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ASPIRATIONS AND INEQUALITY

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ABSTRACT

This paper develops a theory in which society-wide economic outcomes shape individual aspirations, which affect the investment incentives of individuals. Through its impact on investments, aspirations in turn affect ambient social outcomes. We explore this two-way link. A central feature is that aspirations that are moderately above an individual's current standard of living tend to encourage investment, while still higher aspirations may lead to frustration and lower investment. When integrated with the feedback effect from investment, we are led to a theory in which aspirations and income evolve jointly, and the social determinants of preferences play an important role. We examine conditions under which growth is compatible with long-run equality in the distribution of income. More generally, we describe steady state income distributions, which are typically clustered around local poles. Finally, the theory has predictions for the growth rates along the cross-section of income. We use these predictions to calibrate the model so that it fits growth data by income percentile for 43 countries, and back out the implicit aspirations-formation process that underlies these observations.

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1. INTRODUCTION

What individuals want for themselves or for their children is conditioned in fundamental ways by the lives of others. One such pathway of influence is the formation of individual "aspirations" based on society-wide economic outcomes. Existing literature views such reference points as drawn from the past experience of the individual herself. In this paper, we argue that they are also profoundly affected by her *social* environment. Others around us shape our desires and goals. This is a view of individual preferences that isn't standard in economic theory. But it should be.

At the same time, while social outcomes affect aspirations, those very aspirations influence — via the aggregation of individual decisions — the overall development of a society. As a result, aspirations and income (and the distribution of income) evolve together. An examination of this relationship is the subject of our paper.

Any such theory must address three issues. First, there is the question of how aspirations are formed. Second, we must describe how individuals react to the aspirations that they do have. Finally, the theory must aggregate individual behavior to derive society-wide outcomes. The last of these is a standard exercise; we emphasize the first two features.

We define utilities around a "reference point" and interpret that point as an *aspiration*. The use of a reference point is, of course, not new: see, e.g., Kahneman and Tversky (1979), Karandikar et al. (1998), and Kőszegi and Rabin (2006). Our contribution is to emphasize the dependence of such reference points on the ambient income distribution, thereby linking observed social outcomes to individual behavior.[1] For instance, individuals may simply use some common function of the income distribution (such as mean income, or income at the 75th percentile) to form their aspirations. Or they might only look at individuals within a few percentile points of their own economic location. Or they might use the conditional mean of all individuals richer than them.

Next, we relate an individual's aspirations to her incentives to invest and bequeath. We discuss how growth rates of income react to a shift in aspirations brought about, say, through the rise of mass communications media. We argue that the "best" aspirations are those that lie at a moderate distance from the individual's current economic situation standards, large enough to incentivize but not so large as to induce frustration. Our theory draws on Appadurai (2004) and Ray (1998, 2006), who make similar arguments in a more informal setting, as well as our earlier working paper, Genicot and Ray (2009). Our formulation is also in line with evidence from cognitive psychology, sports, and lab experiments (see, e.g., Bernheim (1989), Heath, Larrick, and Wu (1999) and Lockwood and Kunda (1997)) that goals that lie ahead — but not too far ahead — provide the best incentives.[2] The argument captures both encouragement and frustration, and on its own can be used to create an aspirations-based theory of poverty traps.

[1] See Macours and Vakis (2009) for evidence of the importance of social interactions in the formation of aspirations.

[2] To cite just one example from social psychology, LeBoeuf and Estes (2004) find that subjects score *lower* on trivia questions when first primed by self-listing the similarities between them and Einstein (what we might interpret as raising their aspirations), relative to when not primed; and they score *higher* when asked to list the differences between them and Einstein (what we interpret as lowering their aspirations) relative to when not primed.

We also study the behavior of growth rates along the income distribution. Once again, our propositions reflect the idea that aspirations that are too high can serve to frustrate, while aspirations that are too low might breed complacency. It follows that over a zone of incomes that share the same aspirations, individual growth rates should be inverted U-shaped in income.

Finally we embed the theory of aspirations formation into a simple growth model. In equilibrium, the overall income distribution influences individual aspirations, which in turn shape the distribution via individual choices.[3] We study the properties of equilibrium income distributions. We investigate conditions under which perfect or near-perfect equality is unsustainable. Under some conditions, income distributions cannot converge to a degenerate distribution, while in other situations that permit rapid growth, perfect equality may be sustainable. These results are in line with a recent literature that explore various arguments underlying the emergence and persistence of inequality, including nonconvexities (Galor and Zeira (1993), Matsuyama (2004)), occupational choice (Banerjee and Newman (1993), Freeman (1996), Mookherjee and Ray (2003)) and endogenous risk-taking (Becker, Murphy, and Werning (2005), Ray and Robson (2012)). In the case in which aspirations take on at most a finite number of values, depending on which income class the individual belongs to, incomes must cluster into local poles in any steady state. In the special case of commonly held aspirations, typically two poles emerge, in line with the findings of Quah (1993).

As a simple, illustrative empirical exercise, we use the percentile distributions of growth rates available for 43 countries. Using a linear production function, we find the rate of return to investment in the exercise that matches most closely the aggregate growth experiences of these countries, and then employ this information to see which model of aspirations formation appears to fit the data best, in that they come closes to the observed growth incidence curves by percentile. We show that a model of aspirations formation in which individuals use "umbrella-shaped" weights on incomes in some interval around their own incomes comes closest to replicating the data, and this specification captures close to 70% of the observed variation in growth.

There is, of course, a large literature which connects social outcomes to individual behavior. In most part, the link is to an individual's *feasible set*, and not her preferences. For instance, macroeconomic conditions will affect an individual's access to capital or labor markets, and therefore her behavior. We emphasize, in contrast, the effect of social outcomes on what an individual *wants* to do. In this sense, the closest literature would be the one which emphasizes the effect of the ambient distribution on status-seeking and therefore behavior (see, e.g., Clark and Oswald (1996), Corneo and Jeanne (1997), Corneo and Jeanne (1999), Duesenberry (1949), Frank (1985), Hopkins and Kornienko (2006), Ray and Robson (2012), Robson (1992), Schor (1992), Scitovsky (1976), and Veblen (1899)). More closely, our approach is related to Karandikar et al. (1998) and Shalev (2000) who endogenize reference points using realized payoffs of a game. This is a channel that works through preferences for *relative* wealth or income. However, the structure we place on aspirations formation as a reference point, and on the "nonlinear" way in which individuals react to the gap between their aspirations and their current standards of living, makes this a distinct exercise, with its own novel distributional and growth implications.

[3]This approach develops the ideas laid down in an earlier working paper, Genicot and Ray (2009). Following that approach, Bogliacino and Ortoleva (2010) and Dalton, Ghosal, and Mani (2011) also develop models of socially determined aspirations that explore related but distinct issues.

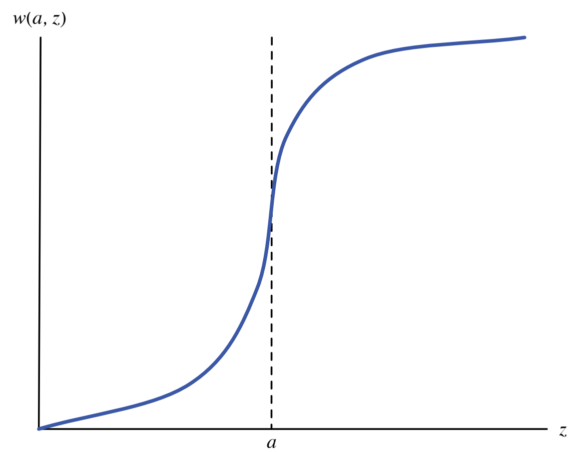


FIGURE 1. THE FUNCTION w

2. ASPIRATIONS, WEALTH DISTRIBUTION AND EQUILIBRIUM

2.1. **An Intertemporal Model With Aspirations.** We study a society populated by a large numberof single-parent single-child families. Each person lives for a single period. A sequence of individuals in a family forms a dynasty. A typical member of any generation has lifetime income (or wealth) y, and allocates y over her lifetime consumption c and investments to affect the lifetime income of her child z, so as to maximize payoff:

$$u(c) + v(z) + \omega(z, a).$$

There are three terms in this payoff. The first is the utility u from own consumption: it is increasing, smooth and strictly concave with unbounded steepness at 0: $u'(0) = \infty$. The second and third terms pertain to the utility derived from the child's wealth. The first of these is an increasing, smooth and strictly concave utility defined on wealth, also with $v'(0) = \infty$; view it as "intrinsic" parental utility derived from the wealth of the child. Together, u and v represent a standard model of intertemporal allocation with altruism.[4]

The last (and new) term is "aspirations utility," the return that parents receive from their child's wealth relative to a certain threshold a, to be thought of as the *aspiration* of the parent. We will measure a in "income units" so that it can also be viewed as an income target and so that the ratio z/a makes sense; see below. We will soon endogenize a, but for now it is akin to a reference point, just as in Karandikar et al. (1998) and Kőszegi and Rabin (2006).

We place additional structure on w to capture two important characteristics. First, we incorporate the feature that the payoff from aspirations is fundamentally relativistic, depending on the *ratio* of achieved income to aspirations. Second, we allow for scale effects, proxied by some function

[4]Because v is exogenously given, we might think of this specification as one of "paternalistic altruism". In contrast, had v been the value function of the child, we would interpret this as a model of "nonpaternalistic altruism". We do not pursue this alternative here.

of a. Specifically, we write $\omega(z, a)$ as

$$\omega(z, a) = s(a)w(z/a),$$

where w picks up the purely relative component and s picks up scale effects. We maintain the following assumptions throughout:

[W.1] $w(x)$ is smooth, with $w'(x) > 0$.

[W.2] $w''(x) > 0$ for $x < 1$ and $w''(x) < 0$ for $x > 1$.

[W.3] $s(a)$ is smooth, $s(a) > 0$ for all $a > 0$, and $s(a)/a$ is nonincreasing.

The first two restrictions are a subset of those imposed in Kőszegi and Rabin (2006). [W.1] presumes that w is increasing in the ratio of income to aspirations. [W.2] implies that the utility gain or loss away from the "baseline" of 1 (which is aspirations-income parity) is *increasing at a decreasing rate in either direction*. If I am far ahead of my aspirations, an extra gain is not going to create much additional satisfaction, and likewise if I am way below my aspirations, an increase or decrease is not going to make much of a difference. It is in the region of the aspiration itself that utility gains are most sensitive to an increase in income. See Figure 1.

Finally, [W.3] allows for scale effects but does not insist on them: $s(a)$ is permitted to be a constant or even decline in a. What *is* important is that there is a restriction on how quickly utility can increase in the scale term, which is captured by the requirement that $s(a)/a$ is weakly decreasing. The important special case in which $s(a)$ is a constant is one of "purely relative" aspirations, in which only the ratio of income to aspirations matters in determining the value of the aspirational component ω.

We make one more assumption on preferences:

[U] For every income $y > 0$ and aspiration a, positive consumption is preferred to zero consumption: there is $c > 0$ such that

$$u(c) + v(f(y - c)) + w(f(y - c), a) > u(0) + v(f(y)) + w(f(y), a).$$

Condition U guarantees the primacy of own-consumption. This condition is actually implied by the unbounded steepness condition $u'(0) = \infty$, unless ω also exhibits unbounded steepness at some interior point, in which case [U] is an additional though entirely intuitive restriction.

2.2. **The Formation of Aspirations.** Two alternative approaches, by no means mutually exclusive, connect aspirations to economic outcomes and so bring the theory full circle. One possibility is to take an entirely *private* viewpoint: one's personal experiences determine future goals, so that each individual can be analyzed as as a self-contained unit. This is the approach taken in Karandikar et al. (1998) and Kőszegi and Rabin (2006) when determining reference points; see also Alonso-Carrera, Caball, and Raurich (2007), Carroll and Weil (1994) and Croix and Michel (2001). In this literature, the loop that runs from reference points to behavior and back to reference points is entirely internal to the individual.

