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ESCAPING THE GREAT RECESSION

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ABSTRACT

High uncertainty is an inherent implication of the zero lower bound, while deflation is not because of inflationary pressure due to uncertainty about how debt will be stabilized. We show that policy uncertainty empirically accounts for the absence of deflation in the US economy. Announcing fiscal austerity is detrimental in the short run, but it preserves macroeconomic stability. On the other hand, a recession can be mitigated by abandoning fiscal discipline, at the cost of increasing macroeconomic instability. The policy trade-off can be resolved by committing to inflating away only the portion of debt accumulated during the recession.

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1 Introduction

The recent financial crisis and the recession that followed led to an increase in macroeconomic uncertainty (Jurado, Ludvigson, and Ng 2013) and to a substantial change in the conduct of monetary policy, with interest rates stuck at the zero lower bound for the past five years. While in a new-Keynesian framework the zero lower bound is associated with deflation, inflation in the data has remained remarkably close to its target value. Following Hall's Presidential Address to the American Economic Association, some researchers have labeled this observation the "Bob Hall puzzle" (Hall 2011). At the same time, the crisis has triggered a widespread policy debate about the best way to mitigate the consequences of a deep recession once monetary policy is constrained by the zero lower bound. While this debate is animated by a wide spectrum of opinions, there seem to be two popular polar views. The first one advocates a discontinuity with respect to the policies of the past, calling for a robust fiscal intervention, perhaps associated with a reduction on the focus on inflation stabilization. The second one strongly opposes the idea of explicitly abandoning policies that have arguably led to a stable macroeconomic environment since the Volcker disinflation. As a result of this debate, policy uncertainty is at historical maxima (Baker, Bloom, and Davis 2011). In this paper, we will show that policy uncertainty can account for both the absence of deflation and the increase in macroeconomic uncertainty.

We construct a dynamic general equilibrium model that captures the policy trade-off that seems to characterize the current economic environment: choosing between mitigating a large recession and preserving a reputation for fiscal discipline. In the model, when the zero lower bound is not binding, policymakers' behavior is characterized by two very distinct policy combinations. Under the Monetary led policy mix, the fiscal authority moves primary surpluses in response to fluctuations in the ratio of public debt to gross domestic product (GDP), while the central bank reacts strongly to deviations of inflation from its target. If agents expect this regime to prevail for a long time, any fiscal imbalance is backed by future fiscal adjustments and reputation for fiscal discipline is strong. Under the Fiscally led policy mix, the fiscal authority does not react strongly enough to debt fluctuations and the central bank disregards the Taylor principle. In this second case, agents understand that policymakers are unlikely to implement the fiscal adjustments necessary to preserve debt stability. Finally, the economy can be hit by a large swing in preferences that induces agents to substantially reduce consumption. In this case, a standard Taylor rule would imply a negative nominal interest rate. This forces policymakers into a zero lower bound regime in which the federal funds rate is restricted to zero and the fiscal authority disregards the level of debt in an attempt to mitigate the resulting deep recession. As in Krugman (1998), Eggertsson and Woodford (2003), and Christiano, Eichen-

¹In the language of Leeper (1991) the Monetary led regime corresponds to Active Monetary policy and Passive Fiscal policy, whereas the Fiscally led regime is associated with Passive Monetary policy and Active Fiscal policy.

baum, and Rebelo (2011), the real interest rate is now too high with respect to what would be desirable. Policymakers would then find it beneficial to induce a jump in inflation expectations in order to cause a drop in real interest rates and push the economy out of the recession.

Given that at the zero lower bound policymakers' behavior is constrained, agents' beliefs about policymakers' behavior once the economy is out of the zero lower bound play a key role in determining macroeconomic outcomes at the zero lower bound. We model this idea by introducing a parameter that controls agents' beliefs about policymakers' exit strategy. We estimate the model and we find that during the recent crisis the probability assigned to a switch to the Fiscally led regime experienced a discrete increase, even if agents still regard a return to the Monetary led regime as more likely (around 92%). Even if the estimated probability of a switch to the Fiscally led regime is relatively low, the inflationary pressure deriving from the large stock of debt is enough to prevent the economy from entering a deflationary state. In order to highlight the importance of policy uncertainty, we use a counterfactual simulation to point out that the US economy would have experienced large deflation if the Monetary led regime had been the only possible one. In other words, absent policy uncertainty, the consequences of the current recession would have been much more severe because the increase in debt would not have implied any inflationary pressure.

We then use counterfactual simulations to point out that while high uncertainty is an inherent implication of the economy entering the zero lower bound, deflation is not. This is because agents are uncertain about the way policymakers will deal with the large stock of debt arising from a severe recession. Specifically, we study the consequences of removing policy uncertainty and introducing explicit announcements about future policymakers' behavior in the aftermath of a large shock. If policymakers announce that as the economy exits the zero lower bound, a prolonged period of fiscal discipline will follow, inflation expectations drop, leading to deflation and a severe recession. Uncertainty remains high because policymakers cannot use monetary policy to mitigate the consequences of the shocks and agents are uncertain about the end of the recession. If instead policymakers announce a prolonged deviation from the Monetary led policy mix, inflation immediately increases because agents expect that debt will be inflated away. This, in turn, leads to a drop in the real interest rate that pushes the economy out of the recession and the economy is able to avoid the zero lower bound. Finally, if policymakers do not make any explicit announcements about the way they will deal with an increasing stock of debt once the economy will be out of the zero lower bound, agents are likely to form expectations that take into account all possible alternative scenarios. In this case, the model is able to rationalize why, despite the time spent at the zero lower bound, we have not observed deflation in the United States. Therefore, Whether or not deflation occurs or not at the zero lower bound depends on which exit strategy agents regard to be the most likely.

Macroeconomic uncertainty turns out to be high as long as the economy remains at the

zero lower bound. This is because of three reasons. First, given that the timing of the end of the recession is unknown, the possibility of a large swing in real activity creates uncertainty. Second, the fact that the federal funds rate (FFR) is stuck at zero implies that policymakers cannot mitigate the consequences of the shocks hitting the macroeconomy. Finally, fiscal shocks have large effects on the macroeconomy because of the increase in policy uncertainty. Modest changes in the relative probability of the different exit strategies do not get rid of macroeconomic uncertainty. If agents expect a return to the Monetary led regime, uncertainty is high because both inflation and real economic activity are far from where they will be once out of the recession. If the probability assigned to the Fiscally led regime increases, inflation and output move closer to their respective steady states, but agents come to realize that all shocks that are neutralized under the Monetary led regime will now be inflated away, causing an increase in macroeconomic uncertainty. These results square well with recent contributions by Baker, Bloom, and Davis (2013), that show that policy uncertainty has increased since the beginning of the recession, Kitsul and Wright (2013), that point out that markets seem to swing between fear of inflation and fear of deflation, and Jurado, Ludvigson, and Ng (2013) and Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2012) that document an overall increase in macroeconomic uncertainty during the recent recession.

Only a clear announcement of a switch to the Fiscally led regime would push the economy out of the zero lower bound. However, announcing a switch to the Fiscally led policy mix would also result in an increase in macroeconomic volatility once the economy is out of the zero lower bound. The two results go together. The announcement is effective if and only if it is able to convince agents that the Fiscally led policy mix will prevail for a long time. In this situation the macroeconomy is not insulated with respect to fiscal disturbances. When policymakers are expected to follow the Monetary led rule for many periods ahead, all the shocks that hit the debt-to-GDP ratio are neutralized by the fiscal authority and the economy is therefore insulated with respect to fiscal disturbances. However, if the Fiscally led regime is expected to be in place most of the time, agents realize that inflation, not taxation, will be used to keep debt on a stable path. Therefore, all the fiscal imbalances that are systematically neutralized under the Monetary led regime will now affect inflation. In the presence of nominal rigidities, inflation volatility translates into output volatility, resulting in a more uncertain macroeconomic environment.

In summary, a policy trade-off arises the moment that a large negative preference shock pushes monetary policy to the zero lower bound. The fact that the Monetary led regime results in a more stable macroeconomic environment in the long run provides support to those who are reluctant to explicitly abandon the policies that prevailed from the Volcker disinflation to the current crisis. Yet, the possibility of mitigating the recession by moving to the Fiscally led regime can explain why some policymakers and economists have suggested discontinuity with respect to the past.

It is then natural to ask if it is possible to go beyond these two polar views. In other words, it would be interesting to see if it is possible to escape the Great Recession by generating an increase in inflation expectations via the fiscal mechanism outlined in this paper and at the same time preserving long-run macroeconomic stability. We show that in fact a way out exists: Policymakers could commit to inflate away only the portion of debt resulting from the exceptionally large recession.² This shock specific rule provides a sort of automatic stabilizer: The large negative preference shock can lead to a deep recession and a corresponding large increase in the debt-to-GDP ratio. The expectation that this extra fiscal burden is going to be inflated away causes a drop in the real interest rate that stimulates demand, reducing the size of the output contraction and the amount of debt that needs to be inflated away. This mechanism can be strong enough to prevent the economy from hitting the zero lower bound. Furthermore, given that the recession is now largely mitigated, the resulting increase in the debt-to-GDP ratio is small and so is the increase in inflation necessary to stabilize it.

At the same time, policymakers never changed their behavior with respect to the pre-crisis stock of debt and in response to other exogenous business cycle disturbances that are unlikely to push the economy to the zero lower bound. This has two very important consequences. First, the level of debt that existed before the crisis is irrelevant for the amount of inflation that is generated because it is still backed by future fiscal adjustments. Second, agents expect that all future fiscal imbalances will still be taken care of by the fiscal authority. Therefore, the proposed policy is successful in mitigating the recession and preserving long-run stability.

This paper is organized as follows. Section 2 reviews the related literature. Section 3 presents the benchmark model. Section 4 shows that policy uncertainty can account for the lack of deflation and high macroeconomic uncertainty. Section 5 outlines the policy trade-off that arises at the zero lower bound: Mitigating a recession at the cost of losing long run macroeconomic stability. Section 6 proposes the shock-specific policy response. Section 7 concludes.

2 Related Literature

Our work is related to the vast theoretical literature on the zero lower bound. Wolman (1998), Fuhrer and Madigan (1994), Krugman (1998), and Orphanides and Wieland (1998, 2000) are among the first to study the zero lower bound and monetary policy in an intertemporal framework. Eggertsson and Woodford (2003) show that optimal monetary policy at the zero lower bound involves a commitment to generate future inflation. Eggertsson (2006) argues that such a policy can suffer from a time-inconsistency problem, while Eggertsson (2008), using a model

²This policy has been advocated by Krugman (2013) and Rogoff (2008) among several others.

in which taxation is costly, shows that President Franklin Delano Roosevelt was able to make the promise of future inflation credible by expanding fiscal deficits. Benhabib, Schmitt-Grohe, and Uribe (2001b) show that active monetary policy rules can lead to a liquidity trap, while Benhabib, Schmitt-Grohe, and Uribe (2002) explain how fiscal and monetary policies can be designed in order to rule out deflationary spirals. Correia, Farhi, Nicolini, and Teles (2012) show how distortionary taxes can be used to replicate the effects of negative nominal interest rates and completely circumvent the zero bound problem. Werning (2012) works in a deterministic environment and shows that the effectiveness of policies at the zero lower bound crucially depends on what agents expect after the constraint is not binding anymore. Fernandez-Villaverde, Guerron-Quintana, and Rubio-Ramirez (2011) show how supply-side policies may play a role in preventing an economy from hitting the zero lower bound. Galí (2014) studies the effects of a money-financed fiscal stimulus. Schmitt-Grohe and Uribe (2012) present a model that can account for a recession associated with a protracted liquidity trap and a jobless recovery.

Our work differs from each of these papers in one or more of the following dimensions. First, we investigate the effects of policy uncertainty at the zero lower bound on macroeconomic uncertainty. In this respect, the paper is related to the literature on the macroeconomic effects of uncertainty (e.g., Bloom 2009; Fernandez-Villaverde, Guerron-Quintana, Rubio-Ramirez, and Martin Uribe 2011; Gilchrist, Sim, and Zakrajsek 2012; Williams 2012; and Basu and Bundick 2012). Second, we work in a stochastic environment (not perfect foresight/deterministic) with a standard new-Keynesian model augmented with a fiscal block. This makes our framework suitable for a quantitative assessment of the different exit strategies. Third, zero lower bound episodes are recurrent, and agents take this into account when forming expectations. In contrast, the literature generally considers situations in which the economy is currently at the zero lower bound and it will never be there again. Moreover, our paper proposes an alternative way for modeling recurrent zero-lower-bound events in microfounded dynamic stochastic general equilibrium (DSGE) models to those of Gust, Lopez-Salido, and Smith (2013) and Aruoba and Schorfheide (2013), and is related to the growing literature that allows for parameter instability in DSGE models (Justiniano and Primiceri 2008). Finally, our results are based on the possibility of generating an increase in inflation expectations through a change in the monetary/fiscal policy combination and do not require the use of distortionary taxation.

Fernandez-Villaverde, Kuester, Guerron-Quintana, and Rubio-Ramirez (2013) and Johannsen (2013) use higher order approximations to study the role of fiscal volatility in slowing down the recovery during the current crisis, but they assume that government debt is always backed by future fiscal surpluses while agents face uncertainty regarding the magnitude of the innovations to the fiscal instruments. In our model, agents instead are uncertain about the rules governing policymakers' behavior. We will show that this kind of policy uncertainty can be beneficial at the zero lower bound because it prevents deflation. An interesting extension for future research

would consist of integrating the two approaches.

Other papers have addressed Bob Hall puzzle (Ball and Mazumder 2011; King and Watson 2011; and Del Negro, Giannoni, and Schorfheide 2013). Unlike those contributions, this paper focuses on the consequences of uncertainty about future policymakers' behavior, showing that policy uncertainty in an otherwise standard New Keynesian model accounts for both the lack of deflation and the high uncertainty observed at the zero lower bound.

