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ASSET PRICES IN A LIFECYCLE ECONOMY

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ABSTRACT

The representative agent model (RA) has dominated macroeconomics for the last thirty years. This model does a reasonably good job of explaining the co-movements of consumption, investment, GDP and employment during normal times. But it cannot easily explain movements in asset prices. Two facts are hard to understand 1) The return to equity is highly volatile and 2) The premium for holding equity, over a safe government bond, is large. This paper constructs a lifecycle model in which agents of different generations have different savings rates and different attitudes to risk and I use this model to account for both a high equity premium and a volatile stochastic discount factor. The model is persuasive, precisely because it explains so much with so few parameters, each of which is pinned down by a few simple facts.

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ASSET PRICES IN A LIFECYCLE ECONOMY

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ABSTRACT. The representative agent model (RA) has dominated macroeconomics for the last thirty years. This model does a reasonably good job of explaining the co-movements of consumption, investment, GDP and employment during normal times. But it cannot easily explain movements in asset prices. Two facts are hard to understand 1) The return to equity is highly volatile and 2) The premium for holding equity, over a safe government bond, is large. This paper constructs a lifecycle model in which agents of different generations have different savings rates and different attitudes to risk and I use this model to account for both a high equity premium and a volatile stochastic discount factor. The model is persuasive, precisely because it explains so much with so few parameters, each of which is pinned down by a few simple facts.

I. INTRODUCTION

The dominant macroeconomic *representative agent*, (*RA*) model of the last thirty years assumes that the co-movements of aggregate consumption, investment, GDP and employment behave *as if* they were chosen by a representative agent who maximizes the expected discounted sum of a stream of future utilities. Although the RA model can capture the volatilities and co-movements of aggregate quantities during ‘normal times’, it cannot easily explain movements in asset prices.¹ Two facts, in particular, are hard to explain.

The first fact is that the stock market is highly volatile and daily movements in the average price of stocks are much too large to be accounted for by movements in future earnings (Leroy and Porter, 1981; Shiller, 1981). To explain this *excess volatility puzzle* the rate at which agents discount the future must be stochastic and highly volatile. In a representative agent economy, the stochastic discount factor is a function of aggregate consumption, and when the utility function of the representative

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¹The Great Depression of the 1930s and the Great Recession of 2008 are clearly not ‘normal times’ and the fact that the RA model does not explain those episodes suggests that we need to rethink the foundations of macroeconomic theory in a more radical way. That topic, which I have discussed in other work (Farmer, 2012), is beyond the scope of the current paper.

agent is calibrated to a reasonable degree of risk aversion, the observed volatility of consumption is not large enough to account for observed price volatility in the data.

The second fact is that the return to holding a risky asset has historically been six hundred basis points higher than the return to holding government bonds (Mehra and Prescott, 1985). To explain the *equity premium puzzle*, the covariance of the rate of return on a risky asset with the agent's marginal utility of consumption, must be large. It is hard for the RA model to explain a six hundred basis point equity premium because asset prices do not covary enough with marginal utility for any reasonable model of preferences.

In this paper I construct a lifecycle model that successfully explains both volatile asset prices and the equity premium puzzle as equilibrium responses to purely non-fundamental shocks.² I am able to explain these two puzzles without invoking exotic preferences, credit constraints, sticky prices or frictions of any kind. The only difference of my model from a conventional representative agent economy is the assumption that there is more than one generation of agent alive in any given period. My work demonstrates that this simple and realistic modification to the representative agent model is all that is required to reconcile it with asset price data.

In my model, agents have logarithmic preferences and a constant discount factor which I set to 0.97. I choose two parameters to match an average working life of 45 years and an average time spent in retirement of 15 years. These three parameters, plus the standard deviation of non-fundamental shocks, are the only parameters of my model economy. Using this tightly parameterized model, I am able to generate an equity premium of 5.5% and a Sharpe ratio of 0.35.³

II. RELATIONSHIP TO THE LITERATURE

There is a large literature on asset price puzzles summarized by Campbell (2003). The current consensus amongst finance economists, as laid out by John Cochrane

²Cass and Shell (1983) use the term *intrinsic uncertainty*, to refer to anything that affects preferences, endowments or technology, and they use the term *extrinsic uncertainty*, or *sunspots*, to refer to anything that doesn't. Azariadis (1981) uses the term *self-fulfilling prophecy* to refer to sunspots. In this paper I will use the term fundamental shock, to refer to intrinsic uncertainty and non-fundamental shock to refer to extrinsic uncertainty or sunspots. These terms are, I believe, more commonly accepted in the literature.

³The Sharpe ratio is the sample average of the difference between the risky rate and safe rate of return divided by the standard deviation of the risky rate. It is a better measure of the return difference between two assets because, unlike the equity premium, the Sharpe ratio cannot be increased by leveraging a risky portfolio through borrowing.

(2007) in his Presidential Address to the American Finance Association, is that most of these puzzles occur because of unexplained movements in *the stochastic discount factor*.

The excess volatility puzzle and the equity premium puzzle are related to each other and both puzzles are connected to our lack of understanding of why the stochastic discount factor moves around so much. A large equity premium implies that the covariance between the marginal utility of consumption and the stochastic discount factor must also be large. Because the covariance of two random variables is bounded by the product of their individual standard deviations, a high equity premium implies that the marginal utility of consumption, or the stochastic discount factor, or both, must have a large standard deviation. The bounds placed on the equity premium by this condition are known in the literature as the Hansen-Jagannathan bounds (Hansen and Jagannathan, 1991).

Constantinides (1990) and Abel (1990) have proposed resolutions of the equity premium puzzle by modifying the representative agent's preferences. By adding 'habits' to the utility function, these authors provide alternative models of preferences in which the representative agent's marginal utility is highly sensitive to movements in consumption. The additional volatility in marginal utility that occurs in habit formation preferences increases the covariance of the marginal utility of consumption with the stochastic discount factor and helps to explain a large equity premium.

In Constantinides' (1990) paper, habits are determined by the previous consumption of a representative agent whereas Abel, (1990), assumes that there are many identical representative agents, each of whom cares about what *other* agents consumed in the past. Although these papers can account for a large equity premium, they do it at the cost of producing a riskless rate that is counterfactually volatile.

Campbell and Cochrane (1999) solve this problem by using the data to determine how the past consumption of other agents, influences marginal utility. Although this manoeuvre succeeds in replicating the facts, the way that marginal utility depends on past consumption is a complicated recursive function and Campbell and Cochrane do not provide a convincing account of why utility should depend on past consumption in this way.

In a separate literature, Rietz (1988), and Barro (2006), argue that there is a small but significant probability of occasional large disasters. Even if a disaster has not occurred in sample, the possibility that it might occur in the future will cause the representative agent to appear excessively cautious. But although the Barro-Reisz

model can explain the equity premium, it cannot explain the excess volatility of stock market returns.

Bansal and Yaron (2004) have shown that persistence of the dividend growth rate, and time varying volatility of asset returns, are important elements of an explanation of a high equity premium. In separate but related work, both Gabaix (2008) and Wachter (2013) modify the rare disaster model by assuming that the probability of rare disasters varies over time. Using that assumption, they are able to explain volatility in the stochastic discount factor in a model that also has a high equity premium. But although the rare disaster explanation is promising, it is not uncontroversial, and Julliard and Ghosh (2012) have argued that rare disasters do not occur often enough in the data for the hypothesis to be plausible.

An alternative explanation for excess volatility was developed by Farmer, Nourry, and Venditti (2012). Building on Farmer (2002a,b), they constructed a two agent version of Blanchard's (1985) perpetual youth model, and they showed that it generates a volatile stochastic discount factor as a consequence of non-fundamental shocks. Cass and Shell (1983) refer to non-fundamental shocks as 'sunspots'. They showed that sunspots can alter the allocations of consumption goods across agents, even when there is a complete set of asset markets. Farmer, Nourry, and Venditti (2012) extended the model of Cass and Shell to a calibrated example with long-lived agents. But although their model succeeds in generating large and volatile fluctuations in the stochastic discount factor, it cannot explain why there is a large equity premium.

Much of the empirical work in the literature on the equity premium does not distinguish between the premium that arises from the fact that a dividend stream is random and the premium that arises from the fact that it is paid at different times. The former is a *risk premium*; the latter a *term premium*. Abel (1999) offers a theoretical framework for understanding the difference between these two concepts and he points out, that in U.S. data, the premium for holding long-term government bonds over short-term government bonds has historically been of the order of 1.7%.

