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ABSTRACT

We construct a microfounded, dynamic version of the IS-LM-Phillips curve model by adding two elements to the money-in-the-utility-function model of Sidrauski (1967). First, real wealth enters the utility function. The resulting Euler equation describes consumption as a decreasing function of the interest rate in steady state–the IS curve. The demand for real money balances describes consumption as an increasing function of the interest rate in steady state–the LM curve. The intersection of the IS and LM curves defines the aggregate demand (AD) curve. Second, matching frictions in the labor market create unemployment. The aggregate supply (AS) curve describes output sold for a given market tightness. Tightness adjusts to equalize AD and AS curve for any price process. With a rigid price process, this steady-state equilibrium captures Keynesian intuitions. Demand and supply shocks affect tightness, unemployment, consumption, and output. Monetary policy affects aggregate demand and can be used for stabilization. Monetary policy is ineffective in a liquidity trap with zero nominal interest rate. In contrast, with a flexible price process, aggregate demand and monetary policy are irrelevant when the nominal interest rate is positive. In a liquidity trap, monetary policy is useful if it can increase inflation. We discuss equilibrium dynamics under a Phillips curve describing the slow adjustment of prices to their flexible level in the long run.

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1 Introduction

This paper constructs a microfounded, dynamic version of the IS-LM-Phillips curve model. Our model also includes unemployment, which is absent from the original IS-LM-Phillips curve model. We use our model to study the effect of macroeconomic shocks and monetary policy on unemployment. In particular, we describe these effects in liquidity traps, either temporary or permanent.

We present the model in Section 2 and analyze its steady-state equilibrium in Section 3. Our business-cycle model starts from the classical money-in-the-utility-function model of Sidrauski [1967]. Individuals maximize an intertemporal utility function that depends on consumption of services and real money balances. They can borrow or save with bonds. The central bank issues money in exchange for bonds through open market operations. Individuals are self-employed and supply a fixed amount of labor. One unit of labor produces one unit of service. We add two ingredients to this framework.

First, as in Kurz [1968], we assume that the utility function also depends on real wealth.[1] This assumption may capture a love for the social status provided by wealth, as described in Veblen [1899] and Frank [1985]. This assumption may also capture a desire to accumulate wealth as an end in itself, as described in Smith [1776], Weber [1930], and Keynes [1919].[2] In the context of a life-cycle model, the assumption that wealth enters the utility function allows Carroll [2000] to explain the finding that the rich save much more than the poor.[3] In addition, the assumption of wealth in the utility may capture a rule of thumb for savings, as wealth can be used for future consumption. With utility for wealth, the consumption Euler equation is modified and defines a downward-sloping relation between consumption and interest rate in steady state, even though there is zero wealth in aggregate. This relation can

[1] For others models in which consumption and wealth enter the utility function, see Zou [1994, 1995] and Bakshi and Chen [1996]. For a model in which consumption, money, and wealth enter the utility function, see Zou [1998]. These models have a different focus then ours. Kurz [1968] and Zou [1994, 1998] study long-term growth. Zou [1995] studies saving over the life cycle. Bakshi and Chen [1996] study portfolio choice and asset pricing. We study business cycles.

[2] Zou [1994] provides the detailed citations.

[3] Dynan, Skinner and Zeldes [2004] find a strong empirical correlation between saving rate and lifetime income, and between marginal propensity to save and lifetime income.

be seen as an IS curve. In contrast, with no utility for wealth, the steady-state real interest rate equals the subjective discount rate generating a flat IS curve in the consumption, interest plane. The demand for real money balances defines an upward-sloping relation between consumption and nominal interest rate. This relation is the LM curve. For a given inflation and real money supply, the intersection of the IS and LM curves defines an aggregate demand (AD) curve and an equilibrium interest rate.

Second, we build on the theory developed by Michaillat and Saez [2013] to link aggregate demand to unemployment. In the market where sellers of services meet consumers of services, matching frictions affect trade.[4] As a result, not all services available are sold and there is unemployment; furthermore, resources are dissipated when shopping for services. The concept of tightness, defined as the ratio of shopping intensity to quantity of services for sale, summarizes the state of the market. The cost of consumption grows with tightness as consumers need to expend more resources to find matches when tightness is high; therefore, the AD curve is downward sloping in tightness. Output grows with tightness as it is easier to sell services when tightness is high. There is a tightness level that maximizes consumption. The economy is slack if tightness is below the consumption-maximizing tightness, and tight if tightness is above it. The relation between the amount of consumption sold and tightness is the aggregate supply (AS) curve. In equilibrium, given a price, tightness adjusts to equalize AD curve and AS curve.

Because of matching frictions, the price of services is indeterminate and we need to choose a price process to select an equilibrium. We focus primarily on two polar cases: a rigid price process and a flexible price process. The rigid price process has a fixed inflation rate and a fixed initial price level that are not affected by other economic variables. This process captures the behavior of prices at business-cycle frequency if prices are slow to adjust and inflation is anchored. With a rigid price process, the model behaves as a Keynesian IS-LM model. The flexible price process adjusts to maintain consumption at its maximum level. With a flexible price process, the model behaves as a Real Business Cycle model.

Sections 4 and 5 study the effects of shocks and monetary policy under the two price pro-

[4] Diamond [1982], Mortensen [1982], and Pissarides [1985] first modeled markets with matching frictions.

cesses. To characterize these effects, we undertake comparative steady states. The comparative steady-states analysis gives an exact qualitative description of the dynamics in response to an unexpected, permanent shock.[5] We believe that the comparative steady-states analysis also gives a good qualitative description of the dynamics under transitory shocks.[6]

Section 4 studies the steady-state equilibrium under rigid price process. Negative aggregate demand shocks are caused by adverse shifts in the IS curve, due for instance to increased preference for wealth or for future consumption. These shocks reduce consumption and tightness and increase unemployment. They also reduce the interest rate. Negative aggregate supply shocks, caused for instance by a reduction in labor supply, reduce consumption and output but increases tightness and hence reduce unemployment. Negative mismatch shocks, defined as reduction in matching efficacy, reduce consumption and increase both tightness and unemployment. The effects of the shocks can be easily understood with the help of diagrams representing the IS and LM curves in a (consumption, interest rate) plane and the AS and AD curves in a (consumption, tightness) plane.

With a positive nominal interest rate, monetary policy can control real money balances (or equivalently the interest rate) and hence affect aggregate demand. An increase in real money supply shifts the LM curve outward and stimulates aggregate demand. Hence, it increases consumption and tightness and reduces unemployment and the interest rate. Therefore, monetary policy can be used for stabilization, defined as maintaining consumption at its maximum level. Negative demand shocks require expansionary monetary policy while negative supply shocks require contractionary monetary policy. However, monetary policy is effective only if real money balances have not reached individuals' bliss point of real money balances. When the bliss point is reached, the LM curve becomes horizontal at a

[5] In response to an unexpected, permanent shock, the equilibrium jumps from the old steady state to the new one. The reason is that in the model, there are no slow-moving variables that cannot jump in response to an unexpected shock. Wealth is a slow-moving variable but it is always zero in aggregate. The price is another slow-moving variable under the rigid price process, but it does not respond to shocks or monetary policy. The price is a jump variable under the flexible price process.

[6] Michaillat [2012] uses numerical methods to show that simulation of a collection of steady-state equilibria and simulation of the exact dynamic equilibrium produce broadly the same results in a matching model of the labor market. We could use the same method in the context of our model to validate our comparative steady-states approach.

zero nominal interest rate because zero is the only nominal interest rate at which individuals are indifferent between money and bonds. In this situation of liquidity trap, changing real money balances can no longer affect aggregate demand. We discuss how other policies such as a wealth tax or a helicopter drop of money can help in a liquidity trap.

Section 5 studies the steady-state equilibrium under flexible price process. Monetary policy is useless for stabilization when the nominal interest rate is positive because the price automatically adjusts so that consumption is maximized. In fact, changes in money supply automatically translate into changes in prices. However, the flexible price economy can also fall in a liquidity trap. In a liquidity trap, even the flexible price process cannot stabilize the economy. Under the flexible price process, if we assume that money growth also increases inflation in a liquidity trap, then monetary policy can stimulate the IS curve and aggregate demand via the inflation channel, and solve the liquidity trap problem. In that case, with flexible prices, liquidity traps can be solved at no cost using monetary policy. Note however that this relies on the strong assumption that money growth continue to affect inflation immediately in a liquidity trap.

In Section 6, we discuss a more general price process that bridges the gap between the rigid price process of Section 4 and the flexible price process of Section 5. In the short run, inflation may be partly rigid. Hence, aggregate demand shocks matter and monetary policy plays a role to stabilize the economy. In the medium run, inflation adjusts such that tightness moves toward its consumption-maximizing level. This price adjustment mechanism constitutes a Phillips curve. In a slack economy away from a liquidity trap, and absent monetary policy stabilization, a reduction in price helps restore efficiency. Slack economies tend to generate lower inflation, creating a positive relationship between output (relative to potential output) and inflation. Monetary policy is effective for stabilization in the short-run but there is no long-run trade-off between inflation and output.

2 The Model

We formulate the model in continuous time to simplify the algebra. To economize on notations, we abstract from stochastic shocks and focus on a deterministic environment. The model could accommodate technology shocks, labor supply shocks, mismatch shocks, preference shocks, or monetary shocks. To introduce these shocks, we would only need to represent the relevant parameters as stochastic processes.

2.1 Market for Services

We model the market for services as in Michaillat and Saez [2013]. A measure 1 of identical self-employed workers sell services on a market with matching frictions. The capacity of each worker is exogenously given by k. Each worker would like to sell k units of services at any point in time. Workers are also consumers of services, but they cannot consume their own services, so they have to trade with other workers.

To purchase services, each consumer visits $o(t)$ workers at time t. The number of trades between consumers and workers at time t is given by a matching function with constant returns to scale:

$$y(t) = h(k, o(t)).$$

The matching function is strictly increasing in both arguments, with diminishing marginal returns in both arguments, and twice differentiable. In addition, the matching function satisfies $0 \leq h(k, o(t)) \leq k$.[7] In each trade, a consumer buys one unit of service at price $p(t) > 0$.

The market tightness at time t is the ratio of visits to capacity:

$$x(t) \equiv \frac{o(t)}{k}.$$

Since the matching function has constant returns to scale, market tightness determines the rate at which services are sold and the rate at which visits lead to a purchase. Workers sell

[7] A matching function satisfying these properties is $h(k, o) = (k^{-\eta} + o^{-\eta})^{-1/\eta}$ with $\eta > 0$.

services at rate

$$f(x(t)) = \frac{y(t)}{k} = h(1, x(t)),$$

and visits lead to a purchase at rate

$$q(x(t)) = \frac{y(t)}{o(t)} = h\left(\frac{1}{x(t)}, 1\right).$$

note that $q(x) = f(x)/x$. Since the matching function is strictly increasing in its two arguments, the function f is strictly increasing in x, and the function q is strictly decreasing in x. In other words, when the market is slacker, a larger fraction of workers' capacity remains unsold while a larger fraction of consumers' visits results in a purchase. We denote by $1-\eta$ and $-\eta$ the elasticities of f and q: $1-\eta \equiv x \cdot f'(x)/f(x) > 0$ and $\eta \equiv -x \cdot q'(x)/q(x) > 0$.

We abstract from randomness at the level of the worker and the consumer: a worker sells $f(x(t)) \cdot k$ units of services at time t with certainty, and a consumer purchases $q(x(t)) \cdot o(t)$ units of services at time t with certainty. Since the matching function is less than k, the selling rate $f(x(t))$ is always between 0 and 1, and workers may not be able to sell all their services at any time. We define the unemployment rate as the fraction of services that are available but not sold at time t: $u(t) = 1 - f(x(t))$. A fraction $u(t)$ of services workers' capacity is unemployed at time t.

A consumer visits $o(t)$ workers to purchase services. The flow cost of visits is $\rho \geq 0$ units of services. The $\rho \cdot o(t)$ units of services for matching are purchased like the $c(t)$ units of services for consumption. The matching cost represents the resources devoted to matching with an appropriate seller. For example, one first needs to purchase a taxicab service to go to a hair salon and purchase a haircut. The taxicab service does not enter the utility function, but the haircut does.

The number of visits is related to consumption and market tightness by $q(x(t)) \cdot o(t) = c(t) + \rho \cdot o(t)$. Hence, the desired level of consumption determines the number of visits: $o(t) = c(t)/(q(x(t)) - \rho)$. Hence, consuming one unit of services requires to purchase $1+$

$(\rho \cdot o(t)/c(t)) = 1 + \tau(x(t))$ units of services where

$$\tau(x(t)) \equiv \frac{\rho}{q(x(t)) - \rho}.$$

The function τ is positive and strictly increasing for all $x \in [0, x^m)$ where $x^m > 0$ satisfies $\rho = q(x^m)$. Furthermore, $\lim_{x \to x^m} \tau(x) = +\infty$. It is easy to show that the elasticity of τ is $x \cdot \tau'(x)/\tau(x) = \eta \cdot (1 + \tau(x))$. Using tightness $x(t)$, we no longer need to use the number of visits $o(t)$ in the rest of the analysis.

We define and characterize the market tightness that maximizes consumption at each point in time. In equilibrium, we have:

$$c(t) = \frac{y(t)}{1 + \tau(x(t))} = \frac{f(x(t))}{1 + \tau(x(t))} \cdot k.$$

Since $1/(1 + \tau(x)) = 1 - \rho/q(x)$ and $q(x) = f(x)/x$, we obtain

$$c(t) = [f(x(t)) - \rho \cdot x(t)] \cdot k. \tag{1}$$

This equation says that $\rho \cdot x(t) \cdot k = \rho \cdot o(t)$ units of services are dissipated in matching frictions.

