AGGREGATE ISSUANCE AND SAVINGS WAVES

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<u>ABSTRACT</u>

We use firms' decisions in the cross-section about their sources and uses of funds in order to make inferences about the aggregate cost of external finance. The basic intuition is as follows: Firms which raise costly external finance can invest the issuance proceeds in productive capital assets, or in liquid financial assets with a low physical rate of return. If firms raise costly external finance and allocate some of the funds to liquid assets, either the cost of external finance is relatively low, or the total return to liquidity accumulation, including its value as a hedging asset, is particularly high. We construct and estimate a quantitative, dynamic model of firms' financing and savings decisions. We then use the model's predictions for variation in firm policies and implied cross sectional moments, along with empirical moments from Compustat, to infer the average cost of external finance per dollar raised in the US time series 1980-2010.

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A data appendix is available at http://www.nber.org/data-appendix/w20442

I. Introduction

We propose and implement a method for using data on firms' decisions in the *cross-section* about their sources and uses of funds in order to make inferences about the *aggregate* cost of external finance. The basic intuition is as follows: Firms which raise costly external finance can invest the issuance proceeds in productive capital assets, or in liquid financial assets with a low physical rate of return. If firms raise costly external finance and allocate some of the funds to liquid assets, either the cost of external finance is relatively low at that time, or the total return to liquidity accumulation, including its value as a hedging asset, is particularly high.

We explore this intuition theoretically and empirically by constructing a dynamic, quantitative model of firms' financing and savings decisions in which both aggregate productivity, and the aggregate cost of external finance, vary over time. In the model, as in the data, firms typically finance investment with operating cash flows. However, when the cost of external finance is low, firms in our model raise external finance, invest some of the proceeds, and save the remainder of proceeds in liquid assets. As a result, when the aggregate cost of external finance is low, firms are more likely to both raise external finance, and to accumulate liquid assets. Consistent with this idea, we show that in the model and in the data, cross sectional moments describing the incidence of firms raising external finance, and importantly, the co-incidence of firms raising external finance and saving the proceeds, are informative about the aggregate cost of external finance. Aggregate issuance and savings waves coincide with a high correlation in the cross section between issuance and saving, and tend to occur when traditional empirical proxies indicate that the aggregate cost of external finance is low. Our study is aimed at providing an estimate of a "revealed preference" measure of the aggregate cost of external finance in the US time series from 1980-2010 based on this intuition and results from our quantitative model.

We begin by documenting three new stylized facts. The first is the strong positive correlation between issuance and savings at the aggregate level. For all but the very largest firms, the aggregate correlation between external finance raised and liquidity accumulation is 0.6. This high correlation is not due to some firms raising, and other firms saving. Conditioning on firms that raise external finance, the aggregate correlation increases to 0.74. The second stylized fact we develop is the strong relationship between firms' issuance and savings decisions, and traditional proxies for the cost of external finance. We show that the cross sectional correlation between external finance raised and liquidity accumulated, which we denote $\rho_{i_l,e}^{xs}$, tends to be low when the default spread, the tightness of lending standards, and consumer sentiment, indicate that external finance is particularly costly, and vice versa. We argue, then, using the results and intuition from our model, and the empirical relationship between $\rho_{i_l,e}^{xs}$ and traditional proxies for

the cost of external finance, that firms' behavior in the cross section contains useful information about the the aggregate cost of external finance. Consistent with this, the third stylized fact we develop is that the difference in investment between high and low productivity firms (controlling for average productivity) is larger when $\rho^{xs}_{i_l,e}$ is high. The difference in investment between high and low productivity firms should be larger when investment is less constrained by the cost of external finance, thus this "difference in differences" result provides further support for $\rho^{xs}_{i_l,e}$ as a measure of the level of the cost of external finance. It also highlights the connection between our research strategy, and the large literature which uses differences in differences to identify the effects of supply shocks on corporate policies. As in that literature, we exploit cross sectional heterogeneity in our identification scheme. We further build on these prior results by using a structural model to enable us to extend our results to a broad sample and to aggregate implications.

Our structural model consists of a panel of firms which face idiosyncratic and aggregate productivity shocks, as well as aggregate shocks to the cost of external finance, and choose their desired stock of physical capital and liquid assets in order to maximize the present discounted value of payouts net of issuance costs. Firms in our model are risk neutral, but discount as if they are averse to the risk of having their sources of funds fall below their desired level of fund usage. Gross payouts are defined as after tax operating profits plus interest on liquid assets less investment in physical capital and liquid assets, and less investment adjustment costs. In our analysis, we pay careful attention to the role of capital adjustment costs, as well as external financing costs, in driving firm issuance, investment, and savings behavior. Our model features a convex cost of investment adjustment, a constant fixed cost of external finance, and a convex cost of external finance, the level of which is determined by an aggregate shock.

Aggregate issuance and savings waves arise when the aggregate cost shock is low. When the aggregate cost of external finance is low, firms which have high enough productivity issue external finance in order to take advantage of the relatively high net return to investment and relatively low cost of raising funds. However, the convex investment adjustment cost incentivizes firms to smooth investment over time. As a result, issuing firms save some of their proceeds in liquid assets in order to both smooth investment, and to avoid repeatedly paying the fixed cost of external finance. When the aggregate cost of external finance is high, high productivity firms will also want to invest, however because the return to investment net of financing costs is lower they will invest less. Moreover, since productivity is persistent over time, these firms will tend to be able to fund this lower level of investment using operating cash flows. And, finally, because investment adjustment costs are smaller at lower investment rates, these firms will be

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able to achieve a smooth enough investment policy without the use of liquid assets.

After developing our structural model and its intuition, we turn to estimating the main parameters of interest using Simulated Method of Moments. With our estimated parameters in hand, we compare the model moments to the data. We show that the model does a good job of matching the main aggregate moments we document, as well as a range of additional aggregate and firm level moments not targeted by our estimation. The model generates aggregate issuance and savings waves in line with the data. The model also generates a strong positive correlation between $\rho_{i_l,e}^{xs}$ and the level of the cost of external finance, consistent with the empirically high correlation between $\rho_{i_l,e}^{xs}$ and proxies such as the default spread, lending standards, and consumer sentiment. Finally, the model is able to replicate the difference in differences in investment between high and low productivity firms conditional on proxies for the level of the cost of external finance. In addition, the model predicts that this difference in differences will be larger for small firms, a fact we confirm empirically.

We also test our model against its main alternative, a model with constant costs of external finance in which aggregate issuance and savings waves can potentially arise from productivity shocks alone. This model is a nested version of the model with a stochastic cost of external finance, and is strongly rejected by the data using a formal J-test. Intuitively, this test shows that the benefit in terms of model fit more than justifies the additional parameters introduced by the full model. The restricted model also fails to generate aggregate issuance and savings waves at the estimated parameter values; the data force the model to compromise on the aggregate correlation between issuance and savings in order to minimize as much as possible the large overall model errors.

Because we cannot feasibly test our model against every alternative, we instead provide substantial evidence that the model we estimate is consistent with empirical moments that we do not target specifically, but that vary with firms' key issuance and savings policies in the way our model predicts. We highlight two key additional implications. First, the model replicates the larger difference in investment between high and low productivity firms when the cross sectional correlation between liquidity accumulation and external finance is high. This is consistent with the findings from the large literature in empirical corporate finance upon which we build which uses difference in differences to show that credit supply shocks impact corporate policies. Second, the moments we show are correlated in the model with the state variable for the level of the cost of external finance are correlated with traditional empirical proxies for the cost of external finance (such as the index for lending standards), but are only weakly correlated with TFP.

After developing and estimating our model, and comparing the implied model moments to

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their empirical counterparts, we turn to using the intuition we develop, along with the estimated version of our structural model, to construct a time series of the average per dollar cost of external finance raised for the US from 1980 to 2010. Our strategy is to use the policy functions from the structural model, along with the estimated parameters, to identify moments from the cross section which vary substantially with the aggregate state. When applied to the data, this variation in cross sectional moments then offers information about a hidden aggregate state variable.

We first construct a continuous series for this average cost using a regression based "cost of external finance index". Specifically, we construct the index weights by running a regression in the model of the cost of external finance on the cross sectional moments describing firms' issuance and savings decisions on that date. We then use the model implied weights, along with the empirical moments from Compustat data to construct an estimate of the empirical cost of external finance in US data. Figure 1 graphs the resulting series, which has an average value of 2.3%, and exhibits substantial and intuitively appealing variation over our sample. We find that our estimated cost is highly correlated with individual proxies for the cost of external finance such as the default spread, index of lending standards, and consumer sentiment, but is most highly correlated with a (normalized) average of these series, suggesting that our measures helps to pick out the common variation in each series. Moreover, our index implied cost does contain new information relative to existing individual measures. For example, the index implied cost predicts that external finance was less costly in 1986 and more costly in 2001 than the default spread seems to imply. We also note that the widely used default spread measure only captures the cost of debt finance, and much of the default spread may be due to a fair return adjustment for risk. Thus, we argue that our index measure of the aggregate cost of external finance, which exploits firms' revealed preferences implied by their financing and savings decisions, is a useful complement to existing measures. One appealing feature of our series is that our estimates generate an average issuance cost at each date, which also allows for quantitative comparisons of the variation in the relative cost over time.

Next, we use Simulated Method of Moments (SMM) to construct a binary measure of the cost of external finance, and we plot this in the bottom panel of Figure 1 graphs the resulting series. Specifically, at each date we find the value of the aggregate state for the level of the cost of external finance which sets the model moments describing issuance and savings behavior in the cross section closest to their empirical counterparts. Thus, we propose two methods for using cross sectional moments, along with a calibrated model, to make inferences about a hidden aggregate state. The advantage of the SMM method is that it takes advantage of more of the

structure of the model, however the cost is that it is constrained to choose states that are part of the estimated process for the cost. Accordingly, the advantages of the regression index method is that it is more flexible, and the resulting estimate is a continuous measure. We show that the results from both methods are mutually consistent, and in particular indicate high costs in the early 1980's, followed by lower costs in the mid 1980's, high costs in the early 1990's, very low costs in the mid 1990's through 2000, high costs in 2001 and low costs thereafter until the onset of the financial crisis.

II.   Related Literature

This paper contributes to the growing literature at the intersection of finance and macroeconomics which studies the interaction between firm financing, savings, and investment decisions, and the macroeconomy. Two recent prominent papers document the cyclical behavior of firm financing. Jermann and Quadrini (forthcoming), and Covas and Den Haan (2011a) both document that debt issuances are highly procyclical, and Covas and Den Haan also report procyclical equity issuances.[1] We are the first to incorporate data on firms' liquidity accumulation, as well as their investment, into a business cycle model aimed at considering the role of pure financing shocks vs. shocks to productivity in explaining firm level and aggregate investment and financing activities.[2] We argue that looking at the joint dynamics of liquidity accumulation and external finance is useful for examining the role of shocks to the cost of external finance, since how firms use funds may help to disentangle financing shocks from shocks that drive investment opportunities. Therefore while previous studies have focused on how external funds are raised, whether by debt or equity financing, our paper shows that how external funds are used is also useful in understanding the cost of external finance.

Several recent papers develop theoretical models which use a shock which originates in the financial sector to better match business cycle facts.[3] Jermann and Quadrini (forthcoming) show how a model with an endogenous credit limit and a shock to capital liquidity can generate realistic business cycles as well as match the procyclical debt issuance and countercyclical equity issuance

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[1]Choe *et al.* (1993), and Korajczyk and Levy (2003) also study issuances over the business cycle. Both find that equity issuance is procyclical. Korajczyk and Levy (2003) report countercyclical debt issuance. Huang and Ritter (2009) provides evidence that active issuance decisions are driven by the relative cost of equity vs. debt.

[2]Eisfeldt and Rampini (2009) builds an aggregate model of internal and external finance to study the implications of corporate liquidity demand for the observed low return on liquid assets, but does not consider shocks to the cost of external finance. Covas and Den Haan (2011a) focus on debt and equity issuances, but they do note that, empirically, firms tend to both accumulate financial assets and invest when they issue external finance.

[3]These papers build on the seminal contributions of Bernanke and Gertler (1989), Kiyotaki and Moore (1997) and Carlstrom and Fuerst (1997) on the role of financial market conditions on firm investment and business cycle dynamics.

which they document using US Flow of Funds data. Covas and Den Haan (2011b) develops a model in which countercyclical equity issuance costs are useful for generating both procyclical equity issuance and a countercyclical default rate. Khan and Thomas (2011) build a quantitative business cycle model in which credit shocks drive aggregate productivity down by inhibiting productive investment reallocation across firms. This effect shows up in our model as well, and we show that estimated TFP is below actual TFP when external finance is costly. Hugonnier *et al.* (2011) build a search theory of external finance and show how idiosyncratic external finance risk affects corporate savings, investment, and payout policy. Bolton *et al.* (2013) develop a dynamic theory of firm finance and risk management with stochastic financing costs, and show analytically that such costs can increase savings and can delink external finance from investment at the firm level in a model with constant investment opportunities. Our model confirms these effects in a calibrated, quantitative model with stochastic investment opportunities, and, importantly we also document their empirical relevance.

We also use our model to estimate the cost of external finance in the US time series. Thus, our paper is most closely related to Jermann and Quadrini (forthcoming), with two key differences. First, Jermann and Quadrini (forthcoming) focus on the distinction between debt vs. equity in their estimation, and estimate a debt financing cost shock, whereas we do not distinguish between sources of external finance and instead incorporate information regarding how all external funds are used into our estimation strategy. Second, Jermann and Quadrini (forthcoming) use an assumed binding constraint to identify their shock. While we cannot solve our model for the cost of external finance shock in closed form, we think that the use of cross sectional moments to identify a hidden aggregate state is a complementary methodology with other potential uses. For example, and building on our work, Belo *et al.* (2014) use a closely related measure that exploits the cross-section of firm policies to study the asset pricing implications of a model of costly external finance. They show that these cost shocks can help explain the cross-section of returns and account for many of the failures of the CAPM.