Our choice of working with regimes gives us the possibility of capturing the consequences of policy uncertainty and to compare different scenarios. Other authors have approached the problem of the zero lower bound from a different angle, i.e., by solving for optimal policies. While such an approach has provided the theoretical foundations of our understanding of the zero lower bound, it does not leave space for comparative analysis or the possibility of allowing for policy uncertainty in the moment that one optimal policy emerges. Accounting for policy uncertainty is important in light of a growing literature that argues that there were in fact changes in policymakers' behavior over the past 60 years (Clarida, Gali, and Gertler 2000; Lubik and Schorfheide 2004; Davig and Doh 2013; Fernandez-Villaverde, Guerron-Quintana and Rubio-Ramirez 2010; and Bianchi 2013). Finally, since the seminal contribution of Sims and Zha (2006), regimes have become a popular way to think about changes in policymakers' behavior in applied work.

This paper is related to a research agenda that aims to understand the role of fiscal policy in explaining changes in the reduced form properties of the macroeconomy. Using a Markov-switching DSGE model, Bianchi and Ilut (2012) show that the rise and fall of US inflation can be explained in light of a change in the monetary/fiscal policy mix that occurred a few years after the appointment of Paul Volcker as Federal Reserve Chairman. Bianchi and Melosi (2013) introduce the notion of dormant shocks, showing that a fiscal imbalance can lead to an increase in inflation many years after it occurred. This paper differs from the two aforementioned contributions across several dimensions. First, we here allow for the zero lower bound and show that policy uncertainty can account for the absence of deflation. Second, we outline that at the zero lower bound a policy trade-off between mitigating a large recession and preserving long run macroeconomic stability emerges. Finally, we show how policymakers can resolve this trade-off using a shock-specific rule.

Our work is then related to the study of the interaction between fiscal and monetary policies in determining inflation dynamics (Sargent and Wallace 1981; Leeper 1991; Sims 1994; Woodford 1994, 1995, 2001; Schmitt-Grohe and Uribe 2000; Cochrane 1998, 2001; among many others) and to the vast literature on fiscal multipliers (Blanchard and Perotti 2002; Mountford and Uhlig 2009; Uhlig 2010; Romer and Romer 2010; Mertens and Ravn 2011, 2013; Leeper, Walker, and Yang 2013; Misra and Surico 2013). Mertens and Ravn (2014) and Drautzburg and Uhlig (2011) use a DSGE model to study the fiscal multiplier when interest rates are stuck

at the zero bound. Coibion, Gorodnichenko, and Wieland (2012) study the optimal inflation target in a new-Keynesian model in which the policy rate occasionally gets constrained by the zero lower bound.

3 The Model

In this section we introduce the model that we will fit to US data in order to quantify the importance of policy uncertainty. The model is based on Bianchi and Ilut (2012) and is obtained by augmenting the prototypical new-Keynesian model used by Clarida, Galí, and Gertler (2000) and Lubik and Schorfheide (2004) with external habits, a maturity structure for debt, and a fiscal block.

3.1 A New-Keynesian Model

Households. The representative household maximizes expected utility:

$$E_0 \left[\sum_{s=0}^{\infty} \beta^t \exp\left(\zeta_t^d\right) \left[\log\left(C_t - \Phi C_{t-1}^A\right) - h_t \right] \right] \tag{1}$$

subject to the budget constraint:

$$P_tC_t + P_t^m B_t^m + P_t^s B_t^s = P_t W_t h_t + B_{t-1}^s + (1 + \rho P_t^m) B_{t-1}^m + P_t D_t - T_t + T R_t$$

where D_t stands for real dividends paid by the firms, C_t is consumption, h_t is hours, W_t is the real wage, T_t is taxes, TR_t stands for transfers, and C_t^A represents the average level of consumption in the economy. The parameter Φ captures the degree of external habit. In line with Cochrane (2001), we recognize the importance of allowing for a maturity structure of government debt. Longer maturities imply important fluctuations in the return on bonds and consequently in the present value of debt. Hall and Sargent (2011) show that these revaluation effects explain a significant fraction of the fluctuations in the debt-to-GDP ratio. Following Eusepi and Preston (2012) and Woodford (2001), we assume that there are two types of government bonds: one-period government debt, B_t^s , in zero net supply with price P_t^s and a more general portfolio of government debt, B_t^m , in non-zero net supply with price P_t^m . The former debt instrument satisfies $P_t^s = R_t^{-1}$. The latter debt instrument has payment structure $\rho^{T-(t+1)}$ for T > t and $0 < \rho < 1$. The value of such an instrument issued in period t in any future period t + j is $P_{t+j}^{m-j} = \rho^j P_{t+j}^m$. The asset can be interpreted as a portfolio of infinitely many bonds, with weights along the maturity structure given by $\rho^{T-(t+1)}$. Varying the parameter ρ varies the average maturity of debt.

The preference shock ζ_d is the sum of a continuous and discrete component: $\zeta_d = d_t + \overline{d}_{\xi_t^d}$.

The continuous component d_t has mean zero and time series representation: $d_t = \rho_d d_{t-1} + \sigma_d \varepsilon_{d,t}$. The discrete component $\overline{d}_{\xi_t^d}$ can assume two values: high or low $(\overline{d}_h \text{ or } \overline{d}_l)$. The variable ξ_t^d controls the regime in place and evolves according to the transition matrix H^d :

$$H^d = \left[egin{array}{cc} p_{hh} & 1-p_{ll} \ 1-p_{hh} & p_{ll} \end{array}
ight],$$

where $p_{ji} = P\left(\xi_{t+1}^d = j | \xi_t^d = i\right)$. The values of H^d , \overline{d}_h , and \overline{d}_l are such that the unconditional mean of the discrete shock $\overline{d}_{\xi_t^d}$ is zero. This specification is in the spirit of Christiano, Eichenbaum, and Rebelo (2011). However, in the current setup shocks to preferences that are able to trigger the zero lower bound are assumed to be recurrent, and agents take into account that these episodes can lead to unusual policymakers' responses, as discussed later on.

Firms. The representative monopolistically competitive firm j faces a downward-sloping demand curve:

$$Y_t(j) = (P_t(j)/P_t)^{-1/\nu_t} Y_t$$
(2)

where the parameter $1/v_t$ is the elasticity of substitution between two differentiated goods. Firms take as given the general price level, P_t , and the level of real activity, Y_t . Whenever a firm changes its price, it faces quadratic adjustment costs represented by an output loss:

$$AC_t(j) = .5\varphi \left(P_t(j) / P_{t-1}(j) - \Pi \right)^2 Y_t(j) P_t(j) / P_t$$
(3)

where $\Pi_t = P_t/P_{t-1}$ is gross inflation at time t and Π is the corresponding steady state. Shocks to the elasticity of substitution imply shocks to the markup $\aleph_t = 1/(1 - v_t)$. We assume that the rescaled markup $\mu_t = \kappa \log (\aleph_t/\aleph)$ follows an autoregressive process, $\mu_t = \rho_\mu \mu_{t-1} + \sigma_\mu \epsilon_{\mu,t}$, where $\kappa \equiv \frac{1-v}{v\varphi\Pi^2}$ is the slope of the Phillips curve. The firm chooses the price $P_t(j)$ to maximize the present value of future profits:

$$E_t \left[\sum_{s=t}^{\infty} Q_s \left(\left[P_s(j) / P_s \right] Y_s(j) - W_s h_s \left(j \right) - A C_s(j) \right) \right]$$

where Q_s is the marginal value of a unit of consumption good. Labor is the only input in the firm production function, $Y_t(j) = A_t^{1-\alpha}h_t(j)$, where total factor productivity A_t evolves according to an exogenous process: $\ln(A_t/A_{t-1}) = \gamma + a_t$, $a_t = \rho_a a_{t-1} + \sigma_a \varepsilon_{a,t}$, $\epsilon_{a,t} \sim N(0,1)$.

Government. Imposing the restriction that one-period debt is in zero net supply, the flow budget constraint of the federal government is given by:

$$P_t^m B_t^m = B_{t-1}^m (1 + \rho P_t^m) - T_t + E_t + T P_t$$

where $P_t^m B_t^m$ is the market value of debt and T_t and E_t represent federal tax revenues and

federal expenditures, respectively. Government expenditure is the sum of federal transfers and goods purchases: $E_t = P_tG_t + TR_t$. The term TP_t is a shock that is meant to capture a series of features that are not explicitly modeled here, such as changes in the maturity structure and the term premium. This shock is necessary to avoid stochastic singularity when estimating the model given that we treat debt, taxes, and expenditures as observables.³ We rewrite the federal government budget constraint in terms of debt-to-GDP ratio $b_t^m = (P_t^m B_t^m) / (P_t Y_t)$:

$$b_t^m = \left(b_{t-1}^m R_{t-1,t}^m\right) / \left(\prod_t Y_t / Y_{t-1}\right) - \tau_t + e_t + t p_t$$

where all the variables are now expressed as a fraction of GDP, $R_{t-1,t}^m = (1 + \rho P_t^m)/P_{t-1}^m$ is the realized return of the maturity bond, and we assume $tp_t = \rho_{tp}tp_{t-1} + \sigma_{tp}\varepsilon_{tp,t}$, $\epsilon_{tp,t} \sim N(0,1)$. It is worth pointing out that in equilibrium revisions of future expected short term interest rates will imply fluctuations in the price of maturity bonds and, consequently, in $R_{t-1,t}^m$ and b_t^m .

The linearized federal transfers as a fraction of GDP tr_t following process:

$$\begin{pmatrix} \widetilde{tr}_{t} - \widetilde{tr}_{t}^{*} \end{pmatrix} = \rho_{tr} \left(\widetilde{tr}_{t-1} - \widetilde{tr}_{t}^{*} \right) + (1 - \rho_{tr}) \phi_{y} \left(\widehat{y}_{t} - \widehat{y}_{t}^{*} \right) + \sigma_{tr} \epsilon_{tr,t}, \ \epsilon_{tr,t} \sim N \left(0, 1 \right) \\
\widetilde{tr}_{t}^{*} = \rho_{tr^{*}} \widetilde{tr}_{t-1}^{*} + \sigma_{tr^{*}} \epsilon_{tr^{*},t}, \ \epsilon_{tr^{*},t} \sim N \left(0, 1 \right)$$

where \widetilde{tr}_t^* represents a long term component that is assumed to be completely exogenous and it is meant to capture the large programs that arise as the result of a political process that is not modeled here. Transfers fluctuate around this trend component as a result of business cycle fluctuations captured by the log-linearized output gap $(\widehat{y}_t - \widehat{y}_t^*)$, where \widehat{y}_t^* is natural output, the level of output that would prevail under flexible prices.

The government also buys a fraction G_t/Y_t of total output, equally divided among the J different goods. We define $g_t = 1/(1 - G_t/Y_t)$ and we assume that $\tilde{g}_t = \ln(g_t/g)$ follows the process:

$$\widetilde{g}_{t} = \rho_{g}\widetilde{g}_{t-1} + \sigma_{g}\epsilon_{g,t}, \ \epsilon_{g,t} \sim N(0,1).$$
 (4)

Fiscal and Monetary Rules. The fiscal authority moves taxes according to the following rule:

$$\widetilde{\tau}_{t} = \rho_{\tau,\xi_{t}^{p}}\widetilde{\tau}_{t-1} + \left(1 - \rho_{\tau,\xi_{t}^{p}}\right) \left[\delta_{b,\xi_{t}^{p}}\widetilde{b}_{t-1}^{m} + \delta_{e}\left(\widetilde{tr}_{t}^{*} + g^{-1}\widetilde{g}_{t}\right) + \delta_{y}\left(\widehat{y}_{t} - \widehat{y}_{t}^{*}\right)\right] + \sigma_{\tau}\epsilon_{\tau,t}, \ \epsilon_{\tau,t} \sim N\left(0,1\right)$$
(5)

where $\tilde{\tau}_t$ is the level of tax revenues with respect to GDP in linear deviations from the steady state. Tax revenues respond to the state of the economy, captured by the output gap, to the sum of long run level of transfers and government purchases, and to the level of debt. The

³Alternative approaches consist of excluding one of the fiscal components or including an observation error. Our results are robust to these alternative specifications.

strength with which the government tries to stabilize the debt-to-GDP ratio is captured by the coefficient $\delta_{b,\xi_{*}^{p}}$ that is allowed to vary over time.

The central bank follows the rule:

$$\frac{R_t}{R} = \left(1 - Z_{\xi_t^d}\right) \left(\frac{R_{t-1}}{R}\right)^{\rho_{R,\xi_t^p}} \left[\left(\frac{\Pi_t}{\Pi}\right)^{\psi_{\pi,\xi_t^p}} \left(\frac{Y_t}{Y_t^*}\right)^{\psi_{y,\xi_t^p}} \right]^{\left(1 - \rho_{R,\xi_t^p}\right)} e^{\sigma_R \epsilon_{R,t}} + Z_{\xi_t^d} \left[\left(\frac{R_{t-1}}{R}\right)^{\rho_{R,Z}} \left(\frac{1}{R}\right)^{\left(1 - \rho_{R,Z}\right)\psi_Z} \right] e^{\sigma_Z \epsilon_{R,t}}$$

where $\epsilon_{R,t} \sim N\left(0,1\right)$, R is the steady-state gross nominal interest rate, Y_t^* is the output target, Π is the target/steady-state level for gross inflation, the variable ξ_t^p captures the monetary/fiscal policy combination that is in place at time t, and the dummy variable $Z_{\xi_t^d}$ controls if the economy is in or out of the zero lower bound. When $\overline{d}_{\xi_t^d} = \overline{d}_h$, the economy is out of the zero lower bound and monetary and fiscal policies are not constrained ($Z_{\xi_t^d} = 0$). In this case the evolution of the policy mix can be described by the two-regime Markov switching process ξ_t^p . The properties of the transition matrix and of the regimes will be described below. When $\overline{d}_{\xi_t^d} = \overline{d}_l$, the zero lower bound is binding, given that a standard Taylor rule would require a negative nominal interest rate.⁴ In this case, policymakers abandon the policy mix that they were following and set the net nominal interest rate close to zero ($Z_{\xi_t^d} = 1$).