In contrast, economic models of status (see the many references in the Introduction) achieve closure by using *social* outcomes external to the individual. In all these theories, an individual's payoff depends on a comparison of her own outcomes with the lives of others. That is the approach we take here. A broad range of possibilities is captured under the specification

$$a = \Psi(y, F), \tag{1}$$

where a stands for the aspiration of an individual, y her lifetime income or wealth, and F the society-wide distribution of lifetime incomes.

It is, of course, also possible to adopt a specification in which the *anticipated* distribution of wealth over future generations drives aspirations.[5] We are comfortable with either model, but adopt the current approach for two reasons: (a) it uses the perhaps more satisfying formulation that goals are derived from an *actual* situation rather than an anticipated state of affairs which may or may not come to pass, and (b) the resulting structure is fully recursive and far more friendly to numerical computation.

We assume that:

[A] Ψ is continuous in y and F,[6] nondecreasing in y, nondecreasing in F with respect to first-order stochastic dominance, and $\Psi(y, F) \in$ Range(F).

It is innocuous to maintain that aspirations are nondecreasing in income, though certainly the gap between the two could narrow. If a is *strictly* increasing in y, then personal experience plays an active role in determining aspirations, something that we find reasonable but do not necessarily insist upon. In addition, it is natural to suppose that individual aspirations are heightened as all incomes in society rise, hence the above restriction with respect to first-order stochastic dominance. Finally, we presume that the reference point does not wander out of the range of actually observed incomes.

Assumptions W, U and A will be presumed to hold throughout the paper, and we will not refer to them explicitly in the statements of any of the propositions.

Consider some particular processes of aspirations formation:

Common Aspirations. This is the simplest case. All individuals have exactly the same aspirations, which are given by some *common* function of the income distribution (e.g., the income of the 75th percentile), and do not depend on the specific value of individual income:

$$\Psi(y, F) = \psi(F).$$

Stratified Aspirations. Aspirations depend both on the income distribution and one's membership in one of n quantiles of that distribution. Define

$$\Psi(y, F) = a_i$$

[5]A previous version of the paper, see Genicot and Ray (2009), discusses and compares the two approaches.

[6]Continuity in F is with respect to the topology of weak convergence on distributions.

for individuals with income y in quantile i, where a_i is a scalar representing some summary statistic of the distribution in that quantile: e.g., the income of the 75th conditional percentile in that quantile.

Upward-Looking Aspirations. Aspirations might always exceed incomes. For instance, suppose that individuals look "upwards" at all families who are richer than them, and that aspirations are the conditional mean of all such incomes:

$$\Psi(y, F) = \frac{\int_y^\infty x dF(x)}{1 - F(y)}.$$

Local Aspirations With Population Neighborhoods. Ray (2006) discusses "aspiration windows," in which people draw upon the experiences of those in some cognitive window around them. For instance, suppose that weight is placed only on the surrounding d (income) percentiles of the population. That is, an individual with income y is cognizant of income y' only if $|F(y') - F(y)| \leq d$, so that

$$\Psi(y, F) = \frac{1}{d} \int_{L(y)}^{H(y)} x dF(x),$$

where $L(y)$ and $H(y)$ are the appropriately defined edges of the cognitive window for a person situated at y.[7]

Local Aspirations With Income Neighborhoods. Or suppose that weight is placed instead only on incomes within an interval $N(y)$ of the individual's income, regardless of percentile distance. Then

$$\Psi(y, F) = \frac{1}{F(N(y))} \int_{N(y)} x dF(x),$$

where $F(N(y))$ has the obvious meaning.

2.3. **Dynastic Equilibrium With Aspirations.** To describe equilibria, we embed our model of aspirations formation into a standard growth model. Suppose that the distribution of current wealth is given by F_t. Then each individual in generation t with wealth y_t has aspirations given by $a_t = \Psi(y_t, F_t)$. She divides her wealth between consumption c_t and a bequest for the future, k_t:

$$y_t = c_t + k_t.$$

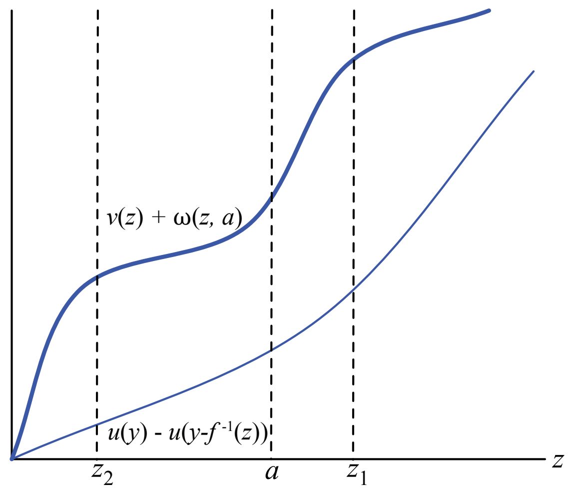
That bequest gives rise to fresh wealth for the next generation:

$$y_{t+1} = f(k_t),$$

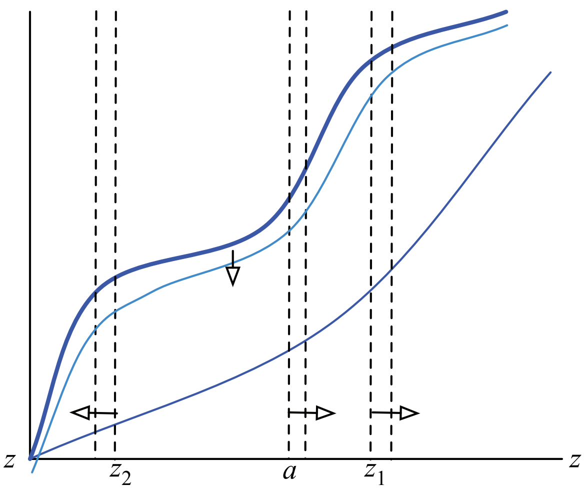
where f is a smooth increasing function. A *policy* ϕ maps current wealth y and aspirations a to wealth z for the next generation.

An *equilibrium* from some initial distribution F_0 is a sequence of income distributions $\{F_t\}$ and a policy ϕ such that

[7]That is, $L(y)$ is the lowest income in the support of F with $F(y) - F(L(y)) \leq d$, and $H(y)$ is the highest income in the support of F with $F(H(y)) - F(y) \leq d$.



(A) Costs and Benefits



(B) Change in a

FIGURE 2. THE CHOICE OF FUTURE WEALTH.

(i) For every t and y in the support of F_t, aspirations are given by $a = \Psi(y, F_t)$, and $z = \phi(y, a)$ maximizes

$$u\left(y - f^{-1}(z)\right) + v\left(z\right) + \omega\left(z, a\right) \tag{2}$$

over $z \in [0, f(y)]$.

(ii) F_{t+1} is generated from F_t and the policy ϕ; that is, for each $z \geq 0$,

$$F_{t+1}(z) = \text{Prob}_t\{y|\phi(y, \Psi(y, F_t)) \leq z\},$$

where Prob_t is the probability measure induced by the distribution function F_t.

Note that *given the aspirations*, there is no particular need for the policy function to be time-dependent; the resulting maximization problem (2) is entirely stationary.

Proposition 1. *An equilibrium exists.*

The proof of this proposition is a simple recursive exercise, starting from any initial distribution of wealth.

3. THE PARTIAL EQUILIBRIUM OF ASPIRATIONS, WEALTH AND GROWTH

As we've already remarked, the properties of the policy function are entirely independent of time or the surrounding distribution of incomes, because that function has both income and aspirations in its domain. It will be useful, then, to begin with a "partial equilibrium" analysis in which we examine the properties of this policy function. Figure 2 shows us how to graphically think about the maximization problem induced by expression (2). The horizontal axis plots the choice of future wealth z, while the vertical axis records various benefits and costs. It will be useful to write the benefits as the payoffs that accrue from next generation's wealth; i.e., as

$$v(z) + \omega(z, a),$$

while the cost is the sacrifice of current utility, which we can write as $u(y) - u\left(y - f^{-1}(z)\right)$. Panel A of Figure 2 plots both these functions. The "cost function" has a standard shape: it is the convex lower curve. The "benefit function" is concave to the right of a; after all both v and ω are concave in z over that region. To the left of a, the function has ambiguous curvature. Given income and aspirations, our maximization problem seeks a continuation income z that produces the largest vertical distance between these two curves.

By the concavity of benefits to the right of a, there can be at most one solution that exceeds a. See point z_1 in Figure 2(A). There could be a number of them below a; one such point is z_2 in Figure 2(A). Finding an optimal solution involves comparing all the continuation incomes for which the interior first-order condition

$$v'(z) + \frac{s(a)}{a} w'(z/a) = u'\left(y - f^{-1}(z)\right) f^{-1'}(z) \tag{3}$$

holds, and picking the one that yields the highest payoff. Generically, the optimal choice of z will be unique, with multiple solutions possible for lower-dimensional subsets of the parameters (y, a).

It will be useful to introduce some terminology to describe the solutions. We'll say that aspirations are *satisfied* if there is an optimal solution which is at least a, and *frustrated* if there is an optimal solution that falls short of a. (The slight ambiguity in this definition will be resolved in Proposition 2 below: there can be at best a single point at which aspirations are both satisfied and frustrated.)

When are aspirations satisfied, and when are they frustrated? We can examine this question by varying aspirations for a fixed level of income, or by varying income for some fixed aspiration. The *joint* variation of incomes and aspirations will depend on the ambient income distribution, which is itself endogenous. Section 4 takes up this general equilibrium question.

3.1. **Changes in Aspirations.** Consider an exogenous change in aspirations for some individual with given income. Such changes don't just constitute an abstract exercise. For instance, the rise of mass media in developing countries (such as television, advertising or the internet) will bring particular socioeconomic groups into focus, thus affecting aspirations upward or downward, often upward.[8] In addition, of course, a change in aspirations can also be fueled by growth or decay in ambient incomes.

For aspirations close to zero, the assumed end-point condition guarantees a solution that strictly exceeds aspirations, and so aspirations are satisfied. As aspirations continue to rise, there comes a threshold when the solution makes a switch from satisfaction to frustration: often, this switch will arrive with a discontinuous fall in investment, as we note below.

[8]See Jensen and Oster (2009) and Ferrara, Chong, and Duryea (2012) for evidence on how the introduction of cable television can expose people to very different lifestyles, thereby affecting their aspirations and fertility preferences.

But higher aspirations can do more than switch individuals from satisfaction to frustration. Once in the "frustration zone," economic growth is actually *lowered* by an increase in a: higher aspirations encourage less investment. To understand why, consult Panel B of Figure 2. In that panel, we shift aspirations upwards. Depending on how payoffs are affected by aspirations the benefit function (which includes $\omega(z,a)$) may shift up or down; in the purely relativistic case with $s(a)$ a constant, it will shift down at every value of z. But there is also a change in slope: as we establish more formally below, the function also becomes *flatter* to the left of a. It follows that every candidate for an optimal solution already below a must decrease still further.

