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TURNOUT ACROSS DEMOCRACIES

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ABSTRACT

World democracies widely differ in electoral rules, as well as in legislative, executive or legal institutions. Different institutional environments induce different mappings from electoral outcomes to the distribution of power. We explore how these mappings affect voters' participation to an election. We show that the effect of such institutional differences on turnout depends on the distribution of voters' preferences for the competing parties. In particular, we uncover a novel contest effect: given the distribution of preferences, turnout increases and then decreases when we move from a more proportional to a less proportional system; turnout is maximized for an intermediate degree of proportionality. Moreover, we generalize the competition effect, common to models of endogenous turnout: given the institutional environment, turnout increases in the ex-ante closeness of the election and peaks when the population is evenly split between the two parties. These results are robust to a wide range of modeling approaches, including ethical voter models, voter mobilization models, and rational voter models.

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1 Introduction

Electoral participation is considered as an indication of the health of a democracy (Rose 1980, Powell 1982) and a pillar of the democratic ideal of political equality (Lijphart 1997). The historical average turnout rate as a percentage of voting age population displays a large variance across world democracies: New Zealand, Portugal, Indonesia, Italy, and Albania have an average turnout rate above 85%, while Senegal, Colombia, Ecuador, Ghana, and the U.S. all have average turnout rates below 50%.[1] A number of empirical papers have attempted to account for cross-national variations in turnout (Powell 1980, 1982, 1986; Crewe 1981; Jackman 1986; Jackman and Miller 1995; Blais and Carty 1990; Black 1991; Franklin 1996; Blais and Dobrzynska 1998). These studies highlighted that political institutions, not only electoral rules but also party systems, are an important factor affecting turnout.

This raises an important question: How do political institutions, that is, electoral rules, bicameralism, judicial review, federalism, separation of powers, committee chair assignments, and all other institutions determining the power of majority and minorities, affect electoral participation in a democracy? A recent theoretical literature in economics and political science has compared turnout in first-past-the-post (FPTP) and proportional representation (PR) electoral systems (Herrera et al. 2014, Kartal, forthcoming, Faravelli and Sanchez-Pages, forthcoming) and has shown that what system promotes higher turnout crucially depends on the closeness of the election. However, this is unsatisfactory as most electoral systems in use around the world are *de jure* or *de facto* rather somewhere in between FPTP and PR. Many countries adopt explicitly mixed systems, where two electoral systems using different formulae run alongside each other. These systems are likely to give results the proportionality of which falls somewhere between that of a FPTP and that of a PR system.[2] Moreover, even when the electoral rule is not mixed, formal and informal institutions in the legislative, executive or judicial branches (for example, the veto power of a "qualified minority" in the U.S., the way margins of victory translate into

[1] Pintor et al. 2012, based on electoral data from all parliamentary elections since 1945.

[2] Usually, one of the two systems is a plurality system, often a single-member district system, and the other a list PR system. We observe two forms of mixed system. When seat allocations at the PR level is dependent on what happens in the SMD seats and compensates for any disproportionality that arises there, the system is called a Mixed Member Proportional (MMP) system (this is the case, for instance, in Germany, New Zealand, Hungary, Bolivia, Venezuela, Scotland, Wales and in Italy between 1994 and 2001). When the two sets of elections are not dependent on each other for seat allocations, the system is called a Parallel or Mixed Member Majoritarian (MMM) system (this is the case, for instance, in Japan, South Korea, Russia, Ukraine, Lithuania, and Croatia). A comparative analysis of these systems is presented in Moser and Scheiner (2004).

committee assignments or guarantee a more powerful mandate) induce a different mapping from electoral outcomes to the distribution of power. This means that, in order to assess the effect of political institutions on turnout it is important to consider the finer details of the electoral rule, as well as the political institutions mediating between seat shares and actual distribution of power, well beyond the crude distinction between majoritarian and proportional elections.

This is exactly what we try to do in this paper. In particular, we develop a formal approach in order to provide a rigorous foundations for the study of the complex relationship between proportionality and turnout. We present a theoretical analysis of the fundamental causes of the variation in turnout based on differences in institutions for political power sharing, or proportionality. Instead of exploring all the formal and informal political institutions mentioned above, we consider all possible determinants of the degree of proportionality in a reduced form, considering as equally important all the different institutional systems impacting the mapping from vote shares to the relative weight of different parties in policy making (henceforth *power shares*). We use the "contest success function"[3] and introduce a power sharing, or proportionality, parameter, γ, that allows us to embed a wide array of electoral systems or power sharing regimes, ranging from a fully proportional power sharing system ($\gamma = 1$) to a pure winner-take-all system ($\gamma = \infty$). This modeling innovation allows us to span continuously across all electoral systems and to analyze them. We study the role of these institutions on electoral participation by characterizing how that vote-shares-to-power-shares mapping affects voters' incentives to vote and parties' campaign efforts. We try to develop a theory that is as robust and general as possible. First, we take into account the distribution of political preferences in the population, a contextual factor that has proven to be crucial in models of endogenous turnout (see, for example, Herrera et al. 2014 and Kartal, forthcoming). Second, we allow for multiple alternative behavioral assumptions about the turnout mechanics, rather than limiting our analysis to one single approach.

We show that the effect of the institutional differences on turnout depends on the distribution of voters' preferences for the competing parties or the "closeness" of the election in a non obvious way.[4] In particular, we uncover a novel *contest*

[3]See Tullock 1980. This function is extensively used in several economic contexts, especially in the contest literature (see among many others Skaperdas 2006) typically as a mapping from efforts or resources to the chance of victory.

[4]Cox (1999), as summary of the analysis of the elite mobilization section, says that "...the argument following Key (1949) says that closeness will (a) boost mobilizational effort and (b) correlate positively with turnout." Our model will qualify these statements for each degree of

effect: given any asymmetric distribution of preferences, as we move from a relatively more proportional to a more majoritarian system, turnout increases first and then decreases. Therefore, turnout is highest for an intermediate degree of proportionality of the electoral system. The particular degree of proportionality depends on the distribution of preferences, but it is always intermediate for any ex-ante uneven election. The intuition is as follows. When γ grows large, the electoral system becomes more and more similar to a winner-take-all system. Hence, turnout drops for any lopsided preference distribution because the underdog side has no chance of winning, and winning is all that matters. When γ is low, the electoral system becomes similar to a proportional system with moderate turnout for all preference distributions, as the incentives to turnout remain even in a very lopsided election: there is always a proportional power gain for turning out more. Finally and crucially, for intermediate γ turnout is the highest. To understand the intuition for this result, imagine that the preference split is 40-60 and think which institutional system would grant the largest turnout in this case. Intuitively, such a system would be one that grants significant power gains around an election outcome close to 40-60. Namely, an intermediate system whose vote-shares-to-power-shares mapping is very steep around a qualified minority of 40%, so that the marginal gain from extra votes can make a significant difference in the powers granted de jure or de facto.[5]

In addition to this, we generalize to all institutional environments the *competition effect*, already well-documented in several models of endogenous turnout: given the institutional environment, turnout increases in the ex-ante closeness of the election and peaks when the population is evenly split between the two parties. Several experimental work has confirmed this theoretical prediction (see Levine and Palfrey 2007, Kartal 2014, Herrera et al. 2014).

We derive our results for the ethical voter model and then show that these results are preserved in other costly voting models. These models are the voter mobilization model, which we fully characterize and the rational voter model, for which we provide numerical simulations supporting the qualitative results of the previous two models. Unlike the rational voter model, the ethical voter model, for which we conduct the core of our analysis, assumes that voters on the two sides overcome the free-rider problem so that each side turns out at the optimal level. This guarantees that turnout remains large in a large election, a desirable property. On the technical side, the decreasing generalized reversed hazard rate (DGRHR) property of the cost function,

proportionality and hence comparatively. We do this not only for the mobilization logic but also from the instrumental and ethical voting perspectives.

[5]So for instance, leadership in committees, ministerial appointments, power to appoint or veto nominations, veto power on some types of legislation, etc.

a regularity condition on probability distributions, turns out to be they key sufficient condition to obtain all the results in all group models. The equilibria from all models feature another well documented property (see, among others, Castanheira 2003), the *underdog effect*. We show that, in all the models we present and for all institutional systems, the underdog effect is *non-full*, which means that the side which enjoys the majority of ex-ante support also obtains the majority of the votes in equilibrium for all γ. This property drives all comparative statics results we obtain, namely, the competition effect and, most importantly, the contest effect described above.

1.1 Related Literature

Our modeling strategy is related to a body of literature that studies voters' turnout in large elections. Our main model is the ethical voting model (Coate and Conlin 2004, Feddersen and Sandroni 2006). We also show the same results hold for mobilization models (Morton 1987, 1991; Cox and Munger 1989, Uhlaner 1989, Shachar and Nalebuff 1999).

Razin (2000) studies the effect of vote shares on policy platforms. He shows that vote shares communicate information to the candidates, who consequently have an incentive to moderate their policy when their margin of victory shrinks. Castanheira (2003) is, to our knowledge, the first paper to consider the effect of "mandates" on turnout in large elections. Its focus is not on comparing the size of mandates per se, but on showing that mandates have in general the effect of dramatically increasing turnout relative to winner-take-all elections.

A recent strand of literature—including, among others, Herrera et al. (2014) and Kartal (forthcoming, 2014)—studies strategic voting in a rational voter framework and analyzes how turnout varies in two extreme electoral systems (fully proportional and fully majoritarian) for all levels of preference splits. These papers cannot say much about intermediate systems. Kartal (forthcoming) focuses more on comparative welfare results than on comparing turnouts. As far as turnout is concerned, Kartal (see also Herrera et al. 2014) shows that full underdog compensation, and hence a close high-turnout election, can occur when the distribution of voting costs is degenerate (see also Goeree and Grosser 2007, Taylor and Yildirim 2010), or is bounded below by a strictly positive minimum voting cost (see also Krasa and Polborn 2009), but not otherwise. Faravelli and Sanchez-Pages (2014) is the first paper comparing turnout and welfare across a wider range of power sharing rules, in the same spirit as we do here. They obtain results in a neighborhood of perfectly even elections ($q = 1/2$) or perfectly biased elections ($q = 1$), the only tractable cases in a rational voter model. Ours is the first paper that studies a continuum of electoral systems for a general distribution of preferences in the population.