To match the behavior of the FFR in the data during the zero-lower-bound period, we need to allow for: (1) small disturbances to the FFR, (2) the fact that the FFR is not exactly zero, and (3) the fact that the Federal Reserve lowered the interest rate gradually, even if quickly. The size of the monetary policy shocks to the FFR at the zero lower bound are controlled by σ_Z . This is assumed to be a tenth of the out of the zero lower bound standard deviation of the monetary policy shocks: $\sigma_Z = \sigma_R/10$. The persistence of changes in the FFR at the zero lower bound is controlled by $\rho_{R,Z}$ and fixed to .2. Finally, the parameter $0 < \psi_Z \le 1$ controls the average level of the FFR when at the zero lower bound. It can be thought as the fraction of the steady state net interest rate. Notice that if we set $\sigma_Z = 0$, $\rho_{R,Z} = 0$, and $\psi_Z = 1$, we would obtain $R_t = 1$ at the zero lower bound. Consequently, the linearized monetary policy rule at the zero lower bound would read $\tilde{R}_t = -\ln(R)$. In other words, the net nominal interest rate would be exactly zero.

⁴We assume that whenever the negative preference shock hits, policymakers move to the zero-lower-bound regime described later on and we choose the parameters values in a way that the zero lower bound is binding with high probability when $\bar{d}_{\xi_t^d} = \bar{d}_l$. Our approach to model the zero lower bound differs from the conventional one (e.g., see Eggertsson and Woodford 2003; Benhabib, Schmitt-Grohe, and Uribe 2002), which implies $R_t = \max(0, R_t^*)$, where R_t^* is the interest rate implied by the Taylor rule. While our approach cannot rule out that there exist some unlikely states of the world in which the nominal rate R_t assumes negative values, it has the advantage of making the model tractable and allows us to study the consequences of policy uncertainty.

	Active Fiscal (AF)	Passive Fiscal (PF)
Active Monetary (AM)	No Solution	Determinacy
Passive Monetary (PM)	Determinacy	Indeterminacy

Table 1: Partition of the parameter space according to existence and uniqueness of a solution (Leeper 1991).

3.2 Regime Changes

To characterize policymakers' behavior out of the zero lower bound, we will make use of the partition of the parameter space introduced by Leeper (1991). We can distinguish four regions (Table 1) based on the properties of the model under fixed coefficients. When the values of model parameters are fixed, the two policy rules are key in determining the existence and uniqueness of a solution. There are two determinacy regions. The first region, Active Monetary/Passive Fiscal (AM/PF), is the most familiar one: The Taylor principle is satisfied and the fiscal authority moves taxes in order to keep debt on a stable path: $\psi_{\pi} > 1$ and $\delta_b > \beta^{-1} - 1$. We will refer to this policy combination as Monetary led regime.

The second determinacy region, Passive Monetary/Active Fiscal (PM/AF), is less familiar and corresponds to the case in which the fiscal authority is not committed to stabilizing the process for debt: $\delta_b < \beta^{-1} - 1$. Now it is the monetary authority that passively accommodates the behavior of the fiscal authority, disregarding the Taylor principle and allowing inflation to move in order to stabilize the process for debt: $\psi_{\pi} < 1$. Under this regime, even in the absence of distortionary taxation, shocks to net taxes can have an impact on the macroeconomy as agents understand that they will not be followed by future offsetting changes in the fiscal variables. We will label this policy combination as Fiscally led regime. Finally, when both authorities are active (AM/AF) no stationary equilibrium exists, whereas when both of them are passive (PM/PF) the economy is subject to multiple equilibria.⁵

In the benchmark model, when the preference shock is high $(\xi_t^d = h)$, the economy is out of the zero lower bound $(Z_{\xi_t^d} = 0)$ and the evolution of policymakers' behavior is captured by a two-regime Markov chain that evolves according to the transition matrix H^p :

$$H^p = \left[egin{array}{cc} p_{MM} & 1 - p_{FF} \ 1 - p_{MM} & p_{FF} \end{array}
ight],$$

where $p_{ji} = P\left(\xi_{t+1}^p = j | \xi_t^p = i\right)$. This transition matrix is supposed to capture the stochastic outcome of a game between the monetary and fiscal authorities that is not explicitly modeled in this paper. Regime M is the *Monetary led regime*, under which the Taylor principle is satisfied and fiscal policy accommodates the behavior of the monetary authority. In terms

⁵Benhabib, Schmitt-Grohe, and Uribe (2001a) show that if money is assumed to enter or not preferences and technology matters for whether a particular monetary/fiscal regime is conducive to determinacy. Our setting is standard in this respect and Leeper's (1991) partition applies.

of policy parameters, this implies that $\psi_{\pi,M} = \psi_{\pi}^A > 1$ and $\delta_{b,M} = \delta_b^P > \beta^{-1} - 1$. Regime F is the *Fiscally led regime*. Under such a regime, the central bank reacts less than one-for-one to inflation and the fiscal authority does not move surpluses in response to movements in government debt: $\psi_{\pi,F} = \psi_{\pi}^P < 1$ and $\delta_{b,F} = \delta_b^A < \beta^{-1} - 1$.

When the low value for the preference shock occurs ($\xi_t^d = l$), the zero lower bound becomes binding ($Z_{\xi_t^d} = 1$), and policymakers' behavior is now constrained. In this third policy combination the nominal interest rate is set to zero and the fiscal authority disregards the level of debt: $\delta_Z = 0$. Notice that the zero-lower-bound policy mix can be considered as an extreme version of the Fiscally led policy mix. However, while out of the zero lower bound, switches to the Fiscally led regime capture deliberate choices of policymakers, the adoption of the zero-lower-bound regime is induced by an exogenous negative preference shock that prompts the fiscal authority to forgo fiscal adjustments to counter the effects of a deep recession. Once the preference shock is back to its high value ($\xi_t^d = h$), policymakers' behavior is not constrained anymore.

It is worth emphasizing that even if the zero lower bound imposes a constraint on policy-makers' behavior, agents' beliefs are not constrained. Therefore, beliefs about the exit strategy and policy uncertainty are going to be key to understand the macroeconomic dynamics at the zero lower bound. To capture this feature, we introduce a parameter controlling the expected exit strategy from the zero lower bound (ZLB). The parameter p_{MZ} represents the probability that once the discrete preference shock will be reabsorbed, the economy will move to the the Monetary led regime.

In summary, the joint evolution of policymakers' behavior and the discrete preference shock is captured by the regime obtained combining the two chains $\xi_t = [\xi_t^p, \xi_t^d]$. The combined chain can assume three values: $\xi_t = \{[M, h], [F, h], [Z, t]\}$. The corresponding transition matrix H is obtained by combining the transition matrix H^d , which describes the evolution of the preference shock, the transition matrix H^p , which describes policymakers' behavior out of the zero lower bound, and the parameter p_{MZ} that controls the probability of moving to the Monetary led regime once the negative preference shock is reabsorbed:

where $p_{FZ} = 1 - p_{MZ}$.

3.3 Solving and Estimating the MS-DSGE Model

The technology process A_t is assumed to have a unit root. The model is then rescaled and linearized around the unique deterministic steady state. The model can be solved with any of the solution methods developed for Markov–switching DSGE models. We use the solution method of Farmer, Waggoner, and Zha (2009). It is worth emphasizing that in our model, agents form expectations while taking into account the possibility of entering the zero lower bound and the possibility of changes in policymakers' behavior. Furthermore, they understand that entering the zero lower bound is an event induced by an exogenous shock that can modify policymakers' behavior even once the constraint stops being binding. In other words, our approach allows us to model recurrent zero-lower-bound episodes and to capture the impact of different exit strategies for policymakers' behavior at the zero lower bound. The solution can be characterized as a regime-switching vector autoregression, of the kind studied by Hamilton (1989), Chib (1996), and Sims and Zha (2006):

$$S_{t} = c(\xi_{t}, \theta, H) + T(\xi_{t}, \theta, H) S_{t-1} + R(\xi_{t}, \theta, H) Q(\theta^{v}) \varepsilon_{t}$$

$$(6)$$

where θ , θ^v , and S_t are vectors that contain the structural parameters, the stochastic volatilities, and all the variables of the model, respectively. Appendix A provides more details about the linearization and the solution algorithm.

It is worth emphasizing that the law of motion of the model depends on the structural parameters (θ) , the regime in place (ξ_t) , and the probability of moving across regimes (H). This means that what happens under one regime does not only depend on the structural parameters describing that particular regime but also on what agents expect is going to happen under alternative regimes and on how likely it is that a regime change will occur in the future (see also Davig and Leeper (2007)). In other words, agents' beliefs about future regime changes matter for the law of motion governing the economy.

4 Policy Uncertainty and the Zero Lower Bound

The law of motion (6) is combined with a system of observation equations. The likelihood is computed with the Kalman filter and then combined with a prior distribution for the parameters to obtain the posterior. As a first step, a block algorithm is used to find the posterior mode, while a Metropolis algorithm is used to draw from the posterior distribution. Appendix B provides evidence for convergence.

We include seven observables spanning the sample 1954:Q4-2014:Q1: real GDP growth, annualized GDP deflator inflation, FFR, annualized debt-to-GDP ratio on a quarterly basis, federal tax revenues to GDP ratio, federal expenditure to GDP ratio, and a transformation of

government purchases to GDP ratio. Appendix C describes the dataset in detail. All variables are expressed on a quarterly basis: This implies that a value of 200% for the debt-to-GDP ratio corresponds to a 50% debt-to-GDP ratio on annual basis, given that in the latter case quarterly GDP would be multiplied by 4.

For tractability we fix the sequence for the out of the zero lower bound regimes based on the estimates obtained by Bianchi and Ilut (2012). Bianchi and Ilut (2012) estimate a model similar to the one described above, but they exclude the recent years and the zero lower bound regime. They find that the fiscal authority was the leading authority in the '60s and '70s. Following their estimates, the fiscally led regime is assumed to be in place from 1957:Q2 to 1981:Q3, while over the remainder of the sample the Monetary led regime is assumed to prevail. The zero lower bound regime starts in 2008:Q4 and remains in place until the end of the sample. We chose 2008:Q4 as the starting date for the zero lower bound based on a model comparison exercise in which we considered all quarters between 2008:Q1 and 2009:Q3 as possible starting dates. The outcome of this exercise is presented in Appendix D.

4.1 Parameters Estimates and Regime Probabilities

Table 2 reports priors and posterior parameter estimates. The priors for the parameters that do not move across regimes are in line with previous contributions in the literature and are relatively loose. As for the parameters of the Taylor rule, the prior for the autoregressive component is symmetric across regimes, whereas we have chosen asymmetric and truncated priors for the responses to inflation and the output gap in line with the theoretical restrictions outlined above: Under the Monetary led regime (M) monetary policy is active, whereas under the Fiscally led regime (F), monetary policy is passive. In a similar way, the priors for the response of taxes to government debt are asymmetric across the two regimes: Under the Fiscally led regime and the ZLB regime, this parameter is restricted to zero, whereas under the Monetary led regime is expected to be fairly large. In order to separate the short and long term components of transfers we restrict the persistence of the long term component ($\rho_{e^L} = .99$) and the standard deviation of its innovations ($\sigma_{e^L} = .1\%$).⁶ We fix the discount factor β to .9985, a value broadly consistent with an annualized 2\% real interest, and the average maturity to 5 years (this is controlled by the parameter ρ). Finally, we chose a loose and symmetric prior for the parameter p_{MZ} , which captures the probability of the Monetary led policy mix conditional on abandoning the ZLB policy. This parameter captures the uncertainty about the policy that will be carried out after the lift of the interest rate from the ZLB. As we shall show, this form of uncertainty is a key element to explain the absence of deflation observed during the Great Recession. Our

⁶This choice imposes a constraint on the amount of macroeconomic volatility that can be explained by the long term component. Our results are confirmed when removing this constraint.

	Mean	5%	95%	Type	Mean	Std
$\psi_{\pi,M}$	1.6019	1.1758	2.0207	N	2.5	0.3
$\psi_{y,M}$	0.5065	0.2980	0.7688	G	0.4	0.2
$\rho_{R,M}$	0.8652	0.8135	0.9108	В	0.5	0.2
$\delta_{b,M}$	0.0712	0.0457	0.1041	G	0.07	0.02
$ ho_{ au,M}$	0.9652	0.9439	0.9816	В	0.5	0.2
$\psi_{\pi,F}$	0.6356	0.5007	0.7546	G	0.8	0.3
$\psi_{y,F}$	0.2709	0.2005	0.3458	G	0.15	0.1
$ ho_{R,F}$	0.6663	0.5927	0.7401	В	0.5	0.2
$\rho_{ au,F}$	0.6874	0.5358	0.8438	В	0.5	0.2
$\overline{\overline{d}}_l$	-0.3662	-0.4827	-0.2789	N	-0.3	0.1
p_{hh}	0.9995	0.9984	0.9999	Dir.	0.96	0.03
p_{ll}	0.9306	0.8936	0.9599	Dir.	0.83	0.10
p_{MM}	0.9923	0.9872	0.9965	Dir.	0.96	0.03
p_{FF}	0.9923	0.9888	0.9951	Dir.	0.96	0.03
p_{MZ}	0.9225	0.8108	0.9861	Dir.	0.50	0.22
$\overline{\psi_Z}$	0.9678	0.9596	0.9761	В	0.95	0.02
κ	0.0073	0.0052	0.0098	G	0.3	0.15
$ ho_{tr}$	0.4599	0.3677	0.5537	В	0.2	0.05
δ_y	0.2766	0.1899	0.3707	N	0.4	0.2
δ_e	0.3661	0.2096	0.5137	В	0.5	0.2
$\phi_{m{y}}$	-0.2910	-0.3433	-0.2435	N	-0.4	0.2
Φ	0.8628	0.8299	0.8928	В	0.5	0.2
ρ_g	0.9796	0.9742	0.9839	В	0.5	0.2
$ ho_a$	0.4053	0.1522	0.6480	В	0.5	0.2
$ ho_d$	0.3944	0.2881	0.4963	В	0.5	0.2
$ ho_{tp}$	0.3267	0.2276	0.4218	В	0.5	0.2
$ ho_{\mu}$	0.4784	0.4053	0.5513	В	0.5	0.2
$100\sigma_R$	0.1934	0.1767	0.2115	IG	0.5	0.5
$100\sigma_g$	0.2826	0.2614	0.3049	IG	1.00	1.00
$100\sigma_a$	0.7708	0.4713	1.1552	IG	1.00	1.00
$100\sigma_{\tau}$	0.4547	0.4174	0.4951	IG	2.00	2.00
$100\sigma_d$	7.6949	6.3336	9.3354	IG	10.00	2.00
$100\sigma_{tr}$	0.3007	0.2750	0.3282	$_{ m IG}$	2.00	2.00
$100\sigma_{tp}$	2.8511	2.6336	3.0922	IG	1.00	1.00
$100\sigma_{\mu}$	0.1774	0.1530	0.2037	IG	1.00	1.00
100π	0.4631	0.3955	0.5369	G	0.5	0.05
100γ	0.4185	0.3523	0.4849	G	0.4	0.05
b^m	1.0689	0.9368	1.1983	N	1	0.1
g	1.0865	1.0779	1.0960	N	1.06	0.04
<u>τ</u>	0.1721	0.1670	0.1779	N	0.18	0.01

Table 2: Posterior means and 90% error bands of the model parameters. For the structural parameters, M stands for Monetary led regime, whereas F stands for Fiscally led regime.

