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**ABSTRACT**

Three mutually uncorrelated economic shocks that we measure empirically explain 85% of the quarterly variation in real stock market wealth since 1952. We use a model to show that they are the observable empirical counterparts to three latent primitive shocks: a total factor productivity shock, a risk aversion shock that is unrelated to aggregate consumption and labor income, and a factors share shock that shifts the rewards of production between workers and shareholders. On a quarterly basis, risk aversion shocks explain roughly 75% of variation in the log difference of stock market wealth, but the near-permanent factors share shocks plays an increasingly important role as the time horizon extends. We find that more than 100% of the increase since 1980 in the deterministically detrended log real value of the stock market, or a rise of 65%, is attributable to the cumulative effects of the factors share shock, which persistently redistributed rewards away from workers and toward shareholders over this period. Indeed, without these shocks, today's stock market would be about 10% lower than it was in 1980. By contrast, technological progress that rewards both workers and shareholders plays a smaller role in historical stock market fluctuations at all horizons. Finally, the risk aversion shocks we identify, which are uncorrelated with consumption or its second moments, largely explain the long-horizon predictability of excess stock market returns found in data. These findings are hard to reconcile with models in which time-varying risk premia arise from habits or stochastic consumption volatility.

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## 1 Introduction

Asset pricing theorists have long been concerned with explaining stock market *expected returns*, typically measured over monthly, quarterly or annual horizons. This is important because empirical evidence suggests that variation in the stock market price-dividend ratio is driven almost entirely by expected excess return variation (i.e., forecastable movements in equity premia).<sup>1</sup> Far less attention has been given to understanding the real (adjusted for inflation) *level* of the stock market, i.e., stock price variation, or the cumulation of returns over many decades. The profession spends a lot of time debating which risk factors drive expected excess returns, but little time investigating why real stock market wealth has evolved to its current level compared to 30 years ago. To understand the latter, it is necessary to probe beyond the role of stationary risk factors and short-run expected returns, to study the primitive economic shocks from which all stock market (*and* risk factor) fluctuations originate.

To see why, consider that some economic shocks may have tiny innovations but permanent or near-permanent effects on cash flows. Under rational expectations, permanent cash flow shocks have no influence on the price-dividend *ratio* (they are incorporated immediately into both prices in the numerator and dividends in the denominator), but they can have a dramatic influence on real stock market wealth as the decades accumulate. Such shocks are the sources of stochastic trends in stock prices that are by definition impossible to predict and not reflected in expected returns. On the other hand, fluctuations in expected returns may be associated with movements in risk premia and can persistently shift the real value of the stock market around its long-term trend. But because these fluctuations are transitory, their impact eventually dies out. Stock market wealth evolves over time in response to the cumulation of both transitory expected return and permanent cash flow shocks. The crucial unanswered questions are, what are the economic sources of these shocks? And what have been their relative roles in evolution of the stock market over time?

The objective of this paper is to address these questions. We begin by identifying three mutually orthogonal observable economic shocks that explain the vast majority (over 85%) of quarterly fluctuations in real stock market wealth since the early 1950s. Econometrically, these shocks are measured as specific orthogonal movements in consumption, labor income, and asset wealth (net worth), identified from a cointegrated vector autoregression (VAR) and extracted using a standard recursive identification procedure. We investigate how these

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<sup>1</sup>Expected dividend growth and expected short-term interest rates play little role empirically in price-dividend ratio variation (Campbell (1991); Cochrane (1991); Cochrane (2005); Cochrane (2008)).

shocks have affected stock market wealth over time, with special attention paid to their relative importance over short versus long time horizons.

We then address the question of what these observable VAR shocks represent economically. Providing an economic interpretation of the shocks requires a theoretical framework. To do so, this paper develops a general equilibrium model of two types of households: shareholders and workers. Economic fluctuations in the model originate from three mutually orthogonal primitive shocks that differentially affect each type: a permanent total factor *productivity* (TFP) shock that governs the state of factor neutral technological progress and propels aggregate (shareholder plus worker) consumption, a near-permanent *factor shares* shock that reallocates the rewards from production between shareholders and workers without affecting the size of those rewards, and an exogenous shock to shareholder *risk aversion* that moves the stochastic discount factor pricing assets independently of stock market fundamentals or real variables such as consumption and labor income. The modifier “exogenous” in reference to shareholder risk aversion refers to the independence in the model of such shocks from stock market fundamentals (dividends or earnings), consumption, and labor income. We argue that this exogeneity is essential for explaining time-variation in the reward for bearing stock market risk, a point we come back to below.

The model is then employed to show that the mutually orthogonal VAR innovations that explain almost all stock market variation are the observable empirical counterparts to the latent primitive shocks in the theoretical framework. Specifically, we show that, if the model generated the data, the total factor productivity shock would be revealed as a *consumption shock* in the VAR, namely a movement in log consumption,  $c_t$ , that contemporaneously affects both log labor income,  $y_t$ , and log asset wealth,  $a_t$ , where all three move in the same direction. The factors share shock would be revealed as a *labor income shock* that moves  $a_t$  in the opposite direction of  $y_t$  but is restricted to have no contemporaneous impact on  $c_t$ . And the risk aversion shock would be revealed as a *wealth shock* that moves  $a_t$  but is restricted to have no contemporaneous impact on either  $c_t$  or  $y_t$ . We show that the dynamic responses to these mutually orthogonal VAR innovations produced from model generated data are remarkably similar to those produced from historical data. We refer to the three mutually orthogonal VAR innovations (consumption, labor income and wealth) interchangeably as productivity or TFP, factors share, and risk aversion innovations, respectively.

How have these shocks affected stock market wealth over time? We find that the vast majority of short- and medium-term stock market fluctuations in historical data are driven by risk aversion shocks, revealed as movements in wealth that are orthogonal to consumption

and labor income, both contemporaneously (an identifying assumption), and at all subsequent horizons (a result). Although transitory, these shocks are quite persistent with a half life of over four years. On a quarterly basis, they explain approximately 75% of variation in the log difference of stock market wealth, but their contribution declines as the horizon extends. These facts are well explained by the model, in which the orthogonal wealth shocks originate from exogenous shifts in investors' willingness to bear risk.

At longer horizons, the relative importance of the shocks changes. The factors share shock explains a negligible fraction of variation in the stock market over shorter time horizons, but because its innovations are nearly permanent, it plays an increasingly important role as the time horizon extends. A spectral decomposition of variance by frequency shows that this shock explains virtually none of the variation in the real level of the stock market over cycles of a quarter or two, but it explains roughly 40% over cycles two to three decades long. These facts are well explained by the model economy, which is subject to small but highly persistent innovations that shift the allocation of rewards between shareholders and workers independently from the magnitude of those rewards.

By contrast, productivity shocks, revealed as consumption shocks empirically, play a small role in the stochastic fluctuations of the stock market at all horizons, once a deterministic trend is removed. The model is consistent with this finding. Given capital's smaller role in the production process, most gains and losses from total factor productivity shocks accrue to workers, so their effect on stock market wealth is smaller than that of the other two shocks.

As an example of the magnitude of these forces for the long-run evolution of the stock market, we decompose the percent change since 1980 in the deterministically detrended real value of stock market wealth that is attributable to each shock. The period since 1980 is an interesting one to consider, in which the cumulative effect of the factor shares shock persistently redistributed rewards away from workers and toward shareholders. (The opposite occurs from the mid 1960s to mid 1980s.) After removing a deterministic trend, we find that the cumulative effects of the factors share shocks have resulted in a 65% increase in real stock market wealth since 1980, an amount equal to 110% of the total increase in stock market wealth over this period. Indeed, without these shocks, today's stock market would be roughly 10% lower than it was in 1980. This finding underscores the extent to which the long-term value of the stock market has been profoundly altered by forces that reallocate the rewards of production, rather than raise or lower all of them.

Our calculations imply that an additional 38% of the increase in the detrended real value of the stock market since 1980, or a rise of 22%, is attributable to the cumulative effects

of risk aversion shocks, which were on average lower in the last 30 years than earlier in the post-war period. By contrast, the cumulative effects of TFP shocks have made a negative contribution to change in stock market wealth since 1980, once a deterministic trend is removed. The importance of the TFP shock is uncharacteristically large over this period, a direct consequence of the string of unusually large negative draws for consumption in the Great Recession years from 2007-2009. These shocks account for -38% of the increase since 1980. Together, the three mutually orthogonal economic shocks we identify explain almost all of the increase in deterministically detrended real stock market wealth since 1980. (Specifically, they account for 110% of the increase, with the remaining -10% accounted for by a residual.)

Finally, our findings speak to the question of why the stock market is predictable. The exogenous risk aversion shocks in the model are associated with forecastable changes in excess stock market returns, consistent with the model implication that they drive changes in equity risk premia over time. We show that this is also true in the data. In particular, the predictive content for long horizon excess stock market returns of common stock market forecasting variables is found to be virtually subsumed by the information in lags of the exogenous risk aversion/wealth shock at all but very long horizons (where both variables have independent forecasting power). At the same time, the exogenous risk aversion shocks we identify are contemporaneously unrelated to consumption, labor income, dividends, earnings, consumption volatility, or broad-based macroeconomic uncertainty, and none of these variables forecast equity premia. These findings are hard to reconcile with models in which time-varying risk premia arise from habits (which vary with innovations in consumption), stochastic consumption volatility, or uncertainty. On the other hand, these features of the data may be well captured by models with time-varying ambiguity aversion, which, much like our time-varying risk aversion framework, are capable of generating predictable variation in excess returns that is disconnected with real activity (e.g., Bianchi, Ilut, and Schneider (2013)). We view time-varying ambiguity and risk aversion as members of the same broad class of models where fluctuations in the reward for bearing risk occur independently of economic risks.

This empirical part of this paper builds on Lettau and Ludvigson (2013). That paper provided empirical evidence in a purely statistical model, studying (a rotation of) the three VAR innovations described here and their relationship to different components of household wealth, consumption and labor income. The main contribution of this paper is to provide an economic interpretation of these innovations and a detailed investigation of their implications

for the stock market. Our model is also related to the work of several recent papers that have emphasized the weak empirical correlation between stock market behavior and innovations to consumption growth or its second moments (Duffee (2005), Albuquerque, Eichenbaum, and Rebelo (2012), Lettau and Ludvigson (2013)). And there is an important earlier literature in asset pricing that identified and distinguished cash-flow from discount rate “shocks” (e.g., Campbell (1991); Cochrane (1991)). This work was central to our understanding of how innovations in stock returns are related to forecastable movements in returns as compared to dividend growth, but it is silent on the underlying *economic* mechanisms that drive these forecastable changes. It is precisely these primitive economic shocks that are the subject of this paper.

The rest of this paper is organized as follows. The next section describes the econometric procedure and data used to identify the three mutually uncorrelated empirical shocks from a VAR on consumption, labor income, and asset wealth. Section 3 describes the theoretical model that we use to interpret these shocks. Section 4 presents our findings, which are of two forms. The first are results on the performance of the model, including summary statistics for standard asset pricing implications. This section also demonstrates that the mutually orthogonal VAR innovations described in Section 3 are the observable empirical counterparts to the latent primitive shocks in the model. The second set of results studies the relative role of the observable shocks in historical stock market fluctuations, with special attention paid to how these roles depend on the time horizon over which one measures a change in stock market wealth. Here we compare the role of each shock in the dynamic responses and variance decompositions of stock market wealth when estimated from historical data with the same statistics when estimated from model-generated data and show that they are remarkably similar along a number of dimensions discussed below. The final subsection investigates the question of why stock returns are predictable. We show that the exogenous risk aversion shocks we identify largely explain why long-horizon excess stock market returns are predictable by common forecasting variables such as the log price-dividend ratio or the consumption-wealth variable  $cay_t$ . Section 6 briefly discusses the link between the factors share shock and economic inequality. Section 7 concludes.

## **2 Econometric Analysis: Three Mutually Orthogonal Shocks**

This section describes the empirical analysis used to identify three mutually orthogonal observable shocks from data on aggregate consumption, labor income and household wealth.

## 2.1 Data

Many empirical details of the estimation that follows are covered in Lettau and Ludvigson (2013). Here we outline the main elements and refer the reader to that paper for more information.<sup>2</sup>

We consider a cointegrated vector of variables in the data, denoted  $\mathbf{x}_t = (c_t, y_t, a_t)'$ , where  $c_t$  is log of real, per capita aggregate consumption,  $y_t$  is log of real, per-capita labor income, and  $a_t$  is log of real, per-capita asset wealth. Throughout this paper we use lower case letters to denote log variables, e.g.,  $\ln(A_t) \equiv a_t$ . Lettau and Ludvigson (2013) provide updated evidence of a cointegrating relation among these variables, which can be motivated by considering the long-run implications of a standard household budget constraint (see Lettau and Ludvigson (2001)).

The Appendix contains a detailed description of the data used in this study. The log of asset wealth,  $a_t$ , is a measure of real, per capita household net worth, which includes all financial wealth, housing wealth, and consumer durables. It is compiled from the flow of funds accounts by the Board of Governors of the Federal Reserve.

We study the implications of the empirical shocks identified from the system  $(c_t, y_t, a_t)'$  and subsequently relate these shocks to stock market wealth. We denote the log of stock market wealth  $s_t$ . Stock market wealth is a component of total asset wealth. Corporate equity was 23% of total asset wealth in 2010, and 29% of net worth. For comparison, we will often also study the implications of these same shocks for the Center for Research in Securities Prices (CRSP) value-weighted stock price index. We denote the log of the CRSP value-weighted stock price index  $p_t$ .

We use the log of real, per capita, expenditures on nondurables and services (excluding shoes and clothing), as a measure of  $c_t$ . From the household's budget constraint, an internally consistent cointegrating relation among  $c_t$ ,  $y_t$ , and  $a_t$ , may then be obtained if we assume that the log of (unobservable) real total flow consumption is cointegrated with the log of real nondurables and services expenditures (Lettau and Ludvigson (2010)). The log of after-tax labor income,  $y_t$ , is also measured in real, per capita terms. Lettau and Ludvigson (2013) presents empirical evidence supportive of a single cointegrating relationships between  $c_t$ ,  $a_t$ , and  $y_t$  in the post-war data used in the study.<sup>3</sup> Our data are quarterly and span the first

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<sup>2</sup>That paper showed how the innovations we study here can be econometrically identified as disturbances distinguished by whether their effects are permanent or transitory. An additional rotation of innovations was required for this interpretation, but the shocks obtained there are perfectly correlated with those obtained here and discussed below.

<sup>3</sup>The data provide no evidence of a second linearly independent cointegrating relation (there can be at



quarter of 1952 to the third quarter of 2012.

## 2.2 Empirical Implementation

Before explaining the details of the estimation, we discuss a practical distinction between the data and the model. The model developed below is intended to focus on the implications of the empirical shocks for stock market wealth. As such, it has one form of risky capital (equity) and a risk-free bond in zero net supply. It follows that, in the model, all wealth is stock market wealth, which is the same as total wealth, which is the same as net worth. In historical data, total wealth contains non-equity forms of wealth, so total wealth and stock market wealth are two different variables. As noted above, stock market wealth accounts for about 23% of total wealth in 2010. In the data we distinguish the two by denoting log of asset wealth (net worth) by  $a_t$  and log of stock market wealth by  $s_t$ . In the model,  $a_t = s_t$ . In constructing the empirical VAR innovations discussed next, we use system of variables that contains consumption, labor income and total asset wealth  $a_t$ , and then subsequently relate these innovations to stock market wealth. We do *not* construct these innovations by restricting analysis to how consumption and labor income move only with stock market wealth. We do this for two reasons. First, a (factor neutral) TFP shock should affect the value of all productive capital, so a system that identifies such a shock from the data should include total wealth. If TFP shocks affect non-stock wealth but these components are omitted from the system, this could lead to spurious estimates of productivity and its dynamics, which would also contaminate estimates of the factors share and risk aversion shocks that are presumed orthogonal to the TFP shock. Second, consumption and labor income are cointegrated with total wealth, as expected from theory (Lettau and Ludvigson (2001)), but there is no implication that these variables should be (or are) cointegrated with stock market wealth by itself, a component of total wealth. It is important to control empirically for these long-run relationships by imposing the restrictions implied by cointegration in a VAR for which wealth shares a common trend with consumption and labor income. When this is done, it will then be an empirical matter how closely the identified VAR shocks are related

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most two). In particular, bivariate log ratios of these variables appear to contain trends in our sample. Economic models often imply that bivariate log ratios (e.g.,  $y_t \square a_t$ ) are stationary, and the model below assumes as much. In finite samples, it is impossible to distinguish a highly persistent but stationary series from a non-stationary one. For the empirical system, we follow the advice of Campbell and Perron (1991) and empirically model only the single, trivariate cointegrating relation for which we find direct statistical evidence of in our sample. For the model, we set parameter values that imply the log ratio  $y_t \square a_t$  is stationary but deviations from the common trend in  $y_t$  and  $a_t$  are so persistent one could not reject a unit root in samples of the size we currently face, consistent with the data.

to stock market wealth, which is the subject of an extensive investigation below. Notice that there is no implication that the shocks identified from the  $c_t$ ,  $y_t$ ,  $a_t$  empirical system should explain all or even most of the variation in stock market wealth  $s_t$ . Although  $s_t$  can be related to these shocks, there will be an unexplained residual that in principal could be quite important, as we explain below.

Identification of the three mutually orthogonal empirical disturbances is achieved in several steps. First, we assume all of the series contained in  $\mathbf{x}_t$  are first order integrated, or I(1), an assumption confirmed by unit root tests, available upon request. The cointegrating coefficient on consumption is normalized to one, and we denote the single cointegrating vector for  $\mathbf{x}_t = [c_t, a_t, y_t]'$  as  $\boldsymbol{\alpha} = (1, \square\alpha_a, \square\alpha_y)'$ . The cointegrating parameters  $\alpha_a$  and  $\alpha_y$  are estimated using dynamic least squares, which generates “superconsistent” estimates of  $\alpha_a$  and  $\alpha_y$  (Stock and Watson, 1993).<sup>4</sup> We estimate  $\widehat{\boldsymbol{\alpha}} = (1, \square 0.18, \square 0.70)'$ . The Newey and West (1987) corrected  $t$ -statistics for these estimates are 20 and 56, respectively.

Second, a cointegrated VAR (or vector error correction mechanism–VECM) representation of  $\mathbf{x}_t$  is estimated taking the form

$$\Delta\mathbf{x}_t = \mathbf{v} + \widehat{\boldsymbol{\alpha}}'\mathbf{x}_{t-1} + \square(L)\Delta\mathbf{x}_{t-1} + \mathbf{u}_t, \quad (1)$$

where  $\Delta\mathbf{x}_t$  is the vector of log first differences,  $(\Delta c_t, \Delta a_t, \Delta y_t)'$ ,  $\mathbf{v}$ , and  $\equiv (c, a, y)'$  are  $(3 \times 1)$  vectors,  $\square(L)$  is a finite order distributed lag operator, and  $\widehat{\boldsymbol{\alpha}} \equiv (1, \square\widehat{\alpha}_a, \square\widehat{\alpha}_y)'$  is the  $(3 \times 1)$  vector of previously estimated cointegrating coefficients.<sup>5</sup> The term  $\widehat{\boldsymbol{\alpha}}'\mathbf{x}_{t-1}$  gives last period’s equilibrium error, or cointegrating residual, a variable we denote with  $cay_t \equiv \widehat{\boldsymbol{\alpha}}'\mathbf{x}_{t-1}$ . The coefficients are the vector of “adjustment” coefficients that tells us which variables subsequently adjust to restore the common trend when a deviation occurs. Throughout this paper, we use “hats” to denote the estimated values of parameters.

