log utility, increased risk lowers the required rate of return through the precautionary saving effect, and raises market values. With log utility, the fall in the interest rate and the rise in the equity premium are of the same magnitude, so that stock values remain invariant. In an ad hoc simulation model, Hendershott (1979) arrived at the conclusion that the effects of increased risk would fall primarily on the interest rate, with stock prices largely unchanged. Without endorsing Hendershott's logic, my model with one tangible asset and log utility provides a formal derivation of his result.

It turns out that there is a striking isomorphism between an increase in rate-of-return uncertainty and a reduction in the expected growth rate, and equity values can also go "the wrong way" in response to decreased productivity growth. If so, we gain an explanation of the drop in real interest rates, but lose our ability to shed light on the fall in the stock market $\stackrel{b}{\text { by }}$ analysis of either disturbance. The model with two tangible assets, in which corporate equity no longer represents all claims on output, largely remedies this problem. The critical value of the curvature parameter beyond which stock prices move perversely is negatively related to the share of equity in the total wealth portfolio, and with reasonable values for $\gamma$ and for the share of the corporate sector, we might well be able to rationalize the behavior seen in the data.

The plan of the paper is as follows. Section I lays out the basic model with a fixed capital
stock, and only one tangible asset. The principal effects are seen here, but perhaps in somewhat misleading proportions. The relation of our general equilibrium model to the comparative statics of Sandmo (1970), Rothschild and Stiglitz (1971) and Dreze and Modigliani (1972) comes through clearly. Section I closes with a discussion of nominal bonds, in contrast with the riskless bonds of the formal theory. Unfortunately, the character of nominal bonds as risky assets depends on the money supply process in such a way that no clearcut results are available.

Section II treats the more subtle extensions. It begins with the important distinction between risk aversion and aversion to intertemporal substitution. I address this using the Ordinal Certainty Equivalence approach pioneered by Selden (1978) and advocated by Hall (1985). Although both play a role, the pivotal parameter in terms of the direction of the effect on stock prices turns out, surprisingly perhaps, to be the intertemporal elasticity of substitution rather than the coefficient of relative risk aversion.

Section II-b treats the all-important extension to two tangible assets. This section considerably strengthens the ability of our analysis to rationalize the appropriate stylized facts. In response to the two shocks we consider, it is plausible that real interest rates fall, stock prices fall, and house prices rise, exactly the pattern seen in the puzzling 1970's.

Section II-c treats a " $q=1$ " version of the model, the case of long-run capital accumulation. Because the relative price of capital in that model is determined by reproduction cost, we cannot discuss fluctuation of stock prices. However, we can study the response of the riskless rate and the equilibrium marginal product of capital to an increase in risk and a decrease in the expected marginal productivity of capital. This helps us to get some intuition for the intermediate case, in which both the price and quantity of capital vary in response to shocks.

Section III contains some brief concluding remarks.

## I. Risk, Growth Rates, and Required Returns in General Equilibrium

In this section, I develop the simplest two-period, two-asset model of general equilibrium asset pricing. I then go on to show how the required returns both to capital and to riskless bonds are affected by: a)increased dispersion in the payoff of capital; and b) a decline in the expected growth rate of the economy.

For simplicity, I work with a representative consumer model. This appears to be appropriate for our purposes, as we are interested in studying the effect of aggregate risk on equilibrium asset returns. The results could also be derived in a model of many consumers with a set of complete markets. The contingent claims markets result in optimal sharing of risks among the consumers. This accomplished, the stock market value reflects the equilibrium price of undiversifable, aggregate risk. On the other hand, a world of incomplete markets raises a multitude of questions which must be considered beyond the scope of this analysis.

In addition to the one-consumer assumption, I begin with the assumption that there is one kind of capital which is in fixed supply, and that all output is produced by this capital. Thus, as in the models of Lucas (1978) and Campbell (1986), among others, a share of stock amounts to a unit claim on the aggregate output of the economy. In later sections, I relax some of these assumptions to treat long-run capital accumulation and to allow claims on output other than those traded in the stock market. The latter, in particular, is crucial for a complete understanding of the issues. This section should be regarded as illustrating the principal effects, in preparation for the more realistic treatment in Section II-b.

## A. Basic Model

Each individual perceives himself as able to freely trade in equity at the market price, and also to buy (or issue) bonds that are riskless in terms of consumption. Agents maximize (subject to their intertemporal budget constraints) time-additive, concave expected utility functions of the form $U\left(C_{1}\right)+\beta E\left[U\left(\tilde{C}_{2}\right)\right]$. Thus the first-order conditions are:

$$
\begin{equation*}
\beta E\left[U^{\prime}\left(\tilde{C}_{2}\right) \tilde{R}\right]=U^{\prime}\left(C_{1}\right) \tag{1}
\end{equation*}
$$

$$
\begin{equation*}
R^{f} \beta E\left[U^{\prime}\left(\tilde{C}_{2}\right)\right]=U^{\prime}\left(C_{1}\right) \tag{2}
\end{equation*}
$$

where $\tilde{R}$ is one plus the random rate of return on equity, and $R^{\rho}$ is one plus the real risk-free rate.
As in Lucas (1978), output is nonstorable and produced by tradeable "trees," which we identify with "equity". The trees are homogeneous and in fixed positive supply, and the riskless asset is in zero net supply. Nonstorability, in addition to the assumption that all ouput is produced by the "trees" allows us immediately to characterize the general equilibrium:

$$
\begin{gather*}
C_{1}=Y_{1} \quad \tilde{C}_{2}=\tilde{Y}_{2}  \tag{3}\\
R^{f}=\frac{1}{p^{\text {bond }}}=\frac{U^{\prime}\left(Y_{1}\right)}{\beta E\left[U^{\prime}\left(\tilde{Y}_{2}\right)\right]}  \tag{4}\\
E[\tilde{R}]=\frac{E\left[\tilde{Y}_{2}\right]}{p^{\text {eq. }}}=\frac{U^{\prime}\left(Y_{1}\right) E\left[\tilde{Y}_{2}\right]}{\beta E\left[U^{\prime}\left(\tilde{Y}_{2}\right) \tilde{Y}_{2}\right]} \tag{5}
\end{gather*}
$$

Equation (4) tells us that the equilibrium interest rate on riskless bonds is just the inverse of an expected marginal rate of substitution. Thus (4) is, for the endowment economy without storage, the generalization to a stochastic world of Irving Fisher's two-period diagrammatic theory of the interest rate. Note that in the present context (4) is a complete theory of the interest rate, since the technology makes consumption exogenous at the aggregate level. More generally, the first-order condition from which (4) is derived must be combined with a production relationship to determine the equilibrium interest rate.

