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ABSTRACT

This paper analyzes the possible inception of rational inflationary bubbles under the assumption that the empirically relevant environment precludes the existence of rational deflationary bubbles. The analysis shows that if a rational inflationary bubble exists, then it must have started on the date of initial issuance of the fiat money. Moreover, the existence of a rational inflationary bubble would imply that, prior to the initial issuance of the fiat money, agents who anticipated its introduction expected a rational inflationary bubble to occur. The analysis also shows that once a rational inflationary bubble bursts it cannot restart. The analysis, however, does not preclude the existence of a rational inflationary bubble that shrinks periodically, but never bursts. The limitations on the inception and existence of rational inflationary bubbles also apply to rational exchange-rate bubbles.

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Flood and Garber (1980)--henceforth F&G--utilize the rational expectations version of Cagan's inflation model to analyze the theoretical and empirical implications of rational inflationary bubbles. This model implies that the logarithm of the market-clearing price level satisfies a first-order linear expectational difference equation with a stochastic forcing term that consists of the variables shifting the demand and supply of money. F&G define the market-fundamentals component of the price level to be the particular solution to this expectational difference equation that is obtained by setting the solution to the associated homogeneous equation equal to zero. They define other solutions to the homogeneous equation to be the rational-bubbles component of the price level.

Defined in this way, the market-fundamentals component relates the current price level uniquely to the parameters of the money demand and supply functions and, except in extreme cases of the forcing processes, to the current and expected future values of the stochastic forcing variables. The existence of a rational-bubbles component would reflect a self-confirming belief that the price level depends on a variable (or a combination of variables) that is intrinsically irrelevant--that is, not part of market fundamentals--or on truly relevant variables in a way that involves parameters that are not part of market fundamentals.

In the F&G model, the assumption that demand for real money balances depends negatively on the expected rate of inflation implies that the eigenvalue of the expectational difference equation governing price fluctuations is greater than unity. This property of the difference equation has two important consequences. First, it guarantees the existence of an economically meaningful (i.e., forward looking) market-fundamentals solution except in extreme cases of the forcing processes. Second, it implies that rational bubbles have explosive conditional expectations. Specifically, the expected value of a rational-bubbles component of the price level either would increase or would decrease geometrically into the infinite future.

This property of explosive conditional expectations notwithstanding, Obstfeld and Rogoff (1983) show that a rational inflationary bubble can exist in an economy with an inconvertible fiat money unless money is essential to the economy in the sense that no finite amount of extra consumption could compensate agents for reducing their money balances to zero. Kingston (1982) shows that the Cagan money demand schedule used by F&G implies that money is not essential to the economy in this sense.

In contrast, the property of explosive conditional expectations provides the basis for various arguments in the literature for ruling out the existence of rational deflationary bubbles and, more generally, of positive rational bubbles in the value of any asset. One such argument is that the existence of a rational deflationary bubble would violate a transversality condition that must hold if agents have infinite planning horizons--see Brock (1974), Gray (1984), and Obstfeld and Rogoff (1986). Specifically, the existence of a rational deflationary bubble would imply that agents expect to gain utility from reducing their money holdings permanently.

Another argument, developed by Tirole (1982), assumes that, even if asset holders have infinite planning horizons, they would not plan to hold forever an overvalued asset, such as a fiat money whose value reflects a deflationary rational bubbles component. Instead, each asset holder would want to realize the capital gain associated with the deflationary rational bubble at some date in the finite future. Consequently, if the number of potential asset holders is finite, a finite future date would exist beyond which no one would plan to hold the overvalued asset. Under these conditions, a backward unraveling argument precludes the existence of a positive rational bubbles component in the value of any asset and, in particular, in the value of a fiat money.

Tirole (1985) and Weil (1986) develop overlapping generations models in which money is a pure store of value and would be worthless according to their definition of market fundamentals. They show that these economies possess equilibria in which money is valuable and refer to such equilibria as bubbles in the value of money. Because the concepts of market fundamentals and bubbles in the analyses of Tirole and Weil do not coincide with the concepts of the present paper, our results are not directly comparable to theirs. Their results, however, suggest that the impossibility of rational deflationary bubbles under all conceivable conditions cannot be taken for granted. Also, Quah (1985) points out that even if agents have infinite planning horizons, if they ignore low probability events, their optimizing decisions are not necessarily inconsistent with the existence of positive rational bubbles in asset values if these rational bubbles will almost surely burst at a date in the finite future.

