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CREATING INTRINSIC MOTIVATION IN SAVINGS AND DEBT REDUCTION

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ABSTRACT

Saving when faced with the immediate option to spend is an unpleasant but not conceptually difficult task. One popular approach contradicts traditional economic theory by suggesting that people in debt should pay off their debts from smallest size to largest regardless of interest rate, to realize quick motivational gains from eliminating debts. We more broadly define this idea as "small victories" and discuss, model, and empirically examine alternative behavioral theories that might explain it. Using a laboratory computer task, we test the validity of these predictions by breaking down this approach into component parts and examining their efficacy. Consistent with the idea of small victories, we find that when a mildly unpleasant task is broken down into parts of unequal size, subjects complete these parts faster when they are arranged in ascending order (i.e, from smallest to largest) rather than descending order (i.e., from largest to smallest). Yet when subjects are given the choice over three different orderings, subjects choose the ascending ordering least often. Given the magnitude of our results, we briefly discuss the possible efficacy of these alternative methods in actual debt repayment scenarios.

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I. Introduction

Savings provides for consumption in retirement, helps households smooth consumption after unexpected job loss, and lowers the utilization of public assistance programs. Despite its importance over the lifecycle, the accumulation of savings is far from universal. According to Bureau of Economic Analysis estimates, personal saving as a percent of disposable personal income dropped to a low of 1.4% in 2005, increasing to 4.2% in 2014 (Bureau of Economic Analysis 2010, 2014). Consumer debt compounds this problem; before many households can make progress on retirement savings, they must first reduce or eliminate personal debt. According to the Survey of Consumer Finances, in 2010, even those nearing the end of their earning years hold significant debt; over 75% of families with heads aged 55-64 held debt, and over 40% held credit card debt. Similarly, for families with heads age 65-74 more than 65% hold any debt and more than 30% hold credit card debt (Bricker et al. 2012).

There are two potential components to suboptimal savings. First, individuals have difficulty comprehending the basic principles of the dynamic optimization and financial markets ("financial literacy" see Lusardi and Mitchell 2007a, b). Second, they have self-control problems and succumb to the immediacy of short term consumption (e.g. Raab et al. 2011, Rick et al. 2008); even when subjects are capable of understanding the complexity of a savings problem, the temptation of immediate rewards can cause them to make sub-optimal longer-term decisions (Brown et al. 2009). With debt-reduction, the need for self-control should be even more pronounced. Unlike optimal saving decisions which require an understanding of economic principles, the underlying concept behind getting out of debt is simple—spend less than you earn and apply the excess towards debt. For individuals with significant debt, behavioral techniques may provide an effective approach towards optimal savings.

This paper explores one method of increasing motivation towards task completion such as debt repayment. We focus on a concept we term "small victories," the idea that people can motivate themselves to greater task completion by first completing an easier related task. That is, a larger project is not only broken up into smaller tasks, but the tasks are ordered to become progressively larger or more difficult. In relation to debt-reduction, this principle suggests there may be an additional motivational benefit for a person paying off his or her smallest debt first, and then paying the rest of his or her debts from smallest to largest. The popular press terms this method of debt reduction the “debt snowball”. This economically sub-optimal strategy has been advocated by self-help books for debt-reduction, including those by radio personality and author Dave Ramsey (1998, 2009).

> “The reason we list [debts from] smallest to largest is to have some quick wins…When you start the Debt Snowball and in the first few days pay off a couple of little debts, trust me, it lights your fire…When you pay off a nagging $52 medical bill or that $122 cell-phone bill from eight months ago, your life is not changed that much mathematically yet. You have however, begun a process that works, and you have seen it work, and you will keep doing it because you will be fired up about the fact that it works.” (Ramsey, 1998, p. 114-117)

Gal and McShane (2012) provide suggestive evidence on the efficacy of this method. Using data on 6000 debtors from a leading debt settlement company, they find that people who use the debt snowball were more likely to eliminate their debt balance controlling for debt size in comparison to other methods. Although they cannot control for selection effects (e.g. who chooses this method) or omitted variables bias (e.g. taking a financial class that encourages debt snowball use), their findings suggest that further study of this method is merited.

In comparison, standard economic models advocate paying off debts in order from highest to lowest interest rates because mechanically this method results in the least amount of money paid to interest, assuming no motivational boost from small victories. Amar et al. (2011) suggest that, when given a choice, individuals reject this economically optimal method in favor of the economically sub-optimal debt snowball approach, paying off their small debts first regardless of interest rate and then progressing to larger debts, even though doing so means that they pay more money in interest. Because their experimental set-up does not allow for motivational boosts, their participants lose money by choosing the snowball approach over the economically optimal approach.

Rather than demonstrating that people are making mistakes, we hypothesize that debt-reduction strategies that utilize “small victories” are more effective at reducing debt and increasing savings than are strategies dictated by traditional economic theory in some situations. The literature on goal setting and task motivation in the field of psychology provides competing theories on how motivation may improve with task completion. One predominant theory, often termed the “goal-gradient” hypothesis or “discrepancy theory,” proposes that the closer one gets to completing a goal, the more motivated one is to complete it (Hull 1932; Heilizer 1977).[1] A second competing theory, termed “proximal-distal” or “social-cognitive,” advocates that successful past task completion provides an emotional boost that pushes one to the next task (Bandura, 1977; 1986).[2] Numerous experiments have supported the predictions of these theories in different settings (e.g. Bandura and Simon, 1977; Bandura and Schunk, 1981; Morgan, 1985; Stock and Cervone, 1990; Latham and Seijts, 1999; Kivetz, Urminsky and Zheng, 2006; Nunes and Dreze, 2006).

To directly connect the small victories approach to these psychological theories, it is important to pinpoint exactly how these theories might cause the small victories approach to work. With this aim in mind, in Section II we develop a formalized model of task completion that allows us to make general predictions about how dividing and ordering tasks into small subtasks will affect overall task completion. Our first Proposition validates the small victories approach: if a task is already divided into a predetermined subtasks (like with many debt situations), as long as social-cognitive theory holds, optimal performance is achieved by completing subtasks in ascending order. If we relax the assumption of predetermined subtasks and allow any configuration of subtasks, a configuration of all subtasks of equal size could be optimal, provided goal-gradient factors are strong enough. Thus, of the two theories, only the social-cognitive theory is necessary for the small-victories approach to work, but both theories have implications on what type of order of subtasks might be optimal for task completion.

To test these predictions, and ultimately the affirm the validity of the small-victories approach, we develop a laboratory experiment that allows us to isolate the underlying mechanics of small victories. Unlike previous laboratory work, we abstract away from the debt repayment scenario in order to make sure that our subjects are uncontaminated with popular suggestions on

[1] Recent research suggests that for certain tasks performance is U-shaped (Bonezzi et al. 2011). That is, individuals exhibit the best performance at the beginning and end of a task, performing worse in the middle. We discuss the implications of this research in more detail in our results section.

[2] This approach is especially beneficial with complex tasks. Completing a small proximal goal before a large distal one can aid people by providing feedback about their current performance, affording individuals an opportunity to learn while completing the task (Lantham and Seijts, 1999). Two studies suggest conditions in which this approach may be detrimental to overall performance. Amir and Ariely (2008) find that having proximal goals may hinder task completion if the task will be completed with near certainty. Koo and Fishbach (2010) find that focusing on past success may cause individuals to be more likely to repeat that past success rather than challenge themselves with a more difficult task. Neither situation corresponds to our stylized debt-repayment scenario or to our experimental environment.

how to repay debt. To simulate the elements of debt repayment, we chose a task that is unpleasant but not conceptually difficult. In 30 minutes, subjects attempt to retype 150 ten-character strings in a Microsoft Excel workbook. The strings are divided over 5 columns where the length of the columns is ascending, descending or even throughout as subjects progress. The completion of each column is framed as a distinct event for each subject. This provides a sterile test for our motivational framework. If, as we find, subjects perform better with the ascending ordering, this provides evidence for the small victories approach in general and underlying social-cognitive theories in particular.

Our results show that subjects complete a tedious task faster when it is broken up into parts in order of ascending length compared to descending or equal lengths. Further analysis—which shows subjects speed up as they approach the end of columns and slow down at the beginning of columns—provides support for goal-gradient theories. However, the fact that ascending length orderings are completed faster than orderings of equal lengths, something that our experimental set-up allows us to test, demonstrates that social-cognitive factors dominate goal-gradient effects in our environment.

In a second study we find when subjects are given the opportunity to choose among all three orders they choose the ascending ordering, the one that provides the most motivational benefit, *least* often. Additionally, regression results suggest there is subject heterogeneity in the benefits of the small victories approach. Those with higher self-control, better critical reasoning skills, and higher risk aversion, as measured by survey results, benefit more from having chosen ascending. We argue a plausible extension of this result suggests the people least in need of this intervention are the ones most likely to benefit from it.

Taken at face value, our results suggest there is some benefit in the form of intrinsic motivation in taking the small victories approach to debt-reduction. While it will take substantial investigation to determine the exact magnitude of this benefit in field situations, we show in Section VI that it will only be useful to borrowers in specific cases of debt-reduction where interest rates between loans do not differ greatly. In the event of large differences in interest rates on loans, it will be best for consumers to pay off debts from largest interest rate to smallest, despite the additional motivational benefit from the small victories approach.

The remainder of the paper is organized as follows: Section II provides an overview of a theoretical framework that explains the small victories phenomenon. Section III describes our experiment. Section IV applies our model to our laboratory environment and gives the resulting theoretical predictions. Section V provides results, Section VI adds greater context to our main result, and Section VII discusses and concludes. A mathematical appendix provides the full formalization and complete proofs of the ideas presented in Section II.

II. Theory

We develop a formalized, theoretical model of task-competition. Our aim is to demonstrate how psychological theories may create a motivational boost like small victories. Our theoretical propositions will provide testable predictions about how these theories will work and whether they may be responsible for the (potential) efficacy of small victories. We illustrate the intuition of the formal theory in the framework of a debt-repayment problem. The full formalization is available in the appendix. It is important to note that even though this discussion is framed in terms of debt repayment, the theory is relevant to any type of task completion where the main task can be perceived as having separate discrete subtasks.[3]

We view the main task as X, the removal of all debt. X is a set consisting of individual payments of x. The individual payments are grouped and ordered by α, a partition of X that divides the payments into individual debts and orders the debts in the order in which they will be paid.

The function $\tau_i(x, \alpha)$ determines the amount of time that it takes an individual, i, to complete an individual debt payment, x. This function is a linear combination of two functions: $h(.)$, a function of how many payments are left in the current individual debt, and $v(.)$ how many individual debts have been completed. These components, $h(.)$ and $v(.)$, effectively represent the goal-gradient and social cognitive theories. Recall, goal-gradient theories suggest individuals speed up as they near the end of a task; this can be represented as h increasing. Similarly, social-cognitive theories suggest that individuals increase performance after past success; this can be represented as v decreasing.

Variation in performance in the time to make a payment depends on the grouping and ordering of the individual debts. The partition $\alpha=(\alpha_1,\ldots,\alpha_k,\ldots,\alpha_m)$ is made up of m ordered sets, each set can be thought of representing an individual debt. Each set α_k ($k=1,\ldots,m$) is composed of

[3] Whether the task is physically divided may not be important. What is important is that the person completing the task recognizes the task's division into subtasks.

