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ABSTRACT

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July 31, 2014

Abstract

Heterogeneously risk-averse individuals who lack access to formal insurance build and use relationships with each other to manage risk. I show that equilibrium relationships under pairwise matching and endogenous group size are determined by the mean-variance trade-off across differentially risky productive opportunities, though output distributions have infinitely-many nonzero moments. I show that the need to manage risk informally influences firm structure and entrepreneurship, and policymakers must account for this. A risk-reduction policy which ignores the equilibrium response of informal institutions may cause the least risk-averse to abandon their roles as informal insurers, exacerbating inequality and hurting the most risk-averse.

1 Introduction

The goal of this paper is to develop a theory of the equilibrium formation and structure of the relationships which poor, risk-averse people build with each other, when they lack access to formal risk management tools. This theory enables us to think rigorously about the emergence and evolution of informal insurance institutions. Existing risk-sharing literature has focused largely on analyzing the insurance agreement reached by a fixed, isolated group of individuals, or by individuals who match with a fixed probability, which precludes an understanding of what groups might actually coexist in the first place. How does the network shape that emerges depend on the alternative shapes the network could have assumed? Endogenizing the structure of informal insurance not only yields insights into the correspondence of different economic environments with different relationship compositions and network shapes, but also sheds light on the connections

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between informal insurance and income inequality, entrepreneurship, and the structure of informal firms.

I develop a model inspired by a lively informal insurance literature.[1] Risk imposes a serious burden on the poor, and the desire to manage risk evidently influences many dimensions of the lives of poor individuals. For example, Rosenzweig and Stark (1989) show that daughters of more risk-averse farmers are married to more distant villages, to minimize the correlation between farming incomes. Stiglitz (1974) suggests that sharecropping arises because landowners layer insurance provision on top of incentives for tenant farmers, and Ackerberg and Botticini (2002) find evidence that heterogeneously risk-averse tenant farmers and landlords in medieval Tuscany strategically formed sharecropping relationships based on differing risk attitudes. Thus, the poor use complex relationships with each other in a variety of creative ways when they lack access to formal insurance and credit institutions.

The model has the following key elements. Risk-averse individuals with exponential utility work together to be productive.[2] For example, in an agricultural village, some individuals own land but would prefer not to farm it themselves, while other, landless individuals have both the willingness and the skill to farm. Alternatively, an investor and an entrepreneur, or two entrepreneurs with different sets of skills and resources, must work together to launch a business. Matching is assumed to be pairwise in the benchmark model, but this is later relaxed to allow group size itself to be endogenous.

There are two key types of heterogeneity: heterogeneity of preferences, and heterogeneity of technology. Individuals vary in their degree of constant absolute risk aversion, and a matched group chooses a joint income distribution by choosing from a set of differentially risky portfolio options, or "projects". Projects with higher expected return come at the cost of higher variance of return, and members of a group share the realized return of their chosen project according to a rule determined *ex ante*. For example, investors seeking to form joint ventures are presented with business proposals of differing riskiness, and a landowner and her tenant farmer face a spectrum of portfolios of crops, land plots, inputs, and farming methods, which each yield different profit distributions. I allow for a large class of symmetric and skewed return distributions. Importantly, distributions may have infinitely many higher order moments (as opposed to just two nonzero moments, as in the case of the normal distribution).

The results reveal that accounting for the endogeneity of informal institutions is essential for policymaking. I show that a policy which reduces aggregate risk is a strict Pareto improvement if informal insurance is assumed to stay fixed. However, accounting for the endogenous network

[1] For background and institutional details, see Alderman and Paxson (1992), Morduch (1995), Dercon (2004), Fafchamps (2008). Empirical risk-sharing papers include Townsend (1994), Dercon and Krishnan (2000), Fafchamps and Lund (2003), and Mazzocco and Saini (2012). Theoretical papers on informal insurance include Ligon et. al. (2002), Genicot and Ray (2003), and Bloch et. al. (2008). Theoretical and empirical papers on risk-sharing networks include Bramoulle and Kranton (2007), Fafchamps and Gubert (2007), and Ambrus et al. (2013).

[2] I show that this model is substantively equivalent to one in which individuals can produce on their own: each chooses her own income distribution, and a matched group shares the pooled realizations. A simplified intuition for the equivalence is that even if individuals can produce on their own, groups are effectively choosing their joint income distribution when they match. Please refer to Online Appendix OA.1 for the details.

response changes this analysis entirely. Those the policy is particularly intended to help, the most risk-averse, are in fact particularly harmed: the change in the risk environment incentivizes the less risk-averse agents to abandon their roles as informal insurers of the more risk-averse, in favor of entrepreneurial pursuits with other less risk-averse agents. This forces the most risk-averse agents to match with each other instead, leaving them strictly worse off through *two* channels. First, they've lost insurance through weakened consumption-smoothing–each member wants her consumption to depend as little as possible on income realization, and nobody is willing to absorb the volatility to provide this. In other words, the gains from trade between less and more risk-averse agents are lost. This is compounded for the more risk-averse by an additional channel: the loss of consumption-smoothing means that the group must rely on income-smoothing to manage risk. But this means selecting projects with particularly low expected return, so that the more risk-averse agents are unable to take advantage of the policy's reduced variance of the higher mean, entrepreneurial projects.

Notably, this is the case *despite* the ability of individuals to commit *ex ante* to a return-contingent sharing rule. This presents an interesting contrast to Attanasio and Rios-Rull (2000), who model informal insurance as being fundamentally about limited commitment, where members in a given group punish those who renege on the sharing rule by consigning them to autarky. They find that aggregate risk reduction resulting from the strengthening of formal insurance may decrease welfare by making autarky more appealing, thereby lessening the punishment for reneging. My results show that studying informal institutions by focusing on the evolving relationships which comprise them yields insights which may be too hastily attributed to a weaker contracting environment.[3]

The results also relate the equilibrium structure and composition of relationships to the set of portfolios in the economy, and to income inequality. To show this formally, I derive a transferable utility representation of the model. Since agents are heterogeneous in risk attitudes, a single unit of output generates a different level of utility for one agent than it does for another. However, I show that *expected utility* is transferable, and seek conditions under which the total certainty-equivalent of a matched group exhibits supermodularity and submodularity in risk attitudes. This approach is inspired by Legros and Newman (2007) and Schulhofer-Wohl (2006), who both observe that models with non-transferable utility may admit a transferable utility representation.[4] They show this for a simpler risk-sharing problem where agents face an exogenous risk and can commit *ex ante* to a return-contingent sharing rule, but are not able to choose what risk they face.

The key observation is that the total certainty-equivalent of a matched group is the product of the cumulant-generating function of the return distribution of the group's chosen project, and the group's representative risk tolerance. I show that, although the return distributions may have infinitely many nonzero moments, unique assortative matching in risk attitude is determined by the

[3] Although the intent of the model is to focus on the endogeneity of informal insurance and shut down commitment problems, I discuss the effect of introducing limited commitment in Online Appendix OA.2.

[4] Legros and Newman (2007) develop a more general method to approach matching problems when utility is not transferable.

first two moments alone, and does not depend on any aspect of the distribution of risk attitudes in the population. Specifically, if the ratio of the mean return to the standard deviation across all possible projects is increasing in the mean (that is, higher mean projects have a higher ratio), then unique positive assortative matching results in the pairwise equilibrium, while if the ratio is decreasing in the mean, unique negative assortative matching results in the pairwise equilibrium. This ratio is known as the Sharpe ratio (Sharpe (1966)); its reciprocal is known as the coefficient of variation.

But when is the Sharpe ratio increasing or decreasing in the mean return? Think of the function describing the variance of project return when the mean return is p as the "cost" of obtaining an expected return p. Then the Sharpe ratio is increasing (decreasing) in mean return if and only if the marginal cost function is concave (convex) in the mean p. Intuitively, equilibrium matching is driven by the trade-off between preference for a similar partner when choosing risk *ex ante*, and preference for a dissimilar partner when sharing risk *ex post*, and the curvature of the marginal cost function captures this trade-off. I show that when the ability to share risk *ex post* is shut down, e.g. because the government passes a wage law which fixes sharing rules, positive assortative matching is always the unique equilibrium. This is supported by the experimental finding of Attanasio et al. (2012), who find that individuals match positive assortatively in risk type when they know each other's types, and when they are able to choose what risky gamble to face, but must equally share returns. By contrast, when the ability to choose risk *ex ante* is shut down, negative assortative matching is always the unique equilibrium. This coincides with the theoretical finding of Legros and Newman (2007), Schulhofer-Wohl (2006), and Chiappori and Reny (2006), and is supported by the empirical finding of Ackerberg and Botticini (2002) discussed earlier that landowners and farmers match negative assortatively.

I then show that for a given population of individuals, income inequality is maximized when individuals match positive assortatively, and minimized when individuals match negative assortatively. In fact, the distribution of wealth when individuals are matched positive assortatively is a mean-preserving spread of the negative assortative case. Thus, the model predicts that entrepreneurship in developing countries goes hand-in-hand with high income inequality, and that this equilibrium arises in a risk environment where the marginal variance cost of projects is concave in mean return, that is, the Sharpe ratio is increasing in mean return across the spectrum of available portfolios.

Finally, I allow group size itself to be endogenous. Using coalitional stability as the equilibrium criterion, I show that, under the condition yielding unique positive assortative matching in the pairwise case, whole-group matching (maximal connectedness) is the unique equilibrium, while under the condition yielding unique negative assortative matching in the pairwise case, negative assortative, pairwise matching (minimal connectedness) continues to be the unique equilibrium. Thus, extremal network shapes are tied to extremal within-group composition, which yields intriguing predictions for the structure of informal firms. The result provides an interesting contrast to Genicot and Ray (2003), who find that, under limited commitment, whole-group matching can never be sustained. In their model, because individuals cannot choose what risk they face and cannot commit to a sharing rule, the marginal benefit of an additional member is eventually negative: an individual's expected income is independent of group size, but she becomes increasingly likely to bear a partner's downside risk as the group grows. In my model, it is possible for the marginal benefit of an additional member to be positive even for arbitrarily large groups, since the trade-off is slightly different: adding a member allows the group to take on additional risk because of the increased ability to share risk *ex post*, which the group can leverage into taking up a higher mean, higher variance project. However, now the realized surplus must be divided across more people. I show that the curvature of the marginal variance cost function again determines which force outweighs.

In the next section, I set up the benchmark model with pairwise matching and describe the main matching results. I then analyze a hypothetical policy and show how calculation of the welfare impact changes drastically after accounting for the response of the network. Following this, I extend the model and allow group size itself is endogenous. Then, I discuss falsifiability of the theory, and show support for the theory in the existing empirical literature. Finally, I conclude. Technical details are relegated to Appendix A and the Online Appendix (OA).

2 The Model

In the benchmark model, matching is restricted to be pairwise. In Section 5, I relax the constraint of partnerships and analyze this model when the sizes of the groups are also endogenous.

2.1 Setup

The population of agents: the economy is populated by two groups of agents, $G1$ and $G2$, where $|G1| = |G2| = Z$, $Z \in \{2, 3, 4, ...\}$.[5] (The case $|G1| \neq |G2|$ does not substantively change the results: the most risk-averse excess individuals simply remain unmatched. See Online Appendix OA.3 for details.) Agents differ in their Arrow-Pratt degree of *absolute risk aversion* r, where an agent i of type r_i derives utility $u_i(x) = -e^{-r_i x}$ from consuming x units of output. Let $r_i > 0$: each individual is risk-averse to some extent. Define agent r_i's degree of *risk tolerance*: $R_i \equiv \frac{1}{r_i}$.[6]

No assumptions are imposed on the distributions of risk preferences within each group, or across groups.

The risky environment: a spectrum of risky projects is available, with return distributions

[5] Matching is assumed to be across two groups rather than within a single group because assortative matching patterns are well-defined under the former, not because it is necessary for the results. To see this, suppose that matching is within a group of four people, $\{r_1, r_2, r_3, r_4\}$, ordered from least to most risk-averse. Then there are two possible positive assortative matchings: $\{(r_1, r_2), (r_3, r_4)\}$, and $\{(r_1, r_3), (r_2, r_4)\}$.

[6] The model can account for kinship and friendship ties. For example, individuals linked by these ties are more likely to know each other's risk types, and are more likely to trust each other, or to monitor and discipline each other. Hence, an individual might first identify a pool of feasible risk-sharing partners, where this pool would be shaped by kinship and friendship ties, due to good information and commitment properties. Individuals would then choose their risk-sharing partners from these pools based on risk attitude.

parameterized by $p \in \Pi \subseteq \mathbb{R}_0^+$. A project p returns output Y_p, a random variable described by:

$$Y_p = p + V(p)^{\frac{1}{2}} Y$$

where Y is a random variable with a well-defined cdf $F_Y : \mathbb{R} \to [0,1]$, and $E(Y) = 0,\ V(Y) = 1$. Note that this allows for a large class of possible distributions for project returns, which may be symmetric or skewed, including the Normal, Laplace, Logistic, Student, and generalized extreme value (e.g. Gumbel) distributions.[7]

Hence:

$$\begin{aligned} E(Y_p) &= p \\ V(Y_p) &= V(p) \end{aligned}$$

The function $V : \Pi \to \mathbb{R}_0^+$ maps $p \in \Pi$ to a nonnegative real number, and describes the variance of output of a project with expected output p. Assume that $\Pi = \mathbb{R}_0^+$, so that there exists an undominated portfolio achieving any $p \geq 0$.

Let $V(\cdot)$ be thrice-differentiable, and:

1. $V(0) = 0$, $V(p) > 0$ for $p > 0$.
2. $V'(0) = 0$, $V'(p) > 0$ for $p > 0$.
3. $V''(p) > 0$.

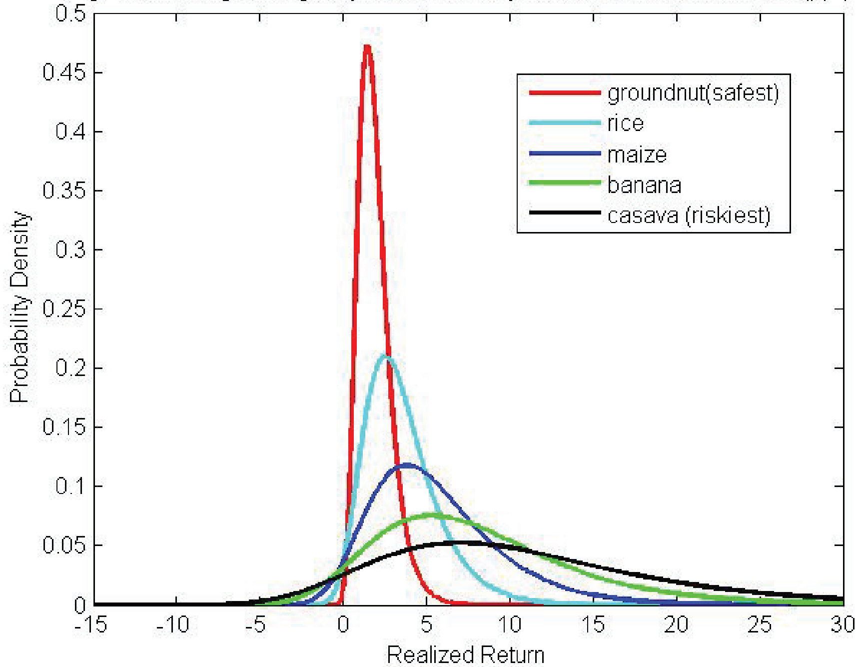
The first property ensures that variance is nonnegative, and that an action which returns 0 with certainty exists (e.g. "do nothing")[8]. The second property, that projects with higher mean return also have higher variance of return, is without loss of generality. To see this, observe that, if given the choice between two projects with the same mean but differing variances, any agent with concave utility will always choose the project with lower variance. Tracing out the set of undominated projects shows that $V'(p) > 0$. Finally, the third property ensures an interior solution for project choice for any agent r.

A subset of the risky projects available might therefore look something like this:

[7] These distributions are also known as location-scale families.

[8] Imagine instead that the variance of a project with mean p were given by $V(p) + c$, where $V(0) = 0$. Then both the level c and the curvature $V'(p)$ affect an individual's project choice. However, the marginal variance difference between two projects p_1 and p_2 is $V(p_1) - c - V(p_2) + c = V(p_1) - V(p_2)$. Thus, I simply assume $c = 0$, which has the effect of a normalization.

Fig. 1: Choosing Among Projects Which Vary in Risk Structure: Gumbel, $V(p)=p^4$



Production: Assume that any project p requires the partnership of two agents, one from $G1$ and one from $G2$. For example, a landowner and a tenant farmer must work together to choose inputs and grow crops, two adults marry to form a productive household, an investor funds an entrepreneur with a business proposal, two entrepreneurs with different skill sets and time constraints work together to form a successful joint venture, and so on. A matched pair (r_1, r_2) jointly selects a project. (See Appendix 1 for a proof that this framework is equivalent to one where each partner individually chooses a project and both share the pooled returns.)

All matched pairs face the same spectrum of projects, each agent can be involved in at most one project, and there are no "project externalities". That is, one pair's project choice does not affect availability or returns of any other pair's project.

To focus on the impact on equilibrium matching of the trade-off in *ex ante* and *ex post* risk management across partnerships of different risk compositions, there is no moral hazard in this model, although it is straightforward to allow for observable, contractible actions. Please refer to Wang (2013b) for an explicit treatment of moral hazard and informal insurance in an endogenous matching problem.