Is the opposite true for every candidate solution larger than a? That is, does an increase in aspirations incentivize growth as long as aspirations remain in the "satisfaction zone"? The answer is: not always. We return to this issue below. First, we formalize the discussion above and add important details that have been omitted:

Proposition 2. *For given y, there is a unique threshold value of aspirations below which aspirations are satisfied, and above which they are frustrated. Once aspirations are frustrated, chosen wealth declines as aspirations continue to grow.*

Proof. By [U] and the end-point condition on v, we have interior solutions for any $y > 0$. So if aspirations are set equal to zero, then aspirations must be satisfied. Because y is fixed, aspirations must be frustrated once a is high enough. So there is certainly a threshold at which a changeover occurs from satisfaction to frustration. Below, we shall prove that such a threshold must be unique. To go further, we employ the following lemma:

Lemma 1. *Consider any selection from the optimality correspondence that links a to an optimal choice z. Then that mapping cannot exhibit a discontinuous upward jump.*

Proof. Suppose that z_1 and z_2 are both optimal choices at a, with $z_2 > z_1$. We claim that

$$(z_2/a)w'(z_2/a) - w(z_2/a) > (z_1/a)w'(z_1/a) - w(z_1/a) \tag{4}$$

To prove this, recall the first-order condition (3) for z_1 and z_2:

$$v'(z_i) + \frac{s(a)}{a}w'(z_i/a) = u'\left(y - f^{-1}(z_i)\right) f^{-1\prime}(z_i) = u'(c_i)/f'(y - c_i) \tag{5}$$

for $i = 1, 2$, where c_i is consumption under z_i. At the same time, by the joint optimality of z_1 and z_2, $u(c_1) + v(z_1) + s(a)w(z_1/a) = u(c_2) + v(z_2) + s(a)w(z_2/a)$, or equivalently,

$$s(a)w(z_2/a) - s(a)w(z_1/a) = [u(c_1) + v(z_1)] - [u(c_2) + v(z_2)]. \tag{6}$$

Multiplying both sides of (5) by $z_i = f(y_i - c_i)$, combining the result with (6), and defining $\Delta_i \equiv (z_i/a)w'(z_i/a) - w(z_i/a)$ for $i = 1, 2$, we see that

$$\begin{aligned} s(a)(\Delta_2 - \Delta_1) &= \left[\frac{u'(c_2)f(y_2 - c_2)}{f'(y - c_2)} + u(c_2)\right] - \left[\frac{u'(c_1)f(y_1 - c_1)}{f'(y - c_1)} + u(c_1)\right] \\ &\quad - [z_2v'(z_2) - v(z_2)] + [z_1v'(z_1) - v(z_1)]. \end{aligned} \tag{7}$$

Simple differentiation plus the strict concavity of u, v (and the concavity of f) show that $zv'(z) - v(z)$ is decreasing in z while $[u'(c)f(y - c)/f'(y - c)] + u(c)$ is decreasing in c (for given y).

Using this information in (7) along with the fact that $z_2 > z_1$ and $c_2 < c_1$, we establish (4), as desired.

Let $z(a)$ be any selection from the optimality correspondence. Suppose, contrary to the lemma, that it jumps up at a. By the upperhemicontinuity of optimal choices, that implies (a) there are z_1^* and z_2^* with $z_2^* > z_1^*$, both optimal at a, (b) z_1^* is a limit point of optimal choices $z(a')$ for $a' < a$, and (c) z_2^* is a limit point of optimal choices $z(a')$ for $a' > a$. Note that (4) holds with $z_i = z_i^*$ for $i = 1, 2$, so that transposing terms,

$$(z_2^*/a)w'\left(z_2^*/a\right) - (z_1^*/a)w'\left(z_1^*/a\right) > w\left(z_2^*/a\right) - w\left(z_1^*/a\right).$$

Because $z^* > z_1^*$ and w is increasing, both terms in the inequality above are positive. So, because $s(a) \geq as'(a)$ by [W.3],[9] we can conclude that

$$\frac{s(a)}{a}\left[(z_2^*/a)w'\left(z_2^*/a\right) - (z_1^*/a)w'\left(z_1^*/a\right)\right] > s'(a)\left[w\left(z_2^*/a\right) - w\left(z_1^*/a\right)\right],$$

so that transposing terms again,

$$s(a)(z_2^*/a^2)w'\left(z_2^*/a\right) - s'(a)w\left(z_2^*/a\right) > s(a)(z_1^*/a^2)w'\left(z_1^*/a\right) - s'(a)w\left(z_1^*/a\right). \tag{8}$$

With (8) in mind, we can pick $a_1 < a$ and $a_2 > a$ (but close enough) along with z_i optimal for a_i and close enough to z_i^* for $i = 1, 2$, such that

$$s(\eta_2)\frac{z_2}{\eta_2^2}w'\left(z_2/\eta_2\right) - s'(\eta_2)w\left(z_2/\eta_2\right) > s(\eta_1)\frac{z_1}{\eta_1^2}w'\left(z_1/\eta_1\right) - s'(\eta_1)w\left(z_1/\eta_1\right) \tag{9}$$

for every η_1 and η_2 in the interval $[a_1, a_2]$.

Viewing $s(a)w(z_i/a)$ as a function of a, and applying the mean-value theorem,

$$s(a_1)w(z_i/a_1) - s(a_2)w(z_i/a_2) = (a_2 - a_1)\left[s(\eta_i)z_i/\eta_i^2 w'(z_i/\eta_i) - s'(\eta_i)w(z_i/\eta_i)\right] \tag{10}$$

for $i = 1, 2$, where η_1 and η_2 are the points in $[a_1, a_2]$ where the relevant mean values are attained. Combining (9) and (10), it follows that

$$s(a_1)w(z_2/a_1) - s(a_2)w(z_2/a_2) > s(a_1)w(z_1/a_1) - s(a_2)w(z_1/a_2). \tag{11}$$

Now, z_2 is an optimal choice at a_2, so in particular we have

$$u(c_2) + v(z_2) + w(z_2/a_2) \geq u(c_1) + v(z_1) + w(z_1/a_2),$$

where c_1 and c_2 are the levels of consumption corresponding to the choices z_1 and z_2. Applying (11) to this inequality, we must conclude that

$$u(c_2) + v(z_2) + w(z_2/a_1) > u(c_1) + v(z_1) + w(z_1/a_1),$$

but this contradicts the fact that z_1 is an optimal choice at a_1. ■

Let's now return to the main proof, and suppose that aspirations are frustrated at a_1: z_1 is an optimal choice with $z_1 < a_1$. Consider an increase from a_1 to a_2, with z_2 optimal at a_2. Then

$$u(c_1) + v(z_1) + \omega(z_1, a_1) \geq u(c_2) + v(z_2) + \omega(z_2, a_1),$$

[9] To see this, simply differentiate $s(a)/a$ with respect to a and use [W.3].

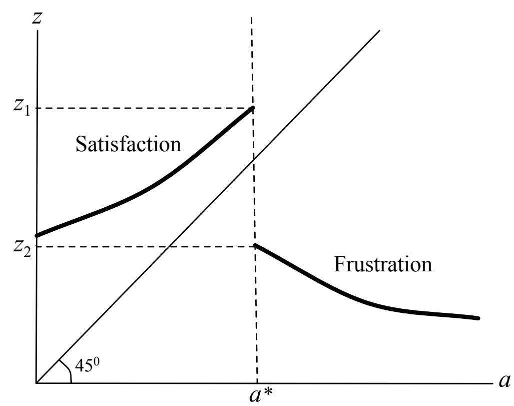


FIGURE 3. SATISFACTION AND FRUSTRATION AS ASPIRATIONS CHANGE.

where c_1 and c_2 are the levels of consumption corresponding to the choices z_1 and z_2, and likewise

$$u(c_2) + v(z_2) + \omega(z_2, a_2) \geq u(c_1) + v(z_1) + \omega(z_1, a_2).$$

Adding both these inequalities and transposing terms, we must conclude that

$$\omega(z_1, a_1) - \omega(z_2, a_1) \geq \omega(z_1, a_2) - \omega(z_2, a_2). \tag{12}$$

For a small increase in aspirations from a_1 to a_2, Lemma 1 implies that $\max\{z_1, z_2\} < a_1 < a_2$ for any optimal choice z_2 at a_2. But over this zone, the cross partial derivative

$$\frac{\partial^2 \omega(z, a)}{\partial z \partial a}$$

is strictly negative (for details, see this footnote).[10] It follows from (12) that z_1 must be no smaller than z_2. Moreover, the first order condition

$$v'(z_1) + \frac{s(a_1)}{a_1} w'(z_1/a_1) = u'\left(y - f^{-1}(z_1)\right) f^{-1'}(z_1)$$

can no longer hold when a_1 increases to a_2, so $z_1 > z_2$.

This argument can obviously be extended to any change in aspirations, small or not, as long as aspirations are frustrated to begin with.

The above argument, coupled with Lemma 1, also proves that the critical threshold of movement from satisfaction to frustration is unique. For once aspirations are frustrated, they can never be satisfied at higher levels of aspirations. ■

[10]We have $\omega(z, a) = s(a)w(z/a)$. Differentiating with respect to z, we see that $\omega_1(z, a) = \phi(a)w'(z/a)$, where $\phi(a) \equiv s(a)/a$. Differentiating the result with respect to a, we see that $\omega_{12}(z, a) = \phi'(a)w'(z/a) - \phi(a)w''(z/a)z/a^2$. We have $\phi'(a) \leq 0$ by [W.3], while $w''(z/a) > 0$ by $z < a$ and [W.2]. Therefore $\omega_{12}(w, a) < 0$, as claimed.

It should be noted that the crossover from satisfaction to frustration will usually (but not always) occur with a discrete fall in investment. This stems from the non-concavity of the benefit function. As illustrated in Figure 3, there will typically be a critical aspiration level a^* where there are two solutions z_1 and z_2 that are both optima, with $z_1 < a^* < z_2$.

As we've already seen, an increase in aspirations unambiguously lowers the marginal utility of wealth in the frustration zone and so discourages wealth accumulation. That drives the last argument in the proof of Proposition 2. However, in the satisfaction zone, the cross-partial above is of ambiguous sign, so that chosen wealth could increase or decline as aspirations rise. One way to see this is to evaluate the sign of the cross-partial derivative $\partial^2\omega/\partial z\partial a$ in the satisfaction zone, which is given by

$$\frac{\partial^2\omega(z,a)}{\partial z\partial a} = \phi'(a)w'(z/a) - \phi(a)w''(z/a)z/a^2,$$

where $\phi(a) = s(a)/a$ (see footnote 10 for details of this derivation).

These results have implications for mobility. If aspirations are commonly held and there is no uncertainty, there is no relative mobility (individuals maintain their rank in the income distribution) but the *shape* of the growth incidence curve over different incomes does have consequences for measures of "upward mobility" which assign higher weight to the growth rate of incomes among poorer individuals (see Genicot and Ray (2009)). The discouragement effect of rising aspirations among the poor — a result emphasized in Proposition 2 — will contribute to a reduction in any such mobility measure. The overall effect on mobility will depend on how investment is affected for the "satisfied." We know this to be ambiguous, so in some cases, the discouragement of the frustrated can be offset if investments rise for the satisfied.

It would be of interest to apply these ideas in social and economic situations that display visible increases in income and wealth, and are yet characterized by a substantial degree of poverty and inequality. Indian liberalization in the 1990s and thereafter present precisely such a picture. At one level, India is a vibrant and growing economy, particularly in sectors that are geared to exports, or contain a sizable foreign exchange component, such as business services. So it is little surprise that the Indian growth story has enjoyed particular visibility in the world at large. Moreover, the veritable explosion of social media, from television to the internet, has undoubtedly raised aspirations everywhere. The rise of an economically powerful urban middle class is certainly consistent with a story of burgeoning aspirations with salubrious effects on investment. But there is a second story to be told, in which large sections of the population are effectively delinked from the growth process. Economic inequality has risen substantially, both across income groups (see Banerjee and Piketty (2005)) and across sectors such as rural and urban, as well as within urban areas; see Deaton and Drèze (2002). A multitude of indicators — literacy rates, infant and child mortality rates, gender imbalances, access to sanitation or electricity — point to India's poor socioeconomic performance, not just relative to the developed world but to other peer groups, such as the BRIC countries or poorer neighbors such as Bangladesh; see, e.g. Drèze and Sen (2013). And along with the success stories that foreign investors so like to hear, there is a subtext of apathy and despair, violence and conflict, driven by increased perceptions of economic inequality coupled with the large displacements of land, capital and labor that are endemic under uneven growth. Whether the potential for frustration caused by rising aspirations plays a central role in this story deserves more investigation and research. But the observations are *prima facie* consistent with such a story.