The results of estimating a first-order specification of (1) are presented in Lettau and Ludvigson (2013). An important result is that, although consumption and labor income are somewhat predictable by lagged consumption and wealth growth, they are not predictable by the cointegrating residual  $\widehat{\boldsymbol{\alpha}}'\mathbf{x}_{t-1}$ . Estimates of  $c$  and  $y$  are economically small and insignificantly different from zero. By contrast, the cointegrating error  $cay_t$  is an economically large and statistically significant determinant of next quarter’s wealth growth:  $a$  is estimated

<sup>4</sup>We use eight leads and lags of the first differences of  $\Delta y_t$  and  $\Delta a_t$  in the dynamic least squares regression. Monte Carlo simulation evidence in both Ng and Perron (1997) and our own suggested that the DLS procedure can be made more precise with larger lag lengths.

<sup>5</sup>Standard errors do not need to be adjusted to account for the use of the “generated regressor,”  $\boldsymbol{\alpha}'\mathbf{x}_t$  in (1) because estimates of the cointegrating parameters converge to their true values at rate  $T$ , rather than at the usual rate  $\sqrt{T}$  (Stock (1987)).

to be 0.20, with a  $t$ -statistic equal to 2.3.<sup>6</sup> Thus, only wealth exhibits error-correction behavior. Wealth is mean reverting and adapts over long-horizons to match the smoothness in consumption and labor income.

Third, the individual series involved in the cointegrating relation can be represented as a reduced-form multivariate Wold representation:

$$\Delta \mathbf{x}_t = \boldsymbol{\delta} + \boldsymbol{\Omega}(L)\mathbf{u}_t, \quad (2)$$

where  $\mathbf{u}_t$  is an  $n \times 1$  vector of innovations, and where  $\boldsymbol{\Omega}(L) \equiv \mathbf{I} + \boldsymbol{\Omega}_1 L + \boldsymbol{\Omega}_2 L^2 + \boldsymbol{\Omega}_3 L^3 + \dots$ . The parameters  $\boldsymbol{\alpha}$  and  $\boldsymbol{\beta}$ , both of rank  $r$ , satisfy  $\boldsymbol{\alpha}'\boldsymbol{\Omega}(1) = 0$  and  $\boldsymbol{\Omega}(1)\boldsymbol{\beta} = 0$  (Engle and Granger, 1987). The “reduced form” disturbances  $\mathbf{u}_t$  are not necessarily mutually uncorrelated. To identify shocks that are mutually uncorrelated, we employ a recursive orthogonalization. Specifically, let  $\mathbf{H}$  be a lower triangular matrix that accomplishes the Cholesky decomposition of  $\text{Cov}(\mathbf{u}_t)$ , and define a set of orthogonal structural disturbances  $\mathbf{e}$  such that

$$\mathbf{e} \equiv \mathbf{H}^{\square 1} \mathbf{u}_t.$$

Also define

$$\mathbf{C}(L) \equiv \boldsymbol{\Omega}(L)\mathbf{H}.$$

Then we may re-write the decomposition of  $\Delta \mathbf{x}_t = (\Delta c_t, \Delta y_t, \Delta a_t)'$  as

$$\Delta \mathbf{x}_t = \boldsymbol{\delta} + \mathbf{C}(L)\mathbf{e}_t, \quad (3)$$

which now yields a vector of mutually uncorrelated innovations  $\mathbf{e}_t$  in consumption, labor income, and asset wealth. We refer to the mutually orthogonal  $\mathbf{e}_t$  shocks as “structural” disturbances, but the specific economic interpretation of these shocks must wait for the model presented below. Denote the individual consumption, labor income and wealth disturbances as  $e_{c,t}$ ,  $e_{y,t}$ , and  $e_{a,t}$ . Note that these shocks are i.i.d.

The ordering of variables will matter for this orthogonalization. We choose a particular ordering, and thus a particular orthogonalization so that these will reveal the effects of the primitive shocks in the model described next. Specifically, we restrict  $\Delta c$  to be first,  $\Delta y$  to be second, and  $\Delta a$  to be last in  $\Delta \mathbf{x}_t$ , as written above. As we show below in the model, with these restrictions, the TFP shock will be revealed as a consumption shock,  $e_{c,t}$ , the

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<sup>6</sup>We also find that the *four*-quarter lagged value of the cointegrating error strongly predicts asset growth. This shows that the forecasting power of the cointegrating residual for future asset growth cannot be attributable to interpolation procedures used to convert annual survey data to a quarterly housing service flow estimate, part of the services component of  $c_t$ .

factors share shock orthogonal to this will be revealed by the labor income shock,  $e_{y,t}$ , and the exogenous risk aversion shock will be revealed by the wealth shock,  $e_{a,t}$ .<sup>7</sup>

### 2.3 Relating Structural Disturbances to Stock Market Fluctuations

The econometric procedure just described is applied to the system of  $\Delta c_t$ ,  $\Delta y_t$ , and  $\Delta a_t$ . Ultimately our goal is to study the role each shock has played in the dynamic behavior of stock market wealth over time. To relate stock market wealth to the structural disturbances  $e_{c,t}$ ,  $e_{y,t}$ , and  $e_{a,t}$ , we estimate empirical relationships taking the form

$$\Delta s_t = \kappa_0 + \kappa_c(L) e_{c,t} + \kappa_y(L) e_{y,t} + \kappa_a(L) e_{a,t} + \eta_t, \quad (4)$$

where  $s_t$  represents the log level of the stock market wealth,  $\kappa_c(L)$ ,  $\kappa_y(L)$ , and  $\kappa_a(L)$  are polynomial lag operators, and  $\eta_t$  is a residual that represents the component of stock wealth that is unexplained by the structural empirical disturbances  $e_{c,t}$ ,  $e_{y,t}$ , and  $e_{a,t}$ . For comparison, we compute the same type of relationship for the CRSP value-weighted stock price index, replacing stock market wealth  $s_t$  with  $p_t$  on the left-hand-side:

$$\Delta p_t = \lambda_0 + \lambda_c(L) e_{c,t} + \lambda_y(L) e_{y,t} + \lambda_a(L) e_{a,t} + \xi_t. \quad (5)$$

Since  $e_{c,t}$ ,  $e_{y,t}$  and  $e_{a,t}$  are mutually uncorrelated and i.i.d., we estimate these equations separately by OLS with  $L = 16$  quarters.

### 2.4 Decomposition of Levels

We will want to study the role that each shock has played in the evolution of the *levels* of stock market wealth over time. To do so, we decompose the log levels into components driven by each structural disturbance. Let us rewrite the decomposition of growth rates for stock market wealth (4) as

$$\begin{aligned} \Delta s_t &= \kappa_0 + \kappa_c(L) e_{c,t} + \kappa_y(L) e_{y,t} + \kappa_a(L) e_{a,t} + \eta_t \\ &\equiv \kappa_0 + \Delta s_t^c + \Delta s_t^y + \Delta s_t^a + \eta_t, \end{aligned} \quad (6)$$

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<sup>7</sup>Lettau and Ludvigson (2013) showed how the innovations we study here can be given an additional interpretation, namely as disturbances distinguished by whether their effects have permanent or transitory effects on the variables in the system. An additional rotation of innovations was required for this interpretation, but the shocks obtained there are perfectly correlated with the  $\mathbf{e}_t$  obtained here. What this shows is that the permanent and transitory shocks in the  $\Delta \mathbf{x}_t = (\Delta c_t, \Delta y_t, \Delta a_t)'$  system are associated with specific orthogonal movements in consumption, labor income and wealth that readily be related to the latent model primitive shocks, as below.

where  $\Delta s_t^c \equiv \kappa_c(L) e_{c,t}$ , and analogously for the other terms. The effect on the log levels of stock wealth of each disturbance is obtained by summing up the effects on the log differences, so that the log level of stock wealth may be decomposed into the following components:

$$\begin{aligned}
s_t &= s_0 + \kappa_0 t + \sum_{k=1}^t \Delta s_k \\
&= s_0 + \kappa_0 t + \sum_{k=1}^t \Delta s_k^c + \sum_{k=1}^t \Delta s_k^y + \sum_{k=1}^t \Delta s_k^a + \sum_{k=1}^t \eta_k \\
&\equiv s_0 + \kappa_0 t + s_t^c + s_t^y + s_t^a + \sum_{k=1}^t \eta_k,
\end{aligned} \tag{7}$$

where  $s_0$  is the initial level of stock market wealth,  $s_t^c$ ,  $s_t^y$ , and  $s_t^a$ , are the components of the level attributable to the (cumulation of) the consumption shock, the labor income shock, and the wealth shock, respectively. The final sum  $\sum_{k=1}^t \eta_k$  is the component of  $s_t$  attributable to the unexplained residual. Note that  $\kappa_0 t$  is the deterministic trend in stock market wealth, which in the model below is attributable to steady state technological progress. For the level decomposition reported below, we remove this trend and normalize the initial observation  $s_0$  to zero in the quarter before the start of our sample. Expressions analogous to (6) and (7) are also computed for the log stock price index  $p_t$ .

This completes our description of the empirical shocks. The next section presents the theoretical model.

### 3 The Model

#### 3.1 Production and Technology

Suppose that aggregate output  $\mathcal{Y}_t$  is given by a constant returns to scale process:

$$\mathcal{Y}_t = \mathcal{A}_t N_t^\alpha K_t^{1-\alpha}, \tag{8}$$

where  $\mathcal{A}_t$  is a factor neutral total factor *productivity* (TFP) shock, and  $N_t$  and  $K_t$  are inputs of labor and capital, respectively. We further assume that labor supply is fixed and that there is no capital accumulation, so that both  $N_t$  and  $K_t$  are constant over time and normalized to unity. Thus  $\mathcal{Y}_t = \mathcal{A}_t$  is driven entirely by technological change.

The economy is populated by two types of representative households, each of whom consume an endowment stream. The first type, “shareholders,” own a claim to shares of the dividend income stream (equity) generated from aggregate output  $\mathcal{Y}_t$ . There is no saving

and no new shares are issued. Shareholders consume the dividend stream. The second type, “workers,” own no assets, inelastically supply labor to produce  $\mathcal{Y}_t$ , and consume their labor income every period. Dividends,  $D_t$ , are equal to output minus a wage bill

$$D_t = \mathcal{Y}_t - W_t N_t, \quad (9)$$

where  $W_t$  is the wage rate paid to workers. With labor supply fixed at  $N_t = 1$ , log labor income, which equals  $\ln(W_t N_t) = \ln(W_t)$ , is alternatively denoted  $y_t$ , to be consistent with the notation above. The total number of shares is normalized to unity.

The wage rate  $W_t$  is given by marginal product of labor, multiplied by a time-varying function  $f(Z_t)$ :

$$\begin{aligned} W_t &= [\alpha \mathcal{A}_t N_t^{\alpha-1} K_t^{1-\alpha}] f(Z_t) \\ &= \alpha \mathcal{A}_t f(Z_t). \end{aligned} \quad (10)$$

The random variable  $Z_t$  over which the function is defined is referred to as a *factors share* shock. We specify  $f(Z_t)$  to be a logistic function

$$f(Z_t) = \frac{1}{1 + \exp(-\beta Z_t)},$$

where  $\beta$  is a constant parameter. The calibration we choose insures that the real wage is equal to its competitive value on average, but can be shifted away from this value by a multiplicative scale factor  $f(Z_t)$  with  $f(Z_t) = 1$  in the non-stochastic steady state. The logistic function insures that the level labor income is never negative and bounded above and below.

Although not modeled explicitly as such,  $f(Z_t)$  could be interpreted as the time-varying bargaining parameter resulting from some underlying wage-bargaining problem that creates deviations from competitive equilibrium. At a more basic level, it is a reduced-form way of capturing shifts in the allocation of rewards from production between shareholders and workers that could occur for a number of reasons (see the conclusion for more discussion). For example, an alternative interpretation of the model that gives identical equilibrium allocations is that labor markets are competitive and  $f(Z_t)$  represents exogenous technological change that alters how labor-intensive production is:

$$\mathcal{Y}_t = \mathcal{A}_t N_t^{\alpha f(Z_t)} K_t^{1-\alpha},$$

with higher values of  $f(Z_t)$  indicating more labor intensive, and lower values indicating more labor saving. If we continue to assume that labor supply and capital are fixed, and with the

same normalization  $K_t = 1$ , fixed labor supply  $N_t = N$  is now a parameter to be calibrated. But with  $\alpha$  already set to match the long-run (steady state) labor's share, we require  $N = 1$  to be consistent with this value. Under these assumptions, fluctuations in  $f(Z_t)$  have no influence of aggregate output, which remains  $\mathcal{Y}_t = \mathcal{A}_t$ , and only affect how the rewards of production are divided, with  $W_t = \alpha \mathcal{A}_t f(Z_t)$  and  $D_t = \mathcal{Y}_t \square W_t$ . This model has equilibrium allocations that are identical to the previous one.

With this specification for wages, dividends are given by

$$\begin{aligned} D_t &= \mathcal{Y}_t \square \alpha \mathcal{A}_t f(Z_t) \\ &= \mathcal{A}_t (1 \square \alpha f(Z_t)), \end{aligned} \tag{11}$$

and log dividends,  $d_t$  take the form

$$d_t = \mathbf{a}_t + \ln(1 \square \alpha f(Z_t)),$$

where  $\mathbf{a}_t \equiv \ln \mathcal{A}_t$ . Notice that log dividends are a non-linear function of the factors share shock.

Aggregate consumption,  $C_t$ , is the sum of shareholder consumption (total dividends) and worker consumption (total labor income):

$$C_t = D_t + W_t = \mathcal{Y}_t \square W_t + W_t = \mathcal{A}_t.$$

Aggregate (shareholder plus worker) consumption is determined solely by the TFP shock.

The log difference in the TFP shock,  $\Delta \mathbf{a}_t$ , is assumed to follow a first-order autoregressive (AR(1)) stochastic process given by

$$\Delta \mathbf{a}_t \square \mu_a = \phi_a (\Delta \mathbf{a}_{t \square 1} \square \mu_a) + \sigma_a \varepsilon_{a,t}, \quad \varepsilon_{a,t} \sim i.i.d. (0, 1). \tag{12}$$

The factors share shock  $Z_t$  is assumed to follow a mean-zero AR(1) processes:

$$Z_t = \phi_z Z_{t \square 1} + \sigma_z \varepsilon_{z,t}, \quad \varepsilon_{z,t} \sim i.i.d. (0, 1). \tag{13}$$

Note that in a non-stochastic steady state,  $Z_t$  is identically zero,  $f(Z_t) = 1$ , and dividends are proportional to productivity:  $D_t = \mathcal{A}_t (1 \square \alpha)$ . The above specification implies that the economy grows non-stochastically in steady state at the gross rate of  $\mathcal{A}_t$ , given by  $1 + \mu_a$ , the deterministic rate of technological progress.

### 3.2 Shareholder Preferences

Worker preferences play no role in asset pricing since they hold no assets. Shareholder preferences determine the pricing of equity. We assume that the economy is populated by a large number of identical shareholders. Let  $C_{it}^s$  denote the consumption of an individual stockholder indexed by  $i$  at time  $t$ . Let  $\beta_t$  be a time-varying subjective discount factor, discussed below. Identical shareholders maximize the function

$$U = E \sum_{t=0}^{\infty} \prod_{k=0}^t \beta_k u(C_{it}^s) \quad (14)$$

with

$$u(C_{it}^s) = \frac{(C_{it}^s)^{1-x_t}}{1-x_t}, \quad (15)$$

and where  $\beta_0 = 1$ .

An important aspect of these preferences is that the parameter  $x_t$  is not constant but instead varies stochastically over time. It is interpreted as a risk aversion shifter, as discussed further below. Shareholder preferences are also subject to an externality in the subjective discount factor  $\beta_t$ , which is assumed to vary over time in a manner dependent on aggregate shareholder consumption (which in equilibrium equal dividends) as follows:

$$\beta_t \equiv \frac{\exp(-r_f)}{E_t \left[ \frac{D_{t+1}^{x_t}}{D_t^{x_{t+1}}} \right]}, \quad (16)$$

where  $r_f$  is a parameter. We discuss this parametrization for  $\beta_t$  below. In equilibrium, identical individuals choose the same level of consumption, equal to per capita aggregate dividends  $D_t$ . We therefore drop the  $i$  subscript and simply denote the consumption of a representative shareholder  $C_t^s = D_t$  from now on. Notice that aggregate shareholder consumption, given by  $D_t$ , is taken as given by individual shareholders and is therefore not internalized in the individual optimization problem.

With these preferences, the intertemporal marginal rate of substitution in stockholder consumption is the stochastic discount factor (SDF) given by:

$$M_{t+1} = \frac{\exp(-r_f) \left( \frac{D_{t+1}}{D_t} \right)^{x_t}}{E_t \left[ \left( \frac{D_{t+1}}{D_t} \right)^{x_t} \right]}. \quad (17)$$



This can be written

$$M_{t+1} = \beta_t \frac{(D_{t+1})^{x_t}}{(D_t)^{x_{t+1}}} = \frac{\exp(\square r_f) \frac{(D_{t+1})^{x_t}}{(D_t)^{x_{t+1}}}}{E_t \left[ \frac{(D_{t+1})^{x_t}}{(D_t)^{x_{t+1}}} \right]}. \quad (18)$$

Multiplying the numerator and denominator of (18) by  $(D_t)^{x_t x_{t+1}}$  gives

$$M_{t+1} = \exp[\square r_f \square \ln E_t \exp(\square x_t \Delta d_{t+1}) \square x_t \Delta d_{t+1}]. \quad (19)$$

We specify the stochastic risk-aversion variable  $x_t$  so that it is always non-negative and bounded from above. Specifically, let  $x_t$  be specified as a logistic function of a stochastic variable  $\tilde{x}$  that itself can take unbounded values:

$$x_t = \frac{\theta}{1 + \exp(\square \tilde{x}_t)},$$

$$\tilde{x}_t \square \mu_{\tilde{x}} = \phi_{\tilde{x}}(\tilde{x}_{t+1} \square \mu_{\tilde{x}}) + \sigma_{\tilde{x}} \varepsilon_{\tilde{x},t}, \quad \varepsilon_{\tilde{x},t} \sim i.i.d. (0, 1). \quad (20)$$

In the above,  $\theta$  is a parameter that controls the maximum value of  $x_t$ , which is a nonlinear function of shocks to a stationary first-order autoregressive stochastic process with mean  $\mu_{\tilde{x}}$ , innovation variance  $\sigma_{\tilde{x}}^2$ , and autoregressive parameter  $0 < \phi_{\tilde{x}} < 1$ .

### 3.3 Pricing the Dividend Claim

We use the dividend claim to model the stock market claim. Let  $P_t$  denote the ex-dividend price of a claim to the dividend stream measured at the end of time  $t$ . The gross return from the end of period  $t$  to the end of  $t + 1$  is defined

$$R_{t+1} = \frac{P_{t+1} + D_{t+1}}{P_t}. \quad (21)$$

The return on a risk-free asset whose value is known with certainty at time  $t$  is given by

$$R_{f,t+1} \equiv (E_t [M_{t+1}])^{\square 1}.$$

We denote the log return on equity as  $\ln(R_{t+1}) \equiv r_{t+1}$ , and the log excess return  $\ln(R_{t+1}/R_{f,t+1}) \equiv r_{t+1}^{ex}$ .

From the shareholder's first-order condition for optimal consumption choice, the price-dividend ratio satisfies

$$\frac{P_t}{D_t}(\mathfrak{s}_t) = E_t \left[ M_{t+1} \left( \frac{P_{t+1}}{D_{t+1}}(\mathfrak{s}_{t+1}) + 1 \right) \frac{D_{t+1}}{D_t} \right] \quad (22)$$

$$= E_t \exp \left( m_{t+1} + \Delta d_{t+1} + \ln \left( \frac{P_{t+1}}{D_{t+1}}(\mathfrak{s}_{t+1}) + 1 \right) \right), \quad (23)$$

where  $\mathfrak{s}_t$  is a vector of state variables,  $\mathfrak{s}_t \equiv (\Delta \mathbf{a}_t, Z_t, x_t)'$ . Even with all shocks normally distributed, there is no closed-form solution to the functional equation (23), due to two sources of nonlinearities on the right-hand-side of (23). First, dividend growth is non-linear in the  $Z_t$  shock. Second, the term  $\ln \left( \frac{P_{t+1}}{D_{t+1}}(\mathfrak{s}_{t+1}) + 1 \right)$  is itself a non-linear function of the shocks. We therefore solve the  $\frac{P_t}{D_t}(\mathfrak{s}_t)$  function numerically on an  $n \times n \times n$  dimensional grid of values for the state variables, replacing the continuous time processes with a discrete Markov approximation following the approach in Rouwenhorst (1995). Further details are given in the Appendix.