Equation (5) implies that the price of an equity claim is the expectation of the random consumption payoff (or "dividend") weighted by the stochastic marginal rate of substitution. The
multi-period generalization of this expression for the stock price appears in Grossman and Shiller (1982) and Shiller (1982). Note that the equilibrium expected return is the ratio of the expected future dividend to the price, and coincides with the conventional dividend-price ratio $Y_{1} / P$ only if the process for $Y$ is a martingale.

## B. An Increase in Risk

Consider an increase in the dispersion of $\tilde{Y}_{2}$, with $E\left[\tilde{Y}_{2}\right]$ held constant. Such a mean-preserving spread is "increasing risk" in the Rothschild-Stiglitz (1970) sense. In order to get explicit expressions for derivatives of the required returns with respect to the increased risk, let us parametrize the risk as multiplicative of the form $1+h \epsilon$, where $\epsilon$ is a zero-mean random variable, $h$ is a scale parameter, and the support of $\epsilon$ is such that $1+h \epsilon$ is non-negative. For instance, $1+h \epsilon$ could describe a family of log-normal distributions, all having a mean of unity, with variance parametrized by $h{ }^{1}$

Differentiating with respect to $h$, we obtain:

$$
\begin{gather*}
\frac{d R^{f}}{d h}=\frac{-R^{f} \bar{Y}_{2} E\left[U^{\prime \prime}\left(\tilde{Y}_{2}\right) \epsilon\right]}{E\left[U^{\prime}\left(\tilde{Y}_{2}\right)\right]}  \tag{6}\\
\frac{d E[\tilde{R}]}{d h}=\frac{-E[\tilde{R}] \bar{Y}_{2} E\left[U^{\prime}\left(\tilde{Y}_{2}\right) \epsilon\right]}{E\left[U^{\prime}\left(\tilde{Y}_{2}\right) \tilde{Y}_{2}\right]}-\frac{E[\tilde{R}] \bar{Y}_{2} E\left[U^{\prime \prime}\left(\tilde{Y}_{2}\right) \tilde{Y}_{2} \epsilon\right]}{E\left[U^{\prime}\left(\tilde{Y}_{2}\right) \tilde{Y}_{2}\right]} \tag{7}
\end{gather*}
$$

Since the sign of $E\left[U^{\prime \prime}\left(\tilde{Y}_{2}\right) \epsilon\right]$ is that of $U^{\prime \prime \prime}(\cdot)$, (6) tells us that a necessary and sufficient condtion for increasing aggregate risk to depress the riskless rate is that $U^{\prime \prime \prime}(\cdot)$ be positive, or that marginal utility be convex in $C$. Thus our negative relationship between risk and the riskless rate is not an implication of risk aversion alone. Convex marginal utility is exactly the condition for the existence of a precautionary demand for saving (Leland, 1968), and is weaker than the widely accepted condition of decreasing absolute risk aversion (Arrow, 1971). Intuitively, in the presence of convex marginal utility, a mean-preserving spread on $\bar{Y}_{2}$ raises expected marginal
utility in the second period, and thus increases the consumer's desire to hold a riskless (in terms of consumption) bond maturing at that time. Note that (6) implies that a necessary and sufficient condition for the equilibrium riskless rate to be unrelated to consumption variability, as appears to be the presumption in much work in applied finance, is that utility be quadratic.

Equation (7) demonstrates that the effect of increased equity risk on required stock returns (and hence equity values) is ambiguous. The first term in (7) is positive by risk aversion alone. This corresponds to the substitution effect in Sandmo (1970). With increased riskiness of the capital asset, the cost-minimizing way to achieve the original level of utility involves greater first period consumption in order to sidestep the risk, and a corresponding fall in the notional demand for equities. However, since the representative consumer must in equilibrium hold his share of the fixed stock of capital, this tends to raise required returns. A straightforward application of the results in Sandmo (1970, p. 35) shows that the second term is negative under decreasing absolute risk aversion. This reflects a precautionary saving effect. The increased risk raises the prospect of very low consumption in the second period, increasing asset demands. With both assets in fixed supply, this exerts downward pressure on the required returns. In general either effect can dominate, and thus an increase in rate-of-return uncertainty can result in either a rise or fall in the required return to equity. For the case of constant relative risk aversion, (7) becomes:

$$
\begin{equation*}
\frac{d E[\tilde{R}]}{d h}=\frac{(\gamma-1) E[\tilde{R}] \bar{Y}_{2} E\left[U^{\prime}\left(\tilde{Y}_{2}\right) \epsilon\right]}{E\left[U^{\prime}\left(\tilde{Y}_{2}\right) \tilde{Y}_{2}\right]} \tag{8}
\end{equation*}
$$

Since $U^{\prime \prime}$ is negative, $E\left[U^{\prime}(\cdot) \epsilon\right]$ is negative as well. Thus the equilibrium expected return on capital rises with increased uncertainty if and only if $\gamma$ is less than one.

