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### MATURITY RATIONING AND COLLECTIVE SHORT-TERMISM

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## **ABSTRACT**

Financing terms and investment decisions are jointly determined. This interdependence links firms' asset and liability sides and can lead to short-termism in investment. In our model, financing frictions increase with the investment horizon, such that financing for long-term projects is relatively expensive and potentially rationed. In response, firms whose first-best investment opportunities are long-term may change their investments towards second-best projects of shorter maturities. This worsens financing terms for firms with shorter maturity projects, inducing them to change their investments as well. In equilibrium, investment is inefficiently short-term. Equilibrium asset-side adjustments by firms can amplify shocks and, while privately optimal, can be socially undesirable.

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# 1 Introduction

Financing terms affect investment decisions and investment decisions affect financing terms. This interdependence creates an intimate link between firms' asset and liability sides. In particular, when financing for long-term projects is relatively expensive or impossible, firms may adjust their investment behavior towards shorter-term projects, even when those are less efficient.

In this paper we develop an integrated equilibrium framework to study how financing frictions that arise on the liability side affect investments on firms' asset sides, and vice versa. In our model, contracting frictions due to limited commitment are more pronounced at longer horizons, which leads to less attractive funding terms and, ultimately, credit rationing, for long-term investment projects. Firms with long-term investment opportunities respond by adjusting their asset-side investments towards alternative, shorter-maturity projects, even if those projects are second best. The central result of our paper is that these asset-side adjustments are self-reinforcing: An individual firm's asset-side decision endogenously determines the financing terms faced by other firms, thereby influencing their investment decision creating an externality.

Consider a firm that seeks funding for the development of a new product that requires substantial investment in long-term R&D. While the development of this innovative product may be efficient from an NPV perspective, the uncertainty associated with the required longterm R&D can make financing such a project difficult. The firm may therefore choose to develop a product that builds on an existing technology and can be brought to market more quickly, even if that product is inferior. Or consider a mining company seeking to fund a longterm exploration project, such as the development of an oil sand.<sup>1</sup> However, the uncertainty associated with long-term exploration makes financing such a project difficult. The mining company may therefore forego the long-term project and settle for a shorter-term investment, for example the development of a shale gas well, even if this is an inferior investment for this particular company.<sup>2</sup> Importantly, in both cases the firm affects funding terms for those firms that have efficient short-term projects, which may now be abandoned in favor of even shorter-term investments. Or consider a financial institution in the aftermath of the Lehman default. Increased uncertainty about the quality of banks made long-term financing hard to come by,<sup>3</sup> thereby pushing financial institutions with good long-term investments into less profitable shorter-term investments.<sup>4</sup> However, through this adjustment also shorter term financing to banks becomes riskier, thereby encouraging other banks to shorten their assets and liabilities as well.

The common thread in these examples is that privately optimal asset-side adjustments lead to a cross-firm externality. Our model shows that in the presence of this externality, the competitive equilibrium exhibits inefficient "collective" short-termism in real investment relative to the constrained optimum. Moreover, due to their self-reinforcing nature, firms' equilibrium asset side adjustments can amplify shocks and, while privately optimal, they can be undesirable from a social perspective.

<sup>&</sup>lt;sup>1</sup>Oil sand projects require large up-front investments in well pads or mines and are therefore long-term projects.

 $<sup>^{2}</sup>$ Shale gas properties, on the other hand, tend to produce out in a few years and are thus shorter-term projects.

 $<sup>^{3}</sup>$ Krishnamurthy (2010) shows that maturities in the commercial paper market, a significant source of funding for financial institutions, shortened significantly after the Lehman default. Kuo et al. (2013) use FedWire data to show that a similar shortening of maturities occurred in the interbank lending market, with a particularly sharp decline in the fraction of loans with a maturity of at least three months.

<sup>&</sup>lt;sup>4</sup>For example, a financial institution with a comparative advantage in making long-term loans may shift its loan portfolio to shorter maturities, where it has less of an advantage. In addition, the shortening of the financial institution's loans may distort the real decisions of the firms funded through these loans.

In our model, firms are born with an initial, positive NPV investment project. Some firms have safe projects, while others have risky projects whose risk, a mean-preserving spread relative to safe projects, increases with the maturity of the project. The main friction is a limited commitment assumption in the spirit of Bolton and Scharfstein (1990) and Hart and Moore (1994): Ex post, successful firms with risky projects can always pretend to have had a safe project and abscond with the difference in cash flow. Firms seek financing from a financial sector that can observe the maturity of an investment project. Financing optimally occurs via a debt contract and, in order for the financier to break even, the interest rate on this debt contract has to increase with maturity in order to reflect the higher risk at longer project horizons, leading to less attractive financing terms for long-term projects. Beyond a certain maturity, the limited commitment friction is so severe that financiers cannot break even, such that lending breaks down and *maturity rationing* arises.

Rationing of long-term projects generates the endogenous asset side adjustments central to this paper. Firms that cannot fund their first-best long-term projects react by adopting second-best projects of shorter maturity, for which financing is available. This maturity adjustment is unobservable to financiers and therefore creates endogenous asymmetric information: The inflow of second-best projects worsens the pool of funded, shorter-maturity projects. This affects the terms of the debt contract offered by financiers who now face a worse pool of borrowers, leading to a negative externality: Funding terms for firms that up to now could receive financing worsen and, because the maximum funded maturity shortens, a number of formerly fundable firms are now rationed. These firms respond by adopting shorter-term projects, thereby inducing an additional inflow of second-best projects into the funded region. The process repeats and a *short-termism spiral* arises (see Fig. 1). Taking into account the interdependence between the asset and liability sides, the equilibrium is thus given by a fixed point: Firms' investment decisions respond optimally to financing frictions on the liability side, while financiers take into account firm's investment decisions when extending financing.

When capital markets are competitive, the resulting equilibrium is *constrained inefficient*: Investment is inefficiently short-term and surplus is strictly lower compared to the case in which financing is offered by a central planner who is subject to the same informational and limited commitment constraints as financiers. The inefficiency of the competitive equilibrium arises because, through their impact on the quality of the pool of firms seeking financing, the asset side adjustments made by individual firms affect the financing terms faced by *all* firms, generating an externality. When this negative externality from the adoption of secondbest project is strong enough, the short-termism spiral can lead to a complete breakdown of financing across *all* maturities. When financing terms are offered by a planner, this negative feedback loop is mitigated. Specifically, a planner subsidizes long-term projects and taxes short-term projects in order to counteract the excessive short-termism of the competitive equilibrium.

Because of their self-reinforcing nature, firms' privately optimal asset side adjustments can amplify shocks. For example, an increase in risk can lead to significantly larger reductions in financed maturities and surplus in a setting where firms can adjust their asset side, relative to the benchmark in which firms' asset sides are held fixed. We also show firms' privately optimal asset-side adjustments can increase or decrease surplus, depending on the severity of the cross-firm externality. At one extreme, when second-best projects are essentially as good as first-best projects, the ability of firms to adjust their asset-side investments increases



Figure 1: Short-termism Spiral. Illustration of the short-termism spiral that emerges from endogenous adjustments on the asset side in response to financing frictions on the liability side.

surplus. In this case, firms that adopt shorter maturity projects do not impose an externality on other firms; the only consequence from their maturity adjustments is an increase in output, as formerly rationed firms find shorter maturity projects that can be funded. On the other hand, when second-best projects are worse than firms' original projects, privately optimal asset-side adjustments can lead to an overall reduction in surplus. This reduction in surplus occurs even when second-best projects have positive NPV and when, as a result of firms' asset side adjustments, more projects get financed such that total lending increases. Total lending is not a sufficient statistic for surplus because the drop in average project quality can outweigh the gains from increased investment.

Our theory generates a number of empirical implications. First, viewing the asset and liability sides as jointly determined leads to a new interpretation of the positive correlation between debt maturity and asset maturity that has been documented in the empirical literature.<sup>5</sup> While this literature mostly interprets this finding as a one-way link from asset maturity to debt maturity, we emphasize a two-way feedback between debt maturities and asset maturities, consistent with recent empirical evidence in Gopalan et al. (2014) and Paligorova and Santos (2013). Second, our model highlights a novel cross-firm externality: While individual firms find it optimal to adopt shorter-term projects in order to receive better financing terms, this worsens funding terms for other firms and thereby imposes a negative externality. Identifying these spillovers poses an interesting empirical challenge for future work on debt and asset maturities. Third, following the macroeconomics literature on dispersion shocks, our model generates predictions regarding the joint dynamics of debt and asset maturities over the business cycle. Consistent with our model, in the recent financial crisis and over the business cycle in general is that maturities shorten both on the *liability* side and the asset side of firms' balance sheets. On the asset side, Dew-Becker (2012) shows that downturns are also associated with drops in the maturity (or duration) of investment. On the liability side, Mian and Santos (2011), Erel et al. (2012), and Chen et al. (2012) document shortening of debt maturities on firms' liability sides during downturns.

Overall, our results underscore the importance of considering firms' asset side decisions jointly with the financing terms of different liabilities, such as short-term and long-term debt. Most of the literature on short-term debt follows Diamond and Dybvig (1983) and Diamond (1991) in focusing on early liquidation of (fixed) investment projects as the main cost of short-term debt.<sup>6</sup> Our analysis draws attention to a different and complementary channel: Short-term financing changes firms' investment behavior and generates inefficient endogenous

<sup>&</sup>lt;sup>5</sup>See, e.g., Morris (1992); Guedes and Opler (1996); Stohs and Mauer (1996); Johnson (2003).

<sup>&</sup>lt;sup>6</sup>See, e.g., Acharya et al. (2011), He and Xiong (2012), and Brunnermeier and Oehmke (2013).

short-termism on the asset side. Unlike other theories that have focused on bad incentives and behavioral biases as sources of short-termism (such as reputation building (Narayanan (1985)), concern with near-term stock prices (Stein (1989)), short investor horizons (Froot et al. (1992)) or speculative investors (Bolton et al. (2006))), in our framework short-termism arises as an equilibrium phenomenon in a fully rational setting. Moreover, short-termism in our model is collective, in the sense that in competitive equilibrium firms privately optimal decisions reinforce the short-termism in investment decisions.

In highlighting this link between short-term financing and short-termism in investment, our paper is related to von Thadden (1995) and Dewatripont and Maskin (1995), who study settings in which firms may adopt short-term projects in fear of being liquidated at an interim date. In contrast to von Thadden (1995), where short-termism is part of a constrained efficient outcome, our framework highlights a cross-firm externality that leads to *constrained inefficient* short-termism. In Dewatripont and Maskin (1995), a related inefficiency can arise, in the sense that the threat of interim liquidation can lead to multiple Pareto-ranked equilibria. Cheng and Milbradt (2012) develop a model in which a firm trades off liquidation costs arising from debt runs against asset side distortions that arise from managerial riskshifting. However, also they focus on a single firm, such that the cross-firm externality that is central to this paper cannot arise.<sup>7</sup> In highlighting how endogenous asymmetric information can lead to cross-firm externalities that can amplify the response of equilibrium prices and quantities to shocks, our paper is related to Eisfeldt (2004), Kurlat (2010) and Bigio (2011), who study amplification through asymmetric information in macroeconomic settings without

<sup>&</sup>lt;sup>7</sup>A recent paper that highlights spillover effects among firms is Bebchuk and Goldstein (2011). In their setup, spillovers arise directly in project payoffs (projects become more attractive as more firms invest, leading to a payoff externality), while in our framework spillovers arise due to endogenous asymmetric information on the financing side (leading to an information externality).

maturity choice. Finally, as a point of departure, our paper builds on the extensive literature on credit rationing.<sup>8</sup>

# 2 Model setup

There is a continuum of firms, each of which is born with an (initial) investment project of maturity t, drawn uniformly from the interval  $[0, \overline{T}]$ .<sup>9</sup> The maturity of a project indicates how long it takes for the project to pay off: A project of maturity  $t \in [0, \overline{T}]$  generates cash flow only at date t and no cash flow beforehand (or after). To undertake their projects, firms seek financing from a financial sector composed of a continuum of competitive, risk-neutral financiers with deep pockets.