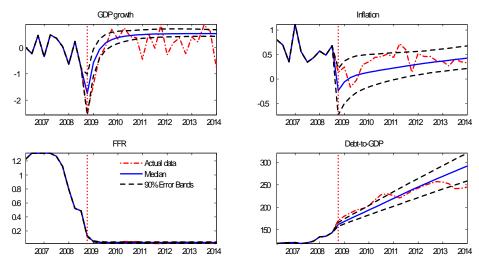


Figure 1: Macroeconomic dynamics at the zero lower bound. Response of GDP growth, inflation, FFR, and debt-to-GDP ratio to a discrete negative preference shock. The red dashed line reports actual data.

symmetric and broad prior implies that we maintain an agnostic view with respect to which exit strategy agents should regard as more likely.

Regarding the parameters of the Taylor rule, under the Monetary led regime the Federal Funds rate reacts strongly to both inflation and the output gap. The opposite occurs under the Fiscally led regime regime. Under the Fiscally led and ZLB regimes the response of taxes to debt is restricted to zero, while under the Monetary led regime it turns out to be significantly larger than the threshold value described in Subsection 3.2 ($\beta^{-1} - 1 = .0015$).

As mentioned above, we fixed the regime sequence. Therefore the estimates of the transition matrix are determined by the model dynamics across the different regimes and not by the frequency of regime changes. It is therefore useful to review the properties of the estimated transition matrix. Both the Monetary led regime and the Fiscally led regime are quite persistent, implying that when one of the two regimes is in place, agents expect to spend a significant amount of time under such a regime. The persistence of the high state for the discrete preference shock is also very high. This implies that when out of the ZLB, agents attach a small weight to the possibility of a large contraction in real activity deriving from the negative preference shock. This result is consistent with the fact that before the recent crisis, the US economy had always been able to avoid the zero lower bound. Finally, when at the ZLB agents regard as more likely that once the negative preference shock is reabsorbed policymakers will move to the Monetary led regime ($p_{MZ} = 92\%$ at the posterior mean). However, it is important to emphasize that this probability is substantially smaller than the estimated persistence of the Monetary led regime (p_{MM} is around 99%). Therefore, our estimates suggest that when the economy entered the ZLB the probability attached to switching to the Fiscally led regime experienced a discrete jump.

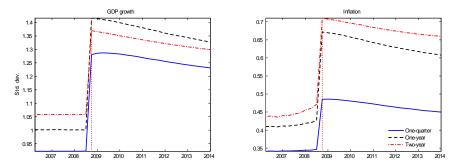


Figure 2: Evolution of uncertainty at the zero lower bound. The figure reports the conditional standard deviations at different horizons for GDP growth and inflation following the discrete negative preference shock.

4.2 Dynamics at the Zero-Lower-Bound

Figure 1 reports the estimated impulse response to a discrete negative preference shock \overline{d}_l . In order to compute the impulse response, we use the actual data in 2008:Q3. The shock occurs in 2008:Q4 and is marked by a vertical line. Using actual data as a starting point serves two purposes. First, it allows us to easily assess the relative importance of the discrete preference shock in explaining the macroeconomic dynamics during the current recession. Second, it allows us to account for the fact that the impulse response is not invariant with respect to the state of the economy. This is because the negative preference shock also implies a change in expectations about future policymakers' behavior. We will soon show that the fiscal situation is important to understand how the economy behaves in presence of this increase in policy uncertainty.

The model is able to replicate the key changes that occurred starting with the 2008 crisis as a result of a single disturbance, the discrete negative preference shock. The economy experiences a drop in real activity, a large increase in the debt ratio, monetary policy enters the zero lower bound, but inflation remains relatively stable. It is also interesting to note that the model is not only able to replicate the absence of deflation, but also the fact that inflation has been trending up. The model rationalizes this behavior as a result of increasing inflationary pressure coming from the large debt accumulation.

Figure 2 reports the model implied evolution of uncertainty at different horizons once the economy enters the zero lower bound. Uncertainty is computed by taking into account the possibility of regime changes and future Gaussian shocks by using the methods described in Bianchi (2014). For a variable X_t and an horizon q, it corresponds to the conditional standard deviation $sd_t(X_{t+q})$. In other words, the figure reports the conditional standard deviations for GDP growth and inflation at different horizons associated with the macroeconomic path presented in Figure 1. Notice that as soon as the negative preference shock occurs, the economy experiences an increase in macroeconomic uncertainty. The increase in uncertainty is explained by a series of factors. First, the economy is currently in a large recession and even if eventually it will do better, the timing of the recovery is uncertain. Second, as explained above, when

the economy is at the zero lower bound, the probability assigned to a switch to the monetary-fiscal policy mix experiences a discrete increase. As it will be shown in Section 5, the Fiscally led regime tends to lead to an increase in macroeconomic uncertainty. Finally, the zero lower bound regime is inherently more volatile because policymakers cannot react to disturbances using the FFR. Summarizing, the estimates suggest that the zero lower bound is associated with a discrete increase in uncertainty, but no deflation.

It is also interesting to notice that output uncertainty becomes hump shaped: Medium run uncertainty (one-year horizon) becomes larger than long run uncertainty (two-year horizon). This is because two forces are at play. On the one hand, events further in the future are harder to predict and are therefore more uncertain. On the other hand, in the long run agents expect the economy to be out of the zero lower bound. This leads to a reduction of uncertainty. As the horizon moves from one quarter to two years, the relative importance of these two channels vary.

4.3 The Role of Fiscal Policy

In order to emphasize that the absence of deflation is tightly linked to uncertainty about future policymakers' behavior, Figure 3 compares the effects of the discrete negative preference shock under the benchmark estimated model with its effects in a counterfactual economy in which the Monetary led policy mix is the only possible regime when out of the zero lower bound (black dashed line). Thus the vertical distance between the two lines captures the effects of uncertainty about the exit strategy on output gap, inflation, and the debt-to-GDP ratio. Notice that under the counterfactual economy the negative preference shock has now a much larger impact on inflation and real activity. The economy experiences a very large and persistent deflation and a much larger contraction in output. Furthermore, the massive increase in the debt-to-GDP ratio does not have any mitigating effect on inflation: Agents expect that the entire debt-to-GDP ratio will eventually be repaid with an increase in taxation.

Therefore, when the possibility of a switch to the fiscally led regime is ruled out, entering the zero lower bound implies a large contraction in real activity and a large deflation, in line with the textbook version of the new-Keynesian model. The fact that the model can in principle reproduce the standard features of the zero lower bound is important: The data could reject the role played by policy uncertainty in explaining inflation dynamics. For example, the estimates could have suggested a very flat Phillips curve, a very small probability of ever moving to the Fiscally led regime, or a small preference shock not large enough to determine deflation. Instead, the estimates suggest that the preference shock was in fact large, in a way to explain the prolonged decline in real activity. The absence of deflation is then rationalized in light of

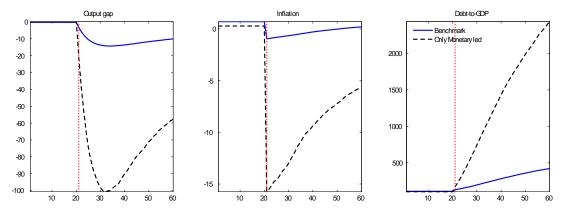


Figure 3: Actual and counterfactual impulse responses to a discrete negative preference shock. The solid blue line corresponds to the estimated model, whereas the black dashed line corresponds to a counterfactual economy in which the Monetary led policy mix is the only possible policy mix.

the possibility of a change in the monetary/fiscal policy mix.⁷ In the following sections we will examine more in detail why entering the zero lower bound could lead to an increase in the probability of a change in the monetary/fiscal policy mix.

It is also important to emphasize that the amount of policy uncertainty required to explain the absence of deflation during the Great Recession is quite moderate. As shown in Table 2, the posterior mean of the probability of moving to the Monetary led policy mix once out of the ZLB regime is $p_{MZ} = 92.25\%$. This implies that while at the zero lower bound agents are inclined to believe that policymakers will eventually resume the same policy mix observed before the Great Recession. Such a belief is plausible in light of the long spell of Monetary led policy mix observed between the Volcker disinflation and the onset of the Great Recession. However, as noted before, the value also implies an increase in the probability of moving away from the Monetary led regime with respect to the pre-crisis period. Also note that in the estimated model the inflationary effects of public uncertainty about the exit strategy are linked to the pre-crisis level of government debt, which is observed to be above its estimated steady-state value (around 170% of quarterly GDP or 42.5% on annual basis). In Figure 3 we assume that the debt-to-GDP ratio is equal to its out of the zero lower bound steady-state value. Hence this graph provides a conservative estimate of the anti-deflationary effects that policy uncertainty is likely to have exerted during the U.S. Great Recession, while in Figure 1 we used the actual data as a starting point.

Figure 4 provides further corroborating evidence for the mechanism proposed in this paper. The figure reports the evolution of the one-year-ahead inflation expectations as implied by the model and compares them with the Michigan surveys. The error bands reflect parameter

⁷In Appendix E we show that our results do not depend on the shock that triggers the zero lower bound. To that end, we consider a prototypical new-Keynesian model in which we can directly shock the natural rate as in Eggertsson and Woodford (2003).

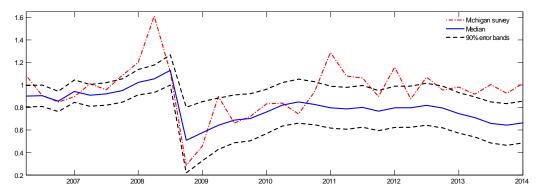


Figure 4: **Inflation expectations.** The figure reports the evolution of the model implied one-year-ahead inflation expectations together with the Michigan surveys (red dashed dotted line).

uncertainty. They are obtained computing the model implied expectations for each draw from the posterior distribution. Even though we do not use inflation expectations for estimation, the model is able to replicate some salient features of the Michigan surveys. First of all, the model captures the upward trend in inflation expectations that is visible right before the recession started and that can be explained in light of an increase in the fiscal burden during those years. In the data, such a trend is somehow more pronounced than in the model toward the early 2008. Second the model captures remarkably well the swing in inflation expectations that occurred once the crisis started. From above trend, inflation expectations quickly moved below trend. However, they never became negative and they quickly recovered. Finally, in line with what predicted by the model, as more time is spent at the zero lower bound inflation expectations show an upward trend as a result of the large debt accumulation.

We find encouraging that the model is able to replicate these key facts even if inflation expectations are not included in the estimates. This result shows that the inflationary pressure coming from the fiscal burden delivers inflation expectations that are very much in line with the data. It is particularly important to emphasize that inflation expectations moved down, as predicted by our model, when the ZLB was encountered, but they never entered the negative territory. In other words, agents were somehow confident that deflation would not have occurred. This implies that the absence of deflation in the United States cannot be easily rationalized ex-post with a series of lucky realizations of inflationary shocks. In other words, if the Monetary led policy mix were the only possible regime, then agents should expect deflation once the economy enters the zero lower bound.

In order to understand why the possibility of changes in the monetary/fiscal policy is important to rationalize the absence of deflation and the increase in macroeconomic uncertainty, Figure 5 reports the impulse responses to an increase in the long term component of transfers under the three different regimes. Impulse responses are computed conditionally on one regime being in place over the entire horizon. Nevertheless, model dynamics reflect the possibility of

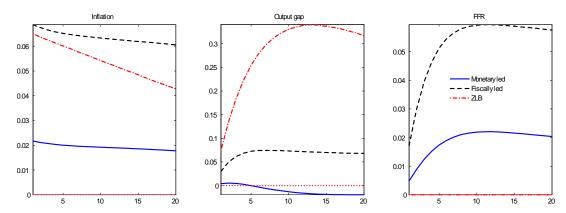


Figure 5: **Impulse responses to a shock to transfers.** The impulse responses are computed assuming a regime in place over the the relevant horizon. However, agents form expectations taking into account the possibility of regime changes.

regime changes. We choose to report impulse responses to a shock to the long term component of transfers for two reasons. First, under the maintained assumption of non-distortionary taxation, shocks to the trend component of transfers would not have any impact on the macroeconomy if the Monetary led policy mix were the only possible regime. In that case the government would be fully committed to move taxes in response to fiscal imbalances. Second, this shock has a direct impact on the debt-to-GDP ratio and, consequently, on the amount of spending that would need to be financed with future taxes. When the Fiscally led regime is in place, agents understand that in the near future the probability of a fiscal adjustment in response to the current increase in the primary deficit is fairly low. This determines an increase in inflation that is made possible by the accommodating behavior of the Monetary authority. Given that the Taylor principle does not hold, the response of the nominal interest rate is less than one-to-one. The resulting decline in the real interest determines an increase in real activity. The debt-to-GDP ratio is then stabilized because of the fall in the real interest rate and the faster growth in real economic activity. The macroeconomy is therefore not insulated with respect to fiscal imbalances even if taxation is non-distortionary.

Under the Monetary led regime the primary deficit shock triggers a much smaller increase in inflation because the fiscal authority is expected to implement the necessary fiscal adjustments. However, the response of inflation is not zero because agents form expectations by taking into account the possibility of moving to the Fiscally led regime. As a result, a high level of debt determines some inflationary pressure even when the Monetary led regime is in place. This feature of the model is in line with the results obtained by Bianchi and Ilut (2012) in an estimated model, Davig and Leeper (2006) in a calibrated model, and Davig, Chung, and Leeper (2007) in an analytical example. Given that the Taylor principle holds, the central bank reacts more than one to one to the increase in inflation. The result is a prolonged period of slightly negative output gaps that last as long as the fiscal imbalance is not fully reabsorbed.