DEFINITION 1. *The **consumption-maximizing tightness** is the tightness that maximizes consumption given the matching frictions:* $x^* = \operatorname{argmax}\{(f(x) - \rho \cdot x) \cdot k\}$. *Since the function f is strictly concave, the consumption-maximizing tightness is uniquely defined by* $f'(x^*) = \rho$.

The consumption-maximizing tightness is the tightness underlying the condition of Hosios [1990] for efficiency in a matching model. The labor market can be in three regimes:

DEFINITION 2. *The labor market is **slack** if a marginal increase in tightness increases consumption, **tight** if a marginal increase in tightness decreases consumption, and **at potential** if a marginal increase in tightness has no effect on consumption. Equivalently, the regimes can*

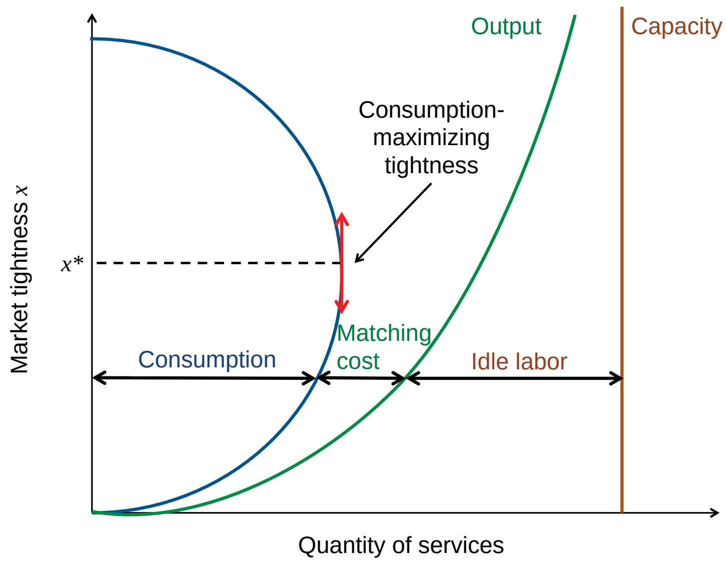


Figure 1: The concepts of capacity, output, and consumption

be described by the departure of actual tightness from the consumption-maximizing tightness. The labor market is slack if $x(t) < x^*$, at potential if $x(t) = x^*$, and tight if $x(t) > x^*$.

We define business cycles as a succession of slack and tight episodes. This definition is unconventional, but it is the natural definition of business cycles in our model.

Figure 1 summarizes the relation between market tightness and the different quantity concepts in the service market. Capacity k is a vertical line, independent of tightness. Output $y = f(x) \cdot k$ is increasing in tightness x as it is easier to sell services when tightness is high. Consumption $c = f(x) \cdot k/[1+\tau(x)] = [f(x) - \rho \cdot x] \cdot k$ first increases and then decreases in tightness. At the consumption-maximizing tightness x^*, the consumption curve is vertical. The difference between output and consumption is $\rho \cdot x \cdot k = \rho \cdot o$ and represents resources dissipated through matching frictions.

2.2 Money and Bonds

Individuals can issue or buy riskless nominal bonds. At time t, an individual holds $B(t)$ bonds. The rate of return on bonds is the nominal interest rate $i(t)$. The government circulates a quantity $M(t)$ of money at time t. Money is the unit of account in the economy. The price level at time t is $p(t)$. The rate of inflation at time t is $\pi(t) = \dot{p}(t)/p(t)$. The quantity of real money in circulation at time t is $m(t) = M(t)/p(t)$.

The government circulates money through open market operations: the government buys bonds issued by individuals with money. In our representative agent model, individuals are net borrowers: $B(t) \leq 0$. At any time t, the quantity of bonds issued equals the quantity of money put in circulation: $-\dot{B}(t) = \dot{M}(t)$. Initially, $-B(0) = M(0)$. Therefore, at any time t,

$$-B(t) = M(t). \tag{2}$$

At time t, the revenue from seignorage is $S(t) = -B(t) \cdot i(t) = i(t) \cdot M(t)$. The government rebates this revenue lump sum to individuals. Without public spending or taxes, the government's budget is therefore balanced at any time.

2.3 Intertemporal Utility Maximization

Consumers use part of their labor income to purchase services and save part of their income as money and bonds. The law of motion of a consumer's assets is given by

$$\dot{B}(t) + \dot{M}(t) = p(t) \cdot f(x(t)) \cdot k - p(t) \cdot (1 + \tau(x(t))) \cdot c(t) + i(t) \cdot B(t) + S(t).$$

Here, $M(t)$ are nominal money balances, $B(t)$ are bond holdings, $p(t)$ is the price of services, $(1 + \tau(x(t))) \cdot c(t)$ is the quantity of services purchased, $f(x(t)) \cdot k$ is the quantity of services sold, and $S(t)$ is lump-sum transfer of seignorage revenue from the government. Let $A(t) = M(t) + B(t)$ denote nominal financial wealth at time t. We rewrite the law of motion of a consumer's assets in terms of nominal money balances and nominal wealth:

$$\dot{A}(t) = p(t) \cdot f(x(t)) \cdot k - p(t) \cdot (1 + \tau(x(t))) \cdot c(t) - i(t) \cdot M(t) + i(t) \cdot A(t) + S(t).$$

Let $a(t) = A(t)/p(t)$ denote real financial wealth at time t and $s(t) = S(t)/p(t)$ denote real transfer of government seignorage. Using $\dot{a}(t) = \dot{A}(t)/p(t) - [\dot{p}(t)/p(t)] \cdot [A(t)/p(t)] =$

$\left[\dot{A}(t)-\pi(t)\cdot A(t)\right]/p(t)$, we rewrite the law of motion of a consumer's assets in real terms:

$$\dot{a}(t)=f(x(t))\cdot k-(1+\tau(x(t)))\cdot c(t)-i(t)\cdot m(t)+r(t)\cdot a(t)+s(t), \tag{3}$$

where $r(t)\equiv i(t)-\pi(t)$ is the real interest rate at time t. This flow budget constraint is standard but for two differences arising from the presence of matching frictions on the service market. First, income k is discounted by a factor $f(x(t))<1$ as only a fraction $f(x(t))$ of k is actually sold. Second, consumption $c(t)$ has a price wedge $1+\tau(x(t))>1$ because resources are dissipated in matching frictions: buying one unit of services for consumption requires buying $1+\tau(x(t))$ units of services. Individuals take $x(t)$ are given, hence the intertemporal choice problem can be solved using the standard method.

Consumers' instantaneous utility function is $v(c(t),m(t),a(t))$, where $c(t)\geq 0$ is consumption of services, $m(t)\geq 0$ are real money balances, and $a(t)$ is real wealth. We assume that the function v is strictly increasing in its three arguments, strictly concave, and twice differentiable. The assumptions that real money balances and real wealth enter the utility function are critical to obtain a nondegenerate IS-LM system. The utility function of a consumer at time 0 is the discounted sum of instantaneous utilities

$$\int_0^{+\infty} e^{-\delta\cdot t}\cdot v(c(t),m(t),a(t))\,dt, \tag{4}$$

where $\delta>0$ is the subjective discount rate.

Concretely, the model can be seen as the Sidrauski [1967] model with two additions. First, wealth $a(t)$ enters the utility function. Second, matching frictions lower labor income k by a factor $f(x(t))$ and increase the effective price of consumption $c(t)$ by a factor $[1+\tau(x(t))]$. Because $x(t)$ is a given for the individuals, the model can be solved exactly as the original Sidrauski [1967] model. The new parameter $x(t)$ provides on extra degree of freedom allowing us to consider various pricing processes as we shall see.

2.4 Equilibrium

Throughout, $[x(t)]_{t=0}^{+\infty}$ denotes the continuous-time path of variable $x(t)$ [Acemoglu, 2009].

DEFINITION 3. *The **representative consumer's problem** is to choose paths for consumption, real money balances, and real wealth $[c(t), m(t), a(t)]_{t=0}^{+\infty}$ to maximize (4) subject to (3), taking as given initial real wealth $a(0) = 0$ and the paths for market tightness, nominal interest rate, inflation, and seignorage $[x(t), i(t), \pi(t), s(t)]_{t=0}^{+\infty}$.*

To solve the consumer's problem, we set up the current-value Hamiltonian:

$$\begin{aligned}\mathscr{H}(t, c(t), m(t), a(t)) =& v(c(t), m(t), a(t)) \\ &+ \mu(t) \cdot [f(x(t)) \cdot k - (1 + \tau(x(t))) \cdot c(t) - i(t) \cdot m(t) + r(t) \cdot a(t) + s(t)]\end{aligned}$$

with control variables $c(t)$ and $m(t)$, state variable $a(t)$, and current-value costate variable $\mu(t)$. Throughout we use subscripts to denote partial derivatives. The necessary conditions for an interior solution to this maximization problem are

$$\begin{aligned}\mathscr{H}_c(t, c(t), m(t), a(t)) &= 0 \\ \mathscr{H}_m(t, c(t), m(t), a(t)) &= 0 \\ \mathscr{H}_a(t, c(t), m(t), a(t)) &= \delta \cdot \mu(t) - \dot{\mu}(t)\end{aligned}$$

together with the transversality condition $\lim_{t \to +\infty} \left[e^{-\delta \cdot t} \cdot \mu(t) \cdot a(t) \right] = 0$. Given that v is concave in (c, m, a) and that $\mathscr{H}$ is the sum of v and a linear function of (c, m, a), $\mathscr{H}$ is concave in (c, m, a) and these conditions are also sufficient.

These three conditions imply that

$$v_c(c(t), m(t), a(t)) = \mu(t) \cdot (1 + \tau(x(t))) \tag{5}$$

$$v_m(c(t), m(t), a(t)) = \mu(t) \cdot i(t) \tag{6}$$

$$v_a(c(t), m(t), a(t)) = (\delta - r(t)) \cdot \mu(t) - \dot{\mu}(t). \tag{7}$$

Equations (5) and (6) imply that the marginal utilities from consumption and real money balances satisfy

$$v_m(c(t),m(t),a(t)) = \frac{i(t)}{1+\tau(x(t))} \cdot v_c(c(t),m(t),a(t)). \tag{8}$$

In steady state, this equation yields the LM curve. It represents a demand for money. The demand for real money is declining with $i(t)$ because $i(t)$ is the implicit price of holding money paying zero nominal interest instead of bonds paying a nominal interest rate $i(t)$.

Equations (5) and (7) imply that the marginal utilities from consumption and real wealth satisfy

$$(1+\tau(x(t))) \cdot \frac{v_a(c(t),m(t),a(t))}{v_c(c(t),m(t),a(t))} + (r(t)-\delta) = -\frac{\dot{\mu}(t)}{\mu(t)}, \tag{9}$$

where $\dot{\mu}(t)/\mu(t)$ can be expressed as a function of $c(t)$, $m(t)$, $a(t)$, $x(t)$, and their derivatives using (5). This is the consumption Euler equation. In steady state, this equation yields the IS curve. It represents a demand for saving in part from intertemporal consumption-smoothing considerations and in part from the utility provided by wealth.

Note that if there are no matching costs ($\rho = 0$ and hence $\tau(x) = 0$) and if the utility only depends on consumption ($v_a = v_m = 0$), this Euler equation reduces to the standard continuous-time consumption Euler equation: $(r(t)-\delta)\cdot\varepsilon = \dot{c}(t)/c(t)$ where $\varepsilon \equiv -v'(c)/[c \cdot v''(c)]$ is the intertemporal elasticity of substitution.

We now define and characterize the equilibrium. The definition and characterization of an equilibrium in presence of matching frictions follow Michaillat and Saez [2013].

DEFINITION 4. *An* ***equilibrium*** *consists of paths for market tightness, consumption, real money balances, nominal money balances, real wealth, nominal interest rate, and price level,* $[x(t),\ c(t),\ m(t),\ M(t),\ a(t),\ i(t), p(t)]_{t=0}^{+\infty}$*, such that the following conditions hold:*

1. $[c(t),m(t),a(t)]_{t=0}^{+\infty}$ *solve the representative consumer's problem;*

2. *monetary policy determines* $[M(t)]_{t=0}^{+\infty}$*;*

3. *the money market clears;*

4. *the bond market clears;*

5. *actual tightness on the market for services equals posted tightness, which is the tightness taken as given by consumers for their optimization problem.*

Condition 1 says that consumers choose consumption, money balances, and wealth to maximize utility taking as given prices and market tightnesses. Condition 2 says that the government determines the amount of nominal money in circulation. Condition 3 says that consumers' money balances equal the amount of money circulated by the government. Condition 4 says that the amount of bonds outstanding for consumers equal the government's demand for bonds, which is equal to the government's supply of nominal money. Condition 5 says that the trading probabilities taken into account by consumers for their optimization problem are realized in equilibrium (the trading probabilities depend only on tightness).[8]

PROPOSITION 1. *An equilibrium consists of paths of market tightness, consumption, real money balances, nominal money balances, real wealth, nominal interest rate, and price level,* $[x(t),c(t),m(t),M(t),a(t),i(t),p(t)]_{t=0}^{+\infty}$*, that satisfy the following conditions:*

1. *equation* (8) *holds;*

2. *equation* (9) *holds;*

3. $m(t)=M(t)/p(t)$ *where* $[M(t)]_{t=0}^{+\infty}$ *is determined by monetary policy;*

4. *aggregate wealth is zero:* $a(t)=0$*;*

5. *equation* (1) *holds.*

These conditions listed in the proposition follow almost immediately from the conditions listed in the definition. In particular, the condition that $a(t)=0$ follows from the condition that the bond market clears, and the condition that equation (1) holds follows from the condition that actual tightness equals posted tightness.