For the case of constant relative aversion combined with a lognormal distribution for $\tilde{Y}_{2}$, we can, following Campbell (1984), derive explicit solutions for riskless interest rates and equity returns. These are illuminating, as they allow an alternative interpretation of the two opposing
forces affecting the required return on capital. This interpretation is based on a decomposition of the impact on $E[\tilde{R}]$ into an effect on the riskless rate and an effect on the equity premium, rather than a decomposition into income and substitution effects. It is useful in this example to define the equity premium as $Z=\log (E[\tilde{R}])-\log R^{f}$. Then:

$$
\begin{gather*}
\log \left(R^{f}\right)=-\log (\beta)+\gamma E\left[\log \left(\tilde{Y}_{2}\right)-\log \left(Y_{1}\right)\right]-\frac{1}{2} \gamma^{2} \operatorname{Var}\left[\log \left(\tilde{Y}_{2}\right)\right]  \tag{9}\\
\log (E[\tilde{R}])=-\log (\beta)+\gamma E\left[\log \left(\tilde{Y}_{2}\right)-\log \left(Y_{1}\right)\right]+\left(\gamma-\frac{1}{2} \gamma^{2}\right) \operatorname{Var}\left[\log \left(\tilde{Y}_{2}\right)\right]  \tag{10}\\
Z=\gamma \operatorname{Var}\left[\log \left(\tilde{Y}_{2}\right)\right]
\end{gather*}
$$

Now consider the effects of increasing risk on the required returns in (9), (10), and (11). Letting $\sigma^{2}$ denote the variance of the normally distributed random variable $\log \left(\tilde{Y}_{2}\right)$, note that increasing risk raises both the mean and variance of $\tilde{Y}_{2}$. To study the effect of applying a mean-preserving spread to $\tilde{Y}_{2}$, we need to differentiate with respect to $\sigma^{2}$, subject to the constraint $d E\left[\log \left(\tilde{Y}_{2}\right)\right]=-\frac{1}{2} d \sigma^{2}$. The resulting expressions are:

$$
\begin{gather*}
\frac{d \log \left(R^{f}\right)}{d \sigma^{2}}=-\frac{1}{2} \gamma(1+\gamma)  \tag{12}\\
\frac{d \log E[\tilde{R}]}{d \sigma^{2}}=\frac{1}{2} \gamma(1-\gamma)  \tag{13}\\
\frac{d Z}{d \sigma^{2}}=\gamma \tag{14}
\end{gather*}
$$

Equation (12) confirms that, under constant relative risk aversion, a mean-preserving spread on $\tilde{Y}_{2}$ unambiguously depresses the risk-free rate. This is as expected, since the CRRA utility
function exhibits a positive third derivative. Equation (14) shows that increasing risk raises the equity premium.

Finally, (13) confirms that increased risk raises required stock returns only if the parameter $\gamma$ is less than one. ${ }^{2}$ For the benchmark case of $\log$ utility, equity values are independent of the dispersion of $\tilde{Y}_{2}$. This is because the increase in the equity premium is exactly offset by the drop in the riskless rate. If $\gamma$ exceeds $\langle\boldsymbol{u n i t y}$, increased risk lowers the riskless rate by more than the increase in the equity premium, causing a fall in the required stock return and a rise in stock prices. In either case, increased dispersion makes stocks less attractive relative to riskless bonds, raising the equity premium in equilibrium. When $\boldsymbol{\gamma}$ exceeds unity, however, the precautionary saving effect is sufficiently large as to depress required returns on both assets.

Equations (7), (10), and (13) undermine the theoretical presumption behind the argument of Pindyck (1984) that the fall in equity values in the 1970's should be attributed to an increase in the risk premium due to increased volatility of the marginal product of capital. Pindyck's unambiguous results depend critically on his treatment of the riskless rate as an exogenous parameter, rather than an equilibrium price endogenously related to the amount of risk in the economy. ${ }^{3}$ In Section II-a, I discuss the empirical interpretation of the parameter $\gamma$, and conclude that available evidence suggest that it is probably greater than unity. However, it is premature to infer at this point that Pindyck's story is necessarily incompatible with the evidence on $\boldsymbol{\gamma}$. Section II-b treats a more complex problem in which corporate equity is only one of the output-yielding "trees" in the economy. For representative values of the share of the corporate sector, the critical value of $\boldsymbol{\gamma}$ becomes considerably greater than unity. There the Pindyck presumption gains in plausibility, although the theoretical ambiguity remains.

A fall in the riskless rate appears to be a more robust implication of increased risk than is a drop in stock prices. Further, if increased aggregate risk has an important effect on the equity premium, it must (at least in the current setting) also have a comparable effect on the riskless
rate, since the two effects were seen to be of the same order of magnitude. ${ }^{4}$ Thus the effect of risk on the riskless rate cannot be dismissed a priori as quantitatively insignificant. The conclusion of Hendershott (1979) that increased risk might depress the interest rate but have little effect on stock prices, though not well justified in that paper, receives some theoretical support from our analysis.

The joint endogeneity of risky and riskless interest rates, in particular the tendency for increased risk to depress the riskless rate, is a central theme of this paper. This point has received inadequate attention in the literature, although it has not gone completely unnoticed. Hirshleifer (1970), in his lucid discussion of state-preference models in the presence of complete markets, noted that the value of a riskless bond (which is just the sum of all the contingent claims prices) is increasing in the dispersion of aggregate output across states when $U^{\prime \prime \prime}$ is positive. Campbell (1984) and Breeden (1985) both drew attention to the appearance of consumption variance with a negative sign in closed-form expressions for the riskless rate (based on constant relative risk aversion utility and log-normal consumption processes), and gave an interpretation in terms of the convexity of the marginal utility function. On quite a different note, the notion of a "flight to safety" in the popular financial press describes the depression of the riskless rate resulting from the process of substitution towards less risky assets in the presence of increasing risk. Such non-academic discussions, however, appear to suggest that this effect is an implication of risk aversion per se, an implication not supported by rigorous analysis. ${ }^{5}$
C. A Reduction in the Rate of Growth

In Section A we interpreted movement of bond prices and equity values in opposite directions as a reflection of differential changes in the required rate of return on the two assets. However, it is possible that at least some movements in dividend yields are indicative not of changes in the expected returns on stocks, but rather changes in expected future dividends relative to current dividends. To take a simple example, imagine a reduction in expected future real dividends with the current dividend and the required rate of return unchanged. The current stock price would
fall and thus the dividend to price ratio would rise, though the discount rate was constant by hypothesis. ${ }^{6}$

Between 1973 and 1978, the average growth rate of output per employed person in the U.S. was 0.38 percent per annum, compared with 2.1 percent for the period 1950 to 1973 (Gordon, 1981, p. 559). An exogenous reduction in the expected growth of productivity in the economy as a whole would probably result in a reduction in expected future dividends relative to current dividends, but it would not leave equilibrium interest rates unaffected. At least in the framework of the Lucas-type asset pricing models considered here, a reduction in the growth rate would also be associated with a lower real rate of interest on both bonds and stocks. If the drop in the rate of interest is not sufficiently large so as to overturn the effect on the stock price of the reduced future dividend, this reduction in productivity growth would yield the pattern seen in the 1970's - real interest rates fall and dividend-price ratios rise. On the other hand, one is lead to ask whether the real interest rate might drop so much that the dividend-price ratio will also fall.