Finally, under the zero lower bound regime the effects of the fiscal shock are quite similar to those that characterize the Fiscally led regime. The increase in spending triggers a fairly large increase in inflation. Given that the FFR is stuck at zero, the resulting drop in the real interest rate is amplified with a consequent large increase in the output gap. Therefore, the zero lower bound regime presents many of the characteristics of a Fiscally led regime, even if the probability assigned to a return to the Monetary led regime is quite large. Furthermore, these results suggest that at the zero lower bound, the effects of the shocks are amplified by the fact that the FFR cannot respond to macroeconomic fluctuations.

In summary, two important lessons can be drawn from this exercise. First, under the Fiscally led regime, the macroeconomy is not insulated with respect to fiscal imbalances. Second, as long as agents are aware of regime changes, even under the Monetary led regime and the zero lower bound regime the macroeconomy is not insulated with respect to fiscal disturbances and fiscal imbalances have inflationary pressure. This inflationary pressure would disappear only if the Monetary led policy mix were the only possible one. This explains the large recession and large deflation in the counterfactual economy presented in Figure 3.

5 Policy Trade-off

In this section we will show that while the Monetary led regime leads to a more stable macroeconomic environment during regular times, extraordinary events can make deviating from such
a regime desirable. One of such events is a significant drop in aggregate demand, which is induced by the discrete preference shock \bar{d}_l . Therefore, a policy trade-off arises at the zero lower
bound: mitigating a large recession or preserving long-run macroeconomic stability. If agents
are uncertain about which one of these goals is more important for policymakers, deflation
is not a necessary implication of entering the zero lower bound. The existence of this policy
trade-off also helps explaining why the probability of a switch to the Fiscally led regime is likely
to increase at the zero lower bound.

5.1 Mitigating the Recession...

In this subsection, we will illustrate that while uncertainty is an inherent implication of entering the zero lower bound, deflation is not. This is because a deep recession leads to a large accumulation of debt that provides inflationary pressure. Furthermore, the recession can be mitigated by inflating away all or part of debt, at the cost of raising long run macroeconomic volatility. As a result, agents are going to be very uncertain about policymakers' behavior and, consequently, future economic outcomes. In what follows, we assume that agents recognize that the zero-lower-bound regime is triggered by an exogenous event over which policymakers

have no control. In this situation, agents pay attention to policy announcements to get some guidance about future policymakers' behavior.

In the past five years, the Federal Reserve has kept the federal funds rate at zero. In the textbook new-Keynesian model once the policy rate hits the zero lower bound, deflation occurs. A large preference shock that induces consumers to drastically reduce demand can lead to this situation. Agents want to save more, so they reduce consumption and demand falls; consequently, real economic activity and inflation fall. If the drop is large enough, the desired policy interest rate becomes negative and the best that the central bank can do is to drive the interest rate to zero. Therefore, the real interest rate is in fact too high compared with what would be desirable, and the economy can experience a very large drop in real economic activity and deflation.

These basic predictions of the textbook new-Keynesian model seem at odds with US data that show that inflation has been fluctuating around the implicit Federal Reserve's inflation target. We have shown that the absence of deflation can be explained in light of the possibility of a change in the monetary/fiscal policy mix. Therefore, the behavior of inflation during this period is consistent with the high uncertainty that surrounds how policymakers will behave in the future. Policymakers do not seem to have outlined a clear exit strategy yet. Arguably, this creates some uncertainty about the way they will deal with the large stock of debt that originated from the current crisis. The outcomes of the policy implemented at the zero lower bound critically depend on what agents expect to happen once the economy has exited the zero lower bound. Here, we analyze the different options that could be followed by policymakers and provide an explanation for why no clear exit strategy has been proposed.

We consider the US economy as it was in 2008:Q3, when the most recent crisis started, and we analyze the effects of a large negative preference shock that can push the economy to the zero lower bound occurring in 2008:Q4. Recall that before the shock occurred, policymakers were following the Monetary led regime. We analyze three different scenarios concerning policymakers' behavior. In the first scenario, we consider the benchmark model in which no announcement is made. In the second and third scenario policymakers make announcements about the exit strategy. Specifically, in the second scenario, policymakers announce that fiscal discipline will be abandoned and that the economy will move to the Fiscally led regime. In the third scenario, policymakers announce that once the economy is out of the zero-lower-bound period, fiscal discipline will be restored, implying that the economy will move back to the Monetary led regime. In the case of announcements, the probability of announcing a return to the Monetary led regime is fixed to p_{MZ} . Appendix A.3 provides details on how to build the transition matrix for the economy with announcements.

Figure 6 reports the responses of the variables in an economy that is hit by the large negative preference shock ($\xi_t^d = l$). It is worth emphasizing that agents are fully aware of the structure

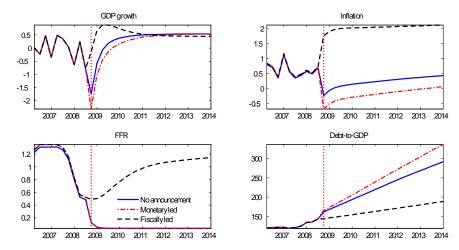


Figure 6: Macroeconomic Dynamics with Coordinated Announcements: The figure reports the effects of coordinated announcements following a large negative preference shock that can force the interest rate to the zero lower bound. Three cases are considered for the exit strategy. In the first case ("Monetary led"), policymakers announce a return to the Monetary led regime; in the second case ("Fiscally led"), a switch to the Fiscally led regime is announced; the third case ("No Announcement"), no announcement about the exit strategy is made. This corresponds to the estimated benchmark model and it is represented by the solid blue line.

of the model. Therefore, they understand that in response to the negative preference shock policymakers can follow one of the two exit strategies outlined above. However, agents do not know when the preference shock will return to the high value ($\xi_t^d = h$) and are aware that in the future zero lower bound episodes might occur again. In other words, unlike previous contributions in the literature, we do not impose perfect foresight or an absorbing state for $\xi_t^d = h$. Finally, agents do not necessarily know which exit strategy will be adopted by policymakers. As a result, announcements by policymakers play a critical role by steering agent's expectations about the likely exit strategy.

If policymakers announce that fiscal discipline will be abandoned (black dashed line) agents expect that the preexisting stock of debt and the additional amount of debt accumulated during the recession will be inflated away. Therefore, they revise upward their inflation expectations and, consequently, inflation increases today through the expectation channel. Notice that the recession is in this case substantially mitigated and the economy is effectively leaving the zero lower bound.

If instead policymakers explicitly announce that the stance toward fiscal discipline has not changed and that after the economy exits the zero lower bound they will resume the same policies that characterized the pre-crisis period, the economy enters a recession and deflation arises (red dashed-dotted line). The outcomes for this case are qualitatively in line with the traditional view about the zero lower bound. However, the drops in real activity and inflation are substantially mitigated with respect to the counterfactual economy presented in Subsection 4.3 in which the Monetary led policy mix is the only possible regime. This is because the

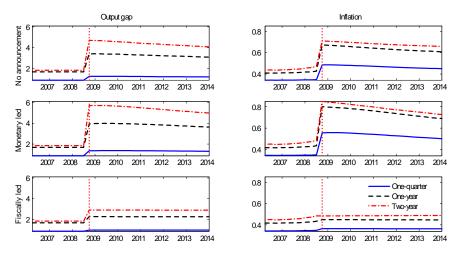


Figure 7: Evolution of uncertainty under coordinated announcements. The graphs report the evolution of uncertainty at different horizons following an adverse discrete preference shock and different announcements about future policy makers' behavior.

expectations of rapid debt accumulation determine inflationary pressures even if agents expect that the Monetary led regime will follow the end of the zero lower bound. As discussed before, agents are aware of regime changes and hence know that there is a non-zero probability that the Fiscally-led policy mix will follow the announced policy shortly after the economy will be out of the zero lower bound.

The most relevant case from an empirical point of view is obviously represented by the third scenario, whose macroeconomic implications are illustrated by the solid blue line. When policymakers do not make any announcement, agents are uncertain about which exit strategy will in fact prevail. The recession is mitigated and inflation remains very close to its target value. This is because agents attach positive probabilities to the two outcomes described above: In one case, agents expect inflation stability to be preserved, while in the other case they expect a large spur of inflation in order to stabilize debt. Note that as debt keeps increasing, inflation slowly goes up in response to the increasing inflationary pressure.

Figure 7 illustrates the evolution of uncertainty across different horizons for the three scenarios presented above and using the same simulation for the preference shocks. We consider three horizons: 1 quarter (solid blue), 1 year (black dashed line), and 2 years (red dashed-dotted line). Notice that as long as the economy is at the zero lower bound uncertainty is high even if policymakers announce that they will eventually return to the Monetary led regime. This is because the zero lower bound still implies uncertainty about the end of the recession and uncertainty caused by the lack of a systematic monetary policy response to the shocks hitting the macroeconomy. Only if policymakers announce that they will move to the Fiscally led regime, uncertainty drops. This is because the economy is in fact able to leave the zero lower bound thanks to the inflationary pressure coming from the fiscal imbalance. Based on these results,

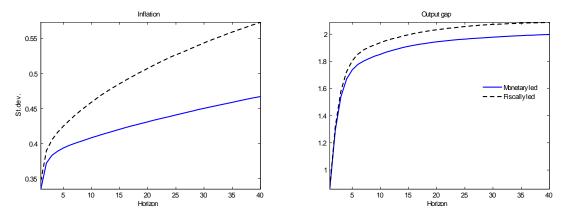


Figure 8: Evolution of uncertainty when out of the zero lower bound. The graphs report the evolution of uncertainty at different horizons conditional on being in specific regime at time 0 taking into account the possibility of future regime changes.

it is then natural to ask why policymakers do not simply announce a switch to the Fiscally led regime. We will analyze the drawbacks of such a policy in the next subsection.

5.2 ...at the Cost of Higher Macroeconomic Uncertainty

In the previous subsection, we showed that policymakers could escape the Great Recession and partially mitigate the associated increase in uncertainty if they were willing to move to the Fiscally led regime. In this subsection, we will present the drawback of such a regime change: While the regime shift would largely mitigate the recession in the short run, it would also imply an increase in macroeconomic uncertainty in the long run. In other words, we will show that the Monetary led regime generally implies a more stable macroeconomic environment when out of the zero lower bound.

Figure 8 reports the evolution of uncertainty at different horizons, from 1 quarter to 10 years, for the Monetary led and the Fiscally led regime. This measure of uncertainty is computed by taking into account the possibility of regime changes and the occurrence of Gaussian shocks. When policymakers follow the Monetary led policy mix, agents anticipate that with high probability future fiscal imbalances will be neutralized through the actions of the fiscal authority. This leads to a reduction in macroeconomic uncertainty. At the same time, the central bank behaves according to the Taylor principle, leading to a further reduction in volatility. If instead policymakers follow the Fiscally led regime, uncertainty increases at all horizons. This is because of two reasons. First, the central bank reacts less aggressively to economic fluctuations given that the Taylor principle is not satisfied. Second, agents anticipate that all fiscal imbalances that are largely neutralized when policymakers follow the Monetary led regime will now strongly affect inflation and real economic activity. Inflation, not taxation, will mainly adjust to stabilize the path for debt. In other words, the macroeconomy is heavily affected by

fiscal imbalances when the policymakers adopt a Fiscally led policy mix. As a result, under this policy mix, uncertainty is higher at every horizon because agents expect all future fiscal imbalances to be largely inflated away.

The level of uncertainty under the Monetary led regime is higher than what would be if the Monetary led regime were the only possible one. This is because of two reasons. First, as shown in subsection 4.3, the macroeconomy is not fully insulated with respect to fiscal imbalances because agents always discount the possibility of a switch to the Fiscally led policy mix. This effect is present at all horizons. Second, uncertainty is computed by taking into account that in the future the economy might in fact switch to the Fiscally led policy mix. This effect is increasing with the time horizon. In fact, as the horizon approaches infinity, the regime probabilities converge to their ergodic values and so does uncertainty.

In summary, our estimates imply that when out of the zero-lower-bound, the Monetary led regime is generally preferable because it leads to a stable macroeconomic environment. We obtain this result because under the Monetary led regime the Taylor principle is satisfied and the macroeconomy is more insulated with respect to fiscal imbalances. Furthermore, a further reduction in macroeconomic uncertainty would occur if agents regarded the Monetary led regime as the only possible one, since the macroeconomy would be completely insulated with respect to fiscal imbalances. To the extent that macroeconomic stability is desirable, countries with a strong reputation for fiscal discipline will benefit from a more favorable outcome during regular times.

This previous result is key to understand the ensuing slow recovery and why many countries, including the US, have not explicitly announced a departure from their long-run policy strategy during the Great Recession. One might correctly point out that when a departure from the Monetary led policy mix is announced, the persistence of the Fiscally led policy mix is critical to explain the increase in inflation expectations and for long-run macroeconomic uncertainty to rise. It is then worth emphasizing that the announcement is effective in mitigating the recession if and only if it is able to convince agents that the Fiscally led policy mix will prevail for a long time. Only under these circumstances agents expect that debt will be inflated away. If the Fiscally led regime had low persistence, the announcement would lead to effects on the output gap, inflation, and the macroeconomic volatility that are very similar to those associated with announcing a return to the Monetary led policy mix. Agents would simply expect a change in the timing of the fiscal adjustments. Therefore, the increase in uncertainty and the reduction in the magnitude of the recession are two sides of the same coin.

It might be argued that many countries, including the US, are now in a situation with large uncertainty about the way policymakers will deal with the large stock of debt that has been accumulated during the recent crisis. Part of the debt is expected to be absorbed by higher growth once the economy is out of the crisis. However, it is quite likely that this factor alone

will not be enough to correct the dynamics of the US sovereign debt in absence of substantial fiscal adjustments. The results presented here highlight how this type of policy uncertainty can lead to a situation in which inflation remains close to the target even if the economy spends a significant amount of time at the zero lower bound. At the same time, the model predicts that policy uncertainty leads to macroeconomic uncertainty.

Our results can also explain why markets swing between the fear of inflation and the fear of deflation in response to policymakers' announcements or decisions, as pointed out by Kitsul and Wright (2013). In our model, any announcement about the exit strategy affects agents' expectations about inflationary or deflationary episodes. Finally, the model is able to replicate a situation in which long-run inflation expectations are low on average, but surrounded by rising uncertainty. Specifically, as shown in Figure 7, even if policymakers announce that after the zero-lower-bound period they will return to the Monetary led regime, the large increase in the stock of debt and the possibility of policymakers revising their decision determine a slow moving increase in inflation and an increase in uncertainty.