Finally, Faravelli, Man and Walsch (2014) study the effect of mandate together with "paternalism". They show, under very general conditions, that the combination of these two factors guarantees positive turnout even in a large elections and in a rational voter framework. In addition, they provide evidence of a mandate effect from U.S. congress elections.

The article is organized as follows. Section 2 presents the general model setup. Section 3 contains the analysis of group voting models—that is, the ethical voter model (Section 3.1) and the mobilization voter model (Section 3.3)— and compares turnout across different institutional environments and preferences distribution. Section 4 shows that the similar results hold if we consider, instead, a rational voter framework. Section 5 offers some concluding remarks and describes potential paths of future research.

2 General Setup

We introduce here a setup common to all models we consider. Consider two parties, A and B, competing for power. Citizens have strict political preferences for one or the other. We denote by $q \in (0,1)$ the preference split, that is, the chance that any citizen is assigned (by Nature) a preference for party A (thus, $1 - q$ is the expected fraction of citizens that prefer party B). Beside partisan preferences, the second dimension along which citizens differ from one another is their cost of voting: each citizen's cost of voting, c, is drawn from a distribution with twice differentiable cumulative distribution function $F(c)$ over the support $c \in [0, \overline{c}]$, with $\overline{c} > 0$. The cost of voting and the partisan preferences are two independent dimensions that determine the type of a voter.

For any vote share V obtained by party A, an institutional system γ determines the mapping to the respective power shares $P^A_\gamma(V) \in [0,1]$ and $P^B_\gamma(V) = 1 - P^A_\gamma(V)$. For normalization purposes, we let the utility from "full power to party i" equal 1 for type i citizens and 0 for the remaining citizens.[6]. Hence the power shares are the reduced form "benefit" components of parties' (respectively, voters') utility functions that will determine the incentives to campaign (respectively, vote) in a given institutional system γ. In a $\gamma-$system, payoffs as a function of the vote share are represented by a standard "contest success function"[7], where γ ranges from 1 to ∞:

[6]This normalization will allow us to match party utility and voters' utilities in a simple way under all the models that will be considered.

[7]See, for instance, Hirshleifer (1989). When nobody votes ($\alpha = \beta = 0$) assume equal shares

$$P^A_\gamma(V) = \frac{V^\gamma}{V^\gamma + (1-V)^\gamma}\ , \quad P^B_\gamma(V) = \frac{(1-V)^\gamma}{V^\gamma + (1-V)^\gamma} \tag{1}$$

This representation can accommodate a wide range of intermediate power sharing rules between pure proportional systems (P) and winner-take-all systems (M) using a single parameter in the payoff function. The two extreme cases correspond to $\gamma = 1$ (P) and $\gamma = \infty$ (M).[8]

As we discussed in the Introduction, intermediate systems which are a mixture of proportional representation and majoritarian systems are very common and have plenty of institutional details we do not model.[9] Intuitively, we just want to capture the fact that the larger γ is the more majoritarian is the system.

Even in a winner-take-all-system like the U.S. Presidential race, a large winning margin carries with it added benefits to the winner due to a "mandate" effect, and larger winning margins for the President can carry over to a larger majority in one or both houses of Congress, via "coattails" effect.[10] Also, the fact that the legislative branch in a M system has leverage over the executive branch and the presidency will tend to smooth out the winner-take-all payoff function in the direction of a proportional system. Similarly, an increase in vote shares might have a disproportional impact on payoffs also in political systems with proportional representation. For example, in parliamentary systems that require the formation of a coalition government, a party that is fortunate to win a clear majority of seats outright has much less incentive (or in some cases none at all) to compromise with other parties in order to govern effectively.

Figure 1 illustrates the power share payoff P^A_γ as a function of the vote share V for three power sharing parameters γ, namely: $\gamma = 1$ (i.e. the P system, dashed line), $\gamma = 5$ (i.e. an intermediate power sharing system, continuous line), and $\gamma \to \infty$ (i.e. a pure M system, dotted line).

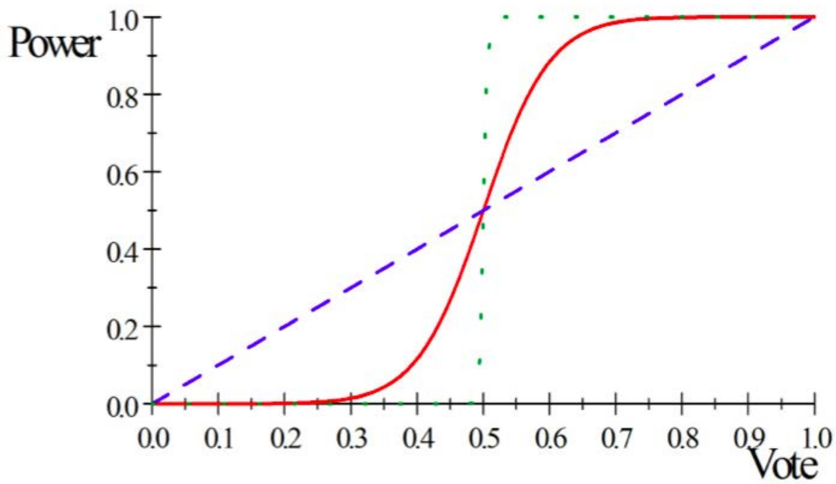
$(V = 1/2)$.

[8]There are other ways to introduce a proportionality parameter. Faravelli and Sanchez-Pages (2014) model it as a linear combination of proportional representation and winner-take-all.

[9]For example, in Germany, voters express two preferences, one for a candidate and one for a party: 299 members of parliament's lower house are directly elected in single member districts; another 299 members are elected from candidate lists until each party's seat share matches the proportion of party votes that it won.

[10]For an empirical analysis of such effects, see Ferejohn and Calvert (1984) and Calvert and Ferejohn (1983). See also Golder (2006).

Figure 1: Power sharing function for different values of γ.



Citizens choose whether to vote for party A, party B, or abstain. If a share α of A types vote for A and a share β of B types vote for B, the expected turnout for party A and B and total turnout are, respectively:

$$T_A := q\alpha, \quad T_B := (1-q)\beta, \quad T := T_A + T_B$$

while the expected vote shares for party A and B are respectively:

$$V = \frac{q\alpha}{T}, \quad 1 - V = \frac{(1-q)\,\beta}{T}$$

Without loss of generality, in the remainder we assume party A is the ex-ante *underdog*, namely, $q \in [0, 1/2]$ where applicable. We also define the preference ratio Q and the turnout ratio R as:

$$Q := \frac{q}{1-q}, \qquad R := \frac{T_A}{T_B}$$

We look for symmetric equilibria. These equilibria can be characterized by a voting cost threshold for each side (c_α, c_β) below which supporters turn out and above which they abstain, hence the share of A (B) supporters that turn out can be expressed by $\alpha = F(c_\alpha)$ $(\beta = F(c_\beta))$. Henceforth, we denote as $f(c) = F'(c)$ the probability density function of the cumulative cost distribution function $F(c)$, and we call G its inverse, namely $G(\alpha) := F^{-1}(\alpha) = c_\alpha$. Moreover, we denote partial derivatives of any function Z with respect to q or γ (our main comparative statics parameters) with the compact notation: $Z_q := \frac{\partial Z}{\partial q}$.

3 Group Voting Models

The basic idea behind these models is that the positive externality of voting among supporters of the same party is internalized, leading to higher turnout. The rationale behind the solution to this collective action problem may differ across group voting models, but the end result is that, contrary to the instrumental voting model (discussed in Section 4 below), the share of voters turning out is high regardless of the size of the population. In group voter models, the two sides compete in an election by turning out their supporters, which have a voting cost to turn out. The population is a continuum of measure one, divided into q A supporters and $(1-q)$ B supporters. In a $\gamma-$power sharing system, the marginal group benefits to the two sides, with respect to (c_α, c_β), can be derived from (**??**) and are, respectively:

$$\frac{dP_\gamma^A}{dc_\alpha} = \frac{dP_\gamma^A}{dV}\left(\frac{(1-q)\,\beta}{T^2}\right) qf\left(c_\alpha\right), \quad \frac{dP_\gamma^A}{dc_\beta} = -\frac{dP_\gamma^B}{dV}\left(\frac{q\alpha}{T^2}\right)(1-q)\,f\left(c_\beta\right) \tag{2}$$

where:

$$\frac{dP_\gamma^A}{dV} = -\frac{dP_\gamma^B}{dV} = \frac{\gamma}{V\,(1-V)}\frac{\left(\frac{V}{1-V}\right)^\gamma}{\left[1+\left(\frac{V}{1-V}\right)^\gamma\right]^2}$$

3.1 Ethical Voter Model

Our main approach to study turnout in elections, which is grounded in group-oriented behavior, is the ethical voter model (Coate and Conlin 2004, Feddersen and Sandroni 2006). This model assumes that citizens are "rule utilitarian" which means that they overcome the free-riding problem and manage to act as one cohesive group. Since the ethical voter model assumes that citizens are "rule utilitarian" all citizens of one group act as one agent. As a consequence, this model involves an equilibrium between two party-planners on each side, A and B. In the solution, each planner looks at the total benefit from the outcome of the election considering the total cost of voting incurred by the supporters of his side, taking the other planner's turnout strategy as given.[11] The cost of turning out the voters for the social planner on side A is the total cost suffered by all the citizens on side A that vote, namely,

$$C\left(c_\alpha\right) := q\int_0^{c_\alpha} cf\left(c\right)dc$$

[11] We assume "collectivism", so the planner on each side, A and B, only looks at the total cost of voting of the voters on his side. The results would not change if we assumed "altruism" as in Feddersen and Sandroni (2006): each planner takes into account the cost of voting of all citizens that vote regardless of their side.