### 3.4 The Risk-free Rate, the Sharpe Ratio and Risk Aversion

It is well known that empirical measures of the risk-free interest rate are extremely stable, with a standard deviation one-tenth (or less) the size of that for an aggregate stock market index return or its premium over a risk-free interest rate. The specification for the subjective time discount factor in (16) is essential for obtaining a stable risk-free rate alone with a volatile equity premium. If instead the subjective time discount factor were itself a constant (as is common), shocks to  $x_t$  and dividend growth would generate counterfactual volatility in the risk-free rate. For simplicity, we calibrate the subjective time-discount factor  $\beta_t$  so that the risk-free rate is constant. Specifically, the externality involving the term  $1/E_t \left[ \frac{D_{t+1}^{x_t}}{D_t^{x_t+1}} \right]$  is a compensating factor in the stochastic discount factor that renders the risk-free rate constant even in the face of large shocks to the price of risk. While this compensating factor may seem unusual, it is in fact a generalization of a familiar compensating Jensen's term that appears in many lognormal models of the stochastic discount factor (e.g., Campbell and Cochrane (1999)) and (Lettau and Wachter (2007)).<sup>8</sup> The generalization given here holds for any

<sup>8</sup>Specifically, in the lognormal model of Lettau and Wachter (2007), the stochastic discount factor takes the form

$$M_{t+1} = \exp \left\{ r^f - \frac{1}{2} x_t^2 - x_t \epsilon_{d,t+1} \right\}$$

and the compensating factor is the Jensen's term  $\frac{1}{2} x_t^2$ .

distribution of shocks, not just lognormal. This specification can be understood intuitively by noting that shocks to  $x_t$  generate changes in risk aversion and a desire for precautionary savings that would (absent offsetting movements in the subjective rate of time-preference for saving) produce counterfactually large movements in the risk-free rate. Specifying the variation in  $\beta_t$  as a function of aggregate shareholder consumption, rather than individual shareholder consumption, greatly simplifies the implications for risk aversion in the model, as discussed below.

From (19) we have

$$\begin{aligned} E_t M_{t+1} &= E_t [\exp(\square r_f) \exp(\square \ln E_t \exp(\square x_t \Delta d_{t+1})) \exp(\square x_t \Delta \ln d_{t+1})] \\ &= \frac{\exp(\square r_f) E_t \exp(\square x_t \Delta \ln d_{t+1})}{\exp(\ln E_t \exp(\square x_t \Delta d_{t+1}))} = \exp(\square r_f). \end{aligned}$$

Denoting the gross risk-free rate  $R_f \equiv \exp(r_f)$ , it follows that

$$\begin{aligned} E_t [M_{t+1}]^{\square 1} &= \exp(r_f) = R_f \Rightarrow \\ \square \ln E_t [M_{t+1}] &= r_f. \end{aligned}$$

Thus the log risk-free rate is equal to the constant parameter  $r_f$ .

Define the price of risk as  $\sqrt{\text{Var}_t(M_{t+1})}/E_t(M_{t+1})$ . This variable is equal to the largest possible Sharpe ratio  $E_t \square R_{t+1}^{ex} / \sqrt{\text{Var}_t \square R_{t+1}^{ex}}$  on an asset with excess return  $R_{t+1}^{ex}$  (Hansen and Jagannathan (1991)). Given (19), the maximal Sharpe ratio can be shown to equal

$$\frac{\sqrt{\text{Var}_t[M_{t+1}]}}{E_t[M_{t+1}]} = \frac{\sqrt{\text{Var}_t[\exp(\square x_t \Delta d_{t+1})]}}{E_t \exp(\square x_t \Delta d_{t+1})}. \quad (24)$$

Because of the nonlinearities in this model, there is no simple analytical expression for (24) as a function of the underlying state variables  $\mathfrak{s}_t$ . Nevertheless, (24) shows that the price of risk will vary with innovations in risk aversion  $x_t$  that multiply shocks to dividend growth.

The computation of risk-aversion in the full stochastic model is quite complicated numerically. However, it is straightforward to calculate risk aversion along a non-stochastic balanced growth path. Define the coefficient of relative risk aversion  $RRA_t$  as

$$RRA_t = \frac{\square A_t E_t V''(A_{t+1})}{E_t V'(A_{t+1})}, \quad (25)$$

where  $V(A_{t+1})$  is the representative shareholder's value function associated with optimal consumption choice, and  $A_t$  is this shareholder's asset wealth. Following the derivation

in Swanson (2012), we show in the Appendix that risk aversion along the non-stochastic balanced growth path,  $RRA$ , is equal to

$$RRA = \frac{C_t^s u''(C_{t+1}^s)}{u'(C_{t+1}^s)} = E(x_t) / (1 + \mu_a),$$

where  $E(x_t)$  is the unconditional mean of  $x_t$  and  $1 + \mu_a$  is the non-stochastic gross growth rate of the economy driven by steady state technological progress  $\mathcal{A}_t$ .<sup>9</sup> This shows that risk aversion in the model is a function of the process  $x_t$ , with its steady state value given by the mean of this process. We refer to  $x_t$  as a *risk aversion* shock. Because its innovations are assumed to be uncorrelated with the productivity and factors share shocks, we refer to this as an *exogenous* risk aversion shock.

### 3.5 Calibration

This section discusses the calibration of the model. We begin with a discussion of the calibration of the primitive shocks.

Calibrating the parameters of the stochastic processes for the primitive shocks in the model is complicated by the fact that they are all shocks to latent variables. This is most obvious for risk aversion, for which we have no direct empirical measure, but it is also true of the TFP and factors share shock processes. It is true that estimates of TFP for the United States exist, with those by Fernald (2009) especially sophisticated. But since the consumption shock in the model is driven entirely by TFP whereas consumption in the data is imperfectly correlated with estimates of TFP, it is axiomatic that the latter cannot be capturing precisely the same parameter specified in the model as TFP.<sup>10</sup> One immediate difference is that the TFP shock in the model is orthogonal to the factors share shock, but the Fernald estimate is significantly negatively correlated with a Bureau of Labor Statistics (BLS) labor share measure. For this same reason, the BLS labor share measure is unlikely to be a good proxy for our factor shares shock, which is orthogonal to TFP. The BLS labor share variable also suffers from other potential problems as a measure of factors shares. It is total worker compensation relative to gross domestic product (GDP), rather than total income. Recent research suggests that GDP may be significantly mismeasured and a poor proxy for the income side of the national accounts (see Aruoba, Diebold, Nalewaik, Schorfheide,

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<sup>9</sup>The  $1 + \mu_a$  term in the expression for risk aversion is a nuisance term that arises because of balanced growth and the beginning of period timing assumption for assets.

<sup>10</sup>In fact, the Fernald measure has the same long-term tendencies as our structural consumption shock (see Lettau and Ludvigson (2013)), but there are important differences at short horizons.

and Song (2013)). These observations highlight the extent to which estimates of TFP or labor’s share are not direct observations on the primitive shocks in the model, but are instead estimates based on modeling assumptions that are likely to differ in important ways from those here. We take the approach of choosing parameters of the model so as to match key empirical moments of asset pricing data.

Table 1 presents a list of all parameters and their calibrated values. The parameter  $\alpha$ , the exponent on labor in the production function, is set to 0.667, a value that is standard in real business cycle modeling. The constant value for the quarterly log risk-free rate is set to match the mean of the quarterly log 3-month Treasury bill rate. We set  $\phi_a = 0$  so that the log level of TFP,  $\mathbf{a}_t$ , follows a unit root stochastic process with drift. The mean and standard deviation of productivity  $\Delta \mathbf{a}_t$  is set to roughly match the mean and standard deviation of the quarterly log difference of consumption in the data. The factor share shock  $Z_t$  is set to be very persistent, with  $\phi_z = 0.995$ , to match the extreme persistence of the empirical labor income shock found in the data. The parameter  $\beta$  in  $f(Z_t)$  is set to  $\beta = 0.5$  so  $f(Z_t)$  lies in the interval  $[0.5, 1.5]$  and equals unity in the non-stochastic steady state. The symmetry of the (normal) distribution for  $Z_t$  insures that the mean of the factor’s share shifter is also unity  $E(f(Z_t)) = 1$ . This calibration, along with the calibration of the volatility of  $Z_t$ , allow the model to roughly match the volatility of dividend growth. Finally, the parameters of the risk-aversion process  $\sigma_{\tilde{x}}$ ,  $\phi_{\tilde{x}}$ , and  $E(x_t)$ , are set to roughly match the volatility, persistence, and mean of the equity premium. This requires  $E(x_t) = 32$ , implying a steady state coefficient of relative risk aversion close to 32, a value that is in between the high values of some calibrated habit-based asset pricing models (e.g., Campbell and Cochrane (1999)) and the lower values obtained in models with long-run consumption risk (Campbell (2003), Bansal and Yaron (2004)).

## 4 Results

### 4.1 Model Summary Statistics

Table 2 presents summary asset pricing statistics of the model and compares them to those in post-war data. The average annual excess return on equity in the model exactly matches that in the data, equal to 5.1%, and the model standard deviation closely matches the annualized standard deviation of this return, which is 18.5% in the data and 17.9% in the model. By construction, the model exactly matches the mean risk-free rate. The model also does a good job of matching the evidence the volatility of dividend growth is much higher than that

of consumption growth; the annual standard deviation of log dividend growth is 13.3% in the data and 15% in the model. Because dividends are subject to the factors share shock, they are more volatile than aggregate consumption. The model also matches the mean and standard deviation of the log price-dividend ratio. The model correctly predicts that labor income growth is more volatile than consumption growth, but puts the standard deviation too high: 6% annually compared to 2% in the data. The somewhat higher volatility of labor income shocks is required to match the evidence presented below on the relative role of the factors share shock in stock market fluctuations over longer periods of time.

We investigate the model’s implications for the dynamic relationship between the log price dividend ratio,  $p_t \square d_t$ , and future long horizon excess equity returns,  $\sum_{j=0}^h r_{t+j+1}^{ex}$ , consumption growth,  $\sum_{j=0}^h \Delta c_{t+j+1}$ , and dividend growth,  $\sum_{j=0}^h \Delta d_{t+j+1}$ . Table 3 reports regression results of one through five year log excess equity returns on the lagged price-dividend ratio, as well as one through five year log differences in consumption growth or dividend growth on the lagged price-dividend ratio. The columns on the right present results from our model; the columns on the left present the corresponding results from historical data. The model results are averages across 1000 simulations of length 238 quarters, the same length as our quarterly historical data.

Table 3 shows that the log price-dividend ratio predicts future excess returns with statistically significant negative coefficients in the model, while the coefficients for consumption growth are statistically indistinguishable from zero at all horizons and those for dividend growth are indistinguishable from zero for all but the 5 year horizon. Persistent but stationary variation in risk aversion in the model generates forecastable variation in equity premia but there is no forecastability of consumption or dividend growth. These implications of the model are consistent with the data: the price-dividend ratio exhibits long-horizon predictive power for equity premia, but not consumption growth or dividend growth, as shown in the left-most columns of Table 3. Moreover, the adjusted R-squared statistics for forecasting excess returns are comparable between model and data: they range from 0.138 to 0.237 in the model for one to five year horizons, and from 0.068 to 0.188 in the data. Thus the model is consistent with the well known “excess volatility” property of stock market returns, namely that fluctuations in stock market valuation ratios are informative about future equity risk premia, but not about future fundamentals on the stock market (i.e., dividend or earnings growth, LeRoy and Porter (1981), Shiller (1981)), or future consumption growth (Lettau and Ludvigson (2001); Lettau and Ludvigson (2004)).

## 4.2 Comparing the Model Primitive Shocks to the Model VAR shocks

We next investigate the connection in the model between the observable VAR shocks and the latent primitive shocks. To do so, we take model simulated data, compute the VAR shocks and compare them to the primitive shocks. It should be emphasized that we do not claim to uncover mutually uncorrelated VAR shocks  $(e_{c,t}, e_{y,t}, e_{a,t})$  that exactly equal the primitive shocks  $(\varepsilon_{a,t}, \varepsilon_{z,t}, \varepsilon_{\tilde{x},t})$ , respectively. Exact equality is impossible because the endogenous variables in the model are nonlinear functions of the primitive shocks, while the VAR imposes a linear relation between these variables and the VAR shocks. We will show, however, that the VAR shocks are an extremely close approximation of the theoretical primitive shocks in the model considered here, and are for all practical purposes the observable empirical counterparts to the primitive shocks. The purpose of this exercise is to show that, if the model generated the data, the observable VAR shocks would (to a very close approximation), reveal the latent primitive shocks.

Figure 1 shows two sets of cumulative dynamic responses of  $\Delta c_t$ ,  $\Delta a_t$ , and  $\Delta y_t$ . The left column shows the cumulative responses of these variables to the three primitive shocks in the model. The top panel shows the responses of these variables to the TFP shock, the middle panel shows the responses to the factors share shock, and the bottom panel shows the responses to the risk aversion shock. These responses are calculated by applying, for each shock one at a time, an increase at time  $t = 0$  and a zero value at all subsequent periods, and then simulating forward using the solved policy functions with the other two shocks set to zero in every period. The right column uses model simulated data to calculate the mutually orthogonal VAR innovations (3) that we estimate on historical data and shows responses to one standard deviation increases in each structural  $\mathbf{e}_t$  shock. The top right panel shows the responses of consumption, labor income and wealth to a consumption shock,  $e_{c,t}$ , the middle right panel shows the responses to a labor income shock,  $e_{y,t}$ , and the bottom right panel shows the responses to a wealth shock,  $e_{a,t}$ .<sup>11</sup>

The key result shown in Figure 1 is that the dynamic responses of aggregate consumption, labor earnings, and asset wealth to the VAR innovations in the right column are virtually identical to the theoretical responses of the same variables to the productivity, factors share, and risk aversion shocks, respectively, in the left column. The small deviations that do exist

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<sup>11</sup>Since we compare the model-based VAR responses to the in-population theoretical responses to primitive shocks, we rid the VAR responses of small sample estimation biases by computing them from a single simulation of the model with very long length (238,000 quarters). The size of the primitive shocks are normalized so that they are the same as the empirical shocks in the right column.

from perfect correlation for some responses are attributable to small numerical errors and to nonlinearities in the model not captured by the linear VAR, as discussed above. But these deviations are small. The responses of  $\Delta c_t$ ,  $\Delta y_t$  and  $\Delta a_t$  to the consumption shock,  $e_{c,t}$ , are all perfectly correlated with the responses of these variables to the TFP shock  $\varepsilon_{a,t}$ ; the response of  $\Delta c_t$  to the labor income shock  $e_{y,t}$  is perfectly correlated with the response of  $\Delta c_t$  to the factors share shock  $\varepsilon_{z,t}$ , and the responses of  $\Delta c_t$ ,  $\Delta y_t$ , and  $\Delta a_t$  to the wealth shock  $e_{a,t}$  are all perfectly correlated with the responses of  $\Delta c_t$ ,  $\Delta y_t$ , and  $\Delta a_t$  to the risk aversion shock  $\varepsilon_{\tilde{x},t}$ . The responses of  $\Delta y_t$  and  $\Delta a_t$  to the labor income shock  $e_{y,t}$  have correlations equal to 0.991 and 0.956, respectively, with the responses of these variables to the factors share shock  $\varepsilon_{z,t}$ . Separately we verify, from a long simulation of the model, that the correlation between the consumption shock  $e_{c,t}$  and the productivity shock  $\varepsilon_{a,t}$  is unity, the correlation between labor income shock  $e_{y,t}$  and first difference of the factors share shifter  $\Delta \ln f(Z_t)$  is unity, and the correlation between the wealth shock  $e_{a,t}$  and the innovation in  $\Delta a_t$  attributable only to risk aversion shocks  $\varepsilon_{\tilde{x},t}$  is 0.97.<sup>12</sup> These findings underscore the close connection between the latent primitive shocks in the model and the observable VAR shocks. The properties of these model-based VAR responses may be compared to those using historical data and are discussed in the next section.

### 4.3 The Role of the Empirical Shocks in Quarterly Stock Market Fluctuations

We now study the role of the empirical shocks for historical stock market data. In each investigation, we compare the outcomes for stock market wealth using historical data with those using model-simulated data. We begin by studying the dynamic responses to shocks.

#### 4.3.1 Impulse Responses

Figure 2 again shows two sets of cumulative dynamic responses of  $\Delta c_t$ ,  $\Delta a_t$ , and  $\Delta y_t$ . The left column uses model simulated data to calculate model-based responses to the mutually orthogonal VAR innovations (3). These are the same responses that are shown in the right panel of Figure 1, except that the responses in Figure 2 are averages across 1000 samples of size 238 quarters, rather than over one very long sample. The right column of Figure 2 shows the cumulative dynamic responses of  $\Delta c_t$ ,  $\Delta a_t$ , and  $\Delta y_t$  in historical data to the VAR innovations (3) estimated from historical data. In each column, the top panel shows the responses of consumption, labor income and wealth to an  $e_c$  shock, the middle right panel

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<sup>12</sup>The innovation in  $\Delta a_t$  attributable to risk aversion shocks is computed as  $\Delta a_t \square E[\Delta a_t | \mathfrak{s}_{t \square 1}, Z_t, \Delta \mathbf{a}_t]$ .



shows the responses to an  $e_y$  shock, and the bottom right panel shows the responses to an  $e_a$  shock.

Figure 2 shows that the cumulative dynamic responses  $\Delta c_t$ ,  $\Delta a_t$ , and  $\Delta y_t$  to the observable VAR shocks are remarkably similar across model and data. These in turn are reasonably comparable to the responses of  $\Delta c_t$ ,  $\Delta a_t$ , and  $\Delta y_t$  to the model primitive shocks in Figure 1. In each case, a positive innovation in the consumption  $e_c$  shock leads to an immediate increase in  $c_t$ ,  $a_t$ , and  $y_t$ , both in the data and the model. From Figure 1, it is clear that this shock in the model reveals the effects of the total factor productivity shock. The model responses of  $c$ ,  $y$ , and  $a$ , to the consumption shock lie on top of each other because the levels of these variables are all proportional to TFP, so the log responses are the same. Moreover, in the model the TFP shock is the innovation to a random walk, so all three variables move immediately to a new, permanently higher level. In the data, this adjustment is not exactly immediate but it is relatively quick: full adjustment of consumption occurs within 3 quarters or less, very close to what one would expect from a random walk shock. Cochrane (1994) makes the same observation when studying a bivariate cointegrated VAR for consumption and GNP and argues that consumption is sufficiently close to a random walk so as to effectively define the stochastic trend in GNP.

The second rows of Figure 2 displays the dynamic responses of  $c_t$ ,  $a_t$ , and  $y_t$  to the labor income shock  $e_{y,t}$ , which Figure 1 shows reveals the effects of the factors share shock in the model. In the data, the response of consumption to this shock is economically negligible. This is true by construction on impact as a result of our identifying assumption. But it is also true in all subsequent periods, an important result that is not part of our identifying assumption. Instead, this shock affects asset wealth and labor income, driving  $a_t$  and  $y_t$  in opposite directions. A positive value for this shock raises asset wealth  $a_t$  and lowers labor income  $y_t$ , both of which are moved to new long-run levels. The effect on labor earnings is large and immediate: labor income jumps to a new lower level within the quarter. Below we present evidence that changes in  $a_t$  resulting from this shock are driven by the stock market. The model dynamic responses in the left column show the same basic patterns: the  $e_{y,t}$  shock is one that has no affect on consumption at any horizon, but drives a near-permanent wedge between asset values and labor income.<sup>13</sup> Lettau and Ludvigson (2013) show that this factor

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<sup>13</sup>The model responses in Figure 2 were obtained in the same way as the data responses, namely from a cointegrated VAR that imposes cointegration among  $c_t$ ,  $a_t$ , and  $y_t$  but does not impose a second linearly independent cointegrating relation between  $y_t$  and  $a_t$ . The model shocks to  $y_t \square a_t$  are very persistent (inheriting the persistence of the  $Z_t$  shock), but ultimately stationary. This degree of persistence implies that we cannot reject a unit root for  $y_t \square a_t$  in either the model or the data in samples of the size we currently

shares shock has almost no effect on housing wealth or non-stock market financial wealth, so that it is effectively a shock to *shareholder* wealth.