Despite this renewed interest, the fact that financial constraints, or shocks originating in the financial sector, are important for either firm level investment, or business cycle dynamics, is not a foregone conclusion amongst economists. While Ivashina and Scharfstein (2010), Duchin *et al.* (2010), Campello *et al.* (2010), Matvos and Seru (2011), Almeida *et al.* (2009), and Chodorow-Reich (2014), provide evidence that the financial crisis hindered external finance, investment, and employment activity at the firm level, Paravisini *et al.* (2011) find only small effects of credit supply shocks on trade. Gomes *et al.* (2006) point out that the shadow cost of external finance is procyclical in a standard business cycle model with agency costs of external finance.

Finally, Chari *et al.* (2008) argue that aggregate data do not support the occurrence of a credit crunch and question the appropriateness of government interventions aimed at improving access to external finance.[4] In contrast to these papers, our paper uses corporate policies from our structural model along with US data to extract information about the level of financing frictions in the US time series.

Our paper is also related to papers which develop dynamic models of corporate saving. The main difference is in focus; these papers are focused on understanding firm level dynamics or making inferences about firm level of financial constraints. In contrast, our paper, which is focused on understanding the dynamics and the effects of the aggregate component of the cost of external finance connects ideas from this literature to the macro finance literature which studies business cycles with financial frictions. Kim *et al.* (1998) develop a three date model and show that cash accumulation is increasing in the cost of external finance, the variance of future cash flows, and the return on future investment opportunities, but decreasing in the return differential between physical capital and cash.[5] Almeida *et al.* (2004) study the cash flow sensitivity of cash and empirically document a link between the propensity to save out of cash flow and financial constraints.[6] Riddick and Whited (2009) construct a fully dynamic model of corporate savings and emphasize the importance of uncertainty for determining corporate savings, and argue that in such a model, the propensity to save is not an accurate measure of financial constraints. Thus, the firm level link between financial constraints and investment in financial assets is also under debate. Note also that because both Riddick and Whited (2009), is focused on firm level moments, the parameter estimates therein do not incorporate any information in aggregate moments. However, our calibration focusing on aggregate moments is not too dissimilar, and supports the applicability of the basic Riddick and Whited (2009) framework for aggregate studies.

A contemporaneous paper with a related focus to ours, but again directed at understanding firm level behavior, is Warusawitharana and Whited (2011), which uses simulated method of moments to show that equity misvaluation shocks can help explain firm level corporate issuance

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[4]Likewise, Chari *et al.* (2007) use business cycle accounting to argue that shocks to the cost of installing capital, or to the return on capital, are only of tertiary importance for explaining the US fluctuations output, investment, and employment. However, papers such as Justiniano *et al.* (2010), Christiano *et al.* (2010), Hall (2011), Shourideh and Zetlin-Jones (2012), and Gilchrist and Zakrajsek (2012b), find that such shocks explain a large fraction of business cycle fluctuations.

[5]For a model which instead focuses on the value of the flexibility of cash for adjusting net leverage, see Gamba and Triantis (2008).

[6]See also Faulkender and Wang (2006) for evidence that cash is more valuable when held by financially constrained firms. Harford *et al.* (2011) argue that firms save to insure against refinancing risk and document an inverse relationship between debt maturity and cash holdings which is stronger when credit market conditions are tighter.

and savings policies. At estimated parameters, our model implies that external finance is, in aggregate, costly on average. However our estimated series also supports the idea that equity issuance (even in aggregate) might have been associated with negative costs briefly in the late 1990's, and equity issuance may more frequently be associated with negative costs at the firm level than in the aggregate. However, including the possibility of negative costs requires additional parameters which prevent unlimited issuance, and so for parsimony we restrict issuance to have a positive cost.

Finally, our paper is related to dynamic models of capital structure. The fact that firms tend to simultaneously raise external finance and accumulate liquidity is at odds with standard static pecking order intuition. Static pecking order theories based on Myers (1984) predict that firms will first draw down cash balances and only once these are exhausted will they seek external finance. Thus, such theories predict a counterfactually negative correlation between liquidity accumulation and external finance. Our dynamic model features a pecking order in the sense that internal funds are less costly than external funds, and generates the observed positive correlation between liquidity accumulation and external finance. This result is similar to the implications of the models in Hennessy and Whited (2005) and Strebulaev (2007) for the trade off theory of capital structure. Those papers show that data which appear to be inconsistent with static trade-off theories of capital structure can be generated by dynamic models in which firms' objectives are based precisely on the trade-off between the tax benefits and distress costs of debt.

III. Stylized Facts

A. Data Description

Our main data set consists of annual firm level data from Compustat from 1980-2010. We focus on Compustat data so that we are able to analyze firm level, as well as aggregate, facts. Our sample selection criterion follows that in Covas and Den Haan (2011a). We also show that using Flow of Funds data to construct aggregate issuance and savings moments yields similar results. The Online Appendix gives a detailed description of the construction of our data.

We use firm level cash flow statements to track corporate flows. We define liquidity accumulation as changes in cash and cash equivalents.[7] We define net external finance raised as the

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[7]We do not use the balance sheet measure of cash since the stock measure is affected by acquisitions. Covas and Den Haan (2011a) instead remove firms involved in mergers which increase sales by more than 50%. We have checked that our findings are similar using stock measures and the non-merger sample. All non-reported robustness checks are

negative of the sum of net flows to debt and net flows to equity. We define flows to debt as debt reduction plus changes in current debt plus interest paid, less debt issuances, and flows to equity as purchase of common stock plus dividends less sale of common stock. Following Covas and Den Haan (2011a), and Fama and French (2005), we also consider using the negative of the change in total liabilities as flows to debt and negative changes in book equity as flows to equity. We find similar results using these stock measures. We focus on the flow measures in the interest of brevity, and since our model does not feature issuances which are not truly "external" like those related to mergers or employee compensation which are emphasized in Fama and French (2005). Finally, we have also verified that the results are similar if we just focus on issuances of debt and equity, rather than the total net flows from these claim holders. We define investment (in physical capital) as capital expenditures. We do not include acquisitions in our investment measure. Firm level acquisitions are very lumpy, which can bias the firm level correlations we compute. Including acquisitions does not change our aggregate results, since the aggregate series smooths out individual firm lumpiness, however we use capital expenditures throughout to keep the data definitions constant.

When computing most aggregate and firm level moments, we normalize firm level variables by current total book assets. When computing aggregate correlations, we instead normalize by the lag of book assets, to avoid inducing spurious correlations. However, book assets are slow moving and fairly acyclical and thus shouldn't induce any cyclical variation. Our results are robust to alternative normalizations, such as aggregate output or aggregate gross-value added from the corporate sector. We use the Hodrick and Prescott (1997) filter to remove any remaining low frequency trends when computing aggregate correlations, since, for example, cash holdings have trended upwards as a share of assets over our sample (Bates *et al.* (2009)). The filter ensures that the empirical series are stationary, consistent with the stationary business cycle model we study, and with our focus on the business cycle dynamics of the cost of external finance.

As in Covas and Den Haan (2011a), our main analysis drops the top 10% of firms by asset size. There are several reasons to do this. First, the very largest firms present unique measurement problems. A significant fraction of the investment for these firms falls under the accounting category "other investments". These other investments are typically long term receivables from unconsolidated subsidiaries. Thus, a large firm may raise funds on behalf of a smaller subsidiary, which in turn may use the funds to build a new factory, or may store the funds as liquid assets. Since we are not able to measure these funds' ultimate use, we are not able to identify accumulated liquidity vs. physical investment, the main goal of this paper. Second, the largest

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available from the authors upon request.

firms tend to have a much larger share of foreign earnings. Cash accumulation for firms with large foreign earnings may be influenced by tax motives and repatriation timing. Third, as Covas and Den Haan (2011a) point out, the timing of external finance for the largest firms is not representative of the rest of the sample. They show in particular that one incidence of AT&T raising equity during a recession in 1983 has implications for the cyclicality of *aggregate* equity issuance. They advocate dropping the top firms because they have an unusually large influence on the aggregate series. Fourth, it is possible that the very largest firms face little or no financial constraints. Finally, we note that in the type of stationary model we study, the distribution of firm sizes will be much less skewed than that in the data. As a result, aggregate model data will not be as heavily driven by the activities of a few large firms and is more readily comparable to our sample which excludes these extremely large firms.

For the Flow of Funds data, we normalize each series by the HP filter implied trend in gross-value added of the corporate sector.[8] If we very narrowly define the accumulation of liquid assets as the net acquisition of financial assets minus trade receivables minus miscellaneous assets, the flow of funds data display a counterfactual decrease over time in this series for liquid assets held within the corporate sector.[9] Thus, the Flow of Funds data do not do a good job of identifying and classifying all corporate investment in marketable securities. There is a large, and growing, category "miscellaneous assets," which contains both marketable and non-marketable assets. To account for this, we also include 1/3 of miscellaneous other assets as liquid.[10]

B. Main Facts

We document three new stylized facts describing aggregate issuance and savings waves. First, the aggregate time series correlation between external finance raised and liquidity accumulation is strongly positive at 0.6. Second, the cross sectional correlation between issuance and saving tends to be low when traditional empirical proxies indicate that the cost of external finance is high. Third, and building on these results, we show that the difference in investment between high productivity firms and low productivity firms is higher when the cross sectional correlation between liquidity accumulation and external finance is high, indicating a low current cost of external finance.

Our first fact is that firms tend to issue and save in aggregate waves. For all but the

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[8]Results using total GDP are similar.

[9]See Bates *et al.* (2009).

[10]The decision to use 1/3 of other miscellaneous assets was based on personal communication with staff at the Board of Governors. Their rough estimate using recent IRS data is that about 1/3 of miscellaneous other assets were marketable securities.

largest 10% of Compustat firms, the aggregate correlation between liquidity accumulation and external finance is 0.60 and is statistically significant at the 5% level. Figure 2 plots the cyclical component of aggregate net liquidity accumulation vs. the cyclical component of aggregate net external finance and clearly illustrates our first stylized fact. This aggregate correlation is higher (0.74) if one conditions on firms that are currently raising external finance. Thus, the positive aggregate correlation does not seem to be driven by some firms saving, and other firms issuing external finance, nor is it driven by the behavior of payouts.

The aggregate correlation is also higher when one excludes more of the largest firms. For the smallest half of firms, the correlation between aggregate external finance raised and liquidity accumulated is 0.84. This is in contrast to conditioning on other measures of financial constraints, such as whether a firm pays no dividends, or has no credit rating, in which case we find correlations close to that for the larger sample (0.68 and 0.56 respectively). This could be due to the importance of fixed costs in accessing external financial markets, or it could be that size is simply a better proxy for financial constraints. Finally, we also find a positive correlation using flow of funds data. If we very narrowly define liquid assets as the net acquisition of financial assets minus trade receivables minus miscellaneous assets, we find a correlation between liquidity accumulation and external finance of 0.33. Including 1/3 of miscellaneous other assets as liquid helps align the flow of funds data with the fact that the net accumulation of liquid assets within the corporate sector has been positive over recent history. Using this measure, we find a correlation of 0.38 which is statistically significant. Table I displays our main aggregate issuance and savings stylized facts.

Table II displays the correlations between liquidity and investment with debt vs. equity separately. While we see that the correlation with liquidity accumulation is stronger for equity (0.69) then debt (0.16), both are positive. Conditional on firms raising external finance, we see both correlations increase to 0.77 and 0.33, respectively, and both are statistically significant. We also note that investment is more correlated with debt (0.60) than equity (-0.15). This fact has been pointed out by DeAngelo *et al.* (2010) who argue that debt might be used more frequently for investment. Also, we note that debt drives most of the variation in external finance, with a correlation with external finance of 0.77 vs. 0.43 for equity. For parsimony, and to match our model, we focus on the overall correlation with external finance and abstract from the debt vs. equity distinction. Studying total external finance allows us to focus on what is new in our work; whereas other studies have focused on variation in firms' *sources* of funds, our study focuses on firms' *uses* of the external finance they raise.

The second main new fact that we document is that in the cross section, firms are more

likely to raise external finance and save the proceeds when the proxies for the cost of external finance are low. We proxy for the cost of external finance with the default spread, index of tightening lending standards, and consumer sentiment. This is consistent with the intuition we illustrate theoretically that when financing costs are high, firms are unlikely to raise costly external finance only to save the proceeds in low-return, liquid, assets. This finding is closely related to the evidence provided by McLean (2010), who shows that share issuance-cash savings are inversely related to microstructure measures of stock market liquidity, and also provides evidence that firms are more likely to issue equity to raise cash for precautionary savings when the economy is growing. We also argue that firms accumulate liquidity for hedging motives when the cost of external finance is low. In addition, we provide a structural model in which one can disentangle the different effects of productivity and cost of external finance shocks on issuance and savings decisions. In particular, firms in our model only raise external finance and save the proceeds when both productivity is relatively higher, and the cost of external finance is low.

At each date, we compute the cross sectional correlation between aggregate net external finance raised and liquidity accumulation (each normalized by lagged book assets), and construct a time series of this cross sectional correlation, which we call "$xsrho$", using the notation $\rho_{i_l,e}^{xs}$. The correlation will be high, when, volatility adjusted, both issuance and savings are high relative to their mean. We then show that the correlation between $\rho_{i_l,e}^{xs}$ and the negative of the Baa-Aaa default spread is 0.64. Similarly, the correlation between $\rho_{i_l,e}^{xs}$ and the negative of the fraction of banks reporting tighter lending standards is 0.58. Both correlations are statistically significant at the 5% level and show that $\rho_{i_l,e}^{xs}$ is picking up common variation in each of these series. Figure 3 illustrates the strong relationship between $\rho_{i_l,e}^{xs}$, the percentage of firms raising external finance, the default spread, the index of consumer sentiment, and lending standards by plotting the time series for $\rho_{i_l,e}^{xs}$ and the percent of firms raising external finance (top panel) along with the negative of the default spread, lending standards, and consumer sentiment (bottom panel). Although all the series are highly correlated, there is independent information in $\rho_{i_l,e}^{xs}$. For example, the high $\rho_{i_l,e}^{xs}$ indicates a low cost of external finance in the boom of 1986, however the default spread was not particularly low then. The tech bust of 2001 is also more apparent in the drop in $\rho_{i_l,e}^{xs}$ than it is in the relatively small increase in the default spread, potentially suggesting that firms faced an increase in the cost of equity issuance at that time which is not captured by the default spread. Finally, and by contrast, we show that the correlation between $\rho_{i_l,e}^{xs}$ and TFP is 0.48, which is considerably lower than its correlation with any of the empirical proxies for the cost of external finance.