The issues are readily apparent in the constant relative risk aversion, lognormal consumption example from Section A. We have:

$$
\begin{gather*}
\log \left(R^{f}\right)=\gamma\left(E\left[\log \left(\tilde{Y}_{2}\right)\right]-\log \left(Y_{1}\right)\right)+\quad \text { inessentials } \\
\log \left(P^{\mathrm{eq}}\right)=(1-\gamma) E\left[\log \left(\tilde{Y}_{2}\right)\right]+\gamma \log \left(Y_{1}\right)+\quad \text { inessentials } \\
\log \left(Y_{1} / P^{e q}\right)=(\gamma-1)\left(E\left[\log \left(\tilde{Y}_{2}\right)\right]-\log \left(Y_{1}\right)\right)+\quad \text { inessentials }
\end{gather*}
$$

Consider the case of a reduction in $E\left[\tilde{Y}_{2}\right]$ with $Y_{1}$ and $\operatorname{Var}\left[\tilde{Y}_{2}\right]$ held constant. From ( $9^{\prime}$ ), we see that $R^{f}$ falls, the extent of the drop being determined by the degree of aversion to intertemporal substitution (measured by the parameter $\boldsymbol{\gamma}$ ). From ( $10^{\prime}$ ), we see that the net effect of decreased growth on the price of equity (and hence on the dividend/price ratio in $11^{\prime}$ ) is ambiguous. Evidently, there are two effects operating here. On the one hand, a fall in expected future dividends exerts
direct downward pressure on the stock price, for a given required rate of return. On the other hand, the fall in the required rate of return which accompanies the reduction in the growth rate of consumption tends to raise stock prices. For $\gamma<1$ the "numerator effect" dominates, while for $\gamma>1$, the percentage drop in the required rate of return exceeds the percentage drop in the expected future dividend, and equity values rise as the growth rate falls.

An increase in risk and a_reduction in the rate of grow.th show a striking symmetry in terms of their effects on asset prices. In response to either shock, riskless bond prices always rise, but stock prices fall when $\gamma<1$, and rise when $\gamma>1$. In the case of stocks, this symmetry is the general equilibrium analogue of a well-known comparative statics result (Sandmo, 1969; Dreze and Modigliani, 1972). When all saving must be in the form of a single risky asset, the parametric effect on individual saving of an increase in rate-of-return uncertainty is the same as that of a reduction in the expected rate of return. Substitution effects (away from saving) dominate when the intertemporal elasticity of substitution is large (i.e. when $\boldsymbol{\gamma}$ is small), and income effects dominate for large $\boldsymbol{\gamma}$. Stock prices rise when the parametric effect on individual saving is positive, and fall when that effect is negative.
D. Nominal Bonds

Unfortunately, we cannot proceed directly from our results for riskiess bonds to inferences concerning the effect of increased risk on the expected real return on nominal bonds. The required return on dollar-denominated bonds will depend crucially on the stochastic process followed by the money supply (and, in general, money demand). Lucas (1982) points out that when the stock market represents all claims on output, the money supply follows any deterministic process, and velocity is constant (due to a Clower constraint, say), a nominal bond is equivalent to equity! This is because (under the above scenario) any shock to output is met one-for-one by a movement in the price level in the opposite direction, and any other price movements are perfectly predictable. Thus the real return on a nominal bond is perfectly correlated with aggregate consumption and
output, and with the equity return.
While the Lucas example powerfully illustrates the importance of the distinction between riskless bonds and nominal bonds, it seems implausible that, in the United States, nominal bonds are generally closer to equity than to index bonds. In fact, two realistic departures from the above example lead me to suspect that the opposite is the case. First, the money supply is, to a very large degree, endogenous. Endogenous procyclicality of the money stock, as discussed by King and Plosser (1982) and Greenwood and Huffman (1986), might largely offset the effect of output supply shocks on the price level (and hence the realized real return on nominal bonds). Second, as I discuss extensively in Section II-B, corporate equity represents only a fraction of total wealth. Then, with a deterministic money supply, the return on nominal bonds will be perfectly correlated with the sum of the payoffs from all sources of wealth, not with the return on equity (narrowly defined). The shocks to total GNP which affect the price level will not be identical with shocks to the stock market.

Fama (1976) and Bodie, Kane, and McDonald (1985) investigate the role of inflation risk in the pricing of short-term bills. Their conclusion is that inflation risk plays a negligible role in short-term interest rate determination, i.e. that the premium for inflation risk embodied in bill rates is close to zero. Mehra and Prescott (1985) also adopt that viewpoint. This suggests that the expected return on short-term nominal assets may be close to the "shadow" riskless rate given by theory. For long term bonds, the relationship may be more tenuous; there appears to be very little basis on which to judge. Thus we must be cautious in extrapolating from the theory of the riskless rate predictions about rates on long-term nominal bonds. The theoretical "shadow" riskless rate, in any event, continues to be of interest in studying the determination of stock prices.

## II. Extensions

## A. Risk Aversion vs. Intertemporal Substitution

The choice of time-separable Von Neumann-Morgenstern utility and the power function for one-period utility implies strong restrictions on preferences. In particular, these choices constrain the intertemporal elasticity of substitution to be the inverse of the (constant) coefficient of relative risk aversion. As Hall $(1982,1985)$ emphasizes, a consequence of this simple representation of preferences is that the effects of risk aversion and aversion to intertemporal substitution are confounded. In this section, I show how the previous analysis can be extended to distinguish between the two effects.

Selden (1978) offers a representation of preferences which he calls "Ordinal Certainty Equivalence", and which considerably generalizes the class of time-additive, Von Neumann-Morgenstern preference orderings. This approach specifies distinct functions governing intertemporal choice and attitudes toward risk, while remaining time-additive.