I will argue, in this paper, that a large part of the equity premium puzzle is a puzzle over the term premium, not the risk premium. I will show, that if the stochastic discount factor is sufficiently volatile, the value of leveraged equity in a company will offer a high and volatile return, even if the dividend payment offered by the firm is safe.

III. MY LIFECYCLE MODEL

In this section I will construct a lifecycle model in which agents of different generations have different savings rates and different attitudes to risk and I will explain how this model can account for both a high equity premium and a volatile stochastic discount factor. Unlike the habit formation models of Constantinides (1990) and Abel (1990), my model generates a smooth path for the safe rate of interest. And unlike the model of Campbell and Cochrane (1999), I do not need to reverse engineer a utility function to explain these two puzzles. My agents are expected utility maximizers with logarithmic preferences. My only modification to a representative agent asset pricing model is to allow these agents to pass through two stages of life; work and retirement.

My model builds on Gertler (1999), who studied an economy with two generations. Following Blanchard (1985), Gertler constructs an economy where agents die each period with an age-invariant probability. He extends Blanchard's framework to allow for a stochastic transition from youth to old age. I use that same idea.

In my model there are two generations of agents. Workers earn a constant endowment stream of 1 unit and retirees earn nothing, and like Gertler (1999), I assume that the transition from work to retirement is stochastic. Gertler does not permit agents to insure against the event of retirement. He exploits a utility function proposed by Farmer (1990) to find a closed form solution to the equilibria of his artificial economy and he studies the welfare consequences of this market incompleteness for the design of optimal social security policies.

I modify the Gertler economy by allowing for complete insurance markets, including complete pension schemes, that insure the worker against the stochastic event of retirement. Unlike Gertler, I use the more conventional assumption, that agents maximize expected utility, rather than Farmer's (1990) non-expected utility formulation of preferences.

Vissing-Jorgenson (2002) has argued that the elasticity of substitution of stock holders is lower than that of bond holders and Vissing-Jorgenson and Attanasio (2003) use that fact to offer a partial resolution of the equity premium puzzle. My work exploits that same idea. The equity premium depends on the covariance of the stochastic discount factor with the consumption of asset holders. And since, in my model, there is more than one type of agent, the consumption growth of individuals can be volatile even when the consumption growth of aggregate consumption is not. Although aggregate consumption in my model is constant, the consumption growth of workers and retirees displays substantial variation.

Two features of my model account for its success in solving the two asset pricing puzzles. The first feature is that shocks are unbounded above. That allows me to construct equilibria in which the stochastic discount factor is extremely volatile.⁴ The second feature is that the lower bound on non-fundamental shocks is state dependent. That provides additional volatility to the stochastic discount factor and generates time varying return volatility, a property that also characterizes the time series data. By exploiting these two features, my lifecycle model is able to explain both excess volatility and the equity premium puzzle with a parsimonious model in which the parameters of the model are pinned down by population demographics.

IV. ASSET PRICING IN A LIFECYCLE MODEL

In this section, I will explain how asset pricing in a lifecycle model differs from asset pricing in a representative agent model. The main difference is that the expression for the pricing kernel no longer depends on aggregate consumption growth. Instead, it depends on the division of resources between workers and retirees. That fact allows me to construct equilibria in a lifecycle model in which asset prices are volatile, even when aggregate consumption growth is not.

Consider an asset that sells for price p and pays 1 unit of a unique consumption good with probability π_1 and 0 units in all future periods, with probability $(1 - \pi_1)$. This asset has three interpretations.

First, it is the human wealth of a worker who retires with probability $(1 - \pi_1)$. Second, it is a bond which pays a coupon that decays at rate π_1 , a theoretical construct suggested by Woodford (2001) to model long duration bonds.⁵ Third, it is a claim to an unleveraged firm that has a dividend stream of 1 unit with probability π_1 and nothing in all future periods with probability $(1 - \pi_1)$.

Let Q' represent the price for delivery of a consumption good next period in state s' , divided by the conditional probability that this state will occur. The price of the asset p is determined by the equation,

$$p = 1 + E[\pi_1 Q' p'] . \tag{1}$$

Because I assume that the aggregate endowment is constant, this is a model with no fundamental aggregate uncertainty. In a representative agent model, the assumption

⁴This differs from previous work by Farmer, Nourry, and Venditti (2012) where non-fundamental shocks have bounded support.

⁵A Woodford bond has a duration of $1/(1 - \pi_1)$ years. By choosing this number appropriately, a Woodford bond can be used to find a tractable formula for the price of a bond of any desired maturity.

that there is no fundamental uncertainty would imply that Q' is constant and equal to β . In the lifecycle model studied here, I will show that Q' can be expressed as a rational polynomial in asset prices

$$Q' = \frac{a_0 + a_1 p}{b_0 + b_1 p'}, \quad (2)$$

where the parameters a_0 , a_1 , b_0 and b_1 are functions of the rate of time preference, the expected duration of a person's working life, and the expected duration of his time spent in retirement. The right side of this expression is equal, in equilibrium, to the ratio of marginal utilities between today and tomorrow for both types; workers and retirees.

I will show, in this paper, that there exist equilibria in which p fluctuates in response to non-fundamental shocks to beliefs. And although the aggregate endowment is constant, the distribution of the endowment will fluctuate between workers and retirees. I will calibrate the model to fit demographic data and I will show that the calibrated model explains both the excess volatility puzzle and the equity premium puzzle.

V. DEMOGRAPHICS

This section explains the assumptions I make about population demographics. I assume a pure exchange economy that consists of a measure n_1 of workers and a measure n_2 of retirees. Population is constant and equal to one,

$$n_1 + n_2 = 1. \quad (3)$$

Time is discrete and retirement and death are modeled as stochastic events. Each period a worker has a probability π_1 , of remaining active and a probability $1 - \pi_1$ of retiring. A retired worker has a probability π_2 of surviving for an extra period and a probability $1 - \pi_2$ of dying.⁶

I assume that the number of deaths equals the number of births and the measures of workers and retirees are constant. Since $n_1(1 - \pi_1)$ workers retire each period there must be $n_1(1 - \pi_1)$ births to replace them. And since I assume a stationary population, the measure of births must equal the measure of deaths,

$$(1 - \pi_1) n_1 = (1 - \pi_2) n_2. \quad (4)$$

⁶I assume for simplicity that workers retire before dying although relaxing that assumption is not difficult and does not materially alter the main results.

Using the assumption that the total population size is one, equations (3) and (4) imply that the stationary measures of workers and retirees are given by the expressions,

$$n_1 = \frac{1 - \pi_2}{2 - \pi_1 - \pi_2}, \quad n_2 = \frac{1 - \pi_1}{2 - \pi_1 - \pi_2}. \quad (5)$$

Since the population of each group is stationary, the inflow of new workers must equal the inflow of new retirees, also equal to the number of deaths. Using the symbol λ to represent this number, we may rewrite the expressions from Equation (4) as follows,

$$\lambda = \frac{(1 - \pi_2)(1 - \pi_1)}{2 - \pi_1 - \pi_2}. \quad (6)$$

VI. PENSIONS AND ANNUITIES

In this section, I explain the institutions that insure workers against the event of retirement and that provide annuities to retirees to support their consumption in old age.

Each worker is endowed with one unit of the endowment in every period in which he remains active. Retirees are endowed with zero units of the endowment and they must live off the assets acquired during their working life. These assumptions provide a demand for two specialized assets; pensions and annuities. I assume that workers can purchase actuarially fair pensions and that retirees can purchase actuarially fair annuities. Consider first, the market for pensions.

There is a large number of competitive risk-neutral pension funds that invest in a riskless security that costs \bar{Q} units at date t and pays 1 unit at date $t + 1$. Each pension fund offers two securities for sale to workers. A type 1 security sells for price p_1 and pays 1 unit in the subsequent period if the worker remains active, and nothing, if he retires. A type 2 security sells for price p_2 and pays nothing next period if the worker remains active, and 1 unit, if he retires. By purchasing a portfolio of these two securities, the worker can provide for any chosen allocation of his future wealth between the states of employment and retirement. Importantly, I assume that the event of retirement is stochastic and cannot be influenced by any decision taken by the worker.