Information and commitment: all agents know each other's risk types and the risk environment.

A given matched pair (r_1, r_2) undertaking project p_{12} observes the realized output $y_{p_{12}}$ of their partnership, and is able to commit *ex ante* to a feasible return-contingent sharing rule $s : \mathbb{R} \to \mathbb{R}$ (there is no limited liability). Denote r_2's share of realized output by $s(y_{p_{12}})$. Feasibility implies that the income r_1 receives must be less than or equal to $y_{p_{12}} - s(y_{p_{12}})$. Since all agents have monotonically increasing utility, r_1's share will be equal to $y_{p_{12}} - s(y_{p_{12}})$.

The equilibrium: An equilibrium is[9]:

1. A match function $\mu : \mathbb{R}^+ \to \mathbb{R}^+$, mapping each agent in group 1 to a single agent in group 2. Thus, r_1's partner is denoted by $\mu(r_1)$, and $\mu(\cdot)$ assigns distinct members of group 1 to distinct partners in group 2.

   Moreover, the matching pattern described by $\mu(\cdot)$ must be stable. It must be that no agent is able to propose a feasible project and sharing rule to an agent not matched to her under μ, such that both agents are happier when matched with each other in this proposed arrangement than they are with the partners assigned by μ (no blocks).[10]

2. A set of sharing rules and project choices, one sharing rule and project for each matched pair, such that no pair can choose a different sharing rule and/or a different project which leaves both partners weakly better off, and at least one partner strictly better off. In other words, the sharing rule and project chosen by a matched pair must be optimal for that pair.

Matching patterns: Let $G_j = \{r_j^1, r_j^2, ..., r_j^Z\}$, $j \in \{1, 2\}$, ordered from least to most risk-averse. Under "positive assortative matching" (PAM), the i^{th} *least* risk-averse person in $G1$ is matched with the i^{th} *least* risk-averse person in $G2$: $\mu(r_1^i) = r_2^i$, $i \in \{1, ..., Z\}$. So, (r_1^1, r_2^1) is a matched pair.

On the other hand, under "negative assortative matching" (NAM), the i^{th} *least* risk-averse person in $G1$ is matched with the i^{th} *most* risk-averse person in $G2$: $\mu(r_1^i) = r_2^{Z-i+1}$, $i \in \{1, ..., Z\}$. So, (r_1^1, r_2^Z) is a matched pair.

To say that the unique equilibrium matching pattern is PAM, for example, is to signify that the only μ which is stable under optimal within-pair sharing rules and projects is the match function which assigns agents to each other positive assortatively in risk attitudes.

The next section discusses the results for this model, with technical details relegated to the Appendix.

3 Results

The first step is to identify a transferable utility representation for this model. The heterogeneity of risk-aversion makes this a model of matching under nontransferable utility: one unit of output yields utility $u_1(1) = -e^{-r_1}$ for an agent with risk aversion r_1, but utility $u_2(1) = -e^{-r_2} \neq u_1(1)$ for an agent with risk aversion r_2. Thus, an individual evaluates potential partners based not only on how much output they can produce together, as in the standard case, but also on how happy the partner is with a given level of output–a partner who produces a lot but then demands a large transfer might be less desirable than a partner who produces less but is satisfied with little.

[9] Existence is assured by Kaneko (1982).

[10] Individual rationality holds, as individuals cannot produce on their own.

Proposition 1 *Expected utility is transferable in this model. Denote a matched pair by* (r_1, r_2) *and their chosen project by* $p^*(r_1, r_2)$*. Let* $CE(r_1, r_2, p^*(r_1, r_2))$ *describe the certainty-equivalent of a matched pair in equilibrium; it is twice continuously differentiable in each argument. Then:*

$$\begin{aligned}\frac{\partial CE(r_1, r_2, p^*(r_1, r_2))}{\partial r_1 r_2} &> 0 \Leftrightarrow \text{ unique } PAM \\ \frac{\partial CE(r_1, r_2, p^*(r_1, r_2))}{\partial r_1 r_2} &< 0 \Leftrightarrow \text{ unique } NAM\end{aligned}$$

Corollary 1 *The equilibrium matching maximizes the sum of certainty-equivalents, and is Pareto efficient.*

A sketch of the proof of Proposition 1 provides a useful understanding of the matching problem, but all technical details for the proofs of the proposition and corollary are relegated to Appendix A.1.

First, we characterize the optimal project and sharing rule chosen by a matched pair (r_1, r_2). Suppose r_1 and r_2 have already selected a project p. Let $v \in \mathbb{R}$ parameterize the division of surplus between the two partners. Then the program below characterizes the equilibrium sharing rule, given v. (Symmetry implies that the program could also have been set up fixing r_1's expected utility.)

$$\max_{s(y_p)} \int_{-\infty}^{\infty} -e^{-r_1[y_p - s(y_p)]} f(y_p|p) dy_p \quad s.t.$$

$$\int_{-\infty}^{\infty} -e^{-r_2 s(y_p)} f(y_p|p) dy_p = -e^{-v}$$

Solving this program shows that the optimal sharing rule is linear, where the more risk-averse partner's transfer is less dependent on realized output y_p. That is, if $r_1 < r_2$ (so r_2 is more risk-averse), then r_2 receives a share $\frac{r_1}{r_1+r_2} < \frac{1}{2}$ of realized output, plus a constant. The division of surplus v affects only the constant part of the total transfer.

Since r_2's expected utility is fixed at v, it's clear that, for each v, both members "agree" on project choice–they want to maximize surplus, given the division.[11] The optimal project can thus be characterized by maximizing r_1's objective function under $s^*(y_p; v)$. Crucially, this shows that there exists a *unique* optimal project for the pair, which depends only their risk tolerances and not on the division v.

We can now express the certainty-equivalent for each member of the pair, given v:

$$\begin{aligned}CE_{r_1}(v) &= -\left(\frac{1}{r_1} + \frac{1}{r_2}\right) \log \int_{-\infty}^{\infty} e^{-\frac{r_1 r_2}{r_1+r_2} y_p} f(y_p|p^*(r_1, r_2)) dy_p - \frac{1}{r_2} v \\ CE_{r_2}(v) &= \frac{1}{r_2} v\end{aligned}$$

[11] Wilson (1968) showed that, as a consequence of CARA utility, any pair acts as a syndicate and "agrees" on project choice.

This makes it clear that r_2 gains one unit of certainty-equivalent at the expense of exactly one unit of certainty-equivalent for r_1. Since the certainty-equivalent is just a monotonic transformation of expected utility, this shows the transferability of expected utility. Hence, conditions under which $CE : G1 \times G2 \to \mathbb{R}$ exhibits increasing and decreasing differences in risk types are necessary and sufficient for unique PAM and NAM, respectively. Proposition 2 gives these conditions.

Proposition 2 *Recall that $V(p)$ describes the variance cost of a project with mean return p. Then $M(p) \equiv V'(p)$ describes the marginal variance cost of a project with mean return p.*

(a) A sufficient condition for PAM to be the unique equilibrium match is $M''(p) < 0$ for $p > 0$ (concavity).

(b) A sufficient condition for NAM to be the unique equilibrium match is $M''(p) > 0$ for $p > 0$ (convexity).

(c) A sufficient condition for any match to be sustainable as an equilibrium is $M''(p) = 0$ for $p > 0$ (linearity).

Note that while the curvature of marginal variance cost is sufficient for assortative matching in a known population of individuals, it is both necessary and sufficient to guarantee assortative matching for any possible population $G1 \times G2 \subset \mathbb{R}^+ \times \mathbb{R}^+$.

In addition to being independent of any assumption about the risk types in the economy, note that Proposition 2 tells us that equilibrium matching depends only on the relationship between the mean and the variance of the risky project returns, even though the return distributions may have infinitely many nonzero higher order moments.

The following corollary provides intuition for this result by relating the curvature of the marginal variance function to properties of the risk environment.

Corollary 2 *Let $SR(p) = \frac{p}{V(p)^{\frac{1}{2}}}$ denote the ratio of a project's mean return to its standard deviation for $p > 0$. Then:*

(a) $SR'(p) > 0 \ \forall p > 0$ iff $M''(p) < 0 \ \forall p > 0$ (PAM)

(b) $SR'(p) < 0 \ \forall p > 0$ iff $M''(p) > 0 \ \forall p > 0$ (NAM)

Recall that $SR(p)$ is the Sharpe ratio of a project p (Sharpe 1966), while its reciprocal is the coefficient of variation. The Sharpe ratio is unitless (dimensionless), and thus a convenient tool for comparing portfolios.

As the proof of Proposition 2 involves a novel approach to portfolio problems with higher order moments, I sketch it here. Details for the proposition and the corollary can be found in Appendix A.2.

Define the *representative risk tolerance* of a matched pair (r_1, r_2):

$$\begin{aligned} R &= R_1 + R_2 \\ &= \frac{1}{r_1} + \frac{1}{r_2} \end{aligned}$$

Then observe that the sum of certainty-equivalents of the pair, $CE(r_1, r_2)$, depends only on each individual's risk tolerance through the representative risk tolerance of the pair:

$$CE(r_1, r_2) = -R \log \int_{-\infty}^{\infty} e^{-\frac{1}{R}y_p} f(y_p|p^*(R))dy_p$$

Moreover, $CE(r_1, r_2)$ exhibits supermodularity (submodularity) in r_1, r_2 precisely when $CE(R)$ exhibits convexity (concavity) in R.

The key is to observe that $CE(R)$ is the product of $-R$ and the *cumulant-generating function* $K_{Y_p}(t)$ of the distribution of project returns Y_p. The cumulant-generating function (cgf) is the log of the moment-generating function (mgf), and the first two cumulants of any distribution are the mean and the variance.

Using the series expansion of the cgf, where $k_n(y)$ is the n^{th} cumulant of Y, we see that:

$$CE(p, R) = p - \sum_{n=2}^{\infty} \frac{(-1)^n}{n!R^{n-1}} V(p)^{\frac{n}{2}} k_n(y)$$

A pair with representative risk tolerance R chooses p to maximize $CE(p, R)$. Thus, the optimal project balances the *marginal benefit* of increased mean with the *marginal cost* of higher "generalized variance" (the aggregated higher-order cumulants which form a polynomial in $V(p)$), where a given level of "generalized variance" is less costly for more risk-tolerant pairs. Hence, the curvature of the marginal variance cost $M(p)$ is at the heart of the curvature of $CE(R)$ and of assortative matching.

Corollary 2 describes how the curvature of $M(p)$ shapes the set of portfolios available to a population, providing further intuition for the connection between $M(p)$ and equilibrium matching. A less risk-averse person enjoys the premium a more risk-averse partner is willing to pay her to smooth his consumption, but acting as the informal insurer and bearing her partner's risk forces the pair to choose a safer project with lower expected return. If she instead matches with a less risk-averse partner, she forgoes the premium from providing insurance, but she and her partner are able to undertake a riskier project with higher expected return. Whether a less risk-averse individual prefers to be an informal insurer or an entrepreneur, and thus whether negative or positive assortative matching results, depends on whether partnerships generate the most value through insurance or production. When the ratio of expected return to standard deviation is lower for riskier projects, the less risk-averse will prefer to be informal insurers; when the ratio is higher for riskier projects, the less risk-averse will prefer to be entrepreneurs.

Thus, the key trade-off is between sharing a given risk, and choosing what risk to share. Propositions 3 and 4 demonstrate this rigorously by shutting down each channel in turn and characterizing the equilibrium match. I show that, when individuals face a fixed risk and partner choice affects only how that risk is shared, negative assortative matching is the unique equilibrium, and the less risk-averse act as informal insurers. When individuals face a fixed sharing rule and partner choice affects only what risk is faced, positive assortative matching is the unique equilibrium, and the less risk-averse are entrepreneurs.

Proposition 3 *Suppose that all agents in $G1$ draw income iid from a distribution M, and all agents in $G2$ draw income iid from a distribution W, so that all possible pairs face the same joint income distribution. Once matched, agents can commit ex ante to a return-contingent sharing rule. Then the unique equilibrium matching is NAM.*

See Online Appendix OA.4 for the proof.

This coincides with the theoretical finding of Legros and Newman (2007), Schulhofer-Wohl (2006), and Chiappori et al. (2006), and with the empirical finding of Ackerberg and Botticini (2002), who find that heterogenously risk-averse landlords and tenant farmers who chose sharing rules but who couldn't choose which crop portfolios to grow matched negative assortatively in medieval Tuscany.

Proposition 4 *Suppose that the slope of the sharing rule $s(R_p) = a + bR_p$ is fixed. For example, a wage law requires a 50-50 split of output. However, a matched pair is able to choose a risky project p, where $Y_p = p + V(p)^{\frac{1}{2}}Y$ as in the benchmark model. Then the unique equilibrium matching is PAM.*

See Online Appendix OA.5 for the proof.

When the dependence of each agent's consumption on realized return is fixed exogenously, individuals can no longer trade comparative advantage: more risk-averse agents are unable to offer the less risk-averse higher premiums to bear their risk. Hence, partnerships are motivated by production, and positive assortative matching is the unique equilibrium. This aligns with the experimental finding of Attanasio et al. (2012), who find that agents match positive assortatively in risk preference when they can choose the riskiness of the gamble they face, but the sharing rule is fixed at equal division.

Finally, the last two results demonstrate that an understanding of the formation of informal relationships yields insights into the broader economy.

Proposition 5 *Let $p^*(r_i, \mu(r_i))$ denote the mean return of the project chosen by a matched pair $(r_i, \mu(r_i))$. Then:*

(a) If $M''(p) < 0$ for $p > 0$, then $p^*(r_i, \mu(r_i))$ is convex in representative risk tolerance $R(r_i, \mu(r_i))$.

(b) If $M''(p) > 0$ for $p > 0$, then $p^*(r_i, \mu(r_i))$ is concave in representative risk tolerance $R(r_i, \mu(r_i))$.

Corollary 3 *For a given population of individuals $G1, G2$ and a given risk environment described by $V(p)$, income inequality, measured by the dispersion of and gap between lowest and highest expected incomes, is maximized when individuals are matched positive assortatively, and minimized when individuals are matched negative assortatively.*

See Appendix A.3 for the proofs.

In words, Proposition 5 tells us that positive (negative) assortative matching corresponds to the convexity (concavity) of the mean joint incomes of matched pairs in the representative risk tolerances of those matched pairs. Crucially, this result suggests a falsifiability test for the theory which requires observing only the *actual* matching and *mean* incomes, and not any data on higher-order moments (such as variance of income), or any counterfactual matching data (such as how agents would have behaved had they been matched with other partners). Falsifiability is discussed in detail in Section 6.

Further, Corollary 3 demonstrates that environments which foster informal entrepreneurship (PAM) also generate higher income inequality, while environments in which informal insurance is strong (NAM) have less income inequality. In particular, the poorest (those below the median) are poorer and the richest (those above the median) are richer under PAM than NAM.

4 Policy

The world's very poor largely depend on agriculture for subsistence. An extensive body of literature documents the economic importance of agricultural sector growth in developing countries (e.g. Cervantes-Godoy and Dewbre (2010)). Stabilization of crop prices is frequently proposed as a tool for alleviating the substantial risk burden shouldered by poor, risk-averse farmers (Dawe (2001)). Notable examples include maize, sorghum, and rice in Venezuela, the Andean Price Band system between Colombia, Ecuador, and Venezuela, bananas and grains in Ethiopia, and many others (Knudsen and Nash (1990), Minot (2010)).

The theory presented in this paper provides a useful framework for evaluating such stabilization policies. I abstract from possible governmental and other frictions in policy implementation, in order to study how accounting for the equilibrium response of informal institutions might substantially alter policy analysis even in an otherwise ideal world.

Suppose a benevolent government contemplates solutions for its many poor farmers, who face an unforgiving risk environment, and lack access to formal insurance. Because a slight increase in profitability of crop portfolio comes at the cost of extremely high variance of profit, farmers are trapped into growing crops that are safer but not very profitable–they forgo innovations for less profitable, traditional methods. To encourage the farming of crops with higher expected profitability, the government places price bands of the form $[p_L, p_H]$ on each crop's price. If the world price of a crop happens to fall within this band, that is the price the farmer faces. However, if the world price falls below the price floor, the farmer is guaranteed to receive p_L, and if the world price is above the price ceiling, the farmer faces p_H. The marginal impact of stabilization is largest for crops with the most volatile prices: the variance of every crop falls, but the variance of the riskiest crops falls by the largest amount. Thus, the policy leads to a change in the curvature of the marginal variance cost $M(p)$ across different crop portfolios.

A numerical example illustrates the effect of the policy concretely. Suppose there are five different crops, denoted by $i \in \{0, 1, 2, 3, 4\}$, where Q_i represents the yield, and P_i the price, of crop i. Crop 0 represents not growing at all. Since the purpose of this short exercise is simply to demonstrate how price bands may change the curvature of $M(p)$, we normalize yield risk and assume that $E(Q_i) = 1,\ V(Q_i) = 0$. Assume that an individual farmer does not affect the world price, so that P_i is independent of Q_i.

Then the mean profitability and variance of profitability for each crop is:

$$\begin{aligned} E(P_iQ_i) &= E(P_i) \\ Var(P_iQ_i) &= V(P_i)[E(Q_i)]^2 + V(Q_i)[E(P_i)]^2 + V(P_i)V(Q_i) \\ &= V(P_i) \end{aligned}$$

In other words, the mean and variance of profitability for crop i is simply the mean and variance of the price of crop i.