The citizens with cost below the planner-chosen cost threshold, c_α, vote because ethical voter models assume citizens get an exogenous benefit D (larger than their private voting cost $c \leq c_\alpha$) for "doing their part" in following the optimal rule established by the planner.

Defining the *generalized reversed hazard rate* as $\frac{cf(c)}{F(c)}$, we introduce the following definition: a distribution satisfies the *decreasing generalized reversed hazard rate* (DGRHR) property if and only if $\frac{cf(c)}{F(c)}$ is decreasing. We call it DGRHR by analogy with the known increasing generalized failure rate (IGFR, see, for instance, Lariviere 2006), which refers to the function $\frac{cf}{1-F}$.[12]

Lemma 1 *If a distribution function satisfies DGRHR, then its inverse satisfies the increasing generalized reversed hazard rate (IGRHR) and vice versa.*

Proof. See Appendix. ■

The above lemma helps to prove that, if the cost distribution satisfies the DGRHR property, we have the following result for all $q \in [0, 1/2]$.

Proposition 1 *The equilibrium exists and it is unique and has the following properties: (1) Partial Underdog Compensation: $\alpha > \beta$, $q\alpha < (1-q)\beta$, hence $R < 1$. (2) Competition Effect: given γ, turnout, T, and turnout ratio, R, increase in q. (3) Contest Effect: given $q < 1/2$, turnout increases and then decreases with γ; it achieves its maximum for intermediate γ. (4) As γ goes to infinity (winner-take-all), turnout goes to one when $q = 1/2$ and goes to zero otherwise.*

Proof. See Appendix. ■

The solution for the ethical voter model for a general γ is not straightforward to derive, because the underdog compensation is strictly partial (rather than zero), so $\alpha \neq \beta$, and the two equations of the system of FOCs do not decouple. It is convenient for the analysis to rewrite the two FOCs compactly as:

$$W = q\alpha G(\alpha) = (1-q)\beta G(\beta)$$

where:

$$W := \gamma \frac{R^\gamma}{[1+R^\gamma]^2}$$

[12] The DGRHR is also used in Che, Dessein and Kartik (2013), which in their Appendix G verify that a variety of familiar classes of distributions, including Pareto distributions, power function distributions (which subsume uniform distributions), Weibull distributions (which subsume exponential distributions), and gamma distributions, satisfy this condition.

The DGRHR property turns out to be key for several reasons, not only to guarantee existence, but also for the competition effect and for the contest effect. It is easy to show non existence if DGRHR is violated, at least in certain parameter ranges. Even when existence is granted, a violation of DGRHR can cause the competition effect to fail, that is, higher equilibrium turnout in more lopsided elections, as we show later in an example. The DGRHR property guarantees some regularity in the cost distribution function, so that, for instance, if the ratio of the proportion of voters turning out from each side—α/β—increases as parameters change, then also the cost threshold ratio—c_α/c_β—increases. The latter implies, among other things, that there is monotonicity between the relative support ex-ante and the relative support ex-post: if q, the relative ex-ante support for A, increases, then the relative turnout for A, $R = T_A/T_B$, does too in equilibrium. For instance if 1 out of 4 citizens prefer A and 1 out of 3 voters actually voted for A (see the partial underdog effect described below), then it cannot be that increasing the former reduces the latter, under DGRHR. We now discuss the properties we derived in order.

(1) What seems to be a common feature across several costly voting models is the underdog effect (see Castanheira 2003 and Levine and Palfrey 2007, among others). Namely, voting models have the general property that the supporters for the underdog side have higher incentive to turn out than leader's supporters. Hence, in equilibrium they vote proportionally more than the leader's supporters. We call this feature "compensation" as it reduces the leader's initial advantage. Moreover, we call this compensation "partial" when the larger turnout of the underdog supporters is not enough to overturn the leader's initial advantage in preferences. In rational voter models, partial compensation seems to be a general feature when voting costs are heterogenous (see also Herrera et al. 2014 and Kartal Forthcoming), the reason being that while the underdog side has higher benefits from turning out, it also bears higher costs, as it needs to turn out more supporters, and hence will only find more supporters with higher costs. In the ethical model analyzed here, though, the compensation effect comes from the cost side not from the benefit side, as in rational voter models. More specifically, the benefit side is in fact identical for the two competing sides. First, because it is a zero sum game so there is a symmetry between the incentives of one side and the incentives of the other. Second, because the left and right derivatives are identical in a continuous model. This means that, from any starting turnout profile, increasing marginally the turnout of one side increases the benefit to that side as much as increasing marginally turnout for the opposite side increases the benefit to the opposite side. Hence, the difference lies on the costs sides alone, namely, for any given cost threshold c_α the underdog party has lower additional cost to turn out those additional voters. In other words, the marginal cost

is

$$qc_{\alpha}f\left(c_{\alpha}\right)$$

and it is lower for the underdog ($q < 1/2$). This compensation, however, is partial so the ex-ante leader remains the ex-post leader, albeit by a lower margin. This happens because of the term c_{α} in the marginal cost, which means that it costs more to turn out additional agents. This is the same logic described above for the rational voter model.

(2) We show that the competition effect is general for the ethical voter model regardless of the electoral system: the closer the ex-ante preference split, the larger the turnout. While the competition effect seems like a very intuitive property, it is in fact not generally true in rational voter models even with a majoritarian system.

(3) We believe the contest effect is novel: previous work has not compared turnout across different electoral systems for all preference splits q (for extreme values of q,see Faravelli and Sanchez-Pages 2014). The intuition is as follows: take any value of the preference split, say for instance a 40-60 preference split ($q = 40\%$). When γ is large the electoral system becomes similar to a winner-take-all system. Hence, turnout should be low because, despite the underdog compensation, the underdog side has a very small chance of winning. When γ is low, the electoral system becomes similar to a proportional system with moderate turnout for all preference splits. For intermediate γ, the marginal gain from extra votes can make the most difference and this is where turnout is highest. For instance, an intermediate γ could model in reduced form an electoral system (and possibly several other institutional details) where extra votes for the underdog around a 40-60 outcome could mean leadership in more committees and/or obtaining veto or filibuster powers for some decisions. Having partial (rather than full) underdog effect is crucial for the contest effect: an election that is not a toss up ex-ante needs to remain such ex-post. If, in equilibrium, we had a 50-50 electoral outcome, then turnout would always be increasing in γ. This is because the slope of the power function is steepest around a 50-50 outcome in all electoral systems (for all $\gamma > 1$).

(4) Lastly, in a winner-take-all system turnout is high and positive only in a very close election, because due to the *partial* underdog compensation, the underdog side has no chance of winning an ex-ante lopsided election. Hence, the underdog supporters will not turn out significantly in such an election.

3.2 Examples

We provide first a simple example which satisfies DGRHR. Second, to show that DGRHR is a tight condition we provide an example that violates it and violates the competition effect.

Closed Form Example Assume the cdf comes from the family (which satisfies weakly DGRHR):

$$F(c) = \left(\frac{c}{\overline{c}}\right)^{1/k}, \quad c \in [0, \overline{c}], \; k > 0 \quad \Longleftrightarrow \quad G(\alpha) = \overline{c}\alpha^k$$

The FOCs are

$$q\overline{c}\alpha^{1+k} = (1-q)\,\overline{c}\beta^{1+k} = W$$

where:

$$R = Q^{\frac{k}{1+k}}, \qquad W = \left(\gamma \frac{Q^{\gamma\frac{k}{1+k}}}{\left(1 + Q^{\gamma\frac{k}{1+k}}\right)^2}\right)$$

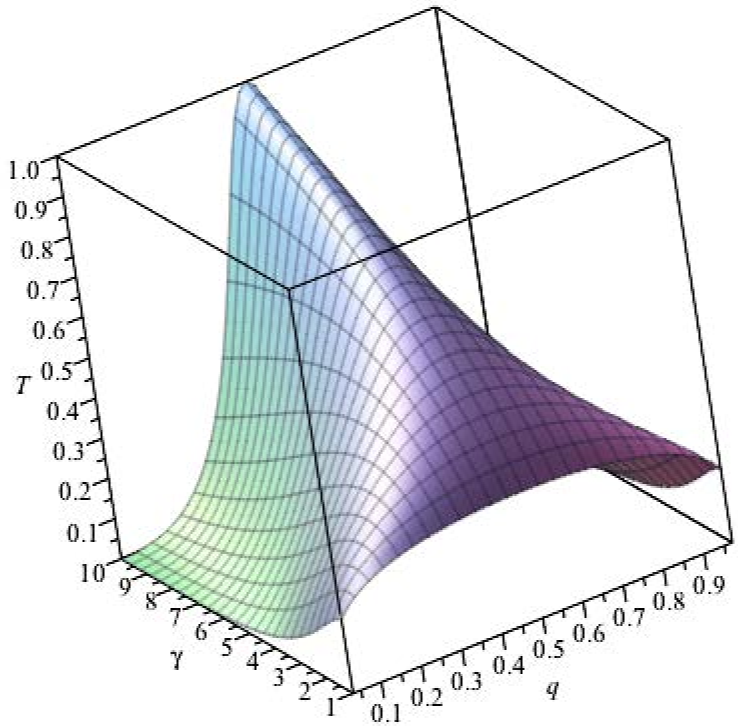
Hence turnouts for each side are:

$$\alpha = \left(\frac{\gamma}{q\overline{c}} \frac{Q^{\gamma\frac{k}{1+k}}}{\left(1 + Q^{\gamma\frac{k}{1+k}}\right)^2}\right)^{\frac{1}{1+k}}, \qquad \beta = \left(\frac{\gamma}{(1-q)\,\overline{c}} \frac{Q^{\gamma\frac{k}{1+k}}}{\left(1 + Q^{\gamma\frac{k}{1+k}}\right)^2}\right)^{\frac{1}{1+k}}$$

Figure 2 shows T as a function of both q and γ for the uniform distribution case $(k = 1)$ with $\overline{c} = 5$.