Formally, consumers maximize $U\left(C_{1}\right)+\beta U\left(\hat{C}_{2}\right)$, where $\hat{C}_{2}$ satisfies $V\left(\hat{C}_{2}\right)=E\left[V\left(\tilde{C}_{2}\right)\right]$, and $U(\cdot)$ and $V(\cdot)$ are concave functions. Here $\hat{C}_{2}$ is the "certainty equivalent" level of consumption, i.e. that nonstochastic level that would provide utility equal to the expected utility of the random second-period consumption. The curvature of $V$ governs risk aversion, while that of $U$ determines willingness to substitute intertemporally.

The first order conditions are:

$$
\begin{align*}
& U^{\prime}\left(C_{1}\right)=\frac{\beta E\left[\tilde{R} V^{\prime}\left(\tilde{C}_{2}\right)\right] U^{\prime}\left(\hat{C}_{2}\right)}{V^{\prime}\left(\hat{C}_{2}\right)}  \tag{15}\\
& U^{\prime}\left(C_{1}\right)=\frac{\beta R^{\prime} E\left[V^{\prime}\left(\tilde{C}_{2}\right)\right] U^{\prime}\left(\hat{C}_{2}\right)}{V^{\prime}\left(\hat{C}_{2}\right)} \tag{16}
\end{align*}
$$

Choosing $U$ and $V$ so that the coefficient of relative risk aversion and the intertemporal elasticity of substitution are constant (though independent) parameters, we have:

$$
U(C)=\frac{C^{1-\gamma}}{1-\gamma}, \quad \text { and } \quad V(C)=\frac{C^{1-\alpha}}{1-\alpha}
$$

where $\alpha$ is relative risk aversion, and $\gamma$ is the inverse of the elasticity of substitution. Equations (15) and (16) become:

$$
\begin{equation*}
\cdot \quad C_{1}^{-\gamma}=\beta E\left[\tilde{R} \tilde{C}_{2}^{-\alpha}\right] \hat{C}_{2}^{\alpha-\gamma} \tag{17}
\end{equation*}
$$

and

$$
\begin{equation*}
C_{1}^{-\gamma}=\beta R^{f} E\left[\tilde{C}_{2}^{-\alpha}\right] \hat{C}_{2}^{\alpha-\gamma} \tag{18}
\end{equation*}
$$

Note that when $\alpha=\gamma$, the first order conditions collapse to those of the simple model of Section I.
Let $\tilde{C}_{2}=\tilde{Y}_{2}$ be conditionally lognormal, and then following Hall (1985), we have:

$$
\begin{equation*}
\log \left(\hat{Y}_{2}\right)=E\left[\log \left(\tilde{Y}_{2}\right)\right]+\frac{1}{2}(1-\alpha) \operatorname{Var}\left[\log \left(\tilde{Y}_{2}\right)\right] . \tag{19}
\end{equation*}
$$

The solutions for the expected return to equity, the risk-free rate, and the equity premium for the lognormal, constant parameter OCE model are:

$$
\begin{gather*}
\log \left(R^{f}\right)=-\log (\beta)+\gamma E\left[\log \left(\tilde{Y}_{2}\right)-\log (Y)\right]-\frac{1}{2}(\gamma-\alpha(1+\gamma)) \operatorname{Var}\left[\log \left(\tilde{Y}_{2}\right)\right]  \tag{20}\\
\log E\left[\tilde{R}^{\mathrm{eq}}\right]=-\log (\beta)+\gamma E\left[\log \left(\tilde{Y}_{2}\right)-\log \left(Y_{1}\right)\right]+\frac{1}{2}(\gamma+\alpha(1-\gamma)) \operatorname{Var}\left[\log \left(\tilde{Y}_{2}\right)\right]  \tag{21}\\
Z=\alpha \operatorname{Var}\left[\log \left(\tilde{Y}_{2}\right)\right] \tag{22}
\end{gather*}
$$

Once again, to study the effects of a mean-preserving spread on the distribution of $\tilde{Y}_{2}$, we differentiate with respect to $\sigma^{2}$, imposing $d E\left[\log \left(\tilde{Y}_{2}\right)\right]=-\frac{1}{2} d \sigma^{2}$.

We obtain:

$$
\begin{align*}
\frac{d \log \left(R^{f}\right)}{d \sigma^{2}} & =-\frac{1}{2} \alpha(\gamma+1)  \tag{23}\\
\frac{d \log E[\tilde{R}]}{d \sigma^{2}} & =\frac{1}{2} \alpha(1-\gamma)  \tag{24}\\
\frac{d Z}{d \sigma^{2}} & =\alpha \tag{25}
\end{align*}
$$

The riskless rate again falls unambiguously as risk increases. The magnitude of this effect is increasing in both risk aversion and aversion to intertemporal substitution. The equity premium depends on risk aversion alone. This is as expected, since the equity premium is well defined even in a model without intertemporal choice. The result (24) may appear surprising to readers unfamiliar with Selden's (1979) paper, which extends Sandmo (1970) to the OCE case. Whether the required return on equity falls or rises with increasing risk depends only on whether the intertemporal sustitution parameter is greater or less than unity. The risk aversion parameter helps to determine the magnitude of the effect (in either direction), but has no bearing on the sign. This is the general equilibrium analogue of Selden's finding that it is the degree of substitutability between first and second period consumption that determines the sign of the effect of rate-of-return uncertainty on first-period saving. The precautionary saving effect dominates when first and second period consumption are strong "complements", i.e. when consumers are very averse to an uneven consumption profile over time.

## B. Two Risky Assets

In this section, I relax the asssumption that the stock market represents all of the assets in the economy. Corporate equities comprise only a fraction of total wealth. Further, while the return on the S\&P 500 is significantly correlated with consumption and with the returns on other assets, these correlations are not overwhelming. Although we would expect increases in general economic uncertainty to be associated with increased stock market volatility, to obtain robust results we want to extend the model to allow risky assets which are not traded in the "stock market", allowing us
to make precise the notion that the corporate sector may be "small", or that its risk may be largely idiosyncratic.