One puzzling aspect of the data impulse responses exhibited by Figure 2 is that stock market wealth responds sluggishly to the factors share shock, suggesting that the information revealed in the innovation is incorporated only slowly into stock prices. This could reflect a composition effect outside of our model, if for example an increasing fraction of firms going public over the sample employ labor saving technologies. Future research is needed to formally investigate the reasons for this.<sup>14</sup>

The third row of Figure 2 shows the effects of a positive wealth shock,  $e_{a,t}$ , which is driven by a decline in risk aversion in the model. In both the data and the model, this shock leads to a sharp increase in asset wealth, but has no impact on consumption and labor earnings at any future horizon. The zero responses of  $c_t$  and  $y_t$  on impact are the result of our identifying assumptions, but the finding that this shock has no subsequent influence on consumption or labor income at any future horizon is a result. The Appendix describes a bootstrap procedure for computing error bands, and shows that these zero responses are within 90% error bands. By contrast, the effect of this risk aversion shock on  $a_t$  is strongly significant over periods from a quarter to many years. Although transitory, the degree of persistence of this shock is quite high. Its influence on  $a_t$  has a half-life of over 4 years in historical data.

This result is central to understanding why the risk aversion shock must be modeled as exogenous to consumption and labor earnings: the large transitory movements in wealth captured by the  $e_{a,t}$  shock (and which we show below drive forecastable variation in equity risk premia) are unrelated to consumption and labor income at all horizons. The response of  $a_t$  to this shock in the model is of the same pattern, but less persistent than in the data. As in the data, the model-based responses in the left column to the  $e_{a,t}$  shock are driven by the risk aversion shock and have big effects on  $a_t$  but no effect on  $c_t$  or  $y_t$ .

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have. Thus the model is consistent with the data in this respect. Given the extreme persistence of  $y_t \square a_t$ , we follow the advice of Campbell and Perron (1991) to *not* impose stationarity for  $y_t \square a_t$  in the estimated cointegrated VAR for either the model or the data.

<sup>14</sup>The left column of Figure 2 shows that, even in the model, there is slight sluggish response of stock market wealth to the factors share shock, although it is far less pronounced than in the data. Interestingly, this is a finite sample effect: the responses in Figure 2 for the model are averages over 1000 simulations of size 238 quarters (the same size as our historical dataset). In many samples of this size, the response appears sluggish, even though the population response displays no sluggishness. This can be seen via a comparison of Figure 2 with the same response in Figure 1, which, unlike the response in Figure 2, is computed from one very long simulation of the model, rather than from averages over many short ones. The response of  $a_t$  to a  $y$  shock in Figure 1 displays no sluggishness (middle row, right column).

Figure 2 showed dynamic responses of asset wealth  $a_t$ . Figure 3 shows the cumulative dynamic responses of stock market wealth  $\Delta s_t$  in historical data to a one-standard deviation innovation in each VAR disturbance, along with 90% error bands computed from the bootstrap procedure described in the Appendix. The responses are constructed using the OLS estimates of (4) for stock wealth or (5) for CRSP stock price  $\Delta p_t$ . It is clear that the responses of stock wealth or stock price to the wealth and labor income shock mimic those of asset wealth to these same shocks, indicating that they are primarily shocks to *shareholder* wealth, not other forms of wealth. This is consistent with the evidence in Lettau and Ludvigson (2013) which finds that other forms of wealth are not closely related to these wealth shocks. To avoid clutter, the responses for the CRSP stock price  $\Delta p_t$  using (5) are not reported, but we confirm that they are very similar to those for stock wealth.

### 4.3.2 Variance Decompositions

The impulse responses tell us about the dynamic effects of each shock on stock market wealth, but are less informative about the quantitative importance of each shock. Using (4), we characterize the relative quantitative importance of each empirical disturbance, as well as that of the residual, for quarterly stock market wealth using a variance decomposition. Table 4 displays the fraction of the unconditional variance in the log difference of stock market wealth that is attributable to each empirical shock. The variance decompositions in the data are also computed for the log difference in the CRSP value-weighted stock price index using (5). The model-based variance decompositions are computed by estimating (4) on simulated data and reporting average statistics over 1000 simulations of size 238 quarters.

Table 4 shows that the wealth shock  $e_a$  explains the largest fraction of quarterly stock wealth growth  $\Delta s_t$  and accounts for 76% of its quarterly volatility (75% of the quarterly variation in  $\Delta p_t$ ). The two other shocks account for very small amounts, 4% and 6% for the labor income and consumption shocks, respectively. Note that the residual explains only 13% of quarterly stock market fluctuations, showing that the vast majority of quarterly stock market fluctuations are explained by these three shocks.

The model is broadly comparable with the data along these lines: the vast bulk of quarterly fluctuations in stock wealth in the model are attributable to the wealth/risk aversion shock, with much smaller roles for the consumption/TFP and labor income/factors share shocks. The model gets the role of the labor income shock almost exactly right, but somewhat overstates the role of the wealth shock and understates the role of the consumption shock. It is important to bear in mind that, by construction, there is no “residual” in the

model version of equation (4), since the productivity, factors share, and risk aversion shocks explain 100% of the variability in stock market wealth in the model. Since there is no residual in the model but there is in the data, the variance decomposition in the model must differ from the data somewhere. It is encouraging that the residual accounts for less than 15% of total variation in stock market wealth in the data, implying that the model implications for the sources of stock market fluctuations are reasonably correct.

#### 4.4 Short- Versus Long-Term Determinants of Stock Market Wealth

Above we examined the role of each shock for quarterly stock market growth. A key question this paper seeks to address is how the sources of stock market fluctuation might vary by the time frame over which a change in stock market wealth is measured. This section examines the short- versus long-term determinants of stock market wealth in two ways.

##### 4.4.1 Frequency Decomposition

One way to examine how the stock wealth is affected at different time horizons is to decompose the variance of the stock wealth by frequency, using a spectral decomposition. This decomposition tells us what proportion of sample variance in  $\Delta s_t$  is attributable to cycles of different lengths. We estimate the population spectrum for the deterministically detrended log difference in stock wealth  $\Delta s_t \square \kappa_0$  or stock price  $\Delta p_t \square \lambda_0$  using (4) or (5). Noting that  $\Delta s_t \square \kappa_0$  in (6) is a function of three components,  $\Delta s_t^c$ ,  $\Delta s_t^y$ ,  $\Delta s_t^a$ , plus an i.i.d. residual  $\eta_t$ , and using the fact that the spectrum of the sum is the sum of the spectra, we estimate the fraction of the total variance in stock market wealth that is attributable to each component at cycles of different lengths, in quarters. This is computed as the estimated spectrum for the component, e.g.,  $\Delta s_t^c$ , divided by the estimated spectrum for  $\Delta s_t \square \kappa_0$ . The Appendix provides additional details. This decomposition is potentially quite informative about relative importance of different sources of variation for the long-run evolution of stock market wealth.

Figure 4 exhibits these decompositions for the model (top panel), and for the data using stock market wealth (middle panel) or stock price (bottom panel). The horizontal axis shows the length of the cycle in quarters. The vertical axis gives the frequency decomposition of variance. Consider the line marked “a” in the middle panel for historical stock market wealth. This line shows that, for short cycles (i.e., periods of a few quarters), the fraction of variance in stock wealth that is attributable to the exogenous risk aversion/wealth shock

is very high, close to 80%. The high frequency, short-horizon variability of the stock market in post-war data is dominated by exogenous shocks to investors' willingness to bear risk. As these cycles become longer, the fraction of variance in stock wealth explained by this shock declines and asymptotes to roughly 40%. Thus the lower frequency, longer horizon variability of the stock market is less dominated by such shocks.

Next consider the line marked “*y*” in the middle panel of Figure 4. This line shows that, for short cycles of a few quarters, the fraction of variance in stock wealth that is attributable to the factors share/labor income shock is very low, close to zero. Thus the high frequency, short horizon, variability of the stock market in post-war data is virtually unrelated to these factors share shocks. But because these innovations are nearly permanent, they play an increasingly important role as the time horizon extends. Figure 4 shows that as the cycle become longer, the fraction of variance in stock wealth explained by the factors share shock steadily rises and asymptotes to roughly 40%, equal in importance to the risk aversion shock at long horizons. Thus the lower frequency, long horizon, variability of the stock market is quite significantly effected by the factors share shock, even though these shocks play virtually not role in quarterly or even annual stock market fluctuations.

Finally in this middle panel, consider the line marked “*c*.” This line shows that, no matter what the length of the cycle, the fraction of variance in stock wealth that is attributable to the TFP/consumption shock is very low, close to zero. The line marked “residual” shows the contribution of component of stock market fluctuations that is unexplained by these three mutually orthogonal innovations. This residual explains less than 20% of the variability in the stock market at all frequencies, and asymptotes to around 10% as the horizon extends. There are no important differences in these results if we instead use stock price in place of stock market wealth (bottom panel).

The model captures this frequency decomposition well. The top panel of Figure 4 shows model-based orthogonal wealth shocks that originate from exogenous shifts in investors' willingness to bear risk explain almost all of the variation in stock market fluctuations over short horizons and less as the horizon extends, consistent with the data. The model-based labor income/factors share shock explains a negligible fraction of variation in the stock market over shorter time horizons, but is equal in importance to the risk aversion shock as the time horizon extends. Indeed these two shocks each explain about 45% of the variation in the stock market over cycles three decades long, a fraction comparable to the 40% found in the data for these shocks. And the model provides an explanation for why the productivity shocks plays such a small role in the stochastic fluctuations of the stock market at all horizons:

Given capital's smaller role in the production process, most gains and losses from total factor productivity shocks accrue to workers, so these shocks are less important for stock market wealth than are the other two sources of variation.

#### 4.4.2 The Level of the Stock Market in the Post-War Period

A second way we investigate the short- versus long-run determinants of stock market wealth is to study the role each shock has played in driving stock market wealth at specific points in our sample. This can be accomplished by examining the levels decomposition of stock market wealth (7) over time. Figure 5 plots the levels decomposition for stock wealth (left column) and stock price (right column) over our sample. The top panels of each column shows the sum of all components, which equals the log level of the variable (stock wealth or stock price) after removing the deterministic trend. The panels below the top panel show the component attributable to the cumulation of the consumption/TFP shocks, the labor/factors share shock, the wealth/risk aversion shock, and the residual. Each component (and the sum) is normalized so that its value in 1952:Q1 (one period prior to the beginning of our sample) is zero.

It is immediately clear from Figure 5 that the TFP component contributes relatively little to the variation in stock market wealth consistently throughout the sample. This component does take a noticeable drop at the end of the sample during and after the recession of 2007-2009, but it is still quite modest compared to the variation in other components. The bottom panel shows that the variation attributable to the unexplained residual is also reasonably small. Instead, the big movers of stock market wealth are the factors share shock and the risk aversion shock. Figure 5 shows that the low frequency movements in the level of stock market wealth are dominated by the cumulative swings in the factor shares component, while shorter-lived peaks and troughs in the stock market have coincided with spikes up or down in the risk aversion component.

The large role of the factors share shock in driving stock market wealth over horizons of many years is particularly striking in this decomposition. Let us consider the last 25 years as an example. The third panel of Figure 5 shows that the cumulative effect of the factors share shock has persistently boosted stock market wealth over this time. From the dynamic responses in Figure 2, it's clear that these very shocks also persistently lowered the level of labor earnings over this period.<sup>15</sup> By contrast, from the mid 1960s to the mid 1980s, the

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<sup>15</sup>Lettau and Ludvigson (2013) study the implications of this shock for labor income and show that there is a stark inverse relationship over time between labor earnings and the stock market that is the result of

cumulative effect of this shock persistently boosted labor earnings and lowered stock market wealth. The large low frequency swings in the stock market over the sample are exactly mirrored in the low frequency swings of the factors share component.

As an example of the quantitative importance of such shocks over long-horizons, we use this levels decomposition to calculate the percentage change since 1980 in the deterministically detrended real value of stock market wealth that is attributable to each shock. This is trivial to compute once we have (7), because the change in the total value of the stock market on the left-hand-side of is the sum of three structural components and a residual on the right-hand-side, all of which are orthogonal to one another. As mentioned, the period since 1980 is an interesting one to study, given the large cumulative effects of the factor shares shock on stock market wealth and labor earnings over this time. The decomposition (7) implies that the cumulative affects of the factors share shock, which persistently boosted stock market wealth at the expense of labor earnings over this period, have resulted in a 65% increase in the deterministically detrended real value of the stock market since 1980, an amount that exceeds 100% of the total increase. (Precisely, these shocks account for 110.5% of the increase.) Indeed, without these shocks, today's stock market would be about 10% lower than it was in 1980. The result provides a timely example of how small but near-permanent shocks to cash-flows can dramatically affect real stock market wealth over long time horizons. It also underscores the extent to which the long-term value of the stock market has been far more influenced by forces that redistribute the rewards of production, rather than by forces that raise or lower the total amount of rewards.

The levels decomposition implies that an additional 38% of the increase in stock market wealth since 1980, or a rise of 22%, is attributable to the cumulative effects of risk aversion shocks, which tended to be lower in the last 30 years than in earlier in the post-war period. By contrast, the cumulative affects of TFP shocks have made a negative contribution to change in stock market wealth since 1980, once a deterministic trend is removed. Although the role of this shock is in general quite modest, over this particular period it is non-trivial because its contribution was close to an all-time high (in our sample) in 1980 and close to an all-time low at the end of the sample. This latter observation is a direct consequence of the string of large negative draws for the consumption/productivity shock in the Great Recession years from 2007-2009. Our calculations imply that the stock market would be 22% higher today compared to 1980 had these productivity shocks been zero. These shocks accounted for -38% of the total increase since 1980. The residual accounts for the remaining

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the cumulative reallocative outcomes of the factor shares shock.

-10.5% of the increase.

The findings are very similar if we instead study the real CRSP value-weighted stock price index plotted in the right-hand column of Figure 5. The decomposition implies that 103% of the increase in the deterministically detrended CRSP price index since 1980, or a rise of 59%, is attributable to the cumulative effects of the factors share shock; 34%, or a rise of 19%, is attributable to the cumulative effects of the risk aversion shock, and -22%, or a rise of -12.6%, is attributable to the cumulative effects of the productivity shock. The residual accounts for the remaining -14.5% of the increase.

We close this section by briefly investigating the relationship between the mutually orthogonal VAR innovations and earnings on the Standard & Poor (S&P) 500 stock market index. To do so, we estimate an empirical relationship analogous to (4), but instead of putting the log difference in stock wealth on the left-hand-side, we put log earnings changes on the left-hand-side. Quarterly S&P earnings are known to be noisy.<sup>16</sup> We therefore use long-term earnings growth (four year log changes in real per capita earnings) rather than quarterly log changes.

Figure 6 shows the dynamic response (not cumulated) of the four-year (16 quarter) log difference in real per capita earnings to the  $\mathbf{e}_t$  shocks. The response of earnings to a consumption/TFP shock is positive after a couple of quarters but much smaller than that the response to a factors share shock. A positive factors share shock (one that shifts rewards *toward* shareholders) leads to a much larger jump in long-run earnings that persists for many quarters. One aspect of these results that is not explained by the model is that there is a temporary response of earnings to the wealth/risk aversion shock: it is small but positive and grows over time before falling back again. This result is reminiscent of empirical findings showing that expected stock market returns and expected dividend growth rates are correlated (Lettau and Ludvigson (2005)). The model proposed above has no mechanism for capturing this.

#### 4.5 Why is the Stock Market Predictable?

We now turn to the subject of stock market predictability. A large and well known body of evidence finds that excess stock returns are forecastable over longer horizons, suggesting

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<sup>16</sup>S&P earnings fluctuate with time-variation in the application of accounting principles that affect the definition of some receipts and expenses and the timing with which some receipts and expenses are recorded (e.g., Petrick (2001)). These fluctuations may be strategic and therefore correlated with the economy or stock prices.



that the reward for bearing risk changes over time.<sup>17</sup> But what type of primitive innovation might be responsible for this predictability? Several theories have been put forth to explain this forecastability, including habit formation (Campbell and Cochrane (1999)), or stochastic consumption volatility (Bansal and Yaron (2004)). So far, however, no direct evidence has been found that these mechanisms actually generate the observed predictability. This section provides evidence on this question and shows that the empirical wealth shocks/exogenous risk aversion shocks we identify explain most of the forecastable variation in excess stock market returns found in historical data and associated with common predictor variables such as the price-dividend ratio or the consumption-wealth variable  $cay_t$  (Lettau and Ludvigson (2001)). The results are presented in Table 5, with the top half showing results from historical data, and the bottom half showing results from the model.

Table 5 has several panels. The left panel reports regression results of one through five year log excess equity returns on the lagged price-dividend ratio alone. Moving rightward, the next panel reports regression results of one through five year log excess equity returns on lagged  $cay_t$  alone. We will also discuss the predictability of equity premia by measures of stochastic consumption volatility and uncertainty—the next panel reports regression results of log excess returns on one measure of stochastic consumption volatility. The panel to the right of this one, headed “ $\mathbf{e}_a$  only,” reports regression results of one through five year log excess equity returns on multiple lags of the i.i.d. wealth/risk aversion innovations  $e_{a,t}$ , alone. The table reports the sum of the coefficients on all lags. The next two columns show results when returns are predicted either by the component of the price-dividend ratio that is *unrelated* to the wealth shocks,  $\hat{u}_{pd,t}$ , or by the component of  $cay_t$  that is driven *only* by the wealth shocks. We motivate these regressions further below.

Several points bear noting. First, as previously shown in Table 3, the log price-dividend ratio predicts future excess returns with statistically significant negative coefficients and adjusted  $R$ -squared statistics ranging from 7% at a one year horizon to 19% at a five year horizon. Similarly, Table 5 shows that  $cay_t$  predicts future excess returns with statistically significant positive coefficients in the model and adjusted  $R$ -squared statistics ranging from 7% at a one year horizon to 25% at a five year horizon.

Second, the third panel to the right in Table 5 shows that time-varying consumption volatility has no predictive power for equity premia at any horizon. Consumption volatility is measured as the conditional (on time  $t$  information) expectation of the squared innovation

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<sup>17</sup>For extensive reviews of this evidence see Campbell, Lo, and MacKinlay (1997), Cochrane (2005), Lettau and Van Nieuwerburgh (2008), and Lettau and Ludvigson (2010).

in consumption growth one quarter ahead.<sup>18</sup> These results are robust to using additional lags of the stochastic volatility measure, to using first differences of the stochastic volatility measure, to using GARCH measures of consumption growth volatility, and to using measures of stochastic consumption growth volatility looking out over horizons greater than one quarter. In addition, broad-based measures of *macroeconomic* uncertainty developed in Jurado, Ludvigson, and Ng (2013) also exhibit no forecasting power for equity premia at any horizon. (These results are omitted to conserve space but are available upon request.) And none of these measures display any significant relationship with the empirical wealth shocks  $e_{a,t}$  or their lags. These results provide no evidence that stock return predictability is driven by time-varying second moments of consumption growth or broad-based macroeconomic uncertainty.