$|\alpha_k|$ debt payments where $l=1,\ldots, |\alpha_k|$ represents the lth payment in debt k. Then individual i's performance in making payment x given debt order α is

$$\tau_i(x,\alpha) = \tau_i(\alpha_{kl}) = \mu_i + h(|\alpha_k| - l) + v(k) + \varepsilon_{i\alpha_{kl}}$$

where μ_i represents individual i's baseline performance, $|\alpha_k| - l$ represents the number of payments to the end of the current debt, k, and $k-1$ is the number of previous debts successfully paid. The total time it takes to remove all debts under α, $T_i(\alpha)$, is the sum of all $\tau_i(x, \alpha)$,

$$T_i(\alpha) = \sum_{x\in X} \tau_i(x,\alpha) = \sum_{k=1}^{m} \sum_{l=1}^{|\alpha_k|} \mu_i + h(|\alpha_k| - l) + v(k) + \varepsilon_{i\alpha_{kl}}.$$

For our first proposition, we will incorporate social-cognitive theory into our model. We will do this by assuming that v, the social-cognitive function, is non-increasing. In other words, after an individual debt is paid off, the debtor is motivated to pay off future individual debts faster or at least as fast as previous debts. If this assumption is true, then the debtor would want to get a quick boost immediately in order to benefit from the increased productivity. If one can only re-order debts, but is unable to change the structure of any debts (i.e., the number of payments inside a debt), then ordering debt from smallest to largest, something we are terming an ascending ordering, would result in the fastest debt repayment, while ordering debts from largest to smallest would be the slowest.

To formalize this idea we define $A(X)$ as the set of all subtask partitions that could be formed from X. We define a class of subtask partitions as a subset of $A(X)$ where all terms differ by only the re-ordering of subtasks. Each possible re-ordering is included in the set. Using this terminology, Proposition 1 demonstrates the superiority of an ascending ordering in this case, the central tenant of the small victories approach.

Proposition 1. *For any i, for a given class of subtask partitions, $\beta \subseteq A(X)$. Define an ascending ordering, α' where*

$$|\alpha'_1| \leq \ldots \leq |\alpha'_k| \leq \ldots \leq |\alpha'_m|,$$

and a descending ordering where

$$|\alpha''_1| \geq \ldots \geq |\alpha''_k| \geq \ldots \geq |\alpha''_m|.$$

Then for any $\alpha \in \beta$,

$$T_i(\alpha') \leq T_i(\alpha) \leq T_i(\alpha'').$$

If v is non-constant and $\alpha' \neq \alpha''$, $T_i(\alpha') < T_i(\alpha'')$.

Proposition 1 shows that all that is required for the small victories approach to be effective in our framework is that the assumptions of social cognitive theory hold. It requires no assumptions about goal-gradient theories; conditions on function *h* are irrelevant in the proof of Proposition 1 available in the appendix. Thus, under social cognitive theory, the small victories approach works: arranging subtasks in ascending order leads to better performance.

Given that ascending orderings are optimal when the assumptions of social-cognitive theories hold, a remaining question is what structures might be ideal for task completion under the competing goal-gradient theory. If goal-gradient theory holds and the effect of social cognitive theories is zero, then an even ordering, that is, a main task divided so that all subtasks are of the same length, will be ideal. In the context of debt-repayment this is equivalent to having our *m* debts have exactly the same number of payments.[4] If we enact the general assumptions of goal-gradient theory,[5] this ordering is optimal relative to any other possible ordering (Proposition 2) with the same number of debts.

Proposition 2. *Suppose v is constant. Then for any X, i, and m, if there exists an* <u>*even ordering*</u> $\alpha' \in A(X)$ *such that* $|\alpha'_k|=c$ *for all* $1 \leq k \leq m$ *then*

$$T_i(\alpha') \leq T_i(\alpha) \qquad \forall \alpha \in A(X) \text{ where } |\alpha| = |\alpha'| = m.$$

Further, if all the subtasks in α are not of the same length and h is strictly increasing, $T_i(\alpha') < T_i(\alpha)$.

In other words, we assume the goal-gradient hypothesis holds and social-cognitive theories have no impact. Here, what is important to debt-repayment productivity is how close you are to paying each individual debt—how close you are to the end of the task.[6]

Unlike Proposition 1, which places no restrictions on its competing theory, Proposition 2 requires the effects of social-cognitive theory to be zero. This added assumption allows Proposition 2 to apply to a much greater domain.[7] It is possible to make equivalent assumptions

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[4] This type of ordering may be impossible to construct in an actual debt-repayment situation. One of the benefits of a laboratory setting is our ability to test the components of this theory.

[5] That is we assume *h* is non-decreasing, meaning motivation increases or at least does not decrease the closer one gets to the end of a debt. We also hold *v* constant because that term is not involved with goal-gradient theory.

[6] This conclusion is more obvious using another task completion example. If instead of debt repayment, we think of port-a-potties at an amusement park, where the distance to the end is the only thing that matters, optimally all lines will be the same size. (Similarly you could think of sorting into "10 items or less" lines at the grocery store.)

[7] Proposition 2 shows an even ordering with *m* subtasks is optimal over *any arrangement* of elements into *m* subtasks. Proposition 1 shows that an ascending ordering with *m* subtasks is optimal over *any arrangement* of those subtasks. The domain in Proposition 2 necessarily contains the domain of Proposition 1.

(i.e., make the goal-gradient effects zero) and show an ascending ordering is optimal across a similar domain. The result is Proposition 3.

Proposition 3. *Suppose h is constant. Then for any X, i, and m, there exists a subtask partition* $\alpha' \in A(X)$*, in ascending order,* $|\alpha'_1| \leq \ldots \leq |\alpha'_k| \leq \ldots \leq |\alpha'_m|$*, for all* $1 \leq k \leq m$*, such that*

$$T_i(\alpha') \leq T_i(\alpha) \qquad \forall \alpha \in A(X) \text{ where } |\alpha| = |\alpha'| = m.$$

Further, if α *is not ascending and v is strictly decreasing,* $T_i(\alpha) < T_i(\alpha')$*.*

Propositions 2 and Proposition 3 are based on different assumptions. With Proposition 2, only the goal-gradient theory holds, meaning completion of past subtasks does not matter, and only distance to the end of those subtasks matters. In such case the even ordering will feature better subject performance than either ascending or descending orderings. Alternatively under social cognitive theory, only successful completion of past subtasks matters, meaning distance to the end of subtasks does not matter. In such case the ascending ordering will feature better subject performance than either even or descending orderings.

Both factors may be present in subject behavior, therefore we would like to have a way to talk about the relative strength of each factor. To this end, we develop the concept of "dominance." For any two subtask partitions, we say the social-cognitive factors *dominate* the goal-gradient if the total differences across the social-cognitive term are greater in magnitude than the total differences across the goal-gradient term.[8] Alternatively, we say the goal-gradient factors *dominate* the social-cognitive if the previous relation is reversed. If both magnitudes are equal, then there is no dominance.

Proposition 4 shows that our definition will be useful in explaining results. Over a given three orderings, ascending, descending and even, subject performance in the ascending ordering will be faster than even *if and only if* social-cognitive factors dominate goal-gradient. Subject performance in the even ordering will be faster than ascending *if and only if* goal-gradient factors dominate social-cognitive. If there is no dominance, the performance of subjects on ascending and even orders should be the same.

Proposition 4. *For a given* Δ*-set,* $\{\alpha^a, \alpha^d, \alpha^e\}$ *where all the subtasks in* α^a *are not of the same length, for any i,*

1. $T_i(\alpha^a) < T_i(\alpha^e)$ *if and only if social-cognitive factors dominate goal-gradient factors.*

[8] See Appendix for a formal definition of dominance.

2. *$T_i(\alpha^a)>T_i(\alpha^e)$ if and only if goal-gradient factors dominate social-cognitive factors.*
3. *$T_i(\alpha^a)=T_i(\alpha^e)$ if and only if there is no dominance between social-cognitive and goal-gradient factors.*

Our theoretical framework is agnostic on whether ascending or even orders will lead to optimal task performance—it depends entirely on the relative strength of the social-cognitive and goal-gradient factors. However, it does have a clear result about the descending ordering in the corollary to Proposition 4.

Corollary. *For a given Δ-set, and any i, $T_i(\alpha^d)\geq T_i(\alpha^a)$, $T_i(\alpha^e)$. Provided all the subtasks of α^d are not of the same length, $T_i(\alpha^d)>T_i(\alpha^a)$ if v is non-constant; $T_i(\alpha^d)>T_i(\alpha^e)$ if either v is non-constant or h is strictly increasing.*

Proposition 1 already tells us that a descending ordering should be completed more slowly than the ascending ordering. The corollary shows that provided *either* the *v* function is non-constant or the *h* function is strictly increasing (either social cognitive factors exist or goal gradient factors exist under certain conditions), the even ordering should be completed faster than the descending ordering. To frame this in terms of debt repayment, completing one's debts in descending order leads to the least motivational gains.[9] Our experiments will compare ascending, descending, and even-length orders to test these predictions. Section IV will discuss this theory in the context of our experiment.

III. The experiment

a. Design

In the experiment, subjects typed ten-character lines of text in a Microsoft Excel worksheet. Subjects would type a line of text in a cell and then click a button on their worksheet. If they typed the line correctly, they would move to the next cell; if they typed it incorrectly, nothing would happen until they typed the line of text correctly. The lines of text included upper and lower case letters, numbers and their shifts[10] (e.g., !#) and had been randomly constructed before so that each subject encountered the same order and same lines of text in cells.

[9] Note that the existence of goal-gradient factors make no prediction on whether ascending or descending should have greater performance. Any superiority of ascending compared to descending comes from social-cognitive factors.

[10] To avoid confusion the characters "I," "l," and "|" were excluded. The sign "@", which produces a hyperlink in Excel, was also excluded.

The experiment had two main parts: a practice session, and one large typing task. Subjects would either complete their tasks or reach the time limits.[11] The practice session had a 5-minute time limit, and the large typing task had a 30-minute time limit. The practice task was the same for all subjects. It consisted of each subject typing ten lines of text. It was designed to familiarize subjects with the experiment as well as to get an estimate of their general skill in these typing tasks.

The larger, 30-minute, tasks varied depending on the environment, but all tasks featured subjects encountering 150 lines to type divided into 5 columns. In the initial study (as opposed to the "choice" study described later), subjects were randomly assigned to three orders. In the "ascending ordering," the columns increased in size. The columns had 10, 20, 30, 40, and 50 cells respectively. In the "descending order," the columns decreased in size and had the reverse ordering of the ascending (50, 40, 30, 20, and 10). In the "even ordering" all columns had 30 cells. Figures 1(a-c) provide screenshots of the initial worksheet under each of the three orders.

Subjects completed their columns in order, left to right. Unless completing a column, every time a subject finished a cell he or she would move down to the next cell below the current cell. To frame column completion as a distinct event, the experimental interface would open a message box every time a subject completed a column. For all columns but the last column, the message said "You have completed X columns. Only 5-X to go!" Once subjects clicked ok on that message, they moved to the cell at the top of the next column to the right. When subjects finished the last column, a similar message informed them informed they had finished the task.

Subjects were paid $10 if they could complete the task in under 30 minutes, plus an additional $0.50 for every minute they finished early, rounded up to the nearest minute. Subjects that did not complete the task were paid $10 minus $0.05 for every cell they left uncompleted. Because this structure guarantees a minimum payment for subjects, there were no additional payments given to subjects (i.e., show up fees).

After examining the results of the initial study of subjects, one remaining question was how subjects might have chosen among the orders if given that choice. This question led to the creation of a second, "choice" study. The study featured the same task and basic design as the

[11] After the typing tasks, subjects had a ten-minute break and then participated in various pilots of future typing tasks. Subjects were aware of this second 30-minute session at the beginning of their experiments. The results of the pilot sessions are not presented here and are not relevant to the data analysis or conclusions of this paper.

first, except that before the experiments began, subjects were allowed to choose whether they would encounter the ascending, descending or even ordering. The experimenter showed pictures (nearly identical to those shown in Figure 1) of each ordering on a screen with randomized names (i.e., “a”, “b” and “c”) and subjects would click on any of three icons corresponding to those names. After the subjects clicked on the icon, they would have a five-minute practice session and a 30-minute typing task. Again, the second half of the experiments was used for piloting other effort tasks.