The assumption that this pension scheme is actuarially fair implies that

$$p_1 = \pi_1 Q, \quad p_2 = (1 - \pi_1) Q. \quad (7)$$

There is a similar scheme for retirees. A retired person pays $\pi_2 Q$ units at date t for a security that pays 1 unit at date $t + 1$, if he remains alive, and nothing, if he dies. In the event of death, the assets of the retiree are returned to the annuities company.

VII. THE DEFINITION OF WEALTH

In this section I am going to introduce three concepts of wealth: The total wealth of a worker that I call W , the human wealth of a worker, that I call h , and the financial assets of a worker or retiree that I call a_i where I use a subscript 1 to denote a worker and 2 to denote a retiree. Using this notation, the wealth of a worker is given in Equation (8) as the sum of financial assets, a_1 and human wealth, h ,

$$W = a_1 + h. \quad (8)$$

There is no subscript on W or h because these terms are used exclusively to refer to workers. Retirees have no human wealth and, as a consequence, their wealth is equal to their net financial assets, a_2 .

Human wealth is defined recursively by the following functional equation,

$$h = 1 + \pi_1 E [Q' h']. \quad (9)$$

This is Equation (2) from Section 2. The human wealth of a worker is equal to the discounted stream of endowments, discounted at rate Q' . In a representative agent economy, this equation would be solved forward after replacing Q' by the constant discount factor, β , to find an expression for h as the discounted sum of all expected future income.

I will show, in this model, that Q' can be expressed, in equilibrium, as a function of h and h' . Replacing Q' by this function in Equation (9) leads to an expression that can be solved backwards to find h' as a function of h . And because the steady state of this equation is stable, there exist equilibria in which stochastic sunspot shocks reallocate resources between workers and retirees.

VIII. BUDGET CONSTRAINTS OF WORKERS AND RETIREES

In this section, I will use the definitions of wealth, human wealth and financial assets to write down the budget constraints of a worker and a retiree. These budget constraints reflect the assumption that there is a complete set of financial assets and that agents can contract against all observable events. Importantly, as in Cass and Shell (1983), there is incomplete participation in asset markets that arises from the generational structure of the demographics.

Consider first, the budget constraint of a representative worker.

$$\alpha_1 + 1 - C_1 = E [\pi_1 Q' \alpha'_1] + (1 - \pi_1) E [Q' \alpha_2^0]. \quad (10)$$

The worker enters the period holding net financial assets a_1 . He receives an endowment of 1 unit and he chooses to consume C_1 units of the consumption good. That explains the left side of Equation (10).

The first term on the right side, $E[\pi_1 Q' \alpha'_1]$, is the worker's purchases of Arrow securities contingent on remaining a worker.⁷ The second term on the right side of Equation (10), $(1 - \pi_1) E[Q' \alpha_2^{0'}]$, is the workers purchases of Arrow securities contingent on retirement. These securities constitute his pension assets.⁸ In the event that the worker retires in period $t + 1$, he receives the retirement assets $\alpha_2^{0'}$. This is a state contingent portfolio that depends on the realization of the shock, s' . I use the subscript 2 because, in the event he retires, the worker joins the ranks of the type 2 agents; the retirees.

A similar expression to Equation (10), holds for retirees,

$$\alpha_2 - C_2 = \pi_2 E[Q'(S') \alpha'_2]. \quad (11)$$

The retiree has no endowment income and his wealth is equal to his financial assets α_2 . His consumption is C_2 and his accumulation of contingent assets is equal to $\pi_2 E[Q'(S') \alpha'_2]$.

IX. DECISION RULES

This section derives the decision rules that would be followed by a worker and a retiree who maximizes utility subject to the budget constraints outlined in Section VIII. I assume that utility is logarithmic and that agents discount the future with discount factor β . Because there are complete markets and the period utility is logarithmic, decision rules are linear in wealth. The form of these rules is summarized in Proposition 1.

⁷Security s'_i costs $\pi_1 \psi(s'_i) Q'(s'_i)$ where π_1 is the discount that the worker receives in exchange for relinquishing his claim to $a'_1(s'_i)$ in the event that he retires in the next period. $\psi(s'_i)$ is the conditional probability that state s'_i will occur and $\psi(s'_i) Q'(s'_i)$ is the price of an s_i security.

⁸Security $a_2^{0'}(s'_j)$ costs $(1 - \pi_1) \psi(s'_j) Q'(s'_j)$ where $(1 - \pi_1)$ is the discount the worker receives for relinquishing his claim to $a_j^{0'}(s'_j)$ in the event that he remains a worker in the subsequent period. The superscript 0 reflects the fact that $\alpha_2^{0'}$ is the wealth of a new retiree in period $t + 1$. This variable is distinct from α'_2 , which refers to the wealth of a retiree in period $t + 1$ who was also retired in period t .

Proposition 1. The solution to the workers' and the retirees' decision problems are represented by a set of linear decisions rules described by equations (12)–(14):

$$C_1 = A^{-1}W, \quad (12)$$

$$Q'\alpha_2^0 = \frac{1 - \beta\pi_1}{1 - \beta\pi_2}W, \quad (13)$$

$$C_2 = B^{-1}\alpha_2, \quad (14)$$

where A and B are positive real numbers, defined by the expressions

$$A = \frac{(1 - \beta\pi_2) + (1 - \pi_1)\beta}{(1 - \beta\pi_1)(1 - \beta\pi_2)}, \quad B = \frac{1}{1 - \beta\pi_2}, \quad (15)$$

where

$$0 < B < A. \quad (16)$$

Proof. See Appendix A. □

Equations (12) and (13) are the decision rules for a young agent. Equation (12) instructs him to consume a fraction A^{-1} of his wealth in every state and Equation (13) provides the rule for how his saving is to be allocated between pension assets and non pension assets. Equation (14) is the decision rule for a retiree.

X. EQUILIBRIUM

In this section I will characterize an equilibrium as the solution to a functional equation in a single state variable, human wealth. The proof proceeds by establishing two lemmas. First I show how the consumption of the young depends on human wealth; then I establish that the stochastic discount factor is a rational polynomial in h and h' .

To establish the first lemma, I use the solution to the decision problems from Proposition 1 together with the asset market clearing and goods market clearing conditions.

Lemma 1. Define the sets $H \equiv [B, A]$ and $\bar{C} = [0, 1]$. There exists an increasing affine function $\phi(\cdot) : H \rightarrow \bar{C}$ such that the aggregate consumption of workers, C_1 , is given by the expression,

$$C_1 = \phi(h) \equiv \frac{h - B}{A - B}. \quad (17)$$

Further, the aggregate consumption of retirees, C_2 , is given by

$$C_2 = 1 - \phi(h) \equiv \frac{A - h}{A - B}. \quad (18)$$

In period 0, human wealth is pinned down by the initial condition

$$h_0 = \frac{(A - B)}{B} \bar{\alpha}_{10} + 1, \quad (19)$$

where $\bar{\alpha}_{10}$ is the net financial assets of workers in period 0.

Proof. See Appendix B. □

Next, I turn to the pricing kernel, Q' . To obtain an expression for this term, I use the first order condition of a worker, together with the asset and goods market clearing conditions.

Lemma 2. There exists a function $\zeta(\cdot) : H \times H \rightarrow R_+$ such that the stochastic discount factor, Q' is given by the expression

$$Q' = \zeta(h, h') \equiv \frac{D_1(h - B)}{D_2 h' - AB}, \quad (20)$$

where

$$D_1 = A\beta\pi_1, \quad D_2 = A(1 - \lambda) + B\lambda, \quad (21)$$

and λ is the measure of newborns from Equation (6).

Proof. See Appendix C □

Using lemmas 1 and 2, I am ready to provide a characterization of competitive equilibria for this economy. From (9) we have

$$h - 1 = E[\pi_1 Q' h']. \quad (22)$$

Replacing Q' in (22) from (20) gives

$$h - 1 = E \left\{ \pi_1 \frac{D_1(h - B)}{D_2 h' - AB} h' \right\}. \quad (23)$$

Equation (23) is a difference equation in h' that characterizes sequences $\{h\}$ which satisfy the goods and asset market clearing conditions,

$$C_1 + C_2 = 1, \quad \alpha_1 + \alpha_2 = 0, \quad (24)$$

and the optimal decision rules, (12) – (14). Any sequence $\{h\}$ that satisfies this equation, together with the feasibility conditions that $Q' > 0$ and $C_1 \in \bar{C}$, and the initial condition, $\alpha_1(0) = \bar{\alpha}_1$, completely characterizes a competitive equilibrium.

