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Francesco Bianchi
Cosmin Ilut
Martin Schneider

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ABSTRACT

This paper studies a DSGE model with endogenous financial asset supply and ambiguity averse investors. An increase in uncertainty about profits leads firms to substitute away from debt and reduce shareholder payout in bad times when the equity premium is high. Regime shifts in volatility generate large low frequency movements in asset prices due to uncertainty premia that are disconnected from the business cycle.

Francesco Bianchi
Department of Economics
Duke University
213 Social Sciences Building
Box 90097
Durham, NC 27708
and CEPR
and also NBER
francesco.bianchi@duke.edu

Cosmin Ilut
Department of Economics
Duke University
223 Social Sciences Building
Box 90097
Durham, NC 27708
and NBER
cosmin.ilut@duke.edu

Martin Schneider
Department of Economics
Stanford University
579 Serra Mall
Stanford, CA 94305-6072
and NBER
schneidr@stanford.edu

1 Introduction

The excess volatility of asset prices is a long standing puzzle in financial economics. Recent research argues that taking into account time variation in uncertainty helps reconcile price volatility and investor behavior. For example, if investors perceive a lot of uncertainty in bad times, they are reluctant to buy equity and prefer bonds. As uncertainty premia on equity rise, stock prices fall until they appear excessively low. Based on this intuition, several studies have now shown that stock prices and dividends can be consistent with investor optimization.[1] The results suggest that accounting for uncertainty shocks might also help improve asset pricing implications of business cycle models.

A fully specified general equilibrium business cycle model makes explicit not only the behavior of investors who demand assets, but also the choices of firms who supply those assets. In particular, the quantity of shares issued and the payout to shareholders are determined jointly with investment and production decisions. Much existing literature on business cycles simplifies here by assuming that payout and leverage follow exogenous decision rules.[2] It is natural to ask, however, how firms optimally choose capital structure and payout in response to uncertainty shocks and to require the model to be consistent with evidence on such choices.

This paper proposes and estimates a DSGE model with endogenous financial asset supply. Firms face frictions in debt and equity markets and choose capital structure as well as the net payout to shareholders in order to maximize shareholder value. Shareholders perceive time varying uncertainty about cost, and hence profit. Their response to uncertainty shocks allows the model to match the joint dynamics of equity prices, interest rates, real activity, as well as corporate sector leverage and payout for the postwar United States. The key mechanism is that an increase in uncertainty about profit increases the equity premium and raises shareholders' concern with future financing constraints. Firms then respond not only on the real side by cutting production and investment, but also reoptimize capital structure by reducing debt and lowering payout.

Our approach allows for two effects of uncertainty shocks: changes in volatility are reflected in the size of realized shocks, while changes in perceived uncertainty can be inferred from agents' actions and their effect on asset prices, in particular the presence of time varying uncertainty premia. While changes in volatility always induce changes in perceived

[1] The typical approach assumes that investors have recursive utility that exhibits strong aversion to persistent shocks (for example, Epstein-Zin utility with risk aversion substantially higher than the inverse of the intertemporal elasticity of substitution) together with time variation in higher moments, for example through stochastic volatility or movements in the probability of a disaster.

[2] For example, studies often equate the value of equity and the value of the capital stock, or impose a fixed leverage ratio.

uncertainty, the latter can occur also if volatility does not move. For example, the arrival of news that raises concerns about the future path of cost may induce firms to act more cautiously even if costs are not more volatile.

We find that both effects of uncertainty shocks are important and that they operate at different frequencies. On the one hand, volatility moves slowly. In particular, a regime of high volatility in cost during the 1970s and 80s was responsible for fluctuations in leverage and payout during that time. Its presence introduces a slow moving component of equity prices that is not connected with the business cycle. On the other hand, perceived uncertainty also tends to decline during booms and increase in recessions, making the stock market rise and fall "in sync" with the business cycle, especially in recent decades.

In our model, equity and corporate debt are priced by a representative agent. The supply of equity and debt, and hence leverage, is endogenously determined. Firms face an upward sloping marginal cost curve for debt: debt is cheaper than equity at low levels of debt, but becomes eventually more expensive as debt increases. Firms also have a preference for dividend smoothing. To maximize shareholder value, they find interior optima for leverage and net shareholder payout. Firm decisions are sensitive to uncertainty since shareholder value reflects the preferences of the representative agent and thus incorporates uncertainty premia.

We capture investors' behavior towards uncertainty by recursive multiple priors utility.[3] Formally, when agents evaluate an uncertain consumption plan, they use a worst case conditional probability drawn from a set of beliefs. A larger set indicates higher perceived uncertainty. Following Ilut and Schneider (2014), we parametrize beliefs about shocks by their means and describe belief sets as intervals for the mean that are centered around zero. The width of the interval then measures the amount of uncertainty about a particular shock. Different degrees of uncertainty perceived about different sources of shocks are accommodated by different widths.

We allow perceived uncertainty to change for two reasons. First, it can increase with the volatility of shocks. It makes sense that people worry more about the future in turbulent times. In existing models, this effect is generated by the interaction of higher volatility and risk aversion; in our setup, higher volatility makes agents act *as if* they face lower means. Either way, higher volatility lowers certainty equivalent consumption and welfare. Second, perceived uncertainty can change in our model even if there is nothing unusual in realized fundamentals. The idea here is that investors may receive intangible information, for

[3]The multiple priors model was introduced by Gilboa and Schmeidler (1989) to describe aversion to ambiguity (or Knightian uncertainty); it was extended to intertemporal choice by Epstein and Wang (1994) and Epstein and Schneider (2003). It has since been used in a number of studies in finance and macroeconomics. See Epstein and Schneider (2010) or Guidolin and Rinaldi (2013) for surveys.

example, worrisome news about the future. The estimation can disentangle the two channels because an increase in volatility is reflected not only in agents' precautionary behavior and asset premia (as is any other change in uncertainty), but also in larger realized innovations.

We estimate the model with postwar US data on five observables. We include three quantities chosen by the nonfinancial corporate sector: investment growth, net payout to shareholders relative to GDP and market leverage. In addition, we include the value of nonfinancial corporate equity relative to GDP. We thus effectively consider also the corporate price/payout ratio, which behaves similarly to the price-dividend ratio. Finally, we include the real short term interest rate. In sum, we ask our model to account for the price and quantity dynamics of equity and debt, along with real investment.

Our estimation assumes two sources of shocks: a shift in the marginal product of capital and an operating cost that affects corporate sector earnings but does not scale with production. The latter cost may reflect, for example, expenses incurred in "packaging" corporate sector earnings in the form of individual firm payout. For example, the cost of reorganization through changes in composition such as mergers, spin-offs or IPO's can vary over time because of changes in financial conditions. More generally, the operating cost may reflect other reorganization activities – changes in compensation or the relationship with the government – that redistribute resources away from shareholders. We emphasize that just two shocks – as well as changes in uncertainty about them – can account for much of the variation in the key price and quantity variables.[4]

We allow changes in uncertainty to exhibit large discrete changes triggered by a finite state Markov chain. The presence of large shocks allows the model to capture asymmetries in business cycle dynamics. A convenient feature of working with multiple priors utility is that linearized decision rules capture the (first order) effect of uncertainty shocks. As a result, the model solution can be represented as a Markov-switching DSGE (MS-DSGE) that can be estimated using the methods described in Bianchi (2013).The specification of uncertainty shocks is flexible enough to allow perceived uncertainty change with either realized volatility or intangible information.[5]

The fit of the model is due in large part to how firms and the investor households respond

[4]This result is related to Greenwald et al. (2014) who decompose changes in stock market wealth into three components that they label productivity, factor share and risk aversion shocks. The latter are responsible for most of the variation in equity prices in their setting and thus play a role similar to changes in perceived uncertainty shocks in our model. At the same time, our specification allows for stochastic volatility and thus relates high equity premia in part to the occurrence of large shocks.

[5]To discipline the size of the latter shocks, we follow Ilut and Schneider (2014) in specifying priors for the ambiguity parameters that bound the range of conditional means for an innovation relative to the variance of that innovation. Intuitively, this assumes that agents should entertain forecasts as part of their belief sets only if those forecasts perform well sufficiently often in the long run.

to uncertainty shocks. Consider first the role of investor responses for asset pricing. Investors' perceived time variation in ambiguity accounts for the fact that stock prices (normalized by dividends) forecast excess stock returns. Intuitively, when investors evaluate an asset *as if* the mean payoff is low, then they are willing to pay only a low price for it. To an econometrician, the return on the asset – actual payoff minus price – will then look unusually high. The more ambiguity investors perceive, the lower is the price and the higher is the subsequent return.

At the same time, the riskless interest rate in our model is determined as usual by the representative agent's first order condition. It varies much less than stock returns because there is less ambiguity about consumption growth than about payout growth. The latter is affected by uncertainty about operating cost, a small share of GDP which is nevertheless important for equity valuation. With interest rates that are stable the price-dividend ratio then helps to forecast *excess* returns on stocks, that is, there are time varying premia on stocks. This allows the model to account for the joint dynamics of prices and excess returns.[6]

Firms' responses to uncertainty shocks account for the fact that changes in debt comove positively with shareholder payout and stock prices. In particular, in good times, when prices are high and uncertainty premia are low, firms increase debt and pay out more to shareholders. The basic intuition here is that firms that try to smooth dividends over time will increase debt when they receive good news about future profits.

A firm that experiences lower uncertainty about profits acts as if mean profits will be higher. In order to smooth payout, it borrows and increases current dividend payout. Uncertainty shocks thus induce positive comovement of payout and debt. At the same time, market leverage is countercyclical even though debt increases in booms; this is because a lower uncertainty also raises equity values in line with the data. In contrast to an uncertainty shock, a shock to realized cash flow has the opposite effect: a firm that faces a temporary shortfall of funds will both pay out less and borrow to make up the shortfall. Cash flow shocks, such as realized changes in operating cost thus generate negative comovement of payout and debt and play a relatively small role for fluctuations.

Relative to the literature, the paper makes three contributions. First it specifies a linear DSGE model that allows for both endogenous asset supply and time varying uncertainty premia. Second, the model can accommodate first order effects of stochastic volatility and hence distinguish between realized volatility and perceived uncertainty. The model nevertheless remain tractable and can be estimated jointly with data on quantities and prices. Finally, the results suggest a prominent role for uncertainty shocks in driving asset

[6]Put differently, if an econometrician were to run a regression of excess return on price (normalized by dividends) on data simulated from our model, he would find a negative coefficient, as in the data.

prices and firm financing decisions jointly.

There are a number of papers that study asset pricing in production economies with aggregate uncertainty shocks. Several authors have studied rational expectations models that allow for time variation in higher moments of the shock distributions. The latter can take the form of time varying disaster risk (Gourio (2012)) or stochastic volatility (Basu and Bundick (2011), Caldara et al. (2012), Malkhozov and Shamloo (2012)). Another line of work investigates uncertainty shocks when agents have a preference for robustness (Cagetti et al. (2002), Bidder and Smith (2012), Jahan-Parvar and Liu (2012)). Most of these papers identify equity with the value of firm capital or introduce leverage exogenously. In contrast, our interest is in how uncertainty shocks drive valuation when leverage responds optimally to those shocks.

Recent work has explored whether the interaction of uncertainty shocks and financial frictions can jointly account for credit spreads and investment. Most of this work considers changes in firm-level volatility (Arellano et al. (2010), Gilchrist et al. (2010), Christiano et al. (2013)). Gourio (2013) incorporates time varying aggregate risk and thus allows risk premia to contribute to spreads. In contrast to our paper, this line of work does not focus on the determination of equity prices.

We also build on a recent literature that tries to jointly understand financial flows and macro quantities. Jermann and Quadrini (2012), Covas and Den Haan (2011, 2012), and Begenau and Salomao (2013) develop evidence on the cyclical behavior of debt and equity flows. Their modeling exercises point out the importance of shocks to firm profits other than productivity, a finding that is confirmed by our results. We also emphasize, however, the role of uncertainty about financial conditions. The latter is essential in order for our estimated model to account for time variation in risk premia on equity.

Glover et al. (2011) and Croce et al. (2012) study the effects of taxation in the presence of uncertainty shocks. Their setups are similar to ours in that they combine a representative household, a trade-off theory of capital structure and aggregate uncertainty shocks (in their case, changes in stochastic volatility under rational expectations). While their interest is in quantifying policy effects, our goal is to assess the overall importance of different uncertainty shocks.

Our estimation strategy follows the literature in using Bayesian techniques for inference

of DSGE models, but also incorporates financial variables and particularly asset prices.[7] We build on the Markov switching model introduced by Hamilton (1989) that is now a popular tool for capturing parameter instability. For example, Sims and Zha (2006) use a Markov-switching vector autoregression to investigate the possibility of structural breaks in the conduct of monetary policy, while Schorfheide (2005), Liu et al. (2011), Davig and Doh (2013), Bianchi (2013), and Baele et al. (2011) estimate MS-DSGE models.[8] We show that the MS-DSGE setup can also accommodate uncertainty averse agents' responses to volatility shocks. Indeed, since switches in volatilities have first order effects on decision rules, agents' responses are reflected in switches of the constants in the MS-DSGE.

The paper is structured as follows. Section 2 presents the model. Section 3 uses first order conditions for households and firms to explain the effect of uncertainty shocks on firm asset supply and asset prices. Section 4 describes our solution and estimation strategy, and then discusses the estimation results.

2 Model

Our model determines investment, production and financing choices of the US nonfinancial corporate sector as well as the pricing of claims on that sector by an infinitely-lived representative household. Firms are owned by the household and maximize shareholder value.

2.1 Technology and accounting

There is a single perishable good that serves as numeraire.

Production

The corporate sector produces numeraire from physical capital K_t according to the production function

$$Y_t = Z_t K_t^{\alpha} \xi^{(1-\alpha)t} \tag{1}$$

[7]Most estimated DSGE models do not include asset prices other than interest rates and do not address facts discussed in the empirical asset pricing literature such as the equity premium. A recent exception is Kliem and Uhlig (2013) who propose a new estimation approach for DSGE models that consists of using an augmented prior distribution in order to account for stylized asset pricing facts which are not directly included in the likelihood. The authors show that their estimated model can replicate stylized business cycle facts even when constrained to match the unconditional Sharpe ratio for equity.

[8]An alternative approach models smooth changes in the parameters (see Fernandez-Villaverde et al. (2010), Fernandez-Villaverde et al. (2011), and Justiniano and Primiceri (2008) for applications in a DSGE model and Primiceri (2005) and Cogley and Sargent (2006) for applications in a VAR).

where ξ is the trend growth rate of the economy and α is the capital share. The shock Z_t to the marginal product of capital accounts for fluctuations in variable factors including labor. We will assume below that realizations of Z_t are correlated with changes in uncertainty about future Z_t. The shock thus captures how firms adjust variable factors in response to changes in uncertainty.[9]

Capital is produced from numeraire and depreciates at rate δ

$$K_{t+1} = (1-\delta)K_t + \left[1 - \frac{S''}{2}\left(I_t/I_{t-1} - \xi\right)^2\right] I_t, \tag{2}$$

Capital accumulation is subject to adjustment costs that are convex in the growth rate of investment I_t, as in Christiano et al. (2005). This functional form captures the idea that the scale of investment affects the organization of the firm. For example, investing at some scale I_t requires allocating the right share of managerial effort to guiding expansion rather than overseeing production. Moving to a different scale entails reallocating managerial effort accordingly.