[8] Interested readers should refer to Michaillat and Saez [2013] for more details on the definition and characterization of the equilibrium, on the link between this equilibrium concept and the concept of Walrasian equilibrium, and on the indeterminacy arising in equilibrium.

Seven variables are determined by six independent conditions, so the equilibrium is indeterminate. As explained in Michaillat and Saez [2013], the indeterminacy arises from the presence of matching frictions on the market for services. As a consequence, we need a criterion to select a unique equilibrium.[9]

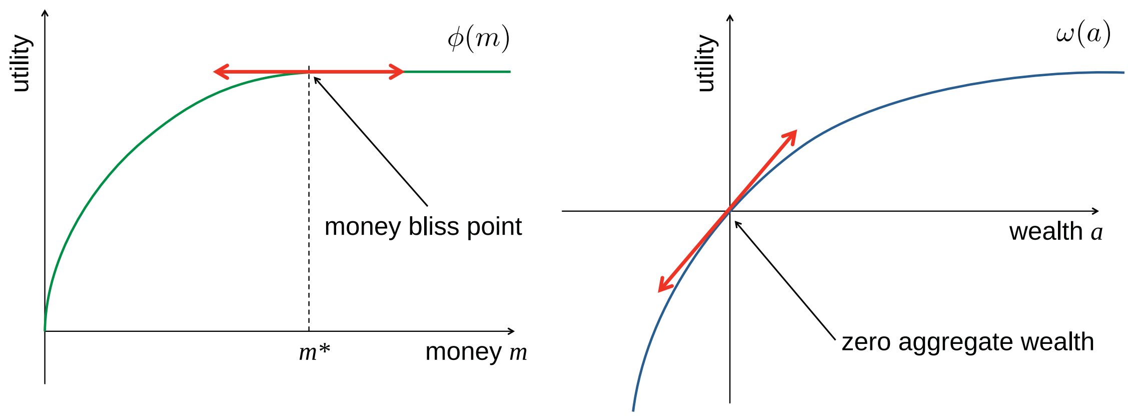
This paper focuses primarily on two particular equilibrium selection criteria. The first equilibrium selection criterion imposes that the price process $[p(t)]_{t=0}^{+\infty}$ is exogenous. The price process responds neither to equilibrium variables nor to monetary policy. This selection criterion allows us to capture Keynesian intuitions. The second equilibrium selection criterion imposes that the price process $[p(t)]_{t=0}^{+\infty}$ immediately adjusts to the state of economy such that consumption is maximized at each instant. We show that this criterion is the natural generalization of the Walrasian market-clearing condition to a market with matching frictions. This selection criterion captures the idea that prices always ensure that the economy is at potential. We define and discuss the first criterion in Section 4 and the second one in Section 5. Before that, Section 3 describes the steady state associated with the equilibrium of Proposition 1.

3 Steady-State Equilibrium

This section shows that the steady-state equilibrium is described by a downward-sloping IS curve in a (consumption, interest rate) plane, an upward-sloping LM curve in a (consumption, interest rate) plane, a downward-sloping AD curve in a (quantity, market tightness) plane, and an upward-sloping AS curve in a (quantity, market tightness) plane. The steady state can be used to analyze the effect of aggregate shocks and monetary policy, and to think about liquidity traps—situations when the nominal interest rate is zero.

To obtain simple closed-form expressions for the curves, we assume that the utility func-

[9]The standard equilibrium selection mechanism in the matching literature is that the price level is the outcome from bargaining between seller and buyers. However, Michaillat and Saez [2013] show this mechanism eliminate the effect of aggregate demand shocks that are the object of our study.



(a) Utility over real money

(b) Utility over wealth

Figure 2: The utility functions over money and wealth

tion is separable in consumption, real money, and real wealth:[10]

$$v(c,m,a) = \frac{\varepsilon}{\varepsilon-1} \cdot c^{\frac{\varepsilon-1}{\varepsilon}} + \phi(m) + \omega(a).$$

The curvature of utility over consumption is measured by $\varepsilon \geq 1$. As depicted on Figure 2(a), the function ϕ is strictly concave and strictly increasing on $[0, m^*]$ and constant on $[m^*, +\infty)$. The quantity $m^* \in (0, \infty]$ is a bliss point in real money balances. As we shall see, a bliss point with zero marginal utility of money above is necessary to model liquidity traps. As depicted on Figure 2(b), the function ω is concave and strictly increasing on $(-\infty, +\infty)$. As wealth is zero in aggregate, the key parameter is the marginal utility of wealth at the origin, $\omega'(0)$. We assume that $\omega'(0) \in [0, +\infty)$. As we shall see, a positive marginal utility of wealth is necessary to model a nondegenerate steady-state IS curve.

[10]We can also analyze the steady state with nonseparable utilities. The IS-LM curves become more complex but the key economic intuitions carry over, hence our focus on the simpler separable case.

3.1 LM and IS Curves

DEFINITION 5. *The* ***LM curve*** *expresses consumption as a function of nominal interest rate, real money balances, and market tightness:*

$$c^{LM}(i,m,x) = \left[\frac{i}{(1+\tau(x))\cdot\phi'(m)}\right]^{\varepsilon}$$

for all $i \in [0,+\infty)$*, all* $m \in [0,m^*)$*, and all* $x \in [0,x^m]$*. Above the money bliss point (*$m \geq m^*$*), the LM curve determines a unique nominal interest rate:*

$$i^{LM}(c,m,x) = 0$$

for all $c \in [0,+\infty)$*, all* $m \in [m^*,+\infty)$*, and all* $x \in [0,x^m]$*.*

The LM curve is the collection of quadruples (c,i,m,x) that solves equation (8). The LM curve is defined separately for real money balances below and above the money bliss point m^* because when real money is at or above the bliss point, $\phi'(m) = 0$ so equation (8) is degenerate and simply imposes $i = 0$. The situation in which real money balances are at or above the bliss point is a liquidity trap.

DEFINITION 6. *The* ***IS curve*** *expresses consumption as a function of nominal interest rate, inflation, and market tightness:*

$$c^{IS}(i,\pi,x) = \left[\frac{\delta+\pi-i}{(1+\tau(x))\cdot\omega'(0)}\right]^{\varepsilon}$$

for all $i \in [0,\delta+\pi]$*, all* $\pi \in [-\delta,+\infty)$ *and all* $x \in [0,x^m]$*. If marginal utility of wealth is zero (*$\omega'(0) = 0$*), the IS curve determines a unique interest rate:*

$$i^{IS}(c,\pi,x) = \pi+\delta$$

for all $c \in [0,+\infty)$*, all* $\pi \in [-\delta,+\infty)$*, and all* $x \in [0,x^m]$*.*

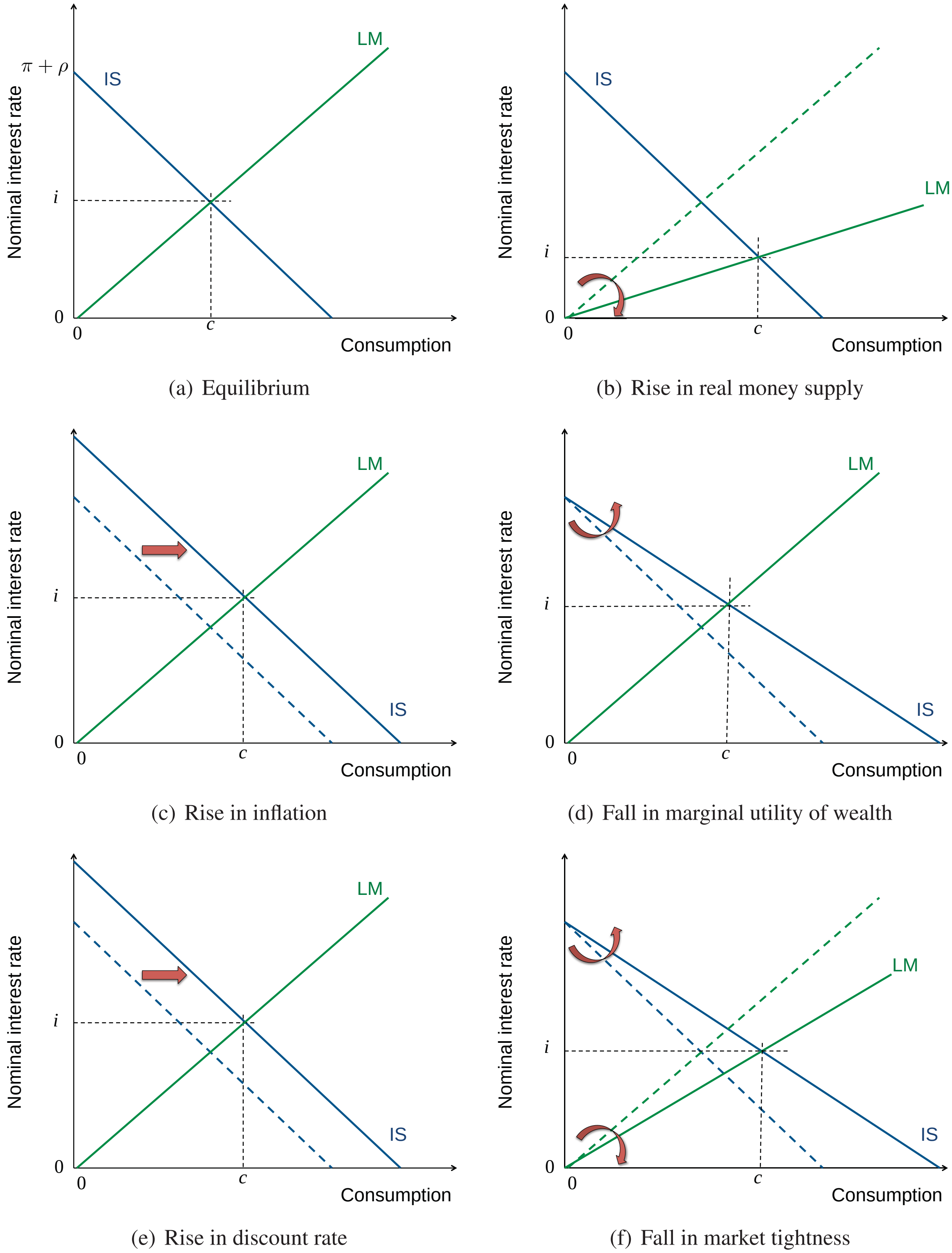
The IS curve is the collection of quadruples (c,i,π,x) that solves equation (9) in steady state, when $\dot{\mu}(t) = 0$. The IS curve is expressed as a function of inflation and nominal interest rate. However, the IS curve only depends on the real interest rate, r, because $r = i - \pi$. The IS curve is defined separately when the marginal utility of wealth is positive or zero. When the marginal utility of wealth is zero, the IS curve yields the usual result that $r = \delta$.

Several important properties of the IS and LM curve are illustrated in Figure 3.[11] First, Figure 3(a) shows that the LM curve is upward sloping in a (c, i) plane. This property follows the standard logic. Demand for real money is decreasing with i as a higher i makes money less attractive relative to bonds. Demand for real money is increasing in c as a higher c reduces marginal utility of consumption, which makes real money more attractive relative to consumption. Given that the real money supply is constant, an increase in i requires an increase in c to maintain equilibrium. Through the same logic, an increase in real money supply naturally shifts the LM curve out, as illustrated in Figure 3(b).

Second, Figure 3(a) shows that in a (c, i) plane, the LM curve imposes that $i \geq 0$ for all $c \geq 0$. The LM curve prevents the nominal interest rate, i, from falling below zero because the marginal utility of money $\phi'(m)$ is nonnegative. If the nominal interest rate were negative, money would strictly dominate bonds. When real money is at or above the bliss point m^*, the LM curve becomes horizontal at $i = 0$, as illustrated in Figure 4(a). An increases in real money supply has no impact on the LM curve. This situation of liquidity trap has important implications to which we will come back.

Third, Figure 3(a) shows that the IS curve is downward sloping in a (c, i) plane. The intuition is the following. A higher interest rate increases the marginal value of savings through bonds via the wealth effect $r \cdot \omega'(0)$. This requires a corresponding increase in the marginal utility of consumption and a lower consumption level. With a higher r, the individual prefers to hold more wealth and spend less on consumption. Wealth stays at zero in equilibrium but consumption declines. This logic also implies that an increase in inflation, which reduces the real interest rate for a given nominal interest rate, shifts the IS curve out in the (c, i) diagram as illustrated in Figure 3(c). Similarly, a decrease in the marginal utility

[11] We draw linear IS and LM curves in Figure 3, which corresponds to the case with log utility over consumption ($\varepsilon = 1$).