It is important to reiterate that these experiments were not designed to mimic debt repayment scenarios, but rather to test the underlying psychological theories of motivation that might affect debt repayment. Like debt repayment, these experiments involve tasks that are mildly unpleasant but not conceptually difficult. However, in order to focus on the most basic features of the “small victories” scenario, the experiment did not feature the full debt-repayment scenario including interest rates, minimum payments, and so on.

b. Procedure

Both studies took place at the Economic Research Laboratory at Texas A&M University. Subjects were recruited from the econdollars website (econdollars.tamu.edu) that uses ORSEE software (Greiner, 2004). Subject earnings averaged $11.25 for the 35-minute session, with subjects in the initial study averaging $10.95 and subjects in the choice study averaging $11.63.[12]

Ninety-one subjects participated in the initial study between December 6, 2011 and March 7, 2012. Subjects completed a demographic survey (from Eckel and Grossman, 2008), the Barratt Impulsivity Test (BIS 11) (Patton et al., 1995), the Zuckerman Sensation-Seeking Scale (SSS-V) (Zuckerman, 1994) and a five-factor personality assessment (John et al., 2008).[13]

Seventy subjects participated in the choice study between January 30-31, 2013. Because the first set of surveys had little explanatory power and helpful suggestions brought other surveys to the attention of the authors of this paper, a second set of surveys was used in the choice study. Subjects completed a demographic survey and non-incentivized risk-preference choice to elicit risk attitudes (both from Eckel and Grossman, 2008), financial literacy questions from the Health

[12] Including the second pilot task, subject earnings averaged $21.67 for 75 minutes, with initial study subjects averaging $20.01 and choice study subjects averaging $23.83.

[13] None of these three personality tests in the initial study were correlated with any subject performance measures (see Appendix, Table 1). We will not discuss these tests in relation to our results. In retrospect, it would have been preferable to replace these surveys with the surveys used in the choice study (see below).

and Retirement Survey (Lusardi and Mitchell 2007b), the Tangney-Baumeister-Boone Scale (Tangney et al. 2004), and the Cognitive Reflection Task (Frederick, 2005).

IV. Theoretical Predictions

Our theoretical framework shows that an ascending ordering is the idea structure for task completion if only social-cognitive factors matter (Proposition 3), an even ordering is the ideal structure if only goal-gradient factors matter (Proposition 2), and as long as social-cognitive factors exist an ascending ordering should do better than a descending ordering (Proposition 1).

Proposition 4 ties these three propositions together in an environment where both social cognitive and goal gradient theories may hold. It uses the term "dominance" to classify whether social-cognitive or goal-gradient factors have greater influence on results. Its corollary also shows that the descending ordering should have the worst performance regardless of the relative strength of these factors. Because it is much like a cumulative proposition in this way, the predictions for this experimental environment can be found entirely in Proposition 4 and its corollary.

Prediction 1a (Proposition 4). If social-cognitive factors dominate goal-gradient factors across our orders, subjects in the ascending orders will finish faster than those in the even orders, and be more likely to complete their tasks in the time allowed. Formally,

$$t_A < t_E, \qquad p_A > p_E.$$

where t is the time to completion and p is the probability of completion.

Prediction 1b (Proposition 4). If instead goal-gradient factors dominate social-cognitive factors, subjects in the even orders should finish faster than those in the ascending orders, and be more likely to complete their tasks in the time allowed. Formally,

$$t_E < t_A, \qquad p_E > p_A.$$

Prediction 2 (Corollary to Proposition 4). Regardless of dominance, subjects in both ascending and even orders should finish faster than those in descending orders, and be more likely to complete their tasks in the time allowed. Formally,

$$t_A, t_E < t_D, \qquad p_A, p_E > p_D.$$

Combining these predictions we have one of two possible predicted orderings,

1a. $t_A < t_E < t_D$ if social-cognitive factors dominate,

1b. $t_E < t_A < t_D$ if goal-gradient factors dominate.

Because small-victories (or in relation to debt, the debt-snowball, Ramsey 1998, 2009) recommends ordering tasks in ascending order to create maximum motivational gains, the advice is most consistent with environments where social-cognitive theories dominate. Thus the greatest validation of small-victories (and social-cognitive theory) would be if prediction 1a held. If, instead, even orderings are faster than both ascending and descending, then goal-gradient factors dominate social cognitive factors. However, in most cases of debt repayment arranging one's debt in an even ordering is not possible, while paying debt in ascending and descending orders is possible. So as long as ascending orders are faster than descending orders we would find validation of the concepts behind the small victories approach (and Proposition 1).[14]

Our underlying psychological theories also make predictions about subject performance in specific columns throughout the experiment. If our assumptions based off social-cognitive theories are valid, we should see subjects speed up as they complete more columns. If our assumptions about goal-gradient theories are valid, subjects should perform faster as they reach the end of each column.

V. Results

a. Time to Completion and Probability of Completion (Predictions 1a and 1b)

In experiment 1, consistent with Prediction 1a and the small-victories approach, subjects performed in the ascending ordering 1.42 seconds per cell faster on average than in the descending ordering (significant at the 5% level), as shown in Table 1 (Panel I). This relationship does not substantially change when ascending is compared to the pooled results of both descending and even orders (1.23 seconds per cell faster on average, two-sided p-value: 0.019). Even subject performance falls between ascending and descending, which is inconsistent with Prediction 1b, but is consistent with Prediction 1a, suggesting social-cognitive factors dominate in this environment. A Kruskal-Wallis test indicates the differences for all three orders are significant at the 10% level (two-tailed p-value: 0.084). Both the Cuzick trend test and the Jonckheere-Terpstra test for ordered alternatives find the ascending-even-descending ordering to

[14] If even orderings do lead to the best subject performance, one could still question the wisdom of the small victories approach. The result would suggest an alternative approach that applies goal-gradient theory rather than social cognitive theory to debt-reduction would likely be more effective. Of course, it is not clear whether such alternative approach could be plausibly applied to these scenarios.

be significant, with p-values of 0.0260 and 0.0265 respectively.[15] As a robustness check, a regression (see Table 3) which controls for subject practice time does not appreciably change results. Additionally, time to complete each cell in seconds during the practice time is strongly correlated with actual time in seconds with a coefficient of 0.133 and a standard error of 0.045 (results not tabled).

Table 1 (Panel II) shows the number of subjects that completed the full task, that is, those who copied all 150 cells in the 30-minute limit. A higher percentage of subjects (71%, 22 of 31) complete the task in ascending than descending (48%, 14 of 29) or even (58%, 18 of 31). A Pearson's chi-square test reveals this difference is meaningful at the 10% level. As before, this result is consistent with Prediction 1a, and the idea of social-cognitive factors dominating in this environment, but not consistent with Prediction 1b and the idea of goal-gradient factors dominating.

b. Relative cell speed during task completion

A crucial assumption behind both predictions 1a and 1b is that subjects complete cells at different speeds depending on their position within the columns that make up the general task. The validity of this assumption can be examined directly. Figure 2 shows subject performance for each ordering relative to average performance. Consistent with the goal-gradient hypothesis, subjects complete cells at a faster rate as they near the completion of a column. While subjects do not appear to start columns immediately faster, their time per cell greatly decreases over the course of completing columns, which is consistent with the social-cognitive theories. Further, note that this decrease is inconsistent with fatigue; subjects are speeding up over time.[16]

To test whether this apparent speed-up shown in the figures is statistically significant, we use t-tests comparing the speed in the first five cells to that in the last five cells. In order to control for a potential effect of subjects performing faster over the course of the experiment,[17] we

[15] We report two-tailed values whenever possible. Literally interpreting our predictions (ascending<even< descending) would result in the one-tailed p-values of 0.0130 and 0.0132 for the Cuzick trend test and the Jonckheere-Terpstra test for ordered alternatives, respectively.

[16] Bonezzi et al. (2011) find that for certain tasks performance is U-shaped: individuals exhibit the best performance at the beginning and end of a task, performing worse in the middle. As a quick look at Figure 2 demonstrates, there is little support for this finding in our results. We do not see an upside-down U-shape either overall or within each column. If anything, the only support for this idea is that subjects appear to slow-down around cell 75, the middle of the task, regardless of treatment modality.

[17] The apparent slow-down around cell 75 (see footnote 13) and the potential for subjects performing faster over the course of the experiment are elements of performance that are not captured in our theory. Because they are

control for a linear acceleration trend by using the residuals from a corrected equation on time. Thus, our coefficients indicate distance from the population mean for each environment across all orders. Next, we created a new variable that indicated a "1" if the cell completed was in the first five cells of a column and a "0" if in the last five cells of a column. Cells that were neither in the first nor last five of any column were coded as missing. These results are presented in Table 2.

As shown in Table 2, participants complete the last five cells of each column on average 1.11 seconds faster than they complete the first five cells of each column. The pattern is similar for descending, but not significant, and attenuated for equal. To bound for a non-linear effect of subjects performing faster over the course of the experiment, we repeated these t-tests with the first five and last five cells overall removed from the analysis. That is, we stared with the last five cells of the first column and ended with the first five cells of the last column. Results, though attenuated compared to the bound in the other direction, were same-signed and still significant and are available from the authors.[18]

Additionally, there may be some concerns that we are over-stating our power, depending on the assumptions of independence of columns or individuals. Therefore, we also present more conservative t-tests and regressions that assume that each set of five cells should be treated as one observation, assume that the first 5 cells across all columns should be treated as one observation per person as should the last 5 cells, or group all orders together. Even the most conservative t-tests only reduce previously significant results from 5% to 10% with magnitudes largely unchanged, as shown in Table 4. We also provide regression results in Table 5 with clustering at the subject, cell, and subject*column levels, allowing for different assumptions about the standard errors. Again, these results are consistent with our t-test results.

As an overall trend, the results presented here are consistent with both goal-gradient and social-cognitive theories. While the results supported prediction 1a and not prediction 1b, this

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independent of ordering, incorporating such elements would not affect any theoretical propositions or experiment predictions. To see this, note that an extra term, say γ_j, could be added to equation $\tau_i(x, \alpha)$ where $1 \leq j \leq |X|$ indicates the jth element completed in the global task X. Since γ_j does not depend on subtask partition it will cancel out in all proofs, much like terms μ_i and $\varepsilon_{i\alpha_{kl}}$ already do.

[18] A related worry is that some participants do not finish the experiment and thus these slower participants may be completing fewer "last" cells than "first" cells depending on where in the sheet they stop. A robustness check that cuts first and last cells until there are an even number of cells on either end (or as close as possible to an even number of cells) finds remarkably similar results to our main results.

only means that the social-cognitive factors were able to dominate the goal-gradient factors in this environment. The relative support for the Corollary to Proposition 4 (descending orders being the worst performing) also is consistent with either theory. Our separate results on speeding up at the end of columns provide support for the validity of goal-gradient theories. The support of prediction 1a gives credence to the idea of the small victories strategy in generalized task completion environments.

c. Choice Study

In a second study, 70 subjects were given the opportunity to choose their ordering. Table 6 shows the results of this choice. Of the 70 subjects, 16 chose ascending, 31 chose even and 23 chose descending. A Pearson's chi-square test reveals these results are different from a random distribution at the 10% level.[19] Strikingly, the ascending ordering—the method that follows the small-victories theory and is shown to lead to the best performance among subjects in the initial study—is the least preferred. Less than one fourth of all subjects (22%) choose that method.[20] This general trend is in contrast to Amar et al. (2011), who find that subjects prefer to pay debts from smallest to largest without considering interests rates, albeit in a very different choice problem. Their environment, unlike ours, transparently resembles debt-repayment. This difference may cause subjects familiar with the debt-snowball strategies to follow the advice directly. Our environment was designed specifically to abstract from the debt-repayment problem so as not to use popular debt-snowball heuristics.