Third, by comparison with *pd* or *cay*, lags of the  $e_{a,t}$  wealth shocks exhibit greater forecasting power than either of these variables at every horizon except the five-year horizon. The adjusted  $R$ -squared statistics range from 14% at a one year horizon to 31% at a 3 year horizon before falling back to 18% at a 5 year horizon. A Wald test strongly rejects the hypothesis that the coefficients on the lags of these shocks are jointly zero. Note that a positive innovation for the wealth shock *increases* asset wealth, so the negative coefficients in this forecasting regression imply that increases in wealth holding fixed consumption and labor income are transitory and therefore forecast lower returns in the future.

Fourth, the predictive content for long horizon excess stock market returns contained in the price-dividend ratio and *cay* <sub>$t$</sub>  is virtually subsumed by the information in lags of the wealth shocks at all but very long horizons, while at the longest horizons both variables have independent forecasting power. This can be seen in the two far right panels of Table 5. Denote the fitted residuals from regressions of the price-dividend ratio on the current and lagged values of the  $e_{a,t}$  shock as  $\hat{u}_{pd,t}$ . These residuals give the component of *pd* that is *not* attributable to the wealth shocks. Moreover, denote the component of *cay* <sub>$t$</sub>  that is driven *only* by the wealth shocks as *cay* <sub>$a,t$</sub> .<sup>19</sup> The next to last column shows that, at all horizons except the five year, the *pd* residual components  $\hat{u}_{pd,t}$  have no statistically significant forecasting power for equity premia. This shows that all of the predictive power of the price-

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<sup>18</sup>A stochastic volatility model is used to estimate  $E_t \left( [\Delta \ln C_{t+h} \square E_t [\Delta \ln C_{t+h}]]^2 \right)$ , for different horizons  $h$ . This estimate is taken from Jurado, Ludvigson, and Ng (2013).

<sup>19</sup>This is computed by inserting  $\boldsymbol{\epsilon}_t \equiv \mathbf{H}\mathbf{e}_t$  into (1), using the initial values of  $\mathbf{x}_t$  and  $\Delta\mathbf{x}_t$  in the sample along with the estimated parameter values, and rolling forward (1) to obtain components of  $c_t$ ,  $a_t$ , and  $y_t$ , attributable to each element of  $\mathbf{e}_t$ . These may then be combined with the estimated cointegrating vector  $\boldsymbol{\alpha}$ , to obtain components of *cay* <sub>$t$</sub>  attributable to each shock.

dividend ratio over these horizons is related to the wealth shocks. Moreover, the adjusted  $R$ -square statistics for the regressions on  $\hat{u}_{pd,t}$  are small and much smaller than the than those for the regressions using  $pd_t$  itself. Even though the fitted residual components are statistically significant at the 5 year horizon, the  $R$ -square statistics are much smaller than those regressions for which the wealth shocks are used as predictor variables. There is no horizon at which the wealth shocks are not strongly statistically marginally significant. This indicates that the wealth shocks are still explaining the bulk of predictive power at the 5 year horizon even though the residual components have some predictive power at the this horizon. The results from the model are remarkably similar along this dimension.

A similar conclusion can be drawn from examining the forecasting regressions using  $cay_{a,t}$  (last column, Table 5). These regressions show that the component of  $cay_t$  that is driven solely by the orthogonal wealth shocks,  $e_{a,t}$ , is entirely responsible for the forecasting power of  $cay_t$  over all horizons except the 5 year. This can be observed by noting that the adjusted  $R^2$  statistic is if anything higher when using  $cay_{a,t}$  rather than  $cay_t$  as a predictor variable to forecast equity premia.

Finally, it should also be noted that the wealth/risk aversion shocks we identify are contemporaneously unrelated to stock market dividend growth in our sample: the correlation between  $e_{a,t}$  and  $\Delta d_t$  is 0.06. The contemporaneous correlation with earnings growth is only slightly higher, 0.127. These results provide little evidence that the wealth/risk aversion shocks we identify that are, by construction, uncorrelated with consumption instead originate from shocks to measures of fundamental stock market value such as dividends or earnings.

In summary, the predictive content of common forecasting variables for long horizon excess stock market returns is largely subsumed by the information in a shock to asset values that is orthogonal to consumption and labor income and only very weakly related to stock market fundamentals. Taken together, these results mean that the observable wealth shocks we identify largely explain *why* common predictor variables forecast excess returns. Changes in the reward for bearing stock market risk must be understood as attributable to sources that are largely unrelated to movements in economic fundamentals alone, including aggregate consumption, labor income, measures of uncertainty or stochastic consumption volatility, dividend growth, or earnings growth. These findings are hard to reconcile with models in which time-varying risk premia arise from habits in consumption or stochastic consumption volatility.

## 5 Economic Inequality

The causes and consequences of the upward trend in economic inequality over the last 30 years are hotly debated (see for example, Heathcote, Perri, and Violante (2010)). The model above has strong implications for this debate and indicates that fluctuations in economic inequality should be closely related to movements in the factors share shock. Figure 7 provides suggestive evidence that the cumulative effects of the factors share shock since 1980 may be associated with the observed rise in consumption inequality over this period. Figure 7 plots the consumption Gini coefficient from Heathcote, Perri, and Violante (2010), which uses data from the Consumer Expenditure Survey (CEX) over the period 1980 to 2006. Along with this series, we plot the cumulated factors share shock from the empirical VAR, and the model-implied consumption Gini obtained by feeding the observed sequence of factors share shocks from 1980 to 2006 into the model. (The Appendix gives the mapping between the consumption Gini and the cumulated factors share shocks in the model.) In the model, the consumption Gini is almost perfectly correlated with the cumulated factors share shocks, both of which rise over the 1980-2006 period as rewards shifted away from workers and toward shareholders.<sup>20</sup> This is not surprising since, in the model, all inequality is between group inequality across shareholders and workers, which is driven by the factors share shock. But there is also a striking low frequency correlation shown in Figure 7 between the rise in consumption inequality in the CEX data and the observed cumulated factors share shock, suggesting that the shift in rewards away from workers and toward shareholders over the last thirty years could be a driving force behind the rise in consumption inequality.

## 6 Conclusion

No comprehension of stock market behavior can be complete without understanding the origins of its fluctuations. Surprisingly little research has been devoted to this question. As a consequence, we have only a dimly lit view of why the real value of stock market wealth has evolved to its current level compared to five, or ten, or thirty years ago.

The starting point of this paper is to demonstrate that three mutually orthogonal economic shocks that we measure empirically explain the vast majority of fluctuations in real stock market wealth since the early 1950s. We then propose a model to interpret these

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<sup>20</sup>This calculation makes the (empirically relevant) assumption that equity holders' share of aggregate consumption is greater than their share in the population so that a shift in rewards toward shareholders increases rather than decreases consumption inequality.

shocks and show that they are the observable empirical counterparts to three latent primitive shocks: a total factor productivity shock that benefits both workers and shareholders, a factors share shock that shifts the rewards of production between workers and shareholders without affecting the size of those rewards, and an exogenous risk-aversion shock that shifts the stochastic discount factor pricing equities but is unrelated to aggregate consumption, labor earnings, or measures of fundamental value in the stock market.

Once we remove deterministic growth attributable to technological progress, the model implies that there are two big drivers of stock market wealth over time. One is a discount rate shock driven by fluctuations in investors' willingness to bear risk that is unrelated to real economic activity, including consumption, labor income, stock market dividends and earnings.<sup>21</sup> The other is a cash-flow innovation that redistributes the rewards of production between shareholders and workers with no change aggregate consumption. The exogenous discount rate shock dominates stock market volatility over periods of several quarters and a few years, while the factors share shock plays an increasingly important role as the time horizon extend into decades. Technological progress that raises aggregate consumption and benefits both workers and shareholders plays a small role in historical stock market fluctuations at all horizons.

A particularly striking example of the long-run implications of these economic shocks is provided by examining the period since 1980, a period in which the cumulative effect of the factor shares shock persistently rewarded shareholders at the expense of workers. After removing a deterministic trend, we find that factors share shocks have resulted in a 65% increase in real stock market wealth since 1980, an amount that exceeds 100% of the increase in stock market wealth over this period. Indeed, without these shocks, today's stock market would be about 10% lower than it was in 1980. This finding underscores the potential for small but near-permanent shocks to dramatically affect equity values over long periods of time. It also implies that the *type* of shocks responsible for big historical movements in stock market wealth are not those that raise or lower aggregate rewards, but are instead those that redistribute a given level of rewards between workers and shareholders. Loosely speaking, this can be understood by noting that a unit increase in total factor productivity raises shareholder income by the fraction one minus the labor share, while a dollar transferred from workers raises shareholder income by 100% of the amount transferred.

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<sup>21</sup>One real variable that in the data is related to the wealth shock is investment (Lettau and Ludvigson (2013)). But this is theoretically consistent with a discount rate shock, which should affect the present discounted value of marginal profits and therefore the optimal rate of investment (e.g., Abel (1983); Cochrane (1996)).

Our final results pertain to the question of why equity premia are forecastable over time. We find that most of the predictive content contained in common forecasting variables such as the price-dividend ratio or  $cay_t$  for future equity premia is subsumed by the information in lags of the empirical wealth shocks we identify. The predictability of excess stock market returns must therefore be understood as originating from a shock to the discount rate that is not sparked by a movement in consumption, labor income, stock market earnings or dividends, measures of stochastic consumption volatility, or broad-based macroeconomic uncertainty. These facts are well explained by the model presented above, in which shareholders exhibit changes in their willingness to bear risk that is largely unrelated to the underlying economic fundamentals represented by these variables. These conclusions echo those recently drawn from completely different data (stock market index options), whereby risk price variation responsible for time-varying equity premia is found to be largely unrelated to, and unidentifiable from, the stochastic evolution of risks in the economy (Andersen, Fusari, and Todorov (2013)).

The model presented here is deliberately stylized on the quantity side of the economy, abstracting from capital accumulation and fluctuations in employment. We have taken this approach in order to embed our analysis into an empirically plausible stock market environment. But there is growing evidence of important changes in the labor market over the last 30 years that have coincided with sharp declines in labor’s share of output. These include the permanent disappearance of “middle skill” occupations not accompanied by a commensurate rise in employment elsewhere (Acemoglu (1999); Autor, Katz, and Kearney (2006); Jaimovich and Siu (2013)). Notably, the disappearance of these jobs does not associated with a decline in output but instead reflects progress in technologies that more cheaply substitute for labor (Autor, Levy, and Murnane (2003)). Although the model we have explored here does not have an explicit role for changes in the level of employment, we consider the factors share shifter a reduced-form way of modeling these phenomena for understanding stock market behavior. Our findings imply that these forces for redistribution between shareholders and workers—whatever their cause—have had a profound effect on stock market wealth over longer periods of time and have been an especially important driver in the last thirty years.

## Appendix

### Data Description

#### CONSUMPTION

Consumption is measured as either total personal consumption expenditure or expenditure on nondurables and services, excluding shoes and clothing. The quarterly data are seasonally adjusted at annual rates, in billions of chain-weighted 2005 dollars. The components are chain-weighted together, and this series is scaled up so that the sample mean matches the sample mean of total personal consumption expenditures. Our source is the U.S. Department of Commerce, Bureau of Economic Analysis.

#### LABOR INCOME

Labor income is defined as wages and salaries + transfer payments + employer contributions for employee pensions and insurance - employee contributions for social insurance - taxes. Taxes are defined as [ wages and salaries/(wages and salaries + proprietors' income with IVA and CCADJ + rental income + personal dividends + personal interest income)] times personal current taxes, where IVA is inventory valuation and CCADJ is capital consumption adjustments. The quarterly data are in current dollars. Our source is the Bureau of Economic Analysis.

#### POPULATION

A measure of population is created by dividing real total disposable income by real per capita disposable income. Our source is the Bureau of Economic Analysis.

#### WEALTH

Total wealth is household net worth in billions of current dollars, measured at the end of the period. A break-down of net worth into its major components is given in the table below. Stock market wealth includes direct household holdings, mutual fund holdings, holdings of private and public pension plans, personal trusts, and insurance companies. Nonstock wealth includes tangible/real estate wealth, nonstock financial assets (all deposits, open market paper, U.S. Treasuries and Agency securities, municipal securities, corporate and foreign bonds and mortgages), and also includes ownership of privately traded companies in noncorporate equity, and other. Subtracted off are liabilities, including mortgage loans and loans made under home equity lines of credit and secured by junior liens, installment consumer debt and other. Wealth is measured at the end of the period. A timing convention for wealth is needed because the level of consumption is a flow during the quarter rather than a point-in-time estimate as is wealth (consumption data are time-averaged). If we

think of a given quarter's consumption data as measuring spending at the beginning of the quarter, then wealth for the quarter should be measured at the beginning of the period. If we think of the consumption data as measuring spending at the end of the quarter, then wealth for the quarter should be measured at the end of the period. None of our main findings discussed below (estimates of the cointegrating parameters, error-correction specification, or permanent-transitory decomposition) are sensitive to this timing convention. Given our finding that most of the variation in wealth is not associated with consumption, this timing convention is conservative in that the use of end-of-period wealth produces a higher contemporaneous correlation between consumption growth and wealth growth. Our source is the Board of Governors of the Federal Reserve System. A complete description of these data may be found at <http://www.federalreserve.gov/releases/Z1/Current/>.

#### STOCK PRICE

The stock price is measured as the Center for Research on Securities Pricing (CRSP) value-weighted stock market index covering stocks on the NASDAQ, AMEX, and NYSE.

#### PRICE DEFLATOR

The nominal after-tax labor income and wealth data are deflated by the personal consumption expenditure chain-type deflator (2005=100), seasonally adjusted. In principle, one would like a measure of the price deflator for total flow consumption here. Since this variable is unobservable, we use the total expenditure deflator as a proxy. Our source is the Bureau of Economic Analysis.

### Risk Aversion Along a Balanced Growth Path

The budget constraint for the representative shareholder can be written

$$A_t = \theta_t (P_t + D_t) + B_t \quad (26)$$

$$C_t^s + B_{t+1}q_t + \theta_{t+1}P_t \leq A_t, \quad (27)$$

where  $A_t$  are period  $t$  assets,  $\theta_t$  are shares held in equity,  $P_t$  is the ex-dividend price of these shares,  $B_t$  is the beginning of period value of bonds held, and  $q_t = 1/(1 + R_f)$  is the risk-free rate paid on bonds. Along the non-stochastic balanced growth path, the equity return is equal to the risk-free bond rate. Rewrite (26) as

$$\begin{aligned} A_{t+1} &= P_t \theta_{t+1} \left( \frac{P_{t+1} + D_{t+1}}{P_t} \right) + B_{t+1} \\ &= P_t \theta_{t+1} R_{t+1} + B_{t+1} \Rightarrow \\ P_t \theta_{t+1} &= \frac{A_{t+1}}{R_{t+1}} \square \frac{B_{t+1}}{R_{t+1}}. \end{aligned} \quad (28)$$



Plugging (28) into (27) and evaluating (27) at the equilibrium value of equality, we obtain

$$\begin{aligned}
A_t &= C_t^s + B_{t+1}q_t + \frac{A_{t+1}}{R_{t+1}} \square \frac{B_{t+1}}{R_{t+1}} \\
&= C_t^s + \frac{A_{t+1}}{R_{t+1}} \\
&= C_t^s + \frac{A_{t+1}}{R_f}
\end{aligned}$$

where the last equality follows because  $q_t = 1/R_f = 1/R_{t+1}$  along the equilibrium balanced growth path. Thus we have a beginning of period assets:

$$A_{t+1} = R_f (A_t \square C_t^s)$$

or solving forward

$$A_t = \sum_{i=0}^{\infty} \left( \frac{1}{R_f} \right)^i C_{t+i}^s. \quad (29)$$

The value function is defined

$$V(A_t) = \max_{C_t^s} \{u(C_t^s) + \beta_t E_t V(A_{t+1})\}$$

or using (27)

$$V(A_t) = \max_{B_{t+1}, \theta_{t+1}} \{u(A_t \square B_{t+1}q_t + \theta_{t+1}P_t) + \beta_t E_t V(A_{t+1})\}.$$

Following the derivation in Swanson (2012), the coefficient of relative risk aversion  $RRA_t$  is

$$RRA_t = \frac{\square A_t E_t V''(A_{t+1})}{E_t V'(A_{t+1})}. \quad (30)$$

It can be shown (see below) that

$$V'(A_{t+1}) = u'(\square C_{t+1}^s) \Rightarrow \quad (31)$$

$$V''(A_{t+1}) = u''(\square C_{t+1}^s) \frac{\partial C_{t+1}^{s*}}{\partial A_{t+1}}, \quad (32)$$

where the notation “\*” denotes the shareholder’s optimal choice of  $C_{t+1}^s$ .

Swanson (2012) derives a relative risk aversion coefficient for dynamic models at a non-stochastic steady state or along a balanced growth path. We use  $G = 1 + \mu_a$  to denote steady state growth. Along a balanced growth path where for  $z \in \{A, C^s, D, C, Y\}$  we have

$z_{t+k} = G^k z_t$ , where  $G \in (0, R_f)$  we have the following derivation in this setting. Note that the steady state value of  $\beta_t \equiv \frac{\exp(\square r_f)}{E_t \left[ \frac{D_{t+1}^{\square x_t}}{D_t^{\square x_{t+1}}} \right]}$  is given by

$$\beta = \frac{\exp(\square r_f)}{G^{\bar{x}}},$$

where the mean of  $x_t$  is denoted  $\bar{x}$ . Using the first order condition for optimal consumption choice in steady state:

$$\begin{aligned} u'(C_t^{s*}) &= \beta R_f u''(C_{t+1}^{s*}) \Rightarrow \\ u''(C_t^{s*}) \frac{\partial C_t^{s*}}{\partial A_t} &= \beta R_f u''(C_{t+1}^{s*}) \frac{\partial C_{t+1}^{s*}}{\partial A_t} \Rightarrow \\ \square \bar{x} (C_t^{s*})^{\square \bar{x} \square 1} \frac{\partial C_t^{s*}}{\partial A_t} &= \frac{\exp(\square r_f)}{G^{\bar{x}}} R_f \left[ \square \bar{x} (G C_t^{s*})^{\square \bar{x} \square 1} \right] \frac{\partial C_{t+1}^{s*}}{\partial A_t} \Rightarrow \\ \frac{\partial C_t^{s*}}{\partial A_t} &= G^{\square 1} \frac{\partial C_{t+1}^{s*}}{\partial A_t}. \end{aligned} \quad (33)$$

Applying the same transformation to the first order condition at time  $t + 1$  we have

$$\begin{aligned} u''(C_{t+1}^{s*}) \frac{\partial C_{t+1}^{s*}}{\partial A_t} &= \beta R_f u''(C_{t+2}^{s*}) \frac{\partial C_{t+2}^{s*}}{\partial A_t} \Rightarrow \\ \frac{\partial C_{t+1}^{s*}}{\partial A_t} &= G^{\square 1} \frac{\partial C_{t+2}^{s*}}{\partial A_t} \end{aligned} \quad (34)$$

and combining (33) and (34) we obtain

$$\begin{aligned} \frac{\partial C_t^{s*}}{\partial A_t} &= G^{\square 2} \frac{\partial C_{t+2}^{s*}}{\partial A_t} \Rightarrow \\ \frac{\partial C_{t+2}^{s*}}{\partial A_t} &= G^2 \frac{\partial C_t^{s*}}{\partial A_t}, \end{aligned}$$

and iterating obtain

$$\frac{\partial C_{t+i}^{s*}}{\partial A_t} = G^i \frac{\partial C_t^{s*}}{\partial A_t}.$$