Projects cost one dollar to set up. Once set up, a project can be of two types. With probability  $\alpha$  the project is safe and, at maturity t, pays off R for sure. With probability  $1 - \alpha$  the project is risky and, at maturity, pays off  $e^{\lambda t}R$  with probability  $e^{-\lambda t}$  and zero otherwise.<sup>10</sup> The risky project therefore has the same expected payoff as the safe project, but it defaults over time with intensity  $\lambda$ . This generates a natural link between project maturity and project risk, which is the key assumption in our model.<sup>11</sup> At the time of

<sup>&</sup>lt;sup>8</sup>For a summary of this literature, a good starting point is the discussion in Bolton and Dewatripont (2005, Chapter 2), Freixas and Rochet (2008, Chapter 5), or the survey on financial contracting by Harris and Raviv (1992). The classic contributions on credit rationing are Jaffee and Modigliani (1969), Jaffee and Russell (1976), Stiglitz and Weiss (1981), and DeMeza and Webb (1987). Bester (1985) and Besanko and Thakor (1987) examine the role of collateral as a screening device in models with credit rationing. Suarez and Sussman (1997) develop an overlapping generations model in which credit rationing can lead to endogenous business cycles.

<sup>&</sup>lt;sup>9</sup>For industrial firms, projects should be interpreted as real investments. For financial institutions, projects should be interpreted as loan portfolios on the financial institution's asset side.

<sup>&</sup>lt;sup>10</sup>The restriction to two project types is for tractability. More generally, one could also assume a distribution of exponentially compensated project risks  $\tilde{\lambda}$  with a density function  $f(\tilde{\lambda})$ . Our model is a special case with mass points at  $\tilde{\lambda} = 0$  and  $\tilde{\lambda} = \lambda$ .

<sup>&</sup>lt;sup>11</sup>Note that a positive relation between time and risk (after conditioning on observable characteristics) is a common feature of many standard models in finance, such as structural models of credit risk (e.g., Merton

contracting, neither firms nor financiers know whether a project is safe or risky.<sup>12</sup>

Firms can freely adjust the maturity of their original project. However, this adjustment is costly in the sense that a maturity-adjusted project is second best: In adjusting the maturity of its original project, a firm deviates from its first-best investment strategy. One interpretation of a second-best project with maturity t' < t is that the firm implements a rushed version of the original project, in which the firm speeds up the required research and development, hurries the construction of plants and equipment, or otherwise cuts corners in the implementation of the project. An alternative interpretation is in adjusting its maturity the firm literally searches for a new, second-best investment project that it will undertake instead of the first-best project it was born with.<sup>13</sup>

Specifically, we assume that when firms adjust the maturity of their project to any  $t' \in [0, \overline{T}]$ , the project is less likely to succeed, which we capture by assuming that replacement projects have an additional probability of default of  $1 - \Delta$  that applies to both safe and risky projects.<sup>14</sup> This additional default risk of adjusted projects can realize at any point after contracting up to maturity t'.<sup>15</sup> In addition, we assume that there is an arbitrarily small

<sup>(1974)</sup> and Leland (1994)) and dynamic agency models (e.g., models in the spirit of Sannikov (2008)).

<sup>&</sup>lt;sup>12</sup>Our results are robust to a number of variations in these assumptions. For example, it is not necessary to assume that the drift of bad projects exactly compensates for the default intensity  $\lambda$ . However, this assumption is convenient because it guarantees that the NPV of bad projects is independent of the project maturity. Hence, our results are driven by differences in financial frictions (arising from limited commitment) across different maturities as opposed to differences in NPV across different maturities. It would also be straightforward to allow for for a positive rate of time preference r > 0 or time dependency in the intensity of default  $\lambda(t)$ .

<sup>&</sup>lt;sup>13</sup>When the firm is a financial institution, a maturity adjustment should be interpreted as a shortening of the financial institution's loan portfolio. The additional default risk should then be interpreted as a deviation from the type of loans where the financial institution has a comparative advantage. In addition, the shortening of the financial institution's loan portfolio may distort the real decisions of the firm funded by such a loan: Because financing is more short-term, this firm may now alter its investment away from its first-best investment strategy, thereby increasing the riskiness of the loan portfolio.

<sup>&</sup>lt;sup>14</sup>The additional default risk  $\Delta$  could also be random, in which case only its mean  $\overline{\Delta}$  would matter.

<sup>&</sup>lt;sup>15</sup>The exact specification when this risk is realized on (0, t] does not matter for our results. They key assumption is that that this risk applies at any point after contracting at time  $t = 0_{-}$ .

probability  $\varepsilon \to 0$  that firms who attempt to adjust their maturity are unsuccessful in doing so and remain stuck at their original maturity with a second-best project.<sup>16</sup> Given these assumptions, the NPV of a project of type  $\theta \in \{1, \Delta\}$  is given by

$$NPV(\theta, t) = \theta \left[ \alpha R + (1 - \alpha) e^{-\lambda t} e^{\lambda t} R \right] - 1 = \theta R - 1, \tag{1}$$

such that the NPV of the original project is given by  $NPV_{original} \equiv NPV(1,t) = R-1 > 0$ , and the NPV of the adjusted project by  $NPV_{adjusted} \equiv NPV(\Delta, t) = \Delta R - 1$ .

We make two key assumptions. First, we assume that while the maturity of a project is commonly observable (such that there is no uncertainty about when a particular project pays off), whether or not a project is an original project or a project with adjusted maturity is private information to the firm. This means that firms' decisions to adjust their project maturity generate endogenous asymmetric information. Second, we assume that the project cash flows that are realized at maturity are not contractible, which introduces a limited commitment friction in the spirit of Bolton and Scharfstein (1990) and Hart and Moore (1994). Specifically, we assume that at maturity it can only be verified whether or not a project succeeded, but not which exact cash flow, (i.e., R or  $e^{\lambda t}R$ ), has realized. Therefore, a successful firm with a risky project, which receives  $e^{\lambda t}R$  at maturity, can always claim to have received only R (the payoff from a safe project), and pocket the difference. This contracting friction limits the amount financiers can extract from a firm with a successful

<sup>&</sup>lt;sup>16</sup>The main role of this assumption is that after the maturity adjustment all maturities are still populated by some firms. This means that we do not have to specify out-of-equilibrium beliefs regarding project maturity choice. In the main paper we focus on the case  $\varepsilon \to 0$ , (i.e., essentially all firms can pick their desired maturity), in which case the assumption is similar to a trembling hand refinement. In the appendix, we also solve the model for  $\varepsilon > 0$ .

project to R.

Because firms' maturity adjustments are not observable, from the financiers' point of view firms with second-best projects are indistinguishable from firms with first-best projects. Moreover, firms with first-best projects have no way to separate themselves from second-best firms.<sup>17</sup> Hence, financing at any maturity t is only possible if financiers can break even on a pooling contract. The terms of this pooling contract depend on the endogenous asymmetric information created by firms' maturity adjustment decisions.

# 3 Competitive equilibrium

We assume that financiers maximize profits and compete by simultaneously offering takeit-or-leave-it funding schedules contingent on the project maturity  $t \in [0, \overline{T}]$ , taking into account firms' equilibrium maturity adjustments and the resulting project quality distribution as a function of project maturity. After funding schedules have been posted, firms make their maturity adjustments and fund themselves at the best rate they can find, if funding is available.

We now derive the pure-strategy Bayesian Nash equilibrium of funding terms and investment decisions given the setup introduced in Section 2.

**Definition 1** A Bayesian Nash equilibrium in competitive capital markets is given by (i) price schedules of funding conditions offered by financiers and (ii) maturity adjustment decisions by firms, such that:

<sup>&</sup>lt;sup>17</sup>There is no scope for signaling in our model. Because no interim information is revealed, firms with second-best projects can costlessly mimic firms with first-best projects.

- 1. Financiers maximize expected profits by deciding which maturities to fund and posting deterministic funding terms for these maturities, taking into account firms' optimal project maturity adjustments.
- 2. After observing funding terms offered by financiers, firms adjust maturities to maximize expected profits.

Three features of this equilibrium warrant brief comment. First, note that we assume that financiers only compete on price and the set of maturities that is funded deterministically. This means that we rule out probabilistic funding at a given maturity t. Hence, for any funded maturity t, firms can fully fund their project at the funding terms offered. Second, limited liability implies that firms always accept the best available funding terms at their ultimate project maturity. Thus, the main decision for firms is whether to adjust the maturity of their project and, if so, what maturity to pick. Third, because financiers act competitively, in equilibrium they have to break even maturity by maturity. Hence, competition rules out cross-subsidization across maturities.

Based on these observations, the equilibrium has to satisfy two conditions, which can be interpreted as individual rationality (IR) and incentive compatibility (IC) constraints for financiers and firms, respectively:

- (IR): Investors break even at each funded maturity.
- *(IC):* Given the offered funding terms, firms that are offered funding at their original maturity have no incentive to adjust the maturity of their projects.

Because cash flows are not verifiable, the optimal financial contract that pools original

and adjusted types takes the from of a debt contract.<sup>18</sup> Consider first debt contracts that match the maturity of the project (as we will show below, this restriction is without loss of generality). Recall that the limited commitment friction implies that raising the face value of debt above R leaves the amount paid back to financiers unchanged because firms with a payoff higher than R would simply claim to have received R. Thus, the maximum "effective" face value D of a matching-maturity debt contract is given by

$$D \le R. \tag{2}$$

As long as (2) holds, financiers can break even and their IR constraint is satisfied.

To determine the face value of debt, suppose that, at a given maturity t, financiers expect a proportion p of projects to be original, first-best projects and a proportion 1-p of projects to be maturity-adjusted, second-best projects. The financiers' break-even constraint, assuming  $D \leq R$ , is then given by

$$1 = p \left[ \alpha D + (1 - \alpha) e^{-\lambda t} D \right] + (1 - p) \Delta \left[ \alpha D + (1 - \alpha) e^{-\lambda t} D \right]$$
  
$$= \beta \left[ \alpha + (1 - \alpha) e^{-\lambda t} \right] D, \qquad (3)$$

where

$$\beta \equiv p + (1 - p)\,\Delta\tag{4}$$

<sup>&</sup>lt;sup>18</sup>Note that because we set the low cash flow to zero, debt contracts can, strictly speaking, also be structured as equity contracts. However, if the payoff in the default state was L > 0, then a debt contract becomes strictly optimal: To relax the incentive constraint in the high state, it is then optimal to extract as much as possible in the low state.

captures the average quality of projects at maturity t.<sup>19</sup> Solving the break-even constraint for D, we have

$$D_c(t,\beta) = \frac{1}{\beta \left[\alpha + (1-\alpha) e^{-\lambda t}\right]},\tag{5}$$

where the subscript "c" denotes that this is the competitive face value. Note that the breakeven face value is increasing in project maturity, reflecting the higher risk of projects of longer maturity, and decreasing in average quality.

Focusing on debt contracts that match the maturity of the project is without loss of generality: Allowing for rollover contracts does not change the equilibrium funding terms or which projects are undertaken in equilibrium. The intuition for this result is that, given our assumptions, rollover contracts do not add to the contracting environment. Because there are no intermediate signals about project type (except for the realization of default), any sequence of rollover contracts is payoff-equivalent to a debt contract that matches the maturity of the project.

**Proposition 1** Any sequence of rollover debt contracts is payoff equivalent to the matching maturity debt contract. The sequence of debt contracts that firms choose to finance their projects is indeterminate. A project can be financed if and only if it can be financed with a matching maturity debt contract.

In addition, if we add a small cost of rolling over debt, then firms match maturities of assets and liabilities in the unique equilibrium that survives the D1 criterion, a standard refinement used in signaling games.<sup>20</sup>

<sup>&</sup>lt;sup>19</sup>This interpretation follows from the observation that the expected NPV of a project of maturity t which has a probability p of being first-best is given by  $pNPV_{original} + (1-p)NPV_{adjusted} = \beta R - 1$ .

 $<sup>^{20}</sup>$ Condition D1 is an equilibrium refinement that requires out-of-equilibrium beliefs to be placed on types that have the most to gain from deviating from a fixed equilibrium (see Banks and Sobel (1987)).

**Proposition 2** Suppose there is a small debt rollover cost  $c \to 0$  and that the additional default risk  $(1 - \Delta)$  realizes continuously over  $[0, \overline{T}]$ . Then, under the D1 refinement, in any equilibrium all firms use maturity-matching debt contracts.

The intuition for this results is straightforward. Firms that have adjusted the maturity of their project default with higher probability. This implies that they end up paying the full sequence of rollover costs c less often, which makes a deviation to rollover financing more attractive for firms with maturity-adjusted projects than for firms that have not adjusted the maturity of their project. Under the D1 criterion, financiers then attribute any deviation to rollover financing to firms with second-best, maturity-adjusted projects, making such a deviation unprofitable. On the other hand, any deviation from a rollover contract to maturity matching is attributed to firms with first-best projects, making such a deviation profitable and thereby ruling out rollover equilibria.

## 3.1 Benchmark: Fixed asset side

Before we characterize the full equilibrium with the endogenous asset side, we briefly consider the equilibrium under the assumption that firms' assets are fixed (i.e., firms cannot adjust the maturity of their project).