Regression estimates suggest that the choice of ascending would help some participants more than others.[21] Table 7 explores the effects of order on performance in the choice study for participants interacted with their answers to survey questions on self-control, critical reasoning

[19] One surprising difference is that subjects complete cells faster in the choice study than with random assignment, and that practice times are faster in the choice experiment. Because the practice time occurred after the choice decision, it is possible that being offered a choice improves efficiency, but we do not have enough evidence to test this possibility and doing so would be outside the scope of this paper. These experiments occurred with the same subject pool in the same lab roughly a year apart. So it is possible the difference in practice time results are due to a year effect but t-tests find no substantial or significant differences between basic demographic characteristics collected for both samples and the authors have no reason to believe anything changed in the subject pool during that time.

[20] The result is, however, reminiscent of Loewenstein and Prelec (1993) who find subjects prefer outcomes that improve over time provided those outcomes are explicitly defined as sequences.

[21] Although participants only participate in one condition, randomization allows us to use interaction terms in the regression analysis to present counterfactuals.

skills, and risk aversion.[22] Participants with higher measures of self-control benefit more from ascending than from equal with a one point increase in the self-control scale leading to a 0.139 second decrease in average cell time compared to those who chose equal, as shown in column (1). Similarly, participants with higher critical reasoning skills benefit more from the ascending choice than do other participants; a one-point increase in critical reasoning skills leads to a one second decrease in average cell time, with significance at the 10% level in column (2). Finally, the interaction between risk aversion and the choice of ascending is also negative; a one-point increase in risk aversion leads to a drop of 0.71 seconds on average cell time, also significant at the 10% level in column (3).

VI. Extension to Field Debt-Reduction Environments

Our main result is that individuals increase their performance in tedious tasks when those tasks are broken down and put in ascending rather than descending order. When directly applied to the field, this suggests there is some benefit in using the small victories approach to debt reduction. While it will take substantial investigation to determine the actual magnitude of this benefit in the field, we can project in what types of debt situations the small victories approach would be effective using the estimated benefit from our experiments.

In the initial study, subjects in the ascending ordering, on average, complete a cell in 11.08 seconds compared to 12.50 seconds in the descending ordering (as seen in Table 1, Panel I). Converted to rates, these values are 325 and 288 cells/hour, for ascending and descending, respectively. Thus, in terms of total performance, our results suggest subjects in the ascending ordering are about 13% more productive than descending.

We caution that these results should not be used to make definitive conclusions about debt-reduction situations without further analysis. The 13% figure is for illustrative purposes in order to show that there will be limits to the small victories approach. The actual number used is unimportant; for any number, there exists a difference in interest rates in which the small victory approach will not be beneficial. Field experiments may provide an exact number in specific

[22] We also control for practice time to make sure that the results are not being driven by differences in ability among the different choice orderings. There is little evidence this is the case. With the exception of suggestive evidence that types that choose ascending over descending are slightly more proficient at the typing task (10% level), we find no other significant correlations in the data. Another issue would be if survey answers were correlated with choice. With the exception of our risk aversion measure, which may predict the choice of ascending over equal at the 10% level in a multinomial logit, there is no significant difference between those who choose ascending and those who choose other orderings.

contexts, but that number could also vary tremendously across different contexts and individuals. We hope that the following exercise can illustrate how such a number could be used to determine the magnitude of the benefits of small victories for faster debt repayment compared to the drawbacks of a higher interest rate.

Suppose an individual has two $10,000 outstanding loans. The first loan is at 10%, and the second has a rate between 10% and 20%. She may make monthly repayments of $300[23] on either loan. Suppose repaying the first loan first triggers the psychological motivations of small victories,[24] and this individual is able to come up with 13% more on each payment, for a total payment of $369.

Figure 3 shows the total difference in months to repay the two loans when following the small victories approach compared to the conventional method. In this example for all interest rates 16% and below, this individual would pay back both loans faster following the small victories method than the conventional economic method. But for rates 17% and higher, the conventional economic method of paying down debts with a higher rate of interest still produces faster debt repayment even though one is paying less per month.

Figure 4 shows the total amount spent on loans in both these cases. For rates 12% and lower, the additional psychological boost of the small victories and subsequent increase in debt repayment leads to a lower amount spent on loans than under the standard economic strategy. For rates between 13% and 16% inclusive, more is spent in total using the small victory method, but that is only when one includes the assumed $69 boost each month from following that method. Depending on whether one believes that money would have been wasted or put to good use, the small victories method may or may not achieve a greater benefit for this individual. For values 17% and above, it is clear the individual is spending more on loans following the small victories method than the conventional method.

While the preceding is only an illustrative example—actual parameters in debt-repayment situations vary greatly—the general lesson should be clear. Even if the small victories approach

[23] While this number was chosen somewhat arbitrarily, note that values much smaller than this (e.g., $100) will never pay off either loan. Values much larger than this (e.g., $1000) pay off the loan too quickly for interest to make much of a difference.

[24] Without significantly changing our results, we could make the first loan worth $9,999.99 and the second loan $10,000.01 just to be consistent with the idea of ascending and descending orderings and the small-victories approach.

can give individuals the ability to save x% more on average, there is a limited range around x% in which the difference between interest rates is overcome by the motivational boost. In general, this method works best when individuals have debts with similar interest rates.

VII. Discussion and Conclusion

We have examined a non-traditional approach to debt payment, advocated by personal finance gurus such as Dave Ramsey, by developing the psychological theories that underlie it into a formal model. We find that if the effects of one set social-cognitive theories (Bandura 1977; 1986) dominate, ordering subtasks in ascending order of difficulty should produce optimal performance. Instead, if the effects of a competing, goal-gradient theory (Hull 1932; Heilizer 1977) dominate, then dividing the task into equal lengths will produce optimal performance, an approach that Ramsey does not advocate (though it may not be possible in a debt-repayment situation). In the initial study of this experiment, subjects who are randomly assigned to the small victories treatment (i.e., the ascending ordering), perform significantly faster and complete a higher percentage of tasks on average than in other orders. This result is supportive of social-cognitive psychological theories of motivation; it also provides support for the debt snow-ball approach provided this increase in motivation overcomes differences in interest rates.

We find additional evidence for both of sets of psychological theories by directly examining our data. Subjects speed up at the ends of columns relative to their performance at the beginning of those columns, consistent with the goal-gradient hypothesis. Additionally, our estimations find that past columns completed is positively correlated with performance, consistent with social-cognitive effects. If both factors are present, Proposition 1 shows that in our environment subjects in ascending ordering should outperform those in the descending ordering. Further, the Corollary to Proposition 4 shows subjects in the even ordering should outperform those in the descending ordering. With both factors present, Proposition 4 shows that ascending subjects will outperform even subjects in this environment if and only if social-cognitive factors dominate goal-gradient. We conclude in our environment this must be the case.

Interestingly, when we allow a new set of subjects to choose which of the three orders they prefer, the ascending ordering is chosen least often. Our regression results indicate that the subjects who benefit most from the ascending ordering are subjects with the highest self-control and reasoning ability. This last finding may suggest a flaw with the debt-snowball approach: the people who would benefit most from small victories may be the ones least likely to be in debt. Obviously, future research will need to look at the issue more carefully, perhaps in more stylized debt-repayment scenarios or with surveys and actual field data.

To conclude, the strength of the small victories effect could be strong enough to overcome small deviations in loan interest rates. However, we must caution those who advocate the debt-snowball strategy. Our results indicate that the "small victory" of paying off the smallest debt first may increase motivation in debt repayment. However, this increase in motivation may not offset the additional interest accrued by not paying off the highest-interest-rate debts first if there are relatively different interest rates across debts.

Future research can determine the full effects of framing debt in these situations. Adding additional factors to our experimental design, commonly found in debt-repayment scenarios, such as interest rates, minimum payments, and actual cash values may aid in determining the appropriate bounds on the motivational improvement of the small victories method. Field experiments that randomly assign repayment strategies to consumers with debt are also a promising future direction.

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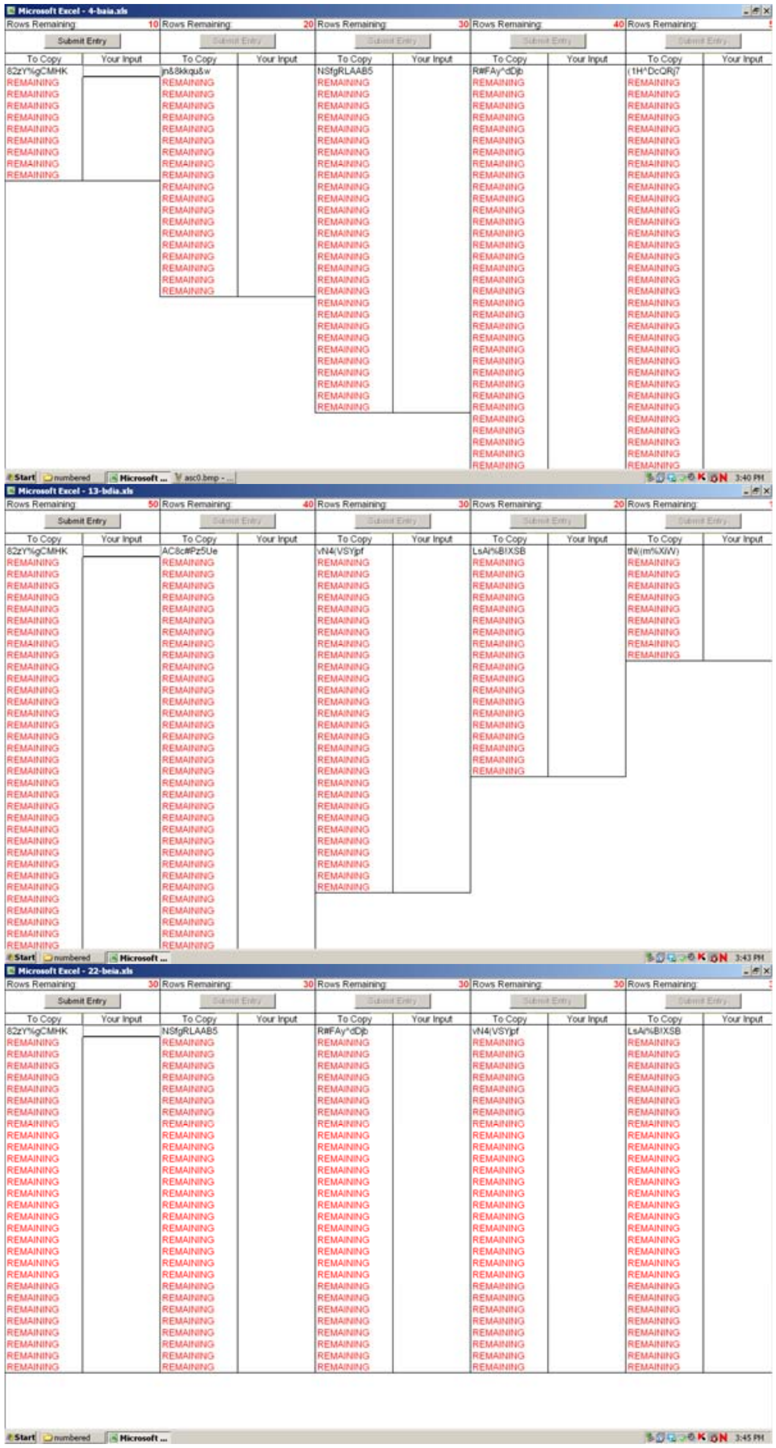
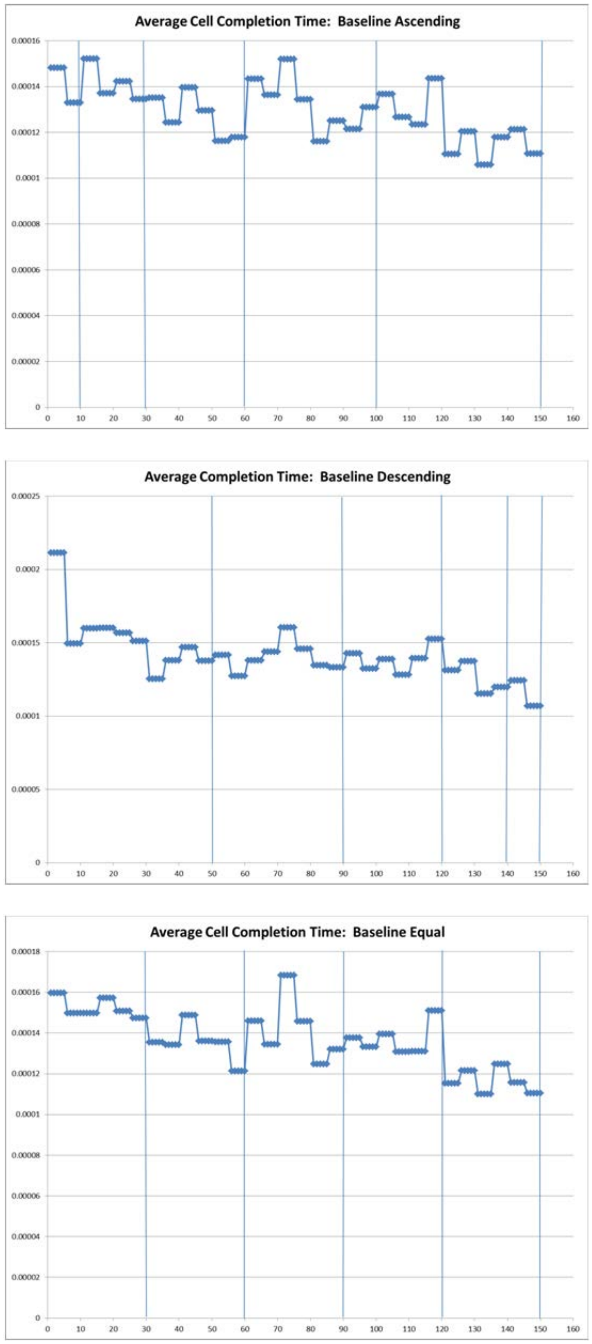