3.2. **Changes in Wealth.** Next, we turn to changes in current wealth y, holding aspirations constant. As far as the optimal choice of next generation's wealth is concerned, the argument is intuitive and uncomplicated:

Proposition 3. *For given aspiration a, there is a unique threshold value of current wealth below which aspirations are frustrated, and above which they are satisfied. Optimally chosen wealth for the next generation is nondecreasing in current wealth.*

The proof of this result follows from a standard single-crossing argument based on revealed preference, and is therefore omitted. In what follows we study how the *growth rate* of wealth varies along the cross-section of "starting wealths". To do so, we provide some more structure on preferences, as well as on the production function. Write $\omega(z, a)$ as $s(a)w(z/a)$, and suppose that u, v and s have the same constant-elasticity functional form; i.e., $u(c) = c^{1-\sigma}/(1-\sigma)$, $v(z) = \rho z^{1-\sigma}/(1-\sigma)$ and $s(a) = a^{1-\sigma}$ for some $\sigma > 0$ and $\rho > 0$.[11] Suppose, moreover, that the production function is linear: $f(k) = (1+r)k$, where r is the rate of return on investment.

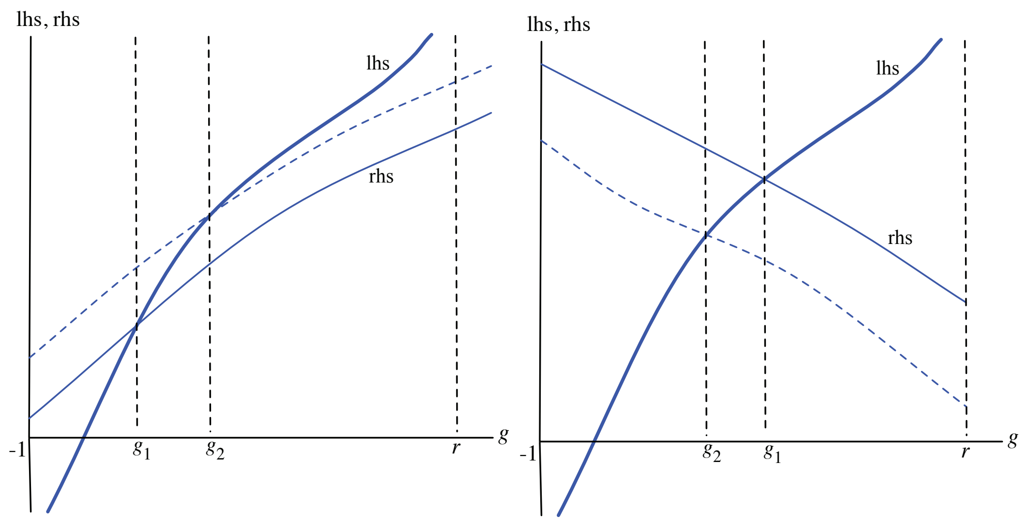
Given constant elasticity, the use of a common elasticity term σ for the utility and aspirations components is all but unavoidable (once we incorporate the notion that aspirations move in tandem with income). To see why, imagine scaling up aspirations and income together, which is what will happen in the sequel when incomes are growing and aspirations are growing along with incomes. If the elasticities are not the same, then at least one of these three terms will either become relatively insignificant or unboundedly dominant. For instance, if $s(a)$ is linear, then as the economy grows the aspirations effect will become all-dominant relative to the intrinsic utility of consumption. The reverse could also occur. If, for instance, $s(a)$ is a constant and $\sigma \in (0, 1)$, then the entire question of aspirations is ultimately unimportant with growth. To retain the relative importance of both intrinsic consumption and aspirations, we use the same elasticity for each of these functions.

The expositional advantage of constant-elasticity utility with linear production is that, in the absence of an aspirations effect, bequests are proportional to wealths and therefore growth rates are constant across the cross-section of current wealths. We can therefore be sure that any cross-sectional variation in the presence of aspirations stems entirely from aspirations alone. We would like to describe the *growth incidence curve*, a relationship that links baseline income to subsequent rates of growth.

Recall the maximand (2), substitute the specific functional forms, and define the growth rate of income as $g \equiv (z/y) - 1$, so that the individual now maximizes

$$(13) \qquad \frac{1}{1-\sigma}\left(y\left[\frac{r-g}{1+r}\right]\right)^{1-\sigma} + \frac{\rho}{1-\sigma}\left([1+g]y\right)^{1-\sigma} + a^{1-\sigma}w([1+g]y/a).$$

[11]It should be noted that we do not divide by $1-\sigma$ in the term $s(a)$. This is as it should be, for we want $\omega(z, a)$ to be always increasing in z, for fixed a.



(A) Frustrated Aspirations (B) Satisfied Aspirations

FIGURE 4. GROWTH RATES AND INCOME.

by choosing g, for each y. As we've already noted, this problem is nonconvex and may exhibit more than one solution. However, any solution is interior in the choice of g and is therefore described by the necessary first-order condition

$$(r-g)^{-\sigma}(1+r)^{\sigma-1} - \rho(1+g)^{-\sigma} = w'\left([1+g]y/a\right)(y/a)^{\sigma}. \tag{14}$$

The chosen growth rate will lie between a minimum of -1 and a maximum of r. Figure 4 describes how the rate of growth g is determined by this first-order condition. The upward-sloping line with unbounded endpoints is the left hand side of (14). The right hand side has different slope depending on whether aspirations are frustrated or exceeded. When $(1+g)y < a$ the right hand side also increases with g (Panel A); once $(1+g)y \geq a$, it declines in g (Panel B). Thus in Panel A, where aspirations are frustrated, there could be several potential solutions, as already observed. The second-order condition assures us, however, that we only need to consider those intersections in which the right hand side cuts the left hand side from above; see the point g_1. (Even that isn't enough to fully pin the solution down, but it is certainly necessary.) In Panel B, there is a unique solution.

Now we conduct the exercise of increasing current wealth y. When aspirations are frustrated, the right hand side of (14) is unambiguously shifted upwards. Panel A of Figure 4 suggests that the new growth rate is higher. While this intuitive argument in quite far from a formal proof, it motivates the following two propositions.[12]

Proposition 4. *Suppose that aspirations are held constant. Consider any income level y at which aspirations are frustrated. Then the growth rate of income declines as incomes fall below y.*

[12]In particular, local second-order conditions are not sufficient for optimality, and a change in y could move the optimal choice to an entirely different location instead of simply precipitating a local change. The proofs of Propositions 4 and 5 therefore employ revealed-preference arguments that are not based on local conditions.

For a proof, see the Appendix.

Propositions 2 and 4 together capture a notion of "failed aspirations". When aspirations are already frustrated, anything that magnifies the aspirations gap creates a perverse reaction. Instead of incentivizing individuals, such high aspirations (relative to income) can lead to a sense of despair, along with lower investments in the future. In particular, both higher aspirations and lower incomes can independently damage the capacity to save. This observation is in line with the arguments in Appadurai (2004) and Ray (1998).

What of initial wealth levels for which aspirations are satisfied? The answer depends on the behavior of the right-hand side of the first-order condition (14) as income rises. Panel B of Figure 4 is drawn on the presumption that the following condition holds:

[W.4] $w'(x)x^{\sigma}$ is declining in x when $x > 1$.

How reasonable is [W.4]? If w has unbounded steepness at 1, the condition *must* hold for some region above 1. Whether it holds more globally will depend on the value of σ, as well as the specific form of w, and in particular on the degree of concavity exhibited by it when $x > 1$.[13]

Our next result is the analogue of Proposition 4 for the case in which aspirations are satisfied.

Proposition 5. *Assume* [W.4]. *Suppose that aspirations are held constant. Then growth rates decline as incomes increase, once aspirations are satisfied.*

In contrast, then, Proposition 5 attempts to deliver a notion of "complacency" when aspirations are satisfied. In this case, and in opposition to Proposition 4, an increase in baseline income lowers subsequent growth. In summary, these propositions together represent a formal statement of our assertion that "attainable" aspirations, which can be met by a round of sustained growth, are the most conducive to investment.

Does it follow that growth is unambiguously inverted-U shaped over the cross-section of incomes, and maximized at "intermediate" levels of income? Certainly, when aspirations are commonly held and [W.4] applies, equilibrium rates of growth rise and then fall over the cross-section, though the relative sizes of the two segments will depend on just where the common aspirations happen to be placed. As in the discussion following Proposition 2, the implications for "upward mobility" are ambiguous. Compared with the same average growth equally distributed among the population, higher growth rates in the middle of the distribution rather than at the top raise upward mobility, but the lower growth rates at the bottom reduce upward mobility.

The inverted-U shape is, however, a consequence of the assumption that aspirations are commonly held. When y also affects aspirations, the theory could make other predictions regarding the shape of growth rates in income. It is possible, for instance, that every income grows at exactly the same rate, once the initial distribution is suitably chosen: the analogue of a "growth steady state" with aspirations. That said, it is nevertheless of conceptual interest to record the "partial effect" of starting income on subsequent growth, which is precisely what Propositions 4 and 5 do. In a more general setting, to which we now turn, both aspirations and income will move in tandem, and affect subsequent rates of growth.

[13]It requires $\frac{-w''(x)x}{w'(x)} > \sigma$.

4. The Joint Evolution of Aspirations and Incomes

In the previous section, we emphasized some partial effects of aspirations and wealth on the subsequent growth of incomes. Once we recognize that aspirations and incomes evolve jointly, these effects intertwine, depending on the precise manner in which aspirations are formed. Recall that for every individual currently at wealth y, aspirations are generated by the equation

$$a = \Psi(y, F),$$

where F is the going distribution of wealth. Such a closure of the model generates a society-wide equilibrium starting from any initial distribution F_0. We therefore obtain a sequence of income distributions $\{F_t\}$ that are linked in the way described in Section 2.3.

4.1. **Growing Paths and Inequality.** There are several questions that one can ask of such a formulation. For instance:

(i) Does the general equilibrium of aspirations and income foster income inequality in "steady state"?

(ii) What is the relationship between equilibrium inequality and growth?

(iii) Is higher productivity more conducive to equality?

There is a trivial sense in which a perfectly equal income distribution is invariably an equilibrium of this model. Consider the maximization problem (2) for a single agent with income $y(0)$ in which aspirations a are also equal to $y(0)$. This problem will have a solution in choice of continuation income z; let $z = y(1)$. Re-do the maximization problem with initial income $y(1)$ and aspirations $y(1)$, and continue in this fashion, *ad infinitum*. Consider the resulting sequence of incomes $\{y(t)\}$, and define a sequence of entirely equal income distributions concentrated on these incomes. Given Assumption A, it is easy to see that such a sequence constitutes an equilibrium.[14]

The problem is that the resulting equilibrium path might have no discernible trend. For example, incomes could perpetually oscillate. While such paths are technically equilibria, they do not appear to be particularly interesting. We rule out such cases by focusing on equilibria with nondecreasing trajectories. This includes the possibility that equilibrium paths might converge, as well as the possibility of unbounded growth. Convergence becomes relevant whenever the production function is strictly concave, satisfying familiar end-point conditions, while sustained growth is germane when the production function is linear.