(a) Equilibrium

(b) Rise in real money supply

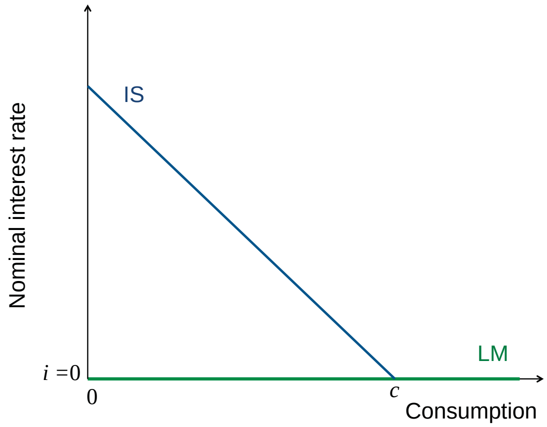
(c) Rise in inflation

(d) Fall in marginal utility of wealth

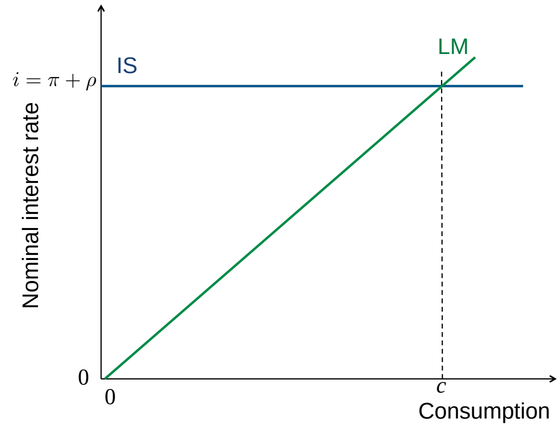
(e) Rise in discount rate

(f) Fall in market tightness

Figure 3: IS curve and LM curve in (c, i) plane



(a) Real money above bliss point



(b) Zero marginal utility of wealth

Figure 4: IS curve and LM curve in special cases

of wealth shifts the IS curve out as shown in Figure 3(d). In the standard case with no utility of wealth, the IS curve becomes horizontal at $i = \delta + \pi$ as depicted in Figure 4(b). The intuition is well known: steady-state consumption is constant so individuals hold bonds only if the return on bonds, r, equals the subjective discount rate, δ, which determines the nominal interest rate for a given inflation. Obtaining a downward-sloping IS curve requires introducing utility of wealth. In that case, $\delta > r$ as individuals also save for wealth.[12]

Fourth, Figure 3(f) shows that the IS and LM curves shift outward when market tightness, x, decreases. However, the nominal interest rate defined by the intersection of the IS and LM curves does not depend on market tightness.[13] The IS and LM curves shift by commensurate amounts such that the equilibrium interest rate remains the same.

[12] Utility of wealth generates a downward-sloping IS curve even though wealth is zero for all individuals in equilibrium.

[13] If $\varepsilon = 1$, we can rewrite the IS and LM curves in terms of output instead of consumption: $y^{LM} = [1 + \tau(x)] \cdot c^{LM}$ and $y^{IS} = [1 + \tau(x)] \cdot c^{IS}$. These two curves are not affected by market tightness. If there are no matching costs ($\rho = 0$ and hence $\tau(x) = 0$), then the IS and LM curves are independent of tightness, as in the textbook model.

3.2 AS and AD Curves

Combining the IS and LM curves defines an AD curve and an equilibrium interest rate. The equality $c^{IS}(i,\pi,x) = c^{LM}(i,m,x)$ implies that the equilibrium nominal interest rate is

$$i = \frac{\phi'(m)}{\phi'(m)+\omega'(0)} \cdot (\delta+\pi) \tag{10}$$

At that interest rate, consumers are indifferent between money and bonds. This equation also defines an equilibrium real interest rate

$$r = \frac{\phi'(m)}{\phi'(m)+\omega'(0)} \cdot \delta - \frac{\omega'(0)}{\phi'(m)+\omega'(0)} \cdot \pi.$$

Four points are worth noting about the equilibrium interest rates. First, both nominal and real interest rate decrease with real money supply and marginal utility of wealth. These results are illustrated in Figures 3(b) and 3(d). Second, the real interest rate decreases with inflation but the nominal interest rate increases with inflation. This result is illustrated in Figure 3(c). Third, the equilibrium interest rates are independent of tightness x. This result is illustrated in Figure 3(f). Fourth, the zero lower bound on nominal interest rate is reached when the real money supply reaches its bliss point m^*, in which case $\phi'(m) = 0$.

Plugging the expression (10) for the equilibrium nominal interest rate into the LM curve defines the AD curve:

DEFINITION 7. *The **AD curve** expresses consumption as a function of real money balances, inflation, and market tightness defined by*

$$c^{AD}(m,\pi,x) = \left[\frac{\delta+\pi}{(1+\tau(x))\cdot(\phi'(m)+\omega'(0))}\right]^{\varepsilon}. \tag{11}$$

for all $\pi \in [-\delta,+\infty)$, all $m \in [0,\infty)$, and all $x \in [0,x^m]$.

As τ increases with x, aggregate demand declines with x as illustrated on Figure 3(f). Aggregate demand also increases with the discount rate δ and decreases with marginal utility of wealth $\omega'(0)$ as these two factors shift the IS curve inward (and hence can be seen as negative aggregate demand shocks as shown in Figures 3(d) and 3(e)). Real money and inflation stimulate aggregate demand via outward shifts in the LM curve and IS curve in the (c,i) diagram as shown in Figures 3(b) and 3(c) respectively.

It is also possible to express the AD curve as a function of the nominal interest rate i by using the fact that $\phi'(m)/\omega'(0) = i/(\delta+\pi-i)$, obtained from (10). We obtain the alternative expression for the AD curve:

$$c^{AD}(i,\pi,x) = \left[\frac{\delta+\pi-i}{(1+\tau(x))\cdot\omega'(0)}\right]^{\varepsilon}.$$

When the nominal interest rate is at its equilibrium value, given by (10), this alternative expression is equivalent to (11). This expression shows explicitly how the nominal interest rate affects aggregate demand. Modern monetary policy directly targets the nominal interest rate instead of the money supply, and the expression is useful in this context. However, this alternative expression cannot be used in the case with no utility of wealth. Indeed, without utility of wealth, both numerator and denominator are zero since $\omega'(0) = 0$ and $\delta = i - \pi$. Without utility of wealth, $r = \delta$; hence, for a given π, i is determined. Changing the real money supply affects aggregate demand but with no effect on i. Therefore, a model with the standard Euler equation cannot meaningfully describe a monetary policy that determines the nominal interest rate in steady state.[14] It is crucial to introduce utility of wealth to obtain a nondegenerate steady state.

To complete the model, we define the AS curve:

DEFINITION 8. *The **AS curve** is a function of tightness defined for all $x \in [0, x^m]$ by*

$$c^{AS}(x) = (f(x) - \rho\cdot x)\cdot k.$$

The AS curve increases with tightness up to the consumption-maximizing tightness and then decreases with tightness. The AS curve also increases with capacity, k. The AS and AD curves are illustrated in Figure 5, together with the output and capacity.

[14]The New Keynesian model generates an IS curve solely as a dynamic adjustment phenomenon. The real

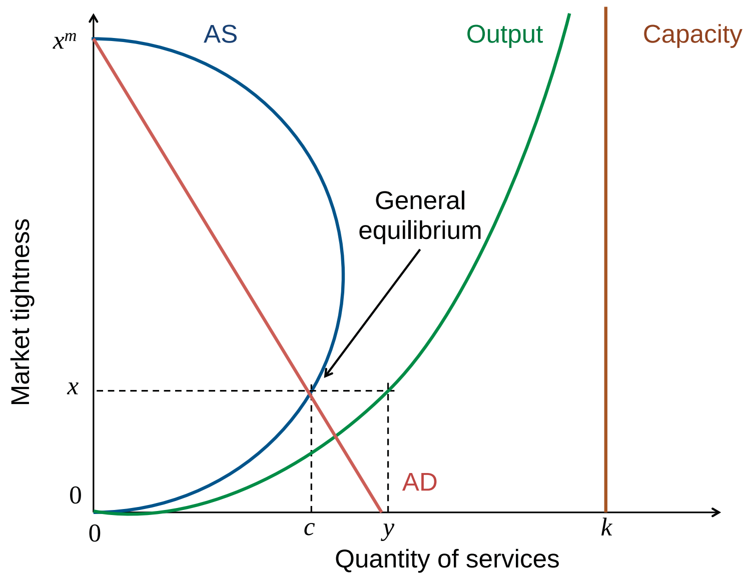


Figure 5: Steady-state equilibrium in a (c,x) plane

3.3 Welfare-Maximizing Steady State and Friedman Rule

Let x be the market tightness arising in a steady-state equilibrium with inflation π and money supply m. In the steady state, consumption is $c = [f(x) - \rho \cdot x] \cdot k$ and aggregate wealth is $a = 0$ so per-period welfare is

$$v([f(x) - \rho \cdot x] \cdot k, m, 0).$$

Per-period welfare is maximized at the consumption-maximizing tightness and bliss point for real money: $x = x^*$ and $m = m^*$. Reaching the bliss point imposes a zero nominal interest rate (see Figure 4(a)) and hence $\pi = r$. Therefore, the optimum is attained when π is such that

$$\pi^* = -\delta + \omega'(0) \cdot (1 + \tau(x^*))^{\frac{\varepsilon-1}{\varepsilon}} \cdot (f(x^*) \cdot k)^{\frac{1}{\varepsilon}}. \tag{12}$$

With no utility of wealth ($\omega'(0) = 0$), equation (12) boils down to the the famous Friedman [1969] rule, $\pi^* = -\delta < 0$. The Friedman rule recommends deflation. With positive marginal utility of wealth ($\omega'(0) > 0$), the Friedman rule may require positive inflation ($\pi^* > 0$).

interest rate equals the discount rate in steady state.

4 Analysis with Rigid Price Process

We focus on the equilibrium arising under a rigid price process, defined as follows:

DEFINITION 9. $[p(t)]_{t=0}^{+\infty}$ *is a* ***rigid price process*** *if the price* $p(t)$ *is a state variable with initial condition* $p(0)$ *and law of motion* $\dot{p}(t) = \pi \cdot p(t)$*, where* π *is a fixed parameter.*

The rigid price process never jumps and it grows at constant inflation rate π. If $\pi = 0$, the price is completely fixed over time. This criterion may be appropriate to describe the short run because inflation responds only slowly to changes in macroeconomic variables. Indeed, empirical evidence for the US suggests that the response of inflation to monetary shocks—a shock to Federal Funds rate or money supply—is much more sluggish than that of GDP.[15]

Let $g = \dot{M}(t)/M(t)$ be the growth rate of nominal money supply and $M(0)$ be initial money supply. Both g and $M(0)$ are set by monetary policy. Under the rigid price process, the steady state is characterized as follows:

PROPOSITION 2. *The steady-state equilibrium selected by the rigid price process parameterized by* $(p(0), \pi)$ *consists of market tightness, consumption, nominal interest rate, level of nominal money supply, growth rate of nominal money supply, and real money balances,* $(x, c, i, M(0), g, m)$*, such that the following conditions hold:*

- i *is determined by* $c^{LM}(i, m, x) = c^{IS}(i, \pi, x)$*;*
- x *is determined by* $c^{AD}(m, \pi, x) = c^{AS}(x)$*;*
- c *is determined by* $c = c^{AS}(x)$*;*
- $M(0)$ *is determined by monetary policy;*
- *the growth rate of nominal money supply is determined by the inflation rate:* $g = \pi$*;*

[15] For instance, Christiano, Eichenbaum and Evans [1999] find that after a contractionary monetary policy shock, the price level remains constant for roughly 18 months before declining slightly. Furthermore, the decline is not statistically significant. See Figure 2 in Christiano, Eichenbaum and Evans [1999]. More generally, they assess that the empirical literature has converged to the consensus that monetary policy shocks barely contribute to movements of the aggregate price level.

- *m is determined by $M(0)$: $m = M(0)/p(0)$.*

The indeterminacy of the equilibrium that we discussed in Section 2.4 is resolved because we impose the equilibrium selection criterion that the price follows a rigid price process. In steady state, the price grows at a constant, exogenous inflation rate π. The nominal money supply, $M(t)$, must also grows at rate π but monetary policy does not control π. Hence, changing the growth rate of $M(t)$ is not within the scope of the steady-state analysis under rigid price process. The price level is unaffected by macroeconomic shocks or monetary policy, while monetary policy controls the level of nominal money supply. Therefore, monetary policy controls real money supply.

4.1 Aggregate Demand and Aggregate Supply Shocks

We use comparative steady states to describe the response of market tightness, consumption, output, and interest rate to aggregate demand and aggregate supply shocks. Since we study the steady-state equilibrium under the rigid price process, the inflation rate π does not respond to shocks. With no change to monetary policy, real money balances does not change either. Table 1 summarizes these comparative steady states and Figure 6 illustrates them.