Suppose P_i takes on value p_i^L with probability $\theta_i \in [0, 1]$, and $p_i^H > p_i^L$ with probability $(1 - \theta_i)$. The table below describes, for each crop i, the bad price outcome (p_i^L), the good outcome (p_i^H), and the probability of the bad outcome (θ_i), which together determine the mean and variance of crop i's price ($\mu_{p_i}, \sigma^2_{p_i}$), and the Sharpe ratio $SR_i = \frac{\mu_{p_i}}{\sigma_{p_i}}$. Note that a crop with higher mean price also has higher variance of price, and that the Sharpe ratio is smaller for crops with higher expected return.

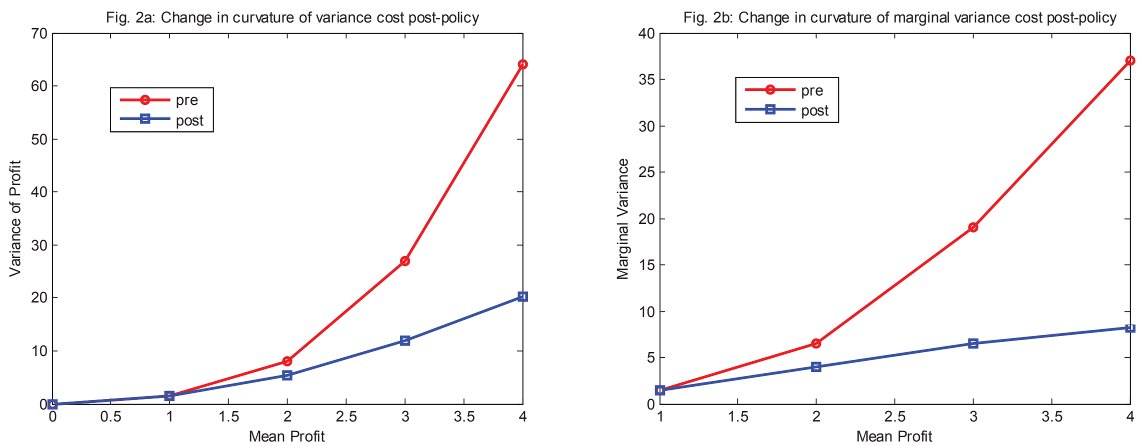
Crop	p_L	p_H	θ	μ_p	σ_p^2	$\frac{\mu_{p_i}}{\sigma_{p_i}}$
Crop 0	0	0	–	**0**	**0**	n/a
Crop 1	0	2.5	$\frac{3}{5}$	**1**	**1.5**	0.82
Crop 2	0	6	$\frac{2}{3}$	**2**	**8**	0.71
Crop 3	0	12	$\frac{3}{4}$	**3**	**27**	0.58
Crop 4	0	20	$\frac{4}{5}$	**4**	**64**	0.5

Now, suppose price bands are placed on each crop to reduce risk for farmers and encourage them to choose riskier but more profitable portfolios: the expected price of each crop is maintained, but the government guarantees a higher lower bound and a lower upper bound for each price. This causes the Sharpe ratio to be bigger for crops with higher expected return. The table below describes each crop post-policy:

Crop	p_L	p_H	θ	μ_p	σ_p^2	$\frac{\mu_{p_i}}{\sigma_{p_i}}$
Crop 0	0	0	–	**0**	**0**	n/a
Crop 1	0.02	2.47	$\frac{3}{5}$	**1**	**1.44**	0.83
Crop 2	0.35	5.3	$\frac{2}{3}$	**2**	**5.45**	0.86
Crop 3	1	9	$\frac{3}{4}$	**3**	**12**	0.87
Crop 4	1.75	13	$\frac{4}{5}$	**4**	**20.25**	0.89

Note that the mean price for each crop remains unchanged, but the variance has fallen, and has fallen more for the most volatile crops. The Sharpe ratio is now larger for crops with higher expected return.

Figures 2a and 2b below show the change in curvature of variance cost and marginal variance cost across the spectrum of crops pre- and post-policy:



Now that we've seen how stabilization of crop prices translates into a change in the curvature of $M(p)$, let us compare the welfare impacts of the risk-reduction policy in a more general environment, ignoring and then accounting for the endogeneity of informal insurance institutions.

For simplicity, assume that the returns of different crop portfolios are distributed normally. In particular, suppose that pre-policy, the profits of a crop with mean p are described by $\pi_p^{pre} \sim N(p, p^{N_1})$, $N_1 > 2$. Thus, the marginal variance cost function is $M_{pre}(p) = N_1 p^{N_1 - 1}$, which is convex in p. The government then implements a stabilization policy to differentially reduce the risk of higher mean portfolios. Post-policy, profits are described by $\pi_p^{post} \sim N(p, p^{N_2})$, where $N_2 \in (1, 2)$, so that $M_{post}(p)$ is now concave in p.[12] According to Proposition 2, the unique equilibrium match is negative assortative pre-policy, and positive assortative post-policy ($N = 2$ is the "tipping point", at which every matching pattern is stable).

Importantly, the risk-reduction policy affects level as well as curvature of risk. Note that reducing the variance of every portfolio to 0 would clearly make everyone happier: the positive impact of the change in levels outweighs any impact the change in curvature might have. However,

[12] Note that modeling $V(p)$ as a power function has one small drawback. It's a natural choice, since we want to analyze a policy that reduces the variance of every project, and particularly reduces the variance of the riskiest projects, which is captured well by "decreasing N". However, when N falls, p^N for $p \in (0, 1)$ actually increases. Hence, I simply assume that the population of risk types in $G1$ and $G2$ is such that no possible pair ever wishes to undertake a project $p \in (0, 1)$ pre-policy:

$$\frac{1}{\max(r_1)} + \frac{1}{\max(r_2)} \geq \frac{N_1}{2}$$

This is simply for convenience, and does not substantively influence the intuition or the policy analysis, since the matching results are free of any assumption on the distribution of risk types. If the above condition did not hold, the policy would be a bad idea without considering informal institutions, since individuals choosing $p < 1$ would be worse off.

policy is generally only able to achieve small reductions in risk levels before returns are affected. Moreover, we know from Proposition 2 that the composition of partnerships is driven by curvature, not levels. Thus, setting $N_1 = 2 + \varepsilon$ and $N_2 = 2 - \varepsilon$, ε small, maximizes the ratio of the change in curvature to the change in levels, and enables us to isolate the impact stemming from the endogenous response of the informal insurance network. For this example, let $\varepsilon = 0.05$.

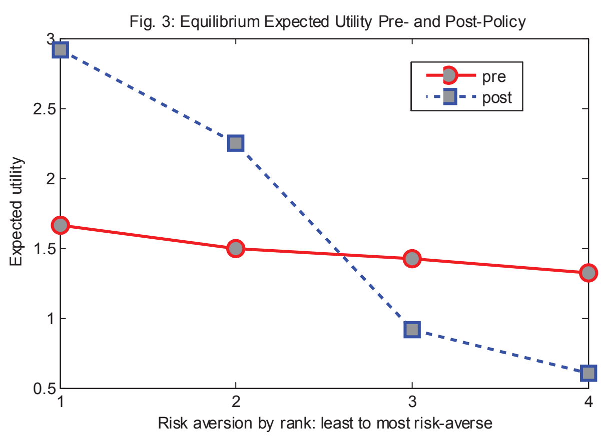
Suppose $G1 = \{0.5, 0.8, 0.9, 1\}$, and $G2 = \{1, 2, 3, 4\}$, so that the agents in the second group are more risk-averse than the first (typically believed to be the case with landowners and tenant farmers, but the matching results and intuition about the policy do not depend on which risk types are chosen). Then the effect of the policy without accounting for the response of informal institutions is described by the lemma below.

Lemma 1 *A policy which reduces the variance of every available project is a strict Pareto improvement if the composition of partnerships does not change.*

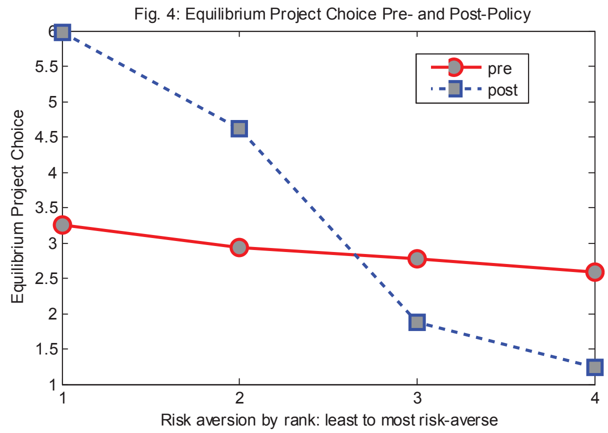
The proof is intuitive: each matched partnership can stay on the same project post-policy, or can choose a different project. If a partnership retains its original project, it is strictly better off, since the project has the same mean as it did before the policy, but a lower variance. If a partnership switches to a different project, then by revealed preference, they must be even better off facing the new project than facing the old project with decreased variance. But this means that each partnership is strictly better off.

Now, let's account for the endogenous re-formation of partnerships triggered by the policy.

What happens to the expected utility generated by the collaboration of each matched pair? The red line with circular markers and blue line with square markers in Figure 3 depict the expected utility pre- and post-policy, respectively, generated by matched partnerships, where "1" on the x-axis represents the least risk-averse pair, and "4" on the x-axis represents the most risk-averse pair. Clearly, the least risk-averse benefit at the cost of the more risk-averse:



The change in crop portfolios chosen in equilibrium is depicted in Figure 4:



Clearly, this risk-reduction policy is *not* a Pareto improvement. We see that the more risk-averse agents are worse off after implementation of the policy, purely as a result of the endogenous network response: the policy causes the least risk-averse agents to abandon their roles as informal insurers of the most risk-averse agents, in favor of entrepreneurial partnerships with fellow less risk-averse agents. The poorest, most risk-averse agents are harmed via two channels: first, they've lost their informal insurers, and this weakens their capacity to smooth consumption, which reduces their welfare. To make matters worse, because the most risk-averse agents, who are now paired with each other, have very little capacity to smooth a given risk (as neither is willing to bear the volatility to smooth her partner's consumption), they must instead manage risk by choosing projects with low variance, which traps them into growing crop portfolios with low mean returns.

On the other hand, the least risk-averse agents, who are now paired with each other, no longer play the role of informal insurer, and this enables them to take advantage of the decreased aggregate risk and undertake the higher mean, entrepreneurial projects (e.g. adopt a new technology). They are better off post-policy. Thus, we see that the emergence of enterpreneurship corresponds to higher income inequality.

These insights provide an interesting complement to existing literature. Attanasio and Rios-Rull (2000) model the introduction of formal insurance as a policy which reduces the aggregate riskiness of the environment. They also find that such a policy may hurt the welfare of the most risk-averse agents. However, their model, which builds off Ligon, Thomas, Worrall (2001), considers a fixed group of risk-sharing members whose informal insurance arrangement is constrained by limited commitment. Two agents sustain informal risk-sharing by threatening credibly to cut off all future ties if someone reneges, that is, does not honor the risk-sharing agreement (e.g. a member keeps her own income realization instead of transferring some of it to an unlucky partner). Thus, anything that lowers the cost of autarky (the state of being alone and unable to share risk with somebody else) will decrease the level of informal insurance that can be sustained, because the punishment has become less costly. Since the introduction of formal insurance reduces aggregate riskiness, such a policy reduces the cost of autarky, and as a consequence informal insurance is weakened.

However, if commitment were perfect in Attanasio and Rios-Rull (2000), the introduction of formal insurance would strictly improve welfare, because lowering the cost of autarky matters only through the punishment of cutting off future ties, which would no longer be relevant. One contribution of this example, then, is to show that, *even when commitment is perfect*, introducing formal insurance might still reduce the welfare of the most risk-averse agents, because the composition of the informal risk-sharing network changes in response. Reducing the riskiness of the environment does increase the value of autarky, but it *also* increases the value of being in a relationship, and increases it heterogeneously across partnerships of different risk compositions.

This example also contrasts with Chiappori et al. (2011), who estimate that the *least* risk-averse individuals are the ones left worse off after the introduction of formal insurance, since they have been displaced as informal insurers. However, this exactly illuminates the need for a model of the *equilibrium network* of relationships–I show that the least risk-averse agents do leave their roles as informal insurers, but only because they prefer to undertake entrepreneurial pursuits instead. It would be interesting to see how their estimation changes after accounting for the endogeneity of matching.

5 Endogenous Group Size

The focus thus far has been on conditions under which assortative matching emerges as the unique equilibrium when groups are constrained to be of size two. However, we know that a matched pair behaves like a single agent with representative risk tolerance R. In fact, the proof of Proposition 2 echoes Wilson (1968) and shows that this is a property of a matched group of *any* size. This suggests defining the representative risk tolerance of a group of N matched people:

$$R = \sum_{i=1}^{N} \frac{1}{r_i}$$

In order to think about equilibrium matching when group size itself is endogenous, we need to make a few adjustments to the benchmark model. Instead of matching across two distinct groups, individuals all belong to one group, G, and match within this group (this simply allows for group size to be an odd number). As before, production requires at least two collaborators (discussed in Appendix 1), and a matched group of agents jointly chooses a risky project $p \geq 0$ from a spectrum of risky projects, where a project p's returns are described by $Y_p = p + V(p)^{\frac{1}{2}} Y$, Y a random variable with well-defined cdf and $E(Y) = 0$, $V(Y) = 1$. Individuals in an N-person group commit to a feasible return-contingent sharing rule $s_2(y_p), s_3(y_p), ..., s_N(y_p)$, which describes the share of realized output each member receives, for each possible level of output.

Define an equilibrium matching to be one that is stable to all *coalitional* deviations. That is, a group is stable if no strict subset of the group is able to break away and choose a project and sharing rule such that every member of the deviating coalition is weakly better off, and at least one member is strictly better off.

Proposition 6 relates familiar conditions on the risk environment to the size and composition of matched groups in equilibrium in this more general model.

Proposition 6 *Let $M(p) \equiv V'(p)$ describe the marginal variance cost of each project p. Then:*

(a) If $M''(p) < 0$, the unique equilibrium is maximal-connectedness: the whole group, G, is matched.

(b) If $M''(p) > 0$, the unique equilibrium is minimal-connectedness. Individuals match in negative assortative pairs: the i^{th} least risk-averse person in G is matched with the i^{th} most risk-averse. (If $|G|$ is odd, then the most risk-averse individual in the population remains unmatched.)

See Appendix A.4 for the details of the proof.

Proposition 6 shows that when the marginal variance cost is concave in expected return, the unique equilibrium matching coincides with the matching when group size is restricted to two. That is, pairwise matching is actually the optimal matching structure in this case, and the network is minimally-connected. On the other hand, when the marginal variance cost is convex in expected return, all individuals match in one big group, and the network is maximally-connected. Thus, the curvature of the marginal variance cost is tied both to extremal match compositions, as well as to extremal structures of the network of matches.

This result reveals an interesting relationship between the strength of formal insurance institutions and the structure of firms in developing economies. Because poor entrepreneurs lack access to formal insurance, their need to share risk informally influences the nature of the firms they build. When achieving a higher expected return comes at an increasingly steep escalation in risk cost, in the sense that the Sharpe ratio (mean to standard deviation of returns) is smaller for projects with higher expected return, a less risk-averse individual provides more value by informally insuring a more risk-averse individual. However, this informal insurance provision, which causes the less risk-averse person to bear most of the risk, comes at the cost of investing in higher mean, higher variance projects. Thus, the economy is characterized by a minimally-connected network of many small, heterogeneously-composed firms, each choosing safe, non-innovative projects with low mean and low variance of return.

On the other hand, when the Sharpe ratio is larger for projects with higher expected return, the less risk-averse individuals can bear some risk and still be able to choose entrepreneurial projects, since the increase in risk is outweighed by the increase in expected return. In particular, accepting a less risk-averse partner enables the take up of a project with much higher mean return, and this benefit outweighs the cost of having to share good outcomes. Moreover, the less risk-averse are also happy to insure the more risk-averse, since projects with higher mean return no longer come at such a steep escalation in risk cost. Thus, the economy is characterized by a maximally-connected network: one large firm choosing a risky, entrepreneurial project.

This result contrasts interestingly with Genicot and Ray (2003), who find that, under limited commitment, whole-group matching can never be sustained as a coalitionally-stable equilibrium in a model where individuals are homogeneously risk-averse and draw their income realization independently from the same exogenously-specified income distribution. In their framework, because individuals cannot choose what risk to face, the marginal benefit of an additional member is inevitably eventually negative: an individual's expected income is independent of group size, but she becomes increasingly likely to bear a partner's downside risk as the group grows, and limited commitment implies that individuals cannot be compelled to bail partners out after some point. Thus, a subgroup will always profitably deviate if the whole group tries to match, when the number of people in the population is large.

6 Falsifiability and Empirical Support for the Theory

While the previous sections are devoted to exploring interesting implications of the theory, this section provides a variety of empirical approaches for investigating the plausibility of the theory.