6 Escaping the Great Recession

We showed that an exceptionally large contraction in demand leads to a deep recession that pushes the nominal interest rate to the zero lower bound. When this happens, policymakers would like to generate an increase in inflation expectations to stimulate the economy in the short run. Policymakers can achieve this goal by embracing the Fiscally led regime. However, in order for such a regime change to have an effect, agents have to perceive it as long lasting. In fact, announcing that a Fiscally led policy mix will be implemented for too short a time after the economy has exited the zero lower bound would lead to virtually the same macroeconomic effects as announcing fiscal discipline. In other words, once out of the recession, policymakers have to follow the Fiscally led policy mix for a prolonged period of time. Since such a prolonged deviation from fiscal discipline leads to a persistent increase in uncertainty at all horizons, policymakers can be rightfully reluctant to abandon the Monetary led regime because this regime guarantees a stable macroeconomic environment during regular times.

In this section, we propose a possible resolution of this policy trade-off. Policymakers can achieve the goal of increasing inflation expectations and at the same preserving long-run macroeconomic stability by committing to inflate away only the amount of debt resulting from the large preference shock itself. At same time, policymakers would commit to fully repay the pre-existing amount of debt and to follow the Monetary led rule in response to all other business cycle shocks. This commitment determines a sort of *automatic stabilizer*. The large preference shock can potentially cause a deep recession and a corresponding large increase in debt. The expectation that this extra amount of debt is going to be inflated away determines

an increase in inflation expectations and a corresponding drop in the real interest rate. This stimulates real economic activity, reducing the size of the output contraction. This mechanism can be strong enough to prevent the economy from hitting the zero lower bound. At the same time, agents understand that the increase in inflation is the result of a well-defined, exceptional contractionary event, which policymakers are not responsible for, while policy strategies to cope with business cycle disturbances are unchanged. Therefore, the level of uncertainty once out of the recession immediately returns to the pre-crisis levels.

To illustrate these points, we modify the model and assume that policymakers behave according to the Monetary led policy mix all the time, except when responding to the discrete preference shock $\bar{d}_{\xi_t^d}$. Specifically, we assume that the response of the nominal interest rate to inflation and of primary surpluses to debt are both zero if movements in these variables result from the discrete preference shock. In response to all the other fluctuations, policymakers instead follow the Monetary led policy mix. In order to implement this policy we construct a shadow economy to keep track of the amount of debt deriving from the discrete preference shock. Policymakers do not react to debt and inflation caused by the discrete preference shock, while they follow the Monetary led policy mix in response to all other shocks. If we denote debt and inflation of the shadow economy in which discrete preference shocks are shut down as b_t^{nd} and π_t^{nd} , we can write the linearized policy rules as:

$$\begin{split} \widetilde{\tau}_t &= \dots \delta_{b,M} \widetilde{b}_{t-1}^{nd} + \delta_{b,E} \left(\widetilde{b}_{t-1} - \widetilde{b}_{t-1}^{nd} \right) + \dots , \\ \widetilde{R}_t &= (1 - \rho_{R,M}) \left(\psi_{\pi,M} \widetilde{\pi}_t^{nd} + \psi_{\pi,E} \left(\widetilde{\pi}_t - \widetilde{\pi}_t^{nd} \right) \right) + \dots , \end{split}$$

where we assume $\delta_{b,E} = \psi_{\pi,E} = 0$ and the letter "E" stands for "Escaping." This implies that future fiscal adjustments are not enough to stabilize the entire stock of debt \tilde{b}_{t-1} , but only \tilde{b}_{t-1}^{nd} : The amount $\tilde{b}_{t-1} - \tilde{b}_{t-1}^{nd}$ is going to be inflated away. At the same time, the central bank accommodates the resulting increase in inflation $\tilde{\pi}_t - \tilde{\pi}_t^{nd}$. This is the increase of inflation necessary in order to inflate away the additional amount of debt resulting from the recession induced by the negative preference shock.

In Figure 9, we consider two scenarios. In the first one (solid blue line), the shock specific rule is implemented. In the second one, we report the behavior of the economy under the estimated benchmark model (black dashed line). Under the shock specific rule, the drop in real activity is substantially smaller than under the estimated benchmark model, and we do not observe deflation. This is due to the mechanism outlined earlier: The increase in expected inflation prevents a large drop in inflation today and determines a decline in the path of the real interest rate. Instead inflation keeps increasing as more time is spent with the negative preference shock in place. However, the increase in inflation is quite modest. This is because

⁸Appendix F explains more thoroughly how we model the shock-specific policy rule.

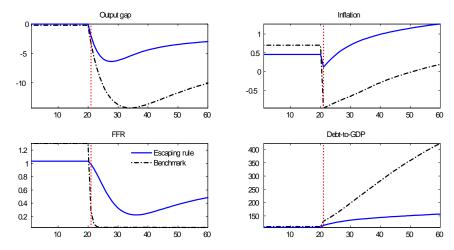


Figure 9: Escaping the Great Recession. The graph reports the evolution of the macroeconomy in response to a discrete negative preference shock for the estimated model (black-dashed line) and for the counterfactual economy in which all debt accumulated during the recession is inflated away.

the recession is largely mitigated, implying that the amount of debt that needs to be inflated away turns out to be small. Furthermore, it is important to point out that the behavior of the macroeconomy does not depend on the level of debt prevailing when the economy entered the zero lower bound because the preexisting amount of debt is always backed by future fiscal adjustments.

It is also worth pointing out that the macroeconomy also behaves differently when out of the zero lower bound because agents form expectations by taking into account all alternative scenarios. As a result, inflation is generally closer to its deterministic steady state when out of the zero lower bound. This result stems from the fact that agents are not concerned about the possibility of large spurs of inflation deriving from a shift in the monetary/fiscal policy mix.

The fact that policymakers inflate away only an amount of debt that can be imputed to the large negative preference shock has important consequences for the level of uncertainty and macroeconomic volatility faced by agents in the model. Figure 10 shows that the shock specific rule leads to a substantial reduction in uncertainty. When out of the zero lower bound uncertainty is lower because business cycle shocks are always stabilized according to the Monetary led policy mix. In correspondence of the negative preference shock, the reduction in uncertainty is even more visible. The low level of uncertainty arises because policymakers are in fact able to avoid the zero lower bound and can keep reacting to the shocks hitting the macroeconomy. The result is that the overall level of uncertainty is lower than in the benchmark case both in and out of the zero lower bound. Policymakers do not have to trade off short-run gains with long-run losses anymore.

Furthermore, given that policymakers always follow the Monetary led policy mix with respect to all business cycle disturbances, we observe a further reduction in inflation volatility

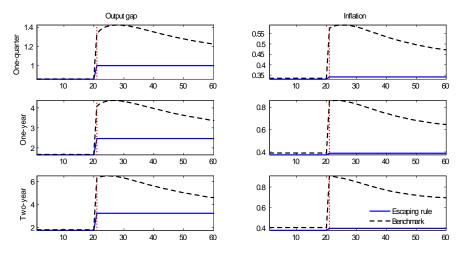


Figure 10: **Evolution of uncertainty.** The graph reports the evolution of uncertainty at different horizons for the estimated model (black-dashed line) and for the counterfactual economy in which all debt accumulated during the recession is inflated away.

with respect to the case in which switches to the Fiscally led policy mix are possible. Under the proposed policy, agents know that policymakers would generate a spur of inflation only in response to a large negative preference shock. Notice that this *reduces* uncertainty because it prevents the possibility of large deflationary states. In this respect, it is interesting to note that the resulting equilibrium path for inflation is in line with the well-established prescriptions of Eggertsson and Woodford (2003) for coping with zero-lower-bound episodes: policymakers should foster a smooth increase of inflation during and after the zero-lower-bound period. However, the mechanism outlined in this paper is quite different.

An important question is how the policy presented here could be implemented in practice. As is common in the literature, we assume in this paper that agents have perfect information and can observe the shocks hitting the economy. In reality, policymakers and agents might not have the possibility of exactly disentangling the contribution of the different shocks to the evolution of the macroeconomy. In that case, a simpler policy would consist of announcing a target for the debt-to-GDP ratio based on the pre-crisis level of debt. Policymakers would commit to raise enough taxes in order to repay the pre-existing level of debt or a projection of this value, but they would not respond to any movement in the debt-to-GDP ratio that occurs during the crisis. The part of debt above the announced target would then be inflated away. Policymakers would then return to the Monetary led regime once the crisis is over. This approach implies that any business cycle shock that occurs during the crisis would also change the level of debt that is going to be inflated away, while in the policy presented in this paper only the amount of debt deriving from the discrete preference shock would be inflated away. As a result this more realistic approach is associated with a slightly higher uncertainty during the crisis than the shock-specific strategy we outlined, but it would have the important advantage

of being easy to implement and to communicate.

Finally, in our model, the shock-specific policy occurs in response to a large negative preference shock. In reality there might be many disturbances that could require a similar change in policy. Furthermore, there might be disagreement among policymakers about whether a realized shock is large enough to trigger the policy change. A simple criterion would consist of following this alternative policy in response to all those disturbances that would drive the nominal interest rate to negative territory under the Monetary led policy mix.

Our results are related to the idea that one can rule out liquidity traps by making them fiscally unsustainable, as first proposed by Woodford (2003) and explored in a perfect foresight setting by Benhabib, Schmitt-Grohe, and Uribe (2002). An important difference is that we study the relation between policy uncertainty, inflation dynamics, and macroeconomic uncertainty at the zero lower bound. A central result of our paper consists of highlighting the trade-off between avoiding deflation and preserving long run macroeconomic stability that seems to characterize the current policy debate. A fiscally led policy mix would allow policymakers to escape the Great Recession, but it would give rise to high macroeconomic uncertainty once the economy is out of the zero lower bound. We have been able to show this result by studying the macroeconomic dynamics at the zero lower bound within a stochastic framework and allowing for policy uncertainty. Our shock specific rule is able to resolve this policy trade-off without abandoning the appeal of simple rules.

In summary, the approach proposed in this section succeeds in mitigating the recession, and at the same time in preserving long-run macroeconomic stability. The proposed policy succeeds in mitigating deep recessions because it modifies agents' beliefs about policymakers' long-run behavior in response to a specific shock. In fact, policymakers are committing to never increase taxes in response to the amount of debt accumulated during these deep recessions and at the same time not to fight the resulting increase in inflation. This policy triggers an increase in short-run inflation expectations and an immediate increase in inflation as large preference shocks hit the economy. At the same time, the proposed policy preserves long-run macroeconomic stability because policymakers are still committed to fully repay any preexisting stock of debt and to fully neutralize all other present and future disturbances affecting the debt-to-GDP ratio.

7 Conclusions

Deflation and a large recession do not necessarily arise when the economy enters the zero lower bound. If agents are uncertain about the way policymakers will deal with a rising stock of debt, inflation remains around its target value and the output loss is substantially mitigated. In this situation, any announcement about policymakers' future behavior can generate large swings in expectations and in the state of the economy. Therefore, at the zero lower bound, a well-defined exit strategy can be very effective. Policymakers can avoid a large collapse in output announcing a prolonged deviation from the Monetary led regime. Such an announcement is effective as long as the deviation is perceived to last for *sufficiently long*. Nonetheless, policymakers might be rightfully reluctant to follow this strategy because it leads to an unstable macroeconomic environment once the economy is out of the zero lower bound.

However, this policy trade-off can be resolved by announcing that only the portion of debt deriving from the exceptionally large shock will be inflated away. This creates a sort of automatic stabilizer: When the negative preference shock hits, agents foresee an increase in spending that in turn translates into an increase in inflation. Inflation starts increasing immediately through the expectation channel. The decline in real interest rates largely mitigates the recession and, consequently, the increase in debt itself. The final outcome is an equilibrium in which a moderate increase in inflation is spread over several quarters. Importantly, macroeconomic volatility returns to the pre-crisis levels as soon as the shock is absorbed because policymakers never changed their behavior with respect to the other disturbances affecting the macroeconomy. Therefore, policymakers succeed in mitigating the recession and preserving a stable macroeconomic environment.

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A Technical details

In what follows, we provide the details for the solution and estimation of the model.

A.1 System of equations

1. Linearized Euler equation:

$$(1 + \Phi M_a^{-1}) \, \widehat{y}_t = -(1 - \Phi M_a^{-1}) \left[\widehat{R}_t - E_t \widetilde{\pi}_{t+1} - (1 - \rho_d) \, d_t - \overline{d}_{\xi_t^d} + E_{\xi_t^d} \overline{d}_{\xi_{t+1}^d} \right] - (\Phi M_a^{-1} - \rho_a) \, a_t + E_t \widehat{y}_{t+1} + (1 - \rho_q + M_a^{-1} \Phi) \, \widetilde{g}_t + M_a^{-1} \Phi \widehat{y}_{t-1}$$

where $M_a = \exp(\gamma)$ and $\bar{d}_{\xi_t^d}$ follows a Markov-switching process governed by the transition matrix H^d . Please refer to the next subsection for details about how to handle the discrete shock.