Figures 1(a-c): The experiment interface featured a task of typing 150 lines of ten character text in a Microsoft Excel Spreadsheet. Figures (a, top),(b, middle), and (c, bottom) show the task with five columns in ascending, descending, and even orderings, respectively.



Figures 2(a)-(c): Average Cell Completion Time by Cell. Note: each group of five cells has been given the average completion time for that group for legibility purposes.

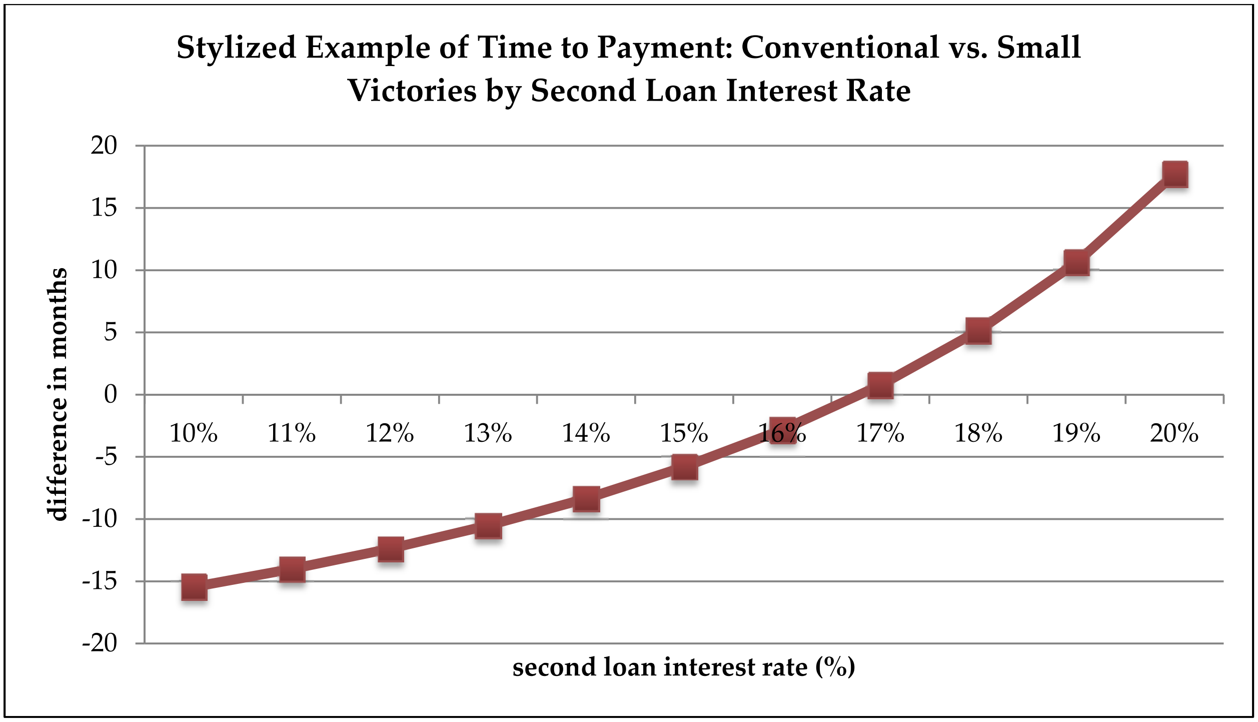


Figure 3: Difference in total time to pay off two loans with one loan at 10% interest and the other at 10%-20%. The standard monthly payment is $300, but the small victories method produces a 13% boost corresponding to a $369 monthly payment.

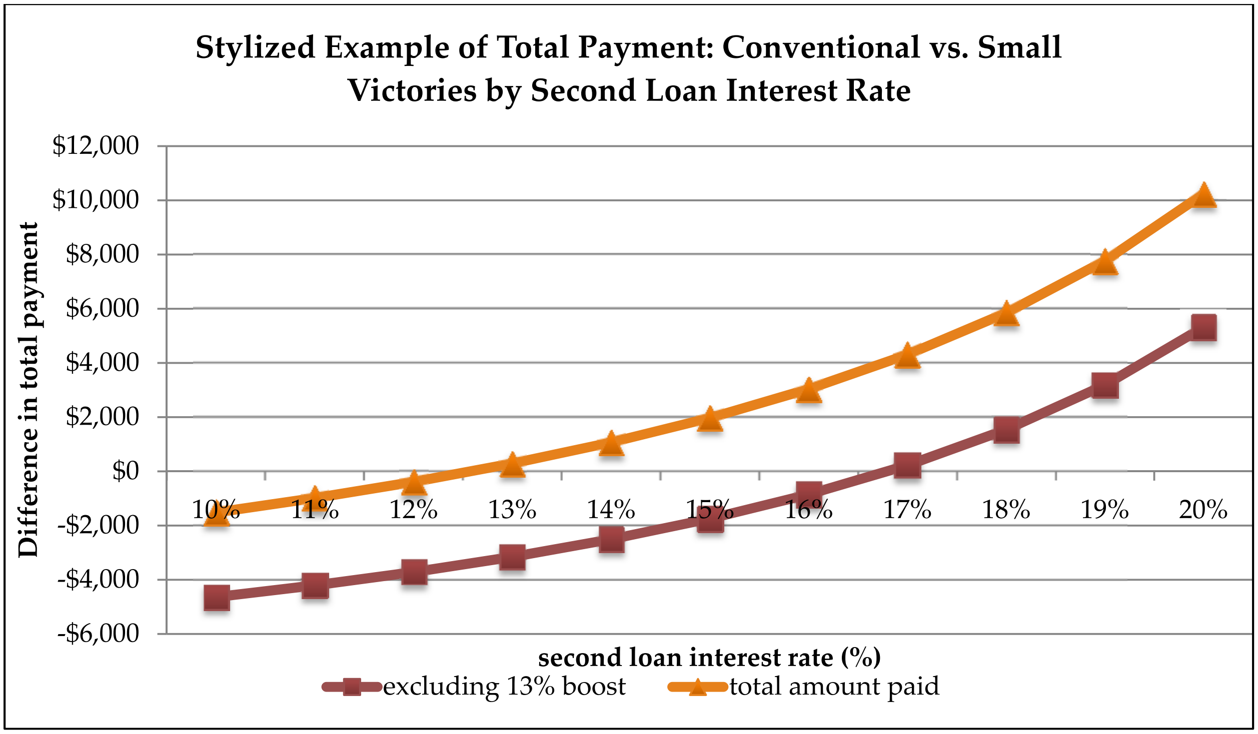


Figure 4: Difference in total amount spent to pay off two loans with one loan at 10% interest and the second at 10%-20%. The standard monthly payment is $300, but the small victories method produces a 13% boost corresponding to a $369 monthly payment. One line shows the total amount spent on the loan, the other shows that amount without the 13% boost (i.e., the extra $69 each month.).

Table 1: Main Results

Panel I: Time to completion				
	Mean (in sec)	N	Asc Difference	p (two-sided)
Ascending	11.08	31		
Not Ascending	12.31	60	1.23	0.019
Even	12.13	31	1.05	0.078
Descending	12.50	29	1.42	0.015
Kruskal Wallis				0.084

Panel II: Completion as an outcome	
Ascending	0.13248
	(0.1222)
Descending	-0.0963
	(0.1280)
Observations	91
Asc-desc chi squared p-value	0.08

Notes: Results in Panel I from separate t-tests on the time to complete each cell in seconds. Panel II provides marginal effects results from a probit regression on whether or not the participant completed the 150 cell task. In Panel II the omitted variable is even.

Table 2: Comparing first 5 cells in each column to last 5 cells

	Mean (residual)	N	Difference	p (two-sided)
Ascending				
In First Five Cells	-0.2660	775		
In Last Five Cells	-1.3763	730	-1.11	0.0000
Descending				
In First Five Cells	1.6540	635		
In Last Five Cells	0.3443	570	-1.31	0.2307
Equal				
In First Five Cells	0.2144	740		
In Last Five Cells	-0.1965	685	-0.41	0.1307

Notes: Magnitudes are measured as difference from the population average.

Table 3

Average time to complete cells

	(1)	(2)
Ascending	-1.0522*	-0.9640*
	(0.5862)	(0.5518)
Descending	0.3718	0.0464
	(0.6496)	(0.5888)
Practice average		0.1266***
		(0.0439)
Observations	91	90
Asc-desc F-test	0.01	0.05

Notes: Robust standard errors in parentheses. F-test value given is for the p-value of the F-statistic. Omitted ordering is equal. One student had technical difficulties with the practice session and is dropped from regressions that control for practice average. *** p<0.01, ** p<0.05, * p<0.1

Table 4: Comparing first 5 cells in each column to last 5 cells

	Mean (residual)	N	Difference	p (two-sided)
Panel I: Collapsed to First/Last				
Ascending				
In First Five Cells	-0.2660	155		
In Last Five Cells	-1.3763	146	-1.11	0.0011
Descending				
In First Five Cells	1.6705	128		
In Last Five Cells	0.3443	114	-1.33	0.2335
Equal				
In First Five Cells	0.2144	148		
In Last Five Cells	-0.1711	138	-0.39	0.2730
Panel II: Collapsed to Person				
Ascending				
In First Five Cells	-0.2660	31		
In Last Five Cells	-1.2599	31	-0.99	0.0699
Descending				
In First Five Cells	2.3895	29		
In Last Five Cells	0.7737	29	-1.62	0.2948
Equal				
In First Five Cells	0.4238	31		
In Last Five Cells	0.1144	31	-0.31	0.6242
Panel III: Collapsed all Firsts/Lasts Initial Study				
In First Five Cells	0.8153	91		
In Last Five Cells	-0.1437	91	-0.96	0.0973

Notes: Panel III groups all firsts and lasts as one observation per person. Magnitudes are measured as difference from the population average for each environment.