Having noted these possibilities, the analysis that follows assumes that in the empirically relevant environment the property of explosive conditional expectations rules out the existence of rational deflationary bubbles. Given this assumption, we can focus on the implications of the impossibility of rational deflationary bubbles for the possible inception of a rational inflationary bubble.

In what follows, Section 1 reviews the basic properties of rational bubbles in the F&G model. Section 2 derives the result that, given the impossibility of rational deflationary bubbles, a rational inflationary bubble can start only on the date of initial issuance of the fiat money. Section 3 derives the further result that rational bubbles cannot burst and simultaneously restart and discusses the possible forms that interesting rational inflationary bubbles could take. Section 4 discusses the relevance of the arguments developed in this paper for the inception of rational bubbles in foreign exchange rates. Section 5 generalizes the analysis for a nonlinear model of the demand for money. Section 6 provides a summary.

1. Properties of Rational Bubbles

F&G analyze the familiar Cagan model of inflation with rational expectations of future inflation replacing Cagan's adaptive expectations. In this model, the current price level satisfies a condition of equality between the real money stock, given by the lhs of equation (1), and the demand for real money balances, given by the rhs of (1):

$$(1) \qquad M_t - P_t = \alpha_t - \beta(E_t P_{t+1} - P_t), \quad \beta > 0,$$

where

- M_t is the logarithm of the nominal money stock at date t,
- P_t is the logarithm of the price level at date t,
- α_t represents all of the variables that influence demand other than expected inflation, and
- β is the semi-elasticity of real money demand with respect to expected inflation.

The conditional expectations operator E_t is based on an information set that contains the current and lagged values of M_t, P_t, and α_t. Equation (1) applies for any date t such that $t \geqslant 0$, where the fiat money was initially issued at date zero.

Rearranging terms in equation (1) leads to the following linear first-order expectational difference equation:

$$(2) \qquad E_t P_{t+1} - (1+\beta^{-1})P_t = \beta^{-1}(\alpha_t - M_t).$$

Because the eigenvalue, $1+\beta^{-1}$, is greater than unity, the forward-looking solution for P_t involves a convergent sum, as long as the sequence $\{E_t(M_{t+i} - \alpha_{t+i})\}_{i=1}^{\infty}$ does not grow at a geometric rate equal to or greater than $1+\beta^{-1}$. The forward-looking solution, denoted by F_t and referred to as the market-fundamentals component of the price level, is

$$(3) \qquad F_t = (1+\beta)^{-1}[(M_t - \alpha_t) + \sum_{i=1}^{\infty} (1+\beta^{-1})^{-i} E_t(M_{t+i} - \alpha_{t+i})].$$

Equation (3) says that F_t is proportionate to a weighted sum of current and expected future realizations of the money supply and the variables that shift money demand.

The general solution to equation (2) for P_t is the sum of the market-fundamentals component, F_t, and the rational-bubbles component, B_t--that is,

$$(4) \qquad P_t = B_t + F_t,$$

where B_t is the solution to the homogeneous expectational difference equation,

$$(5) \qquad E_t B_{t+1} - (1+\beta^{-1}) B_t = 0.$$

A nonzero value of B_t would reflect the existence of a rational bubble at date t--that is, a self-confirming belief that the price level does not conform to the market-fundamentals component, F_t. A positive value of B_t would represent a rational inflationary bubble and would imply that the fiat money is undervalued (relative to market-fundamentals) at date t. A negative value of B_t would represent a rational deflationary bubble and would imply that the fiat money is overvalued at date t.

Solutions to equation (5) satisfy the stochastic difference equation

(6) $$B_{t+1} - (1+\beta^{-1})B_t = z_{t+1},$$

where z_{t+1} is a random variable (or combination of random variables) generated by a stochastic process that satisfies

(7) $$E_{t-j}z_{t+1} = 0 \quad \text{for all} \quad j \geq 0.$$

The key to the relevance of equation (6) for the general solution for P_t is that equation (5) relates B_t to E_tB_{t+1}, rather than to B_{t+1} itself as would be the case in a perfect-foresight model.