Given Assumption A that aspirations lie in the support of the going income distribution, the answer to the question of growth with equality hinges on the behavior of an "auxiliary" problem in which aspirations are perpetually reset to current income. That is, we suppose the agent to

[14] It should be noted that these arguments and the main parts of the arguments to follow do not rely on the literal restriction that aspirations lie in the range of observed incomes, and so must be equal to income in the case of perfect equality. With some expositional clumsiness, we can make the same essential points when (under perfect equality) a is some constant multiple of y.

maximize, by choice of continuation income z,

$$u\left(y - f^{-1}(z)\right) + v(z) + \omega(z, y). \tag{15}$$

The following observation is then immediate from the discussion so far:

Proposition 6. *Perfect equality with nonnegative growth is an equilibrium if and only if the maximand in (15) admits a solution with $z \geq y$ for every y.*

In the three corollaries that follow, we derive the implications of this proposition in some special cases of interest. Consider, first, the classical "Solow setting," in which the production function f is concave and satisfies the end-point conditions: $f(x) > x$ for x small enough, and $f(x) < x$ for all x large enough. Our first corollary states that if marginal incentives to accumulate are strong when aspirations are just about met, then a steady state with income equality is unattainable. The intuition is simple: when income is excessively bunched around a common value, then aspirations are bunched there too, and there is a large gain to be had in accumulating a bit more relative to others. (In our model, this gain comes through the relative ease of meeting and exceeding one's aspirations.) That leads to a race to the top in which all agents accumulate too much. Eventually, the pressure must ease as symmetry is broken by some agents favoring present consumption instead and falling behind in accumulation. The resulting outcome separates near-identical agents, thus destroying equality.

Corollary 1. *Consider the Solow setting and suppose that $\omega_1(a, a) = \infty$ for all a. Then perfect equality with growth cannot be sustained as an equilibrium from any initial $y > 0$.*

This corollary makes use of the restriction that $\omega_1(a, a) = \infty$. That creates a strong force that must break the symmetry of perfect equality. Note that ω_1 does not *literally* have to be infinite; a large enough slope will generally suffice to generate unequal outcomes.

At the same time, we do not wish to give the reader with the impression that growth with equality is plainly impossible in our model, only that the two may be incompatible in some interesting special cases. To underline this qualification, we now allow for the possibility of sustained growth by dropping the Solow assumptions: $f(k) = (1 + r)k$ for some positive rate of return r on capital. We also assume constant-elasticity specification of preferences as introduced above: $u(c) = c^{1-\sigma}/(1 - \sigma)$, $v(z) = \rho z^{1-\sigma}/(1 - \sigma)$ and $s(a) = a^{1-\sigma}$ for some $\sigma > 0$ and $\rho > 0$. Corollary 2 provides conditions for equality with growth in this linear model.

Corollary 2. *With linear production and constant-elasticity preferences, perfect equality with growth can be sustained if and only if some $g \geq 0$ solves the problem:*

$$\max_g \frac{1}{1-\sigma}\left[\frac{r-g}{1+r}\right]^{1-\sigma} + \frac{\rho}{1-\sigma}[1+g]^{1-\sigma} + w(1+g). \tag{16}$$

Linear production permits sustained per-capita growth *rates*, and is therefore more conducive to the retention of symmetry (or equality) across individuals, even when w displays high steepness. Writing down the first-order condition to the maximization problem, we know that for any interior solution of growth rate g,

$$(r-g)^{-\sigma}(1+r)^{\sigma-1} - \rho(1+g)^{-\sigma} = w'(1+g)\,. \tag{17}$$

From (17), it follows that a *necessary* condition for there to be a growth rate $g > 0$ that solves (16) is

$$w'(1) + \rho > \left(\frac{r}{1+r}\right)^{-\sigma} \frac{1}{1+r},$$

while a sufficient condition is given by

$$r(1+r)^{\frac{1-\sigma}{\sigma}} \rho^{\frac{1}{\sigma}} > 1,$$

which guarantees that all solutions to (16) have positive growth rates.

These conditions suggest that societies with higher production capabilities are more able to sustain equitable outcomes that are compatible with continuing growth. In order to formalize this intuition and generalize it to other production technologies, consider a class of production functions of the form $y = Af(k)$, parameterized by the total factor productivity term A. Our next and last corollary to Proposition 6 shows that higher productivity is conducive to the coexistence of equality and growth:

Corollary 3. *If perfect equality with growth is an equilibrium from all initial $y_0 \geq \bar{y}$, where $\bar{y}$ is any threshold, then following an increase in the productivity term A, there is still a growing equilibrium path with perfect equality from every initial $y_0 \geq \bar{y}$.*

Next, we examine the nature of the limit distribution(s) more closely in special cases.

4.2. **Common Aspirations: Equilibrium Inequality and Steady State Comparisons.** Consider first the case of common aspirations in the Solow setting. To avoid complications that might arise from the standard model without aspirations, we are going to impose restrictions that ensure a unique steady state income in that model. That is, consider the artificial benchmark without any aspirations at all, in which the individual chooses z to maximize

$$u(y - k(z)) + v(z)$$

where recall that $k(z)$ is just $f^{-1}(z)$ (for ease of notation). A (non-zero) steady state y in this model is characterized by the condition

$$d(y) \equiv -\frac{u'(y - k(y))}{f'(k(y))} + v'(y) = 0. \tag{18}$$

We will assume in what follows that

[D] $d(y)$ is decreasing in y.

Condition D guarantees a unique steady state in the benchmark model. While this assumption is not implied by the other conditions imposed so far, it is not very restrictive. The second term in the definition of $d(y)$, so it is a question of what happens with the first term. It is easy to see that the first term is decreasing whenever the utility function exhibits high curvature relative to the production function.[15] For instance, if $f(k) = Ak^\alpha$, then the first term in (18) decreases in y whenever $-u''(y)y/u'(y) \leq (1-\alpha)/\alpha$ for all y.

We now return to the model and study steady states with common aspirations.

Proposition 7. *Consider the Solow setting and impose* [D]. *Then a steady state distribution is generically clustered on a discrete set of incomes, with only one income in this collection at which aspirations are satisfied.*

Proof. Suppose that income distributions converge to some limit distribution F, with attendant aspirations a (common to all) within the support of F. Then for each income y in the support of the limit distribution, an individual solves the maximization problem in (15). Proposition 3 informs us that the optimal choice of z must be nondecreasing in y. It follows that if F is a stationary distribution, then y must map into y again. By [U] and the unbounded steepness of v, the solution must be interior for every $y > 0$, and so the (necessary) first-order condition informs us that for every positive y in the support of the limit distribution,

$$D(y, a) \equiv -\frac{u'(y - k(y))}{f'(k(y))} + v'(y) + \phi(a)w'(y/a) = 0 \tag{19}$$

where $\phi(a) = s(a)/a$.

In a steady state with perfect equality $y = a$ and the left hand side of (4.4) becomes $D(y, y) = d(y) + \phi(y)w'(1)$. It follows from [D] and [W3] that $D(y, y)$ is decreasing in y. If $D(y, y) > 0$ for all y (for instance if the conditions of Corollary 1 are satisfied), then there doesn't exist a limit distribution with perfect equality. Otherwise, there is a unique income level y^* that satisfies $D(y^*, y^*) = 0$. For this level of aspirations y^*, there can be no other solution to (4.4). Hence, the above necessary condition must be sufficient, and we have found a limit distribution with perfect equality.

When (4.4) fails at $y = a$, then perfect equality is not achieved, and there *must* be solutions to this equation *both* to the right and the left of a. Generically, there must be finitely many such solutions, so that the steady state distribution develops multiple poles. By the local strict concavity of the maximization problem for continuation incomes that exceed a, there can be only one solution above a, so among the subpopulation for whom aspirations are satisfied, there is full convergence. In contrast, there could be one or more poles in the "frustration zone" below a. ■

Clustering of incomes is a robust feature of the common aspirations model, but it goes without saying that the convergence to degenerate poles is not to be taken literally. When there are stochastic shocks, the distribution will always be dispersed. But the point is that there will be a tendency for the distribution to exhibit local modes: one above the common aspirations level, and one or more modes below it. We illustrate this discussion with an example.

Example 1. Suppose that $u(c) = \ln(c)$ and $v(z) = 0.8\ln(z)$, while ω has the logistic structure

$$\omega(a, z) = \frac{0.8b}{1 + \exp\left[-\kappa(\frac{z}{a} - 1)\right]}. \tag{20}$$

[15] The condition $\frac{-u''(y-k(y))}{u'(y-k(y))} \geq \frac{-f''(k(y))}{1-f'(k(y))}$ is sufficient.

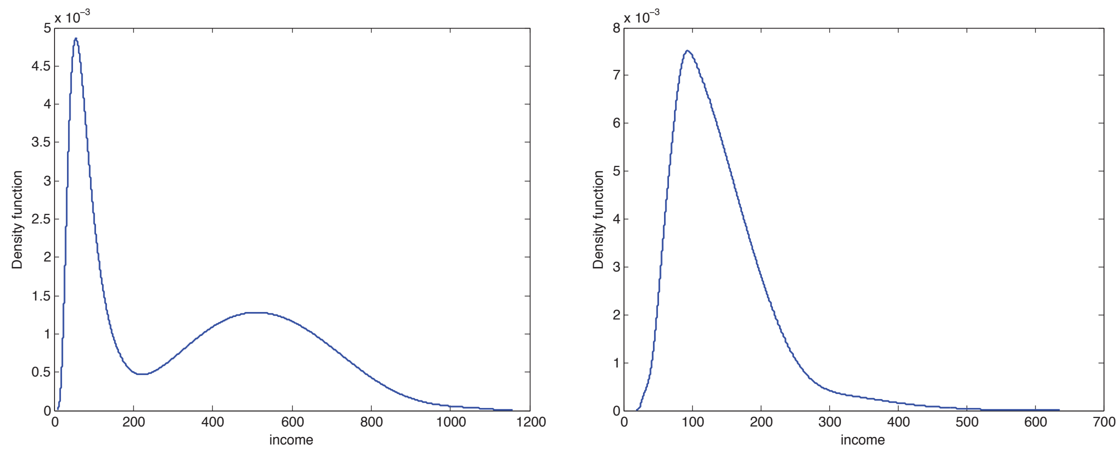


FIGURE 5. POLARIZATION AND COMMON ASPIRATIONS.

For this example set $b = 2$. Note that the parameter κ controls the steepness of w at the point $z = a$, where aspirations are met, while leaving the level of utility w at $z = a$ unaffected. In order to get non-degenerate (and therefore more realistic) distributions of income, we introduce some noise in the production function. We take the production function to be $f(k, \theta) = 5\theta k^{0.8}$, where $\alpha = 0.8$ and θ is a stochastic shock with mean 1.[16] We set aspirations at the median income (but any interior specification would work as well), begin with an initial distribution of income that is uniform over a population of 800 individuals, and iterate the distribution over time. The simulated distributions converge to a steady state (where the only mobility is due to the noise in the production function).

When κ is large enough so that w is suitably steep at $z = a$, equality is impossible and the distribution converges to a bimodal limit, in line with the discussion above. Over time, the distribution of income clusters around two poles. The first panel of Figure 5 illustrates this outcome for $\kappa = 5$.[17]

If the value of κ is lower, then w is relatively flat at $z = a$ and full equality is possible. In the second panel of Figure 5, constructed for $\kappa = 0.5$, we see convergence to a unimodal income distribution where all differences in income are only due to the noise.

These observations can be usefully related to different aspects of the literature on evolving income distributions. The closest relationship is to endogenous inequality, in which high levels of equality are destabilized by forces that tend to move the system away from global clustering. In Freeman (1996) and Mookherjee and Ray (2003), this happens because of imperfect substitutes among factors of productions, so that a variety of occupations with different training costs and returns *must* be populated in equilibrium. Together with imperfect capital markets, this implies that in steady state, there must be persistent inequality, even in the absence of any stochastic

[16] Specifically, we suppose that θ follows a lognormal distribution. The qualitative results do not depend on the magnitude of the noise term, though in general, the degree of clustering must rise as the variance of the shock falls.

[17] In the figures, we smoothed the simulated distribution using the density estimator "ksdensity" for Matlab.

shocks. In related work, Becker, Murphy, and Werning (2005) and Ray and Robson (2012) argue that endogenous risk-taking can also serve to disrupt equality, as relative status-seeking effectively "convexifies" the utility function at high levels of clustering.