The final main fact we document regarding aggregate issuance and savings waves is a fact

about "differences in differences" and exploits the heterogeneous effect of a high cost of external finance on high productivity vs. low productivity firms. The idea is that when the cost of external finance is high, investment will be most constrained for high productivity firms with good investment opportunities because these are precisely the firms with the largest need for external funds. Low productivity firms will not have a strong need for external finance and will thus be less affected by an increase in this cost. In particular, we show that when the cross sectional correlation between liquidity accumulation and external finance is high, indicating that the the cost of external finance is low, the difference in investment by high productivity firms vs. low productivity firms is higher than when external finance is more costly by our measure. This makes sense since costly external finance inhibits investment precisely by those firms with good investment opportunities, and thus this fact provides additional support for $\rho_{il,e}^{xs}$ as a proxy for the level of the cost of external finance. Table VII presents the quantitative regression results, which we discuss further in relation to data from our model in section V.D.

IV. Model Setup and Intuition

A. Model

The intuition in our model for why firms' use of the external finance they raise provides information about the cost of those funds is as follows: The marginal benefit of investing in physical capital is high at low levels, but decreases with the level of investment. Liquidity accumulation displays less decreasing returns to scale, but has a marginal benefit that is on average lower than the marginal benefit of investment in physical capital. As a result, firms will only invest in liquid assets once they have invested enough in physical capital to push the marginal return below that on liquid assets, but will typically invest a positive amount in physical capital. Importantly, the lower the cost of external finance is, the more likely it is that the marginal return to physical capital investment will be pushed down to the marginal return to cash because a lower cost of funds increases investment. We present a simple, two-date counterpart in the associated Online Appendix which we analytically solve to further illustrate this intuition. In our dynamic model, physical capital investment has decreasing marginal returns due to both decreasing returns to scale, as well as to convex adjustment costs. The physical return to liquid assets is fixed at a rate *lower* than the discount rate, and thus positive liquidity accumulation only arises when the total return is endogenously pushed above the discount rate due to the benefit of liquid assets' use as future internal funds for investment.

We study a continuum of risk neutral firms which maximize the expected present discounted

value of their net payouts, taking the interest rate as given. All firms have access to the same production and financing technologies, and are subject to common aggregate shocks. However, firms differ in terms of their idiosyncratic productivity realizations and as a result choose heterogeneous stocks for their physical capital and liquid assets. Thus, we study a panel of heterogeneous firms which experience both idiosyncratic and aggregate shocks. We begin by describing the model, and the implied investment returns, and then turn to its estimation and policy function analysis.

Firms produce output or cash flows using physical capital k according to:

$$y = zk^\theta$$

where z is the level of the firm's productivity and $\theta \in (0, 1)$. Each firm's productivity z is the product of an idiosyncratic shock z_i, and an aggregate shock z_{agg}. The aggregate productivity level, and each idiosyncratic productivity level, follow AR(1) processes in logs with identical persistence parameters, however, we allow for the idiosyncratic and aggregate processes to have different volatilities. These assumptions allow us to construct each firm's productivity shock as follows:

$$
\begin{align}
z &= z_i z_{agg} \tag{1} \\
\ln(z_i') &= \rho \ln(z_i) + \epsilon_i' \tag{2} \\
\ln(z_{agg}') &= \rho \ln(z_{agg}) + \epsilon_{agg}' \tag{3} \\
\ln(z') &= \rho \ln(z) + \epsilon_i' + \epsilon_{agg}'. \tag{4}
\end{align}
$$

The ϵ shocks are both normally distributed with zero mean and standard deviations given by σ_i and σ_{agg}, respectivtly. In our calibration, idiosyncratic shocks will account for the bulk of firms' fluctuations in productivity.

Capital evolves according to the standard law of motion:

$$k' = (1 - \delta)k + i_k, \tag{5}$$

where i_k is investment and $\delta \in (0, 1)$ is the depreciation rate. Investment in physical capital is subject to convex adjustment costs $\phi_i(i_k, k)$ given by:

$$\phi_i(i_k, k) = \frac{a}{2} \left(\frac{i_k}{k} \right)^2 k. \tag{6}$$

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where a determines the slope of the marginal adjustment cost. Liquid assets l evolve according to:

$$l' = (l + i_l)(1 + r(1 - \tau)) \tag{7}$$

where i_l is investment in liquid assets and r is the risk free rate. Thus, corporate payouts are motivated by a tax wedge, $\tau > 0$, as in Riddick and Whited (2009). We note, however, that in practice payout policy is also likely to be driven by agency and information considerations.

Because financing costs will be paid only if payouts gross of financing costs are negative, it is convenient to define a firm's pre-financing payout e as internal cash flows minus investment in physical capital and liquidity accumulation, less investment adjustment costs.

$$e \equiv zk^\theta(1 - \tau) - i_l - i_k - \phi_i(i_k, k), \tag{8}$$

If $e > 0$ the firm is paying out funds and if $e < 0$ the firm is raising external finance. Intuitively, the firm raises external finance if after tax operating profits do not cover the firms' total investment in physical and liquid assets, net of physical adjustment costs. Firms maximize this payout, net of financing costs. Following Gomes (2001), Hennessy and Whited (2005), Hennessy and Whited (2007), and Riddick and Whited (2009), and in order to facilitate estimation, we parameterize the cost of external finance exogenously as follows:

$$\phi_e(e, \xi) = \mathbb{1}_{\{e<0\}}(\lambda_1 + \frac{\xi}{2}\lambda_2 e^2), \tag{9}$$

where $\lambda_1, \lambda_2 > 0$, $\mathbb{1}_{\{e<0\}}$ is an indicator that takes the value 1 when $e < 0$ and 0 otherwise, and $\xi > 0$ denotes the aggregate level of external financing costs. Note that even though the shock ξ only affects the marginal cost of external finance, it will affect firms' propensity to pay the fixed cost because a high marginal cost lowers the value of paying the fixed cost. For this reason, we did not find significantly different results using either this specification, or one in which the fixed cost also varied, and we chose what we believe is the most parsimonious specification. The specification in which only the marginal cost varies with the ξ shock is also closer to what is implied by existing models of the microfoundations of time varying costs of external finance, although no exogenous specification captures the endogenous effects in these models perfectly. Microfoundations of a time varying marginal cost include agency frictions that vary over time, along the lines of Bernanke and Gertler (1989) and Carlstrom and Fuerst (1997), collateral constraints which vary with asset values as in Kiyotaki and Moore (1997), and endogenously time varying adverse selection problems as in Eisfeldt (2004), Kurlat (2011), and Bigio (2013).

The aggregate state of external financing costs, ξ, follows an AR(1) in logs.

$$\ln(\xi') = c + \gamma \ln(\xi) + \eta' \tag{10}$$

where η' is a normal i.i.d. shock with mean zero and standard deviation σ_η. We choose the value c so that ξ, is on average equal to one. The average level of the marginal cost of external finance is then given by λ_2.

To keep things as parsimonious as possible, we employ a standard model of dynamic corporate finance with the minimal elements we found to be quantitatively important in earlier versions of the model. We discuss the role of each element in our results in our discussion of the model's policy functions below. Relative to Riddick and Whited (2009), for example, we have added an aggregate productivity shock and an aggregate shock to the cost of external finance, but stripped out the parts of the investment and external financing costs which we found not to be quantitatively relevant in earlier iterations of the model.[11]

We denote the firm's value function by V and write the firm's problem in recursive form, using the constraints and laws of motion given in equations (1)-(10). Each firm's state is given by its size, k, its liquid asset balance, l, its productivity z (the product of an aggregate and idiosyncratic shock), and the aggregate level of the marginal cost of external finance, ξ. We define $s \equiv \{k, l, z, \xi\}$ to summarize an individual firm's state vector. We use the standard "prime" notation to denote next period values for the state, and a subscript s to denote expectations conditional on the current state.

Each firm then solves the Bellman Equation:

$$V(s) = \max_{k' \in [0, \infty), l' \in [0, \infty)} \left\{ e - \mathbb{1}_{\{e < 0\}} (\lambda_1 + \frac{\xi}{2} \lambda_2 e^2) + \frac{1}{1+r} E_s [V(s')] \right\} \tag{11}$$

subject to:

$$e = (1 - \tau) z k^\theta - \frac{l'}{1 + (1 - \tau)r} + l - k' + (1 - \delta)k - \frac{a}{2} \left(\frac{k' - (1 - \delta)k}{k} \right)^2 k$$

B. Investment Returns

We first illustrate the main intuition for a firm's policy functions by studying the returns to investing in physical capital and in liquid assets, extending the production returns from Cochrane

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[11]Specifically, we do not have fixed investment costs or a linear external finance cost as in Riddick and Whited (2009). As in other studies, we found little role for the linear cost of external finance. Similarly, at the firm level, and in the presence of a fixed cost of external finance, we did not find a quantitatively important role for the fixed cost of investment.

(1991) and Cochrane (1996) to include the effects of financial constraints and to account for firms' investment in more than one asset type. In our partial equilibrium dynamic model, firms adjust their policy functions for investment and liquidity accumulation, and the implied policy for payouts, in order to set the marginal returns on investment and liquidity accumulation, and the marginal cost of external finance, equal to the gross interest rate. Binding non-negativity constraints, and the pseudo-multiplier on the fixed cost of external finance, can lead to wedges between these returns. We highlight the role of an "external finance discount factor", $\mathcal{F}$. Although firms in our economy are risk neutral, $\mathcal{F}$ twists the physical probability distribution to weigh more heavily those states in which internal funds are more valuable. In particular, $\mathcal{F}$ is the ratio of the shadow value of internal funds in the future relative to the current period, which captures the increase (decrease) in the marginal return to investing in either asset when the shadow value of internal funds is expected to be higher (lower) in the future relative to the current period.

We begin by stating the first order conditions and envelope conditions for the program in (11), and then combine these to form investment return Euler equations. In order to account for the fixed cost of external finance, we add a "pseudo constraint" to the dynamic program, requiring that $e \geq 0$. We denote the multiplier on this pseudo constraint ψ_e, and require that $\psi_e > 0$ iff $e = 0$, and $\psi_e = 0$ otherwise. For expositional purposes, we define a new multiplier $\hat{\psi}_e$ which incorporates either the binding multiplier on the pseudo constraint that $e \geq 0$, or the marginal cost of external finance if the firm is raising funds, or zero if the firm is making payouts as follows:

$$\hat{\psi}_e \equiv \begin{cases} 0, & \text{if } e > 0 \\ \psi_e, & \text{if } e = 0 \\ -\lambda_2 \xi e, & \text{if } e < 0 \end{cases} . \tag{12}$$

This pseudo multiplier represents the shadow value of internal funds in excess of the value of payouts in the current period, i.e. the "excess shadow value". If the firm is making payouts, the excess shadow value is equal to zero. If the firm would like to raise external finance, but chooses not to do so because of the fixed cost, the excess shadow value equals the value of the pseudo multiplier, ψ_e. Finally, if the firm is raising external finance, the excess shadow value of internal funds is equal to the marginal cost of external finance at the level of finance the firm chooses to raise.

We denote the multiplier on the constraint that $l' \geq 0$ by $\psi_l \geq 0$, and note that the constraint that $k' \geq 0$ will not bind due to an inada condition. Using subscripts to denote partial derivatives

of the value function, the first order condition for k' is:

$$\left(1 + \hat{\psi}_e\right)\left(1 + \frac{ai_k}{k}\right) = \frac{1}{1 + r}E_s[V_{k'}(s')]. \tag{13}$$

There are three cases, determined by the firm's choice for external finance. If $e > 0$, then $\hat{\psi}_e = 0$. If $e = 0$, then $\hat{\psi}_e = \psi_e > 0$. Finally, if $e < 0$, then $\hat{\psi}_e = -\lambda_2 \xi e > 0$. The right hand side of this first order condition is the marginal value of an additional unit of capital tomorrow. If the firm is paying funds out, the marginal value equals the marginal cost of one plus the adjustment cost. If the firm is raising external finance, the marginal value equals the marginal cost including the marginal cost of an additional unit of external finance, $\lambda_2 \xi e$. Finally, if the firm is neither paying out nor raising funds, the marginal value of an additional unit of capital exceeds the marginal cost, but not by enough to make it worthwhile for the firm to pay the fixed cost of external finance, and this wedge is captured by ψ_e.