Financing and operating costs

In addition to investment, shareholders choose firms' net payout and their level of debt. Two types of frictions are relevant here. First, there are operating costs of running the corporate sector that are unrelated to production and credit. Every period, shareholders pay a cost

$$\phi\left(D_t/D_{t-1}\right) = f_t\xi^t + \frac{\phi''\xi^t}{2}(D_t/D_{t-1} - \xi)^2 \tag{3}$$

where f_t is random. These costs apply at the sectoral level: they represent expenditure incurred by shareholders who own many firms that are managed independently, for example because of limited managerial span of control. Shareholders thus not only choose net aggregate payout, but also make changes to the organization and ownership structure of firms.

The variable component is motivated by costs that occur as the scale of payout is changed, analogously to (2). For example, paying out at a large positive scale (continually repurchasing many shares or paying dividends at a high rate) requires shareholders to pressure managers to relinquish cash flow. In contrast, paying out at a large negative scale (continually raising a lot of new capital) requires the firm to focus more on maintaining relationships with primary

[9]Ilut and Schneider (2014) consider a model with nominal frictions and show that uncertainty about TFP can lead to large fluctuations in labor input over the business cycle. Modeling Z as a *joint* change in marginal product of capital today and uncertainty perceived about the marginal product of capital in the future allows us to accommodate business cycle implications of uncertainty shocks without explicitly modeling nominal frictions.

investors. In both cases, refocusing the firm quickly is difficult.

The fixed component is motivated by expenditure that varies independently from shocks shifting the production technology and that is unrelated to the scale of payout. Examples include costs from restructuring of the corporate sector (due to mergers and spin-offs, for example), reorganization of compensation, compliance with regulation, or lawsuits. In these examples, part of the cost to the corporate sector consists of a transfer payment to some other sector of the economy (the financial sector, households, or the government).

At the same time, we assume that a (possibly small) part of the cost directly lowers the value added of the nonfinancial corporate sector itself. Here we have in mind the labor productivity of management or other employees not directly related to increasing the scale of production (such as the legal or human resources department). For example, reorganization of compensation through negotiation with workers, or dealing with lawsuits implies that management is less productive in its other tasks. A key implication is that shocks like reorganization and lawsuits are not a pure redistribution across sectors, but instead result in a loss of surplus. Even if the loss of surplus is arbitrarily small, a higher fixed cost is then undesirable from the perspective of the representative household – this is the property that matters below.

The second friction arises in the credit market. Firms issue one period noncontingent debt. Let Q_t^b denote the price of a riskless short bond. Suppose the corporate sector issues $Q_{t-1}^b B_{t-1}^f$ worth of bonds at date $t-1$. At date t, it not only repays B_t^f to lenders, but also incurs the financing cost

$$\kappa\left(B_{t-1}^f\right) = \frac{\Psi}{2}\frac{1}{\xi^t}\left(B_{t-1}^f\right)^2$$

The marginal cost of issuing debt is thus upward sloping. This feature naturally arises if there is a idiosyncratic risk at the firm level and costly default. When firms choose capital structure, they trade off this cost against the tax advantage of debt.

Consider the firm's cash flow statement at date t. Denoting the corporate income tax rate by τ_k, we can write net payout as

$$\begin{aligned} D_t = {} & \alpha Y_t - I_t - \kappa(B_{t-1}^f) - \phi\left(D_t, D_{t-1}\right) - \left(B_{t-1}^f - Q_t^b B_t^f\right) \\ & - \tau_k\left[\alpha Y_t - B_{t-1}^f\left(1 - Q_{t-1}^b\right) - \delta Q_{t-1}^k K_{t-1} - I_t\right] \end{aligned} \tag{4}$$

The first line records cash flow in the absence of taxation: payout equals revenue less investment, operating and financing costs as well as net debt repayment. The second line subtracts the corporate income tax bill: the tax rate τ_k is applied to profits, that is, income less interest, depreciation and investment.

Household wealth

We denote the price of aggregate corporate sector equity by P_t. In addition to owning the firm, the household receives an endowment of goods $\pi\xi^t$ and government transfers $t_r\xi^t$. We assume a proportional capital income tax. Moreover, capital gains on equity are taxed immediately at the same rate. The household budget constraint is then

$$\begin{aligned} C_t + P_t\theta_t + Q_t^b B_t^h = (1-\alpha)\, Y_t + \pi\xi^t + t_r\xi^t + B_{t-1}^h + (P_t + D_t)\,\theta_{t-1} \\ - \tau_l \left[(1-\alpha)\, Y_t + (1-Q_{t-1}^b)B_{t-1}^h + D_t\theta_{t-1} + (P_t - P_{t-1})\theta_{t-1} + \pi\xi^t\right] - \tau_c C_t \end{aligned} \tag{5}$$

The first line is the budget in the absence of taxation: consumption plus holdings of equity and bonds – equals labor and endowment income plus the (cum dividend) value of assets. The second line subtracts the tax bill. The income tax rate applies to labor income, interest, dividends as well as capital gains. The consumption tax rate is denoted by τ_c.

We do not explicitly model the government, since we do not include observables that identify its behavior in our estimation. To close the model, one may think of a government that collects taxes based on the rates τ_l, τ_c, τ_k and uses lump sum transfers to follow a Ricardian policy of stabilizing its debt. The market clearing condition then states that $B_t^f = B_t^h$ and that equity holdings $\theta_t = 1$. The model is thus consistent with households owning not only corporate debt but also government debt.

2.2 Uncertainty and preferences

We denote information that becomes available to agents at date t by a vector of random variables ε_t and write $\varepsilon^t = (\varepsilon_t, \varepsilon_{t-1}, ...)$ for the entire information set as of date t. Agents perceive ambiguity about shocks to the marginal product of capital and operating cost. The dynamics of these shocks can be written as

$$\begin{aligned} \log Z_{t+1} &= \tilde{z}\left(\varepsilon^t\right) + \mu_{t,z}^* + \tilde{\sigma}_{t,z}\varepsilon_{t+1}^z + v_{t+1}^z \\ f_{t+1} &= \tilde{f}\left(\varepsilon^t\right) + \mu_{t,f}^* + \tilde{\sigma}_{t,f}\varepsilon_{t+1}^f \end{aligned} \tag{6}$$

where ε_{t+1}^z, ε_{t+1}^f and v_{t+1}^z are iid with $\varepsilon_{t+1}^i \sim \mathcal{N}(0,1)$, $i = z, f$ and $\mu_{t,z}^*$ and $\mu_{t,f}^*$ are deterministic sequences. The components $\tilde{z}(\varepsilon^t)$ and $\tilde{f}(\varepsilon^t)$ capture the time dependence of the shocks, while allowing for a nonnormal innovation v_{t+1}^z in addition to ε_{t+1}^z is helpful for a more flexible specification of business cycle risk, as explained below.[10]

[10]The process for the f shock in (6) is linear in levels which is helpful to allow for large shocks relative to the size of the steady state operating cost.

The decomposition of the innovations into deterministic and random components serves to distinguish between ambiguity and risk, respectively. In particular, changes in risk are modeled in the usual way as changes in realized volatility; we assume throughout that the volatilities are bounded away from zero. For the ambiguous components $\mu^*_{t,i}$, we assume that the long empirical distribution of $\mu^*_{t,i}$ is iid normal with mean zero and variance $\sigma^2_{i,\mu}$ that is independent of the shocks ε^i_t and v^z_t. As a result, it is impossible for an agent (or an econometrician) to learn the sequences $\mu^*_{t,i}$ in (6), even with a large amount of data: the sequence $\mu^*_{t,i}$ cannot be distinguished from the realization ε^i_t.

In our econometric work below, we resolve this uncertainty probabilistically: we work with volatility processes $\sigma^2_{t,i} = \tilde{\sigma}^2_{t,i} + \sigma^2_{i,\mu}$ and an iid innovation process, that is, $\mu^*_{t,i} = 0$. However, the probability we use is not the only one that is consistent with the data – there are many others corresponding to different decompositions of the variance into $\tilde{\sigma}^2_{t,i}$ and $\sigma^2_{i,\mu}$ as well as particular sequences $\mu^*_{t,i}$. We assume that agents in the model treat uncertainty as ambiguity: they do not resolve uncertainty about $\mu^*_{t,i}$ by thinking in terms of a single probability. We describe next the set of probabilities they use for contemplating the next innovation at any given history.

Uncertainty shocks and changes in confidence

Based on date t information, agents consider an interval of conditional means $\mu_{t,i} \in [-a_{t,i}, a_{t,i}]$ for each component i. They are not confident enough to further integrate over alternative forecasts (and so in particular they do not use a single forecast). The vector $a_t = (a_{t,z}, a_{t,f})'$ summarizes ambiguity perceived about Z and f given date t information. It can be thought of as an (inverse) measure of confidence. If $a_{t,i}$ is low, then agents find it relatively easy to forecast the fundamental shock i and their behavior is relatively close to that of expected utility maximizers (who use a single probability when making decisions). In contrast, when $a_{t,i}$ is high, then agents do not feel confident about forecasting.

We allow for two sources of changes in confidence (and thus perceived ambiguity). On the one hand, confidence can depend on observed volatility. It is plausible that in more turbulent times agents find it harder to settle on a forecast of the future. On the other hand, confidence can move with intangible information that is not reflected in current fundamentals or volatility. To accommodate both cases, we let

$$a_{t,i} = \eta_{t,i}\sigma_{t,i}; \qquad i = f, z \tag{7}$$

Here the $\eta_{t,i}$s are stochastic processes that describe changes in confidence due to the arrival of intangible information. Their laws of motion, like those of the volatilities $\sigma_{t,i}$, are known

to agents. The information ε_t received at date t thus includes not only ε_t^z and ε_t^f, but also innovations to $\sigma_{t,i}$ and $\eta_{t,i}$.

We can derive the linear relationship in (7) from the assumption that $\mu_{t,i} \in [-a_{t,i}, a_{t,i}]$ if and only if

$$\frac{\mu_{t,i}^2}{2\sigma_{t,i}^2} \leq \frac{1}{2}\eta_{t,i}^2$$

Here the left hand side is the relative entropy between two normal distributions that share the same standard deviation $\sigma_{t,i}$ but have different means $\mu_{t,i}$ and zero, respectively. The agent thus contemplates only those conditional means that are sufficiently close to the long run average of zero in the sense of conditional relative entropy. The relative entropy distance captures that intuition through the fact that when $\sigma_{t,i}$ increases it is harder to distinguish different models.

Preferences

The representative household has recursive multiple priors utility. A consumption plan is a family of functions $c_t(\varepsilon^t)$. Conditional utilities derived from a given consumption plan c are defined by the recursion

$$U\left(c;\varepsilon^t\right) = \log c_t\left(\varepsilon^t\right) + \beta \min_{\mu_{t,i}\in\times_i[-a_{t,i},a_{t,i}]} E^{\mu}\left[U\left(c;\varepsilon^t,\varepsilon_{t+1}\right)\right], \tag{8}$$

where the conditional distribution over ε_{t+1} uses the means $\mu_{t,i}$ that minimize expected continuation utility. If $a_t = 0$, we obtain standard separable log utility with those conditional beliefs. If $a_t > 0$, then lack of information prevents agents from narrowing down their belief set to a singleton. In response, households take a cautious approach to decision making – they act as if the worst case mean is relevant.[11]

Given the specification of the ambiguous shocks, it is easy to solve the minimization step in (8) at the equilibrium consumption plan: the worst case expected cash flow is low and the worst case expected operating cost is high. Indeed, consumption depends positively on cash flow and negatively on the operating cost. It follows that agents act throughout as if forecasting under the worst case mean $\mu_{f,t} = a_{f,t}$ and $\mu_{z,t} = -a_{z,t}$. This property pins down the representative household's worst case belief after every history and thereby a worst case belief over entire sequences of data. We can thus also compute worst case expectations many periods ahead, which we denote by stars. For example E^*D_{t+k} is the worst case expected

[11] In the expected utility case, time t conditional utility can be represented as as $E_t\left[\sum_{\tau=0}^{\infty}\log c_{t+\tau}\right]$ where the expectation is taken under a conditional probability measure over sequences that is updated by Bayes' rule from a measure that describes time zero beliefs. An analogous representation exists under ambiguity: time t utility can be written as $\min_{\pi\in\mathcal{P}} E_t^{\pi}\left[\sum_{\tau=0}^{\infty}\log c_{t+\tau}\right]$. The time zero set of beliefs $\mathcal{P}$ can be derived from the one step ahead conditionals $\mathcal{P}_t$ as in the Bayesian case; see Epstein and Schneider (2003) for details.

dividend k periods in the future.

3 Uncertainty shocks, firm financing and asset prices

In this section, we describe the main trade-offs faced by investors and firms when pricing assets and deciding asset supply, respectively. To ease notation here, we set the trend growth rate $\log \xi$ equal to zero. The solution of the model with positive growth is provided in the appendix.

Contingent claims prices and shareholder value

To describe t-period ahead contingent claims prices, we define random variables M_0^t that represent prices normalized by conditional *worst case* probabilities. This normalization is convenient for summarizing the properties of prices, which are derived from households' and firms' first order conditions. We also define a one-period-ahead pricing kernel as $M_{t+1} = M_0^{t+1}/M_0^t$. From household utility maximization, we obtain

$$M_{t+1} = \beta \frac{C_t}{C_{t+1}} \frac{1-\tau_l}{1-\tau_l \beta E_t^* \left[C_t/C_{t+1}\right]}$$

The pricing kernel is the marginal rate of substitution, multiplied by a factor that corrects for taxes. Since the qualitative effects we emphasize here do not depend on the level of personal income taxation, we set $\tau_l = 0$ for the remainder of this section.

The formulas for bond and stock prices are then standard, except that expectations are taken under the worst case belief:

$$\begin{aligned} Q_t &= E_t^* \left[M_{t+1}\right] \\ P_t &= E_t^* \left[M_{t+1} \left(P_{t+1} + D_{t+1}\right)\right] \end{aligned} \tag{9}$$

An increase in ambiguity makes the worst case belief worse and thereby changes asset prices. For example, if agents perceive more ambiguity about future consumption, then the bond price rises and the interest rate falls. Similarly, more ambiguity about dividends tends to lower the stock price.

The firm maximizes shareholder value

$$E_0^* \sum_{t=1}^{\infty} M_0^t D_t$$

Shareholder value also depends on worst case expectations. Indeed, state prices determined in

financial markets reflect households' attitudes to uncertainty, as illustrated by the household Euler equations. For example, when there is more ambiguity about future consumption then – other things equal – cash flows are discounted less. When there is more ambiguity about future cash flow, the firm tends to be worth less.