We now allow there to be two "trees" with stochastic second period payoffs. Let $\tilde{Y}_{2}^{1}$ represent the second period payoff from stocks, while $\tilde{Y}_{2}^{2}$ represents the return from all other assets in the economy. Using the first order equation condition (1) from Section I, but with $C_{2}=\tilde{Y}_{2}^{1}+\tilde{Y}_{2}^{2}$, we have:

$$
\begin{equation*}
E[\tilde{R}]=\frac{U^{\prime}\left(Y_{1}\right)}{\beta E\left[U^{\prime}\left(\tilde{Y}_{2}^{1}+\tilde{Y}_{2}^{2}\right)\right]}\left[1-\operatorname{Cov}\left(\frac{\tilde{Y}_{2}^{1}}{p^{\mathrm{eq}}}, \frac{\beta U^{\prime}\left(\tilde{Y}_{2}^{1}+\tilde{Y}_{2}^{2}\right)}{U^{\prime}\left(Y_{1}\right)}\right)\right] \tag{23}
\end{equation*}
$$

which is the "consumption beta" expression for the return on any freely tradeable asset exploited by Grossman and Shiller (1982) .

Taking logs, we have for the required return on equity:

$$
\begin{equation*}
\log E[\tilde{R}]=\log \left(R^{f}\right)+Z, \quad \text { where } \quad Z=\log \left(1-\operatorname{Cov}\left(\frac{\tilde{Y}_{2}^{1}}{p}, \frac{\beta U^{\prime}\left(\tilde{Y}_{2}^{1}+\tilde{Y}_{2}^{2}\right)}{U^{\prime}\left(Y_{1}\right)}\right)\right) \tag{24}
\end{equation*}
$$

Thus, when consumption is no longer perfectly correlated with return on capital, the appropriate risk measure for the equity premium is the covariance of the stock market return with consumption. The variability of consumption remains the relevant notion of risk in determining the riskless rate. Hence the expression for the equilibrium stock return involves both measures of risk. Any shock that raises the variability of consumption will continue to depress the riskless rate. It will raise the equity premium to the extent that it increases the covariation of consumption and the return to "corporate capital". As before, there will still be two opposing forces affecting the required return to equity. Increased risk tends to depress the returns on all assets through the third derivative effect, but raises the premium on those (tradeable) assets in which the increased risk is concentrated.

To obtain explicit comparative statics for the equilibrium stock price in the simple CRRA case, let $\tilde{Y}_{2}^{1}=\bar{Y}_{2}^{1}(1+h \epsilon)$, and $\tilde{Y}_{2}^{2}=\bar{Y}_{2}^{2}(1+k \eta)$, where $\epsilon$ and $\eta$ are independent. The stock price is (up
to a multiplicative factor that doesn't depend on the distribution of $\tilde{Y}_{2}^{1}$ or $\tilde{Y}_{2}^{2}$ )

$$
\begin{equation*}
\left(P^{\mathrm{eq}}\right) \sim E\left[\tilde{Y}_{2}^{1}\left(\tilde{Y}_{2}^{1}+\tilde{Y}_{2}^{2}\right)^{-\gamma}\right]=E\left[\bar{Y}_{2}^{1}(1+h \epsilon)\left(\left(\bar{Y}_{2}^{1}(1+h \epsilon)+\bar{Y}_{2}^{2}(1+k \eta)\right)^{-\gamma}\right]\right. \tag{25}
\end{equation*}
$$

Differentiating with respect to $h$, we obtain:

$$
\begin{equation*}
\frac{d P^{\mathrm{eq}}}{d h} \sim \bar{Y}^{1} \operatorname{Cov}\left[\epsilon,\left(\left(U^{\prime \prime}(\cdot) \tilde{Y}_{2}^{1}+U^{\prime}(\cdot)\right)\right]\right. \tag{26}
\end{equation*}
$$

The sign of the above covariance is determined by the sign of

$$
\frac{d}{d \epsilon}\left(U^{\prime \prime}(\cdot) \tilde{Y}^{1}+U^{\prime}(\cdot)\right)=-\gamma(\cdot)^{-\gamma-1}\left(2-(\gamma+1) \frac{\tilde{Y}_{2}^{1}}{\tilde{Y}_{2}^{1}+\tilde{Y}_{2}^{2}}\right) h \bar{Y}^{1}
$$

Thus increasing risk depresses the stock price if the (stochastic) stock market share $\frac{\tilde{Y}_{2}^{1}}{\tilde{Y}_{2}^{1}+\tilde{Y}_{2}^{2}}<$ $\frac{2}{\gamma+1}$ uniformly, and raises the equity value if $\frac{\tilde{Y}_{2}^{1}}{\tilde{Y}_{1}+\tilde{Y}_{2}}>\frac{2}{\gamma+1}$ uniformly. This result suggests the notion of a critical value of $\gamma$ greater than unity below which the increase in the risk premium dominates the effect on the stock price, and another critical value above which the depression of the riskless rate dominates. $\frac{\tilde{Y}_{2}^{1}}{\tilde{Y}_{2}^{1}+\tilde{Y}_{2}^{2}}$ is best thought of as the share of the corporate sector in total wealth. The numbers in the introduction suggest that a reasonable range for this fraction is $1 / 4$ to $1 / 7$. For instance, if $\frac{1}{7}<\frac{\tilde{Y}_{2}^{1}}{\tilde{Y}_{2}^{1}+\tilde{Y}_{2}^{2}}<\frac{1}{4}$ uniformly, then increasing risk depresses the stock price if $\gamma<7$, and raises it if $\gamma>13$. In the range $7<\gamma<13$, it does not appear possible to sign the effect without further restrictions.

A similar approach modifies the earlier analysis of a reduction in the rate of growth of 'dividends' ${ }^{7}$ to the two-risky-asset case. We have:

$$
\frac{d p^{\mathrm{eq}}}{d \bar{Y}_{2}^{1}} \sim E\left[\left(\tilde{Y}_{2}^{1}+\tilde{Y}_{2}^{2}\right)^{-\gamma}(1+h \epsilon)\left(1-\gamma \frac{\tilde{Y}_{2}^{1}}{\tilde{Y}_{2}^{1}+\tilde{Y}_{2}^{2}}\right)\right]
$$

Thus an increase in expected future dividends raises the stock price if $\frac{\tilde{Y}_{2}^{1}}{\tilde{Y}_{2}^{1}+\tilde{Y}_{2}^{2}}<1 / \gamma$ uniformly, and lowers the stock price if $\gamma \frac{\tilde{Y}_{2}^{1}}{\tilde{Y}_{2}^{1}+\tilde{Y}_{2}^{2}}>1$ uniformly. Thus if $\frac{\tilde{Y}_{2}^{1}}{\tilde{Y}_{2}^{1}+\tilde{Y}_{2}^{2}}$ is bounded between
$\frac{1}{7}$ and $\frac{1}{4}$ with probability one, then the stock price rises with expected future dividends if $\gamma<4$, and falls if $\boldsymbol{\gamma} \boldsymbol{>} \mathbf{7}$.