One such approach is to check directly the conditions of Proposition 2 or Corollary 2. That is, the interested researcher could elicit the risk attitudes of individuals in a population (e.g. using Binswanger (1980) gambles, or a host of other techniques in the literature), as well as record network connections between individuals. The remaining component is the mean and variance of return for each of the risky projects individuals are able to undertake. For example, Online Appendix OA.6 contains tables showing the mean and variance of yield of a variety of crops available to farmers in different regions–this data has been collected extensively for agriculture. Alternatively, the Sharpe ratios of portfolios in the set of investor options are often calculated in finance. This approach also suits a lab experiment, since the researcher is able to design the set of projects offered to different pools of subjects. Hence, a variety of different methods enable the research to construct $V(p)$ (the variance of return associated with a mean return p), or $SR(p)$ (the Sharpe ratio associated with mean return p), and verify or falsify the theoretical prediction regarding the relationship of the curvature of the marginal variance cost, the monotonicity of the Sharpe ratio in expected return, and equilibrium matching patterns.

However, the researcher may not always be able to reliably construct $V(p)$ or $SR(p)$. Proposition 5 suggests another approach. Instead of constructing $V(p)$, it is only necessary to collect the mean incomes of matched groups, in addition to the data on which individuals are matched and their risk attitudes. Then, the theory predicts that positive assortative matching corresponds with convexity of mean incomes in the representative risk tolerances of matched groups, while negative assortative matching corresponds with concavity. Importantly, this approach does not require knowing anything about the set of risky production opportunities available in the economy, does not require knowing any counterfactuals, such as what projects individuals would have chosen had they matched with different partners, and does not require knowing any moments of the distributions of returns beyond the mean. (Note that another interesting application of Proposition 5 is that it enables policymakers to identify when an economy is near a "tipping point" of the kind discussed in the policy section–when mean incomes are close to being linear in risk tolerances, we can infer that even a small change in the risk environment could "tip" the matching from one extreme to the other, and could lead to the sorts of adverse welfare consequences illustrated by the crop price stabilization example.)

To fix ideas, consider the following example. Suppose $G1 = \{r_1^A, r_2^A, r_3^A\}$ and $G2 = \{r_1^B, r_2^B, r_3^B\}$, where $r_1^{A,B} < r_2^{A,B} < r_3^{A,B}$.

Then, suppose the underlying (unobservable) marginal variance cost function is concave: $M''(p) < 0$. Suppose we observe agents matching positive assortatively in risk attitude, as Proposition 2 predicts. Suppose we also observe the mean incomes of each pair, $p(r_1^A, r_1^B)$, $p(r_2^A, r_2^B)$, and $p(r_3^A, r_3^B)$, as well as the risk types of each agent. Note that we aren't able to force the agents to match in different ways, so we cannot observe what they would have chosen with different partners. Furthermore, we do not observe higher-order moments of the income distributions, including the variance.

Hence, we follow the approach suggested by Proposition 5, and use the observed matchings (r_1^A, r_1^B), (r_2^A, r_2^B), and (r_3^A, r_3^B) to calculate the representative risk tolerance $R_i = \frac{1}{r_i^A} + \frac{1}{r_i^B}$ of each matched pair. Then, to check for concavity or convexity of mean incomes in representative risk tolerances, regress $p_i \equiv p(r_i^A, r_i^B)$ on a constant, as well as R_i and R_i^2 (to use a crude second-order polynomial approximation):

$$p_i = \beta_1 + \beta_2 R_i + \beta_3 R_i^2 + \varepsilon_i$$

If $\hat{\beta}_3 > 0$, this suggests that p_i is convex in R_i, and this is evidence supportive of the theory, since we have established the following: (a) individuals are matched positive assortatively in risk preferences, and (b) the mean incomes of the matched pairs are convex in the representative risk tolerances of those pairs. On the other hand, $\hat{\beta}_3 < 0$ would be evidence against the theory, since individuals are matched positive assortatively, but the mean incomes of the matched pairs are concave in the representative risk tolerances. (Of course, more sophisticated techniques may be used to test for concavity or convexity.)

I apply a combination of these approaches to the dataset from Attanasio et al. (2012) to seek preliminary support or falsification of the theory in the existing literature. Attanasio et al. (2012) run a unique experiment with 70 Colombian communities, where they invite individuals to match in risk-sharing groups in a lab setting. Individuals are able to choose what gamble they face from a set of gambles offered by the experimenters, where higher mean gambles come at the cost of higher variance. However, the sharing rule for each risk-sharing group is fixed at equal division. Thus, a group is able to choose what risk to face, but cannot control how to share a given risk. Risk attitudes are elicited by privately asking each subject to select a gamble before the risk-sharing round is played. In addition, data on pre-existing kinship and friendship networks are gathered. Kinship and friendship ties matter for two important reasons: first, individuals are likely to know the risk attitudes of family and friends, and unlikely to know the risk attitudes of strangers. Second, individuals are likely to trust and therefore commit to family and friends over strangers. Indeed, Attanasio et al. find experimentally that family and friends strongly prefer to match with each other rather than with strangers, and that they match positive assortatively in risk attitude.

To provide some theoretical backbone for this finding, Attanasio et al. study a simplified model of their experimental setting. They assume that individuals have CARA utility and either *low* or *high* risk aversion (type is binary). Group size is restricted to pairs. Individuals choose which risky project to undertake from a continuous spectrum of projects, but the sharing rule is fixed at equal division. They show that under the assumptions of this model, individuals match positive assortatively, that is, high types match with other high types while low types match with other low types.

Because Attanasio et al. fix the sharing rule at equal division, the data cannot be used for a full test of the model and theoretical predictions of this paper, which studies risk-sharing groups when individuals choose how to share a given risk as well as what risk to face. However, an interesting partial test is still possible: Propositions 3 and 4 are exactly focused on the "corners" of the model, and describe equilibrium matching when the project choice and sharing rule choice channels are each shut down in turn. Additionally, we can test Proposition 5, which describes the relationship between the curvature of mean incomes in the representative risk tolerances of matched groups, and the curvature of marginal variance cost.

Like the model in Attanasio et al., Proposition 4 predicts unique positive assortative matching when the sharing rule channel is shut down. However, Proposition 4 is proved under more general conditions, including arbitrarily many risk types rather than binary type, and this generality is useful for interpreting the experimental results. In particular, Attanasio et al. assume that the sharing rule is exactly equal division, that is, $s(Y_p) = \frac{1}{2}Y_p$, and provide a variety of reasons why they don't think side transfers were a concern in practice. Proposition 4 shows that, even with side transfers c, as long as the sharing rule is of the form $s(Y_p) = \frac{1}{2}Y_p + c$, the unique equilibrium matching should still be positive assortative, as it is in the experimental findings.

While it is encouraging that the matching pattern predicted by Proposition 4 bears out in the experiment, the predictions of Proposition 5 provide a more rigorous test of the theory. For this, we need to characterize $V(p)$ or $SR(p)$, based on the set of gambles designed by the experimenters. The discrete nature of the gambles suggests drawing upon Corollary 2 and checking for one of the following relationships in the data: either the Sharpe ratio is decreasing in expected return and mean incomes are concave in risk tolerance, or the Sharpe ratio is increasing in expected return and mean incomes are convex in risk tolerance.

The set of gambles in Attanasio et al. is described in the table below (payoffs are in Colombian pesos)[13]. Each gamble had an equally-likely bad and good payoff, where riskier gambles had worse

[13]The standard deviations reported in the table in Attanasio et al. (2012) are the experimental, not analytical, standard deviations. The discrepancy results from altruism on the part of the experimenters, who were sometimes found to give the subject the high payoff even when the subject lost the gamble. Here, I consider the analytical variance, since Attanasio et al. state that subjects were unaware of the bias in probabilities. Hence, subjects should have chosen gambles based on the analytical variance.

bad payoffs but better good payoffs.

Gamble	Mean	Standard Deviation	Sharpe ratio
1	3000	0	∞
2	4200	1500	2.8
3	4800	2400	2
4	5400	3600	1.5
5	6000	5000	1.2

It's clear that the Sharpe ratio is smaller for gambles with higher expected return. Hence, finding that mean incomes are concave in risk tolerance would be evidence for the theory, while finding that mean incomes are convex in risk tolerance would be evidence against the theory.

I regress the mean returns of the chosen gambles on risk tolerance and squared risk tolerance. A positive coefficient β_3 indicates convexity, while a negative β_3 indicates concavity[14]:

$$p_i = \beta_1 + \beta_2 \left(\frac{1}{r_i}\right) + \beta_3 \left(\frac{1}{r_i}\right)^2 + \varepsilon_i$$

The OLS estimates are reported in Table 1:

Table 1: Testing for Concavity of Mean Project Returns in Risk Tolerance

Regressor	OLS Estimates
Sq. risk tolerance	-0.001***
	(-60.73)
Risk tolerance	2.39***
	(2.75)
Constant	5169.18***
	(180.91)

Notes: This table reports the OLS estimates from a regression of mean project returns on squared risk tolerance, risk tolerance, and a constant. Data from Attanasio et. al. (2012).

*significant at 10%; **significant at 5%; ***significant at 1%

In line with the theoretical prediction, we see that mean project returns are concave in risk tolerance. Moreover, individuals with a higher risk tolerance (less risk-averse individuals) choose riskier (higher mean, higher variance) gambles. Hence, the experimental results lend preliminary support to the theory.

[14] There are more precise ways to examine data for convexity or concavity (e.g. local linear regression or nonparametric methods). The purpose here is not to develop the optimal econometric technique for convexity-testing, but rather to describe an approach to the problem and demonstrate its implementability by executing the approach in a simple, intuitive way.

7 Conclusion

This paper enriches our understanding of informal insurance by developing and studying a theory of endogenous relationship formation between heterogeneously risk-averse people who lack access to formal insurance and credit markets and choose both what risk to face as well as how to share a given risk. The strength of informal insurance is thus how well-insured a population of risk-averse individuals is when they must rely only on interactions with fellow risk-averse individuals to manage risk, rather than how well-insured individuals are when the formal contracting environment is weak. Importantly, this approach reveals that what we can learn from endogenizing informal insurance isn't limited to insurance: the need to manage risk informally in an economy influences the strength of entrepreneurship, the correspondence of entrepreneurship with income inequality, and the optimal structure of informal firms.

Existing literature has focused largely on analyzing the insurance agreement reached by a fixed group of individuals, isolated outside of the equilibrium network, or by individuals who match with a fixed probability, which precludes an understanding of what groups would actually coexist in the first place. By contrast, this paper studies how the network shape that emerges in equilibrium is determined by the possibilities of other shapes the network could have assumed, and identifies important connections between the equilibrium network and the risk environment. In particular, while in theory income distributions may have infinitely many nonzero higher-order moments, equilibrium matching is found to be determined by only the first two: the mean-variance trade-off across portfolio choices.

This paper highlights the especial importance of accounting for, rather than abstracting away from, heterogeneity in developing economies. The absence of formal institutions causes individuals to address those needs with their interpersonal relationships. Thus, we see that a natural constraint on informal insurance, apart from any aspect of the formal contracting environment, is simply that all individuals are risk-averse. However, some individuals are more risk-averse than others, and this enables informal insurance and determines its strength. Yet, just like the formal economy, the informal economy too is endogenous: individuals switch between and assume different informal roles in the economy as circumstances change.

Accounting for this endogeneity may substantially alter our design and evaluation of a variety of policies. While crop price stabilization was explicitly discussed in the paper, the framework is also useful for thinking about land reform. For example, Banerjee (2000) points out that the effects of land redistribution cannot be estimated without first understanding the reasons behind the distribution of landholdings in the status quo. If the allocation served a risk-sharing purpose (for example, large landowners provided informal insurance to the landless), then land redistribution could actually decrease welfare.

The model also provides an interesting lens through which to view regulation of wages. We know that employment contracts (such as the sharecrop contract) often balance multiple needs, such as insurance and incentive provision. The results of this paper show that legislation which places restrictions on the set of permissible rental contracts diminishes the power of individuals to share a given risk, which may cause the less risk-averse to cease acting as informal insurers.

Finally, while informal insurance served as the matching motivation in this paper, the ideas of the model are more general and can be applied to other important questions. For example, understanding the equilibrium matching between individuals with different risk attitudes and the risky health behaviors they choose may help us design more effective approaches to encouraging vaccination, boosting health investments and sanitation practices, and preventing and treating HIV.

Many challenges remain. This model can be enriched in a variety of ways–by generalizing preferences, by introducing limited commitment, by adding dynamic re-matching and dynamic investment. I leave these tasks for future research. Developing economies are distinguished by uniquely complex environments and uniquely lofty stakes. That is why they are formidable, and that is why they are important.

8 References

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9 Appendix A

This section contains proofs and discussions referred to in the text of the paper by the convention "A.1", "A.2", and so on. For appendices referred to in the text by "OA.1", "OA.2", and so on, please see the Online Appendix.

9.1 A.1: Proof of Proposition 1 (NTU problem has TU representation)

Proposition 1: *Expected utility is transferable in this model. Denote a matched pair by* (r_1, r_2) *and their chosen project by* $p^*(r_1, r_2)$*. Let* $CE(r_1, r_2, p^*(r_1, r_2))$ *describe the certainty-equivalent of a matched pair in equilibrium; it is twice continuously differentiable in each argument. Then:*

$$\begin{aligned}\frac{\partial CE(r_1, r_2, p^*(r_1, r_2))}{\partial r_1 r_2} &> 0 \Leftrightarrow \text{ unique } PAM \\ \frac{\partial CE(r_1, r_2, p^*(r_1, r_2))}{\partial r_1 r_2} &< 0 \Leftrightarrow \text{ unique } NAM\end{aligned}$$

Proof: The following program characterizes the optimal contract and project choice for a matched pair (r_1, r_2):

$$\max_{s(y_p)} \int_{-\infty}^{\infty} -e^{-r_1[y_p - s(y_p)]} f(y_p|p) dy_p \quad s.t.$$

$$\int_{-\infty}^{\infty} -e^{-r_2 s(y_p)} f(y_p|p) dy_p \geq -e^{-v}$$

An increase in r_2's expected utility strictly corresponds to a decrease in r_1's expected utility, since more output for r_2 means less output for r_1, so the constraint binds.

Solving shows that r_2's share of realized output y_p is:

$$s^*(y_p|p) = \frac{r_1}{r_1 + r_2} y_p + \frac{1}{r_2} \log \int_{-\infty}^{\infty} e^{-\frac{r_1 r_2}{r_1 + r_2} y_p} f(y_p|p) dy_p + \frac{1}{r_2} v$$

Plug this expression into r_1's objective function. Then the optimal project solves

$$\max_{p \in \Pi} -e^{\frac{r_1}{r_2} v} \left[\int_{-\infty}^{\infty} e^{-\frac{r_1 r_2}{r_1 + r_2} y_p} f(y_p|p) dy_p \right]^{1 + \frac{r_1}{r_2}}$$

where $-e^{\frac{r_1}{r_2} v}$ is just a scaling factor.

So, we can write the indirect utility of an agent r_1 who ensures his partner, r_2, a level of expected utility $-e^{-v}$:

$$\phi(r_1, r_2, v) = -e^{\frac{r_1}{r_2} v} \left[\int_{-\infty}^{\infty} e^{-\frac{r_1 r_2}{r_1 + r_2} y_p} f(y_p|p) dy_p \right]^{1 + \frac{r_1}{r_2}}$$

Partner r_2 receives expected utility:

$$\begin{aligned} EU_{r_2}(v) &= -e^{-v} \\ &= -e^{-r_2 \frac{v}{r_2}} \end{aligned}$$

The certainty-equivalent CE_i for an individual r_i facing risky income stream Y_i is:

$$\begin{aligned} -e^{-r_i CE_i} &= E[-e^{-r_i Y_i}] \Rightarrow \\ CE_i &= -\frac{1}{r_i} \log E[e^{-r_i Y_i}] \end{aligned}$$

Clearly, the certainty-equivalent is a monotonic transformation of expected utility. Hence, showing that the slope of the Pareto frontier of certainty-equivalents is -1 shows expected utility is transferable.

The certainty-equivalent of each member is:

$$\begin{aligned} CE_{r_1}(v) &= -\left(\frac{1}{r_1} + \frac{1}{r_2}\right) \log \left[\int_{-\infty}^{\infty} e^{-\frac{r_1 r_2}{r_1 + r_2} Y_p} f(y_p | p^*(r_1, r_2)) dy_p\right] - \frac{v}{r_2} \\ CE_{r_2}(v) &= \frac{v}{r_2} \end{aligned}$$

There is clearly a one-to-one trade-off between the certainty-equivalent of r_1 and the certainty-equivalent of r_2. In other words, the slope of the Pareto frontier of expected utility (modulo a monotonic transformation) is -1. Hence, expected utility is transferable.

The proof that the equilibrium matching maximizes the sum of certainty-equivalents and the sum of expected project returns, and is Pareto efficient, is straightforward. First, if a matching where each pair is engaged in their optimal project and sharing rule does not maximize the sum of certainty-equivalents, there would be at least one profitable blocking. Second, the sum of certainty-equivalents is a social welfare function, so as the matching maximizes this, it must be Pareto efficient. (Another way to see this is by recalling the no-blocking condition of the equilibrium.)

9.2 A.2: Proof of Proposition 2 (main matching result) and Corollary 2 (Sharpe ratio result)

Proposition 2: *Recall that $V(p)$ describes the variance cost of a project with mean return p. Then $M(p) \equiv V'(p)$ describes the marginal variance cost of a project with mean return p.*

(a) A sufficient condition for PAM to be the unique equilibrium match is $M''(p) < 0$ for $p > 0$ (concavity).

(b) A sufficient condition for NAM to be the unique equilibrium match is $M''(p) > 0$ for $p > 0$ (convexity).