This picture summarizes the main insights. For any electoral/institutional system γ, the competition effect is apparent: turnout increases the closer the ex-ante preference split becomes. Fixing the preference split q, turnout is non-monotonic in the electoral/institutional system γ, first increasing and then decreasing. For any $q < 1/2$, the turnout maximizing $\widehat{\gamma}$ is increasing in the competition q: the closer the election, the closer to winner-take-all the electoral system has to be in order to achieve its highest turnout. In other words, if ex-ante preference splits are uneven, then more proportional systems maximize turnout; on the other hand, if ex ante preference splits are even, more majoritarian systems achieve the highest turnout. In sum, the electoral/institutional system that delivers the highest turnout crucially depends on the initial preference split and in a non trivial way. This questions the validity of all cross-country empirical comparisons of turnout which, to the best of our knowledge, lump together electoral turnout results over time in each country, never controlling for the value of q in each election.

Figure 2: T as a function of γ and q, ethical voter model.



Counterexample In general, from the FOCs we obtain turnout

$$T = \gamma \left(G(\alpha) G(\beta)\right)^{\gamma-1} \frac{G(\alpha) + G(\beta)}{\left((G(\alpha))^{\gamma} + (G(\beta))^{\gamma}\right)^2}$$

For $\gamma = 1$ we have:

$$T = \frac{1}{G(\alpha) + G(\beta)}$$

The cdf family

$$\begin{aligned} F(c) &= 1 - (1-c)^{1/m} \qquad \Longleftrightarrow \qquad G(\alpha) = 1 - (1-\alpha)^m \\ c &\in [0,1], \quad m > 0 \end{aligned}$$

violates the DGRHR property, as the GRHR of the inverse $G(\alpha)$ (see Lemma 1) is decreasing:

$$h(\alpha) = \frac{m\alpha (1-\alpha)^{m-1}}{1-(1-\alpha\alpha)^{m}}$$

Take for instance $m = 2$. For $\gamma = 1$ the FOCs are:

$$q\alpha^2 (2-\alpha) = (1-q)\beta^2 (2-\beta) = W$$

where:

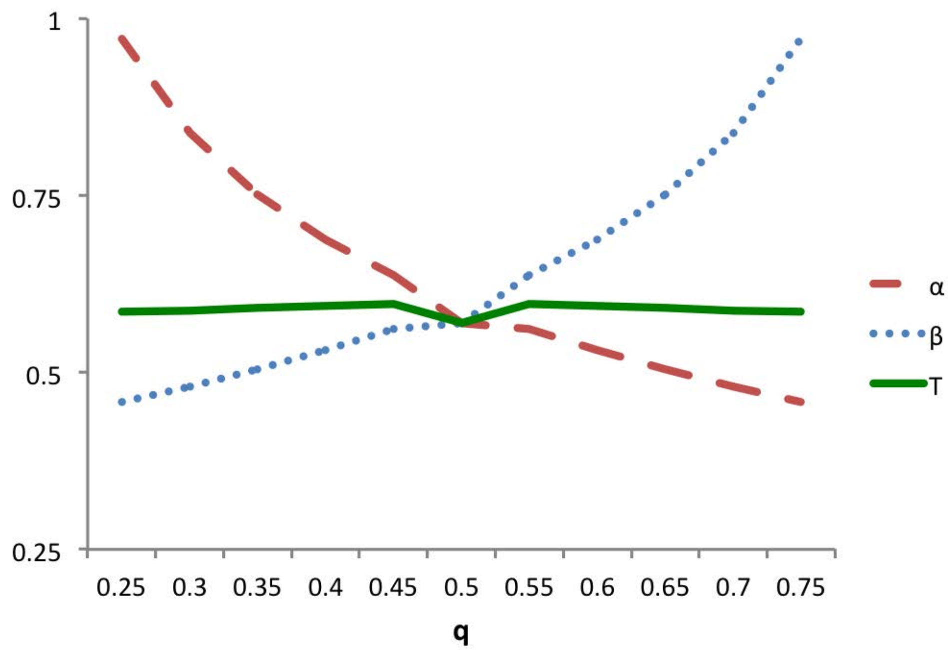
$$R = \frac{\beta(2-\beta)}{\alpha(2-\alpha)}, \quad W = \frac{\alpha(2-\alpha)\beta(2-\beta)}{(\alpha(2-\alpha)+\beta(2-\beta))^2}$$

and turnout is:

$$T = \frac{1}{\alpha(2-\alpha)+\beta(2-\beta)}$$

Figure 3 shows a violation of the competition effect. Namely, despite the presence of the underdog effect, total turnout is not always increasing for $q \in [0, 1/2]$.

Figure 3: T, α, and β as a function of q, ethical voter model.



3.3 Mobilization Model

The main message of this paper is to document the robustness of our comparative statics results of turnout across several well known turnout models. Morton (1987, 1991), Cox and Munger (1989), Shachar and Nalebuff (1999) and others have proposed models based on group mobilization, where parties can mobilize and coordinate citizens to go vote. In major elections, candidates and parties engage in hugely expensive get-out-the-vote drives. Empirical evidence suggests that these drives are effective (Bochel and Denver 1971, Gerber and Green 2000). There is also evidence that mobilization efforts can explain turnout variation across elections and across electoral systems (Patterson and Caldeira 1983, Gray and Caul 2000). We adopt here a group mobilization model a la Shachar and Nalebuff (1999) where parties' campaign efforts and spending are able to mobilize and coordinate citizens to go vote. In this model, each group can "purchase" turnout of its party members by engaging in costly get-out-the-vote efforts. Thus, parties trade off mobilization costs for higher expected vote shares, taking as given the mobilization choice of the other party.

A mobilization model assumes that more campaign spending by a party brings more votes for the party according to an exogenous technology. We consider a very simple version of group mobilization. We assume that the cost a party incurs in order to bring to the polls all its supporters with voting cost below c is $l(c)$, where $c \in [0, \bar{c}]$ and l is an increasing, convex and twice differentiable function. We also assume that it is infinitely costly for a party to turn out all its supporters: $l(\bar{c}) = \infty$. In addition to twice differentiability, we assume $F(c)$ satisfies a (weakly) decreasing reversed hazard rate (DRHR, or log-concavity of F) property.[13]

Under the above conditions, we have the following result, without loss of generality, for all $q \in [0, 1/2]$.

Proposition 2 *In the mobilization model, an equilibrium exists, it is unique and it has the following properties: (1) Zero Underdog Compensation:* $\alpha = \beta$*, hence* $R = Q$ *and* $T = \alpha = \beta$*. (2) Competition Effect: given* γ*, turnout increases in* q *and peaks at* $q = 1/2$*.(3) Contest Effect: given* $q < 1/2$*, turnout increases and then decreases with* γ*; it achieves its maximum for intermediate* γ*. (4) As* γ *goes to infinity (first-past-the-post), turnout goes to one when* $q = 1/2$ *and goes to zero otherwise.*

Proof. See Appendix. ■

[13]Note that the DRHR (also known as log-concavity of F) is weaker than the DGRHR used in the ethical voter model. Hence, the DGRHR property is sufficient for all the results obtained in both models.

The weak DRHR property of the cost distribution roughly means that the relative variation in the number of agent-types does not increase as we span the support of the distribution. In other words, as we increase the cost we do not suddenly find many more agents with a given cost. This guarantees monotonicity and it is essential for a unique interior solution: the cost-benefit ratio of turning agents with marginally higher costs is increasing.

The results for this model and their intuition are very similar to the ethical voter model, with one caveat. The zero underdog compensation obtained in the mobilization model means that, regardless of the electoral system and of the preference split, either side turns out the same proportion of its supporters. The zero underdog compensation is due to the non-rival structure of the campaign spending costs in mobilization models (see, e.g., Morton 1987, 1991, or Schachar and Nalebuff 1999). Namely, it costs the same for either side to mobilize all their supporters below a given voting cost threshold. In particular, it does not cost less to turn out the same share of supporters of the smaller group than of the larger group. This is, for example, the case if one thinks of campaigning as advertising through media, which is in its nature non-rival, but not for other forms of campaigning such as door to door persuasion, which are clearly rival. If the latter were the case, then compensation would be partial and the results would be similar to the ethical voter model.

3.3.1 Example: Uniform Distribution

Assume that costs are uniformly distributed on the unit interval, i.e., $F(c) = c \in [0,1]$. A simple cost function which satisfies our assumption of DGHR is:

$$l(c) = \frac{c}{1-c}$$

Then, the FOC becomes:

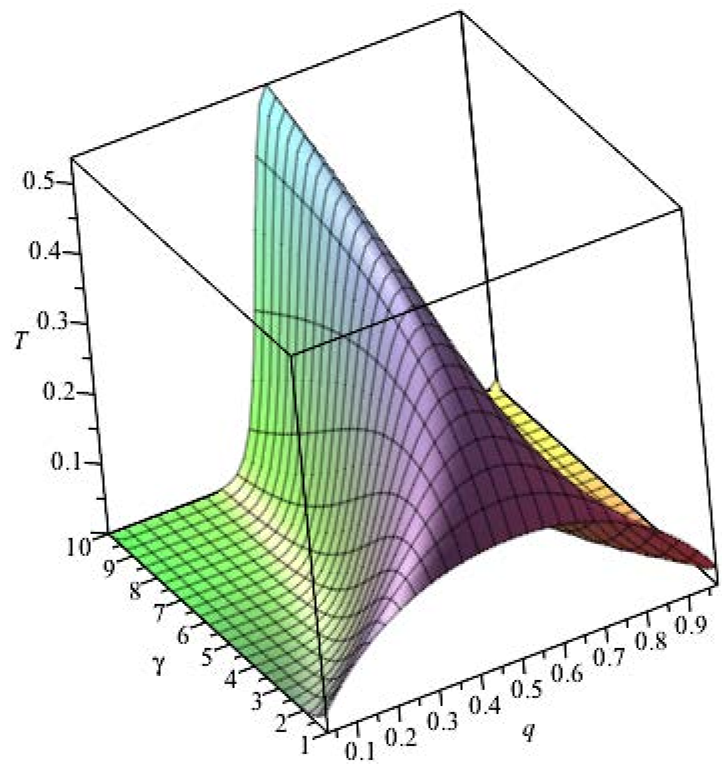
$$\left(MB := \gamma \frac{Q^{\gamma}}{[1+Q^{\gamma}]^2}\right) = \left(\frac{l'(c)}{f(c)/F(c)} = \frac{c}{(1-c)^2}\right)$$

Hence the solution is:

$$T = \alpha = \beta = F(c^*) = c^* = \frac{2(MB) + 1 - \sqrt{4(MB)+1}}{2(MB)}$$

Figure 4 shows T as a function of both q and γ. The similarity with Figure 2 is apparent. Also, for any electoral/institutional system γ the competition effect is clear, as well as the non-monotonic contest effect for any preference split q.