The random variable z_{t+1} is an innovation, comprising new information available at date $t+1$. This information can be intrinsically irrelevant--that is, unrelated to F_{t+1}--or it can be related to truly relevant variables, like α_{t+1} and M_{t+1}, through parameters that are not present in F_{t+1}. The critical property of z_{t+1}, given by equation (7), is that its expected future values are always zero.

The general solution to equation (6) for any date t, $t \geq 0$, is

(8) $$B_t = (1+\beta^{-1})^t B_0 + \sum_{\tau=1}^{t} (1+\beta^{-1})^{t-\tau} z_\tau.$$

Equation (8) expresses the rational-bubbles component at date t as composed of two terms. The first term is the product of the eigenvalue raised to the power t and the value of the rational-bubbles component at date zero. The second term is a weighted sum of realizations of z_τ from $\tau = 1$ to $\tau = t$. The weights are powers of the eigenvalue such that the contribution of z_τ to B_t increases exponentially with the difference between t

and τ. For example, a past realization z_τ, $1 \leq \tau < t$, contributed only the amount z_τ to B_τ, but contributes $(1+\beta^{-1})^{t-\tau}$ to B_t. Blanchard and Watson (1982) suggest, as an empirically interesting specification for a rational-bubbles component, a process in which the analog to z_τ is not covariance stationary and implies that rational bubbles can burst and restart repeatedly.

The assumption of rational expectations implies that in forming $E_t B_{t+j}$, for all $j > 0$, agents behave as if they know that any rational-bubbles component of the price level would conform to equation (5) in all future periods. Accordingly, any solution to equation (5) would have the property

(9) $$E_t B_{t+j} = (1+\beta^{-1})^j B_t \quad \text{for all} \quad j > 0.$$

Equation (9) says that the existence of a nonzero rational-bubbles component at date t would imply that the expected value of the rational-bubbles component at date $t+j$ either increases or decreases with j at the geometric rate $1+\beta^{-1}$. Therefore, the existence of a rational bubble would imply that the expected value of the logarithm of the price level, $\{E_t P_{t+j}\}_{j=1}^{\infty}$, either increases or decreases without bound at approximately the geometric rate $1+\beta^{-1}$. In particular, the existence of a rational deflationary bubble at date t would imply that the expected future value of a unit of fiat money (in units of the consumption good) grows without bound at this increasing proportionate rate. Accordingly, if, as discussed above, in the empirically relevant environment agents cannot rationally expect the value of a unit of fiat money to grow at such a rapid pace, then rational deflationary bubbles cannot exist.

2. The Inception of Rational Inflationary Bubbles

Given that rational deflationary bubbles are not possible, the rational-bubbles component of the price level as given by equation (6) also satisfies $B_{t+1} \geq 0$. Consequently, the realization of z_{t+1} must satisfy

(10) $$z_{t+1} \geq -(1+\beta^{-1})B_t \quad \text{for all} \quad t \geq 0.$$

Equation (10) says that the realization z_{t+1} must be large enough to ensure that equation (6) implies a nonnegative value for B_{t+1}.

Suppose that B_t equals zero. In that case, equation (10) implies that z_{t+1} must be nonnegative. But, equation (7) says that the expected value of z_{t+1} is zero. Thus, if B_t equals zero, then z_{t+1} equals zero with probability one.

This result says that if a rational bubble does not exist at date t, $t \geq 0$, a rational bubble cannot get started at date $t+1$, nor, by extension, at any subsequent date. Therefore, if a rational bubble exists, it must have started at date zero, the date of initial issuance the fiat money, and hence, this fiat money must have always been undervalued relative to market fundamentals. The essential idea underlying this line of argument is that, because the inception of a rational bubble at any date after the introduction of the fiat money would involve an innovation in the price level, the expected initial values of a rational inflationary bubble and a rational deflationary bubble would have to be equal. Accordingly, if a deflationary rational-bubbles component cannot exist, then an inflationary rational-bubbles component also cannot start after the date of initial issuance of a fiat money.