(c) A sufficient condition for any match to be sustainable as an equilibrium is $M''(p) = 0$ for $p > 0$ (linearity).

Proof: Suppose a matched pair (r_1, r_2) has to choose from a family of income distributions defined by $Y_p = p + V(p)^{\frac{1}{2}} Y$, where Y is a random variable with well-defined cumulants: $k_1(y) = E(Y) = 0$, $k_2(y) = V(Y) = 1$, infinite support. (The cumulant-generating function of a random variable is the log of the moment-generating function; the n^{th} order cumulant is the n^{th} derivative of the cumulant-generating function evaluated at 0.)

Recall that the representative risk tolerance of a matched group is the sum of the risk tolerances of the members. So, the representative risk tolerance of (r_1, r_2) is $R = \frac{1}{r_1} + \frac{1}{r_2}$.

We know from Appendix 2 that the expected utility of a group with representative risk aversion r (reciprocal of representative risk tolerance R) which has chosen a project p with risky stream of returns Y_p is:

$$\begin{aligned} E\left[-e^{-r[p+V(p)^{\frac{1}{2}}Y]}\right] &= -e^{-rp}\int e^{-rV(p)^{\frac{1}{2}}y} f(y)dy \\ &= -\int e^{-\frac{1}{R}\left[p+V(p)^{\frac{1}{2}}y\right]} f(y)dy \end{aligned}$$

Divide by (-1), transform by log, and multiply by $(-R)$ for the certainty-equivalent. Since we want to maximize expected utility, this implies we want to choose p to maximize the certainty-equivalent:

$$\begin{aligned} CE(p;R) &= -R\log E\left[e^{-\frac{1}{R}[p+V(p)^{\frac{1}{2}}Y]}\right] \\ &= -R\left[-\frac{1}{R}p + \log E\left[e^{-\frac{1}{R}V(p)^{\frac{1}{2}}Y}\right]\right] \\ &= p - R\left[\sum_{n=2}^{\infty}\frac{(-1)^n}{n!R^n}V(p)^{\frac{n}{2}}k_n(y)\right] \\ &= p - \frac{1}{2R}V(p) - \sum_{n=3}^{\infty}\frac{(-1)^n}{n!R^{n-1}}V(p)^{\frac{n}{2}}k_n(y) \end{aligned}$$

Then:

$$\frac{\partial CE}{\partial R} = \frac{1}{2R^2}V(p^*) + \sum_{n=3}\frac{(-1)^n}{n!}\frac{(n-1)}{R^n}V(p^*)^{\frac{n}{2}}k_n(y)$$

And:

$$
\begin{aligned}
\frac{\partial^2 CE}{\partial R^2} &= -\frac{1}{R^3}V(p^*) + \frac{1}{2R^2}V'(p^*)\frac{\partial p^*}{\partial R} - \sum_{n=3}\frac{(-1)^n}{n!}\frac{n(n-1)}{R^{n+1}}V(p^*)^{\frac{n}{2}}k_n(y) \\
&\quad + \sum_{n=3}\frac{(-1)^n}{n!}\frac{(n-1)}{R^n}\frac{n}{2}V(p^*)^{\frac{n}{2}-1}V'(p^*)\frac{\partial p^*}{\partial R}k_n(y) \\
&= \left[-\frac{2}{R}\frac{V(p^*)}{V'(p^*)} + \frac{\partial p^*}{\partial R}\right]\left[\sum_{n=3}\frac{(-1)^n}{(n-2)!}\frac{1}{2R^n}V(p^*)^{\frac{n}{2}-1}V'(p^*)k_n(y) + \frac{1}{2R^2}V'(p^*)\right]
\end{aligned}
$$

where we know this second bracketed term is positive because it is bounded below by $\frac{1}{R} > 0$. We establish this bound by recalling the first-order condition of the optimization problem:

$$
FOC_p = 0 : -\frac{1}{R} + \frac{1}{2R^2}V(p^*) + \sum_{n=3}\frac{(-1)^n}{(n-1)!}\frac{1}{2R^n}V(p^*)^{\frac{n}{2}-1}V'(p^*)k_n(y) = 0
$$

This implies:

$$
\frac{1}{2R^2}V(p^*) + \sum_{n=3}\frac{(-1)^n}{(n-1)!}\frac{1}{2R^n}V(p^*)^{\frac{n}{2}-1}V'(p^*)k_n(y) = \frac{1}{R}
$$

And the first term of the second bracketed expression:

$$
\sum_{n=3}\frac{(-1)^n}{(n-2)!}\frac{1}{2R^n}V(p^*)^{\frac{n}{2}-1}V'(p^*)k_n(y) > \sum_{n=3}\frac{(-1)^n}{(n-1)!}\frac{1}{2R^n}V(p^*)^{\frac{n}{2}-1}V'(p^*)k_n(y)
$$

Now, we seek conditions such that:

$$
\frac{\partial^2 CE}{\partial R^2} >, < 0
$$

Using the expression for $\frac{d^2CE}{dR^2}$, and the fact that the second bracketed expression is a positive constant, we know that:

$$
\begin{aligned}
\frac{\partial^2 CE}{\partial R^2} &> \quad , < 0 \Leftrightarrow \\
\frac{\partial p^*}{\partial R} &> \quad , < \frac{2}{R}\frac{V(p^*)}{V'(p^*)}
\end{aligned}
$$

Now, find $\frac{\partial p^*}{\partial R}$ by implicitly differentiating $FOC_p = 0$:

$$
FOC_p = 0 : \frac{1}{2R^2}V(p^*) + \sum_{n=3}\frac{(-1)^n}{(n-1)!}\frac{1}{2R^n}V(p^*)^{\frac{n}{2}-1}V'(p^*)k_n(y) = \frac{1}{R}
$$

$$\frac{\partial p^*}{\partial R}: -\frac{1}{R^3}V'(p^*) + \frac{1}{2R^2}V''(p^*)\frac{\partial p^*}{\partial R} - \sum_{n=3}\frac{(-1)^n}{(n-1)!}\frac{n}{R^{n+1}}\frac{1}{2}V(p^*)^{\frac{n}{2}-1}V'(p^*)k_n(y)$$

$$+\sum_{n=3}\frac{(-1)^n}{(n-1)!}\frac{1}{R^n}\frac{1}{2}\left(\frac{n}{2}-1\right)V(p^*)^{\frac{n}{2}-2}V'(p^*)^2k_n\frac{\partial p^*}{\partial R} + \sum_{n=3}\frac{(-1)^n}{(n-1)!}\frac{1}{R^n}\frac{1}{2}V(p^*)^{\frac{n}{2}-1}V''(p^*)k_n(y)\frac{\partial p^*}{\partial R} = -\frac{1}{R^2}$$

Hence:

$$-V'(p^*) + \frac{R}{2}V''(p^*)\frac{\partial p^*}{\partial R} + R^2\sum_{n=3}\frac{(-1)^n}{(n-1)!}\frac{1}{R^n}\frac{n}{2}V(p^*)^{\frac{n}{2}-2}V'(p^*)^2k_n(y)\left[\frac{R}{2}\frac{\partial p^*}{\partial R} - \frac{V(p^*)}{V'(p^*)}\right]$$
$$+R^3\sum_{n=3}\frac{(-1)^n}{(n-1)!}\frac{1}{R^n}\frac{1}{2}V(p^*)^{\frac{n}{2}-2}V'(p^*)k_n(y)\frac{\partial p^*}{\partial R}\left[\frac{V(p^*)V''(p^*)}{V'(p^*)} - V'(p^*)\right] = -R$$

This expression can be rearranged using the first-order condition:

$$-\frac{V'(p^*)}{R} + \frac{\partial p^*}{\partial R}\left(\left[R - \frac{V'(p^*)}{2}\right]\left[\frac{V''(p^*)}{V'(p^*)} - \frac{V'(p^*)}{V(p^*)}\right] + \frac{1}{2}V''(p^*)\right)$$
$$+\frac{R^2}{2}\frac{\partial p^*}{\partial R}\sum_{n=3}\frac{(-1)^n}{(n-1)!}\frac{1}{R^n}\frac{n}{2}V(p^*)^{\frac{n}{2}-2}V'(p^*)^2k_n(y) - R\sum_{n=3}\frac{(-1)^n}{(n-1)!}\frac{1}{R^n}\frac{n}{2}V(p^*)^{\frac{n}{2}-1}V'(p^*)k_n(y) = -1$$

So we see that:

$$\frac{\partial p^*}{\partial R} = \frac{\frac{V'(p^*)}{R} - 1 + R\sum_{n=3}\frac{(-1)^n}{(n-1)!}\frac{1}{R^n}\frac{n}{2}V(p^*)^{\frac{n}{2}-1}V'(p^*)k_n(y)}{R\frac{V''(p^*)}{V(p^*)} - R\frac{V'(p^*)}{V(p^*)} + \frac{V'(p^*)^2}{2V(p^*)} + \frac{R^2}{2}\sum_{n=3}\frac{(-1)^n}{(n-1)!}\frac{1}{R^n}\frac{n}{2}V(p^*)^{\frac{n}{2}-2}V'(p^*)^2k_n(y)}$$

Hence:

$$\begin{aligned}\frac{\partial p^*}{\partial R} &> ,< \frac{2}{R}\frac{V(p^*)}{V'(p^*)} \Leftrightarrow \\ V'(p^*)^2 &> ,< 2V(p^*)V''(p^*)\end{aligned}$$

Call the left-hand side of this inequality LHS, and the right-hand side, RHS. Observe that at $R = 0$ (zero risk tolerance), $LHS = RHS = 0$, since $p = 0$ is the safest project available, so $p^*(0) = 0$, and it is assumed that $V(0) = V'(0) = 0$.

So, compare $\frac{\partial LHS}{\partial R}$ and $\frac{\partial RHS}{\partial R}$: both LHS and RHS are 0 at $R = 0$. As R increases, if LHS either increases faster in R or decreases slower than R at every point R in comparison to RHS, that is, $\frac{\partial LHS}{\partial R} > \frac{\partial RHS}{\partial R}$ for all $R > 0$, then $\frac{\partial^2 CE}{\partial R^2} > 0$ since $LHS > RHS$ in the above inequality. And, vice versa, if $\frac{\partial LHS}{\partial R} < \frac{\partial RHS}{\partial R}$.

Thus:

$$\begin{aligned}V'(p^*)^2 &> ,< 2V(p^*)V''(p^*) \Leftrightarrow \\ 0 &> ,< 2V(p^*)V'''(p^*)\frac{\partial p^*}{\partial R} \Leftrightarrow \\ 0 &> ,< V'''(p^*)\end{aligned}$$

where the last equivalency holds because $2V(p^*)\frac{\partial p^*}{\partial R} > 0$.

Hence, Proposition 5 is proven:

$$\begin{aligned} V'''(p) &< 0 \; \forall p \geq 0 \Leftrightarrow \frac{\partial^2 CE}{\partial R^2} > 0 \; \forall R \geq 0 \Rightarrow \textit{unique PAM} \\ V'''(p) &> 0 \; \forall p \geq 0 \Leftrightarrow \frac{\partial^2 CE}{\partial R^2} < 0 \; \forall R \geq 0 \Rightarrow \textit{unique NAM} \end{aligned}$$

Note that if our focus were not on identifying conditions for assortative matching that are independent of risk type distributions, we could easily write a necessary and sufficient condition for PAM and NAM. Instead of seeking conditions for global convexity and concavity of $CE(R)$ in R, the condition simply has to hold for each set of representative risk tolerances for every possible matching in the given population of risk types.

A quick final note: we need $CE(p; R)$ to be concave in p for a well-defined maximum:

$$\begin{aligned} \frac{\partial^2 CE}{\partial p^2} &= -\frac{1}{2R}V''(p) - \sum_{n=3}^{\infty} \frac{(-1)^n}{n!R^{n-1}} \frac{n}{2} \left[\left(\frac{n}{2} - 1 \right) V(p)^{\frac{n}{2}-2} V'(p)^2 + V(p)^{\frac{n}{2}-1} V''(p) \right] k_n(y) \\ &= -\frac{1}{2} \left[\frac{1}{R} V''(p) + \sum_{n=3}^{\infty} \frac{(-1)^{n-1}}{(n-1)!R^{n-1}} \left[\frac{n}{2} V(p)^{\frac{n}{2}-2} V'(p)^2 + V(p)^{\frac{n}{2}-2} \left(V(p)V''(p) - V'(p)^2 \right) \right] \right] \\ &< 0 \end{aligned}$$

Corollary 2: *Let $SR(p) = \frac{p}{V(p)^{\frac{1}{2}}}$ denote the ratio of a project's mean return to its standard deviation for $p > 0$. Then:*

(a) $SR'(p) > 0 \; \forall p > 0$ iff $M''(p) < 0 \; \forall p > 0$ (PAM)

(b) $SR'(p) < 0 \; \forall p > 0$ iff $M''(p) > 0 \; \forall p > 0$ (NAM)

Proof: We want to show that the Sharpe ratio monotonically increases (decreases) in mean return iff the marginal variance cost is concave (convex) in mean return.

$$\begin{aligned} SR(p) &= \frac{p}{V(p)^{\frac{1}{2}}} \\ SR'(p) &= \frac{V(p)^{\frac{1}{2}} - p\frac{1}{2}V(p)^{-\frac{1}{2}}V'(p)}{V(p)} \end{aligned}$$

Then:

$$\begin{aligned} SR'(p) &> ,< 0 \Leftrightarrow V(p)^{\frac{1}{2}} >,< p\frac{1}{2}V(p)^{-\frac{1}{2}}V'(p) \text{ [since } V(p) > 0 \; \forall p] \\ &\Leftrightarrow V(p) >,< \frac{1}{2}pV'(p) \end{aligned}$$

Then note that $LHS(p = 0) = V(0) = 0 = \frac{1}{2} * 0 * V'(0) = RHS(p = 0)$. So, look at derivatives of LHS and RHS, since the domain is $p \geq 0$.

$$V'(p) >,< pV''(p)$$

But we don't know under what conditions this inequality holds. So we again note that $LHS(p = 0) = V'(0) = 0 = 0 * V''(0) = RHS(p = 0)$, and differentiate LHS, RHS again:

$$\begin{aligned} V''(p) &> ,< V''(p) + pV'''(p) \Leftrightarrow \\ 0 &> ,< pV'''(p) \end{aligned}$$

Since the domain is $p \geq 0$, this means that:

$$\begin{aligned} V'''(p) &> 0 \Leftrightarrow SR'(p) < 0 \ \forall p \geq 0 \\ &\Leftrightarrow \ !NAM \\ &\Leftrightarrow CV'(p) > 0 \\ &\Leftrightarrow \frac{\partial^2 p^*}{\partial R^2} < 0 \end{aligned}$$

and:

$$\begin{aligned} V'''(p) &< 0 \Leftrightarrow SR'(p) > 0 \ \forall p \geq 0 \\ &\Leftrightarrow \ !PAM \\ &\Leftrightarrow CV'(p) < 0 \\ &\Leftrightarrow \frac{\partial^2 p^*}{\partial R^2} > 0 \end{aligned}$$

9.3 A.3: Proof of Proposition 5 (curvature of mean incomes) and Corollary 3 (income inequality)

Proposition 5: *Let $p^*(r_i, \mu(r_i))$ denote the mean return of the project chosen by a matched pair $(r_i, \mu(r_i))$. Then:*

(a) If $M''(p) < 0$ for $p > 0$, then $p^*(r_i, \mu(r_i))$ is convex in representative risk tolerance $R(r_i, \mu(r_i))$.

(b) If $M''(p) > 0$ for $p > 0$, then $p^*(r_i, \mu(r_i))$ is concave in representative risk tolerance $R(r_i, \mu(r_i))$.

Proof: We want to show that the condition for global convexity and concavity of the certainty-equivalent of a group with representative risk tolerance R in R is equivalent to global convexity and concavity of the mean returns of projects chosen by matched pairs in equilibrium in the representative risk tolerances of those matched pairs.

We know from Appendix 3a that:

$$\begin{aligned} \frac{\partial^2 CE}{\partial R^2} &> ,< 0 \ \forall R > 0 \Leftrightarrow \\ \frac{\partial p^*}{\partial R} &> ,< \frac{2}{R} \frac{V(p^*)}{V'(p^*)} \ \forall R > 0 \end{aligned}$$

At $R = 0$, $LHS = RHS = 0$. So, find conditions under which $\frac{\partial LHS}{\partial R} > \frac{\partial RHS}{\partial R}$, and vice versa. Then:

$$\begin{aligned}
\frac{\partial LHS}{\partial R} - \frac{\partial RHS}{\partial R} &> \quad , < 0 \Leftrightarrow \\
\frac{R}{2} V'(p^*) \frac{\partial^2 p^*}{\partial R^2} + \frac{\partial p^*}{\partial R} \left[\frac{R}{2} V''(p^*) \frac{\partial p^*}{\partial R} - \frac{1}{2} V'(p^*) \right] &> \quad , < 0
\end{aligned}$$

The second term is positive or negative depending on whether the bracketed term is positive or negative:

$$\begin{aligned}
\frac{\partial p^*}{\partial R} \left[\frac{R}{2} V''(p^*) \frac{\partial p^*}{\partial R} - \frac{1}{2} V'(p^*) \right] &> \quad , < 0 \Leftrightarrow \\
\frac{\partial p^*}{\partial R} &> \quad , < \frac{1}{R} \frac{V'(p^*)}{V''(p^*)}
\end{aligned}$$

Using the expression for $\frac{\partial p^*}{\partial R}$ we characterized by implicitly differentiating the first-order optimality condition in Appendix 3a, we see that this inequality is equivalent to:

$$2R^2 \left[\frac{1}{2} \sum_{n=3} \frac{(-1)^n}{(n-1)!} \frac{1}{R^n} \frac{n}{2} V(p^*)^{\frac{n}{2}-1} V'(p^*) k_n(y) + \frac{1}{2R^2} V'(p^*) - \frac{1}{R} \right] \left[V''(p^*) - \frac{1}{2} \frac{V'(p^*)^2}{V(p^*)} \right] >, < 0$$

But we know that the first bracketed term is positive, since:

$$\begin{aligned}
\frac{1}{2} \sum_{n=3} \frac{(-1)^n}{(n-1)!} \frac{1}{R^n} \frac{n}{2} V(p^*)^{\frac{n}{2}-1} V'(p^*) k_n(y) &> \frac{1}{2} \sum_{n=3} \frac{(-1)^n}{(n-1)!} \frac{1}{R^n} V(p^*)^{\frac{n}{2}-1} V'(p^*) k_n(y) \\
&= \frac{1}{2R^2} V'(p^*) - \frac{1}{R}
\end{aligned}$$

where the equality holds from $FOC_p = 0$.