One equilibrium sequence is generated by non-stochastic solutions to the implicit difference equation

$$h - 1 = \pi_1 \frac{D_1(h - B)}{D_2 h' - AB} h', \quad h_0 = \frac{(A - B)}{B} \bar{\alpha}_{10} + 1. \quad (25)$$

But this is not the only equilibrium in this economy. Because of incomplete participation in asset markets arising from the generational structure of population demographics, there are also solutions to this equation in which non-fundamental shocks influences outcomes; in the words of Cass and Shell (1983), these are equilibria where sunspots matter. That idea is captured in Proposition 2.

Proposition 2. Define the set $\bar{H} \equiv [B_1, A]$ where

$$B_1 = \frac{AB}{(1-\lambda)A + \lambda B}. \quad (26)$$

There exists a sequence of non-fundamental random variables $\{s'\}$, a function $\bar{S}_1 : \bar{H} \rightarrow [0, 1]$, a set \bar{D} , and pair of functions $g : \bar{D} \rightarrow \bar{H}$, $\psi : \bar{D} \rightarrow R_+$ such that the sequence $\{h\}$ given by the difference equation

$$h' = g(h, s'), \quad (27)$$

with initial condition

$$h_0 = \bar{h}_0 \in \bar{H}, \quad (28)$$

defines a competitive equilibrium. The pricing kernel is given by the expression

$$Q' = \psi(h, s'). \quad (29)$$

The functions $g(\cdot)$ and $\psi(\cdot)$ are given by the expressions

$$g(h, s') = \frac{AB(h-1)s'}{D_2(h-1)s' - \pi_1 D_1(h-B)}, \quad (30)$$

$$\psi(h, s') = \frac{1}{AB\pi_1} [D_2(h-1)s' - \pi_1 D_1(h-B)]. \quad (31)$$

The random variables $\{s'\}$ are drawn from a set of distributions \mathbf{D} which satisfy the following properties. First, the conditional mean of s' is equal to 1;

$$E[s' | h] = 1. \quad (32)$$

Second, the support of s' is unbounded above and has a lower bound $S_1(h)$,

$$s \in \bar{S}(h) \equiv [\bar{S}_1(h), \infty), \quad (33)$$

where $S_1(h)$ is the unique solution to the affine equation

$$g(h, \bar{S}_1(h)) = A. \quad (34)$$

The set \bar{D} is defined by the condition

$$\bar{D} \equiv \{x \in R_+^2 \mid x_1 \in \bar{H}, x_2 \in [\bar{S}_1(x_1), \infty)\}. \quad (35)$$

Proof. See Appendix D. □

XI. THE PROPERTIES OF SHOCKS

This section is technical and can be omitted without losing the flow of my argument. I demonstrate that shocks are unbounded above and I illustrate how to construct a lower bound for the shock distribution that depends on h . The facts that shocks are unbounded above and that the lower bound of the shock distribution is state dependent, are important features of the model that enable it to explain excessive asset price volatility and a large equity premium.

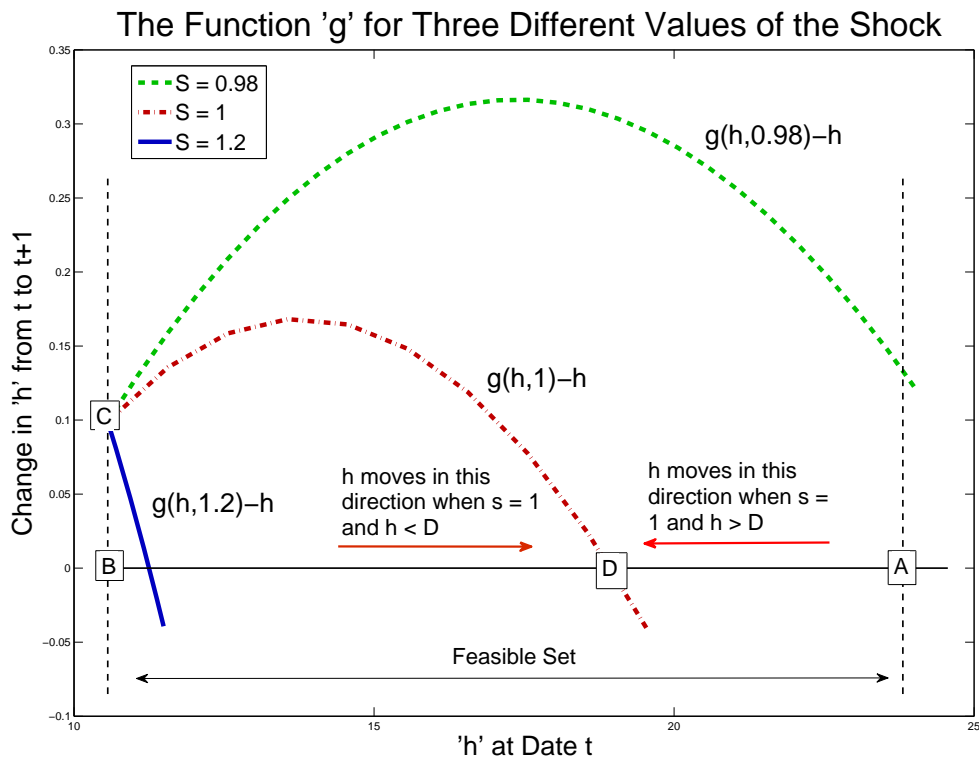


Figure 1

Figure 1 illustrates the properties of the function $g(h, s') - h$ for three different realizations of the shock s' . This function plots the change in h between dates t and $t + 1$ on the vertical axis against the value of h on the horizontal axis. The feasible set is $[B, A]$. When $h = B$, the retirees consume the entire endowment and when $h = A$, the workers consume everything.

The solid blue line represents the function $g(h, s') - h$ for a relatively large value of s' , equal to 1.2. The dashed green line is the function $g(h, s') - h$ for a small value of s' , equal to 0.98; and the dash-dot red line is the value of the this function when the shock is equal to its mean value of 1. The figure illustrates that, if the economy

were to be hit by a constant sequence of shocks, all equal to 1, that h would converge to point D .

The fact that the shock distribution is unbounded above follows from equation (30) which I repeat below

$$g(h, s') = \frac{AB(h-1)s'}{D_2(h-1)s' - \pi_1 D_1(h-B)}. \tag{36}$$

Dividing the numerator and denominator of Equation (36) by s' and taking the limit as $s' \rightarrow \infty$, it follows that $g(h, s')$ converges to a number, B_1 as s' converges to ∞ where,

$$B_1 = \frac{AB}{D_2} \equiv \frac{AB}{(1-\lambda)A + \lambda B}. \tag{37}$$

Notice that the number B_1 is greater than B and less than A and hence $s' = \infty$ is feasible and the support of feasible shocks is unbounded above.

What about a lower bound? Inspection of Equation (36) reveals that small shocks can cause h' to become larger than the maximum feasible value of A and it follows from that fact that the set of feasible shocks is bounded below. But the smallest feasible shock is not state independent; it depends on h .