Analysis of the model with two risky assets considerably raises the plausibility of the view that time-varying risk and changing productivity growth in the corporate sector can account for the failure of bond prices and stock prices to covary closely. Whereas in the model with only one physical asset the anomaly is rationalized only when aversion to intertemporal substitution is implausibly low (i.e. $\gamma<1$ ), we now find that very reasonable values of $\gamma$ are consistent with movements of debt and equity values in opposite directions in response to the two disturbances we have studied. The presumption of Pindyck (1984) that increased stock market volatility would raise the required rate of return and depress stock prices gains in credibility. However, it is still not by any means a foregone conclusion, as values of $\boldsymbol{\gamma}$ consistent with stock price movements in either direction lie in the range consistent with available empirical evidence on intertemporal substitution in consumption. The conclusion that returns on (relatively) riskless assets will bear at least a non-negligible fraction of the burden of adjustment both to increased risk and to declining productivity growth in the corporate sector continues to be robust.

The model with two tangible assets has an additional attractive implication. Suppose we think of the second asset as 'houses.' Then (continuing with the CRRA example) the response of house prices to a mean preserving spread on the payoff to corporate equity is given by

$$
\begin{equation*}
d P^{h} / d h \sim \bar{Y}_{2}^{1} \operatorname{Cov}\left[\epsilon, U^{\prime \prime}(\cdot) \tilde{Y}_{2}^{2}\right] \tag{27}
\end{equation*}
$$

which is easily seen to be positive (again, under the assumption that the two risky assets' returns are uncorrelated), since $U^{\prime \prime \prime}>0$. A corresponding result will hold for the response of house prices to reduced productivity growth in the corporate sector. Thus we have an explanation of the sharp rise in house prices during the 1970's, which is an alternative to that of Poterba (1984) based on tax effects.

In summary, the two-asset model is, for reasonable parameter values, capable of fitting all our major stylized facts about the 1970's.

## C. Capital Accumulation

The exchange models examined so far, in which capital goods are in fixed supply, appear to be a reasonable representation of the short-to-medium run. A year's investment hardly makes a dent in the marginal product of capital. Even over relatively long periods, adjustments to changes in the required rate of return occur through movements in " $q$ ", the relative price of capital, rather than capital accumulation. Indeed, a model in which the supply of capital goods is highly elastic can hardly account for observed fluctuations in stock prices.

Over time, the amount of capital in place is adjusted until, in the long run, the bulk of the adjustment occurs through a change in the quantity (and hence the marginal product) of capital. In this section we examine the polar case of a fully elastic supply of capital, as in standard neoclassical growth models. ${ }^{8}$ Since the relative price of capital in this theoretical long run reflects the reproduction cost of capital, we cannot discuss stock price fluctuations in this context. We can, however, study the effect of increasing risk or a reduction in the rate of technical progress on the riskless rate, and on the equilibrium expected marginal product of capital.

Consider a simple two-period " $q=1$ " model with a diminishing returns production technology. In the first period, the endowment (normalized to unity) can either be consumed, or saved and invested. For ease of solution, I exploit the analogy between the competitive' equilibrium of this well-behaved economy and a planner's problem (Prescott and Mehra, 1980; Brock, 1978 and 1982). The allocation achieved by a planner who chooses $k$ to maximize the utility of the representative consumer will coincide with that resulting from the interaction of atomistic saving behavior and the profit maximization of competitive firms. Once again, the riskless asset is in zero net supply, although individuals see themselves as able to borrow and lend freely.

The technology is represented as $f(k)(1+h \epsilon)$, where $f(k)$ is an ordinary neoclassical production
function, $\epsilon$ is a zero-mean random variable, and $h$ is a scale parameter. The support of $\epsilon$ is such that $(1+h \epsilon)$ is positive. The planner's first-order condition is:

$$
\begin{equation*}
U^{\prime}(1-k)=\beta E\left[U^{\prime}(f(k)(1+h \epsilon)) f^{\prime}(k)(1+h \epsilon)\right] \tag{28}
\end{equation*}
$$

Although no bills are traded in equilibrium, we can still characterize the shadow riskless rate, since the consumer must be indiffecent at the margin between increasing his first-period consumption and purchasing a riskless claim to consumption in the second period. Thus:

$$
\begin{equation*}
R^{f} \beta E\left[U^{\prime}(f(k)(1+h \epsilon))\right]=U^{\prime}(1-k) \tag{29}
\end{equation*}
$$

Differentiating (27) with respect to $h$, we find the effect on the equilibrium capital stock of applying a mean-preserving spread to the return:

$$
\begin{equation*}
d k / d h=-\beta E\left[U^{\prime}(\cdot) \epsilon\right] f^{\prime}(k) / H-f(k) \beta E\left[U^{\prime \prime}(\cdot) \epsilon f^{\prime}(k)(1+h \epsilon)\right] / H, \tag{30}
\end{equation*}
$$

where $H=\beta E\left[U^{\prime \prime}(\cdot)(1+h \epsilon)^{2}\left(f^{\prime}(k)\right)^{2}+U^{\prime}(\cdot)(1+h \epsilon) f^{\prime \prime}(k)\right]+U^{\prime \prime}(1-k)=d / d k\left(\beta E\left[U^{\prime}(\cdot) f^{\prime}(k)(1+\right.\right.$ $\left.h \epsilon)]-U^{\prime}(1-k)\right)$, and thus is negative by the second order condition.