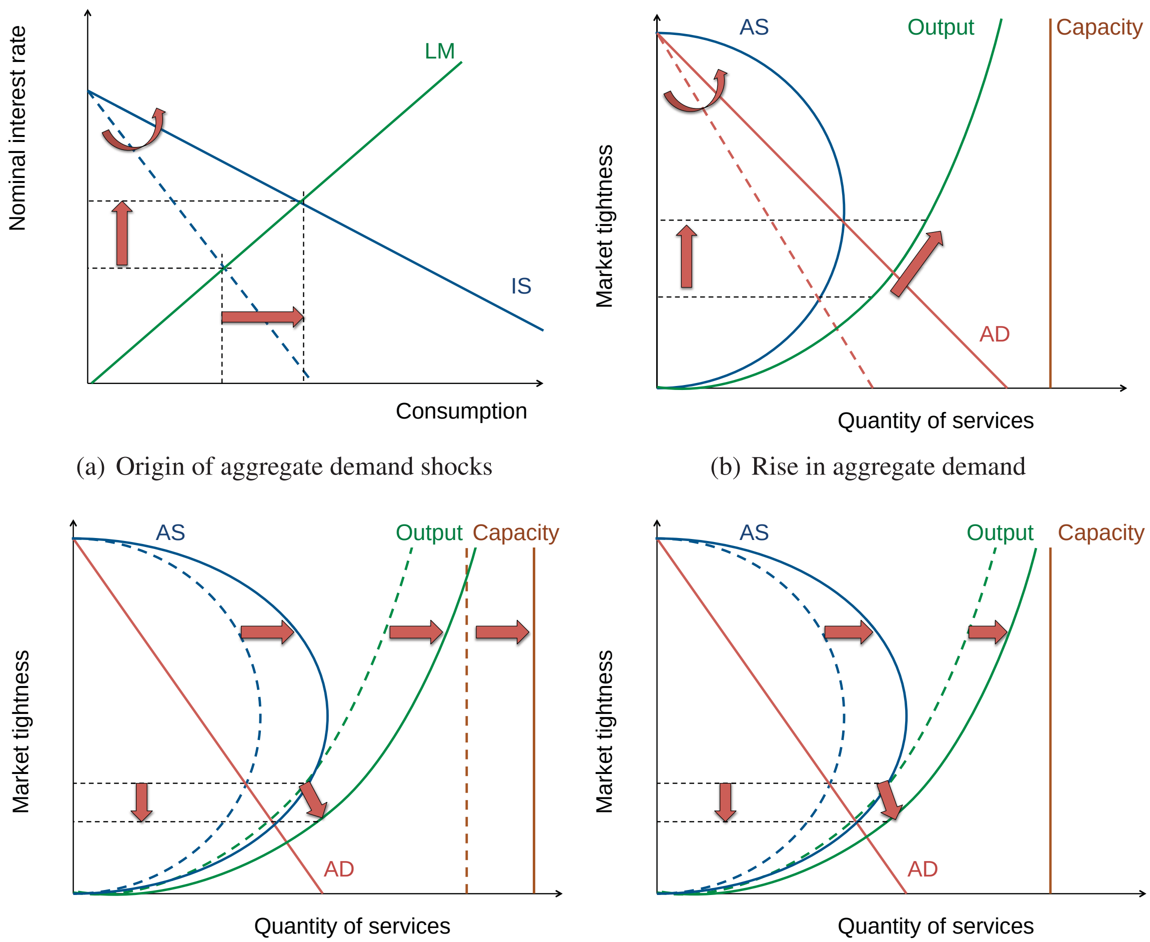
We first consider the effects of aggregate demand shocks. By construction, an aggregate demand shock is caused by a shift in the IS curve due to a change in the discount rate δ or preference for wealth $\omega'(0)$. A positive aggregate demand shock shifts the IS curve out as depicted in Figure 6(a) and hence shifts the AD curve out as depicted in Figure 6(b). This shock leads to increases in interest rate, tightness, and output. In a slack economy ($x < x^*$), consumption increases. In a tight economy ($x > x^*$), consumption decreases. At potential ($x = x^*$), an aggregate demand shock has no first-order effect on consumption.

Next, we consider the effects of aggregate supply shocks. We consider two types of aggregate supply shocks. In Figure 6(c), we consider an increase in capacity, k. This increase shifts out the vertical capacity line, the output curve, and the AS curve. As the AD curve does not change, tightness falls, consumption increases, and unemployment rate also increase. It can also be shown that output increases. In Figure 6(d), we consider a fall in mismatch. We model a fall in mismatch shock as an increase in matching efficiency along with a corresponding increase in matching costs: $h(k,o)$ becomes $\lambda \cdot h(k,o)$ with $\lambda > 1$ and ρ becomes $\lambda \cdot \rho$. The consumption-maximizing tightness is unaffected by a mismatch shock. The function τ is also unaffected so the AD curve does not change. However, a fall in mismatch shifts the AS and output curves. As a result, tightness decreases, consumption increases, and output also increases while capacity stays constant. Hence the unemployment rate falls, in contrast to the unemployment rate response to an increase in capacity.

Panel A in Table 1 summarizes these comparative statics. Panel B presents the alternative case in which monetary policy maintains the nominal interest rate, i, at a constant level. The real money supply must be adjusted to maintain the interest rate constant. This corresponds to actual monetary policy which targets the nominal interest rate. Panel B reports in the last column how real money supply needs to adjust to maintain the nominal interest rate constant. After a rise in aggregate demand, the IS curve shifts outward relative to the LM curve (in addition, both may shift by the same amount because of a change in market tightness). As a consequence, the monetary authority must increase real money supply to keep the interest rate constant. The AD curve rotates outward, and the rise aggregate demand has the same effects on consumption, tightness, and output as under the alternative monetary policy. A rise in aggregate supply has the same effect as under the alternative monetary policy because an aggregate supply shock has no effect on the interest rate.

The effects of shocks are broadly the same at the zero lower bound on nominal interest rate and away from it. This property distinguishes our model from New Keynesian models. In New Keynesian models, aggregate supply shocks have opposite effects at the zero lower bound and away from it. Negative aggregate supply shocks are contractionary away from the zero lower bound, but surprisingly, they are expansionary at the zero lower bound.[16] In our model, at the zero lower bound, the AS curve is exactly the same and the AD curve is flatter, as showed in Figure 7. Since the curves retain the same properties at the zero lower bound,

[16] Eggertsson [2010, 2011, 2012] and Cochrane [2013] discuss the paradoxical effects of aggregate supply shocks at the zero lower bound in New Keynesian model. In the data, unlike what the New Keynesian model predicts, negative aggregate supply shocks seem contractionary at the zero lower bound [Wieland, 2013].



(a) Origin of aggregate demand shocks

(b) Rise in aggregate demand

(c) Supply shock: Rise in capacity

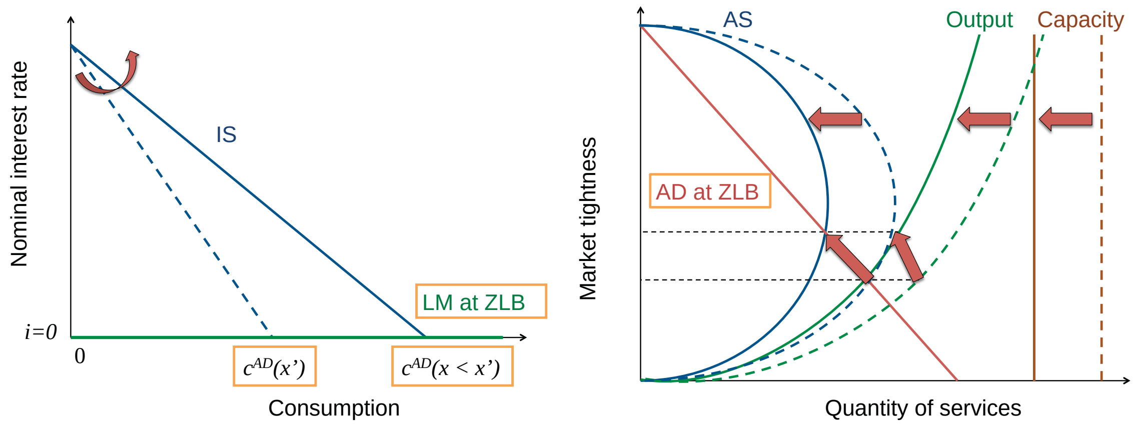
(d) Supply shock: Fall in mismatch

Figure 6: Aggregate demand and aggregate supply shocks with rigid price process

shocks have similar effects. In particular, Figure 7(b) shows that a negative aggregate supply shock is contractionary at the zero lower bound: after a fall in capacity, both consumption and output decrease.

4.2 Monetary Policy

Monetary policy chooses the level of nominal money supply, $M(0)$. A change in $M(0)$ leads to a change in real money supply, m. Monetary policy cannot change the growth rate of $M(t)$, which must satisfy the steady-state requirement that $g = \dot{M}(t)/M(t) = \pi$. We use



(a) Construction of AD curve

(b) Fall in aggregate supply

Figure 7: Aggregate supply shocks at the zero lower bound (ZLB)

comparative steady states to describe the response of market tightness, consumption, output, and interest rate to a change in real money supply. Table 2 summarizes the results.

Away from the Liquidity Trap: $m < m^*$ and $i > 0$. When $m < m^*$, $\phi'(m) > 0$ and an increase in m can stimulate the economy by shifting out the LM curve and hence stimulating aggregate demand: the AD curve, given by (11), shifts outward when m increases. Monetary policy can control m and affects aggregate demand as along as $i > 0$. Monetary policy can fully accommodate shocks that make the economy tighter by reducing m and shocks that make the economic slacker by increasing m as long as the zero lower bound does not bind.

Suppose that the government wants to use monetary policy to keep the economy at potential, where tightness is at its consumption-maximizing level ($x = x^*$). Starting from potential, a negative aggregate demand shock lowers tightness and needs to be accommodated by an increase in m. Symmetrically, a positive aggregate demand shock increases tightness and needs to be accommodated by a decrease in m. A negative aggregate supply shock (or a rise in mismatch) increases tightness and hence needs to be accommodated by a decrease in m. Symmetrically, a positive aggregate supply shock (or a fall in mismatch) needs to be accommodated by an increase in m. In sum, in our model, monetary policy should be guided by tightness rather than output.

Table 1: Effects of aggregate shocks with rigid price process

	Tightness	Consumption	Output	Interest rate
	Panel A: fixed real money supply			
	Tightness	Consumption	Output	Interest rate
Rise in aggregate demand	+	+ (slack) 0 (potential) - (tight)	+	+
Rise in aggregate supply	-	+	+	0
	Panel B: fixed interest rate			
	Tightness	Consumption	Output	Real money
Rise in aggregate demand	+	+ (slack) 0 (potential) - (tight)	+	+
Rise in aggregate supply	-	+	+	0

Notes: Panel A describes comparative steady states when the monetary policy maintains the real money supply, $m = M/p$, at a constant level. Panel B describes comparative steady states when the monetary policy maintains the nominal interest rate, i, at a constant level. A rise in aggregate demand results from a fall in the discount rate, δ, or a decrease in the marginal utility of wealth, $\omega'(0)$. A rise in aggregate supply results from an increase in workers' capacity, k (or a fall in mismatch). Given that inflation is fixed, both nominal and real interest rate move similarly. Their movement is described in the column for the interest rate in panel A. The comparative steady states are explained in Section 4.1.

In a Liquidity Trap: $m \geq m^*$ and $i = 0$. When $m \geq m^*$, $\phi'(m) = 0$ and real money balances does not enter into the AD curve, as seen in (11). Therefore, monetary policy cannot accommodate shocks anymore. Monetary policy becomes ineffective. This is a liquidity trap situation.[17] The economic intuition is the following. When $i = 0$, money dominates bonds because it offers the same return and enters positively the utility function. Hence, demand for money reaches the bliss point m^* where marginal utility of money is zero. In that situa-

[17] A large recent literature discusses the liquidity trap and its impact on the effectiveness of monetary and fiscal policy. Motivated by Japan's experience in the 1990s, Krugman [1998] captures temporary liquidity traps using a modern macroeconomic model. Following Krugman's seminal work, a number of studies have analyzed temporary liquidity traps in the context New Keynesian models. See for instance Eggertsson and Woodford [2003, 2006], Eggertsson [2011], Eggertsson and Krugman [2012], Christiano, Eichenbaum and Rebelo [2011], Fernández-Villaverde, Guerrón-Quintana and Rubio-Ramírez [2011], Coibion, Gorodnichenko and Wieland [2012], Werning [2012], Rendhal [2012], Correia et al. [2013], and Wieland [2013]. Mankiw and Weinzierl [2011] capture liquidity traps in a simpler two-period model.

Table 2: Effects of rise in real money supply with rigid price process

	Tightness	Consumption	Output	Interest rate
Away from liquidity trap ($m < m^*$)	+	+ (slack)	+	+
		0 (potential)		
		- (tight)		
In liquidity trap ($m \geq m^*$)	0	0	0	0

Notes: The comparative steady states are derived in Section 4.2. Given that inflation is fixed, both nominal and real interest rate move similarly. Their movement is described in the column for the interest rate.

tion, adding money in the economy by buying more bonds is a pure wash: individuals issue bonds to purchase the equivalent money asset. Hence, nothing changes in the economy.

Equation (11) shows that, in a liquidity trap, increasing inflation still affects aggregate demand. Hence, monetary policy could be effective if it could change inflation. But monetary policy has no direct effect on inflation under a rigid price process, so monetary policy is indeed ineffective. This is consistent with the empirical fact that liquidity traps can last a long time and that large increases in money supply do not translate into increases in inflation, as in Japan in the 1990s or the United States and the European Union since 2007.

The Friedman rule maximizes welfare but brings the economy in a liquidity trap. Thus, monetary policy cannot accommodate an adverse demand shock under the Friedman rule and a rigid price process. This implies that there is a genuine trade-off between inflation and monetary accommodation. Low inflation is desirable for the Friedman rule reason but it puts the economy at risk of falling into a liquidity trap and hence preventing macroeconomic stabilization through monetary policy. The formalization of this trade-off and the analysis of the optimal π is left for future work.[18]

[18] Coibion, Gorodnichenko and Wieland [2012] discuss, in the context of a New Keynesian model, how central bank should adjust inflation targets when the economy faces a positive probability of falling into a liquidity trap.

4.3 Other Policies in Liquidity Traps

Monetary policy is not effective in a liquidity trap. We present two alternative policies that remain effective in this situation.

Helicopter Drop of Money. Instead of issuing money by buying bonds on the open market, the government could directly print money and give it to individuals.[19] Assume that money comes from two sources: a quantity $M^b(t)$ of money is issued by buying bonds through open market operations and a quantity $M^h(t)$ of money given directly to individuals through a helicopter drop. Let $m^b(t) \equiv M^b(t)/p(t)$ and $m^h(t) \equiv M^h(t)/p(t)$ be the corresponding real quantities of money. The total quantity of money is $M(t) = M^b(t) + M^h(t)$. Open market operations impose that $B(t) = -M^b(t)$.