Table 5: Initial ascending regression results for residual cell time with clustering

	Cluster on subject	Cluster on cell	Cluster on column*subject
	(1)	(2)	(3)
In First Five Cells	1.11	1.11	1.11
	(0.30)	(0.46)	(0.25)
Observations	1505	1505	1505

Notes: Robust clustered standard errors in parentheses. Omitted term is "in last five cells".

Table 6: What they chose

	N	Mean (in sec)
Ascending	16	10.85
Not Ascending	54	11.20
Even	31	11.03
Descending	23	11.44
chi-squared p-value	0.09	

Notes: Results from the choice study.

Table 7: Effect of choice interactions with survey data on average cell time

	Self-control	Cognitive Reflection	Risk Aversion
	(1)	(2)	(3)
X*ascending	-0.139*	-1.060*	-0.710*
	(0.070)	(0.586)	(0.390)
X*descending	-0.026	-0.735*	-0.410
	(0.062)	(0.401)	(0.317)
ascending	6.333*	0.825	2.495
	(3.280)	(0.793)	(1.599)
descending	1.241	0.935	1.178
	(2.600)	(0.663)	(1.013)
X	-0.011	0.073	0.188
	(0.046)	(0.231)	-0.174
practice average	0.257***	0.253***	0.274***
	(0.068)	(0.070)	(0.069)
Observations	64	64	64

Notes: Outcome is average time to complete one cell in seconds. X is the Tangney-Baumeister measure of self-control in column (1) and the Cognitive Reflection Task in Column (2). Robust standard errors in parentheses. Omitted ordering is "equal". Six people with random survey malfunctions were eliminated from these regressions. *** p<0.01, ** p<0.05, * p<0.1

A Theoretical Appendix

Let us define a task, X, that consists of individual discrete elements $x \in X$. Each task X can be framed into subtasks using a partitition α.[1]

Definition. *The subtask partition $\alpha \in A(X)$ is a list of m ordered sets $\alpha = (\alpha_1, \ldots, \alpha_k, \ldots, \alpha_m)$ where $m \leq |X|$. For each $1 \leq k \leq m$, $\alpha_k = (\alpha_{kl})$ for $1 \leq l \leq |\alpha_k|$, such that*

$$\sum_{k=1}^{m} |\alpha_k| = |X| \quad \textit{and} \quad \bigcup_{k=1}^{m} \bigcup_{l=1}^{|\alpha_k|} \alpha_{kl} = X.$$

Note that this implies that no two x's are repeated in α.

The time it takes each individual $i \in N$ to complete a task, X, will be the sum of the time it takes to complete each element, x.[2] This time will strictly depend on each element's position in the subtask partition. The two factors that may matter in the subtask partition are the number of remaining elements in the subtask, and the number of previously completed subtasks. These factors will additively affect time performance for each individual. The function $\tau_i(x, \alpha)$ gives the time it takes individual i to complete element x under subtask partition α.

$$\tau_i(x, \alpha) = \tau_i(\alpha_{kl}) = \mu_i + h(|\alpha_k| - l) + v(k) + \epsilon_{i\alpha_{kl}}.$$

where $x = \alpha_{kl}$ and k and l indicate element x's position in subtask partition α.

Time to complete an element depends on an individual's characteristics μ_i and personal idiosyncratic error with element x, $\epsilon_{ix} = \epsilon_{i\alpha_{kl}}$. Before we continue it is helpful to impose some conditions on this error term, namely it does not vary by partition.[3]

Assumption A.1. *The personal idiosyncratic error term for element x is independent of partition. That is, for any element in any task, $x \in X$, for any α, $\alpha' \in A(X)$, if $x = \alpha_{kl} = \alpha'_{k'l'}$, then $\epsilon_{i\alpha_{kl}} = \epsilon_{ix} = \epsilon_{i\alpha_{k'l'}}$.*

Assumption A.1 requires that any variation in performance due to an element's position in subtask partition is expressed in the terms h and v. Function h expresses how the position of an element within a subtask affects performance, specifically its position from the end of a subtask. Function v expresses how previously completed

[1] In debt-repayment, one could think of each x as a monthly payment, and each subtask as an individual debt. The task X would be the removal of all debt. In our experiment each element is a cell, each subtask in a column, and X is a session.

[2] Alternatively one could say the total cost to complete task X is the sum of the individual costs paid to complete each element x.

[3] A weaker assumption that would work for the analysis in this paper would be that *in expectation* errors for the same element are equal.

subtasks affect performance. Note that the total time it takes individual i to complete a task under subtask partition α is given by

$$T_i(\alpha) = \sum_{x \in X} \tau_i(x, \alpha) = \sum_{k=1}^{m} \sum_{l=1}^{|\alpha_k|} \mu_i + h\left(|\alpha_k| - l\right) + v(k) + \epsilon_{i\alpha_{kl}}.$$

Using the social-cognitive (Bandura, 1977; 1986) and goal-gradient theories (Heilizer 1977; Hull 1932) within psychological literature as our guide, we will impose restrictions on functions h and v.

Assumption A.2 (social-cognitive). *After completing a subtask, individual performance does not decrease. That is, costs or time do not increase with successive subtasks. Formally, v is non-increasing.*

Often subtasks may be already defined, and one may be concerned with the question of how to order the subtasks in a way that will increase performance. For instance, a consumer may have multiple debts owed, and can choose in which order to repay them, but cannot restructure the debts. To fit such cases, we define a class of subtask partitions, or all subtask partitions that have the same structure of elements in each subtask, but the order of the subtasks has been changed.

Definition. *For any given X, the subtask partitions α' and α'' are said to be in the same class of subtask partitions, $\beta \subseteq A(X)$, if and only if $|\alpha'| = |\alpha''| = m$ and for all k, $1 \leq k \leq m$, there exists a k' such that $\alpha'_k = \alpha''_{k'}$. That is, β is the set of all subtask partitions that differ by at most the ordering of subtasks.*

Under assumption A.2 we can deduce a general result about the aggregate performance of tasks under subtask partitions of the same class. To prove the result a helpful lemma is necessary. The lemma is very similar to a standard result of expected utility theory involving first-order stochastic dominance. The proof follows the work of Hadar and Russell (1969), and the more well-known, Rothschild and Stiglitz (1970).

Lemma. *Suppose there are a finite number of distinct values, $j = 1, 2, \ldots n$, $y_{j'} > y_j$ if and only if $j' > j$. Define functions f and g so $f(y_j) = a_j$ and $g(y_j) = b_j$, where $0 \leq a_j, b_j \leq 1$ and $\sum_{j=1}^{n} f(y_j) = \sum_{j=1}^{n} g(y_j) = 1$. Further define*

$$F(y_j) = \sum_{r=1}^{j} f(y_r) \quad \text{and} \quad G(y_j) = \sum_{r=1}^{j} g(y_r). \tag{A.1}$$

For any non-increasing function, $u : \mathbb{R} \to \mathbb{R}$, where u is continuous over $[y_1, y_n]$, and differentiable over all open intervals (y_j, y_{j+1}) for $1 \leq j \leq n-1$. If $G(y_j) \leq F(y_j)$ for $1 \leq j \leq n$ then

$$\sum_{j=1}^{n} u(y_j) g(y_j) \leq \sum_{j=1}^{n} u(y_j) f(y_j). \tag{A.2}$$

The conditions, u is non-constant and $G(y_j) < F(y_j)$ for $1 \leq j \leq n-1$, or u is strictly decreasing and g and f are different, both imply (A.2) *with strict inequality.*

Proof. This proof is largely derived from Hadar and Russell (1969), Theorem 1, and is similar to many others involving first-order stochastic dominance.

For every interval $[y_j, y_{j+1}]$ for $1 \leq j < n$, the Mean Value Theorem shows there exists a $y_j < \xi_j < y_{j+1}$ such that

$$u(y_j) = u(y_{j+1}) - u'(\xi_j)\Delta y_j \quad \text{where} \quad \Delta y_j = y_{j+1} - y_j.$$

Then

$$\begin{aligned}
\sum_{j=1}^{n} u(y_j)f(y_j) - \sum_{j=1}^{n} u(y_j)g(y_j) &= \sum_{j=1}^{n} \left[u(y_n) - \sum_{r=j}^{n-1} u'(\xi_r)\Delta y_r \right] (f(y_j) - g(y_j)) \\
&= \sum_{j=1}^{n} u(y_n) (f(y_j) - g(y_j)) \\
&\quad - \sum_{j=1}^{n} \sum_{r=j}^{n-1} u'(\xi_r)\Delta y_r (f(y_j) - g(y_j)) \\
&= u(y_n) \left[\sum_{j=1}^{n} f(y_j) - \sum_{j=1}^{n} g(y_j) \right] \\
&\quad - \sum_{j=1}^{n} \left[(f(y_j) - g(y_j)) \sum_{r=j}^{n-1} u'(\xi_r)\Delta y_r \right] \\
&= -\sum_{r=1}^{n-1} \sum_{j=1}^{r} (f(y_j) - g(y_j)) u'(\xi)\Delta y_r \\
&= -\sum_{r=1}^{n-1} u'(\xi_r) (F(y_r) - G(y_r)) \Delta y_r \\
&\geq 0.
\end{aligned}$$

By definition, $\Delta y_r > 0$. Non-increasing implies $u'(\xi_r) \leq 0$. With $G(y_r) \leq F(y_r)$, our final result is greater than or equal to 0. If instead, u is strictly decreasing and g and f are different, this implies $u'(\xi_r) < 0$ and that there is some j' where $G(y_{j'}) < F(y_{j'})$. In such case, we have strict inequality. Similarly, if u is non-constant and $G(x_j) < F(x_j)$ for all for $1 \leq j < n$, there is some j' where $u'(\xi_{j'}) < 0$. In such case we also would have strict inequality. □

Proposition 1 (small victories)**.** *For any i, for a given class of subtask partitions, $\beta \subseteq A(X)$. Define an ascending ordering, α' where*

$$|\alpha'_1| \leq \ldots \leq |\alpha'_k| \leq \ldots \leq |\alpha'_m|,$$

and a <u>*descending ordering*</u> *where*

$$|\alpha''_1| \geq \ldots \geq |\alpha''_k| \geq \ldots \geq |\alpha''_m|.$$

Then for any $\alpha \in \beta$,

$$T_i(\alpha') \leq T_i(\alpha) \leq T_i(\alpha'').$$

If v *is non-constant, and all the subtasks in* β *are not of the same length,* $T_i(\alpha') < T_i(\alpha'')$.