There is evidence of multimodality in the income distribution of various countries, including the United States (see Pittau and Zelli (2004), i Martin (2006) and Zhu (2005)). The clustering of incomes into local poles also speaks to the work of Durlauf and Johnson (1995), and Quah (1993, 1996).[18] These authors make a strong case for local clustering in the world income distribution and argue that convergence is a local phenomenon "within the cluster" but not globally. Durlauf and Quah (1999) summarize by writing that there is an "increase in overall spread together with [a] reduction in intra-distributional inequalities by an emergence of distinct peaks in the distribution". This is consistent with a common aspirations model, though we do not mean to suggest that this is the only force at work.

We now turn to a comparison of inequality across steady states.

Proposition 8. *Consider two unequal steady states with common aspirations a and a', with $a < a'$, either due to multiple steady states under the same parameters or because the aspirations formation process was altered. Then every steady state income below a declines. If $s(a) = a$, the unique income level above a must rise.*

Proof. Note that every steady state income y below a exhibits frustrated aspirations, so that by Proposition 2, if $k = f^{-1}(y)$ was an optimal choice at y, the new optimal choice must fall, leading the agent located at y to some lower steady state.

On the other hand, there is just one steady state income h that exceeds a. $D(h, a) = 0$ while $D(y, a) < 0$ for all $y > h$. Now consider h', the highest income at a new steady state with aspiration $a' > a$. If $s(a) = a$, then $\phi(a) = 1$ and $\omega_{za}(z, a) > 0$ when $z > a$. In this case, the increase from a to a' must raise the value of $\omega_1(h, a)$. It follows right away that $h' > h$, and the proof is complete. ∎

By an alteration to the aspiration formation process, we mean a change in the mapping Ψ. An example of such an alteration, shown in Example 2, is a change in the relative weights put on different parts of the income distribution when aspirations consist of a weighted average of the incomes. Notice also that while $s(a) = a$ is a sufficient condition for $\omega_{za}(z, a) > 0$ (which guarantees that the income level of the "satisfied" rises), this cross derivative is positive in many other situations, as illustrated in the following example.

Example 2. In this example, we assume the same production function as in Example 1 but without noise. We consider the same functional form for preferences as in (20) but with $b = 20$ and $\kappa = 5$. Consider an initial income distribution with two levels of income: the poor at $\ell = 10$ and the rich at $h = 40$, and common aspirations set at $a = \pi h + (1 - \pi)\ell$. We iterate the income distribution over time until the simulated distribution settles at a steady state. Consider two cases. In case (a), the weight π on the income of the rich in determining aspirations is 80%, while in case (b), $\pi = 60\%$. For instance, the media might showcase more or less the lifestyle of

[18] See also Henderson, Parmeter, and Russell (2008), Canova (2004) and Pittau, Zelli, and Johnson (2010).

the rich, resulting in these changed weights. The following table shows the income distributions and aspirations at the steady states.

Case	Income of the Rich	Income of the Poor	Aspirations
(a)	93.5	307.1	264.4
(b)	83.5	572.3	376.7

We see that case (b) exhibits higher aspirations than in case (a). At the same time, the poor are poorer, and the rich richer relative to case (a).

4.3. **Stratified Aspirations.** The emergence of clustering — and the number of such clusters or poles — is worth further investigation. One extension of interest is to the case of stratified aspirations, in which different income segments of the economy each harbor common aspirations, but those aspirations vary across segments. For instance, we might think that the economy is divided among the "poor", the "middle class", and the "rich", and each inhabitant of this coarse classification has common aspirations drawn from the going (or anticipated) income distribution.

Recall that under stratified aspirations, the income distribution is segmented into n quantiles with aspirations

$$\Psi(y, F) = a_i$$

for individuals with income y in quantile i, where a_i is a scalar representing some summary statistic of the distribution in that quantile.

Proposition 9. *Under stratified aspirations with n segments, if $\{F_t\}$ is an equilibrium sequence of income distributions converging to some nondegenerate limit distribution F, then F must generically be concentrated on a finite set of points.*

The proof of this proposition is a direct extension of the argument made for common aspirations, and we omit it. In the following numerical example, we contrast stratified and unstratified aspirations and illustrate two interesting features of stratified aspirations. First, narrower aspirations windows tend to reduce inequality by keeping aspirations closer to current incomes and thereby retaining incentives. Second, even in a deterministic model, relative mobility (a reranking of individuals in the income distribution) is possible.

Example 3. In this exercise, we take the preference and production function to be the same as in the "steep" case ($\kappa = 5$) of Example 1, but remove all noise. The thought experiment is as follows. Consider a society in which individuals are "cognitively stratified" into two income classes, perhaps as a result of social or spatial segregation by income. The poorer half of the population draws their aspiration from the median income *among the poor* while, in similar fashion, the richer half use the conditional median income of their group. In this case, the distribution develops multiple poles and exhibits some mobility (despite the absence of noise). In the numerical example illustrated in Figure 6, a group of poor individuals cluster around an income of just below 100, while a group of rich individuals earn around 860. Both experience hardly any mobility. In the middle, groups of individuals earn between 200 and 500 and experience some mobility with their dynasties switching regularly from one class to the other.

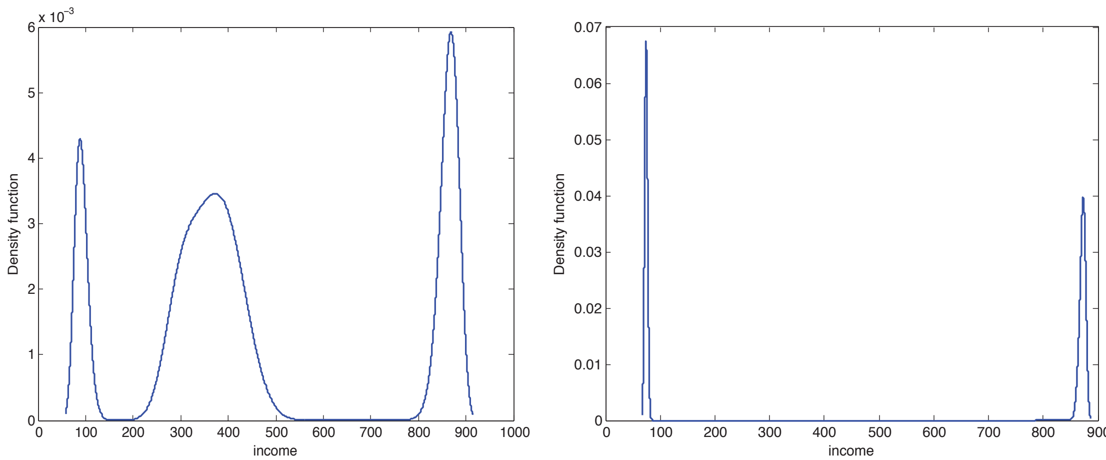


FIGURE 6. STRATIFIED ASPIRATIONS.

It is possible to contrast this outcome with that in a similar society with common aspirations. Say that owing to less segregation or higher media exposure, individuals learn more about the incomes of the entire population and aspirations are commonly tagged at the median income. In this society, the distribution becomes much more polarized and converges to one with twin peaks around 80 and 870 with zero mobility (there is no production noise, in contrast with the previous example). With stratified aspirations, aspirations windows are smaller and aspirations consequently more attainable. This permits the emergence of a middle class, generates an income distribution with less inequality, and points to the psychological importance of maintaining realistic aspirations.

The point of these last two sections is to argue that if aspirations are stratified, or come in discrete steps that pertain to entire segments of income, then the resulting distribution must exhibit clustering, or convergence to local modes (unless there is convergence to global equality). This is not to say that such clustering is an inevitable outcome of an aspirations model. This is what we turn to next.

4.4. **More on Clustering.** Consider any steady state of a dynamic equilibrium with aspirations, with or without growth. If that steady state does *not* exhibit clustering around a finite set of incomes (or balanced-growth income paths), it must be the case that a continuum of paths, indexed by an interval of starting incomes, all solve the individual maximization problem, so that the first order condition (4.4) — modified to allow for growth — holds over that corresponding interval:

$$-\frac{u'(y-k(y,g))}{f'(k(y,g))} + v'(y[1+g]) + \phi(a)w'([1+g]y/a) = 0$$

where $k(y,g)$ is given by $f(k(y,g)) = y(1+g)$. For every g, this requirement necessarily pins down how aspirations must move with initial income y. That function, must, in turn, be generated by the going distribution of income. We conjecture, though have not proven, that for "generic" specifications of the utility and production function, and the aspirations formation process, that this will be impossible. In other words, the clustering into local poles identified in the previous sections is the norm rather than the exception. We leave a fuller exploration of this question for future research.

A remark on exceptions: in special cases, it is indeed possible to construct examples in which a continuum of incomes appear in a steady state with balanced growth. Suppose that the production function f is linear, the utility indicators u and v are logarithmic, and that $\omega(z, a) = w(z/a)$ (so that $s(a) = 1$). Suppose, moreover, that the aspiration of an individual with income y is given by the conditional expectation of all incomes higher than y in the going distribution; that is:

$$a = \Psi(y, F) = \frac{\int_y^\infty x dF(x)}{1 - F(y)}.$$

With these specifications in place, the maximization problem (13) is equivalent to choosing g to maximize

$$\ln\left(\frac{r - g}{1 + r}\right) + \delta \ln(1 + g) + w([1 + g]y/a).$$

for each y. It follows that all individuals will choose the same growth rate, irrespective of their income level, if aspirations move exactly in the same ratio as income, so that y/a is constant over the support of the distribution. With upward-looking aspirations, the Pareto distribution is the only income distribution that keeps this ratio constant. See the appendix for a proof of this observation.

In summary, under these specific functional forms: if incomes at time t follow a Pareto distribution, investment choices result in the same growth rate for all individuals, say g^*. It is then easy to see that incomes at time $t + 1$ follow a Pareto distribution as well, and the argument can be repeated indefinitely.

4.5. **The Welfare Economics of Aspirations.** Begin with a standard model in which aspirations play no role. Consider a dynasty linked by a sequence of altruistic individuals, so that each generation t's utility is given by

$$\Psi(c_t, y_{t+1})) \equiv u(c_t) + v(y_{t+1}) \tag{21}$$

where y_t and c_t are lifetime income and consumption at date t. A social planner who maximizes a utilitarian welfare function of the form

$$\sum_{t=0}^{\infty} \beta^t \Psi(c_t, y_{t+1})$$

will invariably wish to exhort each generation to save more than they do under the specification (21), *even if they are altruistic*. That is because generation t seeks to maximize $\Psi(c_t, y_{t+1})$ subject to the usual constraint $y_t = c_t + f^{-1}(y_{t+1})$, just as in the analysis thus far, but by an envelope argument this is never enough, as long as $\beta > 0$. The argument is quite general and extends to Barro-style dynastic utility as well, as long as such intertemporal altruism is motivated by hedonistic pleasure and is not itself viewed as a social obligation on the part of individuals.[19]

[19]As pointed out by Bernheim (1989) and Ray (2013), it also leads to dynamic inconsistency on the part of the planner, though we ignore this aspect in the present exercise.