In equilibrium, real wealth is

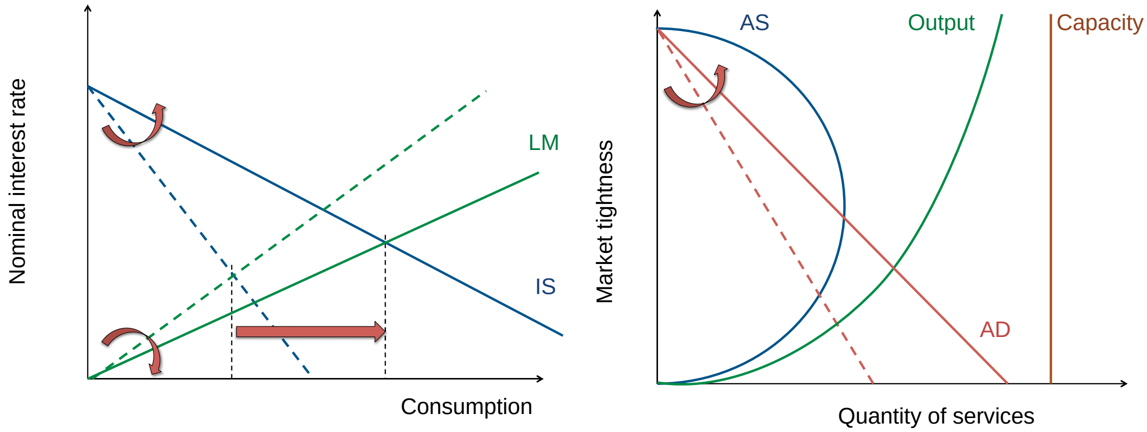
$$a(t) = \frac{B(t)}{p(t)} + \frac{M^b(t)}{p(t)} + \frac{M^h(t)}{p(t)} = \frac{M^h(t)}{p(t)} = m^h(t).$$

Helicopter money contributes to real wealth, and real wealth is no longer zero in equilibrium. With helicopter money, our previous analysis carries over by simply adjusting the marginal utility of wealth from $\omega'(0)$ to $\omega'(m^h)$. The LM curve is unchanged but the IS curve now depends on helicopter money:

$$c^{IS}(i,\pi,x,m^h) = \left[\frac{\delta+\pi-i}{(1+\tau(x))\cdot\omega'(m^h)}\right]^{\varepsilon}.$$

Since the utility of wealth, ω, is concave, an increase in helicopter money shifts the IS curve outward in a (c,i) plane, as showed in Figure 8(a). It also shift the LM curve outward because an increase in helicopter money raises the total money supply. The AD curve now depends

[19] The value of an helicopter drop of money to fight deflation was made originally by Friedman [1969] and has been reiterated by Bernanke [2002].



(a) IS-LM diagram (b) AS-AD diagram

Figure 8: Effects of an helicopter drop of money

on both total money and helicopter money:

$$c^{AD}(m, \pi, x, m^h) = \left[\frac{\delta + \pi}{(1 + \tau(x)) \cdot (\phi'(m) + \omega'(m^h))} \right]^{\varepsilon}.$$

An increase in helicopter money shifts the AD curve outward in a (c, x) plane, as showed in Figure 8(b).

Open-market money stimulates AD through the LM channel when the economy is not in a liquidity trap. In a liquidity trap, the LM curve does not respond to real money supply and open-market money has no effect on the AD curve. Helicopter money stimulates AD through both the LM channel and the IS channel. The IS channel is immune to the liquidity trap so helicopter money is able to bring the economy out of the trap.

The superior efficacy of helicopter money over open-market money relies critically on the curvature of the utility of wealth. With linear utility of wealth, $\omega'(m^h)$ is constant and helicopter money does not shift the IS curve. In that case, helicopter money is not more effective than open-market money.

One drawback of a policy based on helicopter money is that it is harder to reverse than a policy based on open market operations. Effectively, reversing a helicopter drop of money would require taking away money from individuals with no compensation—taxing money held by individuals and destroying the taxed money.

Tax on Wealth. Another way to stimulate the economy is to tax wealth at rate $\tau^a(t)$.[20] The wealth tax applies to the entire wealth (both bond holdings and money balances). The tax raises no revenue as the aggregate wealth is zero. But the tax changes the law of motion of the consumer's wealth and the consumption Euler equation. The law of motion becomes

$$\dot{a}(t) = f(x(t)) \cdot k - (1+\tau(x(t))) \cdot c(t) - i(t) \cdot m(t) + (r(t) - \tau^a(t)) \cdot a(t) + s(t).$$

Therefore, the consumption Euler equation becomes

$$(1+\tau(x(t))) \cdot \frac{v_a(c(t),m(t),a(t))}{v_c(c(t),m(t),a(t))} + (r(t) - \tau^a(t) - \delta) = -\frac{\dot{\mu}(t)}{\mu(t)},$$

and the IS curve admits a new expression:

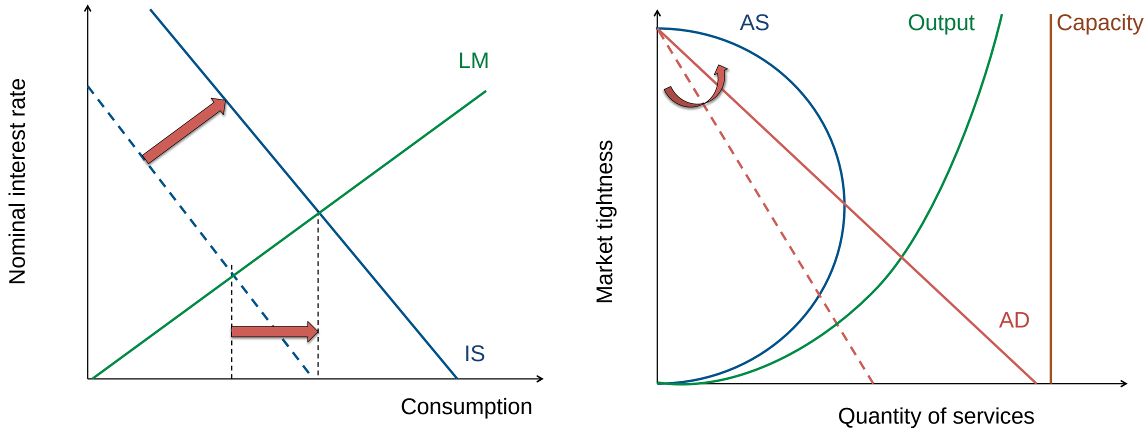
$$c^{IS}(i,\pi,x,\tau^a) = \left[\frac{\delta + \tau^a + \pi - i}{(1+\tau(x)) \cdot \omega'(0)}\right]^{\varepsilon}.$$

A rise in the wealth tax is equivalent to a rise in inflation; hence, a rise in the wealth tax shifts the IS curve outward in a (c,i) plane, as showed in Figure 9(a). The other optimality condition from the consumer's problem, given by equation (8), is not affected by the wealth tax so the LM curve remains the same. The AD curve is now a function of the wealth tax:

$$c^{AD}(m,\pi,x,\tau^a) = \left[\frac{\delta + \tau^a + \pi}{(1+\tau(x)) \cdot (\phi'(m) + \omega'(0))}\right]^{\varepsilon}.$$

A rise in the wealth tax is equivalent to a rise in inflation. Hence, a rise in the wealth tax leads to a positive aggregate demand shock: it shifts the AD curve outward in a (c,i) plane, as showed in Figure 9(b). Since the wealth tax acts on the IS curve and not the LM curve,

[20] Correia et al. [2013] obtain a related result in the New Keynesian model where they show that replacing a labor income tax by a value-added tax can lift the economy of the liquidity trap. Indeed, a new value-added tax is equivalent to a new tax on labor income and a one-time tax on existing wealth (in our simple closed economy model with no real investment). In Correia et al. [2013], a one time wealth tax is sufficient in a temporary liquidity trap. In our model with a permanent liquidity trap, a permanent wealth tax is needed.



(a) IS-LM diagram

(b) AS-AD diagram

Figure 9: Effects of a positive tax on wealth

the wealth tax remains effective in a liquidity trap. The intuition for the effectiveness of the tax is the following: taxing wealth makes wealth and future consumption more costly and hence less desirable, hereby stimulating current consumption.

A wealth tax applied to bonds only has no effect on aggregate demand.[21] A wealth tax applied to money only stimulates aggregate demand. Imposing a wealth tax on money only is economically equivalent to raising inflation.[22]

4.4 Relationship with the Traditional AD Curve

As in our model, the AD curve used in the IS-LM-Phillips curve model is the amount of output given by intersection of the IS and LM curves. Unlike in our model, the goods market is not in equilibrium in the IS-LM-Phillips curve model, and the prevailing amount of output is determined directly by aggregate demand for a given price and inflation.

We can obtain the traditional AD curve as a special case of our model with no matching costs ($\rho = 0$ and thus $\tau(x) = 0$). With no matching costs, consumption equals output and

[21] Such a tax only affects equilibrium interest rates, which respond one-for-one to the tax rate.

[22] There could be practical difficulties to taxing money. Cash in the hands of individuals would be almost impossible to tax. Deposits at financial institutions are monitored and could be taxed. In the real world, there is a continuum of assets between money and bonds. Hence, a tax on both bonds and wealth would be more practical.

the IS and LM curves are independent of tightness. Thus, the IS and LM curves determine equilibrium output as a function of price and inflation. The AD curve becomes

$$c^{AD}(m,\pi) = \left[\frac{\delta+\pi}{\phi'(m)+\omega'(0)}\right]^{\varepsilon}.$$

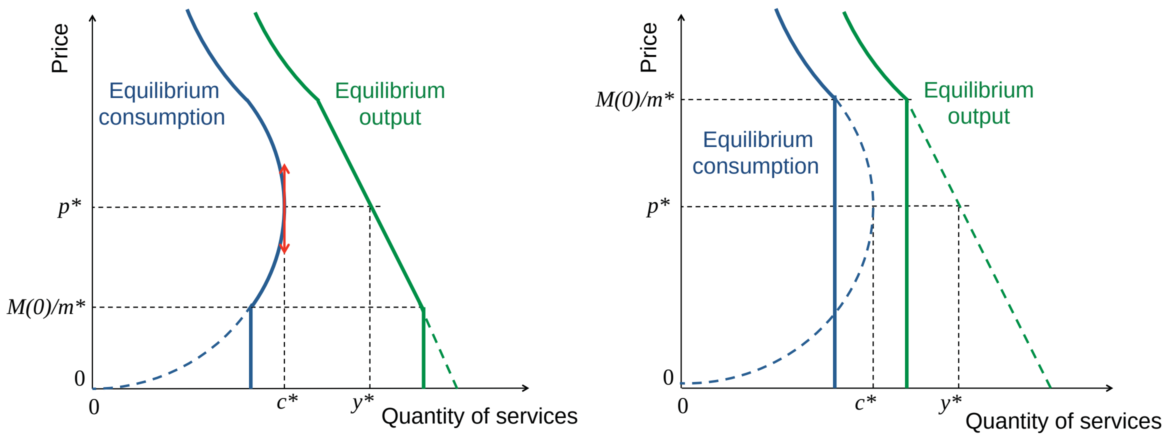
In that case, in a (output,tightness) plane, the AD curve would be perfectly inelastic and the AS curve would be the same as the output curve in Figure 5. The equilibrium condition that aggregate supply equals aggregate demand simply determines equilibrium tightness as a residual; it has no impact on equilibrium output.

In the more general case with positive matching cost, the prevailing amount of output for a given price and inflation is determined by the intersection of aggregate demand and aggregate supply. We denote the levels of consumption and output in steady-state equilibrium under rigid price process with initial price level $p(0) = p$ and inflation rate π by $c^{eq}(p,\pi)$, and $y^{eq}(p,\pi)$. The prevailing amounts of output and consumption for a given price and inflation are $y^{eq}(p,\pi)$ and $c^{eq}(p,\pi)$. In Figure 10(a), we depict $y^{eq}(p,\pi)$ and $c^{eq}(p,\pi)$ in a (quantity, price) plane. The curve $c^{eq}(p,\pi)$ is downward sloping when the economy is slack and then upward sloping when the economy is tight. When the price is below $M(0)/m^{*}$, the economy is in a liquidity trap and the $c^{eq}(p,\pi)$ curve becomes vertical as changes in price no longer affect equilibrium consumption. Output $y^{eq}(p,\pi)$ is always downward sloping until it becomes vertical in the liquidity trap.

The figure also depicts the maximum consumption level $c^{*} = [f(x^{*}) - \rho \cdot x^{*}] \cdot k$ and the price p^{*} that delivers it. At that point, the tangent to the $c^{eq}(p,\pi)$ curve is vertical. The corresponding output level is $y^{*} = f(x^{*}) \cdot k$. Figure 10(b) depicts the same curves when the price p^{*} is below $M(0)/m^{*}$. In that case, the liquidity trap prevents c^{*} and y^{*} to be reached for any price.