3.1 Payout and capital structure choice

Let λ_t denote the multiplier on the firm's date t budget constraint (4), normalized by the contingent claims price M_0^t. In the presence of operating and financing costs, the shadow value of funds inside the firm can be different from one. The firm's first order equation for debt is

$$Q_t^b \lambda_t = E_t^* \left[M_{t+1} \lambda_{t+1} \right] \left(1 - \tau_k \left(1 - Q_t^b \right) + \kappa'(B_t^f) \right) \tag{10}$$

The marginal benefit of issuing an additional dollar of debt is the bond price multiplied by the firm's shadow value of funds. The marginal cost includes not only the present value of a dollar to the firm, but also the tax advantage of debt and the marginal financing cost. The tax advantage implies that marginal cost is typically below marginal benefit at low levels of debt. At the optimal capital structure, it is traded off against the financing cost.

The firm's first order condition for payout is

$$D_t \left(1 - \lambda_t \right) = \lambda_t \tilde{\phi} \left(\frac{D_t}{D_{t-1}} \right) - E_t^* \left[M_{t+1} \lambda_{t+1} \tilde{\phi} \left(\frac{D_{t+1}}{D_t} \right) \right] \tag{11}$$

where the function $\tilde{\phi} \left(D_t / D_{t-1} \right) := \left(D_t / D_{t-1} \right) \phi' \left(D_t / D_{t-1} \right)$ is increasing and satisfies $\tilde{\phi} \left(1 \right) = 0$ since we abstract from growth in this section. It is thus optimal for the firm to stabilize the growth rate of payout in uncertainty adjusted terms. Indeed, at the steady state we have $\lambda_t = 1$. Near a steady state, payout will thus be set to equate the uncertainty adjusted expected growth rate to the realized growth rate.

Consider now the firm's response to an increase in uncertainty. In particular, suppose that, under the worst case belief, future dividends are low so that funds are scarce, that is, the relative shadow value of funds $E_t^* \left[M_{t+1} \lambda_{t+1} \right] / \lambda_t$ increases. From (10), holding fixed the riskless rate, the marginal cost of debt increases and the firm responds by cutting current debt B_t^f. At the same time, (11) suggests that the firm will decrease the payout already at date t in order to smooth the drop in the growth rate of the payout. As a result, uncertainty shocks make the payout and debt move together.

In contrast, consider a shock to the cash flow that temporarily lowers dividends and makes current funds more scarce relative to funds in the future. In this case, (10) suggests

that the firm should borrow temporarily so as to cover the shortfall in funds. Cash flow shocks thus tend to move payout and debt in opposite directions.

3.2 Asset pricing

To see how asset pricing works in our model, we consider an approximate solution that is also used in our estimation approach. The approximation proceeds in three steps. First, we find the "worst case steady state", that is, the state to which the model would converge if there were no shocks and the data were generated by the worst case probability belief. Second, we linearize the model around the worst case steady state. Finally, we derive the true dynamics of the system, taking into account that the exogenous variables follow the data generating process (6).[12]

Linearization around the worst case steady state

At the worst case steady state, the Euler equations (9) imply that the bond price is β and the price dividend ratio is $\beta/(1-\beta)$. These values are the same as in the deterministic perfect foresight steady state. However, the level of consumption and dividends as well as other variables will be lower than in a perfect foresight steady state. This is because they are computed using the worst case mean productivity Z^* and operating cost f^*. These worst-case values are the steady states of the exogenous variables Z_t and f_t when the worst-case one-step mean is $\mu_f = a_f$ and $\mu_z = -a_z$, respectively. Denoting by ρ_z and ρ_f the AR(1) persistence parameters of the two processes we then have that $Z^* = \bar{Z}e^{-a_z/(1-\rho_z)}$ and $f^* = \overline{f} + a_f/(1-\rho_f)$, where $\bar{Z}$ and $\overline{f}$ are the mean values under the econometrician's data generating process.

We mark log deviations from the worst case steady state by both a hat (for log deviation) and a star (to indicate that the perturbation is around the worst case steady state). The loglinearized pricing kernel and the household Euler equation for bonds and equity are

$$\begin{aligned} \hat{m}^*_{t+1} &= \hat{c}^*_t - \hat{c}^*_{t+1} \\ \hat{q}^*_t &= E^*_t[\hat{m}_{t+1}] \\ \hat{p}^*_t &= E^*_t\left[\hat{m}_{t+1} + \beta\hat{p}^*_{t+1} + (1-\beta)\,\hat{d}^*_{t+1}\right] \end{aligned} \tag{12}$$

The short term interest rate is $\hat{r}^*_t = -\hat{q}^*_t = -E^*_t\hat{m}^*_{t+1}$. Linearization implies that asset prices do not reflect risk compensation. However, they still reflect uncertainty premia since

[12]The worst case steady state used in steps 1 and 2 should be viewed as a computational tool that helps describe agents' optimal choices. Agents choose conservative policies in the face of uncertainty, and this looks *as if* the economy were converging to the worst case steady state.

expectations are computed under the worst case mean.

Stock price and interest rate volatilities

We can use the loglinearized Euler equations to understand the relative volatility of stock prices and interest rates in a model with ambiguity shocks. Substituting into the Euler equation for stocks, the price dividend ratio, or more precisely the price payout ratio, can be written as

$$\hat{p}_t^* - \hat{d}_t^* = -\hat{r}_t^* + E_t^* \left[\beta(\hat{p}_{t+1}^* - \hat{d}_{t+1}^*) + \hat{d}_{t+1}^* - \hat{d}_t^* \right] \tag{13}$$

The above relation expresses the price dividend ratio as the worst case expected payoff relative to dividends, discounted at the riskless interest rate. In general equilibrium, an increase in uncertainty can move both the payoff term (if cash flow becomes more uncertain), and the interest rate (if consumption becomes more uncertain). In the data, interest rates are relatively stable whereas the price dividend ratio moves around a lot. As a result, the first effect must dominate the second if uncertainty shocks are to play an important role.

We can solve forward to express the price dividend ratio as the present value of future growth rates in the dividend-consumption ratio

$$\begin{aligned} \hat{p}_t^* \ - \hat{d}_t^* &= E_t^* \left[\beta(\hat{p}_{t+1}^* - \hat{d}_{t+1}^*) + (\hat{d}_{t+1}^* - \hat{c}_{t+1}^*) - \left(\hat{d}_t^* - \hat{c}_t^* \right) \right] \\ &= E_t^* \sum_{\tau=1}^{\infty} \beta^{\tau-1} \left((\hat{d}_{t+\tau}^* - \hat{c}_{t+\tau}^*) - \left(\hat{d}_t^* - \hat{c}_t^* \right) \right) \end{aligned} \tag{14}$$

If dividends are proportional to consumption, then the price dividend ratio is constant – with log utility, income and substitution effects cancel. In contrast, if dividends are a small share of consumption (as in the data), then uncertainty about dividends will tend to dominate and an increase in uncertainty can decrease the price dividend ratio. The formula also shows that the price dividend ratio reflects expected worst case growth rates. If firms smooth these growth rates in response to uncertainty shocks, this tends to contribute to price volatility.

Zero risk steady state and unconditional premia

Unconditional premia predicted by the model depend on the average amount of ambiguity reflected in decisions. Suppose all shocks are equal to zero, but agents still use decision rules that reflect their aversion to ambiguity. In particular, agents perceive constant ambiguity, as in the worst case steady state. We can study this "zero risk" steady state using decision rules derived by linearization around the worst case steady state. From this perspective, the true steady state productivity and operating cost $(\bar{Z}, \bar{f})$ look like a positive deviation from the steady state summarized by the vector $(-\bar{a}_z, \bar{a}_f)$. Mechanically, we are looking at the steady state of a system in which technology is always at $(\bar{Z}, \bar{f})$, but in which agents act *as*

if the economy is on an impulse response towards the worst case steady state (Z^*, f^*).

Consider the impulse response that moves from the zero risk steady state log consumption and dividend, $\left(\bar{C}, \bar{D}\right)$ say, to their worst case counterparts $\left(\bar{C}^*, \bar{D}^*\right)$. We work with loglinearized impulse responses and write $\bar{c} = \log \bar{C} - \log \bar{C}^*$. Along the linearized impulse response, the Euler equations (12) hold deterministically. For example, the steady state log bond price is

$$\bar{Q} = \beta \exp\left(\bar{q}\right) = \beta \exp\left(\bar{c} - \hat{c}_1\right),$$

where $\hat{c}_1$ is the first value along the impulse response. If there is ambiguity about consumption we would expect the impulse response to decline towards the worst case. In this case, the bond price is higher than β, the worst case (as well as rational expectations) steady state bond price. In other words, ambiguity about consumption lowers the interest rate – a precautionary savings effect.

Consider now the steady state price dividend ratio. The log deviation of $\bar{p}^* - \bar{d}^*$ from the worst case value $\beta/\left(1-\beta\right)$ is given by (14), where the sum is over the consumption and dividend path along the linearized impulse response. For example, if the dividend-consumption ratio declines along the impulse response – say because there is a lot of average ambiguity about dividends and dividends are a small share of consumption – then $\bar{p}^* - \bar{d}^*$ is negative, that is, the steady state price dividend ratio $\bar{P}/\bar{D}$ is below $\beta/\left(1-\beta\right)$. The presence of ambiguity thus induces a price discount.

Combining the bond and stock price calculations, the equity premium at the zero risk steady state is[13]

$$\log\left(\bar{P} + \bar{D}\right) - \log \bar{P} + \log \bar{Q} = \left(1-\beta\right)\left(\bar{d}^* - \bar{p}^*\right) - \left(\bar{c} - \hat{c}_1\right)$$

There are two reasons why ambiguity can generate a positive steady state equity premium. First, the average stock return can be higher than under rational expectation because the price dividend ratio is lower. This is the first term. Second, the interest rate can be lower. The second effect is small if dividends are a small share of consumption and ambiguity is largely about dividends. We emphasize the role of the first effect: it says that average equity returns themselves are higher than in the rational expectations steady state. Ambiguity thus does not simply work through low real interest rates.

[13]The log stock return at the zero risk steady state is

$$\log\left(\bar{P} + \bar{D}\right) - \log \bar{P} \approx \left(1-\beta\right)\left(\bar{d} - \bar{p}\right) - \log \beta$$

where we are using the fact that all asset returns are equal to $-\log\beta$ at the worst case steady state.

Predictability of excess returns

A standard measure of uncertainty premia in asset markets is the expected excess return on an asset computed from a regression on a set of predictor variables. The log excess stock return implied by our model can be approximated as

$$\begin{aligned} x^e_{t+1} &= \log(p_{t+1} + d_{t+1}) - \log p_t - \log(i_t) \\ &\approx \beta \hat{p}^*_{t+1} + (1-\beta)\, \hat{d}^*_{t+1} - \hat{p}^*_t + \hat{q}^*_t \\ &= \beta \left(\hat{p}^*_{t+1} - \hat{d}^*_{t+1} - E^*_t[\hat{p}^*_{t+1} - \hat{d}^*_{t+1}] \right) + \hat{d}^*_{t+1} - E^*_t \hat{d}^*_{t+1} \end{aligned}$$

Here the second line is due to loglinearization of the return around the worst case steady state. The third line follows from the household Euler equation for stocks and can easily be derived using equation (13).

Consider now an econometrician who attempts to predict excess stock returns in the model economy. Suppose for concreteness that he has enough predictor variables to actually recover the theoretical conditional expectation of the payoff next period given the state variables of the model. With a large enough sample, he will measure the expected excess return $E_t x^e_{t+1}$, where the expectation is taken with the conditional mean $\mu^*_t = 0$.[14] Using the above expression, we can write the measured risk premium as

$$E_t x^e_{t+1} = \beta (E_t - E^*_t)[\hat{p}^*_{t+1} - \hat{d}^*_{t+1}] + (E_t - E^*_t)\hat{d}^*_{t+1}$$

where $(E_t - E^*_t)$ represents the difference between the expectation under $\mu^*_t = 0$ and the worst case expectation. This is a term that is proportional to ambiguity a_t. This expression suggests an interesting approach to quantify ambiguity in a linear model. Since risk premia must be due to ambiguity, it is possible to learn about ambiguity parameters up front from simple linear regressions without solving the DSGE model fully.

4 Estimation

4.1 Data

Our estimation uses data on investment growth, leverage, the ratios of shareholder payout to GDP and equity value to GDP, as well as the short term real interest rate. The time period is 1959Q1 to 2011Q3. All firm variables are for the US nonfinancial corporate sector.

[14] Indeed, since all unconditional empirical moments converge to those of a process with $\mu^*_t = 0$ by construction, the same is true for conditional moments.

We thus include all nonfinancial firms that are corporations for tax purposes. In terms of value, however, most fluctuations in financial variables are driven by the largest firms, who are also publicly traded (see for example, Covas and Den Haan (2011)).

Investment and GDP numbers come from the National Income and Product Accounts (NIPA), published by the Bureau of Economic Analysis. The nonfinancial corporate sector accounts for about one half of total US GDP. Cyclical behavior of its macro aggregates is familiar from statistics for the economy as a whole. For example, Investment growth is displayed in the bottom left panel of Figure 1, with NBER recessions shaded. The other major sectors contributing domestically in the US are households and noncorporate business (who produce most housing services, for example) and the government.

The NIPA accounts are integrated with the Flow of Funds Accounts (FFA), published by the Federal Reserve Board, from which we take financial variables. The market value of equity for the nonfinancial corporate sector is shown in the top left panel of Figure 1. It exhibits a strong low frequency component with a dip in the 1970s and early 1980s, as well more cyclical behavior recently. It shares both features with other measures of stock prices normalized by real variables, such as the price earnings ratios for the NYSE.

We define net debt issuance as debt issuance less increases in bond holdings. Here we add up over all fixed income instruments listed in the flow of funds accounts. The idea is that all types of bonds are close substitutes, at least compared to equity. To quantify a model that delivers a choice between debt and equity, it thus makes sense to lump all types of bonds together. The market leverage ratio of the nonfinancial corporate sector is defined as outstanding net debt divided by the market value of equity. It is shown as dark blue line in the top left panel of Figure 2 and displays strong countercyclical fluctuations: while there is some cyclical movement in the level of debt, the ratio is mostly driven by stock price fluctuations.

We define shareholder payout as dividends plus share repurchases less issuance of equity. Both debt and equity flows in raw FFA data are highly seasonal. We thus compute four quarter trailing moving averages. There are also two large outliers in bonds acquired by the nonfinancial corporate sector: 14.5% of total GDP in 1977:Q3 and an 9.6% of GDP in 1993:Q4. Both outliers are more than four standard deviations above the mean, whereas the next smallest data point is less than three standard deviations above the mean. Since we cannot expect a business cycle model to account for such sharp changes, and to guard against contamination of our other inference, we linearly interpolate the net debt series for these two quarters. The resulting series for net payout and net debt issuance are shown in the top and bottom right hand panels of Figure 1. In addition to the low frequency dip and upward movement in payout, the most pronounced pattern is the positive comovement at

business cycle frequencies, consistent with Jermann and Quadrini (2012).