2. New-Keynesian Phillips curve:

$$\widetilde{\pi}_{t} = \kappa \left(\left[\frac{1}{1 - \Phi M_{A}^{-1}} + \frac{\alpha}{1 - \alpha} \right] \widehat{y}_{t} - \frac{1}{1 - \Phi M_{A}^{-1}} \widetilde{g}_{t} - \frac{\Phi M_{A}^{-1}}{1 - \Phi M_{A}^{-1}} (\widehat{y}_{t-1} - \widetilde{g}_{t-1} - a_{t}) \right) + \beta E_{t} \left[\widetilde{\pi}_{t+1} \right] + \widetilde{\mu}_{t}$$

where we have used the rescaled markup $\widetilde{\mu}_t = \kappa \left(\frac{v}{1-v}\right)\widetilde{v}_t$

3. No arbitrage condition

$$\widetilde{R}_t = E_t \left[\widetilde{R}_{t,t+1}^m \right]$$

4. Return long term bond

$$\widetilde{R}_{t-1,t}^m = R^{-1} \rho \widetilde{P}_t^m - \widetilde{P}_{t-1}^m$$

5. Government budget constraint:

$$\widetilde{b}_{t}^{m} = \beta^{-1}\widetilde{b}_{t-1}^{m} + b^{m}\beta^{-1}\left(\widehat{R}_{t-1,t}^{m} - \widehat{y}_{t} + \widehat{y}_{t-1} - a_{t} - \widetilde{\pi}_{t}\right)$$
$$-\widetilde{\tau}_{t} + \widetilde{tr}_{t} + g^{-1}\widetilde{g}_{t} + \widetilde{tp}_{t}$$

6. Monetary policy rule

$$\widetilde{R}_{t} = \left[1 - Z_{\xi_{t}^{d}}\right] \left[\rho_{R,\xi_{t}^{p}} \widetilde{R}_{t-1} + (1 - \rho_{R}) \left(\psi_{\pi,\xi_{t}^{p}} \widetilde{\pi}_{t} + \psi_{y,\xi_{t}^{p}} \left[\widehat{y}_{t} - \widehat{y}_{t}^{*}\right]\right) + \sigma_{R} \epsilon_{R,t}\right] + Z_{\xi_{t}^{d}} \left[\rho_{R,Z} \widetilde{R}_{t-1} - \left(1 - \rho_{R,Z}\right) \psi_{Z} \log\left(R\right) + \sigma_{Z} \epsilon_{R,t}\right]$$

7. Fiscal rule

$$\widetilde{\tau}_{t} = \rho_{\tau,\xi_{t}^{p}}\widetilde{\tau}_{t-1} + \left(1 - \rho_{\tau,\xi_{t}^{p}}\right) \left[\delta_{b,\xi_{t}^{p}}\widetilde{b}_{t-1}^{m} + \delta_{e}\left(\widetilde{tr}_{t}^{*} + g^{-1}\widetilde{g}_{t}\right) + \delta_{y}\left(\widehat{y}_{t} - \widehat{y}_{t}^{*}\right)\right] + \sigma_{\tau}\epsilon_{\tau,t}$$

8. Transfers

$$\left(\widetilde{tr}_{t} - \widetilde{tr}_{t}^{*}\right) = \rho_{tr}\left(\widetilde{tr}_{t-1} - \widetilde{tr}_{t}^{*}\right) + \left(1 - \rho_{tr}\right)\phi_{y}\left(\widehat{y}_{t} - \widehat{y}_{t}^{*}\right) + \sigma_{tr}\epsilon_{tr,t}, \ \epsilon_{tr,t} \sim N\left(0,1\right)$$

9. Long term component of transfers

$$\widetilde{tr}_{t}^{*} = \rho_{tr^{*}}\widetilde{tr}_{t-1}^{*} + \sigma_{tr^{*}}\epsilon_{tr^{*},t}, \epsilon_{tr^{*},t} \sim N\left(0,1\right)$$

10. Government purchases $(\widetilde{g}_t = \ln(g_t/g))$:

$$\widetilde{g}_{t} = \rho_{g}\widetilde{g}_{t-1} + \sigma_{g}\epsilon_{g,t}, \ \epsilon_{g,t} \sim N(0,1).$$

11. TFP growth

$$a_t = \rho_a a_{t-1} + \sigma_a \varepsilon_{a,t}$$

12. Term premium

$$tp_t = \rho_{tp} t p_{t-1} + \sigma_{tp} \varepsilon_{tp,t}$$

13. The rescaled markup $\mu_t = \kappa \log (\aleph_t/\aleph)$, where $\aleph_t = 1/(1-\upsilon_t)$, follows an autoregressive process,

$$\mu_t = \rho_\mu \mu_{t-1} + \sigma_\mu \epsilon_{\mu,t}$$

14. Output target

$$\left[\frac{1}{1 - \Phi M_a^{-1}} + \frac{\alpha}{1 - \alpha}\right] \widehat{y}_t^* = \frac{1}{1 - \Phi M_a^{-1}} \widetilde{g}_t + \frac{\Phi M_a^{-1}}{1 - \Phi M_a^{-1}} \left(\widehat{y}_{t-1}^* - \widetilde{g}_{t-1} - a_t\right)$$

A.2 Model solution

As explained in the main text, the Markov-switching process for the discrete preference shock $\bar{d}_{\xi_t^d}$ is defined in a way that its steady state is equal to zero. In order to solve the model with regime changes in the policy rules and a discrete shock, we combine the methods developed by Farmer, Waggoner, and Zha (2009) with the approach used by Schorfheide (2005), Liu, Waggoner, and Zha (2011), and Bianchi, Ilut, and Schneider (2012) to handle discrete shocks. Specifically, we implement the following steps:

- 1. Introduce a dummy variable $e_{\xi_t^d}$ controlling the regime that is in place for the discrete preference shock. Augment the DSGE state vector with this dummy variable.
- 2. Use the aforementioned dummy variable to rewrite all the equations linked to the discrete preference shock. These are the linearized Euler equation and the linearized Taylor rule.
- 3. Solve the model using Farmer, Waggoner, and Zha (2009). This returns a MS-VAR:

$$\widetilde{S}_{t} = \widetilde{T}(\xi_{t}, H, \theta) \widetilde{S}_{t-1} + \widetilde{R}(\xi_{t}, H, \theta) Q \varepsilon_{t}$$

in the augmented state vector \widetilde{S}_t .

4. Extract the column corresponding to the dummy variable $e_{\xi_t^d}$ from the matrix \widetilde{T} and redefine the matrices and the DSGE state vector accordingly. This will return a MS-VAR with a MS constant:

$$S_t = c(\xi_t, H, \theta) + T(\xi_t, H, \theta) S_{t-1} + R(\xi_t, H, \theta) Q\varepsilon_t$$

where Q is a diagonal matrix that contains the standard deviations of the structural shocks and S_t is a vector with all variables of the model.

Unlike other papers that have used the technique described here, our model allows for non-orthogonality between policymakers' behavior and a discrete shock. This allows us to solve a model in which agents take into account that a large preference shock leads to an immediate change in policy, the zero lower bound, and, potentially, to further changes. This proposed method is general and can be applied to other cases in which a shock induces a change in the structural parameters.

A.3 Matrices used in the counterfactual simulations

We here describe the matrices used in the simulations reported in the paper.

A.3.1 Textbook new-Keynesian model: Always Monetary led

In the first counterfactual simulation, policymakers always follow the Monetary led regime when out of the zero lower bound. Furthermore, there is only one zero lower bound regime from which agents expect to return to the Monetary led regime. Therefore, the transition matrix used to solve this counterfactual economy is given by:

$$H^p = 1, \ H^d = \left[egin{array}{cc} p_{hh} & 1 - p_{ll} \ 1 - p_{hh} & p_{ll} \end{array}
ight], \ H = H^d.$$

where p_{hh} and p_{ll} are the estimated parameter values.

A.3.2 Coordinated announcements

In the counterfactual economy with coordinated announcements, at the zero lower bound we distinguish two cases, based on the exit strategy:

- 1. Policymakers announce that they will move to the Monetary led regime once the economy out of the zero lower bound.
- 2. Policymakers announce that they will *immediately* move to the Fiscally led regime.

We assume that the probability of the first scenario is equal to the estimated probability of switching to the Monetary led regime in the benchmark model. In other words, the first scenario is more likely than the second scenario and it has a probability equal to p_{MZ} . Furthermore, their probabilities do not depend on the regime that was in place when the negative preference shock occurred. We then have a total of four regimes $\xi_t = \{[M, h], [F, h], [Z, l], [F, l]\}$ and the corresponding transition matrix is given by:

$$H = \begin{bmatrix} p_{hh}H^p & (1-p_{ll})H^o \\ (1-p_{hh})H^i & p_{ll}H^z \end{bmatrix}$$

$$H^p = \begin{bmatrix} p_{MM} & 1-p_{FF} \\ 1-p_{MM} & p_{FF} \end{bmatrix}, H^o = \begin{bmatrix} 1 & \\ & 1 \end{bmatrix},$$

$$H^i = \begin{bmatrix} p_{MZ} & p_{MZ} \\ 1-p_{MZ} & 1-p_{MZ} \end{bmatrix}, H^z = \begin{bmatrix} 1 & \\ & 1 \end{bmatrix},$$

$$H^d = \begin{bmatrix} p_{hh} & 1-p_{ll} \\ 1-p_{hh} & p_{ll} \end{bmatrix}.$$

B MCMC algorithm and convergence

Draws from the posterior are obtained using a standard Metropolis-Hastings algorithm initialized around the posterior mode. When working with models whose posterior distribution is very complicated in shape it is very important to find the posterior mode. In a MS-DSGE model, this search can turn out to be an extremely time-consuming task, but it is a necessary step to reduce the risk of the algorithm getting stuck in a local peak. Here are the key steps of the Metropolis-Hastings algorithm:

• Step 1: Draw a new set of parameters from the proposal distribution: $\vartheta \sim N\left(\theta_{n-1}, c\overline{\Sigma}\right)$

- Step 2: Compute $\alpha\left(\theta^{m};\vartheta\right)=\min\left\{p\left(\vartheta\right)/p\left(\theta^{m-1}\right),1\right\}$ where $p\left(\theta\right)$ is the posterior evaluated at θ .
- Step 3: Accept the new parameter and set $\theta^m = \vartheta$ if $u < \alpha(\theta^m; \vartheta)$ where $u \sim U([0, 1])$, otherwise set $\theta^m = \theta^{m-1}$
- Step 4: If $m \leq n^{sim}$, stop. Otherwise, go back to step 1

The matrix $\overline{\Sigma}$ corresponds to the inverse of the Hessian computed at the posterior mode $\overline{\theta}$. The parameter c is set to obtain an acceptance rate of around 35%. The posterior is obtained combining the priors with the likelihood computed using the modified Kalman filter described in Kim and Nelson (1999).

Table 3 reports results based on the Brooks-Gelman-Rubin potential reduction scale factor using within and between variances based on the five multiple chains used in the paper. The eight chains consist of 2,100,000 draws each (1 every 3000 draws is saved). The numbers are well below the 1.2 benchmark value used as an upper bound for convergence.

C Dataset

Real GDP, the GDP deflator, and the series for fiscal variables are obtained from the Bureau of Economic Analysis. We follow Leeper, Plante, and Traum (2010) in constructing the fiscal variables. The fiscal series are built using NIPA Table 3.2. (Federal Government Current Receipts and Expenditures). Government purchases (G) are computed as the sum of consumption expenditure (L21), gross government investment (L42), net purchases of non-produced assets (L44), minus consumption of fixed capital (L45), minus wage accruals less disbursements (L33). Transfers are given by the sum of net current transfer payments (L22-L16), subsidies (L32), and net capital transfers (L43-L39). Tax revenues are given by the difference between current receipts (L38) and current transfer receipts (L16). All variables are then expressed as a fraction of GDP. Government purchases are transformed in a way to obtain the variable g_t defined in the model. The series for the FFR is obtained averaging monthly figures downloaded from the St. Louis Fed web-site. Finally, we depart from other papers in the literature that reconstruct the series for government debt using the interest payments reported in the NIPA tables and instead we use the debt series at market values from the Dallas Fed web-site. Hall and Sargent (2011) argue that the interest payments reported by the Government are not consistent with any well defined law of motion for debt. Specifically, the Government reports data that do not fully take into account revaluation effects. Revaluation effects are important in the context of our model that allows for a maturity structure of government debt. However, as explained by

Parameter	PSRF	Parameter	PSRF	Parameter	PSRF	Parameter	PSRF
$\overline{\psi_{\pi,M}}$	1.01	ψ_Z	1	$ ho_d$	1	$100\sigma_R$	1
$\psi_{y,M}$	1.05	κ	1.15	$ ho_{tp}$	1	$100\sigma_g$	1
$ ho_{R,M}$	1.01	$\delta_{b,M}$	1	$ ho_{\mu}$	1.01	$100\sigma_a$	1
$ ho_{ au,M}$	1	$ ho_{tr}$	1.01	100π	1	$100\sigma_{\tau}$	1
$\psi_{\pi,F}$	1.06	δ_y	1.03	100γ	1	$100\sigma_d$	1.04
$\psi_{y,F}$	1.05	Φ	1.04	b^m	1.01	$100\sigma_{tr}$	1
$ ho_{R,F}$	1	δ_e	1.07	$\mid g \mid$	1	$100\sigma_{tp}$	1
$ ho_{ au,F}$	1.02	$ ho_g$	1	au	1.02	$100\sigma_{\mu}$	1
d_{l}	1.01	$ ho_a$	1.01	ϕ_y	1.01	p_{FF}	1.02
p_{hh}	1.09	p_{ll}	1.02	p_{MM}	1.01	p_{MZ}	1.01

Table 3: The table reports the Gelman-Rubin Potential Scale Reduction Factor (PSRF) for eight chains of 540,000 draws each (1 every 200 is stored). Values below 1.2 are regarded as indicative of convergence.

Starting date of ZLB Regime	Likelihood	Posterior
2008:Q1	6,428.5	6,374.2
2008:Q2	$6,\!370.0$	$6,\!376.0$
2008:Q3	6,407.4	$6,\!415.1$
$2008{:}\mathrm{Q}4$	$6,\!522.4$	$6,\!521.1$
2009:Q1	$6,\!496.5$	$6,\!497.7$
2009:Q2	6,490.0	$6,\!487.9$
2009:Q3	$6,\!475.8$	$6,\!476.6$

Table 4: The table shows the value of the posterior and the likelihood at the posterior mode as the starting date of the ZLB regime changes. The results associated with the highest posterior mode are in bold.

Leeper, Plante, and Traum (2010), the two series are highly correlated implying that the choice of the series for debt is going to play only a minor role in the context of a structural estimation.

D Determining the Time of the ZLB Regime

For tractability, we fix the sequence of Markov-switching regimes to estimate the model. To select the date at which the ZLB regime has started, we compute the posterior modes associated with a number of candidate dates. As shown in Table 4, the fourth quarter of 2008 (2008:Q4) attains the highest posterior mode and hence is selected as the date at which the ZLB regime has started (recall that all models only differ in terms of the starting date for the ZLB regime, so they present the same number of parameters).