The envelope condition at the optimal choice for physical capital is:

$$V_k(s) = \left(1 + \hat{\psi}_e\right)\left(\theta(1 - \tau)zk^{\theta - 1} + (1 - \delta) + \frac{ai_k}{k}\left(\frac{1}{2}\frac{i_k}{k} + (1 - \delta)\right)\right), \tag{14}$$

where we again have three cases for e. Combining the first order condition with the envelope condition in the subsequent period, we form the Euler equation which sets the discounted expected return on investment in physical capital equal to the value of a dollar payout today as follows:

$$1 + r = E_s\left[\frac{\left(1 + \hat{\psi}'_e\right)\left(\theta(1 - \tau)z'k'^{\theta - 1} + (1 - \delta) + \frac{ai'_k}{k'}\left(\frac{1}{2}\frac{i'_k}{k'} + (1 - \delta)\right)\right)}{\left(1 + \hat{\psi}_e\right)\left(1 + \frac{ai_k}{k}\right)}\right]. \tag{15}$$

Defining the "external finance discount factor", $\mathcal{F}$, which governs firms' state pricing as

$$\mathcal{F} \equiv \frac{1 + \hat{\psi}'_e}{1 + \hat{\psi}_e}, \tag{16}$$

and using $\hat{R}_k$ to denote the physical return to investing in physical capital, we can write the Euler equation for the return to investing in physical capital as:

$$1 + r = E_s\left[\mathcal{F}\hat{R}_k\right]. \tag{17}$$

The return on capital describes the marginal benefit of increasing capital one unit today relative to the marginal cost. The physical marginal benefit to increasing capital by one unit is: a

marginal increase in output, the value of the additional depreciated capital, and a lower convex cost of investment in the following period. The physical marginal cost of increasing capital by one unit is a dollar, plus adjustment costs. The ratio of the physical marginal benefit to the physical marginal cost is the physical return to capital, $\hat{R}_k$. The total return is then the physical return multiplied by the shadow value of how much a dollar will be worth inside the firm tomorrow relative to today, $\mathcal{F}$. If the firm is raising funds in both periods, this is equal to the marginal cost of funds in the following period relative to the current period. In general, additional capital is more valuable if internal funds are expected to be scarce in the future, i.e. if the firm will be, or will desire to be, raising funds externally. Since the conditional expected return on capital is constant, quantities must adjust for this asset pricing Euler equation to hold. The return to capital is decreasing in investment and increasing in productivity. Thus, the firm will increase investment in high productivity states until this optimality condition holds.

The first order condition for l' is:

$$\left(1 + \hat{\psi}_e\right)\left(\frac{1}{1 + (1 - \tau)r}\right) = \frac{1}{1 + r}E_s[V_{l'}(s')] + \psi_l, \tag{18}$$

where we now have six cases. There are three cases for e as in the first order condition for k'. There are two cases for the choice of future liquid assets. If $l' > 0$ then $\psi_l = 0$ and if $l' = 0$ then $\psi_l > 0$. The envelope condition at the optimal choice of liquid asset balances is:

$$V_l(s) = 1 + \hat{\psi}_e, \tag{19}$$

where again we have the three cases for e. Combining the first order condition with the envelope condition in the subsequent period, we form the return on investment in liquid assets as follows:

$$1 + r = E_s\left[\frac{1 + \hat{\psi}_e' + \psi_l}{1 + \hat{\psi}_e}\left(1 + (1 - \tau)r\right)\right]. \tag{20}$$

Defining the "external finance discount factor", $\mathcal{F}$, as above, and defining the normalized multiplier $\hat{\psi}_l$

$$\hat{\psi}_l \equiv \frac{\psi_l}{1 + \hat{\psi}_e},$$

we can write the return to investing in liquid assets as:

$$1 + r = E_s\left[\left(\mathcal{F} + \hat{\psi}_l\right)\hat{R}_l\right]. \tag{21}$$

where $\hat{R}_l$ is the physical return on liquid assets $(1 + (1 - \tau)r)$. There are eighteen cases for the

investment return for liquid assets, depending on the choices for e and l', and the choices for e', but the general intuition is straightforward. If the firm chooses to make positive payouts in the current and subsequent period, and chooses to carry forward a positive liquid balance, then $\mathcal{F} = 1$, $\psi_l = 0$, and the return to liquid assets equals the pecuniary return. If $\psi_l > 0$ then the true return to investing in liquid assets is less than one, and the multiplier adjusts to set the return equal to one. Finally, the external finance discount factor $\mathcal{F}$ accounts for the relative value of additional internal funds in the subsequent, relative to the current, period. This discount factor equals one if the firm places equal shadow values on funds in both periods. If the firm expects to be raising more finance in the subsequent period (the marginal cost of an additional dollar of external finance, $\mathbb{1}'_{\{e<0\}}\lambda_2\xi'e'$, is large) or expects the pseudo multiplier ψ'_e to be large, the effect through $\mathcal{F}$ will be to generate a higher return to investing in liquid assets. The opposite is true if the firm places a relatively high shadow value on internal funds in the current, relative to the subsequent period, and the effect will be to drive the return to investing in liquid assets down.

Lemma 1. *For each level of capital, liquid assets, and level of the cost of external finance, there is a cutoff for productivity $\hat{z}$ above which the firm finds it optimal to pay the fixed cost and raise external finance.*

Proof. We use a standard variational argument. The full proof appears in the Online Appendix.

Lemma 1 implies that relatively high productivity firms pay the fixed cost, raise external finance, and invest some of the proceeds. A firm which enters the period with high productivity has the highest marginal return to investment in physical capital. As the firm invests, however, the convex cost of investment, and decreasing marginal returns from the concave production function, drive the returns to current investment in capital down. Since productivity is persistent, a high z today makes a high future z more likely. Thus, the returns to future internal funds increase with z, which raises the return to liquidity accumulation. These liquid assets are then used by higher productivity firms to smooth investment over time. We note in particular, it is firms which experience productivity growth which have the highest incentive to invest and to accumulate liquidity from external finance to fund their investment. These firms are smaller than other high productivity firms, and thus they generate fewer internal funds and have a higher marginal marginal product of capital.

C. Firm Policies

We describe the policy functions graphically, using the numerical solution to the model at estimated parameters, in order to provide intuition for our results and to describe firms' decisions. The following section describes the model estimation and the parameters used. Firms construct policies as a function of their four state variables: size k, liquid assets i_l, productivity z and external finance cost level ξ. In the companion Online Appendix to this paper we present complete graphs of the policy functions as a function of the state variables. In Figure 4, we condition on the state for the level of the cost of external finance (left panel is low cost, right panel is high cost), and graph policies as a function of either productivity, z, or size k. Thus, to present firm policies in two dimensions, we average over the other state variables, and this eliminates some of the heterogeneity which leads to the variation in the cross section we exploit empirically. It is important to keep this averaging in mind, for example when interpreting the levels of the variables plotted in Figure 4. However, the main intuition can be illustrated in two dimensions, which are easier to view.

The top panel of Figure 4 plots the percentage of firms raising external finance as a function of firm level productivity from low to high across the six model states, averaging in each state over size and liquid assets. In both the low external finance cost state (left panel) and high external finance cost state (right panel), the fraction of firms raising external finance exhibits a hump-shaped pattern in productivity. The intuition is as follows. Low productivity firms have poor investment opportunities, both currently and likely in the near future due to persistent productivity, so they do not need additional sources of funds. By contrast, high productivity firms potentially have good current and future investment opportunities, however they will also tend to have abundant internal funds from high operating cash flows and are likely to be large and therefore experiencing greater decreasing returns to scale. Similarly, depending on their histories, medium productivity firms can be growing and can thus have good investment opportunities relative to their supply of internal funds. In fact the "high medium productivity" firms are the most likely to raise external finance, and, as can be seen by comparing the left and right panels, medium productivity firms' decisions to raise external finance are also the most sensitive to the cost of external finance. The variation in the marginal cost of external finance as ξ varies has the most important effect on the value of paying the fixed cost of raisng funds for these firms. Moreover, these firms are important in the economy in terms of probability mass, since the productivity shocks are normally distributed.

Comparing the left hand panel to the right hand panel illustrates why $\%raise$ serves as a useful proxy for the level of the cost of external finance, since the area under the curve in the

left panel is clearly larger than that in the right panel. When the cost of external finance is low, the cutoff productivity above which firms raise external finance is lower than when the cost of external finance is high, leading to a larger fraction of firms raising. We note that while the decision to raise external finance exhibits important variation with productivity, as also described in Lemma 1 and as supported by Figure 4, most of the variation in productivity in our model, as well as in the data, comes from idiosyncratic, firm level shocks rather than aggregate fluctuations in TFP. That said, the cross sectional moment describing $\%raise$ is postively correlated with aggregate TFP to a similar extent in both the model and the data, as we will show and discuss below. Higher aggregate TFP, all else equal, will imply firm level TFP will be above average and hence more firms will raise external finance. However, in the model and in the data, and consistent with the top panel of 4, $\%raise$ is even more positively correlated with the aggregate state for the level of the cost of external finance.

The middle panel of Figure 4 graphs liquidity accumulation and net external finance $(-e)$, also as a function of firm level productivity.[12] The policies for the level of net external finance raised illustrate two things: First, $-e$ is negative on average meaning that firms in the model, as in the data, are on average making payouts. However, as the top panel of Figure 4 shows, the average masks important heterogeneity and many firms do raise funds. Second, the shape for the amount of net external finance raised in the middle panel mimics that for $\%raise$ in the top panel, and has the same intuition. Comparing the left and right panels, we see the average net external finance raised is higher when the cost of external finance is low. This shift arises due to an increase in external finance raised rather than a reduction in payouts, since payouts fall, all else equal, in the high cost state when funds are worth more internally rather than in the low cost state. The policy for liquidity accumulation shows that on average, firms tend to invest in physical capital, with its higher rate of return, although the averages again mask individual variation. However, when the cost of external finance is low, high-medium productivity firms raise external finance, and on average save some of the proceeds in liquid assets. These medium productivity firms are also important in terms of their contribution to the overall distribution of firms. Finally, we note that in the low cost of external finance state, the cross sectional pattern across all productivity states for external finance and liquidity accumulation mimic each other closely. In contrast, in the right panel this correlation is low. This tight relationship is why $\rho_{il,e}^{xs}$ serves as a sharp proxy for the cost of external finance.

The bottom panel of Figure 4 plots investment for high and low productivity firms, defined as having above and below the average level of productivity at each date, as a function of firm size for

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[12]A plot of only external financed raised that does not include payouts looks similar in shape but is shifted upwards. We choose to plot the net measure as this variable is most comparable to the main moments we analyze in the data.

the two external finance cost states. Comparing the left and right panels, high productivity firms always invest more than low productivity firms so that the red line is positive, but this difference is larger when the cost of external finance is low and investment is relatively unconstrained. Intuitively, if costs of external finance were zero, there would be no wedge between the shadow values of internal and external funds, so investment would be determined only by the marginal product of capital. In this case differences in investment would be large as productivity varied. In contrast, the more costly external finance is, the more constrained investment will be for high productivity firms and this will tend to narrow the difference in investment. In particular, smaller firms with high marginal products of capital invest up to twice as much in the low cost of external finance state relative to the high cost state. These results motivate our difference in differences exercise in Section V.D., and provide an additional test of our model and our method for identifying the hidden cost of external finance state. The model makes the prediction that the difference in difference in investment will be both positive on average and the effect will be largest for smaller firms who tend to have a higher marginal product of capital. Both of these results are supported in the data.

V.   Estimation and Results

A.   Estimation Procedure

We simultaneously estimate as many of the key structural parameters of the model as is computationally feasible using simulated method of moments (SMM), and calibrate the remainder. We estimate calibrated parameters independently, or use direct estimates from the literature, where possible. SMM estimation, like the Generalized Method of Moments, estimates parameters by choosing values which minimize the weighted sum of squared errors between model moments and their empirical counterparts.[13] SMM is well suited to a model like ours which features substantial non-linearities and for which moments cannot be solved in closed form.

We begin by describing the calibrated parameters which are broadly consistent with values from the existing literature and typically come from independent estimation. Table III displays our calibration and compares our parameter choices to those in the literature. The second to last column offers a comparison to the parameters in Riddick and Whited (2009), (RW). Since our focus is on aggregate moments, where available we also report the parameters used in the real business cycle (RBC) literature as reported in Cooley and Prescott (1995). We set the risk

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[13]See Lee and Ingram (1991), Hansen (1982), and Hansen and Singleton (1982).

free rate to four percent as in the RBC literature, and in RW.[14] We use a standard RBC value for depreciation, 8%. This value also matches the average rate of investment we see in the data, and can thus be thought of as having been estimated independently. For the production function curvature parameter, we specify 0.65, which is consistent with the estimate in Cooper and Halti-wanger (2006). Moreover, we found that higher curvature parameters, i.e. production functions closer to linear such as that in RW, imply too large investment volatilities and disinvestment fre-quencies. We calibrate the persistence of both idiosyncratic and aggregate productivity shocks to be 0.66, which allows us to consolidate idiosyncratic and aggregate productivity into a single state variable. This value is equal to that used by RW for the firm level. Khan and Thomas (2008), page 407, contains a detailed discussion of the disagreement in the literature about this parameter, however based on our reading 0.66 is a modal value from prior studies. Moreover, we also estimated the average industry level persistence in the data from Basu *et al.* (2006) to be 0.65. Similarly, we estimate an aggregate persistence of about 0.62, using two trend breaks as advocated in Fernald (2007) and using the data from Fernald (2009). We set the total volatility of firm level productivity equal to the firm level estimate from Hennessy and Whited (2007), which is also near the value used by Khan and Thomas (2011). We then specify that aggregate volatility is about one fourth of total volatility as in Khan and Thomas (2008) and Cooper and Haltiwanger (2006). Finally, we set τ, the tax rate to 10%. In our model, liquid assets are only accumulated in order to ultimately hedge investment opportunities in physical capital. Moreover, firms can simply over-accumulate physical capital and hedge via the additional cash flows that capital produces. Thus, if the tax rate is too high, liquidity can become a dominated asset, in particular in the nested model we describe below in which the cost of external finance is constant. Furthermore, in this model, and others like it, the only force for payouts is the tax wedge, whereas in practice monitoring and information considerations will also induce firms to make payouts. Taking into account these considerations, we choose a rate of 10%, which leads to positive liquidity accumulation in both the full and restricted models we estimate.

Given these calibrated parameters, we estimate the remaining, key, parameters simultane-ously using SMM. Specifically, we estimate: the fixed cost of raising external finance λ_1, the quadratic cost λ_2, the persistence of the stochastic cost γ, the volatility of the cost shocks σ_η, and the marginal investment adjustment cost parameter a, via Simulated Method of Moments (SMM). The SMM algorithm solves the model and constructs data for a large set of parame-ter combinations, and efficiently searches for the set of parameters which imply moments with

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[14]The Online Appendix provides robustness checks for alternative values of the risk free rate and discusses having a stochastic interest rate. We find that our main results are not substantially affected for values of the risk free rate between 2-6%, with the biggest effect being larger cash balances for smaller levels of the risk free rate.

the lowest weighted sum of squared errors relative to their empirical counterparts. We use a bootstrapping technique to construct standard errors by computing the distributions over the estimates which result from estimating the parameters on samples with limited size.