When firms cannot adjust their maturity, all projects are original, first-best projects, such that pool quality is everywhere  $\beta = 1$ . When  $D_c(\overline{T}, 1) \leq R$ , all projects can be funded. In this case, contractual frictions are small enough, such that the limited commitment constraint never precludes financing at any maturity. When  $D_c(\overline{T}, 1) > R$ , on the other hand, projects beyond some critical maturity  $T_b < \overline{T}$  cannot be funded. For project maturities larger than



Figure 2: Fixed Asset Side: Face value and maturity. The figure illustrates the benchmark case with a fixed asset side. For maturities below 8.4 years, the required face value  $D_c(t, 1)$  lies below R. In this region, a break-even debt contract exists, such that the financiers' IR constraint is satisfied. Beyond a maturity of 8.4 years, the required face value exceeds R, such that a debt contract cannot break even. These maturities are rationed; they cannot be funded in equilibrium.

 $T_b$ , financiers cannot break even because contractual frictions are too severe and maturity rationing arises, as illustrated in Fig. 2.

The main takeaway from this benchmark case with fixed asset side is therefore that, when contractual frictions are sufficiently large, some long-term projects cannot be funded. In the full model with endogenous asset side, this implies that firms with rationed long-term investment projects adopt second-best, shorter-term projects, thereby inducing the shorttermism spiral illustrated in Fig. 1. In the remainder of the paper we concentrate on the case in which contractual frictions are significant and assume that  $D_c(\overline{T}, 1) > R$ .

### 3.2 Full model: Endogenous asset side

We now solve for the equilibrium with endogenous asset side. As discussed above, this requires making sure that both the IC constraint (firms funded at their original maturity have no incentive to adjust their maturity) and the IR constraint (investors break even at each funded maturity) are satisfied. In contrast to the model with fixed asset side, the project quality (i.e., the mix of first- and second-best projects) at any given maturity is now endogenous because it depends on firms' equilibrium asset side adjustments.

We start by making an observation that will significantly simplify our analysis. Denote the set of funded maturities by  $\mathcal{F}$ . Given a face value D, the expected payoff to a firm that funds a project of type  $\theta \in \{1, \Delta\}$  and maturity  $t \in \mathcal{F}$  is then given by

$$\pi (\theta, t, D) = \theta \left[ \alpha (R - D) + (1 - \alpha) e^{-\lambda t} \left( e^{\lambda t} R - D \right) \right]$$
$$= \theta \left[ R - D \left( \alpha + (1 - \alpha) e^{-\lambda t} \right) \right].$$
(6)

Suppose the average quality at maturity t is given by  $\beta_t$ . Then, inserting the expression for the competitive face value  $D_c(t, \beta_t)$ , we see that

$$\pi\left(\theta, t, D_{c}\left(t, \beta_{t}\right)\right) = \theta\left[R - \frac{1}{\beta_{t} + (1 - \beta_{t})\Delta}\right], t \in \mathcal{F}.$$
(7)

Hence, the expected profit conditional on average project quality  $\beta_t$  is independent of project maturity. This is has two important implications. First, conditional on adjusting their maturity, firms must be indifferent between all maturities that are funded in equilibrium. Formally, this requires that

$$\pi\left(\Delta, t', D_c\left(t', \beta_{t'}\right)\right) = \pi\left(\Delta, t, D_c\left(t, \beta_t\right)\right), \forall t, t' \in \mathcal{F},\tag{8}$$

which implies that the average project quality must be constant on the funded set,  $\beta_{t'} = \beta_t = \beta$ , requiring that non-funded firms adjust their maturity uniformly into the funded interval.

Second, combining (7) and (8), we see that it is never optimal for firms to adjust their maturity unless they cannot get funding otherwise: If a firm's original project can get funded, it is more profitable for the firm to fund the original project rather than adopting a second-best project:

$$\min_{t \in \mathcal{F}} \pi \left( 1, t, D\left(t, \beta_t\right) \right) \ge \max_{t \in \mathcal{F}} \pi \left( \Delta, t, D\left(t, \beta_t\right) \right), \tag{9}$$

with strict inequality for  $\Delta < 1$ . We can therefore ignore the IC constraint: In equilibrium, all firms with original project maturities that cannot be funded adjust the maturities of their projects, whereas firms that can obtain funding for their original project do not adjust their project maturity and simply fund their original project.

The unique (pure-strategy) Bayesian Nash equilibrium takes the form of a cut-off equilibrium  $\mathcal{F} = [0, T]$ , which we simply identify by the maximum funded maturity T.

#### **Lemma 1** Equilibrium funding strategies take the form [0, T].

Given this cutoff structure, the proportion of first-best projects (the average pool quality) on the funded set is given by  $p(T) = \frac{T}{\overline{T}}$  so that the average quality on the funded set [0, T]is given by

$$\beta\left(T\right) = \frac{T}{\overline{T}} + \Delta \frac{\overline{T} - T}{\overline{T}} \tag{10}$$

so that  $\Delta \leq \beta(T) \leq 1$  and  $\beta'(T) = \frac{1-\Delta}{\overline{T}} > 0$ . This is illustrated in Fig. 3. The left panel depicts the maturity distribution of projects before maturity adjustment. All projects are original, first-best projects. The right panel depicts the maturity distribution of projects after maturity adjustment. Firms with original projects beyond T = 6 adjust their maturity uniformly into the funded interval. An arbitrarily small fraction of firms gets stuck at their



Figure 3: **Project density and quality** as a function of the project maturity when cutoff is T = 6 and everyone above T adjusts the maturity of their projects. The left panel depicts the maturity distribution of projects before maturity adjustment. All projects are original, first-best projects. The right panel depicts the maturity distribution of projects after maturity adjustment. Firms with original projects beyond T = 6 adjust their maturity uniformly into the funded interval. An arbitrarily small fraction of firms gets stuck at their original maturity and does not receive funding.

original maturity and does not receive funding.

The competitive equilibrium is then given by the largest funding cutoff  $T \in [0, \overline{T}]$  such that the IR constraint  $D_c(T, \beta(T)) \leq R$  is satisfied. This leads to the following proposition:

**Proposition 3** Define  $\mathcal{IR}_c = \{T \in [0,\overline{T}] : D_c(T,\beta(T)) \leq R\}$ . Then the competitive equilibrium is given by the funding cutoff  $T_c = \sup \mathcal{IR}_c$ , unless  $\mathcal{IR}_c = \emptyset$ , in which case funding completely unravels at all maturities. A sufficient condition for complete unraveling is that  $D(\overline{T}, 1) > R$  and  $\Delta < \min\{1 - (1 - \alpha)\lambda\overline{T}R, 1/R\}.$ 

In competitive equilibrium, funding is provided up to the longest maturity for which both the IC and the IR constraints are satisfied. However, because IC constraint is non-binding in the competitive equilibrium, it is sufficient to concentrate on financiers' IR constraint, as stated in Proposition 3. When there is maturity rationing in the benchmark case without asset-side adjustments (i.e.,  $D_c(\overline{T}, 1) > R$ ), then  $T_c$  is the largest maturity on  $[0, \overline{T}]$  for which the financiers' IR constraint holds with equality,  $D_c(T, \beta(T)) = R$ . When there is no funding cutoff that satisfies the financiers' IR constraint, (i.e.,  $\mathcal{IR}_c = \emptyset$ ), then funding completely unravels for all maturities and  $T_c = \emptyset$ . The sufficient condition for complete unraveling has a natural interpretation. There has to be initial rationing, the maturityadjusted project must have sufficiently negative NPV, such that the dilution by maturity adjusted projects is sufficiently strong.

Why is funding provided up to the longest maturity at which the IR constraint can be satisfied? Suppose that funding is instead provided up to a funding cutoff  $T \in I\mathcal{R}_c$  with  $T < T_c$ . Then, there is a profitable deviation in which a financier increases the range of funded maturities he offers. As a result of this deviation, fewer firms adjust their maturity, such that the average project quality the financier is facing improves. This allows the financier to charge a lower interest rate and thereby undercut the other financiers, attracting all firms at a funding rate that is strictly profitable. Financiers compete in this fashion until funding is provided up to  $T_c$ .

Given that the equilibrium requires that the face value at the financing cutoff is smaller than R, it is instructive to consider how the face value at the maximum funded maturity,  $D_c(T, \beta(T))$ , is affected by a small change in the funding threshold T. Writing out the derivative, we have

$$\frac{dD_{c}(T,\beta(T))}{dT} = D_{c}(T,\beta(T))^{2} \times \left[\underbrace{\lambda(1-\alpha)e^{-\lambda T}\beta(T)}_{\text{Maturity effect}} - \underbrace{\left[\alpha + (1-\alpha)e^{-\lambda T}\right]\beta'(T)}_{\text{Dilution effect}}\right].$$
(11)

Thus, a reduction in the maximum funded maturity T has two countervailing effects. The maturity effect leads to a decrease in the required face value because, all else equal, funding shorter maturity projects is less risky. The dilution effect, on the other hand, implies that reducing the maximum funded maturity leads to more maturity adjustment and thereby (weakly) lowers the average project quality on the funded interval (since  $\beta'(T) > 0$ ). This leads to an increase in the required break-even face value.

Finally, we make a brief remarks on the different roles played by financiers and firms in the competitive equilibrium: Financiers take into account that any deviation strategy that offers funding to a range of maturities that was previously unfunded affects the incentives of *all* firms. This is because such a deviation changes the *IC* constraint at every maturity. Hence, financiers internalize the effect of their funding decisions on the average project quality. Firms, on the other hand, ignore the impact of their individual asset-side decisions on the aggregate outcome because they take the average pool quality as given. This difference between firms and financiers is driven by the scale of their impact: Each firm can only undertake one infinitesimal project and thus cannot affect the aggregate. Financiers, on the other hand, can affect the aggregate: Financiers have deep pockets and, therefore, the contracts they offer affect the behavior of a mass of firms.

## 4 Central planner equilibrium

We now contrast the competitive equilibrium derived above with the allocation that would be implemented by a central planner facing the same informational and contractual constraints as the financiers (as well as the same restriction to pure strategies). The main difference between the solution to the constrained planner's problem and the competitive equilibrium is that, in contrast to competitive financiers, the planner can cross-subsidize across maturities: While the planner also has to break even, he faces an aggregate break-even constraint over the entire funded interval  $[0, T_{cp}]$ . Competitive financiers, on the other hand, have to break even maturity by maturity, which rules out cross-subsidization across maturities.<sup>21</sup>

## 4.1 Constrained inefficiency of the competitive equilibrium

We first show that the competitive equilibrium is constrained inefficient. To show this, we demonstrate that a constrained planner can raise surplus by raising face values on short-term projects (effectively taxing them) and using the proceeds to fund more long-term projects, some of them at strictly subsidized rates. Such a cross-subsidization scheme reduces firms' incentive to adopt second-best projects of shorter maturity, leading to an increase in surplus.

To see this, consider the competitive equilibrium allocation, in which funding is provided on the set  $\mathcal{F} = [0, T_c]$ . Recall that in the competitive equilibrium the IR constraint is binding at  $T_c$  (i.e.,  $D_c(T_c, \beta(T_c)) = R$ ) and slack for any  $t < T_c$  (i.e.,  $D_c(t, \beta(T_c)) < R$ ). Also recall that the IC constraint is slack everywhere, which implies that

$$\min_{t \in \mathcal{F}} \pi \left( 1, t, D_c \left( t, \beta \left( T_c \right) \right) \right) > \max_{t \in \mathcal{F}} \pi \left( \Delta, t, D_c \left( t, \beta \left( T_c \right) \right) \right).$$
(12)

Starting from the competitive equilibrium, now consider raising all face values by a factor of  $1 + \eta$  (a proportional tax), except in cases where this would increase the face value beyond

 $<sup>^{21}</sup>$ In contrast to some of the standard explanations of short-termism as resulting from bad incentives or behavioral biases, in our framework, constrained inefficient short-termism emerges as an equilibrium phenomenon in a fully rational setting. We provide a more detailed discussion of our results to the existing academic on policy discussion on short-termism in Section 5.4.