Figure 4: T as a function of γ and q, mobilization model.



4 Rational Voter Model

Another workhorse model for studying turnout in elections is the rational voter model (Palfrey and Rosenthal 1985). Some scholars consider the rational voter model non-satisfactory, as it predicts very low levels of turnout in large electorates. Far from contributing to this debate, our goal here is rather to show that, regardless of the turnout levels predicted, the comparative statics we obtain in the rational voter model across institutional systems and across preference splits is consistent with what we obtained in the, high turnout yielding, group models of turnout we discussed above.

Let N denote the total number of voters. As usual, the symmetric equilibrium is characterized by two cutoff levels, with $\alpha = F(c_\alpha)$ and $\beta = F(c_\beta)$ which solve:

$$MB_A = G(\alpha), \quad MB_B = G(\beta)$$

where MB_A and MB_B are the marginal benefits from voting for an individual of, respectively, group A and group B. Given expected turnout rates in the two parties,

α and β, the expected marginal benefit of voting for a party A and party B citizen are equal to, respectively:

$$\sum_{b=0}^{N}\sum_{a=0}^{N-b}\left[V\left(a+1,b\right)-V\left(a,b\right)\right]\frac{N!}{a!b!\left(N-a-b\right)!}T_A^aT_B^b(1-T_A-T_B)^{N-a-b} \tag{3}$$

$$\sum_{b=0}^{N}\sum_{a=0}^{N-b}\left[V\left(b+1,a\right)-V\left(b,a\right)\right]\frac{N!}{a!b!\left(N-a-b\right)!}T_A^aT_B^b(1-T_A-T_B)^{N-a-b} \tag{4}$$

where:

$$V\left(a,b\right):=\frac{a^\gamma}{a^\gamma+b^\gamma}$$

In equations (3) and (4), the first term in brackets in the summation is the increase in power share as a consequence of an increase in vote shares. The remaining terms represent the probability of the vote share being equal to $\frac{a}{a+b}$ without your vote, given turnout rates α and β.[14] For this model, we can offer analytical proofs for two results, existence of an equilibrium and the presence of a partial underdog effect. To show that the comparative statics discussed for the previous models hold also under these alternative modeling assumptions, we recur to numerical computations.

Existence. Fix N, γ and q. The pair of equilibrium conditions can be written in terms of cost thresholds as:

$$MB_A\left(c_A,c_B\right)=c_A,\qquad MB_B\left(c_A,c_B\right)=c_B$$

Because $\overline{c}>1/2$, MB_A and MB_B are continuous functions of c_A,c_B from $[0,\overline{c}]^2$ into itself, and $[0,\overline{c}]^2$ is a compact convex subset of R^2.[15] Therefore, by Brouwer's theorem there exists a fixed point $(c_A^*,\,c_B^*)$, which satisfies both equations and is an equilibrium.

Underdog Effects. This proof is contained in Kartal (Forthcoming) and Herrera et al. (2014). As pointed out there, in general, the partial underdog effect holds whenever a symmetric power sharing function $V\left(a,b\right)$ (as, for instance, the purely proportional one with $\gamma=1$ in our model: $V=\frac{a}{a+b}$) has the property that an

[14] By convention, we denote $\frac{j}{j+k}=.5$ if $j=k=0$.

[15] The assumption $\overline{c}>1/2$ guarantees that the range of these functions is contained in $[0,\overline{c}]^2$. Existence also holds more generally for any $\overline{c}>0$, with only minor changes in the proof to account for the possibility that $\overline{c}$ is the cutpoint (i.e., 100% turnout) for one or both parties.

additional vote for the underdog has a higher marginal impact for the underdog than an additional vote for the leader has for the leader. In other words, the proof just hinges on following two properties of $W(a,b)$:

$$W(a,b) = -W(b,a), \quad \text{and} \quad W(a,b) > 0 \quad \text{if} \quad a < b$$

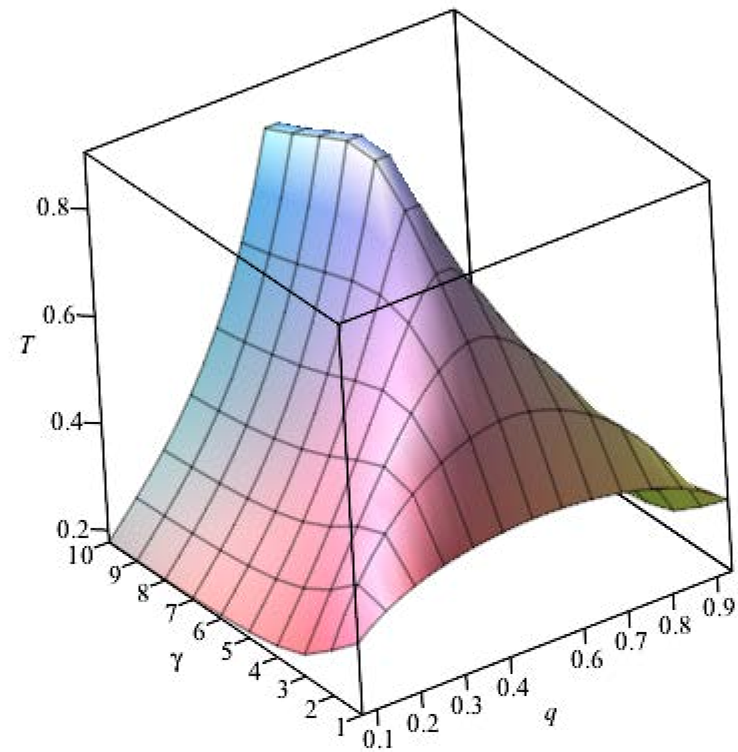
where

$$W(a,b) := (V(a+1,b) - V(a,b)) - (V(b+1,a) - V(b,a))$$

While closed form analytical expressions of the equilibria do not exist, they are easily computed numerically. Figure 5 below shows the equilibrium overall turnout as a function of the institutional environment, γ, and the ex-ante preference split, q, for $N = 30$, $c \sim U[0,1]$, and a benefit from winning the election of 10. This figure is qualitatively similar to the ones we presented for the mobilization and the ethical voter models. Table 1 shows the overall turnout as a function of γ and q for the same parameters. From these numerical computations, we can conclude that comparative statics similar to the one discussed for the other two models hold.

First, given γ, turnout increases in the size of the underdog group and peaks when the preference split is even. This is an analogue of the competition effect we discussed in the previous two models. Second, turnout increases and then decreases as we approach the majoritarian system (that is, as γ grows). In the large majority case, i.e., an uneven preference split (for example, $N_A = 4$ when $N = 30$), turnout is maximized for $\gamma = 1$ and decreases as we increase γ. When preferences are closer (for example, $N_A = 10$ when $N = 30$), turnout initially increases as we increase γ. As the system becomes less proportional, winning the election becomes paramount so competition becomes fiercer. On the other hand, the γ that maximizes turnout is still finite (that is, does not coincide with pure first-past-the-post) . As we approach the majoritarian system, the incentive to vote is reduced: winning becomes all that matters and the underdog, which has a smaller chance of winning (especially when preferences are uneven) turns out less. This is an analogue of the contest effect we discussed in the previous two models. Finally, we see in both cases the presence of a partial underdog effect. The underdog effect in this model comes from the benefit side, not from the cost side. The underdog has a larger benefit to turn out as an additional vote for the underdog brings the election closer to a tie hence raising the stakes, namely the benefit function becomes steeper as we approach a tie. The fact that the underdog effect is partial thought is entirely due to cost heterogeneity, as in the ethical voter model.

Figure 5: T as a function of γ and q, rational voter model with $N = 30$.



5 Concluding Remarks

This article investigated how the endogenous decisions of voters to participate to an election is affected by the degree of proportionality of the political system. We introduce a novel modeling instrument, a generalized context success function, in order to measure the sensitivity of power sharing to vote shares. Contrary to previous theoretical studies linking turnout to political institutions, this allows us to consider a wide array of electoral systems or power sharing regimes, ranging from a perfectly proportional system to a pure winner-take-all system.

We show that turnout depends on the degree of proportionality of influence in the institutional system in a subtle way, and that it is important to control for the interaction of the electoral rules and the relative strength of parties in the electorate. With the exception of the knife-edged case of a perfectly even split of preferences, turnout is highest for an intermediate degree of proportionality. When the distribution of preferences is lopsided, turnout is maximized with a relatively more proportional

Table 1: T as a function of γ and q, rational voter model with $N = 30$ and $c \sim U[0,1]$.