We seek to minimize $g(b)'Wg(b)$, where $g(b)$ is a vector of model errors (the difference between the model moments and data moments), $b = [\lambda_1, \lambda_2, \gamma, \sigma_\eta, a]'$ is our vector of parameters, and W is a weighting matrix. We set the weighting matrix to be the inverse variance-covariance matrix of the moments. This is also known as the efficient weighting matrix, and it places more weight on moments which are more precisely estimated.[15] This typically means that moments involving variances will receive substantially more weight than moments involving means. We use the following seven moments to estimate the vector of parameters b: the pairwise correlations and the volatilities of investment, liquid asset accumulation, and external finance, and the average level of liquid assets to total assets. We target the correlations because our main focus is the co-movement between corporate issuance, savings and investment. Targeting both correlations and volatilities also implies that co-variances will be close to their empirical counterparts.

Identification of our estimation parameters requires that the moments respond to changes in parameter values. In the companion Online Appendix to this paper, we discuss which moments are primarily affected by which parameters by performing comparative statics on the model moments for changes in each of the estimated parameters. We find that the fixed cost of external finance is important in matching the level of cash holdings, and the quadratic cost is helpful in matching the volatility of external finance. The persistence and volatility of the stochastic cost help to match the correlations between liquidity accumulation and external finance as well as between investment and external finance. Without the stochastic cost, the correlation between external finance and investment is very large for *any* reasonable parameter values because firms only raise external finance when there are good productivity shocks and hence good investment opportunities. The stochastic cost breaks this high correlation and simultaneously generates the strong correlation between external finance and liquid assets through market timing by firms.

To solve the model for given parameter values, we use the value function iteration method, described for example in Ljungqvist and Sargent (2012). To discretize the state space, we approximate the realization of the productivity and stochastic cost of external finance shocks using standard Gauss-Hermite quadrature techniques (see Tauchen and Hussey (1991)). There are six productivity states, which govern both idiosyncratic and aggregate productivity, and two aggregate states for ξ. For capital and liquid assets, we choose a large enough grid such that the stationary probabilities of being at the upper bound of the grid are negligible, something we

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[15]See Hansen (1982), Hansen and Singleton (1982), and Lee and Ingram (1991).

verify ex-post.

Given the policy functions implied by our model solution, along with values for the model state variables and the stochastic processes for the exogenous states, we simulate a panel of firms. Specifically, we simulate 1,000 idiosyncratic productivity processes, one aggregate productivity process, and one aggregate stochastic cost process following the persistence and volatility given in Table III. We then create 1,000 total firm productivity shocks by summing each firm specific and aggregate productivity series and taking the exponential. We simulate 600 years of data, throwing away the first 100 years to avoid any dependence on initial values. We then aggregate across firms to form aggregate corporate flows, analogous to our procedure in Compustat.

We report the moments in the data and their counterparts in the model at estimated parameter values in Table IV. We also report a large number of additional moments which our estimation procedure was not explicitly targeted to match. We report parameter estimates and standard errors in the middle panel of Table III. The standard errors show that all parameter values are significant. Intuitively, the fairly small standard errors mean that the moments chosen do indeed respond greatly to changes in parameter values so that the moments could not easily be generated by significantly different parameter values. Our estimation implies an average percentage issuance cost of 2.3%, which is well within empirical estimates. For example, Asquith and Mullins (1986) find that abnormal stock returns around secondary equity offerings are about 3% and Hennessy and Whited (2007) estimate that firms face an issuance cost of 8.3% on the first million dollars raised. We also find an average investment adjustment cost paid of 1.4% per dollar of investment which is relatively low. Relative to RW, our estimated parameters feature a higher fixed cost of raising external funds and lower linear and quadratic components. The higher fixed cost is consistent with the evidence regarding the importance of the variation in the extensive margin of external finance over the business cycle presented in Figure 3. Bazdresch (2005) and Cummins and Nyman (2004) both emphasize importance of lumpy external finance, and provide complementary evidence to ours regarding the importance of fixed issuance costs. Finally, strong empirical evidence for the important role of fixed financing costs is also provided by Welch (2004) and Leary and Roberts (2005), which document a very high level of persistence in corporate capital structures, and emphasize rare active capital structure changes.

We estimate the volatility of the ξ shock to be around 2. Recall that, given this volatility, we choose the mean of the ξ shock ($\exp\left(\mu - \frac{\sigma^2}{2}\right)$) to generate an average credit shock of one. This is without loss of generality, but makes the cost parameter λ_2 more easily interpretable as the average marginal cost, and enables comparison to estimates in prior work. We estimate the persistence of the stochastic cost at 0.22. We note that the annual persistence of the Baa-

Aaa default spread and innovations to the expected default adjusted excess bond spread from Gilchrist and Zakrajsek (2012a), not targeted in our estimation, are 0.4 and 0.35, respectively. We assume the stochastic cost is uncorrelated with the aggregate productivity shock, an assumption consistent with our empirical estimates. The empirical correlation between innovations in the default spread and TFP shocks is -0.2 and is not statistically significantly different from zero. Similarly, the empirical correlation between innovations to the expected default adjusted excess bond spread from Gilchrist and Zakrajsek (2012a) and TFP shocks is -0.1 and is again not statistically significantly different from zero.

B.   Aggregate Analysis

Table IV presents a comparison of aggregate issuance, savings, and investment moments in the data and in the model. The top panel presents a comparison of the moments targeted in the estimation. The model generates a strong, positive correlation between liquidity accumulation and external finance, with an implied correlation of 0.7, similar to the empirical correlation of 0.6. Moreover, the correlation between liquidity accumulation and external finance is higher, as it is in the data, when one conditions on firms raising external finance. The model also does fairly well on matching the other cross correlations, however it overshoots on the correlation between external finance and investment. This is because the only ultimate use for funds is for investment. We note, however, that the ranking of the cross correlations matches the empirical one. Finally, the estimated model produces realistic levels of volatility for investment, savings, and external finance, though the volatility of investment is slightly higher than in the data. The model undershoots on average liquidity accumulation, a moment which does not receive a high weight in the estimation since it is a mean, whereas the other moments are related to variances. In general, because we use the efficient weighting matrix in our estimation, the weights are based on statistical precision with which the empirical moments are estimated. These weights may not always correspond to the weights one would assign based on economic importance within the context of the current study, however this weighting aims for a good statistical fit of the model overall.

The bottom four panels of Table IV present a comparison of the estimated model's implied moments relative to their empirical counterparts for thirteen additional aggregate moments. Considering that none of these additional moments were targeted in the estimation, the model does a good job of matching them. In particular, the model matches the empirical autocorrelation of aggregate investment of 0.38 very well, with an implied estimate of 0.34. This autocorrelation is sensitive to the convex adjustment cost parameter, and the match between these moments

supports the ability of our model to match the persistence of aggregate investment. In the model, as in the data, the correlation of liquidity accumulation and external finance increases when we condition only on firms raising external finance, highlighting that the correlation isn't driven by some firms saving while others are raising. Investment and liquidity accumulation also become more correlated when conditioning on raising firms. The model generates business cycle correlations which match the ranking from data: investment tracks the business cycle most closely, followed by external finance, and then liquidity accumulation. The magnitudes of the correlations are, not surprisingly, higher in the model as it features only two shocks. Importantly, the model also generates the observed, highly positive, correlations between ξ and $\rho_{i_l,e}^{xs}$ and the percentage of firms raising external finance. As illustrated by the firm policy functions in Figure 4, only when the cost of external finance is low do firms find it optimal to issue costly external finance and save the proceeds. We also note that the correlation between TFP and these cross sectional moments describing financing and savings by firms are lower than the correlations with ξ, as is true empirically. When computing TFP in the model, we estimate TFP with a log linear production function, treating financial and investment costs as deadweight costs, which matches the empirical estimation of TFP used in the data. In the model, the correlation between $\rho_{i_l,e}^{xs}$ and TFP is slightly positive due to the positive effect of better resource allocation when ξ is low.

C.   Additional Untargeted Moments: Firm Level Analysis

Although our focus is on aggregate moments, the firm level moments, none of which are estimation targets, provide an additional check on the fit of the model. We use the optimal policy functions for a given firm to compare the model generated firm level moments to those in the data. We report the relevant moments in Table V. Most correlations increase with aggregation, both in the data and in the model. This is not surprising, given that all firms are subject to independent idiosyncratic shocks, and common aggregate shocks. For firm level issuance and savings activity, we find a correlation between external finance and liquidity accumulation of 0.49 in the model, and 0.18 in the data. Thus, the model overshoots on this correlation at the firm level. We note that the lower correlation at the firm level in Compustat data may be due to noise, or accounting statement timing issues.

We find a low correlation between liquidity accumulation and investment in the model (0.12) as well as in the data (-0.06). At the firm level, the correlation between external finance and investment is too high in the model (0.80) relative to the data (0.20). Thus one shortcoming of our model is that the only ultimate use of funds is for investment in physical capital. Liquid asset balances in the model are again low in the model compared to the data at 0.03 vs 0.15.

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The (unconditional) probability of raising funds is about 16% in the model. This probability is lower than in the data (43%), however, we note that the high empirical probability of issuance may be due to a high incidence of small issuances with low costs, such as drawdowns on lines of credit. Consistent with this, Bazdresch (2005) provides evidence that a small fraction of large issuances account for most of firms' external financing activity. Likewise, the average amount of external finance raised is smaller in than in the data. The model does reasonably well in matching the average level of investment and its volatility, and on the standard deviation of external finance. The model produces too low autocorrelation of investment at the firm level, but again this correlation increases with aggregation. Overall, given that our estimation targets aggregate moments, and that our study is focused on understanding aggregate moments, the performance of the model at the firm level seems satisfactory.

D.   Are Stochastic Costs Necessary?

In this section, we provide two important additional lines of evidence in support of our estimated model. First, we show that a restricted version with constant costs, which is main alternative to our model, is rejected in favor of the full model with stochastic costs. Second, we show that key additional predictions of the model, moments that were not targeted in the estimation, are supported by empirical evidence.

We show that stochastic costs are statistically justified by the improvement in model fit, relative to a nested alternative model in which the cost of external finance is constant and the only shock firms face is to their (aggregate and idiosyncratic) productivity. Specifically, we test the improvement that the full model offers over a restricted version in which the volatility of the stochastic cost shock is restricted to be zero (at this point the persistence parameter is no longer identified). The difference in the minimized objective function for the unrestricted version of the model will clearly be smaller, but the J-test evaluates whether the difference in model errors more than justifies the addition of the cost persistence and volatility parameters. The difference between the objective functions, $J_{restr} - J_{unrestr}$, is distributed χ^2 with degrees of freedom equal to 2 (the number of restrictions). We are able to reject the the null hypothesis that the improvement in fit by the full model, relative to the restricted model, does not justify the addition of two additional parameters, at well below the 1% level. Thus, the data strongly prefer the full model with stochastic costs. Table VI presents the results of this test, along with a full comparison of the parameter estimates and implied moments from the full and restricted models. Finally, we also note that the estimate of the volatility of the shock to the cost of external finance in the full model is highly significant.

We can also analyze the individual moments from the nested restricted model in order to understand economically its failures. Table VI shows that a model with constant costs has several important shortcomings. The first is that it has difficulty generating cash balances with liquid assets only making up 1% of total assets. The second is that the only time the firm will want to raise external finance is when it also has a good shock to investment opportunities. Thus the correlation between investment and external finance in this model is essentially 1, whereas it is 0.46 in the data. Indeed, the relatively low empirical correlation between external finance and investment provides some model independent support that these two flows are partially driven by different shocks. Finally, and importantly, at estimated parameter values the constant cost model does not generate aggregate issuance and savings waves in line with the data. The estimation procedure has trouble matching the empirical moments with the restricted model, and is forced to compromise amongst all the moments it aims to match. As a result, at estimated parameters, the correlation between liquidity accumulation and external finance is only weakly positive, with an implied value of 0.28. The model also implies a volatility for liquidity accumulation near zero. In short, in the restricted model liquid assets play very little role as they are low on average and don't vary meaningfully. This means investment is overly correlated with external finance.

The $J - test$ formally confirms that the full model fits the data significantly better than what we consider to be its main alternative. Rather than rule out all other possible alternative explanations, we instead seek to show that important and unique additional implications of the model are supported by the data. To that end, we first note that the high correlation between the cross sectional moments $\rho_{i_l,e}^{xs}$ and $\%raise$, and traditional empirical proxies for the cost of external finance supports the idea that these moments describing corporate policies can help to uncover the actual cost of external finance that firms face. Empirically, we find correlations between the cross sectional correlation between liquidity accumulation and external finance, $\rho_{i_l,e}^{xs}$, and the default spread and lending standards of -0.64 and -0.58, respectively. Figure 3 shows that the percentage of firms raising external finance at any given date, $\%raise_t$, is also highly correlated with these measures.

We next turn to a difference in differences prediction, implied by the model's policy functions, which seems somewhat unique to our model. In particular, our model predicts that the difference in investment between high productivity firms and low productivity firms will be larger when $\rho_{i_l,e}^{xs}$ is high. Consider the policy function in the bottom panel of Figure 4. An implication of the model with a stochastic cost of external finance is that a firm with identical levels of productivity, liquid assets, and capital may have different levels of investment depending on the cost of external finance. Moreover, this difference in investment will be larger when investment

opportunities are better. Comparing the low and high cost of external finance in the left and right panels respectively, we see little difference in investment by low productivity firms, with poor investment opportunities. By contrast, we see higher investment by high productivity firms when the cost of external finance is low. A high cost of external finance substantially constrains investment for these firms. Since firms invest until the marginal product of capital equals the marginal cost of external finance, high productivity firms optimally choose much lower investment when the marginal cost of external finance is high. Finally, we note that the spread between high and low productivity firms' investment is highest for small firms, when the cost of external finance is low.