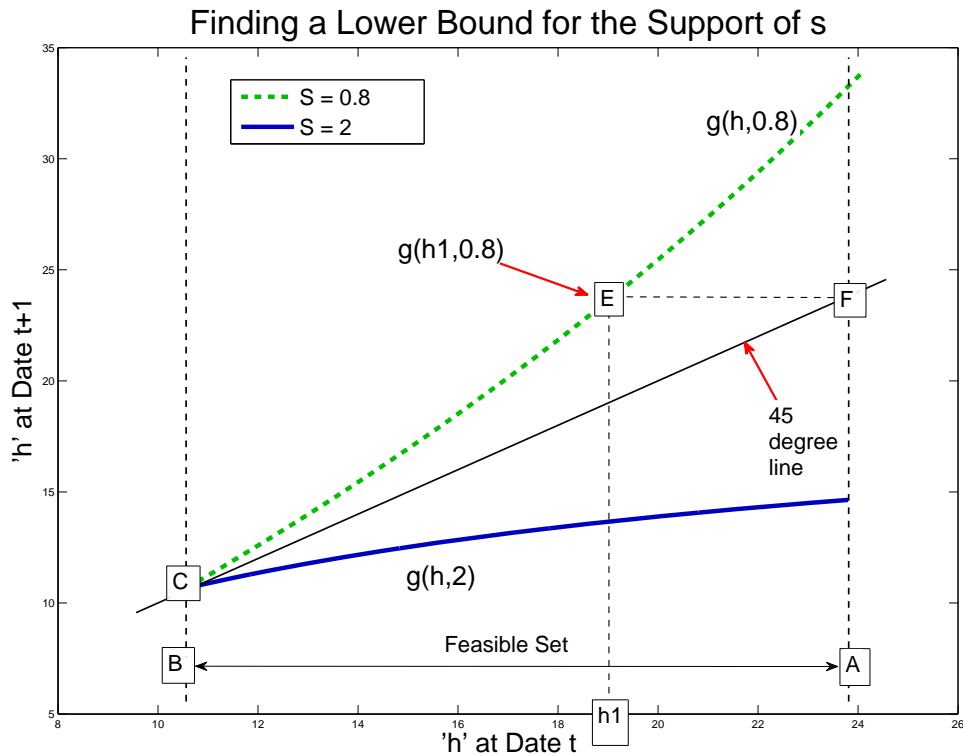


Figure 2

Figure 2 plots h' on the vertical axis and h on the horizontal axis. The figure illustrates the function $g(h, s')$ for two different values of the shock, $s' = 0.8$ and $s' = 2$. When the economy is hit by a large shock, next period's human capital will always remain feasible. That fact is illustrated by the solid blue curve which shows what happens to next period's human capital when the shock s' , is equal to 2.

When the economy is hit by a small shock, next period's human capital will remain feasible if h is small, but not if h is large. That is illustrated on Figure 2 by the dashed green line which represents the function $g(h, s')$ for a value of s' equal to 0.8. The figure shows that for values of h less than h_1 , s' is feasible since $g(h, 0.8)$ is feasible. But if h is greater than h_1 then s' equal to 0.8 is not feasible since $g(h, 0.8)$ will be bigger than A .

To ensure that h' remains feasible for all $s' \in \bar{S}$, we can make the support of s' conditional on h by defining a function $\bar{S}_1(h)$ such that $g(h, \bar{S}_1(h)) = A$. This function is illustrated on Figure 2. The figure shows that $\bar{S}_1(h_1) = 0.8$ since $s' = 0.8$ maps h_1 to A , the boundary of the feasible set.

What else do we know about feasible values of s' ? Not much since economic theory does not place strong restrictions on the distribution of s' . It must obey the property that

$$E(s' | h) = 1 \tag{38}$$

and its support, \bar{S} , must have the property that

$$g(h, \bar{S}) \subset \bar{H} \text{ for all } h \in \bar{H}. \tag{39}$$

In the simulation results, reported below, I used a log normal distribution for $s' - \bar{S}_1(h)$. Specifically, I assumed that

$$s' \sim \exp\left(x\sigma - \frac{\sigma^2}{2} + \log(1 - \bar{S}_1(h))\right) + \bar{S}_1(h),$$

where x is a standard normal random variable and σ is a parameter that controls the standard deviation of s' . It follows from the properties of the lognormal distribution that $E(s') = 1$ and that the support of s' is $\bar{S} \equiv [\bar{S}_1(h), \infty]$.

Is it reasonable to impose a conditional bound on the support of sunspot shocks? I think so. The assumption that shocks have a larger support when h is small, implies that the risky rate will be more volatile when asset prices are low than when asset prices are high. In other words, the model displays conditional heteroskedasticity, a feature that is known to characterize real world asset data.

XII. USING A SIMPLE LIFECYCLE MODEL TO EXPLAIN THE DATA

The lifecycle model I have constructed is extremely simple. There is no fundamental uncertainty and no production, and, there is a complete set of asset markets and two generations of agents. Agents have rational expectations and logarithmic preferences. Nevertheless, the model captures the two anomalies of asset pricing that I referred to in the introduction: it produces volatile asset prices and a substantial risk premium. To illustrate these features of the model, I calibrated it in the way described in Table 1.

Table 1	Value	Implied Value	
β Time preference factor	0.97		
π_1 Pr. of remaining a worker	0.9778	Expected Working Life	45 years
π_2 Pr. of remaining retired	0.9333	Expected Retirement	15 years
σ St. Deviation of Shock	0.5		

Figure 3 compares simulated data on the left two panels with actual U.S. data on the right. The simulated data represents one draw of 60 years using the parameter values from Table 1. The top left panel plots the yield to maturity on a long bond (the dashed red line) and the risk free rate (the solid blue line). The gross yield to maturity for a long bond in simulated data was calculated as

$$R^L = \pi_1 \frac{h}{h-1}, \quad (40)$$

and the gross short rate was calculated as

$$R^s = \frac{1}{E(Q')}. \quad (41)$$

The parameter π_1 enters Equation (40), because the coupon on the bond decays at rate π_1 . I chose this parameter to equal the survival probability of a worker and its value of 0.9778 implies an expected life of 45. Hence, the implied maturity of a long bond in this model is also 45 years and the yield to maturity, r^L , is the yield on a 45 year bond purchased at price h . For the simulated data series depicted in Figure 3, the average yield on a long bond was 1.62% and the average return to a short term asset was equal to 0.69% yielding a sample term premium of 0.93%.

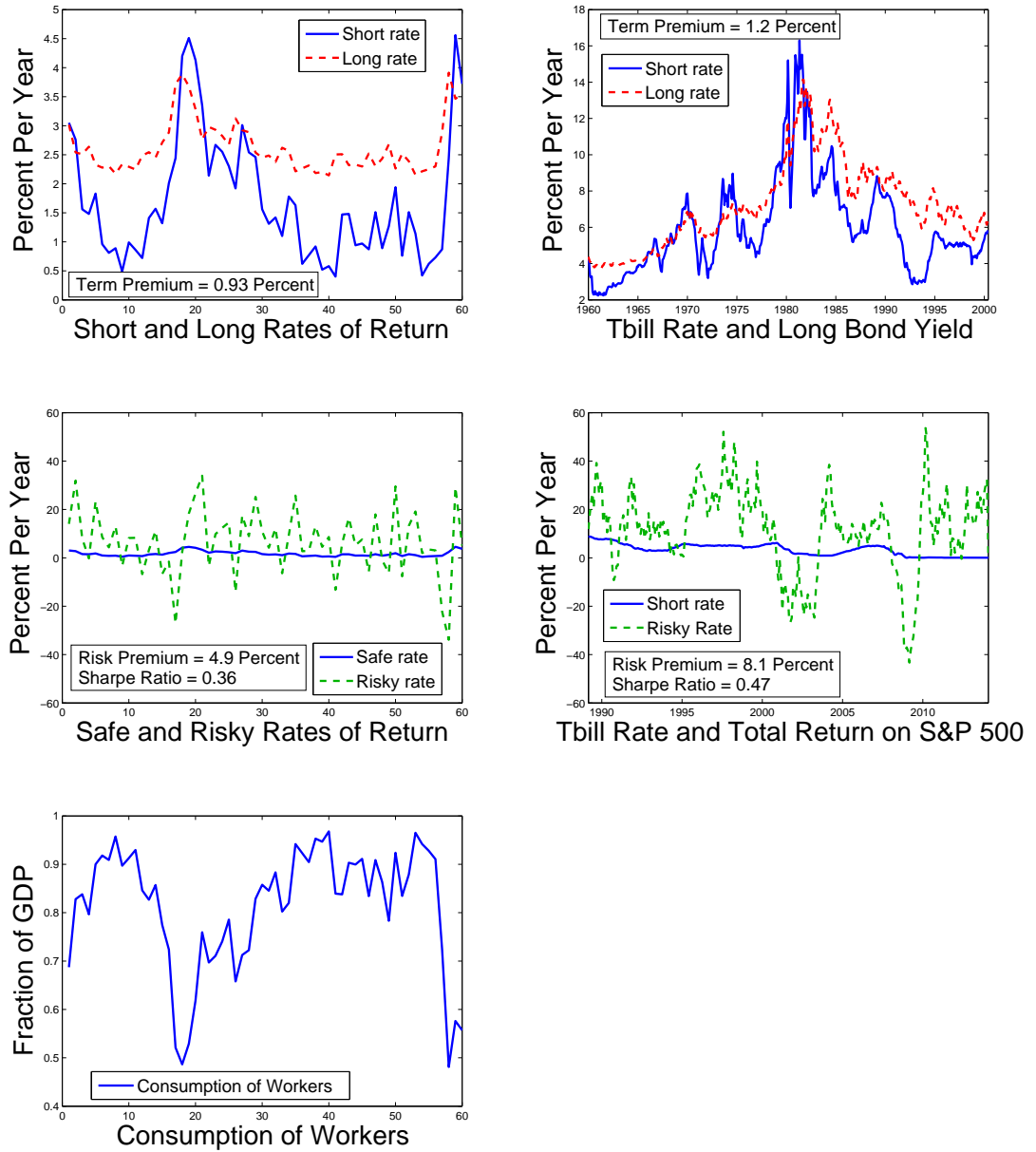


Figure 3

The top right panel of Figure 3 plots the three month treasury bill rate against the yield to maturity of a long term government bond in time series data. The data are monthly at annualized rates and they start in 1960 and end in 2000, when the long bond data were discontinued. This sample includes the great inflation of the late