Now differentiate (29) evaluated along the balanced growth path with respect to  $A_t$ :

$$\begin{aligned} 1 &= \sum_{i=0}^{\infty} \left( \frac{1}{R_f} \right)^i \frac{\partial C_{t+i}^{s*}}{\partial A_t} \\ 1 &= \frac{\partial C_t^{s*}}{\partial A_t} \left[ 1 + \frac{G}{R_f} + \left( \frac{G}{R_f} \right)^2 + \left( \frac{G}{R_f} \right)^3 + \dots \right] \\ &= \frac{\partial C_t^{s*}}{\partial A_t} \left( \frac{R_f}{R_f \square G} \right), \end{aligned}$$

implying

$$\frac{\partial C_t^{s*}}{A_t} = \frac{R_f \square G}{R_f}. \quad (35)$$

Assets  $A_t$  along a non-stochastic balanced growth path are

$$\begin{aligned} A_t &\equiv \sum_{i=0}^{\infty} \left( \frac{1}{R_f} \right)^i C_{t+i}^{s*} \\ &= \sum_{i=0}^{\infty} \left( \frac{G}{R_f} \right)^i C_t^{s*} \\ &= \frac{R_f C_t^{s*}}{(R_f \square G)}. \end{aligned} \quad (36)$$

Plugging (31), (32), (35), and (36) into (30), we obtain a value for risk aversion along a balanced growth path equal to

$$RRA_t = \frac{\square C_t^s u''(\square C_{t+1}^s)}{u'(C_{t+1}^s)} = \bar{x}/G.$$

**Derivation of (31).** First-order condition for  $B_{t+1}$ :

$$\square u'(C_t^s) q_t + \beta_t V'(A_{t+1}) \underbrace{\frac{\partial A_{t+1}}{\partial B_{t+1}}}_{=1 \text{ from (26)}} = 0. \quad (37)$$

First-order condition for  $\theta_{t+1}$ :

$$\square u'(C_t^s) P_t + \beta_t V'(A_{t+1}) (P_{t+1} + D_{t+1}) = 0. \quad (38)$$

Differentiate the value function

$$V(A_t) = \max_{B_{t+1}, \theta_{t+1}} \{u(A_t \square B_{t+1} q_t + \theta_{t+1} P_t) + \beta_t E_t V(A_{t+1})\}$$

with respect to  $A_t$ , keeping in mind

$$C_t^s = A_t \square B_{t+1} q_t + \theta_{t+1} P_t$$

and

$$A_{t+1} = \theta_{t+1} (P_{t+1} + D_{t+1}) + B_{t+1}.$$

We have

$$\begin{aligned} V'(A_t) &= u'(C_t^s) + \left[ \square u'(C_t^s) q_t + \beta_t V'(A_{t+1}) \underbrace{\frac{\partial A_{t+1}}{\partial B_{t+1}}}_{=1} \right] \frac{\partial B_{t+1}}{\partial A_t} \\ &\quad + [\square u'(C_t^s) P_t + \beta_t V'(A_{t+1}) (P_{t+1} + D_{t+1})] \frac{\partial \theta_{t+1}}{\partial A_t}. \end{aligned}$$

Evaluating at the optimum using (37) and (38), the terms in brackets are zero, leaving

$$V'(A_t) = u'(C_t^s).$$

## Numerical Solution

The price-dividend ratio satisfies

$$\begin{aligned} \frac{P_t}{D_t}(s_t) &= E_t \left[ M_{t+1} \left( \frac{P_{t+1}}{D_{t+1}}(s_{t+1}) + 1 \right) \frac{D_{t+1}}{D_t} \right] \\ &= E_t \exp \left( m_{t+1} + \Delta d_{t+1} + \ln \left( \frac{P_{t+1}}{D_{t+1}}(s_{t+1}) + 1 \right) \right), \end{aligned}$$

where  $\mathfrak{s}_t$  is a vector of state variables,  $\mathfrak{s}_t \equiv (\Delta \ln a_t, Z_t, x_t)'$ . We therefore solve the function numerically on an  $n \times n \times n$  dimensional grid of values for the state variables, replacing the continuous time processes with a discrete Markov approximation following the approach in Rouwenhorst (1995). The continuous function  $\frac{P_t}{D_t}(\mathfrak{s}_t)$  is then replaced by the  $n \times n \times n$  functions  $\frac{P_t}{D_t}(i, j, k)$ ,  $i, j, k = 1, \dots, N$ , each representing the price-dividend ratio in state  $\Delta \ln a_i$ ,  $Z_j$ , and  $x_k$ , where the functions are defined recursively by

$$\frac{P}{D}(i, j, k) = \sum_{l=1}^n \sum_{m=1}^n \sum_{n=1}^n \pi_{i,l} \pi_{k,n} \pi_{j,m} \exp \left( m(l, m, n) + \Delta d(l, m, n) + \ln \left( \frac{P}{D}(l, m, n) + 1 \right) \right),$$

where  $m(l, m, n)$  refers to the values  $m_{t+1}$  can take on in each of the states, and analogously for the other terms. We set  $N = 50$ .

## Estimating Population Spectrum for the Level of Stock Market Wealth

Here we discuss the level decomposition of variance based on a spectral decomposition. The reference for this procedure is Hamilton (1994), chapter 6. The procedure may be summarized as follows. First, we estimate (4) and plug the estimated parameters into formulas for the population spectrum for each component in (6)  $\Delta s_t^c$ ,  $\Delta s_t^y$ ,  $\Delta s_t^a$ , and i.i.d. residual  $\eta_t$ . Since  $\Delta s_t \square \kappa_0 = \Delta s_t^c + \Delta s_t^y + \Delta s_t^a + \eta_t$ , the sum of the estimated spectra for each component gives the estimated spectrum for  $\Delta s_t \square \kappa_0$ , denoted  $S_{\Delta s}(\omega)$  as a function of cycles of frequency  $\omega$ . Notice that we remove the deterministic trend from the log level of stock market wealth by subtracting  $\kappa_0$  from  $\Delta s_t$  on the right-hand-side. Thus we have  $S_{\Delta s}(\omega) = S_{\Delta s^c}(\omega) + S_{\Delta s^y}(\omega) + S_{\Delta s^a}(\omega) + S_{\eta}(\omega)$ , where these right hand terms are the spectra for the individual components of  $\Delta s_t \square \kappa_0$ . Roughly speaking, the proportion of sample variance in  $\Delta s_t \square \kappa_0$  attributable to cycles with frequency near  $\omega$  is given by  $S_{\Delta s}(\omega) 4\pi/T$ ,

where  $T$  is the sample size. The fraction of the variance in the  $\Delta s_t \square \kappa_0$  at cycles with frequency near  $\omega$  that is attributable to the consumption shock is

$$\frac{S_{\Delta s^c}(\omega)}{S_{\Delta s}(\omega)}, \quad (39)$$

and fraction of the variance in the  $\Delta s_t \square \kappa_0$  at cycles with frequency  $\omega$  that is attributable to the other components are defined analogously. Recalling that, if the frequency of the cycle is  $\omega$ , the period of the cycle is  $2\pi/\omega$ . Thus we plot (39), which is a function of frequencies  $\omega_j = 2\pi j/T$ , against periods  $2\pi/\omega_j = T/j$  (here in units of quarters), where  $T$  is the sample size.

### Bootstrap Procedure for Error Bands

Confidence intervals for parameters of interest are generated from a bootstrap following Gonzalo and Ng (2001). The procedure is as follows. First, the cointegrating vector is estimated, and conditional on this estimate, the remaining parameters of the VECM and subsequent regressions are estimated. The fitted residuals from the system

$$\begin{aligned} \Delta \mathbf{x}_t &= \hat{\nu} + \hat{\alpha}' \mathbf{x}_{t \square 1} + \hat{\square}(L) \Delta \mathbf{x}_{t \square 1} + \hat{H} e_t \\ \Delta s_t &= \hat{\kappa}_0 + \hat{\kappa}_c(L) e_{c,t} + \hat{\kappa}_y(L) e_{y,t} + \hat{\kappa}_a(L) e_{a,t} + \eta_t, \end{aligned} \quad (40)$$

denoted  $(\hat{e}_{c,t}, \hat{e}_{y,t}, \hat{e}_{a,t}, \hat{\eta}_t)$  are obtained and a new sample of data is constructed (conditional on our initial observations  $\mathbf{x}_{\square 1}, \mathbf{x}_0$  and  $s_0$ .) using the initial VECM and stock wealth OLS parameter estimates by random sampling of  $(\hat{e}_{c,t}, \hat{e}_{y,t}, \hat{e}_{a,t}, \hat{\eta}_t)$  with replacement. Denote the new randomly sampled (via block bootstrap) values for the residuals  $(\tilde{e}_{c,t}, \tilde{e}_{y,t}, \tilde{e}_{a,t}, \tilde{\eta}_t)$  for  $t = 1, \dots, T$ . The new bootstrapped sample of observable data,  $(\tilde{\mathbf{x}}_t, \tilde{s}_t)$ , is constructed from

$$\begin{aligned} \Delta \tilde{\mathbf{x}}_t &= \hat{\nu} + \hat{\alpha}' \mathbf{x}_{t \square 1} + \hat{\square}(L) \Delta \mathbf{x}_{t \square 1} + \hat{H} \tilde{e}_t \\ \Delta \tilde{s}_t &= \hat{\kappa}_0 + \hat{\kappa}_c(L) \tilde{e}_{c,t} + \hat{\kappa}_y(L) \tilde{e}_{y,t} + \hat{\kappa}_a(L) \tilde{e}_{a,t} + \tilde{\eta}_t. \end{aligned}$$

Given this new sample of data, all parameters in (40) (as well as the cointegrating coefficients) are re-estimated, and the impulse responses, variance decompositions, and other statistics of interest stored. This is repeated 5,000 times. The empirical 90% confidence intervals are evaluated from these 5,000 samples of the bootstrapped parameters. The bands for the impulse responses in Figure 2 are reported in Figure A.1. The bands are reasonably tight for most responses except for the response of  $a_t$  to a labor income shock  $e_{y,t}$ . However, Figure 3 shows that the response of stock wealth  $s_t$  (rather than net worth) to an  $e_{y,t}$  shock is

estimated much more precisely, reflecting the fact that the factors share shock affects the stock wealth component of net worth almost exclusively, but shows little relation to other forms of wealth included in net worth.

## Consumption Gini

We explain how the model-implied consumption Gini coefficient is computed over the same sample period as in the empirical consumption Gini series of Heathcote, Perri, and Violante (2010). In the model, inequality is entirely attributable to the division of consumption between shareholders and workers (agents in each group are identical, so there is no within-group inequality). The level of inequality in the model is measured, as in the data, using the Gini coefficient for consumption. To calculate inequality in the model, we assume that the fraction of shareholders is smaller than their share of aggregate consumption, so that shareholders consume a disproportionately large fraction of aggregate consumption. This assumption ensures that a shift of income away from workers toward shareholders (i.e., a positive  $\varepsilon_z$  shock) has the effect of *increasing* consumption inequality. The share of aggregate consumption that accrues to workers is  $\alpha f(Z_t)$ . If we denote  $q$  to be the fraction of the population in the shareholder group, then we assume  $q < 1 - \alpha f(z)$  for all  $z$ .

Under these assumptions, the Gini coefficient takes the simple form

$$G = 1 - q - \alpha f(z). \quad (41)$$

To see this, it is helpful to consider Figure A.2, which shows the consumption distribution in the model. The Gini coefficient is defined to be the ratio  $A/(A + B)$ , where  $A$  and  $B$  are the areas of the relevant labeled areas in Figure A.2. The area  $B$  is the sum of the areas of a triangle with base  $1 - q$  and height  $\alpha f(z)$ , a rectangle with base  $q$  and height  $\alpha f(z)$ , and a triangle with base  $q$  and height  $1 - \alpha f(z)$ . Basic geometry then implies that

$$B = \frac{1}{2}(q + \alpha f(z)).$$

$A + B$  is a triangle with base 1 and height 1, so  $A + B = 1/2$ . Combining results, we obtain that

$$A = \frac{1}{2}(1 - q - \alpha f(z)).$$

Since  $G = A/(A + B)$ , this completes the derivation of (41).

Given this form for the Gini coefficient, it is clear that in the model, the Gini coefficient can be determined up to the constant  $q$  given values for  $f(z)$ . In the model, the  $e_y$  shock is

nearly perfectly correlated with  $\Delta \ln f(Z_t)$ , so that

$$\Delta \ln f(Z_t) \simeq b_1 e_{y,t} \tag{42}$$

for some constant  $b_1$ . The constant is estimated by running the relevant regression using long time series simulated from the model.

Using our estimates  $\widehat{e}_{y,t}$  from the empirical VAR, we can now construct an implied series for the Gini coefficient in the model. Note from (41), we have

$$\ln f(Z_t) = \ln f(Z_0) + b_1 \sum_{i=1}^t e_{y,i}. \tag{43}$$

The consumption gini data from Heathcote, Perri, and Violante (2010) are annual and run from 1980 to 2006. We therefore set  $t = 1$  to 1980 and normalize  $\ln f(Z_0)$  to zero. We take the average quarterly value of  $e_{y,t}$  within a year as the annual observation for  $e_{y,t}$ . Iterating forward on (43) and applying the exponential function to the left-hand-side yields an implied series for  $f(z_t)$ . Finally, plugging this value into (41) yields a model-implied series for  $G$ .

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Table 1: Parameter List and Calibration

No.	Parameter	Description	Calibration
1	$\alpha$	exponent on labor in production function	0.667
2	$\psi$	constant term in $f(Z_t)$	0.5
3	$r^f$	log risk-free rate	0.0038
3	$\theta$	maximum value $x_t$	500
4	$\bar{a}$	mean (quarterly) $\Delta \mathbf{a}_t$	0.0057
5	$f(\bar{Z})$	mean $f(Z_t)$	1
6	$E(x_t)$	mean $E(x_t)$ process	32.4
7	$\phi_{\mathbf{a}}$	autocorrelation $\Delta \mathbf{a}_t$	0
8	$\phi_z$	autocorrelation $z_t$	0.995
9	$\phi_{\tilde{x}}$	autocorrelation $\tilde{x}_t$	0.85
10	$\sigma_{\mathbf{a}}$	standard deviation $\varepsilon_{\mathbf{a},t}$	0.0091
11	$\sigma_z$	standard deviation $\varepsilon_{z,t}$	0.145
12	$\sigma_{\tilde{x}}$	standard deviation $\varepsilon_{\tilde{x},t}$	6.01

Table 2: Simulated and Data Moments

Variable	Data Mean	Data St. Dev.	Model Mean	Model St. Dev.
$\Delta c_t$	0.018	0.014	0.023	0.018
$\Delta y_t$	0.021	0.018	0.023	0.060
$\Delta d_t$	0.020	0.133	0.023	0.151
$r_t^e$	0.065	0.190	0.066	0.179
$r_t^f$	0.015	0.022	0.015	0.000
$r_t^{ex}$	0.051	0.185	0.051	0.179
$p_t - d_t$	3.548	0.376	3.287	0.358

Notes:  $\Delta c_t$  is log differences of real consumption.  $\Delta d_t$  is log differences of dividends obtained from CRSP, where the dividend series is extracted as the difference between the cum-dividend and ex-dividend value-weighted returns. Data for  $r_t^e$  are the cum-dividend value-weighted returns from CRSP. Data for  $r_t^f$  are the constant-maturity 1-year rate on treasury bills from FRED.  $r_t^{ex}$  is the difference between the  $r_t^e$  and  $r_t^f$ . Data for  $p_t - d_t$  is obtained from the CRSP data. The variables  $\Delta d_t$ ,  $r_t^f$ ,  $r_t^e$ , and  $r_t^{ex}$  are adjusted for inflation by subtracting the log difference of realized CPI (all urban consumers) obtained from FRED. All variables are at annual frequency. For all variables except  $p_t - d_t$ , annual observations are obtained by summing over monthly or quarterly variables. Annualization for  $p_t - d_t$  is performed by annualizing dividends using the method just described, and then dividing the final price of the year by the annualized dividend. Coefficients that are statistically significant at 5% level appear in bold. The sample is 1953:4 - 2012:9.

Table 3: Long Horizon Predictability Regressions:  $\sum_{j=0}^h y_{t+j+1} = \beta' X_t + \omega_{t+1,t+h}$

$h$	Data				Model					
	$\Delta c_t$	$\bar{R}^2$	$pd_t$	$\Delta d_t$	$r_t^{ex}$	$\Delta c_t$	$\bar{R}^2$	$\Delta d_t$	$r_t^{ex}$	
4	-0.125 (-0.248)	-0.003	-1.202 (-0.502)	0.001	-0.130 (-2.856)	-0.000 (-0.033)	0.011	0.077 (1.378)	0.035 (-3.528)	-0.262 (-3.901)
8	-0.875 (-0.776)	0.013	-3.354 (-0.780)	0.010	-0.226 (-2.777)	-0.000 (-0.039)	0.025	0.145 (1.547)	0.069	-0.363 (-3.901)
12	-1.423 (-0.849)	0.022	-5.497 (-0.857)	0.019	-0.272 (-3.043)	-0.001 (-0.042)	0.036	0.209 (1.716)	0.099	-0.412 (-4.060)
20	-2.719 (-1.072)	0.050	-4.053 (-0.448)	0.004	-0.361 (-4.023)	-0.001 (-0.060)	0.055	0.318 (2.051)	0.152	-0.455 (-4.083)

Notes: Coefficients are reported for each regression. A constant is included in each regression even though it is not reported in the table. Bolded coefficients indicate significance at the 5 percent or better level. Numbers in parentheses are t-statistics. Test statistics are corrected for serial autocorrelation and heteroskedasticity using a Newey-West estimator with 24 lags.  $\bar{R}^2$  is the adjusted  $R^2$  statistic, corrected for the number of regressors. The sample spans 1952:Q2 - 2012:Q3.

Table 4: Variance Decomposition of Quarterly Log difference in Stock Wealth

	<i>c</i> Shock	<i>y</i> Shock	<i>a</i> Shock	Residual
Model	0.014 (0.002, 0.054)	0.039 (0.010, 0.108)	0.947 (0.861, 0.983)	0.000
Data (Stock Wealth)	0.062 (0.042, 0.135)	0.044 (0.024, 0.101)	0.759 (0.690, 0.815)	0.134
Data (Stock Price)	0.060 (0.043, 0.129)	0.042 (0.025, 0.101)	0.743 (0.678, 0.803)	0.155

Notes: See Table 2. This table reports a variance decomposition of the quarterly log difference in stock market wealth using the OLS regressions of the log difference in stock wealth on contemporaneous and lagged mutually orthogonal VAR innovations. The numbers reported represent the fraction of the  $h = \infty$  step-ahead forecast error in the log difference of stock wealth that is attributable to the shock named in the column heading. The last column reports the share of the variance that is attributable to the residual of the regressions. Model results are calculated as averages over 1,000 simulations of 238 observations each. The numbers in parentheses represent the 5th and 95th percentiles of these statistics from bootstrapped samples using the procedure described in the Appendix. For model results, the numbers in parentheses are the 5th and 95th percentiles of the relevant statistic over the 1,000 simulations. The historical sample spans the period 1952:Q2 - 2012:Q4.