We need a measure of the real three month interest rate over the entire sample period, even though indexed bonds were not traded in the US over most of that period. We rely on a state space model of interest rate and inflation dynamics estimated by Piazzesi et al. (2013). The advantage of a state space approach is that it separates slow moving and transitory components of the inflation process, thus yielding a better fit of inflation dynamics. The resulting real interest rate series is shown as a dark blue line in the bottom left panel of Figure 2. It displays the familiar properties of low real rates in the 1970s, high rates in the early 1980s and procyclical behavior since then.

4.2 Dynamics of shocks, volatility, and ambiguity

We now specialize the fairly general shock dynamics (6) and (7) by choosing functional forms for shocks to the marginal product of capital and the operating cost as well as uncertainty about those shocks.

Volatility and ambiguity regimes

To parsimoniously model correlated changes in uncertainty, and to account for nonlinear dynamics in uncertainty, we use finite state Markov chains. The standard deviations $\sigma_{t,z}$ and $\sigma_{t,f}$ in (6) are driven by a two-state Markov chain s_t^σ with transition matrix H^σ. Each state in s_t^σ represents a "volatility regime"; a regime switch simultaneously moves both standard deviations. The regime is known at date t so that volatility is known one period in advance. Movements in confidence due to intangible information, denoted by $\eta_{t,i}$ also depend on the realization of a two-state Markov chain. This chain, denoted s_t^η, is independent from s_t^σ and is governed by a transition matrix H^η. As for the volatility regimes, each state in s_t^η simultaneously changes both levels $\eta_{t,z}$ and $\eta_{t,f}$.

To derive a loglinear approximation to equilibrium in the presence of stochastic volatility, it is helpful to write the chains as VARs as in Hamilton (1994). For example, s_t^σ can be written as

$$\begin{bmatrix} e_{1,t}^\sigma \\ e_{2,t}^\sigma \end{bmatrix} = H^\sigma \begin{bmatrix} e_{1,t-1}^\sigma \\ e_{2,t-1}^\sigma \end{bmatrix} + \begin{bmatrix} v_{1,t}^\sigma \\ v_{2,t}^\sigma \end{bmatrix} \tag{15}$$

where $e_{j,t}^\sigma = 1_{s_t^\sigma = j}$ is an indicator operator if the volatility regime s_t^σ is in place, and the shock v_t^σ is defined such that $E_{t-1}\left[v_t^\sigma\right] = 0$. A similar VAR representation is available for s_t^η.

Operating cost

Operating cost is modeled as a persistent AR(1) process in levels, with ambiguity driven

by the regimes:

$$\begin{aligned}\tilde{f}\left(\varepsilon^{t}\right) &= \bar{f} + \rho_f\left(f_t - \bar{f}\right), \\ a_{t,f} &= \eta_f(s_t^{\eta})\sigma_f(s_t^{\sigma}),\end{aligned} \tag{16}$$

In the first line, $\tilde{f}$ is the mean function used in (6) and $\bar{f}$ is the steady state operating cost. The second line shows that ambiguity about operating cost follows a four-state Markov chain.

Marginal product of capital

The shock to the marginal product of capital is handled differently because we view Z_t as incorporating the responses of variable inputs to an underlying uncertainty shock. We model shocks to Z_t as a *joint* change in marginal product of capital today and ambiguity perceived about the marginal product of capital in the future.[15] This requires two extensions to the functional form (16). On the one hand, the current shock should depend (negatively) on the current uncertainty regime.

Correlation between uncertainty regime switches and Z_t is due to the nonnormal shock v_{t+1}^z introduced in (6). It is proportional to the forecast error for ambiguity given the current regime. The conditional mean forecast enters into the term $\tilde{z}\left(\varepsilon^{t}\right)$ that describes how Z_t depends on the past. We thus define

$$\begin{aligned}\tilde{z}\left(\varepsilon^{t}\right) &= \log\bar{Z} + \rho_z\left(\log Z_t - \log\bar{Z}\right) - \varkappa E_t\left[\eta_z(s_{t+1}^{\eta})\sigma_z(s_{t+1}^{\sigma})|s_t^{\eta}, s_t^{\sigma}\right] \\ v_{t+1}^z &= -\varkappa\left(\eta_z(s_{t+1}^{\eta})\sigma_z(s_{t+1}^{\sigma}) - E_t\left[\eta_z(s_{t+1}^{\eta})\sigma_z(s_{t+1}^{\sigma})|s_t^{\eta}, s_t^{\sigma}\right]\right)\end{aligned} \tag{17}$$

On the other hand, ambiguity should move continuously (negatively) with the shock. To this effect, we introduce an AR(1) component of a_t with innovations that are perfectly negatively correlated with those to $\log Z_t$:

$$\begin{aligned}a_{t,z} &= \eta_z(s_t^{\eta})\sigma_z(s_t^{\sigma}) + \widetilde{a}_{t,z}^{c} \\ \widetilde{a}_{t,z}^{c} &= \rho_a\widetilde{a}_{t-1,z}^{c} - \sigma_a\sigma_z(s_{t-1}^{\sigma})\varepsilon_t^z\end{aligned} \tag{18}$$

[15] Ilut and Schneider (2014) show that an increase in ambiguity about total factor productivity makes firms and households act cautiously so that hours worked and economic activity can contract even if current labor productivity did not change. We capture similar effects here by making the innovations to real technology negatively correlated with the current innovation to ambiguity about it.

4.3 Markov-switching VAR representation of the equilibrium

We can write the equilibrium representation of our model as a Markov-switching VAR (MS-VAR) in the DSGE state vector containing all the variables of the model. The interval of one-step ahead conditional means given by $[-a_{t,i}, a_{t,i}]$ for each shock i is affected by the product $a_{t,i} = \eta_{t,i}\sigma_{t,i}$ of the two sources of ambiguity. Both uncertainty chains s_t^η and s_t^σ linearly affect $a_{t,i}$ and hence the worst-case conditional expectation. Moreover, the chains are stationary and ergodic and their dynamics are the same under the true and worst case dynamics. The worst case steady state depends on their long run averages $\overline{\eta}_i$ and $\overline{\sigma}_i$. As intangible ambiguity $\eta_{t,i}$ or volatility $\sigma_{t,i}$ fluctuate around their respective long run means, there are "shocks" to ambiguity $a_{t,i}$ and therefore shifts in the constants of the MS-VAR representation.

Formally, we define the vector of linear deviations of the product $\eta_i(s_t^\eta)\sigma_i(s_t^\sigma)$ for $i = z, f$ from its ergodic values of $\overline{\eta}_i\overline{\sigma}_i$ as following a four-state Markov chain, which is obtained by mixing the two independent chains s_t^η and s_t^σ. We can then write the VAR representation of this composite Markov chain as

$$\begin{bmatrix} e^{\eta\sigma}_{1,1,t} \\ e^{\eta\sigma}_{1,2,t} \\ e^{\eta\sigma}_{2,1,t} \\ e^{\eta\sigma}_{2,2,t} \end{bmatrix} = H^{\eta\sigma} \begin{bmatrix} e^{\eta\sigma}_{1,1,t-1} \\ e^{\eta\sigma}_{1,2,t-1} \\ e^{\eta\sigma}_{2,1,t-1} \\ e^{\eta\sigma}_{2,2,t-1} \end{bmatrix} + \begin{bmatrix} \upsilon^{\eta\sigma}_{1,t} \\ \upsilon^{\eta\sigma}_{2,t} \\ \upsilon^{\eta\sigma}_{3,t} \\ \upsilon^{\eta\sigma}_{4,t} \end{bmatrix} \tag{19}$$

where $e^{\eta\sigma}_{m,n,t} = 1_{s_t^\eta = m, s_t^\sigma = n}$ is an indicator operator if at time t the intangible ambiguity regime m and the volatility regime n are in place, where $m, n \in \{1, 2\}$. The realizations of the shock $\upsilon_t^{\eta\sigma}$ are such that $E_{t-1}\left[\upsilon_t^{\eta\sigma}\right] = 0$. The transition matrix is $H^{\eta\sigma} = H^\eta \otimes H^\sigma$.

For each shock i and the four m, n combinations we can define $a_i(m, n) \equiv \eta_i(s_t^\eta = m)\sigma_i(s_t^\sigma = n) - \overline{\eta}_i\overline{\sigma}_i$. For example, when the intangible information regime 1 and volatility regime 1 are in place, $e^{\eta\sigma}_{1,1,t} = 1$ and the rest of the three $e^{\eta\sigma}_{m,m,t} = 0$. This means that our system of equations will load $a_i(1,1)e^{\eta\sigma}_{1,1,t} = a_i(1,1)$ and put zero weight on the other three realizations $a_i(1,2), a_i(2,1)$ and $a_i(2,2)$. In this case, the realization of the $\upsilon_t^{\eta\sigma}$ shock such that $e^{\eta\sigma}_{1,1,t} = 1$ makes the linear deviation $\eta_i(s_t^\eta = 1)\sigma_i(s_t^\sigma = 1) - \overline{\eta}_i\overline{\sigma}_i$ hit the economy as a discrete shock. By augmenting the DSGE state vector with the vector $e_t^{\eta\sigma}$ we control for the first order effects of the shifts in intangible ambiguity and volatility.

Given these first-order shifts, we can then proceed to linearize the rest of the equilibrium conditions of the model. We then use an observational equivalence result according to which our economy behaves *as if* the agent maximizes expected utility under the worst-case belief. Given this equivalence, we use standard perturbation techniques that are a good

approximation of the nonlinear decision rules under expected utility. This allows us to solve the model using standard solution algorithms, such as *gensys* by Sims (2002). The model solution assumes the form of a Markov-switching VAR that allows for changes in the volatility and in the constants:

$$\widetilde{S}_t = C(s_t^\sigma, s_t^\eta) + T\widetilde{S}_{t-1} + R\sigma(s_{t-1}^\sigma)\varepsilon_t$$

where $\widetilde{S}_t$ is a vector containing all the variables of the model. Notice that changes in uncertainty have first order effects captured by changes in the constants of the MS-VAR. Not all variables of the vector $\widetilde{S}_t$ are observable. We then combine the solution with a set of observation equations, obtaining a model in state space form that can be estimated with likelihood based techniques. Further details on the representation of the solution are presented in Appendix 5.1.

4.4 Estimates

We estimate the model using the methods developed in Bianchi (2013). Specifically, we employ a Metropolis-Hastings algorithm initialized around the posterior mode and compute the likelihood using the methods described in Kim and Nelson (1999).[16] Because we only have two continuously distributed shocks but we have five observables, to avoid stochastic singularity we need to introduce three observation errors. We set these errors on the dividend to GDP ratio, real interest rate, and the firm debt to equity value ratio.

There are two key differences from a standard Bayesian estimation of a homoskedastic linear DSGE model. First, we have to account for heteroskedasticity in the shocks of our model, as in the literature on regime-switching volatilities. Second, differently from that literature, the volatility regimes, as well as the intangible ambiguity ones, have first order effects on the endogenous variables of our linear model.

Our approach achieves identification of the volatility and intangible ambiguity regimes through two channels: on the one hand, since they enter as a product in the linearized model, both these uncertainty regimes shift the constant of the Markov-switching DSGE. On the other hand, the two types of regimes can be differentiated through the properties of the fundamental shocks. While the intangible information regime s_t^η leaves unaffected the moments of the shocks, the volatility regime s_t^σ shows up as changes in the second moment of the next period innovations. Through the use of the Kalman smoother, the estimation can then identify how likely it is that a shift in the constant is due to the high volatility or

[16] We also considered an alternative approach based on a Metropolis-within-Gibbs algorithm. The two methods lead to similar results.

the high intangible ambiguity regime.

Choosing ambiguity parameters

The regime-switching dynamics of risk and ambiguity are governed by the Markov chains s_t^σ and s_t^η. There we estimate directly the corresponding two values of $\sigma_{t,i}$ and $\eta_{t,i}$ together with the transition matrices H^σ and H^η. We are then left with choosing parameters σ_a, ρ_a and $\varkappa$. In order for the set $[-a_{t,z}, a_{t,z}]$ to be well-behaved we need the process for $a_{t,z}$ to remain nonnegative. Similarly to the parametrization used in Ilut and Schneider (2014), we then set

$$3\sigma_a = \eta_{z,L}\sqrt{1-\rho_a^2} \tag{20}$$

where $\eta_{z,L}$ is the value for η_z in the low ambiguity regime. This ensures that even in the low ambiguity regime, the probability that $\eta_{t,z}$ becomes negative is .13%, and any negative $\eta_{t,z}$ will be small.

The second consideration is that we want to bound the lack of confidence by the measured variance of the shock that agents perceive as ambiguous. Ilut and Schneider (2014) argue that a reasonable upper bound for $a_{t,i}$ is given by $2\sigma_{t,i}$. Because the ambiguity on the marginal product of capital has a linear component, we cannot enforce the bound exactly. Here we assume that it is violated with probability .13% even in the high ambiguity regime

$$\eta_{z,H} + 3\frac{\sigma_a}{\sqrt{1-\rho_a^2}} \leq 2. \tag{21}$$

We satisfy the constraints in (20) and (21) by imposing that both values $\eta_{z,L}$ and $\eta_{z,H}$ are lower than 1. Finally, we set the proportionality factor $\varkappa$ in (17) equal to σ_a^{-1}. This means that the negative effect on the current $\widehat{z}_t$ of one unit increase in $\widetilde{a}_{t,z}^c$ is the same as that of $\eta_z(s_t^\eta)\sigma_z(s_t^\sigma)$. We then are left with estimating the values $\eta_{z,L}, \eta_{z,H}$ and ρ_a as we can then infer σ_a from (20). Finally, we find that throughout the estimates this bound is not binding for the two estimated values for η_f.

Parameter estimates

We estimate a subset of the parameters, with values reported Table 1. The other parameters are fixed, as reported in Table 2, to calibrated values to match some key ratios from the NIPA accounts. Further details are in Appendix 5.2.1. The priors for the estimated parameters are chosen as follows. We first specify loose independent priors for the different parameters of the model. These priors are described in Table 1. We then specify a prior on the zero-risk steady state of the observables, inducing a joint prior on the model parameters. This second set of priors is described in Table 3.

Our approach is in line with the methods developed by Del Negro and Schorfheide (2008) and allows us to take into account that the zero risk steady state, i.e. the steady state that is observed ex-post by the econometrician, depends on the actual solution of the model. Therefore, unlike in many rational expectations models, the observed steady state is not controlled by a particular parameter or by a simple transformation of a subset of parameters.

4.5 Results

In this subsection we describe the economic forces identified by our estimation through a series of figures. We discuss posterior estimates, the role of measurement error and the patterns, identification and economic effects of the estimated uncertainty regimes.

Posterior estimates

Table 1 provides details on the posterior estimates. Consider the magnitude of the nonstandard parameters. The parameter Ψ, implied by the normalization of Ψ_y, controls the marginal cost of extra debt. When the equilibrium condition for firm's optimal debt, presented in (31), is evaluated at steady state, the parameter Ψ, together with the calibrated tax advantage of debt, determines the steady state level of debt. For our estimation, the implied debt-to-GDP ratio is about 45%.