The testable implication is that the difference in investment between high and low productivity firms should depend largely on the cost of external finance. When the cost is high, differences in investment will tend to be much smaller even when the dispersion in productivity remains large. Our difference in differences exercise is aimed at testing this key result empirically. Specifically, we define firm level productivity as the residual from regressing log operating income on the log capital stock. We then define an indicator for a firm as high or low productivity in each period based on whether they are above or below median productivity. Next, we use our $\rho_{i_l,e}^{xs}$ variable to proxy for the cost of external finance at each date and create a dummy for when this variable is above or below its median value. The coefficient on the interaction term between these two dummies captures the differential effect of the cost of external finance on investment for high and low productivity firms as shown for model policy functions in Figure 4.

Table VII shows the results from the regression. We include controls for the individual firms' state variables. We also include lagged investment in the regression using Compustat data. Whereas in the model the firm's state variables already capture the information in lagged investment, because these state variables are measured noisily in Compustat data, lagged investment contains additional information about the firm's state. Consistent with our model, we find a negative, statistically significant coefficient on the interaction term of -0.24. This implies that the difference in investment rates between high productivity firms and low productivity firms is 0.24% lower on average during times when the cost of external finance is high compared to when it is low. The regression is consistent with the fact that investment is more constrained by the cost of external finance for high productivity firms who have good investment opportunities. According to the Figure 4, we should also see this effect being stronger for small firms both because they are relatively more constrained due to the fixed issuance cost, and because they respond more strongly to productivity shocks because of their steeper marginal product of capital. Accordingly, conditioning on the smallest 50% of firms, the difference in differences interaction

coefficient decreases to -0.36% and conditioning on the smallest 25% of firms it decreases to -0.68%. This is qualitatively in line with the model predictions. These results also support the idea that our measure of the cost of external finance based on $\rho_{i_l,e}^{xs}$ is economically meaningful, since variation in $\rho_{i_l,e}^{xs}$ has an empirically significant impact on investment.

These findings are complementary to, but different from the existing literature that relates shocks to the cost of external finance to investment. The existing literature focuses on a shock to the financial sector that affects some firms more than others, for example due to heterogeneity in firm/bank relationships.[16] The idea is that credit tightens more for certain firms and therefore these firms will reduce investment and employment by more. The identification scheme in these studies is aimed at ensuring that treated and untreated firms have similar investment opportunities on average. In contrast, in our design, and in line with the literature in macroeconomics on the effects of financial frictions, we assume that all firms are hit by an aggregate shock to cost of external finance, but that the shock has a much greater impact on small, high productivity firms which have a greater incentive to invest. Thus, identification comes from the fact that firms are affected differently by common shocks, rather than because the shock is different across firms. This key difference is what allows us to do inference about the aggregate cost of external finance using cross sectional variation in firm policies.

This difference in differences exercise provides unique support for our model with stochastic costs of external finance. The model with constant costs, for example, could not explain these results. In that model the only state variables are capital, liquid assets, and productivity. Controlling for these, investment should not vary. That alternative model is certainly a great simplification relative to the data, however, it is not obvious in what more elaborate alternative model to ours $\rho_{i_l,e}^{xs}$ would capture differing investment policies along the lines we document. We thus take the ability of our model to match this unique feature of the data as important support for the mechanisms in the model.

VI. Estimating The Cost of External Finance

In this section, we use the relationship in the model between financing and savings activity in the cross section, and the aggregate cost of external finance, to estimate the average cost of external finance per dollar raised in the US time series 1980-2010. Specifically, we construct an aggregate external finance index in the model, in which the cost is predicted to be a weighted

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[16]See Ivashina and Scharfstein (2010), Duchin *et al.* (2010), Campello *et al.* (2010), Almeida *et al.* (2009). See also Matvos and Seru (2011) for evidence of financing shocks from their estimation of a structural model comparing resource allocation by diversified and undiversified firms during the financial crisis.

average of cross sectional moments describing firms' issuance and savings. We then use the coefficients from this index, along with the empirical cross sectional moments at each date, to construct our estimates for the US time series. When the cost of external finance is low, firms are both more likely to raise external finance, and also to have large, positive, deviations from the average amount that they raise and the amount that they save. Figure 4 shows these changes in policy functions in the dynamic model as a function of the aggregate cost. Thus, we use these two main cross sectional moments to estimate the aggregate cost of external finance at each date: the fraction of firms raising external finance, and the cross sectional correlation between liquidity accumulation and external finance. These two moments capture the extensive margin of the issuance decision, and the intensive margin of what firms do with the finance they raise. Accordingly, in our model, they are able to explain over ninety percent of the variation in the cost of external finance. As can be seen by comparing Figures 2 and 3, other moments such as the aggregate amount of finance raised, and liquidity accumulated, contain similar information to these moments but do not add to the variation explained once controlling for them. The single variable which explains the most variation in ξ in model data is $\rho_{i_l,e,t}^{xs}$, a fact we document below.

In the model, we can express the average cost of a unit of external finance per dollar raised as $\frac{E[\phi_e(e,\xi)]}{E[e|e<0]}$. This is the average cost of external finance paid across firms at any given date, divided by the average amount raised. We compute this expression at each point in time in the model and then use this series as the dependent variable in the following time series regression estimated on model data:

$$\frac{E[\phi_e(e,\xi)]}{E[e|e<0]}_t = \alpha + \beta_1 \%raise_t + \beta_2 \rho_{i_l,e,t}^{xs} + \varepsilon_t,$$

where we normalize the independent variables, $\%raise_t$ and $\rho_{i_l,e,t}^{xs}$, to have mean zero and unit standard deviation. We report the regression results in Table VIII. We find that both the coefficients, β_1, β_2 are negative, as expected. The coefficient on $\rho_{i_l,e,t}^{xs}$ is -1.3% in magnitude. This implies that a standard deviation increase in $\rho_{i_l,e,t}^{xs}$ is associated with a 1% decrease in the average cost paid per dollar raised. The coefficient on $\%raise_t$ is smaller at -0.3%. The constant term, α, is 2.3% and represents the average cost of external finance paid per dollar raised in the model. Thus, each of these two cross sectional moments is economically important for explaining variation in external financing costs in the model data. Moreover, the R^2 in this regression is around 95%, and thus just these two variables alone do a very good job explaining variation in the cost paid by firms in the model economy. We show that these variables also measure ξ_t, the stochastic cost of external finance "level" shock series, in the model.

We then apply the estimated regression coefficients, $\hat{\alpha}, \hat{\beta}_1, \hat{\beta}_2$, to the analogous empirical moments to form a time series of estimates of the cost of external finance paid per dollar raised for each year from 1980 to 2010. We plot this series in the top panel of Figure 1. Our estimated cost of external finance series has very intuitively appealing properties. The estimated cost is high in the early 1980's, late 1980's, around the 2001 dot com crash, and in the recent financial crisis. Moreover, each spike in this cost is associated with a recession. Economically, the cost per dollar raised varies from essentially zero in the mid to late 1990's, up to over 4% during the recent financial crisis, which is a full 75% larger than the overall average cost of 2.3%. This value seems economically large too, considering the impact of the implied 170 basis point increase on the required return for a firm debating whether to take on an investment project for which it must raise external finance.

Table IX gives correlations of our estimated cost with three proxies of the cost of external finance: the default spread, the index of lending standards, and the Michigan consumer sentiment survey. The default spread provides a market based measures of debt financing costs, while the index of lending standards picks up a measure of bank willingness to lend, and the sentiment index is interesting if one thinks the cost is potentially driven by mispricing. We find that our estimated cost has a correlation of 0.67 with the default spread, 0.58 with the index of lending standards, 0.72 with sentiment, and 0.77 with an equal weighted average of the three (the average is taken after standardizing each series to have zero mean and unit variance). Thus the estimated cost clearly picks up important common variation in these measures. Finally, it is worth noting that the estimated cost is typically more correlated with these variables than either $\%raise_t$ or $\rho_{il,e,t}^{xs}$ alone, meaning that, consistent with the model, the combined information from both of these series does a better job picking up variation in the cost of external finance.

We next turn to a more structural estimation of the cost of external finance. As in the regression method, we use our calibrated model, along with cross sectional moments to make inferences about the aggregate cost of external finance. In particular, at each date in time, we use a version of Simulated Method of Moments (SMM) to infer the value of the stochastic cost that generates simulated cross sectional moments as close as possible to the data. The difference between our estimation exercise and a typical SMM estimation is that we are looking to uncover a hidden state instead of estimating a parameter. This distinction matters since state variables, unlike parameters, influence the model's transition dynamics over time, not just through policy functions, but also directly. However, note that while our estimates in later periods for the state will depend on earlier estimated values, we will still get a consistent and unbiased estimate of the state.

As moments, we choose the correlation between liquidity accumulation and external finance as well as the percentage of firms raising external finance at each date, as these are each informative about this cost in the model. More specifically, we define the vector M_t as follows:

$$
M_t = \left[
\begin{array}{c}
\rho_{xs,\text{mod}}\left(\frac{i_l}{TA}\left(\xi_t\right), \frac{-e}{TA}\left(\xi_t\right)\right) - \rho_{xs,data}\left(\frac{i_l}{TA}, \frac{-e}{TA}\right) \\
E_{xs,\text{mod}}\left[1_{e<0}\left(\xi_t\right)\right] - E_{xs,data}\left[1_{e<0}\right]
\end{array}
\right]
$$

where $\rho_{xs,\text{mod}}(x,y)$ represents the cross sectional correlation between x and y in the model, which is a function of ξ_t, and $\rho_{xs,data}(x,y)$ represents the empirical counterpart. We use the notation xs to emphasize that these are cross sectional, rather than time-series moments.[17]

At every date t we choose the value ξ_t that minimizes deviations of the cross sectional model implied moments and empirical moments. Specifically, we choose ξ_t to minimize the following objective function

$$
\min_{\{\xi_t\}} M_t' W M_t
$$

where we set $W = I_{2x2}$ as the identity matrix which weights all moments equally. We choose equal weights for parsimony, because we do not see a reason for this exercise to weight by the statistical precision with which the two moments are estimated, though using alternative weighting matrices has little effect on the results.

We initialize the series by first starting capital and liquidity at their steady state values, and then feeding in the observed aggregate TFP and the default spread series beginning in 1951 as a proxy for initializing the stochastic cost series. We do this to ensure that the model distribution over capital and liquidity stocks reflects the observed history of TFP and external finance cost shocks. We use the empirical realizations of these series to proxy for $z_{t,agg}$ and ξ_t until 1980 when our Compustat sample begins. Thereafter, we estimate ξ_t date-by-date by setting the value of aggregate productivity to equal to our observed TFP series and choosing ξ_t to minimize our objective function. This allows us to estimate a rolling time series for the stochastic cost of external finance.

The results from the SMM estimation procedure are displayed in the bottom panel of Figure 1, where high cost dates are indicated in red. Comparing these dates to the regression implied estimates, one sees that the estimates from both methods are very similar; the SMM method picks a high ξ when the regression method implies a high average cost. Both methods also estimate high costs during the recessions in our sample, though they are not simply recession

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[17]We have also estimated the series adding two additional moments: $\left(\xi_t - \mu_\xi\right) - \rho\left(\xi_{t-1} - \mu_\xi\right)$ and $\left(\left(\xi_t - \mu_\xi\right) - \rho\left(\xi_{t-1} - \mu_\xi\right)\right)^2 - \sigma_\varepsilon^2$ which help ensure that the series ξ_t follows an AR(1) with the parameters we calibrated. The results are similar.

indicators. The main limitation of the SMM methodology is that the procedure must choose one of the two states specified in the numerical solution of the model, whereas the regression index method estimates form a continuous time series.

In sum, both of our estimation procedures appear to pick up events that our priors suggest might be associated with costly external finance, such as the recession of the early 80's, the crash of the tech boom in 2001, and the recent financial crisis. We take these findings to be additional evidence which supports using moments implied by our model to identify times when the cost of external finance is high or low. Thus, we argue that our model and moment implied cost estimates provide a "revealed preference" measure of the cost of external finance which is complementary to the existing list of empirical proxies.

VII. Conclusion

We document the empirical regularity of aggregate issuance and savings waves, and show that cross sectional moments describing firms's issuance and savings behavior are informative about the aggregate cost of external finance. We document that, empirically, the time series of the cross sectional correlation between external finance and liquidity accumulation, and the time series of the percentage of firms raising external finance, are highly correlated with standard measures of the aggregate state of the cost of external finance, such as the default spread and the fraction of banks tightening lending standards.

We argue that both the observed realization of the correlation between external finance and liquidity accumulation in the cross section, $\rho_{i_l,e}^{xs}$, and our model implied estimate of the level of the cost of external finance in the US time series 1980-2010, are useful measures of the state of the aggregate level of the cost of external finance. Using firms' actual decisions about how much external finance to raise and how they use the proceeds from external finance is a revealed preference method of making inferences about the true cost of external finance. A measure which is based on actual corporate decisions about financing, saving, and investment, seems useful for measurement and for policy guidance.

Understanding the role of a potentially time varying cost of external finance is important for several reasons. First, studying whether shocks to the cost of external finance are important for firm financing, liquidity accumulation, and investment dynamics may help to further understand the role of the financial sector in business cycles. From a theoretical standpoint, many models featuring endogenous variation in the cost of external finance in business cycles feature a tight link between variation in fundamentals such as productivity and variation in the informational

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frictions which make external finance costly. In contrast, shocks to the cost of external finance in recent DSGE models are often only partially correlated with other fundamental shocks. By using our dynamic model as a filter on the US time series for firm financing, liquidity accumulation, and investment, we provide implied estimates of the cost of external finance at each date, allowing us to observe its implied behavior over the last 30 years.

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VIII. Figures, Tables. View in color.