E A Prototypical New-Keynesian Model

The objective of this appendix is to show that the results of Section 4.3 are robust when one considers models that has less bells and whistles and are more agnostic about the nature of shocks than the model we estimated in the paper. Let us a consider a prototypical New-Keynesian DSGE model of the type studied in Woodford (2003) and Galí (2008) This modeling framework is purposely very stylized and follows Eggertsson and Woodford (2003) in considering unanticipated shocks to the natural rate of interest as the cause of ZLB episodes.

The loglinearized equations of the model are as follows.⁹ All the variables henceforth are expressed in log-deviations from their steady-state values with the only exception of the debt-to-output ratio b_t , which is defined in deviation from its steady-state value. The IS equation reads:

$$x_t = E_t x_{t+1} - \sigma^{-1} \left(R_t - E_t \pi_{t+1} - r_t^n \right) \tag{7}$$

where x_t denotes the gap between the actual output and its flexible-price level (henceforth, the output gap), π_t denotes inflation, R_t denotes the nominal interest rate, and r_t^n stands for the natural rate of interest, which is the real interest rate that would be realized if prices were perfectly flexible.

The new-Keynesian Phillips curve is

$$\pi_t = \kappa x_t + \beta E_t \pi_{t+1} \tag{8}$$

The monetary policy reaction function is:

$$R_{t} = \left[1 - Z_{\xi_{t}^{d}}\right] \left[\rho_{R} R_{t-1} + (1 - \rho_{R}) \left(\psi_{\pi, \xi_{t}^{p}} \pi_{t} + \psi_{x} x_{t}\right)\right] - Z_{\xi_{t}^{d}} \ln(R)$$
(9)

where R is the steady-state value of the nominal interest rate R_t . Note that the monetary authority follows the Taylor rule when $Z_{\xi_t^d} = 0$ or set its (net) nominal rate equal to its zero lower bound when $Z_{\xi_t^d} = 1$. It should be noted that $Z_{\xi_t^d}$ is a dummy variable assuming value 0 and 1 depending on the realization of an exogenous discrete Markov-switching process ξ_t^d . As we shall discuss below, this process determines the natural rate of interest r_t^n , implying that ZLB episodes are caused by unanticipated and recurrent, exogenously-driven falls in the natural rate of interest. Furthermore, when the economy is out of the ZLB, the value of the policy parameter ψ_{π,ξ_t^p} , which controls how strongly the central bank adjusts the nominal interest rate to inflation, are affected by the exogenous discrete Markov-switching process ξ_t^p .

The natural rate of interest is linked to the (exogenous) dynamics of the natural output

⁹The model can be derived from first principle. We direct the interested readers to Woodford (2003) and Galí (2008) and to Bianchi and Ilut (2012) for the derivation of the fiscal block.

though the IS equation under flexible prices:

$$r_t^n = \sigma \left(E_t \Delta y_{t+1}^n \right) \tag{10}$$

where Δy_t^n stands for the growth rate of natural output, whose value at any time is assumed to depend on the realization of a discrete Markov-switching process ξ_t^d .

The fiscal rule that determines the primary surplus τ_t

$$\tau_t = \delta_{b,\xi_t^p} b_{t-1} + \delta_x x_t \tag{11}$$

where b_t stands for the government debt-to-output ratio. Note that the response of the primary surplus to the last period's debt-to-output ratio is given by δ_{b,ξ_t^p} whose value depends on the realization of the Markov-switching process ξ_t^p that also determines the central bank's response to inflation in the Taylor rule. Hence, the process ξ_t^p captures the monetary-fiscal policy mix out of the zero lower bound

The government's budget constraint is driven by

$$b_t = \beta^{-1}b_{t-1} + b\beta^{-1} \left(R_{t-1} - \pi_t - \Delta x_t - \Delta y_t^n \right) - \tau_t \tag{12}$$

There are two exogenous Markov-switching processes: ξ_t^p and ξ_t^d . The former captures monetary and fiscal authority's response to their targets out of the zero lower bound. More specifically we assume that there are two monetary and fiscal policy mix: a Monetary led regime ($\xi_t^p = M$) and a Fiscally-led regime ($\xi_t^p = F$). Under the monetary-led regime the monetary authority responds strongly to inflation $\psi_{\pi,\xi_t^p} > 1$ and the fiscal authority promptly adjusts the primary surplus to changes in the debt-to-output ratio $\delta_{b,\xi_t^p} > (\beta^{-1} - 1)$. Under the fiscally-led policy regime the monetary authority adjusts the nominal interest rate R_t less vigorously to inflation $\psi_{\pi,\xi_t^p} \leq 1$ and the fiscal authority pays less attention to the dynamics of its debt-to-output ratio $\delta_{b,\xi_t^p} \leq (\beta^{-1} - 1)$. The transition matrix driving the policy regime out of the zero lower bound ξ_t^p is given by the following matrix

$$H^p = \left[\begin{array}{cc} p_{MM} & 1 - p_{FF} \\ 1 - p_{MM} & p_{FF} \end{array} \right]$$

The non-Gaussian process ξ^d_t determines the growth rate of natural output and hence the natural interest rate through equation (10). The growth rate of natural output $\Delta y^n_t \in \{\Delta y^n_H, \Delta y^n_L\}$, where $\Delta y^n_H > \Delta y^n_L$, and these two states evolve according to the transition ma-

 $^{^{10}}$ See Leeper (1991) for the derivation of this cut-off values for the policy parameters defining the monetary-led and the fiscally-led policy regimes.

Parameters	Values	Parameters	Values
$\overline{\psi_{\pi,M}}$	2.00	p_{hh}	0.98
$\delta_{b,M}$	0.03	p_{ll}	0.95
$\psi_{\pi,F}$	0.80	p_{MM}	0.99
$\delta_{b,F}$	0.00	p_{FF}	0.99
$\delta_{b,Z}$	0.00	p_{MZ}	0.50
κ	0.03	100π	0.5
σ	1.00	b	0.30
ψ_x	0.10	β	0.995
$ ho_i$	0.85	$\Delta y_t^n \left(\xi_t^d = h \right)$	5.30
δ_x	0.5	$\Delta y_t^n (\xi_t^d = l)$	-21.33

Table 5: Parameters used for the prototypical new-Keynesian model.

trix:

$$H^d = \left[egin{array}{cc} p_{hh} & 1-p_{ll} \ 1-p_{hh} & p_{ll} \end{array}
ight]$$

When the growth rate of natural output is low, the natural rate is low, and the policymakers are assumed to engage in the *ZLB policy regime*, which is characterized by a nominal interest rate set to zero and no adjustment of primary surplus to changes in the debt-to-output ratio.

In summary, the joint evolution of policymakers' behavior and the shock to the natural rate is captured by the regime obtained combining the two chains $\xi_t = [\xi_t^p, \xi_t^d]$. The combined chain can assume three values: $\xi_t = \{[M, h], [F, h], Z, l\}$. The corresponding transition matrix H is obtained by combining the transition matrix H^d , which describes the evolution of the preference shock; the transition matrix H^p , which describes policymakers' behavior out of the zero lower bound, and the parameter p_{MZ} that controls the probability of moving to the Monetary led regime once the negative preference shock is reabsorbed:

$$H = \left[egin{array}{cc} p_{hh}H^p & (1-p_{ll}) \left[egin{array}{c} p_{MZ} \ 1-p_{MZ} \end{array}
ight] \ (1-p_{hh}) \cdot [1,1] & p_{ll} \end{array}
ight]$$

Table 5 reports the parameter values we will use to study the property of this stylized model. The parameters π , and b denote the steady-state inflation and the steady-state value of the government debt-to-output ratio.

The exogenous drop in the growth rate of natural output is chosen so that to induce an annualized natural rate of -20% during the ZLB periods. This number is consistent with what Barsky, Justiniano, and Melosi (2014) find during the Great Recession and the ensuing slow recovery in the US when the Federal Funds rate hit its lower bound. In the benchmark calibration, we set the probability of moving to the Monetary led policy mix after the ZLB

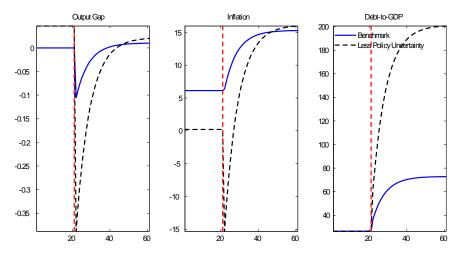


Figure 11: **Prototypical new-Keynesian model.** The figure reports the impulse responses to a discrete shock to the natural interest rate. In the Benchmark model there is high policy uncertainty, while in the counterfactual economy agents think that they are more likely to move to the Monetary led regime.

episode equal to $p_{MZ} = 50\%$ so as to capture a situation of sizable uncertainty about the policymakers' behaviors when the economy will exit the ZLB.

Figure 11 shows the dynamics of the output gap, inflation, and debt-to-GDP ratio in the aftermath of a discrete shock to the natural rate. We consider the benchmark case with parameter values reported in Table 5 and a counterfactual case in which agents are much more certain that the policy mix out of the ZLB will be Monetary led ($p_{MZ} = 85\%$). Both economies are hit by a negative shock to the natural rate at time 6.11

It should be observed that larger policy uncertainty causes absence of deflation in presence of a negative output gap as the economy hits the ZLB. Furthermore, policy uncertainty about future policymakers' behavior largely mitigates the output gap. These results are qualitatively in line with the ones obtained from our estimated model in Section 4.3. The exercise made in this section makes it clear that the results analyzed in the paper are not driven by the type of shock we chose to trigger the ZLB episode or by the more articulated nature of the model used for estimation.

F Shock-Specific Policy Rules

In this appendix, we detail the DSGE model used to perform the analysis of Section 6, in which policymakers do not respond to movements in debt deriving from the discrete preference shocks ξ_t^d . For the sake of exposition, we consider a simplified version of the DSGE model used in the estimates. This DSGE model can be expressed as follows:

¹¹Both economies are assumed to be at their respective out-of-ZLB steady-state equilibrium. However, the starting level of the debt-to-GDP ratio in the counterfactual economy is set to be equal to that in the benchmark so as to ease the comparison.

THE ACTUAL ECONOMY:

$$\widetilde{\pi}_t = \beta E_t(\widetilde{\pi}_{t+1}) + \kappa(\widetilde{y}_t - a_t),$$
(13)

$$\widetilde{y}_{t} = E_{t}(\widetilde{y}_{t+1}) - \left(\widetilde{R}_{t} - E_{t}(\widetilde{y}_{t+1})\right) + d_{t} - \mathbf{E}_{t}(d_{t+1}), \qquad (14)$$

$$\widetilde{b}_{t} = \beta^{-1} \widetilde{b}_{t-1} + b \beta^{-1} \left(\widetilde{R}_{t-1} - \widetilde{\pi}_{t} - \Delta \widetilde{y}_{t} \right) - \widetilde{\tau}_{t}, \tag{15}$$

$$\widetilde{\tau}_{t} = \delta_{b,M} b_{t-1}^{nd} + \delta_{b,E} \left(b_{t-1} - b_{t-1}^{nd} \right) + \delta_{y,M} \left(\widetilde{y}_{t}^{nd} - \widetilde{y}_{t}^{*,nd} \right) + \delta_{y,E} \left(\widetilde{y}_{t} - \widetilde{y}_{t}^{nd} \right), \tag{16}$$

$$\widetilde{R}_{t} = \rho_{R}\widetilde{R}_{t-1} + (1 - \rho_{R}) \begin{pmatrix} \psi_{\pi,M} \pi_{t}^{nd} + \psi_{\pi,E} \left(\pi_{t} - \pi_{t}^{nd} \right) \\ + \psi_{y,M} \left[\widetilde{y}_{t}^{nd} - \widetilde{y}_{t}^{*,nd} \right] + \psi_{y,E} \left[\widetilde{y}_{t} - \widetilde{y}_{t}^{nd} \right] \end{pmatrix} + \sigma_{R} \epsilon_{R,t}.$$
 (17)

where we have used the fact that $\widetilde{y}_t^* = \widetilde{y}_t^{*,nd}$.

THE SHADOW ECONOMY

$$\widetilde{\pi}_t^{nd} = \beta E_t(\widetilde{\pi}_{t+1}^{nd}) + \kappa(\widetilde{y}_t^{nd} - a_t),$$
(18)

$$\widetilde{y}_{t}^{nd} = E_{t}\left(\widetilde{y}_{t+1}^{nd}\right) - \left(\widetilde{R}_{t}^{nd} - E_{t}\left(\widetilde{y}_{t+1}^{nd}\right)\right), \tag{19}$$

$$\widetilde{b}_t^{nd} = \beta^{-1} \widetilde{b}_{t-1}^{nd} + b\beta^{-1} \left(\widetilde{R}_{t-1}^{nd} - \widetilde{\pi}_t^{nd} - \Delta \widetilde{y}_t^{nd} \right) - \widetilde{s}_t^{nd}, \tag{20}$$

$$\widetilde{s}_t^{nd} = \delta_b^{nd} \widetilde{b}_{t-1}^{nd} + \delta_y \left(\widetilde{y}_t^{nd} - a_t \right) + x_t, \tag{21}$$

$$\widetilde{R}_{t}^{nd} = \rho_{R} \widetilde{R}_{t-1}^{nd} + (1 - \rho_{R}) \left(\psi_{\pi,M} \widetilde{\pi}_{t}^{nd} + \psi_{y,M} \left[\widetilde{y}_{t}^{nd} - a_{t} \right] \right) + \sigma_{R} \epsilon_{R,t}.$$
(22)

Exogenous Processes

$$d_t = \overline{d}_{\xi_t^d},\tag{23}$$

$$a_t = \rho_a a_{t-1} + \sigma_a \epsilon_{a,t}, \tag{24}$$

It should be noted that the equations governing the behavior of the shadow economy (18)-(22) differ from those of the actual economy (13)-(17) in only one dimension: While the actual economy is buffeted by all types of shocks (i.e., ξ_t^d and $\epsilon_{a,t}$), the shadow economy is not hit by the discrete preference shock ξ_t^d . The equations of the shadow economy work as a device to keep track of the changes in the policy targets (i.e., the stock of debt b_{t-1}^{nd} and the rate of inflation π_t^{nd}) in equations (16) and (17). Finally, it is important to point out that equations (13)-(24) constitute a system of linear rational expectations equations with fixed coefficients that can be easily solved using one of the many solvers available (e.g., Gensys by Sims, 2002).