Equation (11) is closely related to equation (12) in Sandmo (1970), who considered the effect of rate-of-return uncertainty on individual saving. The analytical difference, of course, is that in my analysis the planner, in accumulating more capital, lowers its marginal product. The first term is negative by risk aversion alone. This is the substitution effect, whereby increased riskiness of the capital asset leads the planner to increase first-period consumption in order to sidestep some of the risk. A straightforward application of the results in Sandmo (1970, p. 35) shows that the second term is positive under decreasing absolute risk aversion. This reflects a precautionary saving effect; capital accumulation is the only channel of saving open to the planner. In general, either effect can dominate, and thus an increase in rate-of-return uncertainty can result in either more or less capital.

For the constant relative risk aversion case, (30) becomes:

$$
\begin{equation*}
d k / d h=(\gamma-1) f^{\prime}(k) \beta E\left[U^{\prime}(\cdot) \epsilon\right] / H \tag{31}
\end{equation*}
$$

Since $U^{\prime \prime}$ is negative, $E\left[U^{\prime}(\cdot) \epsilon\right]$ is negative as well. Thus the equilibrium capital stock falls with increased uncertainty if and only if $\gamma$ is less than one. The implication for the required rate of return to capital corresponds"exactly to that obtained for the exchange economy. If $\boldsymbol{\gamma}$ exceeds one, the required return to capital (which now equals the expected marginal product) falls with increased return variability.

The implication of increased riskiness of capital for the riskless rate in the " $q=1$ " model is obtained by differentiating (29), and substituting (30) for $d k / d h$ :

$$
\begin{equation*}
\beta E\left[U^{\prime}(\cdot)\right] d R^{f} / d h=-R^{f} f(k) \beta E\left[U^{\prime \prime}(\cdot) \epsilon\right]-J d k / d h \tag{32}
\end{equation*}
$$

$\qquad$
where $J=U^{\prime \prime}(1-k)+R^{f} \beta E\left[U^{\prime \prime}(\cdot)(1+h \epsilon)\right] f^{\prime}(k)$ and $J<0$ Note that the first term on the righthand side of (32) corresponds exactly to the "full answer" in the exchange model. It is negative as long as $U^{\prime \prime \prime}$ is positive. Since $\frac{J^{\prime}}{\beta E\left|U^{\prime}(\cdot)\right|}=\frac{\partial}{\partial k}\left(\frac{U^{\prime}(1-k)}{\left.\beta E \mid U^{\prime}(\cdot)\right)}\right)$ the second term in (32) reflects the effect on the marginal rate of substitution of the change in the time profile of consumption resulting from the adjustment of $k$.

When $d k / d h<0$, the response of the capital stock to increased risk reinforces the tendency for the riskless rate to fall. The drop in $k$ means greater present consumption and (in expectation) less future consumption, raising the expected marginal rate of substitution by more than the initial effect of the mean-preserving spread. Thus, in the $\gamma<1$ case, the "long run" (i.e. variable capital stock) drop in the riskless rate exceeds the impact effect. By way of contrast, when $d k / d h>0$, the adjustment of the capital stock mitigates the depressing effect of increased risk on the riskless rate by steepening the expected consumption profile.

The response to a decrease in the expected marginal productivity of capital is entirely analogous. For $\gamma<1$, the substitution effect dominates - agents consume more today, and have less expected consumption tomorrow. This illustrates the point that "bad news" may increase current consumption, if that news heralds a worsening of investment opportunities. The riskless rate in the $\gamma<1$ case falls more in the " $q=1$ " model than in the model with a fixed capital stock. Conversely, when $\gamma>1$, agents reduce current consumption and accumulate more capital in response to the lower marginal productivity. This, of course, reflects the strong "smoothing" tendencies of agents with low intertemporal elasticities of substitution. The adjustment of the capital stock in this case mitigates the effect of the reduced marginal productivity on the riskless rate.

## III. Conclusion

I have tried to air a number of the theoretical issues that arise in interpreting the independent movement of bond and stock prices as a consequence of increased risk or decreased productivity growth. We have found that the effects of these disturbances on the riskless rate are unambiguous, but that stock prices may rise or fall, contrary to the assumptions of much previous applied work. Indeed, in the model with only one tangible asset, it appears likely that stock prices should rise in response to increased risk or decreased growth. On the other hand, a two-asset model shows promise in that, for reasonable parameter values, the disturbances we consider can simultaneously account for a fall in stock prices, a drop in real interest rates, and a rise in the price of housing.

## Footnotes

${ }^{1}$ The results could be obtained for a completely general mean-preserving spread by checking the concavity of the expressions in $Y_{2}$, the approach taken by Rothschild and Stiglitz (1971). The parametric approach taken here, however, allows explicit decomposition of the changes in expected returns into income and substitution effects, as well as direct comparison with the results in Section III in the presence of capital afcumulation.
${ }^{2}$ This result appears at first to conflict with the implication of equation (13) in Campbell (1986) that the critical value of $\gamma$ is 2 . The reconciliation is that Campbell treats an increase in $\sigma^{2}$ without the compensating reduction in $E\left[\log \left(\tilde{Y}_{2}\right)\right]$ so that his perturbation is not a mean-preserving spread on $\tilde{Y}_{2}$.
${ }^{3}$ On reading an earlier version of this paper, Franco Modigliani noted that this endogenous nature of the riskless rate has long been a part of the oral tradition exemplified in his lectures.
${ }^{4}$ However, note the important qualification of this claim in Section II-B.
${ }^{5}$ One might think that the "Wall Street Journal" view would perhaps be better formalized in the context of a conventional CAPM model. The CAPM, however, only determines return differentials. The two-period intertemporal setup given here, which simultaneously treats the saving decision and portfolio choice, is the minimal one capable of determining all rates of return.
${ }^{6}$ Although Blanchard and Summers (1984) and Mankiw, Romer, and Shapiro (1985) suggest that real dividends are very close to a random walk, Campbell and Shiller (1986) find that high dividend-price ratios do indeed predict lower dividends in the future. Even small changes in the long run growth rate, which might be hard to detect in the data, could have major effects on stock prices.
${ }^{7}$ Inour two-period framework, $\tilde{Y}_{2}^{1}$ includes the scrap value of the capital (the fruit and the trees).
${ }^{8}$ The intermediate case, in which capital has a finite elasticity with respect to $q$, would be a
far more ambitious undertaking.

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