	γ									
q	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
0.07	0.311	0.262	0.218	0.199	0.189	0.184	0.181	0.179	0.178	0.178
0.10	0.333	0.318	0.269	0.241	0.223	0.219	0.214	0.211	0.209	0.208
0.13	0.351	0.364	0.318	0.285	0.266	0.255	0.249	0.244	0.241	0.239
0.17	0.365	0.404	0.367	0.331	0.308	0.294	0.285	0.279	0.275	0.272
0.20	0.376	0.437	0.415	0.379	0.352	0.336	0.324	0.316	0.311	0.307
0.23	0.385	0.466	0.462	0.429	0.401	0.381	0.367	0.358	0.351	0.346
0.27	0.393	0.490	0.501	0.482	0.453	0.431	0.415	0.403	0.395	0.389
0.30	0.399	0.510	0.546	0.536	0.511	0.487	0.469	0.455	0.445	0.437
0.33	0.404	0.527	0.583	0.591	0.573	0.550	0.529	0.514	0.502	0.492
0.37	0.408	0.541	0.615	0.644	0.640	0.621	0.600	0.582	0.567	0.556
0.40	0.411	0.551	0.641	0.694	0.711	0.702	0.682	0.662	0.645	0.631
0.43	0.414	0.558	0.660	0.736	0.783	0.795	0.781	0.759	0.738	0.720
0.47	0.415	0.563	0.672	0.765	0.847	0.905	0.906	0.878	0.849	0.823

system; when the distribution of preferences is close to even, turnout is maximized with a relatively more majoritarian system. The fact that in a more majoritarian system the underdog is unlikely to win a large election when partisan preferences are lopsided strongly discourages turnout. On the other hand, with a more proportional power sharing, some competition remains even when the election is not close, and therefore the effect of relative party size on turnout is small. These theoretical results are robust to a wide range of alternative assumptions about the role of parties and about the rationality of voters.

There are many possible directions for the next steps in this research. The theoretical results we obtain from all models, from instrumental voting to mobilization models, depend on a key variable, namely, the expected winning margin or the closeness of the election. While there is some empirical evidence about the relationship between ex ante closeness and turnout (Blais 2000, Cox and Munger 1989), we are not aware of any empirical work focusing on the interaction effect of the expected closeness and the degree of power sharing of the institutional system: most empirical work comparing turnout in different electoral systems does not control for election closeness. As we try to prove in this paper, election closeness is a key variable which should not be overlooked. While closeness has been conjectured to play an important role, at least in plurality elections, little is understood about the effect of closeness in proportional or partially proportional systems. In this paper, we identify theoretically how proportionality and turnout interact with election closeness, and find that these interaction effects are important, yet quite subtle. On the theoretical side, it would be interesting to study voters' turnout decisions when elections have a common value dimension and how system proportionality affects political platforms and the endogenous entry of parties.

A Appendix

A.1 Proof of Lemma 1

Omitting the arguments of the functions in our notation, the derivative of the inverse function is well-known to be:

$$G' = \frac{1}{F'} \quad \Longleftrightarrow \quad g = \frac{1}{f}$$

using the chain rule of the above expression we can obtain the second derivative of the inverse function, i.e.:

$$G'' = -\frac{F''}{(F')^2}G' = -\frac{F''}{(F')^3} \quad \Longleftrightarrow \quad g' = -\frac{f'}{(f)^3}$$

The IGRHR property for G, namely

$$\left(\frac{\alpha g}{G}\right)' > 0 \quad \Longleftrightarrow \quad 1 + \frac{\alpha g'}{g} > \frac{\alpha g}{G}$$

translates, substituting the above expressions, to:

$$-\frac{f'}{(f)^3}Ff + 1 > \frac{F}{cf} \quad \Longleftrightarrow \quad \frac{cf}{F} > 1 + \frac{cf'}{f}$$

which is precisely the DGRHR property for F, and vice versa.

A.2 Proof of Proposition 1

Partial Underdog Compensation Before proving existence and uniqueness (below), we show that any solution must satisfy the partial underdog effect property.

The marginal group benefits to the two parties, with respect to (c_α, c_β), are given by (**??**). The first order conditions are:

$$\begin{aligned} \frac{dP_\gamma^A}{dV}\left(\frac{(1-q)\,\beta}{T^2}\right) qf\left(c_\alpha\right) &= qc_\alpha f\left(c_\alpha\right) \\ \frac{dP_\gamma^A}{dV}\left(\frac{q\alpha}{T^2}\right)(1-q)\,f\left(c_\beta\right) &= (1-q)\,c_\beta f\left(c_\beta\right) \end{aligned}$$

which gives the condition

$$q\alpha G\left(\alpha\right) = (1-q)\,\beta G\left(\beta\right)$$

If a solution exists, the above condition implies *partial underdog compensation*, namely:

$$q < 1/2 \quad\Longrightarrow\quad a > \beta, \quad q\alpha < (1-q)\,\beta$$

The preference group that is smaller in expectation (that is, the underdog) turns out a larger fraction of its members than the larger group, that is, $\alpha > \beta$. However, this is not enough to compensate the initial disadvantage in the population, that is, $q\alpha < (1-q)\beta$.

Existence and Uniqueness We can write:

$$R = \frac{G\left(\beta\right)}{G\left(\alpha\right)}, \quad W = \gamma\frac{\left(G\left(\alpha\right)\right)^\gamma \left(G\left(\beta\right)\right)^\gamma}{\left(\left(G\left(\alpha\right)\right)^\gamma + \left(G\left(\beta\right)\right)^\gamma\right)^2}$$

and the two FOCs compactly as:

$$(1-q)\,\beta G\left(\beta\right) = q\alpha G\left(\alpha\right) = W$$

The first equality above implicitly defines the increasing function $\beta\left(\alpha\right)$. Taking derivatives, the relative variation of both expressions in the first equality gives:

$$\begin{aligned} \frac{d\beta}{\beta} + \frac{dG\left(\beta\right)}{G\left(\beta\right)} &= \frac{d\alpha}{\alpha} + \frac{dG\left(\alpha\right)}{G\left(\alpha\right)} \\ \left(1 + \frac{\beta g\left(\beta\right)}{G\left(\beta\right)}\right)\frac{d\beta}{\beta} &= \left(1 + \frac{\alpha g\left(\alpha\right)}{G\left(\alpha\right)}\right)\frac{d\alpha}{\alpha} \end{aligned}$$

since for $q < 1/2$ we have $\alpha > \beta$, the above implies $\frac{d\beta}{\beta} > \frac{d\alpha}{\alpha}$ under IGRHR. Hence, the function $\frac{G(\alpha)}{G(\beta)}$ is increasing in α as its relative variation is:

$$\frac{d\left(\frac{G(\alpha)}{G(\beta)}\right)}{\frac{G(\alpha)}{G(\beta)}}=\frac{dG\left(\alpha\right)}{G\left(\alpha\right)}-\frac{dG\left(\beta\right)}{G\left(\beta\right)}=\frac{d\beta}{\beta}-\frac{d\alpha}{\alpha}>0$$

The second equality in the FOCs above can be written as:

$$q\alpha G\left(\alpha\right)=\gamma\frac{\left(\frac{G(\alpha)}{G(\beta)}\right)^{\gamma}}{\left(\left(\frac{G(\alpha)}{G(\beta)}\right)^{\gamma}+1\right)^{2}}$$

To show uniqueness, note that the LHS is strictly increasing in α while the RHS is strictly decreasing because for all $\gamma \geq 1$:

$$\frac{d}{dx}\left(\gamma\frac{x^{\gamma}}{\left(x^{\gamma}+1\right)^{2}}\right)<0 \quad \text{for} \quad x>1$$

To show existence, note that for $\alpha = 0$ the LHS is zero while the RHS is bounded by 1 as

$$\frac{G\left(\alpha\right)}{G\left(\beta\right)}>1 \quad \Longrightarrow \quad \lim_{\alpha\to 0}\frac{G\left(\alpha\right)}{G\left(\beta\right)}\geq 1$$

On the other hand, for $\alpha = 1$, the LHS is equal to q while, given that $\frac{G(\alpha)}{G(\beta)}=\frac{(1-q)\beta}{q}$, the RHS is bounded by:

$$\gamma\frac{\left(\frac{G(\alpha)}{G(\beta)}\right)^{\gamma}}{\left(\left(\frac{G(\alpha)}{G(\beta)}\right)^{\gamma}+1\right)^{2}}=\gamma\frac{q^{\gamma}}{\left(1+\left(\frac{q}{(1-q)\beta}\right)^{\gamma}\right)^{2}}<\gamma\frac{q^{\gamma}}{\left(1+\left(\frac{q}{(1-q)}\right)^{\gamma}\right)^{2}}\leq q$$

where the latter inequality is true for all $\gamma \geq 1$.

Competition Effect Take wlog $q < 1/2$, hence $\alpha > \beta$ and $R < 1$. We now show that under IGRHR, if q increases then both R and T increase in the equilibrium solution.