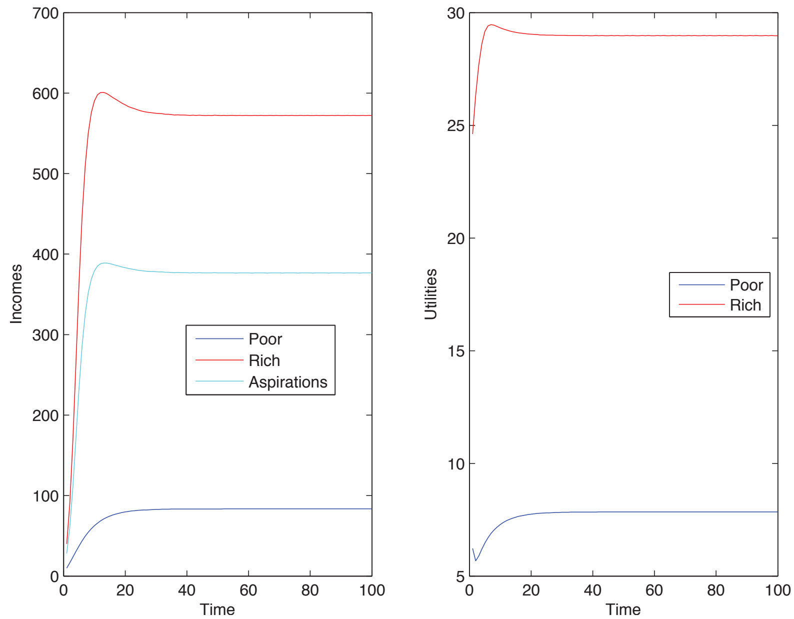


FIGURE 7. FRUSTRATION DUE TO RISING ASPIRATIONS.

We provide this background to motivate the welfare economics of aspirations formation. One might view the additional aspirational term $\omega(z, a)$ as an instrument to exhort additional savings from each generation, in the interests of future generations. An individual may herself be unhappy in the process, as the raising of a will lower her own lifetime utility (both in the model and in real life, higher goals are never intrinsically rewarding, only and at best the *fulfillment* of those goals). But when higher aspirations encourage investment (see Section 3.1 for the comparative statics), her descendants will be happier as a result. Thus a social planner may want to raise aspirations despite the lifetime utility costs that such aspirations impose on individuals. But this is a delicate business: a crossing of the frustration threshold is bad all around, both from the viewpoint of current utility as well as investments for the future, as our analysis has already shown.

This utility cost of raising aspirations — and the associated risk that high enough aspirations will result in despair and even conflict — is related to the *paradox of relative frustration* highlighted by Alexis de Tocqueville. Studying the French Revolution, de Tocqueville (1856) remarked that frustration increased as social conditions improved and living standards rose: “So it would appear that the French found their condition the more insupportable in proportion to its improvement.” (de Tocqueville (1856), p. 214). This type of situation arises in Example 2. Figure 7 illustrates the evolution of incomes and utilities of the poor and rich of Example 2, in case (b). We see that at times both the rich and the poor see their incomes rise while their welfare declines due to rising aspirations. Frustration and better living conditions may grow together. The welfare economics of aspirations presents interesting conceptual angles for future research.

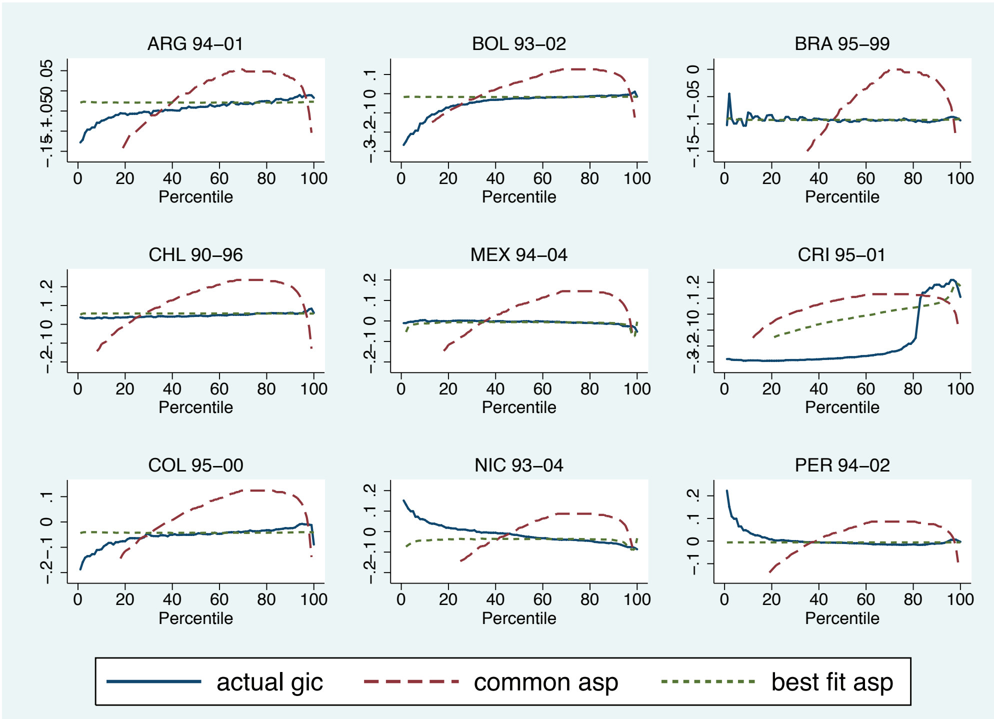


FIGURE 8. GROWTH INCIDENCE CURVES

5. AN ILLUSTRATIVE EMPIRICAL EXERCISE

In this section, we use percentile distributions available for 43 countries over at least two distinct years to illustrate the growth incidence curves predicted by different models of aspirations formation.[20] Throughout, we take $\omega(z, a)$ to have a CARA shape in z/a above 1 and be symmetric around 1,[21] and set $\rho = 0.8$.

In our first exercise, we presume that there are common aspirations tagged to societal mean income. For any distribution, a given return to capital r generates specific individual growth rates as a function of income. These incomes and growth rates imply a specific aggregate growth rate for each country. In this exercise, we find the country-specific returns to capital r so that our model generates the *actual* aggregate annual rate growth observed in each country. Figure 8 shows the resulting growth incidence curves for nine Latin American countries in the 1990s.[22] Propositions 4 and 5 predict that with common aspirations these growth rates follow an inverted U-shape in income, and indeed the same is true of this specific exercise.

[20] Special thanks are due to Claudio Montenegro at the Development Research Group, Poverty Unit, The World Bank.

[21] Specifically, we presume that $\omega(z, a) = 10 - e^{-20(z/a-1)}$ for all $z \geq a$, and $= 8 + e^{-20*(1-z/a))}$ for all $z < a$.

[22] The graphs for the remaining countries are available on request.

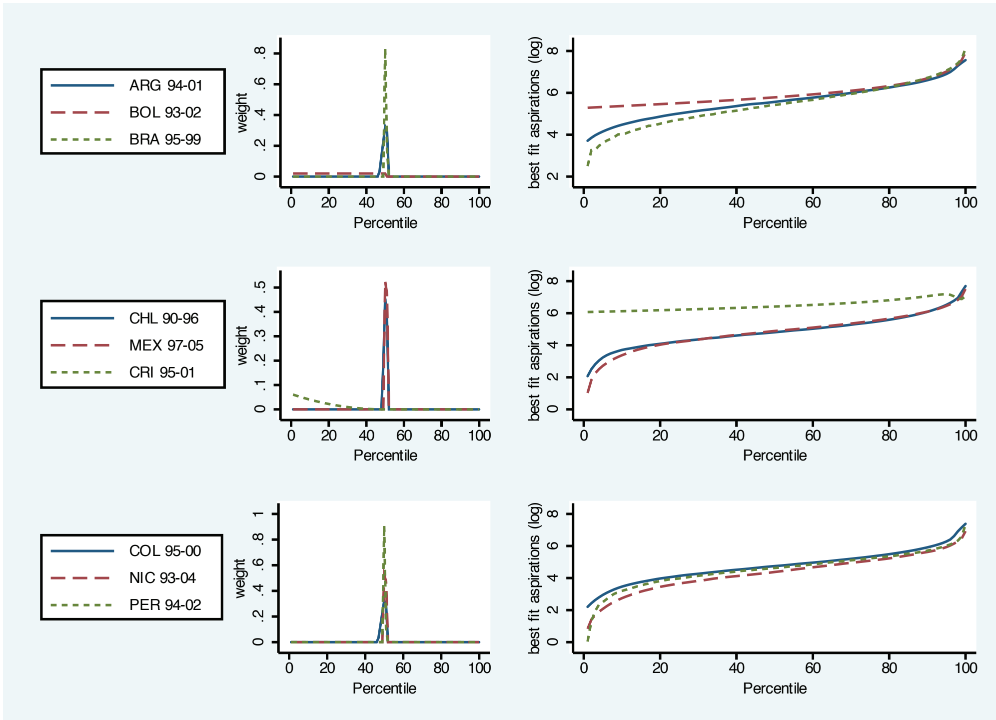


FIGURE 9. WEIGHTS AND ASPIRATIONS LEVELS

Clearly, the observed growth pattern by percentiles in these countries has, in most cases, a different shape. This is not surprising as our simple model does not even come close to capture the many factors that drive growth rates by income classes. But in addition to that, there is no reason to believe that aspirations are commonly held. That suggests the following thought experiment: using these data and our model, we can estimate the aspirations formation process that *best fits* the actual growth incidence curves. Our second exercise does just this.

Our data consists of 55 percentile distribution and growth incidence curves. For each of these, we search among a class of aspiration formation processes where (1) the aspirational weight placed by percentile i on another percentile income j only depends on the percentile distance $d_{ij} = |i - j|$ and on whether i is richer or poorer than j; and (2) these weights have a quadratic shape in the percentile distance on either side of i (but are not necessarily symmetric around i).[23] This class includes among others common and upward-looking aspirations formation processes. As before, the return to capital r in any country is selected to match the actual aggregate annual rate growth observed for that country.

The growth incidence curves predicted by our "best fit" aspirations are illustrated in Figure 8. Although they are not perfect matches for the observed curves, we see that they come quite close.

[23]More precisely, let the term α_{ij} equal $\max\{0, 1 + p(d_{ij}) + q(d_{ij})^2\}$ when $j \geq i$, and $\max\{0, 1 + r(d_{ij}) + s(d_{ij})^2\}$ when $j < i$, for constants p, q, r and s. For each country, the aspirational weight put by percentile i on another percentile income j is then given by $\gamma_{ij} \equiv \frac{\alpha_{ij}}{\sum_j \alpha_{ij}}$, and percentile i's aspiration level is given by $\sum_j \gamma_{ij} y_j$.

Our specification captures 69% of the observed variation in growth (as opposed to 3% for the common aspirations model).

What the estimates generate are "umbrella-shaped" aspirations with two properties. First, they are centered: for 85% of the countries (47 out of 55), individuals place the most weight in forming their aspirations on the income in their own percentile. Second, the windows are narrow: in more than half the countries, individuals put no weight in forming their aspirations on the incomes that are more than two percentiles away from themselves. This is shown in Figure 9 for the same nine countries studied earlier. On the left-hand-side of the picture, we see the weights that the median percentile puts on the neighboring percentiles when forming its aspirations. On the right-hand-side of the pictures we see the resulting level of aspirations (in log) for the various percentiles. Note that, though narrow, our 'best fit" aspirations perform much better than purely "self-centered aspirations" (in which the reference level is purely internal to the agent): that would captures only 19% of the observed variation in growth.

Although the limitations of this exercise are obvious, it suggests a fair amount of stratification in the aspiration formation process. This raises the growth rates of the poorest percentiles but reduces the growth rates in the upper middle range of the distribution.

6. Conclusion

This paper builds a theory of aspirations formation that emphasizes the social foundations of individual aspirations, and relates those aspirations in turn to investment and growth. Following a familiar lead from behavioral economics (see, e.g., Kahneman and Tversky (1979), Karandikar et al. (1998), and Kőszegi and Rabin (2006)), we define utilities around a "reference point", and interpret that reference point as an *aspiration*. Our main departure from this literature is in the determination of aspirations: rather than emphasizing the past experiences of the individual herself in shaping aspirations, we stress the social basis of aspirations formation. We argue that aspirations are likely to depend not only on one's own historical living standards, as commonly assumed, but also on the experience and lifestyle of others.