The estimated operating cost $\overline{f}$ is about 0.89% of the zero-risk steady state of GDP, denoted further by Y_{GDP}.[17] The estimated ambiguity about operating cost at the zero risk steady state is $a_f = 0.022\overline{f}$. Given the persistence in the operating cost process, this implies a worst-case steady state value of f^* of 2.8% of Y_{GDP}. Operating cost matters because it affects payout both directly and indirectly through the endogeneity of the payout choice. As a ratio of Y_{GDP}, payout in the zero risk steady state is 5.39% and investors behave *as if* it converges to 1.22% in the worst-case steady state.

Ambiguity due to intangible information differs across regimes for both marginal product of capital and, more importantly, for the operating cost. Indeed, for the operating cost, we obtain $\eta_{f,L} = 0.0142$ and $\eta_{f,H} = 0.339$, while for the marginal product of capital the values are $\eta_{z,L} = 0.929$ and $\eta_{z,H} = 0.979$. From (7), we can interpret the range of means entertained by investors as the current volatility multiplied by the η relevant for the current regime. In particular, the worst case mean induced by the regimes is never more than one standard deviation below the mean observed by the econometrician. The degree of ambiguity induced by the regimes is thus well below the upper bound of two standard deviations away argued as reasonable in Ilut and Schneider (2014).

[17] Appendix 5.2.1 discusses in detail how to map the estimated parameters f_y, ς_f and $\eta_{f,r}$ into values for $\overline{f}, f^*, \eta_{f,L}$ and $\eta_{f,H}$.

Intangible ambiguity matters only occasionally when the regime s_t^η selects the high value $\eta_{f,H}$. Indeed, in the regime with low ambiguity about cost – which has an estimated ergodic probability of 88% – there is a negligible effect of ambiguity on decision rules, because η_f is very close to zero. This is true regardless of whether the regime coincides with high or low realized volatility. However, intangible ambiguity about cost emerges as a relevant force when ambiguity due to intangible information is high.

The role of measurement error

Figure 2 shows our three observables on which we have measurement error together with their smoothed model implied counterparts. The model exactly matches investment and the equity-gdp-ratio, which are plotted in Figure 1. For the three series where we include measurement error, the latter is small especially for leverage and the dividend-gdp ratio. The model generates movements in those variables both at low frequency and business cycle frequencies. It misses some of the low frequency movements in the real interest rate in the 1970s and 80s. The behavior of the real interest rate over these years was probably affected by events like oil shocks and changes in the monetary policy regime that we do not speak to directly. A key takeaway for our purposes is that the model implied real interest rate is not very volatile, as in the data. The model can therefore explain the excess volatility of equity prices without also making bond prices excessively volatile.

Regime shifts

There are four sources of exogenous variation: the two shocks Z_t and f_t as well as the regimes for volatility and intangible ambiguity. The top and bottom panels of Figure 3 display, for each sample date, the smoothed probabilities at the posterior mode of the high volatility and high ambiguity regimes. The high volatility regime was most likely in place from the mid 1970s to the beginning of the '90s. It is worth pointing out that a similar pattern for breaks in volatility has been obtained by papers that studied the source of the so called "Great Moderation", the period of remarkable macroeconomic stability that preceded the recent financial crisis.[18] Table 1 shows that the volatility regimes differ mostly in the volatility of operating cost.

Identification of the volatility regimes is driven by the two channels through which volatility operates in the model. Consider first the direct effect of volatility on the size of innovations. The leverage ratio is quite volatile from the mid-1970s to beginning of the '90s. This increase in volatility calls for an increase in the volatility of shocks that

[18]See for example Justiniano and Primiceri (2008), Fernandez-Villaverde et al. (2010) and Bianchi (2013). These papers generally find that the return to low volatility occurred a bit earlier, in the mid-80s. However, in our case identification comes from both changes in volatility and in levels, as it will become clear later on.

drive fluctuations in financial variables, such as the operating cost. In contrast, investment growth displays a more complex pattern, with violent contractions from the early '70s to the mid-80s, but even larger ones in the last two recessions. The model captures a significant fraction of these downturns with the discrete shocks to intangible ambiguity. As a result, breaks in the volatility of the Gaussian innovations to productivity are not necessary to explain investment.

The second channel is that volatility has first-order effects on decisions. Here there are two main differential effects of changes in volatility about Z_t against those produced by f_t that explain identification. On the one hand, a significantly larger volatility in Z_t, lasting for more than a decade, would have produced a counterfactual deep and prolonged recession together with very small asset price changes. In contrast, an increase in the volatility of f_t does not imply large changes in real activity but is able to generate a large drop in equity value, a pattern we also see in the data. Both channels thus point toward a substantial break in the volatility of the operating cost shock, while keeping fairly constant the volatility of the marginal product of capital innovations.

The high intangible ambiguity regime is characterized by lower confidence about both the shock to the marginal product of capital Z_t and the operating cost f_t, with a stronger effect for the latter. In terms of time-variation, we find pronounced decreases in confidence around recessions. In particular, there is strong evidence that the drop in prices associated with the last recession can be in part explained by a loss of confidence and that the fluctuations following that drop are due to switches between the high and low confidence regimes. However, our estimation is not conclusive about the regime in place at the very end of the sample. We interpret this as an effect of the lack of sufficient post- crisis history to separate the end-of-sample movements in stock prices into uncertainty premia or measured cash-flow changes.

The volatility regimes are substantially more persistent. This is reflected in the estimates of the diagonal elements of the transition matrix H^σ, substantially larger than the corresponding elements of H^η. However, it is worth pointing out that identification of these parameters does not only come from the frequency of regime changes, but also from the impact that they have on agents expectations. In fact, the estimated high persistence of the volatility regimes also reflect the fact that changes in volatility seem to have a very large impact on asset prices, as it will be illustrated later on: The larger the persistence, the larger the revision of agents' expectations in response to a regime change.

Real uncertainty and the business cycle

Movements in business cycle quantities are mostly accounted for by Z_t, the joint shock to the marginal product of capital and uncertainty. Figure 4 shows the contributions of

different sources of variation to year-on-year investment growth. Each panel focuses on a different source of exogenous variation: the top panel looks at Z_t, the middle panel at the operating cost f_t, and the bottom panel at the regime shifts. For each source of variation, its panel shows the data series as a red dash-dotted line. The solid blue line is what the model would predict if all variation came from the source considered in the panel. The bulk of the variation in investment is clearly due to movements in Z_t, although shifts in regimes also play a role.

Intuitively, movements in Z_t have two effects. On the one hand, they move the marginal product of capital. It is natural that a decrease in Z_t lowers output and investment, as it would in a standard real business cycle model. On the other hand, the change in the *current* marginal product of capital comoves negatively with ambiguity about future capital. This further lowers the return on investment. In addition, it induces ambiguity about future consumption and lowers the real interest rate.

Figure 5 shows the contribution of Z_t to the other observables. In addition to its cyclical effect on investment, it also plays an important role for dividends and the real interest rate. In particular it helps account for sharp drops in the real interest rate in both recent recessions. This effect would not occur if Z_t were purely a TFP shock: mean reversion in TFP would then tend to raise interest rates in recessions. While the real uncertainty shock matters for real quantities, its effect on the stock price is relatively small.

Uncertainty about financial conditions

Figure 6 displays the impulse response to a boost in confidence due to intangible information. This type of shift affects mostly confidence about financial conditions; the estimation suggests it took place for example in the mid 1980s and late 1990s. The figure presents the impulse response by stripping out the contemporaneous negative correlation between a drop in ambiguity about future capital and a higher current marginal product of capital. This helps us isolate the pure effects of changes in uncertainty and thus also link easier the responses based on this quantitative model to the intuition presented in subsection 3.1, where such a negative correlation was absent. Furthermore, the figure plots not only our five observables, but adds also the ratio of debt to GDP so as to make the effects on capital structure more transparent. In each panel, the model is initially in the conditional steady state for the high ambiguity regime. It then experiences a regime shift to the low ambiguity regime in period 20.

A boost in confidence leads to a joint increase in payout and debt (see the middle panels in the top and bottom rows). At the same time, the stock market rises. The price effect is strong enough that leverage of the corporate sector falls even as debt expands. An initial drop in investment – accompanied by a short term upward spike in the interest rate – quickly

turns into an investment boom.

The intuition for the comovement of financial quantities follows from the firm's optimal policies discussed in subsection 3.1. Shareholders would like to issue debt to exploit the tax advantage, but they worry that an increase in operating costs might make internal funds scarce. When they become more confident that funds will be cheap, they effectively substitute away from equity financing by issuing debt and paying dividends to themselves.

The increase in the equity-gdp ratio is stronger than that of the dividend-gdp ratio (see the first two panels in the top row). In other words, the boost in confidence increases the price dividend ratio. As discussed in subsection 3.2, this is due in part because a decline in ambiguity reduces the uncertainty premium on stocks. The stock market boom is not due to a decline in interest rates – in fact the real rate rises as confidence goes up.

Figures 7 displays the impulse response to an increase in volatility. The effect on financial quantities and the stock price are essentially the opposite as in Figure 6. This is to be expected since the increase in volatility affects decision rules by increasing ambiguity. Another similarity is that the effects are quite drawn out over time. This raises the question of what a sequence of regime changes contributes to our interpretation of the data.

To address this question, Figure 8 starts the economy as it was at the beginning of the sample and shocks it with a sequence of regime changes consistent with our posterior mode estimates. Specifically, we make draws for the regime sequence based on the regime probabilities at the posterior mode reported in Figure 3. Therefore, the resulting series show how the economy would have behaved if only the Markov-switching regime changes had occurred, while all the Gaussian shocks had been absent. The panels report the contribution of the regimes to financial quantities, investment and prices. Of particular interest is the fact that breaks in the volatilities of the shocks can account for the low frequency movements of the financial variables, while breaks in ambiguity play an important role at higher frequencies.

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5 Appendix

5.1 Solution method for a model with ambiguity and MS volatility

Here we describe our approach to solving the model with regime switching ambiguity and volatility. The steps of the solution are the following:

1. Describe the law of motion for the shocks

   (a) The perceived law of motion for the continuous shocks:

   $$\widehat{z}_{t+1} = \rho_z \widehat{z}_t - \varkappa \eta_z(s^\eta_{t+1})\sigma_z(s^\sigma_{t+1}) + \mu^*_{t,z} + \sigma_z(s^\sigma_t)\varepsilon^z_{t+1}$$
   $$\widehat{f}_{t+1} = \rho_f \widehat{f}_t + \mu^*_{t,f} + \sigma_f(s^\sigma_t)\varepsilon^f_{t+1}$$

   where each element i in the vector μ_t belongs to a set

   $$\mu_{t,f} \in [-a_{t,f}, a_{t,f}]$$
   $$\mu_{t,z} \in [-a_{t,z}, a_{t,z}]$$

   (b) Volatility follows a two-state Markov chain s^σ_t with transition matrix H^σ.

   (c) There is an independent two-state Markov chain s^η_t that governs intangible ambiguity for operating cost. Thus the ambiguity process $a_{t,f}$ follows:

   $$a_{t,f} = \eta_f(s^\eta_t)\sigma_f(s^\sigma_t)$$

   For the shock to the marginal product of capital, ambiguity follows the process

   $$a_{t,z} = \eta_z(s^\eta_t)\sigma_z(s^\sigma_t) + \widetilde{a}^c_{t,z}$$
   $$\widetilde{a}^c_{t,z} = \rho_a \widetilde{a}^c_{t-1,z} - \sigma_a \sigma_z(s^\sigma_{t-1})\varepsilon^z_t$$

2. Guess and verify the worst-case scenario. As discussed in detail in Ilut and Schneider (2014), the solution to the equilibrium dynamics of the model can be found through a guess-and-verify approach. To solve for the worst-case belief that minimizes expected continuation utility over the i sets in (8), we propose the following procedure:

   (a) guess the worst case belief μ^0

   (b) solve the model assuming that agents have expected utility and beliefs μ^0.

   (c) compute the agent's value function V

(d) verify that the guess μ^0 indeed achieves the minima.

The following steps detail the point 2.b) above. Here we use an observational equivalence result saying that our economy can be solved *as if* the agent maximizes expected utility under the belief μ^0. Given this equivalence, we can use standard perturbation techniques that are a good approximation of the nonlinear decision rules under expected utility. In particular, we will use linearization. When we refer to the guess below, we use $\mu^*_{z,t} = -a_{z,t}$ and $\mu^*_{f,t} = a_{f,t}$.

3. Compute worst-case steady states

   (a) Compute the ergodic values $\overline{\eta}_i$ and $\overline{\sigma}_i$.

   (b) Based on the guess above compute the worst-case steady states for the shocks, denoted by $\tau^* = (z^*,\, f^*)$.

   (c) Compute the worst-case steady state Y^* of the endogenous variables. For this, use the FOCs of the economy based on their deterministic version in which the one step ahead expectations are computed under the guessed worst-case belief.

4. Dynamics:

   (a) Linearize around Y^* and τ^* by finding the coefficient matrices from linearizing the FOCs. Here use that

   $$a_{t,i} - \overline{a_i} = \eta_{t,i}\sigma_{t,i} - \overline{\eta}_i\overline{\sigma}_i$$

   and define a composite Markov-chain for the product $\eta_{t,i}\sigma_{t,i}$ as in equation (19). The linearized FOCs can be written in the canonical form used for solving rational expectations models:

   $$\Gamma_0\widehat{S}^*_t = \Gamma_1\widehat{S}^*_{t-1} + \Psi\left[\varepsilon'_t, \upsilon^{\eta\sigma\prime}_t\right]' + \Pi\omega_t$$

   where $\widehat{S}^*_t$ is the DSGE state vector that includes the dummy variables controlling the regime in place. This vector represents deviations around the worst-case steady state S^*, which contains Y^* and τ^*.

   (b) Given that the shock v_t is defined such that $E_{t-1}\left[\upsilon^{\eta\sigma}_t\right] = 0$, a standard solution method to solve rational expectations general equilibrium models can be employed. The solution can then be rewritten as a VAR with stochastic volatility

   $$\widehat{S}^*_t = T^*\widehat{S}^*_{t-1} + R^*\sigma(s^\sigma_{t-1})\left[\varepsilon'_t, \upsilon^{\eta\sigma\prime}_t\right]' \tag{22}$$

(c) Verify that the guess μ^0 indeed achieves the minima of the time t expected continuation utility over the sets in (8).

5. Equilibrium dynamics under the true data generating process (DGP). The above equilibrium was derived under the worst-case beliefs. We need to characterize the economy under the econometrician's law of motion. There are two objects of interest: the zero-risk steady state of our economy and the dynamics around that steady state.

   (a) The zero-risk steady state, denoted by $\overline{S}$. This is characterized by shocks, including the volatility regimes, being set to their ergodic values under the true DGP. $\overline{S}$ can then be found by looking directly at the linearized solution, adding $R_z^*\overline{\eta}_z\overline{\sigma}_z$ and subtracting $R_f^*\overline{\eta}_f\overline{\sigma}_f$:

$$\overline{S} - S^* = T^*\left(\overline{S} - S^*\right) + R_z^*\overline{\eta}_z\overline{\sigma}_z - R_f^*\overline{\eta}_f\overline{\sigma}_f \tag{23}$$

   where R_z^* and R_f^* are the equilibrium response to positive innovations to $\widehat{z}_t$ and $\widehat{f}_t$ respectively.