5 Analysis with Flexible Price Process

We focus on the equilibrium arising under a flexible price process, defined as follows:



(a) Small liquidity-trap interval

(b) Large liquidity-trap interval

Figure 10: Steady-state equilibria for different price levels under rigid price process

DEFINITION 10. $[p(t)]_{t=0}^{+\infty}$ *is a* ***flexible price process*** *if* $p(t)$ *jumps so that market tightness always maximizes consumption. The growth rate of* $p(t)$ *always equals the growth rate of nominal money supply.*

Under this criterion, the price may jump in response to unexpected shocks to ensure that tightness always maximizes consumption. This criterion could be justified in two ways. First, it can be seen as the extension of the market-clearing price from Walrasian theory. With no matching costs ($\rho = 0$), infinite tightness x^* maximizes consumption. But infinite tightness also ensures that sellers can sell all their services (as $\lim_{x\to+\infty} f(x) = 1$). Therefore, the consumption-maximizing tightness is attained when the market clears and there is no unemployment. Second, market forces should drive price and tightness toward consumption maximization. If tightness does not maximize consumption, both buyers and sellers would be better off with a price adjustment and a corresponding tightness adjustment assuming that real money supply stays constant. This would be the case if the price and tightness change happens only in a small sector of the economy so that real money balances are not affected by the change.[23] In other words, the market force driving price and tightness toward efficiency cannot internalize the effect on real money balances.[24] Under the flexible price process, we

[23]The idea is that consumers shop from many different sectors and that matching costs occur within sectors.

[24]The same happens in the model of Sidrauski [1967]. The clearing price is not necessarily welfare maxi-

also assume that the growth rate of nominal money supply determines the inflation rate.[25] In sum, this criterion represents the ideal market where prices adjust immediately to the state of the economy to maximize consumption. It could be a good representation of the medium or long run.

Let $g = \dot{M}(t)/M(t)$ be the growth rate of nominal money supply and $M(0)$ be initial money supply. Both g and $M(0)$ are set by monetary policy. Under the flexible price process, the steady state is characterized as follows:

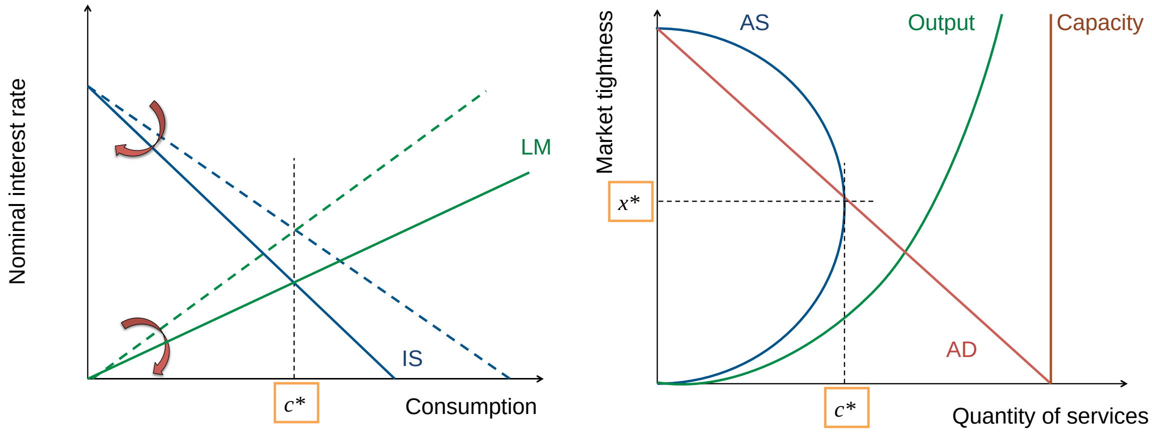
PROPOSITION 3. *The steady-state equilibrium selected by the flexible price process consists of market tightness, consumption, nominal interest rate, level of price, inflation rate, level of nominal money supply, growth rate of nominal money supply, and real money balances, $(x, c, i, p(0), \pi, M(0), g, m)$, such that the following conditions hold:*

- *$M(0)$ and g are determined by monetary policy;*
- *the inflation rate is determined by the growth rate of nominal money supply: $\pi = g$;*
- *i is determined by $c^{LM}(i, m, x) = c^{IS}(i, \pi, x)$;*
- *c is determined by $c = c^{AS}(x)$;*
- *if $c^{AD}(m^*, \pi, x^*) > c^*$, $x = x^*$ and m is determined by $c^{AD}(m, \pi, x^*) = c^*$;*
- *if $c^{AD}(m^*, \pi, x^*) < c^*$, $m = m^*$ and x is determined by $c^{AD}(m^*, \pi, x) = c^{AS}(x)$;*
- *$p(0)$ is determined by $m = M(0)/p(0)$.*

Consumption is maximized when $x = x^*$; therefore, with a flexible price process, $p(0)$ adjusts so that $m = M(0)/p(0)$ meets the condition $c^{AD}(m, \pi, x^*) = c^*$ where c^* is maximum consumption. This equilibrium is possible whenever the resulting m is below the bliss point m^* and the nominal interest rate is positive. If $c^{AD}(m^*, \pi, x^*) < c^*$, then any additional price

mizing in the sense that the Friedman rule does not automatically hold.

[25] This assumption could be replaced by the alternative assumption that the inflation rate determines the growth rate of nominal money supply. As discussed below, this alternative assumption modifies our findings only in a liquidity trap.



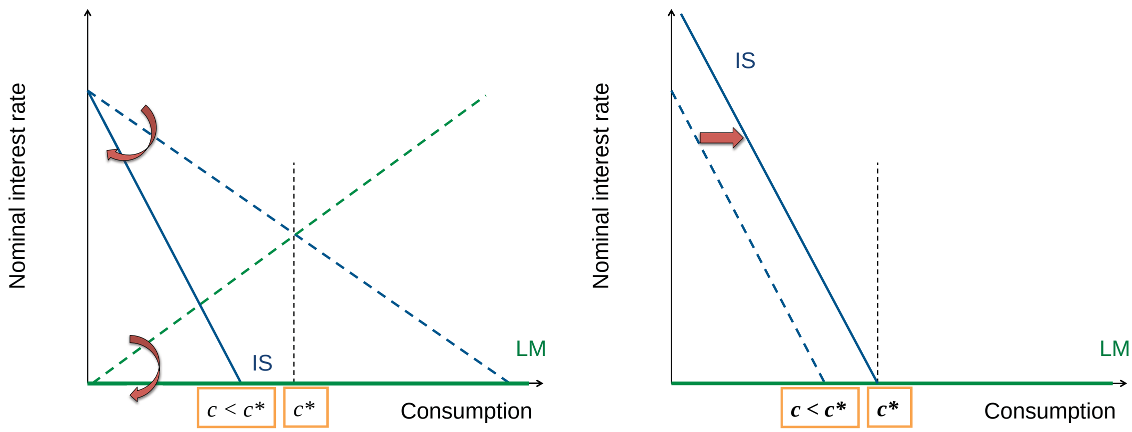
(a) IS-LM diagram (b) AS-AD diagram

Figure 11: Effects of a small fall in aggregate demand with flexible price process

level adjustment cannot bring aggregate demand to c^*. In that case, we assume that the price level adjusts so that $m = m^*$ as any further price drop would not stimulate demand further. This is a strong assumption as, in a liquidity trap, the price level has no effect on the economy. Therefore, there is no deep reason why it should adjust to keep m at m^*. As we shall see, this has important consequences for the efficacy of monetary policy in a liquidity trap.

5.1 Away from the Liquidity Trap

Away from the liquidity trap and under a flexible price process, aggregate demand shocks have zero impact on c and x while aggregate supply shocks have an impact on c but not on x. Figure 11 illustrates the effects of a small fall in aggregate demand under flexible price process. The fall in aggregate demand leads to small shift inward of the IS curve, as showed in Figure 11(a). The fall in aggregate demand is small in the sense that the intersection of the IS curve with the x-axis in the (c,i) plane is larger than c^*. After the shift of the IS curve, the price p falls such that real money balances $m = M(0)/p(0)$ rise and the LM curve rotates outward. The response of the LM curve exactly balances the initial shift of the IS curve such that consumption remains at c^* for the underlying tightness level, x^*. Since the



(a) Without monetary policy accommodation

(b) With an increase in money growth rate

Figure 12: Effects of a large fall in aggregate demand with flexible price process

price adjustment fully absorbs the aggregate demand shock, nothing happens in the AS-AD diagram, as showed in Figure 11(b).

Similarly, an increase in nominal money supply has no impact on real money balances because it is fully absorbed by a price increase. An increase in the money growth rate translates into an increase in inflation which shifts the IS curve out. However, the shift of the IS curve is exactly balanced by a response of the LM curve. When inflation increases, the price jumps up, real money balances fall, and the LM curve rotates inward such that consumption remains at the consumption-maximizing level c^*.

Effectively, money is a veil and aggregate demand is irrelevant with a flexible price process. Therefore, while the IS-LM curves can still be defined, they are irrelevant for understanding the economy.

5.2 In a Liquidity Trap

Figure 12 illustrates the effects of a large fall in aggregate demand under flexible price process. The fall in aggregate demand leads to a large shift inward of the IS curve, as showed in Figure 12(a). The fall in aggregate demand is large in the sense that the intersection of the IS curve with the x-axis in the (c,i) plane is smaller than c^* so that the economy falls in the liquidity trap. After the shift of the IS curve, the price $p(0)$ falls such that real money balances $m = M(0)/p(0)$ rise and the LM curve rotates outward. The LM curve rotates entirely until it becomes horizontal. At that point, the economy is in the liquidity trap and aggregate demand shocks have real effects. The response of the LM curve cannot offset the initial shift of the IS curve; therefore, consumption remains below c^* and tightness remains below x^*. The economy is slack.

Monetary policy is required to stimulate the economy in a permanent liquidity trap.[26] With flexible prices, we have assumed that monetary policy can affect inflation immediately: the growth rate of nominal money supply, $M(t)$, determines inflation, π. Increasing the growth rate of $M(t)$ immediately leads to a higher π which can pull the economy out of the liquidity trap, as illustrated in Figure 12(b). Higher inflation shifts the IS curve outward, which stimulates aggregate demand.

Hence, with a flexible price process, it is always welfare maximizing to follow the Friedman rule of zero nominal interest rate. The Friedman rule has the following implications for monetary policy, as showed by (12). Reductions in aggregate demand (a fall in δ or an increase in $\omega'(0)$) are accommodated with a higher money growth rate and hence higher inflation. Conversely, increases in aggregate demand are accommodated with lower money growth rate and hence lower inflation. Reductions in aggregate supply (a fall in k) are accommodated with a lower money growth rate and hence lower inflation. Increases in aggregate supply are accommodated with higher money growth rate and hence higher inflation.

These results are of course modified if we do not make the strong assumption that money growth affects inflation immediately in a liquidity trap. Indeed, the flexible price adjustment is predicated on the idea that a change in price can benefit both the seller and the buyer. In a liquidity trap, a price level change has no effect. An increase in inflation does stimulate aggregate demand but there is no obvious market mechanism that can translate a higher money growth rate into a higher inflation. If monetary policy cannot affect inflation in a

[26] Our model of a permanent liquidity trap contrasts with the modern New Keynesian literature, which considers transitory liquidity traps. With flexible prices, a sufficiently large drop in price followed by inflation can lift the economy out of a transitory liquidity trap [Krugman, 1998]. In our model, this inflation solution cannot be sustained in steady state without a corresponding change in the growth rate of the nominal money supply.

liquidity trap then conventional monetary policy cannot pull the economy out of the liquidity trap.

5.3 Relationship with the Neoclassical Model

Our framework nest the neoclassical model as a special case. By "neoclassical model", we mean a model with a Walrasian market for services. Indeed, our framework with the flexible price process as equilibrium selection criterion and zero matching cost ($\rho = 0$ and hence $\tau(x) = 0$) corresponds exactly to the neoclassical model.

With zero matching cost, consumption equals output and the consumption-maximizing tightness is infinite ($x^* = \infty$). Since $\lim_{x \to +\infty} f(x) = 1$, all services are sold and there is no unemployment at x^*. Since the flexible price process guarantees that tightness is always at $x^* = \infty$, it guarantees that the market for services clears in the sense that all the services supplied by workers are sold: $c = k$ and unemployment is zero. To summarize, our flexible-price equilibrium selection criterion boils down to a Walrasian market-clearing criterion when matching costs are zero.

6 Phillips Curve

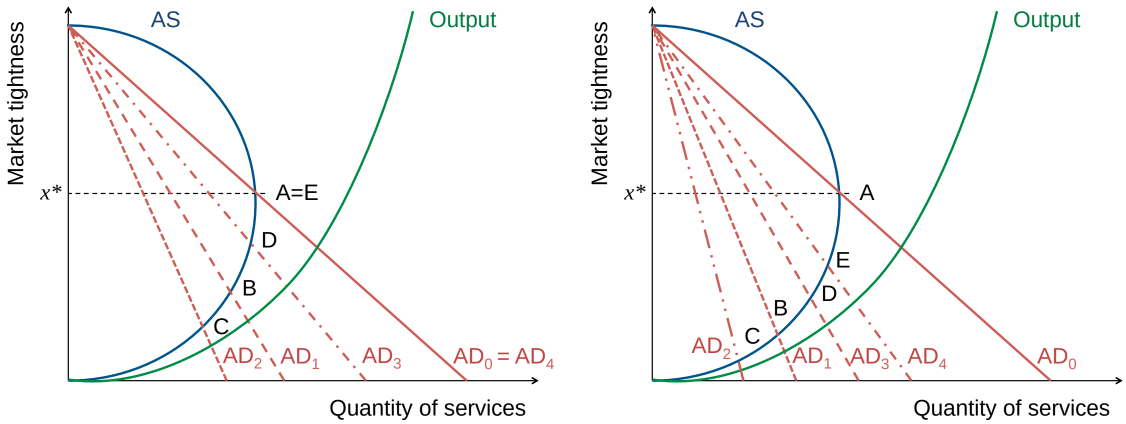
The Phillips curve is a differential equation for the path of prices $[p(t)]_{t=0}^{+\infty}$. The traditional specification of the Phillips curve relates inflation to unemployment and other macroeconomic variables. It can be seen as a first-order differential equation in $p(t)$. The accelerationist Phillips curve relates changes in inflation to unemployment and other macroeconomic variables. It can be seen as a second-order differential equation in $p(t)$. Since the Phillips curves specifies a path of prices $[p(t)]_{t=0}^{+\infty}$, a given Phillips curve selects one equilibrium of the model. The rigid price and flexible price process of Sections 4 and 5 are special forms of Phillips curves, which likely describe well the short run and the long run respectively. In this section, we provide an informal discussion of the possible path of prices over time, from their current position toward the flexible-price position.