Table 5: Long Horizon Return Regressions:  $\sum_{j=0}^h r_{t+j+1}^{cx} = \beta' X_t + \omega_{t+1,t+h}$

DATA												
$h$	$pd$		$cay$		$U_3^C$		$e_a$		$\hat{u}_{pd}$		$cay_a$	
	$pd_t$	$\bar{R}^2$	$cay_t$	$\bar{R}^2$	$U_{3,t}^C$	$\bar{R}^2$	$e_{a,t}$	$\bar{R}^2$	$\hat{e}_{pd,t}$	$\bar{R}^2$	$cay_{a,t}$	$\bar{R}^2$
4	<b>-0.130</b>	0.068	<b>2.722</b>	0.074	-0.047	0.005	<b>-0.401</b>	0.137	-0.083	0.023	<b>-7.357</b>	0.128
	(-2.856)		(2.936)		(-1.080)		(135.490)		(-1.571)		(-3.684)	
8	<b>-0.226</b>	0.120	<b>4.864</b>	0.133	-0.027	-0.003	<b>-0.673</b>	0.262	-0.118	0.029	<b>-13.736</b>	0.261
	(-2.777)		(3.326)		(-0.348)		(209.833)		(-1.363)		(-5.575)	
12	<b>-0.272</b>	0.144	<b>6.667</b>	0.197	-0.013	-0.005	<b>-0.678</b>	0.314	-0.135	0.031	<b>-16.423</b>	0.310
	(-3.043)		(4.098)		(-0.143)		(611.000)		(-1.276)		(-7.052)	
20	<b>-0.361</b>	0.188	<b>9.042</b>	0.253	-0.049	-0.002	<b>-0.536</b>	0.179	<b>-0.216</b>	0.061	<b>-15.142</b>	0.178
	(-4.023)		(4.721)		(-0.403)		(124.327)		(-2.179)		(-3.376)	

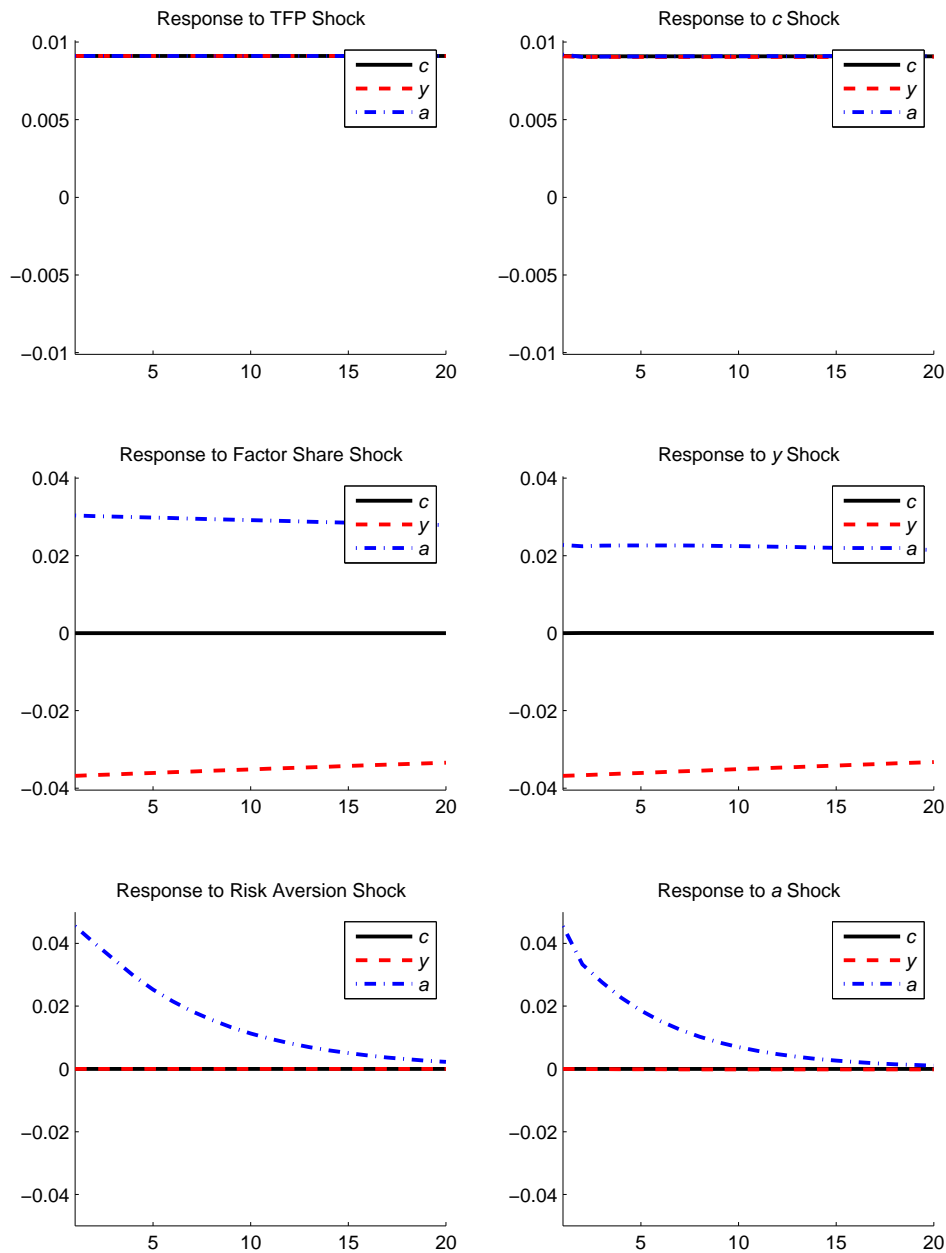
  

MODEL												
$h$	$pd$		$cay$		$U_3^C$		$e_a$		$\hat{u}_{pd}$		$cay_a$	
	$pd_t$	$\bar{R}^2$	$cay_t$	$\bar{R}^2$	$U_{3,t}^C$	$\bar{R}^2$	$e_{a,t}$	$\bar{R}^2$	$\hat{e}_{pd,t}$	$\bar{R}^2$	$cay_{a,t}$	$\bar{R}^2$
4	<b>-0.262</b>	0.138	<b>1.270</b>	0.260			<b>-0.308</b>	0.279	-0.091	0.015	<b>-1.120</b>	0.276
	(-3.728)		(6.162)				(327.230)		(-0.250)		(-6.354)	
8	<b>-0.363</b>	0.192	<b>1.734</b>	0.342			<b>-0.432</b>	0.355	-0.148	0.027	<b>-1.473</b>	0.361
	(-4.046)		(7.043)				(422.588)		(-0.392)		(-7.350)	
12	<b>-0.412</b>	0.217	<b>1.930</b>	0.363			<b>-0.481</b>	0.375	-0.195	0.037	<b>-1.685</b>	0.382
	(-4.159)		(7.052)				(452.639)		(-0.525)		(-7.333)	
20	<b>-0.455</b>	0.237	<b>2.025</b>	0.342			<b>-0.489</b>	0.357	-0.264	0.056	<b>-1.807</b>	0.364
	(-4.131)		(6.294)				(419.722)		(-0.720)		(-6.616)	

Notes: Coefficients are reported for each regression.  $pd$  is the log price-dividend ratio (source: CRSP).  $cay$  is the consumption-wealth variable from Lettau and Ludvigson (2001), while  $cay_a$  is the component of  $cay$  driven by shocks to  $e_{a,t}$ .  $\sigma_t(\Delta c_{t+1} - E_t \Delta c_{t+1})$  is the square root of the conditional expectation of the squared consumption innovation  $\Delta c_{t+1} - E_t \Delta c_{t+1}$ , one quarter ahead, computed using a stochastic volatility model in Jurado, Ludvigson, and Ng (2013). The VAR innovation  $e_{a,t}$  is a shock that raises  $\Delta \alpha_t$ , holding fixed  $\Delta c_t$  and  $\Delta y_t$ . The variable  $\hat{u}_{pd,t}$  is the fitted residual from a regression of  $pd$  on contemporaneous and 19 lagged values of  $e_{a,t}$ . For  $e_{a,t} = (e_{a,t}, \dots, e_{a,t-19})$ , the coefficient reported is the sum of the individual regression coefficients  $\sum_{j=0}^{19} \beta_{e_{a,j}}$ , where  $\beta_{e_{a,j}}$  is the coefficient on  $e_{a,t-j}$ . For the variable  $e_{a,t}$ , the statistic reported in parentheses is a Wald statistic for the null hypothesis that the individual regression coefficients  $\beta_{e_{a,1}}, \dots, \beta_{e_{a,23}}$  are identically zero. For the variables  $pd_t$  and  $cay_t$ , the statistics reported in parentheses are  $t$ -statistics for the null hypothesis that the regression coefficient is zero. A constant is included in each regression even though it is not reported in the table. Bolded coefficients indicate significance at the 5 percent or better level. Test statistics are corrected for serial autocorrelation and heteroskedasticity using a Newey-West estimator with 24 lags.  $\bar{R}^2$  is the adjusted  $R^2$  statistic. For the  $\sigma_t(\Delta c_{t+1} - E_t \Delta c_{t+1})$  regression, the sample spans 1960:Q4 - 2012:Q1. For all other regressions, the sample spans 1952:Q4 - 2012:Q3.

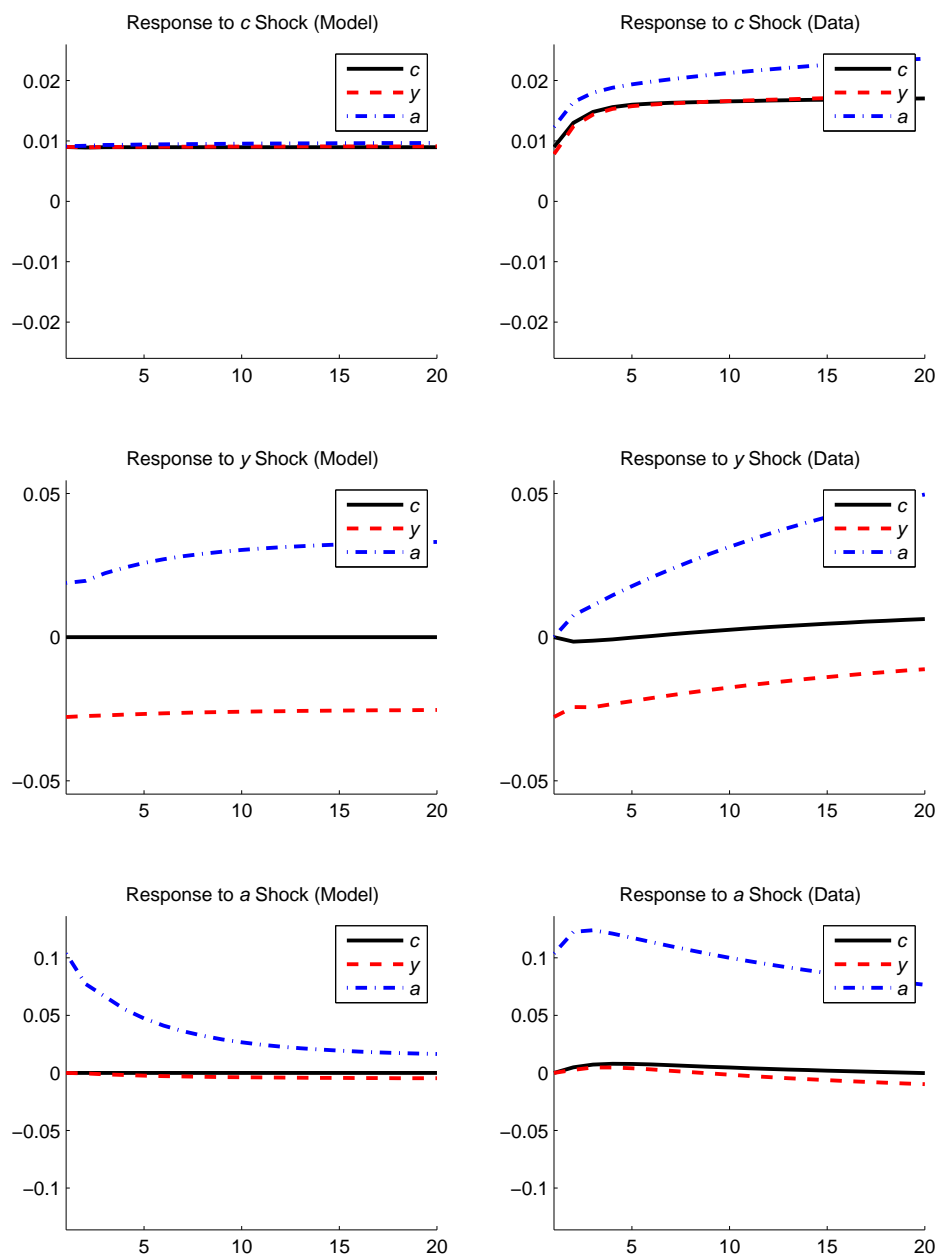


Figure 1: Model-Based Responses: Primitive vs. VAR Shocks



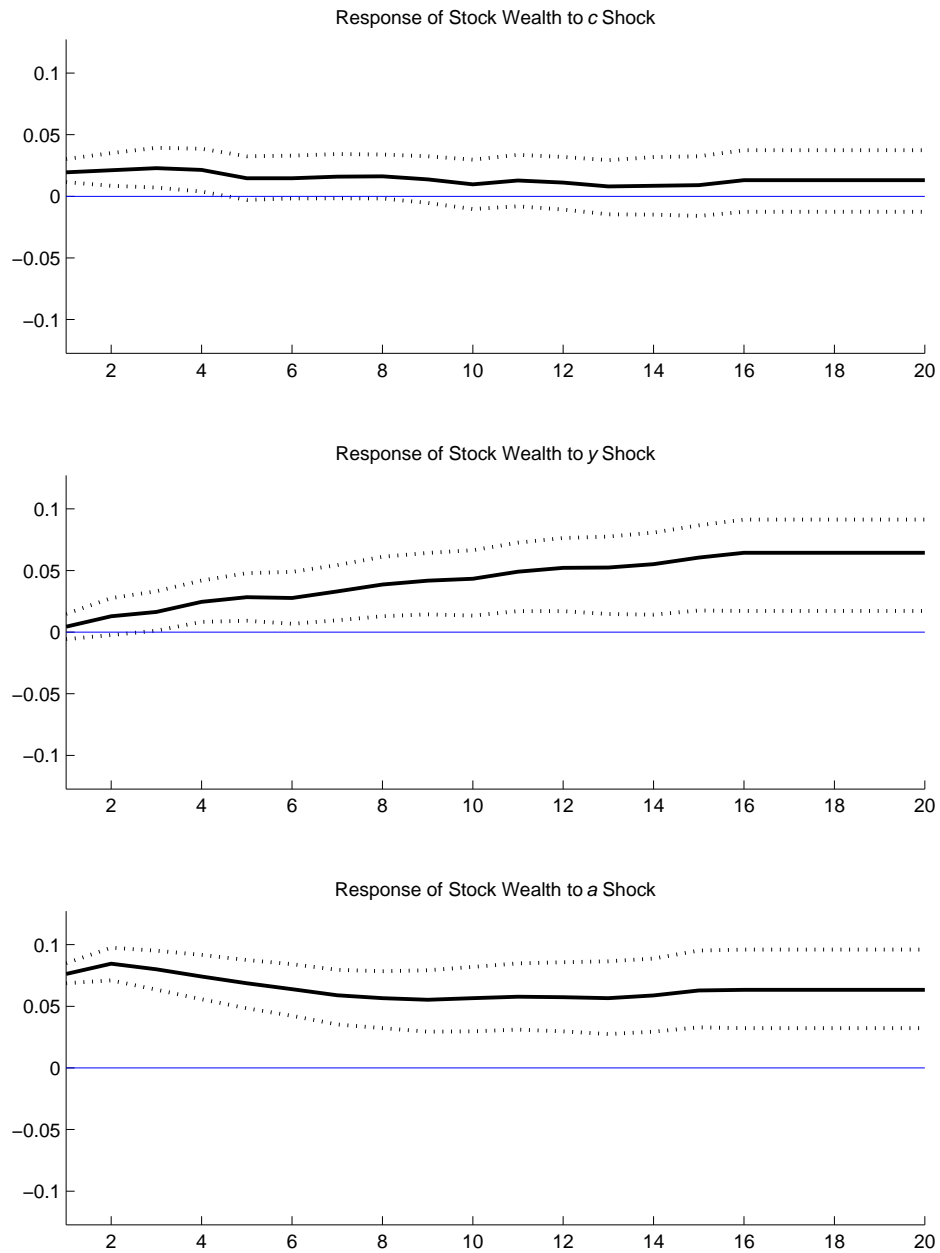
Notes: The left column plots the model impulse responses to the primitive shock named in the sub-graph title. The right column plots the impulse responses implied by the model to the orthogonalized VAR shocks using data simulated from the model (right column). Impulse responses to primitive shocks are obtained by applying a single shock at  $t = 0$ , and simulating the model forward with all other shocks set to zero. Impulse responses to the VAR shocks are obtained by simulating the model over one very long sample (equal to 238,000 observations), estimating a cointegrated VAR, inverting to Wold representation and computing the responses to orthogonalized  $c$ ,  $y$ , and  $a$  shocks with that ordering in the VAR. The size of the shocks are normalized so that the initial response of a variable to its own shock in the right panel is the same as the response of that variable to the corresponding primitive shock in the left panel.

Figure 2: VAR Impulse Responses (Model vs. Data)



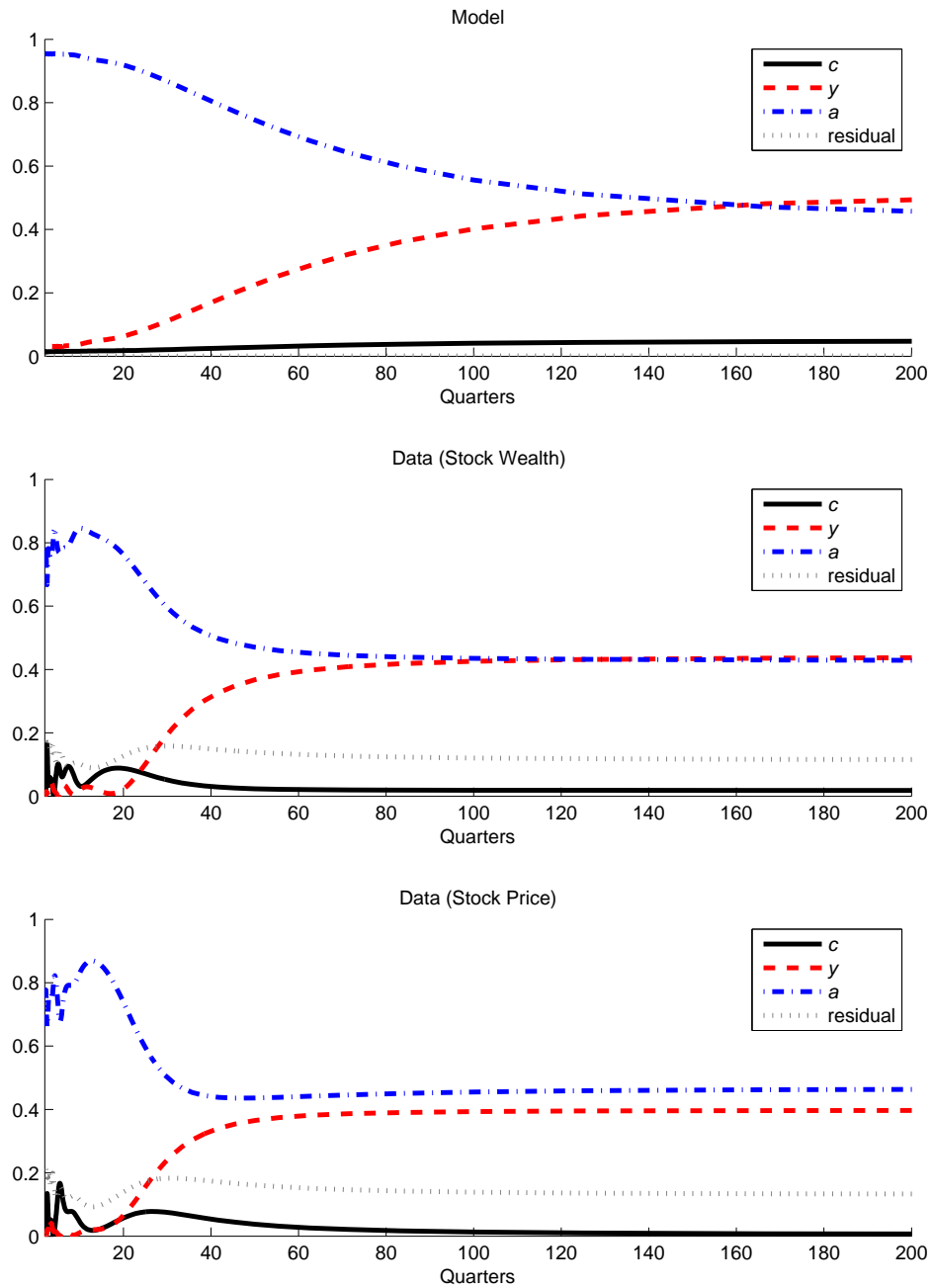
Notes: The figure plots impulse response functions to the VAR shocks in both the model (left column) and data (right column). true structural shocks to the orthogonalized shocks, obtained from the VECM regression using data simulated from the model (left column), and actual data (right column). In both cases, impulse responses are obtained by estimating a cointegrated VAR, inverting to Wold representation and computing the responses to orthogonalized  $c$ ,  $y$ , and  $a$  shocks with that ordering in the VAR. Model results are calculated as averages over 1,000 simulations of 238 observations each. The historical sample spans the period 1952:Q2 - 2012:Q4.