   (b) Dynamics. The law of motion in (22) needs to take into account that expectations are under the worst-case beliefs which differ from the true DGP. Then, we define $\widehat{S}_t \equiv S_t - \overline{S}$ and use (22) together with (23) to obtain:

$$\begin{aligned}\widehat{S}_t = {} & T^*\widehat{S}_{t-1} + R^*\sigma(s_{t-1}^\sigma)\left[\varepsilon_t', \upsilon_t^{\eta\sigma\prime}\right]' + R_z^*(\eta_z(s_{t-1}^\eta)\sigma_z(s_{t-1}^\sigma) - \overline{\eta}_z\overline{\sigma}_z + \widetilde{a}_{t-1,z}^c) - \\ & - R_f^*\left(\eta_f(s_{t-1}^\eta)\sigma_f(s_{t-1}^\sigma) - \overline{\eta}_f\overline{\sigma}_f\right)\end{aligned} \tag{24}$$

   By defining the matrix T accordingly, we represent the law of motion in (24) as

$$\widehat{S}_t = T\widehat{S}_{t-1} + R\sigma(s_{t-1}^\sigma)\left[\varepsilon_t', \upsilon_t^{\eta\sigma\prime}\right]' \tag{25}$$

   where we have also used the notation $R = R^*$. Intuitively, under the econometrician's DGP the economy responds to the exogenous state variables controlling uncertainty differently since the implied worst-case expectations are not materialized in the current shock processes.

   Finally, we can partition out the state vector $\widehat{S}_t$ in a way that the Markov-switching regimes show up as time-varying constants in the law of motion described by (25). In particular, this change allows us to rewrite (25) as

$$\widetilde{S}_t = C(s_t^\sigma, s_t^\eta) + T\widetilde{S}_{t-1} + R\sigma(s_{t-1}^\sigma)\varepsilon_t \tag{26}$$

where $\widetilde{S}_t$ contains the same state variables of $\widehat{S}_t$, except $e^{\eta\sigma}_{1,1,t}, e^{\eta\sigma}_{1,2,t}, e^{\eta\sigma}_{2,1,t}$, and $e^{\eta\sigma}_{2,2,t}$ which have been replaced with the MS constant $C(s^{\sigma}_t, s^{\eta}_t)$. To avoid further notation, the matrices T and R in (26) are the appropriate partitions of the corresponding objects in (25) so that they only capture the effect of $\widetilde{S}_t$. Thus, the DSGE model is represented as a Markov-switching VAR (MS-VAR), where the changes in the constant arise from the first order effects of the composite regimes of stochastic volatility and ambiguity.

5.2 Equilibrium conditions for the estimated model

Here we describe the equations that characterize the equilibrium of the estimated model in Section 4. To solve the model, we first scale the variables in order to induce stationarity. The variables are scaled as follows:

$$c_t = \frac{C_t}{\xi^t}, y_t = \frac{Y_t}{\xi^t}, k_t = \frac{K_t}{\xi^t}, i_t = \frac{I_t}{\xi^t}$$

Financial variables:

$$p_t = \frac{P_t}{\xi^t}, d_t = \frac{D_t}{\xi^t}, b^i_t = \frac{B^i_t}{\xi^t}; i = f, h$$

The borrowing costs:

$$\frac{\kappa\left(B^f_{t-1}\right)}{\xi^t} = \frac{\Psi}{2}\frac{1}{\xi^2}\left(b^f_{t-1}\right)^2; \ \frac{\phi\left(D_t, D_{t-1}\right)}{\xi^t} = f_t + \frac{\phi''\xi^2}{2}\left(\frac{d_t}{d_{t-1}} - 1\right)^2$$

We now present the nonlinear equilibrium conditions characterizing the model, in scaled form. The expectation operator in these equations, denoted by E^*_t, is the one-step ahead conditional expectation under the worst case belief μ^0. According to our model, the worst case is that future z_{t+1} is low, and that the financing cost f_{t+1} is high.

The firm problem is

$$\max E^*_0 \sum M^f_{0.t} D_t$$

subject to the budget constraint

$$\begin{aligned} d_t = (1-\tau_k)\left[\alpha y_t - \frac{b^f_{t-1}}{\xi}\left(1 - Q^b_{t-1}\right) - \frac{\phi''\xi^2}{2}\left(\frac{d_t}{d_{t-1}} - 1\right)^2 - i_t\right] - \\ - f_t - \frac{\Psi}{2}\frac{1}{\xi^2}\left(b^f_{t-1}\right)^2 + \delta\tau_k q^k_{t-1}\frac{k_{t-1}}{\xi} - \frac{b^f_{t-1}}{\xi}Q^b_{t-1} + b^f_t Q^b_t \end{aligned} \tag{27}$$

and the capital accumulation equation

$$k_t = \frac{(1-\delta)k_{t-1}}{\xi} + \left[1 - \left(\frac{S''}{2}\frac{i_t\xi}{i_{t-1}} - \xi\right)^2\right] i_t \tag{28}$$

Let the LM on the budget constraint be $\lambda_t M_{0.t}^f \epsilon_t$ and on the capital accumulation be $\mu_t M_{0.t}^f \epsilon_t$. Then the scaled pricing kernel is

$$m_{t+1}^f \equiv M_{t+1}\frac{\xi^{t+1}}{\xi^t} = \beta\frac{c_t}{c_{t+1}}\frac{1-\tau_l}{1-\tau_l\beta E_t^*\left[c_t/\left(c_{t+1}\xi\right)\right]}. \tag{29}$$

The FOCs associated with the firm problem are then:

1. Dividends:

$$1 = \lambda_t\left[1 + (1-\tau_k)\phi''\xi^2\frac{1}{d_{t-1}}\left(\frac{d_t}{d_{t-1}} - 1\right)\right] - E_t m_{t+1}^f\lambda_{t+1}\left[(1-\tau_k)\phi''\xi^2\frac{d_{t+1}}{d_t^2}\left(\frac{d_{t+1}}{d_t} - 1\right)\right] \tag{30}$$

2. Bonds:

$$Q_t^b\lambda_t = E_t^* m_{t+1}^f\lambda_{t+1}\frac{1}{\xi}\left[1 - \tau_k\left(1 - Q_t^b\right) + \frac{\Psi}{\xi}b_t^f\right] \tag{31}$$

3. Investment:

$$\begin{aligned} 1 = {} & \frac{q_t^k}{(1-\tau_k)}\left[1 - \frac{S''}{2}\xi^2\left(\frac{i_t}{i_{t-1}} - 1\right)^2 - S''\xi^2\left(\frac{i_t}{i_{t-1}} - 1\right)\frac{1}{i_{t-1}}\right] + \\ & + E_t^* m_{t+1}^f\frac{\lambda_{t+1}}{\lambda_t}q_{t+1}^k S''\xi^2\frac{i_{t+1}^2}{i_t^2}\left(\frac{i_{t+1}}{i_t} - 1\right) \end{aligned} \tag{32}$$

where

$$q_t^k \equiv \frac{\mu_t}{\lambda_t}$$

4. Capital:

$$1 = E_t^* m_{t+1}^f\frac{\lambda_{t+1}}{\xi\lambda_t}R_{t+1}^K \tag{33}$$

$$R_{t+1}^k \equiv \frac{(1-\tau_k)\alpha\left(\frac{k_t}{\xi}\right)^{\alpha-1}L^{1-\alpha} + (1-\delta)q_{t+1}^k}{q_t^k} + \delta\tau_k$$

The household problem is as follows:

$$\begin{aligned} &\max E_0^* \sum \beta^t \log c_t \\ (1+\tau_c)c_t + p_t\theta_t = (1-\tau_l)&\left[(1-\alpha)y_t + \pi + d_t\theta_{t-1} + \frac{b_{t-1}^h}{\xi}\left(1-Q_{t-1}^b\right)\right] + \\ &+ p_t\theta_{t-1} - \tau_l(p_t - \frac{1}{\xi}p_{t-1})\theta_{t-1} + \frac{b_{t-1}^h}{\xi}Q_{t-1}^b - b_t^h Q_t^b + t_r \end{aligned} \tag{34}$$

Thus, the FOCs associated to the household problem are:

1. Bond demand:

$$Q_t^b = \beta E_t^* \frac{c_t}{\xi c_{t+1}}\left[1-\tau_l\left(1-Q_t^b\right)\right] \tag{35}$$

2. Equity holding:

$$p_t = \beta E_t^* \frac{c_t}{c_{t+1}}\left[(1-\tau_l)\left(p_{t+1}+d_{t+1}\right) + \frac{\tau_l}{\xi}p_t\right] \tag{36}$$

The market clearing conditions characterizing this economy are:

$$b_t^h = b_t^f \tag{37}$$

$$c_t + i_t + \frac{\phi''\xi^2}{2}\left(\frac{d_t}{d_{t-1}}-1\right)^2 + f_t + \frac{\Psi}{2}\frac{1}{\xi^2}\left(b_{t-1}^f\right)^2 = y_t + \pi \tag{38}$$

$$\theta_t = 1$$

corresponding to the market for bonds, goods and equity shares, respectively.

Thus, we have the following 11 unknowns:

$$k_t, i_t, b_t^f, b_t^h, Q_t^b, p_t, c_t, d_t, q_t^k, \lambda_t, m_t^f$$

The equations (28), (29), (30), (31), (32), (33), (35), (36), (37),(38) give us 10 equations. By Walras' law, we can then use one out of the two budget constraints in (27) and (34) (using $\theta_t = 1$). This gives us a total of 11 equations.

5.2.1 Parametrization

Rescaling and calibrating parameters

We estimate a subset of the parameters, with values reported in Table 1. The other parameters are fixed, as reported in Table 2, to match some key ratios from the NIPA accounts.

For the steady state calculation of the model it is helpful to rescale some parameters. Specifically, denote by y^*_{GDP} the worst-case steady state measured GDP, i.e. total goods $y + \pi$ minus financing costs. Then, define the following ratios:

$$\pi_y = \frac{\pi}{y^*_{GDP}}, t_y = \frac{t_r}{y^*_{GDP}}; \Psi_y = \frac{\Psi}{y^*_{GDP}} \tag{39}$$

$$f_y = \frac{\overline{f}}{y^*_{GDP}}; 1 + \varsigma_f = \frac{f^*}{\overline{f}}; \eta_{f,r} = \frac{\eta_{f,L}}{\overline{\eta}_f} \tag{40}$$

Let us first discuss the normalizations in (39). Total measured GDP in our model, denoted here by y_{GDP}, corresponds to the non-financial corporate sector (NFB) output plus goods produced by the other productive sectors- financial, non-corporate and household. We associate the firm in our model with the NFB sector and thus π_y is set to match goods produced by other productive sectors divided by y_{GDP}. The ratio t_y equals government transfers (including social security and medicare) plus after-tax government wages divided by y_{GDP}. Finally the parameter Ψ_y is estimated and after computing the level of the economy it then implies a value of Ψ.

The tax parameters are computed as follows: τ_l equals total personal taxes and social security contributions divided by total income, where the latter is defined as total wages plus dividends. τ_k equals NFB taxes divided by NFB profits and τ_c equals NFB sales taxes divided by NFB output. The government spending ratio g equals government net purchases from other sectors plus net exports divided by y_{GDP}.

The parameters in (40) are estimated and control the size of the operating cost. First, f_y determines $\overline{f}$, the value of the steady state cost under the true DGP. The parameter ς_f controls by how much higher is f^*, the worst-case steady state operating cost, compared to $\overline{f}$. This normalization is helpful because it allows us to use meaningful priors on this ratio in terms of the type and size of prior mean. Since

$$f^* = \overline{f} + \frac{\overline{\eta}_f \overline{\sigma}_f}{(1 - \rho_f)}$$

then based on ς_f we compute:

$$\overline{\eta}_f = \overline{\sigma}_f^{-1} (1 - \rho_f) \varsigma_f \overline{f}$$

where $\overline{f}$ is obtained after solving for the level of the economy. Finally, the parameter $\eta_{f,r}$, which has a beta prior, determines the value of η_f in the Low ambiguity regime compared to its ergodic value of $\overline{\eta}_f$:

$$\eta_{f,L} = \eta_{f,r} \overline{\eta}_f$$

Based on the estimated transition matrix for the ambiguity regimes, the value of η_f in the High ambiguity regime is then easily computed as:

$$\eta_{f,H} = \frac{\overline{\eta}_f - p_{\eta,L}\eta_{f,L}}{1 - p_{\eta,L}}$$

where $p_{\eta,L}$ is the ergodic probability of the Low ambiguity regime computed using the estimated transition matrix H^{η}.

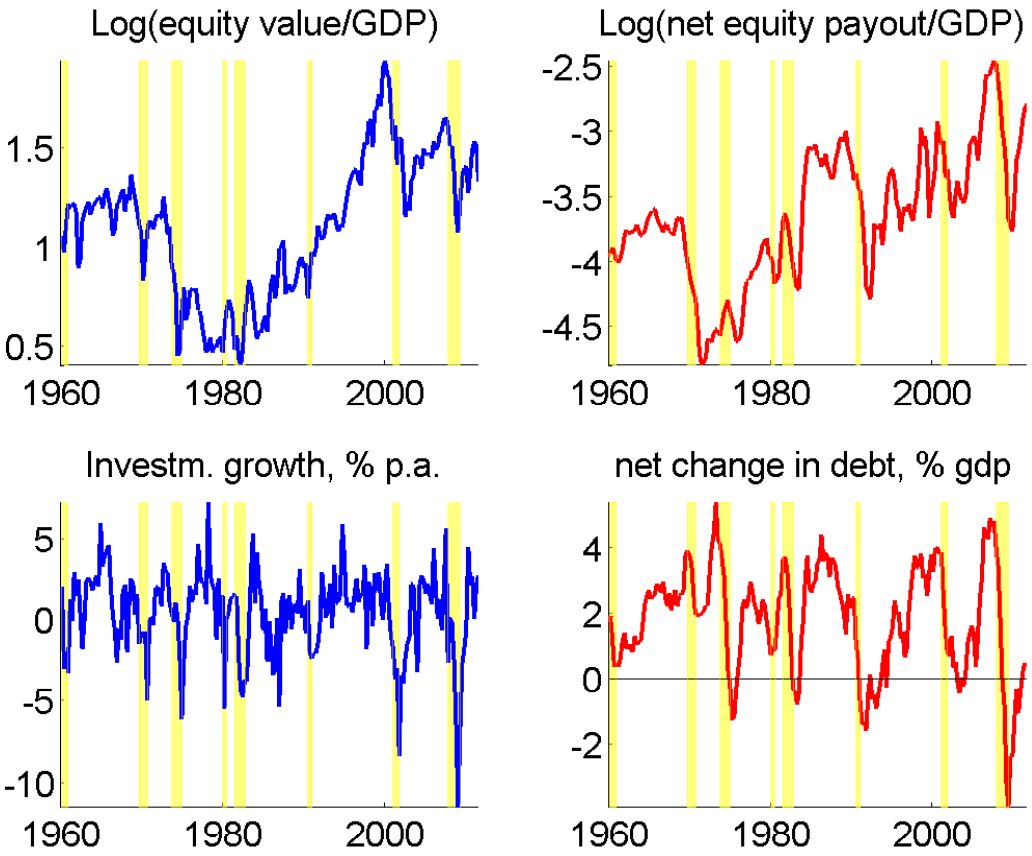

Figure 1: This figure shows the net change in non-financial corporate debt, as a percent of GDP, together with three of the observables used for our estimation: log(equity value/GDP), investment growth and log(net equity payout/GDP). NBER recessions are marked as yellow shades. The other two series used for the estimation, namely firm debt to equity value ratio and the short term real interest rate, are plotted in Figure 2. See Section 4.1 for details on data construction.