Hence:

$$\begin{gathered}
\frac{\partial p^*}{\partial R} >, < \frac{1}{R} \frac{V'(p^*)}{V''(p^*)} \Leftrightarrow \\
V''(p^*) - \frac{1}{2} \frac{V'(p^*)^2}{V(p^*)} >, < 0 \Leftrightarrow \\
2V(p^*) V''(p^*) >, < V'(p^*)^2
\end{gathered}$$

But we know from Appendix 3a that:

$$
\begin{gathered}
2V(p^*)V''(p^*) >, < V'(p^*)^2 \Leftrightarrow \\
V'''(p^*) >, < 0 \Leftrightarrow \\
\frac{\partial^2 CE}{\partial R^2} <, > 0 \Leftrightarrow \\
\frac{R}{2}V'(p^*)\frac{\partial^2 p^*}{\partial R^2} + \frac{\partial p^*}{\partial R}\left[\frac{R}{2}V''(p^*)\frac{\partial p^*}{\partial R} - \frac{1}{2}V'(p^*)\right] >, < 0
\end{gathered}
$$

Hence:

$$
\begin{aligned}
V'''(p^*) &> \quad , < 0 \ \forall p > 0 \Leftrightarrow \\
\frac{\partial p^*}{\partial R}\left[\frac{R}{2}V''(p^*)\frac{\partial p^*}{\partial R} - \frac{1}{2}V'(p^*)\right] &> \quad , < 0 \ and \ \frac{\partial^2 CE}{\partial R^2} <, > 0 \ \forall R > 0 \Leftrightarrow \\
\frac{R}{2}V'(p^*)\frac{\partial^2 p^*}{\partial R^2} &< \quad , > 0 \ \forall R > 0 \Leftrightarrow \\
\frac{\partial^2 p^*}{\partial R^2} &< \quad , > 0 \ \forall R > 0
\end{aligned}
$$

Thus, while $V'''(p)$ (or any higher-order characteristics of the distributions of income streams) might be difficult to measure or estimate in practice, this result suggests a more empirically feasible approach: the *mean* incomes of matched groups in *equilibrium* are convex or concave in their representative risk tolerance if and only if $V'''(p)$ is negative or positive.

Corollary 3: *For a given population of individuals $G1, G2$ and a given risk environment described by $V(p)$, income inequality, measured by the dispersion of and gap between lowest and highest expected incomes, is maximized when individuals are matched positive assortatively, and minimized when individuals are matched negative assortatively.*

Proof: We want to show that, fixing a population of individuals and a risk environment, income inequality (variance of individual incomes) is maximized under positive assortative matching, and minimized under negative assortative matching.

First observe that, given a population, PAM maximizes the variance of the representative risk tolerances of matched pairs, while NAM minimizes the variance.

Let the population of individuals be represented by: $G1 = \{x_1, ..., x_N\}$, $G2 = \{y_1, ..., y_N\}$, ordered from smallest to largest (least to most risk-averse).

Then the set of representative risk tolerances under PAM is $\{\frac{1}{x_1} + \frac{1}{y_1}, \frac{1}{x_2} + \frac{1}{y_2}, ..., \frac{1}{x_N} + \frac{1}{y_N}\}$, and under NAM is $\{\frac{1}{x_1} + \frac{1}{y_N}, \frac{1}{x_2} + \frac{1}{y_{N-1}}, ..., \frac{1}{x_N} + \frac{1}{y_1}\}$.

The variance of the representative risk tolerances of PAM and NAM pairs is:

$$
\begin{aligned}
Var_{PAM} &= \frac{1}{x_1 y_1} + \frac{1}{x_2 y_2} + ... + \frac{1}{x_N y_N} + \sum \frac{1}{x_i^2} + \sum \frac{1}{y_i^2} \\
Var_{NAM} &= \frac{1}{x_1 y_N} + \frac{1}{x_2 y_{N-1}} + ... + \frac{1}{x_N y_1} + \sum \frac{1}{x_i^2} + \sum \frac{1}{y_i^2}
\end{aligned}
$$

where $\sum \frac{1}{x_i^2} + \sum \frac{1}{y_i^2}$ appears in the variance of any matching.

So, focus on the previous terms in Var_{PAM}. Switch an arbitary pair–WLOG (x_1, y_2) and (x_2, y_1) instead of $(x_1, y_1), (x_2, y_2)$. But, since $x_1 < x_2$ and $y_1 < y_2$:

$$\frac{1}{x_1 y_1} + \frac{1}{x_2 y_2} > \frac{1}{x_1 y_2} + \frac{1}{x_2 y_1}$$

So, any change in PAM leads to a decrease in variance. Similarly, switching an arbitrary pair in NAM shows that any change in NAM leads to an increase in variance.

In fact, the representative risk tolerances under PAM are a mean-preserving spread of those under NAM. For example, suppose $N = 3$ (odd).

Then, let $p^*(R_{ij})$ denote the expected income for a matched pair with representative risk tolerance R_{ij}. Since we know $p^*(R)$ increases in R (from Appendix 3a), and the variance of the set of representative risk tolerances is maximized under PAM and minimized under NAM, it must be that the variance of the set of expected joint incomes is maximized under PAM and minimized under NAM. Moreover, expected mean incomes are more dispersed under PAM than under NAM–the poorest are poorer and the richest are richer under PAM.

9.4 A.4: Proof of Proposition 6 (endogenous group size)

Proposition 6: *Let $M(p) \equiv V'(p)$ describe the marginal variance cost of each project p. Then:*

(a) If $M''(p) < 0$, the unique equilibrium is maximal-connectedness: the whole group, G, is matched.

(b) If $M''(p) > 0$, the unique equilibrium is minimal-connectedness. Individuals match in negative assortative pairs: the i^{th} least risk-averse person in G is matched with the i^{th} most risk-averse. (If $|G|$ is odd, then the most risk-averse individual in the population remains unmatched.)

Proof: We know from the proof of Proposition 2 that:

$$\begin{aligned} M''(p) &> 0 \Leftrightarrow \frac{\partial^2 CE}{\partial R^2} < 0 \\ M''(p) &< 0 \Leftrightarrow \frac{\partial^2 CE}{\partial R^2} > 0 \end{aligned}$$

Recall the following property of a function $f : \mathbb{R}_0^+ \to \mathbb{R}$:

$$\begin{aligned} f(0) &\geq 0 \textit{ and } f(\cdot) \textit{ concave} \Rightarrow \forall x, y \in \mathbb{R}_0^+,\ f(x+y) < f(x) + f(y) \\ f(0) &\leq 0 \textit{ and } f(\cdot) \textit{ convex} \Rightarrow \forall x, y \in \mathbb{R}_0^+,\ f(x+y) > f(x) + f(y) \end{aligned}$$

That is, given $f(0) \geq 0$, a concave function is subadditive, and given $f(0) \leq 0$, a convex function is superadditive.

Recall that $CE(\cdot)$ is a function of the risk tolerance R of a matched group. Since $CE(0) = 0$, we know that:

$$\begin{aligned} M''(p) &> 0 \; \forall p \geq 0 \Leftrightarrow CE(R_1 + R_2) < CE(R_1) + CE(R_2) \; \forall R_1, R_2 \in \mathbb{R}_0^+ \\ M''(p) &< 0 \; \forall p \geq 0 \Leftrightarrow CE(R_1 + R_2) > CE(R_1) + CE(R_2) \; \forall R_1, R_2 \in \mathbb{R}_0^+ \end{aligned}$$

But representative risk tolerance is additive: the representative risk tolerance of a matched group is the sum of the individual risk tolerances. This implies that, when $M''(p) > 0$ and $CE(\cdot)$ is concave in risk tolerance R, the matching that maximizes the total sum of certainty-equivalents is unique and is the *minimal* matching–that is, individuals match in partnerships (since production requires at least two collaborators), and we know from Proposition 2 that these partnerships will be negative assortative. And, when $M''(p) < 0$ and $CE(\cdot)$ is convex in risk tolerance R, the matching that maximizes the total sum of certainty-equivalents is unique and is the *maximal* matching–that is, all individuals match in one big group.

Online Appendix (for online publication): Endogenous Informal Insurance Relationships

July 31, 2014

This section contains proofs and discussions referred to in the text of the paper by the convention "$OA.1$", "$OA.2$", and so on. For appendices referred to in the text by "$A.1$", "$A.2$", and so on, please see Appendix A (the section directly following References).

1 OA.1: Equivalence with a model of individual project choice

This discussion clarifies the relationship between a model where matched individuals jointly choose which income distribution to face, and then share the realized return, versus a model where individuals choose their own income distribution, then match and share the pooled realized returns.

Suppose that, as in the benchmark model, a spectrum of projects $p \geq 0$ is available, where $Y_p = p + V(p)^{\frac{1}{2}}Y$, $E(Y) = 0$, $V(Y) = 1$, and $V(p) > 0$, $V'(p) > 0$, and $V''(p) > 0$. Each individual chooses a personal income distribution from the spectrum Y_p, and pools realized returns with her partner. An individual can always choose autarky if she prefers this to matching with any possible partner. A matched pair commits *ex ante* to a sharing rule contingent on pooled income, $s(Y_{p_1} + Y_{p_2})$. (For ease of notation, let $Y_k \equiv Y_{p_k}$.)

Assume that incomes are correlated. Recall that $Var(Y_1 + Y_2) = Var(Y_1) + Var(Y_2) + 2Cov(Y_1, Y_2)$, where $Cov(Y_1, Y_2) = EY_1Y_2 - EY_1EY_2 = EY_1Y_2 - p_1p_2$.

Since $Y_1 = p_1 + V(p_1)^{\frac{1}{2}}Y$ and $Y_2 = p_2 + V(p_2)^{\frac{1}{2}}Y$:

$$\begin{aligned} E(Y_1Y_2) &= E(p_1p_2 + p_1V(p_2)^{\frac{1}{2}}Y + p_2V(p_1)^{\frac{1}{2}}Y + V(p_1)^{\frac{1}{2}}V(p_2)^{\frac{1}{2}}Y^2) \\ &= p_1p_2 + E\left[V(p_1)^{\frac{1}{2}}V(p_2)^{\frac{1}{2}}Y^2\right] \end{aligned}$$

And:

$$\begin{aligned} E(Y^2) &= V(Y) - (EY)^2 \\ &= 1 \end{aligned}$$

So:

$$\begin{aligned} E(Y_1Y_2) &= p_1p_2 + V(p_1)^{\frac{1}{2}}V(p_2)^{\frac{1}{2}} \\ Cov(Y_1, Y_2) &= p_1p_2 + V(p_1)^{\frac{1}{2}}V(p_2)^{\frac{1}{2}} - p_1p_2 \\ Var(Y_1 + Y_2) &= V(p_1) + V(p_2) + 2V(p_1)^{\frac{1}{2}}V(p_2)^{\frac{1}{2}} \end{aligned}$$

Thus, members of a matched pair can choose individual income distributions p_1 and p_2 in a number of ways to achieve mean pooled income $p = p_1 + p_2$ for any $p > 0$. The optimal strategy for the partners, given that they want expected joint income to be p, is to choose p_1 and p_2 to minimze the variance of joint income, given $p_1 + p_2 = p$.

Solving:

$$\min_{p_1,p_2} V(p_1) + V(p_2) + 2V(p_1)^{\frac{1}{2}}V(p_2)^{\frac{1}{2}} \ s.t.$$

$$p_1 + p_2 = p$$

it is straightforward to see that $p_1^* = p_2^* = \frac{p}{2}$, so that $V(p|p_1^*, p_2^*) = 2V\left(\frac{p}{2}\right) + 4V\left(\frac{p}{2}\right)^{\frac{1}{2}}$.

Hence, matched pairs choose from a frontier of joint income distributions described by: $Y_p = p + [2V\left(\frac{p}{2}\right) + 4V\left(\frac{p}{2}\right)^{\frac{1}{2}}]^{\frac{1}{2}}Y$, where $E(Y) = 0$, $V(Y) = 1$.

But this is substantively equivalent to the benchmark model–although individuals ostensibly choose own income in this model, they are actually choosing a distribution of pooled income as a matched pair. In this paper, I work with the benchmark model because it illustrates the dependence of the equilibrium matching on the trade-off between production and insurance more cleanly.

2 OA.2: Limited commitment

While I leave a rigorous treatment of limited commitment in this setting to future work, I discuss some preliminary analysis here.

The basic elements of the model with limited commitment are:

1. (a) 2 periods. At the beginning of each period, each member of a matched group (or as an individual, if in autarky) chooses a risky project.
   (b) Assume that eternal autarky is the punishment for reneging, and that incomes are independent across time and people, since we know that when incomes are independent, autarky is never optimal. Thus, imposing autarky is a legitimate punishment.
   (c) Basic timing of the game:
      i. A matched pair agrees to a return-contingent sharing rule for their pooled incomes at the beginning of pd. 1.
      ii. Each individual in a matched pair observes boht income realizations, and so knows what the transfer specified by the rule is. Each can choose to uphold that transfer,

or to renege (do something other than what the rule specifies). If at least one member reneges, then both members are consigned to autarky in pd. 2. (Assume no re-matching.)

iii. If both uphold the transfer, then the pair stays matched in pd. 2. Assume that in pd. 2, pairs that successfully cooperated in pd. 1 can implement the efficient outcome in pd. 2 (for example, because they've successfully won each other's trust).

Then, what do we know about the equilibrium?

In pd. 2, for non-reneging pairs, we know that NAM conditions imply that the less risk-averse differentially prefer to bid more for the more risk-averse, while PAM conditions imply that the less risk-averse differentially prefer to bid more for the less risk-averse.

So, under NAM conditions, a less risk-averse person finds autarky in pd. 2 more costly with a more risk-averse partner than with a less risk-averse partner. Under PAM, a less risk-averse person finds autarky in pd. 2 more costly with a less risk-averse partner than a more risk-averse partner.

So far, imperfect commitment hasn't changed when PAM is optimal vs. NAM is optimal.

Now, think about the effect on pd. 1 choices. Because of limited commitment, in pd. 1, a matched pair can't always implement the efficient risk-sharing rule. The equilibrium sharing rule in pd. 1 for a matched pair (r_i, r_j) is instead:

1. If the income realizations are such that the efficient risk-sharing rule happens to be such that both partners find $U(\text{keep my own income realization}) - U(\frac{r_j}{r_i+r_j} Y_{p(i,j)}) < U(\frac{r_j}{r_i+r_j} Y_{p\_optimal}) - U(aut)$, then the transfer is the efficient one.

2. If the realizations are outside of this interval (which will loosely be the case when one partner has a good realization and the other has a bad one), then the better off partner should make a transfer to her partner that makes her indifferent between paying that transfer and keeping everything and facing autarky in pd. 2. (Note that if reneging is better than cooperating, the optimal reneging is to keep your own realization.)

Note that this rule is much less smooth than the efficient risk-sharing rule, which is a linear split of pooled output. Now we have a linear split of pooled income for some joint realizations, and near-autarky for others.

More risk-averse people find autarky worse, as well as the volatility of the sharing rule in pd. 1 under limited commitment. So even though under NAM conditions, a more risk-averse person prefers a less risk-averse partner in pd. 2, the volatility of their corresponding pd. 1 sharing rule (and the fact that the less risk-averse person finds autarky a less scary threat than the more risk-averse person), means that more risk-averse people prefer to match with other more risk-averse people *more strongl*y than they do without limited commitment. On the other hand, the less risk-averse really value a more risk-averse partner, because a more risk-averse partner is willing to receive a small transfer and is also the best partner in pd. 2 (NAM conditions). However, limited commitment constrains the credible "bid" a less risk-averse partner can make in pd. 1. That is, a less risk-averse person would like to promise a more risk-averse person more than the transfer that makes her indifferent between honoring the transfer and keeping her own income and facing autarky in pd. 2, but she can't do this credibly.