R:

$$D_{\eta}(t,\beta) = \min\{(1+\eta) D_{c}(t,\beta), R\}.$$
(13)

By charging more than the competitive face value, a planner offering this face value schedule makes *strictly* positive profits on the interval  $[0, T_c]$ , without violating the IC constraint (for small enough  $\eta$ , firms located at  $T_c$  still strictly prefer to fund their original project). The planner can then use these profits to offer funding to firms located on  $(T_c, T_c + \delta]$ . Because the IC constraint is slack at  $T_c$ , by continuity there exist  $\eta > 0$  and  $\delta > 0$  such that the IC constraint is also satisfied on  $(T_c, T_c + \delta]$  and the central planner makes non-negative profits in aggregate. Under the assumption that  $T_c \in [0, \overline{T})$ , such a funding scheme leads to a welfare improvement because an additional measure  $\frac{\delta}{T} > 0$  of first best projects are funded. In the competitive allocation, these firms would have adopted second-best projects of shorter maturity, thereby inducing a loss of surplus. Hence, for any  $T_c \in [0, \overline{T})$ , a constrained planner funds a larger set of maturities and thereby a larger number of first-best projects than the competitive market. Some of the additionally funded maturities receive funding on strictly subsidized terms.

**Proposition 4** Assume  $T_c \in [0, \overline{T})$ . The competitive equilibrium is constrained inefficient. A constrained central planner always funds more maturities than the competitive market, *i.e.*,  $T_{cp} > T_c$ .

## 4.2 Optimal funding schedules

We now derive the optimal funding terms offered by (i) a constrained central planner and (ii) a monopolistic financier. To do so, we first establish a result on the optimal implementation of funding, which we then use to characterize the funding terms offered by a central planner and a monopolist.

Consider funding up to a maturity threshold T with the possibility of cross-subsidizing across maturities. Such a scheme requires picking a funding schedule  $D_T(t)$ , subject to the IC constraint (9) and the maximum face-value constraint (2). There is also a new maturity adjustment constraint,

$$d_{T}(t) = \delta_{t_{\max}}(t) \frac{\overline{T} - T}{\overline{T}}, \quad t_{\max} \equiv \sup\left\{\arg\max_{t \in [0,T]} \pi\left(\Delta, t, D_{T}(t)\right)\right\},\tag{14}$$

where  $d_T(t)$  is the density of second best projects that locate at maturity t and  $\delta_{t_{\text{max}}}(t)$  is the Dirac Delta function.<sup>22</sup> The maturity adjustment constraint (14) captures the behavior of firms that choose to adjust the maturity of their project: These firms optimally choose the funded maturity that gives them the highest expected profit  $\pi$ , as identified by the argmax operator.<sup>23</sup>

The overall profit to the financier is then given by

$$\Pi\left(T, d_{T}\left(t\right)\right) = \int_{0}^{T} \left\{ \left(\frac{1}{\overline{T}} + d_{T}\left(t\right)\Delta\right) \left[\alpha + (1-\alpha)e^{-\lambda t}\right] D_{T}\left(t\right) - 1\left(\frac{1}{\overline{T}} + d_{T}\left(t\right)\right) \right\} dt \quad (15)$$

**Lemma 2** The maximum profit funding schedule for a funding interval [0, T] is given by the

<sup>&</sup>lt;sup>22</sup>The Dirac Delta function  $\delta_t(s)$  concentrates all probability mass at a point t, e.g.,  $\int_0^\infty f(s) \, \delta_t(s) \, ds = f(t)$ .

<sup>&</sup>lt;sup>23</sup>Recall that the NPV of the adjusted project is constant. Hence, if the expected profit to firms is constant on some set of maturities, then the expected profit to the financier also has to be constant on that set (the two have to add up to the NPV of the project). If the argmax set above is not a singleton, we can therefore, without loss of generality, assume that firms pick the highest maturity  $t_{\text{max}}$  in the argmax set without affecting any of the expected payoffs.

continuous function

$$D_T(t) = \min \{ C(T) D_c(t, 1), R \}.$$
(16)

where  $C(T) \equiv R\left(\alpha + (1 - \alpha)\left[(1 - \Delta) + \Delta e^{-\lambda T}\right]\right)$ . Define  $T_C(T) \in [0, \overline{T}]$  to be the maturity at which  $C(T) D_c(t, 1) = R$ . Then the IC constraint is binding on  $[0, T_C(T)]$  and the IR constraint is binding on  $[T_C(T), T]$ .

Inserting the maximum profit funding schedule  $D_T(t)$  into  $\Pi$  we obtain a profit function  $\Pi(T)$ , that satisfies  $\Pi(0) = \Delta R - 1 = NPV_{adjusted}$ . Optimality of the funding schedule implies that for any break-even competitive cutoff  $T \in \mathcal{IR}_c \neq \emptyset$ , the optimal scheme at least breaks even,  $\Pi(T) \ge 0$ .

**Proposition 5** Let  $\mathcal{IR}_{cp,m}$  be the set of funding cutoffs T for which the optimal funding scheme at least breaks even, i.e.,  $\mathcal{IR}_{cp,m} = \{T \in [0,\overline{T}] : \Pi(T) \ge 0\}$ . Then the set of cutoffs that can be funded is larger under the optimal scheme than under the competitive scheme:  $\mathcal{IR}_c \subset \mathcal{IR}_{cp,m}$ . No funding is possible under the optimal funding schedule,  $\mathcal{IC}_{cp,m} = \emptyset$ , if and only if max  $\{\Pi(0), \Pi(\overline{T})\} < 0$ .

**Central planner.** We first characterize the optimal funding schedule offered by a central planner. The central planner's objective is to maximize surplus. As we saw above, in doing so the central planner effectively taxes shorter maturity projects and subsidizes longer maturity projects, in order to keep firms with first-best projects of longer maturity from adopting second-best shorter-term projects. In choosing the funding cutoff T, the central planner picks the split between how many first-best project  $(\frac{T}{T})$  and how many second-best

projects  $(\overline{T}-T)$  are funded. Because  $NPV_{original} > NPV_{adjusted}$ , the central planner picks the maximum T that still fulfills the aggregate break-even constraint across all funded maturities, i.e.,  $\Pi(T_{cp}) \ge 0$ :

**Proposition 6** Assume  $\mathcal{IR}_{cp,m} \neq \emptyset$ . The central planner picks the largest funding threshold  $T_{cp} = \sup \mathcal{IR}_{cp,m}$ . Further,

- 1. if  $\Pi\left(\overline{T}\right) \geq 0$ , all maturities are funded:  $T_{cp} = \overline{T}$ ,
- 2. if  $\Pi\left(\overline{T}\right) < 0$ , there is limited funding  $T_{cp} \in \left(T_c, \overline{T}\right)$ .

Monopolist. Now consider funding by a monopolist. Like the central planner, the monopolist can cross-subsidize across maturities. Unlike the central planner, the monopolist maximizes profits, not surplus, and thus does not necessarily pick the highest T at which profits are non-negative.

The monopolist's optimal funding scheme features a trade-off between the appropriability of surplus and the amount of surplus generated: Because project risk increases in maturity, all else equal the monopolist can extract less profit from longer-maturity projects, whereas the monopolist can extract all surplus from projects of extremely short maturity (i.e., as project maturity approaches 0). At the same time, a higher funding threshold T increases the proportion of good projects funded, and thus increases total surplus. This tradeoff results in a corner solution:

**Proposition 7** Assume  $\mathcal{IR}_{cp,m} \neq \emptyset$ . The monopolist financier picks a corner solution,  $T_m \in \{0, \overline{T}\}$ . Further,

1. if  $\Pi(0) > \Pi(\overline{T})$ , the monopolist only funds extremely short-term projects:  $T_m = 0$ ,

2. if  $\Pi\left(\overline{T}\right) > \Pi\left(0\right)$ , the monopolist offers funding for all maturities:  $T_m = \overline{T}$ .

Hence, when it is difficult to extract surplus at longer maturities (case 1), the monopolist only offers funding for extremely short-term projects  $(T_m = 0)$ . In response, all firms adopt projects of extremely short maturity and the monopolist appropriates all surplus from these projects. Given that  $\Pi(0) > 0$  implies that  $D_c(0, \beta(0)) < R$ , competitive financiers would finance a strictly larger set of maturities, thereby generating higher overall surplus than the monopolist. The central planner offers an even higher funding cutoff,  $T_{cp} \ge T_c > T_m = 0$ , as shown in Proposition 7.

When the monopolist can appropriate sufficient surplus at longer horizons (case 2), he offers funding for all maturities  $(T_m = \overline{T})$ . In this case, it is possible that, due to the monopolist's ability to cross-subsidize, surplus under monopolistic financing is higher than under competitive financing. Whenever the monopolist offers funding for all maturities, the central planner would do the same. However, the reverse is not generally true: full funding by the central planner  $(T_{cp} = \overline{T})$  does not necessarily imply full funding by the monopolist: When  $\Pi(0) > \Pi(\overline{T}) > 0$ , then the central planner funds all maturities,  $T_{cp} = \overline{T}$ , whereas the monopolist only offers funding for short-term projects,  $T_m = 0$ . We summarize the above discussion in the following corollary:

**Corollary 1** The monopolist always funds a weakly smaller set of maturities than the central planner:  $T_m \leq T_{cp}$ . Specifically,

1. if 
$$T_{cp} = \overline{T}$$
, then  $T_m = 0$  if  $\Pi(0) > \Pi(\overline{T})$  and  $T_m = \overline{T}$  otherwise  
2. if  $T_{cp} \in [0, \overline{T})$ , then  $\overline{T} > T_{cp} \ge T_c > T_m = 0$ .

Conversely,

- 1. if  $T_m = \overline{T}$ , then  $T_{cp} = \overline{T}$ ,
- 2. if  $T_m = 0$ , then  $\overline{T} \ge T_{cp} \ge T_c > T_m = 0$ .

Finally, if  $T_c \in [0, \overline{T}]$  then  $T_{cp} \ge T_c \ge 0$  and  $T_m \in \{0, \overline{T}\}$ .

## 5 Discussion

## 5.1 Amplification of shocks through collective short-termism

One important implication of our model is that endogenous asset side adjustments may amplify shocks compared to the benchmark case without asset side adjustments. Below, we illustrate this amplification by investigating the comparative statics of our equilibrium in response to changes in  $\lambda$ . Recall that  $\lambda$  parametrizes the risk inherent in risky projects and therefore captures the severity of the the financing friction arising from limited commitment: An increase in  $\lambda$  increases the amount successful firms with risky projects can abscond with.

The comparative statics with respect to  $\lambda$  are illustrated in Fig. 4. The left panel illustrates the maximum funded maturity as a function of  $\lambda$ . The dashed line depicts the benchmark case in which firms cannot adjust their investment decisions. The solid line depicts the equilibrium maximum funded maturity after privately optimal asset-side adjustments by firms. In the benchmark case, all maturities receive financing when  $\lambda$  is sufficiently low. However, once  $\lambda$  crosses a threshold, some long maturities cannot be financed. This captures the rationing of maturities in the benchmark model, as illustrated in Fig. 2. The solid line in Fig. 4 depicts the full equilibrium with asset-side adjustments. Relative to the benchmark case, we see that due to the endogenous maturity adjustments by firms, the range of funded maturities drops significantly faster than in the benchmark case. The figure also illustrates that a constrained planner funds a strictly larger set of maturities than competitive financiers.<sup>24</sup>

The right panel in Fig. 4 depicts the percentage change in surplus (aggregate NPV) that arises from firms' asset-side adjustments for two values of the dilution parameter  $\Delta$ . The figure shows that, over a large range values for  $\lambda$ , firms' privately optimal maturity adjustments amplify shocks to  $\lambda$ , in the sense that they exacerbate reductions in surplus relative to benchmark case without maturity adjustments. In this region, overall surplus is lower despite the fact that free maturity choice allows all firms to invest. This is because firms' privately optimal maturity adjustments reduce the quality of the average project that is financed, leading to an overall decrease in surplus. Note, however, that this can reverse for high values of  $\lambda$ : When limited commitment frictions in the benchmark case are sufficiently large, then the increase in investment under free maturity choice outweighs the reduction in the average quality of funded projects. In this region, firms' maturity adjustments dampen shocks to  $\lambda$ . The region where the reduction in average quality dominates (and maturity adjustments amplify shocks) is larger the stronger the dilution from second-best projects (lower  $\Delta$ ).

 $<sup>^{24}</sup>$ In addition, note that there is a region where the constrained planner is able to use his ability to cross-subsidize to fund a larger range of maturities than in the benchmark case.



Figure 4: Equilibrium with (i) exogenous and (ii) endogenous asset side as a function of  $\lambda$ . The left panel depicts maximum funded maturities in the benchmark model without asset side adjustments ( $T_b$ , dashed line), the competitive equilibrium with endogenous asset side ( $T_c$ , solid line), and the equilibrium in the presence of a constrained central planner ( $T_{cp}$ , dash-dotted line). The right panel depicts the percentage change in total surplus that results from asset side adjustments in the competitive equilibrium relative to the case with exogenous asset side, ( $W_c - W_b$ )/ $W_b$ . The solid line depicts the welfare resulting from the  $\Delta = .85$  used in the left panel, whereas the dotted line depicts the welfare resulting from a lower  $\Delta = .81$ .