1970s and early 1980s and I have not attempted to correct either series for inflation.⁹ The mean of the long bond yield in these data is 7.2% and the mean of the Tbill rate is 6% giving a term premium of 1.2%.

To compute the risky return on a leveraged claim, I first computed the unleveraged gross risky rate, R^R as

$$R^R = \frac{\pi_1 h'}{h - 1}. \quad (42)$$

The numerator of the risky rate R^R , is $\pi_1 h'$ as opposed to the numerator of the long bond yield, which is $\pi_1 h$. This reflects the calculation used to construct the yield to maturity on a long bond which asks; what rate will the investor earn over the life of the bond if he purchases it at price h ; and holds it to maturity?

The center left panel of Figure 3 plots the safe rate and the leveraged return to equity. In this simulated data series, the equity return was equal to 5.6% and the equity premium was equal to 4.9%. The standard deviation of the leveraged equity claim was equal to 13.53 which implies that the Sharpe ratio for this simulation was 0.36. The calculation used to construct the stock market return assumes that the investor holds the stock for one year and sells it for price h' . That fact partly accounts for the volatility of the simulated stock return in this figure.

There is a second factor generating volatility in simulated stock returns. To compute data for the risky return, I assumed that the stock is leveraged and I computed the equity return using Equation (43).

$$R^E = \frac{1}{1 - \mu} R^R - \frac{\mu}{1 - \mu} R^S. \quad (43)$$

Leverage increases the mean return of a risky asset; but it also increases its volatility. If the debt to equity ratio is represented by μ , then a movement of 1% in the value of the underlying risky asset causes the value of the firm to move by $1/(1 - \mu)\%$. In the U.S., the average value of μ across all U.S. firms is roughly 0.6 which implies that the realized value of a leveraged claim should swing by approximately two and half times as much as the value of the underlying asset.¹⁰

Although the model delivers a high equity premium and a volatile risky rate, aggregate consumption in the model is constant and equal to 1. Agents are willing to

⁹An inflation premium will be included in both series and, although these premia need not be identical, it is not obvious how the two series should be adjusted to account for these differences. That caveat should be taken into account when comparing the term premium from the model with the term premium in the data.

¹⁰Data on the average leverage ratio was taken from Damodaran (2014), Debt-Ratio Trade-Off Table for the U.S. File, dbtfund.xls.

pay a high premium on a risky asset because the stochastic discount factor has a high covariance with the consumption growth of individual agents. That fact is reflected in the lower left panel of Figure 3 which plots the consumption of all workers.

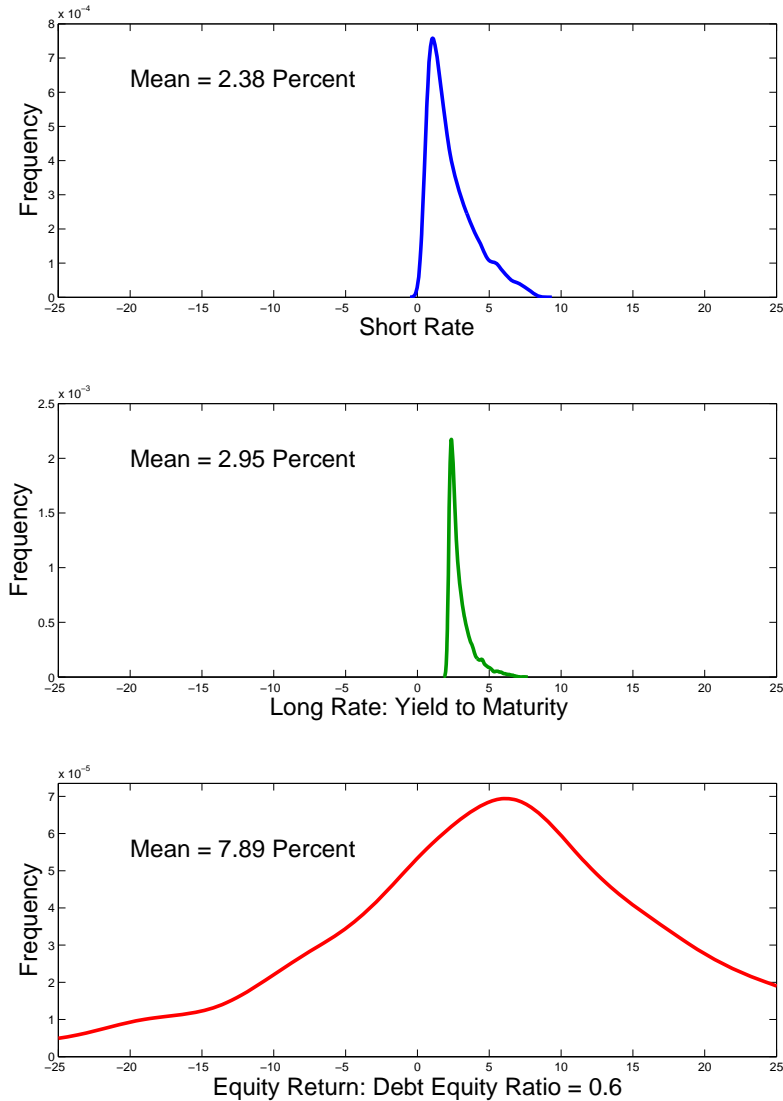


Figure 4

On average, workers as a group consumed 81% of GDP each year in this sample; but that average reflects considerable year to year variation. In a bad year for the working population, consumption in this sample fell as low as 48% of GDP and in a good year, it was as high as 96%. Since 75% of the population are workers and

25% retirees, these numbers imply that in an average year, a retiree consumed 70% as much as a worker, but that fraction fluctuated with a low of 12% and high of 325%.

Although one draw of sixty years of data displays a high equity premium, one is entitled to ask if the realization was a statistical fluke. Since the standard deviation of equity returns is so high, there is a significant probability of observing a high equity premium simply as a result of sampling error. Figure 4 investigates that possibility. The figure plots the empirical densities of the riskless rate, the long bond yield and the leveraged equity rate in a simulated series of 5,000 observations using the parameters from Table 1. The figure illustrates that the distribution of risky rates has a higher mean and a higher variance than the distribution of safe rates; it also has fat tails with a significant probability of very high or very low rates of return.

Figure 4 reports the means of the three rates of return, equal to 2.38, for the safe rate, 2.95 for the long bond yield and 7.89 for the rate of return on leveraged equity.

XIII. CONCLUSION

The lifecycle model I have constructed in this paper is simple and elegant. There is no fundamental uncertainty and no production, and, there is a complete set of asset markets and two generations of agents. Nevertheless, my model is able to capture significant features of the behavior of real world asset markets: asset prices are volatile and the return to equity is significantly higher than the return to government bonds.

The model is persuasive, precisely because it explains so much with so few parameters, each of which is pinned down by a few simple facts, and it offers an elegant way to explain asset prices without resorting to exotic assumptions about preferences. My work offers applied researchers in finance, a new class of pricing kernels, in which the stochastic discount factor is modeled as a rational polynomial in aggregate wealth. This model has testable implications for many other areas of empirical finance including foreign exchange markets, the term structure of interest rates and the role of government asset purchases in influencing the term premium.