Suppose that, prior to the issuance of a new fiat money, agents anticipate its introduction and form an expectation about the initial price level. Suppose further that this expectation coincides with market fundamentals--that is,

(11) $$E_{-1}B_0 = E_{-1}P_0 - E_{-1}F_0 = 0.$$

Equation (11) would imply that B_0 is a random variable with mean zero. Accordingly, given the nonnegativity condition $B_0 \geq 0$, B_0 would equal zero with probability one. This observation implies that if a rational inflationary bubble exists, agents who anticipated the introduction of the new fiat money expected it to be undervalued relative to market fundamentals.

3. Can Rational Inflationary Bubbles Burst and Restart?

Consider the following model of the innovations z_{t+1}:

$$z_{t+1} = [\theta_{t+1} - (1+\beta^{-1})]B_t + \varepsilon_{t+1}, \tag{12}$$

where θ_{t+1} and ε_{t+1} are mutually and serially independent random variables. If the processes generating θ_{t+1} and ε_{t+1} satisfy

$$E_{t-j}\theta_{t+1} = 1+\beta^{-1} \quad \text{for all } j \geq 0 \text{ and} \tag{13}$$

$$E_{t-j}\varepsilon_{t+1} = 0 \quad \text{for all } j \geq 0, \tag{14}$$

then z_{t+1} as given by equation (12) satisfies equation (7).

Substituting for z_{t+1} in equation (6) from equation (12) gives

$$B_{t+1} = \theta_{t+1}B_t + \varepsilon_{t+1}. \tag{15}$$

Quah (1985) suggests the model of the rational-bubbles component given by equation (15) as a generalization of the specification assumed by Blanchard and Watson (1982). Equation (15) says that, with z_{t+1} given by equation (12), an existing rational-bubbles component, B_t, will burst next period if the event $\theta_{t+1} = 0$ occurs. If this event has positive probability, then any rational-bubbles component would burst at a random, but almost surely finite, future date. Specifically, if the probability associated with $\theta_{t+1} = 0$ is Π, $0 < \Pi < 1$, then the expected duration of a rational-bubbles component is Π^{-1} periods and the probability that B_t will not burst by date T $(T > t)$ is $(1-\Pi)^{T-t}$, which tends to zero as T approaches infinity.

Given that realizations of θ_{t+1} and ε_{t+1} are mutually and serially independent and also independent of B_0, then ε_{t+1} is independent of B_t for all $t \geqslant 0$. In this case, if the event $\theta_{t+1} = 0$ were by chance to coincide with a positive realization of ε_{t+1}, then, according to equation (15), as an existing rational-bubbles component bursts, a new rational-bubbles component, which is independent of all existing and past rational-bubbles components, would simultaneously start.

In this model, the impossibility of rational deflationary bubbles would imply that, in addition to satisfying equation (15), the rational-bubbles component satisfies $B_{t+1} \geqslant 0$. Therefore, the event $\theta_{t+1} = 0$ cannot coincide with a negative realization of ε_{t+1}. Accordingly, given that the event $\theta_{t+1} = 0$ has positive probability and that the random variables ε_{t+1} and θ_{t+1} are independent, ε_{t+1} must be nonnegative. But, equation (14) says that the expected value of ε_{t+1} is zero. Therefore, ε_{t+1} equals zero with probability one and the chance coincidence of $\theta_{t+1} = 0$ and $\varepsilon_{t+1} > 0$ has zero probability.

This result says that the impossibility of rational deflationary bubbles, in addition to implying that an inflationary rational-bubbles component that burst could not restart at a later date, also precludes the possibility that a new independent inflationary rational bubble could simultaneously start when an existing inflationary rational bubble bursts. In sum, the analysis of Sections 2 and 3 has shown that, given the impossibility of rational deflationary bubbles, an inflationary rational-bubbles component can start only on the date of initial issuance of the fiat money and must either continue to exist forever or, as in Blanchard's (1979) specification, burst at a date in the finite future and never restart.