## 5.2 Are firms' privately optimal maturity adjustments efficient?

As illustrated above, whether firms' privately optimal maturity adjustments are socially desirable depends on the degree to which second-best projects are inferior to first best projects, which is captured by the dilution parameter  $\Delta$ . To see this more clearly, we can decompose the change in total surplus, into two components, a *direct effect* and a *dilution effect*:

$$W_c - W_b = \underbrace{\left(\frac{\overline{T} - T_b}{\overline{T}}\right) NPV_{adjusted}}_{\text{Direct effect}} - \underbrace{\frac{T_b - T_c}{\overline{T}} \left[NPV_{original} - NPV_{adjusted}\right]}_{\text{Dilution effect}}, \qquad (17)$$

where  $T_c \leq T_b$  denote the equilibrium funding cutoffs in the full model with endogenous asset side and in the benchmark model without maturity adjustments, respectively. The direct effect measures the effect of firms' maturity adjustments, holding the funding threshold fixed at  $T_b$ . When the funding threshold is held fixed, the ability to adjust project maturity means that firms with initial project maturities on  $(T_b, \overline{T}]$  adjust the maturity of their project and find funding on  $[0, T_b]$ . This effect leads to an increase in surplus whenever second-best projects have positive NPV (i.e., when  $NPV_{adjusted} = \Delta R - 1 > 0$ ). However, as we saw above, the inflow of second-best projects dilutes the pool of funded projects and, via the short-termism spiral illustrated in Fig. 1, decreases the funding threshold from  $T_b$  to  $T_c$ . The dilution effect captures the loss of surplus that arises because firms with original project maturities in  $(T_c, T_b]$  cannot fund their original project and choosing a second-best shorterterm project. Firms privately optimal maturity adjustments increase surplus if and only if the direct effect outweighs the dilution effect.

**Proposition 8** Assume that  $T_b \ge 0$  and  $T_c \ge 0$ . Firms' privately optimal maturity adjustments reduce welfare if and only if

$$NPV_{original} (T_b - T_c) \ge NPV_{adjusted} (\overline{T} - T_c).$$
 (18)

To see the two effects at work, consider two special cases. First, when  $\Delta = 1$  maturityadjusted projects are just as good as firms' original projects. In this case, firms' maturity adjustments do not affect the average quality of projects, such that the cutoff remains unchanged,  $T_c = T_b$ . Hence, only the direct effect is present: Firms that were unable to receive financing at their original maturity can now finance an equally attractive positive NPV project of shorter maturity. This results in an unambiguous increase in welfare. Firms' ability to adjust the maturity of their projects helps them circumvent financing frictions for long-term projects without imposing externalities on other firms.

Second, consider the case in which  $\Delta$  is such that maturity-adjusted projects have zero



Figure 5: Equilibrium with (i) exogenous and (ii) endogenous asset side as a function of the dilution parameter  $\Delta$ . The left panel illustrates the equilibrium maximum funded maturity. The dashed line depicts the maximum maturity in the benchmark model with exogenous asset side,  $T_b$ , which does not depend on the dilution parameter  $\Delta$ . The solid line depicts the maximum funded maturity in the competitive equilibrium,  $T_c$  with endogenous asset side, which illustrates how dilution through second-best projects shrinks the range of financed maturities. The dash-dotted line illustrates the maximum maturity funded by a constrained central planner,  $T_{cp}$ . The right panel depicts the percentage change in surplus  $(W_c - W_b)/W_b$  that results from firms' privately optimal asset side adjustments (solid line). The vertical line depicts the value of  $\Delta$  such that  $NPV_{adjusted} = 0$ .

NPV (i.e.,  $\Delta = 1/R$ ). In this case, a firm that manages to obtain funding by adjusting its maturity adds nothing to the aggregate NPV produced. Hence, the direct effect is zero, and only the indirect effect is present: Through its maturity adjustment decision, the firm reduces the average quality of projects, which leads to a reduction in the maximum funded maturity, such that some firms that were able to fund there original project are now forced to also adjust their maturity. Hence, when second-best projects have zero NPV (or worse), privately optimal maturity adjustment decisions lead to an unambiguous reduction in surplus and are therefore socially undesirable. For values of  $\Delta$  that lie in between these two polar cases,  $\Delta \in (\frac{1}{R}, 1)$ , both the direct and indirect effects are present, such that the net effect depends on their relative size.

We illustrate these results in Fig. 5, which plots maximum funded maturities (left panel)

and the percentage change in surplus that results from firms' privately optimal asset side adjustments (right panel) as a function of the dilution parameter  $\Delta$ . The left panel illustrates that firms' ability to adjust their maturity leads to a reduction in the maximum funded maturity relative to the benchmark case, where the the maximum funded maturity is independent of  $\Delta$ . The reduction in the maximum funded maturity is larger, the stronger the dilution effect of second-best projects (smaller  $\Delta$ ). For the parameters in the figure, funding completely unravels when second best projects become negative NPV, as indicated by the vertical line in the graph. In contrast to the competitive equilibrium, the maximum maturity implemented by the central planner increases (weakly) as  $\Delta$  decreases. This is the case because lower  $\Delta$  relaxes the IC constraint (9), which allows the central planner to cross-subsidize more aggressive.<sup>25</sup>

In the right panel, the solid line depicts the percentage change in surplus (aggregate NPV) that results from firms' privately optimal maturity adjustment decisions. When there is no or very little quality difference between first-best and second-best projects (i.e.,  $\Delta$  close to 1), surplus increases when firms can adjust their project maturity. Thus, for high  $\Delta$ , the direct effect outweighs the dilution effect: While firms' maturity adjustments reduce the average quality of funded projects, this negative quality effect is initially outweighed by the increase in the number of projects that can attract financing. In this region, firms' privately optimal maturity adjustments are socially desirable. For low  $\Delta$ , on the other hand, overall surplus decreases relative to the benchmark case, despite the fact that total lending increases.

<sup>&</sup>lt;sup>25</sup>Note that this implies that welfare under a constrained central planner is non-monotone in  $\Delta$ . For low values of  $\Delta$ , all maturities are funded due to aggressive cross-subsidization of maturities. As  $\Delta$  and the IC constraint starts binding increases,  $T_{cp} < \overline{T}$ , such that welfare decreases. However, as  $\Delta$  increases towards 1, the fact that the IC constraint restricts the funding threshold  $T_{cp}$  matters less and less, because second-best projects become closer to first-best projects.

Hence, an increase in lending is *not* a sufficient condition for an increase in surplus. Finally, the figure illustrates that overall surplus can decrease even when second-best projects have positive NPV (which, in the figure, is the case to the right of the vertical line). Hence, while a second-best project may have positive NPV when seen in isolation, because of the dilution, the adoption of positive NPV second-best projects can lead to a decrease in overall surplus. This result illustrates the importance of taking into account cross-firm externalities when evaluating firms' investment choices.

## 5.3 Empirical implications

In this section, we briefly discuss the empirical implications of our model and relate our findings to the stylized facts documented in the empirical literature. The main novelty of our analysis is to highlight the joint determination of financing maturities on the liability side and project maturities on the asset side and the cross-firm externalities that can arise in such a setting.

First, viewing the asset and liability sides as jointly determined leads to a new interpretation of the positive correlation between debt maturity and asset maturity that has been documented in the empirical literature on the determinants of debt maturity (Morris (1992); Guedes and Opler (1996); Stohs and Mauer (1996); Johnson (2003)). This literature mostly interprets this finding as a one-way link from asset maturity to debt maturity. In contrast, our model emphasizes a two-way feedback between debt maturities and asset maturities. In fact, recent evidence by Gopalan et al. (2014) suggests that taking into account the feedback from debt maturity to asset maturity may be a first-order consideration: Taking advantage of a natural experiment (the creation of debt recovery tribunals in India), they show that increased availability of long-term financing does indeed lead to an increase in asset maturity among affected firms, which is consistent with the predictions of our model. In similar
spirit, Paligorova and Santos (2013) show that banks tend to shorten the average maturity of their their loan portfolios (i.e., their asset side) when they rely more heavily on short-term financing.

Second, our model highlights a novel cross-firm externality: While individual firms find it optimal to adopt shorter-term projects in order to receive better financing terms, this privately optimal action affects funding terms for other firms and thereby imposes a negative externality. From an empirical perspective, our model thus implies that there are crossfirm spillovers both in funding maturities and investment horizons of firms and financial institutions. Identifying these spillovers poses an interesting empirical challenge for future work on debt and asset maturities.

Third, our model generates comparative static predictions regarding the joint dynamics of debt and asset maturities over the business cycle. In particular, a recent strand of literature in macroeconomics (Bloom (2009); Bloom et al. (2012); Christiano et al. (2012)) points out that the dispersion of revenues and profits increases during recessions and that this increase in risk can be a first-order driver of the business cycle. In our model, this view of dispersion-driven business cycles translates into a comparative static exercise with respect to the dispersion parameter  $\lambda$ . In addition to predicting a reduction in output in response to a dispersion shock—which has been the focus of many of the macro papers on this issue—our model makes the further prediction that investment horizons (asset maturities) shorten during recessions. Such asset maturity shortening should occur both for firms (the maturities of projects firms invest in) and for banks (the maturities of bank loans issued). This prediction is supported in the data. On the firm side, Dew-Becker (2012) documents that downturns are, indeed, associated with more short-term investment: lower GDP and higher unemployment predict lower duration investments by firms. Because of the cross-firm externality that arises in our model, such a shift in the composition (rather than just the quantity) of investment may constitute a significant amplification channel of the business cycle. Similarly, the maturities of bank loans (and other debt issues) shrink during recessions, as documented by Mian and Santos (2011), Erel et al. (2012), and Chen et al. (2012).

## 5.4 Collective short-termism as an equilibrium phenomenon

In this section, we briefly discuss the implications of our model in the context of the larger debate on short-termism in economic activity. Specifically, whether competitive capital markets inherently lead to short-term behavior has been a major debate at least since the 1980s, when a number of commentators and scholars compared the market-based U.S. economy with the less market-based and supposedly more long-term system in Japan (Corbett (1987), Jacobs (1993), and Porter (1992)). Recently, this debate has resurfaced as part of the discussion of whether advanced and emerging economies will be able to make long-term investments in infrastructure, research and development, and innovation that are required for sustainable long-term growth (World Economic Forum (2011); European Commission (2013); Group of Thirty (2013); OECD (2013)).

While many standard explanations of short-termism rely on adverse incentives are created by managerial reputation building (Narayanan (1985)), concern with short-term stock prices (Stein (1989)), investors with short horizons (Froot et al. (1992)) or speculative investors (Bolton et al. (2006)), in our framework short-termism emerges as an equilibrium phenomenon in a fully rational setting where managers do not face incentive problems. Our model thereby highlights that in the presence of contractual incompleteness, competitive capital markets alone can lead to inefficient short-termism because they rule out the crosssubsidization required to induce long-term investment. The resulting competitive equilibrium is constrained inefficient in the sense that a planner subject to identical contractual and informational frictions could raise surplus. Moreover, short-termism is collective in the sense that in competitive equilibrium firms privately optimal decisions reinforce the short-termism in investment decisions. This suggests that even in the absence of managerial myopia or other short-term biases, renewed focus on fostering an environment that is conducive to long-term investment may be desirable.

## 6 Conclusion

This paper provides a framework to analyze how financing frictions that arise on the liability side affect firms' investment decisions on the asset side, and vice versa. In our model, financing frictions resulting from limited commitment are more pronounced at longer horizons, leading to credit rationing for long-term investment projects. Firms respond by adjusting their asset-side investments and adopt alternative projects of shorter maturities, even if those projects are second best. Because individual firms' asset-side decisions endogenously determine the magnitude of an asymmetric information friction faced by all firms, an externality arises, which leads to inefficient short-termism. Firms' equilibrium asset side adjustments amplify shocks and, while privately optimal, can be socially undesirable. These results highlight the importance of explicitly taking into account the asset side when analyzing the effect of liability side frictions, such as pressure toward short-term financing.

At a broader level, while the focus of our paper has been the maturity of firms' asset and liability side choices, the mechanism behind our results is potentially more general. In our framework, project maturity is systematically related to risk, which, in the presence of contractual frictions, makes long-term projects harder to finance. Firms react to this by attempting to minimize the observable characteristic linked to risk (here: maturity) by increasing the unobservable portion of their riskiness (here: average quality of the project). This increase in unobservable risk is what triggers the adverse selection that ultimately leads to the cross-firm externalities and investment distortions at the heart of the paper. Similar distortions may occur along other observable dimensions that are systematically related to risk and that can be chosen by firms. Potential examples of rationing and cross-firm spillovers along other dimensions could include investment size or the choice of industry. We leave these implications for future research.