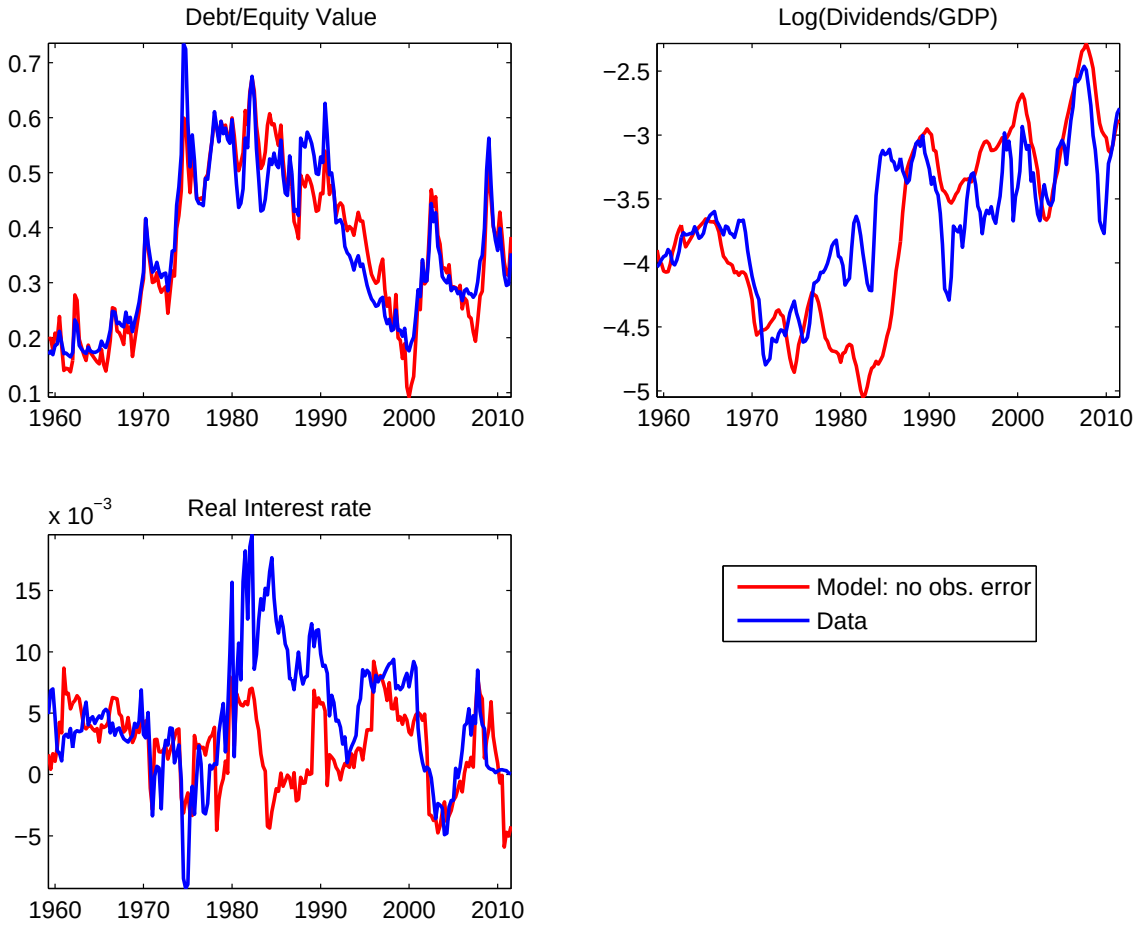

Figure 2: Variables used for estimation that allow for measurement error. Red line is the smoothed model-implied path substracting the estimated observation error. The blue line represents the data.

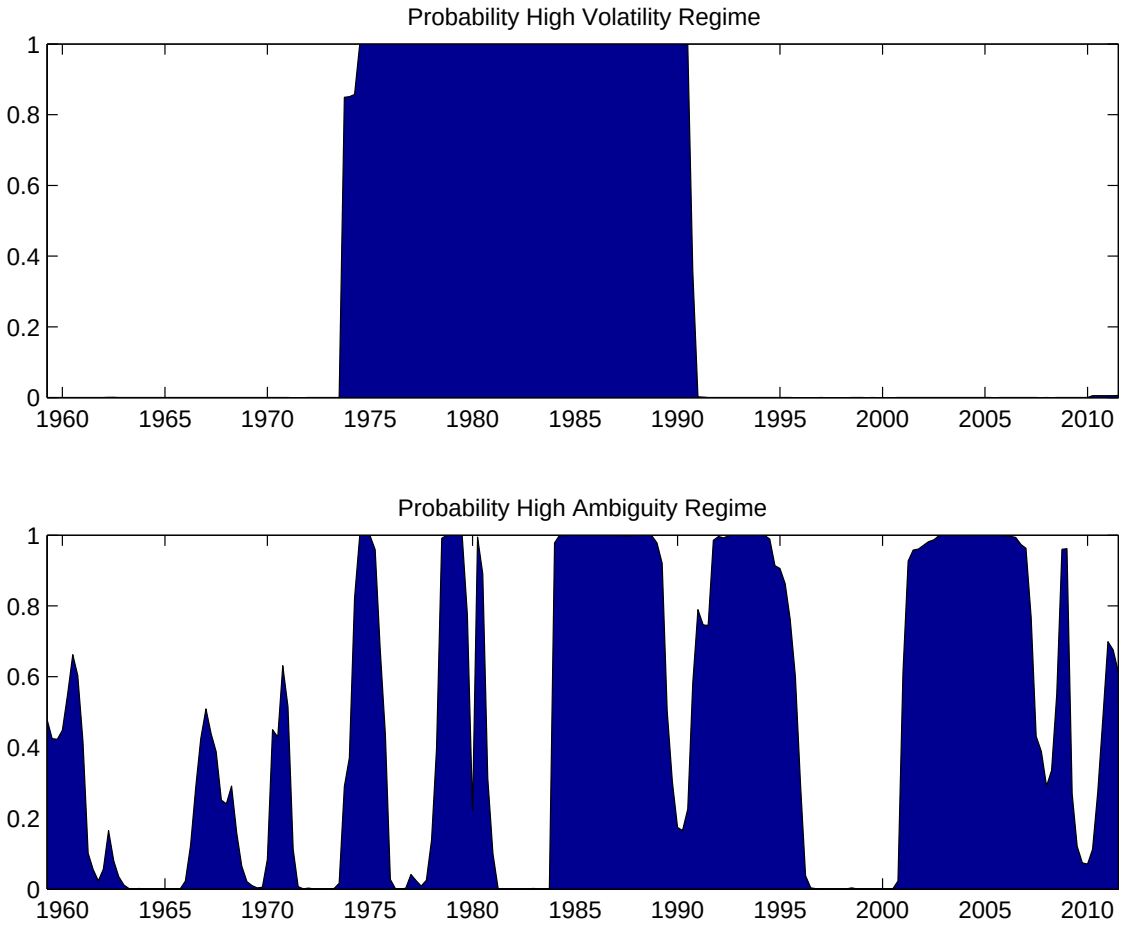

Figure 3: Smoothed regime probabilities. Top panel refers to volatility regime number 1, which we refer to as the High volatility regime. Bottom panel refers to intangible ambiguity regime number 2, which we refer to as the High ambiguity regime. Estimates for the values across regimes are shown in Table 1. Further discussion on normalizations is in Appendix 5.2.1

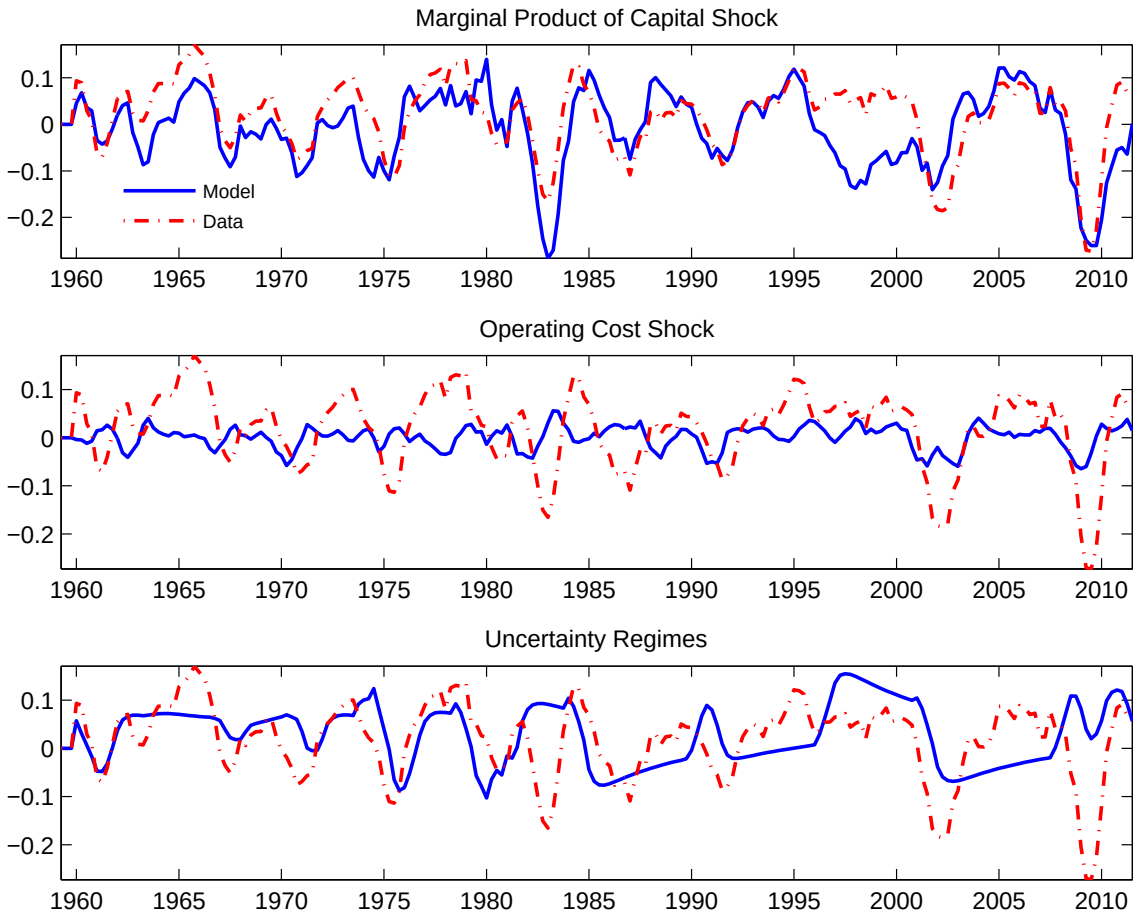

Figure 4: Contribution of different sources of variation on the year-on-year investment growth. Blue line is the counterfactual model-implied evolution based only on that source.

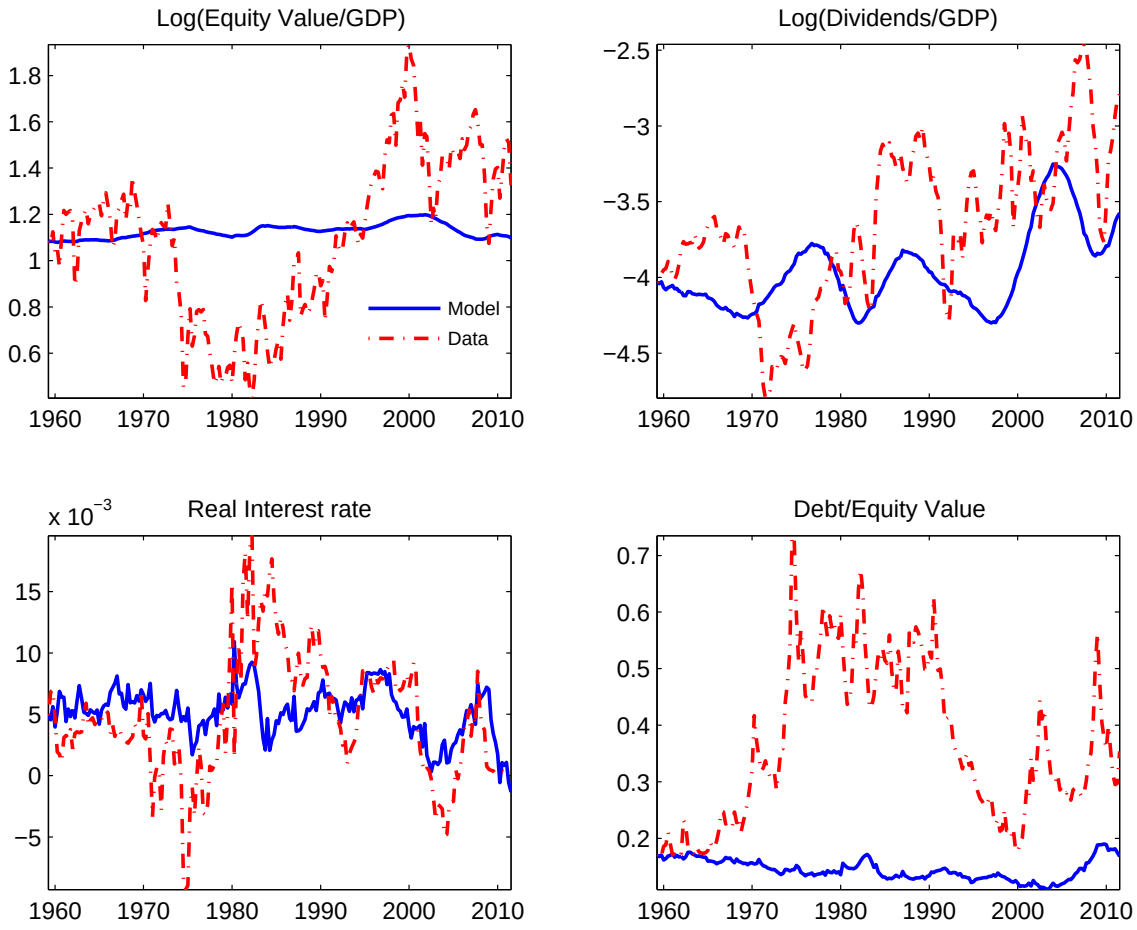

Figure 5: Contribution of the marginal product of capital shock to different financial variables. The blue line is the model-implied counterfactual evolution based only on that shock.