Proof. Choose any α, α', $\alpha'' \in \beta$. First we will show that for a class of subtask partitions, for any $\alpha \in \beta$, the value $\sum_{k=1}^{m}\sum_{l=1}^{|\alpha_k|} h(|\alpha_k| - l)$ is equal. Consider any $x^* \in X$. Since all subtask partitions defined on X must contain one unique x^*, let us define $\alpha_{kl} = x$, $\alpha'_{k'l'} = x$, and $\alpha''_{k''l''} = x$, where $\alpha_{kl} \in \alpha$, $\alpha'_{k'l'} \in \alpha'$, $\alpha''_{k''l''} \in \alpha''$. Since α, α', α'' are all in the same class of subtask partitions, by definition the ordered sets α_k, $\alpha'_{k'}$, $\alpha''_{k''}$ must be equal. It follows that

$$\sum_{k=1}^{m}\sum_{l=1}^{|\alpha_k|} h(|\alpha_k| - l) = \sum_{k=1}^{m}\sum_{l=1}^{|\alpha'_k|} h(|\alpha'_k| - l) = \sum_{k=1}^{m}\sum_{l=1}^{|\alpha''_k|} h(|\alpha''_k| - l).$$

Next we will define the following functions.

$$\bar{v}(y) = \begin{cases} v(y) & \text{if } y \in \mathbb{Z}, \\ (\lceil y \rceil - y)\, v(\lfloor y \rfloor) + (y - \lfloor y \rfloor)\, v(\lceil y \rceil) & \text{otherwise.} \end{cases} \quad \text{(A.3)}$$

$$f_\alpha(k) = \begin{cases} |\alpha_k| \Big/ \sum_{j=1}^{m} |\alpha_j| & \text{if } k = 1, 2, \ldots, m, \\ 0 & \text{otherwise.} \end{cases} \quad \text{(A.4)}$$

It follows that $0 \leq f_\alpha(k),\ f_{\alpha'}(k),\ f_{\alpha''}(k) \leq 1$ for $k = 1, 2, \ldots, m$. Additionally $\sum_{k=1}^{m} f_\alpha(k) = \sum_{k=1}^{m} f_{\alpha'}(k) = \sum_{k=1}^{m} f_{\alpha''}(k) = 1$. Note also that $\bar{v}$ is continuous. It is differentiable over every open interval between integers. It is also non-increasing by Assumption A.2.

Define $F_\alpha(n) = \sum_{k=1}^{n} f_\alpha(k)$ and $F_{\alpha'}$, $F_{\alpha''}$ in the same way. Note that for any $1 \leq n < m$, $F_{\alpha'}(n)$ and $F_{\alpha''}(n)$ contain the sums of the lengths of the n shortest subtasks and the n longest subtasks, respectively. Thus $F_{\alpha'}(n) \leq F_\alpha(n) \leq F_{\alpha''}(n)$. If all the subtasks are not of the same length, $F_{\alpha'}(n) < F_{\alpha''}(n)$.

By our Lemma, $\sum_{j=1}^{m} f_{\alpha'}(j)\bar{v}(j) \leq \sum_{j=1}^{m} f_\alpha(j)\bar{v}(j) \leq \sum_{j=1}^{m} f_{\alpha''}(j)\bar{v}(j)$ which implies

$$\sum_{k=1}^{m}\sum_{l=1}^{|\alpha'_k|} v(|\alpha'_k|) \leq \sum_{k=1}^{m}\sum_{l=1}^{|\alpha_k|} v(|\alpha_k|) \leq \sum_{k=1}^{m}\sum_{l=1}^{|\alpha''_k|} v(|\alpha''_k|).$$

Since the other terms in T_i are independent of the subtask partition or already shown to be constant in summation, we have $T_i(\alpha') \leq T_i(\alpha) \leq T_i(\alpha'')$. If all the subtasks are not of the same length, we would have $F'_\alpha(n) < F''_\alpha(n)$ for $1 \leq n < m$, so by our Lemma, we would have $T_i(\alpha') < T_i(\alpha'')$. □

Proposition 1 states that if people perform better after completing a subtask, and only the ordering of subtasks can be changed, putting subtasks in ascending order leads to optimal performance (or, equivalently, minimal costs), while descending order leads to the worst performance (or, equivalently, maximal costs). As long as all subtasks are not of equal length there should be a difference between these two extremes.

Note that Proposition 1 requires no structure on function h, the function that concerns performance relative to the end of the subtask. Thus, the findings of social-cognitive theories, when applied to our model, suggest an optimal debt-repayment (or any general task-completion strategy) that is consistent with the debt snowball (or small victories) approach.

We now consider the goal-gradient hypothesis and corresponding restrictions it places on function h. The theory suggests distance to the end of a subtask affects performance.

Assumption A.3 (goal-gradient)**.** *As individuals move closer to the end of a subtask, their performance does not decrease. That is, costs or time do not increase the closer one comes to the end of a subtask. Formally, h is non-decreasing.*

If the goal-gradient term, h is all that matters, and the social-cognitive term v is constant (effectively zero), we have a much different result about the optimal structure of subtask partitions.

Proposition 2. *Suppose v is constant. Then for any X, i, and m, if there exists an <u>even ordering,</u> $\alpha' \in A(X)$ such that $|\alpha'| = c$ for all $1 \leq k \leq m$, then*

$$T_i(\alpha') \leq T_i(\alpha) \quad \forall \alpha \in A(X) \text{ where } |\alpha'| = |\alpha| = m.$$

Further, if all the subtasks in α are not of the same length and h is strictly increasing, $T_i(\alpha') < T_i(\alpha)$.

Proof. Choose any $\alpha \in A(X)$. If v is constant, the only term in T_i that changes with α is h. We must show

$$\sum_{k=1}^{m} \sum_{l=1}^{|\alpha'_k|} h(|\alpha'_k| - l) \leq \sum_{k=1}^{m} \sum_{l=1}^{|\alpha_k|} h(|\alpha_k| - l), \tag{A.5}$$

with strict inequality if all the subtasks in α are not of the same length and h is strictly increasing. Next we will define the following functions.

$$\bar{h}(y) = \begin{cases} h(y) & \text{if } y \in \mathbb{Z}, \\ (\lceil y \rceil - y)\, h\left(\lfloor y \rfloor\right) + (y - \lfloor y \rfloor)\, h\left(\lceil y \rceil\right) & \text{otherwise.} \end{cases} \tag{A.6}$$

$$g_\alpha(n) = \begin{cases} \left|\{\alpha_{kl} \in \alpha : |\alpha_k| - l = n\}\right| / |X| & \text{if } n = 0, 1, 2, \ldots \\ 0 & \text{otherwise.} \end{cases} \tag{A.7}$$

Let l^* be the length of the longest subtask in either α or α'. It follows that $0 \leq g_\alpha(n), g_{\alpha'}(n) \leq 1$ for $n = 1, 2, \ldots, l^* - 1$. Additionally $\sum_{n=0}^{l^*-1} g_\alpha(n) = \sum_{n=0}^{l^*-1} g_{\alpha'}(n) = 1$. Note also that $-\bar{h}$ is continuous. It is differentiable over every open interval between integers. The function, $-\bar{h}$, is also non-decreasing by Assumption A.3.

Define $G_\alpha(n) = \sum_{k=1}^{n} g_\alpha(k)$ and $G_{\alpha'}$ in the same way. Note that for every $n = 0, 1, 2, \ldots, c-1$, the function $g_{\alpha'}(n) = m/|X|$. Since there are only m subtasks, we must have $g_\alpha(n) \leq m/|X|$. Then $G_\alpha(n) \leq G_{\alpha'}(n)$ for $n = 0, 1, 2, \ldots, c-1$. Since $G_{\alpha'}(c-1) = 1$, $G_\alpha \leq G_{\alpha'}$. If α contains subtasks of different lengths, let l' be the length of the shortest subtask in α. Then $g_\alpha(l') < m/|X|$, so g_α and $g_{\alpha'}$ are different.

By our Lemma, $\sum_{n=1}^{l^*-1} g_{\alpha'}(j)\left(-\bar{h}(j)\right) \leq \sum_{n=1}^{l^*-1} g_\alpha(j)\left(-\bar{h}(j)\right)$ which implies (A.5). If all the subtasks in α are not of the same length, we have already shown g_α and $g_{\alpha'}$ are different. If in addition, h is strictly increasing, then we have $\sum_{n=1}^{l^*-1} g_{\alpha'}(j)\left(-\bar{h}(j)\right) < \sum_{n=1}^{l^*-1} g_\alpha(j)\left(-\bar{h}(j)\right)$ which implies (A.5) with strict inequality. □

An similar statement can be made about ascending orderings.

Proposition 3. *Suppose h is constant. Then for any X, i, and m, there exists a subtask partition $\alpha' \in A(X)$, in ascending order, $|\alpha'_1| \leq \ldots \leq |\alpha'_k| \leq \ldots \leq |\alpha'_m|$, for all $1 \leq k \leq m$, such that*

$$T_i(\alpha') \leq T_i(\alpha) \quad \forall \alpha \in A(X) \text{ where } |\alpha'| = |\alpha| = m.$$

Further, if α is not ascending and v is strictly decreasing, $T_i(\alpha') < T_i(\alpha)$.

Proof. Choose any $\alpha \in A(X)$. Consider the ascending subtask partition $\hat{\alpha}$ where $|\hat{\alpha}_k| = 1$ for all $k < m$ and $|\hat{\alpha}_m| = |X| - m + 1$. Define functions $\bar{v}(y)$ and $f_\alpha(k)$ as in (A.3) and (A.4). It follows that $0 \leq f_\alpha(k), f_{\hat{\alpha}}(k) \leq 1$ for $k = 1, 2, \ldots, m$. Additionally $\sum_{k=1}^{m} f_\alpha(k) = \sum_{k=1}^{m} f_{\hat{\alpha}}(k) = 1$. Note also that $\bar{v}$ is continuous. It is differentiable over every open interval between integers. It is also non-increasing by Assumption A.2.

Define $F_\alpha(n) = \sum_{k=1}^{n} f_\alpha(k)$ and $F_{\hat{\alpha}}$ in the same way. Note that for any $k < m$, $f_{\hat{\alpha}}(k) = 1/|X|$. Since every subtask must contain at least one element and there are m subtasks, $1/|X| \leq f_\alpha(k) \leq (|X| - m + 1)/|X|$. It follows that $F_{\hat{\alpha}} \leq F_\alpha$. If α is not ascending, there exist k' and k'' such that $k' < k''$ and $|\alpha_{k'}| > |\alpha_{k''}|$. We have $k' < m$ and $|\alpha_{k'}| > 1$. Then $f_\alpha(k') > f_{\hat{\alpha}}(k')$, and f_α and $f_{\hat{\alpha}}$ are different.

By our Lemma, $\sum_{j=1}^{m} f_{\hat{\alpha}}(j)\bar{v}(j) \leq \sum_{j=1}^{m} f_\alpha(j)\bar{v}(j)$ which implies $\sum_{k=1}^{m} \sum_{l=1}^{|\hat{\alpha}_k|} v(|\alpha'_k|) \leq \sum_{k=1}^{m} \sum_{l=1}^{|\alpha_k|} v(|\alpha_k|)$. Since the other terms do not differ from $\hat{\alpha}$ to α, it follows that $T_i(\alpha') \leq T_i(\alpha)$. If α is not ascending, we have already shown f_α and $f_{\hat{\alpha}}$ are different. If in addition, v is strictly decreasing, our Lemma implies $\sum_{j=1}^{m} f_{\hat{\alpha}}(j)\bar{v}(j) < \sum_{j=1}^{m} f_\alpha(j)\bar{v}(j)$. By identical reasoning, we would have $T_i(\alpha') < T_i(\alpha)$. □

It should be noted that the proceeding two Propositions, unlike Proposition 1, apply to all subtasks partitions possible under X, not just rearrangements of subtasks. However, they require stronger restrictions about our functions h and v than our Proposition 1. Specifically, each requires that one of the functions be constant. Under such assumptions, Propositions 2 and 3 provide two different answers on which ordering, ascending or even, is optimal.

The remainder of this section will be concerned with developing general definitions and a proposition about optimal orderings when both social-cognitive and goal-gradient forces are present. First, we need a general way to compare the effects of the two terms.