Nevertheless, a rational inflationary bubble that began on the first date of circulation and will never burst can periodically shrink. An example of such a rational-bubbles component, which is consistent with the preceding analysis, would be

$$(16) \qquad B_{t+1} = \begin{cases} \delta B_t + \varepsilon_{t+1} & \text{with probability } \Pi \\ (1-\Pi)^{-1}(1+\beta^{-1} - \delta\Pi)B_t + \varepsilon_{t+1} & \text{with probability } 1-\Pi, \end{cases}$$

where δ is a small positive constant and where $E_t \varepsilon_{t+1} = 0$ and $B_0 > 0$. This specification corresponds to setting θ_{t+1} in equation (15) equal to δ with probability Π and equal to $(1-\Pi)^{-1}(1+\beta^{-1} - \delta\Pi)$ with probability $1-\Pi$ and allowing ε_{t+1} to depend on B_t and θ_{t+1} in such a way that B_{t+1} remains nonnegative with probability one. In particular, given $\theta_{t+1} = \delta$, realizations of ε_{t+1} must satisfy $\varepsilon_{t+1} \geq -\delta B_t$. Equation (16) specifies an inflationary rational-bubbles component that starts on the first date of trading, that collapses with probability Π in any period, but that, given δ greater than zero and the appropriate restriction on the realizations of ε_{t+1}, always remains positive.

4. Rational Bubbles in Exchange Rates?

Although the analysis in this paper focuses on the determination of the value of a fiat money in units of goods and services, it also has implications for the determination of this value in units of foreign currency. Utilizing a model that is formally identical to the model discussed in Section 1, Meese (1986) suggests that rational bubbles that burst and restart occurred in foreign exchange rates during the 1970's and

1980's. Woo (1985) also suggests that in this period episodes during which exchange rates conformed to market fundamentals alternated with episodes during which rational bubbles were present.

As Singleton (1987) points out, any rational bubble in an exchange rate would have to be reflected either in a rational bubble in the price level at home or abroad or in a rational bubble in the deviation from purchasing power parity. But a rational bubble in the deviation from purchasing power parity cannot exist, because agents cannot expect unexploited potential profits from commodity arbitrage to grow geometrically without bound. Accordingly, given the impossibility of rational deflationary bubbles, any rational bubble in exchange rates would have to coincide with a rational inflationary bubble in the depreciating currency.

The analysis in the preceding sections thus implies that the inception of a rational exchange rate bubble can only occur at the first date of circulation of a fiat money. In particular, the rational-bubbles component of the value of a currency could not burst and restart repeatedly--as in Meese's specification--or only exist during certain periods--as in Woo's specification. As Hamilton and Whiteman (1985) demonstrate, the existence of rational bubbles is empirically indistinguishable from misspecification of market fundamentals. Accordingly, the correct interpretation of the econometric findings of Meese and Woo would seem to be that the models they study misspecify the market-fundamentals component of the exchange rate.

5. A Nonlinear Model of the Demand for Money

The expectational difference equation governing price fluctuations in the F&G model--equation (2) above--is linear. The linearity of this equation makes explicit characterization of the market-fundamentals and rational-bubbles components of the price level possible without assuming that the money supply and other forcing variables are constant over time or grow at a constant rate. The linearity of the difference equation (2) was also convenient for developing the analysis of the inception of a rational inflationary bubble. Specifically, equation (2) implied that a rational-bubbles component in the F&G model would have to satisfy the linear stochastic difference equation (6). Setting B_t equal to zero, then, shows that the inception of a rational bubbles component at any date after the introduction of a new fiat money must involve an innovation in the price level.

In a nonlinear model, equation (6) has no counterpart. The following analysis demonstrates, however, that the inception of a rational-bubbles component in the more general (nonlinear) model of Brock (1974, 1975) would also involve an innovation in the price level. Accordingly, an argument analogous to that of the preceding section would also limit the inception of a rational inflationary bubble in Brock's model.

Assume that a representative household maximizes expected utility over an infinite horizon,

$$(17) \qquad E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t}[u(c_\tau) + v(x_\tau)], \quad 0 < \beta < 1,$$

where c_τ and x_τ represent, respectively, consumption of a single perishable good and holdings of real money balances at date τ, and β is a discount factor. The functions $u(\cdot)$ and $v(\cdot)$ are monotone increasing, strictly concave and continuously differentiable on $(0,\infty)$. In addition, the function $v(\cdot)$ satisfies the Inada conditions: $\lim_{x \to 0} v'(x) = \infty$ and $\lim_{x \to \infty} v'(x) = 0$.