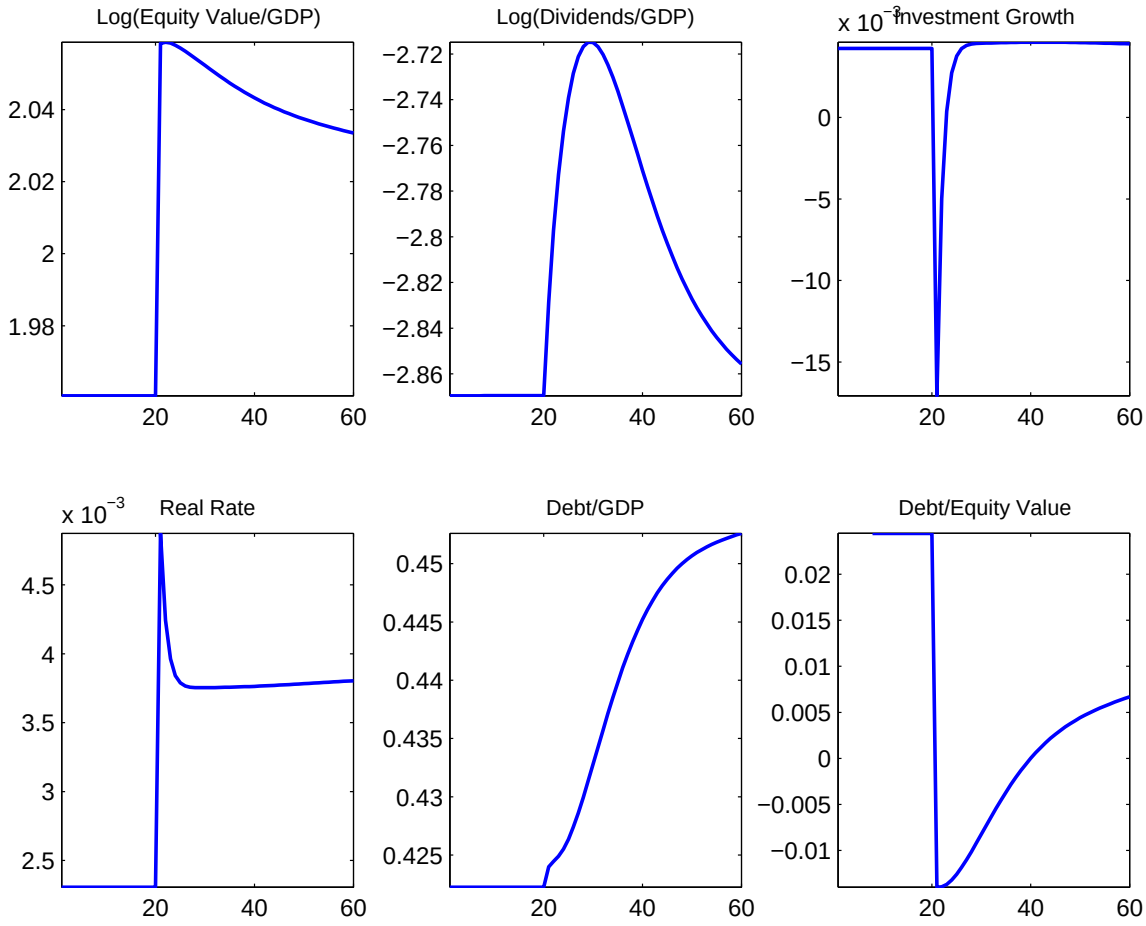

Figure 6: Impulse response for a switch from a high ambiguity-low volatility regime to a low ambiguity-low volatility regime. The switch occurs in period 20.

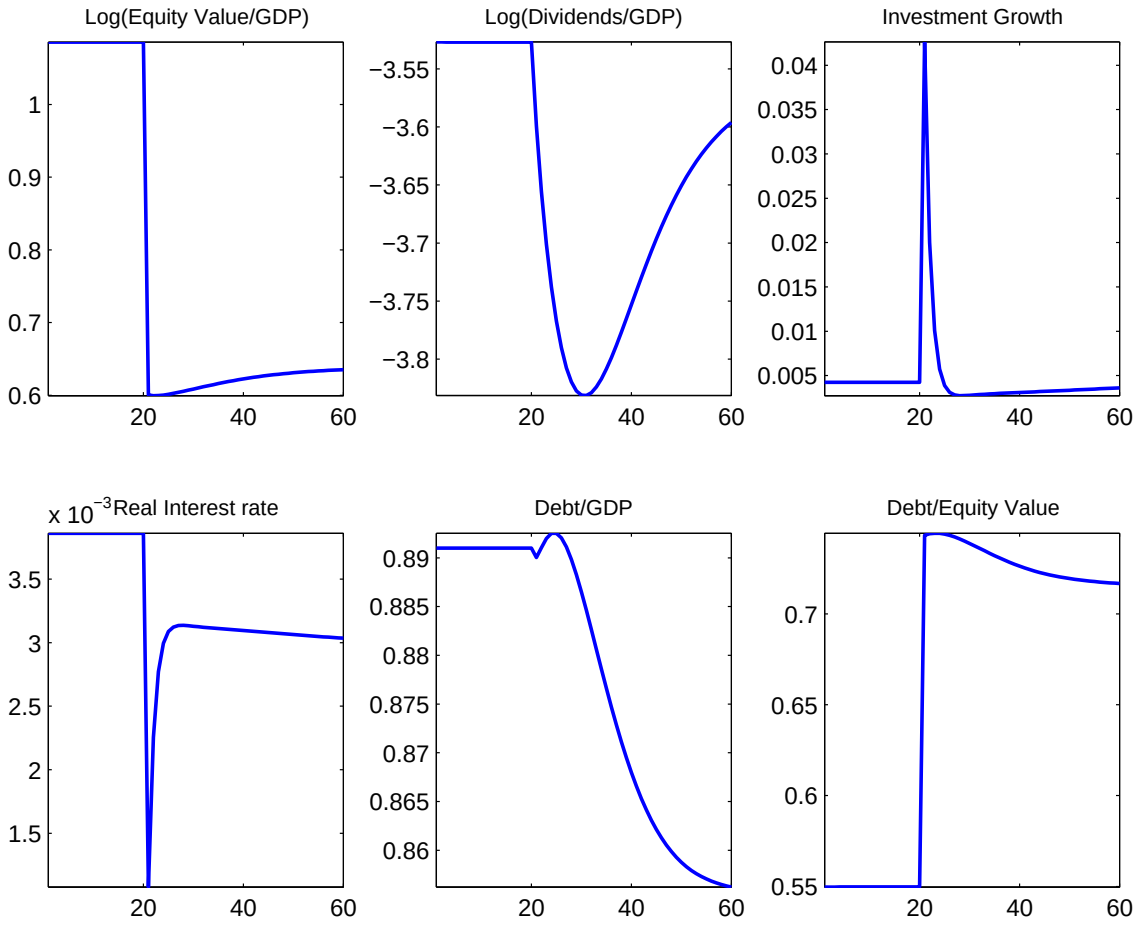

Figure 7: Impulse response for a switch from a high ambiguity-low volatility regime to a high ambiguity-high volatility regime. The switch occurs in period 20.

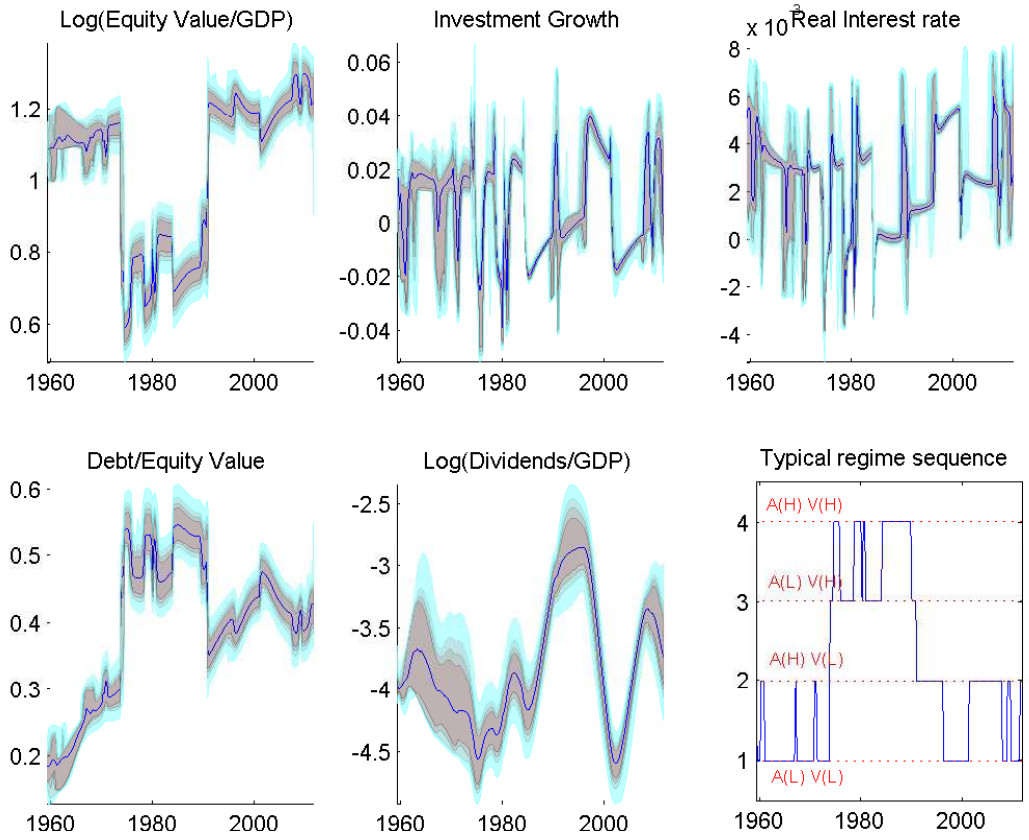

Figure 8: Evolution induced by draws for the regime sequence based on the smooth probabilities of Figure 3.

Table 1: Modes, means, 90% error bands, and priors of the DSGE parameters

Parameter	$Mode$	$Mean$	5%	95%	$Type$	$Para$ 1	$Para$ 2
$\eta_{z,L}$	0.9290	0.9107	0.8654	0.9431	B	0.5000	0.2500
$\eta_{z,H}$	0.9796	0.9581	0.9103	0.9921	B	0.5000	0.2500
$\eta_{f,r}$	0.2583	0.2898	0.0948	0.5150	B	0.5000	0.2000
ς_f	2.1584	2.1747	1.8542	2.5777	G	1.0000	0.5000
ϕ''	0.0020	0.0021	0.0017	0.0025	G	0.1000	0.0800
Ψ_y	0.0022	0.0022	0.0020	0.0024	G	0.0050	0.0040
$100\,(\xi - 1)$	0.4243	0.4375	0.3802	0.4921	G	0.3000	0.0500
$100 f_y$	1.3261	1.3110	1.1571	1.4783	G	2.0000	1.5000
δ	0.0053	0.0053	0.0046	0.0060	B	0.0250	0.0100
α	0.2335	0.2333	0.2216	0.2454	B	0.3500	0.0500
S''	0.1346	0.1484	0.1167	0.1865	G	2.0000	1.0000
$100\,(\beta^{-1} - 1)$	0.5566	0.5581	0.5192	0.5970	G	0.3000	0.2000
ρ_z	0.9714	0.9722	0.9697	0.9747	B	0.5000	0.1500
ρ_f	0.9995	0.9994	0.9990	0.9998	B	0.5000	0.1500
ρ_a	0.9628	0.9634	0.9600	0.9666	B	0.5000	0.2500
$100\sigma_z\,(1)$	1.2874	1.3242	1.2196	1.4254	U	0.01	10
$100\sigma_f\,(1)$	0.7064	0.6870	0.5581	0.8230	U	0.01	10
$100\sigma_z\,(2)$	1.2857	1.3227	1.2185	1.4234	U	0.01	10
$100\sigma_f\,(2)$	0.4301	0.4004	0.3278	0.4780	U	0.01	10
H_{11}^{σ}	0.9990	0.9985	0.9971	0.9995	D	0.8333	0.1034
H_{22}^{σ}	0.9994	0.9992	0.9985	0.9997	D	0.8333	0.1034
H_{11}^{η}	0.9678	0.9689	0.9570	0.9798	D	0.8333	0.1034
H_{22}^{η}	0.7755	0.7730	0.7292	0.8160	D	0.8333	0.1034
$100\sigma_{oe,D/Y}$	18.3919	18.9841	17.4483	20.4196	IG	5.0500	1.0100
$100\sigma_{oe,Debt/P}$	3.7053	3.7835	3.4775	4.1194	IG	1.3943	0.2789
$100\sigma_{oe,RIR}$	0.2676	0.2733	0.2591	0.2892	IG	0.0503	0.0101

B refers to Beta, G to Gamma, IG to Inverse-gamma, U to uniform and D to the Dirichlet distribution. Para 1 and Para 2 denote the mean and standard deviation for all prior distributions except for the uniform distribution. In this last case they denote the two bounds of the distribution.

Table 2: Calibrated parameters

τ_l	τ_k	τ_c	π_y	t_y	g
0.189	0.193	0.09	0.3	0.21	0.05

Table 3: Priors and posteriors for zero-risk steady state

Variable	$Mode$	$Mean$	5%	95%	$Type$	$Mean$	$st.dev.$
$100\Delta I$	0.4234	0.4366	0.3795	0.4909	N	0.47	0.23
D/GDP	0.0539	0.0538	0.0494	0.0589	N	0.06	0.02
SP/GDP	6.5040	6.4572	6.1489	6.7745	N	5	1
B/SP	0.0686	0.0681	0.0617	0.0748	IG	0.2	0.06
$100RIR$	0.3604	0.3568	0.2817	0.4273	N	0.47	0.23

Priors and posteriors for the zero-risk steady state of our observables: investment growth (ΔI), dividend to GDP ratio (D/GDP), equity price to GDP ratio (SP/GDP), firm debt to equity value ratio (B/SP) and the short term real interest rate (RIR). The priors for variables D/GDP and SP/GDP are expressed here in levels.

Table 4: MCMC convergence statistics

Parameter	PSRF	Parameter	PSRF	Parameter	PSRF
$\eta_{z,L}$	1.03	$100\left(\beta^{-1}-1\right)$	1.05	$100\sigma_f\left(2\right)$	1.09
$\eta_{z,H}$	1.04	ρ_z	1.07	H_{11}^{σ}	1.02
$100f_y$	1.13	ρ_f	1.00	H_{22}^{σ}	1.01
ϕ''	1.01	ς_f	1.15	H_{11}^{η}	1.03
Ψ_y	1.04	$\eta_{f,r}$	1.07	H_{22}^{η}	1.01
$100\left(\xi-1\right)$	1.04	ρ_a	1.00	$100\sigma_{oe,D/Y}$	1.13
δ	1.01	$100\sigma_z\left(1\right)$	1.10	$100\sigma_{oe,Debt/P}$	1.00
α	1.02	$100\sigma_f\left(1\right)$	1.02	$100\sigma_{oe,RIR}$	1.01
S''	1.01	$100\sigma_z\left(2\right)$	1.10		

The table reports the Brooks-Gelman-Rubin Potential Scale Reduction Factor (PSRF) for each parameter. Values below 1.2 denote convergence.