Definition. *For any two subtask partitions $\alpha, \alpha' \in A(X)$, with $|\alpha| = |\alpha'| = m$, for the following relation,*

$$\left|\sum_{k=1}^{m}\left(|\alpha_k| - |\alpha'_k|\right)v(k)\right| - \left|\sum_{k=1}^{m}\sum_{l=1}^{|\alpha_k|} h(|\alpha_k| - l) - \sum_{l=1}^{|\alpha'_k|} h(|\alpha'_k| - l)\right| \gtreqless 0, \qquad \text{(A.8)}$$

we say the social-cognitive factors <u>*dominate*</u> *the goal-gradient factors if and only if* (A.8) *is greater than 0. Alternatively the goal-gradient factors* <u>*dominate*</u> *the social-cognitive factors if* (A.8) *is less than 0. There is* <u>*no dominance*</u> *between goal-gradient and social cognitive factors if and only if* (A.8) *is equal to 0.*

Next, we need to restrict our focus to tasks where both types of orderings are possible.

Definition. *Consider any X with α^e, α^a, $\alpha^d \in A(X)$ where α^e, α^a, α^d are even, ascending, and descending orderings, respectively. Further restrict α^a and α^d to be in the same class of subtask partitions, and α^e to have the same number of ordered sets as α^a and α^d. That is, there exists a β such that $\alpha^a, \alpha^d \in \beta$ and $|\alpha^e| = |\alpha^a| = |\alpha^d| = m$. We refer to any set $\{\alpha^e, \alpha^a, \alpha^d\}$ as a* <u>*Δ-set*</u>.

Note that a given Δ-set may not be unique for a given task X or even a given number of subtasks m. Since our experiment involves a particular Δ-set, we find it useful to make comparisons across the three subtask partitions.

Proposition 4. *For a given Δ-set, $\{\alpha^a, \alpha^d, \alpha^e\}$, where all the subtasks in α^a are not of the same length, for any i,*

1. *$T_i(\alpha^a) < T_i(\alpha^e)$ if and only if social-cognitive factors dominate goal-gradient factors.*

2. *$T_i(\alpha^a) > T_i(\alpha^e)$ if and only if goal-gradient factors dominate social-cognitive factors.*

3. *$T_i(\alpha^a) = T_i(\alpha^e)$ if and only if there is no dominance between social-cognitive factors and goal-gradient factors.*

Proof. For a given i, $T_i(\alpha^a) - T_i(\alpha^e) =$

$$\sum_{k=1}^{m} (|\alpha_k^a| - |\alpha_k^e|)\, v(k) + \sum_{k=1}^{m} \left[\sum_{l=1}^{|\alpha_k^a|} h(|\alpha_k^a| - l) - \sum_{l=1}^{|\alpha_k^e|} h(|\alpha_k^e| - l). \right] \tag{A.9}$$

Define $f_{\alpha^a}(k)$, $f_{\alpha^e}(k)$ and $\bar{v}(y)$ as in (A.3) and (A.4). It follows that $0 \leq f_{\alpha^a}(k)$, $f_{\alpha^e}(k) \leq 1$ for $k = 1, 2, \ldots, m$. Additionally $\sum_{k=1}^{m} f_{\alpha^a}(k) = \sum_{k=1}^{m} f_{\alpha^e}(k) = 1$. Note also that $\bar{v}$ is continuous. It is differentiable over every open interval between integers. It is also non-increasing by Assumption A.2.

Define $F_{\alpha^a}(n) = \sum_{k=1}^{n} f_{\alpha^a}(k)$ and F_{α^e} in the same way. Since α^a has subtasks of different lengths, we must have $|\alpha_1^a| < |\alpha_m^a|$. Since $|\alpha^a|$ has exactly $|X|$ elements and α_1^a and α_m^a are its smallest and largest subtasks respectively, we must have $|\alpha_1^a| < |X|/m < |\alpha_m^a|$. Let k' denote the smallest k where $|\alpha_k^a| > |X|/m$. We have $f_{\alpha^a}(k) \leq f_{\alpha^e}(k)$ for all $k < k'$, so $F_{\alpha^a}(k) \leq F_{\alpha^e}(k)$ for all $k < k'$. For $k' < k < m$, the identity, $F_{\alpha^a}(k) + \sum_{j=k+1}^{m} f_{\alpha^a}(j) = F_{\alpha^e}(k) + \sum_{j=k+1}^{m} f_{\alpha^e}(j) = 1$, implies $F_{\alpha^a}(k) \leq F_{\alpha^e}(k)$ because $\sum_{j=k+1}^{m} f_{\alpha^a}(j) > \sum_{j=k+1}^{m} f_{\alpha^e}(j)$. Since $F_{\alpha^a}(m) = F_{\alpha^e}(m)$, we have $F_{\alpha^a} \leq F_{\alpha^e}$. By our Lemma,

$$\begin{aligned} \sum_{k=1}^{m} \sum_{l=1}^{|\alpha_k^a|} v(|\alpha_k^a|) - \sum_{k=1}^{m} \sum_{l=1}^{|\alpha_k^e|} v(|\alpha_e|) &\leq 0 \\ \sum_{k=1}^{m} (|\alpha_k^a| - |\alpha_k^e|)\, v(k) &\leq 0. \end{aligned} \tag{A.10}$$

Define $g_{\alpha^a}(k)$, $g_{\alpha^e}(k)$ and $\bar{h}(y)$ as in (A.6) and (A.7). An identical argument to the proof of Proposition 2 reveals

$$\sum_{k=1}^{m} \left[\sum_{l=1}^{|\alpha_k^a|} h(|\alpha_k^a| - l) - \sum_{l=1}^{|\alpha_k^e|} h(|\alpha_k^e| - l) \right] \geq 0. \tag{A.11}$$

Thus (A.9) is negative if and only if the magnitude of (A.10) is greater than the magnitude of (A.11), that is, when social-cognitive dominate goal-gradient factors. Similiarly, (A.9) is positive if and only if the magnitude of (A.10) is less than the magnitude of (A.11), that is, when goal-gradient dominate social-cognitive factors. This leaves (A.9) equal to zero if and only if the magnitudes of the two parts are equal, when there is no dominance between social-cognitive and goal-gradient factors. □

Corollary. *For a given* Δ*-set and any* i, $T_i(\alpha^d) \geq T_i(\alpha^a), T_i(\alpha^e)$. *Provided all the subtasks of* α^d *are not of the same length,* $T_i(\alpha^d) > T_i(\alpha^a)$, *if* v *is non-constant;* $T_i(\alpha^d) > T_i(\alpha^e)$ *if either* v *is non-constant or* h *is strictly increasing.*

Proof. The relation $T_i(\alpha^d) \geq T_i(\alpha^a)$ and $T_i(\alpha^d) > T_i(\alpha^a)$ if not all the subtasks of α_d are of the same length and v is non-constant follow directly from Proposition 1 since α^a and α^d are in the same class of subtask partitions by definition of Δ-set.

The relation $T_i(\alpha^d) \geq T_i(\alpha^e)$ and $T_i(\alpha^d) > T_i(\alpha^e)$ if not all the subtasks of α_d are of the same length and h is strictly increasing follow directly from Proposition 2 since $|\alpha^d| = |\alpha^e| = m$ by definition of Δ-set.

Now for a given i, $T_i(\alpha^d) - T_i(\alpha^e) =$

$$\sum_{k=1}^{m} \left(|\alpha_k^d| - |\alpha_k^e|\right) v(k) + \sum_{k=1}^{m} \left[\sum_{l=1}^{|\alpha_k^d|} h(|\alpha_k^d| - l) - \sum_{l=1}^{|\alpha_k^e|} h(|\alpha_k^e| - l) \right].$$

Define $f_{\alpha^e}(k)$, $f_{\alpha^d}(k)$ and $\bar{v}(y)$ as in (A.3) and (A.4). It follows that $0 \leq f_{\alpha^e}(k)$, $f_{\alpha^d}(k) \leq 1$ for $k = 1, 2, \ldots, m$. Additionally, $\sum_{k=1}^{m} f_{\alpha^e}(k) = \sum_{k=1}^{m} f_{\alpha^d}(k) = 1$. The function $\bar{v}$ is continuous, differentiable over every open interval between integers, and non-increasing by Assumption A.2.

Define $F_{\alpha^e}(n) = \sum_{k=1}^{n} f_{\alpha^e}(k)$ and F_{α^d} in the same way. Since α^d has subtasks of different lengths, we must have $|\alpha_1^d| > |\alpha_m^d|$. Since α^d has exactly $|X|$ elements and α_1^d and α_m^d are its smallest and largest subtasks respectively, we must have $|\alpha_1^d| > |X|/m > |\alpha_m^d|$. Let k' denote the largest k where $|\alpha_k^d| < |X|/m$. We have $f_{\alpha^e}(k) < f_{\alpha^d}(k)$ for all $k \leq k'$, so $F_{\alpha^e}(k) < F_{\alpha^d}(k)$ for all $k \leq k'$. For $k' < k < m$, the identity, $F_{\alpha^d}(k) + \sum_{j=k+1}^{m} f_{\alpha^d}(j) = F_{\alpha^e}(k) + \sum_{j=k+1}^{m} f_{\alpha^e}(j) = 1$, implies $F_{\alpha^d}(k) > F_{\alpha^e}(k)$ because $\sum_{j=k+1}^{m} f_{\alpha^e}(j) > \sum_{j=k+1}^{m} f_{\alpha^d}(j)$. Then $F_{\alpha^d} > F_{\alpha^e}$ for $1 \leq k < m$. By our lemma, $\sum_{k=1}^{m} \left(|\alpha_k^e| - |\alpha_k^d|\right) v(k) \leq 0$ with strict inequality if v is non-constant. Since an argument identical to that used in the proof of Proposition 2 shows $\sum_{k=1}^{m} \left[\sum_{l=1}^{|\alpha_k^d|} h(|\alpha_k^d| - l) - \sum_{l=1}^{|\alpha_k^e|} h(|\alpha_k^e| - l) \right] \geq 0$, we have $T_i(\alpha^d) > T_i(\alpha^e)$ if v is non-constant.

□

Appendix Table 1: Effect of initial study interactions with survey data on average cell time

	Barratt	High SSH	Extraversion	Agreeableness	Concientiousness	Openness	Neuroticism
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
X*ascending	0.1943	0.3027	0.1582	0.1429	0.1329	-0.03525	-0.2034*
	(1.2095)	(1.1629)	(0.1176)	(0.1354)	(0.1500)	(0.1070)	(0.1217)
X*descending	-1.1639	-0.6144	-0.06990	0.1686	0.1850	-0.05391	-0.2247*
	(1.3108)	(1.3043)	(0.1472)	(0.1647)	(0.1851)	(0.1341)	(0.1279)
ascending	-1.2418	-1.5411*	-1.2023*	-1.4210**	-1.2196*	-1.0727*	-2.1063**
	(0.8183)	(0.8609)	(0.6056)	(0.6874)	(0.6750)	(0.5807)	(0.9536)
descending	0.9077	0.7421	0.1653	-0.09740	-0.006695	0.4927	-0.8077
	(0.8651)	(0.9751)	(0.6668)	(0.7841)	(0.8339)	(0.6859)	(1.0452)
X	0.399	1.0843	-0.06937	-0.1215	-0.1226	0.1045	0.1408
	(0.9592)	(0.9070)	(0.0920)	(0.1194)	(0.1359)	(0.0634)	(0.1072)
Observations	91	91	91	91	91	91	91

Notes: Outcome is average time to complete one cell in seconds. X is the Barratt Impulsivity measure in column (1), high sensation seeking in Column (2), and columns (3)-(7) are the five factors from the Big Five Inventory. Robust standard errors in parentheses. Omitted ordering is "equal".