## A Appendix

**Proof of Proposition 1.** Consider an arbitrary rollover schedule  $[t_0, t_1, ..., t_n]$ , with  $t_0 = 0$  and  $t_n = t$ , where period *i* is equal to  $(t_i, t_{i+1})$ , so that there are a total of *n* rollover periods. Suppose further that the total adjusted-project survival probability  $\Delta$  can be divided into per-period survival probabilities  $\mathbb{P}[survival on (t_i, t_{i+1})] = \Delta_i$ , so that  $\prod_{i=0}^n \Delta_i = \Delta$ .

Then, consider a situation in which  $p_0 = p$  is the time-0 expectation of the proportion of good/original projects, and  $\alpha_0 = \alpha$  is the time-0 expectation of the proportion of risk-free projects. Then, conditional on no observed defaults, we have

$$\begin{aligned} p_{i+1} &= & \mathbb{P}\left[good|p_i\right] \\ &= & \frac{\left[\alpha_i + (1 - \alpha_i) \, e^{-\lambda(t_{i+1} - t_i)}\right] p_i}{\left[\alpha_i + (1 - \alpha_i) \, e^{-\lambda(t_{i+1} - t_i)}\right] p_i + (1 - p_i) \, \Delta_i \left[\alpha_i + (1 - \alpha_i) \, e^{-\lambda(t_{i+1} - t_i)}\right]} \\ &= & \frac{p_i}{p_i + (1 - p_i) \, \Delta_i}, \end{aligned}$$

so that the quality (and also the probability of survival conditional on the risk-free project) on i is  $\beta_i = p_i + (1 - p_i) \Delta$ . Also, we have

$$\begin{split} \alpha_{i+1} &= & \mathbb{P}\left[risk - free | \alpha_i \right] \\ &= & \frac{\alpha_i \beta_i}{\alpha_i \beta_i + (1 - \alpha_i) e^{-\lambda(t_{i+1} - t_i)} \beta_i} \\ &= & \frac{\alpha_i}{\alpha_i + (1 - \alpha_i) e^{-\lambda(t_{i+1} - t_i)}}. \end{split}$$

Finally, let  $D_i$  denote the face value of debt payable at  $t_i$  (i.e., the face-value agreed at the beginning of period i-1). Then, whenever debt is rolled over from period i to period i+1 at  $t_{i+1}$ , the company needs to raise  $D_{i+1}$  via promising  $D_{i+2}$  to be repaid at  $t_{i+2}$ . with  $D_0 \equiv 1$  (the original investment). Thus, we have

$$D_{i} = [p_{i} + (1 - p_{i}) \Delta_{i}] \left[ \alpha_{i} + (1 - \alpha_{i}) e^{-\lambda(t_{i+1} - t_{i})} \right] D_{i+1}$$

Also, note by inspection that  $D_i < D_{i+1}$ , so that we have the restriction  $D_n \leq R$ . Plugging in repeatedly, we have

$$1 = D_n \prod_{i=0}^{n-1} \left[ p_i + (1-p_i) \Delta_i \right] \left[ \alpha_i + (1-\alpha_i) e^{-\lambda(t_{i+1}-t_i)} \right].$$

Note that

$$\begin{aligned} \beta_i \beta_{i+1} &= \left[ p_i + (1 - p_i) \,\Delta_i \right] \left[ p_{i+1} + (1 - p_{i+1}) \,\Delta_{i+1} \right] \\ &= \left[ p_i + (1 - p_i) \,\Delta_i \right] \left[ \Delta_{i+1} + p_{i+1} \left( 1 - \Delta_{i+1} \right) \right] \\ &= \left[ p_i + (1 - p_i) \,\Delta_i \right] \Delta_{i+1} + p_i \left( 1 - \Delta_{i+1} \right) \\ &= \left[ p_i + (1 - p_i) \,\Delta_i \Delta_{i+1} \right], \end{aligned}$$

so that

$$\prod_{i=0}^{n-1} \beta_i = \prod_{i=0}^{n-1} \left[ p_i + (1-p_i) \,\Delta_i \right] = p_0 + (1-p_0) \prod_{i=0}^{n-1} \Delta_i = p + (1-p) \,\Delta = \beta$$

Similarly, we have

$$\begin{bmatrix} \alpha_i + (1 - \alpha_i) e^{-\lambda(t_{i+1} - t_i)} \end{bmatrix} \begin{bmatrix} \alpha_{i+1} + (1 - \alpha_{i+1}) e^{-\lambda(t_{i+2} - t_{i+1})} \end{bmatrix} \\ = \begin{bmatrix} \alpha_i + (1 - \alpha_i) e^{-\lambda(t_{i+1} - t_i)} \end{bmatrix} \begin{bmatrix} e^{-\lambda(t_{i+2} - t_{i+1})} + \alpha_{i+1} \left(1 - e^{-\lambda(t_{i+2} - t_{i+1})}\right) \end{bmatrix} \\ = \begin{bmatrix} \alpha_i + (1 - \alpha_i) e^{-\lambda(t_{i+1} - t_i)} \end{bmatrix} e^{-\lambda(t_{i+2} - t_{i+1})} + \alpha_i \left(1 - e^{-\lambda(t_{i+2} - t_{i+1})}\right) \\ = \begin{bmatrix} \alpha_i + (1 - \alpha_i) e^{-\lambda(t_{i+1} - t_i)} e^{-\lambda(t_{i+2} - t_{i+1})} \end{bmatrix},$$

so that

$$\prod_{i=0}^{n-1} \left[ \alpha_i + (1-\alpha_i) e^{-\lambda(t_{i+1}-t_i)} \right] = \alpha_0 + (1-\alpha_0) \prod_{i=0}^{n-1} e^{-\lambda(t_{i+1}-t_i)} = \alpha + (1-\alpha) e^{-\lambda t}.$$

Thus, we can write

$$1 = D_n \prod_{i=0}^{n-1} \beta_i \left[ \alpha_i + (1 - \alpha_i) e^{-\lambda(t_{i+1} - t_i)} \right] = D_n \beta \left[ \alpha + (1 - \alpha) e^{-\lambda t} \right],$$

which implies that

$$D_n = D_c(t,\beta) = \frac{1}{\beta \left[\alpha + (1-\alpha) e^{-\lambda t}\right]}$$

Hence, the terminal face value  $D_n$  is the same as in the no-rollover case, such that the firm cannot gain from a rollover strategy.

The above irrelevance proposition establishes an indeterminacy result for the maturity of the liability side of firms. We now break this indeterminacy by introducing a small cost of rolling over debt. This small rollover cost, in combination with an equilibrium refinement, induces firms to match the maturities of their assets and liabilities. The equilibrium refinement we use is the relatively powerful D1 criterion, which requires out-of-equilibrium beliefs to be placed on types that have the most to gain from deviating from a conjectured equilibrium (see Banks and Sobel (1987)). This is a common equilibrium refinement used to achieve uniqueness in signaling games. To make this proof without loss of generality, we make the additional assumption that the additional default risk  $1 - \Delta$  is continuously distributed over [0, t] (i.e., the additional default risk has full support over the lifetime of the project).

**Proof of Proposition 2.** Suppose that every time a firm rolls over its debt, it incurs a small fixed cost c. Now conjecture an equilibrium in which firms chose a strictly positive number of rollover dates, n > 0. Because in this conjectured equilibrium no firms match maturities of debt and assets, we have to formulate out-of-equilibrium beliefs regarding the type of a firm that deviates to a strategy of matching maturities. We now show that any conjectured equilibrium of this form does not survive the D1 refinement. Consequently, the unique equilibrium is one where firms match maturities of assets and liabilities.

To see this, note first that the original type ( $\theta = 1$ ) cannot use more frequent rollover as a signaling device, because any rollover strategy can be imitated by the  $\Delta$  type (firms that have adjusted their maturity) at a smaller expected cost.<sup>26</sup> The reason is that the  $\Delta$  type defaults with a higher probability than the 1 type and thus pays the full sequence of rollover costs less frequently. Hence, for a given rollover strategy, the  $\Delta$  type always incurs lower expected rollover costs than the 1 type. Now note that according to this logic, under the D1 criterion any deviation to more frequent rollover has to be attributed to the  $\Delta$  type, while any deviation to less frequent rollover must be attributed to the 1 type. Thus, starting from any equilibrium with a positive number of rollover dates, the out-of-equilibrium beliefs that firms that match maturities of assets and liabilities (that is n = 0) are of type  $\Delta$  are not consistent with the D1 refinement, since any such deviation must be attributed to a firm of type 1. However, this out-of-equilibrium belief makes a deviation

 $<sup>^{26}</sup>$ This differs from Flannery (1986), where the ability to use rollover debt can lead to a separating equilibrium when rolling over has a cost. The main difference relative to our setup is that in Flannery (1986) there is an observable interim signal about the firm's quality, which makes rollover debt relatively more costly for bad firms.

to a no-rollover strategy profitable. This, of course, implies that all equilibria with a strictly positive number of rollover dates n > 0 can be ruled out via a profitable deviation to a no-rollover strategy. The no-rollover equilibrium (n = 0), on the other hand, survives, because every deviation from it is attributed to a firm of type  $\Delta$ .

For generality, we write the equations below explicitly taking into account the probability  $\varepsilon$  that a firm that attempts to change the maturity of its project gets stuck at its original maturity (recall that in the main text we focus on the limiting case  $\epsilon \to 0$ ). Also, for ease of exposition, we define  $\pi(t, D) \equiv \pi(1, t, D)$ so that  $\pi(\theta, t, D) = \theta \pi(t, D)$ . With  $\varepsilon > 0$ , the IC constraint is easier to satisfy because the probability of successfully adjusting the maturity is now only  $1 - \varepsilon$ :

$$\min_{t \in \mathcal{F}} \pi\left(t, D\right) \geq \max_{t \in \mathcal{F}} \Delta \pi\left(t, D\right) \geq \max_{t \in \mathcal{F}} \left(1 - \varepsilon\right) \Delta \pi\left(t, D\right) = \max_{t \in \mathcal{F}} \Delta_{\varepsilon} \pi\left(t, D\right),$$

where we defined

$$\Delta_{\varepsilon} \equiv (1 - \varepsilon) \,\Delta.$$

Further, let

$$p_{\varepsilon}(T) = \frac{T}{T + (1 - \varepsilon)(\overline{T} - T)}$$
$$p'_{\varepsilon}(T) = \frac{\overline{T}(1 - \varepsilon)}{\left[T + (1 - \varepsilon)(\overline{T} - T)\right]^{2}} > 0.$$

Because in the competitive equilibrium the IC constraint is slack, the only effect of allowing for  $\varepsilon > 0$  is that the equilibrium proportion of original projects is now given by  $p_{\varepsilon}(T)$ . All equations and proofs therefore follow immediately with a simple substitution of  $p_{\varepsilon}(T)$  for p(T) and  $\beta_{\varepsilon}(T)$  form  $\beta(T)$ , where

$$\beta_{\varepsilon}(T) = p_{\varepsilon}(T) + [1 - p_{\varepsilon}(T)] \Delta.$$

**Proof of Lemma 1.** Consider, wlog, the following candidate equilibrium funding profile  $\mathcal{F} = [0, T_1] \cup [T_2, T_3]$  with  $T_1 < T_2$ . Equilibrium implies that both the IR and IC constraint hold for all  $t \in \mathcal{F}$ , and additionally that  $\beta$  is constant on  $\mathcal{F}$ . We now show that a single financier can offer funding on  $\mathcal{H} = (T_1, T_2)$  such that he only faces original projects and makes strict profits. To do this, the offers a contract

$$D_{hole}\left(t\right) = \left[R\left(1 - \Delta_{\varepsilon}\right) + \frac{\Delta_{\varepsilon}}{\beta}\right] \frac{1}{\alpha + (1 - \alpha)e^{-\lambda t}}, t \in \mathcal{H},$$

so that the payoff to a firm on  $\mathcal{H}$  that does not adjust its maturity is given by  $\pi(1, t, D_{hole}(t))$  that fulfills

$$\pi (1, t', D_c (t', \beta)), t' \in \mathcal{F}$$

$$= \left[ R - \frac{1}{\beta} \right]$$

$$> \pi (1, t, D_{hole} (t)), t \in \mathcal{H}$$

$$= \left[ R - D_{hole} (t) \left( \alpha + (1 - \alpha) e^{-\lambda t} \right) \right], t \in \mathcal{H}$$

$$= \Delta_{\varepsilon} \left[ R - \frac{1}{\beta} \right]$$

$$= \pi (\Delta_{\varepsilon}, t', D_c (t', \beta)), t' \in \mathcal{F}$$

$$> \pi (\Delta_{\varepsilon}, t, D_{hole} (t)), t \in \mathcal{H}$$

Offering funding on  $\mathcal{H}$  generates strict profits because  $D_{hole}(t) > D(t, 1)$ , which follows from

$$R(1 - \Delta_{\varepsilon}) + \frac{\Delta_{\varepsilon}}{\beta} > R(1 - \Delta_{\varepsilon}) + \Delta_{\varepsilon} > 1,$$

as  $\beta \leq 1$ . Thus, given the contract  $D_{hole}(t)$ , any firm with an original project of maturity  $t \in \mathcal{H}$  has no

strict incentive to adjust its maturity, any firm  $t \in \mathcal{F}$  has no strict incentive to adjust its maturity, and any unfunded firm  $t \in (T_3, \overline{T}]$  strictly wants to adjust into  $\mathcal{F}$  only.