3 OA.3: Differently-sized groups

We know by now (see, e.g., A.1) that the sum of certainty-equivalents for a given pair (r_1, r_2) with representative risk tolerance $R = \frac{1}{r_1} + \frac{1}{r_2}$ in the benchmark case is given by:

$$\begin{aligned} CE(R) &= -R \log K_{Y_{p^*(R)}} \\ &= p^* - \sum_{n=2}^{\infty} \frac{(-1)^n}{n! R^{n-1}} V(p^*)^{\frac{n}{2}} k_n(y) \end{aligned}$$

Then:

$$\begin{aligned} \frac{\partial CE}{\partial R} &= \sum_{n=2}^{\infty} \frac{(-1)^n}{n!} \frac{(n-1)}{R^n} V(p^*)^{\frac{n}{2}} k_n(y) \\ \frac{\partial R}{\partial r_2} &= -\frac{1}{r_2^2} \end{aligned}$$

But we know that the risk premium for any risk-averse individual facing any risky project p is positive:

$$\begin{aligned} RP(p) &= \sum_{n=2}^{\infty} \frac{(-1)^n}{n! R^{n-1}} V(p)^{\frac{n}{2}} k_n(y) > 0 \;\Leftrightarrow \\ \frac{1}{R} RP(p) &= \frac{1}{R} \sum_{n=2}^{\infty} \frac{(-1)^n}{n! R^{n-1}} V(p)^{\frac{n}{2}} k_n(y) > 0 \end{aligned}$$

Hence:

$$\begin{aligned} \frac{\partial CE}{\partial R} &= \sum_{n=2}^{\infty} \frac{(-1)^n}{n!} \frac{(n-1)}{R^n} V(p^*)^{\frac{n}{2}} k_n(y) \\ &> \frac{1}{R} \sum_{n=2}^{\infty} \frac{(-1)^n}{n! R^{n-1}} V(p)^{\frac{n}{2}} k_n(y) \\ &> 0 \end{aligned}$$

But:

$$\begin{aligned} \frac{\partial CE}{\partial r_2} &= \frac{\partial CE}{\partial R} \frac{\partial R}{\partial r_2} \\ &< 0 \end{aligned}$$

Hence, $CE(r_1, r_2)$ is decreasing in the risk aversion of any partner, since $CE(\cdot, \cdot)$ is symmetric in each of its arguments.

Thus, if $|G1| < |G2|$, for example, it is the most risk-averse excess agents of $G2$ who will remain unmatched in equilibrium. For example, if $G1 = \{r_1^A, r_2^A\}$ and $G2 = \{r_1^B, r_2^B, r_3^B\}$, where agents are ordered from least to most risk-averse, r_3 will be unmatched in equilibrium. Then the main matching results apply to the remaining agents, $G1 = \{r_1^A, r_2^A\}$ and $G2 = \{r_1^B, r_2^B\}$.

4 OA.4: Proof of Proposition 3 (NAM when income-smoothing is shut down)

Proposition 3: *Suppose that all agents in $G1$ draw income iid from a distribution M, and all agents in $G2$ draw income iid from a distribution W, so that all possible pairs face the same joint income distribution. Once matched, agents can commit ex ante to a return-contingent sharing rule. Then the unique equilibrium matching is NAM.*

Proof: Suppose that all pairs must undertake the same project, p. For instance, the government mandates that all farmers must grow rice. This effectively shuts down the income-smoothing channel.

Differentiate $CE(r_1, r_2)$ with respect to r_1 and r_2 when there is no project choice, so that all pairs (r_1, r_2) face the same risky income stream $f(Y_p|p)$. The cross-partial $\frac{d^2 CE(r_1,r_2)}{dr_1 dr_2}$ is:

$$-\frac{r_1 r_2}{(r_1+r_2)^3} \frac{\int e^{-\frac{r_1 r_2}{r_1+r_2} y_p} f(y_p|p) dy_p \int y_p^2 e^{-\frac{r_1 r_2}{r_1+r_2} y_p} f(y_p|p) dy_p - \left[\int y_p e^{-\frac{r_1 r_2}{r_1+r_2} y_p} f(y_p|p) dy_p\right]^2}{\left[\int e^{-\frac{r_1 r_2}{r_1+r_2} y_p} f(y_p|p) dy_p\right]^2}$$

But we know that:

$$\int e^{-\frac{r_1 r_2}{r_1+r_2} y_p} f(y_p|p) dy_p \int y_p^2 e^{-\frac{r_1 r_2}{r_1+r_2} y_p} f(y_p|p) dy_p > \left[\int y_p e^{-\frac{r_1 r_2}{r_1+r_2} y_p} f(y_p|p) dy_p\right]^2$$

since we know variance is always positive. Therefore:

$$\int f(y_p|p) dy_p \int y_p^2 f(y_p|p) dy_p > \left[\int y_p f(y_p|p) dy_p\right]^2$$

and $g(Y_p) = e^{-\frac{r_1 r_2}{r_1+r_2} Y_p}$ is a convex function.
Hence:

$$\frac{\partial^2 CE(r_1, r_2)}{\partial r_1 \partial r_2} < 0$$

and negative assortative matching therefore results as the unique equilibrium.
This corresponds with the result from Chiappori and Reny (2006) and Schulhofer-Wohl (2006).

5 OA.5: Proof of Proposition 4 (PAM when consumption-smoothing is shut down)

Proposition 4: *Suppose that the slope of the sharing rule $s(R_p) = a + bR_p$ is fixed. For example, a wage law requires a 50-50 split of output. However, a matched pair is able to choose a risky project* p, where $Y_p = p + V(p)^{\frac{1}{2}}Y$ as in the benchmark model. *Then the unique equilibrium matching is PAM.*

Proof: We know from the main matching result that the optimal sharing rule is linear. Suppose the slope of the sharing rule $s(Y_p) = a + bY_p$ is fixed at b for all possible pairs of risk types. (For example, the government mandates an equal split of the output, but is unable to prevent partners from making fixed transfers to one another.) Recall from the set up that $Y_p = p + V(p)^{\frac{1}{2}}Y$, $E(Y) = 0$, $V(Y) = 1$.

Fixing b removes consumption-smoothing as an effective risk management tool (since transfers can no longer be conditioned on the realized return), leaving only income-smoothing (project choice). What happens to equilibrium risk-sharing relationships?

A matched pair (r_1, r_2) chooses the relationship-specific transfer a and project p:

$$\max_{a,p} \int -e^{-r_1[-a+(1-b)y_p]} f(y_p|p)dy_p \text{ s.t.}$$

$$\int -e^{-r_2[a+by_p]} f(y_p|p)dy_p \geq -e^{-v}$$

Using the structure on Y_p:

$$\max_{a,p} \int -e^{-r_1[-a+(1-b)(p+V(p)^{\frac{1}{2}}y)]} f(y)dy \text{ s.t.}$$

$$\int -e^{-r_2[a+b(p+V(p)^{\frac{1}{2}}y)]} f(y)dy \geq -e^{-v}$$

The transfer a is chosen to satisfy the constraint (which clearly binds in equilibrium), since the division of output is fixed at b by the government. Basically, the transfer a that r_1 must make to a partner r_2 to ensure her each level of expected utility v measures the value of that relationship.

Therefore:

$$\begin{aligned}
\int e^{-r_2[a+b(p+V(p)^{\frac{1}{2}}y)]} f(y)dy &= e^{-v} \Leftrightarrow \\
e^{-r_2 a} \int e^{-r_2 b(p+V(p)^{\frac{1}{2}}y)} f(y)dy &= e^{-v} \Leftrightarrow \\
-\frac{1}{r_2}\left[-v + r_2 bp - \log \int e^{-r_2 bV(p)^{\frac{1}{2}}y} f(y)dy\right] &= a
\end{aligned}$$

Then the equilibrium project selected is:

$$
\begin{aligned}
p^*(r_1, r_2) &= \arg\max_{\tilde{p}\in\Pi} \int -e^{-r_1\left[\frac{1}{r_2}\left[-v+r_2 b\tilde{p}-\log\int e^{-r_2 bV(\tilde{p})^{\frac{1}{2}}y} f(y)dy\right]+(1-b)(\tilde{p}+V(\tilde{p})^{\frac{1}{2}}y)\right]} f(y)dy \\
&= \arg\max_{\tilde{p}\in\Pi} -e^{\frac{r_1}{r_2}v - r_1\tilde{p}+\frac{r_1}{r_2}\log\int e^{-r_2 bV(\tilde{p})^{\frac{1}{2}}y} f(y)dy} \int e^{-r_1(1-b)V(\tilde{p})^{\frac{1}{2}}y} f(y)dy
\end{aligned}
$$

Transform the objective function by dividing by -1 and taking logs:

$$
\max_{\tilde{p}\in\Pi} \frac{r_1}{r_2}v - r_1\tilde{p} + \frac{r_1}{r_2}\log\int e^{-r_2 bV(\tilde{p})^{\frac{1}{2}}y} f(y)dy + \log\int e^{-r_1(1-b)V(\tilde{p})^{\frac{1}{2}}y} f(y)dy
$$

Then the first-order condition characterizing equilibrium project choice is:

$$
-1 - \frac{b\frac{1}{2}V(p^*)^{\frac{1}{2}}V'(p^*)\int ye^{-r_2 bV(\tilde{p})^{\frac{1}{2}}y} f(y)dy}{\int e^{-r_2 bV(\tilde{p})^{\frac{1}{2}}y} f(y)dy} - \frac{(1-b)\frac{1}{2}V(p^*)^{\frac{1}{2}}V'(p^*)\int ye^{-r_1(1-b)V(\tilde{p})^{\frac{1}{2}}y} f(y)dy}{\int e^{-r_1(1-b)V(\tilde{p})^{\frac{1}{2}}y} f(y)dy} = 0
$$

Then the certainty-equivalent of r_1 is:

$$
\begin{aligned}
-e^{-r_1 CE} &= -e^{\frac{r_1}{r_2}v - r_1 p^* + \frac{r_1}{r_2}\log\int e^{-r_2 bV(p^*)^{\frac{1}{2}}y} f(y)dy} \int e^{-r_1(1-b)V(p^*)^{\frac{1}{2}}y} f(y)dy \Leftrightarrow \\
CE_{r_1} &= -\frac{1}{r_2}v + p^* - \frac{1}{r_2}\log\int e^{-r_2 bV(p^*)^{\frac{1}{2}}y} f(y)dy - \frac{1}{r_1}\log\int e^{-r_1(1-b)V(p^*)^{\frac{1}{2}}y} f(y)dy
\end{aligned}
$$

Then the sum of certainty-equivalents for (r_1, r_2) is:

$$
CE(r_1, r_2) = p^* - \frac{1}{r_2}\log\int e^{-r_2 bV(p^*)^{\frac{1}{2}}y} f(y)dy - \frac{1}{r_1}\log\int e^{-r_1(1-b)V(p^*)^{\frac{1}{2}}y} f(y)dy
$$

Using the first-order condition:

$$
\begin{aligned}
\frac{\partial CE}{\partial r_1} &= \frac{\partial p^*}{\partial r_1} - \frac{1}{r_2}\frac{-r_2 b\frac{1}{2}V(p^*)^{\frac{1}{2}}V'(p^*)\frac{\partial p^*}{\partial r_1}\int ye^{-r_2 bV(p^*)^{\frac{1}{2}}y} f(y)dy}{\int e^{-r_2 bV(p^*)^{\frac{1}{2}}y} f(y)dy} + \frac{1}{r_1^2}\log\int e^{-r_1(1-b)V(p^*)^{\frac{1}{2}}y} f(y)dy \\
&\quad - \frac{1}{r_1}\frac{\left[-(1-b)V(p^*)^{\frac{1}{2}} - r_1(1-b)\frac{1}{2}V(p^*)^{\frac{1}{2}}V'(p^*)\frac{\partial p^*}{\partial r_1}\right]\int ye^{-r_1(1-b)V(p^*)^{\frac{1}{2}}y} f(y)dy}{\int e^{-r_1(1-b)V(p^*)^{\frac{1}{2}}y} f(y)dy} \\
&= \frac{1}{r_1^2}\log\int e^{-r_1(1-b)V(p^*)^{\frac{1}{2}}y} f(y)dy + \frac{(1-b)V(p^*)^{\frac{1}{2}}}{r_1}\frac{\int ye^{-r_1(1-b)V(p^*)^{\frac{1}{2}}y} f(y)dy}{\int e^{-r_1(1-b)V(p^*)^{\frac{1}{2}}y} f(y)dy}
\end{aligned}
$$

And:

$$\frac{\partial CE}{\partial r_2 \partial r_1} = -\frac{(1-b)}{r_1}\frac{1}{2}V(p^*)^{-\frac{1}{2}}V'(p^*)\frac{\partial p^*}{\partial r_2}\frac{\int ye^{-r_1(1-b)V(p^*)^{\frac{1}{2}}y}f(y)dy}{\int e^{-r_1(1-b)V(p^*)^{\frac{1}{2}}y}f(y)dy}$$

$$+\frac{(1-b)}{r_1}\frac{1}{2}V(p^*)^{-\frac{1}{2}}V'(p^*)\frac{\partial p^*}{\partial r_2}\frac{\int ye^{-r_1(1-b)V(p^*)^{\frac{1}{2}}y}f(y)dy}{\int e^{-r_1(1-b)V(p^*)^{\frac{1}{2}}y}f(y)dy}$$

$$-\frac{1}{2}(1-b)^2V'(p^*)\frac{\partial p^*}{\partial r_2}\frac{\int e^{-r_1(1-b)V(p^*)^{\frac{1}{2}}y}f(y)dy\int y^2e^{-r_1(1-b)V(p^*)^{\frac{1}{2}}y}f(y)dy-\left[\int ye^{-r_1(1-b)V(p^*)^{\frac{1}{2}}y}f(y)dy\right]^2}{\left[\int e^{-r_1(1-b)V(p^*)^{\frac{1}{2}}y}f(y)dy\right]^2}$$

$$=-\frac{1}{2}(1-b)^2V'(p^*)\frac{\partial p^*}{\partial r_2}\frac{\int e^{-r_1(1-b)V(p^*)^{\frac{1}{2}}y}f(y)dy\int y^2e^{-r_1(1-b)V(p^*)^{\frac{1}{2}}y}f(y)dy-\left[\int ye^{-r_1(1-b)V(p^*)^{\frac{1}{2}}y}f(y)dy\right]^2}{\left[\int e^{-r_1(1-b)V(p^*)^{\frac{1}{2}}y}f(y)dy\right]^2}$$

$$>0$$

since $\frac{\partial p^*}{\partial r_2} < 0$ and the fraction term is positive. Since the variance of Y is positive:

$$\int f(y)dy \int y^2 f(y)dy > \left[\int yf(y)dy\right]^2$$

and $g(y) = e^{-r_1(1-b)V(p^*)^{\frac{1}{2}}y}$ is a convex function.

Hence, positive assortative matching arises as the unique equilibrium.

So, if the government regulates wages by fixing the slope of the sharing rule at some $b \in [0,1]$, where pairs can still make within-pair state-independent transfers, the unique equilibrium matching pattern is always positive assortative, verifying our intuition that, because consumption-smoothing is held fixed, the "similarity of decisionmaking framework" dominates and people match with people who are like them because they will agree about project choice. This can be thought of as the counterpoint to holding income-smoothing fixed (Appendix 4), as in Chiappori and Reny (2006) and Schulhofer-Wohl (2006).

What are some implications of this understanding? The government may be motivated by equality concerns to specify an equal division of output in every relationship, but this may actually generate even more inequality by weakening the informal risk-management toolkit available to individuals, which then triggers endogenous change in risk-sharing networks. Specifically, if agents had been matched negative assortatively in the status quo (because the "cost function" of project mean is quite convex, say), then this imposition of wage equality leads to positive assortative matching, which may actually exacerbate inequality: there is a bigger spread in projects, with less risk-averse agents on projects with much higher expected returns while more risk-averse agents are on projects with much smaller expected returns, and less risk-averse agents abandon their roles as informal insurers, and more risk-averse agents wind up bearing more risk.

6 OA.6: Background and Empirical Context

The purpose of this section is to provide further empirical context for the model. First, I will discuss the substantial role played by informal insurance motivations in building relationships in risky environments with missing formal insurance and credit markets. Additionally, I will show that risk attitudes are a significant determinant of risk-sharing partner choice.

Next, I will provide evidence that there is a great deal of heterogeneity in risk aversion across individuals in a wide range of settings.

Finally, I will provide evidence of heterogeneity in the riskiness of activities available to individuals, as well as heterogeneity in the relative riskiness of these activities across different environments.

To fix ideas, it may be helpful to envision an agricultural setting, which captures nicely the key elements of the model. Much of the literature discussed in this section is drawn from an agricultural context, where landowners and farmers are heterogeneous in the extent of their risk aversion, and landowners must decide which farmers to work with. Different crops have different yield and profit distributions: some crops are very robust to drought but correspondingly tend to produce low yields on average ("safe" crops), while other crops have the potential for very high yields, but are extremely sensitive to rainfall and other inputs, and blight easily ("risky" crops). In addition to crop portfolio and plot locations, fertilizer and other inputs, irrigation, planting times, and general farming methods and technologies must also be chosen.

Furthermore, the yield and profit distribution of each crop varies across agroclimactic region. Different parts of the world experience different levels of rainfall, soil quality, irrigation, elevation, heat, and other such ecological characteristics, and this influences the stochastic yield and profit of each crop. It is no surprise, then, that equilibrium cropping methods, crop mixes, and contracting institutions vary so widely across region. A goal of this paper is to advance the understanding of these differences.