Figure 3: Stock Market Impulse Responses



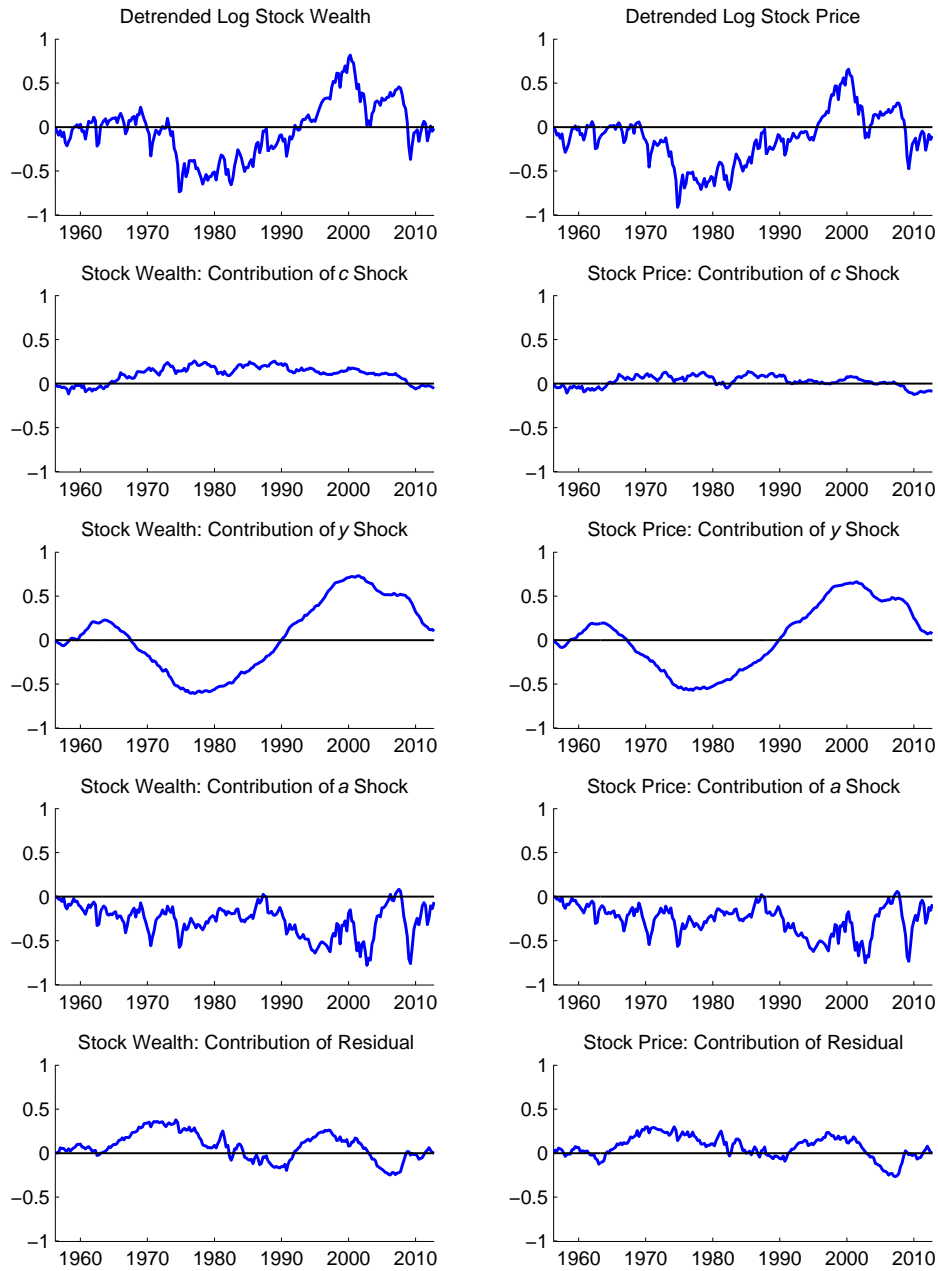
Notes: The figure plots impulse responses of stock wealth to the shocks obtained from the VECM regression. Dotted lines are 90% error bands obtained using the bootstrap procedure described in the Appendix. The historical sample spans the period 1952:Q2 - 2012:Q4.

Figure 4: Decomposition of Spectrum (Model vs. Data)



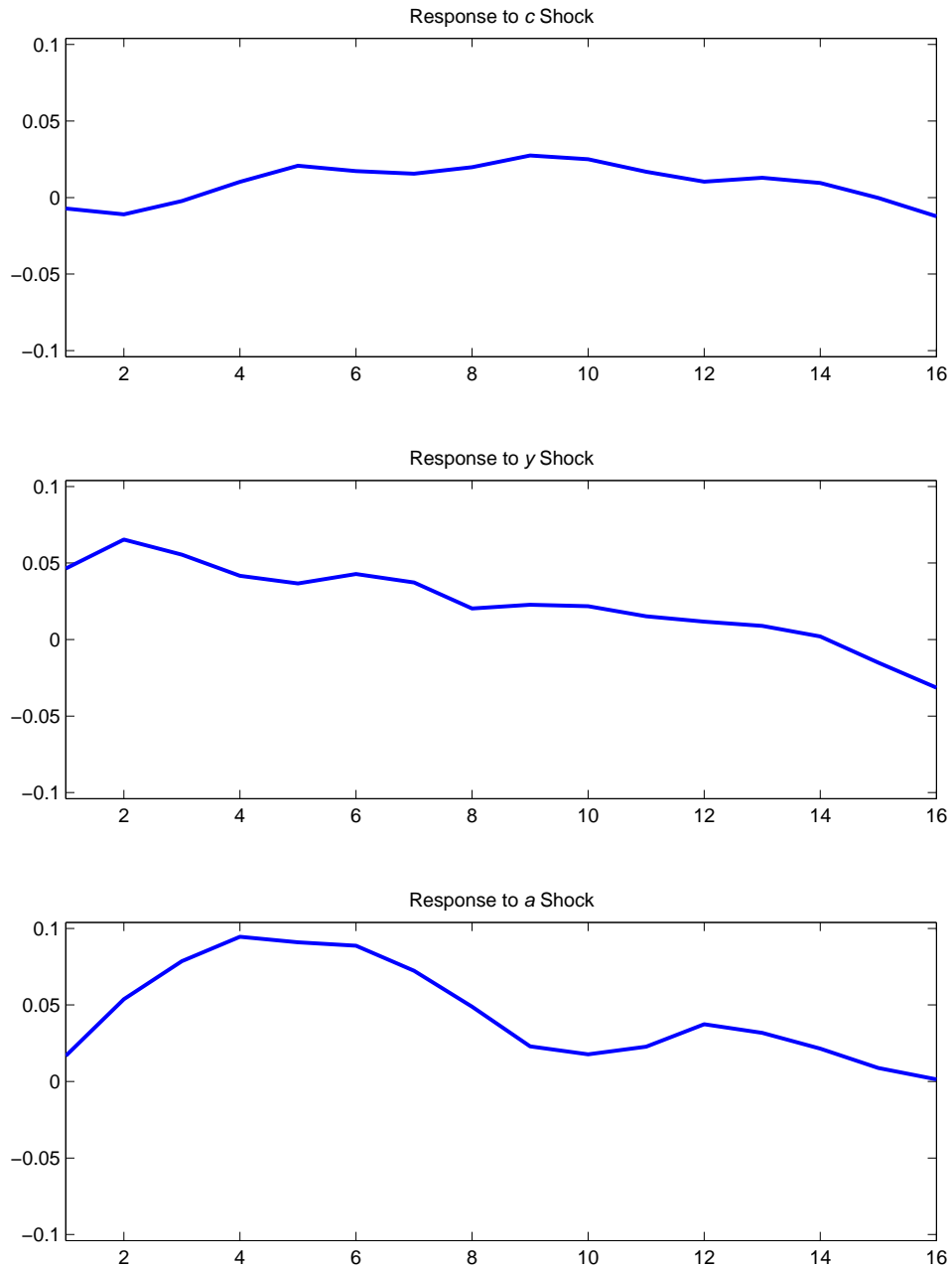
Notes: The figure shows the decomposition of spectra at different frequencies into components driven by each of the orthogonalized shocks of the consumption, labor income and wealth VAR. Results for the model are computed from averages over 1,000 simulations of 238 observations. Results from historical data appear in the two bottom panels. The historical sample spans the period 1952:Q2 - 2012:Q4.

Figure 5: Level Decomposition (Data)



Notes: See Table 2. The figure shows the decomposition of the log level of stock wealth into components driven by the orthogonalized  $c$ ,  $y$ , and  $a$  shocks obtained from the VECM regression. A deterministic trend is removed from the log level of stock wealth by removing the mean in log differences before cumulating. The components plus the residual sum to the log level of detrended stock wealth and detrended stock price in the left and right panels, respectively. Each component and the sum are normalized so that the value in 1952:Q1 is zero. The sample spans the period 1952:Q2 - 2012:Q4.

Figure 6: Earnings Impulse Responses

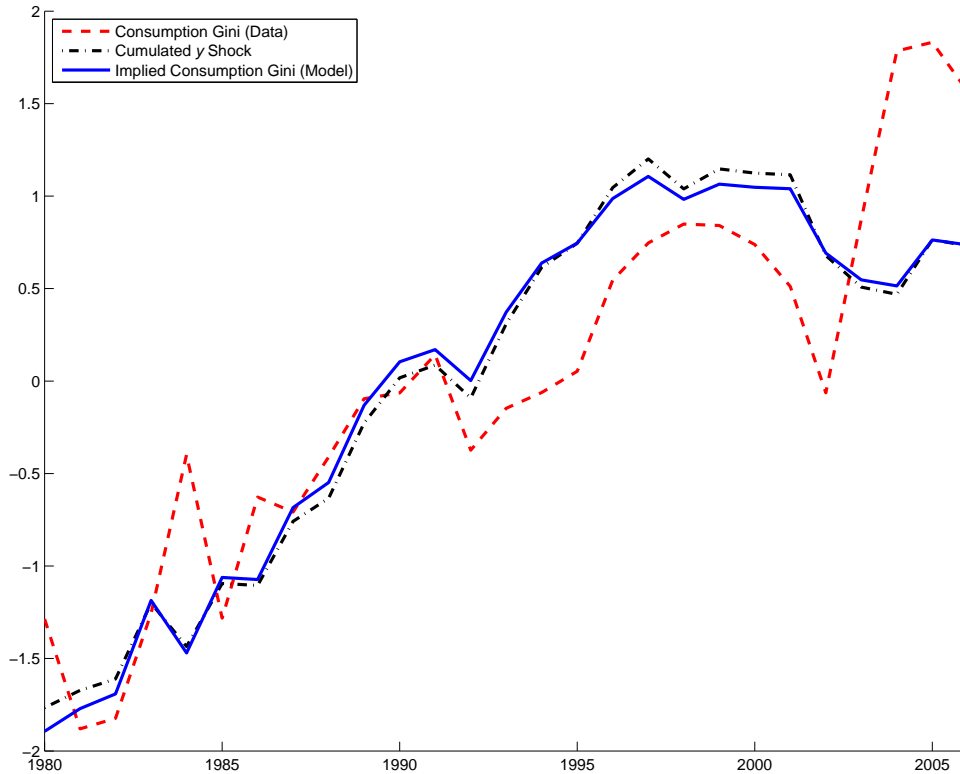


Notes: The figure plots the responses of 16Q real per-capita S&P 500 earnings growth to contemporaneous and lagged values of the shocks obtained from the VECM regression. The IRFs are computed as the coefficients  $A$ ,  $B$ , and  $C$  from the earnings regression

$$\sum_{j=0}^{15} \Delta \ln(EA_{t-15+j}) = \alpha_0 + A(L)e_{c,t} + B(L)e_{y,t} + C(L)e_{a,t} + \nu_t.$$

Each lag polynomial includes sixteen terms total (one contemporaneous and fifteen lag terms).  $EA_t$  is the four quarter moving average of real reported S&P earnings obtained from the website of Robert Shiller. The sample spans the period 1952:Q2 - 2012:Q4.

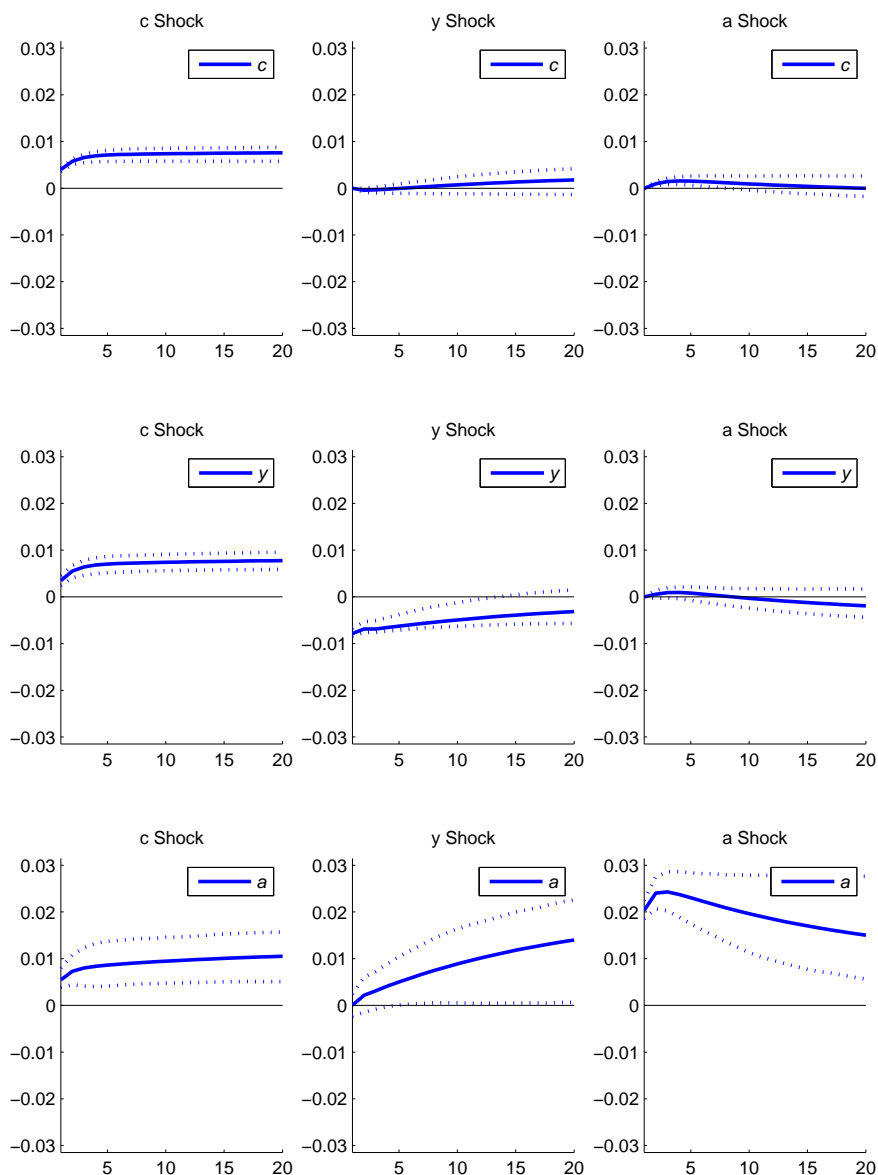
Figure 7: Gini Coefficient and Cumulated  $e_y$  Shock



Notes: The consumption Gini (data) series is the Gini coefficient of inequality for nondurable consumption (source: Heathcote, Perri and Violante (2010)). The cumulated  $y$  shock series is the running total of all the  $e_y$  shocks to date:  $\sum_{j=1}^t e_j$ . The implied consumption Gini (model) series uses the model to calculate the implied Gini coefficient for consumption based on the observed sequence of  $y$  shocks (see appendix for details). All series are presented at annual frequency. For the cumulated  $y$  shock and model-implied Gini series, annual observations are averages over the calendar year. All three series are normalized to have zero mean and unit standard deviation in the sample.

## A Additional Figures

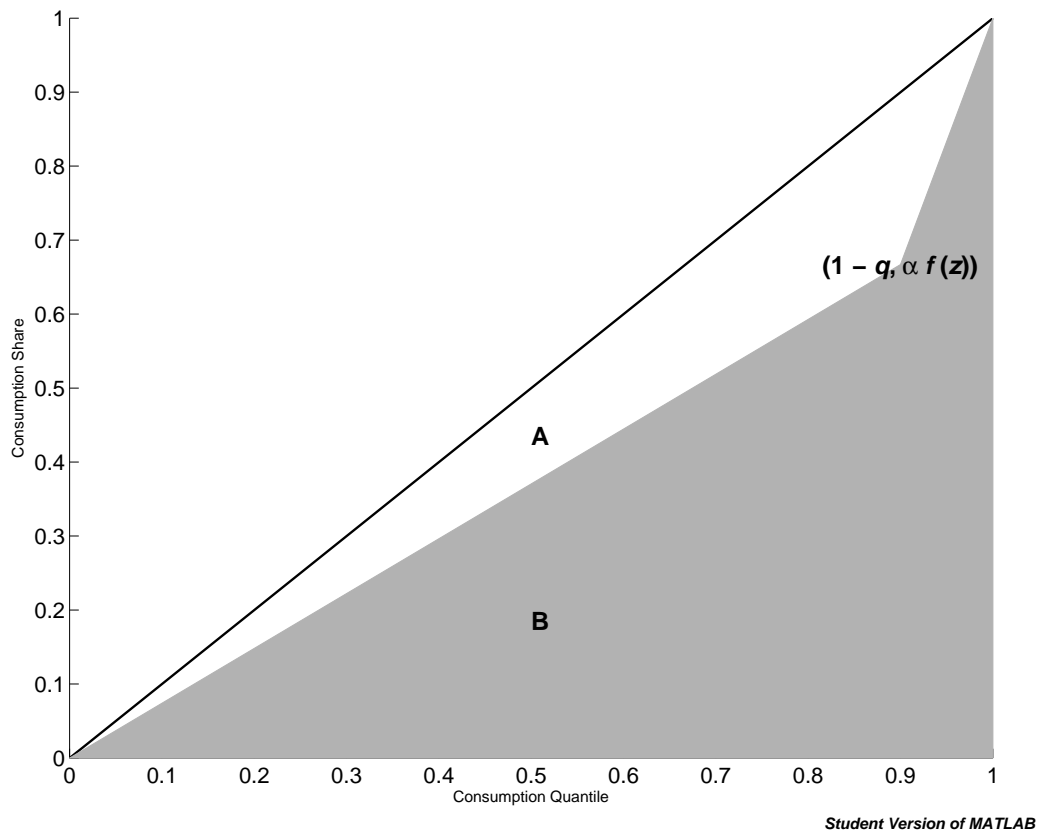
Figure A.1: VAR Impulse Responses with Error Bands



Notes: The figure plots impulse response functions to the VAR shocks obtained from the VECM regression using data. Impulse responses are obtained by estimating a cointegrated VAR in MATLAB to Wold representation and computing the responses to orthogonalized  $c$ ,  $y$ , and  $a$  shocks with that ordering in the VAR. The dotted lines are 90% error bands obtained using the bootstrap procedure described in the Appendix. The historical sample spans the period 1952:Q2 - 2012:Q4.



Figure A.2: Model Gini Coefficient



Notes: This diagram plots the distribution of consumption in the model.  $q$  is the proportion of shareholders. The Gini coefficient of consumption is defined by  $G = A/(A + B)$ , where  $A$  and  $B$  are the areas of the relevant regions.