**Proof of Proposition 3.** What remains to be shown for the first part of the proof is that there is no profitable deviation for projects on  $(T_c, \overline{T}]$ . The off-equilibrium refinement we will use is similar to a trembling-hand refinement. Recall that a small fraction  $\varepsilon$  of projects get stuck at their original maturity when attempting to adjust the maturity (i.e., they become  $\Delta$  types without the benefit of maturity choice). We then note that all projects on  $(T_c, \overline{T}]$  are adjusted projects (type  $\Delta$ ). By the definition of  $T_c$ , i.e.,  $D(T_c, \beta_{\varepsilon}(T_C)) = R$  these projects, for any  $\varepsilon \in [0, 1]$ , cannot attract financing because

$$D(t, \Delta) > D(t, \beta_{\varepsilon}(T_c)) > D(T_c, \beta_{\varepsilon}(T_c)) = R, \forall t \in (T_c, \overline{T}].$$

The first inequality stems from  $\beta_{\varepsilon}(T_c) > \Delta$  and the observation that the face value is decreasing in average quality. The second inequality stems from  $t > T_C$  and the observation that the face value is increasing in maturity. We conclude that, because their outside option of staying put is given by 0, all firms on  $(T_c, \overline{T})$ adjust their maturity.

For the second part of the proof, we differentiate (11) to get

$$\frac{dD_{c}\left(T,\beta\left(T\right)\right)}{dT^{2}} = \frac{2}{D_{c}\left(T,\beta\left(T\right)\right)} \left(\frac{dD_{c}\left(T,\beta\left(T\right)\right)}{dT}\right)^{2} -\lambda \frac{dD_{c}\left(T,\beta\left(T\right)\right)}{dT} + \lambda D_{c}\left(T,\beta\left(T\right)\right)^{2} \left[\left(1-\alpha\right)e^{-\lambda T} - \alpha\right]\beta'\left(T\right).$$

Thus, if there exist a  $T_{extremal} \in [0, \overline{T}]$  such that

$$\frac{dD_c\left(T,\beta\left(T\right)\right)}{dT}\bigg|_{T=T_{extremal}} = 0 \iff \lambda\left(1-\alpha\right)e^{-\lambda T_{extremal}}\left[\left(1-\Delta\right)T + \Delta\overline{T}\right] = \left[\alpha + \left(1-\alpha\right)e^{-\lambda T_{extremal}}\right]\left(1-\Delta\right) + \frac{dT_{extremal}}{dT}\left[\left(1-\alpha\right)T + \Delta\overline{T}\right] = \left[\alpha + \left(1-\alpha\right)e^{-\lambda T_{extremal}}\right]\left(1-\Delta\right) + \frac{dT_{extremal}}{dT}\left[\left(1-\alpha\right)T + \Delta\overline{T}\right] = \left[\alpha + \left(1-\alpha\right)e^{-\lambda T_{extremal}}\right]\left(1-\Delta\right) + \frac{dT_{extremal}}{dT}\left[\left(1-\alpha\right)T + \Delta\overline{T}\right] = \left[\alpha + \left(1-\alpha\right)e^{-\lambda T_{extremal}}\right]\left(1-\Delta\right) + \frac{dT_{extremal}}{dT}\left[\left(1-\alpha\right)T + \Delta\overline{T}\right] = \left[\alpha + \left(1-\alpha\right)e^{-\lambda T_{extremal}}\right]\left(1-\Delta\right) + \frac{dT_{extremal}}{dT}\left(1-\alpha\right)E^{-\lambda T_{extremal}}\left[\left(1-\alpha\right)T + \Delta\overline{T}\right] = \left[\alpha + \left(1-\alpha\right)e^{-\lambda T_{extremal}}\right]\left(1-\Delta\right)E^{-\lambda T_{extremal}}\left[\left(1-\alpha\right)T + \Delta\overline{T}\right]\right]$$

then  $T_{extremal}$  is a maximum if and only if

$$\frac{dD_{c}\left(T,\beta\left(T\right)\right)}{dT^{2}}\bigg|_{T=T_{extremal}} < 0 \iff (1-\alpha) e^{-\lambda T_{extremal}} < \alpha.$$

Suppose there exits a sequence of extremal points  $0 < T_{e1} < T_{e2} < ... < T_{en} < ....$  Consider first the case  $\frac{dD_c(T,\beta(T))}{dT}\Big|_{T=0} > 0 \iff \overline{T} > \frac{1-\Delta}{\lambda(1-\alpha)\Delta}$ . Then we know that the first extremal point  $T_{e1} > 0$  has to be a maximum. But this immediately implies that

$$(1-\alpha) e^{-\lambda T_{e2}} < (1-\alpha) e^{-\lambda T_{e1}} < \alpha$$

and thus  $T_{e_2}$  also has to be a maximum, which by continuity is impossible. Consider now the case in which  $\frac{dD_c(T,\beta(T))}{dT}\Big|_{T=0} < 0 \iff \overline{T} < \frac{1-\Delta}{\lambda(1-\alpha)\Delta}$ . Then we know that the first extremal point  $T_{e_1} > 0$  has to be a minimum and  $T_{e_2} > T_{e_1}$  has to be a maximum. But this immediately implies that

$$(1-\alpha) e^{-\lambda T_{e^3}} < (1-\alpha) e^{-\lambda T_{e^2}} < \alpha < (1-\alpha) e^{-\lambda T_{e^1}}$$

and thus  $T_{e3}$  also has to be a maximum, which by continuity is impossible. We therefore conclude that at most one minimum and one maximum exist on  $[0,\infty)$  with the minimum always smaller than the maximum.

We can derive two sufficient conditions. First, a sufficient condition for complete unraveling of funding is that

$$\min\left\{D_{c}\left(0,\Delta\right), D_{c}\left(\overline{T},1\right)\right\} > R$$

and that  $\frac{dD_c(T,\beta(T))}{dT}\Big|_{T=0} > 0 \iff \Delta > \frac{1}{1+(1-\alpha)\lambda T}$ , because only one maximum exists. Second, in case we have  $\frac{dD_c(T,\beta(T))}{dT}\Big|_{T=0} < 0$ , let  $T_{e1}$  be the first extremal point, so that the condition

becomes min  $\{D_c(T_{e1}, \beta(T_{e1})), D_c(\overline{T}, 1)\} > R$ . From the FOC, we know that

$$\lambda (1-\alpha) e^{-\lambda T_{e1}} \frac{\overline{T}}{1-\Delta} \beta (T_{e1}) = \left[ \alpha + (1-\alpha) e^{-\lambda T_{e1}} \right],$$

so that

$$D_{c}\left(T_{e1}, \beta\left(T_{e1}\right)\right) = \frac{\left(1 - \Delta\right)e^{\lambda T_{e1}}}{\beta\left(T_{e1}\right)^{2}\overline{T}\lambda\left(1 - \alpha\right)}$$

This face value takes the smallest value in the numerator for  $T_{e1} = 0$  and in the denominator for  $T_{e1} = \overline{T}$ . Thus, we have

$$\frac{(1-\Delta)}{\overline{T}\lambda\left(1-\alpha\right)} > R \iff 1 - (1-\alpha)\,\lambda\overline{T}R > \Delta$$

as a sufficient condition that is not based on the slope at 0. This is the sufficient condition we show in the text. The interpretation of this condition is that funding completely unravels when second-best projects are sufficiently bad.  $\blacksquare$ 

**Proof of Lemma 2.** Consider funding up to a threshold T, so that  $\mathcal{F} = [0, T]$ , without the restriction of no cross-subsidization that arises in the competitive case, and an arbitrary funding schedule  $D_T(t)$ , subject to the IC constraint

$$\min_{t \in [0,T]} \pi \left(1, t, D_T \left(t\right)\right) \ge \max_{t \in [0,T]} \left(1 - \varepsilon\right) \pi \left(\Delta, t, D_T \left(t\right)\right) = \max_{t \in [0,T]} \pi \left(\Delta_{\varepsilon}, t, D_T \left(t\right)\right),$$

the maximum face-value constraint,

$$D_T(t) \le R, \forall t \in [0, T],$$

and the maturity adjustment constraint

$$d_{T}(t) = \delta_{t_{\max}}(t) \, \overline{\overline{T} - T}(1 - \varepsilon) \,, \quad t_{\max} \equiv \sup \left\{ \arg \max_{t \in [0,T]} \pi\left(\Delta_{\varepsilon}, t, D_{T}\left(t\right)\right) \right\},$$

where, as a tie-breaking rule, we assumed that the maximum maturity is chosen in the argmax set. In case the argmax is not a singleton, we note that the payoff to the investors and the financiers is constant on this set, and thus picking the maximum maturity is without loss of generality. The overall profit is then given by

$$\Pi = \int_0^T \left\{ \left( \frac{1}{\overline{T}} + d_T(t) \Delta \right) \left[ \alpha + (1 - \alpha) e^{-\lambda t} \right] D_T(t) - 1 \left( \frac{1}{\overline{T}} + d(t) \right) \right\} dt$$

$$= \frac{1}{\overline{T}} \int_0^T \left( \left[ \alpha + (1 - \alpha) e^{-\lambda t} \right] D_T(t) - 1 \right) dt + \frac{\overline{T} - T}{\overline{T}} (1 - \varepsilon) \left( \Delta \left[ \alpha + (1 - \alpha) e^{-\lambda t_{\max}} \right] D_T(t_{\max}) - 1 \right)$$

$$= \frac{1}{\overline{T}} \int_0^T \left( \left[ \alpha + (1 - \alpha) e^{-\lambda t} \right] D_T(t) - 1 \right) dt + \frac{\overline{T} - T}{\overline{T}} \left( \Delta_\varepsilon \left[ \alpha + (1 - \alpha) e^{-\lambda t_{\max}} \right] D_T(t_{\max}) - 1 + \varepsilon \right).$$

We will no establish a sequence of results. First, the expected profit to firms has to be weakly increasing in project maturity t. Let us consider two cases. First, suppose  $D_t = R$ . Then we know that  $\pi$  is mechanically increasing in t because

$$\pi \left(\theta, t, R\right) = \theta R \left(1 - \alpha\right) \left(1 - e^{-\lambda t}\right).$$

Thus, whenever the IR constraint is binding, the expected profit to firms is increasing in project maturity. Second, suppose that  $D_T(t) < R$  and that  $\pi(\theta, t, D_T(t))$  is not flat with respect to t. We know that  $\pi(\theta, t, D)$  is decreasing in D. As  $\pi(\theta, t, D_T(t))$  is not flat, there exists a maximum on [0, T], and for maturities t' away from this maximum the IC constraint is not binding. Then, we can increase  $D_T(t')$  and thereby decrease  $\pi$  at those maturities t' for which  $\pi(1, t', D_T(t')) \ge \min_{t \in [0,T]} \pi(1, t, D_T(t))$  and  $D_T(t') < R$  without affecting the IC or maturity adjustment constraint, but increasing the financier's profit for each maturity t'.

Second, in an optimal funding scheme, on the set where the IR constraint is not binding the IC constraint should be uniformly binding. Moreover, at the highest funded maturity the IR constraint should be binding, i.e.,  $D_T(T) = R$ . The latter is easy to show. As  $\pi$  is weakly increasing in t, setting  $D_T(T) = R$  relaxes the

IC constraint while raising the highest amount of revenue. In other words, it leaves the minimum amount to the original and maturity-adjusted firms. Second, suppose the IC constraint is not uniformly binding on the set on which the IR constraint is slack. Then we can increase  $D_T(t)$  maturity by maturity on this set to make the IC constraint uniformly binding, noting that  $t_{\text{max}} = T$  and  $D_T(T) = R$ , so that the RHS of the maturity adjustment constraint (14) is unaffected.