Figure 1: This figure describes the average cost of external finance paid per dollar of external finance raised in the US time series estimated using cross sectional moments and the estimation procedure described in Section VI. The top graph uses the regression index method. The bottom graph uses the two state SMM procedure, and indicates a high cost for years in red.
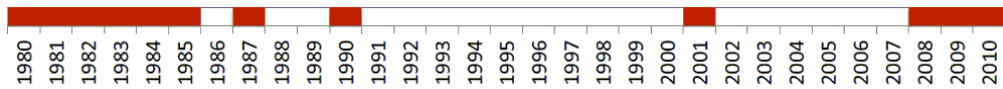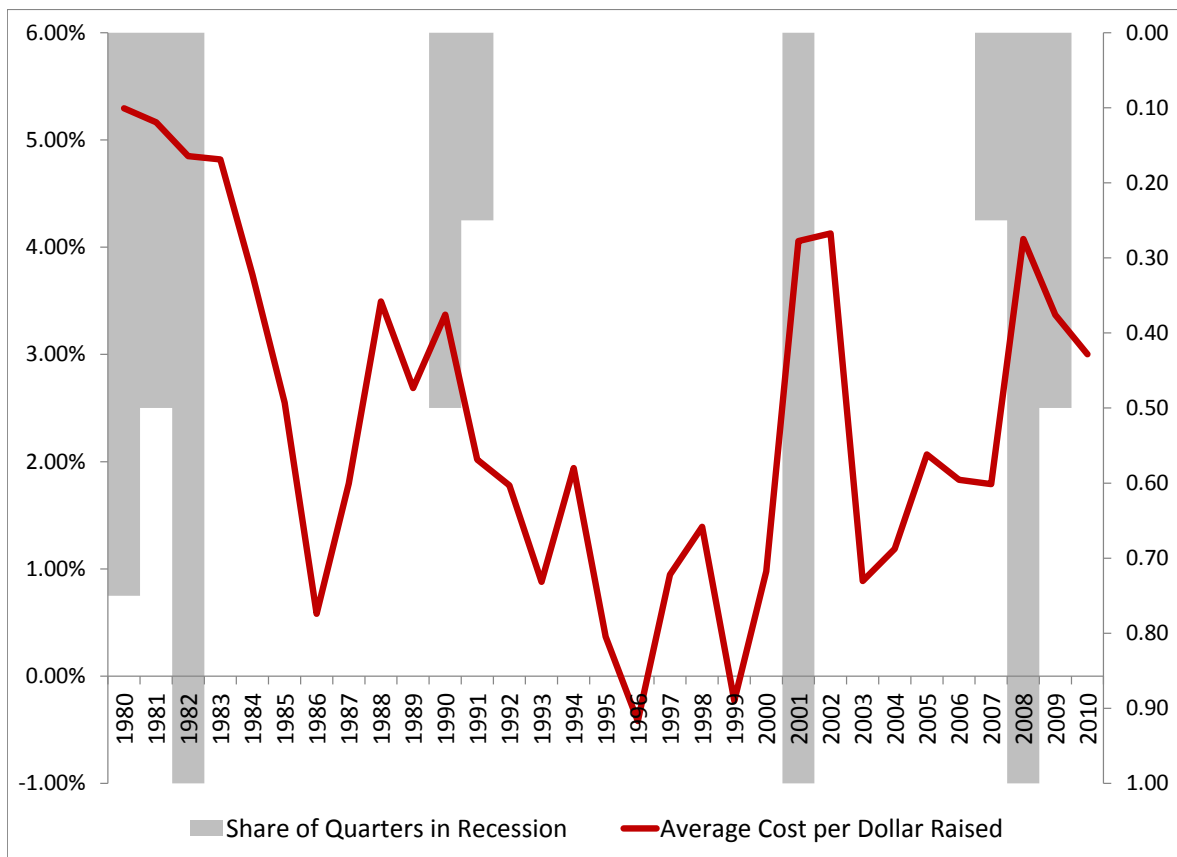
Figure 2: We plot aggregate accumulation of liquid assets against aggregate external finance. Our sample excludes the largest 10% of firms. Plotted data are normalized by lagged assets, HP-filtered, and then scaled to have unit variance. Gray bars indicate fraction of quarters economy is in a recession in the given year (right axis). The correlation between liquidity accumulation and external finance is 0.6. A univariate regression of liquidity accumulation on external finance yields an R^2 of 0.36 and a regression coefficient estimate of 0.19.
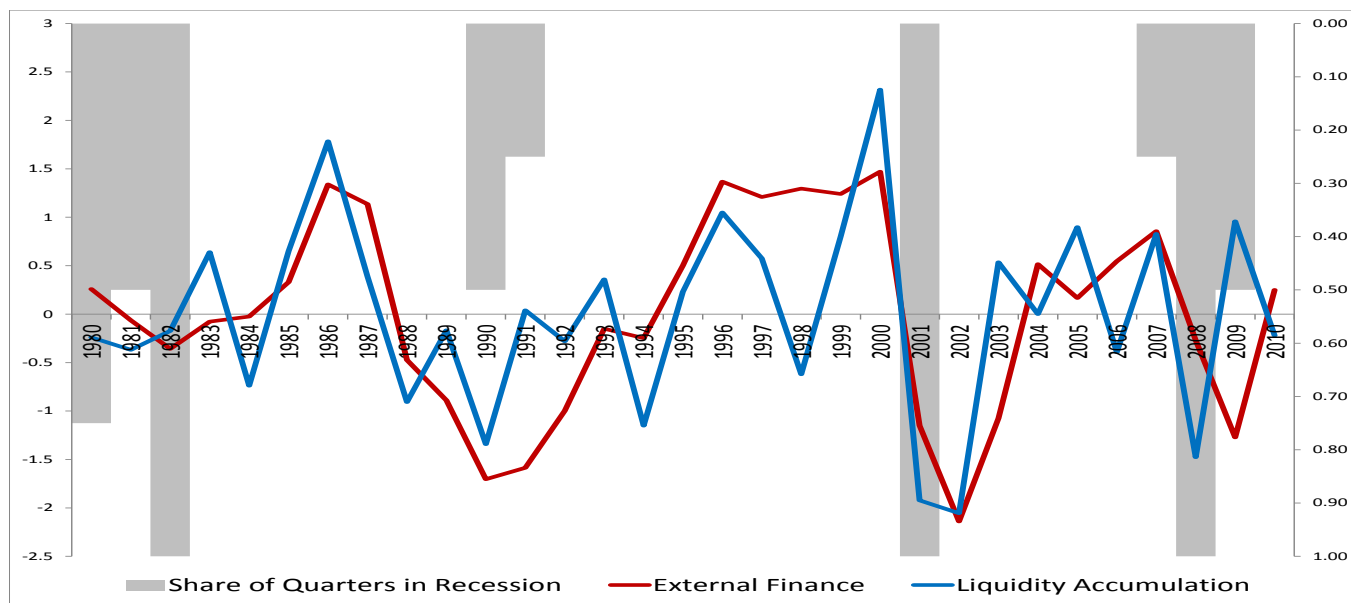
Figure 3: We plot the the cross sectional correlation between liquidity accumulation and external finance and % of firms raising external finance in the top panel. These moments are highly correlated with the shock to the cost of external finance in our quantitative model. The bottom panel plots the negative of the default spread, the net % of banks tightening lending standards, and the sentiment index. Gray bars indicate fraction of quarters economy is in a recession in the given year (right axis). All series are standardized to have mean zero and unit variance.
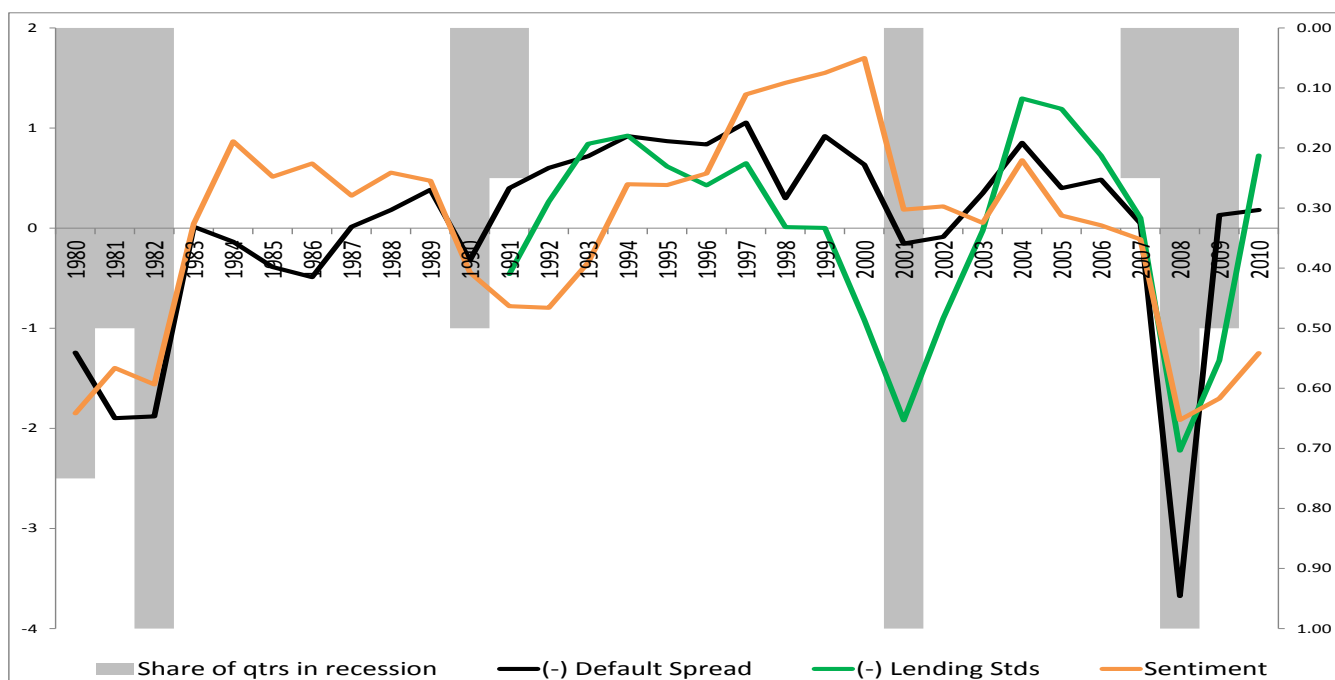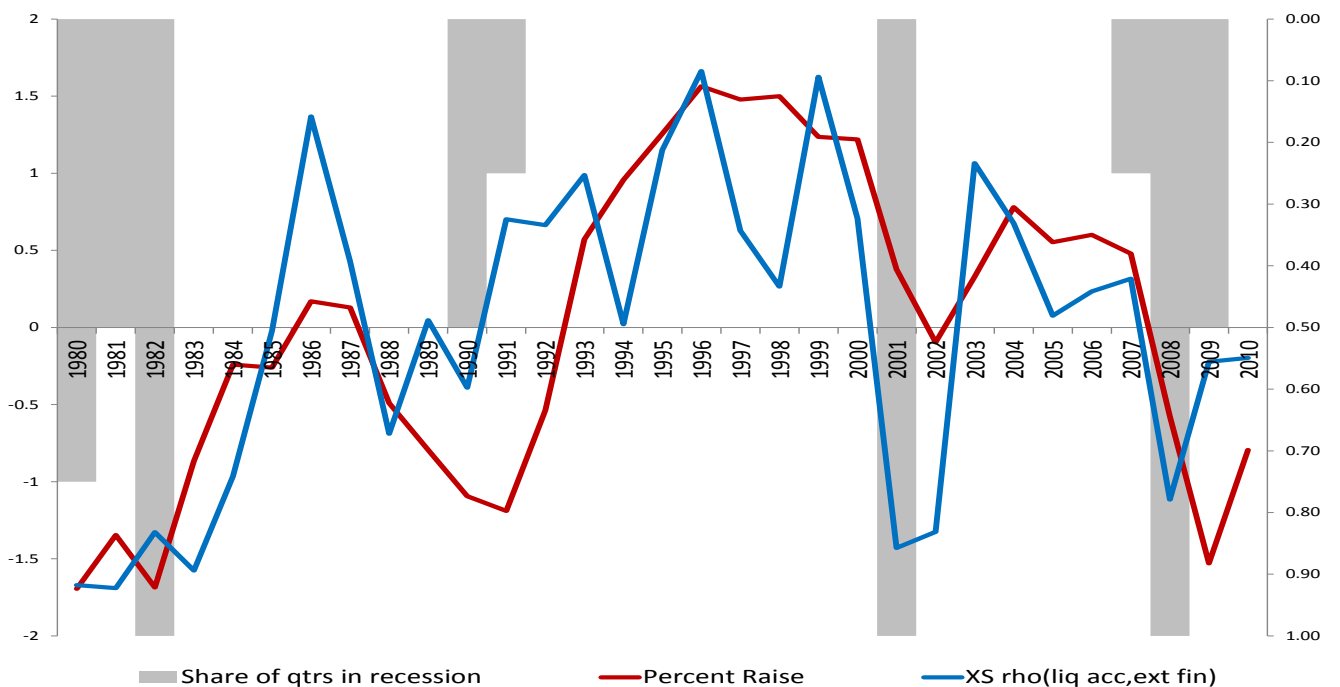
Figure 4: This figure plots firm policy functions across low vs. high cost of external finance states in the left and right panels, respectively. The top panel plots the percentage of firms raising external finance (or the probability of raising for a given firm) conditional on firm level productivity. The middle panel plots external finance (blue line) and investment in liquid assets (green line) as a function of productivity. The bottom panel plots investment in physical capital as a function of both productivity and firm size. The blue line plots investment for high productivity firms as a function of firm size while the green plots investment for low productivity firms. The red triangle line indicates the difference in investment across high and low productivity firms. This comparison supports our difference in difference estimation in Section V.D..
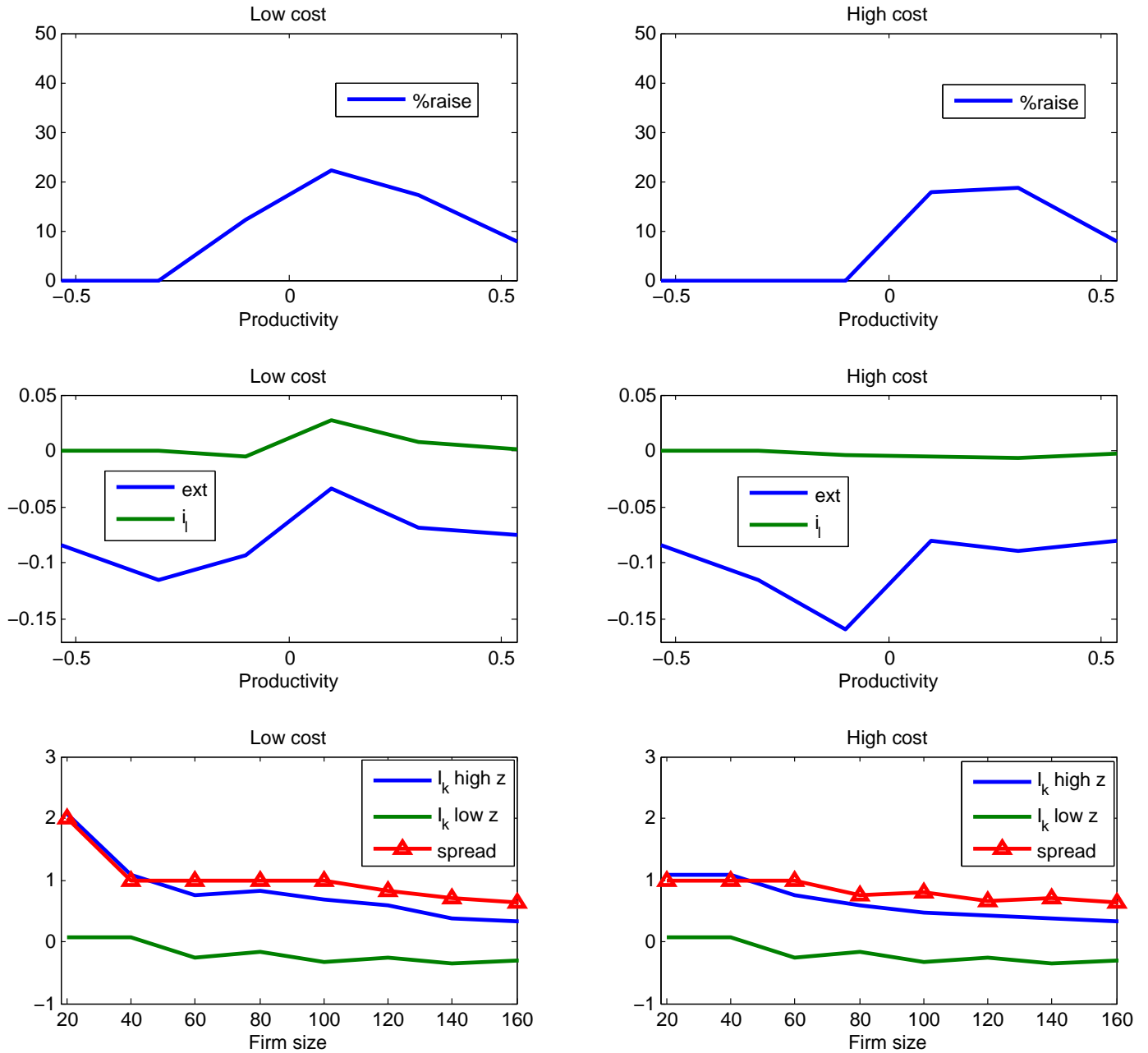
Table I: This table displays the main aggregate issuance and savings waves facts. Except where noted, we use annual Compustat data from 1980-2010. We normalize aggregate series by the lag of total assets and hp-filter. Size bins are determined by total asset size. The main results in the paper use the [0,90]% sample. Flow of funds data are normalized by the trend in gross value added for the corporate sector. Narrow liquidity is the net acquisition of financial assets minus trade receivables minus miscellaneous assets. Broader liquidity also includes 1/3 of miscellaneous other assets as liquid assets. * indicates significance at a 5% level. $\rho_{i_l,e,t}^{xs}$ is $\rho_t\left(\frac{-e_i}{TA_i}, \frac{i_{l,i}}{TA_i}\right)$