We postulate that if consumption is not maximized, prices adjust to increase the level of consumption. However, we postulate that this adjustment is not immediate. In other words, prices adjust sluggishly to maximize consumption. Hence, the price is a state variable and the evolution of prices over time is described by a Phillips curve where prices increase relative to trend when the economy is tight and prices decrease relative to trend when the economy is slack. Hence, inflation is higher relative to its steady-state level when the economy is tight and lower when the economy is slack. Naturally, once the economy enters the liquidity trap, further adjustments in prices have no effect on consumption so we assume that price adjustments just tend toward keeping real money at the bliss point m^* in that case. The long-run inflation rate π^* is given by the long-run rate of money growth. There is no long-run trade-off between inflation and unemployment because unemployment reverts to its natural rate, $1 - f(x^*)$, no matter what the long-run inflation rate π^* is.

In what follows, we describe informally how the economy responds to aggregate demand and supply shock with a price process described by such a Philips curve. The formal description would require solving the system of ordinary differential equations that describe the equilibrium under a general price process, which we plan to do in the future. We start from the consumption-maximizing equilibrium with tightness x^* and then consider a specific shock. We first assume that there is no monetary accommodation: the nominal money supply, $M(t)$, passively grows at a constant rate π^*. Next, we discuss the effects of active monetary policy for stabilization.

6.1 Aggregate Demand Shocks

Suppose that there is a negative aggregate demand shock, as illustrated in Figure 13(a). On impact, the price does not adjust so the AD curve rotates inward from AD_0 to AD_1. The equilibrium moves from point A to point B, and both tightness and output fall. The Phillips curve implies that there is a downward pressure on inflation because the economy is slack ($x(t) < x^*$). Three effects take place consecutively. First, the reduction in inflation exacerbates the initial negative aggregate demand shock by shifting the IS curve inward, following



(a) No liquidity trap (b) Liquidity trap

Figure 13: Aggregate demand shocks under Phillips curve

the mechanism showed in Figure 3(c). This shift in the IS curve further depresses the AD curve, which rotates from AD_1 to AD_2. The equilibrium moves from point B to point C, and both tightness and output fall further. Second, the fall in price relative to nominal money balances slowly increases real money balances. This increase in real money supply progressively shifts the LM curve outward, as in Figure 3(b). This shift in LM curve stimulates the AD curve, which slowly rotates from AD_2 to AD_3. This rotation moves the equilibrium from point C to point D and raises tightness and output. Third, once the price has completed its adjustment, inflation jumps back to its initial value and the IS curve shifts back outward. This shift in IS curve stimulates the AD curve, which rotates from AD_3 to AD_4. The equilibrium moves from point D to point E as the economy comes back to potential. A positive aggregate demand shock would have symmetrical effects.

Suppose now that the negative aggregate demand shock is large enough to push the economy into a liquidity trap, as illustrated in Figure 13(b). On impact, the AD curve rotates inward from AD_0 to AD_1 and the equilibrium point moves from A to B. Note that the AD curve shifts further inward in Figure 13(b) than in Figure 13(a) because the shock is larger. Since tightness is below x^*, inflation falls, which depresses aggregate demand. The AD curve rotates inward from AD_1 to AD_2 and the equilibrium point moves from B to C. Next, the price falls and real money balances increase, shifting the LM curve outward until the economy enters the liquidity trap, reaching the situation depicted in Figure 4(a). The AD curve rotates outward from AD_2 to AD_3 and the equilibrium point moves from C to D. Once real money balances have reached the bliss point m^*, inflation jumps back to normal, which stimulates aggregate demand. The AD curve rotates further outward from AD_3 to AD_4 and the equilibrium point moves from D to E. At that point, tightness and output are stuck at the intersection of the IS curve and x-axis and cannot reach the consumption-maximizing level of tightness, x^*.[27] This feature of our model is consistent with the absence of deflation in liquidity trap episodes. The Japan experience since the 1990s is characterized by zero inflation rather than deflation. The liquidity traps in Europe and the United States since the Great Recession 2008 have also been characterized by stable and very low inflation rather than deflation. Deflation also stopped in the US Great Depression in 1933 once the zero lower bound for interest rates was reached.[28]

With monetary policy, an adjustment in the level of $M(t)$ could make $m(t)$ jump to absorb fully the aggregate demand shock so that tightness stays at x^*. In that case, no price adjustment is needed and hence inflation stays constant. Monetary can achieve perfect stabilization as long as the economy is away from the liquidity trap. In the liquidity trap situation, a higher growth rate of money will eventually build up inflation expectations and push the economy out of the liquidity trap. This monetary solution could however take time if inflation adjusts very slowly to the rate of growth of money.

6.2 Aggregate Supply Shocks

On impact, the price does not adjust so the AS curve shrinks from AS_0 to AS_1. The equilibrium moves from point A to point B, and tightness increases while output falls. The Phillips curve implies that there is a upward pressure on inflation because the economy is

[27] This sequence corresponds roughly to the first few years of the Great Depression in the United States where price inflation declined sharply from 1929 to 1932, turning into significant deflation and high real interest rates, contributing to make the Great Depression worse, at a time where monetary policy was mostly passive.

[28] In contrast, the textbook IS-LM-Phillips curve model predicts deflation in a liquidity trap: "the puzzle is *why* deflation ended in 1933" [Blanchard, 2009, p. 482]. Similarly, the New Keynesian model predicts deflation in a liquidity trap [for example, Cochrane, 2013; Werning, 2012].

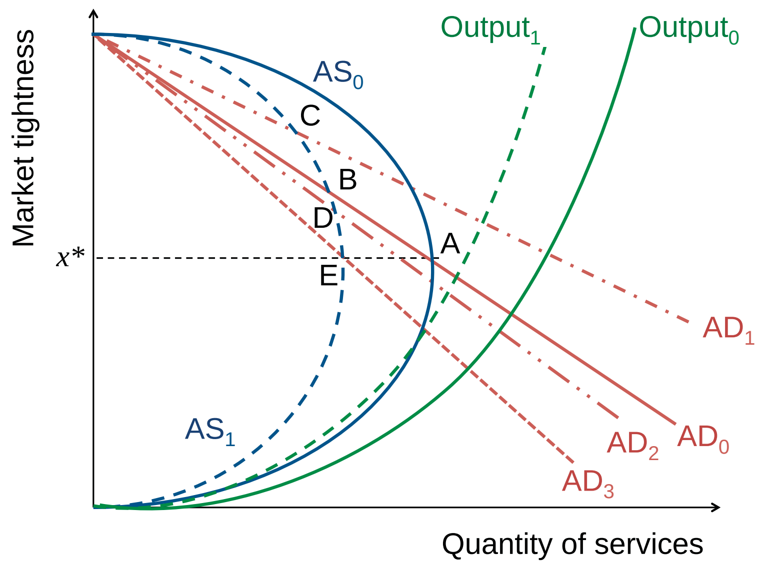


Figure 14: Aggregate supply shocks under Phillips curve

tight ($x(t) > x^*$). Three effects take place consecutively. First, the increase in inflation stimulates aggregate demand by shifting the IS curve outward, as in Figure 3(c). The AD curve rotates outward from AD_0 to AD_1. The equilibrium moves from point B to point C, and both tightness and output increase. Second, the rise in price relative to nominal money balances increases real money balances, shifting the LM curve inward following the mechanism showed in Figure 3(b), and depressing aggregate demand. The AD curve rotates inward from AD_1 to AD_2. This rotation moves the equilibrium from point C to point D, and both tightness and output fall. The economy enters a stagflation: inflation is higher than normal but output falls slowly. As tightness falls back to its consumption-maximizing level, inflation comes back to its initial level. The AD curve rotates from AD_2 to AD_3. The equilibrium moves from point D to point E as the economy comes back to potential. A negative mismatch shock has exactly the same effects as the negative aggregate supply shock. A positive aggregate supply shock would have symmetrical effects. As sufficiently large positive aggregate supply shock could make the economy fall in a liquidity trap.

With monetary policy, an adjustment in the level of $M(t)$ could make $m(t)$ jump to absorb fully the aggregate supply shock so that tightness stays at x^*. In that case, no price adjustment is needed and hence inflation stays constant. A monetary contraction following a negative supply shock creates a faster and actually desirable fall in output. It spares the economy of the need for inflation and hence eliminates the stagflation period. Symmetrically, a monetary expansion is needed to accommodate a positive supply shock.

6.3 Destabilizing Inflation

Our informal analysis shows clearly that there is a conflict between price adjustment and the inflation adjustment. The price adjustment requires an inflation change that further destabilizes the economy, making the recession worse or exacerbating the overheating. This suggests that, even if price adjustments are fairly fast, the temporary inflation changes could amplify short-run fluctuations in tightness and output.[29] We leave for future research a complete numerical simulation.

7 Conclusion

This section concludes by summarizing our findings and discussing possible extensions the model and the analysis.

7.1 Summary

We construct a microfounded, dynamic version of the IS-LM-Phillips curve model by adding two elements to the money-in-the-utility-function model of Sidrauski [1967]. First, real wealth enters the utility function. Consequently, the consumption Euler equation defines in steady state a downward-sloping IS curve in a (consumption, interest rate) plane, instead of the standard horizontal IS curve imposing that the real interest rate equals the subjective discount rate. A downward-sloping IS curve is a critical element to conduct a meaningful monetary policy analysis in steady state. Second, we introduce matching frictions in the

[29] This tension between price-level and inflation adjustments was known in the Keynesian literature and is discussed in detail by Tobin [1993].

labor market following the theory developed by Michaillat and Saez [2013]. The matching frictions allow us to link the aggregate demand coming from the IS-LM system to unemployment. Critically, in presence of matching frictions, equilibrium requires that market tightness equalizes aggregate demand to aggregate supply given a price process. Hence, the model generates a collection of equilibria, one for each possible price process, and it is necessary to specify a price process—a Phillips curve—to select an equilibrium. With a rigid price process, the model captures most of the Keynesian intuitions derived from the old IS-LM model. Aggregate demand shocks affect tightness, unemployment, and consumption. With a positive nominal interest rate, monetary policy affects aggregate demand and can be used for stabilization. When the nominal interest rate hits the zero lower bound, the economy is in a liquidity trap and monetary policy loses its effectiveness. With a flexible price process, aggregate demand shocks and monetary policy have no real effect when the interest rate is positive. In a liquidity trap, aggregate demand matters but monetary policy can restore efficiency if higher money growth generates inflation immediately.

7.2 Possible Extensions

It would be important to introduce firms into the model to make it closer to standard macroeconomic models, with distinct labor and product markets. Following Michaillat and Saez [2013], we could split the service market into a product market and a labor market, and introduce firms who hire workers on the labor market and sell production on the product market. The labor and product market would have the same structure as the service market. This extension would introduce a concept of unemployment that is closer to what statistical agencies measure: the model would feature workers searching for a job and unable to find one instead of self-employed workers unable to sell their services. This extension would also provide a more sophisticated theory of aggregate supply: capacity k, would not not be exogenous but would be determined endogenously by the production decision of firms. Introducing firms would also allow us to introduce productive capital and investment, and would therefore allow us to study the impact of investment decisions on aggregate demand.

Investment is an important determinant of aggregate demand for Keynes [1936], but it is absent from our current model. Last, introducing firms would allow us to consider a broader range of aggregate supply shocks, such as technology or labor force participation shocks.

The model is extremely stylized in other aspects, and it could be fruitfully extended along several dimensions. Fiscal policies, such as public good spending, social insurance, and taxation, should be taken into account. It would also be important to introduce heterogeneity across individuals to capture the fact that some individuals have higher propensity to consume than others. Heterogeneity in preferences would lead to heterogeneity in wealth and consumption. In that context, redistributive tax and transfer policies are likely to generate aggregate demand effects. To understand the effect of financial crises on aggregate demand and unemployment, it would be valuable to add a financial sector to the model. Finally, we conjecture that our methodology could be applied to an open economy as well.

Our analysis of a more general price process in Section 6 is only informal. It would be interesting to estimate empirically a general price process. Estimating the price process is equivalent to estimating a Phillips curve, an endeavor that has already attracted a lot of effort. However, our theory suggests that a key variable to be included in the estimation is the gap between actual tightness and consumption-maximizing tightness. Estimating a Phillips curve with this novel specification may yield new results. These empirical estimates would also provide guidelines to construct a realistic price-setting mechanism that captures the adjustment process of prices in the short and long run.

Our analysis with the general price process has focused almost completely on the analysis of the steady-state equilibria without explicitly considering dynamics. It would be interesting to analyse formally the system of ordinary differential equations that characterize the adjustment from an initial equilibrium to the steady-state equilibrium under a general price process. This analysis would yield theoretical results about the path followed by the equilibrium over time. It would also be interesting to calibrate and simulate the model with a general price process to quantify its dynamics under a variety of shocks. The calibration would exploit the empirical estimates of the price process. To determine whether the model provides a good description of the data, we could simulate the moments of key variables and compare them to their empirical counterparts, and we could simulate impulse responses and compare them to impulse responses estimated empirically.

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