Third, analogous to the proof of Lemma 1, funding on non-monotone sets is inefficient.

We can then establish the following result. A funding cutoff T uniquely pins down the face-value schedule that extracts the most value while fulfilling both the IR and IC constraints. This schedule is given by the continuous function

$$D_T(t) = \begin{cases} C \frac{1}{\alpha + (1 - \alpha)e^{-\lambda t}} & , t < T_C \\ R & , t \ge T_C \end{cases}$$

where C(T) is given by

$$\pi (1, 0, D_T (0)) = \pi (\Delta_{\varepsilon}, T, R)$$
  

$$\iff R - C = \Delta_{\varepsilon} R (1 - \alpha) (1 - e^{-\lambda T})$$
  

$$\iff C (T) = R [1 - \Delta_{\varepsilon} (1 - \alpha)] + \Delta_{\varepsilon} R (1 - \alpha) e^{-\lambda T}$$
  

$$= R (\alpha + (1 - \alpha) [(1 - \Delta_{\varepsilon}) + \Delta_{\varepsilon} e^{-\lambda T}]),$$

which is strictly less than R for T > 0.  $T_C$ , the time at which the IR constraint becomes binding, is given by

$$0 \le T_C(T) = \frac{1}{\lambda} \log \left[ \frac{R(1-\alpha)}{C(T) - \alpha R} \right] = -\frac{1}{\lambda} \log \left[ (1-\Delta_{\varepsilon}) + \Delta_{\varepsilon} e^{-\lambda T} \right] \le T,$$

with equality only for  $\Delta_{\varepsilon} = 1$ . This implies that the IC constraint is binding on  $[0, T_C(T)]$  and the IR constraint is binding on  $[T_C(T), T]$ .

Note that by  $C'(T) = -\lambda R (1-\alpha) \Delta_{\varepsilon} e^{-\lambda T} < 0$  and  $T'_C(T) = \frac{\Delta_{\varepsilon} e^{-\lambda T}}{(1-\Delta_{\varepsilon}) + \Delta_{\varepsilon} e^{-\lambda T}} > 0$ , the higher the (arbitrary) funding cutoff T, the lower starting face value  $D_T(0)$  and the later the IR constraint becomes binding.

We establish the following auxiliary result on the maximized profit function  $\Pi(T)$ :

**Lemma 3** Any point  $T_{extremal}$  such that  $\Pi'(T_{extremal}) = 0$  is a minimum of  $\Pi(T)$ , so that the slope of  $\Pi(T)$  never changes from positive to negative. Thus, the maximum of  $\Pi(T)$  on  $[0,\overline{T}]$  occurs at the boundaries  $T \in \{0,\overline{T}\}$ .

Proof of Lemma 3. Plugging in, we have

$$\begin{split} \Pi(T) &= \frac{1}{\overline{T}} \int_{0}^{T_{C}(T)} \left[ C\left(T\right) - 1 \right] dt + \frac{1}{\overline{T}} \int_{T_{C}(T)}^{T} \left( \left[ \alpha + (1 - \alpha) e^{-\lambda t} \right] R - 1 \right) dt \\ &+ \frac{\overline{T} - T}{\overline{T}} \left( \Delta_{\varepsilon} \left[ \alpha + (1 - \alpha) e^{-\lambda T} \right] R - 1 + \varepsilon \right) \\ &= \frac{T_{C}(T)}{\overline{T}} \left[ R \left( \alpha + (1 - \alpha) \left[ (1 - \Delta_{\varepsilon}) + \Delta_{\varepsilon} e^{-\lambda T} \right] \right) - 1 \right] + \frac{T - T_{C}(T)}{\overline{T}} \left( \alpha R - 1 \right) \\ &+ \frac{1}{\overline{T}} \left( 1 - \alpha \right) R \left[ \frac{e^{-\lambda T_{C}(T)} - e^{-\lambda T}}{\lambda} \right] + \frac{\overline{T} - T}{\overline{T}} \left( \Delta_{\varepsilon} \left[ \alpha + (1 - \alpha) e^{-\lambda T} \right] R - 1 + \varepsilon \right) \\ &= \frac{T_{C}(T)}{\overline{T}} \left( 1 - \alpha \right) \left[ (1 - \Delta_{\varepsilon}) + \Delta_{\varepsilon} e^{-\lambda T} \right] R + \frac{T}{\overline{T}} \left( \alpha R - 1 \right) \\ &+ \frac{1}{\overline{T}} \left( 1 - \alpha \right) R \left[ \frac{\left[ (1 - \Delta_{\varepsilon}) + \Delta_{\varepsilon} e^{-\lambda T} \right] - e^{-\lambda T}}{\lambda} \right] + \frac{\overline{T} - T}{\overline{T}} \left( \Delta_{\varepsilon} \left[ \alpha + (1 - \alpha) e^{-\lambda T} \right] R - 1 + \varepsilon \right) \\ &= \frac{T_{C}(T)}{\overline{T}} \left( 1 - \alpha \right) \left[ (1 - \Delta_{\varepsilon}) + \Delta_{\varepsilon} e^{-\lambda T} \right] R + \frac{T}{\overline{T}} \left( \alpha R - 1 \right) \\ &+ \frac{1}{\overline{T}} \left( 1 - \alpha \right) \left[ (1 - \Delta_{\varepsilon}) + \Delta_{\varepsilon} e^{-\lambda T} \right] R + \frac{T}{\overline{T}} \left( \alpha R - 1 \right) \\ &+ \frac{1}{\overline{T}} \left( 1 - \alpha \right) \left[ (1 - \Delta_{\varepsilon}) + \Delta_{\varepsilon} e^{-\lambda T} \right] R + \frac{T}{\overline{T}} \left( \alpha R - 1 \right) \\ &+ \frac{1}{\overline{T}} \left( 1 - \alpha \right) \left[ (1 - \Delta_{\varepsilon}) + \Delta_{\varepsilon} e^{-\lambda T} \right] R + \frac{T}{\overline{T}} \left( \alpha R - 1 \right) \\ &+ \frac{1}{\overline{T}} \left( 1 - \alpha \right) \left[ (1 - \Delta_{\varepsilon}) \left( 1 - e^{-\lambda T} \right) R + \frac{T}{\overline{T}} \left( \alpha R - 1 \right) \\ &+ \frac{1}{\overline{T}} \left( 1 - \alpha \right) R \left( \frac{1 - \Delta_{\varepsilon} \left( 1 - e^{-\lambda T} \right) R + \frac{T}{\overline{T}} \left( \alpha R - 1 \right) \\ &+ \frac{1}{\overline{T}} \left( 1 - \alpha \right) R \left( \frac{1 - \Delta_{\varepsilon} \left( 1 - e^{-\lambda T} \right) R + \frac{T}{\overline{T}} \left( \alpha R - 1 \right) \\ &+ \frac{1}{\overline{T}} \left( 1 - \alpha \right) R \left( \frac{1 - \Delta_{\varepsilon} \left( 1 - e^{-\lambda T} \right) R + \frac{T}{\overline{T}} \left( \Delta_{\varepsilon} \left[ \alpha + (1 - \alpha) e^{-\lambda T} \right] R - 1 + \varepsilon \right) , \end{split} \right)$$

with

$$\Pi\left(0\right) = \Delta_{\varepsilon}R - 1$$

Now take the derivative of  $\Pi(T)$  w.r.t. T, where  $T_C(T)$  and C(T) are given in the proof of Lemma 2. This yields

$$\Pi'(T) = \frac{1}{\overline{T}} \int_0^{T_C} dt \cdot C'(T) + \frac{1}{\overline{T}} \left\{ (1 - \Delta_{\varepsilon}) \left[ \alpha + (1 - \alpha) e^{-\lambda T} \right] R \right\}$$

$$-\lambda \frac{\overline{T} - T}{\overline{T}} \Delta_{\varepsilon} (1 - \alpha) e^{-\lambda T} R - \frac{\varepsilon}{\overline{T}}$$

$$= \frac{T_C(T)}{\overline{T}} R (1 - \alpha) (-\lambda) \Delta_{\varepsilon} e^{-\lambda T} + \frac{1}{\overline{T}} (1 - \Delta_{\varepsilon}) \alpha R$$

$$+ \frac{1}{\overline{T}} (1 - \alpha) e^{-\lambda T} R \left[ (1 - \Delta_{\varepsilon}) - \lambda \Delta_{\varepsilon} (\overline{T} - T) \right] - \frac{\varepsilon}{\overline{T}}$$

$$= \frac{1}{\overline{T}} (1 - \Delta_{\varepsilon}) \alpha R + \frac{1}{\overline{T}} (1 - \alpha) e^{-\lambda T} R \left\{ (1 - \Delta_{\varepsilon}) - \lambda \Delta_{\varepsilon} (\overline{T} - [T - T_C(T)]) \right\} - \frac{\varepsilon}{\overline{T}} .$$

Note that  $\Pi'(0) = \frac{1}{\overline{T}} \left[ (1 - \Delta_{\varepsilon}) - (1 - \alpha) \Delta_{\varepsilon} \lambda \overline{T} \right] R - \frac{\varepsilon}{\overline{T}}$ . Further, note that

$$T > [T - T_C(T)] = \frac{1}{\lambda} \log\left[\frac{1}{e^{-\lambda T}}\right] - T_C(T) = \frac{1}{\lambda} \log\left[(1 - \Delta_{\varepsilon})e^{\lambda T} + \Delta_{\varepsilon}\right] > 0.$$

Taking the second derivative of  $\Pi(T)$  w.r.t. T, and plugging in  $T - C_C(T)$  from above, we have

$$\Pi''(T) = -\lambda \frac{1}{\overline{T}} (1-\alpha) e^{-\lambda T} R \left\{ (1-\Delta_{\varepsilon}) - \lambda \Delta_{\varepsilon} \left( \overline{T} - [T-T_{C}(T)] \right) \right\} + \frac{1}{\overline{T}} (1-\alpha) e^{-\lambda T} R \left\{ \lambda \Delta_{\varepsilon} \left[ T - T_{C}(T) \right]' \right\} + \frac{\lambda \varepsilon}{\overline{T}} - \frac{\lambda \varepsilon}{\overline{T}} = -\lambda \Pi'(T) + \lambda \frac{1}{\overline{T}} \left[ (1-\Delta_{\varepsilon}) \alpha R - \varepsilon \right] + \frac{1}{\overline{T}} (1-\alpha) e^{-\lambda T} R \left\{ \lambda \Delta_{\varepsilon} \left[ T - T_{C}(T) \right]' \right\}.$$

Since  $[T - T_C(T)]' = \frac{(1-\Delta_{\varepsilon})}{(1-\Delta_{\varepsilon})+\Delta_{\varepsilon}e^{-\lambda T}} > 0$ , for small enough  $\varepsilon$  (e.g., for  $\frac{(1-\Delta)\alpha R}{1-\Delta\alpha R} > \varepsilon$ ), the second and third term are positive. Thus, if there exists a  $T_{extremal}$  such that  $\Pi'(T_{extremal}) = 0$  then we have  $\Pi''(T_{extremal}) > 1$ 

0. This implies that if ever  $\Pi'(t) > 0$  for  $t \ge 0$ , then  $\Pi'(t') > 0, t' > t$ . In words, if the slope of the profit function ever turns positive, it remains positive thereafter. If  $\Pi'(0) > 0$ , then  $\Pi(T)$  is monotonically increasing in T, and either no funding is offered or all maturities are funded. If in addition  $\Pi(0) > 0$ , then funding is provided up to  $\overline{T}$ . If  $\Pi'(0) < 0$ , then there is a possibility that  $\Pi'(T)$  switches sign once to positive, and if an extremal point  $\Pi'(T_{extremal}) = 0$  exists it is a minimum. Thus, the maximum profit arises either from  $\Pi(0)$  or  $\Pi(\overline{T})$ . The only interesting situation in comparing the monopolist and a central planner arises when  $\Pi(0) > 0 > \Pi'(0)$  and there exists a point  $T_{cp}$  such that  $\Pi(T_{cp}) = 0 > \Pi'(T_{cp})$ . Finally, for completeness we note that

$$\Pi\left(\overline{T}\right) = \left\{ \alpha + (1-\alpha) \left[ \frac{T_C\left(\overline{T}\right)}{\overline{T}} \left(1 - \Delta_{\varepsilon}\right) + \Delta_{\varepsilon} \frac{T_C\left(\overline{T}\right)}{\overline{T}} e^{-\lambda \overline{T}} + \frac{\left(1 - \Delta_{\varepsilon}\right) \left(1 - e^{-\lambda \overline{T}}\right)}{\lambda \overline{T}} \right] \right\} R - 1.$$

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