Aggregate Issuance and Savings	
$\rho\left(\frac{\Sigma(-e)_t}{\Sigma TA_{t-1}}, \frac{\Sigma i_{l,t}}{\Sigma TA_{t-1}}\right)$	
[0,50]%	0.84*
[0,90]%	**0.60***
[0,100]%	0.12
Conditional on Raising funds: e<0	0.74*
No Dividends	0.68*
No Rating	0.56*
Flow of Funds: Narrow Liquidity	0.33
Flow of Funds: Broader Liquidity	0.38*

$\rho\left(\rho_{i_l,e,t}^{xs}, \text{Aggregate State}_t\right)$	
Minus Baa-Aaa Spread	0.64*
Minus Lending Standards	0.58*
TFP Shock	0.48*

Table II: This table displays moments for debt and equity separately using our Compustat sample. We normalize each aggregate series by lagged aggregate assets and apply the hp-filter. * indicates significance at 5% level.

<div align="center">Aggregate Compustat Moments: Debt vs. Equity</div>

	Unconditional	Conditional on $e < 0$
$\rho\left(liqacc, debt\right)$	0.16	0.33*
$\rho\left(liqacc, equity\right)$	0.69*	0.77*
$\rho\left(inv, debt\right)$	0.60*	
$\rho\left(inv, equity\right)$	-0.15	

Table III: We give our calibrated parameters below along with those in Riddick and Whited (RW) and the standard business cycle literature (RBC). The label e.c.f. denotes external cost of finance and i.a. denotes investment adjustment costs. The lower panel gives the implied average costs of issuance and investment firms pay with the given parameters. For example, the implied average cost of issuance gives the average cost paid for a firm raising external finance as a fraction of the amount of funds raised.

Calibrated Parameters

Symbol	Description	Model	RW	RBC
τ	tax rate	0.1	0.20	——
δ	depreciation	0.08	0.15	0.08
θ	curvature	0.65	0.75	0.33
ρ	persistence	0.66	0.66	0.9
σ	total vol of prod	0.121	0.121	——
σ_i	idiosyncratic vol	0.11	——	——
σ_{agg}	aggregate vol	0.03	——	0.022
r	risk-free	0.04	0.04	0.04

Estimated Parameters

Symbol	Description	Estimate	Standard Error
a	invest adj cost	0.26	0.02
λ_1	e.c.f. fixed	0.15	0.01
λ_2	e.c.f. quad	0.0004	0.00006
σ_η	vol cost of funds	2.01	0.22
γ	persistence	0.22	0.06

Implied Average Costs Paid

Symbol	Description	Model
$E\left[\frac{\phi_e(e)}{e}\right]$	implied average issuance cost	0.023
$E\left[\frac{\phi_i(i_k,k)}{i_k}\right]$	implied average investment cost	0.014

Table IV: This table displays aggregate moments from the model using a simulated panel of firms. We compare these moments with those from annual Compustat data, 1980-2010. For correlations, we normalize each series by lagged assets and apply the hp-filter. All other series are normalized by current assets. $\rho_{i_l,e,t}^{xs}$ is $\rho_t\left(\frac{-e_i}{TA_i}, \frac{i_{l,i}}{TA_i}\right)$. TFP are TFP level shocks. We use the Baa-Aaa default spread as an empirical proxy for ξ. * indicates significance at 5% level. We use notation from the model: $-e$ represents external finance (negative of payouts), i_l liquidity accumulation, i_k investment in physical capital, and l liquid balances. We normalize each series by total assets except investment which is normalized by physical captial k.

Estimation Moments		
Moment	Data	Model
$E[l]$	0.11	0.03
$\sigma(i_l)$	0.01	0.01
$\sigma(i_k)$	0.01	0.02
$\sigma(-e)$	0.03	0.03
$\rho(i_l, -e)$	0.60*	0.70
$\rho(i_l, i_k)$	0.12	0.18
$\rho(-e, i_k)$	0.46*	0.84

Additional Moments		
Moment	Data	Model
$E[i_k]$	0.07	0.08
$\rho(i_{k,t}, i_{k,t-1})$	0.38*	0.34
$E[-e]$	-0.01	-0.10
$prob(-e > 0)$	0.05	0.02

Moments Conditional on e<0		
Moment	Data	Model
$\rho(i_l, -e)$	0.74*	0.96
$\rho(i_l, i_k)$	0.37*	0.39

Business Cycle Correlations		
Moment	Data	Model
$\rho(i_l, gdp)$	0.00	0.06
$\rho(-e, gdp)$	0.28*	0.71
$\rho(i_k, gdp)$	0.47*	0.94

TFP vs. ξ Moments		
	Data	Model
$\rho(xsrho_{i_l,e}, \xi)$	0.64*	0.92
$\rho(\%raise, \xi)$	0.59*	0.47
$\rho(xsrho_{i_l,e}, TFP)$	0.48*	0.04
$\rho(\%raise, TFP)$	0.25	0.32

Table V: Firm Level Facts. The table gives firm level moments. We compute the relevant moment for the entire panel of firms and then take a median across firms. We use our simulated panel of data (Model column) and Compustat (Data column). We normalize the series by total book assets. * indicates significance at 5% level. We use notation from the model: $-e$ represents external finance (negative of payouts), i_l liquidity accumulation, i_k investment in physical capital, and l liquid balances. We normalize each series by total assets except investment which is normalized by physical captial k.

<div align="center">

Firm Level Moments

Moment	Data	Model
$E[l]$	0.15	0.03
$\sigma(i_l)$	0.10	0.07
$E[i_k]$	0.06	0.07
$\sigma(i_k)$	0.07	0.15
$\rho(i_{k,t}, i_{k,t-1})$	0.29*	0.21
$E[-e]$	-0.02	-0.11
$\sigma(-e)$	0.17	0.17
$\rho(i_l, -e)$	0.18*	0.49
$\rho(i_l, i_k)$	-0.06*	0.12
$\rho(-e, i_k)$	0.20*	0.80
$\%raise$	0.43	0.16

</div>

Table VI: This table compares the model with stochastic costs to a model with a constant cost of external finance (restricted model). We compare the moments across models as well as the estimated parameter values. * indicates significance of parameter values. The bottom panel formally tests the restrictions and rejects the restricted model.

Aggregate Moments

Moment	Data	Model	Restricted Model
$E\left[\frac{l}{l+k}\right]$	0.11	0.03	0.01
$\sigma\left(i_l\right)$	0.01	0.01	0.00
$\sigma\left(i_k\right)$	0.01	0.02	0.03
$\sigma\left(-e\right)$	0.03	0.03	0.03
$\rho(i_l, -e)$	0.60*	0.70	0.28
$\rho(i_k, i_l)$	0.12	0.18	0.34
$\rho(i_k, -e)$	0.46*	0.84	0.99

Model Parameters

Parameter	Model	Restricted Model
a	0.26*	0.25*
λ_1	0.15*	0.17*
λ_2	0.0004*	0.0005*
σ_η	2.01*	—
γ	0.22*	—

Formal Test of Restricted Model

Symbol	P-value
$JT_{rest} - JT \sim \chi^2(2)$	0.00

Test of over-identifying restrictions

Table VII: This table presents the diff in diff results to estimate the effect of a high cost of external finance on investment. * indicates statistical significance at 10%. Standard errors are clustered by firm.

Regression: $\left(\frac{i_k}{k}\right)_{n,t} = \alpha_t + \beta X_{n,t} + \delta_1 1_{(z_{n,t}>0)} + \delta_2 1_{(z_{n,t}>0),(\rho^n_{(i_l,e)t}<0)} + \varepsilon_{n,t}$

Variable	Model	Data	Data (smallest 50%)	Data (smallest 25%)
Capital stock	-0.3	0.2*	0.4*	0.4
Liquid assets	-0.1	-0.2*	-0.2*	-0.1*
Productivity	0.6	0.8*	0.5*	0.4*
Lagged investment		0.5*	0.3*	0.2*
High Prod	0.9	0.7*	0.9*	0.9*
High Prod, High Cost	**-0.5**	**-0.2***	**-0.4***	**-0.7***
Time fixed effects	Y	Y	Y	Y

Table VIII: This table estimates the cost of external finance in the model using regressions of cost measures on $\rho^{xs}_{i_l,e}$, the cross sectional correlation between liquidity accumulation and external finance, and, $\%raise$, the percentage of firms raising external finance. We measure the cost of external finance in two ways: first, as the average cost paid per dollar of external finance raised, $E_N\left[\frac{\phi_e}{e}\right]$, and second as the level of the stochastic cost series ξ.

Infering the Cost of External Finance via Regressions

y	Constant	$\rho^{xs}_{i_l,e}$	$\%raise$	R^2
$E_N\left[\frac{\phi_e}{e}\right]$	0.023***	-0.013***	-0.003***	93.5%
ξ	0.00	-0.92***	-0.30***	94.3%

Table IX: This table gives correlations of our estimated cost of external finance estimated using the cross-section of firm policies, ξ, with three other empirical proxies for the cost of external finance: the Moody's BaaAaa default spread, the index of tightening lending standards, and the consumer sentiment index. The final row in each panel gives the correlation with an average of the three proxies. When taking the average, we first normalize each series to have zero mean and unit variance.

Correlation of estimated cost with:	
Default spread	0.67*
Index of lending standards	0.58*
Sentiment index	0.72*
Average of three	0.77*

Correlation of $\rho_{i_l,e}^{xs}$ with:	
Default spread	0.64*
Index of lending standards	0.58*
Sentiment index	0.50*
Average of three	0.66*

Correlation of % raise with:	
Default spread	0.59*
Index of lending standards	0.42*
Sentiment index	0.79*
Average of three	0.73*