The household receives an initial endowment, m, of (nominal) money balances before date zero and a constant endowment, y, of the consumption good each period beginning at date zero. The household takes the price level, p_τ, as given and chooses consumption, c_τ, and nominal money balances, m_τ, subject to the budget constraint

(18) $$p_\tau c_\tau + m_\tau - m_{\tau-1} \leq p_\tau y.$$

The first-order conditions for the household's utility maximization problem is

(19) $$\beta E_t \left[\frac{u'(c_{t+1})}{p_{t+1}}\right] = \frac{u'(c_t)}{p_t} - \frac{v'(x_t)}{p_t}.$$

Incorporating the market-clearing conditions, $m_t = m$ and $c_t = y$ for all $t \geq 0$, in equation (19) and multiplying both sides of this equation by m yields

(20) $$\beta E_t x_{t+1} = [1 - \frac{v'(x_t)}{u'(y)}]x_t.$$

Equation (20) is a nonlinear first-order expectational difference equation in real money balances.

Define the market-fundamentals component, f_t, of real money balances to be the positive nonstochastic steady-state solution to equation (20)--that is,

(21) $$f_t = x^*,$$

where x^* is the unique solution to

(22) $$v'(x^*) = (1-\beta)u'(y).$$

Define the rational-bubbles component of real money balances, b_t, as any divergence from f_t--that is,

(23) $$b_t = x_t - f_t.$$

In equation (23), a positive rational bubbles component, which would represent a rational deflationary bubble, could not exist for any plausible specification of the function $v(\cdot)$. Specifically, as Gray (1984) argues, a plausible model of how money enhances utility, would imply that, for a given level of consumption, the utility derived from holding real money balances is bounded from above. Boundedness of the function $v(\cdot)$, in turn, would imply that a transversality condition, necessary for the optimality of the household's decisions, rules out the existence of a rational deflationary bubble. In fact, weaker conditions than boundedness of the function $v(\cdot)$ are sufficient for ruling out rational deflationary bubbles--see Brock (1974) and Obstfeld and Rogoff (1986).

Consider now the implications of the impossibility of rational deflationary bubbles in this model for the possible inception of a rational inflationary bubble. Specifically, assume that real money balances conform to market fundamentals -- that is, $x_t = f_t = x^*$ -- for some $t \geq 0$. Equations (20) and (22) would then imply

(24) $$E_t x_{t+1} = x^*.$$

Equation (24) shows that, given $b_t = 0$, the expected value of the rational-bubbles component at date $t+1$, $E_t b_{t+1} = E_t x_{t+1} - x^*$, equals zero. In other words, the inception of a rational-bubbles component after date zero would involve an innovation in real money balances. Accordingly, as the analysis of the linear model demonstrated, the impossibility of rational deflationary bubbles would imply that a rational inflationary bubble cannot exist at any date unless it existed during all previous dates since the initial issuance of the fiat money. Moreover, an argument similar to that for the linear model would show that the existence of a rational inflationary bubble at any date implies that prior to the introduction of the fiat money, agents who anticipated its introduction expected a rational inflationary bubble to occur.

6. Summary

The inception of a rational-bubbles component after the date of initial issuance of a fiat money would involve an innovation in the price level. Accordingly, any rational-bubbles component that starts after the introduction of fiat money has an expected initial value of zero. But, given that the empirically relevant environment precludes the existence of rational deflationary bubbles, this initial value must be nonnegative and therefore, in order to have a mean of zero, must equal zero with probability one.

This argument means that the impossibility of rational deflationary bubbles also rules out the inception of a rational inflationary bubble except at the date of initial issuance of a fiat money. Thus, the existence of a rational inflationary bubble at any date, implies that a rational inflationary bubble has been present since the introduction of the fiat money. Moreover, the existence of a rational inflationary bubble implies that prior to the issuance of the fiat money, agents who anticipated its introduction expected a rational inflationary bubble to occur.

This analysis also implies that once a rational inflationary bubble bursts it cannot restart. In particular, rational inflationary bubbles cannot conform to the specification suggested by Blanchard and Watson (1982) in which rational bubbles start, burst, and restart repeatedly. This analysis, however, does not preclude the existence of a rational inflationary bubble that begins on the first date of circulation of the fiat money and shrinks periodically, but never bursts. Finally, because a rational bubble in exchange rates would imply the existence of a rational inflationary bubble in the depreciating currency, the same restrictions apply to the inception and existence of rational exchange-rate bubbles.

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