The theory we propose has three segments. First, individual aspirations determine one's incentives to invest and bequeath. Second, aspirations are determined by the going distribution of income. Finally, individual behavior is aggregated to derive the social distribution of income, thus closing the model.

A central ingredient of our setup is that aspirations can serve both to incentivize and to frustrate. We argue that aspirations that are above — but not too far — from current incomes can encourage high investment, while aspirations that are too high may discourage it. This insight has implications for growth rates across a cross-section of aspirations for a given starting income, as well as for growth rates across a cross-section of incomes, for a given level of aspirations.

Our main results concern equilibrium income distributions, which connect the above observations by endogenizing the formation of aspirations. We discuss when growth is compatible with equality, and also provide conditions under which equality may be unstable, with incomes converging to a nondegenerate distribution even without any noise or uncertainty. We argue that in many cases, steady state income distributions are likely to display local poles of convergence, but with a multiplicity of poles. For instance, if aspirations are common or stratified, we show that in any steady state, incomes generally exhibit a multimodal distribution. We also remark on the welfare economics of endogenous aspirations formation.

The theoretical results are complemented by an empirical exercise, which uses percentile distributions of growth rates available for 43 countries. We use aggregate growth experiences to estimate a rate of return to individual investment, and then employ the model to see which aspirations formation structure appears to fit the data best, in the sense of approximating the observed growth incidence curves by percentile. We show that a model of aspirations formation in which individuals use umbrella-shaped weights on incomes in some interval around their own incomes comes closest to replicating the data, and that a large fraction of the observed variation is indeed "explained" by our specification.

The goal of this paper has been to take a modest step towards thinking about the social determinants of aspirations or reference points. As in the case of any model with social effects on individual behavior, which are then aggregated to yield those social outcomes, there are difficulties in undertaking a full-blown dynamic analysis, and this paper is no exception. It would be of great interest to fully describe income-distribution dynamics for different models of aspirations formation. In the same spirit, one might ask for a more comprehensive structural exercise which would allow us to exploit the model to uncover more fully the process of aspirations formation. We believe that this approach will shed new and complementary light on the endogenous emergence of inequality.

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Define a function L(g) by

$$L(g) \equiv \frac{1}{1-\sigma}\left(\frac{r-g}{1+r}\right)^{1-\sigma} + \rho\frac{1}{1-\sigma}(1+g)^{1-\sigma}.$$

Then it should be clear from (13) that for each y, the optimal choice of g maximizes

$$L(g) + y^{\sigma-1}w([1+g]y, a).$$

Let g_1 and g_2 be optimal choices at y_1 and y_2 respectively. For additional simplicity of notation, let $G_1 \equiv (1+g_1)$ and $G_2 \equiv (1+g_2)$, and let $h(z) \equiv \omega(z,a)$. A standard revealed preference argument tells us that

$$L(g_1) + y_1^{\sigma-1}h(G_1y_1) \geq L(g_2) + y_1^{\sigma-1}h(G_2y_1)$$

and

$$L(g_2) + y_2^{\sigma-1}h(G_2y_2) \geq L(g_1) + y_2^{\sigma-1}h(G_1y_2).$$

Combing these two inequalities, we must conclude that

$$\Phi(y_1) \geq L(g_2) - L(g_1) \geq \Phi(y_2), \tag{22}$$

where the function Φ is defined by

$$\Phi(y) \equiv y^{\sigma-1}\left[h\left(G_1y\right) - h\left(G_2y\right)\right].$$

Proof of Proposition 4. Suppose that $y_1 < y$, and that aspirations are unattained at y: $(1+g)y < a$. Suppose, contrary to our assertion, that $g_1 > g$. Then, with a little work, we can find $y_2 > y_1$ such that $g_1 > g_2$ *and* $(1+g_1)y_2 < a$.[24]

Now, simple differentiation of Φ tells us that

$$\begin{aligned}\Phi'(y) &= y^{\sigma-2}\left[\left\{h'\left(G_1y\right)G_1y - h'\left(G_2y\right)G_2y\right\} + (\sigma-1)\left\{h\left(G_1y\right) - h\left(G_2y\right)\right\}\right] \\ &\geq y^{\sigma-2}\left[\left\{h'\left(G_1y\right)G_1y - h\left(G_1y\right)\right\} - \left\{h'\left(G_2y\right)G_2y - h\left(G_2y\right)\right\}\right]. \quad (23)\end{aligned}$$

At the same time, the function $h'(z)z - h(z)$ is strictly increasing in z for $z \in [G_2y_1, G_1y_2]$, because $z < a$ over this entire range.[25] Using this information in (23), and recalling that $G_1 > G_2$ by assumption, we must conclude that $\Phi'(y) > 0$ for all $y \in [y_1, y_2]$, which contradicts (22). ■

[24]Suppose that $y_1 < y$ and $g < g_1$. A standard argument establishes the monotonicity of aggregate investment in initial income, so that aspirations are unattained for all $y' < y$. Define $y^* \equiv \inf\{y' < y | g' < g_1\}$. If $y^* = y_1$, we are done by choosing y_2 slightly above y_1: because $(1+g_1)y_1 < a$, we will have $(1+g_1)y_2 < a$ and $g_1 > g_2$, as desired. If $y^* > y_1$, then $g' \geq g_1$ for all $y' \in [y_1, y^*)$, so that once again, y choosing y_2 slightly above y^*, we have $(1+g_1)y_2 < a$ and $g_1 > g_2$.

[25]To see this, differentiate to see that $\frac{d}{dz}[f'(z)z - f(z)] = zf''(z) > 0$.

Proof of Proposition 5. Suppose that $y_1 < y_2$, and that aspirations are exceeded at y_1: $(1+g_1)y_1 > a$. Suppose, contrary to our assertion, that $g_2 > g_1$. Just as in the proof of Proposition 4, we know that

$$\Phi'(y) \quad = \quad y^{\sigma-2}\left[\left\{h'\left(G_1 y\right)G_1 y + (\sigma-1)h\left(G_1 y\right)\right\} - \left\{h'\left(G_2 y\right)G_2 y + (\sigma-1)h\left(G_2 y\right)\right\}\right].$$

But condition [W′] informs us that the function $h'(z)z + (\sigma-1)h(z)$ is strictly decreasing in z for $z \in [G_1 y_1, G_2 y_2]$, because $z > a$ over this entire range.[26] We must therefore conclude that $\Phi'(y) > 0$ for all $y \in [y_1, y_2]$, which contradicts (22). ■

Proof of Corollary 1. Suppose, contrary to our assertion, that sustained nonnegative growth is possible from some $y > 0$, along with perfect equality. Let y_t be the common wealth level at date t. Then $y_t = a_t$ for all t by [A], and y_t must converge to some $y^* > 0$, with associated aspiration $a^* = y^*$. Proposition 6 implies that along the sequence, each individual solves the common maximization problem

$$\max_k u(y_t - k) + v(f(k)) + \omega(f(k), y_t)$$

with an optimal solution achieved at k_t, where $y_{t+1} = f(k_t)$. Given Condition U and $v'(0) = \infty$, the interior first-order condition for a maximum holds along the entire sequence $\{y_t\}$, so that

$$u'(y_t - k_t) = f'(k_t)\left[v'(y_{t+1}) + \omega_1(y_{t+1}, y_t)\right]$$

for all t. Let k^* be a limit point of $\{k_t\}$. Then $y^* = f(k^*)$, and by upperhemicontinuity of the maximum choice, k^* is an optimal solution at y^*. Moreover,

$$u'(y^* - k^*) = f'(k^*)\left[v'(y^*) + \omega_1(y^*, y^*)\right]. \tag{24}$$

By our assumption, $\omega_1(y^*, y^*) = \infty$, so (24) implies that $y^* - k^* = 0$. But this contradicts Condition U at y^*. ■

Proof of Corollary 2. Recall (13), which writes the resulting maximization problem as the choice of a particular rate of growth $g = (z/y) - 1$; we rewrite it here with aspirations set equal to income:

$$\frac{1}{1-\sigma}\left(y\left[\frac{r-g}{1+r}\right]\right)^{1-\sigma} + \frac{\rho}{1-\sigma}\left([1+g]y\right)^{1-\sigma} + y^{1-\sigma}w(1+g).$$

Dividing through by $y^{1-\sigma}$, we see that the agent equivalently chooses g to maximize

$$\frac{1}{1-\sigma}\left[\frac{r-g}{1+r}\right]^{1-\sigma} + \frac{\rho}{1-\sigma}[1+g]^{1-\sigma} + w(1+g),$$

which is (16). Now apply Proposition 6. ■

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[26] To see this, differentiate to see that $\frac{d}{dz}[h'(z)z + (\sigma-1)h(z)] = zh''(z) + \sigma h'(z) < 0$, by [W′].

Proof of Corollary 3. By Proposition 6, if there is a growing equilibrium path with equality from every initial $y_0 \geq \bar{y}$, then for each $y \geq \bar{y}$ there is $z(y) \geq y$ with

$$u\left(y - f^{-1}(z(y)/A)\right) + v(z(y)) + \omega(z(y), y) \geq u\left(y - f^{-1}(z/A)\right) + v(z) + \omega(z, y)$$

for all $z \neq z(y)$. Now increase A to $\tilde{A}$. For any y, consider any choice of z that maximizes $u\left(y - f^{-1}(z/\tilde{A})\right) + v(z) + \omega(z, y)$; call it $\tilde{z}(y)$. We claim that $\tilde{z}(y) \geq z(y)$. Suppose, on the contrary, that there exists y with $z(y) > \tilde{z}(y)$. The optimality of $\tilde{z}(y)$ under $\tilde{A}$ implies that

$$v(z(y)) + \omega(z(y), y) - v(\tilde{z}(y)) - \omega(\tilde{z}(y), y) \leq u\left(y - f^{-1}(\tilde{z}(y)/\tilde{A})\right) - u\left(y - f^{-1}(z(y)/\tilde{A})\right)$$

while the optimality of $z(y)$ under A means that

$$v(z(y)) + \omega(z(y), y) - v(\tilde{z}(y)) - \omega(\tilde{z}(y), y) \geq u\left(y - f^{-1}(\tilde{z}(y)/A)\right) - u\left(y - f^{-1}(z(y)/A)\right),$$

but this yields a contradiction since the right hand side decreases as we move from A to $\tilde{A}$. It follows that $\tilde{z}(y) \geq z(y) \geq y$ for all $y \geq \bar{y}$. Applying Proposition 6 again, we conclude that for every $y_0 \geq \bar{y}$, there is a growing equilibrium path from y_0 with perfect equality, under $\tilde{A}$. ■

Proof of the Assertion in Section 4.4.[27] Recall that with upward-looking aspirations,

$$a = \Psi(y, F) = \frac{\int_y^\infty x dF(x)}{1 - F(y)},$$

so the constancy of y/a (as y and a change) implies that the mapping

$$\phi(y) \equiv \frac{1}{y(1 - F(y))} \int_y^\infty x dF(x)$$

is constant in y. In particular, $\phi'(y) = 0$, and so

$$\phi'(y) = -h(y) - \frac{1}{y}(1 - yh(y))\phi(y) = 0,$$

where $h(y) \equiv f(y)/[1 - F(y)]$. This condition can be rewritten as

$$\frac{\partial \ln(1 - F(y))}{\partial y}(1 - \phi(y)) = \frac{\partial \ln y}{\partial y}\phi(y).$$

Substituting in the constant value of $\phi(y)$ — call it d — yields

$$\frac{\partial}{\partial y}(\ln(1 - F(y))) = \frac{\partial}{\partial y}\left(\ln y^{\frac{d}{1-d}}\right).$$

Therefore,

$$1 - F(y) = Ay^{\frac{d}{1-d}},$$

which implies that F is a Pareto distribution. ■

[27]We are grateful to Joan Esteban for this argument.