Fixing γ, W increases with R as for the partial derivative of W we have:

$$W_R = R^{\gamma-1}\gamma^2\frac{1-R^{\gamma}}{\left(R^{\gamma}+1\right)^{3}}>0 \quad \text{for} \quad R<1$$

Suppose by contradiction that if q increases R decreases in equilibrium, then $W = q\alpha G\left(\alpha\right) = \left(1-q\right)\beta G\left(\beta\right)$ decreases which implies that α decreases. Hence,

β decreases too as $R = \frac{G(\beta)}{G(\alpha)} = \frac{q\alpha}{(1-q)\beta}$, which in turn implies that $\frac{\alpha}{\beta}$ is decreasing and $\frac{G(\alpha)}{G(\beta)}$ increasing, a contradiction under IGRHR. In fact, the IGRHR property guarantees or $\alpha > \beta$, that:

$$\begin{aligned} d\left(\frac{\alpha}{\beta}\right) &> 0 \quad \Longleftrightarrow \quad \frac{d\alpha}{\alpha} > \frac{d\beta}{\beta} \\ d\left(\frac{G(\alpha)}{G(\beta)}\right) &> 0 \quad \Longleftrightarrow \quad h(\alpha)\left(\frac{d\alpha}{\alpha}\right) > h(\beta)\left(\frac{d\beta}{\beta}\right) \end{aligned}$$

which implies the two ratios (the turnout ratio α/β and the cost threshold ratio c_α/c_β) must move in the same direction, namely:

$$\begin{aligned} d\left(\frac{\alpha}{\beta}\right) &> 0 \quad \Longrightarrow \quad d\left(\frac{G(\alpha)}{G(\beta)}\right) > 0 \\ d\left(\frac{G(\alpha)}{G(\beta)}\right) &< 0 \quad \Longrightarrow \quad d\left(\frac{\alpha}{\beta}\right) < 0 \end{aligned}$$

We have shown that R cannot decrease when q increases. As a consequence $W = q\alpha G(\alpha) = (1-q)\beta G(\beta)$ increases (which means that β increases, that is, the front-runner group turns out more when his ex-ante lead shrinks). In formulas this implies that:

$$\begin{aligned} 0 &< \frac{d(q\alpha G(\alpha))}{dq} = \alpha G(\alpha) + q(G(\alpha) + \alpha g(\alpha))\alpha_q \\ &\Longleftrightarrow \quad q\alpha_q > -\frac{\alpha}{1+h(\alpha)} \end{aligned}$$

and

$$\begin{aligned} 0 &< \frac{d((1-q)\beta G(\beta))}{dq} = -\beta G(\beta) + (1-q)(G(\beta) + \beta g(\beta))\beta_q \\ &\Longleftrightarrow \quad (1-q)\beta_q > \frac{\beta}{1+h(\beta)} \end{aligned}$$

Hence the variation of turnout with competition is:

$$T_q = \left(\alpha + q\alpha_q - \beta + (1-q)\beta_q\right) > \alpha\left(1 - \frac{1}{1+h(\alpha)}\right) - \beta\left(1 - \frac{1}{1+h(\beta)}\right)$$

So turnout is increasing if the function h is increasing. In sum under IGRHR for $q \in [0, 1/2]$ we have:

$$T_q > 0, \qquad R_q > 0$$

Contest Effect Fixing the preference split $q < 1/2$, we study turnout as the contest becomes more competitive, namely increasing γ from 1 (proportional power sharing) to infinity (winner-take-all).

Taking the total derivative with respect to γ of the first order conditions, we have:

$$W_\gamma = q\left(G\left(\alpha\right) + \alpha g\left(\alpha\right)\right)\alpha_\gamma = \left(1-q\right)\left(G\left(\beta\right) + \beta g\left(\beta\right)\right)\beta_\gamma$$

This implies that W_γ, α_γ and β_γ and hence also the variation of turnout

$$T_\gamma = q\alpha_\gamma + \left(1-q\right)\beta_\gamma$$

always have the same sign and (if applicable) are maximized for the same value $\widehat{\gamma}\left(q\right)$. In sum, T, α and β are increasing in γ if and only if $W_\gamma > 0$, so it suffices to study when the latter is the case. Defining

$$\begin{aligned} z &: = R^\gamma \in [0,1] \\ z_\gamma &= \left(\ln R\right) R^\gamma R_\gamma = \left(-\ln R\right) R^\gamma \left(R\left(\frac{\beta_\gamma}{\beta} - \frac{\alpha_\gamma}{\alpha}\right)\right) \end{aligned}$$

Taking the total derivative of W with respect to γ we have

$$\begin{aligned} W_\gamma &= z\frac{\left(\ln z\right)\left(1-z\right)+1+z}{\left(1+z\right)^3} z_\gamma \\ &= \left(z^2\frac{\left(\ln z\right)\left(1-z\right)+1+z}{\left(1+z\right)^3}\right)\left(\left(-\ln R\right)R\left(\frac{\beta_\gamma}{\beta} - \frac{\alpha_\gamma}{\alpha}\right)\right) \end{aligned}$$

The second bracket on the RHS is positive because $R < 1$ and under the IGRHR we have $\beta_\gamma/\beta > \alpha_\gamma/\alpha$. Namely, manipulating the total derivative of the first order conditions we obtain:

$$\begin{aligned} q\left(\alpha G\left(\alpha\right) + \alpha^2 g\left(\alpha\right)\right)\frac{\alpha_\gamma}{\alpha} &= \left(1-q\right)\left(\beta G\left(\beta\right) + \beta^2 g\left(\beta\right)\right)\frac{\beta_\gamma}{\beta} \\ \frac{\beta_\gamma}{\beta}/\frac{\alpha_\gamma}{\alpha} &= \frac{q\alpha G\left(\alpha\right)}{\left(1-q\right)\beta G\left(\beta\right)}\frac{1+\frac{\alpha g(\alpha)}{G(\alpha)}}{1+\frac{\beta g(\beta)}{G(\beta)}} = \frac{1+\frac{\alpha g(\alpha)}{G(\alpha)}}{1+\frac{\beta g(\beta)}{G(\beta)}} > 1 \end{aligned}$$

Hence the sign of W_γ depends on the first factor:

$$W_\gamma > 0 \iff \left(z^2\frac{\left(\ln z\right)\left(1-z\right)+1+z}{\left(1+z\right)^3} = 0\right) > 0 \iff z > z^* \simeq 0.21$$

Hence it suffices to study when the function z is above the threshold z^*. Namely, turnout is increasing for all values (q,γ) for which

$$z = R^\gamma = \left(\frac{q\alpha}{(1-q)\,\beta}\right)^\gamma > z^*$$

We can equivalently define the function

$$Z(\gamma,q) := \ln R^\gamma = -\gamma\left(\ln\frac{(1-q)\,\beta}{q\alpha}\right)$$

and explore when:

$$Z(\gamma,q) > \ln z^* \simeq -1.5$$

The function Z has the following properties

$$Z(1,0) = -\infty < \ln z^* < -1 = Z(1,1/2)\,, \qquad Z(\infty,q) = -\infty < \ln z^*$$

$$Z_\gamma(\gamma,q) = -\ln\frac{(1-q)\,\beta}{q\alpha} - \gamma\left(\frac{\beta_\gamma}{\beta} - \frac{\alpha_\gamma}{\alpha}\right) < 0, \qquad Z_q(\gamma,q) > 0$$

Namely, Z decreases in $\gamma \in [1,+\infty)$ for all $q \in [0,1/2]$ and, given γ, increases in q (as $R_q > 0$). Hence, for low enough $q \in (0,1/2)$ we have $Z < \ln z^*$ and turnout decreases for all $\gamma \in [1,+\infty)$. As q increases eventually we have $Z < \ln z^*$ and turnout increases in γ, is highest for intermediate system $\widehat{\gamma}(q)$ and then drops as $Z(\infty,q) < \ln z^*$.Lastly, for $q = 1/2$, we have $z = 1 > z^*$, so turnout is always increasing in γ

First Past the Post Case For $q = 1/2$, as $\gamma \to +\infty$, we have $\alpha G(\alpha) = \beta G(\beta) = 2W = \gamma/2$, which gives the corner solution: $\alpha = \beta = 1$. For $q < 1/2$, as $\gamma \to +\infty$ given that $R < 1$ (partial underdog effect), we have: $W = \alpha = \beta = 0$. Hence, in first-past-the-post we have full turnout in an evenly split election and zero turnout otherwise.

A.3 Proof of Proposition 2

Existence and Uniqueness Given (**??**), the first order conditions that characterize the solution are:

$$\frac{dP_\gamma^A}{dV}\left(\frac{(1-q)\,\beta}{T^2}\right) qf(c_\alpha) = l'(c_\alpha)\,, \quad \frac{dP_\gamma^A}{dV}\left(\frac{q\alpha}{T^2}\right)(1-q)\,f(c_\beta) = l'(c_\beta)$$

taking the ratio of the two we obtain

$$\frac{l'(c_\alpha)}{f(c_\alpha)/F(c_\alpha)} = \frac{l'(c_\beta)}{f(c_\beta)/F(c_\alpha)}$$

Hence, assuming that F satisfies DRHR (decreasing reversed hazard rate, aka log-concavity of F) is sufficient to obtain the following *zero underdog compensation* condition

$$c_\alpha = c_\beta \quad \Longrightarrow \quad \alpha = \beta = T$$

that is, both parties turn out an identical proportion of their supporters regardless of the preference split q.

The mobilization model reduces therefore to one equation in one unknown, equating marginal benefit (MB) and marginal cost (MC):

$$\left(MB := \gamma \frac{Q^\gamma}{[1+Q^\gamma]^2}\right) = \left(\frac{l'(c_\alpha)}{f(c_\alpha)/F(c_\alpha)} := MC\right) \tag{5}$$

The solution $c_\alpha \in [0, \bar{c}]$ exists and is unique because $MC(c_\alpha)$ is increasing, $l(\bar{c}) = \infty$ and l is convex. Therefore, $l'(\bar{c}) = MC(\bar{c}) = \infty$, and $MC(0) = F(0) = 0$.

Competition Effect For any fixed cost distribution and cost mobilization function turnout depends only on the marginal benefit (henceforth MB) and increases with it. Hence, in what follows we study MB as a proxy for turnout T. Fixing the institutional setting γ, we study turnout as we increase the ex-ante preference split q from zero (landslide) to 1/2 (close election). We have $\frac{dMB}{dq} = \frac{1}{(1-q)^2}\frac{dMB}{dQ}$, hence we can focus only on the sign of the derivative with respect to Q:

$$\frac{dMB}{dQ} = \frac{d}{dQ}\frac{\gamma Q^\gamma}{[1+Q^\gamma]^2} = \gamma^2 Q^{\gamma-1}\frac{1-Q^\gamma}{(1+Q^\gamma)^3} > 0$$

Hence, regardless of the institutional setting, as the preference split becomes tighter the marginal benefit of voting (MB) and turnout T increase.