Although the emphasis of the paper is on positive economics, my work also has normative implications. Although the equilibria I describe are competitive, and although I assume that there is a complete set of financial assets; the allocations supported by a sunspot equilibrium are not Pareto optimal (Cass and Shell, 1983). As I have argued elsewhere, (Farmer, 2013b) individuals choosing from behind the Rawlsian veil of ignorance would authorize a government agency to intervene in the asset markets

to smooth the effects of non-fundamental uncertainty on asset prices.¹¹ In other work (Farmer, 2013a) I have shown that asset market volatility can feedback and affect employment. Once one allows for feedback effects of this kind, the case for asset market intervention by a government agency becomes compelling.

¹¹Rawls (1999) argues that a just distribution of social resources should respect the choices that we would each make if we were forced to choose social decision rules before knowing the station in life to which we would be born. Rawls calls this the ‘veil of ignorance’. Rawls’ concept of social justice is reflected in what Cass and Shell (1983) refer to as ex ante Pareto Optimality. They argue that an allocation is suboptimal if an alternative allocation is available that smooths the consumption of at least one agent, across states into which he might be born, without reducing the consumption of any other agent. The sunspot equilibria discussed in this paper are both ex ante Pareto suboptimal and unjust from the Rawlsian perspective.

APPENDIX A. PROOF OF PROPOSITION 1

Proof. The worker and the retiree solve the following problems. Let $J(W)$ represent the value function of a worker and let $V(\alpha_2)$ be the value of function of a retiree. These functions are defined as follows,

$$J(W) = \max_{0 \leq C_1 \leq W_1} \left\{ \log(W - \pi_1 E[Q'W(S')] - (1 - \pi_1) E[Q'\alpha_2^0(S')]) \right. \\ \left. + \beta \pi_1 E J[W(S')] + \beta (1 - \pi_1) E V[\alpha_2^0(S')] \right\}, \quad (\text{A1})$$

$$V(\alpha_2) = \max_{0 \leq C_2 \leq \alpha_2} \left\{ \log(\alpha_2 - \pi_2 E[Q'\alpha_2(S')]) + \beta \pi_2 E V[\alpha_2(S')] \right\}. \quad (\text{A2})$$

Summing Equations (9) and (10) and using the identity, (8), leads to an alternative expression for the budget constraint in terms of W instead of α_1 ,

$$W - C_1 = \pi_1 E[Q'W'] + (1 - \pi_1) E[Q'\alpha_2^0]. \quad (\text{A3})$$

The unknown functions $J(W)$ and $V(\alpha_2)$ must satisfy the following envelope conditions,

$$J_W[W(S)] = \frac{1}{C_1}, \quad (\text{A4})$$

$$V_\alpha[V(\alpha_2)] = \frac{1}{C_2}. \quad (\text{A5})$$

In addition, they must satisfy the three Euler equations for the transfer of wealth into the future,

$$-\frac{\pi_1 Q'(S')}{C_1(S)} + \beta \pi_1 J_W[W(S')] = 0, \quad (\text{A6})$$

$$-\frac{(1 - \pi_1) Q'(S')}{C_1(S)} + \beta (1 - \pi_1) V_\alpha[\alpha_2^0(S')] = 0, \quad (\text{A7})$$

$$-\frac{\pi_2 Q'(S')}{C_2(S)} + \beta \pi_2 V_\alpha[\alpha_2(S')] = 0. \quad (\text{A8})$$

Since this is a logarithmic problem with complete markets we will guess that the functions take the form

$$J(W) = A \log(W), \quad V(\alpha) = B \log(\alpha), \quad (\text{A9})$$

and verify this conjecture by finding values for the numbers A and B such that equations (A4) – (A8) hold. By replacing the unknown functions $J(\cdot)$ and $V(\cdot)$ with their conjectured functional forms from Equation (A9) we arrive at Equations (A10) – (A14).

$$C_1(S) = \frac{W(S)}{A}, \quad (\text{A10})$$

$$C_2(S) = \frac{\alpha_2(S)}{B}, \quad (\text{A11})$$

$$C_1(S) = \frac{Q'(S')W(S')}{\beta A}, \quad (\text{A12})$$

$$C_1(S) = \frac{Q'(S')\alpha_2^0(S')}{\beta B}, \quad (\text{A13})$$

$$C_2(S) = \frac{Q'(S')\alpha_2(S')}{\beta B}. \quad (\text{A14})$$

The two budget equations, (A3) and (11) and the five first order conditions (A10) – (A14) constitute seven equations in the seven unknowns, W' , α_2 , α_2^0 , C_1 , C_2 , A and B . We next solve these equations to find the values of the parameters A and B in terms of the fundamental parameters β , π_1 and π_2 . First, from (A3), we have

$$W - C_1 = \pi_1 E[Q'W'] + (1 - \pi_1) E[Q'\alpha_2^0], \quad (\text{A15})$$

substituting for C_1 , $Q'W'$ and α_2^0 from (A10), (A12) and (A13) we have

$$AC_1 - C_1 = \pi_1 \beta AC_1 + (1 - \pi_1) \beta BC_1, \quad (\text{A16})$$

and canceling C_1 leads to the following expression in the two unknowns, A and B ,

$$A(1 - \beta\pi_1) - (1 - \pi_1)\beta B = 1. \quad (\text{A17})$$

To find a second equation in A and B , we use the budget constraint of a retiree, Equation (11)

$$\alpha_2 - C_2 = \pi_2 E[Q'(S')\alpha_2'], \quad (\text{A18})$$

and replace α_2 and $Q'\alpha_2'$ from (A11) and (A14),

$$BC_2 - C_2 = \pi\beta BC_2, \quad (\text{A19})$$

which, canceling C_2 implies,

$$B = \frac{1}{1 - \beta\pi_2}. \quad (\text{A20})$$

Substituting (A20) into (A17) gives the solution for A

$$A = \frac{(1 - \beta\pi_2) + (1 - \pi_1)\beta}{(1 - \beta\pi_1)(1 - \beta\pi_2)}. \quad (\text{A21})$$

To establish that $A > B$ we need to show that

$$\begin{aligned}
A &= \frac{(1 - \beta\pi_2) + (1 - \pi_1)\beta}{(1 - \beta\pi_1)(1 - \beta\pi_2)} > \frac{1}{(1 - \beta\pi_2)} = B, \\
&\implies \frac{(1 - \beta\pi_2) + (1 - \pi_1)\beta}{(1 - \beta\pi_1)} > 1, \\
&\implies (1 - \beta\pi_2) + (1 - \pi_1)\beta > (1 - \beta\pi_1), \\
&\implies \beta > \beta\pi_2,
\end{aligned} \tag{A22}$$

which follows from the fact that $\pi_2 < 1$. Using (A21), the decisions rules for consumption of workers and retirees and the asset allocation rule for workers can be represented by equations (A23),

$$C_1 = \frac{(1 - \beta\pi_1)(1 - \beta\pi_2)}{(1 - \beta\pi_2) + (1 - \pi_1)\beta}W, \quad C_2 = (1 - \beta\pi_2)\alpha_2, \quad Q'\alpha_2^0 = \frac{1 - \beta\pi_1}{1 - \beta\pi_2}W. \tag{A23}$$

This establishes the form for equations (12) – (14) in Proposition 1. \square

APPENDIX B. PROOF OF LEMMA 1

Proof. Using the decision rules, (12) and (14) and the definition of wealth, (8), we obtain the following equations for the financial wealth of workers and retirees,

$$\alpha_1 = AC_1 - h, \tag{B1}$$

and from Equation (14),

$$\alpha_2 = BC_2. \tag{B2}$$

Since this is a closed economy with no government sector, asset market clearing and goods market clearing give the following two equations linking α_1 to α_2 and C_1 to C_2

$$\alpha_1 + \alpha_2 = 0, \tag{B3}$$

$$C_1 + C_2 = 1. \tag{B4}$$

Combining equations (B1) – (B4), gives the expression we seek,

$$C_1 = \frac{h - B}{A - B}. \tag{B5}$$

It follows from the definitions of A and B that $B \leq A$ and Since $C_1 \in [0, 1]$, we must have that $h \in [B, A]$ for the consumption of workers to be feasible. This gives the domain of $\phi(\cdot)$ as $H \equiv [B, A]$. The expression for C_2 follows from the goods market clearing condition, Equation (B3). The initial condition for h comes from combining equations (17) and (B1) in period 0. \square