6.1 Risk Attitude and Informal Insurance Relationships

An abundance of work discusses the considerable role of informal insurance concerns in network formation. People rely on each other to smooth consumption risk and income risk in a wide variety of ways (Alderman and Paxson (1992), Morduch (1995)). A very prevalent consumption-smoothing technique between people is transfers and remittances, and much work has been done to study the nature of the transfers that can be sustained given a risk-sharing group, the shapes of equilibrium networks holding fixed some transfer rule, and who is empirically observed to make transfers with whom (Townsend (1994), Fafchamps and Lund (2003), Genicot and Ray (2003), Bramoulle and Kranton (2007), and Ambrus et al. (2013), to name a few). A general message these papers convey is that the need to manage risk in the absence of formal insurance institutions has huge effects on interpersonal relationships among the poor.

In fact, risk management can affect relationship formation in very specific ways. Rosenzweig and Stark (1989) show that daughters are often strategically married to villages located in environmentally dissimilar regions with minimally correlated farming incomes, for the purposes of consumption-smoothing; households exposed to more income risk are more likely to invest in longer-distance marriage arrangements. Ligon et al. [cite], Fafchamps [1999], and Kocherlakota [1996], among many others, analyze a pure risk-sharing relationship between two heterogeneously risk-averse households who perfectly observe each other's income. Ackerberg and Botticini [cite] study agricultural contracting in medieval Tuscany, and find evidence that heterogeneously risk-averse tenant farmers and landlords strategically formed sharecropping relationships based on differing risk attitudes, motivated by risk management concerns. Hence, informal insurance motivations play a substantial role in the formation of relationships.

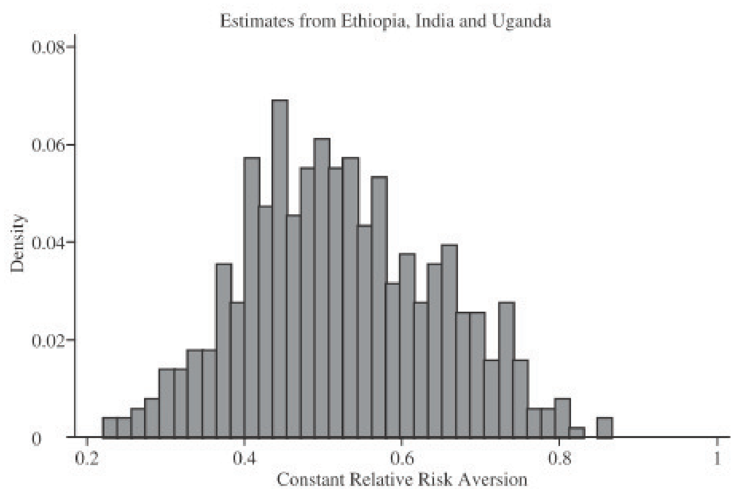
But how much do individuals care about the risk attitudes of potential partners when forming risk-sharing groups? Naturally, there are many other reasons people might match with each other, but the point of the model is to focus on one important determinant of risk-sharing relationship formation, and to study how equilibrium matching patterns shift along that dimension. Furthermore, there is a great deal of evidence that the risk attitudes of partners are indeed a primary determinant of risk-sharing partner choice. Ackerberg and Botticini (2002) provide empirical evidence supporting the presence of endogeneity of matching along risk attitude of landowners and sharecroppers in medieval Tuscany. In their data, they find that share contracts were associated with the safer crop of cereal, while fixed rent (residual claimancy) contracts were associated with the riskier crop of vines. They argue that this is the outcome of endogenous matching–risk-neutral tenants may have been assigned to the riskier crops, resulting in fixed rent contracts for vines, while risk-averse tenants may have been assigned to the safer crops, resulting in share contracts on cereals.

Additional evidence for the importance of risk attitudes as a determinant of risk-sharing relationships is found in Gine et al. (2010) and Attanasio et al. (2012). Gine et al. (2010) run an experiment on small-scale entrepreneurs in urban Peru and allow joint liability groups to form endogenously in a microfinance setting. They find strong evidence of assortative matching along risk attitude. Attanasio et al. (2012) run a unique experiment with 70 Colombian communities. They gather data about risk attitudes and pre-existing kinship/friendship networks, and then allow individuals to form risk-sharing groups of any size. Attanasio et al. find that, when members know each other's risk types, and trust each other (family and friends are in the same group), conditioning on all other potential reasons for matching which they are able to account for (gender, age, geography), there is strong evidence of positive assortative matching along risk aversion.

To further emphasize the significant role of risk attitude in determining risk-sharing relationship formation, I use the dataset from Attanasio et al. (2010) to calculate the proportion of formed links that involved at least one family or one friendship tie, for each municipality. The mean of these proportions is 0.005, or 0.5%. Since it's possible that there were very few family and friend ties reported in the entire dataset to begin with, I also calculate the proportion of all possible links that could have involved at least one family or friendship tie, for each municipality. The mean of this number is 0.05. Hence, this back-of-the-envelope calculation suggests that, in this setting, only about 10% of all possible risk-sharing relationships which could have involved a family or friendship tie, actually did involve such a tie. Hence, while one might expect kinship and friendship to be major influences in partner choice, there is strong evidence that risk attitude is the more prominent consideration when the partner is being chosen specifically for the purposes of informal insurance. In particular, the family and friendship tie is likely to influence the pool of potential partners (because individuals are less likely to know the risk attitudes of strangers, or to trust them), but the choice of partner from this pool for the purposes of insurance is primarily determined by risk attitudes.

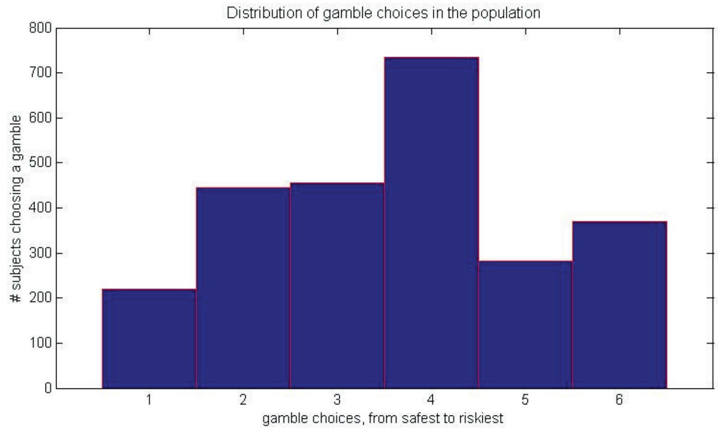
6.2 Heterogeneity in Risk Aversion

The second key piece of the model is heterogeneity in risk attitudes across individuals. There is plenty of evidence that people are risk-averse and that they are heterogeneous in their risk-aversion. Experiments which elicit risk attitudes by asking subjects to choose from a set of gambles differing in riskiness find much variation in gamble choice. For example, Harrison et al. (2010) asked 531 experimental subjects drawn from India, Ethiopia, and Uganda to choose a gamble from a set of gambles varying in riskiness (a riskier gamble has higher mean but correspondingly higher variance), in a similar spirit as the seminal study by Binswanger (1980), and estimated the density of CRRA risk attitudes:



It's clear that there is a substantial amount of variation, and almost every point in the $[0, 1]$ range is represented.

In another experiment involving over 2,000 people living in 70 Colombian communities, where 66% live in rural areas, Attanasio et al. (2012) observes the following distribution (gamble 1 is the safest gamble, while gamble 6 is the riskiest):



Chiappori et al. (2010) use two distinct methods to measure heterogeneity in risk preferences in Thai villages, where these villages are spread across several regions in Thailand. The first method is based off the co-movement of individual consumption with aggregate consumption, and the second is based off of optimal portfolio choice theory. Using both methods, they find substantial heterogeneity in risk attitudes in each village. Moreover, this heterogeneity varies across villages and regions. The following table reports the means of risk tolerance for each of 16 villages, and the test statistic for heterogeneity:

		risk aversion γ_i			risk tolerance $1/\gamma_i$		
village	households	mean	χ^2	p-value	mean	χ^2	p-value
			Chachoengsao				
2	13	2.00	277.29	0.0000	1.56	3543.60	0.0000
4	21	0.79	78.44	0.0000	2.47	1646.42	0.0000
7	6	0.98	6.69	0.3509	1.28	32.21	0.0000
8	14	0.61	31.11	0.0053	5.11	7986.64	0.0000
			Buriram				
2	18	0.62	12.54	0.8184	2.97	368.59	0.0000
10	8	0.34	5.87	0.6618	4.02	147.64	0.0000
13	10	0.41	14.27	0.1610	7.61	2255.00	0.0000
14	15	0.84	73.55	0.0000	3.55	4209.49	0.0000
			Lop Buri				
1	19	1.20	96.08	0.0000	1.36	1011.17	0.0000
3	8	2.12	348.07	0.0000	1.33	3981.73	0.0000
4	27	1.40	173.59	0.0000	1.29	2061.54	0.0000
6	24	1.82	485.27	0.0000	1.29	3074.97	0.0000
			Sisaket				
1	22	0.43	21.94	0.4633	3.78	457.10	0.0000
6	34	0.78	117.07	0.0000	1.85	2010.67	0.0000
9	22	0.76	33.96	0.0495	3.24	2141.48	0.0000
10	13	0.47	9.68	0.7199	2.90	36.03	0.0006
			pooled				
-	274	0.98	1358.43	0.0000	2.64	77568.89	0.0000

Again, it is clear that there is widespread variation in the degree of risk aversion across households.

6.3 Heterogeneity in Risky Activities and Settings

Finally, agents in a given setting have a wide variety of investment options and household decisions to make, which vary in riskiness. For example, a farmer must choose a spatial distribution of his plots, what lumpy purchases to make (e.g. bullocks), and when and how to plant his crop. A microentrepreneur must decide what kind of business he wants to start. Parents must decide how to invest household resources, and whom their children will marry. Individuals face a diversity of choices, and how much diversity, as well as the relative riskiness of one decision compared to another, varies across settings.

For example, Rosenzweig and Binswanger (1993) consider the equilibrium crop portfolio choices of heterogeneously risk-averse farmers living in six ICRISAT villages located across three distinct agroclimactic regions in India. The first region is characterized by low levels of erratically distributed rainfall and soils with limited water storage capacity (this is the riskiest environment), the second region by similarly erratic rainfall and irrigation but better soil storage capacity, and the third region by low levels of more reliable rainfall with reasonable soil storage capacity (this is the safest environment). The principal crops grown are sorghum, pigeon peas, pearl millet, chickpeas, and groundnuts, and their yield distributions vary across environment. They show that differences in risk aversion do translate into differences in choice of risky investments. Individuals are influenced by risk-reduction when choosing income streams, particularly in response to limitations on *ex post* consumption-smoothing, and the degree to which they are influenced depends on their risk aversion.

Dercon (1996) also studies the variation in riskiness of agricultural investment decisions by heterogeneously risk-averse rural households. His data is drawn from Tanzania, a country with very underdeveloped credit markets (in 1989, only 5% of commercial bank lending went to the private sector, and less than 10% of this lending went to individual farmers). The UN Food and Agriculture Organization provides an interesting look at the vast heterogeneity in crop yield distributions and equilibrium crop mix across regions in Tanzania in 1998. The following table shows the area, yield, and production of each of five crops across ten agroclimactically heterogeneous regions in Tanzania[1]:

[1] Of course, in addition to levels and fluctuations of crop yields, farmers care about the levels and fluctuations of crop prices, as they care ultimately about the distribution of profits.

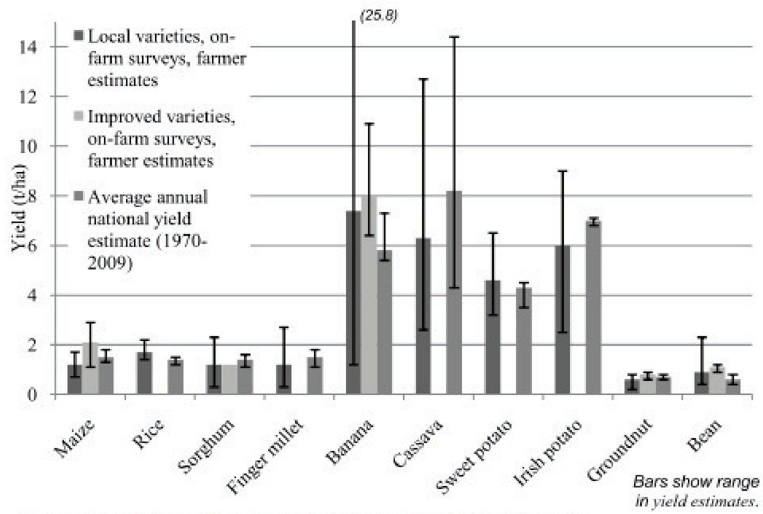
Table 2: Tanzania - 1998 Vuli Crop Production in Cereal Equivalent by Region

	Maize			Sorghum			Paddy			Millet			Wheat		
Region	**Area ('000 ha)**	**Yield (kg/ha)**	**Production (tonnes)**	**Area ('000 ha)**	**Yield (kg/ha)**	**Production (tonnes)**	**Area ('000 ha)**	**Yield (kg/ha)**	**Production (tonnes)**	**Area ('000 ha)**	**Yield (kg/ha)**	**Production (tonnes)**	**Area ('000 ha)**	**Yield (kg/ha)**	**Production (tonnes)**
Mara	13.0	1 700	22.1	20.0	1 300	26.0	1.0	1 600	1.6	0.5	1 100	0.6	0.0	0	0.0
Arusha	50.0	1 600	80.0	10.0	1 200	12.0	2.6	2 300	5.98	3.4	1 000	3.4	14.0	1 700	23.8
Kilimanjaro	30.0	1 500	45.0	1.5	1 300	2.0	3.2	2 500	8	-	-	-	1.2	1 400	1.7
Tanga	20.0	1 500	30.0	-	-	-	2.1	1 900	3.99	-	-	-	0.7	1 000	0.7
Morogoro	40.0	1 400	56.0	8.0	1 200	9.6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Mbeya	20.0	1 600	32.0	0.0	0.0	0.0	1.0	1 800	1.8	0.0	0.0	0.0	0.0	0.0	0.0
Coast/DSM	11.0	1 100	12.1	-	-	-	-	-	-	-	-	-	-	-	-
Kagera	51.0	1 500	76.5	2.7	1 300	3.5	0.1	2 000	0.2	-	-	-	-	-	-
Kigoma	40.0	1 800	72.0	3.0	1 500	4.5	3.0	2 000	6	0.1	1 000	0.1	-	-	-
Mwanza	100.0	1 200	120.0	110.0	1 100	121.0	10.0	1 800	18	1.0	1 000	1.0			
Total	**375**		**545.7**	**155.2**		**178.6**	**23.0**		**45.6**	**5.0**		**5.1**	**15.9**		**26.2**

Unfortunately, this table excludes estimates of the *variance* of yield of each of these crops across regions. Dercon (1996) provides a discussion of this in his paper. He describes a multiplicity of soils and irrigation systems in Tanzania, which support different crops. Paddy, a crop which can yield a high return, is restricted only to specific soils and areas close to a river, and is the least drought and locust resistant. Despite the potential for high returns, only 11% of the total cultivation sample grew paddy. On the other hand, sorghum yields only a low-moderate return, but all soils can sustain it, and it is more resistant to drought and pests. Even though it had a lower mean return, it was grown by all but two households in the sample.

Uganda and Ethiopia are similar to Tanzania in the set of crops grown, though the actual crop mix grown differs due to differences in environmental conditions. An IFPRI report from 2011 estimating crop yields in Uganda provides a useful illustration of how the variance of crop yields differs across crops, and the typical relationship of the variance with the mean:

Figure A.1—Average yield and yield range of selected unfertilized crops observed in on-farm surveys (farmer recall) in Uganda, by variety, type, and average of annual national yield estimates (1970–2000)



Source: Compiled by authors based on database of crop yields in Uganda.

There is a clear positive relationship between mean yield and variance of yield. Groundnuts have low mean yields and correspondingly low fluctuation of yields, making it a "safer" crop, while banana has much higher mean yields but correspondingly higher fluctuation of yields, making it a "riskier" crop.

Abebe et al. (2010) provide a similar graphic for Ethiopia:

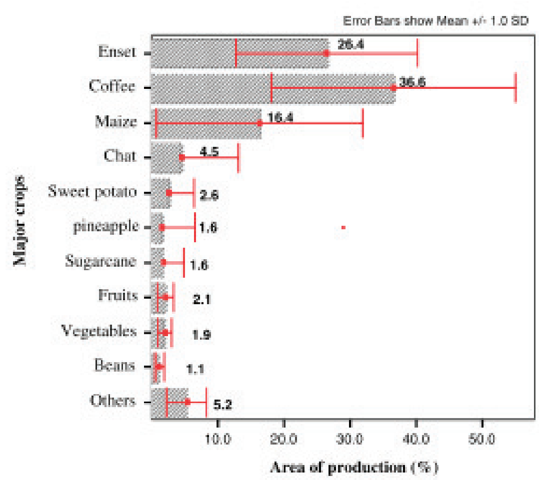


Fig. 3 Mean area share (in percentage of the farm area) of the major crops in farms. All 144 farms are used here. *Error bars* indicate one standard deviation

(Enset is a type of banana.)

Again, we get a general sense that higher mean yield crops have a higher variance of yield, while lower mean yield crops have a smaller variance of yield. Comparing across Uganda and Ethiopia, we see that maize is a safer crop relative to sweet potato in Uganda, but the opposite is true in Ethiopia. Thus, the same set of crops have very different yield distributions in different settings, and furthermore, each crop's relative riskiness with the other crops also varies across setting.