Contest Effect Fixing the preference split q, we study turnout as the contest becomes more competitive, namely, if we increase γ from 1 (proportional power sharing) to infinity (winner-take-all). We have:

$$\frac{dMB}{d\gamma} = \frac{d}{d\gamma}\frac{\gamma Q^\gamma}{[1+Q^\gamma]^2} = \left(\frac{Q^\gamma(1-Q^\gamma)}{(1+Q^\gamma)^3}\right)\left(\frac{1+Q^\gamma}{1-Q^\gamma} + \ln Q^\gamma\right) > 0$$

For any $Q < 1$, the first factor is always positive so does not change the sign of the slope of MB nor its maximum, namely the above condition is equivalent to

$$\Omega := \left(\frac{1 + Q^\gamma}{1 - Q^\gamma} + \ln Q^\gamma \right) > 0$$

where the function Ω is decreasing in γ, as

$$\Omega_\gamma = \ln Q \frac{Q^{2\gamma} + 1}{\left(Q^\gamma - 1\right)^2} < 0$$

hence Ω can cross zero only once, where turnout is highest, i.e. for:

$$\widehat{\gamma}\left(q\right) : \Omega\left(\widehat{\gamma}\right) = 0$$

In sum, for any given preference split $q < 1$, turnout is highest for an intermediate system $\widehat{\gamma}\left(q\right)$

$$\widehat{\gamma}_Q = -\left(\frac{\partial \Omega}{\partial Q}\right) / \left(\frac{\partial \Omega}{\partial \gamma}\right) > 0$$

where:

$$\Omega_Q = \frac{\gamma}{Q} \frac{1 + Q^{2\gamma}}{\left(1 - Q^\gamma\right)^2} > 0$$

First Past the Post Case Lastly, it is easy to see that:

$$\lim_{\gamma \to \infty} MB = \lim_{\gamma \to \infty} \left(\gamma \frac{Q^\gamma}{\left[1 + Q^\gamma\right]^2} \right) = \begin{cases} +\infty & \text{if } Q = 1 \\ 0 & \text{otherwise} \end{cases}$$

Hence, in first-past-the-post we have full turnout in an evenly split election and zero turnout otherwise.

References

[1] Black, J. (1991): "Reforming the Context of the Voting Process in Canada: Lessons from Other Democracies," in *Voter Turnout in Canada,* edited by Herman Bakvis, Toronto: Dundurn Press, 61-176.

[2] Blais, A. (2000): *To Vote or Not to Vote: The Merits and Limits of Rational Choice Theory*, Pittsburgh University Press.

[3] Blais, A. and K. Carty (1990): “Does Proportional Representation Foster Voter Turnout?” *European Journal of Political Research*, 18: 168–181.

[4] Blais, A. and Dobrzynska A. (1998): “Turnout in Electoral Democracies,” *European Journal of Political Research*33: 239-61.

[5] Bochel, J. M. and D. T. Denver (1971): “Canvassing, Turnout and Party Support: An Experiment,” *British Journal of Political Science*, 1(3): 257-269.

[6] Calvert, T. R., and J. A. Ferejohn (1983): ”Coattail Voting in Recent Presidential Elections,” *American Political Science Review*, 77: 407-416.

[7] Canstanheira, M. (2003): “Victory Margins and the Paradox of Voting,” *European Journal of Political Economy*, 25: 152-162.

[8] Che, Y. K., Wouter D., and Navin K. (2013): ”Pandering to Persuade.” *American Economic Review*, 103(1): 47-79.

[9] Coate, S. and M. Conlin (2004): “A Group Rule-Utilitarian Approach to Voter Turnout: Theory and Evidence,” *American Economic Review*, 94(5): 1476-1504.

[10] Cox, G. (1999): “Electoral Rules and the Calculus of Mobilization,” *Legislative Studies Quarterly*, 24(3): 387-419.

[11] Cox, G. and M. C. Munger (1989): “Closeness, Expenditures, and Turnout in the 1982 U.S. House Elections,” *American Political Science Review,* 83: 217-31.

[12] Crewe, I. (1981): “Electoral Participation,” in *Democracy at the Polls: A Comparative Study of Competitive National Elections*, edited by David Butler, Howard R. Penniman, and Austin Ranney, Washington, D.C.: American Enterprise Institute.

[13] Faravelli, M., P. Man and R. Walsh (2013): “Mandate and Paternalism: A Theory of Large Elections”, Working Paper, University of Queensland.

[14] Faravelli, M. and S. Sanchez-Pages (Forthcoming): “(Don’t) Make My Vote Count”, *Journal of Theoretical Politics*.

[15] Feddersen, T. and A.Sandroni (2006): “A Theory of Participation in Elections,” *American Economic Review*, 96(4): 1271-1282.

[16] Ferejohn, J. A., and R. L.Calvert (1984): ”Presidential Coattails in Historical Perspective,” *American Journal of Political Science,* 28:127-146.

[17] Franklin, M. (1996): "Electoral Participation," in *Comparing Democracies: Elections and Voting in Global Perspective*, edited by Lawrence LeDuc, Richard G.Niemi, and Pippa Norris. Thousand Oaks, Calf.: Sage.

[18] Gerber A. S., and D. P. Green (2000): "The Effects of Canvassing, Telephone Calls, and Direct Mail on Voter Turnout: A Field Experiment," *American Political Science Review*, 94(3): 653-663.

[19] Goeree, J. K. and J. Grosser (2007): "Welfare Reducing Polls," *Economic Theory*, 31: 51-68.

[20] Golder, M. (2006): "Presidential Coattails and Legislative Fragmentation," *American Journal of Political Science,* 50: 34-48.

[21] Gray, M. and M. Caul (2000): "Declining Voter Turnout in Advanced Industrial Democracies, 1950 to 1997: The Effects of Declining Group Mobilization," *Comparative Political Studies,* 33(9): 1091-1122.

[22] Herrera, H., M. Morelli and T. R. Palfrey (2014): "Turnout and Power Sharing," *The Economic Journal*, 124 (February), F131-F162.

[23] Hirshleifer, J. (1989): "Conflict and Rent-Seeking Success Functions: Ratio vs. Difference Models of Relative Success," *Public Choice*, 63: 101-112.

[24] Jackman, R. (1987): "Political Institutions and Voter Turnout in the Industrial Democracies," *American Political Science Review*, 81: 405-23.

[25] Jackman, R. and R. A. Miller (1995): "Voter Turnout in the Industrial Democracies During the 1980s," *Comparative Political Studies*, 27: 467–492.

[26] Kartal, M. (Forthcoming): "A Comparative Welfare Analysis of Electoral Systems with Endogenous Turnout," *The Economic Journal.*

[27] Kartal, M. (Forthcoming): "Laboratory Elections with Endogenous Turnout: Proportional Representation versus Majoritarian Rule," *Experimental Economics.*

[28] Key, V. O., Jr. (1949): "Southern Politics in State and Nation," New York: Alfred A. Knopf.

[29] Krasa, S. and M. Polborn (2009): "Is Mandatory Voting Better than Voluntary Voting," *Games and Economic Behavior,* 66:1, 275-291.

[30] Lariviere, M. A. (2006): "A Note on Probability Distributions with Increasing Generalized Failure Rates," *Operations Research*, 2006, 54(3): 602-604.

[31] Levine, D. and T. R. Palfrey (2007): "The Paradox of Voter Participation: A Laboratory Study," *American Political Science Review*, 101: 143-158.

[32] Lijphart, A. (1997): "Unequal Participation: Democracy's Unresolved Dilemma." *American Political Science Review* 91(1): 1–14.

[33] Morton, R. (1987): "A Group Majority Model of Voting," *Social Choice and Welfare,* 4(1): 17-31.

[34] Morton, R. (1991): "Groups in Rational Turnout Models," *American Journal of Political Science,* 35: 758-76.

[35] Moser R. G. and E. Scheiner (2004): "Mixed Electoral Systems and Electoral System Effects: Controlled Comparison and Cross-National Analysis," *Electoral Studies,* 23: 575-599.

[36] Palfrey, T. R. and H. Rosenthal (1985): "Voter Participation and Strategic Uncertainty," *American Political Science Review,* 79: 62-78.

[37] Patterson, S. C. and G. A. Caldeira (1983): "Getting Out the Vote: Participation in Gubernatorial Elections," *American Political Science Review,* 675-689.

[38] Pintor, Rafael, Maria Gratschew, and Kate Sullivan (2012): "Voter Turnout Rates from a Comparative Perspective," Institute for Democracy and Electoral Assistance, June 9, 2012.

[39] Powell, G.B. (1980): "Voting Turnout in Thirty Democracies,*" in Electoral Participation,* edited by Richard Rose. Beverly Hills, Calif.: Sage.

[40] Powell, G.B. (1982): *Comparative Democracies: Participation, Stability and Violence,* Cambridge, MA: Harvard University Press.

[41] Powell, G.B. (1986): "American Voter Turnout in Comparative Perspective," *American Political Science Review,* 80: 17-45.

[42] Razin, R. (2003): "Signaling and Election Motivations in a Voting Model with Common Values and Responsive Candidates," *Econometrica*, 71(4): 1083-1119.

[43] Rose, R. (1980): *Electoral Participation: A Comparative Analysis*, London: Sage.

[44] Shachar R. and B. Nalebuff (1999): "Follow the Leader: Theory and Evidence on Political Participation," *American Economic Review,* 57: 637-660.

[45] Skaperdas, S. (1996) "Contest Success Functions," *Economic Theory*, 7: 283-290.

[46] Taylor C.R. and H. Yildirim (2010): "Public Information and Electoral Bias," *Games and Economic Behavior*, 68(1): 353-75.

[47] Tullock, G. (1980): "Efficient Rent Seeking," in *Toward a Theory of the Rent-Seeking Society,* edited by James M. Buchanan, Robert D. Tollison, and Gordon Tullock, College Station, TX: Texas A&M University Press, 97-112.

[48] Uhlaner, C. (1989): "Rational Turnout: The Neglected Role of Groups," *American Journal of Political Science,* 33: 390-422.