APPENDIX C. PROOF OF LEMMA 2

Proof. From the first order condition of a representative worker,

$$Q' = \frac{\beta C_1^\tau}{C_1^{\tau'}}, \quad (\text{C1})$$

where C_1^τ is the consumption of worker τ at date t and $C_1^{\tau'}$ is the consumption of the same worker at date $t + 1$ in history s' . Aggregating this condition over all workers alive at date t gives

$$Q' = \frac{\beta C_1}{\tilde{C}'_1}, \quad (\text{C2})$$

where C_1 is the aggregate consumption of all workers at date t and \tilde{C}'_1 is the aggregate consumption of all workers alive at date t who are still alive at date $t + 1$. There is a measure π_1 of these agents and their consumption, plus the consumption of newborns, is equal to aggregate consumption of workers at date $t + 1$;

$$\pi_1 \tilde{C}'_1 + C_1^{Y'} = C'_1. \quad (\text{C3})$$

Here, C'_1 is consumption of all workers at date $t + 1$ and $C_1^{Y'}$ is the consumption of all new born agents at date $t + 1$. Replacing (C3) in (C2), using (17) and the fact that there is a measure λ of newborns who consumption a fraction A^{-1} of aggregate human wealth,

$$C_1^{Y'} = \lambda A^{-1} h', \quad (\text{C4})$$

leads to equation (20). \square

APPENDIX D. PROOF OF PROPOSITION 2

Proof. Equation (30) follows from multiplying the left side of Equation (23) by s' and taking expectations. Solving the resulting expression for h' leads to the definition of $g(\cdot)$. The function $\psi(\cdot)$ is found by substituting (30) into (20) and rearranging terms. The fact that s' is unbounded above follows from the fact that

$$\lim_{s' \rightarrow \infty} g(h, s') = \frac{AB}{D_2} = \frac{AB}{(1 - \lambda)A + \lambda B} \equiv B_1. \quad (\text{D1})$$

Further, since $0 < \lambda < 1$, it follows that B_1 is feasible;

$$B < \frac{AB}{(1 - \lambda)A + \lambda B} = B_1 < A \in H. \quad (\text{D2})$$

These facts demonstrate that there is no upper bound on the set of feasible values of s' for any $h \in \bar{H}$. To derive a conditional lower bound on s' , note that $g(\cdot)$ is decreasing in s' for $s' > \bar{S}_1(h)$ where $\bar{S}_1(h)$ solves (34). For all values of $s' \geq \bar{S}_1(h)$,

$g(h, s') \in \bar{H}$ for all h . Shocks smaller than $S_1(h)$ can lead to infeasible values for h' . \square

REFERENCES

- ABEL, A. B. (1990): "Asset Prices under Habit Formation and Catching up with the Joneses," *The American Economic Review; Papers and Proceedings*, 80(2), 38–42.
- (1999): "Risk premia and term premia in general equilibrium," *Journal of Monetary Economics*, 43(1), 3–33.
- AZARIADIS, C. (1981): "Self-fulfilling Prophecies," *Journal of Economic Theory*, 25(3), 380–396.
- BANSAL, R., AND A. YARON (2004): "Risks for the Long Run: A Potential Resolution of Asset Pricing Puzzles," *The Journal of Finance*, 59(4), 1481–1509.
- BARRO, R. J. (2006): "Rare Disasters and Asset Markets in the Twentieth Century," *Quarterly Journal of Economics*, 121(3), 823–866.
- BLANCHARD, O. J. (1985): "Debt, Deficits, and Finite Horizons," *Journal of Political Economy*, 93(April), 223–247.
- CAMPBELL, J. Y. (2003): "Consumption-based asset pricing," in *Handbook of the Economics of Finance*, in: *G.M. Constantinides & M. Harris & R. M. Stulz (ed.)*, *Handbook of the Economics of Finance, edition 1, volume 1, chapter 13, pages 803–887 Elsevier.*, ed. by M. H. . R. M. S. G.M. Constantinides, vol. 1, chap. 13, pp. 803–887. *Handbook of the Economics of Finance*, 1 edn.
- CAMPBELL, J. Y., AND J. H. COCHRANE (1999): "By force of habit: A consumption-based explanation of of aggregate stock market behavior," *Journal of Political Economy*, 107, 205–251.
- CASS, D., AND K. SHELL (1983): "Do Sunspots Matter?," *Journal of Political Economy*, 91, 193–227.
- COCHRANE, J. (2007): "Inflation Determination with Taylor Rules: A Critical Review," *NBER WP 13410*.
- CONSTANTINIDES, G. M. (1990): "Habit Formation: A Resolution of the Equity Premium Puzzle," *Journal of Political Economy*, 98(3), 519–543.
- DAMODARAN, A. (2014): "Online Data Source," <http://pages.stern.nyu.edu/~adamodar/>.
- FARMER, R. E. A. (1990): "RINCE Preferences," *Quarterly Journal of Economics*, 105(1), 43–60.
- (2002a): "Business Cycles with Heterogenous Agents," *UCLA mimeo*, Paper prepared for a conference at GREQAM on "New Perspectives of (In)Stability, the Role of Heterogeneity", Marseilles, June 2001.
- (2002b): "Fiscal Policy, Equity Premia, and Heterogenous Agents," *UCLA mimeo*, Paper presented to the conference "New Developments in Fiscal Policy

- Analysis" at Universitat Pompeu Fabra, Barcelona May 2002.
- (2012): "Confidence, Crashes and Animal Spirits," *Economic Journal*, 122(559).
- (2013a): "Animal Spirits, Financial Crises and Persistent Unemployment," *Economic Journal*, 123(568), 317–340.
- (2013b): "Qualitative Easing; a New Tool for the Stabiization of Financial Markets," *Bank of England Quarterly Bulletin*, Q4, 405–413.
- FARMER, R. E. A., C. NOURRY, AND A. VENDITTI (2012): "The Inefficient Markets Hypothesis: Why Financial Markets do not Work Well in the Real World," *NBER working paper 18647*.
- GABAIX, X. (2008): "Linearity-generating processes: A modelling tool yielding closed forms for asset prices," *Working paper, New York University*.
- GERTLER, M. (1999): "Government debt and social security in a lif-cycle economy," *Carnegie Rochester Conference Series on Public Policy*, 50, 61–110.
- HANSEN, L. P., AND R. JAGANNATHAN (1991): "Restrictions on intertemporal marginal rates of substitution implied by asset returns," *Journal of Political Economy*, 99, 225–262.
- JULLIARD, C., AND A. GHOSH (2012): "Can Rare Events Explain the Equity Premium Puzzle?," *Review of Financial Studies*, 25(10), 3037–3076.
- LEROY, S., AND R. PORTER (1981): "Stock Price Volatitlity: A Test based on Implied Variance Bounds," *Econometrica*, 49, 97–113.
- MEHRA, R., AND E. C. PRESCOTT (1985): "The Equity Premium: A Puzzle," *Journal of Monetary Economics*, 15, 145–161.
- RAWLS, J. (1999): *A Theory of Justice*. Harvard University Press, Cambridge MA, second edn., first edition, 1971.
- RIETZ, T. A. (1988): "The Equity Risk Premium: A Solution," *Journal of Monetary Economics*, 22(1), 117–131.
- SHILLER, R. J. (1981): "Do stock prices move too much to be justified by subsequent changes in dividends?," *American Economic Review*, 71, 421–436.
- VISSING-JORGENSEN, A. (2002): "Limited Asset Market Participation and the Elasticity of Intertemporal Substitution," *Journal of Political Economy*, 110(4), 825–853.
- VISSING-JORGENSEN, A., AND O. P. ATTANASIO (2003): "Stock-Market Participation, Intertemporal Substitution, and Risk-Aversion," *American Economic Review, Papers and Proceedings*, 93(2), 383–391.

- WACHTER, J. A. (2013): “Can Time-Varying Risk of Rare Disasters Explain Aggregate Stock Market Volatility?,” *The Journal of Finance*, 68(3), 987–1035.
- WOODFORD, M. (2001): “Fiscal Requirements for Price Stability,” *Journal of Money, Credit and Banking*, 33(3), 669–728.

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